Doctoral Thesis

Rotation of Particles by Ultrasonic Manipulation

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Rotation of Particles by Ultrasonic Manipulation

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presented by

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Zurich, December 2013
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Abstract

This study is aimed at the theoretical analysis of the acoustic torque and the experimental realization of a controlled rotation of spherical and non-spherical particles by ultrasound. Ultrasonic manipulation of particles exploits the acoustic radiation force to provide a contactless handling method for particles suspended in a fluid. In an ultrasonic standing wave field, the viscous torque or the acoustic radiation torque induces the rotation of an object. Beside the translation of particles due to the acoustic radiation force an additional controlled degree of freedom (rotation) is offered. Therefore, there is an increasing interest in extending the field of application of ultrasonic particle manipulation to the deposition of micro and nanowires, realization of ultrasonically-driven micro-machines and for the assembly of micro objects.

Currently, the analytical solutions of the acoustic force and torque are limited to simple cases of object shape and acoustic field. In the theoretical part of this study a finite element model was developed and validated to calculate the acoustic radiation force and torque on a micro fiber. The influence of different parameters such as the frequency, fiber size, position and orientation of the fiber in 1D and 2D standing wave fields was evaluated. The rotational motion of a non-spherical particle and the resulting drag torque were analyzed with a numerical simulation. This allowed the calculation of the angular velocity for a fiber with different parameters.

Various rotation methods for non-spherical particles with the acoustic radiation torque were developed. The equilibrium position and orientation of a fiber shorter than a quarter wavelength is at the pressure nodes, aligned with the nodal pressure line. Therefore a varying pressure field was necessary where the orientation of the nodal pressure line was influenced. All developed methods were tested experimentally with a micro device at frequencies in the MHz range. Particle clumps (copolymer particles) and micro glass fibers were used as rotation objects.

First, a hexagonal chamber was designed to successively change the wave propagation direction and therefore the orientation of the nodal pressure line in 60° steps.
Abstract

Three additional rotation methods were developed which allowed for a continuous rotation and alignment at defined orientations. All methods were characterized by the modulation of one single parameter (amplitude, phase, frequency) over time. First, the amplitude modulation of two orthogonal ultrasonic modes led to a local rotation of the nodal pressure line. The evaluation of the pressure field provided the different modes inducing rotation and the characteristic of the excitation to achieve a uniform rotation. Second, the phase modulation of degenerated modes can lead to a local rotation of the pressure field. Two degenerated modes slightly separated in frequency were needed to induce the rotation. A numerical model was used to show the separation of the modes and to develop an analytical model for the excited pressure fields. Third, the rotation with frequency modulation, based on two separated modes was realized. The modes can be split by a small difference in the length of the edge of a nearly square chamber.

The rotation of a micro fiber (length 200 µm, diameter 15 µm) was successfully realized with the amplitude modulation. A maximum average rotational speed of 40 rpm was observed at an excitation frequency of 1085 kHz. The acoustic radiation torque and the pressure amplitude were estimated using the drag torque. A pressure of 0.18 MPa and an acoustic radiation torque of $1.84 \times 10^{-14}$ Nm were determined. For a reasonable pressure amplitude in micro devices of 0.5 MPa, a perfectly excited mode and a levitating fiber, a radiation torque of $3.6 \times 10^{-13}$ Nm and rotational speeds up to 780 rpm are theoretically predicted.

Moreover, the viscous torque, generated by two orthogonal standing waves shifted in phase was studied. An induced boundary streaming spins the axisymmetric object. The viscous torque and drag torque allow for calculating the angular velocity of the sphere. Experiments using a macro device showed the location and phase dependency of the rotation direction. The rotational speed of the particle was defined by the pressure amplitude and depended on the particle size. This relation was shown experimentally by measuring the angular velocity of different particle sizes and fitting the curve with a pressure amplitude. The experiments led to a viscous torque of $1.2 \times 10^{-13}$ Nm for the observed rotation of a small particle with 1200 rpm and a radius of 35.5 µm. For the large particles with a radius of 223 µm and a rotational speed of 110 rpm a torque of $3.2 \times 10^{-12}$ Nm was determined. The actuation frequency was 770 kHz and a pressure of 0.18 MPa was estimated.

This study presents the first realizations of continuous rotation methods of micro sized spherical and non-spherical particles with ultrasonic standing waves. The gained knowledge should be a starting point for further investigations and development of applications.
Zusammenfassung


mikrofluidischen Bauteilen bei Frequenzen im MHz-Bereich getestet. Dabei fanden Partikelklumpen (Copolymer Partikel) und Mikroglasfasern als Rotationsobjekte Verwendung. Eine hexagonale Kammer wurde entwickelt, um schrittweise die Wellenausbreitungsrichtung und damit die Orientierung der Druckknotenlinie in 60°-Schritten zu verändern.


Die Rotation einer Mikrofaser (Länge 200 µm, Durchmesser 15 µm) wurde unter anderem mit der Amplitudenmodulation durchgeführt. Eine maximale mittlere Rotationsgeschwindigkeit von 40 U/min wurde bei einer Anregungsfrequenz von 1085 kHz erreicht. Das Schallstrahlungsmoment und die Druckamplitude wurden mit Hilfe des Strömungswiderstandsmomenten bestimmt. Dabei konnte ein Druck von 0,18 MPa und ein Moment von $1,84 \times 10^{-14}$ Nm ermittelt werden. Für eine realisierbare Druckamplitude in einer derartigen mikrofluidischen Kammer von 0,5 MPa, einem perfekten Resonanz-Mode und einer frei schwebenden Faser wurden ein theorethisches Moment von $3,6 \times 10^{-13}$ Nm und eine Rotationsgeschwindigkeit von bis zu 780 U/min ermittelt.

Des Weiteren wurde das Moment durch Randschichtströmung untersucht. Es entsteht durch zwei orthogonale, stehende Wellen mit einem Phasenunterschied. Die induzierte Randschichtströmung rotiert ein axialsymmetrisches Objekt. Das Moment durch Randschichtströmung und das Störungswiderstandsmoment ermöglichen die Berechnung der Rotationsgeschwindigkeit einer Kugel. Mit Experimenten in einem makroskopischen Resonator konnte die Phasen- und Lageabhängigkeit der Rotationsrichtung gezeigt werden. Es konnte nachgewiesen werden, dass die Rotationsgeschwindigkeit durch die Druckamplitu-
Die Größe und die Partikelgröße bestimmt wird. Die Experimente mit kleinen Partikeln (Radius $35.5\,\mu m$) zeigten ein Drehmoment von $1.2 \times 10^{-13}\,Nm$ und eine Rotationsgeschwindigkeit von $1200\,U/min$. Für größere Partikel (Radius $223\,\mu m$) konnte ein Drehmoment von $3.2 \times 10^{-13}\,Nm$ und eine Rotationsgeschwindigkeit von $110\,U/min$ ermittelt werden.

Diese Arbeit präsentiert die erste Realisierung von Rotationsmethoden mit stehenden Ultraschallwellen für kugelförmige oder stäbchenförmige Partikel im $\mu m$-Bereich. Die erlangten Erkenntnisse sind Basis für weiterführende Untersuchungen und die Entwicklung von entsprechenden Anwendungen.
### List of symbols and acronyms

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Amplitude (pressure)</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Angular position of fiber</td>
<td>[°]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Angular position of nodal pressure line</td>
<td>[°]</td>
</tr>
<tr>
<td>$c$</td>
<td>Speed of sound</td>
<td>[m/s]</td>
</tr>
<tr>
<td>$C$</td>
<td>Stiffness matrix</td>
<td>[Pa]</td>
</tr>
<tr>
<td>CNT</td>
<td>Carbon nanotube</td>
<td></td>
</tr>
<tr>
<td>$d_f$</td>
<td>Fiber diameter</td>
<td>[m]</td>
</tr>
<tr>
<td>$D$</td>
<td>Drag force coefficient</td>
<td>[kg/s]</td>
</tr>
<tr>
<td>$\tilde{D}$</td>
<td>Drag torque coefficient</td>
<td>[kg m²/s]</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Boundary layer thickness</td>
<td>[m]</td>
</tr>
<tr>
<td>$\Delta f$</td>
<td>Frequency difference</td>
<td>[Hz]</td>
</tr>
<tr>
<td>$\Delta \varphi$</td>
<td>Phase shift</td>
<td>[rad], [°]</td>
</tr>
<tr>
<td>DEP</td>
<td>Dielectrophoresis</td>
<td></td>
</tr>
<tr>
<td>DNA</td>
<td>Deoxyribonucleic acid</td>
<td></td>
</tr>
<tr>
<td>$e$</td>
<td>Coupling matrix of piezoelectric material</td>
<td>[C/m²]</td>
</tr>
<tr>
<td>$E$</td>
<td>Fiber Young’s modulus</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>Relative permittivity</td>
<td>[-]</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Dynamic viscosity</td>
<td>[Pa s]</td>
</tr>
<tr>
<td>$F^{\text{rad}}$</td>
<td>Acoustic radiation force</td>
<td>[N]</td>
</tr>
<tr>
<td>$f$</td>
<td>Frequency</td>
<td>[Hz]</td>
</tr>
<tr>
<td>$f_1$</td>
<td>Compressibility factor in Gor’kov force potential</td>
<td>[-]</td>
</tr>
<tr>
<td>$f_2$</td>
<td>Density factor in Gor’kov force potential</td>
<td>[-]</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method</td>
<td></td>
</tr>
<tr>
<td>$h_f$</td>
<td>Distance between fiber surface and wall</td>
<td>[m]</td>
</tr>
<tr>
<td>HMDS</td>
<td>Hexamethyldisilazane</td>
<td></td>
</tr>
<tr>
<td>$I$</td>
<td>Moment of inertia</td>
<td>[kg m²]</td>
</tr>
<tr>
<td>$i$</td>
<td>Unit imaginary number</td>
<td></td>
</tr>
<tr>
<td>$k$</td>
<td>Wavenumber</td>
<td>[rad/m]</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Compressibility</td>
<td>[Pa⁻¹]</td>
</tr>
<tr>
<td>$L$</td>
<td>General length of object or cavity</td>
<td>[m]</td>
</tr>
<tr>
<td>$l_f$</td>
<td>Fiber length</td>
<td>[m]</td>
</tr>
<tr>
<td>$l^*_f$</td>
<td>Projected length of the fiber in wave propagation direction</td>
<td>[m]</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wavelength</td>
<td>[m]</td>
</tr>
<tr>
<td>$n$</td>
<td>Normal vector</td>
<td>[-]</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>MEMS</td>
<td>Micro-electro-mechanical system</td>
<td></td>
</tr>
<tr>
<td>ν</td>
<td>Poisson’s ratio</td>
<td>[-]</td>
</tr>
<tr>
<td>ω</td>
<td>Angular frequency</td>
<td>[rad/s]</td>
</tr>
<tr>
<td>ωM</td>
<td>Modulation frequency</td>
<td>[rad/s]</td>
</tr>
<tr>
<td>Ω</td>
<td>Angular velocity of particle</td>
<td>[rad/s], [rpm]</td>
</tr>
<tr>
<td>p</td>
<td>Acoustic pressure</td>
<td>[Pa]</td>
</tr>
<tr>
<td>⟨p^2⟩</td>
<td>Time averaged and squared first order pressure</td>
<td>[Pa^2]</td>
</tr>
<tr>
<td>Pa</td>
<td>Pressure amplitude</td>
<td>[Pa]</td>
</tr>
<tr>
<td>φ</td>
<td>Velocity potential</td>
<td>[m^2/s]</td>
</tr>
<tr>
<td>Q</td>
<td>Q-factor (quality factor)</td>
<td>[-]</td>
</tr>
<tr>
<td>r</td>
<td>Position vector from center of mass to the surface position</td>
<td>[m]</td>
</tr>
<tr>
<td>rs</td>
<td>Radius of spherical particle</td>
<td>[m]</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number</td>
<td>[-]</td>
</tr>
<tr>
<td>ρ</td>
<td>Density</td>
<td>[kg/m^3]</td>
</tr>
<tr>
<td>Sa</td>
<td>Sphere surface</td>
<td>[m^2]</td>
</tr>
<tr>
<td>SAW</td>
<td>Surface acoustic waves</td>
<td></td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
<td>[s]</td>
</tr>
<tr>
<td>T^drag</td>
<td>Drag torque</td>
<td>[Nm]</td>
</tr>
<tr>
<td>T^rad</td>
<td>Acoustic radiation torque</td>
<td>[Nm]</td>
</tr>
<tr>
<td>T^vis</td>
<td>Acoustic viscous torque</td>
<td>[Nm]</td>
</tr>
<tr>
<td>T_M</td>
<td>Time for complete rotation of 360°</td>
<td>[s]</td>
</tr>
<tr>
<td>θ</td>
<td>Phase difference between two modes</td>
<td>[rad]</td>
</tr>
<tr>
<td>U^rad</td>
<td>Gor’kov force potential</td>
<td>[J]</td>
</tr>
<tr>
<td>v</td>
<td>Fluid velocity</td>
<td>[m/s]</td>
</tr>
<tr>
<td>⟨v^2⟩</td>
<td>Time averaged and squared first order velocity</td>
<td>[m^2/s^2]</td>
</tr>
<tr>
<td>V_rms</td>
<td>Root mean square of excitation voltage</td>
<td>[V]</td>
</tr>
<tr>
<td>X₀</td>
<td>Position of the fiber in the pressure field (x-direction)</td>
<td>[m]</td>
</tr>
<tr>
<td>Y</td>
<td>Admittance</td>
<td>[S]</td>
</tr>
<tr>
<td>Y₀</td>
<td>Position of the fiber in the pressure field (y-direction)</td>
<td>[m]</td>
</tr>
<tr>
<td>Y_st</td>
<td>Dimensionless radiation force function</td>
<td>[-]</td>
</tr>
<tr>
<td>⟨ ⟩</td>
<td>Time averaging</td>
<td></td>
</tr>
<tr>
<td>x, y, z</td>
<td>Spatial coordinate</td>
<td>[m]</td>
</tr>
</tbody>
</table>
1 Introduction

Ultrasonic manipulation of particles exploits the acoustic radiation force to provide a contactless handling method for particles suspended in a fluid. It is also referred to as acoustophoresis, acoustic trapping or more general acoustofluidics. Recently a tutorial series \[1\] was published, giving detailed fundamental insights into the theory of the acoustic radiation force, the acoustic streaming, the device design and a state of the art review of the various application fields and experimental works in the area of microfluidics.

The forces on particles in sound fields have been known for more than hundred years. In the middle of the 20th century the theoretical basis for the acoustophoresis has been developed. With the development of micro-fluidic-systems and the concept of lab-on-a-chip technology acoustophoresis gained a strong interest in the last two decades. Additionally, the computational power and software improved and allows the exploration of more complicated and complex problems such as arbitrary shaped objects \[2\].

Ultrasonic manipulation allows the handling of a wide variety of particles such as synthesized, functionalized solid micro beads, droplets, bubbles and biological cells. The important attribute is the material property difference (characteristic acoustic impedance) between the particle and the surrounding fluid to create a scatterer in the fluid. The typical frequency range of ultrasonic standing waves is in the 1 MHz range and μm sized particles are manipulated.

Different manipulation strategies and techniques have been accomplished such as particle separation, focusing, concentration, removal, trapping, mixing and moving \[2,5\]. These manipulation techniques are required for particle handling in lab-on-a-chip devices and micro-total analysis systems to realize different kinds of analysis steps.

1.1 Motivation

In an ultrasonic standing wave field, the viscous torque or the acoustic radiation torque induces rotation of an object. This offers in addition to the translation of particles due
to the acoustic radiation force a new controllable degree of freedom for the movement of particles. Therefore, the application field of the ultrasonic particle manipulation is extended. For other particle manipulation techniques such as dielectrophoresis, magnetic, or optical manipulation the rotation and alignment of objects has been intensively investigated. The implementations, experimental results and targeted applications are a motivation and inspiration. A state of the art for the different manipulation techniques and their applications is presented in Sec. 1.3.

There is a high demand for controlled alignment and deposition of non-spherical objects such as micro- and nanowires [6]. Moreover, the alignment of biological fibers such as collagen is of interest [7,8].

In lab-on-a-chip applications micro motors, stirrer or valves can be realized with ultrasonically-driven micro-machines. There is a growing interest for micro assembly techniques [9,10]. The manufacturing of complex 3D microsystems, the combination of different materials or the combination of micro parts where the manufacturing processes are not compatible depend on controllable positioning and orientation of objects. The ultrasonic manipulation is a useful tool as it allows the positioning of objects and provides levitation to overcome surface forces. It is a step towards micro robotics with acoustics or an element of microassembly.

In a previous project [11] at the ETH Zurich a microgripper was combined with an ultrasonic manipulator. The particles have been positioned with acoustic radiation forces and the gripper was used to remove single particles out of a fluid channel. The positioning of the particles in predictable locations allowed for the automatization of the gripper movement. A realization of the alignment of objects due to the acoustic radiation torque, will offer additionally the automated processing of non-spherical particles.

Moreover, the long term study of biological cells is of interest [12]. Ultrasonic particle manipulation is well known to be biocompatible [13]. The ability to control the orientation of a cell or cell cluster will help in the investigation of biological processes.

### 1.2 Rotational manipulation by acoustic fields

**Acoustic radiation torque**

A non-spherical particle in an ultrasonic standing wave is subjected to the acoustic radiation force and additionally to a torque. This leads to a change of the angular orientation of the particle. There are only few publications concerning experimental work with the
acoustic radiation torque. The focus there is on the development of composite materials with non-spherical particles and using ultrasonic standing waves for the arrangement and alignment.

Brodeur [14] studied 1991 the acoustic layering and reorientation as a function of fiber dimension with optical monitoring. For the experimental part, paper-making fibers suspended in water have been used. The fibers had a length between 0.2 and 3 mm and the excitation frequency was 72 kHz. Brodeur observed in experiments alternating regions of increased and decreased fiber concentration due to the acoustic radiation force and the reorientation of the fibers parallel to the formed layer planes. The layering and reorientation have different characteristic times. Brodeur verified experimentally that acoustic reorientation is a faster process than acoustic displacement. The cylinder first rotates quickly such that its principal axis becomes parallel to the wave front. The model is based on Wu [15] and Puttermann [16] and derives the migration velocity due to the acoustic radiation force and the drag force for a rigid cylinder oriented with its principle axis parallel to the wave front. The reorientation velocity is roughly approximated with scaling laws for the acoustic radiation torque and drag torque. The velocities derived for one cylinder have been used to describe the behavior of randomly distributed cylinders over time. The spatial and angular distribution functions have been compared and fitted to the experimental data. The measurement of scattered light during experiments allows to monitor the layering and reorientation effects. A full discrimination between these two effects from the monitored data was not possible for the used setup.

Saito et al. [17] studied in 1998 the fabrication of polymer composites by solidification of a particle suspension in ultrasonic standing waves. The motivation was to develop composite materials with periodic structures which are of interest due to anisotropy in mechanical, electrical and optical properties. The fabrication of two or three dimensional lattice structures was proposed. Experiments have been carried out with rod shaped particles such as glass rods with a diameter of 10 µm and the arrangement at the nodes with orientation of the axis along the nodal planes was observed. The excitation frequency was at 3 MHz. Polysiloxane resin was used as a medium for the solidification of the particle suspension. The ultrasonic actuation was continued for 8 h at a constant frequency of 8 MHz until the solution was solidified. There was no significant change in the sound velocity recognized and therefore the particle pattern stayed nearly constant.

Yamahira et al. [18] studied in 2000 the behavior of polystyrene fibers in a one-dimensional standing wave. The application is the directional control of reinforcing fibers in composite materials. The equations of motion for a fiber have been derived by evaluation of the force
and torque generated by the radiation pressure, the buoyancy force, and the drag force. In a simplified model the fiber was represented as a chain of small spheres. Experiments have been carried out with polystyrene fibers with a diameter of 0.5 mm and length between 5 and 20 mm at frequencies of 25 and 46 kHz. The motion of the fiber, as well as the influence of the length and position, was compared with the simplified model. Fibers shorter than a quarter wavelength were constrained at the pressure nodes and were oriented perpendicular to the wave propagation direction.

**Acoustic viscous torque**

The viscous torque is generated by two orthogonal standing waves shifted in phase and the resulting near boundary streaming inside the viscous boundary layer spins an axisymmetric object. This phenomenon has first been experimentally observed by Wang et al. [19] in 1977. A cylinder with diameter of 25.4 mm was supported with air bearings and passed through a box. The excitation was done with 2 orthogonal loudspeakers fixed at the side walls of the box with a frequency of 1.62 kHz. A rotational speed of 375 rpm has been measured in dependence of the phase difference between both excitations. Additionally the torque on the cylinder has been measured with a torsion fiber. A series of patents have been filed from 1975 [20] to 1989 [21]. Different systems are presented for a stable and controlled rotation and solutions for a very slow rotation to define the position of an object. It is mentioned that non spherical particles rotate slower due to the higher drag torque and need a large phase difference to induce rotation. This can be used to evaluate the sphericity of an object. Additionally it is mentioned that a non-spherical object can be reoriented in a range of 90° by adjusting the pressure ratio. The motivation for the rotation with the viscous torque was to perform a wide spectrum of experiments in space in a microgravity environment [19]. Applications of these experiments are the study of drop dynamics, processing in space and fusion.

**Acoustic vortex beam**

An acoustic vortex beam (Bessel vortex beam) carries orbital angular momentum along the propagation direction. This angular momentum can be transferred to a particle depending on the object scattering and absorbing properties [22]. This is analog to the transfer of optical orbital angular momentum. The vortex beam is characterized by a screw phase dislocation of the wave around its propagation axis. The first quantitative experimental data for a rotation was presented by Anhäuser et al. [23]. A disk made of
1.2. Rotational manipulation by acoustic fields

Sound absorbing material with a diameter of 3.15 mm and a thickness of 0.51 mm was suspended at the interface of two fluids (glycerol aqueous solution, silicone oil). The acoustic vortex beam was generated by a piezoelectric transducer with 8 independently actuated sectors with a phase delay between each sector and an actuation frequency of 2.25 MHz. More than 99% of the beam power was absorbed by the disk. A rotational speed of 60 rpm has been observed and the corresponding acoustic torque is $6.5 \times 10^{-9}$ Nm. The application field is the contactless, *in situ* rheology similar to the optical microrheology [24]. Other larger experimental setups consist of a 1000 element matrix piezoelectric transducer array [25] or an array of 4 or 8 loudspeakers and measurements of the torque on an absorbing disk suspended in air with a diameter of 6 to 16 cm [26].

**Surface acoustic wave**

Surface acoustic waves (SAW) are generated by interdigital transducers where a comb-shaped electrode array excites different kinds of surface acoustic waves due to the piezoelectric effect of the substrate. Various SAW motors have been developed for Lab-on-a-chip and micro-electro-mechanical systems (MEMS) applications [27,28]. A disc (5 mm in diameter) on a fluid film is rotated by inducing acoustic streaming in the fluid with SAW [29]. Rotational speeds up to 2500 rpm and a torque of $60 \times 10^{-9}$ Nm was realized. Another method rotates a disc (1 mm in diameter) with frictional forces due to the direct contact with the substrate [29]. A rotational speed of 6000 rpm and a torque of $4 \times 10^{-9}$ Nm has been achieved. SAWs have been used for the alignment of carbon nanotubes (CNT) [30,31]. No acoustic radiation forces have been involved in the alignment process. The induced acoustic streaming by SAWs and the electric fields from the SAW actuation apparently led to the alignment of the CNT.

**Acoustic needle**

An acoustic needle [32,33] was used to rotate trapped particles around its tip. The needle was surrounded by water or air and vibrating in a flexural mode. A revolution speed of 300 rpm was observed with seeds. It is assumed that an asymmetry in the sound field may cause an asymmetry of the four eddies around the vibrating needle and leads to the rotation of particles.
1.3 Other rotational manipulation techniques

In addition to ultrasonic manipulation there exist other particle manipulation techniques such as ac electrokinetics, magnetic, or optical manipulation with quite a number of publications dealing with the rotation of micro objects.

Electrokinetics

An introduction to electrokinetics can be found in [34–36]. Dielectrophoresis is the movement of an electrically polarized particle in a non-uniform ac electric field. The non-uniform fields are created by electrode patterns and the devices are fabricated with standard microelectronic and micro-system technology. The frequency range of the ac fields are 100 Hz to 100 MHz. The force acting on the particle depends on the gradient of the applied electric field, the particle size and a polarizability factor. The polarizability factor (Clausius-Mossotti factor) is a function of the complex permittivity of the particle and the suspending medium which are all functions of the frequency especially for biological samples. When a particle is more polarizable than the medium, the particle is attracted to high intensity electric field regions (positive DEP). If the particle is less polarizable than the medium, the particle is forced away from high intensity electric field regions (negative DEP).

To excite a torque, the particle has to be electrically lossy, non-spherical, or offer a permanent dipole moment [34]. For a lossy particle, a finite phase delay between the applied electric field and the establishment of the dipole moment exists. This leads to the rotation when the field vector is changing direction and the vector of the dipole moment tries to follow the change [35]. For the electrorotation, a rotating uniform electric field, created by electrode structures and corresponding multiphase ac voltage signals, leads to a rotation of the particle. The torque depends on the imaginary part of the polarizability factor. The particle will rotate with or in opposite direction of the electric field depending if the charge relaxation time constant is smaller or larger than that of the medium. A non-spherical particle can be aligned in a uniform electric field. The particle tends to align with one of its axes parallel to the electric field but only the longest axis parallel to the electric field is a stable position. For a lossy ellipsoid the alignment and stability position is a function of frequency [34].

A main application is the study and detection of biological samples. Different cell types, cells at various stages of maturation and cells exposed to toxic agents have a characteristic electrorotation signature [36]. This means that the polarizability factor as function of
frequency is strongly varying depending on cell type and cell condition. In \cite{37} it was shown with electrorotation that in a certain frequency range viable and nonviable cysts rotate in opposite directions or at different rotation rates. The presence of toxins can be detected with the selected behavior of micro-organisms under electrorotation. Other applications are the study of the torque generated by bacterial flagellar motors or the test of a toxic agent which reduces the operation of the flagellar motor and thereby increases the amount of rotating bacteria due to electrorotation.

The rotation of nanowires was realized by \cite{38} and possible applications are suggested such as a micromotor for microfluidic devices, a microstirrer, MEMS or for validation of the drag torque. For an Au nanowire with a length of 15 µm and a diameter of 300 nm, rotating around an axis perpendicular to the cylinder axis, a rotational speed of at least 1800 rpm was measured. The nanowire rotation can be instantly switched on and off with precisely controlled total angle of rotation due to the dominating drag force over the inertial force (very small Reynolds number). Furthermore, the alignment of nano- or micro-wires (carbon nanotubes, zinc oxide nanowires) is of interest to determine properties, construct electronic circuits, sensors and biological-electronic interfaces \cite{39,40}.

**Magnetic manipulation**

Magnetic fields can be used for the motion and rotation of particles. The particle or segments of the particle have to be magnetically coated and a rotating external magnetic field generates a driving torque. The rotation of a Ni nanowire with a length of 12 µm, a radius of 100 nm suspended in ethylene glycol ($\eta = 16 \times 10^{-3}$ Pa s) and an external magnetic field of $2.3 \times 10^{-4}$ T led to a maximal rotational speed of 100 rpm \cite{41}. A simple actuation can be a three coil system which is driven with a 120° phase shifted current and allows for flexible actuation from outside of the fluidic channel \cite{42}. This tool provides the possible implementation of a motor into a lab-on-a-chip system to realize a micro-stirrer, -pump or -valve \cite{43}. The magnetic manipulation offers an alignment technique for nano and micro particles and wires which can be useful for the evaluation of particles and wires and implementation of those into devices \cite{6}. Additionally, so called magnetic tweezers are able to apply and measure the torque at the molecular level, for example with a DNA molecule fixed to a magnetic bead and a glass wall \cite{44,45}.
Optical manipulation

Optical tweezers are used to manipulate particles by a gradient force due to the intensity gradient near the focus of a laser. The force arises for particles with different refractive indices to the surroundings and the particle will be confined in the beam focus [46]. Additionally there is a scattering force for example due to reflection of light and therefore a change in the momentum of the particle. A torque can be excited first by the angular momentum of the light and second by the shape of the object. Approaches to generate a torque are intensity shaped beams, circularly polarized beams or helically shaped objects [47]. With optical angular momentum transfer a particle (calcite fragment, 1 μm) rotation up to 350 Hz and therefore 21 000 rpm has been observed [48]. An optical torque of $2 \times 10^{-17}$ Nm was determined and it was claimed that a micron-sized element can be driven with an optical torque in the order of $10^{-15}$ Nm. Applications of the rotational control in optical tweezers are lab-on-a-chip integration as pumps and actuators (valve flaps, stirrer, gripper) [49]. A variety of optically driven micro machines have been presented such as micro gear systems [46]. The precise measurement of the fluid viscosity is possible with the drag torque of a sphere and the measured rotation rate [24]. The optical tweezer is used to generate a torque and simultaneously measure the rotation rate of the particle. Optical tweezers are used for the handling of biological samples and studying of biological macromolecules (DNA, proteins) by applying and measuring forces and torques. By attaching DNA molecules to a quartz cylinder, molecules can be stretched and twisted [50]. A dual optical tweezer was used to rotate chloroplasts inside a cell membrane to allow observation from different angles [51]. A different approach was to use the spinning of particles to induce motion and tumbling on chloroplasts inside a cell membrane.

1.4 Scope and content of this thesis

This study aimed at the development of different rotation techniques for spherical and non-spherical particles with ultrasonic standing waves. The existing theoretical work for the acoustic radiation torque is limited to simple cases of an elliptical cylinder in plane standing waves [52] or spheroids with a length much smaller compared to the wavelength [53]. Additionally, there exist simplified models where a non-spherical object is regarded as a chain of spheres [18]. The theoretical existing work is discussed in more detail in Sec. 2.3. In this study a finite element model was adapted from Philipp Hahn (IMES, ETH Zurich) and validated, which allows the modeling of arbitrarily shaped objects in arbitrary pressure fields. A study with a micro fiber was done to evaluate the influencing
parameters. The results allowed the estimation of the pressure in the experimental part or the prediction of the angular velocity of a particle.

The existing experimental work with the acoustic radiation torque is limited to the alignment of fibers in 1D standing waves. Currently to our knowledge, there exists no publication concerning the continuous rotation of objects with the acoustic radiation torque. This study focused on the development and experimental testing of different rotation techniques. The optimization of this rotation techniques concerning the magnitude of the torque or rotational speed was not part of the thesis. Four different rotation techniques will be presented and discussed in Sec. 3. The experimental work in this thesis is based on the study of Stefano Oberti \[54\] and Adrian Neild \[55\] on two dimensional pressure fields in micro devices.

Moreover, this study covers the rotational manipulation of spherical particles with viscous torque (see Sec. 4) in order to present a complete overview for the rotation in standing wave fields. The theoretical background for this experimental study was provided by \[56\] and by Andreas Lamprecht and Jingtao Wang (IMES, ETH Zurich) \[57\]. Existing experimental work with the viscous torque is limited to large cylinders in air \[19\]. The focus of this study was on the experimental investigation of the viscous torque effect on micro particles and the evaluation of the theoretical predictions.

**Chapter 2:** This chapter provides the theoretical framework for the rotational manipulation with ultrasonic standing waves. A short introduction to acoustic waves and resonances in fluidic cavities, the theory of the acoustic radiation force and torque is presented. A finite element model for the acoustic radiation torque on a micro fiber was developed and the influence of different parameters was evaluated. The rotational motion and influencing parameters are presented including the modeling of the drag torque. The phenomena of the acoustic viscous torque and the calculation of the resulting angular velocity for a sphere are presented.

**Chapter 3:** The topic of this chapter is the development and experimental investigation of different rotation methods for non-spherical particles with the acoustic radiation torque. All methods have been experimentally tested with a micro device. The device function, manufacturing and modeling is presented. Particle clumps and micro glass fibers have been used as rotation objects at excitation frequencies in the MHz range. The first rotation technique is based on the successive changing of the wave propagation direction with a hexagonal chamber. Three additional rotation methods were developed for a
continuous rotation. The common approach is the modulation of a single parameter such as amplitude, phase or frequency.

Chapter 4: The viscous torque was used to realize rotation of spherical micro particles and evaluate the theoretical predictions from the theory chapter. The excited pressure field, the corresponding force potential field and the macro device are presented followed by the experimental results and the evaluation of the theory.

Appendix A: The topic of this chapter is the result of a collaboration with the Institute of Microbiology at ETH Zurich. The method presented here exploits the advantage to simultaneously move bacteria away from a surface by means of acoustic radiation forces. A planar resonator was designed and a transfer-matrix-model was validated and used for optimization. The resonator has been experimentally tested with polystyrene particles and first preliminary tests with *Salmonella Thyphimurium* have been done.
# 2 Theory of the acoustic rotation of particles

If a propagating wave is scattered on an obstacle, the acoustic radiation force is induced. The force arises as a second order effect. This chapter gives an introduction to the theory of this effect. It is the basis for the development and design of the acoustophoresis devices and evaluation of the force and torque of arbitrary shaped objects. In Sec. 2.1 the wave equation and the governing equations of a simple acoustic resonator are discussed. The knowledge of the resonance mode in a fluidic cavity and the corresponding pressure distribution is used for the development of various rotation methods within this thesis. The theory of the acoustic radiation force, especially for the simple case of a spherical particle and a cylinder is the subject in Sec. 2.2. A non-spherical object experiences, in addition to the acoustic radiation force, a torque in an ultrasonic standing wave, this is the topic of Sec. 2.3. As the analytical solutions of the acoustic force and torque are limited to simple cases of object shape and acoustic field a finite element model (see Sec. 2.4) was developed and validated to calculate the acoustic radiation force and torque on a glass fiber such as used in the experimental part of this work. The model was used to evaluate the influence of the frequency, fiber size, position and orientation of the fiber in 1D and 2D standing wave fields. The rotational motion of a non-spherical particle and the resulting drag torque is the topic of Sec. 2.5. A finite element simulation for the drag torque of a glass fiber is presented. The influence of the fiber size, angular velocity and the proximity of a wall are evaluated. The influence of the acoustic radiation torque and drag torque on the angular velocity for a fiber with different parameters is discussed as well. The acoustic viscous torque is a time-averaged effect which leads to the rotation of axisymmetric objects. An induced streaming inside the viscous boundary layer is spinning the object. The calculation of the viscous torque and the resulting angular velocity is topic of Sec. 2.6.
Chapter 2. Theory of the acoustic rotation of particles

2.1 Acoustic waves and fluidic cavity resonances

The linear wave equation is the basis for the calculation of resonance modes in fluidic chambers used for acoustophoresis. An introduction to the perturbation theory and the derivation of the linear wave equation is presented in detail by Bruus [58][59]. It is shortly presented here to derive the resonance frequency of an acoustic cavity, which is later used to design devices and describe different methods for the rotational manipulation of particles.

