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Adaptive model predictive control for constrained MIMO systems ⋆

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Abstract: An adaptive output feedback control algorithm for constrained multiple input multiple output linear systems is proposed, able to cope with input and output constraints, output disturbances and measurement noise. The approach relies on a real-time set membership identification algorithm to provide bounds on the predicted plant outputs. These bounds are exploited in a receding horizon control strategy that guarantees recursive satisfaction of constraints. The algorithm yields offset free reference tracking in case of constant output disturbance and zero measurement noise. The effectiveness of the proposed approach is illustrated on a numerical example.

1. INTRODUCTION

Despite the fact that a well established theory for adaptive control has been developed, there are few results on adaptive control of constrained multiple input multiple output (MIMO) systems (Landau et al. [2011]). This fact limits the use of adaptive control techniques in applications where MIMO plants subject to constraints have to be controlled.

Two main approaches to adaptive control of systems with input constraints are pole placement control with anti-windup compensation (Walgama and Sternby [1993]) and one-step-ahead predictive control (Cheng and Wang [2003]), combined with recursive least square identification. However, these techniques can not be used for handling output constraints and their application to MIMO systems is difficult.

Model predictive control (MPC) is a powerful technique for controlling constrained MIMO systems (Goodwin et al. [2005]). However, the topic of adaptive MPC for constrained MIMO system has received little attention due to difficulties in guaranteeing stability and recursive feasibility under adaptation (Kim [2010]).

Adaptive MPC for input constrained MIMO systems was considered by Maniar et al. [1997]. Kim and Sugie [2008] proposed an adaptive MPC algorithm based on modified recursive least squares identification and tube-like robust MPC. This algorithm guarantees stability and recursive feasibility, but the condition for persistence of excitation is not considered and noise free measurements of the plant states are required, which might be a significant limitation. Nonlinear adaptive MPC for a specific class of systems was considered by Adetola et al. [2009]. Set membership (SM) identification was used for adaptive MPC by Nikolakopoulos et al. [2006], where an explicit MPC law is repeatedly re-calculated off-line when new information on the controlled plant becomes available.

Recently, a novel adaptive control algorithm based on real time SM identification and MPC has been proposed for single input, single output linear systems by Tanaskovic et al. [2013]. Here we present a generalization of this control algorithm to MIMO systems. In addition, the algorithm is extended so that output disturbances can be treated and an integral action is introduced in the control algorithm. We consider a class of linear MIMO systems that are time invariant, but uncertain. The only initial information required by the algorithm are some (eventually very loose) bounds on the impulse response coefficients and bounds on the output disturbance and measurement noise magnitudes. Then, the sets of possible plant impulse response coefficients that are consistent with the initial information are refined on-line by using a real-time SM algorithm. These sets are used to design a receding horizon MPC controller that ensures offset free reference tracking, while at the same time guaranteeing satisfaction of both input and output constraints. After presenting the main aspects of the approach, we show its features through a numerical example.

2. PROBLEM STATEMENT

We consider a square MIMO, discrete time, strictly proper linear time invariant (LTI) system S with n inputs and outputs, for which the influence of the input i to the output j can be modeled by a finite impulse response (FIR):

$$H_{S_{ji}} = [h_{S_{ji}}(1) \dots h_{S_{ji}}(m_{ji})]^T, \ j, i = 1, \dots, n, \qquad (1)$$

where ^{T} denotes the standard matrix transpose operator, $h_{S_{ji}}$ are the impulse response coefficients and m_{ji} is the length of the FIR model from input i to output j. At any time step $t \in \mathbb{Z}$, the system output $y(t) = [y_1(t), \dots, y_n(t)]^T$ is given by:

