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Simulations of Enhanced Geothermal Systems with an Adaptive Hierarchical Fracture Representation

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Simulations of Enhanced Geothermal Systems with an Adaptive Hierarchical Fracture Representation

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Dissertation
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Geothermal energy and Enhanced Geothermal Systems (EGS) in particular are expected to play a vital role in future energy scenarios. In order to enhance the efficiency of the power plant, cold working fluid (usually water) is initially injected to increase the conductivity of existing fractures and to create new ones. To model single phase flow and transport in such dynamically changing, highly fractured reservoirs, a new approach is presented and is used for developing an efficient and accurate EGS simulator, i.e. HFR-Sim.

It is based on an adaptive hierarchical fracture representation (a-HFR) that results in a network of multiple dominant fractures, through which most of the mass flow occurs. These large fractures have a discrete representation, i.e. each fracture is represented by a lower dimensional continuum. A single continuum representation with appropriate effective properties is also employed for the damaged matrix, which consists of many small and medium sized fractures. The main advantage of the new approach is that no expensive re-meshing of the domain is required, when new fractures are added. Dynamically changing fracture networks can be simulated with a grid that effectively adapts itself to changes. Moreover, tiny volumes that reduce the time-step size are avoided.

The a-HFR approach is the cornerstone of the EGS algorithmic framework HFR-Sim, designed for EGS simulations of a wide range of problems. HFR-Sim meshes EGS models and solves for flow and transport based on a-HFR. It offers communication interfaces for coupling with software that model mechanical and chemical effects, such that thermal-hydro-mechanical-chemical simulations can be performed. HFR-Sim is not yet coupled with geomechanics and geochemical modules. However, it is used for a demonstration scenario that shows how EGS simulations can assist in assessing the potentials of different EGS cycles. Simulations are performed for studying the electrical power production of EGS from classical designs with a deep reservoir and combinations of deep and shallow reservoirs, in which the shallow one acts as a preheater.
Modeling flow and heat transport inside fractured rocks is a challenging task. The length scales of the rock discontinuities are many orders of magnitude smaller than the rock domain. With conventional discrete fracture models, very fine grids need to be employed and for simulating complex fracture geometries very tiny time-steps are required. Here, a result of flow and heat advection in an outcrop of a real fracture network, obtained with the a-HFR approach with larger time-steps for finer matrix grids, is presented. Cold water (blue color) is injected from the left side and pushes out the stored hot water (red color) to the right. Volumes with mean temperature between the two extrema are colored accordingly.

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Simulations of Enhanced Geothermal Systems with an Adaptive Hierarchical Fracture Representation

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2013
Geothermal energy and Enhanced Geothermal Systems (EGS) in particular are expected to play a vital role in future energy scenarios. In order to enhance the efficiency of the power plant, cold working fluid (usually water) is initially injected to increase the conductivity of existing fractures and to create new ones. To model single phase flow and transport in such dynamically changing, highly fractured reservoirs, a new approach is presented and is used for developing an efficient and accurate EGS simulator, i.e. HFR-Sim.

It is based on an adaptive hierarchical fracture representation (a-HFR) that results in a network of multiple dominant fractures, through which most of the mass flow occurs. These large fractures have a discrete representation, i.e. each fracture is represented by a lower dimensional continuum. A single continuum representation with appropriate effective properties is also employed for the damaged matrix, which consists of many small and medium sized fractures. The main advantage of the new approach is that no expensive re-meshing of the domain is required, when new fractures are added. Dynamically changing fracture networks can be simulated with a grid that effectively adapts itself to changes. Moreover, tiny volumes that reduce the time-step size are avoided.

The a-HFR approach is the cornerstone of the EGS algorithmic framework HFR-Sim, designed for EGS simulations of a wide range of problems. HFR-Sim meshes EGS models and solves for flow and transport based on a-HFR. It offers communication interfaces for coupling with software that model mechanical and chemical effects, such that thermal-hydraulic-geomechanical-chemical simulations can be performed.

HFR-Sim is not yet coupled with geomechanics and geochemical modules. However, it is used for a demonstration scenario that shows how EGS simulations can assist in assessing the potentials of different EGS cycles. Simulations are performed for studying the electrical power production of
EGS from classical designs with a deep reservoir and combinations of deep and shallow reservoirs, in which the shallow one acts as a preheater.
Zusammenfassung


Der a-HFR-Ansatz stellt das Kernstück des algorithmischen Simulationstools HFR-Sim dar, welches für viele verschiedene Problemstellungen eingesetzt werden kann. In einem ersten Schritt vernetzt HFR-Sim die EGS-Modelle und berechnet daraufhin das Strömungs- und Transportfeld basierend auf dem a-HFR-Ansatz. Es stehen verschiedene Schnittstellen zur Verfügung, mit welcher HFR-Sim mit Programmen zur Berechnung von mechanischen und chemischen Effekten gekoppelt werden kann, sodass thermal-hydro-
mechanical-chemical Simulationen durchgeführt werden können.

Anhand der Simulation eines Testfalls mit HFR-Sim wird gezeigt, wie die Simulation von EGS zur Bestimmung des Potentials verschiedener EGS-Zyklen verwendet werden kann. Die Simulationen werden für klassische Tiefenreservoirs und für die Kombination aus einem Tiefenreservoir und einem oberflächennahem Reservoir, welches als Vorwärmer dient, durchgeführt. Für beide Szenarien, wird die Leistungsfähigkeit der Stromerzeugung analysiert.
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1 Introduction: Enhanced Geothermal Systems

Geothermal energy, which can be utilized for base load electrical power production, is expected to play a vital role in future energy scenarios. Currently, an installed capacity of approximately 48 GWt of thermal power is exploited for heating globally [57] and the electrical power production increases steadily. The electrical power production is expected to rise from 10 GWe to 70 GWe by the year 2050, and a forecast relying on successful development of Enhanced Geothermal Systems (EGS) predicts an installed capacity of up to 140 GWe [7]. Contrary to most of the conventional geothermal technologies for electrical power production, potential sites for EGS plants are less rare, since the thermal energy is extracted from reservoirs located at drillable depths and that do not have significant quantities of water in place.

EGS (or Petrothermal) technology relies on artificially enhancing the permeability of fractured reservoirs through a stimulation process (creation phase). During reservoir stimulation, cold fluid is injected through each well into the fractured rock and mainly due to its high pressure and the thermal rock contraction, rock failure mostly in shear occurs. The shearing of the existing fractures creates irreversible new flow paths that reduce the impedance between wells. A decrease of reservoir impedance is a necessary prerequisite for efficient production of thermal energy during the subsequent production phase, where cold working fluid is injected at high rates (approximately 40 to 50 l/s) into the reservoir (at typical depths between 3 and 6 km), heated up and then forced back to the surface through production wells. Finally, some of the thermal energy transferred by the working fluid from the deep rock to the surface is converted to electrical power.

An EGS site consists of a deep reservoir, the injection and production wells that penetrate the reservoir, and the infrastructure on the surface that converts the extracted thermal energy into electrical power or directly utilizes it for heating purposes. In order to reduce the EGS costs, these
surface infrastructures can convert thermal energy from more than one deep reservoir into electricity. A typical sketch of an EGS power plant is presented in Fig. 1.1.

The foundation for EGS was first set by the experimental studies at Los Alamos National Laboratory, where the Hot Dry Rock (HDR) concept was investigated [42]. The hydraulic fracturing performed there and in the subsequent experimental sites [42] gave valuable insight for the EGS stimulation process. The potential of this novel method have been summarized by Murphy [66]. However, the seismic activity induced during the creation phase still requires better control. Moderate to large earthquake events may occur, as was the case in Basel, Switzerland, where a 3.4 magnitude earthquake was induced in 2006 [51]. Since then, public awareness for applying this new technology close to an urban environment has increased, but since these are the locations of prime interest (to use waste heat for heating), EGS development has decelerated [34].

EGS simulation not only is an invaluable design tool towards financially attractive EGS technologies, but can also assist in critical decision making. Trustful predictions of the event magnitudes during and after stimulation can guide the operators in choosing a successful reservoir creation strategy that does not cause public awareness. History matching and reservoir characterization can also be performed with the assistance of EGS simulations. The thermal revenues of a potential reservoir can be estimated and optimal positions for new wells can be allocated. To sum up, EGS simulations are important for the further development of this low CO$_2$ emissions technology.

1.1 Modeling of flow and transport in fractured media

For simulations of the “creation phase”, thermal-hydro-mechanical (THM) calculations need to be performed. Coupling mass and heat transport solvers sequentially with a software that performs geomechanical calculations is a typical approach for this kind of problem.

For the simulation of the “production phase”, thermal-hydro-chemical (THC) calculations are essential and it is desirable, if not necessary, to model the mechanical phenomena too (THMC). Accurate single phase flow and heat transport modeling is needed in order to model the transport of chemical species, the interaction of the fluid chemicals with the rock, the resulting changes of the hydrological reservoir properties, and thus, possible new flow distributions.

It is clear, that both the creation and production phases require accurate
1.1 Modeling of flow and transport in fractured media

Figure 1.1 – Sketch of an EGS. Main components of an EGS are the injection and production wells that penetrate the reservoir from which thermal energy is extracted. The required energy conversion infrastructure is located on the surface. Initially, the EGS reservoir has a high impedance (left), but after the stimulation procedure (or creation phase) its impedance is significantly reduced due to the created fracture network (right). The extracted thermal energy is converted to electricity and the waste heat can be used for heating by the local communities.
1 INTRODUCTION: ENHANCED GEOTHERMAL SYSTEMS

and efficient modeling of flow and heat transport in a fractured rock of unsteady geometry and properties.

1.1.1 Mathematical formulations and definitions

An EGS reservoir is usually composed of a porous medium with very low permeability (often less than $10^{-16} \, \text{m}^2$) and with very low porosity (typically of the order of 1%) that is penetrated by a large number of fractures of variable size and aperture distribution.

The porosity $\phi$ of the porous medium is defined as the ratio of void volume per unit volume of the medium. The permeability tensor $k$ is a property of the porous medium and is used in single phase problems in combination with Darcy’s law \[6\], according to which the fluid velocity inside the porous medium equals

$$u = -\frac{k}{\mu} \cdot (\nabla p - \rho g \nabla z), \quad (1.1)$$

where $\mu$ is the dynamic fluid viscosity, $p$ the fluid pressure, $\rho$ the fluid density, $g$ the gravitational acceleration and $z$ the depth.

For such large reservoirs it is unfeasible to maintain accuracy at the scale of fracture roughness, thus large fractures are treated as much smoother parallel planes. For laminar single phase flow, the average fluid velocity $u$ within those parallel planes can also be approximated as in equation \(1.1\) with a permeability tensor $k$ such that

$$k = R \cdot \begin{pmatrix} \frac{b^2}{12} & 0 & 0 \\ 0 & \frac{b^2}{12} & 0 \\ 0 & 0 & k_n \end{pmatrix} \cdot R^T, \quad (1.2)$$

where $b$ is the hydraulic aperture of a fracture, $k_n$ is a permeability value across the fracture that takes into account the fracture aperture and $R$ is the matrix that rotates the fracture parallel to the coordinate system. The hydraulic aperture of a fracture is smaller than the geometrical aperture and converges to the latter only when the assumptions for steady flow between parallel plates are satisfied. However in many occasions and due to the uncertainty regarding the fracture properties, these two values are treated as equals. With the only exception of the regions around the EGS wells, where turbulence may occur \[66\], the above-presented law is considered an acceptable approximation for the flow within a fracture \[92\].
1.1 Modeling of flow and transport in fractured media

In subsurface flows the Oberbeck-Boussinesq approximation is considered acceptable [21]. According to the Oberbeck-Boussinesq, which is usually simply called “Boussinesq” assumption, density variations can be neglected, if they are not multiplied with the gravitational acceleration. Mass conservation in a Cartesian grid can now be expressed as

$$\frac{\partial \phi}{\partial t} - \nabla \cdot \left( \frac{k}{\mu} \cdot (\nabla p - \rho g \nabla z) \right) = q,$$  \hspace{1cm} (1.3)

where $q$ is a source term. The permeability tensor $k$ is a highly anisotropic and heterogeneous tensor that makes the numerical solution of equation (1.3) a very challenging task. The density of the working fluid $\rho$ is a function of temperature, of pressure and of the fluid’s chemical composition. From these properties, temperature variations have the largest impact in density and thus, the effect of the rest can be often neglected.

For the same coordinate system, energy conservation can be expressed as

$$\frac{\partial h}{\partial t} - \nabla \cdot (\lambda \cdot \nabla T) + \nabla \cdot (uh) = w,$$  \hspace{1cm} (1.4)

where $h$ denotes the specific enthalpy, $\lambda$ the heat conductivity of the material and $w$ a heat source term. In general, within the void space the advection term dominates and heat conduction dominates everywhere else.

It is noted that both the porosity $\phi$ and the permeability tensor $k$ are functions of pressure, temperature, stress within the rock, and of the chemical species deposition. However, the work presented here only focuses on flow and transport simulations and simple proxies are used whenever mechanical or chemical calculations are considered. These proxies are only used for demonstrating how the presented models can be coupled with mechanical and chemical software and not for deriving conclusions regarding EGS processes, where mechanical and chemical phenomena play an important role.

1.1.2 Challenges and needs for modeling flow and transport

The increase in computational efficiency only recently enabled the development of complex models that include fracture shapes, treat some fractures discretely, and simulate non-linear problems with a large number of unknowns within acceptable computational time. As a result, the desirable features of a modern EGS simulator were revised by Sanyal et al. [80]. There, besides of a review on the current state of the art in EGS simula-
tion, it is explained that future EGS simulators should not only account for geomechanical and chemical phenomena, but should also

1. employ an unstructured grid with discrete and realistic representation of 3D fracture networks,
2. account for flow in the matrix,
3. treat flow rates as a function of the aperture,
4. be capable of modeling heat and tracer transport,
5. be applicable of dealing with multi-phase flow problems, and
6. account for channel flow within fractures.

Obviously, a desired feature for all simulations is the acceptable computational cost. As mentioned earlier, the numerical solution of the equations that govern flow and heat transport are two tasks that become very challenging because of the high anisotropy, the very high heterogeneity and the non-linear behavior of the permeability tensor $k$. The rock discontinuities not only occur at very fine length scales, but also have a very complex geometry in macro-scale. For explicitly representing such complex geometries, fine discretizations are required that favor implicit methods for solving equation (1.3). Although implicit approaches require more computational effort per time-step, they are stable, allow for larger time-steps and in overall reduce the computational cost.

A further desirable feature for EGS simulators is the adaptability to changes of the reservoir properties. During the creation phase existing fractures shear and irreversibly increase both their aperture and size, while the creation of totally new fractures is not excluded. Updating fracture geometry and reservoir properties may require re-meshing, which is expensive and can further reduce the time-step size.

1.1.3 Current practice for numerical simulations of EGS

Reviews of geothermal reservoir modeling [72] and of HDR modeling approaches [90] have previously been published. In the latter review, HDR and consequently EGS models can be classified into reduced and realistic ones. Reduced models stands for those that need to simplify the geometry of fractures by altering either their position, their orientation or their shape, while realistic ones are considered those that respect these fracture properties.
1.1 Modeling of flow and transport in fractured media

Among the reduced models that have been applied on EGS/HDR problems is the GEOTH3D simulation code, which treats fractured networks like a medium and employs an implicit finite difference method for hydro-thermal (TH) scenarios [93].

In FRACSIM-3D, discrete fractures are treated as a 3D network of 2D planes, from which a representative porous medium of equivalent properties is derived in order to solve for flow and heat transfer [90, 91, 43].

The code FRACTure [49], which includes turbulent flow laws, and its subsequent developments, i.e. the codes HEX and FRACHEM, are capable of performing THM [48, 50] and THC simulations [3, 1]. In FRACTure, the finite element method is applied with a grid conforming to the lower dimensional manifolds which represent the fractures. In HEX, a mapping procedure is performed that modifies the properties of the existing grid by introducing anisotropic properties over elements intersecting with the discrete fractures.

Among the first codes for HDR simulations, which can account for realistic geometries, was FRACAS which was developed by Bruel et al.[11, 12]. There, fractures are treated as two dimensional disc-shaped planes that intersect with each other and thus create a 3D fracture network inside a completely impermeable rock.

The Displacement Discontinuity method has been coupled by McClure and Horne with a finite element based flow simulator [62]. The unstructured mesh used there conforms to the lower dimensional fracture manifolds. Their approach is similar to the modeling approach developed by Karimi-Fard et al. [45] for modeling flow and transport problems.

More general purpose hydrothermal software have also been used for simulating EGS scenarios. FEHM and ROCKFLOW are used for THM simulations [10, 63]. FEHM applies a control-volume finite element method on a dual continuum model [4, 89] and ROCKFLOW offers finite element analysis on adaptive unstructured grids. Finally, the TOUGHREACT simulator, which uses integral finite differences for a dual continuum model, is successfully coupled with the mechanical framework of FLAC and has been used for THMC simulations [86, 85].

1.1.4 Other discrete fracture models

Simulations of fractured reservoirs are of prime importance for many other applications besides EGS (and HDR) and therefore a number of discrete fracture models (DFM) can be found in literature.
The DFM approaches, contrary to the continuum fracture models (CFM) that are based on the the dual continuum method suggested by Barenblatt et al. [4] and Warren and Root [89], use less assumptions regarding the shape of the modeled fractures. Although CFMs have been further developed and successfully applied in a wide range of applications [47, 56, 19], often they cannot capture phenomena that occur at length scales as small as the fracture aperture and the results obtained from them are smoothed.

Among the first DFMs are the approaches that treat fracture networks as tube networks [14, 13, 20]. However, neither flow within the matrix is simulated nor are the fractures discretized into finer volume, since each volume represents either a fracture center or a fracture-fracture intersection. Those tube networks neglect the flow within the matrix due to the fact that the number of fractures inside such reservoirs is huge and therefore it is unfeasible to represent all of them discretely and also account for flow within the matrix in the meantime. Instead, hierarchical approaches have been developed where only fractures, for which the accuracy gain is expected to outbalance the additional computational cost, are treated discretely and the rest of them are upscaled. Such hierarchical DFM approaches have shown close agreement to computationally expensive DFMs, where all fractures are treated discretely [17, 53, 54, 40]. The advances in the extensively used CFMs are not neglected but instead are complementary to the discrete representation of large fractures.

The DFMs that both treat fractures discretely and account for flow inside the matrix can be categorized into embedded and conforming DFMs. This classification is based on how the matrix that surrounds discrete fractures is discretized. In embedded approaches the boundaries of the matrix grid cells do not have to coincide with the shape of the fracture and thus a matrix grid cell may surround a fracture from both sides. Conforming DFMs, such as the DFM employed by FRACTure, include approaches where the surrounding matrix grid cells conform with the shape of the discrete fractures, independent of how realistic the shape of the modeled fracture is. Each DFM can further be classified as structured and unstructured DFM, according to the type of grid employed for discretizing the surrounding matrix. In Fig. 1.2 a number of exemplary sketches of embedded and conforming DFMs are presented.

The first embedded structured DFM approach is the hierarchical DFM (EDFM) presented by Lee et al. [53], which was later extended for multiphase flows by Li and Lee [54] and recently applied for 3D problems [64]. In these models, large (and therefore assumed important) fractures are treated
1.1 Modeling of flow and transport in fractured media

(a) Rock mass discontinuity

(b) Conforming unstructured
(fracture is not lower dimensional)

(c) Conforming unstructured

(d) Conforming structured DFM

(e) Embedded unstructured

(f) Embedded structured

Figure 1.2 – Exemplary grids of various DFM approaches.
discretely as lower dimensional manifolds that are embedded inside a continuum. This continuum, which is here called the “damaged matrix”, is discretized by a structured mesh and captures the effect of the matrix and of the fractures that are not treated discretely. Eventually, the resulting grid consists of volumes that are either a matrix volume or a fracture segment within a matrix grid-cell. The same principle regarding volume sizes is applied by Vitel and Souche [88] for the creation of a tube network, which practically coincides with the EDFM grid. This tube network represents possible flow paths in a grid identical to the one produced by the EDFM approach and which is treated as a graph in order to upscale the fracture network into a simpler model. A similar embedded structured approach is employed by Hajibeygi et al. [36], where the fracture boundaries need to conform with the fracture-fracture intersections and the corresponding fracture segments can be discretized without further need for conformity.

An embedded and unstructured DFM has been suggested by Sarda et al. [81], where the matrix grid is shaped according to the fracture geometry. With imaging techniques, the closest fracture segment for each matrix point/pixel is located and each fracture segment is treated as if it was embedded inside a matrix grid-cell that consists of its associated pixels. However, the programming effort for implementing such an unstructured embedded DFM in three dimensional EGS models is very high. For that and for efficiency reasons, the applications of the latter DFM are currently restricted to simulations of naturally fractured reservoirs (NFR), where fractures are vertical to the layers of the reservoir [68].

For the computations of mass transfer from the damaged matrix to fractures and vice versa, both Sarda et al., and Li and Lee [81, 54] assume that close to the fractures the matrix pressure gradient is constant and has a direction perpendicular to the fracture orientation. This way a transport index is derived similar to the productivity index of a well by Peaceman [74]. Previously, Lee et al. [53] treated fractures as singularities that are distributed over a surface and their single phase simulation results showed convergence to a unique solution for very high transport indices. Due to the fact that the mass flux exchange between the matrix and a fracture is primarily dominated by the matrix permeability using overestimated transport indices remains consistent.