The wave equation is derived from the combination of the equation of state, the continuity equation and the Navier-Stokes equation. The analytical solution of a set of coupled, non-linear, partial differential equations is only possible for simplified problems. Therefore the perturbation theory is applied [59]. In perturbation theory the important variables for an acoustic problem, the pressure $p$, the density $\rho$ and the velocity field $v$ are considered as small perturbations of the initial state $p_0$ and $\rho_0$:

\[
p = p_0 + p_1 + ...
\]
\[
\rho = \rho_0 + \rho_1 + ...
\]
\[
v = 0 + v_1 + ...
\]

where the subscript 1 represent the first order perturbation. For first order perturbation and neglecting the viscosity the continuity equation and the Navier-Stokes equation become:

\[
\frac{\partial \rho_1}{\partial t} = -\rho_0 \nabla \cdot v_1 \tag{2.2}
\]
\[
\rho_0 \frac{\partial v_1}{\partial t} = -\nabla p_1 \tag{2.3}
\]

From the isentropic expansion of the equation of state $p(\rho) = p_0 + (\partial p / \partial \rho)_{is} \rho_1$ follows $p_1 = c^2 \rho_1$ where $c$ is the speed of sound in the liquid [58]. The time derivative of Eq. (2.2) and the insertion of Eq. (2.3) gives:

\[
\frac{\partial^2 p_1}{\partial t^2} = c^2 \nabla^2 p_1 \tag{2.4}
\]

The acoustic field variables (pressure $p$, density $\rho$, velocity $v$) are assumed as harmonic time dependent. This is expressed by the complex phase $e^{-i\omega t}$, where $\omega = 2\pi f$ is the
angular frequency and \( f \) the frequency. Then Eq. (2.4) can be expressed as the Helmholtz equation:

\[
\nabla^2 p_1 + k^2 p_1 = 0
\] (2.5)

where \( k = \omega/c \) is the wavenumber. When viscosity is neglected (\( \eta = 0 \)) the velocity \( \mathbf{v} \) can be expressed as the gradient of a potential \( \phi \). This can be shown by inserting the harmonic time dependency of \( \mathbf{v}_1 \) into Eq. (2.3) \[58\]:

\[
\mathbf{v}_1 = -\frac{i}{\rho_0 \omega} \nabla p_1 = \nabla \phi_1
\] (2.6)

The pressure perturbation \( p_1 \) can now be used to calculate the density \( \rho_1 \) and velocity \( \mathbf{v}_1 \) via the velocity potential \( \phi_1 \). The pressure itself can be determined by the Helmholtz equation and the corresponding boundary conditions of a certain acoustic problem. In typical acoustophoresis applications the fluid cavity is driven at its resonance frequency. A piezoelectric element is used for the excitation and is therefore attached to the carrier layer surrounding the fluidic cavity. The vibration is coupled from the piezoelectric element through the carrier layer into the fluidic cavity. The different resonances of the fluid cavity can be calculated directly with the Helmholtz equation and the appropriate boundary condition. The surrounding of the fluid cavity in a typical device is made of silicon or steel which has a much higher characteristic acoustic impedance as compared to water. For a first rough approximation, the walls of the cavity can be treated as completely rigid and the boundary condition is:

\[
\frac{\partial p}{\partial n} = 0
\] (2.7)

where \( n \) is the normal to the surface of a wall. The solution of the Helmholtz equation with the boundary condition from Eq. (2.7) for a rectangular cuboid has the following form \[58\]:

\[
p(x, y, z, t) = A \cos (k_x x) \cos (k_y y) \cos (k_z z) \sin (\omega t)
\] (2.8)

\[
k_x = \frac{n_x \pi}{L_x}, \quad k_y = \frac{n_y \pi}{L_y}, \quad k_z = \frac{n_z \pi}{L_z}
\]

where \( k_x, k_y, k_z \) are the wavenumbers for each of the three spatial directions and \( n_x, n_y, n_z = 0, 1, 2, 3, \ldots \), are the numbers of half wavelength along the \( x-, y-, z- \)direction, respectively. The dimensions of the cavity are \( L_x, L_y, L_z \). The wavenumbers are related by \( k^2 = k_x^2 + k_y^2 + k_z^2 \) and the resonance frequency \( f \) is:

\[
f = \frac{c}{2\pi} \sqrt{k_x^2 + k_y^2 + k_z^2}.
\] (2.9)
Chapter 2. Theory of the acoustic rotation of particles

The knowledge of the resonance modes and their pressure distributions in the cavity can be used for the calculation of the acoustic radiation force field and for the development of different ultrasonic particle manipulation techniques such as the formation of lines, clumps or rotation.

2.2 Acoustic radiation force

The acoustic radiation force arises when a wave is scattered by an object. The shape of the object and the material properties in comparison to the ones of the surrounding medium are important as well as the properties of the acoustic field such as pressure amplitude and frequency. The acoustic radiation force can be easily observed experimentally because it is a time averaged effect. The timescale is in the range of ms to s, compared to the acoustic field with a frequency $f$ in the MHz range and therefore a timescale of $\mu$s. This effect occurs for a traveling wave and standing waves, whereas for the standing wave the force is higher.

A complete history of the publications concerning the acoustic radiation force is given in [2, 60]. Relevant for this thesis are the derivation of the acoustic radiation force on a compressible spherical particle in a plane acoustic wave, published by Yosioka and Kawasima [61] in 1955. This theory was extended to the case of a small particle suspended in an inviscid fluid for arbitrary waves by Gor’kov [62] in 1962. A complete derivation of the acoustic radiation force based on Gor’kov is presented by Bruus [60] and was extended to the case of a slightly viscous fluid [63]. The basis of the derivation is the perturbation theory.

The acoustic radiation force is a time averaged effect. The perturbations of Eq. (2.1) have to be expanded up to second order [59]. In the linear theory the term $\langle \sin(\omega t) \rangle$ would always lead to a zero time average over a complete oscillation cycle. Therefore the non-linear terms of the Navier-Stokes and the continuity equation are necessary to get products of two first-order factors which will lead to a non zero time average as $\langle \sin^2(\omega t) \rangle = 1/2$. The incoming and the scattered velocity potential field have to be determined. The acoustic radiation force $F^{\text{rad}}$ on an object can be calculated as the surface integral of the time averaged second-order pressure and the momentum flux tensor at the object surface $S_0$ [60]:
2.2. Acoustic radiation force

\[
F_{\text{rad}} = - \int_{S_0} \left[ \langle p_2 \rangle \mathbf{n} + \rho_0 \langle (\mathbf{n} \cdot \mathbf{v}_1) \mathbf{v}_1 \rangle \right] dS
\]

\[
= - \int_{S_0} \left[ \left( \frac{1}{2} \frac{1}{\rho_0 c_0^2} \langle p_1^2 \rangle - \frac{1}{2} \rho_0 \langle v_1^2 \rangle \right) \mathbf{n} + \rho_0 \langle (\mathbf{n} \cdot \mathbf{v}_1) \mathbf{v}_1 \rangle \right] dS
\]

This formula is suitable for an arbitrary shaped particle and will be used for the calculation of the force on a non-spherical particle with the finite element method described in Sec. 2.4.

The analytical calculation of the scattered field is in general very complicated. Gor’kov \cite{62} derived the radiation force on a small compressible sphere. The first-order pressure and velocity of the incoming acoustic field at the position where the particle is located are sufficient for the calculation. This fact yields a very useful and simple equation which helps to understand the characteristic and influence parameters of the acoustic radiation force. The theory is valid for a compressible spherical particle suspended in an infinite, inviscid fluid and exposed to an arbitrary pressure field. Additionally the following condition has to be fulfilled: \( r_s << \lambda \), where \( r_s \) is the particle radius and \( \lambda \) the wavelength of the acoustic field. The acoustic radiation force \( F_{\text{rad}} \) can be expressed as the gradient of the force potential field \( U_{\text{rad}} \):

\[
F_{\text{rad}} = - \nabla U_{\text{rad}}
\]

\[
U_{\text{rad}} = \frac{4}{3} \pi r_s^3 \left[ f_1 \frac{1}{2} \kappa_0 \langle p_1^2 \rangle - f_2 \frac{3}{4} \rho_0 \langle v_1^2 \rangle \right]
\]

with \( f_1 = 1 - \kappa_s / \kappa_0 \) and \( f_2 = 2 (\rho_s - \rho_0) / (2 \rho_s + \rho_0) \), where \( \rho_0 \) and \( \kappa_0 \) denote the density and the compressibility of the fluid, respectively, and \( \rho_s \) and \( \kappa_s \) the density and compressibility of the particle, respectively \cite{60}.

For the case of a one dimensional pressure field, Eq. \((2.11)\) was simplified by Bruus \cite{60} and led to the result of Yosioka and Kawasima \cite{61}. The pressure field for a 1D standing wave in x-direction is:

\[
p = P_a \cos (k_0 x) \sin (\omega t)
\]

where \( P_a \) is the time invariant pressure amplitude and \( k_0 \) the wavenumber. The acoustic radiation force is:

\[
F_{\text{rad}}^x = 4 \pi \left[ \frac{1}{3} f_1 + \frac{1}{2} f_2 \right] k_0 r_s^3 \frac{P_a^2}{4 \rho_0 c_0^2} \sin (2k_0 x)
\]

\[
= \pi r_s^3 P_a^2 k_0 \left[ \frac{1}{3} \left( 1 - \frac{\kappa_s}{\kappa_0} \right) + \frac{(\rho_s - \rho_0)}{(2 \rho_s + \rho_0)} \right] \sin (2k_0 x)
\]

15
The term in the square brackets determines the direction of the force and therefore the aggregation position of a particle. Particles with a higher density and speed of sound as compared to the surrounding medium will be forced to the pressure nodes. This is valid for copolymer or glass particles like the ones used in the experimental part of this thesis.

Besides spherical particles the acoustic radiation force on cylindrical objects is of interest for this thesis as most experiments have been performed with glass fibers which could be approximated as cylinders with a small diameter. Wu [15] derived the acoustic radiation force on a rigid long circular cylinder in a plane standing wave field. The diameter of the cylinder has to be small compared to the wavelength and the axis of the cylinder has to be perpendicular to the direction of wave propagation. Experiments confirmed the derived expression with good agreement. Haydock [64] derived the radiation force for a circular cylinder which is free to move in the acoustic field for an inviscid fluid. Haydock claimed to have a more accurate solution, which can easily be evaluated with a standard numerical software. Wei [65] derived the radiation force for a compressible circular cylinder in a standing wave. For a 1D standing wave in x-direction the radiation force per unit length is:

\[
\frac{F_{x}^{\text{rad}}}{L} = \frac{R}{4} P_2 \kappa_0 \sin (2k_0 x) Y_{st} \\
Y_{st} = \pi k_0 R \left[ 1 - \frac{\kappa_0}{\kappa_s} \right] + 2 \left( \frac{\rho_s - \rho_0}{\rho_s + \rho_0} \right)
\]

where \( R, \rho_s \) and \( \kappa_s \) denote radius, density and compressibility of the cylinder, respectively, \( \rho_0 \) and \( \kappa_0 \) the density and the compressibility of the fluid, respectively. \( Y_{st} \) is the dimensionless radiation force function which has been derived in [65] considering only monopole and dipole scattering terms. \( F_{x}^{\text{rad}} \) will always be directed towards a pressure node if \( Y_{st} \) is positive. A cylinder with a higher density and speed of sound as compared to the surrounding medium will lead to a positive \( Y_{st} \). The glass fibers which have been used within this thesis have a positive \( Y_{st} \) and will therefore be forced to the pressure node if the standing wave is perpendicular to the fiber axis. No simple analytical solution exists for arbitrarily oriented fibers. To get the force on an arbitrarily oriented and shaped object a numerical simulation has to be done which is covered in Sec. 2.4.

Beside the acoustic radiation force also referred to as primary force, there exist the secondary forces. This topic is discussed in more detail by Haake [66]. The Bjerknes force is acting between two compressible particles with similar material properties and causes a merging of the particles to a clump. The Bjerknes force is small compared to the maximum primary force, but at the nodal pressure plane where the primary force vanishes
2.3. Acoustic radiation torque

The Bjerknes force might become dominant and leads to the formation of particle clumps. The secondary forces are neglected throughout this thesis even though it is present in the formation and rotation of particle clumps.

2.3 Acoustic radiation torque

A spherical particle experiences an acoustic radiation force in an ultrasonic standing wave. A non-spherical particle is subjected additionally to an acoustic radiation torque. This leads to a change of the angular orientation of the particle. The alignment of fibers by ultrasonic standing waves using the acoustic radiation torque was experimentally examined by Ref. [14, 17, 18]. Brodeur [14] noticed that fibers shorter than one-fourth of the acoustic wavelength are migrating toward stable equilibrium position and reorient to stable equilibrium angular positions. Beside the alignment, the torque can be used for a continuous rotation of objects. Different approaches to realize a rotation were developed and are discussed within this thesis in Chap. [3]. Here the theory of the acoustic radiation torque is discussed.

The acoustic radiation torque is a nonlinear acoustic effect caused by the angular momentum transfer from an acoustic field to a scatterer [53]. For the calculation of the torque the incoming and the scattered velocity potential field have to be determined. The acoustic radiation torque \( T_{\text{rad}} \) on an object can be calculated as the surface integral of the moment of time averaged second-order pressure and the moment of momentum flux tensor at the object surface \( S_0 \):

\[
T_{\text{rad}} = -\int_{S_0} \mathbf{r} \times \left[ \frac{1}{2} \frac{1}{\rho_0 c_0^2} \left\langle p_1^2 \right\rangle - \frac{1}{2} \rho_0 \left\langle \mathbf{v}_1^2 \right\rangle \right] n + \rho_0 \left\langle (\mathbf{n} \cdot \mathbf{v}_1) \mathbf{v}_1 \right\rangle \, dS \tag{2.15}
\]

where \( \mathbf{r} \) is the position vector from the center of mass to the surface position of the scatterer. The main challenge is the calculation of the scattering field. This can be done numerically with the finite element method and will be discussed in the next section. An analytical solution comparable to the practical Gor’kov theory, where the first order pressure and velocity of the incoming acoustic field are sufficient, does not exist for the torque.

Maidanik [67] published in 1958 a theory about the torque due to acoustical radiation pressure. He used the theory of the force due to acoustic radiation pressure which relies on the linear momentum theorem and extended it to the torque by making use of the angular momentum theorem. The general expression was used to derive the torque on a plane...
disk of arbitrary shape excited by a plane progressive wave. Fan et al. \cite{53} derived the acoustic radiation torque on an irregularly shaped scatterer for an arbitrary sound field. The calculation is limited to a simple geometry and simple sound field, e.g., a spheroid in a plane standing-wave field. An analytical result for objects with a high aspect ratio like a fiber (long cylinder) is not possible. Additionally the scatterer has to be small compared to the wavelength of the sound field. The general theory \cite{53,67} is restricted to the case of objects immersed in an inviscid fluid. The approximation is useful when the streaming is weak and the viscous boundary layer is small compared to the object size \cite{68}.

The acoustic radiation force on rigid cylinders in plane progressive waves is studied quite often, for the case where no torque is excited as the wave propagation direction is perpendicular to the cylinder axis. Hasheminejad \cite{52} claimed that there exist no analytical or numerical simulation dealing with the acoustic radiation torque on solid elliptical cylinders. Hasheminejad \cite{52} derived an exact expression for the acoustic radiation torque and force on an elastic cylinder with elliptic cross section. The cylinder is immersed in an ideal fluid and exposed to a standing wave field. An analytical expression with infinite series of Mathieu functions was developed. The influence of the ellipticity and the angle of wave incidence has been investigated on a stainless steel cylinder immersed in water. Wang \cite{69} derived also an analytical solution for rigid cylinders with elliptical cross section based on Mathieu functions and compared the results with a numerical simulation based on the finite volume method. This results have been used later to validate the finite element simulation in Sec. 2.4.

Brodeur \cite{14} derived for his study of paper fibers a rough estimation for the torque on a cylinder in a 1D standing wave. Based on Putterman et al. \cite{16} the torque for a non-spherical object in a standing wave should be proportional to its volume and the mean acoustic energy density. Another rough estimation for the torque on a fiber is given by Yamahira et al. \cite{18}. The fiber was there represented as a chain of small spheres and the expression of the acoustic radiation pressure on a small sphere from Yosioka and Kawasima \cite{61} was applied.

The model has been adapted here and expanded to two-dimensional acoustic fields for a better understanding of the simulation results in Sec. 2.4 and to predict the equilibrium position of elongated particle clumps in arbitrary standing wave fields. The configuration can be seen in Fig. 2.1(a). The chain consists of \( n \) identical spheres with the radius \( r_s \) which gives a fiber length \( l = 2nr_s \). The middle of the chain C gives the position of the fiber in the standing wave with \( X_0 \) and \( Y_0 \). The forces \( F_x^{\text{rad}} \) and \( F_y^{\text{rad}} \) on each sphere are calculated with the Gor’kov potential (Eq. 2.11), therefore the position of each sphere
2.3. Acoustic radiation torque

\textbf{Figure 2.1:} (a) Model for representing the fiber as a chain of spheres for the calculation of the resulting torque $T_{C}^{\text{rad}}$ and the prediction of the general behavior in an arbitrary standing wave field. (b) Model for the calculation of the resulting torque $T_{C}^{\text{rad}}$ with only two spheres separated by a certain distance.

and the acoustic pressure field have to be known. The position of the center of each sphere is given by:

\begin{align*}
  x_i &= X_0 + \left( \frac{n + 1}{2} - i \right) 2r_s \cos(\alpha) \\
  y_i &= Y_0 + \left( \frac{n + 1}{2} - i \right) 2r_s \sin(\alpha) 
\end{align*}

(2.16)

where $i$ is the index of the sphere. The torque is the sum of the forces times the corresponding lever arm.

\begin{equation}
  T_{C}^{\text{rad}} = \sum_{i=1}^{n} T_{i}^{\text{rad}} = -(y_i - Y_0)F_{x_i}^{\text{rad}} + (x_i - X_0)F_{y_i}^{\text{rad}}
\end{equation}

(2.17)

The model can be even more simplified with only two connected spheres, which can be seen in Fig. 2.1(b). The overall length of the object is $l$. For the calculation of the sphere forces with the Gor’kov potential the positions of the spheres are needed.

\begin{align*}
  x_1 &= X_0 + \left( \frac{l}{2} - r_s \right) \cos(\alpha) & y_1 &= Y_0 + \left( \frac{l}{2} - r_s \right) \sin(\alpha) \\
  x_2 &= X_0 - \left( \frac{l}{2} - r_s \right) \cos(\alpha) & y_2 &= Y_0 - \left( \frac{l}{2} - r_s \right) \sin(\alpha)
\end{align*}

(2.18)

The torque $T_{C}^{\text{rad}}$ at the point $C$ can be calculated with Eq. (2.17).
The general behavior of a fiber in a standing wave field can be roughly described with these simple approximations but the accurate calculation of the torque is not possible. The applied Gor’kov theory is only valid for a single spherical particle and for the long wavelength limit. The interaction between neighboring spheres is not considered and a fiber will have a different scattering field compared to a single sphere.

2.4 Numerical simulation of the acoustic radiation force and torque

The previous sections about the acoustic radiation force (Sec. 2.2) and torque (Sec. 2.3) have shown that analytical solutions are restricted to simple cases. The size of the scatterer has to be small compared to the wavelength and the shape is limited to spherical objects or circular cylinders. For the cylindrical objects the propagation direction of the incident acoustic wave is limited to simple cases. This is the motivation for a numerical simulation of the acoustic radiation force and torque on a non-spherical object such as a fiber. The finite element model that is presented was mainly developed by Philipp Hahn (IMES, ETH Zurich) for the simulation of hollow spherical particles and adapted here for the modeling of a micro-fiber. In general any kind of particle with no restrictions concerning the size, shape and material properties can be modeled at any position and orientation in an arbitrary acoustic field. The viscosity of the fluid is neglected which is valid when the viscous boundary layer is small compared to the object size \[68\]. Also the influence of a nearby wall or another particle can be considered when the viscous boundary layer is small compared to the distance.

2.4.1 Finite element model

For the finite element simulation COMSOL Multiphysics 4.2 has been used. This program allows the connection between different physical modules and the full coupling through boundary conditions in order to model the interaction between the modules e.g. fluid structure interaction. The simulation of the force and torque on a particle is done with the pressure acoustics “acpr” and the solid mechanics “solid” module. 2D as well as 3D simulations have been performed. The 2D simulations are preferred when an extremely fine mesh is required e.g. for high aspect ratios of the scattering object in order to keep the computational time reasonable. Also the 2D case is preferred when the simulation is done for a large parameter sweep e.g. different positions and orientations of the scattering
2.4. Numerical simulation of the acoustic radiation force and torque

object in the acoustic wave field. The following paragraph refers to a 2D problem but
can easily be extended to a 3D simulation. The geometry of the model can be seen in
Fig. 2.2(a). The scattering object (e.g. fiber) is placed in the middle of the model with its
centroid at the origin of the coordinate system and has a certain angular position \( \alpha \). The
length of the fiber is \( l_f \) and the diameter is \( d_f \). The fiber was assumed to have spherical
ends in order to avoid sharp edges where a very high discretization would be necessary.

![Figure 2.2:](a) Geometry and domain classification of the 2D model for the simulation of the
force and torque on a fiber. (b) Triangular element mesh with very fine spatial discretization
near the fiber surface. (c) A one dimensional background pressure field \( p_b \) with variation in
\( x \)-direction, where the fiber is placed in the pressure node.

The fiber is modeled as an elastic solid using the solid mechanics module. In the solid
domain the dependent variable is the displacement field \( u \) and the wave equation for a
linear elastic continuum is solved for the given material parameters and boundary condi-
tions which is more complex compared to a fluid due to shear stresses \[70\]. The following
material parameters have been used: The solid glass fiber \[71\] has a Young’s modulus of
73 GPa, a Poisson’s ratio of 0.18 and a density of 2600 kg/m\(^3\). Damping of the fiber was
considered with a complex Young’s modulus \[72\] of 73 (1 + i/400) GPa to reduce ampli-
tudes at fiber resonances. The boundary condition on the surface of the fiber is a load
from the fluid and it is implemented with a boundary load as acoustic load per unit area
denoted by “acpr/pam1” in COMSOL. This is equal to \(-p n\), where \( n \) is the outward
pointing normal vector of the fiber surface and \( p \) the pressure from the acoustic domain.

The surrounding of the fiber (the fluid) is modeled with the pressure acoustics module.
The dependent variable in the acoustic domain is the pressure \( p \) and the Helmholtz wave
equation (Eq. (2.5)) is solved for the given material parameters and boundary conditions.
The size of the domain is mainly specified by the fiber size. It is independent of the
wavelength of the acoustic field. Usually the length of the fluid domain was twice the
fiber length. The required material parameters for water are the density (998 kg/m\(^3\)) and
the speed of sound (1481 m/s). The interaction from the solid to the fluid is modeled as an inward normal acceleration $a_n$ (acceleration denoted by “solid/lemm1” in COMSOL). The outer boundary of the fluid is a perfectly matched layer (PML) which avoids any reflections of the scattered wave back into the acoustic domain and absorbs the wave. An important part is the excitation and the definition of the acoustic field. A background pressure field $p_b$ is therefore very convenient as it allows the creation of arbitrary acoustic fields and defines also the position of the fiber inside the acoustic field. The background pressure field was defined as:

$$p_b = P_{ax} \cos \left( \frac{2\pi f}{c_0} x + k_0 X_0 \right) + P_{ay} \cos \left( \frac{2\pi f}{c_0} y + k_0 Y_0 \right) e^{i\Delta \varphi}$$

(2.19)

where $f$ is the excitation frequency, $P_{ax}$ and $P_{ay}$ are the maximum pressure amplitude for the independent standing waves in $x$- and $y$-direction, respectively. $X_0$ and $Y_0$ describe the position of the fiber in the pressure field. For a one dimensional standing wave in $x$-direction the fiber would be in a pressure node for $k_0 X_0 = \{ \frac{1}{2} \pi; \frac{3}{2} \pi \}$ and in a pressure antinode for $k_0 X_0 = \{ 0; \pi \}$. The additional term $e^{i\Delta \varphi}$ defines a phase shift of the standing wave in $y$-direction in reference to the standing wave in $x$-direction. The harmonic time dependence $e^{i\omega t}$ is omitted. A one dimensional background pressure field in $x$-direction, where the fiber is placed in the pressure node, is depicted in Fig. 2.2(c).

The mesh consists of triangular elements. The element size is determined by the wavelength in the fluid, therefore at least 10 elements per wavelength should be used. Especially important is the very fine meshing of the fiber surface to map the geometry accurately, as the solution will be integrated at this surface. A possible mesh and the decrease of the element size near the fiber is shown in Fig. 2.2(b). A mesh convergence test has been performed to ensure that the spatial discretization especially at the fiber surface is sufficient. For the 2D model, 100000 to 300000 elements have been used, depending on the length and aspect ratio of the fiber. A time harmonic analysis for a certain frequency $f$ was performed using the PARDISO solver. For the calculation of the force and torque the values of the pressure $p$ and velocity $v$ are needed to perform the integration along the surface. The equations for the acoustic radiation force (Eq. (2.10)) and the acoustic radiation torque (Eq. (2.15)) can be implemented in COMSOL and a line integration for the 2D case and a surface integration for the 3D case is done. The equations transformed into the COMSOL notation can be found in Appendix B.

The validation of the model was done with a simpler problem for which an analytical solution exists. The torque on a rigid ellipse fixed in the pressure node of a one-dimensional standing wave for different frequencies was calculated. The maximum pressure amplitude
$P_a$ was set to $1 \times 10^5$ Pa. The length of the semi-major axis $a$ is 100 µm and that of the semi-minor axis $b$ is 20 µm. The angular position of the ellipse is $\alpha = 45^\circ$ which leads to a maximal torque. Jingtao Wang (IMES, ETH Zurich) calculated with MATHEMATICA the analytical solution for the rigid ellipse based on Mathieu functions [69]. Due to the rigid ellipse only the pressure acoustics module was needed for the simulation. The boundary condition at the ellipse outline was set to a hard-wall boundary condition. The results of the torque per unit length plotted as function of the frequency can be seen in Fig. 2.3.

![Figure 2.3: Validation of the COMSOL simulation with an analytical solution for the acoustic radiation torque $2D T_{rad}$ (torque per unit length) on a rigid ellipse with $a = 100\mu m$, $b = 20\mu m$ and $\alpha = 45^\circ$. The ellipse is surrounded by water and is located in the pressure node of a one-dimensional standing wave in $x$-direction with a maximum pressure amplitude $P_a$ of $1 \times 10^5$ Pa.](image)

The agreement between the analytical solution and the simulation is perfect for a frequency range from 10 kHz to 20 MHz, which covers the relevant frequency range for the experimental work within this thesis. The percent error is below 0.01%. For frequencies higher than 20 MHz the deviation is strongly increasing. The discrepancy at high frequencies derives probably from the analytical results. For a higher frequencies, more terms should be kept in the solution.

In order to show that the 2D simulation represents the 3D case, the results of both models have been compared. The acoustic radiation torque on a solid glass fiber with a length $l_f = 200 \mu m$ and a diameter $d_f = 15 \mu m$ for an actuation frequency $f = 1$ MHz have been simulated. For the 3D simulation a torque of $T_{rad}^z = 1.33 \times 10^{-14}$ Nm was determined. The 2D simulation led to a torque per unit depth of $2D T_{rad}^z = 3.00 \times 10^{-9}$ N which leads to a torque of $T_{rad}^z = 4.5 \times 10^{-14}$ Nm, when multiplied with the fiber diameter $d_f = 15 \mu m$. The torque for the 2D case is larger by a factor of 3.4. The characteristic of the torque
as function of frequency, fiber length and diameter was determined for the 2D and the 3D simulation. The results agreed very well and the deviation was mostly constant with a factor of 3.4.

2.4.2 Results of the acoustic radiation force and torque for a micro fiber

The FEM simulation has been used to evaluate the behavior of a non-spherical particle in particular a glass fiber as it was used in the experimental parts of this thesis. The influence of the following points on the acoustic force and torque have been investigated:

- Frequency $f$
- Angular position $\alpha$
- Fiber length $l_f$ and diameter $d_f$
- Pressure amplitude $P_a$
- Fiber position and orientation in a 1D standing wave
- Fiber position and orientation in a 2D standing wave

A basic model was build for a 3D simulation of a solid glass fiber with the following parameters: fiber length $l_f = 200 \mu m$, diameter $d_f = 15 \mu m$, Young’s modulus $E_f = 73 (1 + i/400) \text{ GPa}$, Poisson’s ratio $\nu_f = 0.18$ and density $\rho_f = 2600 \text{ kg/m}^3$. The ends of the fiber are spherical. The fiber is surrounded by water ($\rho_0 = 998 \text{ kg/m}^3$, $c_0 = 1481 \text{ m/s}$) and fixed with an angle $\alpha = 45^\circ$ in the pressure node of a standing wave in $x$-direction with an amplitude $P_a$ of $1 \times 10^5 \text{ Pa}$. The orientation of the nodal pressure line $\beta$ is $90^\circ$ related to the $x$-axis. The frequency is $f = 1 \text{ MHz}$ and the corresponding wavelength in the fluid is $\lambda = 1481 \mu m$. The origin of the coordinate system is placed in the center of the fiber. The definition of the angles and coordinates can be seen in Fig. 2.4(a). Various parameters have been varied and the force and torque were analyzed. The unchanged parameters stayed fixed like for the described basic model.

**Frequency**

The influence of the frequency on the torque will be discussed first. The torque on a rigid ellipse as a function of the frequency can be seen in Fig. 2.3. This simulation was used for the validation of the FEM model. The characteristic behavior of the torque on a non-
2.4. Numerical simulation of the acoustic radiation force and torque

spherical particle can be already seen there. For low frequencies where the wavelength is
much larger than the length of the ellipse the torque is nearly constant. At a frequency
where the ellipse length is in the range between a quarter and half a wavelength the
torque reaches a maximum. For higher frequencies the torque decreases and gets zero
for a certain frequency were the length of the ellipse is larger as half a wavelength. The
torque reaches even negative values for higher frequencies and is alternating at a mean
value close to zero for the frequency range where multiple wavelengths cover the ellipse
length.

The basic model for the 3D simulation of an elastic glass fiber has then been used and
the frequency range was varied between 10 kHz and 10 MHz. The configuration and
results can be seen in Fig. 2.4. In the frequency range below 500 kHz, where the wave-
length is at least 10 times larger than the fiber length, the torque stays nearly con-
stant (see Fig. 2.4(c)). It is only slightly increasing between 10 kHz and 500 kHz from
9.7 \times 10^{-15} \text{ N m} to 10.6 \times 10^{-15} \text{ N m}. This characteristic is unexpected as the acoustic
radiation force on a compressible circular cylinder is proportional to the frequency as can
be seen in Eq. (2.14) for the long wavelength approximation and is hereinafter shown in
Fig. 2.6. A strong simplification of a fiber to a chain of spheres or two spheres connected
by a rigid infinite thin bar with a certain length \( l_f \) also shows a decreasing torque when the
frequency is decreased. A similar frequency dependency was expected for the torque at a
glass fiber. No reference was found in the literature but the analytical model for a rigid
ellipse showed the same characteristic (see Fig. 2.3). Experimental investigations within
this thesis or in the literature are not available. The investigation of particle trajectories
at various frequency ranges might be of interest for future work. Brodeur [14] showed that
the reorientation is faster than the displacement of a fiber at a frequency of 72 kHz. As
the acoustic radiation force decreases with decreasing frequency the dynamic behavior of
a fiber in the kHz range might be different from the MHz range concerning the timescale
of the translation and reorientation.

The acoustic radiation torque \( T_{\text{rad}}^z \) as function of the frequency was simulated for the same
fiber and only the density \( \rho_f \) of the fiber was varied. The result is plotted in Fig. 2.5. A
fiber with an equal density to the surrounding water showed not a constant torque in the
kHz range. The torque is even increasing quadratic with the frequency as the fitted curve
is showing. This is in contrast to the glass fiber or a rigid fiber (infinite density). The
effect of an increase of the fiber density of 1% and 10% in reference to the water density
is plotted in Fig. 2.5 as well. The torque is constant at low frequencies and is approaching
the quadratic increase with frequency from the fiber with equal density to water. The
reason for this effect is unknown and needs to be investigated further.
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Figure 2.4: Results of a 3D FEM simulation for the acoustic radiation torque $T_{\text{rad}}^z$ on an elastic glass fiber. (a) Definition of the fiber variables and positioning in the coordinate system. The coordinate origin is in the pressure node of a 1D standing wave in $x$-direction. (b) The graph shows the torque $T_{\text{rad}}^z$ as a function of the ratio $l_f^*/\lambda$, where $l_f^*$ is the projected length of the glass fiber in the wave propagation direction ($x$-direction). (c) Torque $T_{\text{rad}}^z$ on the glass fiber for a frequency range between 10 kHz and 10 MHz. The frequency is plotted in logarithmic scale. For characteristic frequencies the fiber displacement (exaggerated deflection) in the acoustic field is plotted.

