Type Safety of C# and .NET CLR

A dissertation submitted to the
SWISS FEDERAL INSTITUTE OF TECHNOLOGY ZURICH
(ETH ZÜRICH)

for the degree of
Doctor of Technical Sciences

presented by
Nicu Georgian Fruja
Dipl. Math.–University of Bucharest
born January 13, 1977
citizen of Romania

accepted on the recommendation of
Prof. Dr. Thomas R. Gross, examiner
Prof. Dr. Robert Stärk, co-examiner

2007
Abstract

Type safety plays a crucial role in the security enforcement of any typed programming language. This thesis presents a formal proof of C♯’s type safety. For this purpose, we develop an abstract framework for C♯, comprising formal specifications of the language’s grammar, of the statically correct programs, and of the static and operational semantics. Using this framework, we prove that C♯ is type-safe, by showing that the execution of statically correct C♯ programs does not lead to type errors.

The bytecode resulting from compiling C♯ programs is executed on the Common Language Runtime (CLR), the managed execution environment of the .NET Framework. As the bytecode may be transmitted over a network and get corrupted as a result, the bytecode execution may perform malicious operations. Therefore, to guarantee type safety, the CLR employs a bytecode verification, i.e., a static analysis that performs several consistency checks before the bytecode is run. We show type safety of the bytecode language by proving the soundness of the bytecode verification. The abstract framework developed for the proof encompasses formal specifications of the bytecode language’s abstract syntax, of the well-typedness constraints ensured by the bytecode verification, and of the bytecode language’s static and dynamic semantics. Finally, we demonstrate that legal and well-typed bytecode programs do not lead to any type error when executed.

This thesis reveals an important number of relevant ambiguities in the official specifications of C♯ and CLR and, in certain cases, even the absence of a specification at all. Thus, we identify and fill several gaps in the two official documents. We also point out a series of inconsistencies between different implementations of C♯ and CLR and their official specifications.
Zusammenfassung


Acknowledgments

This thesis would have not been possible without the support of many persons. First of all, I am greatly indebted to my first advisor, Prof. Dr. Robert F. Stärk, not only for providing the challenging and exciting topic of this thesis, but also for his advice and support. I want to express my profound acknowledgement to Prof. Dr. Thomas R. Gross who supervised and stimulated my work during my last two years. My gratitude to both supervisors cannot be realistically expressed and extended in any form of words.

It is a pleasure to thank Prof. Dr. Egon Börger for his advice, valuable criticism, invaluable time, and for sharing his profound knowledge in formal methods. Special thanks go also to Nigel Perry, Peter Sestoft and Jon Jagger (members of the C♯ Standardization Committee) and to the Microsoft Corporation employees Jonathan Keljo, Joe Duffy, Chris Brumme, Don Syme and Matt Grice for providing invaluable insights into the design of C♯ and CLR.

I thank the students I supervised during their master and diploma thesis for their contribution to the validation of the formal models in this thesis: Christian Marrocco, Horatiu Jula, and Markus Frauenfelder. For a successful thesis, not only the scientific part was important but also the ambient in the group. I thank my long-term group mate Stanislas Nanchen and all members of the LST group for the nice atmosphere.

I want to express my sincere thanks to my mother and father, Dobrita and Marin Fruja, for their unconditional support, absolute confidence in me, and for giving me all the opportunities in the world to explore my potentials and pursue my dreams. Last but not least, Karin has always been at my side and offered me strong support - I am deeply grateful for that and for her infinite patience that accompanied me along this long journey.
Contents

Abstract iii

Zusammenfassung v

Acknowledgments vii

Contents ix

1 Introduction 1
   1.1 Motivation ......................................................... 1
   1.2 Thesis Statement and Contributions ................................. 3
   1.3 Thesis Road Map .................................................. 4

2 The ASM Method 5
   2.1 A Mathematical Definition of ASMs ................................ 6
       2.1.1 States ...................................................... 6
       2.1.2 Locations and Updates ...................................... 8
       2.1.3 Transition Rules ............................................ 9
       2.1.4 Importing New Elements ..................................... 11
   2.2 Notational Conventions .......................................... 11

3 Type Safety of C$^\#$ 13
   3.1 An Overview of C$^\#_S$ ........................................... 14
   3.2 A Formal Description of C$^\#_S$ .................................. 15
       3.2.1 The Execution Environment .................................. 15
       3.2.2 The Grammar of C$^\#_S$ .................................... 19
   3.3 The Type Constraints ............................................. 22
   3.4 The Definite Assignment Analysis in C$^\#$ .......................... 26
       3.4.1 The Data Flow Equations .................................... 27
       3.4.2 The Maximal Fixed Point Solution .............................. 33
       3.4.3 The Control Flow Graph ...................................... 36
       3.4.4 The Correctness of the Analysis ............................... 41
   3.5 The Semantics of C$^\#_S$ Programs ................................. 47
       3.5.1 The Static Semantics ......................................... 48
       3.5.2 The Dynamic Semantics ....................................... 49
3.5.2.1 The Evaluation of the $C_S^d$ Expressions and Statements 53
3.6 Type Safety ................................. 56
3.7 Related Work ................................. 69
  3.7.1 Java’s Type Safety ......................... 69
  3.7.2 The Definite Assignment Analysis .......... 69
  3.7.3 The ASM Method ........................... 70
3.8 Summary and Future Work ....................... 70

4 Type Safety of CLR ............................ 73
  4.1 The Lightweight CLR .......................... 75
    4.1.1 An Overview of the Lightweight CLR and its Type System .. 75
    4.1.1.1 Reference Types ........................ 75
    4.1.1.2 Value Types ............................ 77
    4.1.2 The Static Semantics ....................... 79
    4.1.3 The Dynamic Semantics ..................... 85
      4.1.3.1 The Semantics of the Bytecode Instructions .......... 89
    4.1.4 The Bytecode Verification .................. 101
      4.1.4.1 An Overview of the Bytecode Verification .......... 101
      4.1.4.2 The Verification Type System and Bytecode Structure .. 102
      4.1.4.3 Verifying the Bytecode Instructions ................. 106
      4.1.4.4 Computing Successor Type States ................... 112
    4.1.5 The Bytecode Verification Algorithm and Well-typed Methods .. 114
    4.1.6 Type Safety ................................ 118
  4.2 The Exceptions .............................. 131
    4.2.1 An Overview of the Exception Handling Mechanism .......... 132
    4.2.2 The Static Semantics ....................... 134
    4.2.3 The Dynamic Semantics ..................... 137
      4.2.3.1 The StackWalk Pass ...................... 138
      4.2.3.2 The Unwind Pass ........................ 142
      4.2.3.3 The Leave Pass .......................... 146
      4.2.3.4 The Semantics of the Exception Specific Instructions .. 148
    4.2.4 The Bytecode Verification .................. 152
      4.2.4.1 The Bytecode Structure .................... 152
      4.2.4.2 Verifying the Bytecode Instructions ................. 152
      4.2.4.3 Computing Successor Type States ................... 154
    4.2.5 Type Safety ................................ 158
  4.3 The Generics ................................ 162
    4.3.1 The Polymorphic Type System ................ 163
    4.3.2 The Bytecode Semantics ..................... 167
    4.3.3 The Bytecode Verification .................. 174
      4.3.3.1 The Verification Type System .................... 174
      4.3.3.2 Verifying the Bytecode Instructions ................. 178
      4.3.3.3 Computing Successor Type States ................... 179
    4.3.4 Type Safety ................................ 179
4.4 Related Work ................................................................. 188
  4.4.1 The CIL ................................................................. 188
  4.4.2 The JVM ............................................................... 190
  4.4.3 The Variance on Generic Parameters ......................... 192
  4.4.4 The ASM Method .................................................. 193
4.5 Summary and Future Work ............................................. 193

5 Conclusion ........................................................................... 195

Bibliography ........................................................................... 197

List of Figures ......................................................................... 207

List of Tables .......................................................................... 209

A Appendix .............................................................................. 211
  A.1 The $\text{C}^\sharp_\text{S}$ Operational Semantics Rules .................... 211
Introduction

Well-typed programs never go wrong.
Robin Milner

1.1 Motivation

Type safety is a central theme in language-based security. Language type safety is relevant to the security of a system since many attacks exploit vulnerabilities in language’s type system, design and implementation, circumventing system security by hijacking the program execution model. Type safety is a formal guarantee that the execution of any program is free of certain forms of erroneous or undesirable program behavior, called type errors. The behaviors classified as type errors by any given programming language are generally those that result from attempts to perform on some value an operation that is not appropriate to its type: For example, a method defined on integers is applied to an object.

The enforcement of type safety can be static, catching potential errors at compile-time, or dynamic, associating type information with values at run-time and consulting them as needed to detect imminent errors, or a combination of both approaches. Employing a conservative strong static type analysis and dynamic checks, e.g., cast checking, is fundamental to modern programming practice and is used in programming languages, like C\textsuperscript{\#} [74], Java [67], and OCaml [12]. This thesis studies C\textsuperscript{\#}’s type safety, primarily since, at the time of this writing, C\textsuperscript{\#} is, besides Java, one of the most popular modern general-purpose programming languages.

C\textsuperscript{\#} is a class-based single-inheritance object-oriented programming language. It has been developed by Microsoft Corporation specifically for the Common Language Runtime (CLR) of the .NET Framework [2, 3], a managed execution environment with Common Intermediate Language (CIL), often referred to as CLR bytecode language. Approved as a standard by ECMA International [44], C\textsuperscript{\#} has a procedural, object-oriented syntax based on C++ that includes several novel features, but also features of other programming languages, most notably of Java, Visual Basic, and Delphi. Of particular interest are features like struct types as user defined value types, call-by-reference parameter passing for methods, unified type system with boxing and unboxing, and delegates – methods as values, also known as closures – for invoking one or multiple methods. It must be emphasized that C\textsuperscript{\#} also provides the ability to write unsafe code with C-style pointers, but only in clearly marked unsafe contexts. For obvious reasons related to their unsafe nature, the C\textsuperscript{\#} language analyzed in this thesis does not include unsafe features.
The C♯ language is intended for writing applications for both hosted and embedded systems, ranging from the very large ones that use elaborated operating systems, down to the very small ones having dedicated functions. It is aimed to be used, in particular, for developing desktop and internet applications and software components suitable for deployment in distributed environments. Given the importance of security for these kinds of applications, an investigation of C♯’s type safety, a key ingredient for security, is justified and necessary.

C♯’s type safety alone is, however, not sufficient to guarantee type-safe execution of bytecode programs on the CLR. The reason for this is that the bytecode might be produced by an illegal (buggy) C♯ compiler or might not even be related to a C♯ program. Therefore, this thesis also analyzes type safety of the bytecode language executed on the CLR.

The CLR, approved as a standard by ECMA International [45], has been designed from the ground up as a target for a wide variety of languages, called .NET compliant languages. A non-comprehensive list of programming languages compiling to CIL ranges from object-oriented languages, like C♯, Visual Basic, Managed C++, Eiffel [46], Smalltalk, Spec♯, and Active Oberon, to functional programming languages, like Haskell [14], Scheme [43], Mercury [11], and AsmL [49], and from pure procedural programming languages, like COBOL and Modula 2, to scripting languages, like JScript, Perl, and Phyton. At the time of this writing, there are 53 .NET compliant languages (see [17] for an up-to-date list), out of which C♯ is the most popular. The CLR makes possible the cross-language compatibility, i.e., the .NET components can interact with each other regardless of the languages they are written in. Due to the wide spectrum of .NET compliant languages, the CLR is required to support various features, such as, for example, tail method calls, pointers (including method pointers), typed references, generics with F-bounded polymorphism, covariance, and contravariance, a structured exception handling model, and a unified type system with boxing and unboxing.

As programs have become more dynamic and distributed, operating systems and browsers have been required more often to execute downloaded or mobile code. This execution can be problematic as there exists malicious programs, intended to cause mischief when executed on unsuspecting hosts. As the CLR has been designed to be suitable for the development of a variety of applications, ranging from web services to web and Windows applications, CIL’s type safety, central to the notion of safe code execution within the CLR, is crucial. Executing non type-safe bytecode programs on the CLR, regardless whether the bytecode was accidentally or maliciously created, can produce erroneous or destructive behavior within the execution system. It is worth mentioning that CLR’s type safety also includes checks other than just type constraints, such as, for example, prohibition of stack underflow/overflow, correct use of the exception handling facilities, and object initialization.

To ensure type safety, the CLR employs a conservative static analysis, called bytecode verification, that performs several consistency checks before the bytecode is executed. Bytecode verification rejects, for example, a bytecode program that performs operations on values of wrong types, accesses an uninitialized object, calls a method with an incorrect number of parameters, calls a method with a parameter of an incorrect type or executes a method which returns a value of an incorrect type. The checks are not only limited to type checks: Bytecode verification prohibits bytecode programs that jump to an invalid code index. Similarly to the C♯’s type analysis, the bytecode verification is conservative, meaning that not only all the faulty programs are rejected, but also programs that would never exhibit a run-time type error. The reasons this conservative analysis are the undecidability of the halting problem and efficiency considerations. It
must be emphasized that, analogously to the C♯'s type analysis, the CLR’s bytecode verification by itself does not guarantee type-safe execution of the bytecode: Many crucial properties of the code still need to be ensured dynamically, e.g., array bound checks and null pointer checks. The purpose of bytecode verification is to shift as many dynamic checks as possible from run-time to verification-time.

For the sake of clarity, this thesis only analyzes type safety of large, yet representative, subsets of C♯ and CIL. Correspondingly, two formal proofs are developed, basically showing the soundness of the C♯’s type analysis and CLR’s bytecode verification defined accordingly to [74] and [45], respectively. For that, we formally define the abstract syntax, type system, type constraints guaranteed by the C♯’s type analysis and by the CLR’s bytecode verification, and the static and dynamic semantics of C♯ and CLR. Moreover, we provide a formal specification of the definite assignment rules that must be satisfied by a C♯ method in order to be accepted by the definite assignment analysis, i.e., a dataflow analysis performed by the C♯ compiler to guarantee that all local variables are assigned to before their value is used. The two type safety proofs basically establish that the run-time execution, according to the semantics models, of legal C♯ and CIL programs satisfying the type constraints and, in case of C♯, the definite assignment rules does not lead to any type error. The proved type safety properties attest that C♯’s and CIL’s type safety are strictly stronger than the memory safety property. In particular, this means that memory is only ever accessed in a well-known and controlled manner through typed references.

## 1.2 Thesis Statement and Contributions

In this thesis, we establish the following results:

- A large and representative subset of C♯ is type-safe.
- A large and representative subset of CIL is type-safe.

Besides the formal proofs of the two statements, this thesis makes several other contributions:

- Two important by-products of this thesis are represented by the formal models developed for CIL and its bytecode verification. The CIL model, including a fairly elaborate model for the CLR exception handling mechanism, can serve, for example, as the basis for building a robust interpreter for the CIL bytecode [92], whereas the bytecode verification model can assist the implementation of a prototype of a CLR bytecode verifier. Note that the C♯ model, on which the C♯’s type safety proof relies, has been defined in [24] and does not constitute a contribution of this thesis.

- This thesis reveals an important number of relevant ambiguities in the official specifications of C♯ [74] and CLR [45] and, in certain cases, even the absence of a specification at all. Thus, we identify and fill several gaps in the two official documents. Annotations corresponding to some of these gaps have been included in the C♯ Annotated Standard [82], a book for which this thesis’ author was an invited guest annotator. We also point out a series of inconsistencies between different implementations of C♯ and CLR and the standards [74] and [45], respectively. Moreover, many explanations presented in this thesis,
alongside the specifications [74] and [45], provide the rationale behind the design, that is often missing from the official specifications.

- Last but not the least, the abstract frameworks defined in this thesis serve not only as a basis for the type safety proofs, but also as starting point for reasoning about a wide variety of aspects of C♯ and CLR. Thus, one can investigate, for example, stronger safety guarantees for C♯ and CIL programs, or ways in which programs can be optimized in a type-safe manner.

1.3 Thesis Road Map

The semantic models of C♯ and CLR, serving as the rigorous basis for the type safety proofs, are defined in terms of abstract interpreters, using the framework of Abstract State Machines (ASMs). To allow the reader a correct understanding of the formal specifications and proofs in Chapters 3 and 4, Chapter 2 includes a brief overview as well as a mathematical definition of the ASMs, based on the definition presented in [115]. The formal specifications and the corresponding type safety proofs of C♯ and CIL are presented in Chapter 3 and 4. The two chapters may be read independently of each other, but each of them presupposes Chapter 2. Chapter 5 concludes. For the reader’s convenience, the evaluation rules of the semantics model for C♯ defined in [24] are included in the Appendix.
The ASM Method

So-called "natural language" is wonderful for the purposes it was created for, such as to be rude in, to tell jokes in, to cheat or to make love in, but it is hopelessly inadequate when we have to deal unambiguously with situations of great intricacy, situations which unavoidably arise in such activities as legislation, arbitration, mathematics, or programming.

Edsger W. Dijkstra

The Abstract State Machines (ASMs) [70, 27] are a mathematical formalism, widely used for high-level design and analysis of computing systems, in particular for the formal definition of programming language semantics. The notion of ASMs captures precisely some basic operational intuitions of computing. The ASMs describe a system by means of a particular syntax and associated semantics and use classical, well-understood, mathematical structures (algebras) to describe the states of a computation. This distinguishes ASMs from informal methodologies. The ASM programs use a very simple and expressive syntax, whose semantics can be understood without any further explanation, simply viewing the ASM programs as "pseudo-code over abstract data structures". In contrast, other specification methods, notably denotational semantics, use complicated syntax whose semantics is more difficult to read and write. This is one of the reasons we used the ASM method for the formal specifications in Chapters 3 and 4. Despite of their abstract nature, the ASM specifications are executable in the .NET ASM Language (AsmL) [49]. Thereby, a way to validate and verify the ASM specifications is provided.

ASMs in a nutshell In the ASM framework, the systems are formalized in a state-based way, by means of states and state transitions. An ASM state is represented by an algebra over a given vocabulary, where data come as abstract objects, that is, as elements of universes (one for each category of data) which are equipped with basic operations (functions) and predicates. The state transitions are defined by transition rules which specify the possible state changes in terms of updates. The notion of ASM run is the well-known notion of computation of transition systems. In a given state, an ASM computation step consists in executing simultaneously all updates prescribed by the applicable transition rules, i.e., the rules whose guard is true in the given state, if there no clashing updates. Simultaneous execution is a convenient way to make use of synchronous parallelism and to abstract from irrelevant sequentiality.

ASMs defining language semantics The semantics of programming languages is described operationally using ASMs based on the abstract syntax trees. Part of the current state is the
current task, a pointer to the currently to be executed node in the abstract syntax tree. During program execution, states are transformed into new states, by that also updating the pointer to the current task. During a state transition, the interpretation of some function symbols may change. To specify the semantics of a language, the abstract syntax tree is assumed to contain attributes defining all continuations, especially for the non-compositional changes of the control flow. The ASM definition models the program counter during program execution, by that using the continuation attributes which might be split up according to the truth value of guards. The semantics of each program node is described by a finite number of transition rules. Typically, the guard of such a transition rule specifies the nodes in the abstract syntax tree for which the transition rule is applicable. The transition rules define updates, thereby employing child nodes as well as statically computed continuations.

2.1 A Mathematical Definition of ASMs

In this section, we present a mathematical definition for the syntax and semantics of ASMs. An interested reader can find two excellent formal definitions of ASMs in [115, §2] and [27, §2.4]. The definition included here follows very closely the definition in [115, §2].

2.1.1 States

The abstract states of the ASMs are given as algebras. An algebra is defined with respect to a vocabulary (sometimes called signature), variables, and terms. Additionally, an interpretation and a variable assignment are considered to allow evaluation of syntactical entities to semantical values.

Definition 2.1.1 (Vocabulary) A vocabulary \( \Sigma \) is given by a finite set of function names, each with a fixed arity. Every vocabulary should contain the nullary functions True, False, and undef.

Functions Classification As the states are mostly characterized in terms of functions, one distinguishes different kinds of functions:

- **Static** functions never change during any run. The nullary functions True, False and undef are considered static functions.
- **Dynamic** functions may change their interpretation during a run.
- **External** functions, also known as oracles, whose interpretation cannot be changed by the system itself but by the outer environment.
- **Derived** functions are defined in terms of other functions, known as basic functions.
- **Relational** functions (also known as characteristic functions or predicates), with the boolean universe as codomain.

The derived functions can be viewed as macros, applied to shorten expressions. The relational functions are typically used to describe sets. The nullary function names of a vocabulary are called constants, though, as we have seen above, the interpretation of dynamic nullary functions can change from one state to the next state.
2.1. A Mathematical Definition of ASMs

Definition 2.1.2 (State) Given a vocabulary \( \Sigma \), a state \( A \) for \( \Sigma \) is given by a non-empty set \( X \) and the interpretations of the function names of \( \Sigma \):

- the interpretation \( c^A \) of a constant \( c \) of \( \Sigma \) is an element of \( X \).
- the interpretation \( f^A \) of a \( n \)-ary function name \( f \) of \( \Sigma \) is a function from \( X^n \) into \( X \).

The set \( X \) in Definition 2.1.2 is called the superuniverse of the state \( A \) and is denoted by \( \|A\| \).

The interpretations of the nullary logic names \( True, False \) and \( \text{undef} \) are distinct elements of \( X \). The boolean operations behave in the usual way on the boolean values \( True \) and \( False \) and produce \( \text{undef} \) if at least one of the arguments is not boolean.

Formally, all the basic functions are total. They may, however, return the special element \( \text{undef} \) if they are not defined at an argument. Let us stress though that \( \text{undef} \) is an ordinary element of the superuniverse. Often, a basic function returns \( \text{undef} \) if at least one argument is \( \text{undef} \).

The terms, typically denoted by \( t \) or \( s \), are defined recursively, as in first-order logic.

Definition 2.1.3 (Term) The terms of a vocabulary \( \Sigma \) are syntactic expressions inductively produced as follows:

- The variables, \( x, y, z, \ldots \), elements of the set \( V \), are terms.
- The constants \( c \) of \( \Sigma \) are terms.
- If \( f \) is an \( n \)-ary function name of \( \Sigma \) and \( t_1, \ldots, t_n \) are terms, then \( f(t_1, \ldots, t_n) \) is a term.

To provide terms with semantics, one needs first to assign values to variables. This is accomplished by a function called variable assignment.

Definition 2.1.4 (Variable assignment) A variable assignment over a state \( A \) is a function

\[
\zeta : \text{Map}(V, \|A\|)
\]

Given a variable assignment \( \zeta \), \( \zeta[x \mapsto a] \) denotes the variable assignment which coincides with \( \zeta \) except that it assigns the element \( a \) to the variable \( x \).

The evaluation of terms is defined with respect to a variable assignment and a state:

Definition 2.1.5 (Term evaluation) Given a state \( A \) of \( \Sigma \), a variable assignment \( \zeta \) over \( A \) and a term \( t \) of \( \Sigma \), the value \( [t]^A_\zeta \) is defined by induction on \( t \)'s length as follows:

\[
\begin{align*}
[t]^A_\zeta &= \zeta(x) \\
[c]^A_\zeta &= c^A \\
[f(t_1, \ldots, t_n)]^A_\zeta &= f^A([t_1]^A_\zeta, \ldots, [t_n]^A_\zeta)
\end{align*}
\]
Similarly to the terms, the formulas can be interpreted in a state, with respect to a variable assignment. Note that the formulas cannot return undef: They are either True or False.

**Definition 2.1.6 (Formula semantics)** For a state $A$ of $\Sigma$, a variable assignment $\zeta$ over $A$ and a formula $\phi$ consisting of terms over $\Sigma$, the truth value $[\phi]^A_\zeta$ is defined by induction on $\phi$'s length as follows:

$$[s = t]^A_\zeta = \begin{cases} 
  \text{True}, & \text{if } [s]^A_\zeta = [t]^A_\zeta; \\
  \text{False}, & \text{otherwise}. 
\end{cases}$$

$$[-\phi]^A_\zeta = \begin{cases} 
  \text{True}, & \text{if } [\phi]^A_\zeta = \text{False}; \\
  \text{False}, & \text{otherwise}. 
\end{cases}$$

$$[\phi \land \psi]^A_\zeta = \begin{cases} 
  \text{True}, & \text{if } [\phi]^A_\zeta = \text{True and } [\psi]^A_\zeta = \text{True}; \\
  \text{False}, & \text{otherwise}. 
\end{cases}$$

$$[\phi \lor \psi]^A_\zeta = \begin{cases} 
  \text{True}, & \text{if } [\phi]^A_\zeta = \text{True or } [\psi]^A_\zeta = \text{True}; \\
  \text{False}, & \text{otherwise}. 
\end{cases}$$

$$[\phi \Rightarrow \psi]^A_\zeta = \begin{cases} 
  \text{True}, & \text{if } [\phi]^A_\zeta = \text{False or } [\psi]^A_\zeta = \text{True}; \\
  \text{False}, & \text{otherwise}. 
\end{cases}$$

$$[\forall x \phi]^A_\zeta = \begin{cases} 
  \text{True}, & \text{if } [\phi]^A_{\zeta[x \mapsto a]} = \text{True for every } a \in \|A\|; \\
  \text{False}, & \text{otherwise}. 
\end{cases}$$

$$[\exists x \phi]^A_\zeta = \begin{cases} 
  \text{True}, & \text{if there exists } a \in \|A\| \text{ with } [\phi]^A_{\zeta[x \mapsto a]} = \text{True}; \\
  \text{False}, & \text{otherwise}. 
\end{cases}$$

### 2.1.2 Locations and Updates

As a state is described through functions and their current interpretation, the change of a state is described in terms of changing its function values at particular points. To capture state changes, the notions of locations and updates are introduced.

**Definition 2.1.7 (Location)** Given a state $A$ of $\Sigma$, a location of $A$ is a pair $(f, (a_1, \ldots, a_n))$, where $f$ is an n-ary function name of $\Sigma$ and $a_1, \ldots, a_n$ are elements of $\|A\|$.

To change the value of a location, the notion of update is used:

**Definition 2.1.8 (Update)** Given a state $A$, an update of $A$ is a pair $(\text{loc}, \text{val})$ where loc is a location of the state $A$ and val is an element of $\|A\|$. A set of updates is known as an update set.

Given the parallelism underlying several transition rules, a prescribed update set might be inconsistent, i.e., it contains at least two clashing updates.
2.1. A Mathematical Definition of ASMs

<table>
<thead>
<tr>
<th>Rule</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>skip</td>
<td>do nothing</td>
</tr>
<tr>
<td>(f(t_1, \ldots, t_n) := s)</td>
<td>update (f) at the arguments (t_1, \ldots, t_n) to (s)</td>
</tr>
<tr>
<td>(P \mathbf{Q})</td>
<td>execute (P) and (Q) in parallel</td>
</tr>
<tr>
<td>if (\phi) then (P) else (Q)</td>
<td>if (\phi), then execute (P), else execute (Q)</td>
</tr>
<tr>
<td>forall (x) with (\phi) do (P(x))</td>
<td>execute (P(x)) in parallel for each (x) satisfying (\phi)</td>
</tr>
<tr>
<td>choose (x) with (\phi) do (P(x))</td>
<td>choose an arbitrary (x) satisfying (\phi) and then execute (P(x))</td>
</tr>
<tr>
<td>let (x = t) in (P)</td>
<td>assign the value of (t) to (x) and then execute (P)</td>
</tr>
<tr>
<td>(P \mathbf{seq} Q)</td>
<td>execute (P) and then execute (Q)</td>
</tr>
<tr>
<td>(r(t_1, \ldots, t_n))</td>
<td>call (r) with the values of (t_1, \ldots, t_n) as parameters</td>
</tr>
<tr>
<td>(P \mathbf{where} Q)</td>
<td>execute (P) assuming the abbreviations implied by (Q)</td>
</tr>
</tbody>
</table>

Table 2.1: The ASM transition rules.

**Definition 2.1.9 (Consistent update set)** An update set \(U\) is consistent if the following condition is satisfied for any location \(loc\) and elements \(val, val'\):

\[(loc, val) \in U \land (loc, val') \in U \Rightarrow val = val'\]

2.1.3 Transition Rules

A means of updating the abstract states of ASMs is given by transition rules, informally described in Table 2.1. The syntax of ASM programs is defined by these rules (and a rule introduced in Section 2.1.4), typically denoted by \(P\) or \(Q\).

**Definition 2.1.10 (ASM)** An abstract state machine \(M\) is made of a vocabulary \(\Sigma\), a set of initial states for \(\Sigma\), a set of rule declarations, and a special rule name of arity zero, named the main rule of the machine.

In a given state, under a given variable assignment, a transition rule yields an update set. Note that a transition rule can be recursive and consequently produce no update set at all. Therefore, the semantics of transition rules is defined by the calculus in Table 2.2.

**Definition 2.1.11 (Transition rule semantics)** A transition rule \(P\) generates the update set \(U\) in a state \(A\) under a variable assignment \(\zeta\) if and only if \([P]_A^A \triangleright U\) is derivable in the calculus defined in Table 2.2.

Table 2.2 skips the semantics of the where rule, since this is easily reducible to the let construct. In Definition 2.1.11, if \([P]_\zeta^A \triangleright U\) is derivable in the calculus, then the update set \(U\) is typically identified with \([P]_\zeta^A\).

To fire a consistent update set, all its members are fired simultaneously. The result is a new state, with the same vocabulary and superuniverse, where the interpretations of the function names are changed accordingly to the updates in the update set.
where \( \text{loc} = (f, ([t_1]^A, \ldots, [t_n]^A)) \) and \( \text{val} = [s]^A \).

**Table 2.2: The semantics of the ASM rules.**

**Definition 2.1.12 (Firing updates)** Given a consistent update set \( U \) and a state \( A \), the result of firing \( U \) in \( A \), denoted by \( A + U \), is a state \( A' \), where the following conditions are met for every function name \( f \) of \( \Sigma \):

- \( \|A\| = \|A'\|; \)
- \( f^{A'}(a_1, \ldots, a_n) = b \) for every update \( ((f, (a_1, \ldots, a_n)), b) \in U. \)
- if \( f \) is not an external function, then \( f^{A'}(a_1, \ldots, a_n) = f^A(a_1, \ldots, a_n) \), for every \( (a_1, \ldots, a_n) \in \|A\|^n \) for which there exists no \( b \in \|A\| \) with \( ((f, (a_1, \ldots, a_n)), b) \in U. \)
A run of an ASM consists of a sequence of states, each derived from the preceding state by firing the updates yielded by the main rule. If the generated update set is inconsistent or not defined, the ASM reached the final state in the respective run.

**Definition 2.1.13 (ASM run)** Given a variable assignment ζ and an ASM M with vocabulary \( \Sigma \), initial state \( \mathcal{A}_0 \) and main rule \( r \), a run of M is a possibly infinite sequence \( \mathcal{A}_0, \mathcal{A}_1, \ldots \) of states for \( \Sigma \) such that:

- if \( \left[ P \right]^{A_n}_\zeta \) is consistent and defined, then \( \mathcal{A}_{n+1} \) is the state obtained by firing the updates of the set \( \left[ P \right]^{A_n}_\zeta \) in the state \( \mathcal{A}_n \).
- if \( \left[ P \right]^{A_n}_\zeta \) is not consistent or not defined, then \( \mathcal{A}_n \) is the last state of the sequence.

### 2.1.4 Importing New Elements

The transition rules in Table 2.1 suffice for many purposes, but they do not suffice, for instance, for systems that require to increase their working space. For this purpose, as a means to extend the vocabulary of a state, an import construct is considered together with a possibly infinite universe, called reserve, from which new (fresh) elements are imported.

The import construct, whose general form is given below, has the following meaning:

"Choose an element from the reserve, delete it from the reserve, and then execute P where every occurrence of \( x \) is replaced by the reserve element".

\[
\text{import } x \text{ do } P(x)
\]

The precise semantics of this construct, defined in [27, §2.4], goes much beyond the scope of this chapter and therefore is skipped. The different rules specified for the import construct guarantee, as shown in [27, §2.4], that "imports" executed in parallel generate fresh, pairwise different, elements and that permutations of the reserve universe do not change the semantics of ASM rules. Otherwise said, the semantic does not depend on the elements picked up from the reserve universe by the import construct.

### 2.2 Notational Conventions

Before getting started, we should establish some notations. This section includes the most frequently used ones, the majority of which are list operations. Many readers could just skip this section and refer back to it as necessary.

We denote by \( \mathbb{N} \) the set of natural numbers, including 0.

\( \mathcal{P}(A) \) is used to denote the powerset of the set \( A \), i.e., the set of all subsets of \( A \).

We denote by \( \text{Map}(A, B) \) the set of all mappings (functions) from a domain \( A \) to a codomain \( B \).
[x_1, \ldots, x_n] is the list containing the elements x_1, \ldots, x_n; \, [\,] is the empty list.

L(i) is the i-th element of the list L.

length(L) returns the length of the list L.

L \cdot L' is the concatenation of the lists L and L'.

sublist(L, L') is true if the list L is a sublist of the list L'. More exactly, sublist(L, L') holds if there exist two lists L'' and L''' such that L'' \cdot L \cdot L''' = L'.

last(L, L') is true if the last occurrence of the list L in the list L' is at the beginning of L'. More precisely, last(L, L') holds if for every two lists L'' and L''' such that L'' \cdot L \cdot L''' = L', it holds L'' = [\,].

prefix(L, L') returns the sublist of the list L' before the last occurrence of L in L'. More exactly, prefix(L, L') = L'' if there exists a list L''' such that last(L, L \cdot L''') and L'' \cdot L \cdot L''' = L'.

suffix(L, L') returns the sublist of the list L' after the last occurrence of the list L in L'. More precisely, suffix(L, L') = L'' if last(L, L \cdot L'') and there exists a list L''' such that L'' \cdot L \cdot L''' = L'.

top(L) returns the last element of the list L.

pop(L) returns the list L without the last element.

push(L, x) is the result of pushing x to the right of the list L, i.e., the list L \cdot [x].

take(L, n) returns the list consisting of the last n elements of the list L.

drop(L, n) returns the list resulting from dropping the last n elements from the list L.

split(L, n) splits off the last n elements of the list L. More exactly, split(L, n) is the pair (L', L''), where L' \cdot L'' = L and length(L'') = n.

L[y/x] yields the list obtained by replacing all occurrences of x by y in the list L.

L \sqsubseteq_{suf} L' is true if the lists L and L' of lengths m and n, respectively, and the relation \sqsubseteq satisfy the following conditions: m \geq n and L(m - n + i) \sqsubseteq L'(i), for every i = 0, n - 1.

L \sqsubseteq_{len} L' is true if the lists L and L' of lengths m and n, respectively, and the relation \sqsubseteq satisfy the following conditions: m = n and L(i) \sqsubseteq L'(i), for every i = 0, m - 1.

L \sqsubseteq_{suf} \mathcal{L} is true if the list L and the set of lists \mathcal{L} satisfy the following condition: There exists a list L' \in \mathcal{L} such that L \sqsubseteq_{suf} L'.

L \sqsubseteq_{len} \mathcal{L} is true if the lists L and the set of lists \mathcal{L} satisfy the following condition: There exists a list L' \in \mathcal{L} such that L \sqsubseteq_{len} L'.
In this chapter, we formally prove type safety of a large and representative subset of C♯, named C♯S. First, the abstract syntax and the execution environment of C♯S programs are formally specified. We then define the type constraints and the definite assignment analysis dataflow equations that are guaranteed to hold for C♯S programs, upon their successful compilation. The formal specifications of the static and dynamic semantics of C♯S are further provided through an abstract interpreter for C♯S programs, defined as an ASM model. Finally, we prove that C♯S is type-safe, i.e., the execution on the ASM model (for the C♯S semantics) of legal C♯S methods, accepted by the definite assignment analysis and satisfying the type constraints, does not produce any run-time type violations. Moreover, we show that the execution leaves the C♯S programs in a good state, where certain structural constraints are satisfied.

Basis Our type safety proof is built on top of the ASM model for the C♯’s semantics, defined by Stärk et al. [24]. As this model is not a contribution of this thesis, to aid the reader, we describe it separately in Section 3.5 and include its operational semantics rules in the Appendix. We use this model, but with a few minor changes. It is about some differences, pointed out in detail in Section 3.5, between the original model [24] and the C♯ Language Specifications [74], which we were able to fix as a result of considering through our proof all language intricacies.

Challenges C♯S is a large sequential sublanguage of C♯ which includes features that we believe to be important for an investigation of C♯’s type safety. Despite the simple and expressive C♯ syntax, we had to consider a large number of proof cases, mainly due to the C♯’s unified type system including some special value types, called structs. Besides the lengthy proof, we have encountered the following technical difficulties. A key ingredient of C♯S’s type safety, the soundness of the definite assignment analysis, performed by the C♯ compiler to guarantee that every variable is initialized before its value is accessed, is pretty difficult to prove: Firstly, because the data flow equations underlying the definite assignment analysis do not always have a unique solution, and secondly, because the struct type variables are handled specially by the analysis. The “intuitive” invariants, i.e., the invariants apparently expressing C♯S’s type safety,
cannot be proved without considering and proving other "less intuitive" (rather technical) invariants. Due to some circular dependencies, attempting to prove the latter (as separate lemmas) before the type safety theorem is hopeless as they require the proof of the intuitive invariants. Therefore, an important idea for the proof consists in identifying the (least) set of invariants needed for the type safety result. Moreover, it was not trivial to relate analyses performed by the C♯ compiler, e.g., the definite assignment analysis, to run-time properties since the evaluation of certain constructs, e.g, struct fields, might involve several intermediate steps. Furthermore, in the context of C♯’s call-by-reference mechanism, it was tricky to track what locations are initialized as, for example, several field locations of a struct value might be initialized though the struct value location is considered uninitialized.

Features omitted from C♯
To give a flavor of the full C♯, we list here the main features omitted:

- **properties, indexers, and events**: They are implemented in C♯ through syntactic sugar, and therefore can easily be reduced to C♯ S constructs, as shown in [24].

- **arrays and static members**: Including them would only trigger the extension of some definitions and a few more proof cases, but it would not increase the technical complexity of the proof.

- **multi-threading**: The C♯ thread model has been formally specified by Stärk and Börger [112]. The main invariants of this model have then been formally verified by Stärk [111] in the AsmTP system, an interactive proof assistant implemented in Prolog and based on LPTP [110].

- **generics, anonymous methods, and iterators**: The semantics of these features has been formalized by Jula [83]. The anonymous methods and the iterators are implemented through syntactic sugar, as shown in [83].

Chapter outline
The rest of this chapter is organized as follows. Section 3.1 gives an overview of the C♯ S language. A formal description of C♯ S consisting of definitions for the execution environment and the grammar is provided in Section 3.2. Section 3.3 formalizes the type constraints that should be satisfied by every C♯ S program, upon successful compilation. Section 3.4 formally specifies the definite assignment analysis, which is then proved sound. The static and dynamic semantics of C♯ S, as defined in [24], are included in Section 3.5. Finally, Section 3.6 presents a proof of C♯ S’s type safety. Section 3.8 summarizes and gives directions for future work. For the reader’s convenience, the operational semantics rules of [24]’s model are included in the Appendix.

3.1 An Overview of C♯ S

C♯ S is a large and representative sublanguage of C♯ that includes classes, interfaces, instance members (fields, methods, and constructors), inheritance of members, dynamic method dispatch, local variables, and exceptions. Moreover, C♯ S has delegates, structs, call-by-reference (ref/out parameters), boxing, and unboxing.
Delegates  The *delegate types* are reference types, built with the idea of being the type-safe method pointers in C♯. Informally, a *delegate*, i.e., an instance of a delegate type, is an object that points towards an *invocation list* of pairs of *target objects* and *target methods*. Upon the invocation of a delegate with a list of arguments, the methods in its invocation list are invoked *sequentially* with the corresponding target object and the given arguments, returning to the delegate caller the return value of the last method in the list.

Structs  Besides primitive types, C♯ supports some special value types called *structs*. Every struct has the library class *System.ValueType* [10] as the direct base class. In particular, a struct is *sealed*, meaning that it is not possible for a struct to extend another struct. Additionally, a struct can implement one or more interfaces. Similarly to a class, a struct can declare instance fields. A *struct value* is a “self-contained” value. This is so because a *struct instance* can be represented as a *mapping* assigning values to the instance fields of the struct. As the structs are sealed, the instance methods in structs cannot be declared as virtual, since it is just not possible to subtype them and consequently, the definitions of methods cannot be overridden.

Boxing and unboxing  To fill the gap between value types and reference types, i.e., to have an *unified type system*, C♯ supports two special operations called *boxing* and *unboxing*. Through boxing, a *copy* of a value type instance can be ”packed” in an object on the heap, along with the necessary internal data required to create a valid reference object. The inverse process, i.e., converting a reference type to a value type, is called *unboxing*. An unboxing ”back” to a value type returns the value boxed in the object if the object is indeed a boxed value of the given value type.

Although the structs cannot declare virtual methods, the virtual methods defined by *System.ValueType*, *object*, and by the interfaces implemented by a struct (if any) can, however, be called on the struct instances, but *only* if the instances are boxed. This makes sense as the value types have identity only when boxed.

Call-by-reference  In C♯ a *ref* parameter is used for ”by reference” parameter passing in which the parameter acts as an alias for the caller-provided argument. An *out* parameter is similar to a *ref* parameter except that the initial value (if any) of the caller-provided argument is not important. This is because a variable is required to hold a value before it can be passed as a *ref* parameter, whereas a variable need not hold a value before it can be passed as an *out* parameter.

### 3.2 A Formal Description of C♯ₐₜ

In this section, we provide a formal description of C♯ₐₜ along the lines of the C♯ model [24]. Section 3.2.1 specifies the components of the execution environment. The grammar of C♯ₐₜ is defined in Section 3.2.2.

#### 3.2.1 The Execution Environment

Every C♯ₐₜ method runs in an *execution environment* that contains the type hierarchy and the field and method declarations. The classification of the C♯ₐₜ *types* is given in Table 3.1. Thus, a
### Universe of types

<table>
<thead>
<tr>
<th>Type</th>
<th>RefType ∪ ValueType</th>
<th>Typical use</th>
</tr>
</thead>
<tbody>
<tr>
<td>RefType</td>
<td>Class ∪ Interface</td>
<td></td>
</tr>
<tr>
<td>ValueType</td>
<td>Struct ∪ PrimitiveType</td>
<td></td>
</tr>
<tr>
<td>Class</td>
<td></td>
<td>C, C'</td>
</tr>
<tr>
<td>Delegate</td>
<td>⊂ Class</td>
<td>D</td>
</tr>
<tr>
<td>Interface</td>
<td></td>
<td>I, I'</td>
</tr>
<tr>
<td>Struct</td>
<td></td>
<td>S, S'</td>
</tr>
<tr>
<td>PrimitiveType</td>
<td>{bool, int, long, double}</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1: C♯’s types.

### Universe of values

<table>
<thead>
<tr>
<th>Val</th>
<th>ObjRef ∪ SimpleVal ∪ StructVal</th>
<th>Typical use</th>
</tr>
</thead>
<tbody>
<tr>
<td>SimpleVal</td>
<td></td>
<td>val, val', val''</td>
</tr>
<tr>
<td>ObjRef</td>
<td></td>
<td>ref, ref', d</td>
</tr>
<tr>
<td>StructVal</td>
<td>Map(Struct::Field, Val)</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2: C♯’s values.

type is a reference type or a value type. Additionally, void can only be used as a method return type. The reference types are the classes (including the delegate types\(^1\)) and the interfaces. A value type is either a struct or a primitive type.

Corresponding to the above types, there exists the following kinds of values, gathered in Table 3.2: object references, simple values, i.e., values of primitive types, and struct values.

We assume that the execution environment organizes the classes, structs and interfaces into an inheritance hierarchy. The subtype relation, introduced in Definition 3.2.1, depends on this hierarchy.

**Definition 3.2.1 (Compile-time compatibility)** The subtype relation \(\leq\) is the least reflexive and transitive relation such that

- if \(T = \text{int}\) and \(T' \in \{\text{long, double}\}\), or
- if \(T = \text{long}\) and \(T' = \text{double}\), or
- if \(T \in \text{Type}\) and \(T' = \text{object}\), or
- if \(T \in \text{Class}\) extends \(T' \in \text{Class}\), or
- if \(T \in \text{Class} \cup \text{Struct} \cup \text{Interface}\) implements \(T' \in \text{Interface}\), or
- if \(T \in \text{ValueType}\) and \(T' = \text{System.ValueType}\), or

\(^1\)Due to a syntactic sugar defined and explained in Section 3.2.2, we treat the delegate types as special class types. The official documentation [74] is ambiguous in that respect: It defines the delegate types as classes, but treats them as distinct in most places.
3.2. A Formal Description of $\mathbb{C}^d_S$

<table>
<thead>
<tr>
<th>Universe</th>
<th>Typical use</th>
<th>Element name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field</td>
<td>$F$</td>
<td>field name</td>
</tr>
<tr>
<td>MRef</td>
<td>$C::M, S::M, T::M, T'::M, T'::M'$</td>
<td>method reference</td>
</tr>
<tr>
<td>Loc</td>
<td>$loc, loc'$</td>
<td>local variable or parameter</td>
</tr>
</tbody>
</table>

Table 3.3: The environment specific universes.

<table>
<thead>
<tr>
<th>Function definition</th>
<th>Function name</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{instFields}$: $\text{Map}(\text{Class} \cup \text{Struct}, \mathcal{P}(\text{Type}::\text{Field}))$</td>
<td>class/struct instance fields</td>
</tr>
<tr>
<td>$\text{fieldType}$: $\text{Map}(\text{Class}::\text{Field} \cup \text{Struct}::\text{Field}, \text{Type})$</td>
<td>field declared type</td>
</tr>
<tr>
<td>$\text{localVars}$: $\text{Map}(\text{MRef}, \mathcal{P}(\text{Loc}))$</td>
<td>method local variables</td>
</tr>
<tr>
<td>$\text{locType}$: $\text{Map}(\text{MRef} \times \text{Loc}, \text{Type})$</td>
<td>local variable type</td>
</tr>
<tr>
<td>$\text{valueParams}$: $\text{Map}(\text{MRef}, \mathcal{P}(\text{Loc}))$</td>
<td>&quot;by value&quot; parameters</td>
</tr>
<tr>
<td>$\text{refParams}$: $\text{Map}(\text{MRef}, \mathcal{P}(\text{Loc}))$</td>
<td>ref parameters</td>
</tr>
<tr>
<td>$\text{outParams}$: $\text{Map}(\text{MRef}, \mathcal{P}(\text{Loc}))$</td>
<td>out parameters</td>
</tr>
<tr>
<td>$\text{paramType}$: $\text{Map}(\text{MRef} \times \text{Loc}, \text{Type})$</td>
<td>parameter declared type</td>
</tr>
<tr>
<td>$\text{paramIndex}$: $\text{Map}(\text{MRef} \times \text{Loc}, \mathbb{N})$</td>
<td>parameter index</td>
</tr>
<tr>
<td>$\text{retType}$: $\text{Map}(\text{MRef}, \text{Type} \cup {\text{void}})$</td>
<td>method return type</td>
</tr>
<tr>
<td>$\text{callKind}$: $\text{Map}(\text{MRef}, {\text{Virtual, NonVirtual}})$</td>
<td>method kind</td>
</tr>
</tbody>
</table>

Table 3.4: The components of the execution environment.

- if $T \in \text{Delegate}$ and $T' = \text{System.Delegate}$, then $T \preceq T'$.

Incomplete specifications  There are two specification gaps in [74], and each of them would imply the non-transitivity of $\preceq$:

- Concerning the definition of interfaces implementation in [74, §13.4], there are only two cases stated for a class $C$ to implement an interface $I$: The base class list of $C$ includes $I$ or an interface $I'$ which has $I$ as a base interface (not necessarily direct). The following case is missing: $C$ implements $I$ if $C$ extends a class $C'$ and $C'$ implements $I$.

- In [74, §6.1.4], there should also be stated an implicit reference conversions from any delegate type to the class System.Delegate and to the two interfaces implemented by the class System.Delegate, namely, System.ICloneable and System.ISerializable.

Definition 3.2.1 fills these gaps by defining $\preceq$ as a transitive closure.

Each class and struct declares types for a set of instance fields, and each class, struct or interface specifies signatures and return types² for a set of instance methods. Formally, we consider the universes in Table 3.3 and the environment components in Table 3.4.

²Note that, following [74, §1.7.3], the signature of a method does not include its return type.
For any class or struct $T$, $\text{instFields}(T)$ is the set of instance fields of $T$, including the inherited fields\footnote{A struct can only inherit fields from \texttt{System.ValueType} and \texttt{object}. However, as these classes do not declare any instance fields, the set $\text{instFields}$ associated to a struct can only contain instance fields declared by the struct itself.}. For any class or struct $T$ and for any field $T'::F \in \text{instFields}(T)$, $\text{fieldType}(T'::F)$ is the declared type of $T'::F$. For any method $T::M$, $\text{localVars}(T::M)$ returns the set of local variables of $T::M$. The type of a local variable $loc$ declared by a method $T::M$ is maintained in $\text{locType}(T::M, loc)$. The parameters passed "by value" of a method $T::M$ are given by $\text{valueParams}(T::M)$, whereas the parameters passed "by reference", i.e., the \texttt{ref} and \texttt{out} parameters, are maintained in $\text{refParams}(T::M)$ and $\text{outParams}(T::M)$, respectively. The type of a parameter $loc$ of a method $T::M$ is returned by $\text{paramType}(T::M, loc)$, while the index of $loc$ in the parameter list of $T::M$ is given by $\text{paramIndex}(T::M, loc)$. Note that, the parameter indexed with 0 is the \texttt{this} pointer.

The definition of $\text{paramType}$ is subject to the following restriction concerning the type of the \texttt{this} pointer:

\begin{itemize}
  \item [\texttt{this parameter type}] If $T::M \in \text{MRef}$, then $\text{paramType}(T::M, \texttt{this}) = T$.
\end{itemize}

For any method $T::M$, $\text{retType}(T::M)$ returns the possibly \texttt{void} return type of $T::M$. Depending on whether a method $T::M$ is virtual, $\text{callKind}(T::M)$ is \texttt{Virtual} or \texttt{NonVirtual}, respectively.

We define the (derived) function $\text{paramTypes} : \text{Map}(\text{MRef}, \text{List(Type)})$, which, given a method, returns the list of its parameter types.

\begin{definition}
For a method $T::M$ and an index $i = 0, n - 1$, where $n$ is the cardinal of the set $A = \text{valueParams}(T::M) \cup \text{refParams}(T::M) \cup \text{outParams}(T::M)$, we define:

\begin{equation}
\text{paramTypes}(T::M)(i) = \text{paramType}(T::M, loc), \text{ where } loc \in A \text{ such that } \text{paramIndex}(T::M, loc) = i
\end{equation}

\end{definition}

The \texttt{this} variable of a class instance method or constructor behaves as a value parameter. The \texttt{this} variable of a struct instance method is regarded as a \texttt{ref} parameter, whereas the \texttt{this} variable of a struct instance constructor behaves as an \texttt{out} parameter. We express these conditions in terms of restrictions on the definitions of $\text{valueParams}$, $\text{refParams}$, and $\text{outParams}$:

\begin{itemize}
  \item [\texttt{this variable}] Let $T::M \in \text{MRef}$.
  \begin{enumerate}
    \item If $T \in \text{RetType}$, then $\texttt{this} \in \text{valueParams}(T::M)$.
    \item If $T \in \text{Struct}$, then
      \begin{enumerate}
        \item if $T::M$ is an instance method, then $\texttt{this} \in \text{refParams}(T::M)$.
        \item if $T::M$ is an instance constructor, then $\texttt{this} \in \text{outParams}(T::M)$.
      \end{enumerate}
  \end{enumerate}
\end{itemize}

The implementations of virtual methods declared by a class can be \textit{overridden} in methods (marked with the \texttt{C#} keyword \texttt{override}) of derived classes and structs\footnote{A struct can override the virtual methods declared by the classes \texttt{System.ValueType} and \texttt{object}.}. Also, the (virtual)
methods declared by an interface have to be *implemented* in classes or structs that implement the interface. The following conditions should be satisfied when a method overrides/implements another method:

[override/implement] If $T::M \in MRef$ overrides/implements $T'::M \in MRef$, then:

1. $\text{length}(\text{paramTypes}(T::M)) = \text{length}(\text{paramTypes}(T'::M))$
2. for every $i = 1, \text{length}(\text{paramTypes}(T::M)) - 1$,
   
   $\text{paramTypes}(T::M)(i) = \text{paramTypes}(T'::M)(i)$
3. $\text{retType}(T::M) = \text{retType}(T'::M)$

### 3.2.2 The Grammar of $\mathrm{C^\#}_S$

The grammar for the statements and expressions of $\mathrm{C^\#}_S$ is defined in Figure 3.1. This grammar can also be viewed as defining the corresponding ASM domains $\text{Exp}$ and $\text{Stm}$. $\text{Lit}$ denotes the set of *literals*. To handle literals, we consider two functions $\text{valueOfLiteral} : \text{Map}(\text{Lit}, \text{Val})$ and $\text{typeOfLiteral} : \text{Map}(\text{Lit}, \text{Type})$ which, given a literal, return its value and its type, respectively. These functions are defined according to [74, §2.4.4].

The set of *variable expressions*, also known as ”lvalues”, is given by $\text{Vexp}$. The grammar production $\text{RefExp} . \text{Field}$ requires a compile-time check to ensure that the type of the expression $\text{RefExp}$ is a reference type (see Section 3.3 for the compile-time type checks). If the type of the expression is not a reference type, the resulting expression is not considered a variable expression.

The set $\text{Sexp}$ contains the *statement expressions*, *i.e.*, the statements that return results. The *delegate creation expressions* are represented in the grammar by the production rules $\text{new Delegate} (\text{Dexp})$ and $\text{new Delegate} (\text{Exp})$, where $\text{Dexp}$ stands for a special expression consisting of an expression (the value of which gives the target object pointed to by the delegate) and a method (which, as a result of a method lookup, yields the target method pointed to by the delegate) that should have no arguments specified. The delegate calls are defined in the grammar through the production $\text{Exp} (\text{[Args]})$, where $\text{Exp}$ is assumed to return the delegate to be called with the list of arguments produced by $\text{Args}$.

Every *try-catch-finally* statement contains, besides a *try* block, at least one *catch clause*, an optional general *catch clause* (i.e., a clause that can catch every exception), and an optional *finally* block. We assume all the constraints, stipulated by [74], concerning the structure of the *try-catch-finally* statements. In particular, we assume that:

- no *return* occurs in *finally* blocks;
- no *break*, *continue*, or *goto* statement jumps out of a *finally* block;
- the *throw* statements without expression can only occur in *catch* blocks;
- the exception classes in a *Catch* clause should derive (not necessarily directly) from $\text{System.Exception}$ [6] and should appear in a non-decreasing type order.
Figure 3.1 The grammar of expressions and statements.

\[
\begin{align*}
\text{Exp} &::= \text{Lit} | \text{Vexp} | \text{Exp Bop Exp} | \text{Exp ? Exp} : \text{Exp} \\
& | (\text{Type}) \text{ Exp} | \text{Sexp} | (\text{Exp}) | \text{this} | \text{base} | \text{Exp is Type} \\
& | \text{Exp as RefType} | \text{new Delegate (Exp)} | \text{new Delegate (Exp)}
\end{align*}
\]

\[
\begin{align*}
\text{Vexp} &::= \text{Loc} | \text{Field} | \text{Vexp} . \text{Field} | \text{RefExp} . \text{Field}
\end{align*}
\]

\[
\begin{align*}
\text{Sexp} &::= \text{Vexp} = \text{Exp} | \text{Vexp Aop Exp}
\end{align*}
\]

\[
\begin{align*}
\text{Dexp} &::= \text{Exp} . \text{MRef}
\end{align*}
\]

\[
\begin{align*}
\text{Bop} &::= \ast | / | \% | + | - | < | > | <= | >= | !== | === | & | |
\end{align*}
\]

\[
\begin{align*}
\text{Aop} &::= \ast= | /= | \%= | += | -= | &= | |=
\end{align*}
\]

\[
\begin{align*}
\text{Exps} &::= \text{Exp} , \text{Exp}
\end{align*}
\]

\[
\begin{align*}
\text{Stm} &::= \text{Sexp} ; | \text{break} ; | \text{continue} ; | \text{goto } \text{Lab} ; | \text{return } \text{Exp} ; \\
& | \text{return} ; | \text{if} (\text{Exp}) \text{ Stm} \text{ else } \text{Stm} \text{ while} (\text{Exp}) \text{ Stm} \\
& | \text{try Block} \{\text{Catch}\} \text{ [catch Block]} \text{ finally Block} | \text{throw } \text{Exp} ; \\
& | \text{throw} ; \text{ Block}
\end{align*}
\]

\[
\begin{align*}
\text{Catch} &::= \text{catch} (\text{Class} [\text{Loc}]) \text{ Block}
\end{align*}
\]

\[
\begin{align*}
\text{Arg} &::= \text{Exp} | \text{ref Vexp} | \text{out Vexp}
\end{align*}
\]

\[
\begin{align*}
\text{Args} &::= \text{Arg} , \text{Arg}
\end{align*}
\]

\[
\begin{align*}
\text{Sexps} &::= \text{Sexp} , \text{Sexp}
\end{align*}
\]

\[
\begin{align*}
\text{Block} &::= \{\{\text{Bstm}\}\}
\end{align*}
\]

\[
\begin{align*}
\text{Bstm} &::= \text{Type Loc} ; | \text{Lab} : \text{Stm} | \text{Stm}
\end{align*}
\]

<table>
<thead>
<tr>
<th>C# construct</th>
<th>Semantic equivalent C#\text{S} construct</th>
</tr>
</thead>
<tbody>
<tr>
<td>exp &amp;&amp; exp'</td>
<td>exp ? exp' : false</td>
</tr>
<tr>
<td>exp</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.5: Derived language constructs.

Table 3.5 shows how the conditional logical operators \&\&, || can be syntactically reduced to C\#\text{S} constructs. We still, however, include \&\&, || in our formal approach since the definite assignment analysis (see Section 3.4) treats them specially.

To express the sequentiality of the execution of the delegate invocation list elements, [24, §6] translates every delegate declaration to a special class. An inspection of the CIL bytecode to which C\# programs compile shows that this idea closely follows the Microsoft implementation. Thus, the declaration of a delegate type is translated to a class declaration, as described in Figure 3.2. The translation included in Figure 3.2 assumes that T is not void. If T is void, the translation is similar, except that the local variable result is not “needed” anymore, and consequently, the assignment to result in the while loop is replaced by a call of _invoke.
3.2. A Formal Description of $\text{C}^2_\text{S}$

Figure 3.2 The translation scheme of delegate classes.

```
delegate T D(T1 loc1, ..., Tn locn);

\$

sealed class D : System.Delegate
{
  public T Invoke(T1 loc1, ..., Tn locn)
  {
    T result = defVal(T);
    int i = 0;
    while (i < this._length())
    {
      result = this._invoke(i, loc1, ..., locn);
      i += 1;
    }
    return result;
  }
  private extern int _length();
  private extern T _invoke(int i, T1 loc1, ..., Tn locn);
}
```

Remark 3.2.1 It is worth mentioning that the translation in Figure 3.2 slightly differs than the one defined in [24], which does not initialize the variable result upon its declaration. But, if this is the case, the Invoke method would be rejected by the $\text{C}^2$ compiler, as a result of the definite assignment analysis (Section 3.4). Therefore, our translation initializes the variable result (to $T$’s default value).

To keep the formal description straightforward and more modular, we get rid of some syntactic sugar in $\text{C}^2$:

1. base is replaced by $(T)$ this, where $T$ is the direct base class of the enclosing type.
2. If $F$ is a field name, then $F$ is replaced by this.$T::F$, where $F$ occurs in type $T$.
3. If $C$ is a class (other than a delegate), then the instance creation new $C::M$ ( args ) is replaced by new $C::M$ ( args ).
4. If $S$ is a struct, then:
   a) an assignment $vexp = new S::M(args)$ is replaced by $vexp.S::M(args)$, and
   b) every other occurrence of $new S::M(args)$ is replaced by $loc.S::M(args)$, where $loc \in Loc$ is a new temporary local variable of type $S$.
5. Every general clause catch block is replaced by catch (object loc) block, with a new local variable loc.
6. Every try-catch-finally statement is reduced to a try-finally statement, with the try block made of the try block and catch clauses of the original statement and the finally block consisting of the finally block of the original statement.
To guarantee that the splitting in the third transformation reflects the intended meaning of `new T::M(args)`, we assume that the class instance constructors return the value of `this`. The transformation 4a) reflects that `new` applied to a struct triggers no object creation or memory allocation since, anyway, the structs, as value types, get their memory allocated at declaration time. The necessity of the transformation 4b) arises from the fact that the struct instance constructors need a "home", i.e., an address, for `this` (see condition [this variable] in Section 3.2.1). Also, we assume that the struct constructors return the value of `this`.

Whenever, instead of a direct formalization of a construct, [24] uses a syntactical translation to C♯ constructs, one should justify that the translation is correct with respect to the semantics of the construct as intended by the standard [74]. The semantics model yields a basis to formally state and prove the intended equivalence. Such a justification is skipped in [24] since it follows well-known patterns.

We denote the positions in a program by small Greek letters $\alpha$, $\beta$, $\gamma$, etc. One can think of positions either as positions in the source code of the program or as positions in the attributed syntax tree. Positions are displayed as prefixed superscripts, for example, as in $^\alpha$exp or in $^\beta$stm. We denote by Pos the universe of positions. We often refer to expressions and statements using their positions only.

To specify the dynamic semantics in Section 3.5.2, we define the (compiler specific) functions in Table 3.6, that are used during the traversal of the attributed syntax tree (AST). The function $\text{label} : \text{Map}(\text{Pos}, \text{Label})$ decorates the tree nodes with labels, elements of the universe Label. A label is information – in the form of concrete C♯ syntax – that identifies the grammar rule associated to the node. Given the node at a position $\alpha$:

- $\text{first}(\alpha)$ returns the position of the first AST "child" node (if any) of the node at $\alpha$;
- $\text{up}(\alpha)$ returns the position of the "parent" AST node (if any) of the node at $\alpha$;
- $\text{next}(\alpha)$ yields the position of the next AST "brother" node (if any) of the node at $\alpha$;

### Table 3.6: Attributed syntax tree specific functions.

<table>
<thead>
<tr>
<th>Function definition</th>
<th>Function name</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{label} : \text{Map}(\text{Pos}, \text{Label})$</td>
<td>node label</td>
</tr>
<tr>
<td>$\text{first} : \text{Map}(\text{Pos}, \text{Pos} \cup {\text{undef}})$</td>
<td>first child node</td>
</tr>
<tr>
<td>$\text{up} : \text{Map}(\text{Pos}, \text{Pos} \cup {\text{undef}})$</td>
<td>parent node</td>
</tr>
<tr>
<td>$\text{next} : \text{Map}(\text{Pos}, \text{Pos} \cup {\text{undef}})$</td>
<td>next brother node</td>
</tr>
</tbody>
</table>

3.3 The Type Constraints

To enforce type safety, the C♯ compiler has to verify that a program is well-typed. Following the parsing and elaboration, the evaluation tree of a program is annotated with type information, used in the evaluation rules in Section 3.5.2.1. Thus, the compiler infers a static function $\text{ST} : \text{Map}(\text{Pos}, \text{Type})$ – relative to an environment defined as in Section 3.2.1 – which assigns a type to each position in an expression.
We assume that, as a result of field and method resolution [74, §7.4.2] performed by the compiler simultaneously with the type inference, the attributed syntax tree has exact information. In particular, every field reference is of the form \(T::F\), and every method reference has the form \(T::M\). Moreover, as a result of the delegate transformation in Section 3.2.2, we assume that every delegate call \(\alpha_{\exp}(\args)\) (so, \(ST(\alpha) \in \text{Delegate}\)) is replaced by the method call \(\exp.ST(\alpha)::\text{Invoke}(\args)\).

Tables 3.7 and 3.8 contain the type constraints, expressed in terms of conditions for \(ST\), which should be satisfied after the type inference.

For example, for an operation \(\exp \ bop \ \exp'\), it is ensured that the types of the operands \(\exp\) and \(\exp'\) are suitable for the binary operator \(bop\). We assume that the set of types for which a binary operator is defined is maintained in the function \(\text{opTypes}: \text{Map}(\text{Bop}, \mathcal{P}(\text{Type} \times \text{Type}))\) (implicitly) defined in [74, §7]. Also, we consider that, given a binary operator and a pair of types, the function \(\text{opResType}: \text{Map}(\text{Bop} \times \text{Type} \times \text{Type}, \text{Type})\) (implicitly) defined in [74, §7] returns the type of the result of applying \(bop\) to operands of the given types\(^5\).

For a conditional expression \(\exp ? \exp' : \exp''\), either the type of \(\exp'\) is a subtype of the type of \(\exp''\) or vice-versa. The type of the whole expression is determined as the supremum of the operand types.

**Definition 3.3.1 (Supremum)** For two types \(T\) and \(T'\), \(\sup(T, T')\) denotes the supremum of \(T\) and \(T'\) with respect to the compile-time compatibility relation \(\preceq\).

In case of a boxing \((T)\exp\), that is, if \(T\) is a reference type and the type of \(\exp\) is a value type, it is ensured that the value type is a subtype of \(T\).

For a method call, the passing mode of each argument, i.e., ”by value”, \(\text{ref}\) or \(\text{out}\), should be identical to the passing mode of the corresponding parameter. Moreover, for a value parameter, the type of the argument should be a subtype of the declared type of the corresponding parameter, whereas for a \(\text{ref}\) or \(\text{out}\) parameter, the type of the argument should be identical to the declared type of the corresponding parameter. Due to our assumption in Section 3.2.2 that a constructor returns the value of the \(\text{this}\) pointer, it is guaranteed that a constructor of a type \(T\) returns the type \(T\).

In a delegate creation expression of the form \(\text{new} \ D(\exp.T::M)\), the type of \(\exp\) should be a subtype of \(T\) and the method \(T::M\) to which the newly created delegate is going to refer should have a return type and a signature compatible with the delegate type \(D\). This leads to the definition of consistency\(^6\) in Definition 3.3.2 that allows covariance in return type and contravariance in parameter types.

**Definition 3.3.2 (Consistency)** A method \(T::M\) and a delegate type \(D\) are consistent if the following conditions are met:

- for every \(\text{loc} \in \text{valueParams}(T::M)\), there exists \(\text{loc}' \in \text{valueParams}(D::\text{Invoke})\) such that

\(^5\)We suppose that a binary operator overload resolution [74, §7.2.4] has been previously applied to select a specific operator to be invoked.

\(^6\)Initially, the definition of consistency specified in [74] required that the return types and signatures of the method and delegate, respectively, should coincide. However, as one can observe in our type safety proof, the requirement is too strong, and therefore has been weakened in [82].
<table>
<thead>
<tr>
<th>Expression</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>α\text{lit}</td>
<td>ST(α) = \text{typeofLiteral}(lit)</td>
</tr>
<tr>
<td>α\text{loc}</td>
<td>ST(α) = locType(T::M, loc), where T::M is the enclosing method</td>
</tr>
<tr>
<td>α\text{this}</td>
<td>ST(α) = T, where T is the enclosing type</td>
</tr>
<tr>
<td>α(β\text{exp } \text{bop } γ\text{exp}')</td>
<td>(ST(β), ST(γ)) \in \text{opTypes}(\text{bop}), ST(α) = \text{opResType}(\text{bop}, ST(β), ST(γ))</td>
</tr>
<tr>
<td>α(β\text{exp } ? \text{exp}' : δ\text{exp}'')</td>
<td>ST(β) = \text{bool}, ST(γ) \leq ST(δ) \lor ST(δ) \leq ST(γ), ST(α) = \sup(ST(γ), ST(δ))</td>
</tr>
<tr>
<td>α(T β\text{exp})</td>
<td>if ST(β) \in \text{ValueType} and T \in \text{RefType}, then ST(β) \leq T, ST(α) = T</td>
</tr>
<tr>
<td>α(β\text{vexp} = γ\text{exp})</td>
<td>ST(γ) \leq ST(β), \alpha = ST(β)</td>
</tr>
<tr>
<td>α(β\text{vexp } \text{bop } γ\text{exp})</td>
<td>\text{opResType}(\text{bop}, ST(β), ST(γ)) \leq ST(β) or \text{ST}(β) \leq \text{opResType}(\text{bop}, ST(β), ST(γ)), ST(α) = ST(β), ST(β), ST(γ) \in \text{opTypes}(\text{bop})</td>
</tr>
<tr>
<td>α(\text{ref } β\text{vexp})</td>
<td>ST(α) = ST(β)</td>
</tr>
<tr>
<td>α(\text{out } β\text{vexp})</td>
<td>ST(α) = ST(β)</td>
</tr>
<tr>
<td>α(β\text{exp } \text{is } T)</td>
<td>ST(α) = \text{bool}</td>
</tr>
<tr>
<td>α(β\text{exp } \text{as } T)</td>
<td>ST(α) = T, if ST(β) \in \text{ValueType}, then ST(β) \leq T</td>
</tr>
<tr>
<td>α(β\text{exp}.T:F)</td>
<td>ST(α) = \text{fieldType}(T:F), ST(β) \leq T</td>
</tr>
<tr>
<td>α(β\text{exp}.T:M(\ldots, γ\text{exp}_i, \ldots))</td>
<td>ST(β) \leq T, if loc \in \text{valueParams}(T::M) such that \text{paramIndex}(T::M, loc) = i, then ST(γ_i) \leq \text{paramTypes}(T::M)(i) if loc \in \text{refParams}(T::M) \cup \text{outParams}(T::M) such that \text{paramIndex}(T::M, loc) = i, then ST(γ_i) = \text{paramTypes}(T::M)(i) ST(α) = \text{refType}(T::M) if T::M is not a constructor, otherwise ST(α) = T</td>
</tr>
<tr>
<td>α(\text{new } T)</td>
<td>ST(α) = T</td>
</tr>
<tr>
<td>α(\text{new } D(β\text{exp}.T:M))</td>
<td>D \in \text{Delegate}, ST(β) \leq T, T::M and D are consistent, ST(α) = D</td>
</tr>
<tr>
<td>α(\text{new } D(β\text{exp}))</td>
<td>D \in \text{Delegate}, ST(β) = ST(α) = D</td>
</tr>
</tbody>
</table>

Table 3.7: The type constraints for expressions.

\[
\text{paramIndex}(T::M, loc) = \text{paramIndex}(D::\text{Invoke}, loc') \quad \text{and} \\
\text{paramType}(T::M, loc) \leq \text{paramType}(D::\text{Invoke}, loc')
\]

- for every loc \in \text{refParams}(T::M), there exists loc' \in \text{refParams}(D::\text{Invoke}) such that
Table 3.8: The type constraints for statements.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>if (α exp) stm else stm</td>
<td>ST(α) = bool</td>
</tr>
<tr>
<td>while (α exp) stm</td>
<td>ST(α) = bool</td>
</tr>
<tr>
<td>return;</td>
<td>retType(T::M) = void, where T::M is the enclosing method</td>
</tr>
<tr>
<td>return α exp;</td>
<td>ST(α) ≤ retType(T::M), where T::M is the enclosing method</td>
</tr>
<tr>
<td>throw α exp;</td>
<td></td>
</tr>
</tbody>
</table>

We say that a method is well-typed if it satisfies the type checks performed by the compiler.

**Definition 3.3.3 (Well-typed)** A method is well-typed if its body contains only expressions and statements that satisfy the type constraints in Tables 3.7 and 3.8.

After the type constraints in Tables 3.7 and 3.8 are guaranteed, the program transformations defined in Table 3.9 are performed, where necessary. The transformations are executed only if the corresponding conditions in the second column of Table 3.9 (if any) hold.

For example, a conditional expression exp ? exp' : exp'' requires two type casts to be inserted. Thus, both exp' and exp'' are type casted to the supremum of the types of exp' and exp''.

A boxing is inserted in an assignment vexp = exp if the type of exp is a value type, whereas the type of vexp is a reference type. This assignment is replaced by vexp = (T)exp, where T is the type of vexp. A boxing is also added in a method call exp.T::M(args) if the type of exp is a value type and T is a reference type. This is the case when the method is declared by the classes System.ValueType or object or by an interface implemented by the value type. However, if T is not a reference type, and moreover, exp is not a variable expression (so it could not have an address), then a "home" is created for the value of exp (since the this pointer of T::M behaves as a reference parameter). Thus, a new temporary local variable, say loc, of the same type as exp is inserted into the set of local variables of the enclosing method, and the value of exp is assigned to loc. In [24], the temporary variable is created on the fly, during the run-time execution. Our formal specifications follow the Microsoft implementation and creates the variable upon the compilation.
3.4 The Definite Assignment Analysis in C♯

In C♯, local variables are not initialized by default, unlike instance fields. Therefore, in order to ensure type-safety, a C♯ compiler must guarantee that all local variables are assigned to before their value is used. The C♯ compiler enforces this definite assignment rule by a static flow analysis. Since the problem is undecidable in general, [74, §5.3] contains a definition of a decidable subclass of the set of variables that get assigned at run-time. The static analysis guarantees that there is an initialization to a local variable on every possible execution path before the variable is read.