In current embedded models, contrary to the adaptive hierarchical fracture models, fracture volumes are surrounded by a single matrix grid-block and unphysical results may be obtained if perturbations travel perpendicularly to a fracture. Since, fracture volumes can only behave as either a
mass or a sink source term, heat or scalar quantities can be trapped within them if they do not affect the flow. Such an issue does not exist for the conforming DFMs, where fracture volumes are connected to more than one matrix volume.

Structured conforming DFMs [27, 65, 83] capture the effect of discrete fractures by modifying the properties of the intersecting grid cells. As a result, a very fine structured grid is needed both for fractures of complex shape and for reservoirs with high fracture density. Consequently, the computational cost for solving flow increases.

Results of equal high accuracy, but with less computational cost, can be obtained from unstructured conforming approaches that also employ local refinement of the matrix grid around the discrete fractures. The unstructured DFM developed by Karimi et al. [45] employs such an unstructured grid that conforms to the shape of the lower dimensional fractures and the delta star formulation for electrical resistances is applied in order to avoid the creation of tiny volumes between fracture to fracture intersections. An adaptive DFM simulator is also developed by [40, 55] which automatically compromises between geometrical accuracy and numerical efficiency. It creates a convenient unstructured matrix grid that allows for larger time-steps in explicit approaches by surrounding fractures of similar, but slightly altered shape, with the damaged matrix grid.

Darcy-like flows in a lower dimensional system of width equal to a hydraulic-aperture value is considered an acceptable simplification and is used for solving flow and transport in many other conforming DFMs [69, 2, 44, 46]. The code “Complex System Modeling Platform - CSMP++”, developed by Matthai et al. [60, 59, 61, 18] employs a hybrid finite element - finite volume discretization that treats fractures either as lower dimensional manifolds or with the same dimensions as the reservoir. CSMP++ has been successfully used for problems similar to EGS scenarios, such as for studying the multi-phase thermohaline convection, the multi-crack growth, and non-Fourier thermal transport [30, 73, 31]. In general, such computationally complex approaches are preferred for problems where accurate representations of the fracture geometries exist.

1.2 Motivation and outline

The 3.4 magnitude seismic event in Basel revealed the need for further control of induced seismicity during the EGS reservoir stimulation. Towards that goal, GEOTHERM, an interdisciplinary project, had already been initiated
and funded by the Competence Centre for Environment and Sustainability of the ETH-Domain, and by the Swiss Federal Office of Energy (under contract 154278). Among other ends GEOTHERM targeted in the development of a novel EGS simulator that includes all the desired features of modern EGS simulators and that can be used for studying the processes that take place during the EGS creation phase and for performing THMC simulations.

Under this basis, a novel adaptive structured and embedded hierarchical DFM for modeling flow and heat transport has been developed and is presented in part I. This adaptive hierarchical fracture representation (a-HFR) has been especially designed for dynamically changing fractured rocks, such as an EGS reservoir during the stimulation phase. Contrary to conventional reservoir DFMs, a-HFR allows the addition of newly permeable fractures in the computational domain without the need of re-meshing the existing domain. Grid-optimization for avoiding tiny time-steps at regions of high fracture density is less important and unphysical trapping of scalar quantities inside fractures is avoided.

In chapter 2 the governing equations for the a-HFR model are presented. Mass and energy equations are decomposed into a system of similar partial differential equations (PDE), which however, are less heterogeneous, less anisotropic, and therefore easier to be numerically handled. In chapter 3 the finite volume method is explained for the adaptive system of PDEs and issues arising due to the adaptivity of the model are presented and discussed. Verification of the a-HFR for single phase problems, where gravitational effects are neglected, is presented in chapter 4 and for variable density driven flows in chapter 5. Part I ends with the discussion in chapter 6, where the approach is compared to existing DFM approaches and some of its features are further discussed.

The a-HFR approach presented in part I is the cornerstone of the modeling framework HFR-Sim that is presented in part II. HFR-Sim is an EGS modeling framework that models flow and heat transport with the a-HFR DFM and can be coupled with software that models mechanical or chemical phenomena for simulating EGS operational scenarios, both for creation and production phases.

In chapter 7 the algorithmic framework of HFR-Sim is presented and some of its components are further discussed; i.e. the reservoir discretization, the appliance of the a-HFR approach, the wells treatment and its coupling with other software. In chapter 8 exemplary 2D and 3D scenarios are simulated with HFR-Sim and their results are discussed. Simulations that can assist in financial planning, in reservoir characterization and in the
reservoir stimulation are presented. Part II concludes with a demonstration of how EGS simulations can assist in predicting the energy revenues of potential EGS sites and in designing EGS power plants that can produce more energy at the same drilling cost.

This dissertation on EGS simulations finishes with chapters 10 and 11, where the conclusions and the outlook of the presented work are discussed, and future steps for the further development of HFR-Sim are suggested.
Part I

An adaptive hierarchical discrete fracture model
2 Adaptive hierarchical fracture model

Here, the adaptive hierarchical fracture modeling (a-HFR) approach, a discrete fracture model (DFM) which is especially suited for dynamically changing fractured reservoirs, is presented. Large, dominant fractures are treated by a discrete representation and the cumulative effect of the small fractures is captured by a continuum description of the damaged matrix.

The mass and energy conservation for embedded discrete fracture models are initially expressed as a system of coupled partial differential equations (PDE). Discontinuities between fractures and damaged matrix are captured by the here introduced kernel functions that remove singularities and are similar to the ones that have been used for upscaling purposes by Lough et al. [56] previously. This way transfer coefficients are well defined and embedded methods can treat dynamically changing fracture systems. Finally, the importance of accounting for fracture sweeping, which captures transport effects across discrete fractures and is neglected in current embedded fracture models, is demonstrated and discussed.

Without loss of generality, laminar single phase flow is considered and the Boussinesq approximation is employed for modeling buoyant effects due to spatial variations of the fluid density $\rho$. Geomechanics is important for determining how fracture geometries change. At this point however, simple proxies are employed for those calculations and focus solely lies on modeling flow and heat transport in such reservoirs.

Henceforth, we consider a solid rock domain $\Omega^r \in \mathbb{R}^n$, a damaged matrix domain $\Omega^d \in \mathbb{R}^n$, with $\Omega^r \supset \Omega^d$, and $N_f$ overlapping discrete fracture domains $\Omega^i \in \mathbb{R}^{n-1}$ with $i \in \{1, \ldots, N_f\}$; $n \in \{2, 3\}$ denotes the dimensionality of the problem. An example of a two-dimensional matrix with large and small fractures is depicted in the upper plot of Fig. 2.1; the lower plot illustrates a possible splitting, where only the three largest fractures are explicitly represented as one-dimensional manifolds embedded in the damaged matrix.
Figure 2.1 – On the top, the sketch of a highly fractured reservoir $\Omega$ consisting of three dominant large fractures and a large number of smaller ones is presented. In the lower plot, a simple size criteria was applied according to which all the small fractures are upscaled into an equivalent porous medium with domain $\Omega^d$ and each large fracture is treated discretely by a lower dimensional manifold $\Omega^i$ (solid black lines) embedded in this damaged matrix continuum $\Omega^d$. 
2.1 Flow in the damaged matrix

The damaged matrix is characterized by an effective porosity $\phi^d$ and an effective permeability $k^d$, which can change dynamically, e.g. when new small scale fractures are created during the simulation period. According to Darcy’s law, and by both assuming that deformations solely depend on the local fluid pressure and that the Boussinesq approximation can be employed, mass balance leads to the partial differential equation (PDE)

$$\frac{\partial \phi^d}{\partial p^d} = \nabla \cdot \left( \frac{k^d}{\mu} \cdot (\nabla p^d - \rho^d g) \right) = q^d$$

on $\Omega^d$ for the matrix pressure $p^d$; note that $dA(x')$ is an infinitesimal element on $\Omega^i$ around $x'$ with dimension $n - 1$. The dynamic viscosity and the fluid density are denoted by $\mu$ and $\rho^d$, respectively, while $g$ is the vector of the gravitational acceleration with direction towards the center of the earth. $\Psi^{i\rightarrow d}$ accounts for fluid exchange from the discrete fracture $i$ into the damaged matrix (note that $[\Psi^{i\rightarrow d}] = ms^{-1}$), and $\hat{G}(x, x') \geq 0$ with $\int_{\Omega^i} \hat{G}(x, x')dV(x) \equiv 1 \forall x' \in \Omega^d$ is an isotropic kernel. Note that $dV(x)$ is an infinitesimal element with dimensions $n$, that $[\hat{G}] = m^{-n}$ and that the surface area of fracture $i$ can be calculated as $|\Omega^i| = \int_{\Omega^i} \hat{G}(x, x')dA(x')dV(x)$. Note also that here porosity as well as permeability are treated as local functions of $p^d$, which has to be revised, if realistic geomechanical models are employed. Loss of fluid into the pore space as well as injection and production through wells are captured by the volumetric source term $q^d$.

The introduction of the gravity vector $g$ here serves the purpose of accounting for buoyant forces independently of the co-ordinate system. The substitution of depth divergence with the normal vector at the gravity direction is a useful attribute for accounting for buoyancy effects within the lower dimensional fracture manifolds too.

2.2 Flow in large fractures

Since it is problematic to upscale fractures which are much larger than the grid cells of the damaged matrix discretization, those are represented as lower dimensional manifolds and each is discretized independently. Mass
conservation of such a fracture with index \( i \) leads to the equation

\[
d\phi^i b^i \frac{\partial p^i}{\partial t} = - \nabla \cdot \left( \frac{b^i k^i}{\mu} \cdot (\nabla p^i - \rho^i g) \right) + \Psi^{i \rightarrow d} = q^i + \sum_{j=1}^{N_f} \int_{\Omega^i \cap \Omega^j} \Psi^{j \rightarrow i} \hat{G}^i(x, x') ds(x') \quad (2.2)
\]

on \( \Omega^i \), where \( ds(x') \) is an infinitesimal element of \( \Omega^i \cap \Omega^j \) at \( x' \) with dimension \( n - 2 \), and \( \phi^i, b^i, k^i, \rho^i \) and \( p^i \) denote the fracture porosity, aperture, permeability (including the inverse of drag due to wall shear stress divided by \( b^i \)), fluid density and pressure, respectively. Further, the volumetric source term \( q^i \) accounts for exchange with the pore space and wells. Note that the exchange source term \( \Psi^{i \rightarrow d} \) also appears in the damaged matrix pressure equation. The volumetric transfer rate from fractures \( j \) to fracture \( i \) is captured by the last term in equation (2.2), where

\[
\hat{G}^i(x \in \Omega^i, x') = \int_{-\infty}^{\infty} \delta \left( x' - x + x_n \frac{n^i(x)}{|n^i(x)|} \right) dx_n \quad (2.3)
\]

is a kernel function, \( \delta(x) \) the Dirac functional and \( n^i(x) \) the unit normal to \( \Omega^i \) at \( x \). Note that it may occur that multiple disconnected intersection lines between two fractures exist, which can be treated analogously, but for simplicity this scenario is excluded from the discussions presented in this paper.

### 2.3 Matrix-fracture exchange

While the source term \( \Psi^{i \rightarrow d} \) describes the flow rate per unit element of \( \Omega^i \) (dimension \( n - 1 \)) from fracture \( i \) into the damaged matrix, the last term of equation (2.1) is the consistent flow rate into \( \Omega^d \) (dimension \( n \)). In other words, multiplication with the kernel function leads to an effective distribution of the exchange source term in the damaged matrix domain concentrated around the lower dimensional manifold \( \Omega^i \). Here, in line with previous work, we assume a linear relationship between \( \Psi^{i \rightarrow d} \) and the pressure difference \( p^i - p^d \), i.e.

\[
\Psi^{i \rightarrow d} = C^{id} (p^i - p^d) \quad \text{on} \ \Omega^i, \quad (2.4)
\]

where \( C^{id} \) is a connectivity coefficient. Note that \( C^{id} \) depends on the matrix permeability, the fluid viscosity and on the shape of the kernel function. Usually such connectivity coefficients are introduced at the level of
the discretized equations, where the exchange rates of discrete fractures with intersecting matrix grid cells are modeled in a similar way. It is due to the introduction of kernel functions that here modeling of $\Psi_{i \rightarrow d}$ can be addressed independent of numerical considerations. The exchange rate between fractures $i$ and $j$, i.e.

$$
\Psi_{j \rightarrow i} = C_{ji} (p^j - p^i) \quad \text{on } \Omega^i \cap \Omega^j,
$$

is captured in a similar way. If the connectivity coefficient $C_{ji}$ is infinitely large, the intersecting fracture manifolds $\Omega^j$ and $\Omega^i$ share the same pressure at the intersection $\Omega^j \cap \Omega^i$.

### 2.4 Energy conservation

Inside the fracture network including the damaged matrix and large fractures, energy transport is dominated by heat convection. On the other hand, in the rock heat diffusion is the responsible energy transport mechanism. For the damaged matrix energy balance can be expressed as

$$
\frac{\partial \phi^d h^d}{\partial t} + \nabla \cdot (uh^d) = w^d + w^{r \rightarrow d}
$$

$$
+ \sum_{i=1}^{N_f} \int_{\Omega^i} \Xi^{i \rightarrow d}(x') \tilde{G}(x, x') dA(x') \quad (2.6)
$$

for the specific fluid enthalpy $h^d$ on $\Omega^d$ and

$$
\frac{\partial \phi^i b^i h^i}{\partial t} + \nabla \cdot (b^i uh^d) + \Xi^{i \rightarrow d} = w^i + w^{r \rightarrow i}
$$

$$
+ \sum_{j=1}^{N_f} \int_{\Omega^i \cap \Omega^j} \Xi^{j \rightarrow i} \tilde{G}^i(x, x') dS(x') \quad (2.7)
$$

for the specific fluid enthalpy $h^i$ on $\Omega^i$. Similar as $\Psi_{i \rightarrow d}$ and $\Psi_{j \rightarrow i}$ in Eqs. (2.1) and (2.2) for the mass exchange, $\Xi^{i \rightarrow d}$ and $\Xi^{j \rightarrow i}$ represent the specific heat fluxes from the discrete fracture $i$ into the damaged matrix and from fracture $j$ into fracture $i$, respectively. Energy source/sink terms due to loss to the pore space, chemical reactions and wells are captured by the terms $w^d$ and $w^i$ on $\Omega^d$ and $\Omega^i$, respectively. Accordingly, heat flux from the solid rock continuum $\Omega^r$ into $\Omega^d$ and $\Omega^i$ is captured by the terms $w^{r \rightarrow d}$ and $w^{r \rightarrow i}$, respectively. Darcy’s law is assumed for the computation
of the velocity, i.e.

$$u^d = -\frac{k_d}{\mu} \cdot (\nabla p^d - \rho^d \mathbf{g}), \text{ in } \Omega^d \quad (2.8)$$

and

$$u^i = -\frac{k_i}{\mu} \cdot (\nabla p^i - \rho^i \mathbf{g}), \text{ in } \Omega^i \quad (2.9)$$

Similar as in other approaches, a linear relationship is assumed for heat exchange between rock and working fluid, i.e.

$$w^{r \rightarrow d} = K^{rd}(T^r - T^d) \text{ on } \Omega^d \quad (2.10)$$

and

$$w^{r \rightarrow i} = K^{ri}(T^r - T^i) \text{ on } \Omega^i \quad (2.11)$$

with the heat transfer coefficients $K^{rd}$ and $K^{ri}$.

The heat exchange source term $\Xi^{i \rightarrow d}$ on $\Omega^i$ is modeled as

$$\Xi^{i \rightarrow d} = \begin{cases} E_1 & 
H(\Psi^{i \rightarrow d}) \Psi^{i \rightarrow d} h^i + H(-\Psi^{i \rightarrow d}) \Psi^{i \rightarrow d} h^i \\
+ |u^d \cdot \mathbf{n}^i|(h^d - h^i) & E_2 
\end{cases} \quad (2.12)$$

where $H(\cdot)$ is the Heaviside step function. In equation (2.12), term $E_1$ represents the heat exchange rate due to the velocity $\Psi^{i \rightarrow d}$. The term $E_2$, on the other hand, represents heat exchange due to the mass flux component in the matrix perpendicular to fracture $i$. Note that $E_2$ has been ignored so far in hierarchical fracture models, but in the following subsection the importance of this fracture sweeping effect is demonstrated and discussed.

Heat conduction inside the solid continuum is described by the PDE

$$\frac{\partial h^r}{\partial t} = \nabla \cdot (\lambda \cdot \nabla (T^r)) + w^{r \rightarrow d}$$

$$= w^r - \sum_{j=1}^{N_j} \int_{\Omega^i} w^{r \rightarrow i}(x') \hat{G}(x, x') dA(x') \quad (2.13)$$

on $\Omega^r$ for the volumetric rock enthalpy $h^r$. Conductivity tensor, rock temperature and heat sources are denoted as $\lambda$, $T^r$ and $w^r$, respectively.
2.5 Importance of fracture sweeping

To demonstrate the importance of term $E_2$ in equation (2.12), a simple incompressible steady state problem is considered. A homogeneous isotropic damaged matrix with domain $\Omega^d$ and permeability $k^d$ is considered where gravitational effects can be neglected. To drive the flow, Neumann boundary conditions with a constant pressure gradient $(\nabla p)_{BC}$ are applied. A single fracture with domain $\Omega^1 \subset \Omega^d$, aperture $b^1$, and permeability $k^1 = k^d$, which has a pole parallel to the imposed pressure gradient (see Fig. 2.2). Further, a constant fluid velocity $u^d \sim (\nabla p)_{BC}$ is assumed.

Now we consider a planar temperature discontinuity in the damaged matrix parallel to the discrete fracture on its upwind side, which is transported by pure advection, and heat exchange with the rock is neglected. Note that in this scenario the fluid exchange term $\Psi^{1 \rightarrow d}$ is zero, and therefore no contribution by $E_1$ in equation (2.7) can be expected for the energy in the discrete fracture. Obviously this is unphysical, but is corrected by term $E_2$ in equation (2.12). It can easily be verified that the latter ensures asymptotic relaxation of the fracture enthalpy towards the enthalpy in the damaged matrix at a rate proportional to the fracture normal velocity component in the damaged matrix.
Figure 2.2 – Demonstrative test case of the fracture sweeping importance. On the left, a theoretical scenario as the one described in section 2.5 is presented. Neumann boundary conditions with a constant pressure gradient normal to the fracture $\Omega^1$ are applied on the boundaries of $\Omega^d$ and cause a homogeneous fluid velocity $u^d$. The shadowed area represents an area inside the damaged matrix, which has elevated specific enthalpy that gets advected with $u^d$. On the right, the fracture enthalpy is plotted. The dashed line is the theoretical solution when the fracture sweep term is neglected and the solid line is how the solution should look like for an infinitesimal fracture aperture.
In chapter 2, the adaptive hierarchical fracture flow and heat transport model (a-HFR) has been presented. Fractures are divided into two groups according to their size: into discrete ones and the ones that form the damaged matrix continuum. For the damaged matrix and for each discrete fracture one PDE for mass conservation and one for energy conservation was derived. The discrete fractures are treated as lower dimensional manifolds embedded inside the damaged matrix, but with the use of the introduced kernel functions, they appear as "smooth" source terms in the damaged matrix and the rock.

Henceforth, we consider $n^d$ matrix grid cells $\Omega_k^d$ with $\Omega_1^d \cup \Omega_2^d \cup \ldots \cup \Omega_{n^d}^d = \Omega^d$, and $\forall k \neq h : \Omega_k^d \cap \Omega_h^d = \partial \Omega_k^d \cap \partial \Omega_h^d = \Gamma_{kh}^d$. Here, $\partial \Omega$ denotes the surface of domain $\Omega$. Similarly, each fracture domain $\Omega^i$ is divided into $n^i$ cells $\Omega_i^i$ with $\Omega_1^i \cup \Omega_2^i \cup \ldots \cup \Omega_{n^i}^i = \Omega^i$, and $\forall k \neq h : \Omega_k^i \cap \Omega_h^i = \partial \Omega_k^i \cap \partial \Omega_h^i = \Gamma_{ih}^i$. The solid rock domain $\Omega^r$ is divided into $n^r$ rock grid cells $\Omega_m^r$ with $\Omega_1^r \cup \Omega_2^r \cup \ldots \cup \Omega_{n^r}^r = \Omega^r$ and $\forall m \neq h : \Omega_m^r \cap \Omega_h^r = \partial \Omega_m^r \cap \partial \Omega_h^r = \Gamma_{mh}^r$. Figure 3.1 depicts an example of the above domain discretization for the geometry sketched in Fig. 2.1.

Moreover, the intersection between the matrix and fracture cells $\Omega_k^d$ and $\Omega_i^i$ is denoted by $\Gamma_{kh}^{di} = \Omega_k^d \cap \Omega_i^i$, the intersection between the rock and fracture cells $\Omega_m^r$ and $\Omega_i^i$ is denoted by $\Gamma_{mk}^{ri} = \Omega_m^r \cap \Omega_i^i$, between the rock and damaged matrix cells $\Omega_m^r$ and $\Omega_k^d$ as $\Gamma_{mk}^{rd} = \Omega_m^r \cap \Omega_k^d$ and between the fracture cells $\Omega_k^i$ and $\Omega_i^j$ as $\Gamma_{ki}^{ij} = \Omega_k^i \cap \Omega_i^j$.