For frequencies above 1 MHz the ratio of wavelength $\lambda$ to the fiber length $l_f$ becomes relevant. Figure 2.4(b) shows the torque $T_{\text{rad}}^z$ as a function of $l_f^*/\lambda$, where $l_f^*$ is the projected length of the glass fiber in the wave propagation direction ($x$-direction). There is a maximum of the torque, which has been already observed for the case of the rigid ellipse and it is at $l_f^*/\lambda = 0.42$ or 4.35 MHz. The magnitude is about 4 times higher compared to the torque at low frequencies. The highest forces in a standing wave acting on a cylinder (see Eq. (2.14)) will be in between the pressure node and the anti-node. Due to that it could be roughly expected that the maximum torque is reached when the fiber has a length of a quarter wavelength and therefore the fiber ends would be in the region of maximum force. But the fiber length can be in the range of half a wavelength for a maximum torque. This can be explained, if the fiber is modeled as a chain of spheres.
2.4. Numerical simulation of the acoustic radiation force and torque

Figure 2.5: Results of a 3D FEM simulation for the acoustic radiation torque $T_{\text{rad}}^z$ on an elastic glass fiber as a function of the frequency in logarithmic scales. The fiber density ($\rho_f = 998$ kg/m$^3$) is set to the density of water (black line) and the fiber density was increased by 1% and 10% (gray line). For comparison, the result of a glass fiber with density of $\rho_f = 2600$ kg/m$^3$ (bright gray line) is plotted as well.

(see Sec. 2.3). The multiplication of the forces with the distance of the sphere from the rotation axis and the summation gives the overall torque. The maximum torque can be found at $l^*_f/\lambda = 0.44$. The reason that the maximum torque is not at a chain length of a quarter wavelength is that a decrease of the wavelength means an increase of the frequency which leads to higher forces at the spheres and is resulting in a higher torque. For a ratio $l^*_f/\lambda$ higher than 0.5 the forces on the spheres at the end of the chain are changing sign and therefore direction. The point where the torque vanishes due to that is at a ratio $l^*_f/\lambda$ of about 0.71. For even higher ratios the torque gets negative. The FEM simulation results are showing a similar behavior. The torque vanishes as well at a ratio of 0.71. In conclusion, the sphere chain model can be used to explain roughly the qualitative behavior of a fiber in a standing wave. Even the magnitude of the torque is in the same order in the range of the maximum torque. For the case of long and short wavelengths the magnitude and qualitative behavior is not predictable with a chain of spheres.

The acoustic field excites also resonances of an elastic particle. The acoustic radiation force and torque will strongly change in the range of the resonance. The resonance will lead to an increase and a decrease of the force and torque depending on the phase shift between particle displacement and exciting acoustic field which will influence the scattering field. For an elongated object like a fiber, bending resonance modes occur at low frequencies compared to the longitudinal modes. A modal analysis of the fiber was done to evaluate the different mode shapes. The surrounding water was included to consider the additional
mass loading. The same model as for the calculation of the acoustic radiation torque in COMSOL Multiphysics 4.2 has been used and an eigenfrequency study was performed. The background pressure field and the perfectly matched layer surrounding the fluid have been deleted. A matched boundary condition was applied at the outer fluid boundaries. The fluid domain was at least 20 times larger than the fiber diameter. The first, second and third bending modes appear at 1606 kHz, 4343 kHz and 8275 kHz, respectively. The results were verified with the equation for the natural frequencies of a free-free supported Euler-Bernoulli beam given in [73]. The first three natural frequencies are 1769 kHz, 4876 kHz and 9559 kHz. The order of magnitude of the simulated resonance frequencies is correct and the frequencies are lower due to the additional mass loading of the water.

The fiber displacement in the acoustic field for the first bending mode and the corresponding peak in the torque graph can be seen in Fig. 2.4. The second bending mode which would be according to the modal analysis at 4343 kHz, is not excited by the acoustic field. This is not due to the fact that the second bending mode coincides with the maximum torque, also fibers with another aspect ratio have been modeled and the second bending mode was never excited. As only symmetric modes are excited, the excitation has to be symmetric and is the result of a first order effect. The pressure gradient is symmetric at the pressure node. Additionally a FEM simulation for a rigid fiber (no resonance modes) showed that the scattered pressure field along the fiber surface is symmetric regarding the fiber center. Therefore only symmetric modes such as the first and third bending modes are excited. The peak with the high amplitude at 8275 kHz belongs to the third bending mode. The first longitudinal mode is at a frequency of 13.41 MHz and is not excited at this fiber position.

The frequency dependency of the force and torque was also simulated for the position of the fiber in between a pressure node and an anti-node, where the force has a maximum. This simulation is not relevant for the rotation of particles as they are free to move and it is not an equilibrium position. The result can be seen in Fig. 2.6. The force is increasing linearly with the frequency as $F_{\text{rad}} \propto f$ as long as the object is small compared to the wavelength. The peaks in the graph belong to excited bending modes. The force is going to zero at a frequency of 5130 kHz where the projected length of the glass fiber $l'$ is equal to half a wavelength. For higher frequencies the force gets negative. The torque is constant at frequencies below 1 MHz. There is no broad peak for a certain wavelength to fiber length ratio as for the pressure nodal position. Instead the torque is decreasing from its initial value, except at the resonances. In contrast to the pressure node position all bending modes and even the longitudinal modes (not depicted) are excited. The pressure gradient is asymmetric at this fiber position. The simulation for a rigid fiber showed
that the scattered pressure field along the fiber surface is asymmetric regarding the fiber center.

![Diagram of Force and Torque](image)

**Figure 2.6:** Force $|F_{rad}^x|$ and torque $T_{rad}^z$ on an elastic glass fiber as a function of frequency. The fiber with $l_f = 200\,\mu m$ and $d_f = 15\,\mu m$ is positioned with the center in between a pressure node and anti-node of a 1D standing wave in $x$-direction. The force plot is in logarithmic scale and the torque plot is semi-logarithmic. For characteristic frequencies, the fiber displacement (exaggerated deflection) in the acoustic field is plotted.

### Angular position

The dependency of the torque on the angular position $\alpha$ is more relevant for the rotation and the alignment. The torque $T_{rad}^z$ on a fiber was simulated for various angular positions $\alpha$ and fiber lengths $l_f$. The fiber is fixed in the pressure node of a 1D standing wave in $x$-direction with a wavelength $\lambda$ of $1481\,\mu m$ which corresponds to a frequency of $1\,MHz$ in water. The diameter $d_f$ of the fiber is $15\,\mu m$ and lengths between $50\,\mu m$ and $1200\,\mu m$ have been analyzed. The results can be seen in Fig. 2.7. The angular position is only evaluated...
from $0^\circ$ to $180^\circ$ as the fiber is axisymmetric with respect to the $z$-axis and therefore the results will be periodic. The angular positions of $0^\circ$ and $180^\circ$ are identical.

A fiber which is much shorter than the wavelength shows a perfect sinusoidal characteristic. This is the case for the $50 \mu m$ fiber which is more than 10 times shorter than the wavelength. The torque has a maximum at the positions of $45^\circ$ and $135^\circ$ and is oriented positive or negative, respectively. At the positions of $0^\circ$, $90^\circ$ and $180^\circ$ the torque is zero. Only the position of $90^\circ$ will be a stable position for a particle which is free to rotate as the torque is always directed towards the $90^\circ$ position. On the other hand at the $0^\circ$ position, the fiber will rotate in clockwise direction for $\alpha < 0^\circ$ and rotate counter-clockwise for $\alpha > 0^\circ$. When the fiber length is increased and the fiber length to wavelength ratio ($l_f/\lambda$) is larger than 0.2 the maximum of the torque is no longer at $45^\circ$ and is shifted closer to $90^\circ$. There are two aspects which are important for the position of the maximum torque. The effective lever arm which is increasing when approaching $90^\circ$ and the maximum force which is in between the pressure node and anti-node. For a fiber length of about $l_f > 1000 \mu m$ and therefore a ratio ($l_f/\lambda$) > 0.68 the characteristic changes significantly. The angular positions of $0^\circ$ and $180^\circ$ become stable positions and two additional unstable positions where the torque is zero appear. The angle $\alpha$ of the additional unstable positions depends on the length of the fiber.

The characteristic of the torque and angular position can be explained best by the simplified model, where the fiber is reduced to two spheres which are separated by a rigid thin bar of the length $l_f$ (see Fig. 2.1(b)). The acoustic radiation force on the spheres and the connecting bar which acts as a lever arm are resulting in a torque. The effective lever arm is increasing towards $90^\circ$ with $(l/2 \sin \alpha)$. The highest force on the sphere will be in
2.4. Numerical simulation of the acoustic radiation force and torque

between the pressure node and anti-node and therefore in a distance of $\lambda/8$ from the fiber center C. For a ratio $(l_t/\lambda) < 0.1$ the maximum torque is at $45^\circ$ as $(\sin \alpha \cdot \cos \alpha)$ have a maximum at $45^\circ$. When the length $l_t$ is increased the torque will be increased due to a longer lever arm. For a certain length the force on the spheres decreases as the point of the highest force is exceeded and the effective lever arm which is increasing towards $90^\circ$ becomes important.

**Fiber length and diameter**

The influence of the fiber length and diameter on the resulting torque was examined next. This indicates the importance of the aspect ratio. A 2D simulation was done to allow a large variation of the length and diameter parameter. When the aspect ratio of the fiber gets very large the number of elements in the simulation increases strongly and a 3D simulation is not feasible anymore. For both simulations the parameters from the basic model (see p. 24) have been used and only the length $l_t$ or diameter $d_t$ have been varied.

The fiber length $l_t$ was varied from 15 $\mu$m to 1500 $\mu$m. The results can be seen in Fig. 2.8. At a fiber length of 15 $\mu$m the resulting torque is zero as the object is circular since diameter and length are equal in size. For a range of the fiber length from 50 $\mu$m to 800 $\mu$m the curve can be fitted with a power-law function and the proportionality is $T^\text{rad}_z \propto l_t^{3/2}$. The maximum torque is at a length of 1020 $\mu$m which belongs to a ratio $l_t/\lambda$

![Figure 2.8](image-url)

**Figure 2.8:** (a) Torque per unit depth $T^\text{rad}_z$ as a function of the fiber length $l_t$ plotted in logarithmic scale. The diameter $d_t$ of the 2D elastic glass fiber is 15 $\mu$m. The fiber is fixed in the pressure node of a standing wave in $x$-direction with an orientation $\alpha = 45^\circ$. A curve fitting (gray line) for the range 50 $\mu$m to 800 $\mu$m is plotted additionally. (b) Torque per unit depth $T^\text{rad}_z$ as a function of the ratio $l_t^*/\lambda$, where $l_t^*$ is the projected length of the glass fiber in the wave propagation direction ($x$-direction).
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of about 0.49. A further increase of the fiber length strongly decreases the torque. The
peaks in the graph belong to bending modes of the fiber which are excited by the acoustic
field. The mode at 235 µm, 516 µm, 798 µm, 1079 µm, 1360 µm belong to the first, third,
fifth, seventh and ninth bending mode, respectively.

The influence of the diameter on the torque can be seen in Fig. 2.9. The diameter
was varied from 0.1 µm to 200 µm. For thin fibers in the range of 0.1 µm to 20 µm the
torque per unit depth shows a linear behavior in the logarithmic plot and can be fitted
with a power-law function. The torque per unit depth is approximately proportional to
the diameter. The depth of the fiber correlates with the fiber diameter. Therefore the
torque dependency on the diameter will be quadratic ($T_{rad} \propto d_f^2$). This has been checked
also with a 3D simulation of the fiber. For a diameter of 80 µm the torque reaches a
maximum, there the ratio $d_f/l_f$ is 0.4. For larger diameters the torque is decreasing as the
fiber becomes more circular than elongated. For a diameter of 200 µm the fiber is circular
and the resulting torque is zero. The peaks in the graph belong to the bending modes of
the fiber. The strong peak at 11 µm belongs to the first bending mode and the peak at
2.9 µm to the third bending mode. All other peaks at very small diameters correspond to
higher order bending modes. The amplitudes are very small due to the thin fiber.

**Figure 2.9:** (a) Torque per unit depth $T_{rad}$ as a function of the fiber diameter $d_f$ plotted in
logarithmic scale. The length $l_f$ of the 2D elastic glass fiber is 200 µm. The fiber is fixed in the
pressure node of a standing wave in x-direction. A curve fitting (gray line) for the range 0.1 µm
to 20 µm is plotted additionally. (b) Torque per unit depth $T_{rad}$ as a function of the ratio $d_f/l_f$. 

0.2 0.4 0.6 0.8 1
-5
0
5
10
10
-9
10
-7
10
-6
10
-5
10
-4
10
-12
10
-11
10
-10
10
-9
10
-8

0.2 0.4 0.6 0.8 1
-5
0
5
10
10
-9
10
-7
10
-6
10
-5
10
-4
2.4. Numerical simulation of the acoustic radiation force and torque

**Pressure amplitude**

The influence of the pressure amplitude of the standing wave on the torque can be seen in Fig. 2.10. A 3D simulation was performed. The plot in logarithmic scale shows a constant slope which leads to a quadratic relation between the torque and the pressure amplitude $T_{rad} \propto P_a^2$, as expected.

![Figure 2.10: Torque on a glass fiber as a function of the pressure amplitude $P_a$ of a 1D standing wave in logarithmic scale and corresponding curve fit.](image)

**Fiber position and orientation in a 1D standing wave**

The acoustic radiation force and torque depend on the position and orientation of the fiber in the acoustic field. For the development of rotation techniques it is of interest to know the equilibrium position and positions of maximum torque. At first the more simple case of a 1D-standing wave in $x$-direction was analyzed. The definition of the standing pressure wave and the parameters of the fiber are shown in Fig. 2.11(a). The background pressure field is defined as in Eq. (2.19) with only a wave in $x$-direction with an amplitude $P_{ax}$ of $1 \times 10^5$ Pa and a frequency of 1 MHz. The position of the center of the fiber in the standing wave is defined by $k_0 X_0$ and can be varied in the range of 0 to $2\pi$ to displace the fiber center within a complete wavelength. For every position and orientation $\alpha$ a simulation was performed, due to the high amount of simulations a 2D model was chosen. The results of the force per unit length and torque per unit length are shown in Fig. 2.11(b) and (c). The fiber has a diameter of 15 $\mu$m, a length of 200 $\mu$m and is therefore shorter than a quarter wavelength. The orientation of the fiber was varied in a range of 0° to 180°.

The plot of the force shows the equilibrium positions of the fiber which are independent of the fiber orientation for fibers shorter than half a wavelength. The force is zero at pressure nodes ($k_0 X_0 = \frac{1}{2}\pi, \frac{3}{2}\pi$) and pressure anti-nodes ($k_0 X_0 = 0, \pi$). Only the pressure nodes are stable positions as the force vectors are directed towards these positions. The fiber orientation has an influence on the maximum force arising. As expected, the highest force
Figure 2.11: Influence on the acoustic radiation force and torque of a 2D glass fiber depending on the position in the standing wave and orientation $\alpha$. The fiber has a length of 200 $\mu$m and a diameter of 15 $\mu$m. (a) The standing pressure wave is shown and the definition for the position of the fiber. The pressure amplitude $P_{ax}$ is $1 \times 10^5$ Pa. (b) Acoustic radiation force depending on both, the position of the fiber center and orientation $\alpha$ of the fiber in a 1D standing wave. (c) Acoustic radiation torque depending on both, the position of the fiber center and orientation $\alpha$ of the fiber in a 1D standing wave. The black cross indicates the equilibrium position and orientation.

is at an orientation of 90° where the largest surface area of the fiber is perpendicular to the wave propagation direction. The torque depends strongly on position and orientation. It is maximum at the stable equilibrium positions at the pressure nodes and the torque there will be positive for an orientation of $< 90^\circ$ and negative for an orientation of $> 90^\circ$. The sinusoidal characteristic of the torque for different orientations at the pressure node position has been shown already in Fig. 2.7. The torque is zero at an orientation of 0°, 90° and 180° at all positions and additionally very small for the position at the pressure
anti-node. The equilibrium positions and orientations for a fiber shorter than a quarter wavelength are at the pressure nodes \( k_0 X_0 = \frac{1}{2} \pi, \frac{3}{2} \pi \) aligned perpendicular to the wave propagation direction \( (\alpha = 90^\circ) \) and are marked with black crosses in Fig. 2.11.

The influence of the fiber length on the equilibrium position and orientation is shown in Fig. 2.12. The parameters are the same as stated for the previous problem. The simulation results in Fig. 2.12(a) are for a fiber length of 600 µm and therefore a fiber length to wavelength ratio \( l_f/\lambda \) of 0.4. The stable equilibrium position is the pressure node independent of the orientation such as for shorter fiber length. The maximum force increases, but additionally the force is very weak for orientations of 0° or 180° independent of position. The torque characteristic changes also compared to a fiber which is shorter than a quarter wavelength. There is additionally a non zero torque at the pressure anti-

![Figure 2.12: Acoustic radiation force and torque on a 2D glass fiber for two different lengths. The position of the fiber in the standing wave \( k_0 X_0 \) has been varied between 0 and \( \pi \) and the orientation \( \alpha \) between 0° and 180°. The equilibrium positions and orientations are indicated with a black cross. (a) For a fiber length of 600 µm and therefore \( \lambda/4 < l_f > \lambda/2 \). (b) For a fiber length of 1200 µm and therefore \( \lambda/2 < l_f > \lambda \).]
nodes which leads to an equilibrium orientation of 0° or 180°. Therefore a fiber at the pressure anti-node will reorient to 0° or 180° and stay there. This was already observed in the experiments and the model of Yamahira et al. [18]. The equilibrium positions and orientations are indicated with a black cross.

In Fig. 2.12(b) the results for a fiber length of 1200 µm and therefore a fiber length to wavelength ratio (lf/λ) of 0.81 are shown. The force is zero at the pressure nodes, anti-nodes and additionally for all positions at orientations of 52° or 128°. The torque is zero at 90°, 38° and 142°. These orientations will determine the equilibrium positions and orientations indicated with a black cross. For fibers larger than half a wavelength the force and torque characteristic is complicated and depends strongly on the fiber length.

All considerations have been done for objects which are denser and stiffer than the surrounding fluid. It is expected that for other material combinations the sign of the force and torque may change as known from spherical particles.

**Fiber position and orientation in a 2D standing wave**

The behavior of a fiber in a 2D pressure field was simulated and the results are plotted in Fig. 2.13. The definition of the standing pressure field and the parameters of the fiber are shown in Fig. 2.13(a). The background pressure field is defined as in Eq. (2.19) with a standing wave in x- and y-direction, an amplitude P_{ax} and P_{ay} of 1 × 10^5 Pa and a frequency of 1 MHz. The position of the center of the fiber in the pressure field is defined by k_0X_0 and k_0Y_0 and was varied in the range of 0 to π. The squared and time averaged first order pressure ⟨p^2⟩ and velocity ⟨v^2⟩ of the background acoustic field are plotted in Fig. 2.13(a). The minimum of ⟨p^2⟩ represents the nodal pressure line. The pressure field and the velocity field have a different characteristic and the influence can be seen in the acoustic radiation force and torque. The fiber has a diameter of 15 µm, a length of 200 µm and is therefore shorter than a quarter wavelength. The orientation of the fiber α was varied and two characteristic orientations are plotted in Fig. 2.13. In Fig. 2.13(b) the orientation α is 90°. The fiber is forced to the position \{k_0X_0; k_0Y_0\} = \{1/2π; 1/2π\} and the torque is positive. In Fig. 2.13(c) the orientation α is 135° and the fiber is parallel to the nodal pressure plane. The acoustic radiation force is directed to the position \{1/2π; 1/2π\} and for this position and orientation the torque is zero. For positions away from the nodal pressure line the torque can be positive or negative. The conclusion of this simulation is that for all orientations the center of the fiber is forced to the position \{1/2π; 1/2π\} where a nodal pressure line is and the velocity term ⟨v^2⟩ has a maximum. The torque for a fiber in the equilibrium position is always directed towards α = 135° or 315°. Here only fibers
2.4. Numerical simulation of the acoustic radiation force and torque

Figure 2.13: Influence of the acoustic radiation force and torque on a 2D glass fiber depending on the position and angular orientation in a 2D standing wave. The fiber has a length of 200 \( \mu \)m and a diameter of 15 \( \mu \)m. (a) The definition for the position of the fiber in the 2D standing wave is shown. The acoustic field is a standing wave in \( x \)- and \( y \)-direction with the same amplitude and phase. The squared and time averaged first order pressure \( \langle p^2 \rangle \) and velocity \( \langle v^2 \rangle \) of the acoustic field are plotted. (b) Contour plot of the acoustic radiation torque depending on the position of the fiber center for an angular orientation of \( \alpha = 90^\circ \). The arrows represent the acoustic radiation force acting on the fiber. (c) Acoustic radiation force and torque for an orientation of \( \alpha = 135^\circ \).

shorter than a quarter wavelength are considered. For longer fibers the behavior gets even more complex as already shown in a 1D standing wave. Beside the length also the material properties of the fiber are influencing the behavior where the \( \langle p^2 \rangle \) or \( \langle v^2 \rangle \) field can be dominating. For a fiber with equal density as the surrounding fluid the velocity field can be neglected and the behavior is only influenced by the \( \langle p^2 \rangle \) term.
2.5 Rotational motion of non-spherical particles

In order to describe the rotational motion of a particle, Newton’s second law for rotational motion about a fixed axis can be used. For simplicity, a plane rotation is assumed here. Applying this to the rotation of a fiber with acoustic radiation torque leads to:

\[ I_z \frac{\partial \Omega}{\partial t} + T_z^{\text{drag}}(\Omega) + T_z^{\text{misc}} = T_z^{\text{rad}} \]  

(2.20)

where \( I_z \) is the moment of inertia of a fiber for rotation about the z-axis and \( T_z^{\text{drag}}(\Omega) \) is the drag torque of a fiber which is a function of the angular velocity \( \Omega \) and \( T_z^{\text{rad}} \) is the driving torque of the rotation. The variable \( T_z^{\text{misc}} \) represents all unknown effects which are influencing the rotation. These effects might be buoyancy, gravitation, friction due to contact with a wall or acoustic streaming. For the further calculations and discussions these effects are neglected. It is assumed that buoyancy and gravitation have no influence due to the setup and the symmetric fiber. The influence of the acoustic streaming is difficult to estimate. This phenomenon is presented theoretically in [74] and the typical streaming patterns in acoustic cavities can be found in [75]. It is believed that the resulting torque is zero for a symmetric streaming pattern and the symmetric rotating fiber. A torque due to the friction of the fiber at the cavity bottom might be present. This influence is discussed in more detail in the experimental section (see Sec. 3.2 and 3.3).

The motion of micrometer-sized particles is dominated by the viscous forces and the inertial forces can be neglected. Bruus [76] derived the acceleration time of a spherical particle moving in a viscous fluid. For a 15 \( \mu \)m particle moving in water the acceleration time is in the range of 0.2 ms and the steady-state motion is reached therefore almost instantaneously. By neglecting the inertial terms the balance of drag force and acoustic force is responsible for a steady state motion of the particle. For a rotating micro fiber the same principles apply. The driving torque (acoustic radiation torque) \( T_z^{\text{rad}} \) is balanced with the drag torque \( T_z^{\text{drag}} \) and determines therefore the angular velocity \( \Omega \) as the drag torque depends linearly on the angular velocity. This assumption can be made for a steady state rotation or when the inertial terms can be neglected. It allows for the calculation of parameters such as the pressure amplitude or the maximum angular velocity. At first a finite element model was developed to simulate the drag torque and gain knowledge of all influencing parameters. At the end of this section the different influencing parameters of the acoustic radiation torque and the drag torque are compared and discussed for a micro fiber.
2.5. Rotational motion of non-spherical particles

2.5.1 Theory and finite element modeling of the drag torque

The drag force is a resistive force acting on a particle moving in a fluid. It strongly depends on the shape of the object, the velocity difference and the viscosity of the fluid. For a rotating fiber the drag torque is the limiting factor for the rotational speed of the fiber. The calculation of the drag torque and the knowledge of the influence parameters is important. Due to the velocity variation along the fiber length during rotation, no analytical solution is available to derive the drag torque. Therefore a FEM simulation was performed to evaluate the torque and the influence of parameters such as fiber length, diameter and distance to a wall. There exist different approximations for the calculation of the drag torque of a cylinder or fiber, which are discussed in the following and have been used for the validation of the FEM simulation. Furthermore the analytical solutions for the drag force and torque on a sphere have been used for validation.

The general governing equation for a fluid flow problem is the non-linear Navier-Stokes equation. For a very low Reynolds number the equation reduces to the linear Stokes equation [77]. The size of the rotating fiber in the µm range and the occurring maximum velocities of about 2 mm s\(^{-1}\) result in a very low Reynolds number. The Reynolds number is defined as

\[
Re = \frac{\rho v L}{\eta} = \frac{1000 \text{kg} \cdot \text{m} \cdot \text{s}^{-1} \cdot \text{m}^{15} \cdot 10^{-6} \text{m}}{1 \cdot 10^{-3} \text{Pa} \cdot \text{s}} = 0.03 \ll 1 \tag{2.21}
\]

where \(\rho\) is the fluid density, \(v\) the relative velocity of the object in the fluid, \(L\) the characteristic length dimension and \(\eta\) the dynamic viscosity of the fluid. The characteristic length for a rotating fiber is the diameter. The Reynolds number is much smaller than 1 and therefore the viscous forces dominate over the inertial forces. The flow will be laminar and no turbulence will occur. For laminar flow the drag force is directly proportional to the velocity \(v\) and given by:

\[
F_{\text{drag}} = D v \tag{2.22}
\]

where \(D\) is the drag coefficient and depends on the fluid viscosity and the shape and dimensions of the object. The drag coefficient for a prolate spheroid with a high aspect ratio is given by [78]

\[
D = \frac{4 \pi \eta L}{\ln \left( \frac{4}{R} \right) + 0.5} \tag{2.23}
\]

where \(L\) and \(R\) are the length and radius of the spheroid, respectively. Edwards et al. [79] estimated the drag torque on a nano-wire based on the drag force of a prolate spheroid. The nanowire with total length \(l\) was modeled as two spheroids \((L = l/2)\) and
the representative velocity and force for each spheroid was determined at a distance of \( l/4 \) from the center of rotation. The drag torque is given by

\[
T_{\text{drag}} = \frac{\pi \eta l^3}{4 \left( \ln \frac{l/2}{R} + 0.5 \right)} \Omega \tag{2.24}
\]

where \( \omega \) is the angular velocity. Keshoju et al. [41] divided a nanowire into \( N \) segments of prolate spheroids and derived the drag torque by summation over all segments. This is only useful for fibers with a very high aspect ratio as the prolate spheroid requires already a high aspect ratio.

An accurate model is presented by Tirado and Garcia de la Torre [80] to calculate the rotational friction coefficient of cylinders over a wide range of length to diameter ratios. The circular cylinder is modeled as stack of rings composed of touching beads and extrapolated to zero bead size. The drag torque is given by

\[
T_{\text{drag}} = \frac{\pi \eta l^3}{3 (\ln p + \delta)} \Omega \tag{2.25}
\]

where \( p \) is the length to diameter ratio \( (l/d) \) and \( \delta \) the end-effect correction. \( \delta \) depends on \( p \) and is given for \( p = 10 \) with \( \delta = -0.571 \) and for \( p = 20 \) with \( \delta = -0.616 \).

A finite element simulation was done to observe additionally the influence of a wall nearby the rotating fiber and to provide a model to handle different object shapes and aspect ratios such as the model for the acoustic radiation torque. For the simulation COMSOL Multiphysics 4.2 has been used. The creeping flow module solves the Stokes equation for the stationary solution. The geometry is a cubic box with dimensions 8 times the length of the fiber to reduce the influence from the outer boundaries. The rigid fiber is fixed in the middle of the box, which is also the origin of the coordinate system, similar to the simulation of the acoustic radiation torque (see Sec. 2.4). The material properties of the box surrounding the fiber is water with a density of 998 kg/m\(^3\) and a dynamic viscosity of \( 1 \times 10^{-3} \) Pa.s. The boundary conditions on the walls of the box are set to open boundary with zero normal stress. It might be beneficial to set all box edges to a zero pressure with a pressure point constraint and one wall to a slip boundary condition in order to avoid convergence problems. The rotation of the fiber is modeled by defining the velocity components of the fiber surface for a fraction of the rotation. The resulting velocity on the surface is perpendicular to the position vector. A moving wall boundary condition was applied to the fiber surface with the following velocity components \( u_x = \Omega y, u_y = -\Omega x \) and \( u_z = 0 \), where \( \Omega \) is the angular velocity. The mesh consists of triangular elements.
and 700000 to 1200000 elements have been used. The meshing was extremely fine at the fiber surface and coarse at the box walls. Additionally boundary elements have been used at the fiber surface. For the calculation of the drag force and torque the total stress is needed to perform the integration over the fiber surface. The total stress consists of the pressure stress and the viscous stress. The equations in form of the COMSOL notation can be found in Appendix B. The validation of the model was done with a spherical object where analytical solutions exist. The drag force was validated with a 10 μm sphere moving with a velocity of 1 mm/s in water. The analytical solution can be found in [77]. The percent error was 0.21%. The drag torque was validated with a 10 μm sphere rotating with an angular velocity of 2π rad/s. The analytical solution can be found in [81] and the percent error was 0.25%.

2.5.2 Results of the drag torque for a micro fiber

The FEM model is used for a study on the drag torque of a rotating fiber with spherical ends. The results have been compared with existing analytical approximations and combined with the simulation results of the acoustic radiation torque to estimate the pressure in the experiments or the maximal possible angular velocities. The basic parameters of the simulation are a fiber length \( l_f \) of 200 μm, a diameter \( d_f \) of 15 μm, spherical fiber ends, an angular velocity \( \Omega \) of 2π rad/s and the fluid properties of water with density \( \rho = 998 \text{ kg/m}^3 \) and dynamic viscosity \( \eta = 1 \times 10^{-3} \text{ Pa.s} \). A drag torque \( T_{\text{drag}} \) of \( 2.464 \times 10^{-14} \text{ N.m} \) has been simulated. The model derived by Tirado (see Eq. (2.25)) gives a drag torque of \( 2.6362 \times 10^{-14} \text{ N.m} \) and a difference of only 6.7%. The modeling of the fiber with two prolate spheroids (see Eq. (2.24)) leads to a drag torque of only \( 1.278 \times 10^{-14} \text{ N.m} \) and a difference of 63%.

The influence of the fiber length on the drag torque is shown in Fig. 2.14(a) in a logarithmic plot. The drag torque is strongly increasing with the fiber length and can be approximated with a power-law function, which reveals the following proportionality: \( T_{\text{drag}} \propto l_f^{2.6} \). This fit is only valid for a fiber length > 50 μm. For a short fiber the proportionality changes as the object approaches the shape of a sphere and at a length of 15 μm the drag torque of a rotating sphere is reached. The model of Tirado shows a very good agreement with the simulation for a fiber length > 100 μm. The model of Tirado is for a cylinder and the simulation is done for a fiber with spherical end caps. When the fiber length decreases the influence of the spherical end caps increases and the fiber can not be approximated as a cylinder.
Figure 2.14: (a) Drag torque as function of the fiber length $l_f$ plotted in logarithmic scale. The simulation result (black line) is fitted with a power-law function (gray line). The model of Tirado (dashed line) and the approximation with two prolate spheroids (dash-dot line) are plotted as well. The fiber is suspended in water, the fiber diameter is 15 $\mu$m and the angular velocity $2\pi$ rad/s. (b) Drag torque as function of the fiber diameter $d_f$ plotted in semi-logarithmic scale. The fiber length is 200 $\mu$m and the angular velocity $2\pi$ rad/s. (c) Drag torque as function of the angular velocity $\Omega$ plotted in logarithmic scale for a fiber with length of 200 $\mu$m and diameter of 15 $\mu$m.

The diameter $d_f$ was varied in a range of 1 $\mu$m to 100 $\mu$m and the result is shown in Fig. 2.14(b) as a semi-logarithmic plot. The influence of the diameter on the drag torque is small. For a diameter $> 10 \mu$m a linear fit is possible. For fibers with a small diameter $< 10 \mu$m and therefore a high aspect ratio the model of Tirado fits the simulation results.
The influence of the angular velocity $\Omega$ on the drag torque is shown in Fig. 2.14(c). As expected from the theory the proportionality is linear. The influence of the viscosity of the fluid will also be linear.

The simulation and the model of Tirado showed a very good agreement for a fiber with an aspect ratio $l_f/d_f$ of at least 4. The restriction to the aspect ratio is only due to the spherical ends of the fiber in the simulation. The simple model of two prolate spheroids is only useful to estimate the influence of the parameters but gives no accurate value for the drag torque.