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$$y_j(t) = \sum_{i=1}^{n}\sum_{l=1}^{m_{ji}} u_i(t-l)h_{S_{ji}}(l) + d_j(t) \doteq \sum_{i=1}^{n} U_{ji}(t)H_{S_{ji}} + d_j(t),\ j = 1,\dots,n \tag{2}$$

and the measured system output is:

$$\tilde{y}(t) = y(t) + v(t) \tag{3}$$

where $u(t) = [u_1(t),\dots,u_n(t)]^T \in \mathbb{R}^n$ is the control input, $d(t) = [d_1(t),\dots,d_n(t)]^T \in \mathbb{R}^n$ is the output disturbance, $v(t) = [v_1(t),\dots,v_n(t)]^T \in \mathbb{R}^n$ is the measurement noise and $U_{ji}(t) = [u_i(t-1),\dots u_i(t-m_{ji})]$. The output disturbance and the measurement noise are characterized by the next assumption.

Assumption 1. (Prior assumption on disturbance and noise) d and v are bounded as:

$$\begin{array}{l}|d_j(t)| \le \epsilon_{d_j}\\ |v_j(t)| \le \epsilon_{v_j}\end{array},\ \forall t \in \mathbb{Z},\ \forall j = 1,\dots,n, \tag{4}$$

where ϵ_{d_j} and ϵ_{v_j} are positive scalars. ■

Moreover, we assume that the true system is not exactly known, but it belongs to the following class of systems.

Assumption 2. (Prior assumption on the system)

$$\forall j, i = 1,\dots,n,\ H_{S_{ji}} \in \mathcal{K}_{ji}(L_{ji},\rho_{ji},\mu_{ji}), \tag{5}$$

where, for given ρ_{ji}, $L_{ji} \in \mathbb{R}$: $\rho_{ji} \in (0,1)$, $L_{ji} > 0$ and $m_{ji} \ge \mu_{ji} \in \mathbb{N}$:

$$\mathcal{K}_{ji}(L_{ji},\rho_{ji},\mu_{ji}) \doteq \left\{ H_{ji} \in \mathbb{R}^{m_{ji}} : \begin{array}{ll} |h_{ji}(l)| \le L_{ji} & l = 1,\dots,\mu_{ji} \\ |h_{ji}(l)| \le L_{ji}\rho_{ji}^{l-\mu_{ji}} & l = \mu_{ji}+1,\dots,m_{ji} \end{array} \right\}. \tag{6}$$

■

Under these assumptions, the goal is to design a controller to track a known desired output $y_{\text{des}}(t) \in \mathbb{R}^n$, while satisfying input and output constraints of the form:

$$\begin{array}{ll} Cu(t) & \le g,\ \forall t \in \mathbb{Z} \\ E\Delta u(t) & \le f,\ \forall t \in \mathbb{Z} \\ Qy(t) & \le p,\ \forall t \in \mathbb{Z}, \end{array} \tag{7}$$

where $\Delta u(t) = u(t) - u(t-1)$ is the rate of change of the control input, and the inequalities in (7) are element wise inequalities forming convex sets, defined by the matrices $C \in \mathbb{R}^{n_u \times n}$, $E \in \mathbb{R}^{n_{\Delta_u} \times n}$, $Q \in \mathbb{R}^{n_o \times n}$ and vectors $g \in \mathbb{R}^{n_u}$, $f \in \mathbb{R}^{n_{\Delta_u}}$, $p \in \mathbb{R}^{n_o}$, where n_u, n_{Δ_u} and n_o are the number of linear constraints on the inputs, input rates and outputs, respectively. We assume that the set defining the constraints on $\Delta u(t)$ contains the origin and that the constraint set of $u(t)$ is bounded.

In order to track the desired output reference signal while coping with model uncertainty, we propose using an adaptive controller that consists of a recursive SM identification algorithm and a receding horizon predictive controller. At each time step, based on the newly measured plant output, the proposed adaptive controller refines the sets of all the impulse response coefficients that are consistent with the prior information and with the collected input-output data. From this set, a candidate model is selected as an estimate of the true system. The control input is then calculated by solving an optimal control problem that minimizes the weighted l_2 norm of the tracking error for the candidate model over a finite horizon, while at the same time satisfying constraints (7) for all the plants in the model sets and therefore enforcing robust constraint satisfaction. Only the first element in the sequence of the predicted control inputs is applied, and the calculation is repeated in a receding horizon fashion. In the next sections, the main features of this approach will be described in details.