In this section, we provide a formal specification of the definite assignment analysis in C♯ and prove its correctness. The formal specification emphasizes, in particular, the complications caused by the goto and break statements (incompletely specified in [74]) and by method calls with ref/out parameters. The correctness of the analysis is later used in Section 3.6 to prove C♯’s type safety.

To formally define the definite assignment rules, we use data flow equations. For a method body without goto statements, the equations that characterize the sets of definitely assigned variables can be solved in a single pass. If goto statements are present, then the equations defined in our formalization do not uniquely determine the sets of variables that have to be considered definitely assigned. For this reason, a fixed-point computation is performed, and the greatest sets of variables that satisfy the equations of the formalization are computed. Regarding the correctness of the analysis, we prove that these sets of variables represent exactly the sets of variables assigned on all possible execution paths, and in particular, they are a safe approximation. A number of bugs in the Rotor SSCLI (v1.0) [16]’s and Mono (v0.26) [15]’s C♯ compilers were discovered during the attempts to build the formalization of the definite assignment. We only present here three of them.

The rest of the section is organized as follows. Section 3.4.1 introduces the data flow equa-
tions which formalize the \( \text{C}\# \) definite assignment analysis, while Section 3.4.2 shows that there always exists a maximal fixed point solution for the equations. In order to define the execution paths in a method body, the control flow graph is introduced in Section 3.4.3. Section 3.4.4 concludes with the proof of the correctness of the analysis, Theorem 3.4.1.

### 3.4.1 The Data Flow Equations

In this section, we formalize the rules of definite assignment analysis from the \([74, \text{§} 5.3]\) by data flow equations. Since the definite assignment analysis is an intraprocedural analysis, we restrict our formalization only to a given method \( mref \).

To precisely specify all the cases of definite assignment, the functions \( \text{before} \), \( \text{after} \), \( \text{true} \), and \( \text{false} \) are computed at compile-time. Note that \( \text{true} \) and \( \text{false} \) are only defined for boolean expressions. These functions assign sets of variables to each expression or statement \( \alpha \) and have the following meanings:

- \( \text{before}(\alpha) \) contains the variables definitely assigned before the evaluation of \( \alpha \);
- \( \text{after}(\alpha) \) contains the variables definitely assigned after the evaluation of \( \alpha \) when \( \alpha \) completes normally;
- \( \text{true}(\alpha) \) consists of the variables (in the scope of which \( \alpha \) is located) definitely assigned after the evaluation of \( \alpha \) when \( \alpha \) evaluates to \( \text{true} \);
- \( \text{false}(\alpha) \) consists of the variables (in the scope of which \( \alpha \) is located) definitely assigned after the evaluation of \( \alpha \) when \( \alpha \) evaluates to \( \text{false} \);

The sets \( \text{true} \) and \( \text{false} \) are needed because of the conditional operators \&\& and \|\|, as we show in Example 3.4.1. The set \( \text{vars}(\alpha) \) contains the local variables in the scope of which \( \alpha \) is located, \( \text{i.e.} \), the universal set with respect to \( \alpha \).

For the sake of clarity, we skip those language constructs whose analysis is very similar to the constructs dealt with explicitly in our framework\(^7\); examples are the pre- and postfix operators (\( ++ \), \( -- \)).

**Struct type variables** To simplify the proofs, we will treat separately the struct type variables. We point out in Section 3.4.2 how they affect the sets of definitely assigned variables the \( \text{C}\# \) compiler relies on in order to analyze programs. Also, we show in Section 3.4.4 that allowing variables of struct types does not affect the correctness of the analysis. In the rest of Section 3.4, we state explicitly whenever we include struct type variables.

We are now able to state all the data flow equations. A first equation is given by the method’s initial conditions: For the method body \( mb \) of \( mref \), we have \( \text{before}(mb) = \emptyset \). Conceptually, the set \( \text{before}(mb) \) contains the value and reference parameters of \( mref \), since they are assumed to be definitely assigned when \( mref \) is called \([74, \text{§} 5.1]\).

---

\(^7\)Some of the features omitted by the definite assignment analysis presented in this section are investigated in detail in \([56, 57]\).
Expression $\alpha$ | Data flow equations
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$true$</td>
<td>$true(\alpha) = before(\alpha)$, $false(\alpha) = vars(\alpha)$</td>
</tr>
<tr>
<td>$false$</td>
<td>$false(\alpha) = before(\alpha)$, $true(\alpha) = vars(\alpha)$</td>
</tr>
<tr>
<td>$(\beta \ exp ? \ \gamma \ exp' : \ \delta \ exp')$</td>
<td>$before(\beta) = before(\alpha)$, $before(\gamma) = true(\beta)$, $before(\delta) = false(\beta)$, $true(\alpha) = true(\gamma) \cap true(\delta)$, $false(\alpha) = false(\gamma) \cap false(\delta)$</td>
</tr>
<tr>
<td>$(\beta \ &amp; \ &amp; \ \gamma \ exp')$</td>
<td>$before(\beta) = before(\alpha)$, $before(\gamma) = true(\beta)$, $true(\alpha) = true(\gamma)$, $false(\alpha) = false(\beta) \cap false(\gamma)$</td>
</tr>
<tr>
<td>$(\beta \</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.10: Definite assignment for boolean expressions.

For the other expressions and statements in $mb$, instead of explaining how the functions are computed, we simply state the equations they have to satisfy. Table 3.10 contains the equations for boolean expressions (including for completeness the literals $true$ and $false$). If $\alpha$ is the constant $true$, then $false(\alpha) = vars(\alpha)$ as a consequence of the definition of the $false$ set and of the fact that $true$ cannot evaluate to $false$. Similar arguments hold for $true(\alpha) = vars(\alpha)$ when $\alpha$ is the constant $false$. We need the sets $true$ and $false$ since the evaluation of boolean expressions involving the conditional operators $\&\&$ and $||$ does not necessarily require the evaluation of all their subexpressions.

**Example 3.4.1** Consider the following expression:

$$\alpha(b \ \&\& (i = 1) >= 0) ? true : \gamma \ i > 0$$

If $b$ evaluates to $false$, then the test $(b \ \&\& (i = 1) >= 0)$ immediately evaluates to $false$, and its second operand, i.e., $(i = 1) >= 0$ is never evaluated. So, in this case $i$ is not assigned; on the other hand, a necessary condition for the test to be evaluated to $true$ is that both operands of $\alpha$ are evaluated. Therefore, the $C^\sharp$ compiler is sure that $i$ is assigned only if $\alpha$ evaluates to $true$. Formally, this means $i \notin false(\alpha)$ and $i \in true(\alpha)$. Consequently, $i$ is not considered definitely assigned before evaluating $\gamma$, and the compiler should reject this example. Surprisingly, the Mono (v0.26) [15]'s $C^\sharp$ compiler incorrectly accepts it, as it also does for the other conditional operator, $||$.

For all expressions in Table 3.10, we have the equation $after(\alpha) = true(\alpha) \cap false(\alpha)$. For any boolean expression $\alpha$ which is not an instance of one of the expressions in Table 3.10, we have $true(\alpha) = after(\alpha)$ and $false(\alpha) = after(\alpha)$.

Table 3.11 lists the equations specific to arbitrary expressions, where $loc$ stands for a local variable and $lit$ for a literal. Note that the table contains another equation for the conditional expression when its value is not a boolean. The equation for the explicitly boolean expression collects additional information. If the boolean conditional is treated as an arbitrary expression, then the equation for $after(\alpha)$ would still be correct — it can be derived from the other equations for the boolean expressions.
The \texttt{ref} arguments must be definitely assigned before the method call, while the \texttt{out} arguments are not necessarily assigned before the method is called. However, the \texttt{out} arguments must be definitely assigned when the method returns. Note the equation for $after(\alpha)$ of a method call in Table 3.11: The variables passed as \texttt{out} arguments, elements of the set $OutParams(arg_1, \ldots, arg_k)$, get definitely assigned. It does not matter if the calls are recursive, since the definite assignment analysis is an intraprocedural analysis.

For expressions that do not appear in Tables 3.10 and 3.11 (e.g., $\lnot \exp$, $\exp' \land \exp''$, $\exp + \exp'$), if $\alpha \exp$ is an expression with \textit{direct subexpressions} $\beta \exp_1, \ldots, \beta_n \exp_n$, then the left-to-right evaluation scheme yields the following general \textit{data flow equations}:

\[
\begin{align*}
before(\beta_1) &= before(\alpha), & before(\beta_{i+1}) &= after(\beta_i), & i = 1, n - 1, & after(\alpha) &= after(\beta_n)
\end{align*}
\]

The equations specific for statements can be found in Table 3.12. For a block of statements $\alpha$, we have the equation $after(\alpha) = after(\beta_n) \cap vars(\alpha)$: The local variables definitely assigned after the normal execution of the block are the variables which are definitely assigned after the execution of the last statement of the block. However, the variables must still be in the scope of a declaration.

\textbf{Example 3.4.2} Consider the example:

\[
\{\alpha\{\text{int } i; \; i = 1;\} \ {\text{int } i; \; i = 2 * \beta i; \}}\}
\]

The variable $i$ is not in $after(\alpha)$, since at the end of $\alpha$, $i$ is not in the scope of a declaration. Thus, $i \not\in before(\beta)$, and the block is rejected.
For the equation $\text{before}(\beta_{i+1}) = \text{after}(\beta_i) \cap \text{goto}(\beta_{i+1})$, special attention is given to the case when $\beta_{i+1}$ is a labelled statement. A key point is that if a `goto` embedded in a `try` block (of a `try-finally` statement) points to a labelled statement which is not embedded in the `try` block, then the `finally` block has to be executed (before the labelled statement). Thus, the set of variables definitely assigned before executing a labelled statement consists of the variables definitely assigned both after the previous statement and before each corresponding `goto` statement, or after any of the `finally` blocks of `try-finally` statements in which the `goto` is embedded (if any).

**Definition 3.4.1** For two statements $\alpha$ and $\beta$, $\text{Fin}(\alpha, \beta)$ is defined as the list $[\gamma_1, \ldots, \gamma_n]$ of `finally` blocks of all `try-finally` statements in the innermost to outermost order from $\alpha$.
to $\beta$.

Then we define the set $\text{JoinFin}$ of variables definitely assigned after the execution of all the $\text{finally}$ blocks in the list $\text{Fin}$.

**Definition 3.4.2** For two statements $\alpha$ and $\beta$, we define

$$\text{JoinFin}(\alpha, \beta) = \bigcup_{\gamma \in \text{Fin}(\alpha, \beta)} \text{after} (\gamma)$$

**Definition 3.4.3** Given a statement $\beta$, we define:

$$\text{goto} (\beta) = \begin{cases} \bigcap_{\alpha \text{goto} L;} \text{in the scope of } \beta (\text{before}(\alpha) \cup \text{JoinFin}(\alpha, \beta)), & \text{if } \beta \text{ is a labelled statement } ^{\beta L : \text{stm}}; \\ \text{vars}(\beta), & \text{otherwise}. \end{cases}$$

We are now able to state the equation $\text{before} (\beta_{i+1}) = \text{after} (\beta_i) \cap \text{goto} (\beta_{i+1})$ from Table 3.12. In the case of a labelled statement, the equation captures the idea stated above, while for a non-labelled statement, this equations reduces simply to $\text{before} (\beta_{i+1}) = \text{after} (\beta_i)$.

**Example 3.4.3** The following example is a simplification of an example in [74, §5.3.3.15]:

```csharp
int i;
try {
    goto L;
} finally {
    i = 3;
}
\text{L} : \text{Console.WriteLine}(i);
```

[74] claims that the variable $i$ is definitely assigned before $\beta$, i.e., $i \in \text{before}(\beta)$. Our equation $\text{before}(\beta) = \text{after}(\delta) \cap \text{goto}(\beta)$ leads us to the same conclusion. To compute the set $\text{goto}(\beta)$, we need the list $\text{Fin}(\alpha, \beta) = [\gamma]$ and the set $\text{JoinFin}(\alpha, \beta) = \text{after}(\gamma)$. We have:

$$\text{goto} (\beta) = \text{before}(\alpha) \cup \text{JoinFin}(\alpha, \beta) = \text{before}(\alpha) \cup \text{after}(\gamma)$$

and $i \in \text{after}(\gamma) \subseteq \text{after}(\delta)$ (see the equations for a try-finally in Table 3.12). This means that $i \in \text{after}(\delta) \cap \text{goto}(\beta) = \text{before}(\beta)$. Surprisingly, the example is rejected by .NET Framework (v1.0) [2] and Rotor SSCLI (v1.0)’s C$^\#$ compilers: We get the error that $i$ is unassigned. In the meantime, the problem has been fixed in .NET Framework (v1.1) [2].

**Example 3.4.4** Although in the next method body $i$ should be considered definitely assigned before $\beta$, the example is rejected by the Mono (v0.26) [15]’s C$^\#$ compiler.
int i;
bool b = false;
if (b)
{
  i = 1;
  goto L;
}
return;
L: Console.WriteLine(i);

We have \( i \in before(\beta) \), since the set \( before(\beta) \) is computed as follows:

\[
before(\beta) = after(\delta) \cap goto(\beta) = vars(\delta) \cap before(\alpha) = \{i, b\} \cap \{i\} = \{i\}
\]

The idea for the equation which computes \( after(\alpha) \) of a \textit{while} statement \( \alpha \) is the same as for a labelled statement. Similarly as for the set \( goto \), we define the set \( break(\alpha) \) needed for the equation of \( after(\alpha) \) to be the set of variables definitely assigned before all associated \textit{break} statements (and possibly after appropriate \textit{finally} blocks).

**Definition 3.4.4** For a \textit{while} statement \( \alpha \), we define

\[
break(\alpha) = \begin{cases}
\bigcap^\alpha_{\text{break; } \alpha \text{ is the nearest enclosing while for } \beta} (before(\beta) \cup JoinFin(\beta, \alpha)), & \text{if } \alpha \text{ has break; } \text{s associated;}
\vars(\alpha), & \text{otherwise.}
\end{cases}
\]

With this definition of \( break(\alpha) \), we have the equation for \( after(\alpha) \) as stated in Table 3.12.

**Abnormal termination** Finally, we consider the equations for abnormally terminating statements. We now want to state the equation for \( after \) of a jump statement.

**Example 3.4.5** Let \( \alpha \) be the following statement:

\[
if (b) \gamma\{ i = 1; \} else \delta \text{return;}
\]

It is clear that the variables definitely assigned after \( \alpha \) are the variables definitely assigned after the \textit{then} branch, and since our equation takes the intersection of \( after(\gamma) \) and \( after(\delta) \), it is obvious that one has to require the set-intersection identity for \( after(\delta) \).

Considerations of the example above lead to the convention that \( after(\alpha) \) should be considered the universal set \( vars(\alpha) \) for any jump statement \( \alpha \).

**Example 3.4.6** Consider the next method body:
3.4. THE DEFINITE ASSIGNMENT ANALYSIS IN C\textsuperscript{\#}

\[\delta \text{int } i = 1;\]
\[\beta \text{L::; try \{ int } j = 2;\]
\[\text{try } \{ ^{\alpha} \text{goto L; } \}
\[\text{finally }^{\gamma_1} \{ \text{int } k = 3; ^{\omega} \text{throw new Exception(); } \} \]
\[\text{finally }^{\gamma_2} \{ \} \]

In our formalization, we get:

\[
\text{before}(\beta) = \text{after}(\delta) \cap \text{goto}(\beta)
\]
\[
= \text{after}(\delta) \cap (\text{before}(\alpha) \cup \text{JoinFin}(\alpha, \beta))
\]
\[
= \text{after}(\delta) \cap (\text{before}(\alpha) \cup \text{after}(\gamma_1) \cup \text{after}(\gamma_2))
\]

Note that in the computation of JoinFin(\(\alpha, \beta\)), it does not matter whether \(\gamma_1\) and \(\gamma_2\) complete normally. In our case, \(\gamma_1\) does not complete normally, but we still perform our computations with \(\gamma_1\) and \(\gamma_2\). The set after(\(\gamma_1\)) is \(\text{vars}(\omega) \cap \text{vars}(\gamma_1) = \{i, j, k\} \cap \{i, j\} = \{i, j\}\). Note also that after(\(\gamma_1\)) involved in the equation of before(\(\beta\)) also contains \(j\), while \(\beta\) is not in the scope of a declaration of \(j\), i.e., \(j \notin \text{vars}(\beta)\). There is no worry since in the equation of before(\(\beta\)) all the sets that might contain variables declared in “deeper” scopes (like \(j\)) are intersected with after(\(\delta\)), which is supposed to contain only variables from \(\text{vars}(\delta) = \text{vars}(\beta) = \{i\}\) (\(\beta\) and \(\delta\) are at the same nesting level).

The details of Examples 3.4.5 and 3.4.6 become more clear in Lemmas 3.4.4 and 3.4.5 in Section 3.4.4. Whenever a statement does not complete normally, the set of variables considered definitely assigned after its evaluation will be a universal set (see also the proofs of Theorems 3.4.1 and 3.4.2 from Section 3.4.4).

3.4.2 The Maximal Fixed Point Solution

The computation of the sets of definitely assigned variables from the data flow equations described in Section 3.4.1 is relatively straightforward. However, the goto statement brings more complexity to the analysis. Since the goto statement allows to encode loops, the system of data flow equations does not have always a unique solution.

Example 3.4.7 Consider a method (with no parameters) that has the following body:

\[\{ ^{\alpha} \text{int } i = 1; ^{\beta} \text{L:: goto L;} \}\]

We have the following equations after(\(\alpha\)) = \(\{i\}\), before(\(\beta\)) = after(\(\alpha\)) \cap before(\(\gamma\)) and before(\(\gamma\)) = before(\(\beta\)). After some simplification, we find that before(\(\beta\)) = \(\{i\}\) \cap before(\(\beta\)). Therefore, we get two solutions for before(\(\beta\)) (and also for before(\(\gamma\))): \(\emptyset\) and \(\{i\}\). This is the reason we perform a fixed point iteration. The set of variables definitely assigned after \(\alpha\) is \(\{i\}\); since \(\beta\) does not ”unassign” \(i\), \(i\) is obviously assigned when we enter \(\beta\).

The above example and the definition of definitely assigned indicate that the most informative solution is considered, and therefore the solution we require is the maximal fixed point MFP. For computing this solution, various algorithms exist (see e.g., [97]).
Remark 3.4.1 Although the statements $L : \text{goto } L; \text{ and } \text{while (true)}$; are behaviorally similar, they are treated differently by the definite assignment analysis. If $\alpha$ denotes the labelled statement, then the equation for before$(\alpha)$ implies recursion (as noticed above). If $\alpha$ is the while statement above, then no equation corresponding to $\alpha$ involves recursion. The set after$(\alpha)$ of the above while statement can be computed according to the equations in a single step (i.e., with no fixed point iteration) as follows after$(\alpha) = \text{false}(\alpha) \cap \text{break}(\alpha) = \text{vars}(\alpha) \cap \text{vars}(\alpha) = \text{vars}(\alpha)$. The set before$(\alpha)$ is determined as for any regular statement using only the after set of the previous statement. Even if a while statement $\alpha$ has an associated continue statement $\gamma$, the equation for before$(\alpha)$ does not involve the continue, but only the previous statement $\beta$. The reason is that, at the time the analysis is performed, the compiler is sure that the continue is embedded in the while body, and therefore the set before$(\gamma)$ includes the variables in after$(\beta)$ (if the continue is executed, then $\beta$ should have already been executed). This is not always the case for a labelled statement since the associated gotos are not necessarily embedded in the labelled block (see the last example of Section 3.4.1). That is why they are involved in the equation for before$(\alpha)$.

In the rest of this section, we show that there always exists a maximal fixed point for our data flow equations. In order to prove its existence, we first define the function $F$, which encapsulates the equations. For the domain and codomain of this function, we use the set $\text{localVars}(\text{mref})$ of all local variables from the method body $\text{mb}$. A simple inspection of the equations shows that they all have at the left side either a before, after, true or a false set and at the right side a combination of these kinds of sets and $\text{vars}$ sets. We define the function $F : \text{Map}(D,D)$, with $D = \mathcal{P}(\text{localVars}(\text{mref}))^r$, such that $F(X_1, \ldots, X_r) = (Y_1, \ldots, Y_r)$, where $r$ is the number of equations and the sets $Y_i$ are defined by the data flow equations, with the $\text{vars}$ sets interpreted as constants. For example, in the case of an if-then-else statement, if the equation for the after set of this statement is the $i$-th data flow equation, then the set of variables $Y_i$ is defined by $Y_i = X_j \cap X_k$, where $j$ and $k$ are the indices of the equations for the after sets of the then and the else branch, respectively.

We now define the relation $\ll$ on $D$ to be the pointwise set inclusion relation:

Definition 3.4.5 If $(X_1, \ldots, X_r)$ and $(X'_1, \ldots, X'_r)$ are elements of the set $D$, then we have $(X_1, \ldots, X_r) \ll (X'_1, \ldots, X'_r)$, if $X_i \subseteq X'_i$ for all $i = 1, r$.

We are now able to prove the following result:

Lemma 3.4.1 $(D, \ll)$ is a finite lattice.

Proof. $D$ is finite since for a given method body we have a finite number of equations and local variables. $D$ is a lattice since it is a product of lattices: $(\mathcal{P}(\text{localVars}(\text{mref})), \subseteq)$ is a poset, since the set inclusion is a partial order, and for every two sets $X, Y \in \mathcal{P}(\text{localVars}(\text{mref}))$, there exists a lower bound (e.g., $X \cap Y$) and an upper bound (e.g., $X \cup Y$). □

The following result will help us conclude the existence of the maximal fixed point.

Lemma 3.4.2 The function $F$ is monotonic on $(D, \ll)$.
3.4. The Definite Assignment Analysis in C♯

**Proof.** To prove the monotonicity of $F = (F_1, \ldots, F_r)$, it suffices to observe that the components $F_i$ are monotonic functions. This holds since they only consist of set intersections and unions, which are monotonic (see the form of the equations).

The next lemma guarantees the existence of the maximal fixed point solution for our data flow equations.

**Lemma 3.4.3** The function $F$ has a unique maximal fixed point $MFP \in D$.

**Proof.** By Lemma 3.4.1, we know that $(D, \ll)$ is a finite lattice, and therefore a complete lattice. But in a complete lattice, every monotonic function has a unique maximal fixed point, also known as the greatest fixed point. In our case, $F$ is monotonic (by Lemma 3.4.2), and the maximal fixed point $MFP$ is given by $\bigcap_k F^{(k)}(1_D)$. Here, $1_D$ is the $r$-tuple $(localVars(mref), \ldots, localVars(mref))$, i.e., the top element of the lattice $D$.

From now on, for an expression or statement $\alpha$, we denote by $MFP_{b}(\alpha)$, $MFP_{a}(\alpha)$, $MFP_{t}(\alpha)$, and $MFP_{f}(\alpha)$ the components of the maximal fixed point $MFP$ corresponding to before($\alpha$), after($\alpha$), true($\alpha$), and false($\alpha$), respectively.

**Struct type variables** So far, we have not considered struct type variables. However, our analysis can easily be extended to struct type variables. Firstly, we observe that a struct type variable is considered definitely assigned if and only if all its instance fields are definitely assigned [74, §5.3] (this is because a struct value can be seen as a tuple consisting of its instance fields).

If a local variable $loc$, whose declared type is a struct, gets assigned, then all its instance fields are considered definitely assigned, and if there are struct type instance fields, then their instance fields get assigned as well, and so on.

**Example 3.4.8** Let us consider an instance method $C::M$, which takes an out parameter of type the struct $S$. The instance field $y$ of $p.x$ is definitely assigned before it is printed, since $p$ gets definitely assigned after the method call.

```csharp
struct S {  
    public S' x;
}
class Test {  
    static void Main() {
        S p;
        new C().M(out p);
        Console.WriteLine(p.x.y);
    }
}
```

According to our formalization, the C♯ compiler relies on the set $MFP_{a}(\alpha) = after(\alpha) = \{p\}$, with $^a(new C().M(out p))$, when checking the status of the variable $p.x.y$. But, as observed above, after $\alpha$ is evaluated, $p.x$ and $p.x.y$ are considered definitely assigned as well. So, allowing struct type variables requires the compiler to rely on “expanded” sets: The set $\{p\}$ is “expanded” to $\{p, p.x, p.x.y\}$, to also include the instance fields of $p$. 
Further, if a local variable \( \text{loc}' \), which is an instance field of a struct type variable \( \text{loc} \), gets assigned, and each instance field of \( \text{loc} \) except \( \text{loc}' \) either is already considered definitely assigned or gets assigned at the same time as \( \text{loc}' \) (this happens, for example, in case of the \textbf{out} arguments following a method call), then the variable \( \text{loc} \) gets assigned as well, and this “lookup” procedure is repeated with \( \text{loc} \) instead of \( \text{loc}' \).

**Example 3.4.9** In the following example, the struct type field \( \text{p.y} \) gets assigned when its instance field \( \text{v} \) gets assigned. Next, the instance variable \( \text{p} \) gets assigned when its instance field \( \text{p.x} \) gets assigned since \( \text{p.y} \) is already assigned.

\[
\begin{align*}
\text{struct } S & \{ \\
\text{public int } x; \\
\text{public } S’ y; \\
\} \\
\text{struct } S’ & \{ \\
\text{public int } u; \\
\text{public int } v; \\
\} \\
\end{align*}
\]

According to the formalization, the compiler relies on the set \( \text{MFP}_a(\alpha) = \text{after}(\alpha) = \{p.y.u, p.y.v, p.x\} \) with \( \alpha(p.x = 1) \) when verifying whether \( \text{p} \) is definitely assigned before evaluating the assignment to \( \text{r} \). Considering also struct type variables makes the compiler rely on the expanded set of \( \{p.y.u, p.y.v, p.x\} \), i.e., \( \{p.y.u, p.y.v, p.y, p.x\} \).

Hence, we conclude that after the \( \text{C#} \) compiler determines the MFP sets, it relies on the expanded MFP sets in order to reject/accept programs. We will say that an “expansion” function is applied to the MFP sets to propagate the definitely assigned status.

### 3.4.3 The Control Flow Graph

So far, we have seen the equations used for the analysis and we have proven that the fixed point iteration for these equations is well-defined. The result we want to prove is that the outcome of the analysis is correct: For an arbitrary expression or statement, the sets of local variables \( \text{MFP}_b, \text{MFP}_a \) (and \( \text{MFP}_t, \text{MFP}_f \) for boolean expressions) correspond indeed to sets of definitely assigned variables, i.e., variables which are assigned on every possible execution path to the appropriate point.

The considered paths are based on the control flow graph \( \text{CFG} \). The nodes of the graph are actually points associated with every expression and statement. We suppose that every expression or statement \( \alpha \) is characterized by an entry point \( B(\alpha) \) and an end point \( A(\alpha) \). Beside these two points a boolean expression \( \alpha \) has two more points: a true point \( T(\alpha) \) (used when \( \alpha \) evaluates to \text{true} \) and a false point \( F(\alpha) \) (used when \( \alpha \) evaluates to \text{false} \). The edges of the graph are given by the control transfer defined in [74, §8]. Tables 3.13 and 3.14 show the edges specific to each boolean and arbitrary expression, respectively. If the expression \( \alpha \) is not an instance of one expression in these tables (e.g., \( \text{exp | exp’} \)) and has the direct subexpressions \( \beta_1, \ldots, \beta_n \), then the left-to-right evaluation scheme also adds the following edges to the flow graph:
<table>
<thead>
<tr>
<th>Expression $\alpha$</th>
<th>Edges</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{true}$</td>
<td>$(B(\alpha), T(\alpha))$</td>
</tr>
<tr>
<td>$\text{false}$</td>
<td>$(B(\alpha), F(\alpha))$</td>
</tr>
<tr>
<td>$(^3 \exp ? \gamma \exp' : ^5 \exp'')$</td>
<td>$(B(\alpha), B(\beta)), (T(\beta), B(\gamma)), (F(\beta), B(\delta)), (T(\gamma), T(\alpha)), (F(\gamma), F(\alpha))$</td>
</tr>
<tr>
<td>$(^3 \exp &amp; &amp; \gamma \exp')$</td>
<td>$(B(\alpha), B(\beta)), (T(\beta), B(\gamma)), (F(\beta), F(\alpha)), (T(\gamma), B(\gamma)), (T(\gamma), T(\alpha)), (F(\gamma), F(\alpha))$</td>
</tr>
<tr>
<td>$(^3 \exp</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.13: Control flow for boolean expressions.

<table>
<thead>
<tr>
<th>Expression $\alpha$</th>
<th>Edges</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{loc}$</td>
<td>$(B(\alpha), \mathcal{A}(\alpha))$</td>
</tr>
<tr>
<td>$\text{lit}$</td>
<td>$(B(\alpha), \mathcal{A}(\alpha))$</td>
</tr>
<tr>
<td>$(\text{loc} = \beta e)$</td>
<td>$(B(\alpha), B(\beta)), (\mathcal{A}(\beta), \mathcal{A}(\alpha))$</td>
</tr>
<tr>
<td>$(\text{loc} \ bop = \beta e)$</td>
<td>$(B(\alpha), B(\beta)), (\mathcal{A}(\beta), \mathcal{A}(\alpha))$</td>
</tr>
<tr>
<td>$(^3 \exp ? \gamma \exp' : ^5 \exp'')$</td>
<td>$(B(\alpha), B(\beta)), (T(\beta), B(\gamma)), (F(\beta), B(\delta)), (\mathcal{A}(\gamma), \mathcal{A}(\alpha)), (\mathcal{A}(\delta), \mathcal{A}(\alpha))$</td>
</tr>
<tr>
<td>$T::F$</td>
<td>$(B(\alpha), \mathcal{A}(\alpha))$</td>
</tr>
<tr>
<td>$\text{ref}^\beta \exp$</td>
<td>$(B(\alpha), B(\beta)), (\mathcal{A}(\beta), \mathcal{A}(\alpha))$</td>
</tr>
<tr>
<td>$\text{out}^\beta \exp$</td>
<td>$(B(\alpha), B(\beta)), (\mathcal{A}(\beta), \mathcal{A}(\alpha))$</td>
</tr>
<tr>
<td>$T::M^{(\beta_1, \ldots, \beta_k \ 	ext{arg}_k)}$</td>
<td>$(B(\alpha), B(\beta_1)), (\mathcal{A}(\beta_1), \mathcal{A}(\alpha)), (\mathcal{A}(\beta_1), B(\beta_{i+1})), i = 1, k - 1$</td>
</tr>
</tbody>
</table>

Table 3.14: Control flow for arbitrary expressions.

For every boolean expression $\alpha$ in Table 3.13, we define the supplementary edges $(T(\alpha), \mathcal{A}(\alpha))$ and $(F(\alpha), \mathcal{A}(\alpha))$, which connect the boolean points of $\alpha$ to the end point of $\alpha$. These edges are necessary for control transfer in cases when it does not matter whether $\alpha$ evaluates to $\text{true}$ or $\text{false}$.

**Example 3.4.10** If $\beta$ is the method call $T::M(\text{true})$ and $\alpha$ is the argument $\text{true}$, then control is transferred from the end point of the last argument — that is, $\mathcal{A}(\alpha)$ — to the end point of the method call — that is, $\mathcal{A}(\beta)$. But, since in Table 3.13 we have no edge leading to $\mathcal{A}(\alpha)$, we need to define the supplementary edge $(T(\alpha), \mathcal{A}(\alpha))$. 
For a boolean expression \( \alpha \) which is not an instance of any expression from Table 3.13, we add the edges \((A(\alpha), T(\alpha)), (A(\alpha), F(\alpha))\) to the graph. They are needed if control is transferred from a boolean expression \( \alpha \) to different points depending on whether \( \alpha \) evaluates to \textit{true} or \textit{false}.

**Example 3.4.11** If \( \alpha \) is of the form \( \text{exp} | \text{exp}' \) and occurs in \((\text{exp} | \text{exp}') \ ? \ \beta \ \text{exp}'' : \ \gamma \ \text{exp}'''\), then control is transferred from \( F(\alpha) \) to \( B(\beta) \) (if \( \alpha \) evaluates to \textit{false}) or from \( T(\alpha) \) to \( B(\gamma) \) (if \( \alpha \) evaluates to \textit{true}). The necessity of the edges \((A(\alpha), T(\alpha)), (A(\alpha), F(\alpha))\) arises since so far we have defined for \( \text{exp} | \text{exp}' \) only edges to \( A(\alpha) \).

Table 3.15 introduces the edges of the control flow graph for each statement. We assume that the boolean constant expressions are replaced by \textit{true} or \textit{false} in the abstract syntax tree.

**Example 3.4.12** Thus, we consider that \textit{true} \textbf{||} \textit{b} is replaced by \textit{true} in the following \textit{if} statement:

\[
\begin{align*}
\text{\textbf{if}} & \begin{cases}
\beta (\text{true} \textbf{||} \textit{b}) & \delta i = 1; \\
\text{\textit{else}} & \gamma \{ \text{int} \ j = i; \}
\end{cases}
\end{align*}
\]

Although the new test, i.e., \textit{true}, cannot evaluate to \textit{false}, we still add to the graph the edge \((F(\beta), B(\gamma))\): The false point of \textit{true} is, however, never reachable (see Table 3.13).

In the presence of \textit{finally} blocks, the jump statements \textit{goto}, \textit{continue} and \textit{break} bring more complexity to the graph. When a jump statement exits a \textit{try} block, control is transferred first to the innermost \textit{finally} block. If control reaches the end point of that \textit{finally} block, then it is transferred to the next innermost \textit{finally} block and so on. If control reaches the end point of the outermost \textit{finally} block, then it is transferred to the target of the jump statement. For these control transfers, we have special edges in our graph. But one needs to take care of some important details: These special edges cannot be used for paths other than those which connect the jump statement with its target. In other words, if a path uses such an edge, then the path necessarily contains the entry point of the jump statement. For this reason, we say that an edge \( e \) is \textit{conditioned} by a point \( i \) with the meaning that \( e \) can be used only in paths that contain \( i \).

**Example 3.4.13** Let us assume that we do not institute the above restriction. Then the list \([B(mb)B(\alpha_1)B(\alpha_2)B(\alpha_3)B(\alpha_4)A(\alpha_5)B(\alpha_6)]\) would be a possible execution path to the labelled statement in the following method body

\[
\begin{align*}
\text{\textit{try}} & \begin{cases}
\alpha_1 \{ \\
\alpha_3 (\alpha_4 \ (i = 3 \ / \ j);) \\
\alpha \text{\textit{goto}} \ L; \\
\} \ \text{\textit{finally}} & \alpha_5 \{ \}
\end{cases} \\
\alpha_6 \ L : \ \text{Console.WriteLine}(i);
\end{align*}
\]

in case the evaluation of \( \alpha_4 \) would throw an exception. But, this does not match the control transfer described in [74].
Table 3.15: Control flow for statements.

The following sets introduce the above described edges. If $\alpha$ and $\beta$ are two statements and $\text{Fin}(\alpha, \beta)$ is the list $[\gamma_1, \ldots, \gamma_n]$, then the set $\text{ThroughFin}_{\delta}(\alpha, \beta)$ consists of the edges $(B(\alpha), B(\gamma_i)), (A(\gamma_i), B(\beta)), (A(\gamma_i), B(\beta_{i+1})), i = 1, n - 1$, all conditioned by $B(\alpha)$, while the set $\text{ThroughFin}_{\alpha}(\alpha, \beta)$ contains the edges $(B(\alpha), B(\gamma_i)), (A(\gamma_i), A(\beta)), (A(\gamma_i), B(\beta_{i+1})), i = 1, n - 1$, all conditioned by $B(\alpha)$. If $\text{Fin}(\alpha, \beta)$ is empty, then the set $\text{ThroughFin}_{\delta}(\alpha, \beta)$ contains only the edge $(B(\alpha), B(\beta))$, while $\text{ThroughFin}_{\alpha}(\alpha, \beta)$ refers to the edge $(B(\alpha), A(\beta))$.

**Example 3.4.14** Consider again the try-finally statement from Example 3.4.13. The list
\( \text{Fin}(\alpha, \alpha_6) \) is given by \([\alpha_5]\), while the set \( \text{ThroughFin}_{\beta}(\alpha, \alpha_6) \) contains the edges \((\mathcal{B}(\alpha), \mathcal{B}(\alpha_5)), (\mathcal{A}(\alpha_5), \mathcal{B}(\alpha_6))\) conditioned by \( \mathcal{B}(\alpha) \).

Note that in Table 3.15, for \textit{goto} and \textit{continue}, the set of edges \( \text{ThroughFin}_{\beta} \) is added to the graph, since after executing the \textit{finally} blocks control is transferred to the \textit{entry point} of the labelled statement and \textit{while} statement, respectively. However, in case of \textit{break} the set \( \text{ThroughFin}_{\alpha} \) is considered, since at the end, control is transferred to the \textit{end point} of the \textit{while} statement.

There are two more remarks concerning the \textit{try} statement. First of all, since a reason for abruption (e.g., an exception) can occur anytime in a \textit{try} block, we should have edges from every point in a \textit{try} block to: every associated \textit{catch} block, every \textit{catch} of enclosing \textit{try} statements (if the \textit{catch} clause matches the type of the exception) and to every associated \textit{finally} block (if none of the \textit{catch} clauses matches the type of the exception). We do not consider all these edges since the definite assignment analysis is an “over all paths” analysis. It is equivalent to consider only one edge to the \textit{entry points} of the \textit{catch} and \textit{finally} blocks — from the \textit{entry point} of the \textit{try} block (see Table 3.15).

The following remark concerns the \textit{end point} \( \mathcal{A}(\alpha) \) of a \textit{try-finally} statement \( \alpha \). [74, §8.10] states that \( \mathcal{A}(\alpha) \) is reachable only if both \textit{end points} of the \textit{try} block \( \beta \) and \textit{finally} block \( \gamma \) are reachable. The only edge to \( \mathcal{A}(\alpha) \) is \( (\mathcal{A}(\gamma), \mathcal{A}(\alpha)) \), and we know that the \textit{finally} block can be reached either through a jump or through a normal completion of the \textit{try} block. In the case of a jump, if control reaches the \textit{end point} \( \mathcal{A}(\gamma) \) of the \textit{finally}, then it is transferred further to the target of the statement which generated the jump and not to \( \mathcal{A}(\alpha) \). This means that all paths to \( \mathcal{A}(\alpha) \) contain also the \textit{end point} \( \mathcal{A}(\beta) \) of the \textit{try} block. That is why we require that the edge \( (\mathcal{A}(\gamma), \mathcal{A}(\alpha)) \) is conditioned by \( \mathcal{A}(\beta) \) (see Table 3.15).

**Example 3.4.15** If the edge \( (\mathcal{A}(\gamma), \mathcal{A}(\alpha)) \) is not conditioned by \( \mathcal{A}(\beta) \), then \( \mathcal{A}(\alpha) \) would be reachable in our specifications in the following example (under the assumption that \( \mathcal{B}(\alpha) \) is reachable):

\[
\alpha \text{ try} \beta \{ \text{goto} L; \} \text{ finally} \gamma \{\}
\]

We will not consider all the paths in the graph, but only the \textit{valid} paths, i.e., the paths \( p \) for which the following condition is fulfilled: If \( p \) uses a \textit{conditioned} edge, then \( p \) also includes the point which conditions the edge.

**Definition 3.4.6** For a path \( [\alpha_1, \ldots, \alpha_n] \), we define

\[ \text{valid}([\alpha_1, \ldots, \alpha_n]) \Leftrightarrow \forall \text{ conditioned edge } (\alpha_i, \alpha_{i+1}) \exists j < i : (\alpha_i, \alpha_{i+1}) \text{ is conditioned by } \alpha_j \]

If \( \alpha \) is an expression or a statement, then \( \text{path}_b(\alpha) \) is the set of all valid paths from the \textit{entry point} of the method body \( \mathcal{B}(\text{mb}) \) to the \textit{entry point} \( \mathcal{B}(\alpha) \) of \( \alpha \):

\[
\text{path}_b(\alpha) = \{ [\alpha_1, \ldots, \alpha_n] \mid \alpha_1 = \mathcal{B}(\text{mb}), \alpha_n = \mathcal{B}(\alpha), (\alpha_j, \alpha_{j+1}) \in \text{CFG}, i = 1, n - 1 \text{ and valid}([\alpha_1, \ldots, \alpha_n]) \} \]
3.4. THE DEFINITE ASSIGNMENT ANALYSIS IN C♯

Similarly, \( \text{path}_b(\alpha) \) is defined to be the set of all valid paths from the **entry point** of the method body \( B(mb) \) to the **end point** \( A(\alpha) \) of \( \alpha \), while if \( \alpha \) is a boolean expression, \( \text{path}_t(\alpha) \) and \( \text{path}_f(\alpha) \) are the sets of all valid paths from \( B(mb) \) to the **true point** \( T(\alpha) \) and to the **false point** \( F(\alpha) \) of \( \alpha \), respectively.

In the proofs in the next section, we use the following two notations. If \( p \) is a path, then \( p[i,j] \) is the subpath of \( p \), which connects the point \( i \) with the point \( j \). Over the set of all paths, we consider the operation \( \oplus \) to be the path concatenation (also defined for infinite paths).

### 3.4.4 The Correctness of the Analysis

We prove that when a C♯ compiler relies on the sets \( \text{MFP}_b, \text{MFP}_a, \text{MFP}_t, \) and \( \text{MFP}_f \), derived from the maximal fixed point of the data flow equations in Section 3.4.1 (or on their expanded sets if we allow struct type variables), then all accesses to the value of a local variable occur after the variable is initialized. In other words, the correctness of the analysis means that if a local variable is in one of the four sets — that is the analysis infers the variable as definitely assigned at a certain program point — then this variable will actually be assigned at that point during every execution path of the program. A variable \( \text{loc} \) is **assigned** on a path if the path contains an *initialization* of \( \text{loc} \) or a **catch** clause, whose exception variable is \( \text{loc} \). We describe in the following definition what we mean by initialization.

**Definition 3.4.7** A path \( p \) contains an initialization of a local variable \( \text{loc} \) if at least one of the following conditions is true:

- \( p \) contains a simple assignment (as opposed to a compound assignment) to \( \text{loc} \), or
- \( p \) contains a method call, for which \( \text{loc} \) is an **out** argument.

**Struct type variables** The definition above has to be refined if we want to allow variables of struct types. Thus, a path \( p \) contains an initialization of a variable \( \text{loc} \) also in one of the following cases:

- \( \text{loc} \) is an instance field of a struct type variable \( \text{loc'} \), and \( p \) contains an initialization of \( \text{loc'} \), or
- \( \text{loc} \) is of a struct type, and \( p \) contains initializations for each instance field of \( \text{loc} \).

We actually prove more than the correctness. We show that the components of the maximal fixed point \( \text{MFP} \) are exactly the sets of variables which are assigned on every possible execution path to the appropriate point (and not only a safe-approximation). To specify this, we define the following sets. If \( \alpha \) is an arbitrary expression or statement, then \( \text{AP}_b(\alpha) \) denotes the set of local variables in \( \text{vars}(\alpha) \) (the variables in the scope of which \( \alpha \) is) assigned on every path in \( \text{path}_b(\alpha) \):
AP_b(\alpha) = \{x \in \text{vars}(\alpha) \mid x \text{ is assigned on every path } p \in \text{path}_b(\alpha)\}

AP_a(\alpha) is the set of variables in \text{vars}(\alpha), which are assigned on every path in \text{path}_b(\alpha), while for a boolean expression \alpha, the sets AP_t(\alpha) and AP_f(\alpha) are defined similarly as above, but with respect to paths in \text{path}_t(\alpha) and \text{path}_f(\alpha), respectively.

**Struct type variables** If we also consider variables of struct types, the definition of “is assigned on” is extended as pointed out above. The definitions of the sets AP_b, AP_a, AP_t and AP_f are also adapted. But considering the new definition for “is assigned”, one can easily observe that the definitions of the AP sets to include struct type variables are nothing else than their expanded sets. So, the same “expansion” function mentioned in Section 3.4.2 is also applied to the AP sets in order to include struct type variables.

The following result is used to prove Lemma 3.4.5.

**Lemma 3.4.4** For every expression or statement \alpha, if MFP_b(\alpha) \subseteq \text{vars}(\alpha) holds, then we have MFP_a(\alpha) \subseteq \text{vars}(\alpha). Moreover, if \alpha is a boolean expression, then we have also MFP_t(\alpha) \subseteq \text{vars}(\alpha) and MFP_f(\alpha) \subseteq \text{vars}(\alpha).

**Proof.** The proof proceeds by induction over the structure of expressions and statements. Thus, we first prove the base cases of the induction, i.e., the above stated implications for all possible leaves of the AST of our method body. The expressions which are leaves in the AST are the following: \text{true}, \text{false}, \text{loc}, \text{lit}, and \text{T::F}. Since MFP is in particular a solution of the data flow equations, it is obvious that the implications stated in our lemma are satisfied. The statements considered leaves in the AST are the \text{empty-statement}, \text{goto} L, \text{break}, \text{continue}, \text{return}, and \text{throw}. For the last five, from the equations above we obviously have MFP_a(\alpha) \subseteq \text{vars}(\alpha).

For the \text{empty-statement}, this is true as well since our hypothesis is MFP_b(\alpha) \subseteq \text{vars}(\alpha).

In the induction step, the implications for each expression and statement are proved under the assumption that their “children” (subexpressions/substatements) satisfy the implications. ☐

The next lemma is used in the proof of the correctness theorem (Theorem 3.4.1). It claims that the MFP sets of an expression or statement \alpha consist of variables in the scope of which \alpha is located.

**Lemma 3.4.5** For every expression or statement \alpha, we have MFP_b(\alpha) \subseteq \text{vars}(\alpha) and MFP_a(\alpha) \subseteq \text{vars}(\alpha). Moreover, if \alpha is a boolean expression, then we have MFP_t(\alpha) \subseteq \text{vars}(\alpha) and MFP_f(\alpha) \subseteq \text{vars}(\alpha).

**Proof.** We show the above inclusions for all expressions and statements by an induction over the AST, starting at the root, i.e., the method body (the base case of the induction). Notice that the induction schema is in the opposite direction compared to that in Lemma 3.4.4. Therefore, the induction step is: Under the assumption that a node of the AST satisfies the inclusions, all its “children” (subexpressions/substatements) satisfy the inclusions as well.
According to Lemma 3.4.4, it is enough to prove for all labels \( \alpha \): \( \text{MFP}_b(\alpha) \subseteq \text{vars}(\alpha) \). For our method body, this is trivial: The relation \( \text{MFP}_b(mb) \subseteq \text{vars}(mb) \) holds since \( \text{MFP}_b(mb) = \emptyset \). Lemma 3.4.4 is applied again in the next step of the proof, which consists in showing for each expression and statement that under the assumption \( \text{MFP}_b(\alpha) \subseteq \text{vars}(\alpha) \), each of its direct subexpressions/substatements \( \beta \) satisfies \( \text{MFP}_b(\beta) \subseteq \text{vars}(\beta) \).

The correctness of the definite assignment analysis in \( \mathbb{C}^2 \) is proved in the following theorem, which claims that the analysis is a safe approximation.

**Theorem 3.4.1 (Definite assignment analysis correctness)** If \( \alpha \) is an expression or a statement, then the following relations are true: \( \text{MFP}_b(\alpha) \subseteq \text{AP}_b(\alpha) \) and \( \text{MFP}_a(\alpha) \subseteq \text{AP}_a(\alpha) \). Moreover, if \( \alpha \) is a boolean expression, then \( \text{MFP}_t(\alpha) \subseteq \text{AP}_t(\alpha) \) and \( \text{MFP}_f(\alpha) \subseteq \text{AP}_f(\alpha) \).

**Proof.** We consider the following definitions. The set \( \text{AP}_b^0(\alpha) \) is defined in the same way as \( \text{AP}_b(\alpha) \), except that we consider only paths of length less than \( n \). Similarly, we also define the sets \( \text{AP}_a^0(\alpha), \text{AP}_t^0(\alpha), \text{AP}_f^0(\alpha) \) (analogously, we have definitions for the sets of paths \( \text{path}_b^0, \text{path}_a^0, \text{path}_t^0, \text{path}_f^0 \)). According to these definitions, the following set equalities hold for every \( \alpha \):

\[
\text{AP}_b(\alpha) = \bigcap_n \text{AP}_b^n(\alpha), \quad \text{AP}_a(\alpha) = \bigcap_n \text{AP}_a^n(\alpha)
\]

If \( \alpha \) is a boolean expression, then similar equalities hold for \( \text{AP}_t(\alpha) \) and \( \text{AP}_f(\alpha) \).

Therefore, to complete the proof, it suffices to show for every \( n \): If \( \alpha \) is an expression or statement, then \( \text{MFP}_b(\alpha) \subseteq \text{AP}_b^n(\alpha) \) and \( \text{MFP}_a(\alpha) \subseteq \text{AP}_a^n(\alpha) \). In addition, if \( \alpha \) is a boolean expression, then \( \text{MFP}_t(\alpha) \subseteq \text{AP}_t^n(\alpha) \) and \( \text{MFP}_f(\alpha) \subseteq \text{AP}_f^n(\alpha) \). This is done by induction on \( n \).

**Base case** \([B(mb)]\) is the only path of length 1 (the entry point of the method body). Obviously, no local variable is assigned on this path and therefore we have \( \text{AP}_b^1(mb) = \emptyset \) which satisfies \( \text{MFP}_b(mb) \subseteq \text{AP}_b^1(mb) \) since from the equations \( \text{MFP}_b(mb) = \emptyset \). From the definition of \( \text{AP}_b^* \), we get \( \text{AP}_a^0(mb) = \text{vars}(mb) = \emptyset \) and from the equations of a block, we derive also \( \text{MFP}_a(mb) \subseteq \text{vars}(mb) \) and implicitly \( \text{MFP}_a(mb) \subseteq \text{AP}_a^1(mb) \). If \( \alpha \neq mb \), then \( \text{AP}_b^1(\alpha) = \text{AP}_a^1(\alpha) = \text{vars}(\alpha) \) and if \( \alpha \) is a boolean expression \( \text{AP}_f^1(\alpha) = \text{AP}_f^1(\alpha) = \text{vars}(\alpha) \). If we apply Lemma 3.4.5, then the basis of the induction is complete.

**Induction step** The proof has the following pattern: We show for every expression or statement \( \alpha \) from Tables 3.10, 3.11, and 3.12 the relation for \( \text{MFP}_a(\alpha) \), and where applicable for \( \text{MFP}_t(\alpha) \) and \( \text{MFP}_f(\alpha) \) and, for every direct subexpression/substatement of \( \alpha \), the relations for \( \text{MFP}_b \). In this way, all the relations for all expressions/statements are proved except \( \text{MFP}_b(mb) \subseteq \text{AP}_b^{n+1}(mb) \) (since \( mb \) has no “superstatement”) which holds anyway, since \( \text{MFP}_b(mb) = \emptyset \).

We only consider two critical cases: the block of statements and the try-finally statement.

**Case 1** \( \alpha = \{^β_i \text{stm}_1 \ldots ^β_n \text{stm}_n\} \).

We prove \( \text{MFP}_b(\beta_{i+1}) \subseteq \text{AP}_b^{n+1}(\beta_{i+1}) \) for an embedded statement \( \beta_{i+1} \). If we arbitrarily choose a local variable \( \text{loc} \) in \( \text{MFP}_b(\beta_{i+1}) \), then we obtain \( \text{loc} \in \text{MFP}_a(\beta_i) \) and \( \text{loc} \in \text{goto}(\beta_{i+1}) \) (MFP
is a solution of the flow equations). Note that, at this point, \( \text{goto}(\beta_{i+1}) \) depends only on MFP sets and on the control flow graph. From the induction hypothesis, we get \( \text{loc} \in \text{AP}^n_b(\beta_i) \). In particular, this means \( \text{loc} \in \text{vars}(\beta_i) = \text{vars}(\beta_{i+1}) \), i.e., \( \beta_{i+1} \) is in the scope of a declaration of \( \text{loc} \).

**Subcase 1.1** \( \beta_{i+1} \) is not a labelled statement.

In this case, we have \( \text{goto}(\beta_{i+1}) = \text{vars}(\beta_{i+1}) \) from the definition of the \( \text{goto} \) set.

**Subcase 1.1.1** \( \beta_{i+1} \) is not a \textbf{while} statement.

There exists in the \textit{CFG} only one edge to \( B(\beta_{i+1}) \), namely \((A(\beta_i), B(\beta_{i+1}))\). This means that

\[ \text{path}^{n+1}_b(\beta_{i+1}) = \text{path}^n_b(\beta_i) \oplus B(\beta_{i+1}) \]

and implicitly \( \text{loc} \in \text{AP}^{n+1}_b(\beta_{i+1}) \) since \( \text{loc} \in \text{AP}^n_b(\beta_i) \) (induction hypothesis). Remember that we derived earlier that \( \beta_{i+1} \) is in the scope of \( \text{loc} \), i.e., \( \text{loc} \in \text{vars}(\beta_{i+1}) \).

**Subcase 1.1.2** \( \beta_{i+1} \) is a \textbf{while} statement.

There could be many edges to \( B(\beta_{i+1}) \) in the \textit{CFG} (if the \textbf{while} has associated \textbf{continue} statements). If there are no \textbf{continue} statements corresponding to our \textbf{while} statement, then the proof is as in **Subcase 1.1.1**.

If there are \textbf{continue} statements, we would like to show that \( \text{loc} \) is assigned on every path \( p \) to \( B(\beta_{i+1}) \) of length at most \( n + 1 \). If \( p \) contains no \textbf{continue} statements, then \( p \) has the last edge \((A(\beta_i), B(\beta_{i+1}))\), i.e., \( p \) passes through \( A(\beta_i) \). If \( p \) contains a \textbf{continue} statement associated to our \textbf{while} and eventually passes through finally blocks associated to \textbf{try} blocks that contain the \textbf{continue} statement, then \( p \) necessarily passes through \( A(\beta_i) \) because the \textit{CFG} shows that it is not possible to “jump” into the \textbf{while} body (in which our \textbf{continue} is embedded). On the other hand, since \( \text{loc} \in \text{MFP}_a(\beta_i) \), we get \( \text{loc} \in \text{AP}^n_b(\beta_i) \) from the induction hypothesis, i.e., \( \text{loc} \) is assigned on every path to \( A(\beta_i) \) of length at most \( n \). Consequently, \( p \) assigns \( \text{loc} \) since \( p \) passes through \( A(\beta_i) \). We are now sure that \( \text{loc} \) is assigned on every path to \( B(\beta_{i+1}) \) of length at most \( n + 1 \), and therefore we can conclude \( \text{loc} \in \text{AP}^{n+1}_b(\beta_{i+1}) \).

**Subcase 1.2** \( \beta_{i+1} \) is a labelled statement \( L : \text{stm} \).

As in the case of a \textbf{while} statement, there could be many edges to \( B(\beta_{i+1}) \) in the \textit{CFG} (if there are \textbf{goto} statements pointing to our labelled statement). If there are no associated \textbf{goto} statements, then the proof is the same as in the case of a \textbf{while} statement with no associated \textbf{continue} statements.

We want to show that \( \text{loc} \) is assigned on every path \( p \) to \( B(\beta_{i+1}) \) of length at most \( n + 1 \). If \( p \) contains no \textbf{goto} \( L \) statements, then \( p \) necessarily passes through \( A(\beta_i) \). And since \( \text{loc} \) is assigned on every path to \( A(\beta_i) \) of length at most \( n \) (because of \( \text{loc} \in \text{AP}^n_a(\beta_i) \)), we are sure that \( p \) assigns \( \text{loc} \).

Suppose that \( p \) passes through a \( \gamma \text{goto} L \) statement and eventually through finally blocks associated to \textbf{try} blocks in which \( \gamma \) is embedded. Considering the definition of the \textbf{goto} set, we can derive \( \text{loc} \in \text{MFP}_a(\gamma) \cup \text{JoinFin}(\gamma, \beta_{i+1}) \) since \( \text{loc} \in \text{goto}(\beta_{i+1}) \).

If there are no finally blocks in the list \( \text{Fin}(\gamma, \beta_{i+1}) \), then \( \text{JoinFin}(\gamma, \beta_{i+1}) = \emptyset \) and implicitly \( \text{loc} \in \text{MFP}_a(\gamma) \). By the induction hypothesis, we obtain \( \text{loc} \in \text{AP}^n_b(\gamma) \). It means that \( p \), which is of length at most \( n + 1 \) and contains \( B(\gamma) \), assigns \( \text{loc} \).
Let us now suppose that the list $\text{Fin}(\gamma, \beta_{i+1})$ is non-empty: $\text{Fin} = [\gamma_1, \ldots, \gamma_k]$. By the definition of the set $\text{JoinFin}(\gamma, \beta_{i+1})$, we get

$$\text{loc} \in \text{MFP}_b(\gamma) \cup \bigcup_{j=1}^{k} \text{MFP}_a(\gamma_j)$$

If $\text{loc} \in \text{MFP}_b(\gamma)$, then, by the induction hypothesis, we derive $\text{loc} \in \text{AP}_b^n(\gamma)$, and we are sure that $p$, which passes through $B(\gamma)$, assigns $\text{loc}$.

If there is a finally block $\gamma_j$ such that $\text{loc} \in \text{MFP}_a(\gamma_j)$, then the induction hypothesis implies $\text{loc} \in \text{AP}_a^n(\gamma_j)$. And since $p$ necessarily passes through $A(\gamma_j)$, we are sure that $p$ assigns $\text{loc}$.

Thus, we have analyzed every possible path to $B(\beta_{i+1})$ of length at most $n + 1$ and we showed that each such a path assigns $\text{loc}$, i.e., $\text{loc} \in \text{AP}_b^{n+1}(\beta_{i+1})$.

**Case 2** $\alpha$ is try $\beta$ block finally $\gamma$ block.

Here we do the proof for $\text{MFP}_b(\gamma) \subseteq \text{AP}_b^{n+1}(\gamma)$. Let $\text{loc}$ be a local variable in the set $\text{MFP}_b(\gamma)$. Following the data flow equations, we get $\text{loc} \in \text{MFP}_b(\alpha)$. The induction hypothesis implies then $\text{loc} \in \text{AP}_b^n(\alpha)$ and, in particular, $\text{loc} \in \text{vars}(\alpha)$.

It is important to notice that in the $\text{CFG}$ there could be many edges to $B(\gamma)$: from the entry point of the try-finally statement $(B(\alpha), B(\gamma))$, from the end point of the try block $(A(\beta), B(\gamma))$, from a goto, break or continue statement $(B(\delta), B(\gamma))$ (within a conditioned path), and from the end point of another finally block $(A(\omega), B(\gamma))$ (within a conditioned path). We claim that independent of the last edge of a path $p$ to $B(\gamma)$, $p$ passes through the entry point $B(\alpha)$ of the try-finally statement.

- If the last edge of $p$ is $(B(\alpha), B(\gamma))$, then there is nothing to prove.
- If the last edge is $(A(\beta), B(\gamma))$, then the claim holds since the end point $A(\beta)$ can be reached only through $B(\alpha)$ (according to the $\text{CFG}$, it is not possible to “jump” into the try block).
- If the last edge is $(B(\delta), B(\gamma))$, then the claim can be justified in the same way as above, because the respective jump statements are supposed to be embedded in the try block.
- If the last edge of $p$ is a conditioned edge $(A(\omega), B(\gamma))$, then necessarily the finally block $\omega$ (as well as the jump statement which triggered the conditioning) is embedded in our try block. This means that in order to justify the claim, we can apply the same argument as above.

So, all the paths to $B(\gamma)$ should pass through $B(\alpha)$, and since $\text{loc} \in \text{AP}_b^n(\alpha)$, we can be sure that $\text{loc} \in \text{vars}(\gamma) = \text{vars}(\alpha)$ is assigned on every path to $B(\gamma)$ of length at most $n + 1$, i.e., $\text{loc} \in \text{AP}_b^{n+1}(\gamma)$, and the proof of the considered relation is complete.  

As explained above, we can actually prove more: The $\text{MFP}$ solution is not only an approximation of $\text{AP}$, but it is a perfect solution (Theorem 3.4.3). For this, we also use the following theorem that states that the $\text{MFP}$ solution contains the local variables which are initialized over all possible paths.
Theorem 3.4.2 If $\alpha$ is an expression or a statement, then the following relations are true: $\text{AP}_b(\alpha) \subseteq \text{MFP}_b(\alpha)$ and $\text{AP}_a(\alpha) \subseteq \text{MFP}_a(\alpha)$. Moreover, if $\alpha$ is a boolean expression, then $\text{AP}_i(\alpha) \subseteq \text{MFP}_i(\alpha)$ and $\text{AP}_f(\alpha) \subseteq \text{MFP}_f(\alpha)$.

Proof. Tarski’s fixed point theorem [119] states that $\text{MFP}$ is the lowest upper bound (with respect to $\ll$) of the set $\text{Ext}(F) = \{ X \in D | X \ll F(X) \}$. It then suffices to show that the $r$-tuple consisting of the $\text{AP}$ sets is an element of $\text{Ext}(F)$ since $\text{MFP}$ is in particular an upper bound of this set. Since $\ll$ is the pointwise subset relation, the idea is to prove the left-to-right subset relations for the data flow equations in Tables 3.10, 3.11, and 3.12, where instead of the sets before, after, true and false, we have the sets $\text{AP}_b, \text{AP}_a, \text{AP}_i,$ and $\text{AP}_f$, respectively.

We only consider here one critical case, namely the case of a block of statements: Assuming that $\beta_{i+1}$ is a labelled statement $L : \text{stm}$ in a block $\alpha$ given by $\{ \beta_1 \text{stm}_1 \ldots \beta_n \text{stm}_n \}$, we want to prove $\text{AP}_b(\beta_{i+1}) \subseteq \text{AP}_a(\beta_i) \cap \text{goto}(\beta_{i+1})$. Note that here, $\text{goto}(\beta_{i+1})$ depends only on the $\text{AP}$ sets and on the control flow graph $\text{CFG}$.

Let $\text{loc}$ be a variable in $\text{AP}_b(\beta_{i+1})$, i.e., $\text{loc}$ is assigned on every path to $B(\beta_{i+1})$. An immediate consequence is $\text{loc} \in \text{AP}_a(\beta_i)$, since all the paths to $A(\beta_i)$ are — “modulo” the edge $(A(\beta_i), B(\beta_{i+1}))$ — also paths to $B(\beta_{i+1})$, and no variable is assigned on this edge.

To show $\text{loc} \in \text{goto}(\beta_{i+1})$, we need to prove that the variable $\text{loc}$ is in the set $\text{AP}_b(\gamma) \cup \text{JoinFin}(\gamma, \beta_{i+1})$ for every $\gamma \text{goto} L$ whose target is our labelled statement.

If there is no such $\text{goto}$ statement, then we obviously have $\text{loc} \in \text{goto}(\beta_{i+1})$ since in this case $\text{goto}(\beta_{i+1}) = \text{vars}(\beta_{i+1})$ and $\text{loc} \in \text{AP}_b(\beta_{i+1}) \subseteq \text{vars}(\beta_{i+1})$. Let us now suppose that there exists at least one $\text{goto}$ statement $\gamma$ pointing to $\beta_{i+1}$.

Case 1 $B(\gamma)$ is not reachable.

Then $\text{path}_b(\gamma)$ is empty, and consequently, we get $\text{loc} \in \text{AP}_b(\gamma) \cup \text{JoinFin}(\gamma, \beta_{i+1})$ because $\text{AP}_b(\gamma) = \text{vars}(\gamma) \supseteq \text{vars}(\beta_{i+1})$ and $\text{loc} \in \text{vars}(\beta_{i+1})$. The last subset relation holds because, in C$, a \text{goto}$ statement should be always in the scope of the corresponding labelled statement.

Case 2 $B(\gamma)$ is reachable.

Let $p$ be an arbitrary path to $B(\gamma)$. Here we will only consider the case when there are finally blocks from $\gamma$ to $\beta_{i+1}$, i.e., $\text{Fin}(\gamma, \beta_{i+1}) = [\gamma_1, \ldots, \gamma_k]$ (the proof of the case where there are no finally blocks is much simpler). Accordingly, also the edges

$$(B(\gamma), B(\gamma_1)), (A(\gamma_k), B(\beta_{i+1})), (A(\gamma_j), B(\gamma_{j+1})), j = 1, k - 1$$

defined by the set $\text{ThroughFin}_b(\gamma, \beta_{i+1})$ are added to the $\text{CFG}$.

We prove $\text{loc} \in \text{AP}_b(\gamma) \cup \text{JoinFin}(\gamma, \beta_{i+1})$ by reductio ad absurdum. Let us assume that $\text{loc} \notin \text{AP}_b(\gamma) \cup \text{JoinFin}(\gamma, \beta_{i+1})$. This is equivalent to $\text{loc} \notin \text{AP}_b(\gamma)$ and $\text{loc} \notin \text{AP}_a(\gamma_j)$ for all $j = 1, k$. This means that the paths $p_0 \in \text{path}_b(\gamma)$ and $p_j \in \text{path}_a(\gamma_j)$ for $j = 1, k$ exist, such that $\text{loc}$ is not assigned on any of these paths. A simple inspection of the $\text{CFG}$ shows that the point $B(\gamma_j)$ necessarily occurs on the path $p_j$ for every $j = 1, k$ since it is not possible to “jump” into a finally block. We now want to prove that the following list

$$q := p_0 \oplus p_1[B(\gamma_1), A(\gamma_1)] \oplus \ldots \oplus p_k[B(\gamma_k), A(\gamma_k)] \oplus B(\beta_{i+1})$$
3.5. The Semantics of $\text{C}^\sharp_S$ Programs

represents a valid path to $B(\beta_{i+1})$. The only problem that could arise is concerning the *conditioned* edges. Remember that the edges conditioned by a certain *goto*, *break* or *continue* statement can only be used in paths (or subpaths) that contain the *entry point* of the respective jump statement. The use of edges $(B(\gamma), B(\gamma_1)), \ldots, (A(\gamma_k), B(\beta_{i+1}))$ is correct as long as our path $q$ contains $B(\gamma)$.

Let us consider one of the subpaths $p_j[B(\gamma_j), A(\gamma_j)]$ used in $q$. If this subpath contains a conditioned edge, then since the conditioned edges connect jump statements with *finally* blocks we would be sure that these *finally* blocks are embedded in our *finally* block $\gamma_j$. The respective jump statement is embedded into the *try* blocks (associated to the conditioned “connected” *finally* blocks) which necessarily should be in $\gamma_j$ (this is an immediate consequence of the $\text{C}^\sharp$ grammar). So, the jump statement is necessarily embedded in the *finally* block $\gamma_j$. Considering that the subpath $p_j[B(\gamma_j), A(\gamma_j)]$ contains conditioned edges, we get that also $p_j$ uses the same conditioned edges and since we assumed that $p_j$ is a valid path, necessarily $p_j$ should contain the *entry point* of the respective jump statement which, as we proved above, is embedded in our *finally* block, and consequently appears in the subpath $p_j[B(\gamma_j), A(\gamma_j)]$. It means that this subpath is valid. Obviously, this is true for all the considered subpaths in $q$.

The above defined path $q$ is a valid path to $B(\beta_{i+1})$ which does not assign $\mathit{loc}$. Obviously, this contradicts $\mathit{loc} \in \mathit{AP}_{b}(\beta_{i+1})$ and therefore our assumption is wrong. Hence, we obtain the desired $\mathit{loc} \in \mathit{AP}_{b}(\gamma) \cup \mathit{JoinFin}(\gamma, \beta_{i+1})$.

The following result is then an obvious consequence of Theorems 3.4.1 and 3.4.2:

**Theorem 3.4.3** The maximal fixed point solution of the data flow equations in Tables 3.10, 3.11, and 3.12 represents the sets of local variables which are assigned over all possible execution paths. More exactly, for every expression or statement $\alpha$, the following are true: $\mathit{AP}_{b}(\alpha) = \mathit{MFP}_{b}(\alpha)$ and $\mathit{AP}_{a}(\alpha) = \mathit{MFP}_{a}(\alpha)$. Moreover, if $\alpha$ is a boolean expression: $\mathit{AP}_{t}(\alpha) = \mathit{MFP}_{t}(\alpha)$ and $\mathit{AP}_{f}(\alpha) = \mathit{MFP}_{f}(\alpha)$.

**Struct type variables** Suppose now that we include for the analysis also variables of struct types. In this case, the $\text{C}^\sharp$ compiler relies on the expanded MFP sets and the correctness of the analysis would mean that the expanded MFP sets are a safe approximation of the expanded AP sets. On the other hand, we proved in Theorem 3.4.3, that the MFP and AP sets coincide. Then, also the expanded MFP sets will coincide with the expanded AP sets; so they are, in particular, a safe approximation. This means that, allowing variables of struct types does not affect the correctness of the definite assignment analysis. One can justify the correctness also by applying Theorem 3.4.1 and by observing that the “expansion” function is monotonic.

3.5 The Semantics of $\text{C}^\sharp_S$ Programs

In this section, the static and dynamic semantics of $\text{C}^\sharp_S$ are formally specified, along the lines of [24].
3.5.1 The Static Semantics

Besides the execution environment and the grammar defined in Section 3.2, the static semantics of C# also comprises the static functions in Table 3.16.

For any two values \( \text{val} \) and \( \text{val}' \) and binary operator \( \text{bop} \), \( \text{opResVal}(\text{bop}, \text{val}, \text{val}') \) returns the result of applying \( \text{bop} \) to operands \( \text{val} \) and \( \text{val}' \) (assuming that the operation is valid). For any primitive type \( T \) and simple value \( \text{val} \), \( \text{convertVal}(T, \text{val}) \) yields the result of converting \( \text{val} \) to \( T \).

Given a type \( T \), \( \text{defVal}(T) : \text{Map}(\text{Type}, \text{Val}) \) gives the default value, i.e., the ”zero”, of type \( T \). It is worth mentioning that the default value of a struct is the same as the value computed by the struct’s default constructor, namely the tuple with all the components initialized to default values.

**Definition 3.5.1 (Default value)** For a type \( T \), we define

\[
\text{defVal}(T) = \begin{cases} 
\text{False}, & \text{if } T = \text{bool}; \\
0, & \text{if } T = \text{int}; \\
0l, & \text{if } T = \text{long}; \\
0d, & \text{if } T = \text{double}; \\
\text{null}, & \text{if } T \in \text{RefType}; \\
\{T::F \mapsto \text{defVal}(\text{fieldType}(T::F)) \mid T::F \in \text{instFields}(T)\}, & \text{if } T \in \text{Struct}.
\end{cases}
\]

**Remark 3.5.1** The definition of \text{defVal} is indeed recursive, but it is well-founded in any case, in the sense that there is no type \( T \) such that \text{defVal}(T) is defined in terms of \text{defVal}(T) itself. This is justified in particular by the key fact that a struct cannot be recursive, i.e., the declaration of a struct \( S \) is not allowed to use the name \( S \) in the types of its instance fields.

A virtual method call is resolved dynamically, i.e., at run-time. More exactly, the method to be invoked at run-time is determined based on the type of the object the method is called on. This is known as dynamic method binding [74, §10.5.3], and Definition 3.5.2 specifies it as a derived function \( \text{lookUp} : \text{Map}(\text{Type} \times \text{MRef}, \text{MRef}) \). Thus, given the run-time type \( T \) (see the definition in Section 3.5.2) of an object and a virtual method \( T'::M \), \( \text{lookUp}(T, T'::M) \) yields the method \( T''::M \), where \( T''::M \) is the first implementation of \( T'::M \) declared by a supertype of \( T \), starting with \( T \) itself.

**Definition 3.5.2 (Method lookup)** For a type \( T \) and a method \( T'::M \), we define:
3.5. THE SEMANTICS OF C♯ PROGRAMS

The semantics of C♯ programs is defined in a way that it is not overly complex. The function `lookUp(T, T':M)` returns a value based on the method declared in `T`. If `T` declares a non-abstract method `M`, then the function checks if `T` is equal to `T'` or if `T'` is a class, then it checks if `T'` overrides `T::M`. If `T'` is an interface, then it checks if `T::M` implements `T'::M`. If none of these conditions are met, it returns `object` and otherwise, it recursively calls itself with the `T''` as the direct base class of `T`.

```plaintext
lookUp(T, T':M) =
  if T declares a non-abstract method M ∧
  ((T = T') ∨ (T' ∈ Class ⇒ T'::M overrides T::M)) ∧
  (T' ∈ Interface ⇒ T::M implements T'::M)
  then T::M
  elseif T = object then undef
  else lookUp(T'', T'::M) where T'' is the direct base class of T
```

3.5.2 The Dynamic Semantics

The dynamic semantics does not work directly on C♯ syntax, but on the abstract terms produced by the grammar in Figure 3.1. The dynamic state of C♯ programs is given by the dynamic functions in Table 3.18. Some operational semantics rules (that enable the transition between states) are explained in Section 3.5.2.1.

The dynamic semantics specification assumes that the static semantics is correct. Its definition proceeds by walking through the AST of the current method `meth` and determining at each node the effect of the C♯ construct corresponding to the node. This walk is formalized by means of a cursor `▶`, for example, as in `▶ exp` or `▶ stm`, whose position in the tree – represented by an abstract program counter `pos` – is updated using the static tree functions defined in Table 3.6. The counter `pos` points to the current expression or statement to be executed. The moves of `pos` implicitly contain the C♯’s control flow graph.