An additional advantage of the finite element model is that the influence of a wall near the rotating fiber can easily be implemented. In the experiments the height of the fiber above the cavity bottom is unknown. Due to the higher density of the glass fiber it is expected that the fiber is located close to the chamber ground. In the simulation a no slip boundary condition was set for the wall and the height $h_f$ of the fiber to a wall was varied in a range of 1 $\mu$m to 150 $\mu$m. The result is shown in Fig. 2.15. The drag torque will increase when the distance $h_f$ decreases. For a height $< 10$ $\mu$m the curve can be fitted

![Figure 2.15](image)

**Figure 2.15:** (a) Simulation of the drag torque as function of the height $h_f$ of the fiber to a wall. The wall is located parallel to the fiber axis. The fiber has a length of 200 $\mu$m, diameter of 15 $\mu$m and angular velocity of $2\pi$ rad/s. The simulation result is fitted with a power-law function. (b) The drag torque as function of the ratio $h_f/d_f$ showing the decrease of the wall influence.
Chapter 2. Theory of the acoustic rotation of particles

with a power-law function and shows a proportionality $T_{\text{drag}} \propto h_f^{-0.41}$. For a height which
corresponds to 10 times the ratio $h_f/d_f$ the influence of the wall is negligible.

2.5.3 Discussion on the rotational motion of a micro fiber

In this section the different influencing parameters of the acoustic radiation torque (see
Sec. 2.4.2) and the drag torque (Sec. 2.5.2) are compared and discussed for a micro fiber.
The acoustic radiation torque and the drag torque can be set equal:

$$T_{\text{rad}} = T_{\text{drag}}$$ (2.26)

under the following assumptions: The fiber is performing a steady state rotation (angular
velocity $\Omega$ is constant) where the drag torque is the only resistive torque and the acoustic
radiation torque is the only driving torque. Here, only the rotation around the $z$-axis at
the center of the fiber is considered. As $T_{\text{drag}} \propto \Omega$ the maximal angular velocity can be
increased by reducing $T_{\text{drag}}$ or increasing $T_{\text{rad}}$.

The acoustic radiation torque can be strongly increased when the pressure amplitude
is increased as $T_{\text{rad}} \propto P_a^2$. The pressure amplitude can be increased by increasing the
applied peak-to-peak voltage of the exciting piezoelectric element: $P_a \propto U_{pp}$ as shown
in [76]. The frequency has also an influence on the acoustic radiation torque. When the
wavelength is much larger than the fiber length the influence can be neglected as the
acoustic radiation torque is nearly constant. For a projected fiber length to wavelength
ratio of $0.1 < l_f^*/\lambda < 0.3$ the acoustic radiation torque increases linearly with the frequency
and a maximum is reached at $l_f^*/\lambda = 0.44$. The acoustic radiation torque is varying
sinusoidally with the angular orientation of the fiber to the nodal pressure plane. The
maximum is reached at an angle of $45^\circ$ for a fiber shorter than a quarter wavelength.
The influence of the angular orientation depends also on the rotation method. It will be
discussed in detail for each rotation method in Sec. 3.

The drag torque can be decreased by increasing the distance to a cavity wall. For a height
to diameter ratio $h_f/d_f = 4$ the influence of the wall is weak and for a ratio of 10 negligible.

The fiber length $l_f$ is influencing the acoustic radiation torque as well as the drag torque.
The following influence has been found for fibers of a ratio $l_f/d_f > 3$ and $l_f^*/\lambda < 0.4$:
$T_{\text{rad}} \propto l_f^{1.5}$ and $T_{\text{drag}} \propto l_f^{2.6}$. Therefore a longer fiber has a slower maximal angular
velocity.
2.6 Acoustic viscous torque

The fiber diameter is also influencing both torques. For a diameter of 0.1 \( \mu m \) to 20 \( \mu m \) or a ratio of \( l_t/d_t \) > 10 the influence on the acoustic radiation torque is \( T_{rad} \propto d_t^2 \). For the drag torque the proportionality is more complicated but it can be assumed that it is less than linear. Therefore a larger diameter leads to a higher maximal angular velocity.

2.6 Acoustic viscous torque

The acoustic viscous torque is a time-averaged acoustic effect. It is excited by two orthogonal standing waves shifted in phase. The resulting near boundary streaming inside the viscous boundary layer spins an axisymmetric object about the third axis. The characteristic thickness of the boundary layer is \( \delta = \sqrt{2 \nu/\omega} \) where \( \nu \) is the kinematic viscosity and \( \omega \) the angular frequency. This effect has been observed first experimentally with a rotatable cylinder surrounded by air [19]. Theoretical investigations were done to calculate a torque on spheres, cylinders and circular plates [82]. A very specific analytical information of the viscous torque on a sphere is given by Lee and Wang [56] on the basis of the boundary streaming provided by Nyborg [83].

A sphere with radius \( r_s \) is placed in \( X_0, Y_0, Z_0 \) far away from the walls. The incident pressure fields are defined by:

\[
\begin{align*}
    p_x &= A_x \cos(kx) \sin(\omega t) \\
    p_y &= A_y \cos(ky) \sin(\omega t + \Delta \varphi)
\end{align*}
\]  

(2.27)

where \( A_x \) and \( A_y \) are the pressure amplitudes, \( k = \omega/c \) is the wavenumber in which \( c \) is the speed of sound, \( \Delta \varphi \) is the phase difference between the orthogonal waves. Additionally the sphere has to be small compared to the wavelength with \( kr \ll 1 \). The time averaged flow on the surface of the sphere was derived including the incident plane standing waves and their scattered wave fields due to the sphere. The shear stress on the sphere due to the viscous boundary layer, induced by the time averaged flow, was integrated over the surface of the sphere and results in the viscous torque:

\[
T_{vis}^{z} = \frac{3}{4} \delta S_s A_x A_y \frac{1}{\rho_0 c^2} \sin(\Delta \varphi) \sin(kX_0) \sin(kY_0)
\]  

(2.28)

where \( S_s = 4\pi r_s^2 \) is the sphere surface area. The sign of the torque and therefore the direction of rotation depends on the phase shift \( \Delta \varphi \) and the position of the sphere in the pressure field. The maximum torque acts on a sphere for a phase shift of \( \frac{1}{2}\pi \) and \( \frac{3}{2}\pi \) and when the sphere is located in the pressure nodes of the standing wave.
The viscous torque given in Eq. (2.28) is valid for a fixed sphere. When a particle fixed in space but rotatable is regarded, the viscous torque might be influenced by the rotation due to the additional flow field around the sphere. This has been investigated analytically by Lamprecht et al. [57]. A sphere rotating in a viscous fluid will lead to a Stokes flow because of the non-slip condition between the fluid and sphere surface. Therefore the acoustic streaming velocity was extended by a background Stokes flow formed by the sphere rotation. The influence of the additional flow field on the force related to the Reynolds stress was determined. This force leads to the time averaged flow on the surface of the sphere which results in a viscous torque. It was found that the background flow has no influence on the components of the force related to the Reynolds stress. Therefore the time averaged flow on the surface of the sphere is identical as for the fixed sphere presented by Lee and Wang [56] and the viscous torque results in Eq. (2.28).

The angular velocity $\Omega$ of the sphere can now be calculated with Newton’s second law for rotational motion:

$$I_z \frac{\partial \Omega}{\partial t} + T_{\text{drag}}^z(\Omega) = T_{\text{vis}}^z \tag{2.29}$$

where $I_z$ is the moment of inertia for a sphere rotating around the $z$-axis and $T_{\text{drag}}^z(\Omega)$ is the drag torque for a rotating sphere which depend on $\Omega$. The rotation of a micro sphere will be dominated by the viscous terms and therefore the inertial terms can be neglected as discussed in Sec. 2.5. Additionally only the steady state angular velocity is of interest. This leads to $T_{\text{drag}}^z(\Omega) = T_{\text{vis}}^z$.

The drag torque of a rotating sphere assuming a low Reynolds number ($\text{Re} < 10$) is given by [81]:

$$T_{\text{drag}}^z = \tilde{D}\Omega = 8\pi\eta_0 r_s^3 \Omega \tag{2.30}$$

where $\tilde{D}$ is the drag torque coefficient. The Reynolds number is defined by $\text{Re} = \rho_0 r_s^2 \Omega / \eta_s$, with $\rho_0$ being the fluid density and $\eta_s$ the dynamic viscosity. This allows to predict the steady angular velocity for the rotation of a spherical particle driven by the time-averaged acoustic viscous torque. The angular velocity $\Omega$ is proportional to $1/r_s$ as $\tilde{D} \propto r_s^3$ and $T_{\text{vis}}^z \propto r_s^2$. Therefore smaller particles will rotate faster. For a given particle the angular velocity can be controlled best by the pressure amplitudes $A_x$, $A_y$ and the phase shift $\Delta \varphi$ of the excited acoustic field. The parameters influencing the rotation are a topic of the experimental part in Sec. 4.
3 Rotational manipulation by acoustic radiation torque

The topic of this chapter is the development and experimental investigation of different rotation methods for non-spherical particles with the acoustic radiation torque. It is possible to use the acoustic radiation torque not only for alignment but also for the continuous and controlled rotation of objects. Therefore a varying pressure field is necessary where the orientation of the nodal pressure line can be influenced. All methods have been experimentally tested with a micro device and at frequencies in the MHz range. Particle clumps and micro glass fibers have been used as rotation objects. In the first section (Sec. 3.1) the micro device is presented, including the manufacturing, modeling, function principle and experimental setup. The first rotation technique consists in a change of the propagation direction of one-dimensional standing waves (Sec. 3.2). A hexagonal chamber has been used to successively change in 60° steps the wave propagation direction and therefore the orientation of the nodal pressure line. The other rotation methods are for a continuous rotation and alignment at arbitrary orientations. The common approach is the modulation of a single parameter, where modulation is understood here as a slow variation of a parameter over time. The amplitude modulation is presented in section 3.3. A slow variation of the amplitude of two orthogonal ultrasonic modes over time leads to a local rotation of the nodal pressure line. The pressure field has been used to evaluate the different modes to achieve rotation and the characteristic of different excitations is determined. The next method is the phase modulation of slightly separated degenerated modes (Sec. 3.4). In this section the theory of mode separation is treated with a finite element simulation and an analytical model was developed to discuss the different influencing parameters for the rotation with phase modulation. The last rotation technique is the frequency modulation of slightly separated modes (Sec. 3.5).
Chapter 3. Rotational manipulation by acoustic radiation torque

3.1 Micro devices and experimental setup

The experiments for the rotation using the acoustic radiation torque have been performed with micro devices based on silicon. The basic device design was developed and presented by Neild et al. [55] and Oberti et al. [84]. The two dimensional pattern formation of particles using orthogonal standing waves [84] was the basis for the development of the rotation methods with amplitude, phase and frequency modulation. The device has been adapted and slightly modified to fit the experiments with non-spherical particles such as micro glass fibers. The photolithographic structuring of the silicon offers a high flexibility for the device design. It allows to produce a variety of different fluidic cavity sizes and shapes simultaneously in one production step. This led to the development of other related rotation techniques where the chamber shape was modified to a hexagonal shape to realize the rotation by changing the wave propagation direction of the standing wave (Sec. 3.2). In the following section the structure, assembly, working principle and modeling of such micro devices is described. Only the basic device design (square fluidic chamber) will be discussed here and all relevant device modifications will be discussed in the section of the corresponding rotation technique.

3.1.1 System description and functional principle

The device consists of three main layers: the piezoelectric transducer, the silicon plate and the glass plate. An exploded view and pictures of the assembled device can be seen in Fig. 3.1. The main part of the system is a 500 µm thick silicon plate 10 mm \times 20 mm where a chamber and channels are etched to a depth of $h = 200$ µm. Mainly square chambers have been used with edge lengths of 2 mm, 3 mm or 4 mm depending on the frequency range and particles used in the experiment. The chamber is covered by a 500 µm thick glass plate with anodic bonding, which is sealing the fluidic cavity. The glass plate participates in the acoustic wave reflection and propagation and allows the observation of the fluid filled chamber from the top with a microscope. The reservoirs, which are connected to the inlet channels of the fluidic cavity are not covered by a glass plate. The chamber is loaded by applying a droplet of fluid to the reservoir leaving the other reservoir empty. The chamber will be filled by capillary forces. Most experiments are done with deionized (DI-) water and particles (copolymer, glass). During the experiment, evaporation takes place and the reservoir has to be kept filled to avoid air bubbles entering the channel or even the cavity. Larger air bubbles inside the cavity will strongly influence the pressure field and induce acoustic streaming. The inlet channel width is small compared to the
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Figure 3.1: (a) Exploded view of the micro manipulation device (b) Detailed view of the bottom electrode of the piezoelectric transducer with its strip electrodes. (c) Front side of the device showing the cone shaped inlet channels and the fluidic chamber (3 mm × 3 mm). (d) Back side of the device showing the piezoelectric transducer with the strip electrodes and the wire connections.

cavity width in order to minimize the influence on the pressure field inside the fluidic cavity. A width of 50 µm to 200 µm has been used depending on the particle and cavity sizes. For experiments with glass fibers, cone shaped inlet channels have been used. They have a very small influence on the standing pressure field inside the chamber due to a small connection width (100 µm) to the chamber and the cone shape helps the 200 µm long fibers to align with the flow during the filling process and entering of the chamber.

The actuation of the system is done with a 0.5 mm thick piezoelectric plate (Pz26, Ferroperm Piezoceramics), fixed with conductive epoxy (EPO-TEK H20E, Epoxy Technology) at the bottom of the silicon plate, directly underneath the chamber. The edge length of the piezoelectric element has been varied between 2 mm and 4 mm. The smaller the transducer the more complicated is the wiring and also the acoustic energy is lower for the same applied voltage. The size does not have to be equal to the cavity size, therefore an edge length of 4 mm was generally used. The electrodes of the 4 mm × 4 mm piezo-
cereal plate are divided into four strip electrodes (width 0.7 mm). The strip electrodes allow an asymmetric excitation of the system, which results in a larger number of excitable modes compared to a full plate excitation. The electrodes can be oriented orthogonal which allows the excitation of complex pressure fields. The excited waves are traveling along the piezoelectric and silicon plate. The waves are coupled through the silicon into the water filled cavity. In combination with the reflection of the glass cover a standing wave between the chamber side walls can be set up for a matching frequency. The rectangular walls and the high characteristic acoustic impedance difference between the silicon and the water provide a very good reflection to build up a standing wave. A standing pressure wave is excited in the $xy$ plane of the fluidic chamber. The depth of the chamber $h$ is with $h < \lambda/2$ smaller than half the acoustic wavelength $\lambda/2$ in the fluid for an actuation frequency of $f < 4$ MHz, hence the pressure field will not vary substantially in the $z$-direction. Therefore all treatments concerning the pressure field for the rotation will be two-dimensional in the $xy$ plane. Compared to a planar resonator (see Sec. A.2) the nodal pressure planes are not parallel to the piezoelectric element and therefore the experiments can be observed from above through the glass plate.

**Experimental setup**

The experimental setup can be seen in Fig. 3.2. The observation of the experiments is done with a microscope (SZH, Olympus) and a camera (Imperx IPX-2M30-L, Lynx) connected to a computer. The micro device is fixed with 2 small strips of a 2 mm thick double-faced adhesive tape to a standard microscope glass slide. The adhesive tape supports the micro device only on the outer edge and the high characteristic acoustic impedance difference

![Figure 3.2: Experimental setup for rotation of particles with ultrasonic standing waves. The micro device is located below a microscope, a detailed view is showing the mounting of the device on a standard glass slide. A camera connected to a PC records images and videos. The excitation of the strip electrodes 1 and 2 is achieved with function generators and amplifiers. The function generators are synchronized and connected with the PC for the control via LabVIEW.](image-url)
ensures a minimal energy transfer to the device holder. The piezoelectric element is not covered and can move freely. The standard glass slide with the micro device is clamped under the microscope. The excitation of the piezoelectric transducer is achieved with a function generator (DS345, Stanford Research Systems) and an amplifier (2100 RF power amplifier, ENI). If a second strip electrode has to be excited with a different amplitude, frequency or phase, a second function generator and amplifier are necessary. The function generators are connected with each other to share the same time base and therefore exactly the same frequency. The not excited strip electrodes are unconnected and therefore set to float potential.

**Preliminary experimental results**

For the characterization of the devices, experiments have been done with copolymer particles (Duke Scientific) with a diameter of 17 µm. The density of a copolymer particle is \( \rho_s = 1050 \text{kg/m}^3 \) and the compressibility is \( \kappa_s = 1.058 \times 10^{-10} \text{Pa}^{-1} \). The particles will be attracted to the pressure nodes of the standing wave. If a high concentration of particles is used, the change of the pressure field can be observed in the whole fluidic chamber. This was helpful to observe the behavior and different modes of the device in order to find a useful frequency for the rotation. The principle of the excitation and the superposition of two orthogonal standing waves can be seen in Fig. 3.3.

![Figure 3.3](image.png)

*Figure 3.3:* (a) Configuration of the strip electrodes (b) Copolymer particles (17 µm) in a 3 mm × 3 mm fluidic chamber. Excitation of electrode 1 with a voltage \( V_{\text{rms}} \) of 18 V at a frequency of 1700 kHz. (c) Excitation of electrode 2 at a frequency of 1707 kHz. (d) Excitation of electrode 1 and 2 showing the superposition of two orthogonal standing waves at a frequency of 1698 kHz.

In Fig. 3.3(b) the strip electrode 1 (see Fig. 3.3(a)) was actuated with a voltage \( V_{\text{rms}} \) of 18 V at a frequency of 1700 kHz. The mode \((7, 0)\) is excited and the particles are forming 7 lines perpendicular to the \( x \)-direction inside the cavity. The actuation of strip electrode 2 at a frequency of 1707 kHz can be seen in Fig. 3.3(c). There the mode \((0, 7)\) is excited which leads to 7 lines perpendicular to the \( y \)-direction. The superposition of the two modes \((7, 0)\) and \((0, 7)\) can be seen in Fig. 3.3(d). The two strip electrodes 1 and 2 are actuated simultaneously with \( V_{\text{rms}} = 18 \text{V} \) and a frequency of 1698 kHz. The characteristic
pattern for two in-phase orthogonal standing waves can be seen with 3.5 wavelengths in $x$- and $y$-direction. When the pressure amplitudes and the phase are equal, the particle clumps will have an angle of $45^\circ$ or $-45^\circ$ relative to the $x$-axis. Due to manufacturing errors there can be a variation of the pressure amplitudes for the different directions $x$ and $y$, even for the same excitation voltage.

The experimental observation of the modes with a high amount of particles in the fluid is the best method to characterize the device and to find working frequencies for the rotation. A problem arises when only single particles are used, such as for the rotation of a micro fiber, where the overview of the whole mode in the fluidic cavity is missing. Due to temperature variations or different filling conditions the resonance frequency might shift and the new resonance frequency is difficult to find with a single particle inside the cavity. The following methods might be helpful to find the resonance frequency. The schlieren method allows the observation of the whole pressure field during the experiment due to visualization of spatial variations in the refractive index of the fluid. The acoustic waves cause variations in the density which are related to the refractive index. A schlieren setup and experimental results for a mm-sized acoustophoresis device has been presented by Möller et al. [85]. As the device is located in the optical path of the setup, it has to be optically transparent at the top and bottom of the cavity. Therefore the silicon plate has to be replaced by glass and the piezoelectric transducer has to be positioned differently.

The measurement of the electrical admittance for the determination of fluid resonances in a micro-device is only of limited suitability. There exist a lot of peaks due to the complex structure and the fluid resonances are difficult to detect [86]. The influence of the fluid resonance on the piezoelectric element is very small due to the design and complex coupling of the waves into the fluid. For a planar resonator the situation is different and an admittance measurement is more suitable (see Appendix. A.3). The displacement measurements with an interferometer on the underside of a piezoelectric element for a micro device has been shown by Neild et al. [55]. The conclusion was that a resonance peak in the displacement measurement does not necessarily represent the best operational frequency. Many fluid resonances showed only a little displacement on the underside of the piezoelectric plate. The interferometer measurements are therefore as the admittance measurements more suitable for the validation of a finite element model than for the detection of suitable fluid resonances.
3.1.2 Manufacturing and assembly

The structuring of the silicon plate with the channels and cavity was done in a clean-
room facility using standard micro-machining processes. The detailed run-sheet with all parameters can be found in Appendix C. The basic substrate was a 4 inch, 500 µm, single side polished Si wafer. The wafer was first cleaned in an ultrasonic bath with acetone and isopropanol followed by a rinsing bath of DI-water and dried in a spin-dryer. The surface of the wafer was treated with HMDS (hexamethyldisilazane) to promote the adhesion of the photoresist. The positive photoresist AZ4562 (Clariant) was spin coated on the Si surface with a thickness of approximately 10 µm followed by a short soft bake process to remove residual liquid solvent in the photoresist. A mask-aligner (MB6, Karl Süss) was used to expose the uncovered features to UV radiation (700 mJ/cm²). The creation of bubbles in the photoresist was avoided by splitting the exposure time into several cycles of illumination and rest periods. A high quality printed plastic film laminated to a glass panel was used as mask. This offers a convenient, fast and flexible mask for applications with a feature size greater than 5 - 10 µm. The exposed resist was removed with the developer (MF351) in a small bath supported by slow movement of the wafer to remove any exposed resist. Afterwards the wafer was cleaned in a rinsing bath of DI-water and dried in a spin-dryer. The photolithography process was ended with a post bake to fix the resist. For the structuring of the substrate a dry etching process was used. The inductive coupled plasma (ICP) enables the manufacturing of vertical walls due to the alternation of etching and passivation cycles. The use of a passivation layer avoids the further etching of the side walls during the downward directed etching process due to ion bombardment. After a few etching cycles the etch rate was measured and the number of cycles was defined to obtain a final depth of 200 µm ± 10 µm. The photoresist is finally stripped off with acetone and isopropanol in an ultrasonic bath, followed by a rinsing bath of DI-water and the wafer was dried in a spin-dryer.

The cavity and channels of the micro device were covered with a 500 µm thick glass (Borofloat, Schott). A 4 inch glass wafer was therefore bonded with anodic bonding (SB6, Karl Süss) on the Si wafer. This bond offers a perfect sealing of the cavity and the glass is non-detachably connected to the silicon plate. The blocked reservoirs for the filling process were opened in the next step with a wafer saw (model 8003, ESEC). An adhesive bonding should only be used if there is no possibility to open the reservoirs with the wafer saw or a drilling process. The glass can be attached with a two component epoxy (Araldit) to the silicon plate, but there is a high risk of creeping epoxy into the cavity and channels.
A wafer saw (model 8003, ESEC) was used for the dicing of the devices. First the devices were separated and diced to the final shape and size (10 mm × 20 mm). Afterwards the reservoirs were cut open. The dicing blade had a width of 200 µm and was used to cut slices of glass away by setting the cut depth to the glass thickness.

The actuation of the system is done with a 0.5 mm thick piezoelectric plate (Pz26, Ferroperm Piezoceramics) which was cut to a size of 4 mm × 4 mm with the wafer saw. The piezoelectric elements have evaporated aluminum electrodes at the upper and lower surface. One of these electrodes was structured with the wafer saw to create the strip electrodes as can be seen in Fig. 3.1. The electrode was cut to a depth of about 30 µm to reach electrical separation and a 200 µm wide trench was created due to the dicing blade width. The strip electrode width is 0.7 mm and the length depends on the size of the piezoelectric element. For a 4 mm × 4 mm element the length is 2.2 mm. The piezoelectric element was fixed with conductive epoxy (EPO-TEK H20E, Epoxy Technology) at the bottom of the silicon plate, centered underneath the chamber and cured in an oven at 120 °C for 15 min. The assembly of the piezoelectric element was done manually and was thereby a source of inaccuracy. The position, orientation and epoxy thickness are parameters which may vary and influence the performance of the device. The 4 strip electrodes and the ground electrode (in contact with silicon plate) were connected with wires by applying conductive silver and instant adhesive for the fixation. The electrodes which are not connected via wire can either be connected with conductive silver to ground or left on floating potential. In the experiments no significant difference was observed.

### 3.1.3 Micro device modeling

The finite element method is a useful tool for the simulation of complex systems. It can be used for the design process of a device or the understanding of the actuation. A simple model where the boundaries of the fluidic cavity are assumed as rigid walls is useful to describe the acoustic modes and for the description of rotation methods such as the amplitude modulation (see Sec. 3.3). Not all rotation methods can be described by the simple model of rigid walls such as the rotation with phase modulation (see Sec. 3.4). Important for the modeling of the device are the surrounding structure of the cavity and the excitation. In the finite element modeling some simplifications have to be done, otherwise the model would be too complex. The inlet channels are neglected in most of the simulations and the size of the silicon plate is also reduced. The model described here is a general 3D model which was used to predict the general behavior of the device and especially the rotation with phase modulation. A 2D model is only useful for micro
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The aim of the simulation is to obtain the pressure field in the fluidic chamber for a certain frequency and input voltage. Therefore a time harmonic analysis is performed.

The pressure acoustic module \texttt{acpr} with the dependent variable \( p \) (pressure) represents the fluidic cavity. The piezoelectric devices module \texttt{pzd} with the dependent variables \( u,v,w \) as the displacement field components and the electric potential \( V \) is used for the piezoelectric, silicon and glass layer. For each layer a material model has to be defined. The piezoelectric material model is used for the piezoelectric element. The linear elastic material model is used for the silicon and glass layer, where the variable of the electric potential \( V \) is neglected.

**Geometry**

The model, which can be seen in Fig. 3.4(a), consists of four simple geometric parts. The fluidic cavity (3 mm \( \times \) 3 mm \( \times \) 0.2 mm), surrounded by the silicon plate (20 mm \( \times \) 10 mm \( \times \) 0.5 mm) and covered with the glass plate (20 mm \( \times \) 10 mm \( \times \) 0.5 mm). The piezoelectric element (4 mm \( \times \) 4 mm \( \times \) 0.5 mm) is placed on the underside of the silicon plate centered underneath the fluidic cavity. The inlet channels and the reservoirs are neglected to simplify the model and save computational time. The inlet channels are influencing the pressure field inside the cavity especially at the connections but for small inlet channels (width \( < \) 100 \( \mu \)m) this can be neglected.

**Material properties**

To every geometry element, a material with its properties must be assigned. The following material properties have been used: The fluid in the cavity is water with a density of 998 kg/m\(^3\) and a speed of sound of 1481 \([1+i/(2\cdot100)]\) m/s. Damping has been included as complex wave speed for the fluid and complex stiffness parameters for solids \([72]\). Silicon is an anisotropic material with a symmetric stiffness matrix \( C \) with the components \( c_{11} = c_{22} = c_{33} = 165.7 \) GPa, \( c_{12} = c_{13} = c_{23} = 63.9 \) GPa, \( c_{44} = c_{55} = c_{66} = 79.6 \) GPa and a density of 2330 kg/m\(^3\). The damping of silicon has been neglected as it is very small compared to the damping of the other used materials. The glass has a Young’s
modulus of $63 \left(1 + \frac{i}{400}\right)$ GPa, a Poisson’s ratio of $\nu = 0.2$ and a density of $2220$ kg/m$^3$. The piezoelectric material is Pz26 (Ferroperm Piezoceramics) with the parameters given by the manufacturer: a stiffness matrix $C_E$ with the components $c_{E11} = c_{E22} = 168 \left(1 + \frac{i}{100}\right)$ GPa, $c_{E33} = 123 \left(1 + \frac{i}{100}\right)$ GPa, $c_{E44} = c_{E55} = 30.1 \left(1 + i/100\right)$ GPa, $c_{E66} = 28.8 \left(1 + i/100\right)$ GPa, $c_{E12} = c_{E21} = 110 \left(1 + i/100\right)$ GPa, $c_{E13} = c_{E23} = c_{E31} = c_{E32} = 99.9 \left(1 + i/100\right)$ GPa a coupling matrix $e$ with the components $e_{15} = e_{24} = 9.86$ C/m$^2$, $e_{31} = e_{32} = -2.8$ C/m$^2$ and $e_{33} = 14.7$ C/m$^2$, a relative permittivity $\epsilon_{rS}$ with the components $\epsilon_{rS11} = \epsilon_{rS22} = 828 \left(1 - i0.003\right)$ and $\epsilon_{rS33} = 700 \left(1 - i0.003\right)$ and the density is $7700$ kg/m$^3$.

**Boundary conditions**

The fluid-structure interaction at the cavity walls is implemented with boundary conditions. The interaction from the solid to the fluid is modeled as an inward normal acceleration $a_n$ (acceleration denoted by “$pzd/pzd$” in COMSOL). The interaction from the fluid on the structure is implemented with a boundary load as acoustic load per unit area (denoted by “$acpr/pam1$” in COMSOL). At all outside boundaries a free displacement was implemented. The electrical boundary conditions of the piezoelectric element are the following: The surface in contact with the silicon was set to ground. The side walls of the piezoelectric element and the trenches between the strip electrodes are set to zero charge. The active strip electrode was set to a harmonic electric potential of $\sqrt{2} \cdot 18$ V. All other strip electrodes and the middle electrode are set to a floating potential. The excitation of two or more strip electrodes at the same time with equal or different amplitudes is possible. The simulation with a phase shift between two excitations can be done by adding to the electric potential of one strip electrode the term $e^{i\Delta \varphi}$, where $\Delta \varphi$ is the phase shift.

Automatic meshing with triangular elements is used. The mesh density is varied depending on the frequency and speed of sound of the material. At least 8 elements per wavelength should be used. The models are ranging between 50000 and 250000 elements depending on frequency and model dimensions. In the simulation the PARDISO solver is used and the option “fully coupled” is applied as both fields (acoustic, solid) are influencing each other.

**Post-processing**

The post-processing allows the analysis of the displacement of the device structure and the pressure field inside the fluidic cavity. The Gor’kov potential can be implemented for a
certain kind of spherical particles (radius, density and compressibility) as the pressure and velocity of the fluid are simulation results. The simulation can be used to find resonance frequencies and study the system behavior. A design optimization can be performed by varying different geometrical or material parameters.

Simulation results can be seen in Fig. 3.4(b)-(d). The plots of the pressure inside the fluidic cavity in the $xy$ plane in the middle of the cavity depth correspond to the experiments shown in Fig. 3.3. The copolymer particles moved in the experiment to the pressure nodes which are plotted green in the simulation results. The position and number of the nodal pressure planes for the experiment and the simulation agree for all three excitation cases. In the simulation, the resonance frequency for 3.5 wavelengths in the cavity is at 1668 kHz. The strongest modes observed during the experiment depended on the excited electrode and were in the range of 1689 kHz to 1707 kHz. The relative error is below 2.5% for the largest deviation of simulation and experiment. The accuracy is good especially if

![Diagram of micro device model](image_url)

**Figure 3.4:** (a) Sectional view of the micro device model and detail of the piezoelectric element underside with selected boundary conditions. (b) Plot of the pressure $p$ inside the fluidic cavity in the $xz$ and $xy$ plane in the middle of the cavity. The excitation is a voltage of $\sqrt{2} \cdot 18$ V with a frequency of 1668 kHz at strip electrode 1. (c) Plot of the pressure $p$ in the $xy$ plane for the same simulation parameters but for excitation of electrode 2 (d) Same simulation parameters but for excitation of electrode 1 and 2.
the scatter in the experiments and the simplifications and assumptions of the model are taken into account. The resonance frequency in the experiment will vary in a range of about ±10 kHz depending on the reservoir filling, particle concentration and temperature. There are manufacturing deviations especially for the piezoelectric element positioning. The simulation model is reduced in size by neglecting the inlet channels and reservoirs as well as the fixation. The epoxy glue layer between the silicon and the piezoelectric element is also influencing the behavior and was not considered in the model. This thin layer can have a strong influence on the model accuracy which is shown in Appendix A.3 for a planar resonator. The material parameters are also a limitation for the accuracy of the model. This has been shown by Neild et al. \cite{55} by measuring and modeling the dispersion curve of a piezoelectric plate. The exact damping parameters of the different materials are unknown and have been adopted from Gröschl \cite{72} and later modified to fit the experimental results. In Sec. 3 (p. 85) the pressure was roughly determined to be in the range of 0.18 - 0.4 MPa for a (4, 2) mode at a frequency of 1085 kHz. This mode was found in the simulation at a frequency of 1040 kHz and the pressure amplitude was 0.3 MPa for the same excitation voltage of $V_{\text{rms}} = 20 \text{ V}$ at electrode 1. The Q-factor of water was therefore set from 1000 to 100. This value is low but accounts also for the neglected inlet channels and energy losses at the device mounting. A comparable low Q-factor for water was found for the model of a planar resonator validated with admittance measurements in Appendix A.3 or in a recent publication by Courtney et al. \cite{88}. The overall Q-factor of the excited modes will be in the range of 100 which fits to the experimentally determined 10 - 20 kHz bandwidth where the mode patterns can be observed with particles for resonance frequencies in the MHz range.

Barnkob et al. \cite{89} measured the pressure amplitude and Q-factor for a micro device designed and actuated similarly to the device used here. For two different modes a pressure amplitude of 0.16 MPa and 0.37 MPa and a Q-factor of 200 and 500 was determined for a low voltage experiment with $V_{\text{pp}} = 1.5 \text{ V}$ at a frequency of 2 MHz. The comparable pressure amplitude but much lower voltage can be attributed to the much larger actuated piezoelectric electrode.

The exact prediction of the resonance frequencies and pressure amplitudes is not the primary aim of this finite element model. The model helped to understand the complex actuation of the system and was useful for testing design variations. It was especially used for the understanding of the rotation with phase modulation.
3.2 Changing of the propagation direction of one dimensional standing waves

A non-spherical particle aligns in a one-dimensional standing wave perpendicular to the wave propagation direction due to the acoustic radiation torque. After alignment at the equilibrium position a further rotation of the object can only be induced by rotating the nodal pressure plane of the 1D standing wave to a new direction and therefore changing the propagation direction of the one-dimensional standing wave.