Further, the average distance $\Delta(\hat{\Omega}, \hat{\Gamma})$ of all points in domain $\hat{\Omega}$ from domain $\hat{\Gamma} \in \hat{\Omega}$ is defined as

$$\Delta(\hat{\Omega}, \hat{\Gamma}) = \frac{1}{|\hat{\Omega}|} \int_{\hat{\Omega}} \min_{x' \in \hat{\Gamma} \cap \hat{\Omega}} |x - x'| dV(x). \quad (3.1)$$

Figure 3.2a sketches examples of intersecting fracture and matrix cells.
Figure 3.1 – Example of domain space discretization for a-HFR. In (a) the domain $\Omega$ that remains from the hierarchical fracture representation is depicted. It consists of three dominant large fractures, the damaged matrix and the solid rock. Here, the solid rock domain coincides with the damaged matrix domain. In (b) a possible discretization is presented. The damaged matrix $\Omega^d$ is discretized by a rather coarse structured grid that consists of $n^d$ grid-blocks of orientation parallel to the principal axis of the damaged matrix permeability tensor, while the three discrete fractures are divided into smaller volumes ($n^1 = 6$, $n^2 = 4$, and $n^3 = 4$). The solid rock is discretized by any structured orthogonal grid with $n^r$ volumes (not depicted here).
Figure 3.2 – An exemplary sketch of the discretized hierarchical fracture representation. In (a), the matrix volume \(\Omega_k^d\) intersects with \(\Omega_h^d\), \(\Omega_l^d\), and \(\Omega_i^d\) and the intersections between \(\Omega_k^d\) and \(\Omega_h^d\), \(\Omega_k^d\) and \(\Omega_l^d\), \(\Omega_l^d\) and \(\Omega_h^d\), and the vectors \(n_i^d\) and \(n_k^d\) are shown. In (b), the non-zero transmissibilities are depicted as arrows. In (c) it is shown how an additional fracture volume \(\Omega_h^z\) can be considered without affecting the existing discretization and without the need of re-meshing.
3.1 Discretized flow equations

With these definitions, integration of eq. (2.1) over an individual matrix grid cell $\Omega^d_k$ and employing Gauss’ theorem leads to the equation

$$\frac{\partial V^d_k}{\partial t} + \sum_{h=1}^{n^d} C^d_{kh}(P^d_k - P^d_h) + \sum_{i=1}^{n^i} \sum_{h=1}^{n_h} C^i_{hk}(P^i_k - P^i_h) = Q^d_k |\Omega^d_k| + \sum_{h=1}^{n^d} C^d_{kh} \rho^d_h |\Omega^d_k| + \rho^d_h |\Omega^d_h| g |(Z^d_k - Z^d_h)|$$

$$+ \sum_{i=1}^{N_f} \sum_{h=1}^{n^i} C^i_{hk} \rho^i_h |\Omega^i_h| + \rho^i_h |\Omega^i_h| b^i_h |g|(Z^i_k - Z^i_h)$$

(3.2)

for the mean matrix pressure $P^d_k$ in cell $\Omega^d_k$ with

$$(P^d_k, V^d_k, Q^d_k, Z^d_k, \rho^d_k)^T = \frac{\int_{\Omega^d_k} \left( p^d(x), \phi^d(x), q^d(x), \frac{x \cdot g}{|g|}, \rho^d(x) \right)^T dV(x)}{|\Omega^d_k|}.$$ 

The transmissibilities $C^d_{kh}$ and $C^i_{hk}$ quantify the volumetric flow rate per unit mean pressure drop from matrix cell $\Omega^d_k$ to matrix cell $\Omega^d_h$ and from fracture cell $\Omega^i_h$ to matrix cell $\Omega^d_k$, respectively.

Analogously, integration of eq. (2.2) over a fracture cell $\Omega^i_h$ leads to

$$\frac{\partial V^i_h |\Omega^i_h| b^i_h}{\partial t} + \sum_{k=1}^{n^i} C^i_{hk}(P^i_h - P^i_k) + \sum_{k=1}^{n^d} C^i_{hk}(P^i_k - P^d_k)$$

$$+ \sum_{j=1}^{N_f} \sum_{k=1}^{n^i} C^i_{hk} (P^i_h - P^j_k)$$

$$= Q^i_h |\Omega^i_h| b^i_h + \sum_{k=1}^{n^i} C^i_{hk} \rho^i_h |\Omega^i_k| b^i_k + \rho^i_h |\Omega^i_h| b^i_h |g|(Z^i_h - Z^i_k)$$

$$+ \sum_{k=1}^{n^d} C^i_{hk} \rho^d_k |\Omega^d_k| + \rho^d_h |\Omega^d_h| b^i_k |g|(Z^i_h - Z^d_k)$$

$$+ \sum_{j=1}^{N_f} \sum_{k=1}^{n^i} C^i_{hk} \rho^i_h |\Omega^i_k| b^i_k + \rho^i_h |\Omega^i_h| b^i_h |g|(Z^i_h - Z^j_k)$$

(3.3)
for the mean fracture pressure \( P_h^i \) in cell \( \Omega_h^i \) with

\[
(P_h^i, V_h^i, Q_h^i, Z_h^i, \rho_h^i)_T = \frac{\int_{\Omega_h^i} \left( p^i(x), \phi^i(x), q^i(x), \frac{x\cdot g}{|g|}, \rho^i(x) \right)^T dA(x)}{|\Omega_h^i|},
\]

Here, the transmissibilities are modeled as

\[
C_k^{dh} = \frac{\left| \Gamma_k^{dh} \right| |\mathbf{k}_k \cdot \mathbf{n}_{k_h}^d||\mathbf{k}_h \cdot \mathbf{n}_{k_h}^d|(|\Omega_k^d| + |\Omega_h^d|)}{\Delta_k^{dh} \mu |\mathbf{k}_k \cdot \mathbf{n}_{k_h}^d||\Omega_k^d| + |\mathbf{k}_h \cdot \mathbf{n}_{k_h}^d||\Omega_h^d|},
\]

\[
C_h^{id} = 2\frac{\left| \Gamma_h^{id} \right| |\mathbf{k}_h \cdot \mathbf{n}_i^d|}{\Delta_h^{id} \mu |\mathbf{k}_h \cdot \mathbf{n}_i^d||\Omega_h^i| + |\mathbf{k}_i \cdot \mathbf{n}_i^d||\Omega_i^h|},
\]

\[
C_k^{ii} = \frac{\left| \Gamma_k^{ii} \right| (b_k^i + b_h^i) |\mathbf{k}_k \cdot \mathbf{n}_{k_h}^i||\mathbf{k}_h \cdot \mathbf{n}_{k_h}^i|(|\Omega_k^i| + |\Omega_h^i|)}{2\Delta_k^{ii} \mu |\mathbf{k}_k \cdot \mathbf{n}_{k_h}^i||\Omega_k^i| + |\mathbf{k}_h \cdot \mathbf{n}_{k_h}^i||\Omega_h^i|},
\]

and

\[
C_k^{ij} = \frac{2|\Gamma_k^{ij}|}{\mu} \left( \frac{\Delta(\Omega_k^i, \Gamma_k^{ij}) + \Delta(\Omega_h^j, \Gamma_k^{ij})}{|\mathbf{k}_k \cdot \mathbf{n}_{k_h}^j||b_k^i|} \right)^{-1},
\]

where \( \mathbf{n}_{k_h}^d \) is the unit normal of \( \Gamma_k^{dh} \) pointing from cell \( \Omega_k^d \) to \( \Omega_h^d \), \( \mathbf{n}_{k_h}^i \) the unit normal of \( \Gamma_k^{ii} \) pointing from cell \( \Omega_k^i \) to \( \Omega_h^i \), \( \mathbf{n}_h^i \) the unit normal to \( \Omega_h^i \) and \( \mathbf{n}_{k_h}^{ij} \) the unit normal of \( \Gamma_k^{ij} \) pointing from cell \( \Omega_k^i \) to \( \Omega_h^j \). Further, \( \Delta_k^{dh} = (\Delta(\Omega_k^d, \Gamma_k^{dh}) + \Delta(\Omega_h^d, \Gamma_k^{dh})) \), \( \Delta_k^{id} = (\Delta(\Omega_k^i, \Gamma_k^{id}) + \Delta(\Omega_h^d, \Gamma_k^{id})) \), and \( \Delta_k^{ij} = (\Delta(\Omega_k^i, \Gamma_k^{ij}) + \Delta(\Omega_h^j, \Gamma_k^{ij})) \).

Without loss of generality, in our model, a simple structured grid with \( n_d \) cells is employed for the damaged matrix. Note that the upscaled permeability \( k^d \) is given as an input, i.e. upscaling of the damaged matrix is not part of the discussion here. Further, by aligning the damaged matrix grid with the principle axes of \( k^d \) (Fig. 3.1), the complication of multi-point flux approximation is avoided, i.e. without further justification two point flux approximation is always applied here.

Similarly, for each fracture \( i \) a structured grid consisting of \( n^i \) cells is employed. Cubic law can be assumed for the flux within each fracture \( i \) of hydraulic aperture \( b_h^i \), i.e. \( k^i = I (b_h^i)^2/12 \), where \( I \) is a unity matrix in the dimensions of \( \Omega^i \).

The grid employed for the fractures does not have to conform to the boundaries of the damaged-matrix grid-cells, i.e. a damaged matrix volume may be penetrated by more than one cells of the same fracture. This creates new flow paths parallel to the fracture that leads to artifacts that may have to be treated. To examine this, we consider the transmissibilities
connecting two neighboring fracture volumes $\Omega^i_l$ and $\Omega^i_h$ with one matrix cell $\Omega^d_k$. By treating transmissibilities similarly as electrical conductors, it can be verified that in this case one has to subtract $(C^id_{lk}C^id_{hk})/(C^id_{lk} + C^id_{hk})$ from the transmissibility between $\Omega^i_l$ and $\Omega^i_h$ in order to obtain the correct flow rate in the fracture. Similar to electrical networks where electrical current goes through parallel resistors due to the applied voltage difference, here mass flow along parallel paths occurs due to the pressure difference $(P^i_l - P^i_h)$. One of the two parallel paths corresponds to transmissibility $C^id_{lh}$ that connects the two fracture volumes and the other consists of the transmissibilities $C^id_{lk}$ and $C^id_{hk}$ that are serially connected. In Fig. 3.3 an exemplary sketch of two neighboring fracture volumes $\Omega^i_l$ and $\Omega^i_h$ that intersect the same damaged matrix volume $\Omega^d_k$ is depicted; together with a corresponding electrical network with resistors.

The derivation and further discussion of the transmissibilities are presented in appendix A.

At this point geomechanics is excluded. Instead, fluid-structure interac-
Discretized energy equations

Here, we first introduce the quantities

\[
\begin{align*}
(\hat{H}_d^k, T_d^k, W_d^k)^T &= \frac{1}{|\Omega_d^k|} \int_{\Omega_d^k} (h^d(x), T^d(x), w^d(x))^T \, dV(x) \quad (3.8) \\
(\hat{H}_i^h, T_i^h, W_i^h)^T &= \frac{1}{|\Omega_i^h|} \int_{\Omega_i^h} (h^i(x), T^i(x), w^i(x))^T \, dA(x) \quad (3.9) \\
\text{and} \\
(\hat{H}_r^l, T_r^l, W_r^l)^T &= \frac{1}{|\Omega_r^l|} \int_{\Omega_r^l} (h^r(x), T^r(x), w^r(x))^T \, dV(x), (3.10)
\end{align*}
\]

which respectively represent the mean quantities of enthalpy, temperature and heat source in cells \( \Omega_d^k \), \( \Omega_i^h \) and \( \Omega_r^l \).

At this point we assume constant specific heat capacities. Similar as in section 3.1, we integrate Eqs. (2.6) and (2.7) over individual matrix grid cells, employ Gauss’ theorem and apply upwinding for heat advection. We then obtain the equations

\[
\begin{align*}
\frac{\partial V_d^k}{\partial t} \hat{H}_d^k |\Omega_d^k| + \sum_{h=1}^{n_d} F_{kh}^d \{ H(F_{kh}^d \hat{H}_d^k) + H(-F_{kh}^d \hat{H}_d^h) \} \\
= W_d^k |\Omega_d^k| + \sum_{m=1}^{n_r} K_{mk}^{rd} (T_m^r - T_d^k) \\
- \sum_{i=1}^{N_f} \sum_{h=1}^{n_i} \hat{F}_{hk}^{id} \{ \hat{H}_d^k - \hat{H}_h^i \} \\
+ \sum_{i=1}^{N_f} \sum_{h=1}^{n_i} F_{hk}^{id} \{ H(F_{hk}^{id} \hat{H}_h^i) + H(-F_{hk}^{id} \hat{H}_d^k) \}, (3.11)
\end{align*}
\]
for any damaged matrix cell $\Omega^d_k$ and to equation

$$\frac{\partial V^i_h}{\partial t} \hat{H}^i_h | \Omega^i_h | b^i_h + \sum_{h=1}^{n^i} F^i_{hk} \{ H(\hat{F}^i_{hk}) \hat{H}^i_h + H(-\hat{F}^i_{hk}) \hat{H}^i_k \} - \sum_{k=1}^{n^d} \hat{F}^{id}_{hk} \{ \hat{H}^d_h - \hat{H}^i_h \} + \sum_{j=1}^{n^r} \sum_{k=1}^{n^d} F^{ij}_{hk} \{ H(F^{ij}_{hk}) \hat{H}^i_h + H(-F^{ij}_{hk}) \hat{H}^j_k \} = W^i_h | \Omega^i_h | + \sum_{m=1}^{n^r} K^{ri}_{mh} \{ T^r_m - T^i_h \} - \sum_{k=1}^{n^d} F^{id}_{hk} \{ H(F^{id}_{hk}) \hat{H}^i_h + H(-F^{id}_{hk}) \hat{H}^d_k \}, \quad (3.12)$$

for any fracture cell $\Omega^i_h$. Here, $F^d_{kh}$, $F^{id}_{hk}$, $F^i_{kh}$, and $F^{ij}_{kh}$ quantify the fluxes between volumes $\Omega^d_k$ and $\Omega^d_h$, $\Omega^i_k$ and $\Omega^i_h$, $\Omega^i_k$ and $\Omega^d_h$, and $\Omega^i_h$ and $\Omega^i_k$, respectively:

$$F^d_{kh} = C^d_{kh} (P^d_k - P^d_h - \rho^d_{kh} | \Omega^d_k | + \rho^d_h | \Omega^d_h | |g|(Z^d_k - Z^d_h)), \quad (3.13)$$

$$F^{id}_{hk} = C^{id}_{hk} (P^i_k - P^i_h - \rho^i_{kh} | \Omega^i_k | b^i_h + \rho^i_h | \Omega^i_h | |g|(Z^i_k - Z^d_h)), \quad (3.14)$$

$$F^i_{kh} = C^i_{kh} (P^i_k - P^i_h - \rho^i_{kh} | \Omega^i_k | + \rho^i_h | \Omega^i_h | |g|(Z^i_k - Z^i_h)), \quad (3.15)$$

$$F^{ij}_{kh} = C^{ij}_{kh} (P^i_k - P^i_h - \rho^i_{kh} | \Omega^i_k | b^i_k + \rho^i_h | \Omega^i_h | b^i_h |g|(Z^i_k - Z^i_h)). \quad (3.16)$$

Moreover, $\hat{F}^{id}_{kh}$ is the heat sweep flux of fracture $i$; it is modeled as

$$\hat{F}^{id}_{kh} = \left( \frac{k^d_{kh} \cdot n^i_h}{\mu | \Omega^d_h |} \right) \cdot | \Gamma^{d}_{hm} | n^d_{hm} \left( \frac{| \Omega^d_h | P^d_h + | \Omega^d_m | P^d_m }{| \Omega^d_h | + | \Omega^d_m |} \right) \cdot \left( \frac{\rho^d_{kh} g + \sum_{m=1}^{n^d} | \Gamma^{d}_{hm} | n^d_{hm} \left( | \Omega^d_h | P^d_h + | \Omega^d_m | P^d_m \right)}{| \Omega^d_h | + | \Omega^d_m |} \right). \quad (3.17)$$

Further, $K^{rd}_{km}$ and $K^{ri}_{hm}$ are effective coefficients to quantify energy fluxes. To obtain their numerical value, upscaling has to be performed (as
for \( k^d \)). Here, since upscaling is not topic of this paper, we simply use

\[
K_{mk}^{rd} = \frac{1}{|\Gamma_{mk}^r|} \int_{\Gamma_{mk}^r} K^{rd} dV(x) \quad \text{and} \quad K_{mh}^{ri} = \frac{1}{|\Gamma_{mh}^{ri}|} \int_{\Gamma_{mh}^{ri}} K^{ri} dA(x),
\]

(3.18)

being aware that this represents a very crude approximation.

Conductive heat flux is treated by integrating equation (2.13) over each discrete rock volume \( \Omega_m^r \) and subsequently applying Gauss’ theorem, we obtain

\[
\frac{\partial \hat{H}_m^r |\Omega_m^r|}{\partial t} + \sum_{h=1}^{n_r} K_{mh}^r \{T_m^r - T_h^r\} = W_m^r |\Omega_m^r| - \sum_{k=1}^{n_d} K_{mk}^{rd} \{T_m^r - T_k^d\} - \sum_{i=1}^{N_f} \sum_{h=1}^{n_i} K_{mh}^{ri} \{T_m^r - T_h^i\},
\]

(3.19)

where

\[
K_{mh}^r = \frac{|\Gamma_{mh}^r|}{\Delta_{mh}^r} \left| \lambda_{m} \cdot n_{mh}^r \right| |\Omega_m^r| + \left| \lambda_{h} \cdot n_{mh}^r \right| |\Omega_h^r| \quad |\Omega_m^r| + |\Omega_h^r|
\]

(3.20)

is another effective coefficient to quantify heat diffusion from rock volume \( m \) to volume \( h \).
4 Verification of a-HFR

In this chapter, verification results for the discretized equations of the a-HFR model are presented. Mechanical (and chemical) effects, although they are important for EGS simulations, are not modeled here. Gravitational effects are not treated here and the approach is verified for variable density driven flows in chapter 5 that follows.

In order to compute the transmissibility $C_{hk}^{id}$ between a damaged matrix volume $\Omega_k^d$ and a fracture volume $\Omega_h^i$, the accurate calculation of the length $\Delta_{kh}^{di}$ is required. This calculation is a straight forward task for simple geometries [36], but for more complicated shapes numerical approaches, such as the Monte Carlo integration, are required and the computational cost can significantly increase in 3D problems. But, since it is the matrix permeability that dominates the single phase mass exchange rate between a fracture and its surrounding matrix, the transmissibility $C_{hk}^{id}$ can be overestimated and still to converge to a unique solution [53].

Here, 2D equidistant orthogonal structured grids of variable grid-block size $\delta x$ are always employed for the damaged matrix. In order to overestimate the fracture-matrix transmissibilities, the length $\Delta_{kh}^{di}$ is assumed constant and equal to $\Delta_{kh}^{di} = \delta x/(3\sqrt{2})$. This is the analytical value of $\Delta_{kh}^{di}$ when the 1D fracture volume $\Omega_h^i$ coincides with one of the diagonals of the square shaped matrix volume $\Omega_k^d$ and it is the minimum value that $\Delta_{kh}^{di}$ can have for a 1D fracture volume that is a straight line. This way, the matrix-fracture transmissibilities are always overestimated, and in the mean time the conductivity between fracture volumes that intersect the same matrix cell does not significantly increase.

4.1 Grid convergence analysis

Due to the nature of the studied problems reliable experimental results that could serve as benchmark solutions for DFM verification are not available. For that reason, comparisons with existing software and challenging test
cases are preferred. Such a test case is the fracture network simulated here, which is an outcrop of a fracture network from the Devonian basin of Hornelen, in western Norway [9, 70]. The fracture network modeled and simulated in the following subsections is a hand-mapped highly fractured sandstone bed in two dimensions, which has a size of $18\text{m} \times 18\text{m}$ and is depicted in Fig. 4.1a.

A Dirichlet boundary pressure of 1 MPa is applied on the left boundary ($x = -9\text{m}$) and of 0 Pa on the right boundary ($x = 9\text{m}$). No flow boundary conditions are applied on the top and bottom boundaries. The single phase permeability of the matrix is here assumed isotropic and equal to $k^d = 10^{-12}\text{m}^2$, while for each 1D fracture $i$ is assumed that $b^i k^i = 10^{-9}\text{m}^3$. Figure 4.1b depicts a sketch of the boundary pressure conditions applied to the models employed for the grid convergence analysis both for the flow and the heat transport equations.

### 4.1.1 Flow equations

The conforming and unstructured DFM employed by CSMP++, is a well established software for flow in discrete fractures. It is based on a hybrid finite-volume finite-element method [59] and has accurately solved
many challenging problems; with some of them similar to EGS scenarios [60, 30, 59, 61, 18, 73, 31]. Fractures there are treated discretely, the grid surrounding the fractures conforms to the geometry of the fracture network and the damaged matrix is locally refined around the fractures. The benchmark CSMP++ model of the Hornelen fracture geometry that is used here for verifying the flow equations consists of 100100 triangular elements for the matrix continuum and of 19881 volumes for the discrete fractures. The pressure distribution obtained by the CSMP++ model is presented in Fig. 4.2a.

The a-HFR approach is employed for five different a-HFR models of the Hornelen fracture network. In all of those a-HFR models, a structured orthogonal and equidistant grid is employed for the damaged matrix. Upscaling of small fractures has not been performed here and since all fractures are treated discretely, the effective permeability of the damaged matrix equals the permeability of the porous medium. The coarse a-HFR models discretize the fractures with 9229 volumes each and the finest a-HFR model uses 77929 fracture volumes, with lengths close to the damaged matrix grid block lengths.