The addresses of the local variables and parameters are maintained in the function `locAdr`. Thus, for a local variable or parameter `loc`, `locAdr(loc)` is the address allocated for `loc` in the current method (if `loc` is a local variable or a value parameter) or the address passed to the current method for `loc` (if `loc` is a ref/out parameter). The function `values` is used to store intermediate evaluation results, elements of the universe `Result` defined in Table 3.17. Thus, given the node at a position `α`, `values(α)` is eventually assigned a value, an address, a reason for abruption (element of the universe `Abr` defined in Table 3.17), `undef`, or the constant `Norm` (used to denote the normal completion of statements). Unlike [24], we do not consider the addresses to be values. For the clarity of the type safety proof, the universes `Adr` and `Val` are regarded as disjoint in our semantics model.

A reason for abruption, as opposed to normal completion, represents an abruption of the control flow, which can happen due to one of the statements `break`, `continue`, `goto`, `return`, and `throw`.

Method invocations produce (activation) frames, elements of the universe `Frame` defined in Table 3.17. The current frame is given by the 4-tuple (`meth, pos, locAdr, values`). The currently still to be executed frames on the stack are maintained by the function `frameStack`.

For accessing/updating the values of (managed) memory locations, e.g., local variables, parameters, fields, the function `mem` is considered. This function assigns a simple value or object reference to every memory address.

The function `fieldAdr` assigns to every instance field of an object reference or of a struct

---

*In our abstract memory model, we assume that a single address can hold either a simple value or an object reference, whereas a struct value is held by a block of addresses.*
instance stored at a given address its allocated address. More exactly, the address of an instance field $C::F$ of an object pointed to by a reference $ref$ is given by $\text{fieldAdr}(ref, C::F)$. Also, the address of an instance field $S::F$ of a struct instance stored at the address $adr$ is $\text{fieldAdr}(adr, S::F)$. For a reference $ref$ pointing to a boxed value, $\text{addressOf}(ref)$ returns the address of the value being boxed.

Every object, i.e., class instance or boxed value type, has an initial part that represents its type, usually called the run-time type, by opposition to the compile-time type $ST$. For every object reference $ref$, $\text{actualTypeOf}(ref)$ returns the run-time type of $ref$. The run-time type of an instance of a class $C$ is set to $C$ upon the instance creation, whereas the run-time type of a boxed value of type $T$ is set to $T$ upon the boxing. Note that, although the boxing returns objects, the dynamic type checks, e.g., triggered by the $\text{is/as}$ operators, can directly reference the value type $T$.

A $\text{throw}$ statement without expression, allowed in a $\text{catch}$ block only, is applied to rethrow the exception handled in the corresponding $\text{catch}$ block. To specify the semantics of these statements, a stack $\text{excStack}$ of exception references is required to record exceptions which are to be re-thrown.

The invocation list of a delegate is maintained in $\text{invocationList}$. So, for a delegate reference $d$, $\text{invocationList}(d)$ keeps the list of pairs consisting of a target object and a target method pointed to by the delegate $d$.

**Initial State** The following conditions should be satisfied in the initial state of the operational semantics model by the dynamic functions in Table 3.18:

<table>
<thead>
<tr>
<th>$\text{meth}$</th>
<th>$\text{fieldAdr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Main}$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$\text{pos}$</td>
<td>$\alpha$, where $\alpha \in \text{Pos}$ with $\wedge \text{body}(\text{Main})$</td>
</tr>
<tr>
<td>$\text{locAdr}$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$\text{values}$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$\text{frameStack}$</td>
<td>$[]$</td>
</tr>
<tr>
<td>$\text{mem}(adr)$</td>
<td>$\text{undef}$ for every $adr \in \text{Adr}$</td>
</tr>
</tbody>
</table>

As $\text{mem}$ is applied to "read" from the memory only simple values and object references, it is considered a (derived) function $\text{memVal} : \text{Map}(\text{Adr} \times \text{Type}, \text{Val} \cup \{\text{undef}\})$, which in addition "reads" struct values. The definition below of $\text{memVal}$ relies on the fact that the struct values
### Table 3.18: The basic dynamic functions.

<table>
<thead>
<tr>
<th>Function name</th>
<th>Function definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>meth</code></td>
<td><code>MRef</code></td>
</tr>
<tr>
<td><code>pos</code></td>
<td><code>Pos</code></td>
</tr>
<tr>
<td><code>locAdr</code></td>
<td><code>Map(Loc, Adr)</code></td>
</tr>
<tr>
<td><code>values</code></td>
<td><code>Map(Pos, Result)</code></td>
</tr>
<tr>
<td><code>frameStack</code></td>
<td><code>List(Frame)</code></td>
</tr>
<tr>
<td><code>mem</code></td>
<td><code>Map(Adr, SimpleVal ∪ ObjRef ∪ {undef})</code></td>
</tr>
<tr>
<td><code>fieldAdr</code></td>
<td><code>Map((ObjRef ∪ Adr) × Type::Field, Adr)</code></td>
</tr>
<tr>
<td><code>actualTypeOf</code></td>
<td><code>Map(ObjRef, Class ∪ ValueType)</code></td>
</tr>
<tr>
<td><code>excStack</code></td>
<td><code>List(ObjRef)</code></td>
</tr>
<tr>
<td><code>invocationList</code></td>
<td><code>Map(ObjRef, List(ObjRef × MRef))</code></td>
</tr>
</tbody>
</table>

are defined as mappings. Thus, for an address `adr` and a type `T`, `memVal(adr, T)` is the value stored in the memory block at `adr`, sufficient to hold values of type `T`.

**Definition 3.5.3** For an address `adr` and a type `T`, we define:

\[
\text{memVal}(\text{adr}, T) = \\
\begin{cases} 
\text{if } T ∈ \text{PrimitiveType} ∪ \text{RefType} & \text{then mem}(\text{adr}) \\
\text{elseif } T ∈ \text{Struct} & \text{then} \\
\forall T::\text{F} ∈ \text{instFields}(T) \quad \{T::\text{F} \mapsto \text{memVal(fieldAdr(adr, T::\text{F}), fieldType(T::\text{F}))} | T::\text{F} ∈ \text{instFields}(T)\}
\end{cases}
\]

The “write” in memory of values, including struct values, is accomplished with the below defined macro `WRITEMEM`. Thus, given an address `adr`, a type `T` and a value `val`, `WRITEMEM(adr, T, val)` stores `val` in the memory block at `adr`, sufficient to hold values of type `T`.

**Definition 3.5.4** Given an address `adr`, a type `T` and a value `val`, we define:

\[
\text{WRITEMEM}(\text{adr}, T, val) ≡ \\
\begin{cases} 
\text{if } T ∈ \text{PrimitiveType} ∪ \text{RefType} & \text{then mem}(\text{adr}) := val \\
\text{elseif } T ∈ \text{Struct} & \text{then} \\
\forall T::\text{F} ∈ \text{instFields}(T) \quad \text{WRITEMEM(fieldAdr(adr, T::\text{F}), fieldType(T::\text{F}), val(T::\text{F}))}
\end{cases}
\]

Note that the definitions of `memVal` and `WRITEMEM` are well-founded since a struct cannot be recursive, i.e., the declaration of a struct type is not allowed to use the struct name in the types of its instance fields.

**Example 3.5.1** In order to aid the reader’s understanding of the definitions of `memVal` and `WRITEMEM`, we consider the following example. Let `S` and `S’` be two structs. We assume that `S`
declares two instance fields S::F and S::F’ of declared types S’ and object, respectively. Moreover, S’ has two instance fields S’::F” and S’::F’” of declared types object and int, respectively.

When a value val of type S is created (upon evaluating a new expression), a memory block at an address adr, sufficient to hold values of type S, is allocated for val. As val is a composite value, addresses adr’ and adr” are also allocated for the values which compose val, i.e., for val’s instance fields S::F and S::F’ (see the evaluation rule of the new expressions in Section 3.5.2.1):

\[ \text{fieldAdr}(adr, S::F) = adr’ \quad \text{fieldAdr}(adr, S::F’) = adr” \]

Similarly, as the field S::F is of struct type S’, addresses adr”’ and adr(4) are allocated for the instance fields S’::F” and S’::F’” of the struct instance stored at adr’:

\[ \text{fieldAdr}(adr’, S’::F”) = adr”’ \quad \text{fieldAdr}(adr’, S’::F’”) = adr(4) \]

The value val can then be recursively computed as the value of type S stored at the address adr:

\[
\begin{align*}
\text{memVal}(adr, S) &= \{S::F \mapsto \text{memVal}(\text{fieldAdr}(adr, S::F), S’), \\
&\quad S::F’ \mapsto \text{memVal}(\text{fieldAdr}(adr, S::F’), \text{object})\} \\
&= \{S::F \mapsto \text{memVal}(adr’, S’), S::F’ \mapsto \text{memVal}(adr”’, \text{object})\} \\
&= \{S::F \mapsto \{S’::F” \mapsto \text{memVal}(\text{fieldAdr}(adr’, S’::F”), \text{object}), \\
&\quad S’::F’” \mapsto \text{memVal}(\text{fieldAdr}(adr’, S’::F’”), \text{int})\}, \\
&\quad S::F’ \mapsto \text{memVal}(adr”’, \text{object})\} \\
&= \{S::F \mapsto \{S’::F” \mapsto \text{memVal}(adr”’, \text{object}), S’::F’” \mapsto \text{memVal}(adr(4), \text{int})\}, \\
&\quad S::F’ \mapsto \text{memVal}(adr”’, \text{object})\} \\
&= \{S::F \mapsto \{S’::F” \mapsto \text{mem}(adr”’), S’::F’” \mapsto \text{mem}(adr(4))\}, S::F’ \mapsto \text{mem}(adr”)\} \\
\end{align*}
\]

If one has, for example, to update val’s instance field S::F with the value

\[
\text{val’} = \{S’::F” \mapsto \text{val”}, S’::F’” \mapsto \text{val”’}\}
\]

then val’ is stored at adr’. This is accomplished through WRITEMEM(adr’, S’, val’). This macro simply updates the instance fields S’::F” and S’::F’” with val” and val”’, respectively. Concerning type safety, the following question arises: Is the updated value val still of type S?

Not surprisingly, if a value val is stored in a memory block, sufficient to hold values of a type T, then the value ”read” from that block is val.

**Lemma 3.5.1** After WRITEMEM(adr, T, val) is executed, \( \text{memVal}(adr, T) = \text{val} \).

**Proof.** By Definitions 3.5.3 and 3.5.4. □
Variable expressions vs. Address positions  One key aspect in the modelling is represented by the "indirection through memory addresses". The idea of separately defining the universe $Vexp$ of variable expressions is to isolate the expressions whose evaluation can produce addresses. The addresses are needed for the call-by-reference mechanism with ref and out parameters. Thus, as we will see in Section 3.5.2.1 and in the Appendix, if a variable expression is passed "by reference" in a method call, it is the address of the variable expression which is passed as argument to the method. This means that, depending on the context, the evaluation of a variable expression $vexp$ is required to yield its address or its value. In other words, if $vexp$ is at an address position, then the address of $vexp$ is needed, otherwise the value of $vexp$. An address position (relative to a given attributed syntax tree) is the position in a ref/out argument, or the position which is assigned to in a compound assignment, or the position of an access (other than in a delegate creation) to a variable expression of a struct type. Definition 3.5.5 makes precise what we mean by an address position.

**Definition 3.5.5** For a position $\alpha \in Pos$, we define:

\[
\text{isAddressPos}(\alpha) : \iff \\
\text{first}(\text{up}(\alpha)) = \alpha \land \\
(\text{label}(\text{up}(\alpha)) \in \{\text{ref}, \text{out}\} \cup Aop \lor \\
(\text{label}(\text{up}(\alpha)) = '. ' \land \text{up}(\alpha) \notin Dexp \land \alpha \in Vexp \land ST(\alpha) \in \text{Struct}))
\]

**Remark 3.5.2** The definition of isAddressPos in [24] considers an address position the position of the target object expression in a delegate creation expression, i.e., when $\text{up}(\text{pos}) \in Dexp$, though the target object cannot be an address – see also the description of delegate creations in Section 3.5.2.1.

The following lemma says that only variable expressions can occur at address positions.

**Lemma 3.5.2** If isAddressPos($\alpha$), then $\alpha \in Vexp$.

**Proof.** By Definition 3.5.5 and $C^{\#}_S$'s grammar in Figure 3.1. \qed

### 3.5.2.1 The Evaluation of the $C^{\#}_S$ Expressions and Statements

In this section, we briefly describe the evaluation rules for a few expressions, whose formal specifications are defined in [24]. For the reader’s convenience, we include these rules in the Appendix. The evaluation rules extensively make use of the macros defined and informally described in Table 3.19. The definitions of these macros assume an infinite memory space $Adr$, assumption that is often made when specifying the semantics of programming languages.

**Compound assignment** In $vexp \ bop = \ exp$, $vexp$ is evaluated before $exp$. Since $vexp$ occurs at an address position (see Definition 3.5.5), its evaluation should produce an address, say $adr$. Let $val$ be the value $exp$ evaluates to. After $vexp$ and $exp$ are evaluated, the assignment is in the form $adr \ bop = \ val$, and $pos$ points to the position of $val$. The result $val''$ of $bop$ having as operands the value stored at $adr$ and $val$ is stored with WRITEMEM at $adr$ and yielded at the parent position of $pos$. To yield a result at a given position, the macro YIELD is used:
let ref = new(ObjRef, C) in P ≡
import ref do
  ObjRef(ref) := True
  ALLOCFIELDS(ref, instFields(C))
seq P

Returns a reference to a newly allocated object of a given type.

let ref = new(ObjRef) in P ≡
import ref do
  ObjRef(ref) := True
seq P

Returns a new object reference.

let adr = new(Adr, T) in P ≡
import adr do
  ALLOCADR(adr, T)
seq P

Returns the address of a newly allocated memory block, sufficient to hold values a given type.

\[
\text{ALLOCFIELDS}(x, A) \equiv \\
\text{forall } T::F \in A \text{ do import } adr \text{ do} \\
  \text{fieldAdr}(x, T::F) := adr \\
  \text{ALLOCADR}(adr, \text{fieldType}(T::F))
\]

Allocates the instance fields in a given set.

\[
\text{ALLOCADR}(adr, T) \equiv \\
  \text{Adr}(adr) := True \\
  \text{if } T \in \text{Struct } \text{then} \\
  \text{ALLOCFIELDS}(adr, \text{instFields}(T))
\]

Allocates the instance fields of a value of a given type, stored at a given address.

<table>
<thead>
<tr>
<th>YIELD(res, α) ≡</th>
<th>values(α) := res</th>
<th>pos := α</th>
</tr>
</thead>
</table>

This macro is applied in two forms:

| YIELD(res) ≡ YIELD(res, pos) | YIELDUP(res) ≡ YIELD(res, up(pos)) |

If the binary operation is a division by zero, then a DivideByZeroException is raised. This case is omitted in \([24]\)'s semantics model.

**Boxing and unboxing**  The boxing occurs, for example, when evaluating a type cast (\(T\) \(exp\), where \(T\) is a reference type and the compile-time type \(ST\) of \(exp\) is a value type. Note that the type constraints in Table 3.7 guarantee that the compile-time type of \(exp\) is a subtype of \(T\).
For boxing, the macro \textit{NewBox} is defined. Thus, for a type \(T\), a value type \(\text{val}\), \(\text{NewBox}(T, \text{val})\) returns a reference to a newly allocated object which embeds \(\text{val}\):

\[
\text{let } \text{ref} = \text{NewBox}(T, \text{val}) \text{ in } P \equiv \\
\text{let } \text{ref} = \text{new}(\text{ObjRef}) \text{ and } \text{adr} = \text{new}(\text{Adr}, T) \text{ in } \\
\text{actualTypeOf}(\text{ref}) := T \\
\text{addressOf}(\text{ref}) := \text{adr} \\
\text{WRITEMEM}(\text{adr}, T, \text{val}) \\
\text{seq } P
\]

The unboxing is triggered by the evaluation of an expression of the form \((T) \text{exp}\), where \(T\) is a value type and the compile-time type of \(\text{exp}\) is a reference type. The value embedded into the boxed object is yielded up if its type is \(T\). Otherwise, an \textit{InvalidCastException} is raised.

\textbf{Field access} Assume that in \(\text{exp} . T::F\), \(\text{exp}\) is evaluated to \(\text{valadr}^9\), possibly an address due to the position of \(\text{exp}\). If \(\text{valadr}\) is a struct value, as opposed to an address where a struct value is stored, then its field \(T::F\) is computed as \(\text{valadr}(T::F)\) (remember that the struct values are defined as mappings). If \(\text{valadr}\) is \texttt{null}, a \textit{NullReferenceException} is thrown\(^10\). If \(\text{valadr}\) is not a struct value or \texttt{null}, then \(\text{valadr}\) is either an address or an object reference. In either case, the address of \(T::F\) is \(\text{fieldAdr}(\text{valadr}, T::F)\). Depending on whether \(\text{up}(\text{pos})\) is an address position, the address of \(T::F\) or the value stored at this address is yielded up, respectively. For this purpose, the macro \textit{YIELDINDIRECT} is defined:

\[
\text{YIELDINDIRECT}(\text{adr}, \alpha) \equiv \\
\text{if isAddressPos}(\alpha) \text{ then } \text{YIELD}(\text{adr}, \alpha) \\
\text{else } \text{YIELD}(\text{memVal}(\text{adr}, ST(\alpha)), \alpha)
\]

Similarly to \textit{YIELD}, we also have two forms for \textit{YIELDINDIRECT}: \textit{YIELDINDIRECT}(\text{adr}) and \textit{YIELDUPINDIRECT}(\text{adr}).

\textbf{Method call} In the expression \(\text{exp} . T::M(\text{args})\), the expression \(\text{exp}\) is evaluated before the list of arguments \(\text{args}\). Let \(\text{valadr}\) and \(\text{valadrs}\) be the value/address and the list of values/addresses \(\text{exp}\) and \(\text{args}\) evaluate to, respectively. If \(\text{valadr}\) is a \texttt{null} reference, then a \textit{NullReferenceException} is raised. Otherwise, the method \(T::M\) is called with the arguments \(\text{valadr}\) (as the \texttt{this} pointer) and \(\text{valadrs}\). To call methods, we define the macro \textit{VIRTCall}. Thus, for a method \(T::M\) called with the arguments \(\text{valadr}\) (as the \texttt{this} pointer) and \(\text{valadrs}\), \textit{VIRTCall}(\(T::M, \text{valadr}, \text{valadrs}\)) is defined as follows\(^11\):

\(^9\)We typically use the name \(\text{valadr}\) for an evaluation result from \(\text{Val} \cup \text{Adr}\), which, depending on the context, is a value or an address.

\(^{10}\)As observed in \([55]\), the semantics (related to the timing of the \texttt{null} check) of field accesses involved in assignments is violated by an optimization of the \(\texttt{C}^\ast\) compiler of \([3]\).

\(^{11}\)The macro \texttt{INVOKEMETHOD}, defined in the Appendix, is used to invoke a method with a given list of arguments.
\[ \text{VirtCall}(T::M, \text{valadr}, \text{valadrs}) \equiv \]
\[
\text{if callKind}(T::M) = \text{Virtual} \text{ then }
\begin{align*}
& \text{let } T''::M = \text{lookUp}(\text{actualTypeOf(\text{valadr})}, T::M) \text{ in } \hfill \\
& \text{let } \text{this} = \text{if } T'' \in \text{Struct} \text{ then } \text{addressOf(\text{valadr}) } \text{ else } \text{valadr} \text{ in } \\
& \text{INVOKEMETHOD}(T''::M, [\text{this}] \cdot \text{valadrs}) \\
& \text{else } \text{INVOKEMETHOD}(T::M, [\text{valadr}] \cdot \text{valadrs})
\end{align*}
\]

In `VirtCall(T::M, valadr, valadrs)`, if `T::M` is virtual, the method to be invoked at run-time is determined with `lookUp`. The type of `valadr` considered by `lookUp` is `actualTypeOf(valadr)`. Depending on whether the method to be invoked at run-time is declared by a class or by a struct, the `this` pointer of the method to be called is `valadr` or the address `addressOf(valadr)` of the value inside the boxed object (remember that the `this` pointer of a struct method call is passed "by reference"), respectively.

**Delegate creation** We only describe here creations of the form `new D(exp.T::M)`. Let `val` be the value to which `exp` evaluates. If `val` is `null`, then a `NullReferenceException` is thrown. Otherwise, it is proceeded according to the following case distinction.

If `T::M` is virtual, the target method to be inserted in `invocationList(d)`, where `d` is the newly allocated delegate, is determined via `lookUp`, exactly in the same way as the method to be invoked in the case of a method call. The target object is set to `val`.

**Remark 3.5.3** [24]'s semantics model does not perform a method lookup at the delegate creation time, but at the time when the delegate is called. This is, however, not according to [74]. Therefore, we changed the model to correctly reflect the Microsoft implementation.

If `T::M` is not virtual, `T::M` is inserted in `invocationList(d)`. If `T` is a struct, then `val` (assumed to be a struct value) is boxed and considered as the target object. Otherwise, `val` becomes the target object.

**Remark 3.5.4** The case when the target object is a boxed struct value is omitted by [24]'s semantics model. Note that, this case is also possible if `T::M` is virtual.

### 3.6 Type Safety

In this section, we discuss our type safety theorem together with its crucial lemmas. The properties ensured for legal, well-typed methods accepted by the definite assignment analysis are the following:

**Type safety** If an expression evaluates to a value, then the value is compatible with the compile-time type of the expression’s position. The value parameters, local variables, instance fields always contain values of the declared types. The values inside boxed objects are compatible with the corresponding value types.
3.6. Type Safety

**Lvalue evaluation consistency** If an expression evaluates to an address, then the address is a valid address for the compile-time type of the expression’s position. The `ref/out` parameters always point to valid addresses for the declared types. The addresses always occur at address positions, and the expressions in address positions always evaluate to addresses.

**Statement execution consistency** Run-time abruptions always occur in accordance with the corresponding compile-time constraints for jumps, method returns, or exceptions.

Any address appearing in the evaluation tree should be a valid (managed) address, i.e., the address of a managed memory block as opposite to the address of a random memory block. In other words, the addresses occurring in the evaluation tree should only be memory locations the program is authorized to access. To ensure that, we have to answer the question: "If an address `adr` appears in the evaluation tree at a position `α`, where it originates from?". Intuitively, in general, there should be a variable expression `vexp` at `α` such that `adr` is "the address" of `vexp`, where "the address" of `vexp` might have been previously allocated for a variable (not necessarily `vexp`) to which `vexp` points (this is possible because of the call-by-reference mechanism). This natural answer is, however, not simple to formally reason about, since several evaluation steps might be required to produce the address of a variable expression. Moreover, it is not always the case that, if an address appears in the tree, then there exists an expression at the same position whose evaluation produced the address. A counterexample for this is given by the `ref/out` arguments: The argument `ref loc` at a position `α` is evaluated in two steps to the address of `loc`.

Due to the call-by-reference mechanism, it is possible that updating a variable `vexp` in a frame `fr` implies another variable `vexp'` in another frame `fr'` (below `fr` in the frame stack) to be updated. One has to carefully analyze these updates since `vexp'` does not necessarily point directly to `vexp`, but it could also point to its "interior", e.g., `vexp'` is a field of a field of `vexp`. In both cases, one has to ensure that the update of `vexp'` preserves the typing of `vexp` with respect to `fr`. Otherwise, C#’s type safety would be violated when `fr` becomes (again) the current frame.

If a local variable `loc` is not defined in the local environment, then an access to `loc` yields the constant `undef`. This constant is then propagated in the evaluation tree, and eventually the operational semantics model stops, and the execution fails. Our type safety result ensures that the constant `undef` never occurs in the evaluation tree. This is because in the real C# implementation, there is no constant `undef`. In the real implementation, if the uninitialized `loc` is accessed, the execution proceeds with whatever value was in the memory at the address of `loc`. This value could be of any type, and thus C#’s type safety would be violated. The fact that `undef` does not show up in the evaluation tree implies, in particular, that there no dangling pointers. To ensure that `undef` does not appear during the evaluation is not trivial. Not every address occurring in the evaluation tree holds a value. For example, consider an argument `α(out β loc)`, where the local variable is not definitely assigned. The address of the uninitialized `loc` is produced at `β` and then propagated to `α`. It is then crucial that the memory block pointed to by such an address is not "read". Note that, a memory block might be regarded as uninitialized, i.e., the value "read" from the block is `undef`, though several memory blocks "in" the given block are initialized. Consider, for example, a local variable `loc` of type a struct `S`, where `S` has at least two instance fields. Let `S::F` be one of them. If `loc` is not initialized, then the value "read" from the memory block pointed to by `locAdr(loc)`, sufficient to hold values of type `S`, is `undef`. However, if there
was an assignment to \( \text{loc}.S::F \), then the memory block pointed to by the address of \( S::F \) holds a value though \( \text{loc} \) is regarded as uninitialized.

When defining the typing of values, one key point is how to distinguish between values of value types and boxed values of value types. Note that, when a value \( \text{val} \) of a value type \( T \) is boxed, we considered the type \( T \) as the run-time type used in dynamic type-checks. For this reason, we introduce boxed types, as reference types used only for the purpose of verifying type safety:

\[
\text{BoxedType} = \{ \text{boxed}(T) \mid T \in \text{ValueType} \}
\]

Thus, we consider a boxed value of a value type \( T \) to be typable with the type \( \text{boxed}(T) \).

To define the typing of values, we introduce the notion of run-time compatibility for C\(^\#\)'s types, boxed types, and \texttt{Null} (the type of the \texttt{null} references).

**Definition 3.6.1 (Run-time compatibility)** The run-time compatibility relation \( \sqsubseteq \) is the least reflexive and transitive relation such that:

- if \( T'' \in \text{ValueType} \) extends / implements \( T' \) and \( \text{boxed}(T'') = T \), or
- \( T, T' \in \text{RefType} \) and \( T \preceq T' \), or
- \( T = \text{Null} \) and \( T' \in \text{RefType} \),

then \( T \sqsubseteq T' \).

The following lemma is applied in the type safety proof:

**Lemma 3.6.1** If \( T \sqsubseteq T' \) and \( T' \preceq T'' \), then \( T \sqsubseteq T'' \).

**Proof.** By case distinction where Definitions 3.2.1 and 3.6.1 are applied. \( \square \)

The typing of values takes advantage of the fact that the values carry their type at run-time. In other words, every value can be viewed as a tagged value. The tag types are the value types and \texttt{ref}. The tag \texttt{ref} is used for object references independently of their actual type. The values are constrained by the following well-formedness conditions:

**[value well-formedness]** For any tagged value \( \langle \text{val}, T \rangle \), it holds:

\[
(T \in \text{PrimitiveType} \Rightarrow \text{val} \in \text{SimpleVal}) \land
(T \in \text{Struct} \Rightarrow \text{val} \in \text{StructVal}) \land
(T = \texttt{ref} \Rightarrow \text{val} \in \text{ObjRef})
\]

To type values, we recursively define the function \( \text{type} : \text{Map}(\text{Val, Type} \cup \text{BoxedType}) \), which, given a value, returns the least type (with respect to the run-time type compatibility \( \sqsubseteq \)) the value can be typable with. One crucial point is how to define what it means that a struct
value is of that struct type. The idea is that, since a struct value is a tuple consisting of the
values of its instance fields, every instance field should hold a value typed according to the
(corresponding declaration type. Another point is that the function type should also distinguish
between a boxed object and the value inside a boxed object.

**Definition 3.6.2** For a tagged value \(\langle \text{val}, T \rangle\), we define:

\[
type(\langle \text{val}, T \rangle) = \begin{cases} 
T, & \text{if } T \in \text{PrimitiveType}; \\
T, & \text{if } T \in \text{Struct and } \\
\text{type}(\text{val}(T::F)) \sqsubseteq \text{fieldType}(T::F) & \text{for every } T::F \in \text{instFields}(T); \\
\text{actualTypeOf}(\text{val}), & \text{if } T = \text{ref and } \text{actualTypeOf}(\text{val}) \in \text{Class}; \\
\text{boxed}(\text{actualTypeOf}(\text{val})), & \text{if } T = \text{ref and } \\
\text{actualTypeOf}(\text{val}) \in \text{ValueType}.
\end{cases}
\]

Note that Definition 3.6.2 is correct as the declared type of a struct instance field cannot be the
struct type itself [74].

We assume that the results returned by the static semantics functions are typed correctly:

**[static functions’ correct typing]**

1. for every \(\text{lit} \in \text{Lit}\), \(\text{type}(\text{valueOfLiteral}(\text{lit})) = \text{typeOfLiteral}(\text{lit})\);
2. for every \(T \in \text{Type}\), \(\text{type}(\text{defVal}(T)) = T\);
3. for every \(T \in \text{PrimitiveType} \text{ and } \text{val} \in \text{SimpleVal}, \text{type}(\text{convertVal}(T, \text{val})) = T\);

The following lemma determines the kind of each value depending on the types the value is
typable with.

**Lemma 3.6.2** Assume \(\text{type}(\text{val}) \sqsubseteq T\).

1. If \(T \in \text{PrimitiveType}\), then \(\text{val} \in \text{SimpleVal}\).
2. If \(T \in \text{Struct}\), then \(\text{val} \in \text{StructVal}\).
3. If \(T \in \text{RefType}\), then \(\text{val} \in \text{ObjRef}\).

**Proof.** By the condition [value well-formedness] and Definitions 3.6.2 and 3.6.1. \(\square\)

Lemma 3.6.3 is used to show that the fields of a struct value (so, a defined value) are defined.

**Lemma 3.6.3** If \(\text{val} \neq \text{undef} \text{ and } \text{type}(\text{val}) \sqsubseteq S\text{ where } S \in \text{Struct}, \text{then } \text{val}(S::F) \neq \text{undef for every } S::F \in \text{instFields}(S)\).
Proof. By reductio ad absurdum and making use of Lemma 3.6.2 and Definition 3.6.2.

An interesting part consists in defining what addresses are valid addresses, i.e., addresses of managed memory blocks. The key idea is that, if a memory block is allocated to store a struct value, then it is considered that also the memory blocks for storing the instance fields of the struct value are allocated. Therefore, we introduce the predicate isAddressIn with the following meaning. For two addresses \(adr, adr'\) and two types \(T, T'\), \(\text{isAddressIn}(adr, T, adr', T')\) holds if a value of type \(T\) is stored at the address \(adr\) which "is an address in" the block of memory pointed to by the address \(adr'\) where a value of type \(T\) is stored.

**Definition 3.6.3 ("is address in")** Given the addresses \(adr, adr'\) and the types \(T, T'\), we define:

\[
isAddressIn(adr, T, adr', T') :\iff (adr = adr' \land T = T') \lor (T' \in \text{Struct} \land \exists T':F \in \text{instFields}(T') : \text{isAddressIn}(adr, T, \text{fieldAdr}(adr', T':F), \text{fieldType}(T':F)))
\]

If the address of a struct value "is an address in" a memory block, then also the address of each instance field of the struct value "is an address in" that memory block.

**Lemma 3.6.4** If \(T \in \text{Struct}\) and \(\text{isAddressIn}(adr, T, adr', T')\), then

\[
\text{isAddressIn}(\text{fieldAdr}(adr, T::F), \text{fieldType}(T::F), adr', T')
\]

for every \(T::F \in \text{instFields}(T)\).

Proof. By case distinction, where Definition 3.6.3 is applied.

**Lemma 3.6.5** There exists no types \(T, T'\), object reference \(ref\), address \(adr\) and instance field \(T''::F \in \text{instFields}(\text{actualTypeOf}(ref))\) such that

\[
\text{isAddressIn}(\text{fieldAdr}(ref, T''::F), T, adr, T')
\]

Proof. By applying Definition 3.6.3 and using the abstract memory allocation defined in Table 3.19.

Definition 3.6.4 makes precise what we mean by a valid address. An address \(adr\) is valid for a type \(T\) with respect to a stack of frames if \(adr\) is the address of a local variable or value parameter of declared type \(T\) of a frame in the frame stack, or the address of a class instance field of declared type \(T\), or the address of a value of a value type \(T\) inside a boxed object, or it "is an address in" a memory block pointed to by the addresses of one of the above locations.
Definition 3.6.4 (Valid address) An address \( \text{adr} \) is valid for a type \( T \) with respect to a stack of frames \( \{fr_1, \ldots, fr_n\} \), denoted \( \{fr_1, \ldots, fr_n\} \vdash \text{validAddress}(\text{adr}, T) \), if one of the following conditions holds:

(va-loc) there exists \( i = 1, n \) and \( loc \in \text{localVars}(\text{meth}(fr_i)) \) such that \( \text{isAddressIn}(\text{adr}, T, \text{locAdr}(fr_i)(loc), \text{locType}(\text{meth}(fr_i), loc)) \) holds;

(va-arg) there exists \( i = 1, n \) and \( loc \in \text{valueParams}(\text{meth}(fr_i)) \) such that \( \text{isAddressIn}(\text{adr}, T, \text{locAdr}(fr_i)(loc), \text{paramType}(\text{meth}(fr_i), loc)) \) holds;

(va-field) there exists \( C \in \text{Class}, C':::F \in \text{instFields}(C) \) and \( \text{ref} \in \text{ObjRef} \) such that \( \text{actualTypeOf}(\text{ref}) = C \) and \( \text{isAddressIn}(\text{adr}, T, \text{fieldAdr}(\text{ref}, C':::F), \text{fieldType}(C':::F)) \) holds;

(va-box) there exists \( T' \in \text{ValueType} \) and \( \text{ref} \in \text{ObjRef} \) such that \( \text{actualTypeOf}(\text{ref}) = T' \) and \( \text{isAddressIn}(\text{adr}, T, \text{addressOf}(\text{ref}), T') \) holds.

If an address is valid for a struct type, then the address of each of its instance fields is valid for the declared type of the respective field.

Lemma 3.6.6 If \( \{fr_1, \ldots, fr_n\} \vdash \text{validAddress}(\text{adr}, S) \) and \( S \in \text{Struct} \), then

\[ \{fr_1, \ldots, fr_n\} \vdash \text{validAddress}(\text{fieldAdr}(\text{adr}, S::F'), \text{fieldType}(S::F)) \]

for every \( S::F \in \text{instFields}(S) \).

Proof. By Definition 3.6.4 and Lemma 3.6.4. \qed

The following lemma states that each valid address with respect to a stack of frames remains valid upon pushing a new frame onto the stack.\(^\text{12}\)

Lemma 3.6.7 If \( \{fr_1, \ldots, fr_n\} \vdash \text{validAddress}(\text{adr}, T) \) and \( fr \in \text{Frame} \), then

\[ \{fr_1, \ldots, fr_n, fr\} \vdash \text{validAddress}(\text{adr}, T) \]

Proof. By Definition 3.6.4. \qed

Lemma 3.6.8 says that if an address \( \text{adr} \) ”is an address in” the memory block pointed to by an address \( \text{adr}' \) where a value of a type \( T' \) is stored, and \( \text{adr} \) is the address of a memory block sufficient to hold values of a type \( T \), then the value read from the memory block pointed to by \( \text{adr} \) is of type \( T \). This lemma is used to prove that if the evaluation of a struct field access \( \text{exp}::S::F \) in the case when \( \text{isAddressPos}(\text{pos}) \) holds is required to produce a value (as opposite to an address), then this value is of the expected (compile-time) type.

\(^{12}\)Note that, the converse implication is false. For example, the address of a local variable of a popped frame is not valid anymore with respect to the new frame stack.
Lemma 3.6.8 If isAddressIn(adr, T, adr', T') and type(memVal(adr', T')) ⊑ T', then
\[
\text{type}(\text{memVal}(\text{adr}, T)) \sqsubseteq T
\]

Proof. By induction on the derivation of isAddressIn in Definition 3.6.3 and making use of Definition 3.6.2. □

Assume that adr'' points to a memory block and in this block adr' is an address where a value of a type T' is stored. Lemma 3.6.9 claims that if the value "read" at adr'' is of a type T'', then, after a value of type T' is stored at adr', the value in the memory block pointed to by adr'' is still of type T''. This lemma is applied, for example, to ensure that, following a ref/out parameter update, the variable to which or to whose "interior" the parameter points has still a value of the expected type.

Lemma 3.6.9 If the types T, T', T'' satisfy T' ⊑ T, type(val) ⊑ T', type(memVal(adr'', T'')) ⊑ T'' and isAddressIn(adr', T', adr'', T''), then type(memVal(adr'', T'')) ⊑ T'' also holds after the macro WRITEMEM(adr', T, val) is executed.

Proof. By induction on the definition of the isAddressIn predicate. The base case of the induction is proved by applying Lemmas 3.5.1 and 3.6.8. □

To prove type safety of method calls, in particular of delegate calls, the following lemma concerning the method lookup is required.

Lemma 3.6.10 If lookUp(T, T'::M) = T''::M, then:

- T ⊑ T'' ⊑ T';
- length(paramTypes(T'::M)) = length(paramTypes(T''::M)) and the following relation holds for every i = 1, length(paramTypes(T'::M)) − 1
  \[
  \text{paramTypes}(T''::M)(i) = \text{paramTypes}(T'::M)(i)
  \]
- retType(T'::M) = retType(T''::M).

Proof. By Definition 3.5.2 and the condition [override/implement]. □

The soundness of the definite assignment analysis is crucial for proving that undef does not show up during the evaluation of C# expressions. The following definition makes precise what variables are guaranteed by the definite assignment analysis to hold a value.

Definition 3.6.5 The variables tracked by the definite assignment analysis are defined by the following grammar:
DefAssVar ::= Loc | StructDefAssVar.Field

where StructDefAssVar represents a DefAssVar of a struct type.

The following lemma states that every variable tracked by the definite assignment analysis is in particular a Vexp.

Lemma 3.6.11 If exp ∈ DefAssVar, then exp ∈ Vexp.

Proof. By Definition 3.6.5 and the definition of Vexp in Figure 3.1.

The recursively defined function adrOfVexp : Map(Frame × DefAssVar, Adr) determines the addresses of the variables tracked by definite assignment analysis.

Definition 3.6.6 Given vexp ∈ DefAssVar in the method executed in a frame fr, we define:

\[
adrOfVexp(fr, vexp) = \begin{cases} 
\text{locAdr}(fr)(loc), & \text{if vexp is a local variable or parameter loc;} \\
\text{fieldAdr}(adrOfVexp(fr, vexp'), ST(\alpha)::F), & \text{if vexp is of the form } \alpha\text{vexp'.F.}
\end{cases}
\]

As the definite assignment analysis is sound (Section 3.4), the methods accepted by this analysis should satisfy the following condition:

[definite assignment analysis] If \(\alpha\text{vexp} \in \text{DefAssVar}\) and \(\alpha\) occurs in the method executed in a frame \(fr\), then

\[-\text{isAddressPos}(\alpha) \lor \text{label}(\text{up}(\alpha)) \in \{\text{Aop, ref}\} \Rightarrow \text{memVal}(\text{adrOfVexp}(fr, vexp), ST(\alpha)) \neq \text{undef}\]

So, a variable is definitely assigned unless the variable occurs at an address position, and it is neither a variable which is assigned to in a compound assignment nor a ref argument. It worths mentioning that, although a variable which is assigned to in a compound assignment or is passed as a ref argument, is at an address position, the variable is required to have a value according to the definite assignment rules [74, §5.3].

Finally, we state the type safety result in Theorem 3.6.1. The invariants of this theorem are categorized into frame invariants, global invariants, and dynamic method chain invariants.

Frame invariants If an expression evaluates to a value, then the value is compatible with the compile-time type associated to the expression position (val). If an expression evaluates to an address, then the address is a valid address for the compile-time type of the expression position with respect to the substack of frames under the current frame (adr). The constant undef never
occurs in the evaluation tree (undef). Expressions and arguments (other than ref/out arguments) not occurring in address positions are ensured to evaluate to values (as opposite to addresses), whereas the ref/out arguments are guaranteed to evaluate to addresses (label1). Expressions occurring in address positions evaluate to addresses (label1). If a statement is executed, then its execution can only complete normally or with an abruption reason (label2). The ref/out parameters can only point to valid addresses for their declaration type with respect to the substack of frames under the current frame (arg1). The value parameters are guaranteed to have values of their declared type (arg2). The local variables and the fields of struct type local variables are either uninitialized, i.e., undef, or have values of their declared type (loc). The catch parameters always hold values of types compatible with the type of exceptions the catch clauses are handling. Every position where a Break or a Continue occurs is within a while statement (abr1). A Goto(lab) can only occur in the scope of the statement labelled with the label lab, i.e., in the block in which the labelled statement occurs (abr2). Every position where Return or Return(val) occurs is within a method body, with a return value of a type compatible with method return type (abr3), (abr4). The types of thrown exceptions are subtypes of object.

Global invariants The instance fields of an object (other than a boxed object) have values of their declared type (field). The value inside a boxed value type is compatible with the value type (box). Every target object in the invocation list of a delegate is an object of an appropriate type: (1) if it is a boxed object, then the value type being boxed is compatible with the type of the corresponding target method; (2) if it is not a boxed object, then the run-time type is compatible with the type that declares the corresponding target method. Every target method is consistent with the delegate type (del). Every exception in the excStack is an object reference (excstack).

Dynamic method chain invariants The current position of a caller frame is either the position of an instance creation or the position of a method call (framestack1). If the return type of the method associated to a callee frame is not void, then the return type is compatible with the compile-time type expected by the caller frame (framestack2). If a callee frame corresponds to an instance constructor call, then its this pointer is the newly allocated address (in case of a struct constructor) or the newly created reference (in case of a class constructor) (framestack3).

Theorem 3.6.1 (Type safety) Assume that frameStack is given by \([fr_1, \ldots, fr_{n-1}]\) and the current frame is a frame \(fr_n\). Moreover, assume that \(\text{meth}(fr_i)\) is well-typed for every \(i = 1, n\). Then the following invariants hold for every frame \(fr_i = (\text{meth}^*, \text{pos}^*, \text{locAdr}^*, \text{values}^*), i = 1, n:\)

\((\text{val})\) If \(\text{label}(\alpha) \in \text{Exp}\) and \(\text{values}^*(\alpha) = \text{val}\) where \(\text{val} \in \text{Val}\), then \(\text{type}(\text{val}) \subseteq ST(\alpha)\).

\((\text{adr})\) If \(\text{label}(\alpha) \in \text{Exp}\) and \(\text{values}^*(\alpha) = \text{adr}\) where \(\text{adr} \in \text{Adr}\), then

\([fr_1, \ldots, fr_i] \vdash \text{validAddress}(\text{adr}, ST(\alpha))\)

\((\text{undef})\) \(\text{values}^*(\alpha) \neq \text{undef}\) for every \(\alpha \in \text{dom}(\text{values}^*)\).
3.6. Type Safety

(label1) If \(\text{label}(\alpha) \in \text{Exp} \cup \text{Arg} \) and \(\alpha \in \text{dom(values}^*)\), then

\[
\begin{align*}
(isAddressPos(\alpha) & \Rightarrow (\text{label}(\alpha) \in \{\text{ref}, \text{out}\} \Rightarrow \text{values}^*(\alpha) \in \text{Adr}) \land \text{label}(\alpha) \notin \{\text{ref}, \text{out}\}) \Rightarrow \text{values}^*(\alpha) \in \text{Val}) \land \text{values}^*(\alpha) \in \text{Adr} )
\end{align*}
\]

(label2) If \(\text{label}(\alpha) \in \text{Stm} \) and \(\alpha \in \text{dom(values}^*)\), then

\[\text{values}^*(\alpha) \in \{\text{Norm} \} \cup \text{Abr}\]

(arg1) If \(\text{loc} \in \text{refParams(meth}^*) \cup \text{outParams(meth}^*)\), then

\[
[fr_1, \ldots, fr_i] \vdash \text{validAddress(locAdr}^*(\text{loc}), \text{paramType(meth}^*, \text{loc}))
\]

(arg2) If \(\text{loc} \in \text{valueParams(meth}^*)\), then

\[
\text{type(memVal(locAdr}^*(\text{loc}), \text{paramType(meth}^*, \text{loc}))) \subseteq \text{paramType(loc, meth}^*)
\]

(loc) If \(\text{loc} \in \text{localVars(meth}^*)\), then

\[
\text{memVal(adr, T)} = \text{undef or type(memVal(adr, T))} \subseteq T
\]

for every \((\text{adr, T}) \in \text{Adr} \times \text{Type} \) with \(\text{isAddressIn(adr, T, locAdr}^*(\text{loc}), \text{locType(meth}^*, \text{loc}))\).

(catch) If \(\text{pos}^* \) is in the scope of a catch parameter \(\text{loc} \) of type \(T\), then

\[\text{type(memVal(locAdr}^*(\text{loc}), T)) \subseteq T\]

(abr1) If \(\text{values}^*(\alpha) \in \{\text{Break, Continue}\}\), then \(\alpha \) is in a while statement.

(abr2) If \(\text{value}^*(\alpha) = \text{Goto(label)}\), then \(\alpha \) is in the scope of a statement labelled with \(\text{lab}\).

(abr3) If \(\text{values}^*(\alpha) = \text{Return}\), then \(\text{retType(meth}^*) = \text{void}\).

(abr4) If \(\text{values}^*(\alpha) = \text{Return(val)}\), then \(\text{val} \in \text{Val} \) and \(\text{type(val)} \subseteq \text{retType(meth}^*)\).

(abr5) If \(\text{values}^*(\alpha) = \text{Exc(ref)}\), then \(\text{type(ref)} \subseteq \text{object}\).

The following global invariants are satisfied:
If ref ∈ ObjRef is such that actualTypeOf(ref) = C where C ∈ Class, then
\[
\text{type}(\text{memVal}(\text{fieldAdr}(\text{ref}, C'::F), \text{fieldType}(C'::F))) \sqsubseteq \text{fieldType}(C'::F)
\]
for every C'::F ∈ instFields(C).

If ref ∈ ObjRef is such that actualTypeOf(ref) = T where T ∈ ValueType, then
\[
\text{type}(\text{memVal}(\text{addressOf}(\text{ref}), T)) \sqsubseteq T
\]

If d ∈ ObjRef is such that actualTypeOf(d) = D where D ∈ Delegate, then the following hold for every (ref, T::M) ∈ invocationList(d):

- ref ∈ ObjRef
- If there exists T' ∈ ValueType such that type(ref) = boxed(T'), then T' ⊑ T. Otherwise, type(ref) ⊑ T.
- T::M and D are consistent.

If ref ∈ excStack, then type(ref) ⊑ object.

If (α, β, values*) is the parent frame of (T::M, α, values*, locAdr*), then the following dynamic method chain invariants are satisfied:

- β is the position of an instance creation or of a method call.
- If retType(T::M) ≠ void, then retType(T::M) ⊑ ST(β).
- If T::M is an instance constructor and values*(β) = valadr.T::M(valadrs), then
  - If T ∈ Struct, then locAdr*(this) = valadr.
  - If T ∈ Class, then memVal(locAdr*(this), T) = valadr.

Proof. For the proof of several invariants, e.g., (val), (adr), (undef), the following (frame) invariants corresponding to the frame fr_i are required:

- If values*(α) = adr where adr ∈ Adr, then
  \[
  \text{memVal}(adr, ST(\alpha)) = \text{undef} \text{ or } \text{type}(\text{memVal}(adr, ST(\alpha))) \sqsubseteq ST(\alpha)
  \]

- If α ∈ Vexp and values*(α) = adr where adr ∈ Adr satisfies (va-loc) for ST(α) with respect to the stack [fr_1, ..., fr_i], then α ∈ DefAssVar and adrOfVexp(fr_i, vexp) = adr, where vexp is the expression at α.
The invariant \((\text{adr1})\) says that if an address appearing in the evaluation tree holds a value, then the value is compatible with the static type associated to the position where the address occurs. The invariant \((\text{vexp})\) establishes a correspondence between certain addresses from the evaluation tree and variables tracked by the definite assignment analysis. Assume that an address \(\text{adr}\) appearing at an address position of a variable expression \(\text{vexp}\) has been allocated as the address of a local variable of a frame in the stack \((\text{not necessarily the current frame})\). Then \(\text{vexp}\) is a variable tracked by the definite assignment analysis, whose address with respect to the current frame is \(\text{adr}\).

The proof of all invariants, including \((\text{adr1})\) and \((\text{vexp})\), proceeds by induction on the number of steps in the run of the operational semantics model.

**Base case**  In the initial state of the semantics model, the invariants are trivially satisfied.

**Induction step**  There is a proof case for each of the rules of the operational semantics model. We only do here one interesting proof case.

**Case**  \(\text{pos}\) is pointing to \(\text{valadr}\) in \(\text{valadr}.T::F\).

Of particular interest is the application of the soundness of the definite assignment analysis in the proof. To illustrate that, we only study here the subcase characterized by

\[
\text{ST}(\text{pos}) \notin \text{ValueType} \lor \text{valadr} \in \text{Adr}
\]

and

\[
\neg \text{isAddressPos}(\text{up}(\text{pos}))
\]

and

\[
\text{valadr} \neq \text{null}
\]  \hspace{1cm} (3.1)

In this case, the operational semantics model fires the updates described by the macro \text{YIELDUPINDIRECT}(\text{fieldAdr}(\text{valadr}, T::F)). More exactly, \(\text{values}(\text{up}(\text{pos}))\) is set to the value \(\text{memVal}(\text{fieldAdr}(\text{valadr}, T::F), \text{fieldType}(T::F))\) (note that the assumption (3.1) implies that \(\neg \text{isAddressPos}(\text{up}(\text{pos}))\), and so, a value – as opposed to an address – is required at \(\text{up}(\text{pos})\)) and \(\text{pos}\) becomes \(\text{up}(\text{pos})\). We distinguish the following cases:

**Subcase 1** \(\neg \text{isAddressPos}(\text{pos})\) is false.

By \(\neg \text{isAddressPos}(\text{pos})\) and the induction hypothesis – that is, the invariant \((\text{label1})\) – we get \(\text{values}(\text{pos}) \in \text{Val}\). By this and the induction hypothesis \((\text{val})\), we have \(\text{type}(\text{valadr}) \sqsubseteq \text{ST}(\text{pos})\).

As \(\text{meth}\) is well-typed, according to the type constraints in Table 3.7, we have \(\text{ST}(\text{pos}) \preceq T\). By (3.1) and \(\text{values}(\text{pos}) \notin \text{Adr}\), we know that \(\text{ST}(\text{pos}) \notin \text{ValueType}\), that is \(\text{ST}(\text{pos}) \notin \text{RefType}\).

This and Lemma 3.6.2 imply that \(\text{valadr} \in \text{ObjRef}\).

That the invariant \((\text{val})\) holds in the next step of the semantics model follows from \(\text{valadr} \in \text{ObjRef}\) and the induction hypothesis \((\text{field})\). So, we have

\[
\text{type}(\text{memVal}(\text{fieldAdr}(\text{valadr}, T::F), \text{fieldType}(T::F))) \sqsubseteq \text{fieldType}(T::F)
\]

By this and Lemma 3.6.2, we get \(\text{values}(\text{up}(\text{pos})) \notin \text{Adr}\). This implies that the invariant \((\text{label1})\) holds in the next step of the semantics model.

The other invariants are trivially satisfied.
Subcase 2  \( \text{isAddressPos}(\text{pos}) \) is true.

By \( \text{isAddressPos}(\text{pos}) \) and the induction hypothesis (label1), we get \( \text{valadr} \in \text{Adr} \). By this, \( \text{values}(\text{pos}) = \text{valadr} \) and the induction hypothesis (adr), we know that \( [\text{fr}_1, \ldots, \text{fr}_i] \vdash \text{validAddress}(\text{adr}, \text{ST}(\text{pos})) \) where \( [\text{fr}_1, \ldots, \text{fr}_i] \) is the current frameStack with the current frame on top. According to Definition 3.6.4, this means that \( \text{valadr} \) satisfies one of the conditions (va-box), (va-field) or (va-loc) with respect to \( \text{ST}(\text{pos}) \). If \( \text{valadr} \) fulfills (va-arg), (va-field) or (va-box), then the fact that \( \text{val} \) holds in the next step of the model follows by Lemma 3.6.8, the type constraints in Table 3.7 and the induction hypothesis, that is, the invariants (arg2), (field), or (box), respectively. In turn, the invariant \( \text{val} \) for \( \text{up}(\text{pos}) \) also implies that (label1) is satisfied in the next step of the semantics model.

Let us now assume that \( \text{valadr} \) satisfies (va-loc), that is, there exists \( j = 1, i \) and \( \text{loc} \in \text{localVars}(\text{meth} (\text{fr}_j)) \) such that

\[
\text{isAddressIn}(\text{valadr}, \text{ST}(\text{pos}), \text{locAdr}(\text{fr}_j)(\text{loc}), \text{locType}(\text{meth} (\text{fr}_j), \text{loc})) \quad (3.2)
\]

By \( \text{isAddressPos}(\text{pos}) \) and Lemma 3.5.2, we get \( \text{pos} \in \text{Vexp} \). By this and the induction hypothesis – that is, by the special invariant (vexp) – we determine that \( \text{pos} \in \text{DefAssVar} \) and \( \text{adrOfVexp}(\text{fr}_i, \text{vexp}) = \text{valadr} \), where \( \text{vexp} \) is the expression at \( \text{pos} \).

By \( \text{isAddressPos}(\text{pos}) \) and Definition 3.5.5, we have \( \text{ST}(\text{pos}) \in \text{Struct} \). It then follows \( \text{ST}(\text{pos}) = T \) by Definition 3.2.1, \( \text{ST}(\text{pos}) \preceq T \) (by the type constraints), and the fact that the \( \text{ST}(\text{pos}) \)’s supertypes that are reference types, i.e., \( \text{System}.\text{ValueType}, \text{object} \) and possible interface types, do not declare instance fields.

By \( \text{ST}(\text{pos}) \in \text{Struct} \) and \( \text{pos} \in \text{DefAssVar} \), we derive that \( \text{up}(\text{pos}) \in \text{DefAssVar} \). We also have \( \text{up}(\text{pos}) \in \text{Vexp} \) by Lemma 3.6.11. Let \( \text{vexp} \) be the variable expression at \( \text{up}(\text{pos}) \).

We can now apply the soundness of the definite assignment analysis. Thus, by [definite assignment analysis] applied for \( \text{up}(\text{pos}) \) and (3.1), we have

\[
\text{memVal}(\text{adrOfVexp}(\text{fr}_i, \text{vexp}), \text{ST}(\text{up}(\text{pos}))) \neq \text{undef} \quad (3.3)
\]

According to Definition 3.6.6, we have

\[
\text{adrOfVexp}(\text{fr}_i, \text{vexp}) = \text{fieldAdr}(\text{adrOfVexp}(\text{fr}_i), \text{vexp}', \text{ST}(\text{pos}):F) = \text{fieldAdr}(\text{valadr}, \text{T}:F).
\]

By this, \( \text{ST}(\text{up}(\text{pos})) = \text{ fieldType}(\text{T}:F) \) (see the type constraints in Table 3.7) and (3.3), we derive

\[
\text{memVal}(\text{fieldAdr}(\text{valadr}, \text{T}:F), \text{fieldType}(\text{T}:F)) \neq \text{undef} \quad (3.4)
\]

By (3.2) and Lemma 3.6.4, we get

\[
\text{isAddressIn}(\text{fieldAdr}(\text{valadr}, \text{T}:F), \text{fieldType}(\text{T}:F), \text{locAdr}(\text{fr}_j)(\text{loc}), \text{locType}(\text{meth} (\text{fr}_j), \text{loc}))
\]

By this, (3.4) and the induction hypothesis (loc), it follows that the invariant (val) is satisfied in the next step of the operational semantics model. The proof of the other invariants is trivial. □
3.7 Related Work

The origin of type safety proofs goes back to the subject reduction theorem for typed $\lambda$-calculus (for details, see [101]), but starts in earnest with Milner’s slogan “Well-typed programs never go wrong” [94] in the context of ML. The first statically typed language proved to have a type-hole in its type system is Eiffel [37]. Since then, designers of object-oriented programming languages aimed to prove type safety of their languages (see, for example, the papers [30, 31]).

To the best of our knowledge, our work is the first type safety proof for a fragment of C$\#$.

Section 3.7.2 reports on formal specifications of the definite assignment analysis performed by the Java compiler. A brief evaluation of the ASMs applications in the area of programming languages is included in Section 3.7.3.

3.7.1 Java’s Type Safety

Closely related to our work is the considerable research for proving Java’s type safety [40, 118, 98, 41, 121, 77, 115]. An interested reader can find in [73] an excellent and comprehensive evaluation of this research topic. The most closely related, Stäkr et al. [115]’s type safety proof is the one that had the most impact on our work. Börger and Stäkr [28] provide a concrete comparison of the Java semantics model [115] and C$\#$ semantics model [24] and formulate in a precise technical manner where and in which respect the two languages differ among each other and from other programming languages – methodologically, semantically, and pragmatically.

C$\#$ introduces several new features with respect to Java, e.g., structs, call-by-reference, an unified type system, delegates. Due to the rich and unified type system, the C$\#$’s evaluation rules use the type information, and concerning this aspect, C$\#$ contrasts with most of the Java fragments analyzed so far, where the evaluation does not require any type information. Another crucial modelling difference with respect to Java is the “indirection through memory addresses”, needed for the call-by-reference mechanism. This has significantly contributed to the complexity of the C$\#$’s type safety proof: As one can see in Section 3.6, most of the definitions and lemmas required for Theorem 3.6.1 are concerning the handling of addresses. Moreover, the treatment of the addresses gets even trickier in the context of structs and the C$\#$’s unified type system, enabled by boxing and unboxing. On the other hand, as a noticeable difference between Java’s and C$\#$’s type safety proofs, dealing with the unified type system triggers introducing the special ”boxed types”.

3.7.2 The Definite Assignment Analysis

A definite assignment analysis is also performed by the Java compiler. This analysis has been formally specified in terms of data flow equations and type systems by Stäkr et al. [115] and Schirmer [105], respectively. The [115]’s data flow equations approach has inspired us the most. Stäkr et al. [115] have related the definite assignment analysis not only to the type safety proof, but also to the problem of generating verifiable JVM bytecode from legal Java source code programs. Schirmer [105] has carried out the formalization of the definite assignment analysis and the proofs in the theorem prover Isabelle/HOL [100].

The C$\#$ definite assignment analysis differs from the one of Java in many respects. Due to
the C\# `goto` statement, the C\#’s analysis gets tricky as it involves a fixed point iteration, which is not the case in Java. Therefore, Schirmer [105]’s approach using type systems is not a feasible way to prove the analysis’ soundness (Schirmer [105] himself recognizes this). Note that Java has a `break L;` statement, which has no corresponding statement in C\#. Its treatment is similar to the C\# `continue` statement, and so, introduces no complexity. Moreover, the handling of the C\# struct type variables represents another crucial difference with respect to Java’s analysis. As we have seen, determining their definite assignment status requires considering the so-called ”expansion” function, which takes into account the status of the fields of the struct type variables. Another non trivial difference between C\# and Java with respect to the definite assignment analysis is given by the treatment of the C\# ”by reference” arguments: While a Java local variable can only get definitely assigned upon a direct assignment, a C\# local variable can also get definitely assigned following a method call with the variable as an `out` argument.

3.7.3 The ASM Method

An important advantage of using the ASM method is that the ASMs are appropriate to describe operational features of languages. Thus, the ASM method was successfully applied to define the dynamic semantics of many widely used languages. Among these are C (see [71]), C++ (see [122]), C\# (see [24, 112, 55, 84]), Java (see [115]), Occam (see [72, 23]), Oberon (see [90]), Prolog (see [25]), Smalltalk (see [95]).

For work centered around Java, [18] contains a collection of formal-method-approaches to language specification and analysis. Hartel and Moreau make an evaluation of the ASM-based Java investigations [73]. Given that a semantics model for C\# has only been defined by [24], we cannot perform here a similar evaluation.

If appropriately designed, the ASM models are flexible enough to be refined in order to cover other language features. For example, the C\# model [24] has been firstly extended by Stärk and Börger with a thread and memory model [112], and then by Jula with the main new features of C\# (v2.0), in particular generics, anonymous methods, and iterators [83].

Another strength of the ASM models is that they can be validated and verified against the real systems they model. A purely theoretical specification often contains subtle errors, which are sometimes very hard to discover. An implementation of the theoretical specification can reveal such errors, by showing a misbehavior during its execution. Thus, in [84], we have refined the C\# model to .NET executable AsmL code [49], somehow similarly to the AsmGofer refinement developed by Schmid [106, 107] for the Java model [115].

3.8 Summary and Future Work

We proved type safety of a large fragment of C\# which includes delegates, call-by-reference, structs, boxing, and unboxing. To our knowledge, the treatment of these features seems to be new. To prove type safety, we have specified the type constraints that should be satisfied by a method accepted by the C\# compiler. Further, we have formally defined and proved the soundness of the definite assignment analysis, a crucial ingredient for C\#’s type safety. We have then identified and formalized the invariants expressing the type safety result. Finally, we proved that the run-time execution, according to the operational semantics model [24], of legal
C♯ programs accepted by the definite assignment analysis and obeying the type constraints preserves the invariants.

As future work, the language C♯ can be extended with additional features of C♯, e.g., statics and class initialization, asynchronous exceptions\textsuperscript{13} (e.g., `OutOfMemoryException`, `StackOverflowException`), generics, access modifiers, without major complications, though this would trigger a substantial expansion in the length of the proof. Future work also includes the specification and verification of our type safety result in the theorem prover Isabelle/HOL [100].

\textsuperscript{13}So far, we have formally defined the semantics of the class initialization and of the asynchronous exceptions [58].
In this chapter, we present a formal proof of CLR’s type safety. For this, we begin with specifying the static and dynamic semantics of the CLR bytecode language by providing an abstract interpreter, in terms of an ASM model, which executes arbitrary bytecode programs. We then formally define the structural and type consistency checks performed by the bytecode verification. On top of the type checks, we provide a definition of well-typed methods and of the bytecode verification algorithm. We then prove type safety of well-typed methods, i.e., the execution according to the semantics models of legal and well-typed methods does not lead to any run-time type violations and leaves the program in a good state, where certain structural constraints hold. Finally, to prove CLR’s type safety, we show that the verification algorithm is sound, i.e., the methods accepted by the verification are well-typed, and complete, i.e., the well-typed methods are accepted by the verification.

**Bytecode language decomposition** To keep the size of the semantics model manageable and thus to simplify the development and exposition of the type safety proof, we structure the CLR into three layered modules of orthogonal language features, namely a lightweight CLR (with object classes, value classes, pointers, typed references, inheritance, objects, delegates, dynamic dispatched method calls, tail calls, boxing, unboxing), exception handling, and generics. This decomposition yields a sequence of bytecode sublanguages, which altogether describe almost the entire bytecode language. Each sublanguage extends its predecessor, and for each one’s semantics, we define an ASM model which is a conservative (purely incremental) extension of its predecessor. Thus, for the lightweight CLR, we build the model CLR$_E$, which is then refined (in the sense of ASM refinements [22]) by CLR$_G$ to cover the exception handling. In turn, CLR$_E$ is refined through CLR$_G$ to deal with generic types and generic methods. The verification type systems and type consistency checks performed by the bytecode verification are defined by stepwise refinement, similarly to the layering of the semantics models. For each sublanguage, we develop a type safety proof.

**Features omitted from CLR** The focus of this chapter is not a full model of the CLR that conforms exactly to the CLR specifications [45] down to every technical detail. Such a model
has obviously its merits, but as in every formal development, a balance between abstraction and detail should be determined. As the goal of this thesis is CLR’s type safety, besides omitting the unverifiable code, the analysis presented here also abstracts as far as possible from everything that does not concern type safety. The bytecode language that we consider does not include static members, arrays, and threads. However, we still consider four static methods: entrypoint, i.e., the method that is executed when the code is first run, and, for handling delegates, three static methods declared by the class System.Delegate [5]. The omitted features only contribute to the complexity of the system through the number of additional cases they introduce, but they do not introduce any challenging problems.

Chapter outline  The rest of this chapter is organized as follows. Section 4.1 specifies the semantics and bytecode verification of the lightweight CLR and proves its type safety. Section 4.2 extends the analysis of the lightweight CLR with exception handling. Finally, the generics are added in Section 4.3, and the resulting bytecode language is showed to be type-safe. The related work is discussed in Section 4.4. Section 4.5 concludes and gives directions for future work.
4.1 The Lightweight CLR

This section analyzes the lightweight CLR and proves its type safety. An informal overview of the lightweight CLR and its type system is provided in Section 4.1.1. Sections 4.1.2 and 4.1.3 define the static and dynamic semantics of the lightweight bytecode language. Bytecode verification is described and formally defined in Section 4.1.4. The bytecode verification algorithm and the definition of well-typed methods are given in Section 4.1.5. Section 4.1.6 introduces the typing rules and proves type safety of the lightweight CLR.

4.1.1 An Overview of the Lightweight CLR and its Type System

The CLR is an abstract stack-based machine, with automatic memory management, for CIL bytecode programs. It encompasses a managed heap, for storing objects, and a stack for method calls, for capturing information, in the form of frames, about the still active methods.

A new frame is put on the frame stack every time a method is called. Such a frame comprises a program counter, any local variables, a local memory pool (for dynamic allocations), the arguments passed to the method (if any), and an evaluation stack (for intermediate results of the method’s computations).

A method body is a list of bytecode instructions. The instructions manipulate the heap, the local variables, the arguments, and the evaluation stack. Mainly, there are three kinds of instructions: instructions that load values onto the evaluation stack, instructions that store values from the evaluation stack into memory locations, and instructions for method calls.

In the rest of this section, we give a detailed informal description of the CLR type system.

4.1.1.1 Reference Types

The CLR supports reference types such as object classes, interfaces, pointers, and delegates. A value of a reference type is always allocated on the managed heap.

Object classes An object class is a reference type of a “self-describing” value, i.e., a value whose representation and applicable operations are unambiguously defined. The object classes, supporting for example the C♯, J♯, or Smalltalk classes, can declare instance fields and instance methods (virtual and non-virtual). A value of an object class, also known as an object class instance, can be referred to by object references and can be created with a constructor. The constructors are special methods, named .ctor.

Interfaces Unlike the object classes which fully describe their values, interfaces are reference types that are only a partial description. For this reason, an interface cannot declare fields and methods for the values of the interface, i.e., it cannot define instance fields and non-virtual instance methods. However, an interface can declare (abstract) virtual methods.

As there are .NET compliant languages (e.g., Scheme [43], Haskell [14], Mercury [11, 38]) where recursion is the only way to express repetition, the CLR needs to support tail method calls. When executing a tail call, the CLR discards the caller’s frame prior making the call. This means, for example, that a method which calls itself tail can implement a desired repetition.
Regarding type safety, the tail calls require attention because of two main reasons. Firstly, because their execution results in situations when the method invoked tail does not return to the (in the meantime discarded) frame that contained the tail call, but to another frame. Secondly, because although a call is marked tail, it could happen that the tail prefix is ignored when the bytecode is jitted, and consequently the call would be interpreted as a non-tail call. The reason for that is because the JIT compiler views this prefix as an optional directive. However, it should be ensured that the call is type safe, regardless of whether the call has been interpreted as tail or not.

Pointers There exists two kinds of pointers: managed pointers and method pointers. The CLR also supports unmanaged pointers. However, since their use is deemed unverifiable, this thesis does not consider them.

A managed pointer is a reference to a managed memory block that has been allocated from the CLR’s memory heap. Unlike a reference to an object class instance, a managed pointer can point to the interior of the instance (e.g., to an instance field), rather than to point to the ”entry” of the instance. Usually, managed pointers are generated for method arguments that are passed "by-reference", e.g., the C\^\# ref/out parameters. The CLR provides two type-safe operations on managed pointer types: loading a value from (operation known as “indirect load”) and writing a value (operation known as “indirect store”) to the address referenced by a pointer.

A key ingredient in the type safety proof, identifying the typing rules for pointers is fairly complex, somehow comparable with the notion of valid address in the C\^\#’s type safety proof. To type pointers, the notion of ”is an address in” introduced in Chapter 3 is reconsidered. Also, ensuring type-safe tail calls can be problematic especially in the presence of pointers, as the tail calls’ execution can easily generate dangling pointers.

A method pointer is a pointer to a method entry point. It reliably identifies the method assigned to the pointer. Thus, if one knows the method entry point, one can determine, for example, the method signature, the return type\(^1\), the local variable types. Therefore, one can see a method pointer as a method identifier. This identifier can be regarded as pointing to an address or directly to the corresponding method reference. The only operations on method pointers are the call of a method via the corresponding method pointer and the creation of delegates. As the bytecode verification does not track method pointer types, the first operation, also known as indirect call, is regarded as unverifiable by [45] and therefore not considered for our analysis.

Delegate classes Similarly as in C\^\#, the delegates were built with the idea of being the verifiable method pointers. Informally, a delegate wraps a list of method pointers called invocation list. Exactly as in C\^\#, the methods pointed to in the invocation list can be invoked sequentially, in the order in which they appear through a delegate call. More formally, a delegate is simply an instance of a delegate class. Figure 4.1 shows the general form of a delegate class. A delegate class is an object class whose definition obeys certain conditions:

- It should be declared as a sealed subclass of \texttt{System.Delegate}.
- It should declare exactly two members: a \texttt{ctor} and a method \texttt{Invoke}. As the implementations of these two methods are provided by the run-time environment, the methods should have empty bodies.

\(^1\)In CLR, the signature of a method reference does not include method return type.
4.1. THE LIGHTWEIGHT CLR

Figure 4.1 The general form of delegate classes.

```csharp
.class private auto ansi sealed DC extends System.Delegate
{
    .method public hidebysig specialname rtspecialname
        instance void .ctor(object o, native int m) runtime managed
    {
        // end of method DC::.ctor
    }
    .method public hidebysig
        instance T Invoke(T1 x1, ..., Tn xn) runtime managed
    {
        // end of method DC::Invoke
    }
} // end of class DC
```

- The `.ctor` should have exactly two parameters: the first of type `object` and the second of type `native int` (see Section 4.1.1.2 for details on this type). When the constructor is invoked, the first argument – known as the `target object` – is an instance of the class or one of its subclasses that declares the target method, and the second argument is a method pointer to the method – known as the `target method` – to be invoked. The second parameter of the `.ctor` should be of type `native int`. This is so since the method pointers are regarded as values of the type `native int`.

- The `Invoke` method should have the signature `compatible` with the target method.

Similarly as in C♯, each call to an `Invoke` method of a delegate class is turned into a synchronous call of the methods pointed to in the invocation list. The fact that `Invoke` and the target method do not necessarily have the same signature might be problematic in ensuring type safety, and therefore Definition 4.1.8 of compatible signatures requires attention. By only allowing two methods, the delegates can, on one hand, be easier controlled. Because the target object and the target method are passed to the delegate constructor, there exists the information to ensure that run-time (type) errors caused by mismatched parameters and return types do not occur. On the other hand, specifying the delegates semantics is not an easy task because of their “opaque” implementation: Both the creation and invocation of a delegate are accomplished through methods whose bodies, assumed to be empty, are automatically generated by the run-time system. Concerning the creation of a delegate instance, there are also verification rules which bound to a certain bytecode sequence.

4.1.1.2 Value Types

The `value types` supported by the CLR are the primitive types and the value classes. Unlike the values of reference types, the values of a value type are usually allocated “in place”, i.e., on the evaluation stack. A value type can also be allocated on the heap, but only within (e.g., as a field of) an object class instance.

**Primitive types** We consider only the following primitive types: `int32`, `int64`, `float64`, and `native int`. The type `native int` is mapped at run-time to the natural size of the specific architecture: `int32` on a 32-bits architecture and `int64` on a 64-bits architecture. It becomes efficient when the target machine architecture is not known until run-time.
Figure 4.2 Example: typed references and methods with variable number of arguments.

```csharp
// the __arglist keyword represents
// a method with a variable number of arguments
public static void WriteAtConsole(__arglist)
{
    // an ArgIterator object is used to loop through the arguments
    ArgIterator iterator = new ArgIterator(__arglist);

    // each item in the ArgIterator object is a typed reference,
    // which can be converted to an object using the
    // static method TypedReference.ToObject()
    while (iterator.GetRemainingCount() > 0)
    {
        TypedReference tr = iterator.GetNextArg();
        Console.WriteLine(TypedReference.ToObject(tr));
    }
}

// the Main method calls the method with
// a variable number of arguments
public static void Main()
{
    // define arguments of different types
    double z = 2.5;
    int x = 31;
    string y = "hello";

    // call the variable argument method, passing
    // the arguments in, using the __arglist keyword
    WriteAtConsole(__arglist(x, y, z));
}
```

Value classes The value classes are the CLR support not only for the C♯ structs, but also for C++ structs, Pascal and Modula 2 record types. A value class is a value type whose values are “self-contained” values. More exactly, these values – also known as instances of the value class – are represented as mappings assigning values to the instance fields of the value class. Every value class should have the library object class `System.ValueType` as the direct base class. In particular, it is not possible for a value class to extend another value class. Additionally, a value class can implement one or more interfaces. Similarly to an object class, a value class can declare instance fields. It can only declare non-virtual instance methods.

Just in the same manner as C♯, the CLR supports boxing to fill the gap between value types and reference types.