The most intuitive setup for the change of the propagation direction is a movable transducer. Haake and Dual [90] presented a device where a glass plate was excited to bending vibrations with piezoelectric elements and emitted sound waves into a fluid layer. A rigid surface below the device acted as a reflector and determined the depth of the fluid gap where the particle manipulation took place. A one-dimensional bending wave in the glass led to a two-dimensional pressure field in the fluid (varying in the bending wave direction and the fluid depth). Particles with a density higher than the fluid were located on the rigid surface and were concentrated to lines perpendicular to the propagation direction when the top glass layer was excited. The spacing between these lines is half a wavelength of the plate vibration. The piezo-glass unit was mounted on a positioning stage and moved continuously in the horizontal direction. The particles followed this displacement as they were trapped in the potential field. A problem of this method is that the shape of the sound field changed during the displacement, which caused unwanted movement of the particles. The experiments focused on the rectilinear motion. It would also be possible to rotate the piezo-glass unit and the particle lines or a non-spherical particle would follow this rotation.

Here in this section a realization with a micro device is presented. Instead of a movable transducer unit, a hexagonal fluidic chamber and three piezoelectric transducers for excitation are used. The propagation direction of a one-dimensional standing wave can be shifted in 60° steps, allowing a discrete rotation of a non-spherical object. The implemented method, the device and experimental results are shown in the following section. Parts of this section have been developed in collaboration with Guillaume Petit-Pierre (master thesis) [91] and presented in [92].
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3.2.1 Method

The working principle is based on the alternating generation of ultrasonic standing waves with different propagation directions. The hexagonal configuration of the chamber allows to set up standing waves with a wave vector oriented along three different directions in the $xy$ plane. An alternating excitation of three actuators allows the complete rotation of an object in $60^\circ$ steps. Fig. 3.5 shows schematically the three standing waves created by one of the active piezoelectric actuators (marked red). A nodal pressure line is assumed to be in the middle of the parallel chamber walls with the orientation $\beta$. A free fiber (black arrow) is moving to the pressure node and aligns perpendicular to the wave propagation direction. The orientation of the object $\alpha$ equals the orientation of the nodal pressure line $\beta$. By switching from excitation 1 to excitation 2 or 3 the fiber rotates $60^\circ$ clockwise or counter-clockwise, respectively. For a complete rotation of $360^\circ$ in clockwise direction the sequence $1 - 2 - 3$ has to be excited two times. For a counter-clockwise rotation the sequence is $1 - 3 - 2$. The rotation speed for a complete rotation of $360^\circ$ is defined by the switching frequency between the actuators.

![Figure 3.5: Schematic depiction of the alternating excitation of three actuators in combination with a hexagonal chamber for rotation of non-spherical objects. (a) The active piezoelectric actuator 1 excites a 1D standing wave in $x$-direction with a pressure node in the middle of the chamber. A fiber (black arrow) will align with an angular position $\alpha = 90^\circ$. (b) By switching to actuator 2 the wave propagation direction changes by $60^\circ$ and the fiber aligns perpendicular to the new direction. (c) The actuator 3 aligns the fiber at an orientation of $\alpha = -30^\circ$ and the fiber has done a rotation of $120^\circ$ compared to the orientation when actuator 1 is active.](image)

The fiber rotation can be stopped only at discrete angular positions defined by the chamber. For a hexagonal chamber the possible angular positions $\alpha$ of the fiber are $30^\circ$, $90^\circ$, $150^\circ$, $210^\circ$, $270^\circ$ and $330^\circ$. It might be possible to apply a mode switching technique presented by Glynne-Jones et al. [93] for the arbitrary orientation of particles. The excitation has to be switched rapidly (100 Hz) between two transducers and depending on the duty cycle and amplitude ratio an average angular position should be achieved. Also the excitation of two transducers at the same time and varying of the amplitude ratio should lead to an arbitrary angular position. This has been implemented with the amplitude
3.2. Changing of the propagation direction of one dimensional standing waves

Modulation of orthogonal modes in square chambers and is topic in Sec. 3.3. Here we focus only on 1D standing waves.

**Cavity design**

For the variation of the propagation direction of a one-dimensional standing wave in a fluidic cavity the hexagonal configuration is the simplest implementation concerning the excitation and design. A square chamber is not possible as the change of the propagation direction has to be smaller than 90°. For exactly 90° the torque on the object is zero. This is an unstable equilibrium orientation and the direction of rotation is uncertain. This can be seen in Fig. 2.7 (p. 30) where the torque on a fiber is plotted as a function of the angle $\alpha$ between fiber orientation and wave propagation direction. For $\alpha = 90^\circ$ a stable equilibrium exists and for $\alpha = 0^\circ$ or $180^\circ$ an unstable equilibrium orientation exist.

The hexagonal chamber provides parallel walls and a change of the propagation direction of 60°. Chamber designs with a higher number of parallel walls such as an octagonal chamber lead to a change of the propagation direction of 45°. The four necessary actuators lead to a more complex system and the interference between different propagation directions increases. Recently, a heptagonal and an octagonal cavity design have been reported. The combination of different active transducers and the controlled phase delay between them allowed the creation of different potential fields for the patterning of particles (lines, squares or more complex patterns) and the translation of particles.

**Maximal non-spherical object size**

The system can be operated with half a wavelength between two parallel walls of the fluidic chamber. The equilibrium position of the non-spherical object is in the middle of the chamber and the maximum length is limited by the distance between two parallel walls. An excitation with more than half a wavelength is also possible, in this case a pressure node in the middle of the chamber has to exist to allow a rotation along the object center. The maximum length of an object is limited by the wavelength and the angle for the change of the propagation direction of the standing wave. The calculation of the maximum length can be done with the finite element simulation presented in Sec. 2.4 to determine the acoustic radiation torque. Here we will concentrate on the maximal length of a glass fiber. The simulation was done for a frequency of 1730 kHz which has been used in the experiments. The fiber length was varied in the simulation from 100 µm to 800 µm and the torque on the fiber was evaluated. The fiber is positioned at an angle.
of 30° with respect to the wave propagation direction. This represents in the experiment the case of switching to a new actuator. The new equilibrium position of the fiber will be at 90° to the propagation direction which belongs to a 60° rotation of the fiber. In order to achieve a rotation of the fiber a non zero torque must exist at an angular position of 30° and the torque has to be directed towards the equilibrium position at 90°. The torque as a function of the fiber length \( l_f \) and the ratio fiber length to wavelength \( (l_f/\lambda) \) are plotted in Fig. 3.6. The two peaks at 193 µm and 439 µm belong to bending modes of the fiber. The torque has a maximum at a fiber length of 470 µm (excluding the high amplitudes at the bending modes). At a length of 678 µm the torque gets zero and this is the limit for the fiber length. This belongs to a fiber length to wavelength ratio \( (l_f/\lambda) \) of 0.79. For a fiber longer than 678 µm the torque gets negative and the new equilibrium position is at 0° which is parallel to the wave propagation direction. Figure 2.7 (p. 30) illustrates also the length limit for this rotation technique. The torque is plotted as a function of the angular position \( \alpha \) for different fiber lengths and a frequency of 1 MHz. The fiber with a length of 1200 µm \( (l_f/\lambda = 0.81) \) has a negative torque at \( \alpha = 30° \).

The ratio limit \( (l_f/\lambda) \) of 0.79 can be used to calculate the maximum length for different actuation frequencies. If the aspect ratio of the fiber changes strongly or for other non-spherical particles the length limit is changing. Also for an octagonal chamber a new simulation is required where the change in propagation direction is 45° and the limiting fiber length will be larger.

![Figure 3.6: Acoustic radiation torque as function of the fiber length \( l_f \) and the ratio fiber length to wavelength \( (l_f/\lambda) \) for a frequency of 1730 kHz to determine the fiber length limit for the rotation in a hexagonal chamber.](image)
3.2. Changing of the propagation direction of one dimensional standing waves

3.2.2 Device and experimental results

The device depicted in Fig. 3.7 is based on the micro devices presented in Sec. 3.1. The main changes are concerning the fluidic chamber and the actuation transducers. The fluidic chamber is a hexagon with a width of 3 mm. The actuation is done with three independent piezoelectric elements with an electrode area of 2.8 mm × 0.7 mm and a thickness of 0.5 mm. The transducers are aligned with the fluidic chamber walls as shown in Fig. 3.7(a) and fixed on the back side of the silicon plate.

The width between parallel walls in the fluidic chamber is with 3 mm for all three directions the same, therefore the frequencies for the standing waves are all the same and do not depend on a direction. An actuation of a standing wave in only one direction is possible by an asymmetric excitation, which is realized by actuating alternately one of the aligned piezoelectric transducers. The three piezoelectric transducers are connected to one signal generator (DS345, Stanford Research Systems) and one amplifier (2100 RF power amplifier, ENI) via a switcher built by Ueli Marti (IMES, ETH Zurich). A micro-controller triggers three reed relays in a predefined sequence and tunable switching frequency to

Figure 3.7: (a) Exploded view of the device with the hexagonal chamber etched into silicon and three separated piezoelectric elements on the back side for actuation. (b) Front side of the device showing the inlet channels and the hexagonal fluidic chamber. (c) Back side of the device showing the three piezoelectric transducers aligned with the hexagonal chamber walls.
connect the input (amplifier) to one of the three outputs (piezoelectric transducers). The disadvantage of this simple actuation setup is, that all transducers are actuated with the same frequency. Due to manufacturing inaccuracy the perfect actuation frequency deviates for each transducer and direction in a range of 30 kHz. Therefore a compromise frequency has to be used where all directions work acceptably but can lead to deviations in the angular alignment of the object. An improvement would be the excitation control with LabVIEW, where the program controls the excitation frequency, switching frequency and sequence.

**Device characterization with particles**

For the characterization of the device, experiments with copolymer particles suspended in DI-water have been performed. By using a high amount of particles the pressure field in the fluidic chamber can be evaluated. The copolymer particles (Duke Scientific) will accumulate in the pressure nodes of the standing wave. By changing the actuation frequency in a range of 500 kHz to 3 MHz for all three transducers, the different modes in the fluidic chamber have been evaluated. A useful mode appears at 620 kHz with 3 nodal pressure lines but it was not possible to excite the mode with all transducers. The modes with 5 and 7 nodal pressure lines have been observed at 1175 kHz and 1730 kHz. The 7 line mode was later used to realize the rotation. The experimental results with copolymer particles actuating this mode for all three transducers are shown in Fig. 3.8. All modes depicted differ slightly in frequency depending on the excited transducer. In general the modes are observable in a range of 20 kHz and the results in Fig. 3.8 represent the best working frequency in this range. This was judged by visual observation during the experiment by searching for fast particle formation and straight particle lines. The formation of straight particle lines is restricted to the area between the parallel walls.

![Figure 3.8: Experiment in hexagonal chamber with copolymer particles (17 µm) for excitation of the mode with 7 nodal pressure lines with (a) transducer 1 at 1719 kHz, (b) transducer 2 at 1734 kHz, (c) transducer 3 at 1726 kHz.](image-url)
3.2. Changing of the propagation direction of one dimensional standing waves

Outside this area, there is a disturbed pressure field, but it is still strong enough to manipulate particles. The lines are also slightly disturbed in the middle section which might be coming from reflections of the side walls.

**Rotation of a micro fiber**

The rotation of a non-spherical particle was realized using the mode with 7 nodal pressure lines. A glass fiber with a length $l_f$ of 205 $\mu$m and a diameter $d_f$ of 15 $\mu$m suspended in DI-water was used. The fiber was introduced into the chamber via the inlet channels. The placement of the fiber in the center of the chamber can be done by using the half wavelength mode. For the presented device this mode could not be observed as it probably has a weak amplitude and is at a low frequency. Therefore the fiber was placed in the middle with the help of the laminar flow. When the fiber was placed in one reservoir, water was removed on the other reservoir which created a slow fluid flow. The process was stopped when the fiber was near the chamber center. The rotation can then be started by switching excitation between the three transducers. A tuning of the frequency in a range of 20 kHz is necessary to find the best compromise frequency for all transducers. The final frequency was set to 1730 kHz and an excitation voltage $V_{\text{rms}}$ of 18 V was used. A sequence of the rotation is shown in Fig. 3.9 with three frames extracted from a video. Each frame represents the excitation of one of the three transducers. The shown rotation is a clockwise rotation. The direction of rotation was determined in the experiments by the connection order at the switcher output.

The orientation and the position of the fiber center have been determined with the video analyzing tool ProAnalyst (Xcitex). A particle tracking tool allows to extract the contour of the fiber and to evaluate the center and angle of an excentric object frame by frame.

![Figure 3.9: Rotation of a glass fiber ($l_f = 205 \mu$m, $d_f = 15 \mu$m) in a hexagonal chamber at a frequency of 1730 kHz. The excitation is switched from (a) transducer 1 to (b) transducer 2 to (c) transducer 3.](image-url)
The most important step for a good tracking is the image filtering. The frames of the video have to be transformed into binary images. The following four filters have been used: a zero border filter to reduce the area which is analyzed, a despeckle filter that removes the noisy background by removing pixel clusters below a certain size and intensity threshold. The fiber has to be detected as one connected pixel cluster in order to evaluate the center position correctly. Therefore a close connections filter was used which connects nearby pixel clusters. The last filter is a binary filter which transforms the frame into a binary image for a certain threshold. The values of each filter has to be adapted depending on the lighting conditions in the video.

A plot of the angular position $\alpha$ of the fiber versus time is shown in Fig. 3.10. Each frame of the video is represented by a black dot. The connecting gray line is for better illustration. The discrete steps of the curve belong to the correspondent actuator noted on the right side of the graph. The plot is for two complete rotations of the glass fiber. The time for one rotation is $6.35\, \text{s}$ and the average rotational speed is therefore $9.45\, \text{rpm}$. This corresponds to a switching frequency between the actuators of $0.945\, \text{Hz}$. The instantaneous rotational speed when the fiber is doing a $60^\circ$ step is much larger than the average rotational speed. The rotation was too fast to be captured properly by the video with a frame rate of $18$ frames/s. The $60^\circ$ rotation occurs in between two frames. Therefore, the instantaneous rotational speed is at least $180\, \text{rpm}$.

The gray dashed lines mark the theoretical angular position for each actuator. The deviation is due to the compromise frequency and slightly curved nodal pressure lines. For the rotation in Fig. 3.10, actuator 3 has the smallest deviation with $0^\circ$ at the beginning and increasing to $8^\circ$. The highest deviation has actuator 2 with $14^\circ$. The deviation is changing over time as parameters such as temperature, fluid level in the reservoirs are changing and influencing the resonance modes. Additional to the deviation in the orientation, the fiber experiences small translational movement as the position along the nodal pressure line is not fixed. The displacement of the fiber center for the two rotation cycles in Fig. 3.10 is approximately in a radius of $25\, \mu\text{m}$ around the chamber center.

The maximal average rotational speed of the fiber for this setup was determined to be approximately $34\, \text{rpm}$ for an excitation frequency of $1730\, \text{kHz}$ and an excitation voltage of $V_{\text{rms}} = 18\, \text{V}$. The rotational speed is limited by the drag torque which balances with the acoustic radiation torque. A discussion on the influence parameters to increase the radiation torque and decrease the drag torque can be found in Sec. 2.5.3. No influence have the settling and decay times of the standing wave due to switching of the actuator as they are below $0.1\, \text{ms}$, assuming a Q-factor of 500 or lower, which is typical for such
3.2. Changing of the propagation direction of one dimensional standing waves

Figure 3.10: Angular orientation $\alpha$ of a glass fiber in a hexagonal chamber rotating in clockwise direction. Each frame of the video (18 frames/s) is represented by a black dot. The connecting gray line is for better illustration. The discrete steps of the curve belong to the correspondent actuator noted on the right side of the graph. The plot is for two complete rotations of the glass fiber. The time for one rotation is 6.35 s which gives an average rotational speed of 9.45 rpm. The instantaneous rotational speed at the 60° step is at least 180 rpm. The gray dashed lines mark the theoretical angular position for each actuator.

For a maximal switching frequency of 3.3 Hz and therefore an actuation time of 0.3 s for each transducer the settling and decay time have a negligible influence.

In addition to the drag torque, adhesion and friction influences the maximal rotation speed. The fiber density is higher as the density of the surrounding fluid and therefore the fiber is probably in contact with the cavity ground. For micro particles the surface forces (adhesion and friction) become important as the surface area-to-volume ratio is significantly larger at smaller length scales. A detailed treatment of this complex topic can be found in [96, 97]. For the rotation with the hexagonal chamber the influence of the friction is highest as the rotation is stopped every step and the fiber has to overcome adhesion and friction. When the fiber is rotating with a steady state angular velocity it might be that the fiber is lifted by the fluid drag force and the interaction with the wall. It might be as well that only parts of the fiber are in contact with the wall. The contact area and surface roughness of the cavity ground are one of the important parameters. All these parameters are unknown and it is more reasonable to avoid the contact with the cavity ground in future experiments instead of modeling the friction. The friction can be avoided by acoustic levitation of the fiber or adjustment of the density difference between fluid and fiber. The stiction of the fiber to the chamber can also be reduced by surface treatments [97].
Chapter 3. Rotational manipulation by acoustic radiation torque

The time $t_{\text{step}}$ the fiber needs to rotate the 60° step can be used to evaluate the pressure amplitude or the maximal theoretical rotational speed. The basis for the calculation can be found in Sec. 2.3 where the equation of motion (Eq. (2.20)) for a fiber is given. The influence of friction, gravitation, etc. are neglected to simplify matters and therefore the variable $T^\text{misc}_z$ is zero. The drag torque $T^\text{drag}(\Omega) = \tilde{D} \Omega$ is depending linearly on the angular velocity $\Omega$ with $\Omega = d\alpha/dt$ and $\tilde{D} = 4.121 \times 10^{-15} \text{Nm/(rad/s)}$ is the drag torque coefficient. The pressure amplitude $P_a$ gives the maximum acoustic radiation torque $\hat{T}^\text{rad} = P_a^2 \times 2.052 \times 10^{-24} \text{Nm/Pa}^2$. Assuming a $\beta$ of 90° the acoustic radiation torque is $T^\text{rad}(\alpha) = \hat{T}^\text{rad} \sin(2\alpha)$. In a first step the moment of inertia is neglected and the resulting differential equation

$$\tilde{D} \frac{d\alpha}{dt} = \hat{T}^\text{rad} \sin(2\alpha)$$

(3.1)

can be solved by separation of the variables. The differential equation and its solution are analog to the case of a moving spherical particle in a 1D standing wave [89]. The solution is:

$$\alpha(t) = \arctan \left( \tan(\alpha(0)) \exp \left[ \frac{2\hat{T}^\text{rad} t}{\tilde{D}} \right] \right)$$

(3.2)

where $\alpha(0)$ is the start angular position of the fiber at $t = 0$ which is in this case 30°.

The step time can be calculated as function of $\hat{T}^\text{rad}$ or the pressure $P_a$ by inserting for $\alpha(t)$ the end orientation of the fiber at $t = t_{\text{step}}$. The results for various $\alpha$ close to 90° are plotted in Fig. 3.11(a). The equation is not valid for $\alpha = 90°$ as the torque is zero and $\tan(90)$ is infinite. The angular position of the fiber after $t = t_{\text{step}}$ can only be estimated because the exact angular orientation of the nodal pressure line $\beta$ is unknown. Therefore an $\alpha$ between 80° - 89° has been used for the estimation of the pressure. It is expected that for $t_{\text{step}} = 0.056$ s the pressure $P_a$ is in the range of 0.2 MPa to 0.3 MPa. This is in the same range as the pressure estimated in square chambers with rotation experiments using the amplitude modulation (see p. 85). Considering the neglected adhesion and friction influences, the pressure will be higher than predicted here. For the evaluation the rotation step at $t = 1$ s in Fig. 3.10 has been used where the actuator 1 is active. Actuator 2 seems to have a smaller pressure amplitude. The time $t_{\text{step}}$ is at least 2 times larger and a pressure amplitude of about 0.15 MPa has been estimated. A more accurate prediction would be possible with an high-speed camera where the angular position of the rotating fiber can be captured over time. This would allow the fitting of the curve $\alpha(t)$ to the experimental data.

A plot of the angular position $\alpha$ over time can be seen in Fig. 3.11(c) where the black dashed line represents Eq. (3.2) for a pressure $P_a$ of 0.25 MPa. The angular velocity is
not constant during the rotation of the fiber. It is derived by taking the derivative of \( \alpha(t) \) in Eq. (3.2). A peak instantaneous angular velocity of 31.1 rad/s or 297 rpm was determined. The maximum acoustic radiation torque was \( \hat{T}_{\text{rad}} = 1.28 \times 10^{-13} \text{Nm} \).

Due to the high accelerations, the inertial term might not be negligible. The following differential equation was solved numerically:

\[
I_z \frac{\partial^2 \alpha}{\partial t^2} + D \frac{d\alpha}{dt} = \hat{T}_{\text{rad}} \sin(2\alpha)
\]  

with the initial conditions \( \dot{\alpha}|_{t=0} = 0 \) and \( \alpha|_{t=0} = 30^\circ \). The main difficulty is the determination of the moment of inertia \( I_z \). The moment of inertia for a rod rotating about a perpendicular axis through the center is \( I_z = \frac{1}{12} \rho \pi r^2 l^3 \), which leads for the fiber to \( I_{\text{fiber}} = 3.299 \times 10^{-19} \text{kg m}^2 \). The added mass of the surrounding water is difficult to determine and is here only considered by multiplying the moment of inertia of a fiber with a factor in order to estimate the effect of the inertial terms. In Fig. 3.11(c) the result of Eq. (3.3) is plotted for \( \alpha \) as function of time \( t \). For \( I_z = I_{\text{fiber}} \) the result is identical to \( I_z = 0 \) and the percent error is below 0.5\%. The percent error for various moment of inertia in reference to \( I_z = 0 \) is plotted in Fig. 3.11(b). A small deviation occurs when \( I_z \) is at least 10 times larger \( I_{\text{fiber}} \). Especially at the beginning of the fiber rotation where the acceleration is high and the influence is noticeable. It is expected that the moment of inertia for a fiber including the additional mass of the water will be in the range of 2 times \( I_{\text{fiber}} \). Therefore the error is below 1\% and the inertial terms can be neglected for the here used fiber and acoustic radiation torque.
Figure 3.11: (a) Plot of Eq. (3.2) with $t_{\text{step}}$ as function of the pressure $P_a$ for various angular end orientations $\alpha$ of a fiber. (b) Plot of the percent error for various moment of inertia in reference to $I_z = 0$. (c) Plot of the angular orientation $\alpha$ of a fiber as function of time $t$. This is the result of the differential Eq. (3.3).
3.3 Amplitude modulation of two orthogonal ultrasonic modes

Contactless rotation of non-spherical particles has been modeled and experimentally achieved using amplitude modulation of two orthogonal ultrasonic modes. A slow variation of the amplitudes over time leads to a local rotation of the nodal pressure line. The resulting pressure field due to amplitude modulation has been used to evaluate different modes to achieve rotation and to evaluate the characteristic of different excitations. Experiments have been performed in micro devices using copolymer particles or a micro glass fiber. A continuous rotation was successfully demonstrated and the method allowed to stop the rotation at arbitrary angular positions. The maximal angular velocity was measured and the influencing parameters are discussed. This Section has been reported by Schwarz et al. in [98, 99].

3.3.1 Method and modeling

The basis of the rotation presented here is the superposition of two orthogonal ultrasonic modes excited by two sources. With the variation of the amplitude of these two modes, the nodal pressure line and therefore the position of an object can be changed. When the amplitudes are varied, a continuous rotation or controlled change in the angular position of an object is possible. The rotation is due to the acoustic radiation torque on a non-spherical particle. The viscous torque is assumed to be negligible as the phase shift between both orthogonal modes is zero. The principle of the superposition of two orthogonal standing waves was already presented by Oberti et al. [84]. There the superposition of two standing waves with the same frequency and different amplitudes has been examined theoretically and experimentally but with only one of the two amplitudes decreasing to show the effect of manufacturing errors or the possibility of merging particles.

To visualize and understand the principle of the rotation method, the squared and time averaged first order pressure \( \langle p^2 \rangle \) can be used. The minimum of \( \langle p^2 \rangle \) represents the nodal pressure line. In Sec. 2.4 it is shown that a fiber shorter than a quarter wavelength moves to the nodal pressure line and aligns parallel to that in a 1D standing wave. In a 2D standing wave the situation is more complicated as the squared and time averaged first order velocity \( \langle v^2 \rangle \) differs in appearance compared to \( \langle p^2 \rangle \). The influence of \( \langle v^2 \rangle \) on the acoustic radiation force and torque depends on the material properties of the object. For a first approximation, \( \langle p^2 \rangle \) is sufficient to model the characteristic behavior of an object.
as shown in Sec. 2.4 for a glass fiber in a 2D standing wave and the influence of \( \langle v^2 \rangle \) is discussed later on in more detail.

**Pressure modes**

For a cavity surrounded by hard boundary walls, the different possible pressure modes can be calculated with Eq. (2.8) (see p. 13). A single mode excited inside the cavity can be named as \((m, n)\), where the first variable stands for the number of nodes of the pressure wave in the \(x\)-direction and the second variable stands for the number of nodes in the \(y\)-direction. Only modes in the \(xy\) plane are considered as the used cavity in the experiments had a depth smaller than half of the acoustic wavelength in the fluid and therefore the pressure field will not vary substantially in the \(z\)-direction [84]. For the rotation with amplitude modulation, the two modes \((m, n)\) and \((n, m)\) have to be excited with two separated excitations which are building up the following two standing pressure fields \(p_{e1}\) and \(p_{e2}\) (assuming rigid walls):

\[
p_{e1} = A_1(t) \cos(k_{x1}x) \cos(k_{y1}y) \sin(\omega_1t) \\
p_{e2} = A_2(t) \cos(k_{x2}x) \cos(k_{y2}y) \sin(\omega_2t)
\]

(3.4)

with the amplitudes of the pressure fields \(A_1(t)\) and \(A_2(t)\), the wavenumber \(k = 2\pi/\lambda\), the angular frequency \(\omega\), and time \(t\). The two amplitudes depend on time \(t\) as they are varied slowly over time (amplitude modulation). The definitions of the wavenumbers are:

\[
k_{x1} = \frac{m\pi}{L_x}, \quad k_{y1} = \frac{n\pi}{L_y}, \quad k_{x2} = \frac{n\pi}{L_x}, \quad k_{y2} = \frac{m\pi}{L_y}
\]

(3.5)

where \(L_x\) and \(L_y\) are the dimensions of the chamber. For the case considered here the two frequencies \(\omega_1 = \omega_2\) are equal and the cavity is a square chamber with \(L_x = L_y\) and therefore \(k_{x1} = k_{y2}\) and \(k_{x2} = k_{y1}\). The variables \(m\) and \(n\) represent the mode of the vibration inside the chamber and determine the frequency \(f\) with

\[
f = \frac{c_0}{2\pi} \sqrt{k_{x1}^2 + k_{y1}^2}
\]

(3.6)
3.3. Amplitude modulation of two orthogonal ultrasonic modes

The squared and time averaged first order pressure for the superimposed $p_{e1}$ and $p_{e2}$ can be derived using Eq. (3.4),

$$\langle p^2 \rangle = \langle (p_{e1} + p_{e2})^2 \rangle = \frac{1}{2} [A_1 \cos(k_{x1}x) \cos(k_{y1}y) + A_2 \cos(k_{x2}x) \cos(k_{y2}y)]^2$$  \hspace{1cm} (3.7)

The squared and time averaged first order velocity is derived using the relation $\partial \phi / \partial t = p / \rho_0$ (see Eq. (2.3) and (2.6)) with $\phi$ being the velocity potential.

$$\langle v^2 \rangle = \left\langle \left( -\frac{\partial \phi}{\partial x} \right)^2 + \left( -\frac{\partial \phi}{\partial y} \right)^2 \right\rangle = \frac{1}{2\rho_0^2\omega^2} \left[ (A_1 \sin(k_{x1}x) \cos(k_{y1}y) + A_2 \sin(k_{x2}x) \cos(k_{y2}y))^2 
+ (A_1 \cos(k_{x1}x) \sin(k_{y1}y) + A_2 \cos(k_{x2}x) \sin(k_{y2}y))^2 \right]$$  \hspace{1cm} (3.8)

**Amplitude modulation of a pressure mode**

As an example, Fig. 3.12 shows the contour plot of $\langle p^2 \rangle$ with $m = 2$ and $n = 0$. All subplots are showing one wavelength in the $x$ and $y$ directions. Each subplot is defined by a certain set of amplitude values $A_1$ and $A_2$. The black lines in the contour plots indicate the minimum of $\langle p^2 \rangle$ and therefore the nodal pressure line and the bright gray lines represent the maximum of $\langle p^2 \rangle$. A fiber will always align with the nodal pressure line as shown in Fig. 3.12 with a gray arrow.

By running the right sequence of amplitude variations (dashed arrow line in Fig. 3.12) the pressure field changes in such a manner that the fiber (gray arrow) will rotate. The sequence in Fig. 3.12 is showing a rotation of 180°. By repeating this sequence twice a 360° rotation cycle is obtained. There exist four spots in a domain $\lambda_x \times \lambda_y$ which are rotating, where $\lambda$ is the wavelength defined here by $\lambda_x = 2\pi / k_{x1}$ and $\lambda_y = 2\pi / k_{y2}$. For the sequence shown in Fig. 3.12 a non-spherical object will rotate at the positions ($\frac{1}{4} \lambda_x; \frac{1}{4} \lambda_y$) and ($\frac{3}{4} \lambda_x; \frac{3}{4} \lambda_y$) in a counter clockwise direction and in a clockwise direction at the other two positions ($\frac{3}{4} \lambda_x; \frac{1}{4} \lambda_y$) and ($\frac{1}{4} \lambda_x; \frac{3}{4} \lambda_y$). The rotation direction can easily be changed by inverting the sequence of the amplitude variation. The average rotation velocity of the object is determined by the time period $T_M$ of the amplitude modulation cycle for a complete rotation of a fiber. The average rotation velocity $\Omega$ of the object is determined with $\Omega = 2\pi / T_M$.

The modes with $m = 1$, $n = 0$ have half a wavelength inside the chamber and therefore only one pressure node is created in the middle of the chamber. In this situation one
Chapter 3. Rotational manipulation by acoustic radiation torque

Figure 3.12: Contour plot sequence of the squared and time averaged first order pressure $\langle p^2 \rangle$ as a result of amplitude change in the $x$- and $y$-direction, resulting from superposition of two in phase cosine functions with amplitudes $A_1$ and $A_2$ and identical frequency. The bright gray lines are the maximum of $\langle p^2 \rangle$, the black lines are indicating the minimum of $\langle p^2 \rangle$. The gray arrow is representing a fiber located at one of the nodal pressure lines. The black and the gray amplitude values result in the same $\langle p^2 \rangle$ field.

Object can be rotated in every direction in the middle of the chamber. In general all modes with $n = 0$, meaning the modes $(m, 0)$ in combination with $(0, m)$, can be used for the rotation. The number of rotation spots is given by $m^2$.

The modes $(m, n)$ with $n = m$ have no rotation spot. The pressure field is not varying by changing the amplitude ratio. One rotation spot can be found for all $(m, m - 1)$ modes in the middle of the cavity. For the other modes no general conclusions are possible, therefore the potential fields have to be evaluated for every combination of modes.

Relation of nodal pressure line orientation and amplitudes

The angle $\beta$ of the nodal pressure line at one of the four rotation locations is derived from the pressure field as a function of $A_1$ and $A_2$. The angle $\alpha$ of the object is identical with $\beta$ if the $\langle v^2 \rangle$ field can be neglected and the amplitudes are constant or the rotation is very slow. Here only the modes with $n = 0$ are considered. At the nodal pressure lines,
3.3. Amplitude modulation of two orthogonal ultrasonic modes

the sum of both standing pressure waves from Eq. (3.4) is zero. For \( n = 0 \) this can be simplified to

\[
0 = A_1 \cos(kx) + A_2 \cos(ky) (3.9)
\]

The slope and therefore angle of Eq. (3.9) can be calculated for a certain position

\[
\beta = \arctan \left( \frac{dy}{dx} \right) = \arctan \left( -\frac{A_1}{A_2} \frac{\sin(kx)}{\sqrt{1 - \frac{A_1^2}{A_2^2} \cos^2(kx)}} \right) (3.10)
\]

with \( k = k_x = k_y \).

Here, the position will be one of the rotation locations \((\frac{1}{4} \lambda_x; \frac{1}{4} \lambda_y)\), i.e., \( kx = 2\pi/\lambda_x \cdot \frac{1}{4} \lambda_x \) and therefore the angle \( \beta \) is

\[
\beta = \arctan \left( -\frac{A_1}{A_2} \right) (3.11)
\]

As a next step, a continuous rotation of the nodal pressure line (or the particle) is produced by a suitable modulation of \( A_1 \) and \( A_2 \). A full rotation is performed over the rotation time \( T_M = 2\pi/\omega_M \), where \( \omega_M \) is the angular frequency of the modulation and defines also the average angular velocity of the object. One possibility for the time history of the amplitude variation is shown in Fig. 3.13(a). The amplitude of \( A_2 \) (gray) is kept constant at 1 and \( A_1 \) (black) is varied starting at 1 and going down to -1. A negative amplitude means a phase shift of \( \pi \) of the pressure field \( p_{e2} \) compared to the pressure field \( p_{e1} \). The object is performing a 90° rotation. In the next step the amplitude of \( A_1 \) is kept constant at 1 and \( A_2 \) is varied from -1 to 1. This linear amplitude variation is also represented in the sequence in Fig. 3.12 (black dashed arrow line). The relation between the angle \( \beta \) of the nodal pressure line for the position \((\frac{1}{4} \lambda_x; \frac{1}{4} \lambda_y)\) and the amplitudes is plotted in Fig. 3.13(b). There the change of the angle is not linear and leads to a slight variation in the angular velocity.