For each of the a-HFR models, the steady state pressure solution is obtained by an algebraic multigrid (AMG) linear system solver. The results obtained from these five a-HFR models are also presented in Fig. 4.2 with those from CSMP++.

In Table 4.1 the root mean square (rms) of the normalized fracture pressure difference between the a-HFR models and the CSMP++ benchmark solution is given. The fluid pressure is normalized with the injection pressure of 1 MPa. Afterwards, the normalized pressure inside the fractures of the CSMP++ solution is interpolated over the fracture cells of the five a-HFR models and the mean value of their difference is computed.

In Table 4.2 the rms of the matrix pressure difference between the four coarse a-HFR models and the fine one is presented. Bilinear interpolation is used for computing the matrix pressure of the fine grid \((1000 \times 1000)\) at the centers of the coarser damaged volumes and the rms of those difference is computed and again divided by the injection pressure.

For both grid convergence problems it is quantitatively verified that the a-HFR models converge to the fine solution, which is less than 2% different from the CSMP++ benchmark solution. The largest rms is obtained from the coarsest a-HFR model, which consists of one magnitude less grid cells \((11729 \text{ volumes})\); the rms of fracture pressure difference is less than 5%.
4.1 Grid convergence analysis

Figure 4.2 – Steady state pressure distribution for the CSMP++ model (a) and for the five a-HFR models (b)-(f) of the Hornelen fracture network.
Table 4.1 – Grid convergence results for the fracture pressure of the a-HFR models with a CSMP++ as a reference solution.

<table>
<thead>
<tr>
<th>Model</th>
<th>Matrix cells</th>
<th>Fracture cells</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSMP++</td>
<td>100100</td>
<td>19881</td>
<td>-</td>
</tr>
<tr>
<td>a-HFR(_{50})</td>
<td>50 × 50</td>
<td>9229</td>
<td>0.048611</td>
</tr>
<tr>
<td>a-HFR(_{100})</td>
<td>100 × 100</td>
<td>9229</td>
<td>0.033700</td>
</tr>
<tr>
<td>a-HFR(_{200})</td>
<td>200 × 200</td>
<td>9229</td>
<td>0.021719</td>
</tr>
<tr>
<td>a-HFR(_{400})</td>
<td>400 × 400</td>
<td>9229</td>
<td>0.019857</td>
</tr>
<tr>
<td>a-HFR(_{1000})</td>
<td>1000 × 1000</td>
<td>77929</td>
<td>0.016362</td>
</tr>
</tbody>
</table>

Table 4.2 – Grid convergence results for the damaged matrix pressure of four a-HFR models with a fine a-HFR solution as a reference solution.

<table>
<thead>
<tr>
<th>a-HFR model</th>
<th>Matrix cells</th>
<th>Fracture cells</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>a-HFR(_{1000})</td>
<td>1000 × 1000</td>
<td>77929</td>
<td>-</td>
</tr>
<tr>
<td>a-HFR(_{50})</td>
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<td>9229</td>
<td>0.041020</td>
</tr>
<tr>
<td>a-HFR(_{100})</td>
<td>100 × 100</td>
<td>9229</td>
<td>0.034629</td>
</tr>
<tr>
<td>a-HFR(_{200})</td>
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<td>9229</td>
<td>0.014612</td>
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<tr>
<td>a-HFR(_{400})</td>
<td>400 × 400</td>
<td>9229</td>
<td>0.009761</td>
</tr>
</tbody>
</table>

4.1.2 Energy equations

Heat transport Eqs. (2.6) and (2.7) are solved with the coarsest four a-HFR models. As already presented, these models converge to the fine a-HFR solution for the Hornelen fracture network. Here, an initial temperature condition of \(T(t = 0) = 200^\circ C\) is applied everywhere except at the left boundary, where water of temperature \(T(x = -9m) = 30^\circ C\) is injected; warm water is extracted at the right boundary. Three different cases are simulated where the damaged matrix porosity is set equal to \(\phi^d = 0.3\), \(\phi^d = 0.1\) and \(\phi^d = 0.05\). The specific heat capacity of the fluid is constant everywhere and equals \(4.18 \cdot 10^3 J/(kgK)\). Here, the aperture of the fractures is set equal to the hydraulic aperture that has been used for computing their permeability.

The modeled fracture geometry consists of two separate fracture networks. The first network intersects only with the left boundary and the right network only with the right boundary and thus, the injected cold fluid has to flow through the damaged matrix in order to be extracted from the right side. As a result, the mean temperature of the extracted water declines faster as the damaged matrix porosity \(\phi^d\) decreases. The mean temperature
4.2 Transport - sweeping flux

Table 4.3 – RMS of the produced normalized temperature.

<table>
<thead>
<tr>
<th>model</th>
<th>Matrix cells</th>
<th>Fracture cells</th>
<th>( \phi = 0.3 )</th>
<th>( \phi = 0.1 )</th>
<th>( \phi = 0.05 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>a-HFR(_{400})</td>
<td>400 × 400</td>
<td>9229</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>a-HFR(_{50})</td>
<td>50 × 50</td>
<td>9229</td>
<td>0.0292</td>
<td>0.0344</td>
<td>0.0295</td>
</tr>
<tr>
<td>a-HFR(_{100})</td>
<td>100 × 100</td>
<td>9229</td>
<td>0.0111</td>
<td>0.0119</td>
<td>0.0108</td>
</tr>
<tr>
<td>a-HFR(_{200})</td>
<td>200 × 200</td>
<td>9229</td>
<td>0.0104</td>
<td>0.0105</td>
<td>0.0144</td>
</tr>
</tbody>
</table>

of the water extracted from the right boundary is plotted versus time in Fig. 4.3 and the corresponding rms of temperature differences, with the fine grid a-HFR solution as a reference, are presented in Table 4.3. For the computation of that rms difference, the production temperature of each moment is subtracted from the production temperature of the finest a-HFR model (400×400 damaged matrix grid) and then normalized with the temperature difference \( T^f(x, t = 0s) - T^f_{inj} \).

Snapshots of the temperature distributions in the a-HFR\(_{50}\) are shown in Figs. 4.4 until 4.6. Volume cells are colored according to their mean temperature. Blue corresponds to the injection temperature of \( T^f_{inj} = 30^\circ C \) and red to the initial temperature boundary condition \( T^f(t = 0s) = 200^\circ C \). Fluid temperatures within this range are colored accordingly.

4.2 Transport - sweeping flux

A heat advection problem similar to the example demonstrated in subsection 2.5 is solved with the HFR approach and compared to the analytical solution.

A two-dimensional 500\(m\)×500\(m\) domain is assumed with constant single phase permeability \( k^d = 10^{-12}m^2 \) and porosity \( \phi^d = 0.25 \) (Fig. 4.7). Inside the domain lies a single fracture that starts at point \((x, y) = (250m, 100m)\) and ends at point \((250m, 400m)\) with a fracture aperture equal to its hydraulic aperture and equal to \( b^1 = 10^{-3}m \). Dirichlet boundary pressure of 1 MPa is applied on the left boundary \((x = 0 m)\) and 0 Pa on the right \((x = 500 m)\). Fluid viscosity is taken constant and equal to \( 10^{-3}Pas \), while fluid density and heat capacities are considered constant. The analytical solution of this steady state linear problem is given by the pressure function \( p(x, y) = 10^6 \frac{Pa}{m} (500m - x)/500 \) and gives a constant horizontal fluid velocity of \( u^d = 2 \cdot 10^{-6}m/s \) and with direction orthogonal to the discrete fracture.

At time \( t = 0s \) the temperature of the damaged matrix is considered
equal to $T^d = 30^\circ C$ everywhere and the fracture temperature is considered equal to $T^1 = 250^\circ C$. The temperature of the fluid that enters the domain from the left boundary is equal to the damaged matrix temperature and the fracture temperature at later moments is computed with the a-HFR model. Here, heat exchange with the solid rock is neglected and as a result, the energy conservation equation results to a linear hyperbolic PDE.

For the above described hyperbolic problem, the analytical solution of the fracture temperature gives $T^1 = 30^\circ C$ for $t \geq (b^1/u^d = 500s)$. The resulting pressure distribution of the a-HFR model employed is in agreement with the analytical solution. The pressure drops linearly in the $x$-direction and as expected the fracture pressure does not affect the flow, since it is perpendicular to the pressure gradient of the damaged matrix.

Two different simulations of the above described a-HFR model are performed. In the first simulation, the fracture heat sweep term is neglected and as a result the fracture temperature is not altered from its initial value. Such a discontinuity, as explained in section 2.5 is unphysical and practically “traps” such temperature perturbations inside fractures. In the second simulation, the fracture heat sweep term is considered and the fracture
Figure 4.4 – Snapshots of the temperature distribution obtained by a-HFR for $\phi^d = 0.3$. 
Figure 4.5 – Snapshots of the temperature distribution obtained by a-HFR for $\phi^d=0.1$. 
4.2 Transport - sweeping flux

Figure 4.6 – Snapshots of the temperature distribution obtained by a-HFR for $\phi^d=0.05$. 
temperature quickly approaches the analytical value. It is noted that the
discretized heat sweep term has similarities with a simple upwind scheme.
Figure 4.7 – In (a) the assumed geometry is illustrated on the bottom and its pressure distribution is plotted on the top. In (b) the fracture temperature as a function of time is plotted, for the scenario where the fracture heat sweep term is considered and for the scenario where it is neglected.
5 Verification for variable density problems

Variable density flows are important for the operation of geothermal power plants and thus need to be considered during EGS simulations. The variations of the fluid temperature cause buoyant effects that reduce the pumping load, reduce the cost of operation and enable larger flow rates of geothermal fluid without the creation of seismic events. Additionally, buoyant forces in the heterogeneous formations that surround an EGS reservoir can affect the subsurface temperature profile. Numerical modeling of these effects can assist in the characterization of formations and in locating new potential EGS reservoirs.

The verification of the a-HFR approach for fractured media follows a procedure very similar to the method of manufactured solutions. The procedure followed here makes use of the conclusions derived from chapter 4. There, the a-HFR approach has been verified for solving the steady state flow equations without gravitational effects with this basis, manufactured solutions are constructed that use analytical functions combined with the discrete solutions obtained from the verified part of a-HFR.

5.1 Description of the modified method of manufactured solutions

The method of manufactured solutions is a verification technique for numerical schemes and algorithms to solve mathematical problems. Contrary to verification approaches that compare simulation results either with experimental data or with other software, the main aim of the manufactured solution approach is to assess the accuracy of an algorithm by comparing numerical results with analytical solutions.

As a first step for applying the method of manufactured solutions, the system of governing equations $\mathcal{R}(\phi^*) = Q$ needs to be determined. The field $\phi^*$ is the unique solution of the considered equations with right hand
With an analytic value for $\Phi^*$ the right hand side $Q = R(\Phi^*)$ also attains an analytic form. Now, the manufactured solution is ready to be applied. The numerical solution $\Phi^*$ of $R(\phi^*) = R(\Phi^*)$ can then be compared with the analytic solution.

A similar method is applied here for the verification of the steady state flow equations. Here, the unique solution is $\phi^* = \nabla p(x)$, which is the pressure gradient.

According to the mass conservation equations described in chapter 2, the operators $R$ and $Q$ are

$$R(\nabla p) = -\nabla \cdot \left( \frac{k}{\mu} \cdot (\nabla p) \right)$$

and

$$Q = q - \nabla \cdot \left( \frac{k}{\mu} \cdot (\rho g) \right).$$

(5.1)

In fractured media however, the permeability tensor $k$ is heterogeneous, anisotropic and discontinuous due to fractures. As a result, analytically calculating a manufactured solution is not a straight-forward task for fractured media. For this reason, a slightly modified approach is employed here. It takes advantage of the fact that this flow equation is linear and therefore the rule

$$R(\alpha \phi_1^* + \beta \phi_2^*) = \alpha R(\phi_1^*) + \beta R(\phi_2^*)$$

(5.2)

can be applied.

In chapter 4, the a-HFR approach has been verified for steady state flow problems, where gravitational effects are neglected, and it was demonstrated that it leads to acceptable numerical solutions of $R(\nabla p) = q$.

Here, the same right hand side $q$ is considered and the function $f^{MS}(x)$ is defined such that $\nabla f^{MS} = \rho g$, which leads to

$$R(\nabla p) + \nabla \cdot \left( \frac{k}{\mu} \cdot (\rho g) \right) = R(\nabla p) - R(\nabla f^{MS}) = R(\nabla (p - f^{MS})) = q.$$  

(5.3)

Now, the previously verified part of the a-HFR approach can be used for constructing reference solutions. Initially, the a-HFR approach is employed once for solving the problem $R(\nabla p) = q$, for which its performance has been verified. The obtained solution is considered an acceptable approximation of the reference solution $\nabla P_{ref}^*$. According to relation (5.3), the discretized equations of the a-HFR approach are able to numerically solve variable density driven flows, only if $\nabla P_{ref}^* = \nabla (p^* - f)$ for the obtained solution $\nabla p^*$. 


5.2 Implementation of the verification approach

A certain advantage of employing methods such as manufactured solutions is that the analytical functions do not have to be physically realizable, i.e. neither the obtained pressure values nor the assumed fluid density distributions $\rho$ have to be realistic or strictly positive. They only have to be such that their contribution in the manufactured solution is obvious.

Taking the above into consideration, a simple reservoir geometry is assumed. It is a two-dimensional $1 \, km \times 1 \, km$ square shaped reservoir with homogeneous and isotropic damaged matrix permeability $k_d = 10^{-12} m^2$ that is penetrated by a single fracture with domain $\Omega^1$ and $b^1 = 1 \, cm$. The injection rate at the right corner is $1 \, l/s$ and $-1 \, l/s$ at the bottom left corner. Zero flux boundary conditions are applied everywhere else and the fluid viscosity is always considered constant and equal to $\mu = 10^{-3} \, Pa \cdot s$.

To construct the manufactured solution, three a-HFR models are employed that consist of $30 \times 30$, $50 \times 50$ and $100 \times 100$ damaged matrix grids and which treat the fracture of the problem discretely. For the computation of all the flow fields presented here, the same AMG solver as in chapter 4 was applied.

The problem geometry as well as the $P_{ref}^*$ solutions obtained from the a-HFR models are presented in Fig. 5.1. Four different depth-dependent density functions $\rho_i$ are assumed and the method of manufactured solutions is applied for each function and each a-HFR model. These four $\rho_i$ functions and their corresponding $f^{MS}$ integrals are given in Table 5.1. The variable
5.2 Implementation of the verification approach

Table 5.1 – Functions used for constructing the benchmark manufactured solutions.

| i | \( \rho_i \) | \( f_{i}^{MS}/|g| \) |
|---|---|---|
| 1 | \( 6 \cdot 10^3 - 2z \) | \( -6 \cdot 10^3 z + z^2 \) |
| 2 | \( 10^3 - 5z + 0.01z^2 \) | \( -10^3 z + \frac{5}{2} z^2 - \frac{0.01}{3} z^3 \) |
| 3 | \( 10^3 + 10^3 \cos \left( \frac{2\pi(z)}{300} \right) \) | \( -3 \cdot 10^5 \frac{1}{2\pi} \sin \left( \frac{2\pi(z)}{300} \right) \) |
| 4 | \( 10^3 \cos \left( \frac{2\pi(z)}{80} \right) \) | \( -8 \cdot 10^4 \frac{1}{2\pi} \sin \left( \frac{2\pi(z)}{300} \right) \) |

Table 5.2 – Normalized standard deviation obtained for each of the a-HFR models.

<table>
<thead>
<tr>
<th>( \rho \times \rho )</th>
<th>( \rho_1 )</th>
<th>( \rho_2 )</th>
<th>( \rho_3 )</th>
<th>( \rho_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 \times 30</td>
<td>1.238e-4</td>
<td>0.0027</td>
<td>0.0071</td>
<td>0.0206</td>
</tr>
<tr>
<td>50 \times 50</td>
<td>2.772e-5</td>
<td>8.466e-4</td>
<td>0.0022</td>
<td>0.0085</td>
</tr>
<tr>
<td>100 \times 100</td>
<td>4.563e-6</td>
<td>1.782e-4</td>
<td>4.583e-4</td>
<td>0.0017</td>
</tr>
</tbody>
</table>

\( z \) is the height above the lowest point of the reservoir and the gravitational vector is always taken equal to \( g = (0, 0, -9.81)^T \).

As previously analyzed, the value of \( (P_{ref} - p_i^* + f_{i}^{MS}) \) can be any constant real value. For that reason, the computation of the standard deviation of \( (P_{ref} - p_i^* + f_{i}^{MS}) \) from its mean value qualifies as an acceptable measure for the distance of the obtained solution from the semi-numerical manufactured solution. These deviations are also normalized here with the range of values of solution \( P_{ref}^* \).

Figures 5.2 and 5.3 show the considered fluid density functions and the corresponding a-HFR pressure solutions.
Figure 5.2 – Plots of assumed density functions versus depth.
5.2 Implementation of the verification approach

Figure 5.3 – Plots of the a-HFR pressure solutions for each assumed density function assumed.
6 Differences between a-HFR and current DFMs

As presented in chapter 2, with the use of kernel functions one can express mass conservation as a system of PDEs. Similar kernel functions have been previously used by Lough et al. [56] for upscaling purposes, but here they have been introduce to remove singularities.

These kernels model mass exchange between different continua and capture the mean properties along the fracture manifolds. The governing equations presented ensure that transfer coefficients are well defined and simplify the treatment of the governing equations for embedded models such as the a-HFR. The a-HFR approach enables modeling of flow and transport between non-conforming volumes and its convergence to a unique solution has been verified in chapters 4 and 5.

Here, similarities and differences of this approach with other DFMs are discussed. Initially, in section 6.1 the method is compared with the rest of the embedded and unstructured DFMs. Afterwards, in section 6.2 the adaptability of the method with regard to geometrical changes is discussed and compared with other models. Aspects regarding computational efficiency are presented in section 6.3 and issues that arise due to the adaptive nature are discussed in section 6.4.

It is noted that conforming structured DFM approaches resemble the CFM approaches. They show similar behavior and their computational cost increases disproportionally with geometrical accuracy; they will not be compared with a-HFR here. Instead, the following discussion focuses on methods that use both explicit and accurate representations of fractures without the need of a very fine structured grids.

6.1 Similarities with other embedded methods

The a-HFR approach is practically a structured embedded DFM that employs a hierarchical fracture representation. Therefore, it has many similarities with the structured embedded DFM (EDFM) presented by Lee et
6.2 Grid adaptability

One of a-HFR key features is its adaptability to geometrical changes that may occur during simulations of scenarios with deformations. Such changes for example appear while performing hydrofracturing of an EGS reservoir. Conformity among the boundaries of volumes from different continua is not necessary and thus no restrictions regarding volume size arise. Fracture volumes need to conform neither with the damaged matrix grid nor with other fractures. Consequently, a-HFR does not require re-meshing of the reservoir when new fractures are added.

In general, remeshing can be avoided in structured DFMs and cannot be avoided in unstructured ones.

In both the unstructured embedded DFM by Sarda et al. [81] and the various unstructured conforming approaches [45, 60, 40] remeshing is unavoidable, since the unstructured mesh of the surrounding matrix strongly depends on the geometry of the considered fracture volumes. The addition of a new fracture requires re-meshing of a possibly large portion of the unstructured grid. Moreover, the fluid properties in the new volumes have to be re-calculated. For these reasons, if an unstructured DFM is employed, pre-describing all possible and foreseeable large fractures is suggested. This way, re-meshing is avoided at the cost of dealing with a possibly too large...
Figure 6.1 – Here, exemplary grids are shown for the same fracture network and with three different embedded structured DFMs. In all cases, the small fractures have been upscaled and are represented by an equivalent porous medium with domain $\Omega^d$, which is discretized by an orthogonal structured grid. The matrix grid-blocks that overlap with a fracture are highlighted; i.e. light-gray color is for those cells that overlap with a fracture trace that includes a sector that has a small volume (thus, it restricts the time-step size) and dark-gray color is for those that overlap with a large fracture volume. In (a) the resulting volumes for employing the $\alpha$-HFR approach are presented; each large fracture with domain $\Omega^i$ is resolved into finer volumes separately and independently of each other and the boundaries (white circles) can be located anywhere on the fracture. In the EDFM approach (b), each fracture segment can intersect with only one damaged matrix cell. Thus, boundaries of fracture volumes need to conform to the damaged matrix grid. In the 2D DFM suggested by Hajibeygi et al. [36] (c), the fracture boundaries need to conform with the fracture-fracture intersections and the corresponding fracture segments can be discretized without further need for conformity.
and too complicated mesh throughout the simulation.

The addition of new fractures is much simpler in DFMs that use a structured grid for the surrounding matrix (including both conforming or embedded approaches). New embedded fractures simply need to be connected with the intersecting cells and the new transmissibilities have to be calculated.

Remeshing issues only arise when fracture cells need to conform to fracture-fracture intersections, as is the case in the approach adapted by Hajibeygi et al. [36].

### 6.3 Computational efficiency

The computational efficiency of the a-HFR model is discussed here only for flow and heat transport problems.

An implicit scheme is used for the flow equations to ensure numerical stability for large time-steps. To solve the discretized flow equations, an algebraic multigrid method (AMG) is employed. Contrary to simple iterative linear systems solvers, such as Gauss-Seidel and Gauss-Jacobi methods, the convergence rate of an AMG solver increases almost linearly with the size of the linear system, which is an important property if large, realistic problems are to be addressed.