A non-virtual method receives a `this` pointer that is a managed pointer (see below the pointer types) to the (unboxed) value class. One fundamental difference between value classes and object classes is that the formers do not inherit behavior, and therefore cannot declare virtual methods. However, virtual methods defined by `System.ValueType` and by the interfaces implemented by a value class (if any) can be called on that value class instances, but only if the instances are boxed. This makes sense since the value types have identity only when boxed.

`System.TypedReference` is a special value class in the .NET Base Class Library.
4.1. The Lightweight CLR

<table>
<thead>
<tr>
<th>Universe of types</th>
<th>Typical use</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type</strong> = RefType ∪ ValueType</td>
<td><strong>T, T', T'', T'''</strong></td>
</tr>
<tr>
<td><strong>RefType</strong> = ObjClass ∪ Interface ∪ PointerType</td>
<td>–</td>
</tr>
<tr>
<td><strong>ValueType</strong> = ValueClass ∪ PrimitiveType</td>
<td>–</td>
</tr>
<tr>
<td><strong>ObjClass</strong></td>
<td><strong>OC, OC', OC''</strong></td>
</tr>
<tr>
<td><strong>DelegateClass</strong> ⊆ ObjClass</td>
<td><strong>DC</strong></td>
</tr>
<tr>
<td><strong>Interface</strong></td>
<td><strong>I</strong></td>
</tr>
<tr>
<td><strong>PointerType</strong> = {T &amp;</td>
<td>T ∈ ReferentType}</td>
</tr>
<tr>
<td><strong>ReferentType</strong> = ObjClass ∪ Interface ∪ ValueType</td>
<td><strong>VC</strong></td>
</tr>
<tr>
<td><strong>ValueClass</strong></td>
<td>–</td>
</tr>
<tr>
<td><strong>PrimitiveType</strong> = {int32, int64, native int, float64}</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 4.1: CIL’s types.

This class instances are known as typed references. They enable the CLR to provide C++-style support for methods that have a variable number of arguments. Thus, they can be used to represent method arguments in variable argument lists. In general, the typed references support languages that require “by-reference” passing of unboxed values to methods that are not statically restricted as to the type of values they accept. Therefore, a typed reference is regarded as an opaque descriptor of a pointer and a type.

**Example 4.1.1** Figure 4.2 lists a C# example to illustrate the use of typed references. Each of the arguments x, y, and z of the method WriteAtConsole is “packed” into a typed reference. As the declaration of WriteAtConsole does not have a list of parameter types (so, it can have a variable number of arguments), the types of its arguments are determined from the corresponding typed references. For example, the value class System.TypedReference defines a method GetTargetType that “extracts” the type information out of a typed reference.

### 4.1.2 The Static Semantics

The execution environment in which every method runs consists of the type hierarchy and the field and method declarations. Table 4.1 defines the classification of the CIL’s types described in Section 4.1.1. We also denote by Class the set of object classes and value classes:

\[
\text{Class} = \text{ObjClass} \cup \text{ValueClass}
\]

We assume that the classes and interfaces are organized by the execution environment into an inheritance hierarchy, on which the subtype relation \(\preceq\) defined in Definition 4.1.1 depends.

**Definition 4.1.1 (Subtype relation)** The subtype relation \(\preceq\) is the least reflexive and transitive relation such that

1. if \(T \in \text{Class} \cup \text{Interface} \) and \(T'\) is object, or
2. if \(T \in \text{ValueType} \) and \(T'\) is System.ValueType, or
Table 4.2: CIL’s values.

<table>
<thead>
<tr>
<th>Universe</th>
<th>Typical use</th>
<th>Element name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Val</td>
<td>SimpleVal U ObjRef U Map(FRef, Val) U Adr U TypedRef U MethPtr</td>
<td>val, val' – ref, ref', ref'' adr, adr', adr'' tr mp</td>
</tr>
</tbody>
</table>

Table 4.3: Environment specific universes.

- if \( T \in \text{ObjClass} \) and \( T' \) is a base class of \( T \) or an interface implemented by \( T \), or
- if \( T \in \text{Interface} \) and \( T' \) is an interface implemented by \( T \), or
- if \( T \in \text{ValueType} \) and \( T' \) is an interface implemented by \( T \),

then \( T \preceq T' \).

The possible kinds of values corresponding to the CIL’s types are gathered in Table 4.2. Thus, a value can be

- a simple value, element of the universe SimpleVal, i.e., a 32-bit integer\(^2\), a 64-bit integer, a native size integer, or a 64-bit floating point number, or
- an object reference, element of the universe ObjRef, or
- a value class instance, element of Map(FRef, Val), defined as a mapping assigning values to the instance fields of the value class, or
- a pointer (referencing an address), element of the universe Adr, or
- a typed reference, element of TypedRef, or

\(^2\)The type bool is omitted in our approach as booleans are treated as 32-bit integers: 0 is used for False, and 1 for True.
4.1. THE LIGHTWEIGHT CLR

- a method pointer, element of \textit{MethPtr}.

Each class declares types for a set of instance fields, and each class and interface specifies signatures and return types for a set of instance methods. Every method consists of a list of bytecode instructions. Formally, we consider the universes in Table 4.3 and the environment components in Table 4.4 defined as basic static functions.

The list of instructions of a method body is stored in \textit{code}. Thus, \textit{code}(mref)(pos) gives the instruction with the code index \textit{pos} of the method \textit{mref}. The abstract bytecode instructions of the bytecode language that we consider are given in Table 4.8. They are constructors for real CIL instructions: One abstract instruction models one or more real instructions.

If omitted in a static or dynamic function, the method reference is considered to be the method of the currently executed frame. For example, \textit{code}(pos) denotes the instruction with code index \textit{pos} in the current method.

The local variable types are \textit{declared} and \textit{fixed}. They are maintained in the local signature \textit{locTypes}. In the verifiable code, the local variables are assumed to be initialized to default values according to their type. The goal is to simplify the bytecode verification. Due to this decision, the bytecode verification does not need to perform a definite assignment analysis as the C\# compiler does – see Chapter 3, Section 3.4).

For every method, the function \textit{paramTypes} yields the list of parameter types given in the method signature. The list returned by \textit{paramTypes} does not include the type of the \textit{this} pointer used for instance methods. Therefore, we consider the derived function \textit{argTypes} : \textit{Map}(\textit{MRef}, \textit{List}(\textit{Type})) which also includes the type of the \textit{this} pointer\(^4\). For a method declared by an object class (or interface), the \textit{this} pointer is of the class (or interface) type, whereas if the class of the method is a value class, the \textit{this} pointer is a pointer to an instance of the value class.

\textbf{Definition 4.1.2} For a method \textit{T::M}, we define

\[
\text{argTypes}(T::M) = \begin{cases} 
[T] \cdot \text{paramTypes}(T::M), & \text{if } T \in \text{ObjClass} \cup \text{Interface}; \\
[T\&] \cdot \text{paramTypes}(T::M), & \text{if } T \in \text{ValueClass}.
\end{cases}
\]

We denote by \textit{locNo} : \textit{Map}(\textit{MRef}, \mathbb{N}) and \textit{argNo} : \textit{Map}(\textit{MRef}, \mathbb{N}) the length of \textit{locTypes} and \textit{argTypes}, respectively. The return type, the class name, and the name of a method reference are selected by \textit{retType}, \textit{classNm}, and \textit{methNm}, respectively. The set of the fields of a class (including the inherited fields) is maintained in \textit{instFields}. The declared type of any instance field is recorded by \textit{fieldType}.

Besides the execution environment, the static semantics also comprises the universes in Table 4.5 and the basic static functions in Table 4.6.

\(^3\)As the use of a method pointer is very limited, we see the method pointers as elements of the universe \textit{MethPtr} that point to the corresponding method references.

\(^4\)Every time our model exceptionally considers a static method, we make an exception and consider \textit{argTypes} to coincide with \textit{paramTypes}.\)
The values carry their associated CLR types. Any value can be viewed as a pair consisting of the value and a tag representing its CLR type (and not the type tracked by the bytecode verification). The set of tags is denoted by TagType and is defined as follows:

\[
\text{TagType} = \text{PrimitiveType} \cup \text{ValueClass} \cup \{\text{ref}, \&, \text{typedref}\}
\]

Given a value, the function tagType yields the tag type associated to the value.

The stack frames, also known as activation records, are defined as elements of the universe Frame. Every frame is given as a 6-tuple comprising a program counter, local variable addresses, addresses allocated in the local memory pool, argument addresses, an evaluation stack, and a method reference.

\[
\text{Frame} = \text{Pc} \times \text{Map} (\text{Local, Adr}) \times \mathcal{P} (\text{Adr} \times \text{ValueClass}) \times \text{Map} (\text{Arg, Adr}) \times \text{List} (\text{Val}) \times \text{MRef}
\]

Bytecode verification has to ensure that all objects are properly initialized before they are used. To make object initialization accessible to the type safety proof, the semantics model has to be designed in such a way that programs which do not properly initialize objects can be identified. In other words, the run-time representation of partially initialized objects must be distinguishable from that of (fully) initialized objects. For this purpose, we consider the universe InitState. The initialization status and the run-time type of object references are recorded.
as elements of \( \text{InitState} \). An object reference whose run-time type is an object class can be either partially initialized, i.e., its initialization is in progress, or initialized. An object reference whose run-time is a value type can only be initialized.

\[
\text{InitState} = \begin{cases} 
\text{InProgress} (\text{ObjClass}) & \text{initialization in progress} \\
\text{Init} (\text{ObjClass} \cup \text{ValueType}) & \text{initialization done}
\end{cases}
\]

Given an operator, the function \( \text{opNo} \) returns the number of operands necessary to perform the operation indicated by the operator. The resulting value of an operation and the resulting verification type can be determined through the functions \( \text{opResVal} \) and \( \text{opResType} \), respectively. While \( \text{opResVal} \) is a “semantics” function, \( \text{opResType} \) is defined by [45, Partition III, §1.5].

The \text{RefAnyType} instruction applied with a type token argument \( T \) requires a handle, i.e., an opaque descriptor, associated to \( T \). For a type \( T \), we assume that \( \text{handleOf}(T) \) returns a handle, i.e., an instance of the special value class \text{System.RuntimeTypeHandle} [7], corresponding to \( T \).

The below defined universe \text{Switch} depicts the possible execution modes of the operational semantics model CLR\(_E\) defined in Section 4.1.3.

\[
\text{Switch} = \begin{cases} 
\text{Noswitch} & \text{normal execution mode} \\
\text{Invoke} (\text{Bool}, \text{MRef}, \text{List} (\text{Val})) & \text{method invocation mode} \\
\text{Result} (\text{List} (\text{Val})) & \text{method returning mode}
\end{cases}
\]

The model is either in the normal execution mode, i.e., it executes an instruction, or in the method invocation mode, i.e., it invokes a method with a given list of arguments, or in the method returning mode, i.e., it returns from a method possibly with a return value.

Similarly as for the C\#’s static semantics, we define two derived static functions \( \text{defVal} : \text{Map}(\text{RefType} \cup \text{ValueType}, \text{Val}) \) and \( \text{lookUp} : \text{Map}(\text{ObjClass} \cup \text{ValueType}) \times \text{MRef} \rightarrow \text{MRef} \).

Given a type \( T \), \( \text{defVal}(T) \) is the default value, also known as ”zero”, associated to the type \( T \). Note that the default value of a value class is the result of initializing each instance field of the class to its default value.

**Definition 4.1.3 (Default value)** For a type \( T \in \text{RefType} \cup \text{ValueType} \), we define
\[
\text{defVal}(T) = \begin{cases} 
0, & \text{if } T \in \text{PrimitiveType} \setminus \{\text{float64}\}; \\
0.0d, & \text{if } T = \text{float64}; \\
\text{null}, & \text{if } T \in \text{RefType}; \\
\{T::F \mapsto \text{defVal(fieldType}(T::F)) | T::F \in \text{instFields}(T)\}, & \text{if } T \in \text{ValueClass}. 
\end{cases}
\]

To resolve dynamic dispatched method calls, we consider the function \(\text{lookUp}\), defined similarly as for C\#\textsuperscript{♯}. We use the same terminology as in Chapter 3: The method argument of a virtual call is called, whereas the method selected for the run-time execution is invoked. For non-virtual call cases, we use the two notions interchangeably.

**Definition 4.1.4 (Method lookup)** For a type \(T\) and a method \(T'::M\), we define

\[
\text{lookUp}(T, T'::M) = \begin{cases} 
\text{if } T \text{ declares a non-abstract method } M \land \\
\left((T = T') \lor (T' \in \text{ObjClass} \Rightarrow T::M \text{ overrides } T'::M)\right) \land \\
(T' \in \text{Interface} \Rightarrow T::M \text{ implements } T'::M) \\
\text{then } T::M \\
\text{elseif } T = \text{object} \text{ then } \text{undef} \\
\text{else } \text{lookUp}(T'', T'::M) \text{ where } T'' \text{ is the direct base class of } T
\end{cases}
\]

The conditions [override/implement], defined for C\#\textsuperscript{♯} in Chapter 3, Section 3.5.1, should also be satisfied in the CLR:

**[override/implement]** If \(T::M \in \text{MRef}\) overrides/implements \(T'::M \in \text{MRef}\), then:

\(\textbf{(at)}\) argNo\((T::M) = \argNo(T'::M)\) and for every \(i = 1, \argNo(T::M) - 1,\)

\[
\argTypes(T::M)(i) = \argTypes(T'::M)(i)
\]

\(\textbf{(rt)}\) retType\((T::M) = \text{retType}(T'::M)\)

The instructions dealing directly with pointers, \(i.e., \text{LoadInd} \text{ and } \text{StoreInd}\), require the addresses referenced by the pointers to be valid. An address is a valid address if it not null, it is in the range of \(\text{Adr}\), and it is "naturally aligned" for the target architecture. To distinguish between invalid addresses and valid addresses, we define the external function validAdr : \(\text{Map(Adr, Bool)}\) which, given an address, indicates the validity of the address. This function is external as its definition depends on the target architecture. However, the addresses of local variables, arguments, fields and value type instances inside boxed objects are regarded as valid.

**Delegate classes** As the two methods declared by a delegate class are not allowed to have a body, the semantics model should treat them specially. We proceed as follows. Every delegate
4.1. THE LIGHTWEIGHT CLR

ctor is implemented via ASM rules. Similarly as for C♯, every delegate’s Invoke method is provided with the body, resulted upon compiling the corresponding C♯ Invoke method, that expresses the sequentiality of the execution of delegate invocation list elements. To implement that body, we insert two more methods, implemented via ASM rules, in every delegate class. The method _length is supposed to return the length of the delegate invocation list, while the method _invoke is used to invoke a certain method in the delegate invocation list. Thus, every delegate class is translated as shown in Figure 4.3 (for an Invoke method with a non-void return type).

So, basically, our model considers the delegates implemented, in particular created, through ASM rules because their CLR implementation is not transparent. This modelling decision has an impact on the members declared by System.Delegate, whose implementation relies on the delegates implementation. Therefore, the implementation of these members should be accomplished via ASM rules, too. A full specification of the CLR would consist of ASM rules for each member declared by System.Delegate. As such an approach would significantly blow up the size of the exposition, we consider that System.Delegate declares only three members, i.e., the static methods Combine, Remove, and op_Equality.

4.1.3 The Dynamic Semantics

The dynamic state of the machine CLR_L is given by the basic dynamic functions of Table 4.7.

The function mem is applied to read and to store values other than value class instances from and into the memory, respectively. This function can return an undefined value. The stack of call frames frameStack is defined as a list of frames. A call frame comprises a program counter pc, local variable addresses locAdr, local memory pool locPool, argument addresses argAdr, an evaluation stack evalStack, and a method reference meth.

The local memory pool locPool is a (possibly empty) set that consists of pairs of the form (adr, T), where adr is the address of a memory block allocated for an instance of a value class T. In the lightweight CLR, an address adr is considered in locPool in a single case: if the address is allocated for a value class instance created upon a constructor invocation (see the semantics of the NewObj instruction in Section 4.1.3.1). The memory allocated in the local memory pool is reclaimed upon method context termination. Modelling the local memory pool is an important contribution of our formal specification. The reason behind modelling this pool concerns the type safety proof, more precisely, the typing of pointers (see Remark 4.1.10 in Section 4.1.6).

The derived nullary function frame : Frame denotes the currently executed frame. It is determined as a 6-tuple consisting of pc, locAdr, locPool, argAdr, evalStack, and meth.

\[
\text{frame} = (pc, \text{locAdr}, \text{locPool}, \text{argAdr}, \text{evalStack}, \text{meth})
\]

Accordingly, pc gives the program counter of the current frame, locAdr the local variable addresses of the current frame, etc. To simplify the technical presentation, we separate the current frame from the stack of call frames, i.e., frame is not contained in frameStack.

The function fieldAdr is defined analogously as for C♯. It assigns to every instance field of a reference or of a value class instance stored at a given address its allocated address. More

\footnote{Upon adding generics in Section 4.3, there will be another case.}
Figure 4.3 The translation scheme of delegate classes.

```csharp
.class private auto ansi sealed DC extends System.Delegate
{
    .method public hidebysig specialname rtspecialname
        instance void .ctor(object o, native int m) runtime managed
    {
    } // end of method DC::.ctor
    .method public hidebysig instance T Invoke(T x1,...,T n x n) runtime managed
    {
    } // end of method DC::Invoke
} // end of class DC

↓

.class private auto ansi sealed DC extends System.Delegate
{
    .method public hidebysig specialname rtspecialname
        instance void .ctor(object o, native int m) runtime managed
    {
    } // end of method DC::.ctor
    .method public hidebysig instance T Invoke(T x1,...,T n x n) cil managed
    {
        .maxstack n+2
        .locals init (T V_0, int32 V_1)
        0:          br n+9
        1:          ldarg 0 // load the delegate instance
        2:          ldloc 1 // load the current index in the invocation list
        3:          ldarg 1 // load the first argument of the delegate call
                      ...
        n+2:        ldarg n  // load the n-th argument of the delegate call
        n+3:        call instance T DC::__invoke(int32, T1,..., T n)
        n+4:        stloc 0
        n+5:        ldloc 1
        n+6:        ldc.i4 1
        n+7:        add
        n+8:        stloc 1
        n+9:        ldloc 1
        n+10:       ldarg 0
        n+11:       call instance int32 DC::__length()
        n+12:       clt
        n+13:       brtrue 1
        n+14:       ldloc 0
        n+15:       ret
    } // end of method DC::Invoke

    .method private instance int32 __length() runtime managed
    {
    } // end of method DC::__length

    .method private instance T __invoke(int32 i,T1 x1,...,T n x n) runtime managed
    {
    } // end of method DC::__invoke
}
```
4.1. THE LIGHTWEIGHT CLR

<table>
<thead>
<tr>
<th>Function definition</th>
<th>Function name</th>
</tr>
</thead>
<tbody>
<tr>
<td>mem : (\text{Map}(\text{Adr}, \text{Val} \cup {\text{undef}}))</td>
<td>memory function</td>
</tr>
<tr>
<td>frameStack : (\text{List}(\text{Frame}))</td>
<td>stack of call frames</td>
</tr>
<tr>
<td>pc : (\text{Pc})</td>
<td>current program counter</td>
</tr>
<tr>
<td>locAdr : (\text{Map}(\text{Local}, \text{Adr}))</td>
<td>local variable addresses</td>
</tr>
<tr>
<td>locPool : (\mathcal{P}(\text{Adr} \times \text{ValueClass}))</td>
<td>local memory pool</td>
</tr>
<tr>
<td>argAdr : (\text{Map}(\text{Arg}, \text{Adr}))</td>
<td>argument addresses</td>
</tr>
<tr>
<td>evalStack : (\text{List} (\text{Val}))</td>
<td>evaluation stack</td>
</tr>
<tr>
<td>meth : (\text{MRef})</td>
<td>current method</td>
</tr>
<tr>
<td>fieldAdr : (\text{Map}(\text{ObjRef} \cup \text{Adr}) \times \text{FRef}, \text{Adr})</td>
<td>instance field addresses</td>
</tr>
<tr>
<td>initState : (\text{Map}(\text{ObjRef}, \text{InitState}))</td>
<td>initialization status</td>
</tr>
<tr>
<td>addressOf : (\text{Map}(\text{ObjRef}, \text{Adr}))</td>
<td>boxed value address</td>
</tr>
<tr>
<td>typedRefAdr : (\text{Map}(\text{TypedRef}, \text{Adr}))</td>
<td>typed reference address</td>
</tr>
<tr>
<td>typedRefType : (\text{Map}(\text{TypedRef}, \text{Type}))</td>
<td>typed reference type</td>
</tr>
<tr>
<td>methodOf : (\text{Map}(\text{MethPtr}, \text{MRef}))</td>
<td>method pointer method</td>
</tr>
<tr>
<td>invocationList : (\text{List}(\text{ObjRef} \times \text{MRef}))</td>
<td>invocation list of a delegate</td>
</tr>
<tr>
<td>switch : (\text{Switch})</td>
<td>current execution mode</td>
</tr>
</tbody>
</table>

Table 4.7: The basic dynamic functions.

precisely, the address of an instance field \(C::F\) of an object \(\text{ref}\) is given by \(\text{fieldAdr}(\text{ref}, C::F)\). Also, the address of an instance field \(C::F\) of a value class instance stored at an address \(\text{adr}\) is \(\text{fieldAdr}(\text{adr}, C::F)\).

The function \(\text{initState}\) records the run-time type and the initialization status of the object references. Given an object reference \(\text{ref}\), if \(\text{initState}(\text{ref}) = \text{InProgress}(T)\), where \(T\) is an object class, the meaning is that the run-time type of \(\text{ref}\) is \(T\), and the initialization of \(\text{ref}\) is in progress. If \(\text{initState}(\text{ref}) = \text{Init}(T)\), the run-time type of \(\text{ref}\) is \(T\), and \(\text{ref}\) is initialized. The values of value types can be “boxed” in the heap and then addressed by object references. Such a reference is always considered initialized. Formally, if \(\text{ref}\) is a boxed value of a value type \(T\), then it always holds \(\text{initState}(\text{ref}) = \text{Init}(T)\).

\[
\begin{align*}
\text{InitState} & = \text{InProgress}(\text{ObjClass}) & \text{initialization in progress} \\
& \quad | \text{Init}(\text{ObjClass} \cup \text{ValueType}) & \text{initialization done}
\end{align*}
\]

To simplify the exposition of the semantics rules, we define the derived (selector) function \(\text{actualTypeOf} : \text{Map}(\text{ObjRef}, \text{Type})\), which assigns to every object reference its run-time type. Given a reference \(\text{ref}\), \(\text{actualTypeOf}(\text{ref}) = T\) if there exists \(T \in \text{ObjClass} \cup \text{ValueType}\) such that \(\text{initState}(\text{ref}) = \text{InProgress}(T)\) or \(\text{initState}(\text{ref}) = \text{Init}(T)\).

A boxed value on the heap embeds the value type and a list of instance field addresses. When “unboxing” a “boxed” value, one needs its address on the heap. This address is maintained in \(\text{addressOf}\), and it is set when the value is boxed (see the semantics of the \text{Box} instruction in Section 4.1.3.1).

The function \(\text{typedRefAdr}\) returns the address embedded in a typed reference, while the type transmitted with a typed reference is recorded by the function \(\text{typedRefType}\). For a
method pointer, \textit{methodOf} returns the method reference pointed to by the pointer. Exactly as in the C\# model, the invocation list each delegate instance is equipped with upon its creation is maintained by \textit{invocationList}. As in C\#, each invocation list is a per delegate instance immutable. The function \textit{methodOf} is only accessed for initializing the \textit{invocationList} upon calling a delegate \texttt{.ctor}. 

The current execution mode of the semantics model defined in Section 4.1.3.1 is maintained by \texttt{switch}.

**Initial Constraints** For the initial state of CLR\_E, the following conditions are satisfied by the basic dynamic functions:

| mem(adr)    | = undef, for every \(adr \in \text{Adr}\) | fieldAdr = \(\emptyset\) |
| frameStack | = [ ] | initState = \(\emptyset\) |
| pc         | = 0 | addressOf = \(\emptyset\) |
| locAdr     | = \(\emptyset\) | typedRefAdr = \(\emptyset\) |
| locPool    | = \(\emptyset\) | typedRefType = \(\emptyset\) |
| argAdr     | = \(\emptyset\) | methodOf = \(\emptyset\) |
| evalStack  | = [ ] | invocationList = \(\emptyset\) |
| meth       | = \texttt{.entrypoint} | switch = \texttt{Noswitch} |

Similarly as in the C\# model, we consider \textit{memVal} : \(\text{Map(Adr} \times \text{Type}, \text{Val} \cup \{\text{undef}\})\) and \textit{WRITEMEM} to “read from” and to “write in” memory, respectively. Apart from substituting \texttt{Struct} by \textit{ValueClass}, the definitions are the same.

**Definition 4.1.5** For an address \(adr\) and a type \(T\), we define

\[
\text{memVal}(adr, T) =
\begin{cases} 
\text{if} \ T \in \text{ValueClass then} \\
\{T::F \mapsto \text{memVal(fieldAdr(adr, T::F), fieldType(T::F))} \mid T::F \in \text{instFields}(T)\} \\
\text{else} \ \text{mem}(adr)
\end{cases}
\]

**Definition 4.1.6** For an address \(adr\), a type \(T\) and a type \(val\), we define

\[
\text{WRITEMEM}(adr, T, val) \equiv
\begin{cases} 
\text{if} \ T \in \text{ValueClass then} \\
\text{forall} \ T::F \in \text{instFields}(T) \ \text{do} \\
\text{WRITEMEM(fieldAdr(adr, T::F), fieldType(T::F), val(T::F))} \\
\text{else} \ \text{mem}(adr) := \text{val}
\end{cases}
\]

Obviously, after storing a value \(val\) at an address of a memory block, sufficient to hold values of a type \(T\), the value read from the memory block is \(val\).

**Lemma 4.1.1** After \texttt{WRITEMEM(adr, T, val)} is executed, \textit{memVal}(adr, \(T\)) = \(val\).
4.1. The Lightweight CLR

Proof. By induction on the structure of the possibly value class type $T$. □

To determine the local variable values and argument values of the current frame, we define the derived functions $locVal : Map(Local, Value)$ and $argVal : Map(Arg, Value)$, respectively. The value of a local variable is determined as the value stored in the memory block at the local variable address, sufficient to hold values of the local variable declared type. Similarly, the value of an argument is determined in terms of the argument address and argument declared type.

\[
locVal(n) = \text{memVal}(\text{locAdr}(n), \text{locTypes}(n)), \text{ for every } n = 0, \text{locNo}(\text{meth}) - 1
\]

\[
argVal(n) = \text{memVal}(\text{argAdr}(n), \text{argTypes}(n)), \text{ for every } n = 0, \text{argNo}(\text{meth}) - 1
\]

4.1.3.1 The Semantics of the Bytecode Instructions

This section defines the effect of the instructions in Table 4.8 on the dynamic state of the virtual machine. The operational semantics model – given in terms of the ASM interpreter named CLR – executes the macro execScheme which is parameterized by the submachines EXECCLR and SWITCHCLR. This macro gives control to these submachines depending on the current execution mode switch of the machine CLR.

The macro execScheme is defined as follows. If switch is Noswitch, i.e., the machine is in the normal execution mode, then EXECCLR defined in Figures 4.4, 4.5, and 4.6 takes control and executes the current instruction code(pc). If switch is not Noswitch, i.e., the machine either invokes a method or returns from a method, execScheme gives control to the machine SWITCHCLR defined in Figure 4.7.

\[
\text{CLR}_E \equiv \text{execScheme}(\text{EXECCLR}, \text{SWITCHCLR})
\]

\[
\text{execScheme}(\text{EXECCLR}, \text{SWITCHCLR}) \equiv
\begin{align*}
& \text{if switch = Noswitch then EXECCLR(code(pc))} \\
& \text{else SWITCHCLR}
\end{align*}
\]

The Dup instruction duplicates the top element of the stack, while the Pop instruction removes the top element from the stack. The instruction Const($T$, lit) pushes the constant (literal) lit, of type $T$, onto the evalStack.

The instruction LoadLoc($n$) loads on the evalStack the value locVal($n$) of the $n$-th local variable of the current frame. The address locAdr($n$) of the $n$-th local variable is pushed onto the stack by the instruction LoadLocA($n$). The instruction StoreLoc($n$) stores with WRITEMEM the topmost value of evalStack at the variable address locAdr($n$).

The Execute instruction is a “constructor” (abstraction) of several real CLR instructions. The instruction models, for example, the generic (read: with no specified data type) instructions add, div, mul, and sub, but also instructions performing conversions such as
Instruction | Informal description
--- | ---
**Dup** | Duplicates the top value of the stack.
**Pop** | Removes the top element of the stack.
**Const(T, lit)** | Loads the literal *lit* of type *T* onto the stack.
**LoadLoc(n)** | Loads the *n*-th local variable onto the stack.
**StoreLoc(n)** | Pops the top stack element, and stores it in the *n*-th local variable.
**Execute(op)** | Executes the operation indicated by the operator *op*.
**Cond(op, target)** | Executes *op* and transfers control to *target* if *op* returns true.
**LoadArg(n)** | Loads the *n*-th argument onto the stack.
**StoreArg(n)** | Pops the top stack element and stores it in the *n*-th argument.
**Call(tail, T, mref)** | Invokes the method *mref* of return type *T*; the boolean *tail* indicates whether the call is tail.
**CallVirt(tail, T, mref)** | Calls the late bound method *mref* of return type *T* associated, at run-time, with an object; *tail* indicates if the call is tail.
**Return** | Returns from the current method.
**NewObj(mref)** | Creates a new object, and invokes the instance constructor *mref* with the object.
**LoadField(T, C::F)** | Loads the field *C::F*, of declared type *T*, onto the stack.
**StoreField(T, C::F)** | Stores the top stack element into the field *C::F*, of declared type *T*.
**LoadArgA(n)** | Loads the address of the *n*-th argument onto the stack.
**LoadFieldA(T, C::F)** | Loads the address of the field *C::F*, of declared type *T*, onto the stack.
**LoadInd(T)** | Loads value of type *T* indirect onto the stack.
**StoreInd(T)** | Stores value of type *T* indirect from the stack.
**Box(T)** | Converts value of type *T* to object reference (boxed value).
**Unbox(T)** | Converts boxed value of value type *T* to a pointer to its unboxed form.
**Unbox.Any(T)** | Converts boxed value to its unboxed form.
**MkRefAny(T)** | Loads a typed reference pointing to type *T* onto the stack.
**RefAnyType** | Loads the handle to the type embedded in a typed reference.
**RefAnyVal(T)** | Loads the address out of a typed reference onto the stack.
**CastClass(T)** | Attempts to cast the top stack element to object class or interface *T*.
**IsInstance(T)** | Tests if the top stack element is an instance of the object class or interface *T*.
**LoadFtn(mref)** | Loads the (method) pointer associated to the method *mref*.
**LoadVirtFtn(mref)** | Loads the (method) pointer associated to the virtual method *mref*.

Table 4.8: The considered CIL instructions.

*conv.i4, conv.i8, and conv.r8.* The execution of the operation underlying a generic instruction requires the operands’ types. This is one reason why the values are carrying their types. Given an operator *op*, the instruction *Execute(op)* pops a number *opNo(op)* of operands from the *evalStack*. If the operation does not throw an exception, the result of the operation is pushed onto the stack. The resulting value is obtained with *opResVal*. The below defined predicate *exceptionCase* captures the cases when an exception is thrown. These are the following:

- division by zero for operators of integral types;
- operations that perform an overflow check and whose results cannot be represented in the result type;
- values that are not “normal” numbers are checked for finiteness or division/remainder operations are executed for a minimal value of an integral type and −1.
exceptionCase(op, vals) \iff divByZeroCase(op, vals) \lor overflowCase(op, vals) \\
\lor invNrCase(op, vals)

\[
\text{divByZeroCase}(op, vals) \iff op \in \{\text{div, rem}\} \land vals(1) = 0 \\
\land \text{tagType}(vals(0)) \in \{\text{int32, int64, native int}\} \\
\land \text{tagType}(vals(1)) \in \{\text{int32, int64, native int}\}
\]

\[
\text{overflowCase}(op, vals) \iff op \in \{\text{add.ovf, conv.ovf, mul.ovf, sub.ovf}\} \\
\land \text{opResVal}(op, vals) < \text{MIN}([\text{opResType}(op, [\text{tagType}(vals(0)), \text{tagType}(vals(1))])]) \\
\land \text{opResVal}(op, vals) > \text{MAX}([\text{opResType}(op, [\text{tagType}(vals(0)), \text{tagType}(vals(1))])])
\]

\[
\text{invNrCase}(op, vals) \iff (vals(0) \in \{\text{NaN, +infinity, -infinity}\} \\
\land op = \text{ckfinite}) \lor (op \in \{\text{div, rem}\} \\
\land vals(0) = \text{MIN}(\text{tagType}(vals(0))) \land vals(1) = -1 \\
\land \text{tagType}(vals(0)) \in \{\text{int32, int64, native int}\} \\
\land \text{tagType}(vals(1)) \in \{\text{int32, int64, native int}\})
\]

**Example 4.1.2** In the following bytecode fragment, the instruction `Execute(div)` (the abstract instruction corresponding to the real CIL instruction `div`) computes the result of dividing the first local variable by the second local variable.

```
0: ldloc 0
1: ldloc 1
2: div
```

The “type” of the division depends on the operands’ types: For example, the 32-bit integer division by zero throws an exception, whereas the 64-bit floating division by zero returns the special value NaN (read: ”not a number”). Therefore, every value has assigned a tag type. In the above example, the types carried by the values of the first two local variables determine the exact “type” of division.

Similarly as `Execute`, the `Cond` instruction is a constructor of several real CIL instructions, e.g., conditional branch instructions such as `beq`, `bge`, and `blt`. Given an operator `op` and a code index target, `Cond(op, target)` pops a number `opNo(op)` of operands from the `evalStack`. The `pc` is set to `target` if `opResVal` yields true for the operator `op` and the popped operands, and otherwise, the `pc` is incremented by 1.

The value `argVal(n)` of the `n`-th argument of the current `frame` is pushed onto the stack by `LoadArg(n)`. The instruction `LoadArgA(n)` loads the address `argAdr(n)`. The instruction `StoreArg(n)` writes at `argAdr(n)` with WRITEMEM the topmost value of the `evalStack`.

The `Call` instruction can be used to call one of the three methods declared by the class `System.Delegate`, or one of the two methods added to every delegate class definition, or a regular method.
**Figure 4.4** The operational semantics rules.

\[
\begin{align*}
\text{EXECCLR}(\text{instr}) & \equiv \text{match instr} \\
\text{Dup} & \rightarrow \text{let } (\text{evalStack}', [\text{val}]) = \text{split(\text{evalStack}, 1)} \text{ in} \\
& \quad \text{evalStack} := \text{evalStack}' \cdot [\text{val}, \text{val}] \\
& \quad \text{pc} := \text{pc} + 1 \\
\text{Pop} & \rightarrow \text{let } \text{evalStack}' = \text{pop(\text{evalStack}) in} \\
& \quad \text{evalStack} := \text{evalStack}' \\
& \quad \text{pc} := \text{pc} + 1 \\
\text{Const}(\_, \text{lit}) & \rightarrow \text{evalStack} := \text{evalStack} \cdot \text{[lit]} \\
& \quad \text{pc} := \text{pc} + 1 \\
\text{LoadLoc} (\text{n}) & \rightarrow \text{evalStack} := \text{evalStack} \cdot \text{[locVal(n)]} \\
& \quad \text{pc} := \text{pc} + 1 \\
\text{StoreLoc} (\text{n}) & \rightarrow \text{let } (\text{evalStack}', [\text{val}]) = \text{split(\text{evalStack}, 1)} \text{ in} \\
& \quad \text{WRITEMEM(locAdr(n), locTypes(n), val)} \\
& \quad \text{evalStack} := \text{evalStack}' \\
& \quad \text{pc} := \text{pc} + 1 \\
\text{Execute(\text{op})} & \rightarrow \text{let } (\text{evalStack}', \text{vals}) = \text{split(\text{evalStack}, opNo(\text{op})) in} \\
& \quad \text{if } \neg \text{exceptionCase(op, vals)} \text{ then} \\
& \quad \quad \text{let } \text{val} = \text{opResVal(op, vals)} \text{ in} \\
& \quad \quad \text{evalStack} := \text{evalStack}' \cdot [\text{val}] \\
& \quad \quad \text{pc} := \text{pc} + 1 \\
\text{Cond(\text{op}, \text{target})} & \rightarrow \text{let } (\text{evalStack}', [\text{val}]) = \text{split(\text{evalStack}, 1)} \text{ in} \\
& \quad \text{WRITEMEM(argAdr(n), argTypes(n), val)} \\
& \quad \text{evalStack} := \text{evalStack}' \\
& \quad \text{pc} := \text{if } \text{opResVal(op, vals)} \text{ then } \text{target else } \text{pc} + 1 \\
\text{LoadArg(\text{n})} & \rightarrow \text{evalStack} := \text{evalStack} \cdot \text{[argVal(n)]} \\
& \quad \text{pc} := \text{pc} + 1 \\
\text{StoreArg(\text{n})} & \rightarrow \text{let } (\text{evalStack}', [\text{val}]) = \text{split(\text{evalStack}, 1)} \text{ in} \\
& \quad \text{WRITEMEM(locAdr(n), locTypes(n), val)} \\
& \quad \text{evalStack} := \text{evalStack}' \\
& \quad \text{pc} := \text{pc} + 1 \\
\text{Call(\text{tail}, \_, \text{T::M})} & \rightarrow \text{if } \text{T} = \text{System.Delegate then} \\
& \quad \text{let } (\text{ref}, \text{ref'}) = \text{take(\text{evalStack}, \text{argNo(\text{T::M})}) in} \\
& \quad \quad \text{if } \text{M} = \text{Combine then } \text{DELEGATECOMBINE(\text{T}, \text{ref}, \text{ref'})} \\
& \quad \quad \text{if } \text{M} = \text{Remove then } \text{DELEGATEREMOVE(\text{T}, \text{ref}, \text{ref'})} \\
& \quad \quad \text{if } \text{M} = \text{op\text{\_Equality then } DELEGATEEQUAL(\text{ref}, \text{ref'})} \\
& \quad \quad \text{else let } (\text{evalStack}', [\text{ref}] \cdot \text{vals}) = \text{split(\text{evalStack}, \text{argNo(\text{T::M})}) in} \\
& \quad \quad \text{if } \text{ref} \neq \text{null then} \\
& \quad \quad \quad \text{if } \text{T} \in \text{DelegateClass then} \\
& \quad \quad \quad \quad \text{if } \text{M} = \_\text{length then} \\
& \quad \quad \quad \quad \quad \text{evalStack} := \text{evalStack}' \cdot [\text{length(\text{invocationList(ref)})}] \\
& \quad \quad \quad \quad \text{if } \text{M} = \_\text{invoke then} \\
& \quad \quad \quad \quad \quad \text{DELEGATECALL(\text{ref}, \text{vals})} \\
& \quad \quad \quad \quad \text{else} \\
& \quad \quad \quad \quad \quad \text{evalStack} := \text{evalStack}' \\
& \quad \quad \quad \quad \quad \text{switch} \quad := \text{Invoke(\text{tail}, \text{T::M}, [\text{ref}] \cdot \text{vals})} \\
\text{Call\text{\_}virt(\text{tail}, \_, \text{T::M})} & \rightarrow \text{let } (\text{evalStack}', \text{vals}) = \text{split(\text{evalStack}, \text{argNo(\text{T::M})}) in} \\
& \quad \text{evalStack} := \text{evalStack}' \\
& \quad \text{VIRT\text{\_}CALL(\text{tail}, \text{T::M}, \text{vals})}
\end{align*}
\]
EXECCLR(instr) ≡ match instr

: Return → if retType(meth) = void then switch := Result([])
     else switch := Result(top(evalStack))

NewObj(C::ctor) → let vals = take(evalStack, argNo(C::ctor) - 1) in
    if C ∈ DelegateClass then DELEGATECREATE(C, vals)
    elseif C ∈ ObjClass then OBJECTCREATE(C::ctor, vals)
    else VALUECLASSCREATE(C::ctor, vals)

LoadField(T, C::F) → let (evalStack', [val]) = split(evalStack, 1) in
    if val ≠ null then
        evalStack := evalStack' · [memVal(fieldAdr(val, C::F), T)]
        pc := pc + 1

StoreField(T, C::F) → let (evalStack', [val, val']) = split(evalStack, 2) in
    if val ≠ null then
        WRITEMEM(fieldAdr(val, C::F), T, val')
        evalStack := evalStack'
        pc := pc + 1

LoadLocA(n) → evalStack := evalStack · [locadr(n)]
    pc := pc + 1

LoadArgA(n) → evalStack := evalStack · [argadr(n)]
    pc := pc + 1

LoadFieldA(_, C::F) → let (evalStack', [val]) = split(evalStack, 1) in
    if val ≠ null then
        evalStack := evalStack' · [fieldadr(val, C::F)]
        pc := pc + 1

LoadInd(T) → let (evalStack', [adr]) = split(evalStack, 1) in
    if validAdr(adr) then
        evalStack := evalStack' · [memVal(adr, T)]
        pc := pc + 1

StoreInd(T) → let (evalStack', [adr, val]) = split(evalStack, 2) in
    if validAdr(adr) then
        WRITEMEM(adr, T, val)
        evalStack := evalStack'
        pc := pc + 1

Box(T) → let (evalStack', [val]) = split(evalStack, 1) in
    let ref = NewBox(val, T) in
    evalStack := evalStack' · [ref]
    pc := pc + 1

Unbox(T) → let (evalStack', [ref]) = split(evalStack, 1) in
    if ref ≠ null ∧ T = actualTypeOf(ref) then
        evalStack := evalStack' · [addressOf(ref)]
        pc := pc + 1

Unbox.Any(T) → let (evalStack', [ref]) = split(evalStack, 1) in
    if T ∈ ValueType then
        if ref ≠ null ∧ actualTypeOf(ref) = T then
            evalStack := evalStack' · [memVal(addressOf(ref), T)]
            pc := pc + 1
        elseif ref = null ∨ actualTypeOf(ref) ≤ T then
            pc := pc + 1

Figure 4.5 The operational semantics rules (continued).
System.Delegate’s method calls  If the Call instruction’s method token argument is System.Delegate::Combine, two delegate references are popped from the evalStack, and their invocation lists are concatenated through the macro DELEGATECOMBINE (defined similarly as in the C♯ model):

\[
\text{DELEGATECOMBINE}(DC, \text{ref}, \text{ref'}) \equiv \\
\begin{align*}
\text{if } \text{ref} \neq \text{null} \text{ then} & \quad \text{evalStack} := \text{drop}(\text{evalStack}, 2) \cdot [\text{ref'}] \\
\text{elseif } \text{ref'} \neq \text{null} \text{ then} & \quad \text{evalStack} := \text{drop}(\text{evalStack}, 2) \cdot [\text{ref}] \\
\text{else let } \text{ref}'' = \text{new(ObjRef, DC)} \text{ in} & \\
\quad \text{initState}(\text{ref}’’) := \text{Init}(DC) \\
\quad \text{invocationList}(\text{ref}’’) := \text{invocationList}(\text{ref}) \cdot \text{invocationList}(\text{ref'}) \\
\quad \text{evalStack} := \text{drop}(\text{evalStack}, 2) \cdot [\text{ref}'']
\end{align*}
\]

If the called method is System.Delegate::Remove, two delegates are popped from the evalStack, and the macro DELEGATEREMOVE (defined similarly as in the C♯ model) is applied to remove the last occurrence of the invocation list of the second delegate from the invocation list of the first delegate.

\[
\text{DELEGATEREMOVE}(DC, \text{ref}, \text{ref'}) \equiv \\
\begin{align*}
\text{if } \text{ref} = \text{null} \text{ then} & \quad \text{evalStack} := \text{drop}(\text{evalStack}, 2) \cdot [\text{null}] \\
\text{elseif } \text{ref'} = \text{null} \text{ then} & \quad \text{evalStack} := \text{drop}(\text{evalStack}, 2) \cdot [\text{ref}] \\
\text{else let } L = \text{invocationList}(\text{ref}) \text{ and } L' = \text{invocationList}(\text{ref'}) \text{ in} & \\
\quad \text{if } L = L' \text{ then} & \quad \text{evalStack} := \text{drop}(\text{evalStack}, 2) \cdot [\text{null}] \\
\text{elseif sublist}(L', L) \text{ then} & \quad \text{let } \text{ref}'' = \text{new(ObjRef, DC)} \text{ in} \\
\quad \quad \text{initState}(\text{ref}’’) := \text{Init}(DC) \\
\quad \quad \text{invocationList}(\text{ref}’’) := \text{prefix}(L', L) \cdot \text{suffix}(L', L) \\
\quad \quad \text{evalStack} := \text{drop}(\text{evalStack}, 2) \cdot [\text{ref}''] \\
\text{else } \text{evalStack} := \text{drop}(\text{evalStack}, 2) \cdot [\text{ref}']
\end{align*}
\]

Finally, if System.Delegate::op_Equality is called, the macro DELEGATEEQUAL (defined similarly as in the C♯ model) determines whether the two delegates on top of the evalStack have identical invocation lists.

\[
\text{DELEGATEEQUAL}(\text{ref}, \text{ref'}) \equiv \\
\begin{align*}
\text{if } \text{ref} = \text{null} \lor \text{ref'} = \text{null} \text{ then} & \quad \text{evalStack} := \text{drop}(\text{evalStack}, 2) \cdot [\text{ref} = \text{ref'}] \\
\text{else let } L = \text{invocationList}(\text{ref}) \text{ and } L' = \text{invocationList}(\text{ref'}) \text{ in} & \\
\quad \text{let } \text{val} = (\text{length}(L) = \text{length}(L')) \land \forall i = 0, \text{length}(L) - 1 : L(i) = L'(i) \text{ in} \\
\quad \text{evalStack} := \text{drop}(\text{evalStack}, 2) \cdot [\text{val}]
\end{align*}
\]
Delegate class method calls  If the method token argument of a \textit{Call} instruction is the method 
\_length of a delegate class, a delegate instance is popped from the evalStack, and the length of its invocation list is loaded. If the called method is the \_invoke method of a delegate class, then a delegate reference, an index in the invocation list of the delegate and a list of arguments are popped from the evalStack. The macro \texttt{DELEGATECALL} (defined similarly as in the C\# model) calls the method in the invocation list pointed to by the index with the given arguments.

\[
\text{DELEGATECALL}(\text{ref}, [i] \cdot \text{vals}) \equiv \\
\quad \text{let } (\text{ref}', T::M) = \text{invocationList}(\text{ref})(i) \text{ in} \\
\quad \text{let } \text{this} = \text{if } T \in \text{ValueClass} \quad \text{then } \text{addressOf}(\text{ref}') \quad \text{else } \text{ref}' \text{ in} \\
\quad \text{switch} := \text{Invoke}(False, T::M, [\text{this}] \cdot \text{vals})
\]

As a result of the delegate classes translation in Figure 4.3, the call of a delegate class \textit{Invoke} method is regarded as a regular call.

Regular method calls  The instruction \textit{Call} takes a number \text{argNo}(T::M) of arguments from the evalStack, and if the target reference (denoting the instance whose method is invoked) is not \texttt{null}, the method \textit{T::M}, of return type \textit{T'}, is called with the arguments popped from the evalStack by setting switch to the appropriate method invocation mode. Note that although [3] performs a \texttt{null} check for the target reference, [45] omits to mention it.

In contrast with the \textit{Call} instruction, the \textit{CallVirt} instruction’s method token argument is late bound, \textit{i.e.}, the method to be invoked is looked up dynamically by means of the \texttt{lookUp} function. Furthermore, if the \textit{CallVirt} instruction finds a boxed instance of a value type on the stack and is applied to a virtual method inherited from System.ValueType or from an interface implemented by the value type, and the value type overrides or implements that method, then the boxed instance is “unboxed” and passed as the \texttt{this} pointer to the implementing method, \textit{i.e.}, the method determined through the \texttt{lookUp} function. More exactly, the address of the boxed instance is passed as the \texttt{this} pointer. Another difference between the \textit{Call} and \textit{CallVirt} instructions is that the latter cannot have a \texttt{.ctor} as method token argument. However, this detail is omitted by [45].

\[
\text{VIRTCELL}(\text{tail}, T::M, [\text{ref}] \cdot \text{vals}) \equiv \\
\quad \text{if } \text{ref} \neq \texttt{null} \text{ then} \\
\quad \quad \text{let } T'::M = \text{lookUp}(\text{actualTypeOf}(\text{ref}), T::M) \text{ in} \\
\quad \quad \text{if } \text{actualTypeOf}(\text{ref}) = T' \in \text{ValueClass} \text{ then} \\
\quad \quad \quad \text{switch} := \text{Invoke}(\text{tail}, T'::M, [\text{addressOf}(\text{ref})] \cdot \text{vals}) \\
\quad \quad \text{else } \text{switch} := \text{Invoke}(\text{tail}, T'::M, [\text{ref}] \cdot \text{vals})
\]

The \textit{Return} instruction takes from the evalStack no or one value, depending on the return type of the current method. It then returns this value (if any) by setting switch to the appropriate method returning mode.

The \textit{NewObj} instruction can be used for three purposes: to construct an object (other than a delegate), to construct a delegate, or a to create a value class instance.
Object creation  If \( OC \) is an object class (other than a delegate class), the instruction \( \text{NewObj}(OC::<:\text{ctor}) \) first allocates\(^6\) an object on the heap for a fresh object reference \( ref \). It sets \( \text{initState}(ref) \) to \( \text{InProgress}(OC) \) and pushes the reference \( ref \) onto the \( \text{evalStack} \). The reference gets fully initialized, i.e., \( \text{initState}(ref) \) becomes \( \text{Init}(OC) \), upon invoking the instance constructor \( \text{object::<ctor} \) (see the object initialization rules in Section 4.1.4.3). It then invokes the constructor \( OC::<:\text{ctor} \) with \( ref \) and the arguments on the \( \text{evalStack} \) by setting \( \text{switch} \) to the appropriate method invocation mode. Finally, the reference \( ref \) is pushed onto the stack.

\[
\text{OBJECTCREATE}(OC::<\text{ctor},vals) \equiv \\
\text{let } ref = \text{new}(\text{ObjRef},OC) \text{ in} \\
\text{evalStack} := \text{drop}(\text{evalStack},\text{argNo}(OC::<\text{ctor}) - 1) \cdot [\text{ref}]
\]

Delegated creation  If the \(<\text{ctor}> \) reference token argument of the \( \text{NewObj} \) instruction is declared by a delegater class, an object and a method pointer are popped from the \( \text{evalStack} \). A new delegate is then created and loaded onto the \( \text{evalStack} \). Its invocation list is initialized with the object on the stack and the method reference referred to by the method pointer.

\[
\text{DELEGATECREATE}(DC,[\text{ref},m]) \equiv \\
\text{let } ref' = \text{new}(\text{ObjRef},DC) \text{ in} \\
\text{initState}(ref') := \text{Init}(DC)
\]

Value class instance creation  If \( VC \) is a value class, the instruction \( \text{NewObj}(VC::<\text{ctor}) \) first allocates a block of memory where the new value class instance is stored. The address of the memory block is considered in the local memory pool \( \text{locPool} \) of the current frame. The constructor is then invoked with the address of the allocated memory block and the necessary arguments (assumed to be on the \( \text{evalStack} \)). Finally, the value class instance is pushed onto the stack.

\[
\text{VALUECLASSCREATE}(VC::<\text{ctor},vals) \equiv \\
\text{let } adr = \text{new}(\text{Addr},VC) \text{ in} \\
\text{locPool} := \text{locPool} \cup \{(adr,VC)\}
\]

\(^6\)The macros \( \text{new}(\text{ObjRef},C) \), \( \text{new}(\text{ObjRef}) \) and \( \text{new}(\text{Addr},T) \), defined in Chapter 3 Table 3.19 to allocate object references and memory blocks, are also applied throughout the current chapter with a trivial change in the definition: \( \text{ValueClass} \) instead of \( \text{Struct} \).
The instructions \(\text{LoadField}(T, C::F)\) and \(\text{StoreField}(T, C::F)\) loads and updates the value of the field \(C::F\) of declared type \(T\), respectively. The address of the field \(C::F\), of declared type \(T\) is addressable through the instruction \(\text{LoadFieldA}(T, C::F)\). Both instructions \(\text{LoadField}(T, C::F)\) and \(\text{LoadFieldA}(T, C::F)\) pop the topmost value of the \(\text{evalStack}\), which is an object reference \(\text{ref}\) (if \(C\) is an object class) or a pointer \(\text{adr}\) to a value class instance (if \(C\) is a value class). The \(\text{LoadField}\) instruction computes the field value with \(\text{memVal}\): It is the value of type \(\text{fieldType}(C::F)\) stored at \(\text{fieldAdr}(\text{ref}, C::F)\) (if \(C\) is an object class) or \(\text{fieldAdr}(\text{adr}, C::F)\) (if \(C\) is a value class). The instruction \(\text{LoadFieldA}\) loads on the \(\text{evalStack}\) the address \(\text{fieldAdr}(\text{ref}, C::F)\) (if \(C\) is an object class) or \(\text{fieldAdr}(\text{adr}, C::F)\) (if \(C\) is a value class). The instruction \(\text{StoreField}(T, C::F)\) takes from the \(\text{evalStack}\) the two topmost values: The first is a reference \(\text{ref}\) (if \(C\) is an object class) or a pointer \(\text{adr}\) to a value class instance (if \(C\) is a value class), and the second is a value \(\text{val}\). The instruction stores \(\text{val}\) at the address given by \(\text{fieldAdr}(\text{ref}, C::F)\) (if \(C\) is an object class) or \(\text{fieldAdr}(\text{adr}, C::F)\) (if \(C\) is a value class).

The instruction \(\text{LoadInd}(T)\) pops the topmost value of the \(\text{evalStack}\). This is supposed to be a pointer pointing to an address, say \(\text{adr}\). If \(\text{adr}\) is a valid address, then the value of type \(T\) stored at \(\text{adr}\) is computed. Finally, the value is loaded on the \(\text{evalStack}\). The instruction \(\text{StoreInd}(T)\) takes the two topmost values of the \(\text{evalStack}\) which are supposed to be a pointer pointing to an address, say \(\text{adr}\), and a value, say \(\text{val}\). If \(\text{kadr}\) is a valid address, then \(\text{val}\) is stored at the address \(\text{adr}\).

According to [45], to load an object reference, the \(\text{LoadInd}\) instruction should be used in the form \(\text{LoadInd}(\text{object})\). Similarly, to store an object reference, the \(\text{StoreInd}\) instruction should be used in the form \(\text{StoreInd}(\text{object})\).

The \(\text{Box}\) instruction turns a value type instance into a heap-allocated object “by copying”, while \(\text{Unbox}\) performs the inverse coercion. Applied to a value type, \(\text{Box}\) copies the data from the value type instance into a newly allocated object, operation accomplished by means of the macro \(\text{NewBox}\), defined exactly as for \(\text{C}^{\text{d}}\). The \(\text{Box}\) instruction does nothing if it is applied to a reference type\(^7\).

The macro \(\text{NewBox}(\text{val}, T)\) picks up an unallocated address \(\text{adr}\) from the heap and writes the top stack value \(\text{val}\) (assumed to be of type \(T\)) at the memory block at \(\text{adr}\) sufficient to hold values of type \(T\). The run-time type, used, for example, by the instructions \(\text{CastClass}\) and \(\text{IsInstance}\), is set to \(T\).

\[
\begin{align*}
\text{let } \text{ref} &= \text{NewBox}(\text{val}, T) \text{ in } P \\
\text{let } \text{ref} &= \text{new}(\text{ObjRef}) \text{ and } \text{adr} = \text{new}(\text{Addr}, T) \text{ in } P \\
\text{WRITEMEM}(\text{adr}, T, \text{val}) \\
\text{addressOf} (\text{ref}) &:= \text{adr} \\
\text{initState}(\text{ref}) &:= \text{Init}(T) \\
\text{seq } P
\end{align*}
\]

The \(\text{Unbox}\) instruction takes from the \(\text{evalStack}\) an object reference to a boxed value and extracts the value type instance out it, assuming that the value type coincides with the instruction’s type token argument. However, the value pushed onto the stack is a pointer representing the address (given by \(\text{addressOf}\)) of the value type instance embedded into the boxed value.

\(^7\)In \(.NET\) Framework (v1.1) and (v1.0), the \(\text{Box}\) instruction was only applicable to value types. Upon introducing the generics in (v2.0), the applicability of \(\text{Box}\) has been extended to reference types.
Figure 4.6 The operational semantics rules (continued).

\[
\text{EXEC CLR(\textit{instr})} \equiv \text{match instr}
\]

: \\
\text{MkRefAny}(T) \rightarrow \text{let } (\text{evalStack}', [adr]) = \text{split(.evalStack, 1)} \text{ in}
\text{let } tr = \text{new(TypedRef)} \text{ in}
\text{typedRefAdr(tr)} := adr
\text{typedRefType(tr)} := T
\text{evalStack} := \text{evalStack}' \cdot [tr]
\text{pc} := \text{pc} + 1
\\
\text{RefAnyType} \rightarrow \text{let } (\text{evalStack}', [tr]) = \text{split(.evalStack, 1)} \text{ in}
\text{let } T = \text{typedRefType(tr)} \text{ in}
\text{evalStack} := \text{evalStack}' \cdot [\text{handleOf}(T)]
\text{pc} := \text{pc} + 1
\\
\text{RefAnyVal}(T) \rightarrow \text{let } (\text{evalStack}', [tr]) = \text{split(.evalStack, 1)} \text{ in}
\text{if } T = \text{typedRefType(tr)} \text{ then}
\text{evalStack} := \text{evalStack}' \cdot [\text{typedRefAdr(tr)}]
\text{pc} := \text{pc} + 1
\\
\text{CastClass}(T) \rightarrow \text{let ref} = \text{top(.evalStack)} \text{ in}
\text{if ref} = \text{null} \lor \text{actualTypeOf(ref)} \preceq T \text{ then}
\text{pc} := \text{pc} + 1
\\
\text{IsInstance}(T) \rightarrow \text{let } (\text{evalStack}', [ref]) = \text{split(.evalStack, 1)} \text{ in}
\text{pc} := \text{pc} + 1
\text{if ref} \neq \text{null} \wedge \text{actualTypeOf(ref)} \not\preceq T \text{ then}
\text{evalStack} := \text{evalStack}' \cdot [\text{null}]
\\
\text{LoadFtn}(T::M) \rightarrow \text{let } mp = \text{new(MethPtr, T::M)} \text{ in}
\text{evalStack} := \text{evalStack} \cdot [mp]
\text{pc} := \text{pc} + 1
\\
\text{LoadVirtFtn}(T::M) \rightarrow \text{let } (\text{evalStack}', [ref]) = \text{split(.evalStack, 1)} \text{ in}
\text{if ref} \neq \text{null} \text{ then}
\text{let } T' = \text{actualTypeOf(ref)} \text{ in}
\text{let } mp = \text{new(MethPtr, lookUp(T', T::M))} \text{ in}
\text{evalStack} := \text{evalStack}' \cdot [mp]
\text{pc} := \text{pc} + 1
\\
For value types, the \textit{Unbox.Any} instruction, unlike \textit{Unbox}, leaves the value, not the address of the value, on the \textit{evalStack}. Moreover, while the \textit{Unbox}'s type token argument can only represent value types, the \textit{Unbox.Any} instruction can also be applied to reference types, in which case has the same effect as the \textit{CastClass} instruction below, namely it casts the object reference on top of the stack to the reference type.

A typed reference is created and loaded onto the stack using the \textit{MkRefAny} instruction. The new typed reference, imported from the universe \textit{TypedRef}, embeds the pointer which is on top of the \textit{evalStack} and the instruction’s type token argument.

\[
\begin{align*}
\text{let } tr = \text{new(TypedRef)} \text{ in } P & \equiv \\
& \text{import } tr \text{ do}
& \quad \text{TypedRef}(tr) := \text{True}
& \text{seq } P
\end{align*}
\]
4.1. The Lightweight CLR

The RefAnyType instruction takes a typed reference from the evalStack and loads not the "type token" embedded in the typed reference (as wrongly specified by [45, Partition III, §4.21]), but a handle, i.e., an instance of the value class System.RuntimeTypeHandle corresponding to the type in the typed reference. The handle is obtained by means of the function handleOf. The RefAnyVal instruction retrieves the address embedded in the typed reference on top of the evalStack, assuming that the instruction's type token argument is the same as the type stored in the typed reference.

The instruction CastClass(T) tests whether the topmost value on the evalStack is of type T. If not, an exception is thrown. The instruction IsInstance(T) pops a reference to an (possibly boxed) object from the evalStack. If the actual type of the object is a subtype of T, then it is cast to T, and the result is pushed on the stack, exactly as CastClass(T) had been called. If the object is not an instance of T, null is pushed on the evalStack.

The LoadFtn instruction pushes onto the evalStack a pointer to the instruction's method as a native int value. The method pointer, imported from the universe MethPtr, is mapped by methodOf to the instruction’s method token. A method pointer for a virtual method can be obtained with the LoadVirtFtn instruction. This instruction pops an object reference from the evalStack. It then loads on the evalStack a pointer to the virtual method determined based on the run-time type of the object reference and the given method. It also sets the function methodOf to map the method pointer into the determined method reference.

```plaintext
define mp = new(MethPtr, T::M) in P ≡
  import mp do
    MethPtr(mp) := True
    methodOf(mp) := T::M
  seq P
```

We now define the submachine SWITCHCLR which is responsible for method transfers, i.e., for invoking methods and returning from methods. This submachine takes control whenever switch is not Noswitch.

**Method invocations** If switch = Invoke(tail, mref, vals), the current frame is pushed onto the frameStack, unless the boolean tail indicates a tail call, in which case the current frame is discarded. The new current frame is set up through the below defined macro SETFRAME.

The macro SETFRAME(T::M, vals) sets the frame for invoking the method T::M with the arguments vals: pc is set to 0, locPool to ∅, evalStack to [], meth to T::M. Moreover, by means of the below defined macro ALLOCARGLOC, the arguments vals are stored at the newly allocated argument addresses argAdr, and "zeros" of the appropriate types are stored at the newly allocated local variable addresses locAdr. If the invoked method is a .ctor declared by object, the target reference becomes fully initialized. This is according to the object initialization rules: An object gets fully initialized when a .ctor of the inheritance tree’s root is invoked.
The macro ALLOCARGLOC is defined as follows. For the current method, two lists of types \( L, L' \) and a list of values \( vals \), \( \text{ALLOCARGLOC} (L, L', vals) \) allocates addresses\(^8\) for arguments of the types in \( L \) and for local variables of the types in \( L' \) and then writes the values \( vals \) at the allocated addresses. The definition of \( \text{ALLOCARGLOC} \) is well-founded, since the sum of the lengths of \( L \) and \( L' \) is equal with \( vals \)'s length.

\[
\begin{align*}
\text{ALLOCARGLOC}(L, L', vals) \equiv & \\
& \text{forall } i \in 0, \text{length}(L) - 1 \text{ do} \\
& \quad \text{let } adr_i = \text{new} (\text{Adr}, L(i)) \text{ in} \\
& \quad \text{argAdr}(i) := adr_i \\
& \quad \text{WRITEMEM}(adr_i, L(i), vals(i)) \\
& \text{forall } j \in 0, \text{length}(L') - 1 \text{ do} \\
& \quad \text{let } adr_j = \text{new} (\text{Adr}, L'(j)) \text{ in} \\
& \quad \text{locAdr}(i) := adr_j \\
& \quad \text{WRITEMEM}(adr_j, L'(j), vals(j + \text{length}(L)))
\end{align*}
\]

**Remark 4.1.1** [45] contradicts itself. On one hand, it states that the local variables whose declared types are reference types (so, in particular, also those whose declared types are pointer types) are initialized to null upon the method’s entry [45, Partition II, §15.4.1.3]. On the other hand, it claims that pointers cannot be null [45, Partition III, §1.1.4.2]. The truth is that the pointers can be assigned the value null, but only in a single case: when the local variables of pointer types are automatically initialized upon the method’s entry.

**Remark 4.1.2** It is worth noticing an important difference between the C\(^2\) and CLR models. The C\(^2\) model does not allocate addresses for the ref/out arguments, whereas the CLR model allocates addresses for pointer type arguments (the equivalent of the C\(^2\) by-reference arguments). The modelling for CLR is justified as long as the CLR, unlike C\(^2\), can directly handle pointers, in particular, pointers to pointers.

**Method return** If \( \text{switch} = \text{Return}(vals) \), then, by means of \( \text{POPFRA ME} \), the current frame is discarded, and the new current frame is set to the topmost frame on the \( \text{frameStack} \) with the possibly empty list of return values \( vals \) pushed onto the \( \text{evalStack} \).

\(^8\)All simultaneous allocations, i.e., calls of the macro \( \text{new} \), are supposed to provide pairwise different fresh elements from \( \text{Adr} \); see [62] and for a justification of this assumption.
4.1. The Lightweight CLR

**Figure 4.7** The switchCLR submachine.

```
switchCLR ≡ match switch
  Invoke(tail, T::M, vals) → if ¬tail then frameStack := push(frameStack, frame)
  ∙ setFrame(T::M, vals)
  ∙ switch := Noswitch

Result(vals) → popFrame(1, vals)
  ∙ switch := Noswitch
```

The definition of the macro `popFrame` is as follows. `popFrame(k, vals)` discards the current `frame` by returning to its caller frame the possibly empty list of values `vals` and by incrementing the `pc` by `k`.

```
popFrame(k, vals) ≡
  let (frameStack', [(pc', locAdr', locPool', argAdr', evalStack', meth')]) = split(frameStack, 1) in
  pc := pc' + k
  locAdr := locAdr'
  locPool := locPool'
  argAdr := argAdr'
  evalStack := evalStack' · vals
  meth := meth'
  frameStack := frameStack'
```

4.1.4 The Bytecode Verification

A global view of the CLR bytecode verification is given in Section 4.1.4.1. Section 4.1.4.2 concentrates on the verification type system that we consider. It also states what conditions should be satisfied by the bytecode structure to be verifiable. The type-consistency checks performed by the bytecode verification are specified in Section 4.1.4.3. The function used by the bytecode verification to simulate the execution paths through the bytecode is defined in Section 4.1.4.4.

4.1.4.1 An Overview of the Bytecode Verification

The bytecode verification is performed on a per-method basis. The verification attempts to associate a valid `stack state` with every instruction. The stack state specifies the number of values on the `evalStack` at that point in the code and for each slot of the `evalStack`, a required type that should be present in that slot. Also, in order to decide whether an object reference is fully initialized, the verification tracks in a `.ctor` a special type for the `this` pointer (`zeroth argument`). Given a method, we refer to the type of the zeroth argument\(^9\) and to the stack state as a `type state` of the method.

\(^9\)The type of the zeroth argument is relevant only when verifying a `.ctor.`
Universe of verification types

\[
\text{VerificationType} = \text{Type} \cup \text{BoxedType} \cup \{\text{UnInit, typedref, Null}\}
\]
\[
\text{BoxedType} = \{\text{boxed}(T) \mid T \in \text{ValueType}\}
\]

Table 4.9: CIL’s verification types.

Bytecode verification should simulate all possible control flow paths through the bytecode and ensure that a valid type state exists for every reachable instruction. By a valid type state, we mean a type state that satisfies certain type-consistency checks (see Section 4.1.4.3). The verification uses only type assignments and does not take advantage of any values during the simulation. The verification terminates successfully if all the control paths have been simulated. It finishes unsuccessfully when the verification cannot compute a valid type state for a particular instruction.

At the beginning of the simulation, the stack state is empty and the zeroth argument type is set appropriately (see Definition 4.1.9 in Section 4.1.5). For each instruction, the verification checks that the stack before the instruction execution contains enough slots and these slots are types compatible with the expected types for the instruction. It then simulates the effect of the instruction on the evalStack and zeroth argument (in case of verifying a .ctor). Any type mismatch on instruction arguments, evaluation stack overflow or underflow, or any violation of other verification conditions causes the verification to fail. It then propagates the inferred type state to all possible successors of the instruction. If an instruction is the target of several branches, the verification has to “merge” the type states along these branches. The “merge” for two zeroth argument types succeeds if the types are the same. The “merge” for two stack states is executed slot-by-slot: The merged stack state contains for each entry the supremum, i.e., “least upper bound”, of the corresponding types in the stack states. The simulation algorithm fails if it cannot compute a merged type state. This happens if the two stack states have different lengths or one slot of a stack state has a type non-compatible with the type in the corresponding slot of the other stack state.

While the verification is described above as both computing type states and checking them, we will assume that the information stored in the type states has already been computed prior to the type-consistency checks.

4.1.4.2 The Verification Type System and Bytecode Structure

The verification types are depicted by VerificationType in Table 4.9. A verification type is a CIL type as described in Table 4.1, a boxed type described by BoxedType, the type UnInit, the type typedref, the special type Null (used as the type of the null reference).

For every value type \(T\), there exists a reference type boxed\((T)\), called boxed type. The value of a type boxed\((T)\) is a location where a value of type \(T\) can be stored. Only the bytecode verification has knowledge of the boxed types. In the bytecode, they can only be referred to as object, System.ValueType, or as interfaces implemented by the underlying value.

\[\text{boxed}\((T)\)\]

The situation when two stack states have different lengths corresponds to a program point where the run-time stack can have different heights depending on the path by which the point is reached; such bytecode cannot be proved correct in the framework described in this section, and must be rejected according to [45, Partition III, §1.8.1.1].
4.1. THE LIGHTWEIGHT CLR

The type `UnInit` is a special type used by the verification (only) to track the type of the zeroth argument of a `.ctor`.

We define the *compatibility relation* \( \sqsubseteq \) for verification types.

**Definition 4.1.7 (Compatible verification types)** The relation \( \sqsubseteq \), for verification types, is the least reflexive and transitive relation such that

1. if \( T \) is `Null` and \( T' \in \text{ObjClass} \cup \text{Interface} \cup \text{BoxedType} \), or
2. if \( T \in \text{ObjClass} \cup \text{Interface} \cup \text{BoxedType} \) and \( T' \) is `object`, or
3. if \( T \in \text{BoxedType} \) and \( T' \) is `System.ValueType`, or
4. if \( T \in \text{ObjClass} \) and \( T' \) is a base class of \( T \) or an interface implemented by \( T \), or
5. if \( T \in \text{Interface} \) and \( T' \) is an interface implemented by \( T \), or
6. if \( T \) is boxed(\( T'' \)), \( T'' \in \text{ValueType} \), and \( T' \) is an interface implemented by \( T'' \),

then \( T \sqsubseteq T' \).

The following lemma, required for the type safety proof, claims that a pointer type is only compatible with itself.

**Lemma 4.1.2** If \( T\& \sqsubseteq T' \) or \( T' \sqsubseteq T\& \), then \( T' = T\& \).

**Proof.** Follows immediately from Definition 4.1.7. \( \Box \)

There are a number of restrictions imposed by [45] in using pointer types in verifiable code. Since the typed references embed pointers, there are also restrictions in using the type `typedref`. The restrictions are *sufficient* conditions for the type safety result.

<table>
<thead>
<tr>
<th>Address Of</th>
<th>The declared type of a local variable and argument used with the instruction <code>LoadLocA</code> and <code>LoadArgA</code>, respectively, should not be a pointer type or <code>typedref</code>.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field Type</td>
<td>The declared type of a field should not be a pointer type or <code>typedref</code>.</td>
</tr>
<tr>
<td>Return Type</td>
<td>The return type of a method should not be a pointer type or <code>typedref</code>.</td>
</tr>
<tr>
<td>Tail Call Argument Types</td>
<td>The argument types of a method called tail should not be pointer types or <code>typedref</code>.</td>
</tr>
<tr>
<td>Box Type</td>
<td>The <code>Box</code> instruction should not be applied to <code>typedref</code>.</td>
</tr>
<tr>
<td>MkRefAny Type</td>
<td>The <code>MkRefAny</code> instruction should not be applied to <code>typedref</code>.</td>
</tr>
</tbody>
</table>
Basically, all these constraints aim to prevent the existence of dangling pointers, i.e., pointers which outlive their targets. Consider, for example, **Tail Call Argument Types**. As the tail prefix shortens the lifetime of the caller frame, passing a pointer to the caller frame as an argument produces a dangling pointer. This would happen since the caller frame is discarded just before the tail call.

**Remark 4.1.3** Upon entering a method, the macro `ALLOCARGLOC` allocates memory blocks for all local variables and arguments independent of their declared type, in particular, also for those whose declared type is a pointer type. We took this modelling decision to simplify the exposition. The restriction `Address Of` helps us, in particular, to guarantee that the decision is ”safe” in the sense that the address of a pointer type local variable or argument cannot be addressed.

**Remark 4.1.4** As pointed out in [66], **Field Type** can be relaxed to allow fields of value classes to hold pointers, and yet the type safety result holds. However, this has a downside: the possibility that every value class may be boxed is lost. If a value class has at least one pointer type field, then one should not be allowed to box this value class. Otherwise, dangling pointers could be created, e.g., a pointer in a frame could be stored on the heap in the pointer type field of the boxed value class.