3. REAL-TIME SET MEMBERSHIP IDENTIFICATION

We consider the information given by a finite sequence of past input-output data, measured from an initial time step (taken to be equal to zero without loss of generality) up to a finite time $t \ge m$:

$$\mathcal{M}_0^t : \tilde{\mathcal{U}}_0^{t-1},\ \{\tilde{y}(l)\}_{l=m}^t, \tag{8}$$

where $\tilde{}$ is used to denote measured data, $\tilde{\mathcal{U}}_0^{t-1}$ is s sequence of known, past control inputs, $\{\tilde{y}(l)\}_{l=m}^t$ is its corresponding sequence of measured plant outputs and $m = \max_{j,i=1,\dots,n} m_{ji}$. We define, at a given time step t, the sets $\mathcal{F}_j(\mathcal{M}_0^t)$, $j = 1,\dots,n$, as the sets of FIR coefficients that determine the relation between the plant inputs and the plant output j, and that are consistent with the available prior information and collected data (8):

$$\mathcal{F}_j(\mathcal{M}_0^t) \doteq \left\{ \begin{array}{l} H_{ji} \in \mathcal{K}_{ji}(L_{ji},\rho_{ji},\mu_{ji}), i = 1,\dots,n : \\ \left| \tilde{y}_j(l) - \sum_{i=1}^{n} \tilde{U}_{ji}(l)H_{ji} \right| \le \epsilon_{d_j} + \epsilon_{v_j}, \forall l \in [m,t] \end{array} \right\}, \tag{9}$$

where $\tilde{U}_{ji}(l) = [\tilde{u}_i(l-1)\dots\tilde{u}_i(l-m_{ji})]$. Each inequality in (9) comes from the fact that the discrepancy between the measured and the predicted values of the output j can not exceed the disturbance and noise bounds (4). The sets $\mathcal{F}_j(\mathcal{M}_0^t)$ in (9) are compact sets defined by linear inequalities, i.e. polytopes, that can be described in a compact form by a set of non-redundant inequalities:

$$\mathcal{F}_j(\mathcal{M}_0^t) = \left\{ H_j \in \mathbb{R}^{s_j} : A_j(\mathcal{M}_0^t)H_j \le b_j(\mathcal{M}_0^t) \right\}, \tag{10}$$

where $H_j = [H_{j1}^T \dots H_{jn}^T]^T$, $A_j(\mathcal{M}_0^t) \in \mathbb{R}^{r_j(t) \times s_j}$, $b_j(\mathcal{M}_0^t) \in \mathbb{R}^{r_j(t)}$ and $r_j(t)$ is the number of inequalities.

In the real-time SM identification procedure we propose, each of the polytopes $\mathcal{F}_j(\mathcal{M}_0^t)$ is updated recursively. In general, the polytope $\mathcal{F}_j(\mathcal{M}_0^t)$ can be calculated as the intersection of the polytope $\mathcal{F}_j(\mathcal{M}_0^{t-1})$ and the two half spaces defined by the newly measured plant output $\tilde{y}_j(t)$:

$$\begin{aligned}\mathcal{F}_j(\mathcal{M}_0^t) = & \mathcal{F}_j(\mathcal{M}_0^{t-1}) \\ & \cap \{H_j \in \mathbb{R}^{s_j} : \tilde{\varphi}_j(t)H_j \le \tilde{y}_j(t) + \epsilon_{d_j} + \epsilon_{v_j}\} \\ & \cap \{H_j \in \mathbb{R}^{s_j} : -\tilde{\varphi}_j(t)H_j \le -\tilde{y}_j(t) + \epsilon_{d_j} + \epsilon_{v_j}\},\end{aligned} \tag{11}$$