However, the performance of AMG solvers is not only affected by the size of the problem, but also by the variation of the transmissibilities. Especially in EGS simulations, where the matrix permeability may be several orders of magnitude smaller than the permeability of discrete fractures, highly heterogeneous linear systems are typical. For those scenarios, the adaptability of a-HFR is beneficial for the convergence rate of AMG solvers, e.g. by employing a coarser grid for the discrete fractures. As a result, transmissibilities between fracture volumes reduce and the linear system that needs to be solved becomes less heterogeneous.

An illustrative example is presented in Fig. 6.2. There, an AMG solver has been used for solving the flow equations in two a-HFR models with the same reservoir geometry. Both of them employ the same $20 \times 20$ damaged matrix grid and are subjected to the same pressure boundary conditions. Their only difference is the number of fracture volumes. In one model the number of volumes is $n = 5$ and in the other one it is $n = 20$. The latter results in a larger and more heterogeneous linear system of equations that needs to be solved. On the bottom the convergence rates of the AMG solver are presented for both models. It becomes clear that the AMG solver treats
the smaller model more efficiently.

An unavoidable computational cost for AMG methods is the construction of the linear systems that need to be solved at each multigrid level. In non-linear problems, where either the permeability or the storativity are pressure dependent, these levels need to be re-constructed every few iterations according to the updated matrix-coefficients. This computational cost however is smaller for multi-scale approaches such as the i-MSFV, that have already been applied for structured embedded fracture models [36].

An explicit scheme is used for the discretization of the heat equations. Explicit schemes require less effort per time-step and the advantages of parallel programming can be exploited with less programming effort. As a result, the explicit representation of the heat equations is often preferred.

The maximum time step that satisfies the Courant-Friedrichs-Lewy (CFL) condition is the dominating factor for the computational cost of an explicit method. The CFL condition requires that the numerical domain of influence for each discrete volume is always larger than its physical domain of influence. It is thus concluded that, for a certain velocity field and a certain heat diffusion tensor, it is the size of the control volumes that determines the value for which the CFL condition stands. Larger grid volumes result in larger numerical domains of influence and therefore stability is ensured for larger time-steps.

As already analyzed, due to its adaptive nature the a-HFR avoids the creation of undesired small volumes that could restrict the time-step in time-marching simulations such as the solution of heat advection. The low porosity damage matrix may be represented by a rather coarse grid and the fracture volumes, where most of the flow occurs, are divided into smaller volumes independently of the surrounding grid. The matrix grid neither has to become finer in regions of high fracture density nor are tiny fracture volumes expected. The above feature is also maintained for dynamically changing geometries, since large fracture volumes are maintained, even when a new fracture is added.

6.4 Treatment of issues that arise due to grid-adaptability

The adaptive capability of the modeling approach presented here is facilitated by embedding the discrete fractures in a fixed damaged matrix grid. Among the existing conforming approaches, adaptability with comparable computational cost can be achieved only if new fractures conform to the matrix grid; otherwise re-meshing is required.
6.4 Treatment of issues that arise due to grid-adaptability

Figure 6.2 – On the top, a sketch of the assumed geometry. An identical $20 \times 20$ grid is employed for $\Omega^d$ and $\Omega^1$ is once discretized by $n = 5$ volumes and once by $n = 20$ volumes. On the bottom, the convergence rate of the AMG solver used is presented. The vertical axis is for the residual of the linear system at the end of each AMG cycle and the horizontal axis is for the number of AMG cycle iterations.
The drawback of this enhanced adaptability is the creation of unphysical flow paths. These artifacts result in a decreased impedance of the modeled EGS reservoirs. As explained in section 3.1, these additional flow paths between neighboring volumes can be compensated by decreasing the transmissibility between fracture volumes.

However, it is noted that when transmissibilities between fracture and damaged matrix volumes are overestimated, more than one flow path is enhanced; i.e. the flow path between fracture volumes that are not connected to each but overlap with the same matrix volume, as well as the flow path between matrix volumes intersecting with the same fracture volume. Therefore, too large transmissibilities should be avoided. Instead, transmissibilities that are functions of the damaged matrix permeability, such as the ones employed for the verification test-cases, are suggested.

Another issue is related to the transport problem, which originates from the fact that mass exchange between fracture and matrix volumes does not take into account on which side of the fracture exchange occurs. For example, if the inflow rate on one side of a fracture volume matches the outflow rate on the other side, then a-HFR without the fracture sweeping term predicts no mass exchange between that fracture volume and the matrix. This is wrong, however, and can lead to artificial trapping of cold or hot fluid. This deficiency can be corrected by the fracture sweeping term, which accounts for the mass flow that sweeps a fracture and advects scalars across it. In general, it does not affect the time-step size.

It is noted, that modeling of the fracture sweep flux is not needed in DFM methods where the boundaries of the matrix volumes conform to the discrete fracture geometry [45, 60, 40].
Part II

EGS decision making with HFR-Sim
7 HFR-Sim simulations in hot fractured rocks

To simulate EGS operation scenarios and to support decision processes, accurate modeling of hydrological, thermal, mechanical and chemical phenomena is necessary. A list of desired features for a modern EGS simulator and a short description of current EGS simulators have been presented in chapter 1. In all these simulators either continuum models or conforming DFMs are used for modeling flow and heat transport. None of them relies on embedded DFMs. For that reason, HFR-Sim has been developed and is described here.

HFR-Sim is an algorithmic framework capable of performing EGS simulations utilizing the potential and advantages of the a-HFR approach presented in the previous part. It builds a-HFR models and grids for given geometries and simulates flow and heat transport subjected to spatial boundary conditions. At this point, HFR-Sim does not incorporate geomechanical or geochemical modules; examples discussed in this thesis employ corresponding proxies.

In the following subsection, the algorithmic framework of HFR-Sim is presented and its main components are described and analyzed. Those features presented here are used in all simulation scenarios discussed afterwards.

7.1 Algorithmic framework

The multi-disciplinary nature of the phenomena that need to be modeled for performing THMC EGS simulations favors the development of modular EGS modeling frameworks. This would allow a variety of combinations of modeling approaches for each field to be employed and valuable EGS conclusions derived without neglecting the progress in any one of those different fields.

The algorithmic framework of HFR-Sim respects this need for modularity and provides simple interfaces that facilitate communication with other
software. HFR-Sim, with the use of the a-HFR modeling approach, can compute the history of pressure, velocity, viscosity, temperature and density distributions.

HFR-Sim can be coupled with a vast number of different geomechanics and geochemistry modules. The latter is important to account for the integral effects on the permeability due to chemical species deposition and erosion, while the former describes the effect of solid frame deformation on fracture aperture, and of computing where and when the rock fails.

In our suggested approach, the first step consists of a hierarchical discretization of the reservoir. Once all the required hydraulic and thermal properties are initialized, the main iterative procedure begins. In particular, at each time step an a-HFR model is employed for solving the flow equations; i.e. hydraulic pressure and mass flux are computed. Afterwards, the a-HFR model is used for solving the heat equations, where heat advection and conduction are calculated according to the hydraulic properties obtained in the previous step. Pressure and temperature distributions can then be transferred to the geomechanics and geochemistry modules that in turn will provide all the needed information for updating the fracture network and all required hydraulic properties. Only then can the simulation proceed to the next iteration of a THMC simulation. Obviously, TH simulations that neglect one or more of those modules is possible. A flow chart of the framework of HFR-Sim is shown in Fig. 7.1.

HFR-Sim has been developed in C++ and thus it can either be used as a library by other software or employ libraries that treat geomechanics or geochemistry.

Finally, HFR-Sim employs an efficient linear system solver, i.e. an algebraic multi-grid (AMG) solver with a Gauss-Seidel method as a smoother. As explained in chapter 6, such solvers increase efficiency, since their convergence rate for sparse and symmetric linear systems, which have to be solved here, scales linearly with the problem size.

7.2 Space discretization

A hierarchical fracture representation is employed by HFR-Sim as described in chapters 2 and 3. Upscaling of the damaged matrix must be performed during the pre-processing of the reservoir geometry and is not a topic of the work presented here. HFR-Sim does not favor any of the numerous homogenization, effectivization and upscaling techniques that could be applied. Instead, the damaged matrix properties and information regarding
7.2  Space discretization

EGS reservoir discretization (a-HFR) and initialization of properties for \( t = t_{\text{start}} \)

Model flow for \( t + \Delta t \) (a-HFR)

Model heat transport for \( t + \Delta t \) (a-HFR)

Model chemical phenomena

Model mechanical phenomena

Update model and add new fracture volumes

\( t \leftarrow t + \Delta t \)

Is \( t + \Delta t \leq t_{\text{end}} \) ?

yes

no

End

Figure 7.1 – Flow chart of HFR-Sim.
the discrete are required as an input.

Initially, the grids employed for the solid rock and the damaged matrix are constructed. A rather coarse orthogonal structured grid discretizes the solid rock. Here the same grid is also employed for the damaged matrix but this is not mandatory. The orientation of these two structured grids should be aligned with the damaged matrix anisotropy, such that two-point flux approximation can safely be employed. The structured solid rock grid is composed of $n^r$ volumes and the damaged matrix grid of $n^d$ damaged matrix volumes.

The next step is to resolve the large discrete fractures. HFR-Sim requires information regarding size, shape, hydraulic aperture and aperture distributions of each discrete fracture. Similar to the damaged matrix, for each fracture $i$ a structured grid is laid over its surface and only cells whose center is within the given fracture boundaries is included in the set of $n^i$ fracture volumes. In this way arbitrary fracture shapes can be discretized. In addition, HFR-Sim offers three simple fracture shapes as possible options that have also been integrated into its main gridding algorithms. These shape options are an ellipse, a circle and an orthogonal quadrilateral. In all three cases the user needs to define a fracture center, the lengths and orientations of the minor and the major axis.

Wells are considered as one-dimensional tubes that penetrate multiple damaged matrix and fracture volumes. Currently, HFR-Sim treats only well segments within the EGS reservoir. Besides the shape and position of each well, HFR-Sim further requires that either the injection or extraction flow rates for each well or their wellhead pressure to be specified. These values also serve as flow boundary conditions for each well.

Finally, the hydraulic and thermal properties of each of the above volumes must be initialized before proceeding to any simulations. Necessary inputs are the initial distribution of upscaled permeability $k^d$ and porosity of the damaged matrix $\phi^d$, the physical properties of the working fluid, the properties of the solid rock matrix, and the coefficients required to model heat exchange between domains. The rock matrix is assumed to be impermeable and so it is characterized by thermal conductivity and specific heat that control conductive heat flow.

It is clear that smaller grid-blocks can more accurately capture complex fracture shapes and spatial property variations.
7.3 HFR-Sim tube network for flow and heat transport simulations

To simplify visualization and reduce the programming effort, HFR-Sim builds a graph for each a-HFR model, which here will be called “tube network”. This tube-network concept facilitates vector-based calculations and makes visualization and interpretation of the results easier. Each node of this conceptual tube network represents either a matrix; a fracture; or a well volume, while a separate tube network is created for the solid rock continuum.

Each tube (or edge of this graph), which connects two nodes, represents a possible flow path between those two volumes and therefore a non-zero transmissibility is assigned to it.

For now, the $n^w$ non-deformable well nodes are not taken into account, i.e. only the $n^l = n^d + n^1 + ... + n^N_f$ deformable damaged matrix and fractures nodes are considered here, where $N_f$ is the number of fractures. A new index notation (capital letters) is introduced that is used for indexing the nodes of the tube network. The damaged matrix volume element $\Omega_k^d$ corresponds to the node with index $M = k$ and the fracture volume $\Omega_i^f$ corresponds to the node with index $M = (n^d + \sum_{j=1}^{i-1} n^j + h)$. The transmissibility $C_{M,J}$ can be defined at each tube according to Eqs. (3.4)-
(3.7), such that volume flux from node \( M \) to volume \( J \) equals
\[ F_{M,J} = C_{M,J}(P_M - P_J - (Z_M - Z_J)\bar{\rho}_{M,J}|g|), \]
with \( P_M \) corresponding to the mean volume pressure, \( Z_M \) the average depth of the volume and \( \bar{\rho}_{M,J} \) the average fluid density.

Equations (3.2) and (3.3) result now in a system of \( n^l \) equations where each equation \( M \) is

\[
\frac{\partial V_M}{\partial t} + \sum_{J=1}^{n^l} C_{M,J}(P_M - P_J - (Z_M - Z_J)\bar{\rho}_{M,J}|g|) = Q_M. \tag{7.1}
\]

In deformable scenarios, where both the transmissibilities \( C_{M,J} \) and the volumes \( V_M \) are known functions of the pressure distribution, Eqs. (3.2) and (3.3) can be linearized and the system of equations

\[
(\frac{\partial V_M}{\partial t})^\nu + \sum_{J=1}^{n^l} C_{M,J}^\nu(P_M^{\nu+1} - P_J^{\nu+1} - (Z_M - Z_J)\bar{\rho}_{M,J}|g|) = Q_M, \tag{7.2}
\]

has to be solved for \((\nu + 1)\) iterations, until \( P_M \approx P_M^\nu \approx P_M^{\nu+1} \), while \((\frac{\partial V_M}{\partial t})^\nu \) and \( C_{M,J}^\nu \) are functions of the pressure field obtained after \( \nu \) iterations.

For problems with steady and homogeneous fluid specific heat capacity, Eqs. (3.11) and (3.12) can be explicitly discretized in this conceptual tube network with a system of \( n^l \) equations, where for \( M \leq n^d \)

\[
\frac{\partial \hat{H}_M}{\partial t} + \sum_{J=1}^{n^l} F_{M,J}\{H(F_{M,J})\hat{H}_M + H(-F_{M,J})\hat{H}_J\}
\]

\[= W_M + \sum_{J=\imath n^d+1}^{n^l} \hat{F}_{M,J}(\hat{H}_J - \hat{H}_M) + \sum_{m=1}^{n^r} K_{r,m,M}(T_m^w - T_M), \tag{7.3}
\]
and for \( n^d < M < n^l \)

\[
\frac{\partial \hat{H}_M}{\partial t} + \sum_{J=1}^{n^l} F_{M,J} \{ H(F_{M,J}) \hat{H}_M + H(-F_{M,J}) \hat{H}_J \} = W_M + \sum_{J=1}^{n^d} \hat{F}_{J,M} (\hat{H}_J - \hat{H}_M) + \sum_{m=1}^{n^r} K_{m,M}^r (T_m^r - T_M),
\]

(7.4)

where \( \hat{H}_M \) is the amount of enthalpy stored in volume \( M \), \( \hat{F}_{M,J} \) is the absolute value of the fracture sweep flux between the damaged matrix volume \( M \) and the fracture volume \( J \), and \( K_{m,M}^r \) are the effective coefficients that quantify energy fluxes from the hot rock to the working fluid and which are treated in accordance to chapter 3.

The approach of treating fractured reservoirs as an equivalent tube network is not new. Simpler tube networks have been used for developing DFM\s in the past [14, 13, 20] and their resemblance to graphs also allows the application of graph theory techniques, as e.g. by Vitel and Souche [88] for upscaling purposes.

With the use of the tube network, the adaptive nature of the a-HFR model is more easy to understand. When a new fracture is created, the new nodes and tubes are added to the existing tube-network, while the rest of the nodes and connections are not affected by this addition.

Figure 7.3 shows an example of the 3D irregular tube network created for modeling an EGS reservoir that consists of 2 large discrete fractures. In accordance to the space discretization procedure described in section 7.2, HFR-Sim initially creates the orthogonal and structured tube network for the damaged matrix. Afterwards, structured orthogonal tube grids discretize each fracture manifold. Only nodes within each fracture boundary are preserved and further tubes are added for intersecting fracture nodes. Finally, the discrete fractures and the damaged matrix tube networks are coupled by embedding the fractures and adding tubes that connect the fracture nodes with the intersecting damaged matrix nodes. It should be noted that each node of the grid corresponds to the center of an elemental volume of the a-HFR grid and is not a boundary point, as in Fig. 3.1.
Figure 7.3 – (a) Illustration of the hydraulic coupling between two discrete large 2D fractures and a 3D damaged matrix. (b) The damaged matrix is discretized by a simple orthogonal structured grid that consists of the HFR-Sim damaged matrix tube network. (c) Each large discrete fracture is discretized by a structured 2D tube network and (d) these tubes, as well as the tubes connecting fracture volumes to damaged matrix volumes or volumes from other fractures are then added to the HFR-Sim damaged matrix tube network.
7.4 Wells treatment

Each well $w$ from the $n^w$ wells is treated as a one-dimensional domain $\Omega^w$ that is non-deformable, consists of only one volume (and thus, of only one node) and is not connected to other well nodes. The total number of nodes (deformable and non deformable) in the tube network is therefore $n^l + n^w$.

Each well $w$ corresponds to volume $\Omega_K$, with $K = (n^l + w)$ and mass conservation inside it is modeled as

$$\sum_{M=1}^{n^l} C_{K,M}(P_K - P_M - (Z_K - Z_M)\rho_{K,M}|g|) = Q_K,$$

(7.5)

where the injected or produced flow rate is quantified by the term $Q_K$ and $C_{K,M}$ is the production index of the well with node $M$ that quantifies the volumetric flow rate per unit pressure drop from cell $\Omega_M$ to well volume $\Omega_K$. This production index is modeled as

$$C_{K,M} = \begin{cases} \frac{2\pi ||k_M||\cdot|\Omega_K \cap \Omega_M|}{\mu\ln(0.2058\Delta x_M/r_w)} & \text{if } M \leq n^d, \\ \frac{2\pi ||k_M||\cdot b_M}{\mu\ln(0.2058\Delta x_M/r_w)} & \text{if } n^d < M \leq n^l, \end{cases}$$

where the length $\Delta x_M$ corresponds to the smallest grid-block size of domain $\Omega_M$, $k_M$ is its permeability, $r_w$ is the well radius and $b_M$ is the aperture of a fracture node indexed as $M$. This production index is very similar to the well model presented by Peaceman [74].

7.5 Geomechanics and geochemistry

A geochemistry module is needed for including the integral effects of chemical species deposition and erosion on the permeability. Deposition and dissolution of minerals affects the size of the aperture in long term simulations and thus alters the hydraulic properties of the reservoir. Although the transport and diffusion of chemical species can be modeled similar to the approach presented for modeling heat transport in a-HFR models, their interaction with the solid rock and the rest of the chemical species needs to be calculated by a dedicated geochemical module and is not treated here.

The modular framework and the well defined interfaces also allow the flow and transport solvers presented above to be coupled to a geomechanics module. The unstructured grid that results from the hierarchical fracture representation enables the flow and transport solvers to be coupled with
geomechanics modules that employ either a discrete fracture representation or a continuum approach. These geomechanics modules can be based on pre-decided discrete fractures or on creating "ad-hoc" new ones.

Any of those geomechanical modules will obtain pressure and temperature distributions from the flow and transport simulations performed by HFR-Sim and are expected to return changes of fluid volumes, transmissibilities, heat exchange coefficients and the position of newly created fractures. The expected input for the geochemistry modules consists of the density, the viscosity and the temperature distributions, as well as the fluid velocities inside the reservoir. Such modules should be capable to return information on how apertures and damaged matrix permeability change, as well as the altered species concentrations.

7.6 Updating of EGS models

Before the next simulation time-step begins it is necessary to collect the results obtained from all modules and to update the simulated EGS model accordingly.

At a very first step, the fracture network needs to change according to the geomechanical computations. Newly created fractures need to be discretized and added in the computational domain and the volume size of the existing ones needs to be revised. When an existing permeable fracture fails and shearing occurs, then new fracture cells are added to the computational domain, which are of same size and adjacent to the existing ones. When a new fracture is added or the domain $\Omega^i$ of the $i$-th fracture increases due to shearing, the fluid properties inside these new tube-network volumes are initialized. HFR-Sim currently assumes that new void spaces are immediately filled with working fluid and ensures conservation of mass and energy by making use of a conservative first order scheme for the discretization of the time-derivatives in Eqs. (7.1) and (7.3).

Changes of the hydraulic properties of the updated tube network can be caused by changes of the working fluid’s chemical composition and due to the disposition of minerals that reduce the size of the hydraulic apertures. These changes of the aperture and the working fluid properties cannot be estimated without a geochemical module.

In general, a number of different working fluids can be used for EGS purposes. For example, Pruess et al.[75] studied and suggested the use of liquid CO$_2$ as an EGS working fluid, while chemical species that enhance permeability may also be added.
7.6 Updating of EGS models

The effect of pressure and temperature on the fluid density and viscosity has to be treated by the flow and heat transport modules. For that reason, the state equations for the density and the viscosity of saline water, which is one of the most typical EGS and geothermal fluids, have been integrated in HFR-Sim and can be used for simulations. Since pressure changes have a much smaller effect on the fluid density and viscosity than temperature variations, both fluid density and dynamic viscosity are treated by functions of the fluid temperature only. For simulations with different working fluids other density and viscosity functions have to be provided.

Both the density and viscosity functions that are used by HFR-Sim are presented in Appendix B and they are plotted in Fig. 7.4.

Figure 7.4 – Default density-temperature and viscosity-temperature functions used by HFR-Sim.
8 Exemplary simulations of EGS life-cycle scenarios

The main purpose of developing an EGS simulator is the need for predicting the otherwise unpredictable effects resulting from certain actions on the EGS reservoir. Such a tool can help to optimize operations and the technology to achieve financial sustainability. The algorithmic framework of HFR-Sim allows to make thermal and hydrological predictions for a range of EGS scenarios that take place during the life cycle of an EGS power plant. Some exemplary simulations are presented here.