The characteristic of the angle \( \beta \) is linear over time if the amplitude \( A_1 \) is varied with a cosine function and the amplitude \( A_2 \) with a sine function (see Fig. 3.14(a)). The resulting plot is shown in Fig. 3.14(b) (black line). The influence of unequal amplitudes is also shown in Fig. 3.14(b). The amplitude of \( A_1 \) is set to a reference value of 1. The amplitude \( A_2 \) is reduced from 1 (black) to 1/4 (bright gray). Due to this amplitude mismatch the angular velocity is not constant anymore. Higher velocities are reached at \( \omega_M t = \frac{1}{2} \pi \) where the amplitude \( A_1 \) is zero and the nodal pressure line is parallel to the \( y \)-direction. When the amplitude \( A_1 \) is at its maximum, the angular velocity reaches a minimum.
Chapter 3. Rotational manipulation by acoustic radiation torque

![Figure 3.13](a) Linear amplitude variation of $A_1$ (black) and $A_2$ (gray) for a rotation of $180^\circ$. $A_1$ and $A_2$ are varied over half a rotation cycle $T_M$ between 1 and $-1$. This sequence is used in the experimental part. (b) Other representation of the linear amplitude variation for a complete rotation cycle $T_M$. (c) Orientation of the nodal pressure line $\beta$ and corresponding amplitudes for a variation of one of the amplitudes while the other is set to 1.

![Figure 3.14](a) A sinusoidal amplitude variation is shown, where $A_1$ (black) is a cosine- and $A_2$ (gray) a sine-function and $\omega_M$ is the angular frequency of the modulation. (b) Sinusoidal variation of the amplitudes leads to a linear variation of the angle $\beta$ for equal maximum amplitudes (black). The influence of unbalanced amplitudes is shown with the gray curves.
3.3. Amplitude modulation of two orthogonal ultrasonic modes

Influence of the velocity field

The influence of the squared and time averaged first order velocity $\langle v^2 \rangle$ on the rotation and object orientation $\alpha$ is discussed next. The Gor’kov potential is used for this consideration even if it is only valid for small spherical particles. The equilibrium position of a non-spherical particle can be estimated with the Gor’kov potential. The factors $f_1$ and $f_2$ in the Gor’kov force potential (Eq. (2.11)) for a copolymer ($\rho_s = 1050 \text{ kg/m}^3$, $\kappa_s = 1.058 \times 10^{-10} \text{ Pa}^{-1}$) and water ($\rho_0 = 998 \text{ kg/m}^3$, $\kappa_0 = 4.568 \times 10^{-10} \text{ Pa}^{-1}$) material combination are $f_1 = 0.768$ and $f_2 = 0.034$. The factor $f_2$ is close to zero and the velocity term $\langle v^2 \rangle$ from Eq. (3.8) has only a small influence on the Gor’kov force potential. The dipole coefficient $f_2$ is related to the translational motion of the particle. Therefore the velocity term $\langle v^2 \rangle$ can be neglected for the alignment of neutral buoyancy objects which simplifies the problem.

For a glass ($\rho_s = 2600 \text{ kg/m}^3$, $E = 73 \text{ GPa}$, $\nu = 0.18$, $\kappa_s = 2.630 \times 10^{-11} \text{ Pa}^{-1}$) and water material combination the factors are $f_1 = 0.942$ and $f_2 = 0.517$. The $f_2$ value in the Gor’kov potential is not close to zero, the velocity term $\langle v^2 \rangle$ cannot be neglected and it is difficult to predict exactly the angular position of an object. This is due to the fact that the $\langle p^2 \rangle$ field and the $\langle v^2 \rangle$ field do not have the same characteristics. This is illustrated in Fig. 3.15. The $\langle p^2 \rangle$ field is changing from a diamond shape to a line shape when one of the amplitudes is decreased. This leads to the described rotation. The field $\langle v^2 \rangle$ is changing from a point shape to a line shape. This term is not contributing to the rotation. It is

![Figure 3.15](image)

**Figure 3.15:** (a) Contour plot sequence of the squared and time averaged first order pressure $\langle p^2 \rangle$. (b) Contour plot sequence of the squared and time averaged first order velocity $\langle v^2 \rangle$ for one wavelength in the x- and y-direction. The amplitude $A_1$ is varied from 1 to 0 and $A_2$ is maintained constant at 1.
only slightly influencing the angular position of a non-spherical object in addition to the \(\langle p^2 \rangle\) field. At amplitudes of \(A_1 = 1\) or \(0\) there is no influence. Important are the steps in between. For all amplitudes between \(1\) and \(0\) the angle \(\alpha\) of the fiber will be smaller than expected from the \(\langle p^2 \rangle\) field. Also, the length of the fiber will have an influence. If the fiber is very short the influence from the \(\langle v^2 \rangle\) field can be neglected. For a long fiber (in the range of \(\lambda/4\)) the angle will be smaller than expected from the \(\langle p^2 \rangle\) field only. The exact solution for the angular position for any non-spherical object and every material combination can be derived using the finite element model described in Sec. 2.4. The equilibrium position can be modeled for every amplitude set.

**Maximal non-spherical object size**

The maximal length of a non-spherical object for the rotation with amplitude modulation is difficult to evaluate. In contrast to the shifting of the propagation direction (see Sec. 3.2) the nodal pressure line is rotated continuously. Therefore it is difficult to determine a theoretical length limit. For the rotation of multiple objects, the length should be shorter than half a wavelength to avoid contact between different objects when the nodal pressure line is forming a straight line (\(\beta = 0^\circ\) or \(\beta = 90^\circ\)). If only a single particle is rotated there is no length limit for this angular position of \(\beta = 0^\circ\) or \(\beta = 90^\circ\) as can be seen from Fig. 2.7 (p. 30). Also the positions of \(\beta = 45^\circ\) and \(\beta = 135^\circ\) have no certain length limit as long as the diameter is small compared to the wavelength. More critical are the positions in between such as \(\beta = 112.5^\circ\). It is assumed that the long fiber is located already with its center at one of the rotation spots. Otherwise the length limit is a quarter wavelength to ensure that the only equilibrium position and orientation is one of the rotation spots.

**3.3.2 Experimental results**

The micro device used for the experiments and the setup has been presented in Sec. 3.1. The superposition of two orthogonal pressure fields with variable amplitudes is the most important function of the system to achieve the rotation of non-spherical particles. In the experiment the voltages of the two function generators are controlled via GPIB by a LabVIEW program. The sequence plotted in Fig. 3.13(a) was implemented in LabVIEW. The negative amplitude was realized with a phase shift of \(\pi\) compared to the other excitation signal. The excitation voltage of the piezoelectric element is proportional to the pressure amplitude as shown in \[76\]. The findings of the amplitude in the previous section can directly be transferred to the voltage signal at the electrodes.
3.3. Amplitude modulation of two orthogonal ultrasonic modes

**Rotation of particle clumps**

First, experiments have been done with copolymer particles (Duke Scientific Corp.) with a diameter of 17 µm dispensed in deionized (DI)-water. The density and the compressibility of a copolymer particle are \( \rho_s = 1050 \text{ kg/m}^3 \) and \( \kappa_s = 1.058 \times 10^{-10} \text{ Pa}^{-1} \). The factors of the Gor’kov force potential are \( f_1 = 0.768 \) and \( f_2 = 0.034 \). The particles will therefore be attracted to the pressure nodes of the standing wave and the influence of the \( \langle v^2 \rangle \) field is negligible. If a high concentration of particles is used the change of the pressure field can be observed in the whole fluidic chamber. This was helpful to observe the behavior of the device in order to find a working frequency for the rotation. The principle of the device excitation, the superposition of two orthogonal standing waves, and the formation of particle arrays is described in Sec. 3.1. The rotation of particle clumps is shown in Fig. 3.16 with the modes \((7,0)\) and \((0,7)\) at a frequency of 1689 kHz and a rotational speed of about 44 rpm. The first picture in Fig. 3.16 depicts the whole fluidic chamber seen from the top through the glass plate. The characteristic pattern for two in phase orthogonal standing waves can be seen with 3.5 wavelengths in both the \( x \)- and \( y \)-direction. This is the same pattern as the one shown in Fig. 3.12 for one wavelength.

![Figure 3.16: A 180° rotation of clumps of copolymer particles (\( \varnothing \)17 µm) with amplitude modulation and an excitation frequency of 1689 kHz. The applied voltage \( V_{\text{rms}} \) is 18 V. The pictures (a)-(i) are 0.86 mm × 0.86 mm details of the whole cavity and the elapsed time is given in each part.](image-url)
When the pressure amplitudes are equal, the particle clumps will have an angle of 45° or −45° relative to the x-axis. Due to manufacturing errors there can be a variation of the pressure amplitudes for the different directions x and y, even for the same excitation voltage.

Even though the acoustic radiation torque is not applicable for the rotation of spherical objects, Fig. 3.16 is showing the rotation of clumps formed out of spherical copolymer particles due to the pressure field as shown in Fig. 3.12. The particles are forming an elliptical clump which is rotating around its center. The movement of a single particle inside the clump is undefined. The pictures (a)-(i) in Fig. 3.16 show one wavelength in the x- and y-direction and are related to the plotted pressure field in Fig. 3.12. For the rotational manipulation, the amplitudes were varied linearly as shown in Fig. 3.13(a) with a maximal voltage $V_{\text{rms}}$ of 18 V and a complete rotation time of $T_M = 1.36 \text{s}$. The particle clumps are partially merging as can be seen in Fig. 3.16(c-d) and (g-h). The reason is the straight nodal pressure line when one of the amplitudes is zero.

The angular position $\alpha$ of a particle clump over time is shown in Fig. 3.17 for a complete rotation of 360°. The particle clump marked with a circle in Fig. 3.16 has been used for this analysis. The black dots represent the angular position $\alpha$ of the object readout from each frame of the video with the particle tracking tool of ProAnalyst (Xcitex). The gray line represents the expected average angular position for the rotation time of $T_M = 1.36 \text{s}$ and the corresponding 44 rpm. The time $t = 0 \text{s}$ represents Fig. 3.16(a). One modulation cycle as shown in Fig. 3.13(a) corresponds to a 180° rotation. For a complete rotation of 360° the characteristic of the angular position $\alpha$ is identical for the part from −45° to 135° and 135° to 315°. The angular velocity has a deviation within a full rotation. Especially

![Figure 3.17: Angular position of a particle clump plotted over time for a rotation of 360°. The black dots represent the angle of the clump for each frame in the video. The gray line is the average expected angular position at a rotational speed of 44 rpm (rotation time $T_M = 1.36 \text{s}$).](image-url)
3.3. Amplitude modulation of two orthogonal ultrasonic modes

at the angular positions near 0° and 180° was the angular velocity even close to zero. The linear variation (Fig. 3.13) instead of a sinusoidal amplitude modulation (Fig. 3.14) would only lead to a very small deviation in the angular velocity. The variation of the overall amplitude and therefore also the acoustic radiation torque during a modulation cycle can be responsible for the deviations. But then also the angular velocity near 90° and 270° should be slow. The main part of the deviation is probably coming from the not totally balanced maximum amplitudes $A_1$ and $A_2$ and a not perfectly excited one-dimensional mode. The amplitude $A_2$ seems to be slightly larger as can be concluded from Fig. 3.14(b) for slower velocities near 0° and 180°. The angular difference of approximately 25° is due to the drag torque. During rotation there is an angular difference between the nodal pressure line $\beta$ and the object orientation $\alpha$ which will be maximal 45° for the fastest rotation. This is discussed in more detail for the rotation of a glass fiber. Another reason for the velocity deviation is that the function generators cannot vary the amplitude continuously and in between there have been short interruptions of the excitation signal which are also leading to small jumps in the angular position. The average rotation speed of the particle clumps can be varied between 0 and 50 rpm by setting the corresponding rotation time $T_M$ in the LabVIEW excitation control. For higher velocities the clumps cannot follow the rotating pressure field and the motion becomes a oscillating rotation in an angular range of roughly estimated 45°. The shape of the clumps is than distorted.

**Rotation of a micro fiber**

Further experiments have been performed using non-spherical objects such as a micro glass fiber. The glass fiber with a diameter of 15 µm has been cut to a length of 210 µm from a glass roving [71]. The glass fiber (density $\rho_f = 2600$ kg/m³, Young’s modulus of $E_f = 73$ GPa, Poisson’s ratio $\nu_f = 0.18$) was suspended in DI-water ($f_1 = 0.942$, $f_2 = 0.517$). The results are presented in Fig. 3.18 which is showing a 180° rotation with a rotational speed of about 36 rpm. The actuation frequency was 1085 kHz and the maximum amplitude $V_{\text{rms}}$ was 20 V. The images are taken from a video. They correspond to a 0.5 mm $\times$ 0.5 mm area inside the chamber. Due to the lighting the fiber is not always clearly visible and the white rectangle in Fig. 3.18 helps to visualize the position of the fiber.

The rotational motion of the fiber can be analyzed with considerations of Sec. 2.5. When the angular orientation of the fiber $\alpha$ is equal with the nodal pressure line $\beta$ the acoustic radiation torque on the fiber will be zero. The orientation of the nodal pressure line is changing due to the amplitude modulation. The drag torque, due to the movement in the
Chapter 3. Rotational manipulation by acoustic radiation torque

Figure 3.18: A 180° rotation of a glass fiber with a length of 210 µm and a diameter of 15 µm using amplitude modulation of two ultrasonic modes. The images are taken from a video. They correspond to a 0.5 mm × 0.5 mm area inside the chamber. The actuation frequency is 1085 kHz and the maximum applied voltage $V_{\text{rms}}$ was 20 V.

viscous fluid, arises because of the rotation of the fiber which follows the nodal pressure line. The drag torque is proportional to the angular velocity of the fiber. The acoustic radiation torque is balanced with the drag torque. For a fiber rotation with the maximum angular velocity, the maximal acoustic radiation torque arises on the fiber at an angle difference between $\beta$ and $\alpha$ of 45°. For a very slow rotation the angle difference is nearly zero. For rotations in between the maximum angular velocity and no rotation the angle difference is in between 45° and 0°. This angle difference is important for the analysis of the fiber orientation during rotation. A synchronized data set of the experimental video and the excitation was not available.

In Fig. 3.19 the angular position of the glass fiber is plotted over time for 2 rotation cycles (720°). The black dots represent the angle $\alpha$ of the fiber for each frame in the video, analyzed with the particle tracking tool of ProAnalyst (Xcitex). The gray line represents the expected average angular position for the rotation time $T_M = 1.67$ s and a corresponding rotational speed of 36 rpm. The gray line accentuates the variation in the rotational speed. The time $t = 0$ s corresponds to Fig. 3.18(a). The reasons for the deviation are similar to the ones mentioned for the particle clumps. A part of the deviation might come from unbalanced amplitudes of $A_1$ and $A_2$ and a not perfectly excited mode.
The main part seems to be coming from the variation of the acoustic torque as the overall excitation is varying between one electrode with 20 V and two electrodes with 20 V. This leads to a smaller acoustic radiation torque for orientations of the nodal pressure line $\beta$ at 0° and 90°. The rotation of the fiber is with 36 rpm close to the maximal rotational velocity. The angle difference between $\beta$ and $\alpha$ is approximately 40°. Therefore the slow velocities can be seen 40° below an $\alpha$ of 0°, 90°, 180° and 270°. The contact of the fiber to the cavity bottom leads additionally to deviations in the rotational speed due to friction. As the density of the glass fiber is higher than water and no pressure deviation in the $z$-direction is assumed, the fiber will rotate at the bottom of the cavity.

The actuation frequency of 1085 kHz corresponds to a (4, 2) mode. The corresponding squared and time averaged pressure field $\langle p^2 \rangle$ is plotted in Fig. 3.20. The theoretical position of the fiber center for this mode would be $x = y = 1.125$ mm. The analysis of the video revealed an average position of $x = 1.11$ mm and $y = 1.1$ mm. The maximum deviation from this position was $\pm 50 \mu$m. The fiber can move slightly along the nodal pressure line which leads mainly to the deviation in the position.

The increase of the rotational velocity was possible until about 40 rpm for the used parameters. For higher rotation velocities the fiber cannot follow the rotating pressure field at all times and the motion becomes an oscillating rotation in an angular range of approximately 45°. Recently, Hahn et al. [100] have developed a model for the particle dynamics in acoustofluidics including acoustic radiation force/torque and drag force/torque for arbitrary shaped objects. The simulations for the rotation of a fiber with sinusoidal amplitude modulation have shown, that for high modulation frequencies $\omega_M$, were the fiber can not follow the rotation of the nodal pressure line anymore, the fiber is still doing a slower net...
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Figure 3.20: Contour plot of $\langle p^2 \rangle$ for a (4,2) and (2,4) mode used for rotation of a glass fiber with amplitude modulation. The black arrow is representing a fiber, located at a pressure minimum along a nodal line such as in the experiment.

rotation. For unbalanced amplitudes and too high modulation frequency the fiber shows an oscillating rotation such as observed in the experiments. The analysis of Fig. 3.19 is showing that instantaneous velocities of 150 rpm to 200 rpm are reached during parts of one rotation. For balanced amplitudes and a constant acoustic radiation torque at all times, higher average rotational velocities than 40 rpm should be possible. In general the maximum possible angular velocity depends on the applied acoustic radiation torque and is limited by the drag torque of the rotating fiber. A detailed discussion on the influences of parameters such as pressure amplitude, frequency and fiber size on the acoustic radiation torque and drag torque including the maximal angular velocity can be found in Sec. 2.5.3.

The pressure amplitude in the cavity can be roughly estimated with the experimentally determined angular velocity. The basis for the following discussion can be found in Sec. 2.5. The drag torque of a rotating fiber with the same size as in the experiment and for an angular velocity $\Omega$ of 4.19 rad/s (40 rpm) has been modeled. The drag torque $T_{\text{drag}}$ is $1.836 \times 10^{-14}$ Nm. The acoustic radiation torque has been determined with the model described in Sec. 2.4 for the frequency of 1085 kHz and the mode with $m = 4$ and $n = 2$. The position of the fiber was the same as in the experiment and an orientation of 45° was implemented to reach the maximal torque. The acoustic radiation torque as function of the pressure amplitude is $T_{\text{rad}}(P_a) = P_a^2 \cdot 5.844 \times 10^{-25}$ Nm. Therefore the pressure
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amplitude $P_a$ is 0.18 MPa. The influence of the cavity bottom was neglected. Assuming a wall to fiber distance of 5 µm the drag torque increases to $T_{\text{drag}} = 3.977 \times 10^{-14}$ Nm giving a pressure of 0.26 MPa. The order of magnitude of the pressure amplitude is reasonable. The determination of an exact value is not possible as the distance to the cavity bottom and the influence of possible contact are unknown. The higher instantaneous velocities of 150 rpm - 200 rpm during parts of the rotation allow to assume that the pressure amplitude is higher than calculated above. For these instantaneous velocities a pressure amplitude of 0.34 MPa - 0.4 MPa was determined.

Barnkob et al. [89] measured the pressure amplitude of 0.24 MPa in a low voltage experiment with a micro device and cited results of other authors and devices. Wiklund et al. [101] measured an amplitude of 0.76 - 2.4 MPa and Hultström et al. [102] an amplitude of 0.57 - 0.85 MPa.

When the pressure amplitude in the device is known, the angle difference between $\beta$ and $\alpha$ can be calculated for a constant rotational velocity. The pressure amplitude gives the maximum acoustic radiation torque $\hat{T}_{\text{rad}}$ which is varying sinusoidally with $\alpha$. Assuming a $\beta$ of 90° the acoustic radiation torque is $T_{\text{rad}}(\alpha) = \hat{T}_{\text{rad}} \sin(2\alpha)$. Depending on the rotational speed of the fiber the drag torque can be determined which balances with $T_{\text{rad}}(\alpha)$. The angular difference is then given by $\beta - \alpha$.

Of interest is the theoretical maximal rotational velocity for a glass fiber as used in the experiments. The following assumptions are done. A pressure amplitude of 0.5 MPa is reasonable for a micro device. The frequency is 1 MHz and a mode with $n = 0$ is excited. The fiber is assumed to float in the middle of the cavity without any influence of the walls or other particles. The simulated radiation torque $T_{\text{rad}}$ is $3.57 \times 10^{-13}$ Nm. The drag torque as function of the angular velocity is $T_{\text{drag}}(\Omega) = \Omega \cdot 4.382 \times 10^{-15}$ Nm. This results in a theoretical maximal angular velocity $\Omega$ of 82 rad/s and therefore a rotational speed of 780 rpm. An improvement of the device towards the excitation of two one-dimensional modes with a high pressure amplitude and free floating fiber can result in a very high rotational speed as the influence of the pressure amplitude is quadratic.
3.4 Phase modulation of slightly separated degenerated modes

The phase modulation of two degenerated ultrasonic standing modes leads to a local rotation of the pressure field. Two slightly in frequency separated degenerated modes are needed to induce the rotation. In this section the theory of the phase modulation is treated and experimental results are presented. A finite element simulation has been used to show the separation of the modes and to develop an analytical model for the excited pressure fields. The analytical model has been used to discuss the different influencing parameters on the rotation. Experiments have been performed using copolymer particle clumps and micro glass fibers. Parts of this content have been presented in [103,104].

3.4.1 Method and modeling

For the presentation of this rotation method only the pressure field is considered. For the particles considered here, the velocity field has a small influence on the orientation $\alpha$ of a non-spherical object depending on the material parameters and the object geometry. The driving mechanism for the rotation is coming from the pressure field and for simplicity reasons the velocity field is neglected here.

A rotation cannot be achieved with the simple phase modulation of two orthogonal standing waves in $x$ and $y$ direction. This can be seen in Fig. 4.1 (see p. 106), where the squared and time averaged pressure $\langle p^2 \rangle$ of the superimposed pressure fields from Eq. (4.1) are plotted for different phase shifts $\Delta \varphi$. When the phase is varied, the nodal pressure line is not performing a gradual local rotation but instead it is forming a pattern with points. The pressure field at $\Delta \varphi = \frac{1}{2} \pi$ and $\frac{3}{2} \pi$ is not exciting an acoustic radiation torque. This method was used by Oberti et al. [84] to achieve arrangement of particle clumps.

The rotation with phase modulation is based on slightly separated degenerated modes. In a perfect square chamber with rigid walls, the modes $(m, n)$ and $(n, m)$ as introduced in Sec. 3.3 exist at the exact same frequency. All degenerated modes, meaning the superposition of both modes for various amplitudes, are also at the same frequency. Due to not perfectly rigid walls and therefore interaction of the structure and the fluid the modes can be slightly separated by a small frequency difference. Due to damping of the system both modes are overlapping. Another reason for a slight separation and excitation of degenerated modes might be the method of actuation. The actuation of the micro device described in Sec. 3.1 excites not only one mode in a single direction, for example a single
3.4. Phase modulation of slightly separated degenerated modes

standing wave in \(x\)-direction. The vibration of the piezoelectric element is very complex at higher frequencies and due to the coupling of the vibration into the silicon and the fluidic cavity, the excited modes are also more complex than simple 1D standing waves.

**Modeling of micro device**

The rotation of the pressure field with phase modulation was modeled with FEM for the used micro device. The finite element model of the micro device in Sec. 3.1.3 was adapted and slightly simplified. The reason for the simplification of the model was the reduced computational time and the reduction of the influence parameters for this kind of rotation, thereby facilitating the understanding of the physics. Also the model of the complete device showed a rotating pressure field. The simplified model is depicted in Fig. 3.21(a). It is a 3D model and consist of a piezoelectric element a silicon plate and a fluidic cavity an top. Compared to the complete model of the micro device, the top glass layer and the silicon surrounding of the fluidic cavity have been neglected. The boundary condition for the top and sides of the cavity were simplified to a hard wall. The fluid structure interaction between the silicon layer and the cavity was implemented as described in Sec. 3.1.3. The actuation of the phase modulation is done with two orthogonal separated electrodes as shown in Fig. 3.21(a). At electrode 1 an electric potential of \(V_0\) was applied and at electrode 2 an electric potential of \(V_0e^{i\Delta \varphi}\) including a phase shift \(\Delta \varphi\) compared to electrode 1. The absolute pressure in the \(xy\) plane inside the fluidic cavity for different phase values is shown in Fig. 3.21(b). A time harmonic analysis was performed with a constant frequency of \(1457.3\) kHz. The frequency was chosen to be in the middle of the two separated modes shown in Fig. 3.21(c). A modal analysis was used to get the two modes which occur at slightly different frequencies of \(1456.94\) kHz and \(1457.57\) kHz. A local rotation of the nodal pressure line is observed in Fig. 3.21(b). The rotation by continuously varying the phase over time is not uniform. The pattern where the nodal pressure line is forming straight lines perpendicular to the \(x\)- or \(y\)-direction occur at \(\Delta \varphi = \frac{1}{4}\pi\) and \(\frac{3}{4}\pi\). For a uniform rotation this pattern should be at a phase of \(\Delta \varphi = \frac{1}{2}\pi\) and \(\frac{3}{2}\pi\). This can be influenced by varying the actuation frequency or the separation of the two modes.
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Figure 3.21: (a) The 3D model of a simplified micro device for the observation of the rotation with phase modulation. It consists of a piezoelectric element, a silicon plate and a fluidic cavity. The fluidic cavity is surrounded by hard boundary walls and the top glass layer as well as the surrounding silicon have been neglected. The actuation is done with an electric potential at two electrodes and an additional phase shift $\Delta \varphi$ at one of the electrodes. (b) Absolute pressure inside the fluidic cavity in the $xy$ plane for a time harmonic analysis at a frequency of 1457.3 kHz and varying phase values. (c) Modal analysis of the model showing the 2 modes which occur at slightly different frequencies.

### Simplified 2D model

For a better understanding of this rotation method and to confirm that the degenerated and separated modes are responsible for this rotation, a simple 2D finite element model was developed. The model consisted of a square acoustic domain with a length of 1 mm and was surrounded by a square solid domain with edge length 1.5 mm. Fig. 3.22(a) is showing the model. The dimensions are adapted, to have only one wavelength in the acoustic domain for the same frequency range as the previous model. The material of the acoustic domain was water with the following properties: density of 998 kg/m$^3$ and speed of sound of $1481 \left[1 + i/(2 \cdot 500)\right]$ m s$^{-1}$ including damping. The material of the solid frame was steel with a Young’s modulus of 190 GPa, a Poisson’s ratio of 0.25 and a density
of 7850 kg/m$^3$. The fluid structure interaction between the steel frame and the fluidic cavity was implemented as described for the micro device in Sec. 3.1.3. The actuation of the model was done with a prescribed displacement $\pm u_0$ in $x$-direction and $\pm v_0 e^{i\Delta \varphi}$ in $y$-direction including a phase shift $\Delta \varphi$, as shown in Fig. 3.22(a). The amplitude for the prescribed displacement is with 1 nm equal for the $x$- and $y$-direction. A modal analysis of the model is shown in Fig. 3.22(b). There are two slightly separated modes at 1468.8 kHz and 1471.9 kHz. Even though the system is a perfect square there are two separated modes. This is due to the steel frame. The edges of the steel frame are more stiff compared to the side walls. The mode with pressure anti-nodes at the side walls is therefore at a lower frequency. The frequency difference depends on the geometry of the system such as the thickness of the steel frame. The absolute pressure for a time harmonic analysis with different phase values $\Delta \varphi$ is shown in Fig. 3.22(c). The excitation frequency was 1470.35 kHz which is exactly between the two separated modes. The nodal pressure line is doing a local rotation when the phase is varied. There exist four rotation spots in a domain $\lambda_x \times \lambda_y$ where two spots are doing a clockwise rotation and the other two a counter clockwise rotation. This is identical to the amplitude modulation in Sec. 3.3.

Figure 3.22: (a) 2D model of a cavity filled with water and surrounded by a steel frame. The excitation is done by a prescribed displacement in the $x$- and $y$-direction at the outer side walls of the steel frame. (b) Modal analysis of the model showing the two slightly separated modes. (c) Absolute pressure for a time harmonic analysis at a constant excitation frequency of 1470.35 kHz and for different phase values $\Delta \varphi$. The white arrow is representing the position and orientation of a fiber in the pressure field.
The pressure and the phase of the pressure for two points inside the fluidic cavity are shown in Fig. 3.23. The prescribed displacement $u_0$ was set to 1 nm and the other direction $v_0$ was set to zero. The peak of $P_2$ corresponds only to the first mode at 1468.8 kHz and the peak of $P_1$ only to the second mode at 1471.9 kHz. This can be seen also from the pressure plots of the acoustic cavity in Fig. 3.23 pointing at the peaks. Of special interest is the pressure field in between the two peaks at 1470.3 kHz. The pressure shows a circular pattern as known from two orthogonal modes with a phase shift of 90°, which is shown in Fig. 4.1 on p. 106. The phase difference $\theta$ between the two separated modes is 85° at a frequency of 1470.3 kHz. The damping and the frequency difference between the two modes determines the phase difference $\theta$. In experiments with copolymer particles, the characteristic forming of clumps was observed by the actuation of only one electrode. Even rotating particle clumps due to viscous torque (see Sec. 4) have been observed.

**Figure 3.23:** Pressure and phase plotted as function of frequency for the two points $P_1$ and $P_2$. The prescribed displacement $u_0$ was set to 1 nm and the other direction $v_0$ was set to zero. The pressure plots of the fluidic cavity in the top row belong to three characteristic frequencies.
3.4. Phase modulation of slightly separated degenerated modes

Analytical model for pressure field

The different pressure fields in the fluidic cavity can be described analytically in the following way. The first mode forming the cross shaped pattern in Fig. 3.23 is:

\[ p = A_{11} [\cos(k_1 x) - \cos(k_1 y)] \sin(\omega_1 t) \] (3.12)

It is a degenerated mode of a standing wave in \( x \)- and \( y \)-direction. The second mode with the diamond shape pattern is:

\[ p = A_{12} [\cos(k_2 x) + \cos(k_2 y)] \sin(\omega_2 t) \] (3.13)

with \( k_i = \omega_i / c_0 \). Both modes can be combined at one angular frequency \( \omega \) using the phase difference \( \theta \) between the modes:

\[ p_{e1} = A_{11} [\cos(k x) - \cos(k y)] \sin(\omega t) + A_{12} [\cos(k x) + \cos(k y)] \sin(\omega t + \theta) \] (3.14)

where \( \theta \) depends on damping and frequency difference. For the rotation with phase modulation, a second excitation is necessary which is orthogonal to the first excitation \( p_{e1} \). The pressure field of the second excitation is composed of the same components but the \( x \) and \( y \) variables are interchanged and an additional phase shift \( \Delta \varphi(t) \) is introduced which is slowly varied over time.

\[ p_{e2} = A_{21} [\cos(k y) - \cos(k x)] \sin(\omega t + \varphi(t)) + A_{22} [\cos(k y) + \cos(k x)] \sin(\omega t + \varphi(t) + \theta) \] (3.15)

The squared and time averaged first order pressure \( \langle p^2 \rangle \) for the superimposed \( p_{e1} \) and \( p_{e2} \) can be derived and plotted. The result for a variation of \( \Delta \varphi \), a constant \( \theta = \frac{1}{2} \pi \) and constant and equal amplitudes \( A \) is depicted in Fig. 3.24(a).

Nodal pressure line orientation

For a uniform rotation of the nodal pressure line, the phase shift \( \Delta \varphi(t) \) has to be varied linearly over time. The variation of the phase shift is much slower (in the order of seconds) compared to the periodic time of the excitation (in the order of \( \mu s \)). The angle \( \beta \) of the nodal pressure line can be derived as function of the phase \( \Delta \varphi \). The \( \langle p^2 \rangle \) term is set to zero and solved for \( y \). The slope was derived for a certain position which is one of the
locations of rotation \( \left( \frac{1}{4} \lambda_x; \frac{1}{4} \lambda_y \right) \). The phase difference \( \theta \) between the two separated modes was set to \( \frac{1}{2} \pi \).

\[
\beta = \arctan \left( \frac{dy}{dx} \right) = \arctan \left( \frac{\sin(\Delta \varphi)}{\sin(\Delta \varphi) + \cos(\Delta \varphi)} \right) \quad (3.16)
\]

The non-spherical object is doing a rotation of 180° for a variation of the phase \( \Delta \varphi \) from 0 to 2\( \pi \).

The influence of deviating amplitudes and of the phase difference \( \theta \) between the two separated modes is depicted in Fig. 3.24.

The phase difference \( \theta \) has no influence on the uniformity of the rotation. The pressure field for a \( \theta = \frac{1}{4} \pi \) is shown in Fig. 3.24(b). At \( \varphi = \frac{1}{2} \pi \) and \( \frac{3}{2} \pi \), the difference can be seen best. The two straight pressure lines in (a) are deformed into four spots. This is practical as for the rotation of particles an exchange from one rotation spot to another can be avoided. The \( \theta \) depends mainly on the damping and the separation of both modes and is difficult to influence for a complex device. It is important to notice that the uniformity of the rotation is not influenced by that factor. Only for a \( \theta \) of 0 and \( \pi \) there is no rotation as the pattern at \( \Delta \varphi = \frac{1}{2} \pi \) and \( \frac{3}{2} \pi \) becomes a point shape.

The influence of the amplitudes is shown in Fig. 3.24(c). The amplitudes \( A_{12} \) and \( A_{22} \) are chosen to be only half of \( A_{11} \) and \( A_{21} \). The rotation is getting nonuniform. The rotation at \( \Delta \varphi = 0 \) is faster compared to \( \Delta \varphi = \pi \). The adjustment of equal amplitudes is complicated in the experiment. The main influence parameters are the excitation frequency and the actuation.

The maximal length of a non-spherical object is identical as for the amplitude modulation (see Sec. 3.3). The nodal pressure line is rotated continuously and the pressure field patterns are identical.