Also **Return Type** could be relaxed to allow methods to return a special kind of pointers. It is about the pointers that point to a permanent ”home” that still exists after the method returns, e.g., locations on the heap. When being tracked by the verification, these pointers should, however, be marked distinctly.

There are also constraints on the bytecode structure. [45] states that the following conditions should hold or the bytecode verification will fail. To provide the reader with a complete view of the bytecode verification, we list here all the conditions required by the verification, regardless of whether they are relevant in the type safety proof or aimed to simplify the JIT compilation.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>“This” of .ctor</strong></td>
<td>There should be no instruction <code>StoreArg(0)</code> in the body of a .ctor.</td>
</tr>
<tr>
<td><strong>Tail Call Return Type</strong></td>
<td>If the method <code>mref'</code> calls tail the method <code>mref</code>, then either both <code>mref</code> and <code>mref'</code> have the return type <code>void</code> or <code>retType(mref) ⊑ retType(mref')</code>.</td>
</tr>
<tr>
<td><strong>Tail Call Pattern</strong></td>
<td>The instruction following a tail call should be <code>Return</code>.</td>
</tr>
</tbody>
</table>

The first condition guarantees that the `this` pointer of a .ctor is always pointing to the same location in the heap. This matters, for example, when checking whether the .ctor is called on the same `this` pointer (and not on another object reference) of a .ctor of the same class or the base class (see the object initialization rules in Section 4.1.4.3).

The restriction **Tail Call Return Type** insures that the frame of the method called tail returns a value of the type expected by the caller frame of the discarded frame (the frame which contained the tail call).
The reader might ask what is the reason for requiring Tail Call Pattern. Recall that the tail prefix is an optional directive, which might be ignored when the bytecode is jitted. If the prefix is ignored, the caller frame would continue to execute after the callee’s frame returns. However, because of Tail Call Pattern, the caller frame would immediately (read: in the next step of the operational semantics model) terminate if the callee’s frame returned normally. Thus, the program will execute properly even if the caller’s frame was not discarded before performing the tail call.

While “This” of .ctor and Tail Call Return Type are relevant in the type safety proof, Tail Call Pattern serves to simplify the JIT compilation and is included here for completeness only.

In order to guarantee the type-safe execution of delegates, the verification imposes for a delegate creation the below specified constraint Delegate Pattern. For this, we need first to define the compatibility relation for delegate classes and methods. A delegate class is compatible with a target method if the arguments that can be passed to the delegate’s Invoke method can also be passed to the target method and every value that can be returned by the target method can also be returned by (read: is of the return type of) the delegate’s Invoke method.

Definition 4.1.8 A delegate class DC is compatible with a method mref if the following conditions are satisfied:

- \( \text{argTypes}(DC::\text{Invoke})(i) \subseteq \text{argTypes}(mref)(i) \), for every \( i = 1, \argNo(mref) - 1 \)
- \( \text{retType}(mref) \subseteq \text{retType}(DC::\text{Invoke}) \) or \( \text{retType}(DC::\text{Invoke}) = \text{retType}(mref) = \text{void} \).

Delegate Pattern A delegate creation, i.e., an instruction NewObj(DC::.ctor) with \( DC \in \text{DelegateClass} \), should occur immediately after an instruction LoadFtn(mref) or after an instruction LoadVirtFtn(mref) immediately preceded by the Dup instruction. Moreover, DC should be compatible with mref. Furthermore, no branch target should be within these instruction sequences (other than at the beginning of the sequences).

As the .NET Framework (v2.0) [3]’s bytecode verification does not track method pointer types, requiring Delegate Pattern makes the verification’s job possible. It assures that the NewObj instruction used to construct a delegate always finds a method pointer on the stack. The rationale for the Dup instruction is provided in [45, Partition III, §1.8.1.5.1]: It guarantees that the same object is used as the target object (embedded in the delegate) and to determine the target virtual method (embedded in the delegate).

Example 4.1.3 To understand why the target object should be the same as the object used to determine the target method, we consider the following bytecode fragment which does not obey Delegate Pattern. We assume that the object class \( OC' \) is a subclass of the object class \( OC \), \( OC' \) contains an implementation of the method \( OC::M \), and \( DC \) is a delegate class.
1: `newobj instance void OC::ctor()`  
2: `newobj instance void OC'::ctor()`  
3: `ldvirtftn instance void OC::M(int32)`  
4: `newobj instance void DC::ctor(object, native int)`

Provided that `OC'` implements `OC::M`, the `LoadVirtFtn` instruction loads a method pointer to `OC'::M` on the stack. Consequently, the last `NewObj` instruction creates a delegate with a target object of run-time type `OC` and the target method `OC'::M`. When calling this delegate, `OC'::M` gets invoked on an object of type `OC`. Therefore, in the body of `OC'::M`, the `this` pointer is of run-time type `OC`, but it is used as being of type `OC'`. However, this could lead to memory violations if, for example, `OC'::M` accesses an instance field declared by `OC'`.

### 4.1.4.3 Verifying the Bytecode Instructions

Before simulating the execution of an instruction, the bytecode verification checks whether certain conditions are satisfied. In this section, we formally specify these conditions by means of the predicate `check` defined in Figure 4.8. The checks for a single instruction operate on the stack state `evalStackT : List(VerificationType)` and, in case of verifying a `ctor`\(^{11}\), on the type of the `this` pointer `argZeroT : VerificationType`. Some of the specified checks do not matter for the type safety proof. They are required to simplify the JIT compilation. However, for the sake of completeness, we specify all the conditions imposed by the bytecode verification.

Several instructions, e.g., `Dup`, `Const`, `LoadLoc`, push a value onto the stack, but do not pop off any values. In their case, one has to avoid an overflow of the evaluation stack. More exactly, the number of values on the evaluation stack should not exceed an upper bound computed by every .NET compliant language’s compiler. For this purpose, we consider the function `maxEvalStack` (see Table 4.10) which assigns to every method an upper bound for the number of elements on the stack.

A few instructions, e.g., `Dup`, `LoadField`, `StoreField`, require a minimum number of values on the stack. For them, there is an underflow check.

\[
\text{overflow}(\text{evalStackT}, n) :\leftrightarrow \text{length(\text{evalStackT})} + n > \text{maxEvalStack(meth)}
\]

\[
\text{underflow}(\text{evalStackT}, n) :\leftrightarrow \text{length(\text{evalStackT})} < n
\]

The instructions `Dup` and `Pop` require the stack to have at least one value. Additionally, in case of `Dup`, the duplicated value should not overflow the stack.

Since the local variables in the verifiable CLR bytecode are zeroed upon the method’s entry, the bytecode verification does not need to check for a `LoadLoc` instruction that the corresponding variable has been previously assigned. The `StoreLoc` instruction requires that the topmost value on the stack is of the corresponding variable’s declared type.

The `Execute` instruction expects on the stack values of the types for which the operation indicated by the given operator is verifiable. These types are maintained in the function `verifOpTypes` in Table 4.10, which given an operator, returns the set of lists of types for

\(^{11}\)For technical reasons, we consider the `argZeroT` component for every method.
4.1. The Lightweight CLR

<table>
<thead>
<tr>
<th>Function definition</th>
<th>Function name</th>
</tr>
</thead>
<tbody>
<tr>
<td>maxEvalStack : Map(MRef, N)</td>
<td>the maximum number of elements on the stack</td>
</tr>
<tr>
<td>verifOpTypes : Map(Op, (\mathcal{P}(\text{List(Type)})))</td>
<td>the operands’ types for which an operation is verifiable</td>
</tr>
</tbody>
</table>

Table 4.10: The verification specific functions.

which the corresponding operation is verifiable. The definition of verifOpTypes can be found in [45, Partition III, §1.5].

Example 4.1.4 Consider the addition operator add. According to [45, Partition III, §1.5], this operator is verifiable for several pairs of types. For example, add is verifiable for operands of type int32, but also for operands of type int64. The complete set of type pairs for which add is verifiable is defined through the function verifOpTypes:

\[
\text{verifOpTypes}(\text{add}) = \{[\text{int32, int32}], [\text{int64, int64}], [\text{int32, native int}], [\text{native int, int32}], [\text{native int, native int}], [\text{float64, float64}]\}
\]

The result type for each possible combination of operand types is defined through the function opResType defined in Section 4.1.2. For example, \(\text{opResType}(\text{add}, [\text{int32, int32}]) = \text{int32}\) and \(\text{opResType}(\text{add}, [\text{int32, native int}]) = \text{native int}\).

We will call the elements of the set \(\text{UncondBranchInstr} = \{\text{Return}\}\) unconditional branch instructions\(^{12}\). The evaluation stack before executing an instruction \(\text{instr}\) immediately following an unconditional branch instruction should be empty, unless \(\text{instr}\) is the target of a forward branch instruction\(^{13}\). Therefore, the verification fails if a later branch instruction \(\text{Cond}(\text{op}, \text{target})\) – whose \(\text{target}\) points to \(\text{instr}\) – were to have on the evaluation stack more values than needed for executing \(\text{op}\).

When executing \(\text{op}\), the instruction \(\text{Cond}(\text{op}, \text{target})\) expects on the stack operands of types given by \(\text{verifOpTypes}(\text{op})\).

\[
\text{backBranch}(\text{pos}, \text{target}, \text{evalStackT}, \mathcal{L}) : \iff \\
\begin{align*}
\text{if } & \text{target} < \text{pos} \land \text{code}(\text{target} - 1) \in \text{UncondBranchInstr} \land \\
& \neg \exists \text{pos} \in \text{Pc} : (0 \leq \text{pos} < \text{target} \land \text{code}(\text{pos}) = \text{Cond}(\_\_, \text{target})) \text{ then} \\
& \text{evalStackT} \in_{\text{len}} \mathcal{L} \\
\text{else} & \text{evalStackT} \in_{\text{suf}} \mathcal{L}
\end{align*}
\]

Example 4.1.5 Consider the following bytecode fragment:

\(^{12}\)The set \(\text{UncondBranchInstr}\) consists now of a single element, but it will be extended with exceptions specific instructions in Section 4.2.4.2.

\(^{13}\)This condition is required to ensure that it is possible to infer the state of the evaluation stack at the beginning of each instruction even through a single forward-pass analysis. The constraint guarantees, in particular, that the bytecode can be processed by a simple CIL-to-native-code compiler; see [48] for details.
Figure 4.8 Verifying the bytecode instructions.

\[
\text{check(meth, pos, argZeroT, evalStackT)} \implies \\
\text{match code(pos) \rightarrow} \\
\text{Dup} & \rightarrow \neg\text{underflow}(\text{evalStackT}, 1) \land \neg\text{overflow}(\text{evalStackT}, 1) \\
\text{Pop} & \rightarrow \neg\text{underflow}(\text{evalStackT}, 1) \\
\text{Const(_, _)} & \rightarrow \neg\text{overflow}(\text{evalStackT}, 1) \\
\text{LoadLoc(_)} & \rightarrow \neg\text{overflow}(\text{evalStackT}, 1) \\
\text{StoreLoc(n)} & \rightarrow \text{evalStackT} \sqsubseteq \text{locTypes}(n) \\
\text{Execute(op)} & \rightarrow \text{evalStackT} \sqsubseteq \text{verifOpTypes}(op) \\
\text{Cond(op, target)} & \rightarrow \text{backBranch}(\text{pos, target, evalStackT, verifOpTypes(op)}) \\
\text{LoadArg(_)} & \rightarrow \neg\text{overflow}(\text{evalStackT}, 1) \\
\text{StoreArg(n)} & \rightarrow \text{evalStackT} \sqsubseteq [\text{argTypes}(n)] \\
\text{Call(tail, _\rightarrow T\rightarrow M)} & \rightarrow \begin{cases} \\
\text{cond if then } \text{length}(\text{evalStackT}) = \text{argNo}(T\rightarrow M) \\
\text{else \neg\text{underflow}(\text{evalStackT}, \text{argNo}(T\rightarrow M))} \land \\
\text{let } [T'] \cdot \text{types} = \text{take}(\text{evalStackT}, \text{argNo}(T\rightarrow M)) \text{ in} \\
\text{types} \sqsubseteq \text{paramTypes}(T\rightarrow M) \land \\
\text{if } T \in \text{ObjClass} \land \text{M} = .\text{ctor} \text{ then } \text{initCtorCompat(meth, T', T)} \\
\text{else \text{argZeroCompat}(T', T)} \\
\end{cases} \\
\text{CallVirt(tail, _\rightarrow T\rightarrow M)} & \rightarrow \begin{cases} \\
\text{cond if then } \text{evalStackT} \sqsubseteq \text{len} \text{argTypes}(T\rightarrow M) \\
\text{else } \text{evalStackT} \sqsubseteq \text{argTypes}(T\rightarrow M) \\
\end{cases} \\
\text{Return} & \rightarrow \text{evalStackT} \sqsubseteq \text{len} \text{void}(	ext{retType(meth)}) \land \\
\text{if } \text{methNm(meth)} = .\text{ctor} \land \text{classNm(meth)} \neq \text{object} \text{ then} \\
\text{argZeroT} \neq \text{UnInit} \\
\text{NewObj(C::ctor)} & \rightarrow \text{evalStackT} \sqsubseteq \text{paramTypes}(C::ctor) \land \\
\neg\text{overflow}(\text{evalStackT}, 2 - \text{argNo}(C::ctor)) \land \\
\text{if } C \in \text{DelegateClass} \text{ then} \\
\text{let } [T, _\rightarrow] = \text{take}(\text{evalStackT}, 2) \text{ in } \text{delegateTargetObj(pos, T)} \\
\text{LoadField(_, C::F)} & \rightarrow \neg\text{underflow}(\text{evalStackT}, 1) \land \\
\text{let } T = \text{top}(\text{evalStackT}) \text{ in } \text{initFldCompat(T, classNm(meth), C::F)} \\
\text{StoreField(T, C::F)} & \rightarrow \neg\text{underflow}(\text{evalStackT}, 2) \land \\
\text{let } [T'', T'] = \text{take}(\text{evalStackT}, 2) \text{ in} \\
\text{initFldCompat}(T'', \text{classNm(meth), C::F}) \land T' \sqsubseteq T \\
\text{LoadLocA(_)} & \rightarrow \neg\text{overflow}(\text{evalStackT}, 1) \\
\text{LoadArgA(_)} & \rightarrow \neg\text{overflow}(\text{evalStackT}, 1) \\
\text{LoadFieldA(_, C::F)} & \rightarrow \neg\text{underflow}(\text{evalStackT}, 1) \land \\
\text{let } T = \text{top}(\text{evalStackT}) \text{ in } \text{initFldCompat(T, classNm(meth), C::F)} \\
\text{LoadInd(T)} & \rightarrow \neg\text{underflow}(\text{evalStackT}, 1) \land \\
\text{let } T' = \text{top}(\text{evalStackT}) \text{ in} \\
\exists T'' \in \text{ReferentType} : (T' = T'' \land T' \sqsubseteq T) \\
\text{StoreInd(T)} & \rightarrow \neg\text{underflow}(\text{evalStackT}, 2) \land \\
\text{let } [T'', T'] = \text{take}(\text{evalStackT}, 2) \text{ in} \\
\exists T''' \in \text{ReferentType} : (T'' = T''' \land T' \sqsubseteq T'' \sqsubseteq T) \\
\text{Box(T)} & \rightarrow \text{evalStackT} \sqsubseteq [T] \\
\text{Unbox(_)} & \rightarrow \text{evalStackT} \sqsubseteq [\text{object}] \\
\text{Unbox.Any(_)} & \rightarrow \text{evalStackT} \sqsubseteq [\text{object}] \\
\text{MKRefAny(T)} & \rightarrow \text{evalStackT} \sqsubseteq [T\&] \\
\text{RefAnyType} & \rightarrow \text{evalStackT} \sqsubseteq [\text{typedref}] \\
\text{RefAnyVal(_)} & \rightarrow \text{evalStackT} \sqsubseteq [\text{typedref}] \\
\text{CastClass(_)} & \rightarrow \text{evalStackT} \sqsubseteq [\text{object}] \\
\text{IsInstance(_)} & \rightarrow \text{evalStackT} \sqsubseteq [\text{object}] \\
\text{LoadFin(_)} & \rightarrow \text{True} \\
\text{LoadVirtFin(T::M)} & \rightarrow \text{evalStackT} \sqsubseteq [T] \right)
The instruction LoadLoc(1) is immediately following the unconditional branch instruction Return, and it is not the target of a forward branch instruction. Hence, the stack state before the instruction LoadLoc(1) cannot be determined from the already derived information. Therefore, the bytecode verification considers an empty stack state before LoadLoc(1). To ensure that a single forward-pass suffices for the bytecode simulation, one has to guarantee that the conditional instruction Cond(eq, 1) (corresponding to the real instruction beq 1) has a stack state with only the two values, required by the equality operator eq.

Bytecode verification ensures for the LoadArg instruction that the loaded argument’s value does not overflow the stack. The StoreArg instruction needs the top stack value to be of the corresponding argument’s type.

A Call instruction requires that the argument types, except the type of the this pointer, are compatible with types given in the signature of the invoked method. A tail call requires the stack to be empty except for the arguments necessary to perform the call. Concerning the check for the type of the this pointer, we do a case distinction. If the invoked method $T::M$ is not a .ctor, the type of the this pointer should be compatible with $T$ if $T$ is an object class or an interface, or with $T\&$ if $T$ is a value class.

\[
\text{argZeroCompat}(T', T) :\equiv \\
(T \in \text{ObjClass} \cup \text{Interface} \land T' \subseteq T) \lor \\
(T \in \text{ValueClass} \land T' \subseteq T\&)
\]

For the this pointer of an object class .ctor, there are two special verification rules called object initialization rules:

- A .ctor should not return unless the this pointer is initialized.
- The this pointer is initialized after calling a .ctor of the same class or a base .ctor.

Remark 4.1.5 The object initialization rules are not necessary for type safety of the bytecode language. The fields initialization with default values is sufficient for that purpose. The rules are important because significantly large parts of CLR’s security assume consistent object states. Such a state relies, in particular, on constructors being invoked before objects are used and on base class constructors being invoked before initialization begins.

We now describe the specification of the object initialization rules. For a .ctor, the bytecode verification also tracks the type of the zeroth argument. This is initially set to the special verification type UnInit. An important contribution of our framework consists in adding this type to the verification type system though this is not mentioned in [45]. The only allowed operations on an uninitialized this pointer are loading/storing of and into the corresponding instance fields.
If the \textit{Call} instruction invokes a \texttt{.ctor} of an object class, the type of the \texttt{this} pointer needs to be either \texttt{UnInit} or a subtype of the \texttt{.ctor}'s class, and the \texttt{.ctor} is either of the same class as the caller's method (which is also assumed to be a \texttt{.ctor}) or of its base class.

\begin{tcolorbox}[colback=white]
\textit{initCtorCompat}(T::M, T', T'') :⇔
\begin{align*}
(T' \subseteq T'') \lor (T' = \texttt{UnInit}) \quad \land \\
(T'' = T \lor T'' \text{ is the base class of } T)
\end{align*}
\end{tcolorbox}

We illustrate the object initialization rules in Example 4.1.6 and Example 4.1.7.

\textbf{Example 4.1.6} We consider the following body of a method declared by an object class \texttt{OC}.
We assume that \texttt{OC} declares the fields \texttt{OC::F} and \texttt{OC::F'}, both of type \texttt{int32}, and the instance method \texttt{OC::M}.

\begin{verbatim}
0: ldarg 0
1: dup
2: ldfld int32 OC::F
3: stfld int32 OC::F'
4: ldarg 0
5: dup
6: ldc.i4 2
7: call instance void OC::.ctor(int32)
8: call instance void OC::M()
9: ret
\end{verbatim}

If this method is a not a \texttt{.ctor}, then the instruction \texttt{Call(void, OC::.ctor(int32))} on line 6 would not be allowed since this instruction expects the type of the \texttt{this} pointer to be \texttt{UnInit}. However, this type can only occur in a \texttt{.ctor}. Consequently, a \texttt{.ctor} can be invoked explicitly, i.e., with a \texttt{Call} instruction, only from another \texttt{.ctor}.

Let us now assume that this method is a \texttt{.ctor}. Initially, the type of the zeroth argument \texttt{argZeroT} is \texttt{UnInit}. This type is loaded on the evalStackT by the instruction \texttt{LoadArg(0)} at code index 0. Although the \texttt{this} pointer is uninitialized, the loading of the field \texttt{OC::F} and then the storing into the field \texttt{OC::F'} are verifiable. Also, the constructor call on an uninitialized \texttt{this} pointer at the code index 6 is verifiable. After this call, the \texttt{this} pointer is considered initialized (see Section 4.1.4.4 for the setting of \texttt{argZeroT}), and therefore, the method return in line 8 is verifiable.

\textit{See Example 4.1.7 for a justification why the call at code index 7 is verifiable.}

Note that the object initialization rules are not crucial for type safety. This would be anyway ensured since all fields of the \texttt{this} pointer are zeroed before the \texttt{.ctor} is run. The initialization rules are important to guarantee certain invariants (established by the execution of the \texttt{.ctor}) between the instance fields of the same class objects.

For the \texttt{CallVirt} instruction, one has to guarantee that the types of the arguments on the stack are compatible with the types expected by the method call. In case of a tail call, the evaluation stack should contain no other values than the arguments of the call.

The \texttt{Return} instruction expects the stack to be empty except for the value being returned (if any). To shorten the specification of the underlying check, we use the following notation:
4.1. The Lightweight CLR

\[
\text{void}(T) = \begin{cases} 
[], & \text{if } T \text{ is void;} \\
[T], & \text{otherwise.}
\end{cases}
\]

The \textit{Return} of a \texttt{ctor} ensures that the type of the \texttt{this} pointer, tracked by means of \texttt{argZeroT}, is not \texttt{UnInit}.

Within a \texttt{ctor}, the instructions \textit{LoadField}, \textit{StoreField} and \textit{LoadFieldA} can also be used with an uninitialized \texttt{this} pointer.

\begin{align*}
\text{initFldCompat}(T, T', C::F) :\iff & \quad (C \in \text{ObjClass} \Rightarrow (T \subseteq C \lor T = \text{UnInit} \land T' \subseteq C)) \land \\
& \quad (C \in \text{ValueClass} \Rightarrow T \subseteq Coriously)
\end{align*}

\textbf{Remark 4.1.6} In earlier versions of .NET Framework, the object initialization rules for value classes ensured that each value class instance needed to be initialized by calling the \texttt{ctor} which was supposed to initialize every instance field. Bytecode verification of .NET Framework (v2.0) [3] does not check this anymore.

Since the \texttt{NewObj} instruction invokes a \texttt{ctor} after creating a fresh reference, one has to ensure that the types of the arguments on the stack are compatible with the \texttt{ctor}'s parameter types. The pushed object reference should not overflow the stack from which the \texttt{ctor}'s parameters have been dropped.

We now consider the case when the \texttt{NewObj} instruction creates a delegate, that is \texttt{NewObj} is considered with a delegate class’s \texttt{ctor}. In this case, besides a native \texttt{int} value, the instruction also expects an object reference of a type compatible with the class of the method whose pointer has been loaded by the preceding instruction. Remember that the constraint \textit{Delegate Pattern} given in Section 4.1.4.2 guarantees that the preceding instruction is either a \textit{LoadFtn} or \textit{LoadVirtFtn}. As the value class methods require a ”home” for the \texttt{this} pointer, the value class instances should be boxed before being encapsulated into the delegate.

\begin{align*}
\text{delegateTargetObj}(pos, T) :\iff & \quad \text{if classNm(mref) } \in \text{ValueClass then} \\
& \quad T \subseteq \text{boxed(classNm(mref))} \\
& \quad \text{else } T \subseteq \text{classNm(mref)} \\
& \quad \text{where mref is such that code(pos - 1) } = \text{LoadFtn(mref)} \text{ or code(pos - 1) } = \text{LoadVirtFtn(mref)}
\end{align*}

The delegate’s \texttt{ctor} invoked through a \texttt{NewObj} instruction also expects on the stack a value of type native \texttt{int}. This is used to represent the method pointer which is going to be embedded in the delegate. Bytecode verification of .NET Framework (v2.0) [3] does not track method pointer types. Knowing this, the reader might ask how it is ensured that the native \texttt{int} value represents indeed a method pointer and not an arbitrary native \texttt{int} value. The answer is to be found in the constraint \textit{Delegate Pattern} (defined in Section 4.1.4.2) which guarantees that a method pointer is on top of the stack.
The LoadInd instruction ensures that the top stack value is of a pointer type whose referent type is compatible with the instruction’s type token argument. The StoreInd instruction expects on the stack a pointer and a value to be stored at the address referenced by the pointer. The value should be of a type compatible with the referent type of the pointer type. Moreover, the referent type has to be compatible with the instruction’s type token argument.

The Box instruction expects on the stack an instance of the instruction’s value type token. An object reference is required on the stack by the Unbox and Unbox.Any instructions.

Remark 4.1.7 [45, Partition III, §4.30] says that for the Unbox instruction, the bytecode verification checks whether the object reference on the stack represents a boxed instance of the instruction’s value type token argument. However, in .NET Framework (v2.0) [3], this is not checked by the bytecode verification, but it is the subject of a run-time check.

The MkRefAny instruction expects a pointer to the instruction’s type token argument, while the instructions RefAnyType and RefAnyVal require a typed reference on the stack. The instructions CastClass and IsInstance require an object reference on the stack. No check is required for LoadFtn. The LoadVirtFtn instruction expects on the stack an object of a type compatible with the class of the method given in the instruction.

4.1.4.4 Computing Successor Type States

The type state for an instruction at index pos is constrained by referring to the type states of all instructions that are control-flow successors of pos. In Figure 4.9, we define the function succ which, given an instruction and a type state, computes the type states and the code indices of the instruction’s successors.

For the instructions LoadLoc and LoadLocA, the function succ uses the variable’s declared type maintained in locTypes. For the Execute instruction, the result type of the corresponding operation is determined by means of opResType, based on the given operator and on the operands’ types.

The Cond instruction has two successors. One successor is given by the target instruction to which control is transferred if the operator given in the instruction returns true. The other successor is given by the instruction following the Cond instruction. The stack state of both successors is given by the stack state of the Cond instruction from which the operands of Cond’s operator have been dropped.

If the instructions LoadArg and LoadArgA are applied to the zeroth argument of a .ctor, the succ function uses the type argZeroT tracked for the this pointer. Otherwise, succ uses the argument’s declared type recorded in argTypes.

If the Call instruction is applied to a .ctor, the type tracked for the zeroth argument (assumed to be UnInit) becomes the type denoted by the class of the current method. Also, all occurrences of UnInit in evalStackT are replaced by the fully initialized type, i.e., the class of the current method.

Example 4.1.7 Consider again the method body given in Example 4.1.6. Before the constructor call at code index 6, argZeroT is UnInit, and the stack state evalStackT is [UnInit, UnInit]. After the constructor call,
4.1. The Lightweight CLR

Figure 4.9 The type state successors.

\[
\text{succ}(\text{meth}, \text{pos}, \argZeroT, \text{evalStackT}) =
\]

\[
\text{match code}(\text{pos})
\]

\[
\text{Dup} \rightarrow \{(\text{pos} + 1, \argZeroT, \text{evalStackT} \cdot \text{top}(\text{evalStackT}))\}
\]

\[
\text{Pop} \rightarrow \{(\text{pos} + 1, \argZeroT, \text{pop}(\text{evalStackT}))\}
\]

\[
\text{Const}(\text{T}, \_) \rightarrow \{(\text{pos} + 1, \argZeroT, \text{evalStackT} \cdot \{\text{T}\})\}
\]

\[
\text{LoadLoc}(\text{n}) \rightarrow \{(\text{pos} + 1, \argZeroT, \text{evalStackT} \cdot \{\text{locTypes}(\text{n})\})\}
\]

\[
\text{StoreLoc}(\_) \rightarrow \{(\text{pos} + 1, \argZeroT, \text{pop}(\text{evalStackT}))\}
\]

\[
\text{Execute}(\text{op}) \rightarrow \text{let} (\text{evalStackT}', \text{types}) = \text{split}(\text{evalStackT}, \text{opNo}(\text{op})) \text{ in}
\]

\[
\{(\text{pos} + 1, \argZeroT, \text{evalStackT}' \cdot \{\text{opResNo}(\text{op}, \text{types})\})\}
\]

\[
\text{Cond}(\text{op, target}) \rightarrow \text{let} \text{evalStackT}' = \text{drop}(\text{evalStackT}, \text{opNo}(\text{op})) \text{ in}
\]

\[
\{(\text{target}, \argZeroT, \text{evalStackT}'), (\text{pos} + 1, \argZeroT, \text{evalStackT}')\}
\]

\[
\text{LoadArg}(\text{n}) \rightarrow \text{if} \ n = 0 \land \text{methNm}(\text{meth}) = \text{.ctor} \text{ then}
\]

\[
\{(\text{pos} + 1, \argZeroT, \text{evalStackT} \cdot \{\text{argZeroT}\})\}
\]

\[
\text{else} \{(\text{pos} + 1, \argZeroT, \text{evalStackT} \cdot \{\text{argTypes}(\text{n})\})\}
\]

\[
\text{StoreArg}(\_) \rightarrow \{(\text{pos} + 1, \argZeroT, \text{pop}(\text{evalStackT}))\}
\]

\[
\text{Call}(\text{tail}, \text{T}', \text{T} : \text{M}) \rightarrow \text{if} \ \text{tail} \ \text{then} \ \emptyset
\]

\[
\text{else let} \ \text{evalStackT}' = \text{drop}(\text{evalStackT}, \text{argNo}(\text{T} : \text{M})) \cdot \text{void}(\text{T}') \text{ in}
\]

\[
\text{if} \ M = \text{.ctor} \text{ then}
\]

\[
\text{let} \ \text{evalStackT}'' = \text{evalStackT}' [\text{classNm}(\text{meth}) \text{ / } \text{UnInit}] \text{ in}
\]

\[
\{(\text{pos} + 1, \text{classNm}(\text{meth}), \text{evalStackT}'')\}
\]

\[
\text{else} \{(\text{pos} + 1, \argZeroT, \text{evalStackT}')\}
\]

\[
\text{CallVirt}(\text{tail}, \text{T}', \text{T} : \text{M}) \rightarrow \text{if} \ \text{tail} \ \text{then} \ \emptyset
\]

\[
\text{else let} \ \text{evalStackT}' = \text{drop}(\text{evalStackT}, \text{argNo}(\text{T} : \text{M})) \cdot \text{void}(\text{T}') \text{ in}
\]

\[
\{(\text{pos} + 1, \argZeroT, \text{evalStackT}')\}
\]

\[
\text{Return} \rightarrow \emptyset
\]

\[
\text{NewObj}(\text{C} : \text{.ctor}) \rightarrow \{(\text{pos} + 1, \argZeroT, \text{drop}(\text{evalStackT}, \text{argNo}(\text{C} : \text{.ctor}) - 1) \cdot \{\text{C}\})\}
\]

\[
\text{LoadField}(\text{T}, \_) \rightarrow \{(\text{pos} + 1, \argZeroT, \text{pop}(\text{evalStackT}) \cdot \{\text{T}\})\}
\]

\[
\text{StoreField}(\_, \_) \rightarrow \{(\text{pos} + 1, \argZeroT, \text{drop}(\text{evalStackT}, 2))\}
\]

\[
\text{LoadLocA}(\text{n}) \rightarrow \{(\text{pos} + 1, \argZeroT, \text{evalStackT} \cdot \{\text{locTypes}(\text{n})\})\}
\]

\[
\text{LoadArg}(\text{n}) \rightarrow \text{if} \ n = 0 \land \text{methNm}(\text{meth}) = \text{.ctor} \text{ then}
\]

\[
\{(\text{pos} + 1, \argZeroT, \text{evalStackT} \cdot \{\text{argZeroT}\})\}
\]

\[
\text{else} \{(\text{pos} + 1, \argZeroT, \text{evalStackT} \cdot \{\text{argTypes}(\text{n})\})\}
\]

\[
\text{LoadFieldA}(\text{T}, \_) \rightarrow \{(\text{pos} + 1, \argZeroT, \text{pop}(\text{evalStackT}) \cdot \{\text{T}\})\}
\]

\[
\text{LoadInd}(\text{T}) \rightarrow \text{if} \ T = \text{object} \text{ then}
\]

\[
\text{let} \ T' : \text{C} = \text{top}(\text{evalStackT}) \text{ in}
\]

\[
\{(\text{pos} + 1, \argZeroT, \text{pop}(\text{evalStackT}) \cdot \{\text{T}'\})\}
\]

\[
\text{else} \{(\text{pos} + 1, \argZeroT, \text{pop}(\text{evalStackT}) \cdot \{\text{T}\})\}
\]

\[
\text{StoreInd}(\_) \rightarrow \{(\text{pos} + 1, \argZeroT, \text{pop}(\text{evalStackT}) \cdot \{\text{T}\})\}
\]

\[
\text{Box}(\text{T}) \rightarrow \text{if} \ T \in \text{RefType} \text{ then} \{(\text{pos} + 1, \argZeroT, \text{pop}(\text{evalStackT}) \cdot \{\text{T}\})\}
\]

\[
\text{else} \{(\text{pos} + 1, \argZeroT, \text{pop}(\text{evalStackT}) \cdot \{\text{boxed}(\text{T})\})\}
\]

\[
\text{Unbox}(\text{T}) \rightarrow \{(\text{pos} + 1, \argZeroT, \text{pop}(\text{evalStackT}) \cdot \{\text{T}\})\}
\]

\[
\text{UnboxAny}(\text{T}) \rightarrow \{(\text{pos} + 1, \argZeroT, \text{pop}(\text{evalStackT}) \cdot \{\text{T}\})\}
\]

\[
\text{MKRefAny}(\_) \rightarrow \{(\text{pos} + 1, \argZeroT, \text{pop}(\text{evalStackT}) \cdot \{\text{ typedref} \})\}
\]

\[
\text{RefAnyType} \rightarrow \{(\text{pos} + 1, \argZeroT, \text{pop}(\text{evalStackT}) \cdot \{\text{System.RuntimeTypeHandle} \})\}
\]

\[
\text{RefAnyVal}(\text{T}) \rightarrow \{(\text{pos} + 1, \argZeroT, \text{pop}(\text{evalStackT}) \cdot \{\text{T}\})\}
\]

\[
\text{CastClass}(\text{T}) \rightarrow \{(\text{pos} + 1, \argZeroT, \text{pop}(\text{evalStackT}) \cdot \{\text{T}\})\}
\]

\[
\text{IsInstance}(\text{T}) \rightarrow \{(\text{pos} + 1, \argZeroT, \text{pop}(\text{evalStackT}) \cdot \{\text{T}\})
\]

\[
\text{LoadFn}(\_) \rightarrow \{(\text{pos} + 1, \argZeroT, \text{evalStackT} \cdot \{\text{native int} \})\}
\]

\[
\text{LoadVirtFn}(\_) \rightarrow \{(\text{pos} + 1, \argZeroT, \text{pop}(\text{evalStackT}) \cdot \{\text{native int} \})\}
\]
• \( \text{evalStackT} \) is set to \([\text{OC}]\) since \text{UnInit} is replaced by \text{OC} (therefore, the call at code index 7 is verifiable) and

• \( \text{argZeroT} \) is set to \text{OC} (this guarantees that the return at code index 8 is verifiable)

The successor of a \text{CallVirt} instruction is defined similarly to the successor of a \text{Call} instruction for the case of a non-constructor method (we mentioned in Section 4.1.3.1 that the \text{.ctors} cannot be called with \text{CallVirt}).

If the \text{Call} instruction is used in a tail call, the function \text{succ} contains no successors since the current frame gets immediately discarded. Similarly, the \text{Return} instruction has no successors since its effect is to leave the current method.

The type on top of the stack state in the successor of the \text{NewObj} instruction is the class of the \text{.ctor} given in the instruction. The successors of the instructions \text{LoadField} and \text{LoadFieldA} use the declared type of the field given in the instructions. In our abstract instructions, the declared type is stored as the first parameter in the above instructions.

For the \text{LoadInd} instruction, we make a case distinction. If the instruction is used in the form \text{LoadInd}(\text{object}), \text{i.e.}, for loading an object reference, the type added to the stack state of the successor is the referent type of the topmost type\(^{14}\) of the current stack state. If the instruction is applied with a primitive type, this type is added to the stack state of the successor.

The successor of the instruction \text{Box} has on top of the stack state the boxed type corresponding to the instruction’s type token argument. The successors of the \text{Unbox.Any} and \text{Unbox} instructions have on top of the stack state the type referred to by the instruction’s type token and a pointer type whose referent type is the instruction’s type token, respectively.

A \text{typedref} is pushed onto the stack by the \text{MkRefAny} instruction. The instruction \text{RefAnyType} pushes onto the stack a \text{System.RuntimeTypeHandle}. The successor of the \text{RefAnyVal} instruction has on top of the stack state the pointer type whose referent type is the instruction’s type token. If the \text{RefAnyVal} instruction did not have a type token argument, the type placed on the stack by this instruction would have been unknown. In this case, the instruction would not have been verifiable. In contrast, the \text{RefAnyType} instruction does not require a type token argument as it always loads a type handle on the stack. On the other hand, the \text{RefAnyVal} instruction has to perform a run-time check.

The instruction \text{CastClass}(T) leaves on the stack an object of type \( T \). The instruction \text{IsInstance}(T) has two possible successors: Depending if the class of the object on the stack is a subtype of \( T \), \( T \) or the type \text{Null} are added to the stack state.

The instructions \text{LoadFtn} and \text{LoadVirtFtn} load onto the stack a \text{native int} representing a method pointer.

### 4.1.5 The Bytecode Verification Algorithm and Well-typed Methods

The bytecode verification algorithm, vaguely specified in [45], is described in [65]. Its formal specification, inspired by [115], is given in Figure 4.10. The verifier attempts to determine a valid type state for every instruction of a method. For that, the algorithm uses the dynamic functions described in Table 4.11. The set \( V \) of code indices visited by the algorithm, \text{i.e.}, the

---

\(^{14}\)For \text{LoadInd}, the topmost type of the stack state is assumed to be a pointer type.
Table 4.11: The dynamic functions of the bytecode verification algorithm.

<table>
<thead>
<tr>
<th>Function definition</th>
<th>Function name</th>
</tr>
</thead>
<tbody>
<tr>
<td>isVisited : Map(Pc, Bool)</td>
<td>visited positions</td>
</tr>
<tr>
<td>isPending : Map(Pc, Bool)</td>
<td>not-yet-verified positions</td>
</tr>
<tr>
<td>argZeroV : Map(Pc, VerificationType)</td>
<td>this’ inferred type</td>
</tr>
<tr>
<td>evalStackV : Map(Pc, List(VerificationType))</td>
<td>stack slots’ inferred types</td>
</tr>
</tbody>
</table>

indices that have already a type state assigned, is defined in terms of the function isVisited. The set $P$ of code indices that are still to be explored is defined through the function isPending. The indices in $P$ may be revisited with refined type states.

\[
V = \{ i \in Pc \mid \text{isVisited}(i) = \text{True} \}
\]
\[
P = \{ i \in Pc \mid \text{isPending}(i) = \text{True} \}
\]

**Initial Constraints** The following conditions should be satisfied in the initial state of the bytecode verification algorithm run for a method $mref$:

\[
V = \{0\}
\]
\[
P = \{0\}
\]
\[
\text{argZeroV}_0 = \begin{cases} 
\text{UnInit}, & \text{if classNm}(mref) \in \text{ObjClass} \setminus \{\text{object}\} \text{ and methNm}(mref) = .\text{ctor}; \\
\text{argTypes}(mref)(0), & \text{otherwise.}
\end{cases}
\]
\[
\text{evalStackV}_0 = []
\]

Initially, the type tracked for the zeroth argument of an object class $..ctor$ (other than $\text{object::..ctor}$) is $\text{UnInit}$. The stack state is empty upon the method entry.

When propagated, the type states are merged as described in Section 4.1.4.1: The types tracked for the zeroth arguments are merged to the least common supertype, while the stack states are similarly merged, but component-wise (see also Remark 4.1.8). We assume that as soon as $halt$ is set, the verification algorithm stops and consequently, the verified method is rejected by the algorithm.

**Remark 4.1.8** The least common supertype is not always unique, since an object class or an interface may implement multiple interfaces. Surprisingly, this aspect is not clarified in [45]. An elegant solution, described in detail by Stäck et al. [115, §16.1.2], is to allow finite sets of object types in the bytecode verification process. This approach requires extending the compatibility relation $\subseteq$ to such sets. Two sets are related through the new relation if for every type in the first set, there exists in the second set a supertype of the type in the first set. This relation
**Figure 4.10** The bytecode verification algorithm.

**ONESTEPVERIFICATION** ≡

\[
\text{choose } pos \in \mathcal{P} \text{ do}
\]

\[
\text{if check(mref, pos, argZeroV_{pos}, evalStackV_{pos})} \text{ then}
\]

\[
\text{PROPAGATE(pos)}
\]

\[
isPending(pos) := \text{undef}
\]

\[
\text{else } \text{halt} := \text{"Check failed"}
\]

**PROPAGATE(pos) ≡**

\[
\text{forall } (i, \argZeroT', \evalStackT') \in \text{succ(mref, pos, argZeroV_{pos}, evalStackV_{pos}) do}
\]

\[
\text{PROPAGATESucc}(i, \argZeroT', \evalStackT')
\]

**PROPAGATESucc(i, argZeroT', evalStackT') ≡**

\[
\text{if } \neg \text{isVisited}(i) \text{ then}
\]

\[
\text{if } 0 \leq i \land i < \text{length(code(mref))} \text{ then}
\]

\[
\argZeroV_{i} := \argZeroT'
\]

\[
\evalStackV_{i} := \evalStackT'
\]

\[
isVisited(i) := \text{True}
\]

\[
isPending(i) := \text{True}
\]

\[
\text{else } \text{halt} := \text{"Invalid code index"}
\]

**elseif argZeroT' ⊑ argZeroV_i ∧ evalStackT' ⊑ len evalStackV_i then**

\[
\text{skip}
\]

**elseif argZeroT' and argZeroV_i have a common supertype ∧**

\[
\text{length(evalStackT')} = \text{length(evalStackV_i)} \land
\]

\[
\text{evalStackT'}(k) \text{ and evalStackV}_i(k) \text{ have a common supertype, for every } k = 0, \text{length(evalStackT') − 1}
\]

\[
\text{then}
\]

\[
\argZeroV_{i} := \text{sup}(\argZeroT', \argZeroV_i)
\]

\[
\evalStackV_{i} := [\text{sup}(\text{evalStackT'}(k), \text{evalStackV}_i(k)) \mid k = 0, \text{length(evalStackT') − 1}]
\]

\[
isPending(i) := \text{True}
\]

**else**

\[
\text{halt} := \text{"Propagation not possible"}
\]

could be introduced without harm into our approach. To keep the presentation simple, we do not consider it here explicitly though the bytecode verification "merge" operations mutually assume it. It must, however, be emphasized that the verifier’s soundness and completeness proofs would follow absolutely the same lines if sets of object types were introduced in the verification type system.

**Remark 4.1.9** The bytecode verification algorithm terminates. This can be easily shown by taking into account that the number of elements in the state of the algorithm cannot exceed the total number of bytecode instructions, and with each algorithm iteration the type state of at least one instruction is "increased" according to the ordering ⊑ (upon "merging"), and this ordering is finite.

**Example 4.1.8** To illustrate the merging of stack states, we consider the example in Figure 4.11: a bytecode fragment on the left and stack states on the right. Let us assume that
4.1. The Lightweight CLR

Figure 4.11 Example: merging stack states.

```
.method public hidebysig specialname rtspecialname
    instance void .ctor() cil managed
{
    .maxstack 4
    .locals init (class OC 'V_0, class OC V_1, int32 V_2)
    0: ldarg 0
    1: ldloc 1 [UnInit]
    2: ldloc 2 [OC, UnInit]
    3: ldc.i4 0 [int32, OC, UnInit]
    4: bgt 9 [int32, int32, OC, UnInit]
    5: pop [OC,UnInit]
    6: dup [UnInit]
    7: call instance void System.Object::.ctor() [UnInit,UnInit]
    8: ldloc 0 [OC'']
    9: stfld OC OC''::F [OC,OC'']
   10: ret [OC,OC'']
} // end of method OC''::.ctor
```

This fragment is the body of a constructor OC''::.ctor of an object class OC'', derived from object, which declares a field OC''::F of type the object class OC. Moreover, we assume that OC' is an object class that extends OC.

The instruction at 9 is a merge point. More precisely, the instruction stfld OC OC''::F is the successor of two instructions, namely bgt 9 and ldloc 0. The stack state after the instruction bgt 9 is [OC, UnInit], whereas the stack state following the instruction ldloc 0 is [OC', OC'']. The two stack states cannot be merged since the verification types OC'' and UnInit have no common supertype. So, this constructor violates the object initialization rules, and therefore, it is not considered well-typed. Basically, as the this pointer is not initialized on a flow path to 9, the bytecode verification cannot determine the initialization status at the merge point 9.

If the this pointer were initialized before the branch instruction, the stack states would have been [OC, OC''] and [OC', OC'']. As OC' ⊆ OC, the two stack states can be merged into [OC, OC''], and thus, the constructor would be regarded as well-typed.

We need a characterization of the type properties satisfied by the methods accepted by the bytecode verification. For this purpose, we introduce the notion of well-typed methods in Definition 4.1.9. We say that a method is well-typed if it is possible to assign a valid type state to every instruction of the method. More precisely, a method is well-typed if there exists a type state family that satisfies certain initial conditions, the type-consistency checks defined in Section 4.1.4.3 for all instructions as well as the relations dictated by execution simulation of the bytecode (see the function succ defined in Section 4.1.4.4) and by the rules for merging type states specified in [45, Partition III, §1.8.1.3].

**Definition 4.1.9 (Well-typed)** A method mref is well-typed if there exists a family of type states $(\text{argZeroT}_i, \text{evalStackT}_i)_{i \in \mathcal{D}}$ over the domain $\mathcal{D}$ which satisfies the following conditions:

---

15 The "supertype" is defined in terms of the compatibility relation $\sqsubseteq$ for verification types.
The elements of $D$ are valid code indices of $mref$.

$0 \in D$.

If $\text{classNm}(mref) \in \text{ObjClass} \setminus \{\text{object}\}$ and $\text{methNm}(mref) = .\text{ctor}$, then $\text{argZeroT}_0 = \text{UnInit}$. Otherwise, $\text{argZeroT}_0 = \text{argTypes}(mref)(0)$.

$\text{evalStackT}_0 = []$.

If $i \in D$, then check($mref$, $i$, argZeroT$_i$, evalStackT$_i$) is true.

If $i \in D$ and $(j, \text{argZeroT'}_i, \text{evalStackT'}_i) \in \text{succ}(mref, i, \text{argZeroT}_i, \text{evalStackT}_i)$, then $j \in D$, argZeroT'$_i \sqsubseteq \text{argZeroT}_j$ and evalStackT'$_i \sqsubseteq \text{len evalStackT}_j$.

The domain $D$ of the family denotes the code indices which are reachable from the code index 0. (wt1) states that $D$ consists of valid code indices only and (wt2) says that the code index 0 is in the domain. (wt3) and (wt4) mention conditions for the type state of the code index 0. Thus, the type tracked for the zeroth argument of an object class $\text{ctor}$ (other than object::ctor) is initially considered UnInit. The evaluation stack has to be empty upon the method entry. (wt5) ensures that the type states satisfy all type-consistency checks. (wt6) says that a successor type state has to be more specific than the type state corresponding to the successor index. In particular, this means that the stack state asserted in the successor should be more specific, but of the same length as the stack state associated in the type state. The reasons for these conditions are to be found in the definition of the stack state “merging” described in Section 4.1.4.1.

Given a method, the bytecode verification algorithm should decide whether it is possible to assign a type state to every instruction of the method. In the positive case, the algorithm computes a most specific family of type states the method is well-typable with. Definition 4.1.10, inspired by [115], specifies what we mean by ”more specific”:

**Definition 4.1.10 (More specific type states)** A type state family $(\text{argZeroV}_i, \text{evalStackV}_i)_{i \in V}$ is more specific than a family of type states $(\text{argZeroT}_i, \text{evalStackT}_i)_{i \in D}$ if the following conditions are met:

- $V \subseteq D$,
- $\text{argZeroV}_i \sqsubseteq \text{argZeroT}_i$, for every $i \in V$,
- $\text{evalStackV}_i \sqsubseteq \text{len evalStackT}_i$, for every $i \in V$.

### 4.1.6 Type Safety

If a method is legal, well-typed and, in addition, the bytecode structure satisfies the restrictions stipulated in Section 4.1.4.2, then several properties are guaranteed to hold at its run-time execution:
4.1. The Lightweight CLR

**Type safety:** Every instruction executes with arguments of expected types. The evaluation stack has values of the types assigned in the stack state and the same length as the stack state. The values stored in the local variables and arguments are of the types given in the “local signature” and method signature, respectively. The objects on the heap, including the boxed objects, are of the expected static types. The fields of the objects on the heap have values of the corresponding declared types.

**Object initialization:** Every object on the heap is fully initialized before is used properly.

**Bounded evaluation stack:** The evaluation stack does not exceed the upper bound specified by the method. No instruction attempts to pop a value from an empty evaluation stack.

**Code containment:** The program counter never leaves the code array of the method. In particular, it must not fall off the end of the method’s code.

To reason if a value is typable with a type, we introduce a typing judgment. As pointed out in Section 4.1.2, the values are tagged. The type of a value that is neither a pointer nor a value class nor a typed reference, i.e., a value that is not tagged with & or a value class or typedref, is given by the function type. For a primitive type, type simply returns the tag tagType.

For an object reference, type is given by the function verifType, which, based on the values of initState, assigns to every object reference its verification type.

\[
\text{type}(val) = \text{tagType}(val), \text{if tagType}(val) \notin \{\text{ref, &}\} \cup \text{ValueClass}
\]

\[
\text{type}(val) = \text{verifType}(val), \text{if tagType}(val) = \text{ref}
\]

\[
\text{verifType}(ref) = \begin{cases} 
\text{Null} & \text{if ref} = \text{null} \\
\text{match initState}(ref) & \\
\text{InProgress}(T) \to \text{UnInit} \\
\text{Init}(T) \to T 
\end{cases}
\]

The most interesting part of the typing judgment concerns the pointer types and the type typedref: When is a pointer of type T&? when is a typed reference of type typedref? To answer these questions, we need the predicate isAddressIn, defined similarly as in the C♯’s type safety proof (see Definition 3.6.3 in Chapter 3). Thus, given two types \(T, T'\) and two addresses \(adr\) and \(adr'\), \(\text{isAddressIn}(adr, T, adr', T')\) holds if a value of type \(T\) is stored at \(adr\), which “is an address in” the memory block pointed to by \(adr'\), where a value of type \(T'\) is stored.

**Definition 4.1.11** For the types \(T, T'\) and the addresses \(adr, adr'\), we define

\[
\text{isAddressIn}(adr, T, adr', T') : \equiv \\
(adr = adr' \land T = T') \lor \\
(T' \in \text{ValueClass} \land \\
\exists T'::F \in \text{instFields}(T') : \text{isAddressIn}(adr, T, \text{fieldAdr}(adr', T'::F), \text{fieldType}(T'::F)))
\]
The next definition introduces the typing rules. The typing is with respect to the heap actualTypeOf, the current frame, and the frameStack.

**Definition 4.1.12 (Typing judgment)** Let val be a value, T a verification type and \([fr_1, \ldots, fr_k]\) a list of frames. The typing \([fr_1, \ldots, fr_k] \vdash val : T\) holds if at least one of the following conditions is met:

- \(T \in \text{PrimitiveType} \cup \text{RefType} \cup \{\text{UnInit}, \text{Null}\}\) and type(val) \(\sqsubseteq T\), or
- \(T \in \text{ValueClass}\), val \(\in \text{Map}(\text{instFields}(T), \text{Val})\), and for every \(T::F \in \text{instFields}(T)\), \([fr_1, \ldots, fr_k] \vdash val(T::F) : \text{ fieldType}(T::F)\), or
- val \(\in \text{ObjRef}\) and there exists \(T' \in \text{Value Type}\) such that actualTypeOf(val) = \(T'\) and boxed(T') \(\sqsubseteq T\), or
- \(T\) is typedref and \([fr_1, \ldots, fr_k] \vdash \text{typedRefAdr}(val) : \text{ typedRefType}(val)\&\&\&\,\text{or}\), or
- \(T\) is a pointer type \(T'\&\&\&\) and at least one of the following conditions is satisfied:

\[
\begin{align*}
\text{(tj-null)} & \quad \text{val is null, or} \\
\text{(tj-loc)} & \quad \text{there exists } i = 1, k \text{ and } n = 0, \text{locNo}(\text{meth}(fr_i)) - 1 \text{ such that isAddressIn(val, } T', \text{locAdr}(fr_i)(n), \text{locTypes}(\text{meth}(fr_i))(n) \text{) holds, or} \\
\text{(tj-pool)} & \quad \text{there exists } i = 1, k \text{ and } (val', T'n) \in \text{locPool}(\text{meth}(fr_i)) \text{ such that isAddressIn(val, } T', val', T'n) \text{ holds, or} \\
\text{(tj-arg)} & \quad \text{there exists } i = 1, k \text{ and } n = 0, \text{argNo}(\text{meth}(fr_i)) - 1 \text{ such that isAddressIn(val, } T', \text{argAdr}(fr_i)(n), \text{argTypes}(\text{meth}(fr_i))(n) \text{) holds, or} \\
\text{(tj-field)} & \quad \text{there exists } OC \in \text{ObjClass}, OC'::F \in \text{instFields}(OC) \text{ and ref } \in \text{ObjRef} \text{ such that isAddressIn(val, } T', \text{fieldAdr(ref, } OC'::F), \text{fieldType}(OC'::F) \text{) holds and actualTypeOf(ref) = } OC, \text{ or} \\
\text{(tj-box)} & \quad \text{there exists } T'' \in \text{Value Type} \text{ and ref } \in \text{ObjRef} \text{ such that actualTypeOf(ref) = } T'' \text{ and isAddressIn(val, } T', \text{addressOf(ref), } T'') \text{ holds.}
\end{align*}
\]

A value is typable with a value class if is a mapping of the value class instance fields into values of the field declared types. A typed reference is typable with typedref if the pointer embedded in the typed reference is typable with the pointer type whose referent type is embedded in the typed reference. A pointer \(adr\) is typable with \(T = T'\&\&\&\) if \(adr\) is: null\(^{16}\), the address of a local variable or argument of a frame (the current frame or a frame on the frameStack), or the address of an object class instance field of declared type \(T'\), or the address of a value type in a boxed value on the heap, or an address typable with \(T\) in a memory block pointed to by one of the addresses of the above locations.

The method pointers are not treated specially by Definition 4.1.12 since they are regarded as values tagged with native int. It is then immediate that they are typable with native int.

\(^{16}\)A null pointer can only occur when a local variable, whose declared type is a pointer type, is accessed in its default state, resulted upon the method’s entry. Given the issue pointed out in Remark 4.1.1, this typing ensures the typing of such a local variable.
Remark 4.1.10 Through the definition of the typing judgment, it should become clear why do we have to consider the local memory pool in our framework: Without locPool, the obviously correct typing of the managed pointers to the value class instances (constructed with NewObj) would have got ”lost”. It would then have been impossible to prove the correct typing of the this pointer in a value class .ctor, invoked through a NewObj instruction.

For the type safety proof, we need several lemmas. The next lemma claims that after storing a value of type \( T \) at an address \( \text{adr} \) of a memory block, sufficient to hold values of a \( T' \)’s supertype, the value read at \( \text{adr} \) is of type \( T \).

Lemma 4.1.3 Let \( T \) and \( T' \) be types such that \( T \sqsubseteq T' \). We assume that \( \left[ fr_1, \ldots, fr_k \right] \vdash val : T \) and \( \left[ fr_1, \ldots, fr_k \right] \vdash \text{adr} : T' \&. \) Then, after the macro \( \text{WRITEMEM}(\text{adr}, T', \text{val}) \) is executed, we have \( \left[ fr_1, \ldots, fr_k \right] \vdash \text{memVal}(\text{adr}, T) : T \).

Proof. By induction on the structure of the possibly value class \( T \), using Lemma 4.1.1. \( \square \)

Let \( \text{adr}'' \) be an address of a memory block, sufficient to hold values of a type \( T'' \). Assume that, in this block, \( \text{adr}' \) is an address where a value of type \( T' \) is stored. Storing a value of type \( T' \) at \( \text{adr}' \) raises the following concern: Is the value stored at \( \text{adr}'' \) still of type \( T'' \)? The following lemma addresses this issue.

Lemma 4.1.4 If \( \left[ fr_1, \ldots, fr_k \right] \vdash val : T', \left[ fr_1, \ldots, fr_k \right] \vdash \text{memVal}(\text{adr}'', T'') : T'', T' \sqsubseteq T \) and isAddressIn(\( \text{adr}'', T', \text{adr}'', T'' \)), then the execution of \( \text{WRITEMEM}(\text{adr}', T, \text{val}) \) preserves \( \left[ fr_1, \ldots, fr_k \right] \vdash \text{memVal}(\text{adr}'', T'') : T'' \).

Proof. By induction on the definition of the isAddressIn predicate. The base case of the induction is proved by applying Lemma 4.1.3. \( \square \)

Definition 4.1.12 does not contain a subsumption rule. However, this can be easily derived: If \( \text{val} \) is typable with \( T \) and \( T \) is compatible with \( T' \), then \( \text{val} \) can also be typable with \( T' \).

Lemma 4.1.5 If \( \left[ fr_1, \ldots, fr_k \right] \vdash val : T \) and \( T \sqsubseteq T' \), then \( \left[ fr_1, \ldots, fr_k \right] \vdash val : T' \).

Proof. By induction on the definition of \( \vdash \). \( \square \)

The following lemma establishes that if the address of a value class instance ”is an address in” a memory block, then also the address of each instance field of the value class instance ”is an address in” that memory block.

Lemma 4.1.6 If \( \text{VC} \in \text{ValueClass} \) and isAddressIn(\( \text{adr}, \text{VC}, \text{adr}', T' \)), then

\[ \text{isAddressIn(fieldAdr(adr, VC::F), fieldType(VC::F), adr', T')} \]

for every \( \text{VC::F} \in \text{instFields}(\text{VC}) \).

Proof. By case distinction, applying Definition 4.1.11. \( \square \)
The next lemma says that the typing is preserved upon pushing new frames. The typing with
a non-pointer type other than typedref is also preserved upon popping frames.

**Lemma 4.1.7** Assume that \([fr_1, \ldots, fr_k] \vdash val : T\). If \(fr\) is a frame, then \([fr_1, \ldots, fr_k, fr] \vdash val : T\). If \(T\) is neither a pointer type nor typedref, then \([\cdot] \vdash val : T\) and \([fr_1, \ldots, fr_i] \vdash val : T\) for every \(i = 1, k\).

**Proof.** In Definition 4.1.12, the list of frames is relevant only for \((tj-loc), (tj-arg)\), and the
typing of a typed reference. However, in these cases, \(T\) is a pointer type or typedref. ⊓ ⊔

The following two lemmas are required to ensure type safety of dynamically dispatched
method calls. Lemma 4.1.8 relates the argument types (including the type of the this
pointer) of the called method and the invoked one. Similarly, Lemma 4.1.9 establishes the relation
between return types.

**Lemma 4.1.8** If \(T''::M = lookUp(T, T'::M)\) holds for a type \(T\), then

- If \(T \in \text{ValueType}\), then
  - if \(T'' \neq T\), then \(boxed(T) \subseteq \text{argTypes}(T''::M)(0) \subseteq \text{argTypes}(T'::M)(0)\)
  - if \(T'' = T\), then \(\text{argTypes}(T''::M)(0) = T\&\) and \(boxed(T) \subseteq \text{argTypes}(T'::M)(0)\)

- \(\text{argNo}(T'::M) = \text{argNo}(T''::M)\) and for every \(i = 1, \ldots, \text{argNo}(T'::M) - 1\)

\[
\text{argTypes}(T'::M)(i) = \text{argTypes}(T''::M)(i)
\]

**Proof.** By Definition 4.1.4 and (at) in the group of constraints [override/implement]. ⊓ ⊔

**Lemma 4.1.9** If \(T''::M = lookUp(T, T'::M)\) holds for a type \(T\), then \(\text{retType}(T''::M) = \text{retType}(T'::M)\).

**Proof.** By Definition 4.1.4 and (at) in the group of constraints [override/implement]. ⊓ ⊔

We say that a frame contains an object reference \(ref\) with the assigned type UnInit if \(ref\)
is the this pointer for which the bytecode verification expects the type UnInit or \(ref\) is in an
evaluation stack slot, where the verification expects UnInit.

**Definition 4.1.13** Let \(fr = (pc^*, \text{locAdr}^*, \text{locPool}^*, \text{argAdr}^*, \text{evalStack}^*, \text{meth}^*)\) be a frame and
\((\text{argZeroT}_j, \text{evalStackT}_j)_{j \in D}\) be a family of type states \(\text{meth}^*\) is well-typable with. Let \(\text{argVal}^*\)
be the function \(\text{argVal}\) determined based on \(\text{argAdr}^*\). The frame \(fr\) contains a reference \(ref\) with
the assigned type UnInit if one of the following conditions is fulfilled:

- \(\text{argVal}^*(0) = \text{ref}\) and \(\text{argZeroT}_{pc^*} = \text{UnInit}, or\)
4.1. The Lightweight CLR

- there exists \( i = 0 \), \( \text{length}(\text{evalStack}^*) - 1 \) with \( \text{evalStack}^*(i) = \text{ref} \) and \( \text{evalStack}_{\text{pc}^*}(i) = \text{UnInit} \).

In the theorem asserting type safety of well-typed methods, we prove invariants for frame as well as for the frames on the frameStack. However, an invariant for a frame \( fr \) in the frameStack has to be considered as \( fr \) would be the current frame. But, when \( fr \) becomes the current frame, the initialization status of certain object references as well as the evalStack of \( fr \) might have been changed. This remark triggers the next definitions, inspired by [115].

**Definition 4.1.14 (Succession of a frame)** Let the list \( [fr_1, \ldots, fr_k] \) be the frameStack and \( fr_i = (pc^*, \text{locAdr}^*, \text{locPool}^*, \text{argAdr}^*, \text{evalStack}^*, \text{meth}^*) \) be an arbitrary frame in the frameStack. If \( mref \) is method reference of \( fr_i \)'s callee frame (for \( fr_k \), the callee frame is frame), the succession of \( fr_i \) is defined as follows:

- If \( mref \) returns a value, say \( \text{val} \), with \( [fr_1, \ldots, fr_i] \vdash \text{val} : \text{retType}(mref) \), then \( (pc^*+1, \text{locAdr}^*, \text{locPool}^*, \text{argAdr}^*, \text{evalStack}^*.[\text{val}], \text{meth}^*) \) is called the succession of \( fr_i \) in frameStack.

- If \( \text{retType}(mref) \) is void, then the succession of \( fr_i \) in frameStack is given by \( (pc^* + 1, \text{locAdr}^*, \text{locPool}^*, \text{argAdr}^*, \text{evalStack}^*, \text{meth}^*) \).

**Definition 4.1.15 (Succession of the initialization status)** A function init\( \text{State}^* \) is the succession of init\( \text{State} \) for a frame \( fr \) in the frameStack if init\( \text{State}^* \) is defined as follows.

- If \( fr \) is immediately followed in the frameStack by an object class \( .\text{ctor} \) with the this pointer \( \text{ref} \), then init\( \text{State}^*(\text{ref}') = \text{initState}(\text{ref}) \) for every reference \( \text{ref}' \neq \text{ref} \), and init\( \text{State}^*(\text{ref}) = \text{Init}(OC) \), where \( OC \) is the object class given by init\( \text{State}(\text{ref}) = \text{InProgress}(OC) \).

- If \( fr \) is not followed by an object class \( .\text{ctor} \), then init\( \text{State}^* \) coincides with init\( \text{State} \).

Theorem 4.1.1 proves type safety of legal and well-typed methods. Thus, the following invariants are guaranteed to hold at run-time. The invariant (pc) implies that the program counter is always a valid code index. By (stack1), we know that the evalStack has the same length as the assigned stack state and will never overflow. That the values on the evalStack are of the types assigned in the stack state is ensured by (stack2). By (loc) and (arg), we have that the local variables and arguments contain values of the declared types. In case of a .ctor, the value of the this pointer is of the type assigned in the type state. The invariant (init) makes precise when a value has the type UnInit assigned in the type state. The invariant (field) ensures that the fields of an object class instance are of the declared types. That the value type instance embedded in a boxed value is of the expected value type is guaranteed by the invariant (box). The invariant (del) is similar with corresponding invariant for C# in Theorem 3.6.1. It claims that every target object in the invocation list of a delegate is an object of an appropriate type, i.e., a subtype of the type that declares the corresponding target method. It also guarantees that the delegate class is compatible, according to Definition 4.1.8, with the target method.
Theorem 4.1.1 (Type safety of well-typed methods) Let \( [fr_1', \ldots, fr_{k-1}'] \) be the frameStack and \( fr_k \) be the current frame. For every \( i = 1, k - 1 \), we denote by \( fr_i \) the succession of \( fr_i' \).

We assume that the type system and the bytecode structure of the methods in \( (fr_i) \), satisfy the conditions stated in Section 4.1.4.2. Moreover, we assume that all methods of \( (fr_i) \) are well-typed.

Let \( fr_i = (pc^*, locAdr^*, \_, argAdr^*, evalStack^*, \_meth^*) \) be one of the frames \( (fr_i)^k_{i=1} \). We denote by \( \text{locVal}^* \) and \( \text{argVal}^* \) the functions \( \text{locVal} \) and \( \text{argVal} \) derived based on \( \text{locAdr}^* \) and \( \text{argAdr}^* \), respectively. If \( fr_i \) is the current frame, we consider \( \text{initState}^* \) to be the current initialization status \( \text{initState} \). If \( fr_i \) is not the current frame, we denote by \( \text{initState}^* \) the succession of \( \text{initState} \) for \( fr_i \). Let \( (\text{argZeroT}_j, \text{evalStackT}_j)_{j \in D} \) be a family of type states \( \_meth^* \) is well-typable with.

The following invariants are satisfied at run-time for every frame \( fr_i \) (for the current frame, \( fr_{k-1} \), the invariants \( \text{stack1} \) and \( \text{stack2} \) should only hold if switch = Noswitch):

\( \text{(pc)} \quad pc^* \in D. \)

\( \text{(stack1)} \quad \text{length}(\text{evalStack}^*) = \text{length}(\text{evalStackT}_{pc^*}) \leq \max\text{EvalStack}(\_meth^*). \)

\( \text{(stack2)} \quad [fr_1, \ldots, fr_i] \vdash \text{evalStack}^*(j) : \text{evalStackT}_{pc^*}(j), \quad \text{for every } j = 0, \text{length}(\text{evalStack}^*) - 1. \)

\( \text{(loc)} \quad [fr_1, \ldots, fr_i] \vdash \text{locVal}^*(n) : \text{locTypes}(\_meth^*)(n), \quad \text{for every } n = 0, \text{locNo}(\_meth^*) - 1. \)

\( \text{(arg)} \quad \text{If } \text{classNm}(\_meth^*) \in \text{ObjClass} \text{ and } \text{methNm}(\_meth^*) = \_\text{ctor}, \text{then it holds } [fr_1, \ldots, fr_i] \vdash \text{argVal}^*(0) : \text{argZeroT}_{\_meth^*}; \text{otherwise, } [fr_1, \ldots, fr_i] \vdash \text{argVal}^*(0) : \text{argTypes}(\_meth^*)(0). \)

If \( \text{argNo}(\_meth^*) \geq 2 \), then \( [fr_1, \ldots, fr_i] \vdash \text{argVal}^*(n) : \text{argTypes}(\_meth^*)(n), \text{for every } n = 1, \text{argNo}(\_meth^*) - 1. \)

\( \text{(init)} \quad \text{If } fr_i \text{ contains } \text{ref} \in \text{ObjRef} \text{ with the assigned type UnInit, then } \_meth^* \text{ is an object class } \_\text{ctor} \text{ (not of class } \text{object}), \text{ref} = \text{argVal}^*(0), \text{and } \text{initState}^*(\text{ref}) = \text{InProgress}(\text{OC}), \text{where } \text{OC} \text{ is an object class such that } \text{OC} \preceq \text{classNm}(\_meth^*). \)

\( \text{(field)} \quad \text{If } \text{ref} \in \text{ObjRef} \text{ is such that } \text{actualTypeOf}(\text{ref}) = \text{OC}, \text{where } \text{OC} \in \text{ObjClass}, \text{and } \text{OC}'::F \in \text{instFields}(\text{OC}) \text{ has the declared type } T, \text{then } \)

\[ [\_] \vdash \text{memVal}(\text{fieldAdr}(\text{ref}, \text{OC}'::F), T) : T \]

\( \text{(box)} \quad \text{If } \text{ref} \in \text{ObjRef} \text{ is such that } \text{actualTypeOf}(\text{ref}) = T, \text{where } T \in \text{ValueType}, \text{then } [\_] \vdash \text{memVal}(\text{addressOf}(\text{ref}), T) : T. \)

\( \text{(del)} \quad \text{If } \text{ref} \in \text{ObjRef} \text{ is such that } \text{actualTypeOf}(\text{ref}) = \text{DC}, \text{where } \text{DC} \in \text{DelegateClass}, \text{then the following hold for every } (\text{ref}', T::M) \in \text{invocationList}(\text{ref}): \)
• ref′ ∈ ObjRef
• actualTypeOf(ref′) ⊆ T.
• DC is compatible with T::M.

Proof. The invariants are proved by induction on the run of the operational semantics model defined in Section 4.1.3.1.

Base case In the initial state of the machine CLR_L, the invariants for the single existing frame (of the .entrypoint method) are satisfied as follows. (pc) holds since pc = 0 and 0 ∈ D (from Definition 4.1.9 (wt2)). The invariants (stack1) and (stack2) follow from evalStack = [] and Definition 4.1.9 (wt4). Upon the method’s entry, the virtual machine stores, by means of WRITEMEM, ”zeros” of the appropriate types at the local variable addresses locAdr. By this and Lemma 4.1.3, we derive the invariant (loc). The other invariants obviously hold.

Induction step We assume that all invariants are satisfied in the current state of CLR_L, and we prove that invariants are preserved after CLR_L executes a step. Different cases have to be considered depending on code(pc). For brevity, we only consider here a few cases.

Case 1 code(pc) = StoreInd(T)

By (pc) and Definition 4.1.9 (wt5), there exists evalStackT′, T′ and T′′ such that

\[ \text{evalStackT}' \cdot [T'&; T''] = \text{evalStackT}_pc \]  \quad (4.1)

and

\[ T'' \subseteq T' \subseteq T \]  \quad (4.2)

By (4.1), (4.2), (stack1) and (stack2), we get that the evalStack is of the form evalStack' \cdot [adr, val], where [fr_1, \ldots, fr_k] ⊢ adr : T'&; [fr_1, \ldots, fr_k] ⊢ val : T'' and

\[ [fr_1, \ldots, fr_k] ⊢ \text{evalStack}'(j) : \text{evalStack}'(j), \text{ for every } j = 0, \text{length}(	ext{evalStack}') - 1 \]  \quad (4.3)

Definition 4.1.9 (wt6) implies

\[ \text{evalStackT}' \sqsubseteq_{\text{len}} \text{evalStackT}_pc + 1 \]  \quad (4.4)

When the instruction StoreInd(T) is executed, the new evaluation stack is evalStack' and WRITEMEM(adr, T, val) is fired. The invariants (stack1) and (stack2) are implied by (4.3) and (4.4).

None of the real CIL stind instructions defined in [45, Partition III, §3.62] has a pointer type or typedef as a type token argument. Therefore, T is neither a pointer type nor typedef. By this and Lemma 4.1.2, we get that T'' is neither a pointer type nor typedef. Concerning the typing of adr, by Definition 4.1.12, we have that adr is null (tj-null), or “is an address in” the memory block pointed to by the address of a local variable (tj-loc), or of an
argument \((\text{tj-arg})\), or of an instance field of an object reference \((\text{tj-field})\) or of a value embedded in a boxed value \((\text{tj-box})\), or “is an address in” the memory block pointed to by an address allocated in the local memory pool \((\text{tj-pool})\). Since all cases are similar, we only treat here one. Let this be \((\text{tj-loc})\). It means that there exists a frame \(fr_i\) and a local variable \(n\) in the frame \(fr_i\) such that isAddressIn\((adr,T',locAdr(fr_i))(n),\text{locTypes}(\text{meth}(fr_i))(n))\) holds. Except for \((\text{loc})\) corresponding to the frame \(fr_i\), no invariant is affected by WRITEMEM. Therefore, by the induction hypothesis, all invariants except \((\text{loc})\) hold in the next execution step of CLR\(_C\). For \((\text{loc})\), we prove \([fr_1,...,fr_i]\vdash \text{locVal}(fr_i)(n) : \text{locTypes}(\text{meth}(fr_i))(n)\). By Lemma 4.1.7, we have \([fr_1,...,fr_i]\vdash val : T''\) since \(T''\) is neither a pointer type nor typedref. By (4.2) and Lemma 4.1.5, we get \([fr_1,...,fr_i]\vdash val : T'.\) Lemma 4.1.4 says that the execution of WRITEMEM\((adr,T,val)\) preserves the typing \([fr_1,...,fr_i]\vdash \text{locVal}(fr_i)(n) : \text{locTypes}(\text{meth}(fr_i))(n)\), and this concludes the proof of this case.