The presented scenarios serve the purpose of demonstrating not only the capabilities offered by an adaptive hierarchical fracture model but also the behavior of an EGS reservoir under certain conditions. The latter is important, since in the next chapter HFR-Sim is used for optimizing EGS operational cycles, where more than one of those scenarios occurs.

The simulations are sorted in a chronological order, i.e. according to the stage of the life cycle of an EGS project when they are most likely to appear. For that reason, the complexity of the assumed scenarios also increases gradually. It is assumed that the more mature the exploitation of an EGS reservoir is, the more reliable are the results from its characterization and therefore more accurate and thus complicated models can be developed and need to be simulated.

It is noted that the simulations presented here have been performed for scenarios with similar characteristics as the ones witnessed in real EGS operations, but they are fictional. Their results are not intended for deriving quantitative conclusions, but rather qualitative ones. Also, geochemical and geomechanical effects have either been neglected or treated by simple proxies.
8.1 Assessment of thermal revenues

The need for simulations that can support EGS investment decisions arises very early. Estimation of the thermal revenues of potential EGS sites are prerequisites for the assessment of a more accurate risk analysis that may lead to more EGS investments. The amount of reliable information at this stage of the project is rather poor. Temperature maps of Europe at a depth of a few kilometers have been interpolated from scattered well data [32, 41]. However, the uncertainty regarding the temperature distribution over much of Europe at depths relevant to EGS remains high. For that reason, test-drilling at a prospective EGS site is essential from which information regarding subsurface temperature and fracture characteristics can be acquired. With this information simple 2D or 3D EGS models can be built in order to get a first estimation on the reservoir’s potential.

Here, a simple 2D EGS reservoir geometry is assumed and simulated with HFR-Sim in order to estimate the thermal revenues and the cool-down of the rock. The model consists of 50 large fractures that have been assigned random position, random size, aperture and orientation. The damaged matrix is considered heterogeneous, but hydraulically isotropic, and its porosity

![Figure 8.1 – Damaged matrix permeability distribution for an artificial 2D test case with 50 large fractures (gold lines) and 2 wells (blue lines). Cold water of constant temperature of 30°C is injected through the left well and hot water at reservoir temperature of 160°C is produced at the right well.](image-url)
is $\phi^d = 0.02$. Its boundaries are impermeable and the permeable region has the shape of an ellipse with a permeability decreasing with the distance from the ellipse center. There is one injection and one production well and the injection temperature is $30^\circ C$. The reservoir is initially filled with hot single phase fluid of $160^\circ C$. Constant pressure difference of $1\text{MPa}$ is applied between the two wells forcing the working fluid to circulate through the reservoir. The hydraulic properties, the geometry and the volumes of the large fracture network are assumed constant. Figure 8.1 illustrates the assumed geometry.

The surrounding hot rock is discretized by an orthogonal $60 \times 30$ structured grid identical to the one used for the damaged matrix, exactly as described in section 7.2. On the left and the right boundaries of the reservoir adiabatic conditions are applied and at the top and bottom boundaries of the domain constant rock temperature equal $160^\circ C$ is assumed.

Both density and viscosity are considered constant and do not change during the simulation and neither do the boundary conditions of the flow equations. Consequently, and since chemical effects are not considered here, the steady state linear flow equations are solved once and the obtained velocity field can be used throughout the simulation. A snapshot of the temperature distribution in the tube network employed by HFR-Sim and the temperature distribution in the solid rock continuum are shown in Fig. 8.2. It should be noted that in the tube network each node is colored according to its mean temperature, while for the rock continuum a Delaunay interpolation has been performed.

The main characteristics of the production phase appear in the results of this simplified EGS model. The cold working fluid enters the reservoir through the injection well and flows through the permeable regions. Most of the flux is channeled through the network of large fractures and thermal energy is extracted from the neighboring hot rock. Around the injection well the rock temperature drops at a faster rate and the cold front that is created around the well propagates towards the production well on the right. Eventually, the working fluid with increased temperature and enthalpy is forced out through the production well and a first approximation of the thermal revenues of this EGS model can be derived.

After a test-drilling, the data obtained from the reservoir characterization allow for more reliable estimations regarding the reservoir properties. With this addition of information decisions regarding very expensive drilling can be made with less risk.

The problem of simulating the EGS production phase and estimating
Figure 8.2 – Snapshot of reservoir and rock temperature for an artificial 2D test case with 50 large fractures.
8.2 Pre-stimulation testing

A vital step before the stimulation of an EGS reservoir is its characterization test. Hydraulic tests are conducted to provide information regarding the impedance and storativity/compressibility of the permeable fractures and a geologic model is developed from well-logs and cuttings analysis. These collective data can assist in designing a more efficient reservoir stimulation approach.

The primary purpose of the pre-stimulation test program is the characterization of the reservoir before any hydrofracturing is performed. For that reason, the injection rates are kept sufficiently small so that injection pressures are small, so as to avoid changing the characteristics of the reservoir (i.e. avoid stimulation). HFR-Sim was used for the simulation of such a scenario. The injection rates from the pre-stimulation test performed in the Basel shown in Fig. 8.4 EGS reservoir [37, 71, 35] were used as flow
boundary conditions. Ideally, at this stage of an EGS project, well logs may exist that offer deterministic information regarding position, aperture and orientation of fractures that are penetrated by each well [23, 25, 84]. Here, a simpler 3D reservoir geometry is assumed (Fig. 8.3) and such logs are not considered.

The damaged matrix of this reservoir is assumed homogeneous, isotropic and of very low permeability, since a stimulation operation has not yet been performed. The well is located at the center of the damaged matrix continuum and penetrates the only fracture of the model, which has an aperture of 0.85 mm and is much smaller than the damaged matrix continuum.

The injection rates for the pre-stimulation test are low enough to justify neglecting mechanical and thermal phenomena. The fluid pressure never exceeds values that could cause shearing of the fractures and thermal cracks caused by the injection of cold fluid can be neglected, since the cool-down of the rock occurs at a low rate. Thus, for simulating such a pre-stimulation scenario only hydrological modeling is needed. At each time-step the transient flow equations need to be solved for the applied flow boundary conditions. Here, the rate of fluid injected at the Basel well and Dirichlet pressure boundary conditions are applied to the damaged matrix boundaries, where the far-field pressure (assumed equal to 0 Pa) apply. The considered damaged matrix porosity is equal to 1%.

Fig. 8.4 shows the injection rates and the corresponding wellhead pressures from the Basel three-step pre-stimulation. Also shown is the well curve predicted by the HFR-Sim simulation. While the same injection rate profile was applied, it cannot be expected that the real fracture network is perfectly honored. The simulated pressure follows the same trend as the one observed, but is slightly lower at higher pressures indicating that the impedance of the tube network is too low at the higher pressures. Perhaps indicating pressure-dependent aperture effects or the fracture size is overestimated.

8.3 Reservoir stimulation

Accurate and efficient simulations of the creation phase of a reservoir are of vital importance for the further development of EGS technology. The design of a stimulation strategy that creates an efficient EGS reservoir and does not cause public anxiety through felt seismicity is important for implementing the technology in urban areas where it can be more profitable.

During stimulation, high fluid rates are injected into the existing fracture
Figure 8.4 – The upper frame shows the injection rates of the three-step pre-stimulation test in the Basel EGS reservoir. The lower frame shows a comparison of the corresponding wellhead pressure with the pressure in the well obtained with HFR-Sim for the same injection rates.
network and due to the high fluid pressure micro-seismic events are triggered that cause an irreversible decrease to the reservoir impedance. As Evans et al. [23] note, only shearing can be responsible for irreversible increases of fracture permeability by a factor of up to 3 orders of magnitude. As the fracture permeability increases, new and more permeable paths are created and the high pressure front that is responsible for these events propagates resulting in new failures.

As explained in Part I, the a-HFR modeling approach that is employed by HFR-Sim has been especially designed for modeling flow and heat transport in reservoirs with such a dynamically changing geometry. Here, fractures do not have to be pre-prescribed for solving mass and energy conservation equations during the creation phase, since new fractures can be added in the computational domain without the need of re-meshing.

During creation phase simulations, the geomechanics module must be provided in order to model stress changes in the reservoir given the pressure and temperature distributions. Whenever the failure of a fracture or of a fracture volume is triggered, the tube network simply has to be updated by adding the new nodes, the new connections and by updating the transmissibilities.

To demonstrate the applicability of HFR-Sim for such dynamically changing fracture networks, the cloud of seismic events from the enhancement of an EGS reservoir in Soultz-sous-Forêts in France [25, 24] is employed as a geomechanical proxy and is applied to obtain a reservoir geometry similar to the one assumed for the pre-stimulation test.

Again, the EGS reservoir initially consists of a single injection well GPK1 located at the center of a very large damaged matrix of low permeability. The injection rate increases by 5 l/s every 2 days and Dirichlet pressure boundary condition of 0 Pa is applied at the "far-field" boundaries. At the end of each day, fractures with the shape of an ellipse and with random orientation and size are added to the computational domain and a new pressure field is computed. The center of each fracture is located at the position of the seismic event it associates with at the time of the event. Once all fractures for the next time-step have been added, the pressure solution is computed. Overall, more than 12000 events were located during the stimulation in question, and each has been represented by a discrete fractures.

As expected, the impedance of the reservoir reduces as the fracture network evolves. Mass conservation is preserved during the whole simulation and tiny volumes with working fluid never occur thanks to the employed a-
HFR model. In Fig. 8.5 snapshots of the simulation are provided.

8.4 Simulations of production phase in a characterized EGS reservoir

During the final development stages of an EGS power plant its long term behavior needs to be simulated. Tracer tests between the EGS wells after the reservoir enhancement provide valuable information that assists a rigorous reservoir characterization. Once the reservoir is characterized, more realistic models can be constructed that will lead to more reliable predictions [82, 78].

The result of such a reservoir characterization can lead to highly heterogeneous and thus computationally challenging flow and transport problems. Here, instead of assuming a heterogeneous reservoir geometry, a heterogeneous benchmark test case is used to demonstrate the effect that a highly heterogeneous damaged matrix can have on the flow.

The test case consists of one layer of the tenth SPE comparative test case [16]. The highly heterogeneous permeability field of this case is often used as benchmark for gas-injection problems and employs a continuum representation.

Here, the permeability field of the Tenth SPE Benchmark test case is used as the permeability distribution of an assumed damaged matrix continuum. Inside this continuum 200 discrete fractures of random length and aperture are randomly positioned. A constant pressure difference was applied between the left and the right boundaries and the steady state pressure equation was solved. Gravitational effects were neglected. The resulting transmissibilities and the computed steady state fluxes and pressures are presented in Fig. 8.6.

Afterwards, the heat advection equations were solved based on the computed flux field. Initially, the reservoir was filled with cold fluid of 0°C and hot fluid of 200°C entered from the left boundary. Transfer from the hot rock to the cold working fluid was neglected. In Fig. 8.7 the temperature distributions obtained from HFR-Sim are presented.

8.5 Exemplary 3D production phase simulation

Here, a stochastic fracture network is created and is used for constructing a 3D EGS reservoir geometry, which is then simulated.

The dimensions of the considered solid rock EGS domain \( \Omega^r \) are \( 2.4 \times 1.2 \times 1.8 \ km \) and the maximum considered depth equals 5.6 km. The
Figure 8.5 – Four snapshots of the damaged matrix and the discrete fractures pressure distribution on 2D planes during the imitation of the 3D seismic cloud of events in Soultz.
Figure 8.6 – Results obtained from HFR-Sim for the 10-th SPE benchmark test case with discrete fractures.
Figure 8.7 – Snapshots of temperature distribution from the HFR-Sim solution for the 10-th SPE test case with discrete fractures.
solid rock is assumed to have properties similar to granite with a mass density equal to $\rho_r = 2700 \, \text{kg/m}^3$, heat capacity equal to $920 \, \text{J/kg/°C}$ and an isotropic heat conductivity that equals $\lambda = 3.5 \, \text{W/m/°C}$. For the discretization of the rock domain $\Omega^r$, an orthogonal and equidistant $24 \times 12 \times 18$ mesh is employed.

Three wells that reach at a depth of $5 \, \text{km}$ are considered here. They have a $600 \, \text{m}$ long open casing through which mass can be exchanged. The radius of each well is constant and equals $18 \, \text{cm}$. The position of the wells is such that they form an isosceles triangle, with the two production wells being approximately $600 \, \text{m}$ away from the injection well and at an equal angle of $30^\circ$. Each of the wells is discretized with only one volume and thus pressure losses within the wells are not treated here.

Around the three wells and around the two equal edges of this isosceles triangle, a damaged matrix of low permeability is considered. In particular, around these two triangle edges an ellipsoid region with major and two minor axis of $800 \, \text{m}$, $400 \, \text{m}$ and $500 \, \text{m}$ is considered. The damaged matrix permeability ranges from $k^d = 10^{-16} - 10^{15} \, \text{m}^2$ and equals $0 \, \text{m}^2$ outside of the ellipsoid. The volumes with the maximum damaged matrix permeability are centered around the wells. The damaged matrix domain $\Omega^d$ is discretized with the same grid as the solid rock continuum, but only the 1077 permeable volumes are considered. The injected flow rate is $50 \, \text{l}/\text{s}$.

A total number of 267 discrete fractures $\Omega^i$ is assumed. They are all penny-shaped, they all have a radius of $200 \, \text{m}$ and all of them have an aperture of $50 \, \mu\text{m}$. Each fracture $i$ is discretized with an orthogonal equidistant mesh of grid-block size approximately two times smaller than the damaged-matrix grid-blocks size. The center of those fractures has been allocated stochastically and is always located inside a permeable damaged matrix volume. The final tube network consists of 6960 volumes. Snapshots of the temperature distribution in the fractures and on two surfaces that include the injection and production wells are shown in Fig. 8.8.
8.5 Exemplary 3D production phase simulation

Figure 8.8 – Snapshots of the temperature distribution in a 3D model simulated with HFR-Sim. Just a small fraction of fractures is shown here.
9 Optimization of EGS working cycles

9.1 Introduction

The increase of greenhouse gases in the atmosphere and the resulting concern for undesired climate change has led many industrialized countries to establish an international regulatory framework for the reduction of CO\textsubscript{2} emissions and greenhouse gases. The high associated international cost is compensated by benefits from investments in new electrical base load energy production technologies with reduced emission, such as EGS power plants.

Contrary to most renewable resources, geothermal energy can be utilized on a constant basis and thus, it can cover a portion of the base load energy needs. However, the cost of the electrical power produced from an EGS power plant still remains high and the risks associated with it are not trivial. For making EGS technology financially sustainable, the electrical energy revenues need to increase, the risks associated to it have to be moderated and the construction cost of plants must be reduced.

In general, geothermal power production technologies suffer from increased development costs during periods when the financial risk remains high [29]. Drilling costs are responsible for a large portion of the total investment and they must be covered while the geothermal reservoir’s potentials are still ambiguous. For unconventional geothermal technologies, such as EGS, this risk is even higher. Contrary to conventional approaches, EGS technology is not yet mature enough. No deep EGS that meets commercial targets has yet been developed. Furthermore, several instances where felt seismic events have occurred has raised public awareness of the risk of possible damage. Reducing the cost of energy produced by an EGS would reduce the risk of investments in sites with uncertain potential. Further, better control of the creation phase would enhance production and reduce the risk of strong seismic events.

EGS power plants operate at such deep depths that drilling dominates the overall cost. Tester et al. [42] collected the financial data for completed
9.1 Introduction

Figure 9.1 – The thermal efficiency of conversion cycles is bounded by the efficiency of the Carnot cycle. For the range of EGS operating temperatures, the maximum theoretical efficiency is very low and for EGS it increases linearly with the production temperature.

Regarding energy revenues, the electrical production of an EGS power plant is limited by its low conversion efficiency. An estimation of the thermal efficiency of binary plants that operate at temperatures close to geothermal ones has been given by Tester et al. [42]. They found the net thermal efficiency $\eta_{th}$ was correlated with the production temperature $T_{prod}$ of the power plant, i.e.

$$\eta_{th} = (0.0935 \cdot T_{prod} - 2.3266)/100.$$  \hfill (9.1)

In Fig. 9.1 the thermal efficiency of equation (9.1) is compared with the efficiency of the Carnot cycle for a similar range of temperatures. The low range of temperatures at which EGS power plants operate results in a low thermal efficiency. This efficiency increases linearly with the temperature with which the fluid reaches the surface and therefore, maintaining high temperatures of produced fluid is desirable.
However, maintaining high production temperatures for a longer time period is in conflict with the need for commercially interesting rates of extracted energy, which requires heat to be extracted from the hot rock at higher rates. The total thermal energy $E_{th}$ extracted from an EGS reservoir equals

$$E_{th} = \int_{0}^{t_l} F_{inj}(t) \rho c_p (T_{prod}(t) - T_{inj}) dt,$$

(9.2)

where $t_l$ is the life of the EGS power plant, $F_{inj}$ the production flow rate, $c_p$ the specific heat capacity of the working fluid and $T_{inj}$ the temperature of fluid that gets re-injected.

To reduce the drilling cost per unit of produced electrical energy, the net electrical energy production $E_{el} = \eta_{th} E_{th}$ needs to be maximized. For economic assessments life expectancy $t_l$ is usually assumed to be less than 25 years. Although in general the upper bound of a power plant’s life-expectancy is based on economical criteria, its lower bound strongly depends on the performed cycle during EGS operation. Usually, it coincides with the moment when the produced fluid drops below a critical value.

In general, an increase of the mass flow rate $F$ or a poor heat exchange rate between the circulating fluid and the hot rock have a negative impact on the life expectancy $t_l$. Life expectancy strongly depends on the evolution of the cold temperature front in the rock, as the one presented in section 8.1 (or illustrated in Fig. 8.2). During the production phase, the area surrounding the injection well exchanges heat at the highest rate and cools down the local rock. The cold front that is created in the rock propagates along the flow paths of the working fluid as the exploitation of the EGS reservoir continues. In general, the faster this cold front approaches the production well, the shorter is the reservoir’s life expectancy. It becomes clear that the life expectancy $t_l$ depends on the injection flow rate $F_{inj}$ and on the geometry of the fracture network.

The creation of a reservoir that serves as an efficient and large heat exchanger between the rock and the fluid is the main purpose of the EGS reservoir stimulation process. Low reservoir impedance not only reduces the pumping load and attendant expenses but also increases the maximum flow rate $F_{inj}$ that can be applied without pressures reaching the level where creating too strong seismic event occurs. This maximum mass flow rate corresponds to the maximum operating pressure difference, since injection pressures cannot exceed the minimum principal stress in the rock. Experience has shown that injections of close to the minimum principal stress level leads to reservoir growth and attendant seismicity. Low enough impedance
between injection and production wells ensures that the value of the flow rate $F_{inj}$ that maximizes the thermal revenues in equation (9.2) can be realized without creating too strong seismic events.

Estimations regarding the fracture network inside a reservoir are subjected to high uncertainty and thus, the desired heat exchange efficiency of an enhanced reservoir is not always achieved. The EGS site of Hijiori, Japan is an example of a stimulated reservoir with low thermal recovery efficiency. There, one of the production wells penetrates the fracture system in such a way that a short circuit of fluid is created between the injection and one of the production wells, that is only 90 m distant. Along this circuit the rock temperature decreases rapidly for high (and thus commercial) flow rates. Within 300 days the initial production temperature of 260°C from that well reduced to less than 140°C, implying the existence of this too short flow path. Tenma et al. [87] optimized the operation cycle of the Hijiori EGS power plant with the mass flow rate as a variable and even for the optimized scenario, the net thermal output reduces to one third of its initial value within the first year of operation.

The opposite was the case for the 3.5 km reservoir at Soultz-sous-Forêts, France. There, the flow paths between the injection and the production wells occur at least in part within fracture zones and faults that are both highly permeable and of high porosity [23]. Consequently, large flow rates can circulate inside the EGS reservoir in Soultz and extract thermal energy from a wider hot rock area. However, the location of such fracture systems is hard to predict.

Lower injection temperatures $T_{inj}$ result in both higher thermal revenues and improved conversion efficiency. Due to the latter it is usually treated as one of the optimization parameters for the power generation cycles on the surface [28, 33] and thus, it needs to be treated as a constraint for the overall optimization problem. Only very few and discrete values can be chosen for the working fluid properties such as density $\rho$ and specific heat capacity $c_p$, since the variety of cheap working fluids is very small. From the above it becomes clear that the production temperature $T_{prod}$, the injection flow rate $F_{inj}$ and the life expectancy $t_l$ of a particular EGS reservoir are strongly related to each other and determine the optimal operation cycle. Each combination of these three parameters results in different thermal revenues and their optimal combination needs to be found for minimizing the cost of the EGS electrical energy production $E_{el}$.

Here, the relationship between these three operation parameters is studied. Contrary to traditional EGS cycles, where cold working fluid is injected
directly to the deep reservoir from which it was produced, here it is examined how the shallow geothermal reservoir can be used in combination with a deep one to reduce the drilling costs per unit of energy produced.

9.2 3D HFR-Sim model for combined shallow and deep geothermal exploitation

Here, we test with HFR-Sim how a preheater affects electrical revenues in a 3D EGS model, which for a constant injection rate of 40 $l/s$, suffers from low electrical revenues and short life expectancy.