**Influence of the velocity field**

The \( \langle p^2 \rangle \) field and the \( \langle v^2 \rangle \) field do not have the same characteristics. This is discussed in the Section for the amplitude modulation (see Sec. 3.3) and illustrated in Fig. 3.15. When the \( \langle p^2 \rangle \) field is changing from a diamond shape to a line shape, the field \( \langle v^2 \rangle \) is changing from a point shape to a line shape. This term is not contributing to the rotation. It is only slightly influencing the angular position of a non-spherical object in addition to the \( \langle p^2 \rangle \) field and influencing the magnitude of the acoustic radiation torque. The exact solution for the angular position for any non-spherical object and every material combination can be derived using a finite element analysis as described in Sec. 2.4.
### 3.4. Phase modulation of slightly separated degenerated modes

**Figure 3.24:** Contour plot sequence of the squared and time averaged first order pressure $\langle p^2 \rangle$ as a result of phase modulation (variation of phase $\Delta \varphi$) for the two superimposed pressure fields $p_{e1}$ and $p_{e2}$. The bright gray lines are the maximum of $\langle p^2 \rangle$, the black lines are indicating the minimum of $\langle p^2 \rangle$. (a) For a $\theta$ of $\frac{1}{2} \pi$ and equal amplitudes $A$. The gray arrow is representing the position and orientation of a fiber. (b) For a $\theta$ of $\frac{1}{4} \pi$ and equal amplitudes $A$. (c) For the unequal amplitudes $A_{11} = 2A_{12} = A_{21} = 2A_{22}$ and a $\theta$ of $\frac{1}{2} \pi$. 

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3.4.2 Experimental results

The experiments have been performed using the micro-device presented in Sec. 3.1. For the characterization of the device behavior, first copolymer particles have been used. This allows the observation of the whole cavity by using a high amount of particles, compared to a single or few glass fibers where it is difficult to find a working frequency. The rotation with phase modulation was discovered in experiments first, as it is easy to excite. For the excitation, two possibilities exist. One is the method described in the theory and modeling section, where two electrodes are excited with the exact same frequency and the phase of one signal $\Delta \varphi(t)$ is varied slowly over time. The direction of the rotation is defined by the sign of the phase shift and the rotational velocity by the time $T_M$ for two modulations of the phase $\Delta \varphi$ from 0 to $2\pi$.

Another method is the excitation with two slightly different frequencies $f_1$ and $f_2$. A slight frequency difference between both signals ($\Delta f \approx 1\text{ Hz}$) will lead to a slow linearly varying phase shift over time between both signals. The difference $f_2 - f_1 = \Delta f$ between both frequencies determines the rotational speed. The modulation time $T_M$ for an object rotation of $360^\circ$ is defined as $T_M = 2/|\Delta f|$. The direction of rotation is depending on which of the two frequencies is larger.

In the method and modeling part of this section it was claimed that two degenerated modes with a phase shift $\theta$ are excited with a single electrode. An experimental investigation of this can be seen in Fig. 3.25. The experiments have been done with copolymer particles (17 $\mu$m). Fig. 3.25(a) shows the formation of a cross pattern with excitation of electrode

![Figure 3.25](image)

Figure 3.25: Single electrode excitation with micro devices showing degenerated modes with copolymer particles (17 $\mu$m) (a) Excitation of electrode 1 with an excitation frequency of 1433 kHz leading to a degenerated mode. (b) Excitation of electrode 1 with an excitation frequency of 1444 kHz showing formation of circular clumps, indicating a phase shift $\theta$ of about 90$^\circ$ between orthogonal modes.
3.4. Phase modulation of slightly separated degenerated modes

1 and a frequency of 1433 kHz. This corresponds to the degenerated mode shown in the simulation in Fig. 3.23 at the lowest frequency. An increase of the excitation frequency in the experiment to 1444 kHz for the same electrode 1 leads to the formation of nearly circular clumps (see Fig. 3.25(b)). Moreover, the partial rotation of clumps was observed, which is generated by the acoustic viscous torque (see Sec. 4). This is an indication for a phase shift $\theta$ of 90° between orthogonal modes. The mode shape with a circular pattern can be seen as well in the simulation in Fig. 3.23.

Rotation of particle clumps

The rotation of particle clumps is shown in Fig. 3.26, a rotation of 180° is depicted. The square fluidic cavity of the micro device was filled with copolymer particles with a diameter of 17 µm. The excitation frequency for electrode 1 was $f_1 = 1434$ kHz and $f_2 = 1434$ kHz $+ \Delta f$ for electrode 2. The frequency matches to 3 wavelengths in the $x$- and $y$-direction. This leads to a formation of 36 clumps. A domain of $\lambda_x \times \lambda_y$ is highlighted with a white square to show the change of the pattern from (a)-(e). The different patterns are: (a) diamond shape, (b) lines perpendicular to $x$-direction, (c) cross pattern, (d) lines perpendicular to $y$-direction and (e) diamond shape. The rotation of the clumps is a continuous rotation but not fully uniform as can be seen from the times given for every image of Fig. 3.26. The rotational velocity was controlled by varying the frequency difference $\Delta f$ and the direction of rotation by the sign of $\Delta f$. The $\Delta f$ was approximately 1.12 Hz which leads to a rotation time $T_M$ of 1.79 s and therefore an average rotational speed of 33 rpm.

In contrast to the amplitude modulation, the particle clumps are separated and not merging during the rotation. The reason is that $\theta$ is not $\frac{1}{2}\pi$ as shown in Fig. 3.24(b). When the pattern with straight lines is formed at $\Delta \varphi = \frac{1}{2}\pi$ and $\frac{3}{2}\pi$ the particles stay in clumps.

**Figure 3.26:** A 180° rotation of 36 particle clumps formed out of 17 µm copolymer particles with phase modulation. The excitation frequency for electrode 1 was $f_1 = 1434$ kHz and for electrode 2 $f_2 = 1434$ kHz $+ 1.12$ Hz. The excitation voltage $V_{rms}$ was 18 V. A domain of $\lambda_x \times \lambda_y$ is highlighted with a white square for better observation of the changing pattern shapes.
Rotation of micro glass fibers

The rotational manipulation with phase modulation was also performed with glass fibers and is shown in Fig. [3.27] In the fluidic cavity filled with DI-water there is fiber A and fiber B rotating in different locations. Fiber A consists of two glass fibers sticking together and has a total length of 315 $\mu$m. The fiber B is a single glass fiber with a length of 215 $\mu$m. The actuation frequencies were $f_1 = 1158$ kHz and $f_2 = 1158$ kHz + 0.55 Hz leading to a rotation time of $T_m = 3.64$ s per 360° rotation and a rotational speed of 16.5 rpm. The excitation frequency corresponds to 2.5 wavelengths in the $x$- and $y$-direction. In Fig. [3.27] the orientation $\alpha$ of both fibers is plotted. The fiber positions have been analyzed with the particle tracking tool of ProAnalyst (Xcitex). The fibers both rotate in clockwise direction. Beside the average rotational velocity of 16 rpm, instantaneous angular velocities of 82 rpm can be found. The rotation is not uniform due to unbalanced amplitudes of the modes and possible contact with the cavity floor. This is difficult to evaluate as the frame rate of the video was very slow. It can be seen best for fiber B. The rotational speed is about half of the maximal average angular velocity measured in experiments. Therefore the angle difference between the nodal pressure line $\beta$ and the fiber $\alpha$ is $-30^\circ$. By adding $-30^\circ$ to the fiber orientation $\alpha$ the angular velocities are fast at an angle of 45° and 225°. This angular position belongs to a phase $\Delta \varphi$ of $\pi$. The angular velocities are slow at an angle of 135° and 315° which belongs to a phase $\Delta \varphi$ of 0. In Fig. [3.24(c)] a similar case is shown for unequal amplitudes. In the experiment the amplitude difference is switched which leads to a faster rotation at $\Delta \varphi = \pi$ and a slow rotation at $\Delta \varphi = 0$. The maximum deviation of the position of the average fiber center is for fiber B in $x$- and $y$-direction $\pm 50 \mu$m. For the Fiber B the deviation is even higher as it is sticking to the cavity bottom and not doing a rotation around the center.

The maximum observed average rotational speed of a fiber was 30 rpm. The maximum average angular velocity depends on the applied acoustic radiation torque and is limited by the drag torque and friction force of the fiber at the cavity bottom. A detailed discussion can be found in Sec. [2.5.3] The maximum rotational speed is comparable with the amplitude modulation where an average rotational speed of 40 rpm was observed. It is assumed that the pressure is in the same range as for the amplitude modulation. The cavity and excitation are identical and the rotation techniques are comparable as well. A detailed discussion about the calculation of the pressure can be found in Sec. [3.3] on p. 85.
3.4. Phase modulation of slightly separated degenerated modes

Figure 3.27: Rotational manipulation of two glass fibers with phase modulation. Fiber A (×) consists of 2 glass fibers sticking together and has a total length of 315 µm. The fiber B (○) is a single glass fiber with a length of 215 µm. The actuation frequencies are $f_1 = 1158$ kHz at electrode 1 and $f_2 = 1158$ kHz + 0.55 Hz at electrode 2. The excitation voltage $V_{rms}$ was 20 V. The rotation time $T_M$ is 3.64 s for a 360° rotation. The plot is showing the orientation $\alpha$ of the glass fibers plotted as function of time.
3.5 Frequency modulation of slightly separated modes

Frequency modulation leads to the local rotation of the pressure field when two separated modes are existing. Additionally the phase difference $\theta$ at the middle frequency has to be larger than $90^\circ$. The principle of this method is described with the help of a simple finite element model and the important parameters such as the damping and frequency separation of the modes are discussed. A successful rotation has not been performed with this method but preliminary experimental data is presented.

3.5.1 Method and modeling

This method is related to the rotation with phase modulation (Sec. 3.4) as there is a separation of two modes needed. The phase difference $\theta$ has to be $> 90^\circ$ and $< 180^\circ$ to ensure a rotation. For the rotation with frequency modulation, the modes can be one-dimensional compared to the phase modulation were degenerated modes have been necessary. This method relies strongly on the Q-factor of the device and the frequency difference between the two modes. One method to enforce a separation of two modes is a small asymmetry. A nearly square chamber where one edge length differs slightly from the other length leads to such an asymmetry.

A simple finite element model is presented here to describe this rotation technique. Again, as for all other rotation methods only the pressure field with its nodal pressure lines is considered. A discussion on the influence of the velocity field can be found in Sec. 3.3 for the amplitude modulation. The model consists of only a 3D fluidic cavity modeled with the acoustics module of COMSOL and is depicted in Fig. 3.28(a). The edge lengths are $L_x = 1$ mm, $L_y = 1.005$ mm and $L_z = 0.2$ mm. The cavity is surrounded by hard boundary walls and the excitation is done with a normal acceleration at the bottom of the cavity shown as a red triangle in Fig. 3.28(a). The place and shape of the excitation is arbitrary, important is that all modes are excited therefore the excitation should be asymmetric. There are two points $P_1$ and $P_2$ defined which are used for the evaluation of the pressure and phase.

The material of the acoustic domain is water with the following properties: density of $998 \text{ kg/m}^3$ and speed of sound of $1481 \left[1 + i/(2Q)\right] \text{ m/s}$ where $Q$ is the Q-factor $^{72}$. The Q-factor is the ratio between the stored energy in the resonator and the loss of energy per cycle and therefore contains the damping of a resonator. Alternatively it is described as the resonance frequency divided by the bandwidth and is a measure of how narrow or
3.5. Frequency modulation of slightly separated modes

![Diagram](image)

**Figure 3.28:** (a) Schematic of the finite element model showing the fluidic cavity in the $xy$ plane. The cavity is surrounded by hard boundary walls and the excitation is done with a normal acceleration at the bottom of the cavity shown as a red triangle. (b) Result of a modal analysis showing the modes with one wavelength in the $y$-direction and one wavelength in the $x$-direction at different frequencies.

A broad a resonance peak is. The Q-factor of a system with two separated modes defines the overlapping of the modes.

The result of a modal analysis is shown in Fig. 3.28(b). The two one-dimensional modes occur at slightly different frequencies. The mode in $y$-direction is at a lower frequency as the edge length is chosen to be slightly larger. The larger the length difference of $L_x$ and $L_y$, the larger is the frequency difference.

The results of the absolute pressure and the phase plotted as function of the frequency can be seen in Fig. 3.29. The different graphs belong to the two points $P_1$ and $P_2$ and different Q-factors. The pressure of $P_1$ is maximal for the mode in $y$-direction and the pressure of $P_2$ is maximal for the mode in $x$-direction. For a high Q-factor of 5000 both modes will be completely separated with nearly no overlapping. For a sweep in the frequency range of 1465 kHz to 1490 kHz only the two modes are visible and very weak pressure patterns in between the two modes. Therefore a too high Q-factor is not useful to excite rotation. For a Q-factor of 500 the modes are overlapping. The pressure field for characteristic frequencies is shown in Fig. 3.30(a). The transition of the different patterns can be explained with the phase. At a frequency of 1470 kHz, both modes are in phase excited, leading to a diamond shaped pattern. At 1473.6 kHz the mode in $y$-direction has a high amplitude and is dominating. At 1477.3 kHz are both modes weakly excited and the first mode has a phase shift of nearly 180° compared to the second mode which leads to the cross shaped pattern. For a frequency of 1481 kHz the mode in $x$-direction has a high amplitude and is dominating. At a frequency of 1485 kHz both modes are weakly excited and are in phase again leading to the diamond shaped pattern. With a continuous frequency sweep the nodal pressure...
Chapter 3. Rotational manipulation by acoustic radiation torque

Figure 3.29: Plot of the absolute pressure and phase as function of the frequency. The different graphs belong to the two points $P_1$ and $P_2$ in the fluidic cavity and different $Q$-factors.

line is locally rotating. There exist four rotation spots in a domain $\lambda_x \times \lambda_y$ where two spots are doing a clockwise rotation and the other two a counter clockwise rotation. This is identical as for the amplitude modulation (Sec. 3.3) and phase modulation (Sec. 3.4). One frequency modulation (sweep from start frequency to end frequency) would lead to a rotation of $180^\circ$. The direction of rotation depends on the start frequency of the sweep. The rotational velocity is defined by the time of two frequency sweeps.

When the damping is too high and the overlapping of the peaks increases the rotation is not possible anymore. This can be seen in Fig. 3.30(b) for a $Q$-factor of 200. At the frequency in between the two modes the phase difference of the modes will be $90^\circ$ and a pattern with points is created. Therefore the torque on a non-spherical particle vanishes and a complete rotation is avoided. The object will do an oscillating rotation in both directions instead. The phase difference $\theta$ has to be $> 90^\circ$ to ensure a rotation and $\theta$ has to be $< 180^\circ$ to ensure the overlapping of the two separated modes and a sufficient pressure amplitude in between the two modes.
3.5. Frequency modulation of slightly separated modes

Figure 3.30: Results of the finite element model for different Q-factors. The absolute pressure field inside the cavity is plotted for five characteristic frequencies (a) with a Q-factor of 500, (b) with a Q-factor of 200. The white arrow is representing the position and orientation of a non-spherical object in the pressure field.

This rotation technique is not very uniform as the amplitude is varying strongly during the sweep. But the excitation is simple compared to the other methods as only one excitation is needed and a frequency sweep is very simple to implement. On the other hand precise control over Q-factor is difficult.

3.5.2 Experimental results

Only preliminary experimental results are available for this rotation technique. A complete rotation was not realized. No special device has been constructed for this technique. A micro-device with precise manufacturing and the common technique of excitation might be appropriate. Nevertheless in experiments with devices with a small cavity the different pressure fields were observed using only one excitation. The micro device was constructed as described in Sec. 3.1.

The first experimental results are shown in Fig. 3.31 and the device parameters are the following: The size of the fluidic cavity is $2 \text{mm} \times 2 \text{mm}$. The piezoelectric element $(3 \text{mm} \times 3 \text{mm})$ is divided into 4 strip electrodes as presented in Sec. 3.1. The excitation with electrode 3 is for all images the same. The experiments have been done with copolymer particles (17 $\mu\text{m}$) and the frequency was slowly increased and images have been taken of relevant patterns. The images correlate roughly with the pressure fields shown.
Chapter 3. Rotational manipulation by acoustic radiation torque

Figure 3.31: Experimental results with copolymer particles (17 µm) in a 2 mm × 2 mm fluidic cavity of a micro device. All results are achieved by excitation of electrode 3 at the indicated frequency.

in Fig. 3.30(a). The two patterns with the straight lines are interchanged, which can be modified in the model by interchanging the length of \( L_x \) and \( L_y \). The result for a frequency of 575 kHz might be a different resonance mode as the pattern is circular and the frequency is far below the other experimental results.

The reason for the separation of the two modes for the micro device is unknown but can be also an asymmetry resulting from the manufacturing process. The amplitudes of all modes have been very weak and a rotation with a frequency sweep was not successful. Additionally, other mode patterns might occur in between the shown frequencies and make the rotation impossible.

The second experimental results are shown in Fig. 3.32. The size of the fluidic cavity is only 1 mm × 1 mm and the piezoelectric element has a size of (4 mm × 4 mm). The

Figure 3.32: Experimental results with copolymer particles (17 µm) in a 1 mm × 1 mm fluidic cavity of a micro device. All results are achieved by excitation of electrode 2 at the indicated frequency.
3.5. Frequency modulation of slightly separated modes

excitation with electrode 2 is for all images the same and the experiments have been done with copolymer particles (17 µm) as for the experiment above. The results in Fig. 3.32(a) are for half a wavelength inside the cavity and the results in (b) are for one wavelength in the cavity. Not all necessary patterns have been observed and a rotation was not possible but the formation of the patterns was faster due to higher pressure amplitudes.
4 Rotational manipulation by the viscous torque

The viscous torque has been used to realize the rotation of spherical particles and evaluate the theoretical predictions of Sec. 2.6. In comparison to the rotation techniques using the acoustic radiation torque all parameters such as amplitude, frequency and phase stay constant. The excitation pressure field and the corresponding force potential field is discussed in Sec. 4.1. For the experiments with the viscous torque, single particles with varying size have been used. Therefore a macro device (Sec. 4.2) has been developed which is capable of handling large particles up to 500 µm. Additionally a device design was necessary which allowed a more controlled and stable excitation of orthogonal one-dimensional waves compared to the micro devices. The experimental results and the evaluation of the theory is topic of Sec. 4.3. The presented work was part of the master thesis of Andreas Lamprecht 57,105.

4.1 Method

For the excitation of the viscous torque two orthogonal standing waves with a constant phase shift $\Delta \varphi$ are necessary. The excited two pressure fields are:

\[
\begin{align*}
  p_{e1} &= A_1 \cos(kx) \sin(\omega t) \\
  p_{e2} &= A_2 \cos(ky) \sin(\omega t + \Delta \varphi)
\end{align*}
\]  

(4.1)

where $A_1$, $A_2$ are the pressure amplitudes, $k$ the wavenumber, $\omega$ the angular frequency and $\Delta \varphi$ the phase shift. A spherical particle suspended in a fluid and excited by the pressure field in Eq. (4.1) is exposed to the acoustic radiation force and the viscous torque. The acoustic radiation force defines the position of the particle and the viscous torque lets the particle rotate. The Gor’kov force potential can be used to calculate the equilibrium positions of the particle. Fig. 4.1 shows the squared and time averaged first order pressure
Chapter 4. Rotational manipulation by the viscous torque

\langle p^2 \rangle \) for different phase shifts \( \Delta \phi \). A denser and stiffer particle as used in the experiments will go to the minimum (black lines) of the pressure field which correspond to the nodal pressure lines. Depending on the particle density, the squared and time averaged first order velocity \( \langle v^2 \rangle \) has an influence on the particle position. The velocity field is for all phase shifts a point pattern such as the pressure field at \( \Delta \phi = \frac{1}{2} \pi \). The particles used in the experiment have a density close to water and the influence of the velocity field can be neglected. As can be seen from Fig. 4.1, the position of a particle is only at a phase shift of \( \frac{1}{2} \pi \) and \( \frac{3}{2} \pi \) exactly defined. The positions \( (X_0; Y_0) \) are \((\frac{1}{4} \lambda_x; \frac{1}{4} \lambda_y)\), \((\frac{3}{4} \lambda_x; \frac{3}{4} \lambda_y)\), \((\frac{1}{2} \lambda_x; \frac{3}{2} \lambda_y)\) and \((\frac{3}{4} \lambda_x; \frac{1}{4} \lambda_y)\). For other phase shifts the nodal pressure line increases from a point to a line where the particle is no longer in a well defined trap. A rotation of a free particle is therefore only at a phase shift \( \Delta \phi \) of \( \frac{1}{2} \pi \) and \( \frac{3}{2} \pi \) in the mentioned locations possible. The \([\sin(\Delta \phi)]\) term in the viscous torque equation (see Eq. (2.28)) is 1 or \(-1\). The terms \([\sin(kX_0)]\) and \([\sin(kY_0)]\) which are defined by the position of the particle are 1 or \(-1\) as well, with \( k = 2\pi/\lambda \) and \( X_0, Y_0 \) are one of the four previously mentioned locations. Therefore the viscous torque will have a maximum and the phase and positions are only influencing the direction of rotation. The magnitude of the viscous torque can be controlled by the amplitude of the pressure field. Other fixed values influencing the viscous torque are the sphere radius, thickness of the boundary layer, density and speed of sound of the fluid.

\[ \begin{align*}
\Delta \phi &= 0 & \Delta \phi &= 2\pi \\
\Delta \phi &= \frac{1}{4} \pi & \Delta \phi &= \frac{3}{4} \pi \\
\Delta \phi &= \frac{1}{2} \pi & \Delta \phi &= \frac{3}{2} \pi \\
\Delta \phi &= \frac{3}{4} \pi & \Delta \phi &= \frac{1}{4} \pi
\end{align*} \]

\( \varphi = \pi \)

**Figure 4.1:** Contour plot sequence of the squared and time averaged first order pressure \( \langle p^2 \rangle \) for two orthogonal standing waves in \( x \) and \( y \) direction with identical frequency and amplitudes for a varying phase difference \( \Delta \phi \). The bright gray lines are the maximum of \( \langle p^2 \rangle \), the black lines are indicating the minimum of \( \langle p^2 \rangle \).

### 4.2 Macro device for rotational manipulation

The correct choice of the device design is essential for the investigations of the viscous torque. A homogeneous and stable orthogonal standing wave field has to be formed and the phase shift \( \Delta \phi \) between the two excitations must be controllable. Different device
4.2. Macro device for rotational manipulation

designs have been experimentally tested and supported by finite element modeling. The final design can be seen in Fig. 4.2.

![Figure 4.2: Macro steel device for rotation of particles with the viscous torque. The two crossed fluid channels allow the excitation of two orthogonal standing waves. Each channel is excited with one piezoelectric transducer in direct contact with the fluid and closed by a steel reflector. A removable glass plate (not depicted) covers the fluid channels.](image)

The main body of the device is made of steel and manufactured by milling (Jean-Claude Tomasina, IMES, ETH Zurich). The material offers a high characteristic acoustic impedance difference to water and as the excitation is in direct contact with the fluid there is less energy transfer to the steel structure. The two crossed channels allow in the center the superposition of two orthogonal standing waves. An additional glass plate in the middle section of the crossed channels reduces the channel height to 1 mm. The excitation was done with piezoelectric elements (PZ26, Ferroperm Piezoceramics) with a thickness of 2 mm and an added mass (copper, 4 mm × 2 mm × 1.5 mm). For each direction one transducer was fixed with conductive Epoxy (EPO-TEK H20E, Epoxy Technology) to the channel ends. The other channel end was covered with a steel reflector. A removable glass plate was used to cover the channels and sealed with vacuum grease. A removable glass cover was necessary to introduce large particles. A large inlet would lead to a disturbance and evaporation due to a free water surface. The dimensions of the device have been chosen for frequencies in the range of 500 to 1000 kHz and particles up to 500 µm. First preliminary experiments with a large amount of copolymer particles showed the orthogonal superposition and phase shift depending patterns as seen in Fig. 4.1. The pressure amplitudes of both excitations $A_1$ and $A_2$ have to be equal to form a perfect point pattern for a phase shift of $\frac{1}{2} \pi$. This can be verified with the 45° lines for no phase shift. The experimental setup is similar to the one for a micro device experiment (see Sec. 3.1). The excitation of the transducers was done with two amplifiers (2100 RF power
amplifier, ENI) and one signal generator (Tektronix AFG 3022B) with two outputs and controllable phase shift.

4.3 Experimental results

After proving that the device is able to provide the conditions for an orthogonal standing wave field, the experimental investigations on the viscous torque were possible. The key aspect was to proof the relation $\Omega \propto 1/r_s$ in order to confirm the analytical results from Sec. 2.6. The viscous torque predicted by Lee and Wang [56] is not influenced by the stokes flow due to the rotation of the sphere. Experimental results have been also obtained for the investigation of the following properties influenced by the viscous torque:

- Rotation direction (location dependency, phase dependency)
- Angular velocity $\Omega$ (influence of excitation amplitude, particle shape, particle size)

Rotation direction

Particles placed in the pressure nodes of the orthogonal standing wave with a phase shift $\Delta \varphi = \frac{1}{2} \pi$ will show a change in the rotation direction in each pressure node of the wave field as the signs of $[\sin(kX_0)]$ and $[\sin(kY_0)]$ are changing. The experiments have been done with sodium chloride crystals (size $< 1 \mu$m). It was not possible to occupy the pressure nodes of the standing wave field with single particles. Therefore small particles are used which are forming circular clumps. Compared to small copolymer particles the sodium chloride crystals have a higher density and speed of sound and therefore the acoustic radiation force will be higher and acoustic streaming in the cavity is not dominating the behavior of particles in the size range of $1 \mu$m [75]. The result with the formed clumps and the indicated rotation direction can be seen in Fig. 4.3(a). The change of the phase shift to $\Delta \varphi = \frac{3}{2} \pi$ results in a change of the rotation direction for each particle location as can be seen in Fig. 4.3(b).

Angular velocity $\Omega$

The influence of the angular velocity $\Omega$ on the amplitude of the standing pressure wave was evaluated first. Thinking of future applications the amplitude will be the controllable variable to determine the angular velocity for a given particle. To avoid a change in the position of the particle the amplitude of both directions has to be controlled simultane-
4.3. Experimental results

Figure 4.3: Rotation as function of the location \((X_0, Y_0)\) and phase shift \(\Delta \varphi\). The experiment has been performed with sodium chloride crystals (size < 1 µm) forming large circular clumps. The arrows are indicating the rotation direction of the clumps. The excitation Voltage \(V_{\text{rms}}\) was 30 V and the frequency 846 kHz corresponding to \(\frac{1}{2} \lambda = 875\) µm. Two frames have been extracted from a video where the phase shift \(\Delta \varphi\) was switched from (a) \(\frac{1}{2} \pi\) to (b) \(\frac{3}{2} \pi\).

The variation of the excitation amplitude will cause a quadratic change of \(\Omega\) as \(A_1\) and \(A_2\) are always equal. In the experiment the excitation voltage of the piezoelectric element is varied, which is proportional to the pressure amplitude as shown in [76].

The sodium chloride crystals (size < 1 µm) forming circular clumps (size ≈ 100 µm) have been used again for this experiment as the angular velocity of a clump is much smaller compared to a single spherical particle and only a camera with 60 fps was available. The results for three different particle clumps can be seen in Fig. 4.4.

The excitation Voltage \(V_{\text{rms}}\) has been varied between 23 V and 32 V. The particle clumps have a different angular velocity as they differ slightly in size and shape. Additional deviations occur due to the fact that during the experiment the particle clumps were growing, changing shape and the temperature increased. The graphs show that the rotational

Figure 4.4: Variation of rotational speed \(\Omega\) as function of the excitation voltage \(V_{\text{rms}}\). The three black curves belong to three particle clumps with similar size (diameter of about 100 µm). The gray lines are two quadratic fits of the experimental data.
speed is not varying linear. A quadratic fit of the experimental data is problematic. The rotation of the clumps stopped for excitation amplitudes below 20 V. This was considered in a second fit. Due to the high deviation in the experimental data it is not possible to clearly show the quadratic relation. The experiment has to be redone with single particles and a high-speed camera to reproduce the exact quadratic behavior.

An additional output of this experiment was that the angular velocity depends strongly on the shape of the clump. A spherical clump will have a much higher angular velocity compared to an elliptical or even triangular clump. In a patent [21] it is mentioned that an object with a slight variation in the spherical shape rotates much slower by viscous torque. This effect was observed with a triangular shaped particle clump which rotated with an rotational speed $\Omega = 15\text{ rpm}$. A circular particle clump with a similar characteristic diameter rotated much faster with $66\text{ rpm}$. In the literature no theoretical information was found for the viscous torque on non-axisymmetric particles.

The evaluation of the relation between the sphere size and the angular velocity $\Omega$ was the most important experiment. Due to the high rotational velocities of the particle a high speed camera (Fastec Imaging, HiSpec 1, mono, $1280 \times 1024$ pixels at 506 fps) was necessary. The videos have been recorded with a frame rate of 506 fps and the angular velocity was measured by counting the frames for a full rotation of the sphere. To visualize the rotation of a spherical particle, one half of the sphere has been coated with gold. The particles have been placed on a glass plate and a 20 to 30 nm gold layer was sputtered on the particles. The additional layer on the particle surface did not influence the rotation of the particles. Two types of particles have been used in the experiment depending on the size range. Spherical PMMA (Kisker Biotech) particles in the range of 150 to 650 $\mu$m and copolymer particles (Kisker Biotech) with a diameter of 70 to 140 $\mu$m. The material properties of these particles are similar and therefore the results are comparable. PMMA has a density of 1190 kg/m$^3$ and a compressibility of $1.111 \times 10^{-10}\text{ Pa}^{-1}$ [106]. The properties of a copolymer particle are a density of 1050 kg/m$^3$ and a compressibility of $1.058 \times 10^{-10}\text{ Pa}^{-1}$.

Single particles were manually selected, measured, and placed individually in the fluid chamber to avoid any influence of the other particles. Additionally to the other experiments it was important to avoid air bubbles within the fluid chamber. This was realized by placing the device into a vacuum chamber to degas the fluid. An air bubble will strongly influence the angular velocity of a particle due to additional acoustic streaming induced by the air bubble. The streaming behavior of an acoustically excited air bubble has been described in [107].
All experiments have been performed with an excitation frequency of 770 kHz and a voltage $V_{\text{rms}}$ of 32 V. A continuous rotation of the particles over a time period of 3 s was recorded. The observed experimental data is shown in Fig. 4.5. For every particle 3 to 4 measurements have been done and the average value and the maximum deviation have been calculated and plotted. The $1/r_s$ dependency of the angular velocity as predicted from the analytical theory can be seen in the graph. A larger particle will have a smaller angular velocity. For small particles high rotational velocities with up to 1200 rpm have been measured.

The unknown pressure amplitude in the device can be fitted with the experimental data using Eq. (2.29) and Eq. (2.30). The fitted theoretical calculation is plotted in Fig. 4.5. A pressure amplitude $A$ of 0.18 MPa was determined. The following fluid properties have been used for the calculation. A boundary layer $\delta$ of $6.428 \times 10^{-7}$ m, a density of 998 kg/m$^3$, a speed of sound of 1481 m/s and a dynamic viscosity of $1 \times 10^{-3}$ Pa.s.

The experiments lead to a viscous torque of $1.2 \times 10^{-13}$ Nm for the rotation of a small particle with 1200 rpm and a radius of 35.5 $\mu$m. For the large particles with a radius of $r_s = 223$ $\mu$m and a rotational speed of 110 rpm a torque of $3.2 \times 10^{-12}$ Nm was determined.

It should be mentioned that for the large particles with $r_s = 223$ $\mu$m the long wavelength condition with $kr_s \ll 1$ for the viscous torque (Eq. (2.28)) is not fully fulfilled as $kr_s \approx 0.7$. The Reynolds number limit ($\text{Re} < 10$) for the calculation of the drag torque is still valid. The condition $\text{Re} \ll 1$ is not valid anymore and the inertial terms can not be neglected. However, for the evaluation of a steady state angular velocity such as observed in the

---

**Figure 4.5:** Rotational speed $\Omega$ of spherical particles as function of the particle radius $r_s$. The experiments have been performed with an excitation frequency of 770 kHz and a voltage $V_{\text{rms}}$ of 32 V. The measured data points are plotted as circles. The analytical model is plotted as gray lines for different fitted pressure amplitude $A$ of 0.2 MPa, 0.18 MPa and 0.16 MPa.
Chapter 4. Rotational manipulation by the viscous torque

experiments the inertial term is not necessary. The rotation with viscous torque is due to the quadratic influence of the amplitude and the possibility to fit the curve with different particle sizes a promising tool to evaluate the pressure in devices capable of exciting two dimensional pressure fields.
5 Conclusions and outlook

The rotation of particles was shown caused by the two possible torques arising in ultrasonic standing waves, the acoustic radiation torque and the viscous torque.

Non-spherical particles experience additionally to the acoustic radiation force a torque. The analytical solutions for the acoustic radiation torque on a non-spherical object are limited to simple cases such as a rigid ellipsoid. This was the motivation for a finite element simulation of the acoustic radiation force and torque on arbitrary shaped objects in arbitrary acoustic fields. The validation with a simple analytical model showed perfect agreement. The finite element model was used to predict the equilibrium position and orientation of a micro glass fiber such as used in the experimental part. Moreover, different influencing parameters have been evaluated such as the fiber length, the diameter, the frequency and pressure amplitude. The following aspects were discovered for a micro glass fiber: The acoustic radiation torque stays nearly constant at low frequencies (kHz range) for wavelengths 10 times larger than the fiber. This is in contrast to the acoustic radiation force which is proportional to the frequency. The equilibrium position and orientation of a fiber shorter than a quarter wavelength is at the pressure nodes, aligned perpendicular to the wave propagation direction. For larger fibers additional equilibrium positions occur. The torque varies approximately sinusoidally as function of the fiber orientation $\alpha$ compared to the orientation of the nodal pressure line $\beta$. The position and orientation in a 2D standing wave is more complicated due to the different characteristics of the velocity field. As an approximation a short fiber will align with the nodal pressure line and position where the $\langle p^2 \rangle$ term has a minimum and the $\langle v^2 \rangle$ term a maximum. This knowledge was important for the development of rotation techniques with the acoustic radiation torque.