Case 2  \(\text{code}(pc) = Call(tail,T',T::M)\)

Subcase 2.1  Let us first assume that the call is tail, i.e., \(\text{tail} = True\), and \(T::M\) is not a .ctor. From the invariant \((\text{pc})\) and Definition 4.1.9 (wt5), we obtain that check\((\text{meth},pc,\text{argZeroT}_p,\text{evalStackT}_p)\) holds. Therefore, we have

\[
\text{length(\text{evalStackT}_p) = argNo(\text{meth})} \quad (4.5)
\]

and

\[
\text{evalStackT}_p \sqsubseteq_{\text{argTypes}} \text{argTypes}(T::M) \quad (4.6)
\]

The relation (4.6) follows from the definition of argZeroCompat and the definition of argTypes in terms of paramTypes.

Let \([T_0,...,T_{n-1}]\) be the list of argument types argTypes\((T::M)\). Since the call is tail, we know that none of \((T_j)_{j=0}^{n-1}\) is a pointer type or typedref (see the constraints in Section 4.1.4.2).

The relation (4.5) implies that evalStackT\(_p\) should have the length \(n\). We then derive that evalStackT\(_p\)(\(j\)) \(\sqsubseteq T_j\), for \(j = 0,n - 1\). The invariants (stack1) and (stack2) ensure that the evalStack contains the values \([val_0,...,val_{n-1}]\) such that \([fr_1,...,fr_k]\vdash val_j : \text{evalStackT}_p(j),\) for \(j = 0,n - 1\). By Lemma 4.1.5, we get \([fr_1,...,fr_k]\vdash val_j : T_j,\) for \(j = 0,n - 1\). By this and Lemma 4.1.7, we have

\[
[fr_1,...,fr_{k-1}]\vdash val_j : T_j \text{ for every } j = 0,n - 1 \quad (4.7)
\]

since no \((T_j)_{j=0}^{n-1}\) is a pointer type or typedref.

When the Call instruction is executed, the current frame \(fr_k\) is discarded and not pushed onto the frameStack. The semantics model CLR\(_C\) does one intermediate step between the time the Call instruction is encountered and the set up of the new current frame. This step consists in updating evalStack to [] and switch to Invoke\((True,T::M,(val_j)_{j=0}^{n-1})\). After these updates are fired, (stack1) and (stack2) do not hold anymore since evalStackT\(_p\) remained unchanged, whereas evalStack became []. However, as switch is not Noswitch, one does not have to prove these invariants. Actually, the theorem does not claim that these invariants hold when switch \(\neq\) Noswitch since the above intermediate step is a model specific step, which cannot be observed in the runs of the real CLR (note that switch is a model specific global variable).
After the intermediate step, the new current frame, maintained in frame, is defined as follows: the pc is 0, the evalStack is [], the local variables hold “zeros” of the appropriate types, and the arguments are (val_j)_{j=0}^{n-1}.

The invariants (pc), (stack1) and (stack2) trivially hold for the new current frame. Upon the method’s entry, a WRITEMEM is executed (by means of the macro ALLOCARGLOC) to write the values (val_j)_{j=0}^{n-1} to argAdr. By (4.7) and Lemma 4.1.7, we have \([fr_1, \ldots, fr_{k-1}, frame] \vdash val_j : T_j\), for \(j = 0, n − 1\). It then follows from Lemma 4.1.3 that (arg) holds for frame.

After setting the new current frame, the callee’s frame of \(fr_{k-1}\) is the frame for executing \(T::M\) and not the previously discarded frame. Therefore, we also have to show the invariant (stack2) for the new succession of \(fr_{k-1}'\). If the return type \(T'\) of \(T::M\) is void, (stack2) follows from the induction hypothesis. If \(T'\) is not void, let \(val\) be the return value assumed to satisfy \([fr_1, \ldots, fr_{k-1}] \vdash val : \text{retType}(T::M)\). From Section 4.1.4.2, we know that \(\text{retType}(T::M) \sqsubseteq \text{retType}(\text{meth})\). By this and Lemma 4.1.5, we get \([fr_1, \ldots, fr_{k-1}] \vdash val : \text{retType}(\text{meth})\).

The invariant (stack2) follows then from the induction hypothesis, Definition 4.1.9 (wt6), and Lemma 4.1.5.

**Subcase 2.2** Let us suppose that the call is not tail, \(T\) is an object class, and \(M\) is a .ctor. In this case, the return type \(T'\) is void. By the induction hypothesis (pc) and Definition 4.1.9 (wt5), there exists evalStack\(T'\), \(T''\), and \(L\) such that \(\text{evalStack}T' \cdot [T''] \cdot L = \text{evalStack}_{\text{pc}}, L \sqsubseteq \text{paramTypes}(T::\text{ctor})\), and initCtorCompat(meth, \(T''\), \(T\)). By the definition of initCtorCompat, we obtain that \(T'' = \text{UnInit}\) or \(T'' \sqsubseteq T\) and \(\text{classNm}(\text{meth}) = T\) or \(T\) is the base class of \(\text{classNm}(\text{meth})\). We will only treat here the case \(T'' = \text{UnInit}\).

The invariant (arg) ensures that there exists on the evalStack a reference ref typable with UnInit. By (init), we get that meth is a .ctor (not of class object), \(ref = \text{argVal}(0)\), and initState(ref) = InProgress(OC), where OC is an object class such that \(OC \sqsubseteq \text{classNm}(\text{meth})\).

For the current frame, it only remains to prove (init) since the other invariants can be proved with the same arguments as in case of a tail call (Subcase 2.1). The invariant (init) is preserved since initState(ref) = InProgress(OC) and \(OC \sqsubseteq T\).

We also need to show the invariants for every succession of the topmost frame \(fr_k\), i.e., the current frame before doing the call, and every succession of the initialization status initState. In the succession of \(fr_k\), the pc is incremented by 1. In the succession of the initialization status \(fr_k\), initState(ref) is set to Init(OC), where the reference ref is given by \(ref = \text{argVal}(fr_k)(0)\).

By (pc) and the definition of succ in Figure 4.9, we get that

\[(pc + 1, \text{classNm}(\text{meth}), \text{evalStack}{T''} \circ [\text{classNm}(\text{meth})/\text{UnInit})]\]

is in the set succ(meth, pc, argZero_{\text{pc}}, evalStack_{\text{pc}}), where the list evalStack\(T''\) is given by drop(evalStack_{\text{pc}}, argNo(T::ctor)). By Definition 4.1.9 (wt6), we can deduce that classNm(\text{meth}) \sqsubseteq argZero_{\text{pc} + 1} and

\[\text{evalStack}{T''} \circ [\text{classNm}(\text{meth})/\text{UnInit}] \sqsubseteq \text{ten evalStack}_{\text{pc} + 1}\]  

(4.8)

We have to show the invariant (stack2) for the succession of \(fr_k\). By the induction hypothesis, from (init) we know that if there is on the evaluation stack a value val typable with UnInit, then \(val = \text{argVal}(fr_k)(0) = ref\). Furthermore, in the succession, initState(val) = Init(OC), where \(OC \sqsubseteq \text{classNm}(\text{meth})\). The invariant (stack2) follows then from the induction hypothesis, (4.8), Definition 4.1.12, and Lemma 4.1.5.
Case 3 \(\text{code}(pc) = \text{Return}\)

From \((pc)\) and Definition 4.1.9 (wt5), we obtain \(\text{evalStack}_{pc} \sqsubseteq_{len} \text{void}(\text{retType}(\text{meth}))\) and \(\text{argZero}_{pc} \neq \text{UnInit}\) if the method \(\text{meth}\) is a \(\cdot\text{ctor}\) other than \(\text{object::ctor}\). Upon executing the \text{Return} instruction, the current frame is discarded, and the topmost frame on the \text{frameStack} becomes the new current frame. The \(pc\) of the current frame is incremented by 1. If \(\text{meth}\) returns a value, say \(\text{val}\), this value is pushed onto the new \text{evalStack}. By (stack2) and Lemma 4.1.5, \([fr_1, \ldots, fr_k] \vdash \text{val} : \text{retType}(\text{meth})\). By the constraints stated in Section 4.1.4.2, \(\text{retType}(\text{meth})\) is neither a pointer type nor \text{typedref}. By Lemma 4.1.7, we get

\[
[fr_1, \ldots, fr_{k-1}] \vdash \text{val} : \text{retType}(\text{meth}) \tag{4.9}
\]

So, after executing \text{Return}, the new current frame is exactly \(fr_{k-1}\), i.e., the succession of \(fr'_{k-1}\) according to Definition 4.1.14. The current \text{initState} does not change after executing \text{Return}. If \text{Return} does not correspond to a \(\cdot\text{ctor}\), then \text{initState} is obviously the succession of itself for the frame \(fr_{k-1}\) according to Definition 4.1.15. If \text{Return} corresponds to a \(\cdot\text{ctor}\), then, by \text{argZero}_{pc} \neq \text{UnInit} and (arg), we get that \text{initState}(\text{argVal}(0)) cannot be an \text{InProgress} value. Therefore, \text{initState} is the succession of itself for \(fr'_{k-1}\) according to Definition 4.1.15. Therefore, the induction hypothesis and (4.9) imply that the invariants also hold after executing \text{Return}.

Case 4 \(\text{code}(pc) = \text{MkRefAny}(T)\)

We get \(\text{evalStack} \sqsubseteq_{\text{sub}} \lbrack T\&\rbrack\) from the invariant \((pc)\) and Definition 4.1.9 (wt5). The instruction pops the top value of the \text{evalStack}. Since this value is supposed to be a pointer, let us denote it by \(\text{adr}\). By \(\text{evalStack} \sqsubseteq_{\text{sub}} \lbrack T\&\rbrack\), (stack1), and (stack2), we have that \(\text{adr}\) is a pointer typed with \(T\&\). Formally,

\[
[fr_1, \ldots, fr_k] \vdash \text{adr} : T\& \tag{4.10}
\]

The instruction then pushes a typed reference, say \(tr\), onto the \text{evalStack}. It also sets the functions \text{typedRefAdr} and \text{typedRefType} as follows: \(\text{typedRefAdr}(tr) = \text{adr}\) and \(\text{typedRefType}(tr) = T\). We need to show that the invariant (stack2) is preserved.

From the definition of \text{succ} in Figure 4.9, we have that

\[
(pc + 1, \text{argZero}_{pc}, \text{pop}(\text{evalStack}_{pc}) \cdot [\text{typedref}])
\]

is in the set \(\text{succ}(pc, \text{argZero}_{pc}, \text{evalStack}_{pc})\). From Definition 4.1.9 (wt6), we obtain \(pc + 1 \in D\) (where \(D\) is the domain of the family of type states for the current frame \(fr_k\)) and \(\text{pop}(\text{evalStack}_{pc}) \cdot [\text{typedref}] \sqsubseteq_{\text{len}} \text{evalStack}_{pc+1}\). The first condition ensures that the invariant \((pc)\) is preserved. Based on the second condition and on Definition 4.1.7, we know that in order to prove (stack2), it suffices to show \([fr_1, \ldots, fr_k] \vdash tr : \text{typedref}\). This typing follows from (4.10) and Definition 4.1.12.

\[\square\]

Remark 4.1.11 CLR follows strictly the ECMA Standard [45] and consequently considers that the tail prefix cannot be ignored. The semantics model, bytecode verification and the
4.1. The Lightweight CLR

proof could be, however, easily adapted to take into account the aspect that the prefix can be
generated (when JITting). Thus, in the definition of succ, the tail calls would be treated as regular
calls. More precisely, the tail call would be considered as having a successor (namely a Return
instruction – see Tail Call Pattern), as opposed to the actual definition of succ which does not
define any successors. That the invariants are preserved upon performing a tail call with the tail
prefix ignored would be proved as in the case of a regular call.

It is worth mentioning that the restriction Tail Call Return Type would become redundant
upon redefining the function succ for tail calls. Given that we only deal with well-typed methods,
the above restriction would follow from Tail Call Pattern.

Bytecode verification’s soundness and completeness Well-typed methods’ type safety alone
is, however, not sufficient. We also need to show that (1) the methods accepted by the verification
algorithm are well-typed, and (2) if a method is well-typed, then it is accepted by the verification
algorithm and the algorithm computes a most specific family of type states. In other words, we have to prove the soundness and the completeness of the verification algorithm. These proofs, similar to the corresponding proofs developed in [115], require the following lemmas:\footnote{The proofs of these lemmas and of the theorems asserting the soundness and completeness of the verification algorithm can also be easily extended for CLR\(_E\) and CLR\(_G\), and therefore we will not reconsider them explicitly.}

Lemma 4.1.10 (Monotonicity of checks) Let mref be a method and pos a code index of a
 bytecode instruction of mref. If argZero\(T'\) \(\sqsubseteq\) argZero\(T\), evalStack\(T'\) \(\subseteq_{\text{len}}\) evalStack\(T\) and
check(mref, pos, argZero\(T\), evalStack\(T\)) is true, then check(mref, pos, argZero\(T'\), evalStack\(T'\)) is true.

Proof. By a case distinction on the instruction code(mref)(pos). The transitivity of the relation \(\sqsubseteq\) and Lemma 4.1.2 are applied.

Lemma 4.1.11 (Monotonicity of successors) Let mref be a method and pos a code index of a
 bytecode instruction of mref. Let us assume that we have argZero\(T'\) \(\sqsubseteq\) argZero\(T\), evalStack\(T'\) \(\subseteq_{\text{len}}\) evalStack\(T\) and
check(mref, pos, argZero\(T\), evalStack\(T\)) is true. Then for every \((j, \text{argZero}\(T''\), evalStack\(T''\)) \(\in\) succ(mref, pos, argZero\(T'\), evalStack\(T'\)), there exists
\((j, \text{argZero}\(T'''\), evalStack\(T'''\)) \(\in\) succ(mref, pos, argZero\(T\), evalStack\(T\)) such that

\[
\text{argZero}\(T''\) \(\sqsubseteq\) argZero\(T''\) and evalStack\(T''\) \(\subseteq_{\text{len}}\) evalStack\(T''\)
\]

Proof. By a case distinction on the instruction code(mref)(pos). The transitivity of the relation \(\sqsubseteq\) is used.

Theorem 4.1.2 (Soundness of the verification algorithm) During the bytecode verification
of a method mref, the following conditions are met:

(bv1) \(\mathcal{P} \subseteq \mathcal{V}\) and the elements of \(\mathcal{V}\) are valid code indices of mref.
(bv2) $0 \in \mathcal{V}$.

(bv3) If $\text{classNm}(\text{mref}) \in \text{ObjClass} \setminus \{\text{object}\}$ and $\text{methNm}(\text{mref}) = \text{.ctor}$, then $\text{argZero}V_0 = \text{UnInit}$. Otherwise, $\text{argZero}V_0 = \text{argTypes}(\text{mref})(0)$.

(bv4) $\text{evalStack}V_0 = []$.

(bv5) If $i \in \mathcal{V} \setminus \mathcal{P}$, then $\text{check}(\text{mref}, i, \text{argZero}V_i, \text{evalStack}V_i)$ is true.

(bv6) If $i \in \mathcal{V} \setminus \mathcal{P}$ and $(j, \text{argZero}V', \text{evalStack}V') \in \text{succ}(\text{mref}, i, \text{argZero}V_i, \text{evalStack}V_i)$, then $j \in \mathcal{V}$, $\text{argZero}V' \sqsubseteq \text{argZero}V_j$ and $\text{evalStack}V' \sqsubseteq_{\text{len}} \text{evalStack}V_j$.

Proof. By induction on the run of the bytecode verification algorithm in Figure 4.10. □

One can easily notice that, for $\mathcal{P} = \emptyset$, the conditions (bv1)-(bv6) correspond to the well-typedness constraints (wt1)-(wt6) in Definition 4.1.9. So, if a method is accepted by the verification (in particular, this means that the verification algorithm terminated, i.e., $\mathcal{P} = \emptyset$), then the method is well-typed.

Theorem 4.1.3 (Completeness of the verification algorithm) If $\text{mref}$ is well-typable with a type state family $(\text{argZero}T_i, \text{evalStack}T_i)_{i \in \mathcal{D}}$, then during the verification of $\text{mref}$’s bytecode $(\text{argZero}T'_i, \text{evalStack}T'_i)_{i \in \mathcal{V}}$ is always more specific than $(\text{argZero}T_i, \text{evalStack}T_i)_{i \in \mathcal{D}}$ and halt is not set (to a verification error).

Proof. By induction on the run of the verification algorithm in Figure 4.10, where Lemmas 4.1.10 and 4.1.11 and Definition 4.1.10 are applied. □
4.2 The Exceptions

This section extends the bytecode language introduced in Section 4.1 by exceptions\(^\text{18}\). The ECMA Standard [45] contains only a few yet incomplete paragraphs about the exception handling mechanism. A more detailed description of the mechanism can be found in one of the few existing documents on the CLR exception handling mechanism [32]. The mechanism has its origins in the Windows NT Structured Exception Handling (SEH), described in [102].

We use three different methods to check the faithfulness (with respect to CLR) of the modelling decisions we had to take where [45] exhibits gaps. First of all, we did a series of experiments with CLR, some of which are made available in [54]. They show border cases which have to be considered to get a full understanding and definition of exception handling in CLR. Secondly, some authoritative evidence for the correctness of the modelling ideas we were led to by our experiments has been provided by Jonathan Keljo [86], the CLR Exception System Manager, who essentially confirmed our ideas about the exception mechanism issues left open in [45]. Last but not least, a way is provided to test the internal correctness of the model presented in this section and its conformance to the experiments with CLR, namely by an executable version [92] of the CLR model.

**Challenges** The mechanism handling the exceptions is spectacularly complex, especially due to the handling of exceptions in two passes, but also due to certain exception constructs, known as filter handlers, whose treatment, loosely specified by [45], is clarified in this thesis. The meaning of exception filters is not trivial: A filter allows disregarding some exceptions depending on the result of executing filter code in the context of some caller on the stack. As the handling of an exception is performed in two passes, an important challenge in the study of filters is to deal with exceptions thrown by the filter codes. It turns out that the exception mechanism needs to maintain a stack of passes. On the other hand, an exception specific instruction (leave) may trigger the execution of several exception handlers which in turn may throw exceptions. Consequently, besides the two kinds of exception passes, vaguely specified by [45], we consider a third kind of pass, i.e., Leave passes.

Once the semantics of the exception handling mechanism has been defined, the type safety proof is pretty straightforward. And yet, there are several non-obvious properties concerning the structural correctness of the exception mechanism that have to be proved. Moreover, the complex control flow graph underlying the execution of the exception mechanism complicates the bytecode verification the type safety proof significantly depends on.

**Plan of the section** An overall view of the exception mechanism is provided in Section 4.2.1. Sections 4.2.2 and 4.2.3 define the static and dynamic semantics of the mechanism, respectively. The specification of the bytecode verification is extended in Section 4.2.4 to cover the handling of the exceptions. The bytecode language with exceptions is then proved type-safe in Section 4.2.5.

\(^{18}\)For the sake of clarity, the special System.ThreadAbortException [8] is not treated here as its handling goes much beyond the scope of this thesis. The refinements that should be applied to our formal model in order to also treat the handling of this exception are defined in [61].
4.2.1 An Overview of the Exception Handling Mechanism

Exception handling is supported in CIL through exception objects and exception handlers. The exception handlers are maintained in an exception handling array associated with every method. There are four kinds of exception handlers: catch, filter, finally, and fault. To aid the reader, Figure 4.12 contains an example for each type of exception handlers. We briefly describe below each kind:

- A catch handler consists of a try block and a catch handler region. The handler region handles any exception of the handler specified type or any of its subtypes which occurs in the try block.

- A filter handler is made of a try block, a filter region, and a filter handler region. The filter region is executed to decide if the handler region handles an exception raised in the try block.

Visual Basic and Managed C++ have special catch blocks which can “filter” the exceptions based on the exception type and/or any conditional expression. These are compiled into filter handlers in the CIL bytecode. Figure 4.13 presents an example of a Visual Basic filter.

- A finally handler consists of a try block and a finally handler region. The handler region is executed whenever the try block exits, regardless of whether the try block exits normally or through an unhandled exception.

- A fault handler embodies a try block and a fault handler region. The handler region is executed if the try block is exited by an unhandled exception. Unlike a finally handler region, it should not be executed if the try block is exited normally. Currently, no language (other than CIL) exposes fault handlers directly.

Exit from the blocks and regions of the exception handlers is restricted to be accomplished only through certain bytecode instructions. Thus, every normal execution path of a try block or a catch or filter handler region must end with a leave instruction. Similarly, every normal execution path of a finally or fault handler region must end with the endfinally instruction. Alternatively, every execution path of a try block or a handler region can also terminate with a throw instruction. The last instruction of a filter region should be the endfilter instruction.

We now describe the CLR exception handling mechanism. Every time an exception occurs, control is transferred from “normal” execution to the exception handling mechanism. This mechanism proceeds in two passes. In the first pass, the run-time system runs a “stack walk” searching, in the possibly empty exception handling array associated to the current method, for the first handler that might want to handle the exception:

- a catch handler whose type typeExc is a supertype of the type of the exception, or

- a filter handler – to see whether a filter wants to handle the exception, one has first to execute (in the first pass) the code of the filter region in a separate frame pushed onto the stack of call frames (detail skipped by [45]): If it returns 1, then it is chosen to handle the exception; if it returns 0, this handler is not good to handle the exception.
4.2. The Exceptions

Figure 4.12 Example: exception handlers.

```csharp
.try
{
  2: newobj instance void C::.ctor()
  3: call instance void C::M()
  4: leave 30
} // end try block

filter // decide if handle exception
{
  10: stloc 0
  11: ldc.i4 1
  12: newobj instance void C''::.ctor()
  13: ldloc 0
  14: call instance int32 C''::M(Exception)
  15: ldc.i4 2
  16: beq 19
  17: pop
  18: ldc.i4 0
  19: endfilter
} // end filter region

.catch NullReferenceException
{
  20: stloc 0
  21: newobj instance void C'::.ctor()
  22: ldloc 0
  23: call instance void C'::M(Exception)
  24: leave 30
} // end handler region

... 30: ...

.finally
{
  7: newobj instance void C'::.ctor()
  8: call instance void C'::M()
  9: endfinally
} // end handler region

... 30: ...
```

```csharp
.try
{
  2: newobj instance void C::.ctor()
  3: call instance void C::M()
  4: leave 30
} // end try block

.filter // decide if handle exception
{
  10: stloc 0
  11: ldc.i4 1
  12: newobj instance void C''::.ctor()
  13: ldloc 0
  14: call instance int32 C''::M(object)
  15: ldc.i4 2
  16: beq 19
  17: pop
  18: ldc.i4 0
  19: endfilter
} // end filter region

... 30: ...
```

```csharp
.try
{
  2: newobj instance void C::.ctor()
  3: call instance void C::M()
  4: leave 30
} // end try block

.fault
{
  7: newobj instance void C'::.ctor()
  8: call instance void C'::M()
  9: endfinally
} // end handler region

... 30: ...
```
Figure 4.13 Example: Visual Basic `Catch` with `When` clause.

```
Try
  x = M(y)
Catch exc As Exception When M'(y) < 3
  z = M'(4)
End Try
```

If a match is not found in the faulting frame, i.e., the frame where the exception has been raised, the calling method is searched, and so on. This search eventually terminates since the exception handling array of the `entrypoint` method has as last entry a so-called `backstop entry` placed there by the operating system.

When a match is found, the first pass terminates, and in the second pass, called “unwinding of the stack”, the exception mechanism walks once more through the stack of call frames to the handler determined in the first pass, but this time executing the `finally` and `fault` handlers and popping their frames. It then starts the corresponding exception handler region.

Wrapping non-Exceptions There are .NET compliant languages, for instance Managed C++, where one can throw exceptions of any non-pointer type, as opposed to most of the .NET languages that require that every thrown exception is derived from the class `System.Exception`, the base class of all exception types. To maintain compatibility between languages, the CLR wraps objects that do not derive from `Exception` in a `RuntimeWrappedException` object. One can use a special boolean attribute, called `RuntimeCompatibilityAttribute`, to specify whether exceptions should appear wrapped inside `catch` and `filter` handlers. The CLR still wraps exceptions even if the attribute `RuntimeCompatibilityAttribute` is used to specify that one does not want them wrapped. In this case, exceptions are unwrapped only inside `catch` and `filter` handlers. Independent of the attribute’s value, the new `RuntimeWrappedException` object, with the originally thrown object as an instance field `WrappedException` of declared type `object`, is propagated. Surprisingly, the ECMA Standard [45] does not mention anything about this special semantics.

4.2.2 The Static Semantics

To model the exception handling, the universe `Switch` of execution modes is extended with the value `ExcMech`.

```
Switch = ...
  | ExcMech   exceptional mode
```

If `switch` is `ExcMech`, the semantics model is in the `exceptional mode`. This means that the exception mechanism defined in Section 4.2.3 has control either to handle an exception or to normally exit an exception block.

The exception specific universes are gathered in Table 4.12, while the basic static (exception specific) function are defined in Table 4.13. Thus, every method has assigned a possibly empty
exception handling array maintained in \( \text{excHA} \), and every bytecode program has assigned a flag \( \text{wrapNonExc} \), representing the \text{RuntimeCompatibilityAttribute}.

The elements of \( \text{excHA} \) are known as \textit{handlers} and are defined as elements of the universe \( \text{Exc} \).

Any 7-tuple of the above form describes an exception handler of kind \( \text{clauseKind} \) which “protects” the region that starts at \( \text{tryStart} \) and has the length \( \text{tryLength} \), handles the exception in the handler region that starts at \( \text{handlerStart} \) and has the length \( \text{handlerLength} \); if the handler is a \texttt{catch}, then the type \( \text{typeExc} \) of exceptions it handles is provided, whereas if the handler is a \texttt{filter}, then the first instruction of the \texttt{filter} region is at \( \text{filterStart} \). In case of a \texttt{filter} handler, the handler region starting at \( \text{handlerStart} \) should immediately follow the \texttt{filter} region (this is the reason there is no \texttt{filter} region length provided).

\textbf{Example 4.2.1} Consider the exception handlers from Figure 4.12. They are represented in the exception handling array through the following entries:

\begin{verbatim}
(catch, 2, 3, 20, 5, NullReferenceException, undef )
(filter, 2, 3, 20, 5, undef, 10 )
(finally, 2, 3, 7, 3, undef, undef )
(fault, 2, 3, 7, 3, undef, undef )
\end{verbatim}
The predicates \( \text{isInTry} \), \( \text{isInHandler} \) and \( \text{isInFilter} \) are used to identify those code indices that belong to a \texttt{try} block, handler region, and \texttt{filter} region, respectively.

\[
\text{isInTry}(pos, h) : \iff \text{tryStart}(h) \leq pos < \text{tryStart}(h) + \text{tryLength}(h)
\]

\[
\text{isInHandler}(pos, h) : \iff \text{handlerStart}(h) \leq pos < \text{handlerStart}(h) + \text{handlerLength}(h)
\]

\[
\text{isInFilter}(pos, h) : \iff \text{filterStart}(h) \leq pos < \text{handlerStart}(h)
\]

**Lemma 4.2.1** The predicates \( \text{isInTry} \), \( \text{isInHandler} \) and \( \text{isInFilter} \) are pairwise disjoint.

**Proof.** Based on the lexical nesting constraints of protected blocks specified in [45, Partition I, §12.4.2.7] and the definitions of the predicates. \( \square \)

We assume all the constraints concerning the lexical nesting of handlers specified in [45, Partition I, §12.4.2.7]. The ordering assumption on handlers given in [45, Partition I, §12.4.2.5] is the following:

**Ordering Assumption** If handlers are nested, the most deeply nested handlers should come in the exception handling array before the handlers that enclose them.

We also assume all the syntactic constraints stated in [45, Partition I, §12.4.2.8] concerning the \texttt{Leave} instructions:

- it is not legal to exit with a \texttt{Leave} instruction a \texttt{filter} region, a \texttt{finally/fault} handler region;
- it is not legal to branch with a \texttt{Leave} instruction into a handler region from outside the region;
- it is legal to exit with a \texttt{Leave} a \texttt{catch} handler region and branch to any instruction within the associated \texttt{try} block, so long as that branch target is not protected by yet another \texttt{try} block;
- a \texttt{Leave} instruction is executed only upon the normal exit from a \texttt{try} block or a \texttt{catch/filter} handler region;
- the target of any branch instruction, in particular of \texttt{Leave} target, points to an instruction within the same method as the branch instruction;

Moreover, we assume that the \texttt{entrypoint} has a \texttt{backstop entry}:

**Backstop entry** The \texttt{excHA} of the \texttt{entrypoint} method has as last entry a handler called backstop entry which can handle any exception.

In order to determine the non-\texttt{Exception} object wrapped as an instance field in a \texttt{RuntimeWrappedException} object, we define the (derived) function \( \text{wrappedExc} : \text{Map}(\text{ObjRef}, \text{Val} \cup \{\text{undef}\}) \).
### 4.2. The Exceptions

#### Function definition

<table>
<thead>
<tr>
<th>Function name</th>
<th>Function definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>exc</td>
<td>ObjRef ∪ {undef}</td>
</tr>
<tr>
<td>pass</td>
<td>{StackWalk, Unwind, Leave}</td>
</tr>
<tr>
<td>stackCursor</td>
<td>Frame × N</td>
</tr>
<tr>
<td>handler</td>
<td>Frame × N</td>
</tr>
<tr>
<td>target</td>
<td>Pc</td>
</tr>
<tr>
<td>passRecStack</td>
<td>List(ExcRec ∪ LeaveRec)</td>
</tr>
</tbody>
</table>

#### The current exception

#### The current pass

#### The current stack cursor

#### The suitable handler

#### the target of current Leave pass

#### the pass record stack

---

**Table 4.14: The basic dynamic exception specific functions.**

```plaintext
wrappedExc(ref) = mem(fieldAdr(ref, RuntimeWrappedException::WrappedException))
```

---

#### 4.2.3 The Dynamic Semantics

The *dynamic state* of the exception mechanism is defined by the basic dynamic functions listed in Table 4.14. Thus, to handle an exception, the exception mechanism needs to record:

- the exception reference `exc`,
- the handling `pass`,
- a `stackCursor`, *i.e.*, the position currently reached in the stack of call frames (a frame) and in the exception handling array (an index in `excHA`),
- the suitable `handler` (*also known as the target handler*) determined at the end of the `StackWalk` pass (if any) is the handler that is going to handle the exception in the pass `Unwind` – until the end of the `StackWalk` pass, `handler` is `undef`.

Every normal execution of a `try` block or `catch/filter` handler region must end with an instruction `Leave(target)`. When doing this, every `finally` code up to the `target` has to be executed. Since `finally` code can throw exceptions, and implicitly determine the exception mechanism to execute exception passes, one needs to record for the `Leave` instruction the currently reached `finally` code as well as the `target`. This triggers one significant contribution of our framework: the introduction of `Leave` passes although they are not mentioned in [45]. A `Leave` pass records the `stackCursor` together with the `target` of the `Leave` instruction that triggered the pass.

---


**LeaveRec = LeaveRec ( pass : {Leave}, stackCursor : Frame × N, target : Pc )**
The nesting of passes determines the exception handling mechanism to maintain a stack `passRecStack` of exception or leave records for the passes that are still to be performed. The need for introducing the `passRecStack` is not revealed, and constitutes another important contribution of our framework. Analogously to `frameStack`, the `passRecStack` does not contain the currently handled pass record.

**Initial Constraints**  The stipulations defined in Section 4.1.3 on the initial state of the virtual machine are now extended to also include the basic dynamic exception specific functions:

<table>
<thead>
<tr>
<th><code>passRecStack</code></th>
<th><code>stackCursor</code></th>
<th><code>exc</code></th>
<th><code>handler</code></th>
<th><code>target</code></th>
</tr>
</thead>
<tbody>
<tr>
<td><code>[]</code></td>
<td><code>undef</code></td>
<td><code>undef</code></td>
<td><code>undef</code></td>
<td><code>undef</code></td>
</tr>
</tbody>
</table>

We can now summarize the overall behavior of the exception mechanism, which is analyzed in detail in the following sections, by saying that if there is a handler in the frame defined by `stackCursor`, then the exception mechanism will try to find (when `StackWalking`) or to execute (when `Unwinding`) or to leave (when `Leaveing`) the corresponding handler; otherwise it will continue its work in the caller’s frame, or it ends its `Leave` pass at the `target`.

### 4.2.3.1 The StackWalk Pass

During a `StackWalk` pass, the exception mechanism `EXCCLR` starts in the current `frame` to search for a suitable handler of the current exception in this frame. Such a handler exists if the search position `n` in the current `frame` has not yet reached the last element of the handlers array `excHA` of `frame`’s method `mref`. Formally, there are no more handlers to inspect in a frame pointed to by a `stackCursor` of the form `((__, __, __, __, __, mref), n)` if the below defined predicate evaluates to false.

\[
existsHanWithinFrame((__, __, __, __, __, mref), n) :\iff n < \text{length}(\text{excHA}(mref))
\]

If there are no (more) handlers in the frame pointed to by `stackCursor`, then the search has to be continued at the caller frame. This means to reset the `stackCursor` to point to the caller frame and the index 0 in its exception handling array.

\[
\text{SEARCHINVFRAME}(fr) \equiv \\
\text{let } \cdot [fr', fr] \cdot \_ = \text{frameStack} \cdot [\text{frame}] \text{ in} \\
\text{RESET}(\text{stackCursor}, fr')
\]

There are three groups of possible handlers `h` the exception mechanism is looking for in a given frame during its `StackWalk`:
• a catch handler whose try block protects the program counter \( pc \) of the frame pointed at by \( stackCursor \) and whose type \( typeExc \) is a supertype of the exception type (if the ”wrapping” is turned on) or of the wrapped exception type (if the ”wrapping” is turned off). The handlers of this type are identified through the predicate \( matchCatch \).

\[
matchCatch(pos, T, h) \iff isInTry(pos, h) \land clauseKind(h) = catch \land T \preceq typeExc(h)
\]

• a filter handler whose try block protects the \( pc \) of the frame pointed at by \( stackCursor \). Such handlers are determined with the predicate \( matchFilter \).

\[
matchFilter(pos, h) \iff isInTry(pos, h) \land clauseKind(h) = filter
\]

• a filter handler whose filter region contains the \( pc \) of the frame pointed at by \( stackCursor \) (these handlers are selected through the predicate \( isInFilter \) defined in Section 4.2.2). This corresponds to an outer exception and will be described in more detail below.

The order of the if clauses in the let statement from the rule \( StackWalk \) in Figure 4.2.3.1 is not important. This is justified by the fact that no handler can be of all the above types.

**Lemma 4.2.2** Let \( T \) be a type. The predicates \( matchCatch^T \), \( matchFilter \), and \( isInFilter \) are pairwise disjoint\(^{19}\).

**Proof.** Using the definitions of the three predicates and Lemma 4.2.1. \( \square \)

The handler pointed to by a \( stackCursor \) of the form \( ((\_ , \_ , \_ , \_ , mref), n) \), is defined to be \( excHA(mref)(n) \). If this handler is not of any of the above types, then the \( stackCursor \) is incremented to point to the next handler in the \( excHA \) of the method \( mref \).

The **Ordering Assumption** in Section 4.2.2 and the lexical nesting constraints stated in [45, Partition I, §12.4.2.7] ensure that if \( stackCursor \) points to a handler of one of the above types, then this handler is the first handler in the exception handling array (starting at the position indicated in the \( stackCursor \)) of any of the above types.

**Handler Case 1** If the handler pointed to by the \( stackCursor \) is a matching catch, then this handler becomes the handler to handle the exception in the pass \( Unwind \). The \( stackCursor \) is reset to be reused for the \( Unwind \) pass: It should point to the faulting frame, \( i.e. \), the current frame. Note that during \( StackWalk, frame \) always points to the faulting frame except in case a filter region is executed. However, the frame built to execute a filter is never searched for a handler corresponding to the current exception.

```
FOUNDHANDLER ≡
  pass := Unwind
  handler := stackCursor
```

\(^{19}\)By \( matchCatch^T \) we understand the predicate defined by the set \( \{ (pos, h) \mid matchCatch(pos, T, h) \} \).
**Figure 4.14** The exception handling mechanism EXCCLR.

**EXCCLR** $\equiv$ **match** *pass*

**StackWalk** → if existsHanWithinFrame(stackCursor) then
  let $h = \text{hanWithinFrame}(stackCursor)$ in
  let $T = \begin{cases} \text{wrapNonExc} & \text{if matchCatch}(pos, T, h) \text{ then} \\ \text{actualTypeOf}(\text{exc}) & \text{else} \end{cases}$
  if $\text{matchCatch}(pos, T, h)$ then
    FOUNDHANDLER
    \text{RESET}(stackCursor, frame)
  elseif $\text{matchFilter}(pos, h)$ then
    EXECFILTER(h)
  elseif $\text{isInFilter}(pos, h)$ then
    EXITINNEREXC
  else
    \text{GOToNXTHAND}
  end

end

**Unwind** → if existsHanWithinFrame(stackCursor) then
  let $h = \text{hanWithinFrame}(stackCursor)$ in
  if $\text{matchTargetHan}(\text{handler}, stackCursor)$ then
    EXEC Han(h)
  elseif $\text{matchFinFault}(pc, h)$ then
    EXEC Han(h)
    \text{GOToNXTHAND}
  elseif $\text{isInHandler}(pc, h)$ then
    ABORTPREVPASSREC
    \text{GOToNXTHAND}
  elseif $\text{isInFilter}(pc, h)$ then
    CONTINUEOUTEREXC
    \text{GOToNXTHAND}
  else
    \text{POPFRAME}(0, [])
  end

end

**Leave** → if existsHanWithinFrame(stackCursor) then
  let $h = \text{hanWithinFrame}(stackCursor)$ in
  if $\text{isFinFromTo}(h, pc, target)$ then
    EXEC Han(h)
  elseif $\text{isRealHanFromTo}(h, pc, target)$ then
    ABORTPREVPASSREC
    \text{GOToNXTHAND}
  else
    $pc := target$
    evalStack $:= []$
    POPREC
    switch $:= \text{Noswitch}$

end

\[ \text{RESET}(s, fr) \equiv \]
\[ s := (fr, 0) \]

**Handler Case 2** If the handler is a filter, then its filter region is executed. The execution is performed in a separate frame constructed especially for this purpose (detail omitted by [45]). The current frame becomes the frame for executing the filter region. The faulting frame is pushed onto the frameStack. The current frame points now to the method, local
variables, local memory pool, and arguments of the frame in which \( \text{stackCursor} \) is, it has the exception reference on the \( \text{evalStack} \) (if the ”wrapping” is turned on) or the wrapped exception \( \text{wrappedExc}(\text{exc}) \) (if the ”wrapping” is turned off), and the program counter \( \text{pc} \) set to the beginning \( \text{filterStart} \) of the filter region. Control is switched to normal execution mode by setting \( \text{switch} \) to Noswitch.

\[
\begin{align*}
\text{ExecFilter}(h) \equiv \\
\text{pc} & := \text{filterStart}(h) \\
\text{evalStack} & := \begin{cases} \text{if wrapNonExc then} [\text{exc}] & \text{else \text{wrappedExc}(\text{exc})} \end{cases} \\
\text{locAdr} & := \text{locAdr}' \\
\text{locPool} & := \text{locPool}' \\
\text{argAdr} & := \text{argAdr}' \\
\text{meth} & := \text{meth}' \\
\text{frameStack} & := \text{frameStack} \cdot [\text{frame}] \\
\text{switch} & := \text{Noswitch}
\end{align*}
\]

\textbf{Handler Case 3} The \( \text{stackCursor} \) points to a filter handler whose filter region contains the \( \text{pc} \) of the frame pointed at by \( \text{stackCursor} \).

\textbf{Exceptions in filter region?} It is not clearly documented in [45] what happens if an (inner) exception is thrown while executing the filter region during the StackWalk pass of an outer exception. The following cases are to be considered:

- if the exception is taken care of in the filter region, i.e., it is successfully handled by a \text{catch}/filter handler or it is aborted because it occurred in yet another filter region of a nested handler (see the second case below), then the given filter region continues executing normally (after the exception has been taken care of);

- if the exception is not taken care of in the filter region, then the exception will be discarded (via the \text{CONTINUEOUTEREXC} macro defined in Section 4.2.3.2) after its \text{finally} and \text{fault} handlers have been executed. Therefore, in this case \text{EXCCLR} exits via the macro \text{EXITINNEREXC} the StackWalk pass and starts an Unwind pass, during which all the \text{finally}/fault handlers for the inner exception are executed until the filter region where the inner exception occurred is reached.

\[
\text{ExitInnerExc} \equiv \\
\text{pass} := \text{Unwind} \\
\text{RESET}((\text{stackCursor}, \text{frame}))
\]

\textbf{Example 4.2.2} Consider that the entrypoint has the body in Figure 4.15. According to the Ordering Assumption stated in Section 4.2.2, the \( \text{stackCursor} \) points initially to the filter handler. To decide whether this handler is appropriate to handle the \text{NullReferenceException} thrown at 1, the filter region is executed in a separate frame. So, when the instruction 2 is executed, the frameStack is consisting of
Figure 4.15 Example: discarding exception and leave records in the `entrypoint` method.

```csharp
.try
{
  .try
  {
    .try
    {
      0: newobj instance void NullReferenceException::.ctor()
      1: throw
    }
    filter
    {
      2: newobj instance void InvalidCastException::.ctor()
      3: throw
      4: ldc.i4 1
      5: endfilter
    }
    {
      6: pop
      7: leave 12
    }
  }
  catch NullReferenceException
  {
    8: pop
    9: leave 12
  }
} finally
{
  10: newobj instance void DivideByZeroException::.ctor()
  11: throw
}
12: ret
```

the frame associated to the `entrypoint`. The filter region throws another exception. The context of the just handled outer exception is saved in the passRecStack, and the `InvalidCastException` raised at 3 becomes the current (inner) exception. However, the StackWalk pass of the inner exception is immediately exited since the inner exception cannot be propagated out of the filter region. The Unwind pass for the inner exception is then started.

### 4.2.3.2 The Unwind Pass

As soon as the pass StackWalk terminates, the exception mechanism starts the `Unwind` pass with the stackCursor pointing to the faulting frame. Starting there, one has to walk down to the handler determined in the StackWalk pass, executing on the way every `finally/fault` handler region. This also happens in case handler is `undef`. The handlers of interest for the exception mechanism in the Unwind pass are:
4.2. The Exceptions

- the matching target handler, i.e., the handler determined at the end of the StackWalk pass (if any\(^{20}\)). For that to happen, the two handler and stackCursor frames in question have to coincide. The predicate matchTargetHan decides whether the handler pointed to by the stackCursor is handler. We say that two frames are the same if the address arrays of their local variables and arguments as well as their method names coincide. This is defined by means of the predicate sameFrame.

\[
\text{matchTargetHan}((fr, n), (fr', n')) \iff \text{sameFrame}(fr, fr') \land n = n'
\]

\[
\text{sameFrame}((_, locAdr', _, argAdr', _, meth'), (_, locAdr'', _, argAdr'', _, meth'')) \iff \\
locAdr' = locAdr'' \land argAdr' = argAdr'' \land meth' = meth''
\]

- a matching finally/fault handler whose associated try block protects the pc. The predicate matchFinFault selects the finally/fault handlers whose try blocks protect a given code index.

\[
\text{matchFinFault}(pos, h) \iff \text{isInTry}(pos, h) \land \text{clauseKind}(h) \in \{\text{finally, fault}\}
\]

- a handler whose handler region contains pc: Such handlers are selected with the predicate isInHandler defined in Section 4.2.2;

- a filter handler whose filter region contains pc: These handlers are identified through the predicate isInFilter defined in Section 4.2.2;

The order of the last three if clauses in the let statement of the rule Unwind in Figure 4.2.3.1 is not important since no handler can be of all of the above last three types. It only matters that the first clause is guarded by matchTargetHan.

**Lemma 4.2.3** The predicates matchFinFault, isInHandler and isInFilter are pairwise disjoint.

**Proof.** Using the definitions of the predicates and Lemma 4.2.1. \(\square\)

The Ordering Assumption in Section 4.2.2 and the lexical nesting constraints given in [45, Partition I, §12.4.2.7] ensure that if the stackCursor points to a handler of one of the above types, then this handler is the first handler in the exception handling array (starting at the position indicated in the stackCursor) of any of the above types.

**Handler Case 1** If the handler pointed to by stackCursor is the handler found in the StackWalk, its handler region is executed through EXECHAN: The pc is set to the beginning of the handler region, the exception reference exc (if the ”wrapping” is turned on) or the wrapped exception wrappedExc(exc) (if the ”wrapping” is turned off) is loaded on the evalStack (when EXECHAN is applied for executing finally/fault handler regions, nothing is pushed onto evalStack), and control switches to normal execution mode, i.e., switch is set to Noswitch.

\(^{20}\)Note that handler can be undef if the search in the StackWalk has been exited because the exception was thrown in a filter region.
**Handler Case 2** If the handler pointed to by the stackCursor is a matching \texttt{finally}/\texttt{fault} handler, its handler region is executed with an initially empty evalStack. At the same time, the stackCursor is incremented through \texttt{GOTONXT} to point to the next handler in excHA.

**Handler Case 3** Let us assume that the handler pointed to by stackCursor is an arbitrary handler whose handler region contains \texttt{pc}.

**Exceptions in handler region?** The ECMA standard \cite{ECMA-262} does not specify what should happen if an exception is raised in a handler region. The experimentation in \cite{54} led to the following rules of thumb for exceptions thrown in a handler region, in a way similar to the case of nested exceptions in \texttt{filter} code:

- If the exception is taken care of in the handler region, i.e., it is successfully handled by a \texttt{catch/filter} handler or it is discarded (because it occurred in a \texttt{filter} region of a nested handler), then the handler region continues executing normally (after the exception is taken care of).

- If the exception is not taken care of in the handler region, i.e., it escapes the handler region, then
  - the previous pass of the exception mechanism is aborted through the macro \texttt{ABORTPREVPASSREC} by popping its record from the passRecStack
    \begin{verbatim}
    ABORTPREVPASSREC \equiv
    passRecStack := pop(passRecStack)
    \end{verbatim}
  - the exception is propagated further via \texttt{GOTONXT}\texttt{HAN}, i.e., the Unwind pass continues by setting the stackCursor to the next handler in excHA.

**Remark 4.2.1** An exception can go “unhandled” without taking down the process, namely if an outer exception goes unhandled, but an inner exception is successfully handled. In fact, the execution of a handler region can only occur when the exception mechanism \texttt{EXECLR} runs in the Unwind and Leave passes: In Unwind, handler regions of any kind are executed, whereas in Leave, only \texttt{finally} handler regions are executed. If the raised exception occurred while \texttt{EXECLR} runs an Unwind pass for handling an outer exception, the Unwind pass of the outer exception is stopped, and the corresponding pass record is popped from the passRecStack. If the exception has been thrown while the exception mechanism runs a Leave pass for executing \texttt{finally} handlers “on the way” from a Leave instruction to its target, then this pass is stopped, and its associated pass record is popped from the passRecStack.
Handler Case 4 The handler pointed to by the stackCursor is a filter handler whose filter region contains pc. Then the execution of this filter region must have triggered an inner exception whose StackWalk led to a call of EXITINNEREXC. In this case, the current (inner) exception is aborted, and the filter considered as not providing a handler for the outer exception. Formally, CONTINUEOUTEREXC pops the frame built for executing the filter region, pops from the passRecStack the pass record corresponding to the inner exception and reestablishes the pass context of the outer exception, but with the stackCursor pointing to the handler following the just inspected filter handler. The updates of the stackCursor in POPREC and GOTO NxTHAN are done sequentially such that the update in GOTO NxTHAN overwrites the update in the macro PopRec. Note that by these stipulations, there is no way to exit a filter region with an exception. This ensures that the frame built by EXECFILTER for executing a filter region is used only for this purpose.

\[
\text{CONTINUEOUTEREXC } \equiv \\
\text{PopFRAME}(0, []) \\
\text{PopREC seq GOTO NxTHAN}
\]

\[
\text{PopREC } \equiv \\
\text{if } \text{passRecStack } = [] \text{ then } \\
\text{SETRECUNDEF} \\
\text{switch } := \text{Noswitch} \\
\text{else let } (\text{passRecStack}', [\text{rec}]) = \text{split(} \text{passRecStack}, 1) \text{ in } \\
\text{if } \text{rec } \in \text{ExcRec then } \\
\text{let } (\text{exc}', \text{pass}', \text{stackCursor}', \text{handler}') = \text{rec in } \\
\text{exc } := \text{exc}' \\
\text{pass } := \text{pass}' \\
\text{stackCursor } := \text{stackCursor}' \\
\text{handler } := \text{handler}' \\
\text{if } \text{rec } \in \text{LeaveRec then } \\
\text{let } (\text{pass}', \text{stackCursor}', \text{target}') = \text{rec in } \\
\text{pass } := \text{pass}' \\
\text{stackCursor } := \text{stackCursor}' \\
\text{target } := \text{target}' \\
\text{passRecStack } := \text{passRecStack}'
\]

\[
\text{SETRECUNDEF } \equiv \\
\text{exc } := \text{undef} \\
\text{pass } := \text{undef} \\
\text{stackCursor } := \text{undef} \\
\text{target } := \text{undef} \\
\text{handler } := \text{undef}
\]

Remark 4.2.2 The execution of PopFRAME is safe since the frameStack cannot be empty at the time when CONTINUEOUTEREXC is fired. [45, Partition II, §12.4.2.8.1] states that the control
can be transferred to a filter region only through EXCCLR. Since pc is in a filter region, an ExecFilter should have been already executed. But in this case, a new frame is pushed on the frameStack. Hence, frameStack is not empty when CONTINUEOUTERExc is executed.

**Example 4.2.3** Consider again the entrypoint body listed in Figure 4.15. Since the InvalidCastException raised in the filter region is not taken care of in the filter region, the exception, its Unwind pass, and the frame built to run the filter region are discarded. The NullReferenceException becomes again the current exception, and its StackWalk pass is continued by incrementing its stackCursor. This will point to the catch handler which turns out to be suitable for handling the exception. The StackWalk pass ends by setting handler to point to the catch handler. The Unwind pass starts, and the stackCursor is reset to point to the most deeply nested handler; i.e., the filter handler. The stackCursor is then incremented until it points to a handler of interest for the Unwind pass. The first such handler is exactly handler whose handler region is then executed.

If the handler pointed to by the stackCursor is not of interest for the Unwind pass, the stackCursor is incremented to point to the next handler in excHA.

If the Unwind pass terminated the inspection of all the handlers in the frame indicated by the stackCursor given in the form (frame, n), i.e., the predicate existsHanWithinFrame((frame, n)) evaluates to false, then the current frame is popped from the frameStack, and the Unwind pass continues in the caller frame of the current frame. This continuation is accomplished by setting the stackCursor to point to the new current frame and the index 0 in the excHA of this frame.

**Remark 4.2.3** As long as the exception mechanism is running an Unwind pass, the current stackCursor is always pointing to the current frame. This contrasts with the StackWalk pass where the current stackCursor is usually pointing to a different frame in the frameStack. This remark is proved in Lemma 4.2.4.

**Lemma 4.2.4** Let stackCursor = (fr, _) be the current stackCursor. If pass = Unwind, then fr is the current frame.

*Proof.* By a simple induction on the run of the exception handling mechanism. □

**Remark 4.2.4** The execution of the macros POPFRAME and SEARCHINVF1AME in the else clause is safe since the current frame has a caller frame, i.e., the current frame cannot be the frame of the entrypoint. This is because Backstop entry guarantees that the else clause is not reachable if the current frame is the frame of the entrypoint. The same argument can also be invoked in case of SEARCHINVF1AME in the StackWalk pass.

### 4.2.3.3 The Leave Pass

The exception mechanism gets into the Leave pass when, in the normal execution mode, the virtual machine executes a Leave instruction, which by the Leave constraints (see Section 4.2.2) can only happen upon the normal termination of a try block or of a catch/filter handler region. One has to execute the handler regions of all finally handlers “on the way” from the Leave instruction to the instruction whose program counter is given by the Leave target parameter. The stackCursor used in the Leave pass is initialized by the Leave instruction (see Figure 4.16). In the Leave pass, the exception mechanism EXCCLR searches for
4.2. The Exceptions

- **finally** handlers that are “on the way” from the \( pc \) to the \( target \), and
- **real** handlers, i.e., **catch/filter** handlers whose handler regions are “exited on the way” from the \( pc \) to the \( target \) – more details are given below.

**Handler Case 1** If the handler pointed to by \( stackCursor \) is a **finally** handler \( h \) on the way from \( pc \) to the \( target \) of the current **Leave** pass record, then the handler region of \( h \) is executed (see the first clause of the **Leave** rule in Figure 4.16): \( pc \) is set to \( handlerStart(h) \), \( evalStack \) to \([\ ]\), and control switches to normal execution mode, i.e., \( switch \) is set to \( Noswitch \). We define the predicate \( isFinFromTo \) to identify the **finally** handlers which are “on the way” from a code index to another code index.

\[
isFinFromTo(h, pos, pos') \iff clauseKind(h) = finally \land 
\quad isInTry(pos, h) \land 
\quad \neg isInTry(pos', h) \land \neg isInHandler(pos', h)
\]

**Handler Case 2** If the \( stackCursor \) points to a **catch/filter** handler whose handler region is “exited on the way” from \( pc \) to the \( target \), then the previous pass record, i.e., the top pass record on \( passRecStack \), is popped off (see the second clause of the **Leave** rule). The discarded record can only be referring to an **Unwind** pass for handling an exception. By discarding this record, the mechanism terminates the handling of the corresponding exception. The below defined predicate \( isRealHanFromTo \) is used to determine the **catch/filter** handlers whose handler regions are “exited on the way” from a code index to another code index.

\[
isRealHanFromTo(h, pos, pos') \iff clauseKind(h) \in \{ catch, filter \} \land 
\quad isInHandler(pos, h) \land \neg isInHandler(pos', h)
\]

That the discarded record can only be referring to an **Unwind** pass is proved in the following lemma:

**Lemma 4.2.5** If \( pass = Leave \) and the current \( stackCursor \) points to a **catch/filter** handler, then \( top(passRecStack) = (\_, Unwind, \_, h) \) such that \( matchTarget(stackCursor, h) \).

**Proof.** The proof proceeds by induction on the run of the exception mechanism and takes advantage of the fact that a **catch/filter** handler region can only be entered for handling an exception, i.e., in an **Unwind** pass. \( \square \)

Although the two \( if \) clauses in the \( let \) statement of the **Leave** pass in Figure 4.16 are executed in parallel, it is never the case that the embedded macros \( EXECHAN \) and \( ABORTPREVPASSREC \) are simultaneously executed. The reason is that no exception handler can be of both of the above types.

**Lemma 4.2.6** The predicates \( isFinFromTo \) and \( isRealHanFromTo \) are disjoint.
Table 4.15: The considered exception specific CIL instructions.

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Informal description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Throw</td>
<td>Pops the top stack element, and throws it as an exception.</td>
</tr>
<tr>
<td>EndFilter</td>
<td>Terminates the execution of a filter region.</td>
</tr>
<tr>
<td>EndFinally</td>
<td>Terminates the execution of a finally/fault handler region.</td>
</tr>
<tr>
<td>Leave(target)</td>
<td>Exits a try block or a catch/filter handler region.</td>
</tr>
</tbody>
</table>

Proof. Using the definitions of the predicates. □

For each kind of handler, the exception mechanism EXCCLR also inspects the next handler in excHA. When the handlers in the current method are exhausted, by the Leave constraints this round of EXCCLR is terminated, and the execution proceeds at target: pc is set to target, the context of the previous pass record, i.e., the top pass record on passRecStack, is reestablished, and control is passed to normal execution mode by setting switch to Noswitch (see Figure 4.16).

Example 4.2.4 Consider again the entrypoint body in Figure 4.15. Upon executing the leave instruction at 9, the mechanism enters into a Leave pass with target = 12 and stackCursor initially pointing to the filter handler. The stackCursor is successively incremented until it points to the catch handler whose handler region is “exited on the way” from pc = 9 to target = 12. Therefore, the top pass record of the passRecStack, i.e., the record associated to the NullReferenceException in its Unwind pass, is popped off. The stackCursor is then incremented and points to the finally handler. As the finally handler region raises an exception, the Leave pass is aborted.

4.2.3.4 The Semantics of the Exception Specific Instructions

This section defines the operational semantics model CLR_E which refines CLR_C (given in Section 4.1.3.1) to cover the changes that are needed to model exception handling. In particular, this section defines the operational semantics rules for the exception specific instructions. Table 4.15 contains the informal definitions of these instructions whose precise semantics is defined in terms of the semantics rules in Figure 4.16.

The execution scheme for CLR_E is the one for CLR_C extended with the exception mechanism EXCCLR.

\[
\text{CLR}_E \equiv \text{execScheme}(\text{EXECCLR}, \text{SWITCHCLR}, \text{EXCCLR})
\]

\[
\text{execScheme} \equiv \\
\quad \text{if } \text{switch} = \text{ExcMech} \text{ then} \\
\quad \text{EXCCLR} \\
\quad \text{elseif } \text{switch} = \text{Noswitch} \text{ then} \\
\quad \text{EXECCLR(code(pc))} \\
\text{else SWITCHCLR}
\]
Each of the instructions gives control to the exception mechanism EXCCLR by appropriately setting switch to ExcMech.

The Throw instruction pops the topmost evalStack element (see Remark 4.2.6 below), which is supposed to be an exception reference. It loads with the macro LOADREC to the exception mechanism the pass record associated to the given exception: exc is set to the exception reference (if exc’s type is derived from Exception) or to a RuntimeWrappedException reference pointing to exc (if exc’s type is not derived from Exception), pass to StackWalk, the stackCursor is initialized with (frame, 0), and handler is set to undef.

In order to create a new reference to a RuntimeWrappedException object pointing to a given object reference, we define the macro NewWrapExc. Thus, given an exception reference ref, the macro NewWrapExc(ref) creates a new RuntimeWrappedException object, with the non-Exception as an instance field, and returns a reference to the object.

If the exception mechanism is already working in a pass, i.e., pass is not undef, then the current pass record is pushed with PUSHREC onto passRecStack.
If the exception reference found on top of the evalStack by the Throw instruction is null, a NullReferenceException is thrown. Given a class C, the macro RAISE(C) is defined by the following code template:

RAISE(C) ≡

    NewObj(C::ctor)

    Throw

This macro can be viewed as a static method defined in class object. Calling the macro is then like invoking the corresponding method.

In a filter region, exactly one EndFilter is allowed, namely its last instruction, which is supposed to be the only one used to normally exit the filter region. The EndFilter instruction takes an integer val from the stack that is supposed to be either 0 or 1.

Remark 4.2.5 In [45], 0 and 1 are assimilated with “continue search” and “execute handler”, respectively. There is a discrepancy between [45, Partition I, §12.4.2.5], which states Execution cannot be resumed at the location of the exception, except with a user-filtered handler — therefore a “resume exception” value in addition to 0 and 1 is foreseen allowing CLR to resume the execution at the point where the handled exception has been raised— and [45, Partition III, §3.34] which states that the only possible return values from the filter are “exception_continue_search”(0) and “exception_execute_handler”(1).

If val is 1, then the filter handler to which EndFilter corresponds becomes the handler to handle the current exception in the pass Unwind. Remember that the filter handler is the handler pointed to by the stackCursor. The stackCursor is reset to be used for the pass Unwind:
4.2. THE EXCEPTIONS

It will point to the topmost frame on frameStack which is actually the faulting frame. If val is 0, the stackCursor is incremented to point to the handler following our filter handler. Independently of val, the current frame is discarded to reestablish the context of the faulting frame. Note that we do not explicitly pop val from the evalStack since the global dynamic function evalStack is updated anyway in the next step (through POPFRAME) to the evalStack' of the faulting frame.

The EndFinally instruction terminates (normally) the execution of the handler region of a finally or of a fault handler. It simply transfers control to the exception mechanism EXCCLR. A Leave instruction loads to the exception mechanism a pass record corresponding to a Leave pass which has the target set to the instruction’s code index token argument and the stackCursor to (frame, 0).

Remark 4.2.6 The reader might ask why the instructions Throw and EndFilter do not set the evalStack. The reason is that this set up, i.e., the emptying of evalStack, is supposed to be either a side-effect (the case of the Throw instruction) or ensured for a correct CIL (the case of the EndFilter instruction). Thus, the Throw instruction passes control to EXCCLR which, in a next step, will execute a catch/finally/fault handler region or a filter code or will propagate the exception in another frame. All these “events” will “clear” the evalStack. In case of EndFilter, the evalStack must contain exactly one item (an int32 which is popped off by EndFilter). Note that this has to be checked by the bytecode verification (see Figure 4.18) and not ensured by the exception handling mechanism.

CLR£ also extends the rules where the instructions considered for CLR£ might throw exceptions. The extension is defined through the semantics rules in Figure 4.17.

The rule for the Execute instruction is extended with the three cases when the instruction raises one of the exceptions DivideByZeroException, OverflowException, and ArithmeticException. A NullReferenceException is raised if the target reference of a Call, LoadField, StoreField, CallVirt, LoadFieldA. Accordingly, the macro VIRTCALL defined in Section 4.1.3.1 should be refined.

\[
\text{VIRTCALL}(\text{tail}, T::M, [\text{ref}] \cdot \text{vals}) \equiv
\]
\[
\text{if } \text{ref} \neq \text{null} \text{ then}
\]
\[
\text{let } T'::M = \text{lookUp}(\text{actualTypeOf } (\text{ref}), T::M) \text{ in}
\]
\[
\text{if } \text{actualTypeOf } (\text{ref}) = T' \in \text{ValueType} \text{ then}
\]
\[
\text{switch} := \text{Invoke}(\text{tail}, T'::M, [\text{addressOf } (\text{ref})] \cdot \text{vals})
\]
\[
\text{else switch} := \text{Invoke}(\text{tail}, T'::M, [\text{ref}] \cdot \text{vals})
\]
\[
\text{else RAISE}(\text{NullReferenceException})
\]

A NullReferenceException is raised also if the pointer popped from the evalStack by a LoadInd or StoreInd instruction points to an invalid address, or if the to be unboxed object of an Unbox instruction is the null reference. The Unbox instruction throws

---

21 One can formally prove that there is such a “step” in the further run of the EXCCLR.

22 Some versions of the CLR implementation, e.g., Microsoft .NET Framework (v2.0) [3], are lazy and do not throw a NullReferenceException if one of the instructions LoadFieldA, Call or CallVirt receives null as the target reference, when expecting a managed pointer, as opposed to an object reference. The exception is only raised at the first attempt to dereference the managed pointer. However, being defined according to [45], our semantics model contrasts with these lazy implementations.
an `InvalidCastException` if the object on the `evalStack` is not a boxed object of the instruction’s value type token argument. An `UnboxAny` instruction with a value type token argument throws a `NullReferenceException` if the boxed object is null\(^{23}\) and an `InvalidCastException` if the boxed object is not a boxed instance of the instruction’s type token argument. An exception of the same class is also thrown by a `RefAnyVal` instruction if the instruction’s type token argument is not the type stored in the typed reference. If the top stack element cannot be cast to `CastClass`’ type token argument, an `InvalidCastException` is thrown.

### 4.2.4 The Bytecode Verification

This section is about how the exception handling influences the bytecode verification. The exception specific conditions that should be ensured by the bytecode in order to achieve type safety are depicted in Section 4.2.4.1. The type consistency checks performed for the exception specific instructions are defined in Section 4.2.4.2. The simulation of the execution paths is extended in Section 4.2.4.3 to include the exception handling.

#### 4.2.4.1 The Bytecode Structure

In addition to the constraints stated in Section 4.1.4.2, the bytecode verification requires also that following condition holds:

| Tail Call in Handlers | A tail method call should not be used to transfer control out of a try block, filter region, catch/finally/filter/fault handler region. |

The reason for requiring Tail Call in Handlers is the same as that for Tail Call Pattern in Section 4.1.4.2. If the prefix `tail` is ignored when the bytecode is jitted, the caller frame would continue to execute after the callee’s frame returns. Because of Tail Call in Handlers, the caller frame would not matter for the exception mechanism if the callee’s frame returned with an exception. Although Tail Call in Handlers is not relevant in the type safety proof, we mention it here for completeness only.

#### 4.2.4.2 Verifying the Bytecode Instructions

As mentioned in Section 4.1.4.3, the evaluation stack before an instruction `instr` following an unconditional branch instruction should be empty, unless `instr` is the target of a forward branch instruction. As most of the exception specific instructions are regarded as unconditional branch instruction, the set `UncondBranchInstr` is extended accordingly: `UncondBranchInstr = \{Return, Throw, EndFinally, Leave(-)\}`.

Bytecode verification also checks that the stack is empty upon entering a `try` block. This check can be expressed as a constraint on the first instruction of any `try` block. For this

\(^{23}\)The ECMA Standard [45, Partition III, §4.33] states that a `NullReferenceException` is raised, regardless of the instruction’s type token argument. This is, however, not true as the instruction with reference type token arguments acts as a `CastClass` instruction, and consequently it throws no exception.
4.2. The Exceptions

Figure 4.17 The operational semantics rules for the exception-prone cases.

\[
\text{EXECCLR}(\text{instr}) \equiv \ \text{match} \ \text{instr} \\
\begin{align*}
\text{Execute}(\text{op}) & \rightarrow \ \text{let} \ \text{vals} = \text{take}(\text{evalStack}, \text{opNo}(\text{op})) \ \text{in} \\
& \hspace{1em} \text{if} \ \text{divByZeroCase}(\text{op}, \text{vals}) \ \text{then} \\
& \hspace{2em} \text{RAISE}(\text{DivideByZeroException}) \\
& \hspace{1em} \text{elseif} \ \text{overflowCase}(\text{op}, \text{vals}) \ \text{then} \\
& \hspace{2em} \text{RAISE}(\text{OverflowException}) \\
& \hspace{1em} \text{elseif} \ \text{invalidNrCase}(\text{op}, \text{vals}) \\
& \hspace{2em} \text{RAISE}(\text{ArithmeticException}) \\
\text{Call}(\_, \_, \text{T::M}) & \rightarrow \ \text{if} \ \text{T} \neq \ \text{System.Delegate} \ \text{then} \\
& \hspace{1em} \text{let} [\text{ref}] \cdot \_ = \text{take}(\text{evalStack}, \text{argNo}(\text{T::M})) \ \text{in} \\
& \hspace{2em} \text{if} \ \text{ref} = \text{null} \ \text{then} \ \text{RAISE}(\text{NullReferenceException}) \\
\text{LoadField}(\_, \_) & \rightarrow \ \text{let} \ \text{ref} = \text{top}(\text{evalStack}) \ \text{in} \\
& \hspace{1em} \text{if} \ \text{ref} = \text{null} \ \text{then} \ \text{RAISE}(\text{NullReferenceException}) \\
\text{StoreField}(\_, \_) & \rightarrow \ \text{let} [\text{ref}] \cdot \_ = \text{take}(\text{evalStack}, 2) \ \text{in} \\
& \hspace{1em} \text{if} \ \text{ref} = \text{null} \ \text{then} \ \text{RAISE}(\text{NullReferenceException}) \\
\text{CallVirt}(\_, \_, \text{T::M}) & \rightarrow \ \text{let} [\text{ref}] \cdot \_ = \text{take}(\text{evalStack}, \text{argNo}(\text{T::M})) \ \text{in} \\
& \hspace{1em} \text{if} \ \text{ref} = \text{null} \ \text{then} \ \text{RAISE}(\text{NullReferenceException}) \\
\text{LoadFieldA}(\_, \_) & \rightarrow \ \text{let} \ \text{val} = \text{top}(\text{evalStack}) \ \text{in} \\
& \hspace{1em} \text{if} \ \text{val} = \text{null} \ \text{then} \ \text{RAISE}(\text{NullReferenceException}) \\
\text{LoadInd}(\_) & \rightarrow \ \text{let} \ \text{adr} = \text{top}(\text{evalStack}) \ \text{in} \\
& \hspace{1em} \text{if} \ \neg \ \text{validAdr}(\text{adr}) \ \text{then} \ \text{RAISE}(\text{NullReferenceException}) \\
\text{StoreInd}(\_) & \rightarrow \ \text{let} [\text{adr}] \cdot \_ = \text{take}(\text{evalStack}, 2) \ \text{in} \\
& \hspace{1em} \text{if} \ \neg \ \text{validAdr}(\text{adr}) \ \text{then} \ \text{RAISE}(\text{NullReferenceException}) \\
\text{Unbox}(\text{T}) & \rightarrow \ \text{let} \ \text{ref} = \text{top}(\text{evalStack}) \ \text{in} \\
& \hspace{1em} \text{if} \ \text{ref} = \text{null} \ \text{then} \\
& \hspace{2em} \text{RAISE}(\text{NullReferenceException}) \\
& \hspace{1em} \text{elseif} \ \text{T} \neq \ \text{actualTypeOf}(\text{ref}) \ \text{then} \\
& \hspace{2em} \text{RAISE}(\text{InvalidCastException}) \\
\text{Unbox}.\text{Any}(\text{T}) & \rightarrow \ \text{if} \ \text{T} \in \ \text{ValueType} \ \text{then} \\
& \hspace{1em} \text{let} \ \text{ref} = \text{top}(\text{evalStack}) \ \text{in} \\
& \hspace{2em} \text{if} \ \text{ref} = \text{null} \ \text{then} \ \text{RAISE}(\text{NullReferenceException}) \\
& \hspace{1em} \text{elseif} \ \text{T} \neq \ \text{typedRefType}(\text{tr}) \ \text{then} \\
& \hspace{2em} \text{RAISE}(\text{InvalidCastException}) \\
\text{RefAnyVal}(\text{T}) & \rightarrow \ \text{let} \ \text{tr} = \text{top}(\text{evalStack}) \ \text{in} \\
& \hspace{1em} \text{if} \ \text{T} \neq \ \text{typedRefType}(\text{tr}) \ \text{then} \\
& \hspace{2em} \text{RAISE}(\text{InvalidCastException}) \\
\text{CastClass}(\text{T}) & \rightarrow \ \text{let} \ \text{ref} = \text{top}(\text{evalStack}) \ \text{in} \\
& \hspace{1em} \text{if} \ \text{ref} \neq \text{null} \ \text{and} \ \text{actualTypeOf}(\text{ref}) \nless T \ \text{then} \\
& \hspace{2em} \text{RAISE}(\text{InvalidCastException}) \\
\text{LoadVirtFtn}(\_) & \rightarrow \ \text{let} \ \text{ref} = \text{top}(\text{evalStack}) \ \text{in} \\
& \hspace{1em} \text{if} \ \text{ref} = \text{null} \ \text{then} \ \text{RAISE}(\text{NullReferenceException})
\end{align*}
\]
Figure 4.18 Verifying the exception specific instructions.

\[
\text{check}(\text{meth}, \text{pos}, \text{argZeroT}, \text{evalStackT}) \iff \text{enterTry}(\text{meth}, \text{pos}, \text{evalStackT}) \land \\
\text{match code(pos)} \\
\begin{array}{l}
\text{Throw} \rightarrow \text{evalStackT} \sqsubseteq_{\text{uf}} [\text{object}] \\
\text{EndFilter} \rightarrow \text{evalStackT} \sqsubseteq_{\text{uf}} [\text{int32}] \\
\text{EndFinally} \rightarrow \text{True} \\
\text{Leave}_(-) \rightarrow \text{True}
\end{array}
\]

purpose, we define the predicate \(\text{enterTry}\). Given a method \(mref\), a code index \(\text{pos}\) and a stack state \(\text{evalStackT}\), \(\text{enterTry}(mref, \text{pos}, \text{evalStackT})\) is true if \(\text{evalStackT}\) is empty should exist at least one exception handler in the exception handling array of \(mref\) whose try block has the first instruction at \(\text{pos}\).

\[
\text{enterTry}(mref, \text{pos}, \text{evalStackT}) :\iff \\
\exists h \in \text{excHA}(mref) : (\text{pos} = \text{tryStart}(h)) \Rightarrow \text{evalStackT} = []
\]

This constraint is not relevant in the type safety proof. Its goal is to simplify the JIT compilation scheme. It could happen that the evaluation of an expression is compiled into a sequence of bytecode instructions which is further jitted into a single CPU instruction. If a try block is allowed to be placed in the middle of an expression, the JIT compilation scheme would get complicated, as it would have to break up the expression into protected and non-protected regions. By requiring the stack to be empty upon entering every try block, the bytecode verification does not allow a try block to be placed in the execution of an expression.

The check predicate expressing the type consistency checks is defined in Figure 4.18 also for the exception specific instructions. Moreover, its definition also specifies the above constraint on the first instruction of any try block. The \(\text{Throw}\) instruction expects an object reference on the stack. The \(\text{EndFilter}\) instruction which terminates the execution of a filter region expects an integer on the stack. For the instructions \(\text{EndFinally}\) and \(\text{Leave}\), nothing has to be checked.