In a typical EGS scenario, hot working fluid is produced from a deep reservoir, some of the thermal energy of the fluid is converted into electrical energy at the surface and the cold fluid is injected back into the deep reservoir. An alternative scenario is to inject the cold working fluid into a shallow reservoir, where it warms as it passes through the reservoir, and then bring it back to the surface at elevated temperature. From the surface, this warm water is then injected, into the deep reservoir, where it is heated to the final production temperature and pumped to the surface.

An illustrative sketch of these two EGS scenarios is shown in Fig. 9.2. On the left, a typical EGS scenario as described in chapter 1 is depicted. The right sketch illustrates the proposed combined operation of the deep and shallow reservoirs. In general, as Baria et al. [5] noted, the HDR concept progresses towards larger reservoirs, thus, the combined use of shallow and deep reservoirs may be a realistic future approach.

The deep reservoir and the preheater in the 3D model are considered hydraulically identical. A permeable homogeneous region that has the shape of a horizontal elliptic cylinder and porosity $\phi^d = 0.3$ is assumed. The major axis of the elliptic cylinder is 270 m, the minor axis is 210 m, and the permeable region is penetrated at the edges of the major axis by two parallel horizontal wells, with each one having an open case length of 400 m. The size of the rock domain $\Omega^r$ is 300 m$\times$400 m$\times$300 m and is centered around the permeable region. Figure 9.3 illustrates the assumed geometry. The initial rock temperature profile is assumed to vary linearly with depth; 125°C at 1 km and increases with a rate of 25°C/km. All simulations end, once the production temperature becomes less than 100°C.

The first scenario to be studied here corresponds to a traditional EGS operation. The deep reservoir is located at a depth of 5.5 km and has the characteristics described above. Water is injected at a rate of 40 $l/s$ of constant temperature of 60°C and is produced at the production well.
Figure 9.2 – Typical EGS scenario (on the left) and an alternative EGS scenario for increased recovery of high enthalpy working fluid (right).
at a reservoir temperature of 233.75°C. The simulated life expectancy of the first scenario is approximately 11 years. The production temperature decreases rapidly and asymptotically approaches the injection temperature (Fig. 9.4).

For the second scenario, a preheater in the form of a shallow reservoir centered at 2.35 km is used to warm up the working fluid before it is injected to the deep reservoir at 5.35 km. Water is injected into the the preheater at the same flow rate of 40 l/s and at a temperature of 60°C, and is produced at the same rate at a reservoir temperature of 158.75°C.

The increase of the life expectancy, of the amount of recovered heat and of the electrical energy production can be seen in Fig. 9.4. In addition to the electrical energy production for the two scenarios, the cumulative curves for operating both reservoirs independently are presented. The latter scenario corresponds to the electrical revenues that traditional EGS operation returns at the same cost as the alternative EGS cycle. The alternative scenario not only achieves a life expectancy that is almost 60% longer regardless of the abandonment temperature, but also produces 26% more electrical energy than obtained by exploiting the two reservoirs independently.

Comparing with the traditional scenario of the first simulation, the alter-
native approach has produced almost 68% more electrical energy. According to estimations regarding drilling costs for the assumed depths [39], the additional drilling cost for the described preheater is approximately 20% of the cost of the two deep wells. Thus, since almost 70% more electrical energy has been produced, the alternative scenario achieves almost 28% lower drilling cost per energy unit compared to the traditional scenario for the same EGS model.

9.3 Simplified 1-D modeling of heat transport in an EGS

In the introduction the energy conservation equations were presented for a Cartesian coordinate system and then the governing equations for the embedded fractures were introduced. Here, a 1-D model is described that can be used for quick estimates of the production temperature of an EGS power plant, and then comparisons with results from HFR-Sim are made.

1D models have been previously developed for simple HDR calculations and they have been reviewed in the past [90]. Bodvarsson [8] developed a 1D single fracture model that uses the analytical functions computed by Carslaw and Jaeger [15] for the computation of the outlet water temperature in simple shaped fractures. He also proposed an intergranular 1D model which assumes that water temperature equals rock temperature and neglects heat conduction. Elsworth [22] used a 1D radial model for permeability enhancement studies. Robinson and Kruger [52] made use both of the analytical fracture equations and of a simplified 1D model that splits a reservoir into many identical rock segments inside which 1D heat conduction is solved. The latter model makes use of the same dimensionless equations developed in the 2D model developed of Murphy et al. [67], which is also based on solving a 1D heat diffusion equation inside the rock.

We consider an EGS reservoir with an effective porous domain $\Omega$ of length $L_w$, which equals the distance between injection and production wells. Further, its swept mean surface is $A_w$, such that $|\Omega| = L_w A_w$. Porosity $\phi$ is assumed constant everywhere in $\Omega$ and temperature variations only occur along the domain length $L_w$. For that reason, from now on $\Omega$ is treated as a 1D domain of length $L_w$, cross section $A_w$, with injection occurring at $x = 0$ and fluid being extracted at $x = L_w$. Figure 9.5 illustrates such a 1D domain.
Figure 9.4 – Simulation results for the two 3D EGS models.
9.3 Simplified 1-D modeling of heat transport in an EGS

9.3.1 Governing equations and discretized 1D model

Commercial EGS power plants operate within a range of injection rates $F_{inj}$ that result in a slower rate of heat diffusion inside the rock than the heat exchange rate between the hot rock and the fluid. For that reason, heat diffusion inside domain $\Omega$ can be neglected and the flow rate considered constant and equal to $u = (F_{inj}/A_w)$.

Heat capacities and mass densities are considered constant both for the fluid and the rock and the heat conservation equations read

$$\frac{\partial \phi T}{\partial t} + u \frac{\partial T}{\partial x} = \frac{c^{r \rightarrow f}}{A_w \rho c_p} (T^r - T), \quad (9.3)$$

and

$$\frac{\partial (1 - \phi)T^r}{\partial t} = \frac{c^{r \rightarrow f}}{A_w \rho^r c_p^r} (T - T^r) \quad (9.4)$$

for $0 \leq x \leq L_w$, where $T^r$, $\rho^r$ and $c_p^r$ are the rock temperature, density and heat capacity, respectively, and the coefficient $c^{r \rightarrow f}$ is a heat exchange coefficient that indicates the efficiency with which the fluid exchanges heat with the rock, i.e. quantifies the rate of heat transferred per unit length and per unit temperature difference $(T^r - T)$. Here, this coefficient $c^{r \rightarrow f}$ is treated as a constant that accounts for the geometry of the fracture network and the rock thermal conductivity.

The non-dimensional form of Eqs. (9.3) and (9.4) provides valuable insight for the behavior of the model presented above. For the derivation the reference time $\tau_0 = (L_w \phi)/u$ and the reference length $L_w$ are introduced.

Figure 9.5 – Sketch of the assumed 1D EGS domain.
The reference time corresponds to the time needed by a heat perturbation at the injection well (at \( x = 0 \)) to get advected to the production well (at \( x = L_w \)). The difference between the initial production temperature \( T_{prod}(t = 0) \) and the injection temperature \( T_{inj} \) is the reference temperature difference \( \Delta T_0 = T_{prod}(t = 0) - T_{inj} \). Now, the non-dimensional variables \( \tau = t/t_0, \chi = x/L_w, \theta = (T - T_{inj})/\Delta T_0, \) and \( \theta^r = (T^r - T_{inj})/\Delta T_0 \) can be defined. In non-dimensional form Eqs. (9.3) and (9.4) then become

\[
\frac{\partial \theta}{\partial \tau} + \frac{\partial \theta}{\partial \chi} = \frac{c_l}{F_{inj} \rho c_p} \frac{L_w}{\phi L_w} \left( \theta^r - \theta \right), \tag{9.5}
\]

and

\[
\frac{\partial \theta^r}{\partial \tau} = \frac{c_l}{F_{inj} \phi L_w} \frac{L_w}{(1 - \phi) \rho c_p} \left( \theta - \theta^r \right), \tag{9.6}
\]

for \( 0 \leq \chi \leq 1 \). For each pair of coefficients \( c_l \) and \( c_2 \), a unique solution of Eqs. (9.5) and (9.6) exists corresponding to an infinite number of problems.

The system (9.5) and (9.6) is solved numerically with a finite volume method. The domain is discretized by \( N \) volumes and the mean values of \( \theta_i \) and \( \theta^r_i \) of each volume \( i \) are computed at each time-step as

\[
\theta^{i+1}_f = \theta^f + \Delta \tau N \left( \theta_{i-1}^f - \theta^f_i \right) + \Delta \tau c_1 \left( \theta^r_i - \theta^f_i \right) \tag{9.7}
\]

and

\[
\theta^{i+1}_r = \theta^r_i + \Delta \tau c_2 \left( \theta^r_i - \theta^r_i \right) \tag{9.8}
\]

where \( \Delta \tau \) is the time-step size for which the CFL condition is satisfied and the superscripts \( \nu \) and \( \nu + 1 \) denote old and new time levels respectively.

The thermal efficiency of Tester et al. [42] given by equation (9.1) becomes

\[
\eta_{th}(\theta_{prod}) = (0.0935 \left( \Delta T_0 \theta_{prod} \right) / 100 + \eta_{th0}, \tag{9.9}
\]

where \( \eta_{th0} = (0.0935 \cdot T_{inj} - 2.3266) / 100 \), and \( \theta_{prod} = (T_{prod} - T_{inj}) / (\Delta T_0) \).

The total electrical energy can be approximated as

\[
E_{el} = \int_0^{t_f / \tau_0} \left\{ \eta_{th} F_{inj} \rho c_p \left( T_{prod} - T_{inj} \right) \right\} d\tau \nonumber
\]

\[
= \int_0^{t_f / \tau_0} \left\{ \frac{F_{inj} \rho c_p \Delta T_0 \tau_0}{100} \left( 0.0935 \Delta T_0 \theta^2_{prod} + \eta_{th0} \theta_{prod} \right) \right\} d\tau, \tag{9.10}
\]
where the non-linear relationship between electrical power and $T_{prod}$ can be seen. A decrease in the temperature production $\theta_{prod}$ reduces the produced electrical power quadratically.

### 9.3.2 Verification of simplified 1D model

The continuum 3D model employed in section 9.2 is used here for the verification of the simplified 1D model presented in section 9.3.1. For doing so, a simplified 1D EGS model is found that returns results in close agreement with the results obtained from the 3D model for the $T_{prod}$ when only a deep reservoir is used (typical EGS scenario). Then, the same modeling parameters are used for simulating a 1D scenario, where a preheater is used and the results obtained are compared with the results obtained from the 3D model. The preheater is assumed to be a shallower reservoir with heat exchange coefficients and time reference equal to the deep one.

The two modeling parameters assumed here are the right hand side coefficients $c_1$ and $c_2$. It is noted that the ratio of $c_1$ over $c_2$ is a function of porosity and of the known material properties. Thus, the pair of values of $c_1$ and $c_2$ which gives results in good agreement with the 3D model can also be used for calculating the equivalent porosity $\phi$ that corresponds to the model and a relationship between reference time and domain size $\Omega$ can then be obtained for the 1D model.

The search range for solutions of the 1D model is limited for reference times close to the production temperature breakthrough moment; i.e. $10^7 s \leq \tau_0 \leq 4 \cdot 10^7 s$. The search is also limited to $\phi \leq \phi^d$, where $\phi^d = 0.3$ is the porosity of the damaged matrix in the 3D model.

The set of right hand side coefficients $(c_1, c_2) = (2.7189, 0.3927)$ was found to give a production temperature decline history in good agreement with the 3D model for the typical EGS scenario. These values imply an effective porosity $\phi = 0.079$ and for $F_{inj}=0.04 m^3/s$, and $L_w=400 m$ it can be matched by the 3D model with a reference time $\tau_0 = 3 \cdot 10^7 s$. The same set of coefficients $(c_1, c_2)$ is then used for simulating the scenario where a preheater with initial non dimensional temperature $\theta_p = 0.5683$ is used. The temperature of the preheater corresponds to the non-dimensional value of the preheater in the 3D simulation. The same flow rate $F_{inj} = 0.04 m^3/s$ circulates through the preheater.

In Fig. 9.6, the results obtained from the 1D model are compared with the 3D simulation results for both scenarios. The overestimation of the cold water breakthrough time results from the fact that 3D simulations also account for fluid dispersion in all directions and therefore the production
9.3.3 Numerical studies of combined shallow and deep geothermal exploitation

In this section, the discretized non-dimensional equations of the simplified 1D model are solved for a number of different solution families and the energy revenues are computed and compared.

The simulations can be divided into two sets. The first set consists of simulations of typical EGS operation scenarios that are solved with Eqs. (9.7) and (9.8). The second set of simulations consists of scenarios where a preheater is used.

For both sets of simulations, the deep reservoir has a constant porosity of $\phi = 0.05$, the distance between the wells is $L_w = 1000$ m, the effective area is $A = 10000$ $m^2$ and the injection rate is $F_{inj} = 0.01$ $m^3/s$. The material of the considered rock has properties similar to granite (i.e. $\rho^r = 2700$ $kg/m^3$ and $c_p^r = 920$ $J/kg$) and fresh water ($\rho = 1000$ $kg/m^3$ and $c_p = 4183$ $J/kg$) is used as working fluid. The reference time according to our definition is $\tau_0 = 5 \cdot 10^7 \approx 1.6$ years.

Figure 9.6 – Comparison of simplified 1D and 3D simulations of a typical EGS scenario and one with a preheater.
9.3 Simplified 1-D modeling of heat transport in an EGS

For the traditional EGS model it is assumed that the initial (at $\tau < 0$) temperature in the deep reservoir is $T_{prod}$ and thus $\theta^r = \theta = 1$, and that the injection temperature (at $\chi = 0$) is $T_{inj}$ and thus $\theta = 0$.

For the second set of simulations, it is assumed that the fluid first circulates through the shallow reservoir (also called “preheater”) and then it enters into the same deep reservoir as in the first set of simulations. Here, the deep and the shallow reservoirs are assumed to have the same heat exchange coefficient $c_{r \rightarrow f}$, the same distance between the wells $L_w$ and the same material properties. Therefore, both can be treated as a single 1D non-dimensional domain of length 2. With the non-dimensional initial rock temperature $\theta_p$ of the preheater, initial conditions in the domain are

$$\theta^r(\tau < 0) = \theta(\tau < 0) = \begin{cases} \theta_p & \text{for } 0 \leq \chi < 1 \\ 1 & \text{for } 1 \leq \chi \leq 2 \end{cases} \quad (9.11)$$

and the boundary conditions for $\tau \geq 0$ are $\theta(\chi = 0) = 0$.

Simulations are conducted with the preheater temperature $\theta_p$ set to 0.3, 0.4 and 0.5, i.e. its temperature is such that it potentially can provide 30%, 40% and 50% of the overall temperature gain, respectively.

For each set and for each $\theta_p$, the same suite of 10 heat exchange coefficients $c_{r \rightarrow f}$ are used and the solutions with the corresponding parameters $c_1$ and $c_2$ are computed. Note that here $c_2/c_1 = \phi \rho c_p/((1 - \phi)\rho^r c_p^r) = 0.008863$ and therefore, only the $c_1$ values are listed in Table 9.1. The values employed for $c_{r \rightarrow f}$ and $c_1$ scale logarithmically and the first one without preheater corresponds to a scenario where power production beyond time $\tau_0$ is of almost no commercial interest. It is assumed that only $\theta \geq 0.3$ is of commercial interest, which determines the length of the simulations.

In Fig. 9.7 the non-dimensional temperature production histories for the logarithmically increasing set of $c_1$ coefficients are given for each scenario.

For the traditional EGS operation cycle (top left corner), the least efficient models result in a rapid drop of the production temperature, i.e. the injected fluid does not reach temperature equilibrium with the rock. On the other hand, the use of a shallow reservoir as a preheater results in a significant temperature increase at later times. Using a preheater results in an increase to the effective rock volume out of which thermal energy is extracted. This is the reason for the "plateau" of the $\theta$-curves for $0 < \tau < 2$, e.g. in case of $c_1 \leq 1.5091$. After a period of two reference times the production temperature declines more rapidly.

The most efficient scenarios with high $c_{r \rightarrow f}$ and high $c_1$ values benefit less from the preheater. As a consequence of the high efficiency with which
heat is extracted from the deep reservoir, the influence of the preheater only shows up later, when the production temperature approaches the temperature of the preheater. The produced energy is then similar to the one produced by the preheater alone. For the scenario with $\theta_p = 0.3$, where the shallow reservoir alone does not have commercial potential, its combination with the deep reservoir increases the lifetime of the system by more than 25% for all the $c_1$ values.

Important quantities for determining the amount of produced electrical energy $E_{el}$ are the integrals $I_1 = \int T \, dt$ and $I_2 = \int T^2 \, dt$ that are used in equation (9.10). The produced electrical energy at each time $\tau$ can be computed as

$$E_{el} = \frac{F_{inj} \rho c_p \Delta T_0 \tau_0}{100} (0.0935 \Delta T_0 I_2 + \eta_{th} \psi I_1), \quad (9.12)$$

where the values of $I_1$ and $I_2$ are plotted in Figs. 9.8 and 9.9 for each scenario and each value at $c_1$. In general, the use of preheaters not only increases the life expectancy of the least efficient reservoirs but also leads to higher revenues during the same operation time.

For the following economic study it is assumed that $T_{inj} = 55^\circ C$ and $T_{prod} = 180^\circ C$. The electrical revenues for each heat exchange coefficient and each of the four scenarios (without and with preheater of $\theta_p = 0.3$, $\theta_p = 0.4$, and $\theta_p = 0.5$) are computed. Moreover, for comparison, the revenues for the 2 warmer preheaters alone, i.e. not in combination with the deep reservoir, are computed. Figure 9.10 shows the total electric energy produced for each scenario and for each $c_1$. The lower frame shows the

Table 9.1 – Employed values for coefficient $c_1$.

<table>
<thead>
<tr>
<th>$c_1$</th>
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<tbody>
<tr>
<td>0.3791</td>
</tr>
<tr>
<td>0.6008</td>
</tr>
<tr>
<td>0.9522</td>
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<td>1.5091</td>
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<td>2.3918</td>
</tr>
<tr>
<td>3.7907</td>
</tr>
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<td>6.0079</td>
</tr>
<tr>
<td>9.5218</td>
</tr>
<tr>
<td>15.0911</td>
</tr>
<tr>
<td>23.9177</td>
</tr>
</tbody>
</table>
additional electrical energy produced due to combining the deep reservoir with a preheater. It can be seen that the electrical energy gain for the least efficient models i.e. $c_1 < 2$ is enormous. Apparently, the ratio of drilling costs per produced electrical energy unit is only reduced for $c_1$ values that correspond to an electrical energy gain larger than the additional drilling cost required; e.g. for $c_1 = 15$ a preheater with $\theta_p = 0.3$ would result in a decrease to the ratio of drilling costs per electrical energy produced unit only if the drilling costs for the preheater are less than 19.05% of the drilling costs for the deep reservoir.

In Fig. 9.11 the additional produced electrical energy is quantified for each parameter $c_1$ and is compared to the scenario of producing electrical energy by operating independently the deep and the shallow reservoir. Operating the two reservoirs independently requires almost equal drilling costs. Thus this electrical energy gain can also be considered as a reduction in the drilling costs per produced electrical energy unit. Here, the curve that corresponds to the scenario with $\theta_p = 0.3$ is identical to the one for $\theta_p = 0.3$ in Fig. 9.10, since the preheater is not considered to have high enough temperature for producing electrical energy.
Figure 9.7 – Solution families $\theta_{\text{prod}}$ for four different scenarios and ten different heat exchange efficiencies.
9.3 Simplified 1-D modeling of heat transport in an EGS

Figure 9.8 – Time integrals of $\theta_{prod}$ for each scenario and for each heat exchange efficiency.
Figure 9.9 – Time integrals of $\theta_{prod}^2$ for each scenario and for each heat exchange efficiency.
9.3 Simplified 1-D modeling of heat transport in an EGS

Figure 9.10 – Total electrical energy production for the four scenarios (top) and total gain in electric energy produced compared to the typical EGS scenario (bottom).
Figure 9.11 – Increase in electrical energy production compared to producing electrical energy from the deep and the shallow reservoir independently.
Part III

Conclusion
10 Conclusions and outlook

In part I of this thesis, an adaptive hierarchical fracture representation (a-HFR) for modeling flow and heat transport was introduced. This approach is suitable for simulations of dynamically changing fracture reservoirs, such as in Enhanced Geothermal Systems (EGS). In part II, a-HFR has been used for the development of an algorithmic framework for EGS simulations.

The reservoirs of EGS power plants are deformable fractured impermeable media characterized by high heterogeneity and anisotropy. Inside this unsteady domain, fluid pressure and the temperature distribution have to be accurately and efficiently computed in order to be suitable for the computation of stress distributions inside the EGS reservoir. Due to the huge number of fractures that exist in such reservoirs, representing all fractures discretely is unfeasible. For that reason, a hierarchical fracture representation is employed, in which only fractures larger than a certain size are treated discretely; the effect of the other ones is captured by an equivalent continuum, which is here called “damaged matrix”.

The a-HFR approach is based on a convenient re-expression of the Darcy mass conservation equations. It treats fractures as lower dimensional manifolds and kernel functions are used for capturing the fracture mean properties along it and for removing singularities - in the damaged matrix equations. Each discrete fracture is treated as a separate continuum and the transfer coefficients between different continua are well defined. Large fractures of complex shape can now easily be discretized by a structured grid and can be treated as an embedded lowered dimensional domain inside another continuum, without concern about conforming the boundaries of volumes from different continua. The boundaries of fracture volumes neither have to conform to the faces of the damaged matrix volumes nor to the volume interfaces of other fractures. Consequently, tiny fracture volumes that would reduce the time-step size and increase the computational cost are avoided, and new fractures can be added to the computational domain
without the need of re-meshing. This unnnecessity of re-meshing makes the a-HFR approach efficient for flow and heat transport simulations in dynamically changing fracture geometries.