The first presented rotation method with the acoustic radiation torque was based on the changing of the propagation direction of one dimensional standing waves. A hexagonal cavity design in combination with three piezoelectric transducers was used to change the propagation direction of the standing wave in 60° steps. The rotating object stopped only at discrete angular positions defined by the cavity. The rotation with a micro fiber
(length 205 µm, diameter 15 µm) at an excitation frequency of 1730 kHz showed a maximal average rotational speed of 34 rpm and instantaneous rotational speeds of about 300 rpm. This rotation method was less complicated due to 1D standing waves but was restricted to discrete rotation steps.

Further rotation methods with the acoustic radiation torque rely on the continuous variation of the pressure field. Therefore a uniform rotational velocity is possible and the orientation at arbitrary defined angular positions. The amplitude modulation (slow varying of the amplitudes over time) of two ultrasonic modes leads to a local rotation of the nodal pressure line. In a domain of $\lambda_x \times \lambda_y$ there exist four rotation spots where two showed a clockwise rotation and the other two a counter-clockwise rotation. The rotation speed and direction was controlled by the amplitude modulation (modulation frequency and characteristic). The amplitudes have to be varied sinusoidally for a linear variation of the nodal pressure line $\beta$. The rotation of a micro fiber was successfully realized and a maximum average rotational speed of 40 rpm was observed at an excitation frequency of 1085 MHz. During one rotation, deviations in the angular velocity occurred, leading to instantaneous velocities up to 200 rpm. The reasons were unbalanced amplitudes, not perfectly excited modes, the linear amplitude modulation and contact to the cavity floor.

The phase modulation is based on slightly separated degenerated modes due to influences of the device at fluid resonances (no rigid walls and actuation). The excitation was done with two sources at the frequency in between both modes and a slow variation of the phase difference $\Delta \varphi$ between the excitations. Another method was the excitation with two slightly different frequencies ($\Delta f \approx 1$ Hz) which led to a slow variation of the phase difference $\Delta \varphi$. The rotational speed and the direction of rotation was determined by the magnitude and sign of the frequency difference. A maximum average rotational speed of 30 rpm has been observed with a micro fiber at an excitation frequency of 1158 kHz.

The frequency modulation leads to a local rotation of the nodal pressure line when two separated modes exist. The phase difference $\theta$ between both modes at the middle frequency has to be $> 90^\circ$ and $< 180^\circ$. The modes split by a small difference in the length of the edges of a nearly square chamber. Only one excitation and a frequency sweep were necessary. In the experimental part no rotation was realized with this method, only preliminary results were available.

The rotation with amplitude, phase and frequency modulation were similar in the characteristic of the resulting pressure field. The amplitude modulation offered the best control for a uniform rotation but additional equipment besides a signal generator was necessary. The mechanism behind the phase modulation was more complicated due to the sep-
rated degenerated modes. The excitation was simple but the achievement of a uniform rotation is complicated. As an advantage, the merging of particles rotating at close by positions was avoided. This can be useful for the separated mixing of particle clusters in one chamber. The frequency modulation convinced with only one excitation and a simple frequency sweep. A uniform rotation was very complicated and the Q-factor needs to be precisely controlled. All methods were appropriate for the alignment of objects at arbitrary defined positions.

The acoustic radiation torque and the pressure amplitude were estimated by comparison with the drag torque. Therefore the experimental results of the amplitude modulation were used. A pressure of 0.18 MPa and an acoustic radiation torque of $1.84 \times 10^{-14}$ Nm were determined. For the instantaneous velocities up to 200 rpm a pressure amplitude of 0.4 MPa was determined and a maximal acoustic radiation torque of $9.4 \times 10^{-14}$ Nm. The influence of adhesion and friction of the fiber at the cavity bottom was neglected. Therefore the pressure amplitude in the cavity and the driving torque might be larger. For a reasonable pressure amplitude of 0.5 MPa, a perfectly excited mode and a levitating fiber, a radiation torque of $3.6 \times 10^{-13}$ Nm and rotational speeds of 780 rpm were predicted. The quadratic influence of the pressure led to the strong increase of the torque.

Moreover, the viscous torque on spherical particles was studied. The viscous torque is generated by two orthogonal standing waves shifted in phase. The boundary streaming spins an axisymmetric object. The analytical model for the torque at a fixed sphere was given in the literature. Additionally, Lamprecht et al. 57 investigated the influence of a rotating sphere on the viscous torque due to the additional stokes flow. It was shown that the rotation of the sphere had no influence and the drag torque and viscous torque allowed the calculation of the angular velocity of the sphere. A macro device was designed and used in the experimental work. The location and phase dependency of the rotation direction were shown. A freely movable, spherical particle in an orthogonal standing wave was only rotated for a phase shift of $\Delta \varphi = \frac{1}{2} \pi$ and $\frac{3}{2} \pi$. The rotational speed of the particle is therefore only defined by the pressure amplitudes. Additionally, the rotational speed depended on the particle size with $\Omega \propto \frac{1}{r_s}$. This relation was shown experimentally by measuring the angular velocity of different particle sizes and fitting the curve with an unknown pressure amplitude of 0.18 MPa. The experiments led to a viscous torque of $1.2 \times 10^{-13}$ Nm for the observed rotation of a small particle with 1200 rpm and a radius of 35.5 µm. For the large particles with a radius of $r_s = 223$ µm and a rotational speed of 110 rpm, a torque of $3.2 \times 10^{-12}$ Nm was determined.
Chapter 5. Conclusions and outlook

A comparison of the acoustic torque with other rotational manipulation techniques (see Sec. 1.3) is difficult as the size range of the object depends on the used technique or no torque values were specified. The acoustic torque investigated in this study led to torques of $1 \times 10^{-13}$ Nm for the acoustic radiation torque and $3 \times 10^{-12}$ Nm for the viscous torque for objects in the $\mu$m range. A literature survey of rotation methods showed a torque with acoustic vortex beams of $6.5 \times 10^{-9}$ Nm [23] and a torque with surface acoustic waves of $60 \times 10^{-9}$ Nm [29] for mm sized objects. For the optical manipulation of particles in the nm to $\mu$m range an experimental torque of $2 \times 10^{-17}$ Nm was found and a theoretical torque in the order of $10^{-15}$ Nm [48] was claimed. The elektrokinetics and magnetically manipulation are situated in the nm to $\mu$m range as well. The method of choice depends strongly on the material properties (acoustical, optical, electrical) as well as the size and shape of the particle.

Outlook

The continuous rotation of spherical and non-spherical micro particles were demonstrated and first experimental results and theoretical predictions were done. In the introduction, a few possible future applications were mentioned. There is still a long road ahead until the realization of an application including rotational control. It would be a step towards micro robotics with acoustics or an element of micro assembly.

The controlled and stable excitation of the required pressure modes is difficult for micro devices and an improvement is necessary. A problem arises when only a single particle is used, such as for the rotation of a micro fiber, where the overview of the whole mode in the fluidic cavity is missing. Due to temperature variations or different filling conditions the resonance frequency might shift. Methods such as the schlieren imaging or interferometer measurements might be helpful to improve the experimental setup.

The focus of this thesis was the development and investigation of different rotation principles rather than the optimization of the rotation techniques. The realization of levitation, with an additional standing wave in $z$ direction, to avoid surface contact of the fiber during the rotation is very crucial. A further improvement of the amplitude modulation can lead to rotational speeds up to 1000 rpm. The positioning and alignment accuracy of objects have to be studied especially to further applications in the area of micro assembly.

The accurate measurement of the pressure amplitude inside the cavity with methods such as presented by Barnkob [89] allows for the comparison with the determined pressure amplitude from the rotational experiments. This would lead to the magnitude range of the neglected effects such as friction and acoustic streaming. Also high-speed imaging
and synchronized recording of the excitation parameters would supply insights for the rotational manipulation.

The numerical simulation of the acoustic radiation torque showed a constant torque at low frequencies (kHz range). The theoretical and experimental investigation of the movement of a fiber inside a standing wave as a function of the frequency might be interesting. As the acoustic radiation force decreases with decreasing frequency, the dynamic behavior of a fiber in the kHz range might be different from to the MHz range concerning the timescale of the movement and reorientation. Such theoretical studies are possible with the dynamic model presented by Hahn et al. [100].

The first resonance modes of fibers (length 200 µm) occur at frequencies of 1.6 MHz. This is in the range of the typical excitation frequency for ultrasonic manipulation. The particle resonances lead to high amplitudes in the acoustic radiation force and torque and even sign changes. Experimental investigation of this phenomena would be interesting. This could lead to a very sensitive manipulation and sorting mechanism.

The viscous torque due to the boundary streaming appears either at spherical or non-spherical particles. The cases for non-spherical particles need to be investigated further theoretically as well as experimentally.
A Ultrasonic manipulation of Salmonella in planar resonators

Ultrasonic manipulation is a contactless and gentle method to manipulate a large number of particles. The method presented here exploits the advantage to simultaneously move bacteria away from a surface by means of acoustic radiation forces. The device for the manipulation consists of five layers (transducer, epoxy adhesive layer, carrier, fluid, reflector), stacked like a conventional planar resonator. The resonator behavior was simulated using the transfer matrix method (TMM). Validation of the model was realized with admittance measurements performed over a wide frequency range (100 kHz - 16 MHz). The TMM-model was used to optimize frequency, layer thickness and material of the resonator in order to find a combination with a high force potential gradient pointing away from the reflector surface into the fluid. The resonator has been experimentally tested with polystyrene particles (1 µm in diameter) which revealed a good matching with the TMM-model. First preliminary tests with Salmonella Thyphimurium have been done. The work in this chapter was a collaboration with Prof. Wolf-Dietrich Hardt, Daniel Andritschke and Benjamin Misselwitz from the Institute of Microbiology at ETH Zurich and has been published in [108].

A.1 Introduction

The acoustic radiation force can be used to position particles in prescribed patterns and positions. It is a contactless and gentle method to manipulate particles and cells. This method is perfectly suited for a large number of particles, handled in parallel. The method described here uses these advantages to simultaneously move bacteria away from a surface by means of acoustic radiation forces. Misselwitz et al. [109] performed previous experiments with centrifugation to force bacteria away or towards a surface with cells. This was part of a number of various experiments to study the mechanism of a bacteria finding and docking at target cells. A realization with acoustic radiation forces allows
additionally the observation with a microscope during the experiment. It is known that
the used ultrasound frequencies and amplitudes are not harmful to the bacteria [110],
qualifying acoustic micro manipulation for this type of application.

*Salmonella* are rod shaped bacteria with a length of about 2 µm which have flagella for
propulsion, reaching velocities up to 20 µm/s. Without any external influence *Salmonella*
will typically tend to move along surfaces. The device is designed in such a way to
provide a method to force *Salmonella* away from the surface into the fluid. Therefore a
high acoustic force is needed at the fluid reflector boundary, which is pointing into the
fluid. In further experiments, there will also be the possibility of growing cells at the
reflector surface to observe the interaction between these cells and *Salmonella*.

An ultrasonic standing wave resonator typically has a pressure antinode at the fluid-
structure interface. There the force on particles is zero or very small. The TMM-model
was used to optimize frequency, layer thickness and material of the resonator in order to
find a combination with a high force potential gradient pointing away from the reflector
surface into the fluid.

Forcing particles to a surface is realized in so called quarter wavelength planar devices
[111]. They are normally used to force particles onto the reflector to improve bio-sensing.
This technique has been extended by using two different quarter wave modes [112], one
for forcing particles to the reflector and the other one for forcing particles to the carrier.
The application used here is not restricted to a quarter wavelength. There could exist
more than one nodal pressure plane in the fluid layer.

A number of devices have been reported which focus on the filtration of bacteria [110,113].
The ultrasonic standing waves are used to form clumps of bacteria in order to enable and
enhance their sedimentation. The device presented here focuses only on the fluid-reflector
interface and the formation of clumps should be avoided.

**A.2 Planar resonator device**

The devices for the manipulation of *Salmonella* are planar resonators (see Fig. A.1(a))
and consist of five main layers (transducer, epoxy adhesive, carrier, fluid, reflector). The
observation during experiments is done with a confocal microscope and therefore the
reflector is a microscope cover slide (MENZEL). Two different cover slides have been
used, #1 and #5 with thicknesses of 145 µm and 560 µm, respectively. The two reflectors
lead to a different microscope image quality and operation frequency of the resonator and
therefore the carrier and fluid layers have also different sizes. All dimensions are listed in Table A.1. The lateral size is in the range of a normal microscope slide (75 mm $\times$ 26 mm) to be easily mounted on a confocal-microscope stage. The aluminum carrier is used as a matching layer and for mounting the device. The fluid cavity is milled inside the carrier. The thickness was optimized using the TMM-model. The excitation is done with a piezoelectric transducer (Ferroperm PZ26, 5 mm $\times$ 5 mm) with a thickness of 510 $\mu$m, glued to the carrier with conductive epoxy (Epotek H20E). The resonator is driven near the first and second resonance frequencies at 4.2 MHz and 12.6 MHz of the piezoelectric plate.

<table>
<thead>
<tr>
<th>Layer (Material)</th>
<th>Resonator A</th>
<th>Resonator B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transducer (Ferroperm PZ26)</td>
<td>510 $\mu$m</td>
<td>510 $\mu$m</td>
</tr>
<tr>
<td>Adhesive layer (Epotek H20E)</td>
<td>8 $\mu$m</td>
<td>4 $\mu$m</td>
</tr>
<tr>
<td>Carrier (aluminum)</td>
<td>2890 $\mu$m</td>
<td>2800 $\mu$m</td>
</tr>
<tr>
<td>Fluid (water)</td>
<td>225 $\mu$m</td>
<td>195 $\mu$m</td>
</tr>
<tr>
<td>Reflector (MENZEL cover slide) (#1)</td>
<td>(#1) 145 $\mu$m</td>
<td>(#5) 560 $\mu$m</td>
</tr>
</tbody>
</table>

The reflector is only clamped with a plate (see Fig. A.1(b)) to the carrier to be easily exchangeable. This is needed for further experiments were cells should be grown at the reflector glass. The ambient temperature for the final experiment should be 37 $^\circ$C and the fluid temperature should not rise during the experiment. Therefore, active cooling of the carrier or the piezoelectric transducer needs to be added in the near future.

Figure A.1: (a) Schematic view of the planar resonator and its layers. (b) Picture of the fabricated manipulation device. The transducer is fixed to the middle of the back side of the carrier and is therefore not pictured. The fluid is confined between the carrier and reflector.
A.3 Resonator model and validation

There exist multiple methods for the simulation of resonators. The finite element method is especially useful for three dimensional simulation of complex resonators and complex coupling of the excitation into the fluid [55]. The study of the resonator behavior used here could be simplified to a one dimensional model and therefore the transfer matrix method (TMM) presented in [114] for piezoelectric resonators is used. A detailed explanation of the TMM-model for ultrasonic manipulation, extended with calculations of the acoustic field properties is given in [72]. A straightforward one dimensional model was preferred here instead of a complex FEM-model to reduce the number of possible optimization parameters and to investigate the influence of the resonator layer thicknesses, whilst neglecting all effects from the lateral dimensions. Another possible one dimensional model would be the equivalent-circuit transducer model [115].

The TMM is based on the continuity condition of the electrical (electrical potential) and mechanical (displacement, stress) values at the interfaces of a planar resonator. If the values are known at the beginning of a layer they could be calculated for the end of the layer using the transfer matrix, which depends only on the material properties of the layer itself. A complete planar resonator could then be calculated by multiplying all transfer matrices of the different layers, knowing the boundary values at one side.

The first one-dimensional resonator model used for the present study consists of the four layers: transducer, carrier, fluid and reflector shown in Fig. A.2(a). For the validation process of the model an admittance measurement over a frequency range of 100 kHz to 16 MHz has been used. The measured and modeled admittance have been plotted in Fig. A.3(a). All material properties of the layers have been obtained from the suppliers. The thicknesses have been measured and the missing Q-factors have been fitted with admittance measurements. To this aim, the piezoelectric transducer alone, when attached to the carrier as well as the complete resonator have been measured. The following Q-factors have been determined: piezoelectric transducer 150, carrier 100, fluid 50 and reflector 100. The first model shows large deviations to the admittance measurements. Therefore the model was extended with an adhesive layer and the implementation of the electrical connection. The result is plotted in Fig. A.3(b) and shows a very good agreement between model and measurement. Even if the adhesive layer is very thin in comparison to the other layers, it could not be neglected. The adhesive layer is in fact strongly influencing the Q-factor of the whole device and can lead to a slight change of the resonance frequencies. A Q-factor of 5 was determined for the adhesive layer.
A.3. Resonator model and validation

Figure A.2: (a) Absolute value of admittance $|Y|$ plot of measurement and model for resonator A. The model consists of the four main layers. (b) Absolute value of admittance $|Y|$ plot of measurement and extended model for resonator A. An adhesive layer and the implementation of the electrical connection led to very good agreement between measurement and model.

The Q-factors of the specific layers are smaller than the ones reported in [72]. The size of the transducer in the lateral direction is much smaller than that of the carrier, fluid and reflector layer. This is not represented in the model since in this one-dimensional model only the sound propagation in the $z$-direction is considered. There might be also some deviation in the parallel alignment of the layers. The additional spreading and loss of energy could be considered by using lower Q-factors. Considering the fact that as a consequence the admittance peaks are more damped for higher frequencies than for lower ones a Q-factor in between has to be chosen. The Q-factor is influencing only the height and width of a resonance peak and not the frequency of the resonance.

The electrical connection was also taken into account in the model as it strongly influences the admittance. A simple elementary component of a transmission line consisting of
Appendix A. Ultrasonic manipulation of bacteria in planar resonators

resistance (2 Ω), inductance (690 nH), conductance (50 µS) and capacitance (155 pF) was sufficient to model the peak at about 9 MHz in the admittance plot (Fig. A.3(b)). This peak is a combination of the transducer and the connected 1 m coaxial cable.

The position of all peaks in the measurement matches well with the model, only the peak height deviates due to the compromise with the Q-factor. At frequencies below 1 MHz, small peaks in the measurement arise, which are caused by the lateral resonances of the piezoelectric transducer, neglected in the one dimensional model. The peaks at 4.2 MHz and 13 MHz can be attributed to the piezoelectric transducer surrounded by resonances of the carrier and fluid cavity.

The devices were optimized by searching the maximum acoustic radiation force $F_{rad}$ at the reflector interface pointing into the fluid. The carrier and fluid thickness are free parameters. The thickness of the reflector is limited to the above mentioned cover slides.

The acoustic radiation force in the fluid layer of resonator A plotted in Fig. A.3(a) is derived from the gradient of the Gor’kov potential acting on a 1 µm copolymer particle. The black curve represents the highest absolute force value in the fluid layer. When the force $F_{rad}$ at the reflector is positive the particle will be forced to the reflector. If $F_{rad}$ is negative the particle will be forced into the fluid. The results for resonator B are plotted in Fig. A.3(b).

For resonator A only the frequency range above 12 MHz is of interest for the application. This is due to the thin reflector glass. The frequencies below 6 MHz could be used to force

![Graph A](image1.png) ![Graph B](image2.png)

**Figure A.3:** (a) Maximal force acting on a 1 µm copolymer particle in the fluid layer of resonator A and the force acting on a particle at the reflector fluid interface. (b) Results for resonator B.
A.4 Experimental results

particles at the reflector. For resonator B a frequency of 4 MHz is of interest where the particles are forced into the fluid. Due to the given thicknesses of the cover slides and the piezoelectric transducer it was not possible to increase the negative force at the reflector. To achieve a high force at the interface the reflector should be in between $\frac{1}{4}\lambda$ and $\frac{1}{2}\lambda$ with $\lambda$ being the wavelength in the reflector. The force will be very small at the reflector for $\frac{1}{4}\lambda$ and $\frac{1}{2}\lambda$. At an operation frequency of 4 MHz the reflector thickness should be between 355 - 710 µm and for 13 MHz between 110 - 220 µm. The here presented resonators fulfill these criteria.

A.4 Experimental results

First experiments have been performed with 1µm polystyrene particles (Kisker) for both resonators. Here only results obtained with resonator A are presented. The device was operated with a power amplifier (ENI, 325LA) connected to a function generator (Stanford Research, DS345). The observation was done with a microscope (Zeiss Axio Imager Z1m) through the glass reflector. The focus was set to the reflector water interface (depth of focus > 10 µm). A frequency sweep was preferred compared to a single frequency actuation. When the device is only operated at a single frequency there will be a strong formation of lines or clumps of particles due to the lateral reflections at the fluid cavity boundaries. The force at the reflector interface will also be not uniform. Due to the thin fluid layer and the wide peaks in the force plot (Fig. A.3(a)) the span of the sweep can be up to 1 MHz. Figure A.4 is showing three images taken during an experiment with a fixed focus at the reflector fluid interface. The transducer was driven with a frequency sweep from 12.5 MHz to 13.5 MHz with a rate of 50 Hz and a voltage $V_{\text{rms}}$ of 15 V. Figure A.4(a) represents the state before the amplifier was turned on and all particles were randomly distributed. The following images (Fig. A.4(b) and c) have been taken 0.1 s and 0.4 s after activation. In Fig. A.4(b) already most of the particles moved away from the reflector into the fluid layer and therefore out of focus. The remaining black spots in Fig. A.4(c) are particles which stuck to the reflector surface from older experiments.

Preliminary tests with Salmonella have been performed as well. The experimental results can be seen in Fig. A.5. It has been shown that the Salmonella can be removed from the reflector surface and pushed to the nodal pressure planes. After switching off the ultrasound the Salmonella moved back and along the reflector surface. Resonator B was excited with a frequency sweep of 3 - 5 MHz and a voltage $V_{\text{rms}}$ of 19 V. The observa-
Appendix A. Ultrasonic manipulation of bacteria in planar resonators

Figure A.4: Experiment using resonator A and 1 µm polystyrene particles. The focus was at the reflector-fluid interface. The images have been extracted from a video with (a) before the excitation and (b) 0.1 s and (c) 0.4 s after the start of the excitation.

The experiments with resonator A showed that the reflector was moving out of focus during excitation. The depth of focus in these experiments had been in the range of 2 µm. Interferometer measurements proved that the reflector moves up to 8 µm in z-direction during longer excitations. This might be due to thermal deformations of the whole device. The interferometer measurements for resonator B have shown no movement. The resonator A could be improved by a smaller fluid cavity and temperature stabilization.

Figure A.5: Experiment using resonator B and *Salmonella*. The focus was at the reflector-fluid interface. The images have been extracted from a video. The white circles are indicating the position of moving *Salmonella*. *Salmonella* which were sticking to the glass cover are marked with a black circle. (a) Condition before switching on the excitation, with moving *Salmonella* on the reflector surface. (b) After 1.6 s with ultrasonic excitation no more moving *Salmonella* have been observed. (c) 6 s after switching off the ultrasound excitation, the *Salmonella* were moving along the reflector interface as before.
A.5 Conclusion

The type of planar resonator investigated here is able to force particles and Salmonella away from the reflector surface into the fluid. A TMM-model has been used for the optimization process of the device design. Admittance measurement have shown a good agreement with the model. The experiments with 1 µm polystyrene particles have been successful and correspond to the behavior and frequencies predicted with the TMM-model. The experiments will now focus on the Salmonella manipulation combined with a cell layer at the reflector.
B Acoustic radiation force/torque and drag force/torque in COMSOL

Acoustic radiation force and torque

These are the expressions for the integration on the particle surface in COMSOL for the acoustic radiation force \( F \) and torque \( T \). The expression \( \rho_0 \) is fluid density, \( c_0 \) is the speed of sound of the fluid, \( X_c, Y_c \) and \( Z_c \) are the positions of the center of mass for the \( x, y, z \) directions. All other expressions are defined in COMSOL 4.2. For the 2D case in the \( xy \) plane the terms including \( \text{acpr.vz} \) have to be deleted in \( F_x, F_y \) and \( T_z \).

\[
F_x = \frac{(1/(2*\rho_0*c_0^2)*\text{acpr.p_t*conj(acpr.p_t)/2}}{2} - \frac{1}{2}\rho_0*(\text{acpr.vx*conj(acpr.vx)+acpr.vy*conj(acpr.vy)+acpr.vz*conj(acpr.vz))}{2})*\text{acpr.nx} + \rho_0*\text{real}((\text{acpr.nx*acpr.vx + acpr.ny*acpr.vy+acpr.nz*acpr.vz})*conj(\text{acpr.vx}))/2)
\]

\[
F_y = \frac{(1/(2*\rho_0*c_0^2)*\text{acpr.p_t*conj(acpr.p_t)/2}}{2} - \frac{1}{2}\rho_0*(\text{acpr.vx*conj(acpr.vx)+acpr.vy*conj(acpr.vy)+acpr.vz*conj(acpr.vz))}{2})*\text{acpr.ny} + \rho_0*\text{real}((\text{acpr.nx*acpr.vx + acpr.ny*acpr.vy+acpr.nz*acpr.vz})*conj(\text{acpr.vy}))/2)
\]

\[
F_z = \frac{(1/(2*\rho_0*c_0^2)*\text{acpr.p_t*conj(acpr.p_t)/2}}{2} - \frac{1}{2}\rho_0*(\text{acpr.vx*conj(acpr.vx)+acpr.vy*conj(acpr.vy)+acpr.vz*conj(acpr.vz))}{2})*\text{acpr.nz} + \rho_0*\text{real}((\text{acpr.nx*acpr.vx + acpr.ny*acpr.vy+acpr.nz*acpr.vz})*conj(\text{acpr.vz}))/2)
\]

\[
T_x = (y-Y_c)*((\frac{1}{2}\rho_0*c_0^2)*\text{acpr.p_t*conj(acpr.p_t)/2}}{2} - \frac{1}{2}\rho_0*(\text{acpr.vx*conj(acpr.vx)+acpr.vy*conj(acpr.vy)+acpr.vz*conj(acpr.vz))}{2})*\text{acpr.nx} + \rho_0*\text{real}((\text{acpr.nx*acpr.vx + acpr.ny*acpr.vy+acpr.nz*acpr.vz})*conj(\text{acpr.vx}))/2)
\]

\[
T_y = (z-Z_c)*((\frac{1}{2}\rho_0*c_0^2)*\text{acpr.p_t*conj(acpr.p_t)/2}}{2} - \frac{1}{2}\rho_0*(\text{acpr.vx*conj(acpr.vx)+acpr.vy*conj(acpr.vy)+acpr.vz*conj(acpr.vz))}{2})*\text{acpr.ny} + \rho_0*\text{real}((\text{acpr.nx*acpr.vx + acpr.ny*acpr.vy+acpr.nz*acpr.vz})*conj(\text{acpr.vy}))/2)
\]
\begin{align*}
Ty &= (z-Zc)*((1/(2*rho0*c0^2)*acpr.p_t*conj(acpr.p_t)/2
-1/2*rho0*(acpr.vx*conj(acpr.vx)+acpr.vy*conj(acpr.vy)
+acpr.vz*conj(acpr.vz))/2)*acpr.nx + rho0*real((acpr.nx*acpr.vx
+acpr.ny*acpr.vy+acpr.nz*acpr.vz)*conj(acpr.vx))/2)
-x*(1/(2*rho0*c0^2)*acpr.p_t*conj(acpr.p_t)/2
-1/2*rho0*(acpr.vx*conj(acpr.vx)+acpr.vy*conj(acpr.vy)
+acpr.vz*conj(acpr.vz))/2)*acpr.nz + rho0*real((acpr.nx*acpr.vx
+acpr.ny*acpr.vy+acpr.nz*acpr.vz)*conj(acpr.vz))/2)
\end{align*}

\begin{align*}
Tz &= (x-Xc)*((1/(2*rho0*c0^2)*acpr.p_t*conj(acpr.p_t)/2
-1/2*rho0*(acpr.vx*conj(acpr.vx)+acpr.vy*conj(acpr.vy)
+acpr.vz*conj(acpr.vz))/2)*acpr.ny + rho0*real((acpr.nx*acpr.vx
+acpr.ny*acpr.vy+acpr.nz*acpr.vz)*conj(acpr.vy))/2)
-(y-Yc)*((1/(2*rho0*c0^2)*acpr.p_t*conj(acpr.p_t)/2
-1/2*rho0*(acpr.vx*conj(acpr.vx)+acpr.vy*conj(acpr.vy)
+acpr.vz*conj(acpr.vz))/2)*acpr.nx + rho0*real((acpr.nx*acpr.vx
+acpr.ny*acpr.vy+acpr.nz*acpr.vz)*conj(acpr.vx))/2)
\end{align*}

Drag force and torque

These are the expressions for the integration on the particle surface in COMSOL for the drag force $F$ and torque $T$. All expressions are defined in COMSOL 4.2 using the creeping flow module. The center of rotation of the particle is in the origin of the coordinate system.

\begin{align*}
Fx &= spf.T_stressx = -p*nx+spf.K_stressx \\
Fy &= spf.T_stressy = -p*ny+spf.K_stressy \\
Fz &= spf.T_stressz = -p*nz+spf.K_stressz \\
Tx &= y*(spf.T_stressz) - z*(spf.T_stressy) \\
Ty &= z*(spf.T_stressx) - x*(spf.T_stressz) \\
Tz &= x*(spf.T_stressy) - y*(spf.T_stressx)
\end{align*}
# Micro machining run-sheet

## Photolithography

<table>
<thead>
<tr>
<th>Process</th>
<th>Parameters</th>
<th>Setpoint</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Cleaning</td>
<td>US bath acetone</td>
<td>10 min</td>
</tr>
<tr>
<td></td>
<td>US bath isopropanol</td>
<td>10 min</td>
</tr>
<tr>
<td></td>
<td>QDR (Quick Dump Rinser)</td>
<td>1 cycle</td>
</tr>
<tr>
<td>2. Drying</td>
<td>Rinser dryer</td>
<td>Program 2</td>
</tr>
<tr>
<td>3. HMDS</td>
<td>N2</td>
<td>300 s</td>
</tr>
<tr>
<td></td>
<td>Primer</td>
<td>30 s</td>
</tr>
<tr>
<td></td>
<td>N2</td>
<td>300 s</td>
</tr>
<tr>
<td>4. Spin coater</td>
<td>Resist AZ4562 (Clariant)</td>
<td>3.5 ml</td>
</tr>
<tr>
<td></td>
<td>Program 7</td>
<td>Ramp up 500 rpm/s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5 s at 700 rpm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>35 s at 1700 rpm</td>
</tr>
<tr>
<td>5. Soft bake</td>
<td>Temperature</td>
<td>90 °C</td>
</tr>
<tr>
<td></td>
<td>Time</td>
<td>2 min</td>
</tr>
<tr>
<td>6. Mask aligner</td>
<td>UV radiation</td>
<td>700 mJ/cm²</td>
</tr>
<tr>
<td>MB6 (Karl Süss)</td>
<td>Time/cycles ; cycles</td>
<td>(8.2 s ; 9 cycles)</td>
</tr>
<tr>
<td></td>
<td>Al gap ; Exp gap</td>
<td>50 ; 50</td>
</tr>
<tr>
<td></td>
<td>Wec type ; Exp type</td>
<td>cont ; prox</td>
</tr>
<tr>
<td>7. Develop</td>
<td>Developer</td>
<td>MF351</td>
</tr>
<tr>
<td></td>
<td>Developer dilution</td>
<td>1:5</td>
</tr>
<tr>
<td></td>
<td>Time</td>
<td>3 min (optical inspection)</td>
</tr>
<tr>
<td>8. Cleaning</td>
<td>QDR</td>
<td>1 cycle</td>
</tr>
<tr>
<td>9. Drying</td>
<td>Rinser dryer</td>
<td>Program 2</td>
</tr>
<tr>
<td>10. Post bake</td>
<td>Temperature</td>
<td>80 °C</td>
</tr>
<tr>
<td></td>
<td>Time</td>
<td>2 min</td>
</tr>
</tbody>
</table>
Appendix C. Micro machining run-sheet

Dry etching

Table C.2: Process parameters for dry etching.

<table>
<thead>
<tr>
<th>Process</th>
<th>Parameters</th>
<th>Setpoint</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ICP-DRIE</td>
<td>Recipe</td>
<td>FBA0A</td>
</tr>
<tr>
<td></td>
<td>Etch step</td>
<td>A0A</td>
</tr>
<tr>
<td></td>
<td>Cycles</td>
<td>according to depth</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(130 cycles, 1.38 µm/cycle)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(15 cycles, 1.05 µm/cycle)</td>
</tr>
<tr>
<td>2. Depth measure</td>
<td></td>
<td>determine etch depth, rate</td>
</tr>
</tbody>
</table>

Anodic bonding

Table C.3: Process parameters for anodic bonding.

<table>
<thead>
<tr>
<th>Process</th>
<th>Parameters</th>
<th>Setpoint</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Bonder SB6</td>
<td>Recipe</td>
<td>Anodic STDR</td>
</tr>
<tr>
<td>(Karl Süss)</td>
<td>Bottom temperature</td>
<td>400°C</td>
</tr>
<tr>
<td></td>
<td>Voltage</td>
<td>−1500 V</td>
</tr>
</tbody>
</table>

Dicing process

Table C.4: Process parameters for dicing process.

<table>
<thead>
<tr>
<th>Process</th>
<th>Parameters</th>
<th>Setpoint</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Dicing saw</td>
<td>Blade type</td>
<td>SDC320R13MB01</td>
</tr>
<tr>
<td>model 8003</td>
<td>Rotational speed</td>
<td>30,000 rpm</td>
</tr>
<tr>
<td>(ESEC)</td>
<td>Feed rate</td>
<td>4 mm/s</td>
</tr>
</tbody>
</table>
References


References


List of publications

Journal publications


Conference proceedings


List of publications