where $\tilde{\varphi}_j(t) = [\tilde{U}_{j1}(t)\dots\tilde{U}_{jn}(t)]$ is the regressor vector of the past control input values. With this recursive update the number of faces of $\mathcal{F}_j(\mathcal{M}_0^t)$, $r_j(t)$, can become arbitrarily large, as in general it grows linearly with time, and hence the memory needed to store $A_j(\mathcal{M}_0^t)$ and $b_j(\mathcal{M}_0^t)$ can become impractical. In order to overcome this problem, we propose the use of a polytope update algorithm with bounded complexity, similar to the one proposed by Veres et al. [1999].

In this approach, the polytope $\mathcal{F}_j(\mathcal{M}_0^t)$ is updated by using (11) as long as the number of its faces is lower than a predefined maximum limit L_1. Once this limit is reached, the bounded complexity updating is used, where the update of the polytope $\mathcal{F}_j(\mathcal{M}_0^t)$ is given by the following intersection:

$$\begin{aligned}\mathcal{F}_j(\mathcal{M}_0^t) =&\mathcal{F}_j(\mathcal{M}_0^{t-1})\\ &\cap \{H_j\in\mathbb{R}^{s_j} : v_j^+(t)H_j \le \tilde{y}_j(t) + \delta_j^+(t)\}\\ &\cap \{H_j\in\mathbb{R}^{s_j} : v_j^-(t)H_j \le -\tilde{y}_j(t) + \delta_j^-(t)\},\end{aligned} \tag{12}$$

where $v_j^+(t)$ and $v_j^-(t)$ are selected from a set of L_2 predefined vectors, where L_2 is a chosen positive integer, and $\delta_j^+(t)$ and $\delta_j^-(t)$ are calculated such that the bounded complexity polytope includes the polytope that would be obtained by a normal update (as per (11)). These values can be calculated by solving a single Linear Program (LP). A graphical interpretation of the bounded complexity update algorithm is given in Fig. 1

The polytope obtained with this approach is an outer

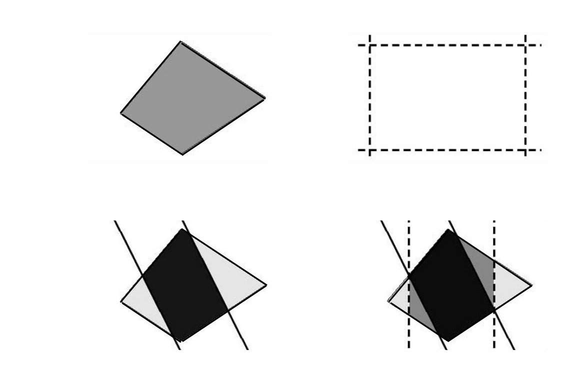

Fig. 1. Illustration of the limited complexity polytopic update algorithm. The upper left plot shows the polytope $\mathcal{F}_j(\mathcal{M}_0^{t-1})$. The upper right plot shows the predefined set of directions for the polytope faces. The lower left plot shows the two new inequalities (solid black lines) and a polytope that would be obtained by using the update (11) (black). The lower right plot shows the polytope obtained by using the limited complexity polytopic update (dark gray). Two new faces of the polytope (dashed lines) are constructed such that their directions are selected as the directions from the set of predefined directions that have maximal dot product with the original faces (solid lines) and that the polytope that would be obtained by nominal update (black) is contained in the newly obtained polytope.

approximation of the polytope (10); by increasing L_1 and L_2, the tightness of such approximation can be increased, at the cost of higher complexity. The procedure for calculating $A_j(\mathcal{M}_0^t)$ and $b_j(\mathcal{M}_0^t)$ amounts to solving an LP for each face of the polytope $\mathcal{F}_j(\mathcal{M}_0^t)$ in order to determine whether it is redundant (see e.g. Mattheiss [1973]). This computation can be parallelized. In addition, the algorithm requires the updating of $A_j(\mathcal{M}_0^t)$ and $b_j(\mathcal{M}_0^t)$ that have dimension bounded by $L_1 + L_2$. All of these features make the described approach computationally efficient and suitable for on-line application. In addition, the algorithm guarantees that $\mathcal{F}_j(\mathcal{M}_0^t) \subseteq \mathcal{F}_j(\mathcal{M}_0^{t-1})$: this property is necessary to obtain recursive output constraint satisfaction.