The kernel functions from the mass conservation equations are also introduced in the energy conservation equations of the a-HFR models. Heat advection due to mass exchange between different continua is the dominant heat transport mechanism for the fluid and conduction is the dominant heat transport mechanism inside the rock. The hot rock continuum is responsible for the heat source term in the fluid energy equations. It increases the fluid enthalpy and couples the energy equations between the rock and other domains.

The a-HFR also copes with the issue of unphysical scalars trapping that may arise inside embedded fractures. Due to the fact that the lower dimensional fractures in embedded models are treated as mass source or sink terms, can result in under-predictions of heat exchange when the total mass exchange between a fracture volume and the surrounding continuum is comparatively low. Due to such cases the fracture sweep flux term that captures the effect of heat advection perpendicularly to the fracture manifolds is introduced in the energy equations.

The discretized a-HFR equations were quantitatively compared to the results given by CSMP++, a well established software, for a realistic 2D benchmark test case and their grid convergence behavior was presented. Grid convergence studies were also performed for the discretized modified energy equations with the same fracture geometry. Code verification for the fracture heat sweep phenomenon was performed by comparing the fracture temperature in a simple scenario, where heat is advected across a fracture.

The modeling of variable density driven flows inside the a-HFR models was verified separately with semi-analytical manufactured solutions that verify that the treatment of the buoyancy terms in the mass conservation equations is not affected by the a-HFR modeling performed for the pressure driven flows. Simple grid convergence analysis was performed also for these semi-analytical manufactured solutions.

In the final chapter of part I, the key features of the a-HFR model, its advantages, and its disadvantages were discussed and compared to other widely used families of discrete fracture models (DFM). Contrary to existing DFMs, the adaptive DFM avoids a significant increase of the computational cost due to dynamically changing systems. Consequently, there is no need for pre-describing all fractures (as most current approaches require), rather the complexity of the mesh incrementally increases as the complexity of the
real problem increases.

In part II, it was shown how some of the advantages of the a-HFR model can be exploited for EGS simulations. HFR-Sim, an algorithmic modeling framework based on the a-HFR modeling approach and which aims at assisting EGS simulations, was introduced.

Initially, the main parts that compose HFR-Sim were presented. HFR-Sim generates 2D and 3D grids for EGS reservoir models according to the resolution needs for the discretized a-HFR equations, i.e. to model flow and heat transport accurately. HFR-Sim builds tube networks for EGS models based on which it solves the a-HFR mass and heat equations employing iterative and multi-grid linear system solvers. A very simple well model is used by HFR-Sim to quantify well production rates. Gravitational effects are also included and communication interfaces that enable the information exchange between HFR-Sim and software that models mechanical and chemical effects, which are also important for EGS simulations.

Afterwards, the framework was qualitatively tested for capturing flow and heat transport phenomena that occur during EGS scenarios and need to be simulated at different moments during the development and the operation of an EGS power plant. Exemplary simulations for the estimation of the thermal revenues from simple EGS models, of the pressure increase during a pre-stimulation three step injection test, of the impedance decrease during the creation phase and of the temperature profiles in highly heterogeneous reservoirs were presented.

Sanyal et al. [80] stated to define desired key properties for an EGS simulator. In general, the a-HFR modeling approach enables HFR-Sim to include most of them and already the current version covers almost all the desired properties related with single phase flow and heat transport modeling. It offers a discrete fractures representation, either in 2D or 3D, and it does not neglect flow inside the matrix. In HFR-Sim, flow inside fractures depends on their hydraulic aperture and channeling of the flow inside them can be modeled by assigning the appropriate heterogeneous and anisotropic permeability field along the fracture. As an embedded and structured DFM the a-HFR approach can potentially handle multi-phase problems [54]. Aperture considerations and thermo-elastic effects could be treated by a geomechanical software applied in each time-step. Here, simple proxy models were included and aperture changes were implicitly computed based on the local pressure field. Similarly, a geochemical module has to be linked to HFR-Sim in order to deal with tracer transport and mineral deposition/dissolution. Since tracer transport is mathematically not that
**Table 10.1** – Desired EGS simulator features offered by HFR-Sim. The "●" symbol indicates that a feature is offered by HFR-Sim and "○" indicates that a feature currently is not offered, but can potentially be included. "(m)" and "(c)" indicate that a feature requires coupling either with a geomechanical or a geochemical software, respectively.

<table>
<thead>
<tr>
<th>Capability</th>
<th>HFR-Sim</th>
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<tbody>
<tr>
<td>Discrete fractures</td>
<td>●</td>
</tr>
<tr>
<td>Aperture function of normal stress</td>
<td>(m) + ○</td>
</tr>
<tr>
<td>Aperture function of shear</td>
<td>(m)</td>
</tr>
<tr>
<td>Flow rate as function of aperture</td>
<td>●</td>
</tr>
<tr>
<td>Channeling</td>
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</tr>
<tr>
<td>Porous flow in matrix</td>
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<tr>
<td>Thermo-elastic effects</td>
<td>(m)</td>
</tr>
<tr>
<td>Tracer transport</td>
<td>(c) + ○</td>
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<tr>
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<tr>
<td>Mineral deposition/dissolution</td>
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Different from heat transport, it will be straightforward to include simple tracer transport equations in HFR-Sim. Table 10.1 summarizes which of the desired EGS simulator features are currently offered by HFR-Sim.

Finally, HFR-Sim was used to study the concept of preheaters. To validate the results obtained from a simple 1D model, where all fractures are represented by the damaged matrix. The results obtained from those simulations were then discussed and the increase of the produced electrical energy due to shallow geothermal preheaters was quantified.

As explained there, circulating the working fluid in a shallow geothermal reservoir has the potential of increasing the electrical power production in less efficient EGS reservoirs for an additional cost that is dis-proportional to the increased revenues. The subsurface temperature above deep EGS reservoirs is of less commercial interest for electrical power production. However, since the drilling costs increase exponentially with the depth, shallow reservoirs at that site can offer a not negligible temperature increase of the working fluid at low cost. By circulating the working fluid first in such a shallow and lukewarm reservoir and then injecting it into the deep reservoir, the cold front in the deep reservoir propagates more slowly and thus high temperatures of the extracted working fluid, which can more efficiently be
converted into electrical energy, can be maintained over larger periods.
11 Closing remarks and future work

When this PhD project started (October 2008) the algorithmic framework for a cerebral blood flow (CBF) simulator existed. That CBF simulator, developed by Reichold et al. [79], served as the basis on which the HFR-Sim framework is built. Issues such as data structure or input/output interfaces already were efficiently treated, which was of great assistance, since focus could be put on the development of the a-HFR approach, which later became the basic concept of HFR-Sim.

The a-HFR modeling approach seems to have the potential for efficient large scale EGS simulations of both the creation and the production phase. However, to also include chemical and mechanical phenomena, HFR-Sim needs to be coupled with corresponding modules.

Chemical reactions are important in order to model the hydraulic long term behavior of the EGS reservoirs [1, 77, 3]. The efficient coupling of HFR-Sim with geochemical modules will increase the accuracy of the predictions regarding thermal revenues and will assist decisions regarding optimal chemical composition of the injected fluid.

Taking into account geomechanical effects will help to better understand mechanisms important for seismicity that is crucial for predicting possible stimulation scenarios [58]. Current deterministic geomechanical software make use either of continuum approaches, which are applied on a fine structured grid, or of conforming unstructured approaches that are coupled with finite element stress solvers. For THM simulations, the coupling between HFR-Sim and any such geomechanical module seems to be a straight forward task. HFR-Sim can interpolate the obtained pressure field at any desired location or compute average pressure values within a given control volume. This information can then be passed to the geomechanical module. In return, HFR-Sim requires the size and apertures of new fractures. Such THM simulations could also be of assistance for problems that are similar to EGS reservoir stimulation [26].
Since the efficiency of the adaptive HFR model is also due to its ability to remain stable for large time-steps, coupling it with a mechanical software that requires small time-steps can result in a dis-proportional increase of the total computational cost. For such scenarios, larger time-steps may be employed by a-HFR and intermediate values required by the geomechanical software may be estimated.

Regarding flow computations, further improvements of the well model and of the non-linear pressure solver can be made. Currently, HFR-Sim uses a very simple well model and assumes Darcy like flow inside both the fractures and the wells. However, as Murphy et al. [66] explained, the flow inside and around the EGS wells very often is non-linear and the assumption of linear flow inside the fractures that penetrate them is not valid. Consequently, the mass conservation equations treated by the current version of HFR-Sim are not representative enough. Such non-linear phenomena can be included, by treating the transmissibilities as functions of the pressure and experimental data for turbulent flow in a single fracture could be used [76]. With the appropriate linearization and iterations, such turbulent flow effects can be captured.

As already explained, the linear system solver based on AMG relies on reconstructing the intermediate levels, whenever the coefficients of the linear system change. This computational cost is significantly lower for the iterative multi-scale finite volume method developed by Hajibeygi et al. [36]. A similar approach for HFR-Sim could significantly increase its efficiency.

As a closing remark, the importance of local refinement around fractures is suggested. When the damaged matrix grid-block size is smaller than the fracture density, then the number of artificial flow paths that need to be corrected, increases. However, employing a fine grid for the whole domain may increase the computational cost dis-proportionally. Local refinement around fractures could reduce the number of such artificial flow paths without a significant increase of the computational cost.
Part IV
Appendices
A Matrix to fracture and fracture to fracture transmissibility

In chapter 2 the volumetric flux between the damaged matrix and a discrete fracture has been expressed with the use of the kernel functions $\Psi^{d\rightarrow i}$. These kernel functions are required in order to avoid singularities in the matrix balance equations. The exchange between a matrix grid cell $\Omega_k^d$ and an embedded discrete fracture $\Omega_i^h$ however, can only be approximately computed, i.e. an effective transmissibility $C_{hk}^{id}$ has to be estimated. Here, it is described, how $C_{hk}^{id}$ can be estimated (see equation (3.5)), gravitational effects are neglected here and the fluid is considered incompressible.

For the derivation, the fracture volume is treated as an infinitely long source manifold in 2D, i.e. a straight line, which penetrates the matrix volume $\Omega_k^d$ splitting it into two smaller volumes $\Omega_k^{d+}$ and $\Omega_k^{d-}$. The fracture unit normal vector $n_i^h$ points into $\Omega_k^{d+}$. The boundary interface between these two sub-volumes is the interface $\Gamma_{hk}^{id} = \Omega_k^d \cap \Omega_i^h$. It is further assumed that mass is transported from the fracture volume into the damaged matrix cell at a constant rate along the fracture and that the total fluid volume exchanged is $F_{hk}^{id}$. Also, the fracture pressure $p^i$ is considered constant inside $\Omega_i^h$ (and thus equal to its mean value $P_i^h$) and the permeability $k_d^i$ is assumed homogeneous.

Assuming that the only flow inside $\Omega_k^d$ is the one induced by $F_{hk}^{id}$ leads to the approximations

$$u^d \approx -\frac{k^d \cdot n_h^i}{\mu} \nabla p^d \approx \kappa n_h^i, \text{ everywhere on } \Omega_k^{d+}$$

and

$$u^d \approx -\frac{k^d \cdot n_h^i}{\mu} \nabla p^d \approx -\kappa n_h^i, \text{ everywhere on } \Omega_k^{d-}, \quad (A.1)$$

where $\kappa$ is a constant real value. Velocity vectors point to the fracture, when the fracture behaves as a sink term ($\kappa < 0$) and away from it otherwise ($\kappa > 0$). Figure A.1 illustrates the above scenario.
With \( x' \in \Omega^i_h \) being any point of the fracture volume, the mean pressure \( P_k^d \) inside volume \( \Omega^d_k \) equals to

\[
P_k^d = \frac{1}{|\Omega^d_k|} \int_{\Omega^d_k} p^d dV
\]

\[
= \frac{1}{|\Omega^d_k|} \left( \int_{\Omega^d_{k+}} p^d dV + \int_{\Omega^d_{k-}} p^d dV \right)
\]

\[
= \frac{1}{|\Omega^d_k|} \left( \int_{\Omega^d_{k+}} p^d + \nabla p^d \cdot (x - x') dV + \int_{\Omega^d_{k-}} p^d + \nabla p^d \cdot (x - x') dV \right)
\]

\[
= P_h^i + \frac{\kappa \mu}{|k^d_k \cdot n^i_h|} \frac{1}{|\Omega^d_k|} \left( \int_{\Omega^d_{k+}} -n^i_h (x - x') dV + \int_{\Omega^d_{k-}} n^i_h (x - x') dV \right)
\]

\[
= P_h^i - \frac{\kappa \mu}{|k^d_k \cdot n^i_h|} \kappa \Delta(\Omega^d_k, \Gamma^d_{kh}),
\]

where equation (3.1) is used for the introduction of the average distance \( \Delta(\Omega^d_k, \Gamma^d_{kh}) \) between volumes \( \Omega^d_k \) and \( \Omega^i_h \).

Since \( F_{hk}^{id} = 2\kappa|\Gamma^id_{hk}| \), it is derived that

\[
C_{hk}^{id} = 2\frac{|\Gamma^id_{hk}|}{\Delta(\Omega^d_k, \Gamma^d_{kh}) \mu |k^d_k \cdot n^i_h|},
\]

which is equation (3.5).

The assumption of constant pressure gradient inside the matrix cell has been previously used by Li and Lee [54] for the derivation of practically the same transport index. The general approach followed here resembles to how Peaceman [74] derived the production index for wells. The same treatment as the one by Li and Lee was later employed in the works on embedded structured DFMS by Hajibeygi et al. [36] and Moinfar et al [64]. A very similar flow rate coefficient is also used in the embedded unstructured DFM of Sarda et al. [81].

For the derivation of the transmissibility \( C_{kh}^{ij} \) in equation (3.7), a similar approach is followed. The volume flux \( F_{kh}^{ij} \) from the fracture volume \( \Omega^i_k \) to fracture volume \( \Omega^j_h \) is assumed to be homogeneously distributed along their intersection \( \Gamma_{kh}^{ij} \). This intersection is considered to be infinitely long (in contrary to the finite fracture volumes) and the velocity component along the intersection is negligible, i.e.

\[
u_k^i \cdot (n_k^i \times n_h^j) = u_h^j \cdot (n_k^i \times n_h^j) = 0.
\]

Therefore, a constant pressure \( P^{ij} \) along the intersection is assumed from now on.
Figure A.1 – Illustration of the simplifications made for the derivation of the transmissibility $C_{d i}^{k h}$ between a damaged matrix volume $\Omega_{k}^{d}$ and a lower dimensional fracture volume $\Omega_{h}^{d}$.

Here, the unit vector $n_{i j} = n_{k}^{i} \times n_{h}^{j}$ is defined, which is parallel to the $\Gamma_{k h}^{i j}$ domain. Also, the $\Gamma_{k h}^{i j}$ intersection is considered to be long enough for dividing volume $\Omega_{k}^{i}$ into the sub-volumes $\Omega_{k+}^{i}$ and $\Omega_{k-}^{i}$ and for dividing volume $\Omega_{h}^{j}$ into the sub-volumes $\Omega_{h+}^{j}$ and $\Omega_{h-}^{j}$.

Now, assuming that the flow inside $\Omega_{k}^{i}$ and $\Omega_{h}^{j}$ is due to $F_{k h}^{i j}$ the steady state mass conservation can be approximated as

$$u_{i}^{i} \approx -\frac{|k_{k}^{i} \cdot (n_{k}^{i} \times n_{i j}^{i})|}{\mu} \nabla p^{i} \approx \frac{\kappa}{b_{k}^{i}} (n_{k}^{i} \times n_{i j}^{i}), \text{ everywhere on } \Omega_{k+}^{i},$$

$$u_{i}^{i} \approx -\frac{|k_{k}^{i} \cdot (n_{k}^{i} \times n_{i j}^{i})|}{\mu} \nabla p^{i} \approx -\frac{\kappa}{b_{k}^{i}} (n_{k}^{i} \times n_{i j}^{i}), \text{ everywhere on } \Omega_{k-}^{i},$$

$$u_{j}^{j} \approx -\frac{|k_{h}^{j} \cdot (n_{h}^{j} \times n_{i j}^{j})|}{\mu} \nabla p^{j} \approx -\frac{\kappa}{b_{h}^{j}} (n_{h}^{j} \times n_{i j}^{j}), \text{ everywhere on } \Omega_{h+}^{j},$$

$$u_{j}^{j} \approx -\frac{|k_{h}^{j} \cdot (n_{h}^{j} \times n_{i j}^{j})|}{\mu} \nabla p^{j} \approx \frac{\kappa}{b_{h}^{j}} (n_{h}^{j} \times n_{i j}^{j}), \text{ everywhere on } \Omega_{h-}^{j},$$

with $\kappa$ again being a constant and the sub-volumes being defined similarly as before, i.e. $\Omega_{k+}^{i}$ and $\Omega_{h+}^{j}$ are the sub-volumes towards which the vectors $n_{k}^{i} \times n_{i j}^{i}$ and $n_{h}^{j} \times n_{i j}^{j}$ point, respectively. It can be shown that the interface
\( \Gamma^{ij}_{kh} \) acts as a sink and source term to volume \( \Omega^i_k \) and \( \Omega^j_h \), respectively if \( F^{ij}_{kh} > 0 \).

Also, as before it can easily be derived that \( F^{ij}_{kh} = 2\kappa|\Gamma^{ij}_{kh}| \) and for the mean pressures \( P^i_k \) and \( P^j_h \) inside the corresponding one gets

\[
\begin{align*}
P^i_k &= P^{ij} - \frac{\mu}{|k^i_k \cdot (n^i_k \times n^{ij}_k)|} \frac{\kappa}{b^i_k} \Delta(\Omega^i_k, \Gamma^{ij}_{kh}) \\
\text{and } P^j_h &= P^{ij} + \frac{\mu}{|k^j_h \cdot (n^j_h \times n^{ij}_h)|} \frac{\kappa}{b^j_h} \Delta(\Omega^j_h, \Gamma^{ij}_{kh})
\end{align*}
\] (A.4)

and by a simple subtraction relation (3.7) is derived, where

\[
C^{ij}_{kh} = \frac{2|\Gamma^{ij}_{kh}|}{\mu} \left( \frac{\Delta(\Omega^i_k, \Gamma^{ij}_{kh})}{|k^i_k \cdot n^{ij}_{kh}|b^i_k} + \frac{\Delta(\Omega^j_h, \Gamma^{ij}_{kh})}{|k^j_h \cdot n^{ij}_{kh}|b^j_h} \right)^{-1}.
\] (A.5)

It is noted that the above relationship also implies that treating the transmissibility between two fractures as the equivalent of two serially connected electrical resistances (which is common practice) is not unjustified.
B State functions for the density and the viscosity of saline water

According to [63] the density of saline water, which is a typical working fluid for geothermal exploitation, can be approximated as a function of pressure $p$, temperature $T$ and salinity $g_s$ with

$$\rho(\psi, \tau) = \frac{p}{RT\psi\gamma_{psi}} + g_s,$$  \hspace{1cm} (B.1)

where $\psi = p/p^*$, $\tau = T^*/T$ and $p^* = 16.53 \text{ MPa}$, $T^* = 1386 \text{ K}$, $R = 0.461526 \text{ KJ/(kgK)}$ and

$$\gamma_{psi} = \sum_{i=1}^{34} n_i l_i (7.1 - \psi)^{l_i} (\tau - 1.222)^{J_i}.$$  \hspace{1cm} (B.2)

The same state function is employed by HFR-Sim for the computation of the water density. Due to the fact that the effect pressure has on the density is smaller than that of temperature, and since the Boussinesq approximation is employed, the default option of HFR-Sim is that $\psi = 1$.

Fluid viscosity is even stronger depended on temperature changes than density. HFR-Sim can use two different functions for the viscosity [63]. For the first one is

$$\mu = \mu^* \left[ \tau^{0.5} \sum_{i=0}^{3} \tau^i \right] \exp \left[ \delta \sum_{i=1}^{19} n_i (\delta - 1)^{l_i} (\tau - 1)^{J_i} \right],$$  \hspace{1cm} (B.3)

with $\mu^* = 5.5071 \times 10^5 \text{ Pas}$, $\tau = 647,226/T$ (temperature $T$ is in Kelvin) and $\delta = \rho/317.763 \text{ kg/m}^3$. The second one, which is also the default option in HFR-Sim, is the dynamic viscosity of pure water with pressure equal to the saturation pressure, i.e.

$$\mu = 343.18 \times 10^{-7} \times 10^{\frac{247.8}{T-249}}.$$  \hspace{1cm} (B.4)

The co-efficients for the presented equations are given in Tables B.1 and B.2
Table B.1 – Coefficients for computing the density of saline water

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### Table B.2 – Coefficients for computing the viscosity of saline water

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References


[9] Olivier Bour. A statistical scaling model for fracture network geometry, with validation on a multiscale mapping of a joint network (Hornelen
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REFERENCES


Curriculum Vitæ

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Date of Birth          23.08.1984

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2002-2003 Scholarship from the State Scholarships Foundation (I.K.Y)
1996-2002 Award of Excellence, issued by the Greek Ministry of National Education and Religious Affairs
Publications