4.2.4.3 Computing Successor Type States

All the instructions can throw exceptions. Although this assumption does not match the semantics rules defined so far, we assume so, since the special \(\text{ExecutionEngineException}\) may be thrown at any time during the execution of a program. Therefore, for an instruction at \(\text{pos}\), the exception handlers \(h\) that protect \(\text{pos}\), i.e., for which \(\text{isInTry}(\text{pos}, h)\) holds, are added by the function \(\text{excHandlers}\) to the set of successor type states defined through the \(\text{succ}\) function. Upon entering a catch handler \(h\), the stack state contains the type \(\text{typeExc}(h)\) of exceptions \(h\) is handling (if \(\text{typeExc}(h)\) is a reference type) or the type \(\text{boxed}(\text{typeExc}(h))\) (if \(\text{typeExc}(h)\) is a value type\(^{24}\)), whereas upon entering a filter, the stack state is \([\text{object}]\). Upon entering a finally or a fault handler, the stack state is \([\text{}]\). By entering the filter region

\(^{24}\)The ECMA Standard [45] does not mention anything concerning the fact that exceptions of value types are considered boxed inside catch handler regions.
of a filter handler \( h \), the successor code index is the index of the first instruction of the filter region, i.e., \( \text{filterStart}(h) \). Otherwise, the successor code index is the index of the first instruction of the handler region of \( h \), i.e., \( \text{handlerStart}(h) \).

Example 4.2.5 Consider the skeleton bytecode of Figure 4.20. To understand the definition of \( \text{excHandlers} \), let us look in detail at the successors via the exception handling array of the instruction at the code index 6. The possible successors are:

- the instruction at 9, i.e., the first instruction of the catch handler region: At run-time, this handler region might be executed in the Unwind pass if an exception of the type NullReferenceException is thrown in the associated try block;

- the instruction at 17, i.e., the first instruction of the fault handler region: At run-time, this handler region might be executed in the Unwind pass if the exception is not of the type NullReferenceException;

- the instruction at 24, i.e., the first instruction of the filter region: At run-time, this region might be executed in the StackWalk pass if the exception is not of the type NullReferenceException;

- the instruction at 27, i.e., the first instruction of the filter handler region: At run-time, this handler region might be executed in the Unwind pass if this filter handler is suitable to handle the exception;
• the instruction at 31, i.e., the first instruction of the finally handler region: At run-
time, this handler region might be executed in the Unwind pass if the exception is not
of the type NullableException and the filter handler is not suitable for
handling the exception;

Note that the code indices 27 and 31 cannot be reached at run-time directly from 6. The reason
is that, the exception handing mechanism has to execute the fault handler region before 27
and 31 are reached. However, to simplify the definition of excHandlers, we consider a con-
servative definition by including more successors than possibly reachable at run-time. This
conservative approach does not affect the set of verifiable methods.

Apart from the successors defined by the function excHandlers, the instructions Throw
and EndFilter have no other successors. The EndFinally instruction terminates the ex-
ecution of a finally/fault handler region. If the associated try block was exited
with a Leave(target) instruction, control is transferred to target if the EndFinally
instruction is the last “on the way” from the Leave instruction to its target. The possible tar-
gets are defined by the set LeaveThroughFin. A code index target is an element of the set
LeaveThroughFin(meth, pos) if the instruction code(pos) is an EndFinally instruction of the last finally handler “on the way” from an instruction Leave(target) embedded into the try
block associated to the EndFinally instruction to the instruction code(target). By this definition,
the set LeaveThroughFin is empty for an EndFinally instruction which corresponds to a fault
handler. This means that the EndFinally instruction which terminates the execution of a fault
handler region has no successors (apart from those defined in excHandlers).

\[
\text{LeaveThroughFin}(\text{meth}, \text{pos}) = \\
\{ \text{target} \in \text{Pc} \mid \exists \text{pos}' \in \text{Pc} \text{ such that code}(\text{pos}') = \text{Leave}(\text{target}) \text{ and} \\
[h' \in \text{excHA}(\text{meth}) \mid \text{isFinFromTo}(h', \text{pos}', \text{target})] = [\ldots, h], \\
\text{where } h \text{ is defined by } [h' \in \text{excHA}(\text{meth}) \mid \text{isInHandler}(\text{pos}, h')] = [h, \ldots] \}
\]

As the EndFinally instruction empties the evaluation stack, the stack state of the successors
of EndFinally is the empty list.

We reason below for the definition of succ in case of the Leave instruction. If there is no finally handler from the Leave instruction to its target, the successor is given by the
target instruction with an empty stack state (this is because the Leave instruction empties the evaluation stack). If there is a finally handler in between, say h, the successor given by the
instruction at handlerStart(h) is already considered in excHandlers.

Example 4.2.6 Consider again the skeleton example from Figure 4.20. We look here in detail
at the successors – others than those defined via the exception handling array – of some of the
Leave and EndFinally instructions.

The Leave instruction at the code index 2 has no successor (except the successors defined
via the exception handling array) since the finally handler region is “on the way” from 2 to
the target 40. The successor of the Leave instruction at the code index 15 is the instruction at
the code index 16 since there is no finally block “on the way” from 15 to the target 16.
**Figure 4.20** Example: successors of *Leave* and *EndFinally* instructions.

```
..try
{  
  ..try
  {  
    ..try
    {  
      ..try
      {  
        2: leave 40  
        ...
        8: leave 23  
      } // end try block  
      catch NullReferenceException
      {  
        9: ...
        ...
        15: leave 16  
      } // end handler region  
    16: ...
    } // end try block  
  fault  
  {  
    17: ...
    ...
    22: endfinally  
  } // end handler region  
  } // end try block  
  23: ...

filter  
{  
  24: ...
  ...
  26: endfilter  
} // end filter region  
{  
  27: ...
  ...
  30: leave 41  
} // end handler region  
} // end try block  

finally  
{  
  31: ...
  ...
  39: endfinally  
} // end handler region  
40: ...
41: ...
```
The successors of the EndFinally instruction at the code index 39 are defined via the set LeaveThroughFin(meth, 39). This set consists of the targets 40 and 41 since the EndFinally instruction at 39 is the last EndFinally instruction “on the way” from the Leave instructions at 2 and 30 to their targets.

4.2.5 Type Safety

This section extends the type safety proof in Section 4.1.6 to include the exception handling. We show that the properties guaranteed by Theorem 4.1.1 are preserved upon adding exceptions. However, besides those properties, we consider one more type safety property:

**Type safety**: Every handled exception is an object reference, whose type is derived from `Exception`.

The definition of well-typed methods in Section 4.1.5 Definition 4.1.9, which serves as the basis for the type safety proof, remains unchanged. The theorem asserting type safety of well-typed methods requires the following lemmas.

Lemma 4.2.7 claims that, after the `stackCursor` reaches in the `Unwind` pass the frame containing the suitable `handler`, `pc` is in the `try` block of `handler`.

**Lemma 4.2.7** Let `stackCursor = (fr, n)` and `handler = (fr', n')` be the stack cursor and the target handler, respectively, where `fr' = (\_ , \_ , \_ , \_ , \_ , mref')`. If `pass = Unwind` and `sameFrame(fr, fr')`, then `isInTry(pc, h)` where `h` is the handler pointed to by `handler`, i.e., `excHA(mref')(n')`.

**Proof.** By the definitions of `matchCatch` and `matchFilter`, we have that `isInTry(pc, h)` holds if `sameFrame(fr, fr')`, `pass = Unwind`, and `n = 0`. That the property is preserved can be easily proved by induction on the run of the exception mechanism EXCCLR.

The following lemma states that if the current program counter is in a `filter` region while the exception mechanism is running in an `Unwind` pass, then there exists at least one more exception whose handling has not been completed yet.

**Lemma 4.2.8** If `pass = Unwind` and there exists `h \in excHA(meth)` such that `isInFilter(pc, h)`, then `passRecStack \neq []`.

**Proof.** By `pass = Unwind`, we derive that `exc` is defined, i.e., the mechanism is handling an (inner) exception. According to [45, Partition I, §12.4], control is never permitted to enter a `filter` region except through the exception mechanism. This can only happen in the `StackWalk` pass of an (outer) exception whose associated record has been saved in `passRecStack`. It then remains to show that this record has not been discarded in the meantime. But a record associated to an exception can only be discarded while the exception is handled in the corresponding suitable handler region. However, this could not have occurred since the (outer) exception is still in the `StackWalk` pass.

**Lemma 4.2.9** shows that if there are no more handlers to be inspected in a `Leave` pass, then there cannot be any `finally` handlers “on the way” from `pc` to the target of the `Leave` pass.
Lemma 4.2.9 If pass = Leave and existsHanWithinFrame(stackCursor) is false, then
\{ h \in \text{excHA}(\text{meth}) \mid \text{isFinFromTo}(h, pc, target) \} = \emptyset

Proof. Let \( \mathcal{A} \) be the set \( \{ h \in \text{excHA}(\text{meth}) \mid \text{isFinFromTo}(h, pc, target) \} \). Let us assume that at the beginning of the Leave pass, i.e., when the stackCursor is pointing to the index 0 in excHA, the set \( \mathcal{A} \) is non-empty. However, for every handler in \( \mathcal{A} \), say \( h \), there will eventually be a step in the run of the exception mechanism which examines \( h \) for the current Leave pass. When \( h \) is going to be inspected, it is actually removed from \( \mathcal{A} \). The reason is that when \( h \)'s handler region is executed, \( pc \) is not anymore in the try block of \( h \).
\( \square \)

We are now ready to state the theorem asserting type safety of well-typed methods. In addition to the invariants considered in Section 4.1.6, an exception specific invariant (exc) is added. This ensures that if the exception mechanism is handling an exception, the exception type is a subtype of Exception, regardless of the flag wrapNonExc.

Theorem 4.2.1 (Type safety of well-typed methods) Let \( [fr'_1, \ldots, fr'_{k-1}] \) be the frameStack and \( fr_k \) be the current frame. For every \( i = 1, k - 1 \), we denote by \( fr_i \) the succession of \( fr'_i \).

We assume that the type system and the bytecode structure of the methods in \( (fr_i) \) satisfy the conditions stated in Section 4.1.4.2 and Section 4.2.4.1. Moreover, we assume that all methods of \( (fr_i) \) are well-typed.

Let \( fr_i = (pc^*, locAdr^*, argAdr^*, \text{evalStack}^*, \text{meth}^*) \) be one of the frames \( (fr_i) \). We denote by \( \text{locVal}^* \) and \( \text{argVal}^* \) the functions \( \text{locVal} \) and \( \text{argVal} \) derived based on \( \text{locAdr}^* \) and \( \text{argAdr}^* \), respectively. If \( fr_i \) is the current frame, we consider \( \text{initState}^* \) to be the current initialization status \( \text{initState} \). If \( fr_i \) is not the current frame, we denote by \( \text{initState}^* \) the succession of \( \text{initState} \) for \( fr_i \). Let \( (\text{argZeroT}_j, \text{evalStackT}_j)_{j \in \mathcal{D}} \) be a family of type states \( \text{meth}^* \) well-typable with.

The invariants in Theorem 4.1.1 and (exc) are satisfied at run-time for every frame \( fr_i \) (for the current frame \( fr_k \), the invariants (stack1) and (stack2) should hold if switch = Noswitch):

(\text{exc}) If ref is an exception reference in passRecStack, then \([\_] \vdash \text{ref} : \text{Exception}. \text{If exc is defined, then } [\_] \vdash \text{exc} : \text{Exception}.

Proof. The proof proceeds similarly with the proof of Theorem 4.1.1, i.e., by induction on the run of the semantics model CLR_E including exception handling. To extend the proof of Theorem 4.1.1, we consider here that the exception mechanism has control, i.e., switch = ExcMech. Different cases have to be considered depending on pass. We only analyze here the cases when pass = Unwind and pass = Leave.

Case 1 Assume that the exception mechanism is currently executing an Unwind pass of an exception, i.e., pass = Unwind.
Subcase 1.1 If there are still handlers to be inspected in the current frame, let $h$ be the handler pointed to by the `stackCursor`.

Subcase 1.1.1 Assume that `handler` is defined and $h$ matches `handler`. By the definition of `matchTargetHan`, we have that `handler` and `stackCursor` are pointing to the same frame, i.e., `sameFrame(pr_1(stackCursor), pr_1(handler))`. By Lemma 4.2.7, we get `isInTry(pc, h)`. This and the definitions of `succ` and `excHandlers` imply:

- If $h$ is a filter handler, then $(handlerStart(h), argZeroT_{pc}, [object])$ is an element of the set $suc(meth, pc, argZeroT_{pc}, evalStackT_{pc})$;
- If $h$ is a catch handler, then $(handlerStart(h), argZeroT_{pc}, [typeExc(h)])$ is in the set $suc(meth, pc, argZeroT_{pc}, evalStackT_{pc})$.

The exception mechanism runs $h$’s handler region by switching to normal execution mode: The new $pc$ is $handlerStart(h)$ (so, $(pc)$ is preserved), the new $evalStack$ is $[exc]$ (if wrapNonExc is True) or $[wrappedExc(exc)]$ (if wrapNonExc is False), and $switch$ becomes Noswitch. Definition 4.1.9 (wt5) and (pc) ensure $handlerStart(h) \in D$ independent of $h$’s kind,

$$[object] \subseteq_{len} evalStackT_{handlerStart(h)} \tag{4.11}$$

if $h$ is a filter, and

$$[typeExc(h)] \subseteq_{len} evalStackT_{handlerStart(h)} \tag{4.12}$$

if $h$ is a catch and $typeExc(h) \in \text{RefType}$, and

$$[boxed(typeExc(h))] \subseteq_{len} evalStackT_{handlerStart(h)} \tag{4.13}$$

if $h$ is a catch and $typeExc(h) \in \text{ValueType}$.

By the induction hypothesis – that is, by $(\text{exc})$ – we have $[\text{exc}] \vdash exc : \text{Exception}$. So, $exc$ is an object reference and, by Definition 4.1.12, $type(exc) \preceq \text{Exception}$. By this, (4.11) and Lemma 4.1.5, we get that $(\text{stack1})$ and $(\text{stack2})$ are preserved if $h$ is a filter handler.

If $h$ is a catch handler, by the definition of `matchCatch`, we have

$$\begin{align*}
\text{wrapNonExc} \Rightarrow & \ \text{actualTypeOf}(\text{exc}) \preceq \text{typeExc}(h) \land \\
\neg \text{wrapNonExc} \Rightarrow & \ \text{actualTypeOf}(\text{wrappedExc}(\text{exc})) \preceq \text{typeExc}(h)
\end{align*} \tag{4.14}$$

The invariants $(\text{stack1})$ and $(\text{stack2})$ follow from Lemma 4.1.5, (4.12), (4.13), and (4.14). Hence, $(\text{stack1})$ and $(\text{stack2})$ are preserved, regardless of $h$’s kind.

Subcase 1.1.2 The case when $h$ is a matching finally is treated similarly to Subcase 2.1.1.1 except that the new $evalStack$ is empty, and therefore the invariants $(\text{stack1})$ and $(\text{stack2})$ are obviously preserved.

Subcase 1.1.3 Assume that $h$ is a filter whose filter region contains $pc$. By Lemma 4.2.8, `passRecStack` cannot be empty. This means that `switch` is still ExcMech after the exception mechanism executes a step. Consequently, nothing has to be proved for the current frame for $(\text{stack1})$ and $(\text{stack2})$. The invariant $(\text{pc})$ simply follows from the induction hypothesis.
4.2. The Exceptions

Subcase 1.1.4  If \( h \) is not of any of the above types, then \( \text{stackCursor} \) is incremented to point to the next handler. The invariants are simply preserved.

Subcase 1.2  If there are no more handlers to be examined in the current frame, \( \text{frame} \) is discarded, and the \( \text{stackCursor} \) is reset to point to the frame’s caller frame. The execution model \( \text{switch} \) is still \( \text{ExcMech} \), and therefore the invariants \((\text{stack1})\) and \((\text{stack2})\) are not supposed to hold. The induction hypothesis implies \((\text{pc})\).

Case 2  Consider now that the exception mechanism is in a \textit{Leave} pass, i.e., \( \text{pass} = \text{Leave} \).

Subcase 2.1  Assume that there are still handlers to be examined in the current frame. Let \( h \) be the handler pointed to by the \( \text{stackCursor} \).

Subcase 2.1.1  If \( h \) is a \textit{finally} handler “on the way” from the \( \text{pc} \) to \( \text{target} \), i.e., \( \text{isFinFromTo}(h, \text{pc}, \text{target}) \), then the exception mechanism runs \( h \)'s handler region by switching to the normal execution mode: The new \( \text{pc} \) is \( \text{handlerStart}(h) \), the new \( \text{evalStack} \) is \([\,]\), and \( \text{switch} \) is \( \text{Noswitch} \). From the definitions of \( \text{succ} \) and \( \text{excHandlers} \), we know that \((\text{handlerStart}(h), \text{argZeroT}_{\text{pc}}, [\,])\) is an element of \( \text{succ}(\text{meth}, \text{pc}, \text{argZeroT}_{\text{pc}}, \text{evalStackT}_{\text{pc}}) \). By this, \((\text{pc}), \) and Definition 4.1.9 (wt6), we derive \( \text{handlerStart}(h) \in \mathcal{D} \) and also \([\,] \subseteq \text{len} \text{evalStackT}_{\text{handlerStart}(h)} \). This means that \((\text{pc}), (\text{stack1}), \) and \((\text{stack2})\) are preserved for the current frame (the last two invariants hold since \( \text{evalStackT}_{\text{handlerStart}(h)} \) is necessarily \([\,]\)).

Subcase 2.1.2  If \( h \) is real handler whose handler region is “exited on the way” from the \( \text{pc} \) to \( \text{target} \), then a pass record is popped from \( \text{passRecStack} \). So, no invariant is affected.

Subcase 2.2.2  Let us assume that there are no more handlers to be inspected in the current frame. The exception mechanism switches to normal execution mode: \( \text{pc} \) is set to \( \text{target} \), \( \text{evalStack} \) to \([\,]\), \( \text{switch} \) to \( \text{Noswitch} \), and the current \textit{Leave} record is popped from \( \text{passRecStack} \). The only invariants that have to be proved are \((\text{pc}), (\text{stack1}), \) and \((\text{stack2})\). By Lemma 4.2.9, \( \{ h \in \text{excHA}(\text{meth}) | \text{isFinFromTo}(h, \text{pc}, \text{target}) \} \) is empty. From the definition of \( \text{succ} \), we have that \((\text{target}, \text{argZeroT}_{\text{pc}}, [\,])\) is in \( \text{succ}(\text{meth}, \text{pc}, \text{argZeroT}_{\text{pc}}, \text{evalStackT}_{\text{pc}}) \). From this, \((\text{pc}), \) and Definition 4.1.9 (wt6), we obtain \( \text{target} \in \mathcal{D} \) and \([\,] \subseteq \text{len} \text{evalStackT}_{\text{target}} \). Therefore, \((\text{pc}), (\text{stack1}), \) and \((\text{stack2})\) are preserved for the current frame (the last two invariants hold since \( \text{evalStackT}_{\text{target}} \) is \([\,]\)).  

\[ \square \]

Remark 4.2.7  As pointed out in the proof of Theorem 4.1.1, \textit{Subcase 2.1}, the invariants \((\text{stack1})\) and \((\text{stack2})\) are only guaranteed to hold if \( \text{switch} = \text{Noswitch} \). The semantics of the exception handling confirms once again this aspect. If, for instance, a method call receives a \texttt{null} as a \texttt{this} pointer, \texttt{null} is popped from the evaluation stack and a "null" check is done. Consequently, a \texttt{NullReferenceException} is then raised. So, while the stack state corresponding to the call instruction remains unchanged, the evaluation stack has one less value. Obviously, \((\text{stack1})\) and \((\text{stack2})\) are not satisfied. However, this is alright since the evaluation stack is not accessed as long as \( \text{switch} = \text{ExcMech} \).
4.3 The Generics

Kennedy and Syme proposed adding support for generics in the CLR [88]. Their proposal was at the basis of the generics implementation in the .NET Framework (v2.0) [3]. Generics are an extension to the CLR that allows to define types and methods parameterized in terms of types. That means, for example, that the same method implementation might be able to deal with both int32 and object, without knowing in advance which type it is working with. Some strengths of the CLR generics are verification-time type safety, efficiency, and clarity.

A class or an interface whose declaration is parameterized by one or more generic parameters is called generic type. A method declared within a type, whose signature includes one or more generic parameters (not present in the type declaration itself) is called generic method. When the generic code is referenced by consumer code, the generic types and methods are instantiated by replacing every generic parameter with a generic argument.

To be useful, generic code needs to be able to call methods other than those defined by object. As the generic code is only valid if it works for every possible constructed generic instantiation, CLR supports a special feature called constraints (also known as F-bounded polymorphism [33]). The constraints are an optional component of a generic type or method definition. A generic type may define any number of constraints, and each constraint applies an arbitrary limitation on the types that can be used as generic arguments. By restricting the types that can be used in the construction of a generic type, the restrictions on the code that references the constrained type parameter are relaxed.

To allow conversions between related (but distinct) instances of the same generic type, the CLR supports variance on generic parameters, in the form of covariant generic arguments and contravariant generic arguments. For simplicity, variance is only permitted on generic interfaces. Covariance means that the subtype relationship of the generic type varies directly with the relationship of the generic argument, whereas the contravariance means that the subtype relationship of the generic type varies inversely with the relationship of the generic argument. An example of a language that provides support for covariance is Eiffel [46]. The contravariance is supported, for instance, by Java (v5.0) [67], Sather [80], Scala [13], and OCaml [12]. The .NET compliant languages not willing to support variance can ignore this feature and treat all generic parameters as non-variant.

Challenges Several versions of Eiffel turned out to be unsafe also due to the variance on generic parameters [37, 75]. Thus, in the context of generic parameter variance, virtual method calls are problematic in ensuring type safety. Let us assume, for example, that the bytecode contains a virtual call of the method T::M. Let T′::M be the method that will be invoked at run-time. The following aspects are critical for type safety:

- due to the variance, the signature of T′::M may not match the signature of T::M; Consequently, the following potential risks for type safety may arise: (1) the arguments of T::M ensured to be of T::M’s argument types might be treated in T′::M as being of another types, namely of T′::M’s argument types; (2) the return value of T′::M guaranteed to be of T′::M’s return type might be handled in the frame containing the virtual call of T::M as being of another type, namely of T::M’s return type.
if \( T::M \) is a generic method, \( T'::M \) should also be generic, but its constraint types may not match the corresponding constraint types of \( T::M \). This is a potential risk for type safety as \( T::M \)'s generic arguments, restricted by \( T::M \)'s constraints, might be treated in \( T'::M \) as being bounded by other constraints, namely by \( T'::M \)'s constraint types. As the latter constraints might not be satisfied by \( T::M \)'s generic arguments, type safety would be violated.

Another critical issue concerns the validity of the run-time instantiation of generic types and methods. Let us assume, for example, that the types \( (U_i)_{i=0}^{n-1} \) satisfy at verification time the constraints associated to the type \( C(U_0, \ldots, U_{n-1}) \). As any \( U_i \) can involve generic parameters, one has to make sure that after \( (U_i)_{i=0}^{n-1} \) get instantiated at run-time with the generic arguments \( (V_i)_{i=0}^{n-1} \), also \( (V_i)_{i=0}^{n-1} \) satisfy the constraints.

Of high risk for type safety turns out to be the boxing of nullable types. Through nullable types, CLR provides support for nullable forms of all value types. A nullable type is a generic value type (that implements no interface) of the form \( \text{Nullable}(T) \), where \( T \) can only be instantiated with value types, other than nullable types. Let \( \text{NullableType} \) denote the (recursively defined) set of nullable types \( \{ \text{Nullable}(T) \mid T \in \text{ValueType} \setminus \text{NullableType} \} \).

A nullable type \( \text{Nullable}(T) \) combines a value of the underlying type \( T \) with a boolean null indicator. More precisely, an instance of a type \( \text{Nullable}(T) \) has two instance fields: \( \text{HasValue}, \) of type \( \text{bool} \) (however, remember that the booleans are treated as \( \text{int32} \) values), indicating whether the underlying value is defined, and \( \text{Value} \), of type \( T \), carrying the defined value (if any). Boxing nullable types results always in situations when the dynamic (run-time) types of the boxed values do not match the static (verification) types predicted by the bytecode verification. Surprisingly, the cause of this problem, i.e., the special semantics of boxing nullable types, is not documented in the ECMA Standard [45]. The only information we have originates from [109, 42].

**Section plan** The remainder of this section is organized as follows. Section 4.3.1 gives a formalization of the extension with generics of the CLR type system. Section 4.3.2 defines CLR\(_G\) by refining the operational semantics model CLR\(_E\). Section 4.3.3 deals with the changes to the bytecode verification. Finally, Section 4.3.4 relates the type system with generics and the operational semantics model CLR\(_G\) in theorem asserting type safety of well-typed methods.

### 4.3.1 The Polymorphic Type System

This section extends the type system defined in Section 4.1.2 by formally introducing the generic types and methods described above. The universe \( \text{Type} \) is extended with generic parameters, elements of \( \text{GenericParam} \), as shown in Table 4.16. Table 4.17 gathers the selector functions which we define to deal with generics\(^{25}\).

In the bytecode, the generic parameters are referred to as \( !i \), whereas the method generic parameters are addressed as \( !!i \). Thus, \( !i \) denotes the \( i \)-th type generic parameter (numbered from left-to-right in the relevant type declaration). Similarly, \( !!i \) designates the \( i \)-th method generic parameter (numbered from left-to-right in the relevant method declaration). The universe \( \text{GenericParam} \) is defined relative to a given (possibly generic) type or method\(^{26}\).
Chapter 4. Type Safety of CLR

Table 4.16: Type system extension with generics.

<table>
<thead>
<tr>
<th>Function definition</th>
<th>Function name</th>
</tr>
</thead>
<tbody>
<tr>
<td>genParamNo : Map(GenericType ∪ MRef, N)</td>
<td>generic type/method’s arity</td>
</tr>
<tr>
<td>genArg : Map(GenericType ∪ MRef, List(Type \ PointerType))</td>
<td>generic argument list</td>
</tr>
<tr>
<td>constr_{i}^{T} : Type \ PointerType</td>
<td>T’s i-th constraint</td>
</tr>
<tr>
<td>constr_{i}^{mref} : Type \ PointerType</td>
<td>mref’s i-th constraint</td>
</tr>
<tr>
<td>(\text{var}_{i}^{mref} )^{n-1} : List({+, -, \emptyset})</td>
<td>I’n’s variances array</td>
</tr>
</tbody>
</table>

Table 4.17: Generics specific selector functions.

Let GenericType stand for the universe of generic types. Only the classes and interfaces can be generic: \(\text{GenericType} \subseteq \text{Class} \cup \text{Interface}\). Typically, \(T’n\) represents the (raw) name of a generic type \(T\) with \(n\) generic parameters. As a convention, we assume that \(\text{GenericType}\) has only instantiated generic types and not generic types’ raw names.

Every generic parameter can have an optional constraint consisting of a type\(^{27}\). Every generic argument which replaces a generic parameter should be compatible when boxed with the type given in the corresponding constraint (see Definition 4.3.4 for the compatibility relation for verification types). This means, for instance, that every value type can be used as generic argument for a generic parameter constrained by \text{object}.

A covariant generic parameter is marked with “+” in the interface declaration, whereas “−” is used to denote a contravariant generic parameter. For the sake of notation, we mark with “\(\emptyset\)” the non-variant parameters.

The subtype relation \(\preceq\) of Definition 4.1.1 is extended to generic types. The relation is defined for both \emph{open generic types}, \ie, instantiations of generic types that have not been supplied all generic arguments for their generic parameters (because they are open to accepting more generic arguments), and \emph{closed generic types}, \ie, instantiations of generic types which have been supplied all of their generic arguments. The definition captures the intuitive notion of generic parameters variance. In this definition, \(T \circ [U_{i}/X_{i}]_{i=0}^{n-1}\) designates the type \(T\), where each generic parameter \(X_{i}\) is substituted by the type \(U_{i}\).

**Definition 4.3.1 (Subtype relation)** The subtype relation \(\preceq\) is the least reflexive and transitive relation such that

- if \(T \in \text{Class} \cup \text{Interface and } T’\ is \text{object}, or

---

\(^{25}\)Several examples illustrating the definitions of the selector functions are included in [59, 60].

\(^{26}\)As a matter of fact, GenericParam should be defined as a map assigning to every (possibly generic) type/method the set of its generic parameters. However, as the parameters’ reference is bounded to the relevant generic type/method bodies, we take the liberty to define GenericParam as a universe, making sure that no confusion can occur.

\(^{27}\)This differs slightly from [45], which allows a constraint to have more than one type. The restriction does not reduce the complexity, but simplifies the exposition.
4.3. The Generics

- if \( T \in \text{ValueType} \) and \( T' \) is \( \text{System.ValueType} \), or
- if \( T \in \text{ObjClass} \setminus \text{GenericType} \) and \( T' \) is a base class of \( T \) or an interface implemented by \( T \), or
- if \( T \in \text{Interface} \setminus \text{GenericType} \) and \( T' \) is an interface implemented by \( T \), or
- if \( T \in \text{ValueClass} \setminus \text{GenericType} \) and \( T' \) is an interface implemented by \( T \), or
- if \( T \) is \( C\langle U_0, \ldots, U_{n-1} \rangle \), where \( C'n \) is a generic object class with the generic parameters \( \langle X_i \rangle_{i=0}^{n-1} \), \( T'' \) is a base class of \( C'n \) or an interface implemented by \( C'n \), and \( T' \) is \( T'' \circ [U_i/X_i]_{i=0}^{n-1} \), or
- if \( T \) is \( I\langle U_0, \ldots, U_{n-1} \rangle \), where \( I'n \) is a generic interface with the generic parameters \( \langle X_i \rangle_{i=0}^{n-1} \), \( T'' \) is an interface implemented by \( I'n \), and \( T' \) is \( T'' \circ [U_i/X_i]_{i=0}^{n-1} \), or
- if \( T \) is \( C\langle U_0, \ldots, U_{n-1} \rangle \), where \( C'n \) is a generic value class with the generic parameters \( \langle X_i \rangle_{i=0}^{n-1} \), \( T'' \) is an interface implemented by \( C'n \), and \( T' \) is \( T'' \circ [U_i/X_i]_{i=0}^{n-1} \), or
- if \( T \) is \( I\langle U_0, \ldots, U_{n-1} \rangle \) and \( T' \) is \( I\langle V_0, \ldots, V_{n-1} \rangle \), and for every \( i = 0, n-1 \) the following conditions hold:
  - if \( \text{var}_i'^{'n} = \emptyset \) or \( V_i \in \text{ValueType} \) or \( V_i \in \text{GenericParam} \), then \( U_i = V_i \);
  - if \( \text{var}_i'^{'n} = + \), then \( U_i \preceq V_i \);
  - if \( \text{var}_i'^{'n} = - \), then \( V_i \preceq U_i \);

then \( T \preceq T' \).

Since it is possible for different methods to have identical signatures when the declaring types are instantiated and consequently ambiguous references, the method references have the return type and parameter types uninstantiated.

**Example 4.3.1** Let us consider the skeleton of the generic class \( C'2 \) in Figure 4.21. One can easily notice that the generic instantiation \( C\langle \text{string, string} \rangle \) contains two generic methods, which can only be distinguished through their uninstantiated signatures, namely
\[
\begin{align*}
0 & C\langle \text{string, string} \rangle :: M(Z)(0, !!0) \\
1 & C\langle \text{string, string} \rangle :: M(Z)(1, !!0)
\end{align*}
\]
To simplify a few aspects of our formal development, we assume that the environment specific functions \( \text{retType} \) and \( \text{paramTypes} \) are defined with respect to these uninstantiated signatures. This means, for instance, that it holds:
\[
\begin{align*}
\text{retType}(0 C\langle \text{string, string} \rangle :: M(Z)(0, !!0)) &= 0 \\
\text{paramTypes}(0 C\langle \text{string, string} \rangle :: M(Z)(0, !!0)) &= [0, !!0]
\end{align*}
\]
To get the instantiated return type and parameter types of a generic method reference, we define \( \text{inst} : \text{Map}(MRef, \{[U_i/X_i]_{i=0}^{n} \mid (U_i, X_i) \in \text{Type} \times \text{GenericParam} \text{ for every } i = 0, n \}) \), which assigns to every generic method the type substitution (or simply substitution when it is obvious that we are talking about types) that should be applied to the method return type and
signature. However, due to technical considerations, the “instantiation” function $\text{inst}$ is also defined for non-generic methods defined by both generic and non-generic types. If the method is not generic and is declared by a non-generic type, $\text{inst}$ returns the empty substitution $\epsilon$. If the method is not generic, but is declared by a generic type, then $\text{inst}$ yields the generic type’s underlying instantiation. Similarly, if the method is generic and declared by a non-generic type, then $\text{inst}$ generates the generic method’s underlying instantiation. Finally, if the method is generic and declared by a generic type, then $\text{inst}$ returns the substitution formed by composing the generic type’s and method’s underlying instantiations. Note that the same generic parameter may occur in both the range and domain of a type substitution.

**Definition 4.3.2** Given a method $T::M$, we define

$$\text{inst}(T::M) = \begin{cases} 
\epsilon, & \text{if } T \notin \text{GenericType and } T::M \text{ is not generic;} \\
\left[\text{genArg}(T)(i)/i\right]_{i=0}^\text{genParamNo}(T)-1, & \text{if } T \in \text{GenericType and } T::M \text{ is not generic;} \\
\left[\text{genArg}(T::M)(i)/i\right]_{i=0}^\text{genParamNo}(T::M)-1, & \text{if } T \notin \text{GenericType and } T::M \text{ is generic;} \\
\left[\text{genArg}(T::M)(i)/i\right]_{i=0}^\text{genParamNo}(T::M)-1, & \text{if } T \in \text{GenericType and } T::M \text{ is generic.}
\end{cases}$$

**Example 4.3.2** To illustrate the intuition behind the definition of $\text{inst}$, consider again the generic class $C'^2$ defined in Figure 4.21. As $C\langle \text{string,string}\rangle::M\langle \text{object}\rangle$ is generic and declared by a generic type, we have:
\[ \text{inst}(\text{!1 } C\langle \text{string,string} \rangle :: M\langle \text{object} \rangle(\text{!1,!!0})) = [\text{string/!0, string/!1}] \circ [\text{object/!!0}] = [\text{string/!0, string/!1, object/!!0}] \]

**Technical Conventions**

Similarly as the return type `retType` and parameter types `paramTypes` (and implicitly also `argTypes`), the local signature `locTypes` is considered uninstantiated in our approach. In contrast, `constr` and `fieldType` are considered to return results instantiated according to the underlying context, unless explicitly stated otherwise.

**Example 4.3.3** Let us consider again the generic class `C\text{′}2` in Figure 4.21. Following the above technical conventions, the constraint type of the generic parameter `!!0` associated to `!0 C\langle \text{string,string} \rangle :: M\langle \text{object} \rangle(\text{!0,!!0})` is `I\langle \text{string} \rangle`, as opposed to `I(\text{!0})`. Also, the declared type of the field `C\langle \text{string,string} \rangle :: F` is `\text{string}`, as opposed to `\text{!0}`.

### 4.3.2 The Bytecode Semantics

In this section, we refine the semantics of the bytecode language with generics. Besides the polymorphic type system and technical conventions stipulated in Section 4.3.1, there is one more extension concerning the static semantics. We consider that the function `code` yields instructions which are not instantiated relative to the underlying context. This is for purely technical reasons. When an instruction is executed at run-time, the possible open generic types referenced by the instruction are priorly instantiated. As we would like to keep the correspondence between the uninstantiated instructions on which the bytecode verification is performed and the instantiated instructions being executed, we consider that `code` returns uninstantiated instructions.

**Example 4.3.4** To illustrate the above assumption, let us have another look at the class `C\text{′}2` in Figure 4.21. In the second method, the instruction corresponding to the index 5 returned by the function `code` is `Box(!!0)`, regardless of the instantiation which is applied to the method. Formally,

\[ \text{code}(C\langle \text{string,string} \rangle :: M\langle \text{object} \rangle(\text{!0,!!0}))(5) = \text{Box}(!!0) \]

The operational semantics model `CLR_E` for the bytecode language with exceptions is refined by `CLR_G` for the bytecode language with generics. The above convention on the function `code` is reflected in the refinement of the execution scheme `execScheme`:

```plaintext
execScheme \equiv
\begin{cases}
\text{if } switch = \text{ExcMech} \text{ then } \\
\text{EXCCLR} \\
\text{elseif } switch = \text{Noswitch} \text{ then } \\
\text{EXECCLR(code(pc) \circ inst(meth))} \\
\text{else SWITCHCLR}
\end{cases}
```
The set of bytecode instructions is extended with the prefix *Constrained*, informally described in Table 4.18, which is permitted only on *CallVirt* instructions. The operational semantics rules for the *CallVirt* instruction prefixed by *Constrained* and the operational semantics rules, defined in Sections 4.1.3.1 and 4.2.3.4, affected by the run-time specialization of generic types/methods are defined, respectively, refined in Figures 4.3.2, 4.3.2, and 4.3.2.

Every instruction is executed under the assumption that the current method *meth* is instantiated (specialized), i.e., every generic parameter of the enclosing type or method is replaced by the corresponding generic argument. Consequently, the substitution given by *inst(meth)* is applied to every instruction. If neither the enclosing class *classNm(meth)* nor *meth* is generic, then *inst(meth)* is, according to Definition 4.3.2, the empty substitution ϵ. It then turns out that the semantics rules defined in this section (except of that for the prefixed *CallVirt* instruction) are refinements for the rules defined in Section 4.1.3.1 and 4.2.3.4.

There are some non-obvious refinements triggered by the extension with generics: the boxing of nullable types, the unboxing to nullable types, and the dynamic method dispatch.

**Boxing nullable types** It occurs when the *Box* instruction’s type token argument is a type Nullable(T) or a generic parameter which is instantiated (by means of *inst(meth)*) with a type Nullable(T). When an instance *val* of a nullable type Nullable(T) is boxed, the below defined macro BOXNULLABLE is executed. If *val* is not defined, i.e., *val(HasValue)* is False, then the result of boxing is a null reference. If *val* is defined, i.e., *val(HasValue)* is True, then *val*’s underlying value *val(Value)* is boxed (through the NewBox macro) and not the Nullable instance *val* itself. In particular, this means that “Nullable” is suppressed in the run-time type of the newly created object. As we will point out in Section 4.3.3, this makes type safety a concern.

```plaintext
BOXNULLABLE(val, Nullable(T)) ≡
    let ref = if *val(HasValue)* then NewBox(*val(Value), T)
    else null in
    evalStack := pop(evalStack) · [ref]
```

**Unboxing back to nullable types** If the type token argument of an *Unbox* or *Unbox.Any* instruction is a nullable type Nullable(T) or a generic parameter instantiated with a type Nullable(T), then CLR allocates and initializes a new Nullable(T) instance:

- if the top stack element, say *ref*, is not null, then Value is set to whatever *T* value has been boxed inside *ref*, and HasValue becomes True.
- if *ref* is the null reference, then HasValue is set to False and Value becomes the default value of type *T*.

If the value *ref* is not a boxed object of type *T*, as opposed to the type Nullable(T), then an InvalidCastException is thrown.

---

28 The reason behind this design is to avoid have a non-null box containing a null, along the same lines as one cannot construct types of the form Nullable(Nullable(T)).
Table 4.18: The generic specific instructions.

As usually, the Unbox instruction pushes a pointer to the newly created Nullable\(\langle T \rangle\) instance onto the stack, whereas the Unbox.Any instruction loads the instance.

To unbox back to nullable types, we define the macro UNBOXNULLABLE. This takes as arguments the to-be-unboxed object, the nullable type, and a boolean indicating whether an address is required to be pushed on the stack (the case of the Unbox instruction) or a value (the case of the Unbox.Any instruction).

Remark 4.3.1 Note that the creation of the nullable instance represents, besides the value class instance creation with NewObj, a new case when the allocated address is considered for the local memory pool locPool. The reason behind this modelling decision is the one pointed out in Remark 4.1.10, namely the risk of “loosing” the typing of the allocated address.

\[
\begin{align*}
\text{UNBOXNULLABLE}(\text{ref}, \text{Nullable}(T), \text{reqAdr}) \equiv \\
\quad \text{if } \text{ref} = \text{null} \lor \text{actualTypeOf}(\text{ref}) \preceq T \text{ then} \\
\quad \quad \text{let } \text{adr} = \text{new}(\text{Adr, Nullable}(T)) \text{ in} \\
\quad \quad \quad \text{locPool} := \text{locPool} \cup \{(\text{adr}, \text{Nullable}(T))\} \\
\quad \quad \quad \text{if } \text{reqAdr} \text{ then } \text{evalStack} := \text{pop}(\text{evalStack}) \cdot [\text{adr}] \\
\quad \quad \quad \text{else } \text{evalStack} := \text{pop}(\text{evalStack}) \cdot \text{memVal}(\text{adr, Nullable}(T)) \\
\quad \quad \quad \text{let } (\text{val}, \text{val}') = \text{if } \text{ref} = \text{null} \text{ then } (0, \text{defVal}(T)) \\
\quad \quad \quad \quad \text{else } (1, \text{memVal}((\text{addressOf}(\text{ref}), T))) \text{ in} \\
\quad \quad \quad \text{WRITEMEM}(\text{fieldAdr}(\text{adr, Nullable}(T)::HasValue), \text{bool}, \text{val}) \\
\quad \quad \quad \text{WRITEMEM}(\text{fieldAdr}(\text{adr, Nullable}(T)::Value), T, \text{val'}) \\
\quad \quad \text{else } \text{RAISE} (\text{InvalidCastException})
\end{align*}
\]

Dynamic method dispatch The ECMA Standard [45] does not specify what is the effect of the generic parameter variance on the dynamical method lookup. The definition of lookUp in Section 4.1.2 is refined to take the variance into account.

Definition 4.3.3 (Method lookup) For a possibly generic method \(T':\langle T_0, \ldots, T_{n-1} \rangle\) and a type \(T\), we define:

\footnote{If the method is not generic, the generic arguments \((T_i)\), should simply be omitted.}
Figure 4.22 The refined operational semantics rules.

\[
\text{EXECCLR}(\text{instr}) \equiv \text{match instr} \\
\begin{align*}
\text{LoadLoc}(n) \circ \text{inst(meth)} & \rightarrow \text{let } T = \text{locTypes}(n) \circ \text{inst(meth)} \text{ in} \\
& \quad \text{evalStack} := \text{evalStack} \cdot [\text{memVal}(\text{locAdr}(n), T)] \\
& \quad pc := pc + 1 \\
\text{StoreLoc}(n) \circ \text{inst(meth)} & \rightarrow \text{let } (\text{evalStack}', [val]) = \text{split(\text{evalStack}, 1)} \text{ in} \\
& \quad \text{WRITEMEM}(\text{locAdr}(n), \text{locTypes}(n) \circ \text{inst(meth)}, val) \\
& \quad \text{evalStack} := \text{evalStack}' \\
& \quad pc := pc + 1 \\
\text{LoadArg}(n) \circ \text{inst(meth)} & \rightarrow \text{let } T = \text{argTypes}(n) \circ \text{inst(meth)} \text{ in} \\
& \quad \text{evalStack} := \text{evalStack} \cdot [\text{memVal}(\text{argAdr}(n), T)] \\
& \quad pc := pc + 1 \\
\text{StoreArg}(n) \circ \text{inst(meth)} & \rightarrow \text{let } (\text{evalStack}', [val]) = \text{split(\text{evalStack}, 1)} \text{ in} \\
& \quad \text{WRITEMEM}(\text{argAdr}(n), \text{argTypes}(n) \circ \text{inst(meth)}, val) \\
& \quad \text{evalStack} := \text{evalStack}' \\
& \quad pc := pc + 1 \\
\text{Call}((\text{tail, _}, T::M)) \circ \text{inst(meth)} & \rightarrow \text{if } T \circ \text{inst(meth)} = \text{System.Delegate} \text{ then} \\
& \quad \text{let } [ref, ref'] = \text{take(\text{evalStack}, \text{argNo}(T::M)) in} \\
& \quad \text{if } M = \text{Combine} \text{ then} \\
& \quad \quad \text{DELEGATECOMBINE}(T \circ \text{inst(meth)}, ref, ref') \\
& \quad \quad \text{if } M = \text{Remove} \text{ then} \\
& \quad \quad \text{DELEGATEREMOVE}(T \circ \text{inst(meth)}, ref, ref') \\
& \quad \quad \text{if } M = \text{op_Equality} \text{ then} \\
& \quad \quad \text{DELEGATEEQUAL}(ref, ref') \\
& \quad \quad \text{else let } (\text{evalStack}', [ref] \cdot \text{vals}) = \text{split(\text{evalStack}, \text{argNo}(T::M)) in} \\
& \quad \quad \text{if } ref \neq \text{null} \text{ then} \\
& \quad \quad \quad \text{if } T \circ \text{inst(meth)} \in \text{DelegateClass} \text{ then} \\
& \quad \quad \quad \quad \text{if } M = \_\text{length} \text{ then} \\
& \quad \quad \quad \quad \quad \text{evalStack} := \text{evalStack}' \cdot [\text{length}((\text{invocationList}(\text{ref}))] \\
& \quad \quad \quad \quad \text{if } M = \_\text{invoke} \text{ then} \\
& \quad \quad \quad \quad \quad \text{DELEGATECALL}(\text{ref}, \text{vals}) \\
& \quad \quad \quad \quad \text{else evalStack} := \text{evalStack}' \\
& \quad \quad \quad \quad \quad \text{switch} := \text{Invoke}(\text{tail}, T::M \circ \text{inst(meth)}, [\text{ref}] \cdot \text{vals}) \\
& \quad \quad \quad \text{else RAISE(NullReferenceException)} \\
& \quad \quad \text{else \text{call(\text{tail, _}, T::M)} \circ \text{inst(meth)}} \rightarrow \text{let } (\text{evalStack}', \text{vals}) = \text{split(\text{evalStack}, \text{argNo}(T::M)) in} \\
& \quad \quad \quad \text{evalStack} := \text{evalStack}' \\
& \quad \quad \quad \text{VIRTCALL(\text{tail, T::M} \circ \text{inst(meth)}, \text{vals})} \\
\text{Constrained}(T''). \quad \quad \text{CallVirt}((\text{tail, _}, T::M)) \circ \text{inst(meth)} \rightarrow \text{let } (\text{evalStack}', \text{vals}) = \text{split(\text{evalStack}, \text{argNo}(T::M)) in} \\
& \quad \text{evalStack} := \text{evalStack}' \\
& \quad \text{if } T'' \circ \text{inst(meth)} \in \text{RefType} \text{ then} \\
& \quad \quad \text{let ref} = \text{memVal(adr, T'' \circ \text{inst(meth)) in} } \\
& \quad \quad \text{VIRTCALL(False, T::M \circ \text{inst(meth)}, [\text{ref}] \cdot \text{vals})} \\
& \quad \quad \text{elseif } T'' \circ \text{inst(meth)} \in \text{ValueClass} \land \\
& \quad \quad \quad T''::M \circ \text{inst(meth)} \text{ implements T::M \circ \text{inst(meth)} then} \\
& \quad \quad \quad \text{switch} := \text{Invoke(False, T''::M \circ \text{inst(meth)}, [\text{adr}] \cdot \text{vals})} \\
& \quad \quad \text{else let ref} = \text{NewBox(\text{memVal(adr, T'' \circ \text{inst(meth))}, T'' \circ \text{inst(meth)) in} } \\
& \quad \quad \text{VIRTCALL(False, T::M \circ \text{inst(meth)}, [\text{ref}] \cdot \text{vals})} \\
\text{Return} \circ \text{inst(meth)} \rightarrow \text{if refType(meth) \circ \text{inst(meth)} = void then } \text{RESULT([]) } \\
& \quad \text{else \text{RESULT([top(\text{evalStack})])}}
\end{align*}
\]
Figure 4.23 The refined operational semantics rules (continued).

\[
\text{EXECCLR}(\text{instr}) \equiv \text{match instr} \\
\vdots
\]

\[
\text{NewObj}(C::\text{ctor}) \circ \text{inst(meth)} \rightarrow \text{let } \text{vals} = \text{take(\text{evalStack}, \text{argNo}(C::\text{ctor}) - 1)} \text{ in} \\
\quad \text{if } C \circ \text{inst(meth)} \in \text{DelegateClass} \text{ then} \\
\quad \quad \text{DELEGATECREATE}(C \circ \text{inst(meth)}, \text{vals}) \\
\quad \text{elseif } C \circ \text{inst(meth)} \in \text{ObjClass} \text{ then} \\
\quad \quad \text{OBJECTCREATE}(C::\text{ctor} \circ \text{inst(meth)}, \text{vals}) \\
\quad \text{else } \text{VALUECLASSCREATE}(C::\text{ctor} \circ \text{inst(meth)}, \text{vals})
\]

\[
\text{LoadField}(T, C::F) \circ \text{inst(meth)} \rightarrow \text{let } (\text{evalStack}', [\text{val}]) = \text{split(\text{evalStack}, 1)} \text{ in} \\
\quad \text{if } \text{val} \neq \text{null} \text{ then} \\
\quad \quad \text{let } \text{val}' = \text{memVal(fieldAdr(val, C::F \circ \text{inst(meth)}), T \circ \text{inst(meth)})} \text{ in} \\
\quad \quad \text{evalStack} := \text{evalStack}' \cdot [\text{val}'] \\
\quad \quad \text{pc} := \text{pc} + 1 \\
\quad \text{else RAISE(NullReferenceException)}
\]

\[
\text{StoreField}(T, C::F) \circ \text{inst(meth)} \rightarrow \text{let } (\text{evalStack}', [\text{val}, \text{val}']) = \text{split(\text{evalStack}, 2)} \text{ in} \\
\quad \text{if } \text{val} \neq \text{null} \text{ then} \\
\quad \quad \text{WRITEMEM(fieldAdr(val, C::F \circ \text{inst(meth)}), T, \text{val}')} \\
\quad \quad \text{evalStack} := \text{evalStack}' \\
\quad \quad \text{pc} := \text{pc} + 1 \\
\quad \text{else RAISE(NullReferenceException)}
\]

\[
\text{LoadFieldA}(_, C::F) \circ \text{inst(meth)} \rightarrow \text{let } (\text{evalStack}', [\text{val}]) = \text{split(\text{evalStack}, 1)} \text{ in} \\
\quad \text{if } \text{val} \neq \text{null} \text{ then} \\
\quad \quad \text{evalStack} := \text{evalStack}' \cdot [\text{fieldAdr(val, C::F \circ \text{inst(meth)})}] \\
\quad \quad \text{pc} := \text{pc} + 1 \\
\quad \text{else RAISE(NullReferenceException)}
\]

\[
\text{MkRefAny}(T) \circ \text{inst(meth)} \rightarrow \text{let } (\text{evalStack}', [\text{adr}]) = \text{split(\text{evalStack}, 1)} \text{ in} \\
\quad \text{let } \text{tr} = \text{new(TypedRef)} \text{ in} \\
\quad \quad \text{typedRefAdr(\text{tr})} := \text{adr} \\
\quad \quad \text{typedRefType(\text{tr})} := T \circ \text{inst(meth)} \\
\quad \quad \text{evalStack} := \text{evalStack}' \cdot [\text{tr}] \\
\quad \quad \text{pc} := \text{pc} + 1
\]

\[
\text{Box}(T) \circ \text{inst(meth)} \rightarrow \text{pc} := \text{pc} + 1 \\
\quad \text{if } T \circ \text{inst(meth)} \in \text{ValueType} \text{ then} \\
\quad \quad \text{let } (\text{evalStack}', [\text{val}]) = \text{split(\text{evalStack}, 1)} \text{ in} \\
\quad \quad \text{if } T \circ \text{inst(meth)} \in \text{NullableType} \text{ then} \\
\quad \quad \quad \text{BOXNULLABLE(\text{val}, T \circ \text{inst(meth)})} \\
\quad \quad \text{else let } \text{ref} = \text{NewBox(\text{val}, T \circ \text{inst(meth)})} \text{ in} \\
\quad \quad \quad \text{evalStack} := \text{evalStack}' \cdot [\text{ref}]
\]

\[
\text{Unbox}(T) \circ \text{inst(meth)} \rightarrow \text{let } (\text{evalStack}', [\text{ref}]) = \text{split(\text{evalStack}, 1)} \text{ in} \\
\quad \text{if } T \circ \text{inst(meth)} \in \text{NullableType} \text{ then} \\
\quad \quad \text{UNBOXNULLABLE(\text{ref}, T \circ \text{inst(meth)}, \text{True})} \\
\quad \text{elseif } \text{ref} \neq \text{null} \land \text{actualTypeOf(ref)} \subseteq T \circ \text{inst(meth)} \text{ then} \\
\quad \quad \quad \text{evalStack} := \text{evalStack}' \cdot [\text{addressOf(ref)}] \\
\quad \quad \quad \text{pc} := \text{pc} + 1 \\
\quad \text{elseif } \text{ref} = \text{null} \text{ then RAISE(NullReferenceException)} \\
\quad \text{else RAISE(InvalidCastException)}
\]
lookUp(\(T, T'::M\langle T_0, \ldots, T_{n-1}\rangle\)) =

if \(T\) declares a non-abstract method \(M\) and

\((T = T') \lor (T' \in ObjClass \Rightarrow T::M\text{ overrides } T'::M)\) \land
\((T' \in Interface \setminus \text{GenericType} \Rightarrow T::M\text{ implements } T'::M)\) \land
\((T' = I\langle U_0, \ldots, U_{m-1}\rangle \in Interface \cap \text{GenericType} \Rightarrow T::M\text{ implements } I\langle V_0, \ldots, V_{m-1}\rangle::M)\) \land
\(\forall j = 0, m - 1 : (\text{var}_{j}^{T'} = + \Rightarrow V_j \leq U_j) \land
(\text{var}_{j}^{T'} = - \Rightarrow U_j \leq V_j) \land
(\text{var}_{j}^{T'} = \emptyset \Rightarrow U_j = V_j)\)

then \(T::M\langle T_0, \ldots, T_{n-1}\rangle\)

elseif \(T = \text{object}\) then undef
else lookUp(\(T''\), \(T'::M\langle T_0, \ldots, T_{n-1}\rangle\)) where \(T''\) is the direct base class of \(T\)

The most significant change occurs when lookUp is applied to a method declared by a generic interface. In such a case, the method selected by lookUp may not override or implement the original method. As one can see in Example 4.3.6, this aspect makes type safety a concern.

As stated at the end of Section 4.3.1, the generic method references have the signatures uninstantiated. Therefore, for example, the return type (included in the signature) of a method which overrides/implements another method may not match the return type (included in the signature) of the overridden/implemented method. To cover this aspect, the conditions [override/implement], defined in Section 4.1.2, are refined below. Moreover, [override/implement] now also includes the conditions that should be satisfied by the constraint types of the overriding/implementing method and overridden/implemented method.

[override/implement] If \(T::M \in MRef\text{ overrides/implements } T'::M \in MRef\), then:

(at) \(\text{argNo}(T::M) = \text{argNo}(T'::M)\) and for every \(i = 1, \text{argNo}(T::M) - 1,\)
\(\text{argTypes}(T::M)(i) \circ \text{inst}(T::M) = \text{argTypes}(T'::M)(i) \circ \text{inst}(T'::M)\)

(rt) \(\text{retType}(T::M) \circ \text{inst}(T::M) = \text{retType}(T'::M) \circ \text{inst}(T'::M)\)

(ct) \(\text{genParamNo}(T::M) = \text{genParamNo}(T'::M)\) and for every \(j = 0, \text{genParamNo}(T'::M) - 1,\)
\(\text{constr}_{j}^{T::M}\text{ is not defined or }\text{constr}_{j}^{T'::M} \preceq \text{constr}_{j}^{T::M}\)

Remark 4.3.2 Concerning (ct), [45, Partition II, §9.9] states that any constraint type specified by the overriding/implementing method should be “no more restrictive” than the corresponding constraint type specified in the overridden/implemented method. However, this does not match the .NET Framework (v2.0) implementation [3], where the bytecode verification checks whether one of the following conditions is satisfied: either the constraint type in the overriding method is not defined, i.e., as it would have been object, or the (assumably instantiated) constraint types coincide.
Let us now look in detail at the Constrained prefix, permitted only on a CallVirt instruction. The Constrained($T''$).CallVirt($T', T :: M$) instruction\(^{30}\) calls the virtual method $T :: M$, of return type $T'$, on a value of a generic parameter $T''$. It pops the necessary number of arguments from the evalStack. The first value is expected to be a pointer, evaluated to an address, say $adr$. There are three cases that can occur. If $T''$ is instantiated with a reference type, then $adr$ is dereferenced and passed as the this pointer. If $T''$ is instantiated with a value class that implements $T :: M$, then $adr$ is passed as the this pointer to the invocation of the method implemented by the

\(^{30}\)A Constrained call cannot be tail as at most one prefix is allowed on the CallVirt instructions.
value class. If \( T'' \) is instantiated with a value class which does not implement \( T::M \)\(^{31} \), then \( \text{adr} \) is dereferenced, boxed, and passed as the \texttt{this} pointer to the virtual call of \( T::M \). Note that, in the macro \texttt{VIRT\_CALL}, the function \texttt{lookUp} is applied to \( T::M \circ \text{inst}(\text{meth}) \), i.e., to the method \( T::M \) where only the generic parameters occurring in \( T \) and possibly in the generic argument list of \( M \) are replaced by the generic arguments indicated in \( \text{inst}(\text{meth}) \).

The \texttt{Constrained} prefix has been introduced to support the compilers generating generic code. As the \texttt{CallVirt} instruction is not allowed on value classes, the compilers should effectively perform the \texttt{this} pointer transformation outlined above at compile time, depending on the type of the pointer and the method being called. One cannot do this transformation at compile time, however, when the pointer is a generic type (which is obviously unknown at compile time). The \texttt{Constrained} prefix is needed to get rid of this inconvenience: It allows compilers to make a virtual call in an uniform way, i.e., regardless of whether the pointer is a value type or reference type.

Example 4.3.5 Figure 4.25 contains a \texttt{C\#} generic class and the bytecode for its method, where the virtual call \( F.ToString() \) is compiled into a \texttt{CallVirt} instruction prefixed by \texttt{Constrained}.

### 4.3.3 The Bytecode Verification

In this section, we refine the bytecode verification specification to accommodate the generics. Section 4.3.3.1 specifies the restrictions on the verification type system, required to guarantee type safety. The verification specific functions \texttt{check} and \texttt{succ} are refined in Sections 4.3.3.2 and 4.3.3.3, respectively.

#### 4.3.3.1 The Verification Type System

The verification types are described by the same universe \texttt{VerificationType} in Table 4.9, where the universe \texttt{Type} has been refined as shown in Table 4.16. The compatibility relation for verification types is refined as follows:

**Definition 4.3.4 (Compatible verification types)** The relation \( \sqsubseteq \), for verification types, is the least reflexive and transitive relation such that

\[ \text{This is the case when the method to be invoked is declared by the base class of the value class, i.e., System.ValueType, or object, and not overridden by the value class.} \]
• if \( T \) is \( \text{Null} \) and \( T' \in \text{ObjClass} \cup \text{Interface} \cup \text{BoxedType} \), or

• if \( T \in \text{ObjClass} \cup \text{Interface} \cup \text{BoxedType} \) and \( T' \) is \( \text{object} \), or

• if \( T \in \text{BoxedType} \) and \( T' \) is \( \text{System.ValueType} \), or

• if \( T \in \text{ObjClass} \setminus \text{GenericType} \) and \( T' \) is a base class of \( T \) or an interface implemented by \( T \), or

• if \( T \in \text{Interface} \setminus \text{GenericType} \) and \( T' \) is an interface implemented by \( T \), or

• if \( T \) is \( \text{boxed}(T'') \), where \( T'' \in \text{ValueType} \setminus \text{GenericType} \) and \( T' \) an interface implemented by \( T'' \), or

• if \( T \) is \( \text{boxed}(X) \), where \( X \) is a generic parameter constrained by \( T' \), or

• if \( T \) is \( C\langle U_0, \ldots, U_{n-1} \rangle \), where \( C'n \) is a generic object class with the generic parameters \( \langle X_i \rangle_{i=0}^{n-1} \), \( T'' \) is a base class of \( C'n \) or an interface implemented by \( C'n \), and \( T' \) is \( T'' \circ [U_i/X_i]_{i=0}^{n-1} \), or

• if \( T \) is \( I\langle U_0, \ldots, U_{n-1} \rangle \), where \( I'n \) is a generic interface with the generic parameters \( \langle X_i \rangle_{i=0}^{n-1} \), \( T'' \) is an interface implemented by \( I'n \), and \( T' \) is \( T'' \circ [U_i/X_i]_{i=0}^{n-1} \), or

• if \( T \) is \( \text{boxed}(C\langle U_0, \ldots, U_{n-1} \rangle) \), where \( C'n \) is a generic value class with the generic parameters \( \langle X_i \rangle_{i=0}^{n-1} \), \( T'' \) is an interface implemented by \( C'n \), and \( T' \) is \( T'' \circ [U_i/X_i]_{i=0}^{n-1} \), or

• if \( T \) is \( I\langle U_0, \ldots, U_{n-1} \rangle \) and \( T' \) is \( I\langle V_0, \ldots, V_{n-1} \rangle \), and for every \( i = 0, n - 1 \) the following conditions hold:
  
  – if \( \text{var}^I_i = \emptyset \) or \( V_i \in \text{ValueType} \) or \( V_i \in \text{GenericParam} \), then \( U_i = V_i \);
  
  – if \( \text{var}^I_i = + \), then \( U_i \preceq V_i \);
  
  – if \( \text{var}^I_i = - \), then \( V_i \preceq U_i \);

then \( T \sqsubseteq T' \).

Definition 4.1.8 of compatible delegate classes and method references is refined as a result of the convention that \( \text{argTypes} \) and \( \text{retType} \) are uninstantiated.

**Definition 4.3.5** A delegate class \( DC \) is compatible with a method \( mref \) if the following conditions are matched:

- \( \text{argTypes}(DC::\text{Invoke})(i) \sqsubseteq \text{argTypes}(mref)(i) \circ \text{inst}(mref) \), for every \( i = 1, \argNo(mref) - 1 \)

- \( \text{retType}(DC::\text{Invoke}) = \text{retType}(mref) = \text{void} \) or \( \text{retType}(mref) \circ \text{inst}(mref) \sqsubseteq \text{retType}(DC::\text{Invoke}) \).
It is generally believed that one has to use contravariance when (static) type safety is required, but that covariance is more natural, flexible and reflects the program designer’s natural wish to model the specialization of the classes. Although it is losing some expressiveness, the CLR manages to combine the two approaches in a way that, as we will prove in Section 4.3.4, guarantees type safety. Thus, [45] imposes several requirements on the instance methods declared by a generic interface which is co-/contra-variant in at least one generic parameter. The requirements should ensure that an overriding method follows the substitution principle, i.e., it has a “covariant” return type, “contravariant” parameter and constraint types. The usage of a “covariant” parameter type, as opposed to a “contravariant” one, cannot be statically verified (see Example 4.3.6) and run-time checks should have been inserted. This is, however, among the issues the generics design was supposed to get rid of.

The above requirements are captured in Definition 4.3.9. This definition necessitates Definitions 4.3.7 and 4.3.8, which in turn require the notion of negation of a variances array \((\text{var}_i)\):

**Definition 4.3.6 (Negated variances array)** The negation \(- (\text{var}_i)\) of an array \((\text{var}_i)\) of variances is defined by

\[
- \text{var}_i = \begin{cases} 
  + , & \text{if } \text{var}_i = - \\
  - , & \text{if } \text{var}_i = + \\
  \emptyset , & \text{if } \text{var}_i = \emptyset 
\end{cases}
\]

The notion of a valid type with respect to a variances array is specified as follows.

**Definition 4.3.7 (Valid type)** The predicate \(\text{validType}\) checks the validity of a type \(T'\) with respect to an array \((\text{var}_i)\) of variances.

\[
\text{validType}(T', (\text{var}_i)) : \Leftrightarrow \\
T' \notin \text{GenericType} \cap \text{GenericParam} \lor \\
T' \text{ is a generic parameter } \forall j \text{ of the enclosing method } \lor \\
T' \text{ is a generic parameter } \forall j \text{ of the enclosing type } \land \text{var}_j \in \{+, \emptyset\} \lor \\
T' = T(U_0, \ldots, U_{n-1}) \in \text{GenericType} \land \\
\forall k = 0, n - 1 : \\
(\text{var}_k^{T'} = + \Rightarrow \text{validType}(U_k, (\text{var}_i))) \land \\
(\text{var}_k^{T'} = - \Rightarrow \text{validType}(U_k, -(\text{var}_i))) \land \\
(\text{var}_k^{T'} = \emptyset \Rightarrow \text{validType}(U_k, (\text{var}_i)) \land \text{validType}(U_k, -(\text{var}_i))))
\]

**Remark 4.3.3** [45, Partition II, §9.7] states that \(T' = T(U_0, \ldots, U_{n-1})\) in the above definition should refer to a “closed” generic type. This does not make a lot of sense, since in this case the definition has nothing to do with the array of variances. This remark and the experiments we have run with CLR indicate that \(T'\) should not necessarily be a closed type.
Definition 4.3.8 stipulates when a method is valid with respect to a variances array. A method is valid if its return type “behaves covariantly”, whereas its argument types and possibly constraint types “behave contravariantly”. In other words, the covariant generic parameters can only appear in "getter" positions in the type definition, i.e., in return types of methods, the contravariant generic parameters can only appear in "setter" positions in the type definition, i.e., in argument and constraint types of methods, and non-variant generic parameters can appear anywhere.

**Definition 4.3.8 (Valid method)** The predicate validMeth checks the validity of the declaration of the method mref with respect to the array \((\text{var}_i)\) of variances.

\[
\text{validMeth}(\text{mref}, (\text{var}_i)) : \iff \\
\text{validType}(\text{retType}(\text{mref}), (\text{var}_i)) \land \\
\forall j = 1, \text{argNo}(\text{mref}) - 1 : \text{validType}(\text{argTypes}(j), -(\text{var}_i)) \land \\
\forall j = 0, \text{genParamNo}(\text{mref}) - 1 : \text{validType}(\text{constr}_{\text{mref}j}, -(\text{var}_i))
\]

Finally, the declaration of a generic interface is valid if all the instance methods declared by the interface and all the implemented interface types are valid with respect to the variances array of the given interface.

**Definition 4.3.9 (Valid interface declaration)** The predicate validDecl checks the validity of the generic interface \(I'\)n declaration.

\[
\text{validDecl}(I'\text{n}) : \iff \\
\forall I'\text{n}::M \in MRef : \text{validMeth}(I'\text{n}::M, (\text{var}'\text{n}_i)) \land \\
\forall J \text{ implemented by } I'\text{n} : \text{validType}(J, (\text{var}'\text{n}_i))
\]

**Remark 4.3.4** [45, Partition II, §9.7] is unclear about the definition of valid interface declarations. It requires that “every inherited interface declaration” should be valid with respect to the variances array. Firstly, the interfaces are not “inherited”, but implemented. Secondly, it is not about the interface “declaration”, but about the interface type present in the extends clause of the given interface.

**Example 4.3.6** Consider the bytecode skeleton in Figure 4.26. The generic interface \(I'1\) is covariant on its generic parameter: \(\text{var}'I1 = [+].\) By this and \(OC' \preceq OC,\) we have \(I\langle OC'\rangle \preceq I\langle OC\rangle\) according to Definition 4.3.1. On the other hand, by the same definition, we get \(OC'' \preceq I\langle OC'\rangle\).

Let us now assume that \(I\langle OC\rangle::M\) is called with the this pointer of type \(OC''\) (allowed since \(OC'' \preceq I\langle OC'\rangle \preceq I\langle OC\rangle\)) and the argument of type \(OC\). The method invoked at runtime is \(OC''::M\) since \(\text{lookUp}(OC'', I\langle OC\rangle::M) = OC''::M\). This means that, the argument of type \(OC\) is used in \(OC''::M\) as an object of type \(OC'\): see, for example, the access at index 5 of the field \(OC'::F\). This is definitely unsafe. Fortunately, such an example is rejected by the bytecode verification as the declaration of \(I'1::M\) is not valid according to Definition 4.3.8: The parameter type \(!0\) is not a valid type with respect to \(\text{var}'I1 = [+].\)
4.3.3.2 Verifying the Bytecode Instructions

The specification by means of check of the type-consistency checks is refined in Figure 4.27. Note that the check predicate is applied to instructions that might involve open generic types, in particular generic parameters, as the verification obviously occurs before the run-time specialization of generics.

Following the technical convention regarding the definition of paramTypes (and implicitly affecting also argTypes), stipulated in Section 4.3.2, the definition of check for the instructions Call and CallVirt is refined accordingly. Concerning the definition of check for an instruction CallVirt(_, T::M) prefixed by Constrained(T''), the following checks are performed. The stack must contain a managed pointer to a value of type T''. Additionally, all the verification rules of the CallVirt instruction apply after the this pointer transformation as described above. That is equivalent to demanding that boxed(T'') must be a subtype of T.'
4.3.3 Computing Successor Type States

Figure 4.28 contains the refinement of the function \( \text{succ} \). Similarly as the \( \text{check} \) function, \( \text{succ} \) is refined for the instructions \( \text{Call} \) and \( \text{CallVirt} \). The definition of \( \text{succ} \) for a \( \text{CallVirt} \) instruction prefixed by \( \text{Constrained} \) is the same as that for a \( \text{CallVirt} \) instruction with no prefix, i.e., with neither \( \text{Tail} \) nor \( \text{Constrained} \).

4.3.4 Type Safety

In this section, we prove that the bytecode language with generics is type safe. Besides the properties guaranteed so far for well-typed methods, we also consider and prove two generic specific type safety properties:

**Type safety**: The generic arguments of any generic type or method satisfy the corresponding declared constraints.
The definition of well-typed methods in Section 4.1.5 Definition 4.1.9 remains unmodified, assuming that the functions check and succ are refined, as indicated in Sections 4.3.3.2 and 4.3.3.3, respectively.

In the sequel, we assume that, besides the conditions on the type system and bytecode structure stipulated in Sections 4.1.4.2 and 4.2.4.1, the following are required to hold:

- every method should be well-typed in the sense of Definition 4.1.9;
- every generic interface declaration should be valid in the sense of Definition 4.3.9;
- the generic arguments of all the generic types and method references should satisfy at verification time the corresponding constraints. In particular, this means that every generic argument used in the inst functions satisfies the corresponding constraints.

Let us now revisit the potential risks for type safety, informally described at the beginning of Section 4.3, involved with the support for generics. Thus, although the signature and constraint types of a virtual called method may not match the signature and constrained types of the invoked method, respectively, type safety is preserved through a conservative assumption, namely by restricting the set of interfaces to those with valid declarations. The validity of the run-time specialization of generic types/methods, in the sense that the generic arguments satisfy the correspond constraints, is proved in Proposition 4.3.1, based on the definition of the verification type compatibility ⊑.

Of particular interest is the mismatch between the run-time type and the verification type which occurs upon boxing a nullable value. Consider that a Box instruction has a generic parameter !0 in the scope as a type token argument. So, the value being boxed should be of some type !0 (perhaps declared by the enclosing method) that cannot be known at verification time. The verification rules state that the result of Box(!0) should be of type boxed(!0) (see the definition of succ for the Box instruction). But, as noticed in Section 4.3.2, if !0 is instantiated with a nullable type Nullable⟨T⟩ as the generic argument for the generic parameter !0, this would not be true. The run-time type actualTypeOf would indicate a boxed object of type T, yet the verification type indicates (indirectly) a boxed object of type Nullable⟨T⟩. Although this looks as a type hole, type safety is maintained, in particular through the design of nullable types as value classes that do not implement interfaces. The key points are that the boxed types are abstract types (used by the bytecode verification) that cannot be referred to by name in the bytecode, and the supertypes of a boxed nullable type are supertypes of any boxed nullable type. Therefore, there is no instance member of the boxed nullable type representing the verification type predicted by the bytecode verification, which is not an instance member of the run-time type.

The type safety proof takes advantage of the following lemmas. Lemma 4.3.1 shows that the subtype relationship of a given type varies directly with the relationship of the covariant generic parameters and inversely with the relationship of the contravariant generic parameters.

**Lemma 4.3.1** If the types \( T, (U_i)_{i=0}^{n-1}, (V_i)_{i=0}^{n-1} \) and the array \( (var_i)_{i=0}^{n-1} \) of variances are such that validType\( (T, (var_i)_{i=0}^{n-1}) \) and

\[
\forall i = 0, n-1 : (var_i = + \Rightarrow V_i \subseteq U_i) \land (var_i = - \Rightarrow U_i \subseteq V_i) \land (var_i = \emptyset \Rightarrow V_i = U_i)
\]
then $T \circ [V_i/\!\!i][i=0]^{n-1}_i \subseteq T \circ [U_i/\!\!i][i=0]^{n-1}_i$.

**Proof.** By induction on the structure of the possibly generic type $T$. Definition 4.3.7 is also applied. 

Lemma 4.3.2 proves that, if $T' \subseteq T''$, then the instantiation of the generic parameters (corresponding to an enclosing generic class and/or method) occurring in $T'$ and $T''$ with generic arguments satisfying the corresponding constraints preserves the subtype relation between $T'$ and $T''$.