Once the polytopes $\mathcal{F}_j(\mathcal{M}_0^t)$ have been updated, for the purpose of control computation, a nominal model of the system is selected. To this end, at each time step we select the nominal model as a collection of vectors $H_{c_j}(t)$ computed as the centers of the maximum volume l_2-norm balls inscribed in the polytopes $\mathcal{F}_j(\mathcal{M}_0^t)$. This can be done by solving n LPs (i.e. one per output), however the solutions might not be unique. Therefore, we introduce an additional regularization term that penalizes the deviation from the previously calculated points $H_{c_j}(t-1)$, giving rise to the following set of LPs:

$$\begin{gathered}\max_{\phi_j(t), H_{c_j}(t)} \phi_j(t) - \alpha \|H_{c_j}(t-1) - H_{c_j}(t)\|_1\\ \text{subject to}\\ a_{ji}(\mathcal{M}_0^t)H_{c_j}(t) + \phi_j(t)\|a_{ji}(\mathcal{M}_0^t)\|_2 \le b_{ji}(\mathcal{M}_0^t),\ \forall i,\end{gathered} \tag{13}$$

where $\phi_j(t) \in \mathbb{R}$ is the radius of the maximum volume ball inscribed in $\mathcal{F}_j(\mathcal{M}_0^t)$, $\alpha > 0$ is a design variable, and $a_{ji}(\mathcal{M}_0^t)$ and $b_{ji}(\mathcal{M}_0^t)$ stand for the i^{th} row of the matrix $A_j(\mathcal{M}_0^t)$ and the vector $b_j(\mathcal{M}_0^t)$. Initially, the vectors $H_{c_j}(0)$ can be formed from arbitrary nonzero points inside the sets $\mathcal{K}_{ji}(L_{ji}, \rho_{ji}, \mu_{ji})$.

The most important feature of the presented identification approach, which is not present in standard techniques, is the capability to calculate, at each time step t, an upper bound on any linear combination of plant outputs given by $qy(t)$, where $q = [q_1 \dots q_n] \in \mathbb{R}^n$ can be an arbitrary nonzero vector. Namely, we define for any given sequence of inputs $\mathcal{U}_{t-m}^{t-1}$, the (local) upper bound of the sets $\mathcal{F}_j(\mathcal{M}_0^t)$ with respect to the vector q as the tightest maximal value of $qy(t)$ that is compatible with the prior information:

$$\overline{y}(q, t, \mathcal{U}_{t-m}^{t-1}) = \max_{H_j \in \mathcal{F}_j(\mathcal{M}_0^t)} \sum_{j=1}^{n} q_j \left(\varphi_j(t)H_j + \epsilon_{d_j}\right), \tag{14}$$

where $\varphi_j(t) = [U_{j1}(t) \dots U_{jn}(t)]$. The bound (14) is "local", because it is referred to a specific control sequence $\mathcal{U}_{t-m}^{t-1}$. In the following, for the sake of notational simplicity, we will denote this upper bound on the predicted output as $\overline{y}(q, t)$, by omitting the dependence on $\mathcal{U}_{t-m}^{t}$, with the knowledge that the bound is indeed local, i.e. related to a specific input sequence.