**Lemma 4.3.2** Let $T'$ and $T''$ be two types such that $T' \subseteq T''$. Assume that they occur in the declaration of a generic type $T'\!':M$, possibly in the declaration of a generic method $T'\!':M$ with $m$ generic parameters. Let $(U_i)^{n-1}_{i=0}$ and $(V_j)^{m-1}_{j=0}$ be generic arguments for $T'\!':n$ and $T'\!':n::M$, respectively, assumed to satisfy the constraints:

\[
\begin{align*}
\boxed(U_i) & \subseteq \text{constr}_{T'\!':n}^{i} \circ [U_i/\!\!i][i=0]^{n-1}_i, & \text{for every } i = 0, n - 1 \\
\boxed(V_j) & \subseteq \text{constr}_{T'\!':n::M}^{j} \circ ([U_i/\!\!i][i=0]^{n-1}_i \cdot [V_j/\!\!j][j=0]^{m-1}_j), & \text{for every } j = 0, m - 1
\end{align*}
\]

It then holds $T' \circ [U_i/\!\!i][i=0]^{n-1}_i \cdot [V_j/\!\!j][j=0]^{m-1}_j \subseteq T'' \circ [U_i/\!\!i][i=0]^{n-1}_i \cdot [V_j/\!\!j][j=0]^{m-1}_j$.

**Proof.** By induction on the structure of the (possibly generic) type $T'$. Definition 4.3.4 is also applied. 

The following two lemmas relate the argument types, respectively the return type of the method of a virtual call, and the argument types, respectively the return type of the method determined through a lookup and invoked at run-time.

**Lemma 4.3.3** If $T''\!::M = \text{lookUp}(T, T'\!::M)$ holds for a type $T$, then

- If $T \in \text{ValueType}$, then
  - if $T'' \neq T$, then $\boxed(T) \subseteq \text{argTypes}(T''\!::M)(0) \subseteq \text{argTypes}(T'\!::M)(0)$
  - if $T'' = T$, then $\text{argTypes}(T''\!::M)(0) = T\&$ and $\boxed(T) \subseteq \text{argTypes}(T'\!::M)(0)$

If $T \in \text{RefType}$, then $T \subseteq \text{argTypes}(T''\!::M)(0) \subseteq \text{argTypes}(T'\!::M)(0)$

- $\text{argNo}(T'\!::M) = \text{argNo}(T''\!::M)$ and for every $i = 1, \text{argNo}(T'\!::M) - 1$

\[
\text{argTypes}(T'\!::M)(i) \circ \text{inst}(T'\!::M) \subseteq \text{argTypes}(T''\!::M)(i) \circ \text{inst}(T''\!::M)
\]

**Proof.** By Definitions 4.3.8, 4.3.9, and 4.3.3, Lemmas 4.3.1 and 4.3.2, and (at).

**Lemma 4.3.4** If $T''\!::M = \text{lookUp}(T, T'\!::M)$ holds for a type $T$, then

\[
\text{retType}(T''\!::M) \circ \text{inst}(T''\!::M) \subseteq \text{retType}(T'\!::M) \circ \text{inst}(T'\!::M)
\]
Proof. By Definitions 4.3.8, 4.3.9, and 4.3.3, Lemmas 4.3.1 and 4.3.2, and (rt).

Lemma 4.3.5 shows that a generic method determined through a lookup has the same generic arguments as the original method and the generic arguments satisfy the corresponding constraints.

Lemma 4.3.5 If $T''::M$ and $T'::M$ are some generic methods and $T$ is some type such that $T''::M = \text{lookUp}(T, T'::M)$ and boxed(genArg($T'::M$))($i$) $\subseteq$ constr$_i^{T'::M}$ holds for every $i = 0$, genParamNo($T'::M$) − 1, then genParamNo($T''::M$) = genParamNo($T'::M$), and for every $i = 0$, genParamNo($T'::M$) − 1

$$\text{boxed}(\text{genArg}(T'::M)(i)) = \text{boxed}(\text{genArg}(T''::M)(i)) \subseteq \text{constr}_i^{T''::M}$$

Proof. By Definitions 4.3.8, 4.3.9, and 4.3.3, Lemmas 4.3.1 and 4.3.2, and (ct).

We assume that, if a generic type or method occurs instantiated or referenced, respectively, the generic arguments satisfy at verification time the corresponding constraints. As the generic arguments might be open generic types, the following question arise: Do they satisfy the constraints also after the run-time instantiation? The following proposition answers positively to this question.

Proposition 4.3.1 (Preservation of constraints under type substitution) We assume that the (not necessarily closed) type $T(T_0, \ldots, T_{n-1})$ occurs in the declaration of a generic type $S'm$, possibly in the declaration of a generic method $S'm::M$ with $p$ generic parameters. Moreover, assume that $(T_i)_{i=0}^{n-1}$ satisfy $T'n$'s constraints.

If $S'm$ and $S'm::M$ are instantiated at run-time with the generic arguments $(U_j)_{j=0}^{m-1}$ and $(V_k)_{k=0}^{p-1}$, assumed to satisfy the constraints of $S'm$ and $S'm::M$, respectively, then $(T_i \circ \rho)_{i=0}^{n-1}$ satisfy $T'n$'s constraints, where $\rho$ is the substitution $\rho = [U_j/!j]_{j=0}^{m-1} \cdot [V_k/!!!k]_{k=0}^{p-1}$ defined in the scopes of $S'm$ and $S'm::M$.

A similar result also holds for a referenced generic method, i.e., $T::M(T_0, \ldots, T_{n-1})$ instead of $T(T_0, \ldots, T_{n-1})$.

Proof. For this proof, we assume that constr$_{T'}^{T'n}$ denotes the uninstantiated (raw) constraints, exactly as given in the declaration of $T'n$. The following relations hold for every $i = 0, n − 1$:

$$\text{boxed}(T_i \circ \rho) = \text{boxed}(T_i) \circ \rho \subseteq (\text{constr}^{T'_n} \circ [T_i/X_i]_{i=0}^{n-1}) \circ \rho = \text{constr}^{T'_n} \circ ([T_i/X_i]_{i=0}^{n-1} \circ \rho) = \text{constr}^{T'_n} \circ [T_i \circ \rho/X_i]_{i=0}^{n-1}$$

The relation $\subseteq$ in the second row above follows from Lemma 4.3.2 and the fact that $(T_i)_{i=0}^{n-1}$ satisfy $T'n$'s constraints. Finally, $(T_i \circ \rho)_{i=0}^{n-1}$ satisfy the constraints of $T'n$.

The proof for the case of a referenced generic method is similar. □
### 4.3. The Generics

The definition of the typing judgment $\vdash$, given in Definition 4.1.12, remains unchanged. However, in the context of boxing nullable types, $\vdash$ does not suffice anymore. Therefore, we introduce a slightly different judgment $\models$ which, unlike $\vdash$, also considers the special cases when the dynamic and static types disagree.

**Definition 4.3.10** Let $\text{val}$ be a value, $T$ a verification type and $[fr_1, \ldots, fr_n]$ a list of frames. The typing $[fr_1, \ldots, fr_n] \models \text{val} : T$ holds if one of the following conditions is satisfied:

- $[fr_1, \ldots, fr_n] \vdash \text{val} : T$;
- there exists $T' \in \text{ValueType}$ such that $\text{actualTypeOf(\text{val})} = T'$ and $\text{boxed} (\text{Nullable}(T')) \sqsubseteq T$

Although Definition 4.3.10 does not explicitly include a subsumption rule for $\models$, this is implied and can be easily proved:

**Lemma 4.3.6** Let $T$ and $T'$ be two types such that $T \sqsubseteq T'$. If $[fr_1, \ldots, fr_n] \models \text{val} : T$, then $[fr_1, \ldots, fr_n] \vdash \text{val} : T'$.

**Proof.** By Definition 4.3.10 and Lemma 4.1.5. $\square$

The next lemma claims that $\vdash$ and $\models$ coincide for each type other than boxed nullable types.

**Lemma 4.3.7** If $T$ is a type such that $[fr_1, \ldots, fr_n] \models \text{val} : T$ and there exists no $T' \in \text{ValueType}$ such that $T = \text{boxed} (\text{Nullable}(T'))$, then $[fr_1, \ldots, fr_n] \vdash \text{val} : T$.

**Proof.** By reductio ad absurdum: Let us assume that $[fr_1, \ldots, fr_n] \not\vdash \text{val} : T$. From Definition 4.3.10, we derive that there exists $T'' \in \text{ValueType}$ such that

$$\text{actualTypeOf(\text{val})} = T'' \text{ and } \text{boxed} (\text{Nullable}(T'')) \sqsubseteq T \quad (4.15)$$

As the nullable types are value classes that do not implement interfaces, by Definition 4.3.4, we know that one of the following conditions should hold: $T = \text{boxed} (\text{Nullable}(T'''))$ or $T = \text{object}$ or $T = \text{System.ValueType}$. The first conditions cannot be satisfied as, by the hypothesis, we know that there exists no $T' \in \text{ValueType}$ such that $T = \text{boxed} (\text{Nullable}(T'))$. If the second or the third condition holds, then, by (4.15), we have $[fr_1, \ldots, fr_n] \not\vdash \text{val} : T$ according to Definition 4.1.12. Finally, the initial assumption is contradicted. $\square$

As the generic methods have the return types uninstantiated, the notion succession of a frame, given in Definition 4.1.14, should be refined.

**Definition 4.3.11 (Succession of a frame)** Let the list $[fr_1, \ldots, fr_n]$ be the frameStack and $fr_i = (pc^*, \text{locAdr}^*, \text{locPool}^*, \text{argAdr}^*, \text{evalStack}^*, \text{meth}^*)$ be an arbitrary frame in the frameStack. We denote by pos and mref the program counter and the method of the callee frame of $fr_i$ (for $fr_n$, the callee frame is frame). The succession of $fr_i$ is defined as follows:
- If `mref` returns a value, say `val`, satisfying \([fr_1, \ldots, fr_i] \vdash val : \text{retType}(mref) \circ \text{inst}(mref), then \((pc^*+1, \text{locAdr}^*, \text{locPool}^*, \text{argAdr}^*, \text{evalStack}^*, [val], \text{meth}^*)\) is called the succession of \(fr_i\) in frameStack.

- If \(\text{retType}(mref)\) is \text{void}, then the succession of \(fr_i\) in frameStack is given by \((pc^* + 1, \text{locAdr}^*, \text{locPool}^*, \text{argAdr}^*, \text{evalStack}^*, \text{meth}^*)\).

The type safety proof for well-typed methods is now extended to generics. We redefine the invariants which are affected by generics, and, additionally, we consider a generics specific invariant (constr). The invariant (stack2) ensures that the values on the evalStack are of the types assigned in the stack state, unless the values are boxed values corresponding to appropriate boxed nullable types. Following the conventions that locTypes and argTypes are uninstantiated, the invariants (loc) and (arg) are reconsidered. The invariant (constr) ensures that the generic arguments of any generic method satisfy the declared constraints.

**Theorem 4.3.1 (Type safety of well-typed methods)** Let \([fr'_1, \ldots, fr'_{k-1}]\) be the frameStack and \(fr_k\) be the current frame. For every \(i = 1, k - 1\), we denote by \(fr_i\) the succession of \(fr'_i\).

We assume that the type system and the bytecode structure of the methods of \((fr_i)_i\) satisfy the conditions stated in Section 4.1.4.2 and Section 4.2.4.1. Moreover, we assume that every generic interface declaration is valid and the generic arguments of all the generic types and method references satisfy at verification time the corresponding constraints. Furthermore, we assume that all methods of \((fr_i)_i^{k=1}\) are well-typed.

Let \(fr_i = (pc^*, \text{locAdr}^*, \ldots, \text{argAdr}^*, \text{evalStack}^*, \text{meth}^*)\) be one of the frames \((fr_i)_i^{k=1}\). We denote by \text{locVal}^* and \text{argVal}^* the functions \text{locVal} and \text{argVal} derived based on \text{locAdr}^* and \text{argAdr}^*, respectively. If \(fr_i\) is the current frame, we consider \text{initState}^* to be the current initialization status \text{initState}. If \(fr_i\) is not the current frame, we denote by \text{initState}^* the succession of \text{initState} for \(fr_i\). Let \((\text{argZeroT}_{j}, \text{evalStackT}_{j})_{j\in D}\) be the family of type states \text{meth}^* is typable with.

The invariants (pc), (stack1), (init), (field) (box) and (del) of Theorem 4.1.1 and the following invariants are satisfied at run-time for every frame \(fr_i\) (for the current frame, \(fr_k\), the invariants (stack1) and (stack2) should only hold if switch = Noswitch):

(stack2) \([fr_1, \ldots, fr_i] \vdash \text{evalStack}^*(j) : \text{evalStackT}_{pc^*}(j) \circ \text{inst}(\text{meth}^*), for every \(j = 0, \text{length}(\text{evalStack}^*) - 1\)."

(loc) \([fr_1, \ldots, fr_i] \vdash \text{locVal}^*(n) : \text{locTypes}(\text{meth}^*)(n) \circ \text{inst}(\text{meth}^*), for every \(n = 0, \text{locNo}(\text{meth}^*) - 1\)."

(arg) If \(\text{classNm}(\text{meth}^*) \in \text{ObjClass} and \text{methNm}(\text{meth}^*) = .ctor, then it holds \([fr_1, \ldots, fr_i] \vdash \text{argVal}^*(0) : \text{argZeroT}_{pc^*}; otherwise, \([fr_1, \ldots, fr_i] \vdash \text{argVal}^*(0) : \text{argTypes}(\text{meth}^*)(0)\).

If \(\text{argNo}(\text{meth}^*) \geq 2\), then \([fr_1, \ldots, fr_i] \vdash \text{argVal}^*(n) : \text{argTypes}(\text{meth}^*)(n) \circ \text{inst}(\text{meth}^*), for every \(n = 1, \text{argNo}(\text{meth}^*) - 1\)."
If \( \text{meth}^* \) is a generic method, \boxed{\text{genArg}(\text{meth}^*)} for every \( n = 0, \text{genParamNo}(\text{meth}^*) - 1 \).

**Proof.** The proof proceeds by induction on the run of the operational semantics model \( \text{CLR}_G \). We only prove here the case of virtual method calls.

**Case** \( \text{code}(\text{meth})(pc) = \text{CallVirt}(T', T::M) \circ \text{inst}(\text{meth}) \).

Since \( \text{meth} \) is well-typed, we get \( \text{check}(\text{meth}, pc, \text{evalStack}_{pc}) \) from Definition 4.19 (wt5). According to the definition of \( \text{check} \) in Figure 4.27, \( \text{evalStack}_{pc} \subseteq_{\text{suf}} \text{argTypes}(T::M) \circ \text{inst}(T::M) \). By (constr) and Lemma 4.3.2, we have \( \text{evalStack}_{pc} \circ \text{inst}(\text{meth}) \subseteq_{\text{suf}} (\text{argTypes}(T::M) \circ \text{inst}(T::M)) \circ \text{inst}(\text{meth}) \) (4.16).

By (4.16) and by the induction hypothesis – that is, by the invariants (stack1) and (stack2) – there exists a list of values \( L \), two lists of types \( L' \) and \( L'' \) and the values \( (\text{val}_j)_{j=0}^{\text{argNo}(T::M)-1} \) such that: 

\[
\text{evalStack} = L \cdot \text{[val}j_{j=0}^{\text{argNo}(T::M)-1}],
\]

\[
\text{evalStack}_{pc} = L' \cdot L'', \text{length}(L'') = \text{argNo}(T::M),
\]

for every \( j = 0, \text{length}(\text{evalStack}) - \text{argNo}(T::M) - 1 \)

\[
[fr_1, \ldots, fr_k] \models L(j) : L'(j) \circ \text{inst}(\text{meth})
\]

and

for every \( j = 0, \text{argNo}(T::M) - 1 \)

\[
[fr_1, \ldots, fr_k] \models \text{val}_j : L''(j) \circ \text{inst}(\text{meth})
\]

By (4.16), (4.17), (4.18) and Lemma 4.3.6, \( [fr_1, \ldots, fr_k] \models \text{val}_j : (\text{argTypes}(T::M)(j) \circ \text{inst}(T::M)) \circ \text{inst}(\text{meth}) \) for every \( j = 0, \text{argNo}(T::M) - 1 \). By this and Lemma 4.3.7, we get for every \( j = 0, \text{argNo}(T::M) - 1 \):

\[
[fr_1, \ldots, fr_k] \models \text{val}_j : (\text{argTypes}(T::M)(j) \circ \text{inst}(T::M)) \circ \text{inst}(\text{meth})
\]

When the bytecode instruction \( \text{CallVirt}(\text{tail}, T', T::M) \circ \text{inst}(\text{meth}) \) is executed, the macro \( \text{VIRTCall}(\text{tail}, T::M \circ \text{inst}(\text{meth}), \text{[val}j_{j=0}^{\text{argNo}(T::M)-1}] \) is invoked. According to [45, Partition I, §12.1.6.2.4], the \( \text{CallVirt} \) instruction is not valid with value class method token arguments. This implies \( T \not\in \text{ValueClass} \). By this and (4.19), we have that \( \text{val}_0 \) is an object reference.

We only analyze here the case when \( \text{val}_0 \) is not \text{null} and \text{tail} is \text{False}. If that is the case, in the frame of new current method \( T''::M = \text{lookUp}(T'''::M, T::M \circ \text{inst}(\text{meth})) \), where \( T''' = \text{actualTypeOf}(\text{val}_0) \), the arguments \( (\text{argVal}(j))_{j=0}^{\text{argNo}(T''::M)-1} \) are set to \( (\text{val}_j)_{j=0}^{\text{argNo}(T''::M)-1} \). Note that, by Lemma 4.3.3, we have \( \text{argNo}(T::M) = \text{argNo}(T''::M) \).

Regarding \( T''::M \), we only focus here on proving the invariants (arg) and (constr) as the proof of the others goes along the lines of the proof **Case 2** in Theorem 4.1.1. Taking into account Lemma 4.3.3, we make the following case distinction.

Let us first assume that \( T'' \in \text{ValueType} \).

---

32 Note that \( \text{evalStack}_{pc} \) might involve generic parameters.
If $T'' \neq T'''$, then \( \text{boxed}(T''') \subseteq \argTypes(T'':M)(0) \). According to the definition of \text{VirtCall}, the \text{this} pointer of $T''':M$ is \text{val}_0. By the definition of \( \vdash \), we have $[fr_1, \ldots, fr_k] \vdash \text{val}_0 : \argTypes(T'':M)(0)$. This and the relation $\argTypes(T'':M)(0) = \argTypes(T''':M)(0) \circ \text{inst}(T''':M)$ (the type of the \text{this} pointer is instantiated, as opposed to the other argument types) imply the invariant \text{(arg)} for the first argument of $T''':M$.

If $T'' = T'''$, then $\argTypes(T'':M)(0) = T'''&$. Conforming with the definition of \text{VirtCall}, the first argument of $T''':M$ is addressOf($\text{val}_0$). By the definition of \( \vdash \) and $\argTypes(T'':M)(0) = T'''&$, we get $[fr_1, \ldots, fr_k] \vdash \text{addressOf}(\text{val}_0) : \argTypes(T'':M)(0)$. Similarly as above, this ensures the invariant \text{(arg)} for the \text{this} pointer of $T''':M$.

If $T'' \in \text{RefType}$, then $T'' \subseteq \argTypes(T'':M)(0)$. The first argument of $T''':M$ is $\text{val}_0$. By the definition of \( \vdash \), we have $[fr_1, \ldots, fr_k] \vdash \text{val}_0 : \argTypes(T'':M)(0)$. This implies the invariant \text{(arg)} for the \text{this} pointer of $T''':M$.

So, in all the cases, the invariant \text{(arg)} for the \text{this} pointer of $T''':M$ is ensured.

By Lemma 4.3.3, the following relations hold for every $j = 1, \argNo(T':M) - 1$:

\[
\begin{align*}
\text{(argTypes}(T':M)(j) \circ \text{inst}(T':M)) \circ \text{inst}(\text{meth}) & = \text{argTypes}(T':M \circ \text{inst}(\text{meth}))(j) \circ \text{inst}(T':M \circ \text{inst}(\text{meth})) \\
& \subseteq \text{argTypes}(T''':M)(j) \circ \text{inst}(T''':M)
\end{align*}
\]

(4.20)

The invariant \text{(arg)} for the other arguments follows from (4.19), (4.20), and Lemma 4.3.6.

Let us now prove \text{(constr)}. We assume that $T'':M$ is a generic method (otherwise \text{(constr)} is obviously satisfied). This implies that also the method $T':M$ is generic. By Lemma 4.3.5, we get $\text{genParamNo}(T'':M) = \text{genParamNo}(T':M)$ and for every $j = 0, \text{genParamNo}(T':M) - 1$:

\[
\text{boxed(\text{genArg}(T':M \circ \text{inst}(\text{meth}))(j))} = \text{boxed(\text{genArg}(T''':M)(i))} \subseteq \text{constr}_{j''':M}
\]

Proposition 4.3.1 applied for the generic method $T':M$ and the above relations imply \text{(constr)}.

We also want to show \text{(stack2)} for the succession, defined according to Definition 4.3.11, of the new current \text{frame} in $[fr'_1, \ldots, fr'_k]$. If $\text{retType}(T'':M) = \text{void}$, then, by Lemma 4.3.4, $\text{retType}(T':M) = \text{void}$, and therefore \text{(stack2)} for the succession follows from the induction hypothesis, i.e., \text{(stack2)} for $fr_k$.

Let us now assume that $\text{retType}(T'':M)$ is not \text{void}. Let \text{val} be the value returned by $T'':M$, assumed to satisfy

\[
[fr'_1, \ldots, fr'_k, fr_k \downarrow \text{frame}] \vdash \text{val} : \text{retType}(T'':M) \circ \text{inst}(T''':M)
\]

(4.21)

Lemma 4.1.7, the constraint \text{Return Type} and (4.21) imply

\[
[fr'_1, \ldots, fr'_k, fr_k] \vdash \text{val} : \text{retType}(T''':M) \circ \text{inst}(T''':M)
\]

(4.22)

To prove \text{(stack2)} for the succession of \text{frame}, it suffices to show

\[
[fr'_1, \ldots, fr'_k, fr_k] \models \text{val} : \text{retType}(T':M) \circ \text{inst}(T':M)
\]

(4.23)
By Lemma 4.3.4, it follows that

\[
\text{retType}(T''::M) \circ \text{inst}(T''::M) \sqsubseteq \text{retType}(T::M) \circ \text{inst}(T::M)
\]  \hspace{1cm} (4.24)

Finally, (4.23), and implicitly also (stack2) for the succession of frame, follows from (4.22), (4.24), Lemma 4.1.5, and Definition 4.3.10. \qed
## 4.4 Related Work

Sections 4.4.1 and 4.4.2 survey the literature on CIL’s formal studies, in particular type safety proofs, and the closely related Java Virtual Machine (JVM) [91]’s type safety, respectively. The application of the ASM method for the specification of programming languages and their implementation is reviewed in Section 4.4.4.

### 4.4.1 The CIL

Regarding CIL’s type safety, there are only two publications we are aware of. Gordon and Syme have developed a type system for a small fragment of CIL, named Baby IL (BIL) [66]. Their work has been partially extended with generics by Yu, Kennedy and Syme for an even smaller CIL subset, called BILG [123]. We provide below a detailed comparison between these papers and our work.

**BIL’s type safety** The main theorem proved by Gordon and Syme [66], based upon Syme’s method [117] for writing functional specifications (subject to theorem proving in HOL), asserts soundness of the BIL’s type system and not the soundness of the bytecode verification. Being focused on the type system, they develop simple evaluation rules, in a big-step style of operational semantics [85]. This approach has, however, some downsides. For example, it seems to be impossible to determine in their framework if and when a location (e.g., a field or argument) is initialized.

Unlike [66], our framework involves a complete (with respect to the considered bytecode language) semantics model, which represents an important by-product of this thesis. To the best of our knowledge, our work is the first that defines a semantics model for the verifiable CIL bytecode.

We briefly mention here the most important aspects omitted in their formal specifications:

- **local variables**: As Gordon and Syme do not provide a formal description for a method frame, leaving out local variables simplifies considerably their approach. As we have seen in Definition 4.1.12, considering local variables increases the complexity of the typing judgment the type safety proof highly relies on.

- **arithmetic operations**: For the semantics of the arithmetic operations, our framework reflects the implementation of the real CLR, where the evaluation stack is strongly typed. Accordingly, we consider the values to be tagged (see Section 4.1.2). As Gordon and Syme abstract away from any details concerning the evaluation stack, the importance of a typed evaluation stack cannot be spotted in their framework.

- **typed references**: Since typed references embed pointers, the verification has to treat them prudently in order to avoid dangling pointers. It is not immediately obvious whether the restrictions imposed in Section 4.1.4.2 guarantee the type-safe execution of programs with typed references.

- **tail method calls**: Tail calls require care, because the verification has to prevent pointers to the current frame being passed as arguments. It is not straightforward at all whether the constraints on the bytecode structure defined Sections 4.1.4.2 and 4.2.4.1 ensure type-safe tail calls.
4.4. RELATED WORK

- **exceptions**: BIL is exception-free. One of our main contributions is the investigation of the fairly elaborate exception handling mechanism, which significantly increased the complexity of our work through the complicated control flow graph underlying the runs of the mechanism and through the nesting of the pass records. The analysis of the exception mechanism’s runs necessitates attention since, as we have seen in Theorem 4.2.1, there are certain runs of the mechanism, characterized by $\text{switch} \neq \text{Noswitch}$, which include steps where the invariants $(\text{stack1})$ and $(\text{stack2})$ are not satisfied. However, these (hidden) steps do not affect CLR’s type safety since the $\text{evalStack}$, involved in $(\text{stack1})$ and $(\text{stack2})$, is not accessed as long as $\text{switch}$ is not $\text{Noswitch}$.

- **delegates**: BIL does not include method pointers. Therefore, the delegates, i.e., the feature that allows for type-safe method pointers, are not treated either. As [66] does not specify the program counter $\text{pc}$, it is not clear whether [66]’s approach can be easily adjusted to specify, for example, the constraint Delegate Pattern in Section 4.1.4.2. Moreover, BIL would need to be considerably extended (e.g., $\text{Dup}$, $\text{LoadFin}$, $\text{LoadVirtFin}$, branch instructions) to cover the delegate handling.

- **boxed types and interfaces**: Although BIL includes instructions for boxing and unboxing, [66], contrary to [45], does not include boxed types. Moreover, without considering boxed types, [66]’s approach is not flexible enough to include interfaces.

The subtleties of the local memory pool are avoided by [66]’s framework. Actually, they cannot be spotted as long as the typing of the this pointer of a .ctor is not an issue in [66]. This is the case since the .ctors considered in [66] are oversimplified and do not have a this pointer.

Gordon and Syme do not take into account details concerning the methods’ evaluation stack. That means, on one hand, that $(\text{stack1})$ and $(\text{stack2})$ cannot be analyzed, and therefore cannot be ensured with their framework. On the other hand, as the exception mechanism deals extensively with evaluation stacks, it is imperatively necessary to specify the $\text{evalStack}$ if one wants to include exceptions.

Another important drawback of [66]’s type system is that it does not analyze the special initialization rules for objects. In this sense, one of our merits is the extension of the verification type system by the type $\text{UnInit}$, though [45, Partition III, §1.8.1.2.1] does not mention it. Another important contribution of our framework is augmenting the bytecode verification with the $\text{argZeroT}$ component to track type information concerning the initialization status of object references. As BIL does not include the Return instruction and assumes, as originally suggested by [76], that each class has exactly one .ctor (which does not even have a this pointer), it would just not be possible to implement the object initialization rules in [66]’s framework according to [45]. Recall that a this pointer of an object class .ctor is fully initialized if a constructor of the same class or of the base class has been invoked.

As the theorem proved by Gordon and Syme applies only to a small bytecode language, they write [66, §1, pag. 3]:

_A soundness proof for the whole of CIL would be an impressive achievement._

Given that we have only omitted a very few, yet non-critical, CIL features, we can certainly claim that this thesis has attained the respective achievement.
**BILG’s type safety** Yu, Kennedy and Syme use judgment forms, in a big-step style operational semantics, to define the generics semantics, in a fashion similar to Featherweight GJ [78]. They focus on aspects of the generics implementation, e.g., specialization of generic code up to data representation, efficient support for run-time types. As the exceptions are not part of BILG, [123] has to specify for certain constructs a different semantics than [45], e.g., the [123]’s `IsInstance` instruction does not return – as it was supposed to – the null pointer if the run-time test fails. Moreover, as their main goal was not BILG’s type safety, the formal specifications do not include the main sources for potential type safety violations: variance on generic parameters and constraint types. The critical issue concerning the mismatch between the static and dynamic types in case of boxing nullable types cannot be uncovered in [123]’s approach as long as the `Box` and `Unbox` instructions are not included. That means, in particular, that the boxed types are not considered either. Consequently, the constraint types cannot be added to their approach.

Bannwart and Müller [20] present a Hoare-style logic (axiomatic semantics) for a sequential kernel bytecode language, similar to CIL. The logic is part of a larger project that aims at developing a proof-transforming compiler (somehow similar to certifying compilers [36] in Proof-Carrying Code [96]), that translates a source program together with a proof of some of its properties to the bytecode level. Their logic considers that the bytecode programs are accepted by the bytecode verification. The logic is sound with respect to the defined operational semantics. They apply the logic in a weakest precondition style and show how source proofs can be translated to the bytecode logic. The bytecode language they consider does not include managed pointers, delegates, value classes, tail calls, typed references, and generics.

### 4.4.2 The JVM

The JVM is the run-time environment designed to execute compiled Java programs. Considerable research effort has been devoted to formally specify the JVM bytecode semantics, verification, and type safety; see, for example, [116, 52, 115, 114, 53, 51, 89, 103, 104, 99, 39]. A detailed review of the huge literature can be found in [73]. Of all these publications, the work by Stärk et al. [115] is the most closely related and had the most impact on this thesis. Besides proving Java’s and JVM’s type safety, [115] (following [113]) also points out a serious problem of the JVM bytecode verification: There are legal Java programs which are correctly compiled (with at that time Sun’s JDK (v1.2) and (v1.3) compilers) into a JVM bytecode that is rejected by the bytecode verification (of JDK (v1.2) and (v1.3)). To fix this problem, [115] (following [114]) propose to restrict the definite assignment analysis rules of the Java compiler such that these programs are no longer allowed. Further, [115] (following [114]) specifies a compilation scheme and proves that the bytecode programs the Java compiler generates, according to the compilation scheme, from the restricted subset of valid Java programs pass the bytecode verification.

As the CLR is designed to support a large diversity of languages, it is understandable that it deals with many constructs that do not have an equivalent in JVM, which, despite of several attempts [93, 69, 1] to be used as target for languages other than Java, has been designed precisely with the Java language in mind. The most important CLR features that are lacking in JVM are the tail method calls, the constructs supporting pointers (including method pointers in the form of delegates), value classes, boxing, unboxing, and typed references. As detailed comparisons
of the two virtual machines already exist [68, 108], we limit ourselves to point out here only a few important aspects concerning the semantics and bytecode verification that differ in JVM and CLR.

The main challenge of the JVM bytecode verification, i.e., the subroutine verification, is not an issue in the CLR. Moreover, in the JVM, the object initialization verification is performed differently than in the CLR.

Subroutine verification  A JVM subroutine can be viewed as a procedure at the bytecode level. If a sequence of bytecode instructions occurs more than once in a method body, the Java compiler can build a subroutine with this sequence and call it at the convenient positions. The main application of the subroutines is for the Java finally blocks. Every such a block is compiled to a subroutine. At each exit point inside a try block and its associated catch handler region, the subroutine corresponding to the finally handler is called. After the finally block completes – as long as it completes by executing past the last statement in the finally, not by throwing an exception or executing a return, continue, or break – the subroutine itself returns. The execution continues just past the point where the subroutine has been called in the first place, so the try block can be exited in the appropriate manner. One problem faced by the JVM bytecode verification when checking subroutines is concerning the polymorphism on local variables. The subroutines may have multiple call points, whose local variables may each hold values of different types. One should expect that the local variables not used inside the subroutine have the same type before and after the execution of the subroutine.

The CLR does not use subroutines to implement the finally handlers. The CLR’s solution seems to be more efficient: As we have seen in Section 4.2.2, the CLR explicitly supports finally handlers in CIL. Moreover, the CLR, unlike JVM, has typed local variables. This approach does not have any downside, except for carrying around some metadata. However, as observed in Section 4.2, the CLR exception handling mechanism becomes very complex by supporting filter handlers. Another construct that has no equivalent in JVM is the CLR fault handler. A semantics model for the JVM exception handling mechanism has been defined in [26, 115].

Object initialization verification  The JVM object creation process is accomplished in two steps as follows. A new reference is created by a new instruction (which does not have a constructor token argument), duplicated, and passed explicitly to a constructor invocation. One reference (to an uninitialized object) remains on the stack, and one reference is ”consumed” as the this pointer by the constructor invocation. After the constructor has been invoked, the reference left on the stack points to a properly initialized object. Bytecode verification is difficult since the constructors operate by side-effect: Instead of taking an uninitialized object and returning an initialized object, they simply take an uninitialized object, leave a reference to uninitialized object on the stack, and return nothing. To find out what references in the current state are ensured to point to the same uninitialized object that is passed as this pointer to a constructor, the JVM verification identifies every uninitialized object by the program counter of the new instruction that created it.

The CLR gets rid of this problem by adding a constructor token argument to the newobj instruction. Thus, in CLR, the same instruction creates a new object reference and initializes it by invoking the constructor pointed to by the token argument. However, in the constructor, the this pointer is considered uninitialized and the CLR verification has to track its initializa-
Another important verification issue, the local variable initialization, is treated differently by JVM and CLR. While the CLR requires that the verifiable methods automatically zeroes the local variables, the JVM bytecode verification performs a definite assignment analysis, in the sense of the one presented in Chapter 3 Section 3.4, and tracks type information for every local variable.

Unlike CLR, the JVM does not have generic instructions. To encode the kind of the performed operation, prefixes are added to the JVM instructions. For example, the JVM instruction \texttt{iadd} performs the addition of two integers, whereas \texttt{dadd} adds two doubles. It is then not necessary for any JVM semantics model to consider tagged values.

The JVM does not provide support for generic types and generic methods though Java (v5.0) supports them. The design of the generics in Java, initially proposed in [29], takes a different approach than the CLR. More precisely, the generics feature is compiled ”away” by the Java compiler. This design has some drawbacks. For instance, generic types can only be instantiated with reference types and not with primitive types, and type information is not preserved at run-time as in the CLR.

### 4.4.3 The Variance on Generic Parameters

The variance on generic parameters has its origins in the POOL-I language presented by America and van der Linden [19]. It has produced a lot of research into the nature of the inheritance relationship and subtyping. Two excellent overviews of the corresponding literature can be found in [35, 79].

There have been proposals to add variance for Java. One of the first designs for generics in Java, the work of Cartwright and Steele Jr. [34] include definition-site variance annotations. Igarashi and Viroli formally specified a system of use-site variance [79]. The latter has been adopted in Java (v5.0) through the wildcard mechanism [120].

As the C\#{\textdegree}’s generic parameters are currently non-variant, Emir et al. [47] propose a system of type-safe variance for C\#{\textdegree} that supports the declaration of covariant and contravariant parameters. They also introduce type constraints that generalize the F-bounded polymorphism [33] of Java [78, 76] and the bounded method generic parameters of Scala and include work on equational constraints [87].

A controversial feature of the Eiffel type system, the variance is in depth analyzed in [63], which also reports on a comprehensive comparison of generics in six programming languages (C++, Standard ML, Haskell, Eiffel, Java, C\#{\textdegree}). Cook [37] identifies some problems in Eiffel, in particular the mismatch of the called and invoked methods’ signatures caused by the variance. Cook also proposes solutions to fix these problems. The solution the above problem is approximately the one that [45] adopted, namely methods are required to have return types behaving ”covariantly” and argument types behaving ”contravariantly”. Howard et al. [75] propose a solution which enables compilers to spot all potential run-time type violations caused by variance and forces programmer to resolve them. The resulting language rules statically guarantee type safety and only require local analysis. However, in the absence of a full semantics model for Eiffel, [75] cannot provide a formal proof that the proposed policy guarantees type safety.
4.5. Summary and Future Work

4.4.4 The ASM Method

This section gives a short critical assessment of the ASM method and provides pointers to publications for further comparisons to alternative approaches. Besides reviewing the work on JVM’s type safety, [73] also includes an evaluation of the ASM based investigations. As a broad comparison of the ASM method with respect to other system design and analysis frameworks has already been done in [21, 64], we only point here some advantages of the ASM method over [66]’s approach.

With Gordon and Syme [66]’s evaluation rules, it is not convenient at all to model irregular control flows, in particular exception handling. To simplify the evaluation and typing rules, [66] decides to create two instructions which assemble certain branch instructions. As a result, a syntax for these two constructs is chosen. This technique is, however, not suitable when considering the whole CIL. As Gordon and Syme [66] mention, “the technique may not scale well to express control flow such as arbitrary branching within a method and exception handling”. In particular, the code containment, i.e., the invariant that pc will always point to a valid code index, cannot be analyzed, and therefore cannot be ensured in [66]’s framework. As demonstrated by this thesis, the ASM method can be successfully applied to model irregular control flows.

Another advantage of using the ASM method is the possibility of validating the corresponding ASM model. Thus, the model can be refined, made executable by means of an implementation in AsmL [49], and then validated by its simulation. To validate our operational semantics model, i.e., to test its internal correctness and its conformance to the Microsoft implementation (v2.0) [3], we have developed an AsmL implementation [92] of our ASM specifications. Furthermore, we have also validated our specifications for the bytecode verification [50].

4.5 Summary and Future Work

Except for static members, arrays and multi-threading – features that do not raise any challenging problems – we have proved type safety of the whole CIL.

We have structured the CIL bytecode language into three bytecode sublanguages, by isolating orthogonal parts of CIL, namely lightweight CIL, exception handling, and generics. Each sublanguage in the sequence extends its predecessor. We proved type safety for each sublanguage as follows. To specify the static and dynamic semantics, for each sublanguage, we defined an ASM interpreter, which is a conservative (purely incremental) extension of its predecessor. We have then provided a formal specification of the static and dynamic consistency checks performed by the bytecode verification. The dynamic checks have been defined by stepwise refinement, analogously to the layering of the semantics models. On top of these checks, we developed precise specifications of the bytecode verification algorithm and of the well-typed methods. We then proved type safety of each sublanguage as follows. We first showed that the well-typed methods are type-safe, i.e., the execution, according to the semantics models, of legal and well-typed methods does not yield any run-time type errors and maintains the program in a good state, where certain structural constraints are satisfied. Finally, we demonstrated that the bytecode verification is sound, i.e., the accepted methods are well-typed, and complete, i.e.,

33Note that denotational semantics is not suitable either to model irregular control flows (see [81] for a solution to cope with this issue).
it accepts every well-typed method and computes a most specific family of type states.

Potential future work includes a few extensions of the bytecode verification. As the bytecode verification is conservative, it might reject non-well-typed programs though their execution is type safe, regardless any input values. To limit the number of these cases, we would like to refine the bytecode verification. One refinement could be, for example, to extend the verification such that it also verifies methods whose local variables are not automatically initialized. For that, the verification should perform a definite assignment analysis similar to the one performed by the JVM verification. Another possible refinement is concerning the method pointers. Thank to the strong constraint Delegate Pattern, the bytecode verification can check the NewObj instructions which create delegates. This restriction can be weakened by inheriting the verification type system VerificationType with an abstract type. This special type would actually be defined as a pair consisting of the following components: a verification type (as defined before) and a method signature. The type component of the pair would be essential when verifying a delegate creation. Thus, for the NewObj instructions applied with delegate .ctor, one would need check that the type of the target object on the stack is compatible with the type component and the delegate class is compatible with the signature component. This extension of the verification type system would also make possible the verification of the currently non-verifiable indirect calls, i.e., calls via method pointers.

Future work might also encompass a formal study of the .NET security model and its properties, but also a correctness proof of a compilation scheme C♯–CIL, similar to the Java–JVM scheme developed in [115]. Also, the automatic verification of our type safety result remains an important challenge for future research.
5

Conclusion

Work for something because it is good, not just because it stands a chance to succeed.

Václav Havel

Until a while ago, C and C++ programming were the real standard in industry, and the types, if present, were not much more than ways to name memory offsets. For instance, a C structure is really just a big sequence of bits with names to access precise offsets, i.e., fields, from the base address. References to structures could be used to point at incompatible instances, data could be indexed into and used freely. Although C++ was already an important step in the right direction, there generally was not any run-time system enforcing that memory access follows the type system rules at run-time. In unmanaged languages, like C and C++, one can easily violate type safety, with direct security implications, through the possibility to easily access memory unimpeded. As this programming approach has proven to be error prone by leading to hard bugs, a trend toward type-safe languages has been started. Thus, over time, type-safe languages, like C♯ and Java, employing a static type analysis and dynamic detections of operations that could lead to type errors, gained popularity. Given the areas of applications these languages are designed for, it is crucial to guarantee that they meet the safety-related expectations, i.e., they are indeed type-safe.

In this thesis, we have formally proved type safety of large and representative subsets of C♯ and of the CIL language to which any .NET compliant language, in particular C♯, compiles. The analyzed subsets are reasonably rich and contain all the features which separately or together might have led to violations of type safety. It must be emphasized that all the formal models the proofs rely on are not only executable in theory: They have been validated through AsmL prototypes [84, 92, 50] generated from the formal specifications.

The abstract frameworks in this thesis have made it possible to clarify a number of important aspects which are ambiguously specified or not handled at all by the official specifications of C♯ and CLR. In this sense, we pointed out and filled several gaps in the two official documents, and we indicated a number of inconsistencies between different implementations of C♯ and CLR and their official specifications. It is important to notice that most of the conditions inherited from the language specifications, in particular even the well-formedness conditions, are actually needed somewhere in the proofs. This inspires, in particular, confidence in the appropriateness of the developed abstract frameworks.

The formal specifications defined in this thesis serve not only as a basis for the type safety proofs, but also as an excellent starting point for further investigations of various aspects of C♯ and CLR. Thus, one can analyze, for instance, stronger safety guarantees for C♯ and CIL
programs, or ways in which one can optimize programs in a type-safe manner. Moreover, the abstract and modular structure of the framework developed for CLR makes it possible to integrate new algorithms as well as new type systems into the existing formalization.

For formal proofs related to other languages, the experience achieved through the type safety proofs in this thesis, especially from the one developed in Chapter 4 for CIL, can be of help. One can learn, for instance:

- how to "divide and conquer" the type system, static and dynamic semantics of a language;
- how to separate the description of conceptually orthogonal constructs by dividing them into sublanguages; modularity is a key issue, since, unless the language changes excessively, the modifications in the formal specifications and proofs tend to be of local nature;
- how to unify and integrate the formal specifications of similar constructs by appropriate parameterizations (e.g., smart grouping of bytecode instructions makes possible factoring out common properties and implicitly reduce the work load significantly);
- how to specify and evaluate variations of specific features (e.g., expression evaluation, boxing and unboxing) by varying macros, rules and/or domains together with their operations.

The research underlying this thesis suggests, in particular, that, for the development of a programming language, the ASM method can be successfully applied for:

- defining an model as executable specification of critical language constructs or layers;
- generating test cases for the implementing code from the abstract model;
- using the abstract model as oracle for test evaluations and for comparing model test runs with code test runs;
- using the abstract model as internal documentation for future language extensions.
Bibliography


[100] Larry Paulson and Tobias Nipkow. Isabelle – Generic Theorem Proving Environment. Cambridge University and Technical University Munich.


## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>The grammar of expressions and statements.</td>
</tr>
<tr>
<td>3.2</td>
<td>The translation scheme of delegate classes.</td>
</tr>
<tr>
<td>4.1</td>
<td>The general form of delegate classes.</td>
</tr>
<tr>
<td>4.2</td>
<td>Example: typed references and methods with variable number of arguments.</td>
</tr>
<tr>
<td>4.3</td>
<td>The translation scheme of delegate classes.</td>
</tr>
<tr>
<td>4.4</td>
<td>The operational semantics rules.</td>
</tr>
<tr>
<td>4.5</td>
<td>The operational semantics rules (continued).</td>
</tr>
<tr>
<td>4.6</td>
<td>The operational semantics rules (continued).</td>
</tr>
<tr>
<td>4.7</td>
<td>The <code>SWITCHCLR</code> submachine.</td>
</tr>
<tr>
<td>4.8</td>
<td>Verifying the bytecode instructions.</td>
</tr>
<tr>
<td>4.9</td>
<td>The type state successors.</td>
</tr>
<tr>
<td>4.10</td>
<td>The bytecode verification algorithm.</td>
</tr>
<tr>
<td>4.11</td>
<td>Example: merging stack states.</td>
</tr>
<tr>
<td>4.12</td>
<td>Example: exception handlers.</td>
</tr>
<tr>
<td>4.13</td>
<td>Example: Visual Basic <code>Catch</code> with <code>When</code> clause.</td>
</tr>
<tr>
<td>4.14</td>
<td>The exception handling mechanism <code>EXCCLR</code>.</td>
</tr>
<tr>
<td>4.15</td>
<td>Example: discarding exception and leave records in the <code>entrypoint</code> method.</td>
</tr>
<tr>
<td>4.16</td>
<td>The operational semantics rules for the exception specific instructions.</td>
</tr>
<tr>
<td>4.17</td>
<td>The operational semantics rules for the exception-prone cases.</td>
</tr>
<tr>
<td>4.18</td>
<td>Verifying the exception specific instructions.</td>
</tr>
<tr>
<td>4.19</td>
<td>The type state successors of the exception specific instructions.</td>
</tr>
<tr>
<td>4.20</td>
<td>Example: successors of <code>Leave</code> and <code>EndFinally</code> instructions.</td>
</tr>
<tr>
<td>4.21</td>
<td>Example: uninstantiated method return type and parameter types.</td>
</tr>
<tr>
<td>4.22</td>
<td>The refined operational semantics rules.</td>
</tr>
<tr>
<td>4.23</td>
<td>The refined operational semantics rules (continued).</td>
</tr>
<tr>
<td>4.24</td>
<td>The refined operational semantics rules (continued).</td>
</tr>
<tr>
<td>4.25</td>
<td>Example: constrained call.</td>
</tr>
<tr>
<td>4.26</td>
<td>Example: unsafe example rejected by the verification.</td>
</tr>
</tbody>
</table>
4.27 The refined instruction verification. ........................................ 179
4.28 The refined computation of type state successors. ..................... 179

A.1 The operational semantics rules for the C♯ S’s expressions. .......... 211
A.2 The operational semantics rules for the C♯ S’s expressions (continued). 212
A.3 The operational semantics rules for the C♯ S’s expressions (continued). 213
A.4 The operational semantics rules for the C♯ S’s statements. ............ 213
A.5 The operational semantics rules for the C♯ S’s statements (continued). 214
List of Tables

2.1 The ASM transition rules. ................................. 9
2.2 The semantics of the ASM rules. ........................ 10
3.1 C♯S’s types. ........................................ 16
3.2 C♯S’s values. .......................................... 16
3.3 The environment specific universes. .................. 17
3.4 The components of the execution environment. .... 17
3.5 Derived language constructs. ........................... 20
3.6 Attributed syntax tree specific functions. .......... 22
3.7 The type constraints for expressions. ................. 24
3.8 The type constraints for statements. .................. 25
3.9 Compile-time transformations. .......................... 26
3.10 Definite assignment for boolean expressions. ...... 28
3.11 Definite assignment for arbitrary expressions. ... 29
3.12 Definite assignment for statements. .................. 30
3.13 Control flow for boolean expressions. ............... 37
3.14 Control flow for arbitrary expressions. ............. 37
3.15 Control flow for statements. .......................... 39
3.16 The basic static functions. ............................ 48
3.17 Semantics specific universes. ........................ 50
3.18 The basic dynamic functions. ........................ 51
3.19 Allocating object references and addresses. ....... 54
4.1 CIL’s types. .......................................... 79
4.2 CIL’s values. .......................................... 80
4.3 Environment specific universes. ....................... 80
4.4 The components of the execution environment. ..... 82
4.5 Semantics specific universes. ........................ 82
4.6 The basic static functions. ............................ 83
4.7   The basic dynamic functions. ........................................ 87
4.8   The considered CIL instructions. .................................. 90
4.9   CIL’s verification types. .............................................. 102
4.10  The verification specific functions. .............................. 107
4.11  The dynamic functions of the bytecode verification algorithm. 115
4.12  Semantics exception specific universes. ....................... 135
4.13  The basic static exception specific functions. ................ 135
4.14  The basic dynamic exception specific functions. ............. 137
4.15  The considered exception specific CIL instructions. .......... 148
4.16  Type system extension with generics. ........................... 164
4.17  Generics specific selector functions. ........................... 164
4.18  The generic specific instructions. ................................. 169
A.1 The C♯S Operational Semantics Rules

The operational semantics model is defined in terms of an ASM interpreter (for C♯S programs), named ExecCSHARP:

\[
\text{ExecCSHARP } \equiv \\
\text{ExecCSHARPExp} \\
\text{ExecCSHARPSTM}
\]

The main transition rule consists of two subrules executed in parallel: ExecCSHARPExp defines one execution step in the evaluation of expressions, and ExecCSHARPSTM specifies one execution step in the execution of statements. Upon executing one of the subrules, the updates implied by the first evaluation rule matching the current context are fired.

The macro context(pos) is used to define the context of the currently to be handled expression or statement or intermediate result, which has to be matched against the syntactically possible cases, in the textual order of the rules, to select the appropriate computation step.

**Figure A.1** The operational semantics rules for the C♯S’s expressions.

\[
\text{ExecCSHARPExp } \equiv \text{match context(pos)} \\
\quad \text{lit} \quad \rightarrow \text{YIELD(valueOfLiteral(lit))} \\
\quad \text{loc} \quad \rightarrow \text{YIELDINDIRECT(locAdr(loc))} \\
\quad \alpha\exp \bop \beta\exp' \quad \rightarrow \pos \leftarrow \alpha \\
\quad \up\val\bop\beta\exp' \quad \rightarrow \pos \leftarrow \beta \\
\quad \alpha\val\bop\up\val' \quad \rightarrow \text{if } \ST\pos \notin \text{Delegate then} \\
\quad \quad \text{if } \text{divisionByZero(bop, val')} \text{ then } \text{FAILUP}(\text{DivideByZeroException}) \\
\quad \quad \text{else } \text{YIELDUP}(\text{opResVal(bop, val, val')}) \\
\quad \text{elseif } bop = + \text{ then } \text{DELEGATECOMBINE}(\ST\pos, \val, \val') \\
\quad \text{elseif } bop = - \text{ then } \text{DELEGATEREMOVE}(\ST\pos, \val, \val') \\
\quad \text{elseif } bop = = = \text{ then } \text{DELEGATEEQUAL}(\ST\pos, \val, \val') \\
\quad \alpha\exp ? \beta\exp' : \gamma\exp'' \quad \rightarrow \pos \leftarrow \alpha \\
\quad \up\val ? \beta\exp' : \gamma\exp'' \quad \rightarrow \text{if } \val \text{ then } \pos \leftarrow \beta \text{ else } \pos \leftarrow \gamma \\
\quad \true ? \up\val : \gamma\exp'' \quad \rightarrow \text{YIELDUP}(\val) \\
\quad \false ? \beta\exp' : \up\val \rightarrow \text{YIELDUP}(\val)
\]
Figure A.2 The operational semantics rules for the C\textsuperscript{5}_S’s expressions (continued).

\[
\begin{align*}
\text{loc} &= \alpha \text{exp} \quad \rightarrow \text{pos} := \alpha \\
\text{loc} &= \bullet \text{val} \quad \rightarrow \text{WRITEMEM}(\text{locAdr(loc)}, \text{locType(loc)}, \text{val})
\end{align*}
\]
\[
\begin{align*}
\alpha \text{exp} \diamond \text{bop} = \beta \text{exp} \quad &\rightarrow \text{pos} := \alpha \\
\bullet \text{adr} \diamond \text{bop} = \beta \text{exp} \quad &\rightarrow \text{pos} := \beta \\
\alpha \text{adr} \diamond \text{bop} = \bullet \text{val} \quad &\rightarrow \text{if } ST(\alpha) \notin \text{Delegate} \text{ then } \\
&\quad \text{let } \text{val}' = \text{memVal}(\text{adr}, ST(\alpha)) \text{ in } \\\n&\quad \text{if } \text{divisionByZero}(\text{bop}, \text{val}) \text{ then } \text{FAILUP}(\text{ DivideByZeroException}) \\
&\quad \text{else let } \text{val}'' = \text{convertVal}(ST(\alpha), \text{opResVal}(\text{bop}, \text{val}', \text{val})) \text{ in } \\\n&\quad \text{WRITEMEM}(\text{adr}, ST(\alpha), \text{val}'') \quad \text{YIELDUP}(\text{val}'') \\
&\quad \text{elseif } \text{bop} = + \text{ then } \text{DELEGATECOMBINE}(ST(\alpha), \text{val}', \text{val}) \\
&\quad \text{elseif } \text{bop} = - \text{ then } \text{DELEGATEREMOVE}(ST(\alpha), \text{val}', \text{val})
\end{align*}
\]
\[
\begin{align*}
\text{ref} \alpha \text{exp} \quad &\rightarrow \text{pos} := \alpha \\
\text{ref} \bullet \text{adr} \quad &\rightarrow \text{YIELDUP}(\text{adr}) \\
\text{out} \alpha \text{exp} \quad &\rightarrow \text{pos} := \alpha \\
\text{out} \bullet \text{adr} \quad &\rightarrow \text{YIELDUP}(\text{adr})
\end{align*}
\]
\[
\begin{align*}
(\ldots \alpha \text{valadr}_1, \ldots, \alpha \text{valadr}_n) \quad &\rightarrow \text{pos} := \alpha_1 \\
(\ldots \bullet \text{valadr}_1, \ldots, \bullet \text{valadr}_n) \quad &\rightarrow \text{pos} := \alpha_{i+1}
\end{align*}
\]
\[
\begin{align*}
\alpha \text{exp} \text{ as } T \quad &\rightarrow \text{pos} := \alpha \\
\bullet \text{val} \text{ as } T \quad &\rightarrow \text{if } ST(\text{pos}) \in \text{ValueType} \text{ then } \text{YIELDUP}(ST(\text{pos}) \leq T) \\
&\quad \text{else } \text{YIELDUP}(\text{val} \neq \text{null} \land \text{actualTypeOf}(\text{val}) \leq T)
\end{align*}
\]
\[
\begin{align*}
\alpha \text{exp} \text{ as } T \quad &\rightarrow \text{pos} := \alpha \\
\bullet \text{val} \text{ as } T \quad &\rightarrow \text{if } ST(\text{pos}) \in \text{ValueType} \text{ then } \\\n&\quad \text{elseif } (\text{val} \neq \text{null} \land \text{actualTypeOf}(\text{val}) \leq T) \text{ then } \text{YIELDUP}(\text{val}) \\
&\quad \text{else } \text{YIELDUP}(\text{null})
\end{align*}
\]
\[
\begin{align*}
(T) \alpha \text{exp} \quad &\rightarrow \text{pos} := \alpha \\
(T) \bullet \text{val} \quad &\rightarrow \text{if } ST(\text{pos}) \in \text{ValueType} \text{ then } \\\n&\quad \text{if } ST(\text{pos}) \in \text{PrimitiveType} \land T \in \text{PrimitiveType} \text{ then } \text{} \\
&\quad \text{YIELDUP}(\text{convertVal}(T, \text{val})) \quad \text{elseif } T = ST(\text{pos}) \text{ then } \text{YIELDUP}(\text{val}) \\
&\quad \text{if } T \in \text{RefType} \text{ then } \text{YIELDUPBOX}(ST(\text{pos}), \text{val}) \quad \text{elseif } T \in \text{RefType} \text{ then } \\\n&\quad \text{if } \text{val} = \text{null} \lor \text{actualTypeOf}(\text{val}) \leq T \text{ then } \text{YIELDUP}(\text{val}) \quad \text{else } \text{FAILUP}(\text{InvalidCastException}) \\
&\quad \text{else } \text{FAILUP}(\text{InvalidCastException})
\end{align*}
\]
\[
\begin{align*}
\alpha \text{exp.T:F} \quad &\rightarrow \text{pos} := \alpha \\
\bullet \text{valadr.T:F} \quad &\rightarrow \text{if } ST(\text{pos}) \in \text{ValueType} \land \text{valadr} \notin \text{Adr} \text{ then } \text{YIELDUP}(\text{valadr}(T:F)) \\
&\quad \text{elseif } \text{valadr} \neq \text{null} \text{ then } \text{YIELDUPINDIRECT}(\text{fieldAdr}(\text{valadr}, T:F)) \quad \text{else } \text{FAILUP}(\text{NullReferenceException})
\end{align*}
\]
\[
\begin{align*}
\alpha \text{exp.T:F} = \beta \text{exp}' \quad &\rightarrow \text{pos} := \alpha \\
\bullet \text{valadr.T:F} = \beta \text{exp}' \quad &\rightarrow \text{pos} := \beta \\
\alpha \text{valadr.T:F} = \bullet \text{val}' \quad &\rightarrow \text{if } \text{valadr} \neq \text{null} \text{ then } \text{} \\
&\quad \text{WRITEMEM}(\text{fieldAdr}(\text{valadr}, T:F), \text{fieldType}(T:F), \text{val}') \quad \text{YIELDUP}(\text{val}') \quad \text{else } \text{FAILUP}(\text{NullReferenceException})
\end{align*}
\]
Figure A.3 The operational semantics rules for the C$^d_S$’s expressions (continued).

\[\text{new } C \text{ } \rightarrow \text{ let ref } = \text{ new(ObjRef, } C \text{) in} \]
\[\text{actualTypeOf(ref) } := \text{ C} \]
\[\text{for all } C::F \in \text{instFields(C) do} \]
\[\text{let addr } = \text{ fieldAddr(ref, } C::F \text{) and } T = \text{ fieldType(C::F) in} \]
\[\text{WRITEMEM(addr, T, defVal(T))} \]
\[\text{YIELD(ref)} \]

\[\alpha \text{exp.T} :: \beta(\text{args} ) \rightarrow \text{pos } := \alpha \]
\[\text{#elif } \alpha \text{valadr.T} :: \beta(\text{args}) \rightarrow \text{pos } := \beta \]
\[\text{else if } \text{valadr } \neq \text{null then VIRTcall(T::M, valadr, valadrs)} \]
\[\text{else FAILUP(NullReferenceException)} \]

\[\text{new D(\alpha \text{exp.T} :: M)} \rightarrow \text{pos } := \alpha \]
\[\text{new D(\text{ref})} \rightarrow \text{if ref } = \text{null then FAILUP(NullReferenceException)} \]
\[\text{else let } d = \text{new(ObjRef, } D \text{) in} \]
\[\text{actualTypeOf(d) } := \text{D} \]
\[\text{if callKind(T::M) } = \text{Virtual then} \]
\[\text{let } T'::M = \text{lookUp(actualTypeOf(val), T::M) in} \]
\[\text{invocationList(d) } := \text{[val, T'::M]} \]
\[\text{else let } ref' = \text{if } T \in \text{Struct then NewBox(ST(pos), val) else val in} \]
\[\text{invocationList(d) } := \text{[ref', T::M]} \]
\[\text{YIELDUP(d)} \]

Figure A.4 The operational semantics rules for the C$^d_S$’s statements.

\[\text{EXECSharpSTM } \equiv \text{match context(pos)} \]
\[\alpha \text{exp; } \rightarrow \text{pos } := \alpha \]
\[\text{while } \rightarrow \text{YIELDUP(Norm)} \]
\[\text{break; } \rightarrow \text{YIELD(Break)} \]
\[\text{continue; } \rightarrow \text{YIELD(Continue)} \]
\[\text{goto lab; } \rightarrow \text{YIELD(Goto(lab))} \]
\[\text{if } (\alpha \text{exp}) \beta \text{stm else } \gamma \text{stm' } \rightarrow \text{pos } := \alpha \]
\[\text{if } (\text{val}) \beta \text{stm else } \gamma \text{stm' } \rightarrow \text{if val then pos } := \beta \text{ else pos } := \gamma \]
\[\text{while } (\alpha \text{true}) \beta \text{stm } \rightarrow \text{if val then pos } := \beta \]
\[\text{while } (\text{true}) \beta \text{stm } \rightarrow \text{else YIELDUP(Norm)} \]
\[\text{while } (\alpha \text{true}) \beta \text{stm } \rightarrow \text{pos } := \text{up(pos)} \]
\[\text{while } (\text{true}) \beta \text{stm } \rightarrow \text{CLEARVALUES(up(pos))} \]
\[\text{while } (\alpha \text{true}) \beta \text{stm } \rightarrow \text{pos } := \text{up(pos)} \]
\[\text{while } (\text{true}) \beta \text{stm } \rightarrow \text{CLEARVALUES(up(pos))} \]
\[\text{while } (\alpha \text{true}) \beta \text{stm } \rightarrow \text{pos } := \text{up(pos)} \]
\[\text{while } (\text{true}) \beta \text{stm } \rightarrow \text{CLEARVALUES(up(pos))} \]
\[\text{while } (\alpha \text{true}) \beta \text{stm } \rightarrow \text{YIELDUP(abr)} \]
Figure A.5 The operational semantics rules for the C#’s statements (continued).

\[
\begin{align*}
T \text{ loc;} & \rightarrow \text{YIELD(Norm)} \\
lab : \alpha \text{stm} & \rightarrow \text{pos := } \alpha \\
lab : \triangledown \text{Norm} & \rightarrow \text{YIELDUp(Norm)} \\
\{ \} & \rightarrow \text{YIELD(Norm)} \\
\{ \alpha \text{stm} \ldots \} & \rightarrow \text{pos := } \alpha \\
\{ \ldots \triangledown \text{Norm} \} & \rightarrow \text{YIELDUp(Norm)} \\
\{ \ldots \triangledown \text{Norm} \alpha \text{stm} \ldots \} & \rightarrow \text{pos := } \alpha \\
\{ \ldots \triangledown \text{Goto}(L) \ldots \} & \rightarrow \text{let } \alpha = \text{GOTOTARGET()} \rightarrow \text{YIELDUp(Goto(L))}
\end{align*}
\]

\[
\begin{align*}
\text{return} \alpha \text{exp}; & \rightarrow \text{pos := } \alpha \\
\text{return} \triangledown \text{val}; & \rightarrow \text{YIELDUp(Return(val))} \\
\text{return}; & \rightarrow \text{YIELD(Return)} \\
\text{Return} & \rightarrow \text{if } (\text{pos is the position of meth’s body}) \land \text{frameStack} \neq [] \text{ then} \\
& \hspace{1cm} \text{EXITMETHOD(Norm)} \\
\text{Return(val)} & \rightarrow \text{if } (\text{pos is the position of meth’s body}) \land \text{frameStack} \neq [] \text{ then} \\
& \hspace{1cm} \text{EXITMETHOD(val)} \\
\triangledown \text{Norm}; & \rightarrow \text{YIELDUp(Norm)} \\
\text{throw} \alpha \text{exp}; & \rightarrow \text{pos := } \alpha \\
\text{throw} \triangledown \text{ref}; & \rightarrow \text{if ref = null then FAILUP(NullReferenceException)} \\
& \hspace{1cm} \text{else YIELDUp(Exc(ref))} \\
\text{throw}; & \rightarrow \text{YIELD}(\text{Exc(top(excStack)))} \\
\text{try} \alpha \text{block catch (E x) stm} & \rightarrow \text{pos := } \alpha \\
\text{try} \triangledown \text{Norm catch (E x) stm} & \rightarrow \text{YIELDUp(Norm)} \\
\text{try} \triangledown \text{Exc(ref)} & \rightarrow \text{if } \exists \ i = 1, n : \text{actualTypeOf}(\text{ref}) \leq E_i \text{ then} \\
\text{catch}(E_1 x_1) \alpha \text{stm}_1 & \hspace{1cm} \text{let } j = \min \{i = 1, n \mid \text{actualTypeOf}(\text{ref}) \leq E_i\} \text{ in} \\
& \hspace{1cm} \text{pos := } \alpha_j \\
\ldots \\
\text{catch}(E_n x_n) \alpha \text{stm}_n & \hspace{1cm} \text{excStack := push(excStack, ref)} \\
& \hspace{1cm} \text{WRITEMEM(locAdr(x_j), object, ref)} \\
& \hspace{1cm} \text{else YIELDUp(Exc(ref))} \\
\text{try} \triangledown \text{ab} & \rightarrow \text{YIELDUp(abr)} \\
\text{catch}(E_1 x_1) \text{stm}_1 & \ldots \\
\text{catch}(E_n x_n) \text{stm}_n & \ldots \\
\text{try} \text{Exc(ref) catch(E x) } \triangledown \text{res} & \rightarrow \text{excStack := pop(excStack)} \\
& \rightarrow \text{YIELDUp(res)} \\
\text{try} \alpha \text{block finally } \beta \text{block'} & \rightarrow \text{pos := } \alpha \\
\text{try} \triangledown \text{res finally } \beta \text{block'} & \rightarrow \text{pos := } \beta \\
\text{try} \text{res finally } \triangledown \text{Norm} & \rightarrow \text{YIELDUp(res)} \\
\text{try} \text{res finally } \triangledown \text{Exc(ref)} & \rightarrow \text{YIELDUp(Exc(ref))} \\
\text{Exc(ref)} & \rightarrow \text{if } (\text{pos is the position of meth’s body}) \land \text{frameStack} \neq [] \text{ then} \\
& \hspace{1cm} \text{EXITMETHOD(Exc(ref))} \\
\{ \ldots \triangledown \text{ab} \ldots \} & \rightarrow \text{YIELDUp(abr)} \\
\ldots \triangledown \text{ab} \ldots & \rightarrow \text{if up(pos) } \neq \text{undef } \land \text{propagatesAbr(up(pos)) then} \\
& \hspace{1cm} \text{YIELDUp(abr)}
\end{align*}
\]
The macro \texttt{FAILUp} is applied to throw exceptions. Given an exception class \(E\), it executes the statement \texttt{throw new \(E\)}(\); at the parent position. It accomplishes this by invoking an internal method, named \texttt{ThrowE} (whose body is assumed to contain only the above \texttt{throw} statement), with that effect for each \(E\).

\[
\text{FAILUp}(E) \equiv \text{InvokeMethod}(\text{ExcSupport::ThrowE}, [])
\]

\[
\text{DELEGATECOMBINE}(D, \text{ref}, \text{ref}') \equiv \\
\begin{array}{l}
\text{if ref = null then } \text{YIELDUp}(\text{ref}') \\
\text{elseif ref' = null then } \text{YIELDUp}(\text{ref}) \\
\text{else let } d = \text{new}(\text{ObjRef}, D) \text{ in } \\
\quad \text{actualTypeOf}(d) := D \\
\quad \text{invocationList}(d) := \text{invocationList}(\text{ref}) \cdot \text{invocationList}(\text{ref}') \\
\quad \text{YIELDUp}(d)
\end{array}
\]

\[
\text{DELEGATEREMOVE}(D, \text{ref}, \text{ref}') \equiv \\
\begin{array}{l}
\text{if ref = null then } \text{YIELDUp}(\text{null}) \\
\text{elseif ref' = null then } \text{YIELDUp}(\text{ref}) \\
\text{else let } L = \text{invocationList}(\text{ref}) \text{ and } L' = \text{invocationList}(\text{ref}') \text{ in } \\
\quad \text{if } L = L' \text{ then } \text{YIELDUp}(\text{null}) \\
\quad \text{elseif } \text{sublist}(L', L) \text{ then } \\
\quad \quad \text{let } d = \text{new}(\text{ObjRef}, D) \text{ in } \\
\quad \quad \text{actualTypeOf}(d) := D \\
\quad \quad \text{invocationList}(d) := \text{prefix}(L', L) \cdot \text{suffix}(L', L) \\
\quad \quad \text{YIELDUp}(d) \\
\quad \text{else } \text{YIELDUp}(\text{ref})
\end{array}
\]

\[
\text{DELEGATEEQUAL}(\text{ref}, \text{ref}') \equiv \\
\begin{array}{l}
\text{if ref = null } \lor \text{ref' = null then } \text{YIELDUp}(\text{ref} = \text{ref}') \\
\text{else let } L = \text{invocationList}(\text{ref}) \text{ and } L' = \text{invocationList}(\text{ref}') \text{ in } \\
\quad \text{YIELDUp}(\text{length}(L) = \text{length}(L') \land \forall i = 0, \text{length}(L) - 1 : (L(i) = L'(i)))
\end{array}
\]

\[
\text{YIELDUpBox}(T, \text{val}) \equiv \\
\begin{array}{l}
\text{let ref = newBox}(T, \text{val}) \text{ in } \\
\quad \text{YIELDUp}(\text{ref})
\end{array}
\]

\[
\text{InvokeMethod}(T::M, \text{valadrs}) \equiv \\
\begin{array}{l}
\text{if } T \in \text{Delegate then } \\
\quad \text{if } M = \_\text{length} \text{ then } \text{YIELDUp}(\text{length}(\text{invocationList}(\text{valadrs}(0)))) \\
\quad \text{if } M = \_\text{invoke} \text{ then } \text{CALLDELEGATE}(\text{valadrs}(0), \text{valadrs}(1), \text{drop}(\text{valadrs}, 2)) \\
\quad \text{else } \\
\quad \quad \text{frameStack} := \text{push}(\text{frameStack}, (\text{meth}, \text{up}(\text{pos}), \text{locAdr}, \text{values})) \\
\quad \quad \text{meth} := T::M \\
\quad \quad \text{pos} := \alpha, \text{ where } \alpha^\text{body}(T::M) \\
\quad \quad \text{values} := \emptyset \\
\quad \quad \text{INITLOCALS}(T::M, \text{valadrs})
\end{array}
\]

\[
\text{CALLDELEGATE}(\text{ref}, i, \text{valadrs}) \equiv \\
\begin{array}{l}
\text{let } (\text{ref'}, T::M) = \text{invocationList}(\text{ref})(i) \text{ in } \\
\quad \text{let this = if } T \in \text{Struct then } \text{valueAdr}(\text{ref'}) \text{ else } \text{ref'} \text{ in } \\
\quad \quad \text{InvokeMethod}(T::M, [\text{this} \cdot \text{valadrs})}
\end{array}
\]
INITLOCALS($T::M, valadrs$) ≡
for all $loc \in localVars(T::M)$ do
  let $adr = new(Adr, locType(T::M, loc))$ in
  $locAdr(loc) := adr$
for all $loc \in valueParams(T::M)$ do
  let $adr = new(Adr, paramType(T::M, loc))$ in
  $locAdr(loc) := adr$
  WRITEMEM(adr, paramType(T::M, loc), valadrs(paramIndex(T::M, loc)))
for all $loc \in refParams(T::M) \cup outParams(T::M)$ do
  $locAdr(loc) := valadrs(paramIndex(T::M, loc))$

CLEARVALUES($\alpha$) ≡
values($\alpha$) := undef
if first($\alpha$) $\neq$ undef then CLEARVALUESSEQ(first($\alpha$))

CLEARVALUESSEQ($\alpha$) ≡
  CLEARVALUES($\alpha$)
if next($\alpha$) $\neq$ undef then CLEARVALUESSEQ(next($\alpha$))
propagatesAbr($\alpha$) $\Leftrightarrow$ label($\alpha$) $\in \{\text{Block, while}\}$

GOTOTARGET($\alpha, lab$) =
  if label($\alpha$) = Lab(lab) then $\alpha$
  elseif next($\alpha$) $\neq$ undef then undef
  else GOTOTARGET(next($\alpha$), lab)

EXITMETHOD(res) ≡
let (oldMeth, oldPos, oldLocals, oldValues) = top(frameStack) in
  meth := oldMeth
  pos := oldPos
  locAdr := oldLocals
  frameStack := pop(frameStack)
  values := oldValues $\oplus$ {oldPos $\mapsto$ res}
# Curriculum Vitae

Nicu Georgian Fruja

<table>
<thead>
<tr>
<th>Year</th>
<th>Education/Experience</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 13, 1977</td>
<td>Born in Ramnicu-Sarat, Romania</td>
</tr>
<tr>
<td>1983 – 1987</td>
<td>Primary school, Ramnicu-Sarat</td>
</tr>
<tr>
<td>1987 – 1991</td>
<td>Secondary school, Ramnicu-Sarat</td>
</tr>
<tr>
<td>1991 – 1995</td>
<td>High school, Ramnicu-Sarat</td>
</tr>
<tr>
<td>1995 – 1999</td>
<td>Studies in Applied Mathematics, University of Bucharest, Romania</td>
</tr>
<tr>
<td>1999 – 2001</td>
<td>Master Studies in Applied Statistics and Optimization, University of Bucharest, Romania</td>
</tr>
<tr>
<td>2000 – 2001</td>
<td>Teaching Assistant, Department of Probabilities, Statistics, and Operations Research, University of Bucharest, Romania</td>
</tr>
<tr>
<td>2001 – 2006</td>
<td>Research and Teaching Assistant, Computer Science Department, ETH Zurich, Switzerland</td>
</tr>
</tbody>
</table>