4. CONSTRAINED ADAPTIVE MODEL PREDICTIVE CONTROL

Following an MPC approach, at each time step t a sequence of $N + m - 1$, $N \ge 1$ future inputs is calculated, according to an optimality criterion that accounts for the aim of the control problem at hand, subject to the input and output constraints (7) and taking the uncertainty described by the sets $\mathcal{F}_j(\mathcal{M}_0^t)$ into account. Then, the first element in the optimal sequence is applied as the actual control input, and the procedure is repeated at the next sampling time, in a receding horizon fashion. More specifically, let $u_i(k|t)$, $k \in [t, t+N+m-m_i-1]$, $i = 1, \dots, n$ be the predicted control moves, where the notation $k|t$ indicates the prediction at step $k \ge t$ given the information at the current step t and $m_i = \max_{j=1,\dots,n} m_{ji}$. We define the vectors of predicted control inputs $u(k|t)$, $k \in [t, t+N+m-2]$ as:

$$u(k|t) = [u_1(l_1|t) \dots u_n(l_n|t)]^T, \tag{15}$$

where

$$l_i = \begin{cases} k & \text{if } k \leq t+N+m-m_i-1 \\ t+N+m-m_i-1 & \text{if } k \geq t+N+m-m_i-1 \end{cases} \quad (16)$$

The predicted control input vectors are constructed such that the last m_i predicted values for each of the input channels i remain constant. This is necessary for guaranteeing constraint satisfaction $\forall t$ with a receding horizon controller. Similarly, we define the vectors of predicted input increments $\Delta u(k|t)$, $k \in [t, t+N-m-2]$ as:

$$\Delta u(k|t) = \begin{cases} u(t|t) - \tilde{u}(t-1) & \text{if } k = t \\ u(k|t) - u(k-1|t) & \text{if } k > t \end{cases} \quad (17)$$

Moreover, we define the vectors $V_j(k|t) \in \mathbb{R}^{s_j}$, $j = 1, \ldots, n$, $k \in [t, t+N+m-2]$ that are to be used for calculating the system output predictions and that consist of past known inputs $\tilde{\mathcal{U}}_{t-m}^{t-1}$ and of predicted control inputs as follows:

$$V_j(k|t) = [V_{j1}(k|t) \ldots V_{jn}(k|t)],$$

where

$$V_{ji}(k|t) = \{v_i(l|t)\}_{l=k-m_{ji}+1}^{k},\ k = t, \ldots, t+N+m-2,$$

and

$$v_i(l|t) = \begin{cases} \tilde{u}_i(l) & \text{if } l < t \\ u_i(l|t) & \text{if } t \leq l \leq t+N+m-m_i-1 \\ u_i(t+N+m-m_i-1|t) & \text{if } l > t+N+m-m_i-1 \end{cases}$$

In addition, we define the prediction error vector as the difference between the measured and the predicted plant outputs at time step t:

$$\hat{d}(t) = \tilde{y}(t) - \begin{bmatrix} \tilde{\varphi}_1(t) H_{c_1}(t) \\ \vdots \\ \tilde{\varphi}_n(t) H_{c_n}(t) \end{bmatrix}. \quad (18)$$

Then, we consider the following cost function:

$$J(U, \tilde{y}(t), \tilde{\mathcal{U}}_{t-m}^{t-1}) \doteq \sum_{k=t}^{t+N+m-2} (\hat{y}(k+1|t) - y_{des}(k+1|t))^T \xi_T$$
$$(\hat{y}(k+1|t) - y_{des}(k+1|t)) + \Delta u(k|t)^T \xi_C \Delta u(k|t), \quad (19)$$

where $\hat{y}(k|t)$ denote the predicted output values of the selected nominal model, assuming that the prediction error remains constant along the horizon:

$$\hat{y}(k+1|t) = \begin{bmatrix} V_1(k|t) H_{c_1}(t) \\ \vdots \\ V_n(k|t) H_{c_n}(t) \end{bmatrix} + \hat{d}(t). \quad (20)$$

In (19), $U = [u(t|t) \ldots u(t+N+m-2|t)]$ are the decision variables, while $\tilde{y}(t)$ and $\tilde{\mathcal{U}}_{t-m}^{t-1}$ are known parameters. $y_{\text{des}}(k|t)$, $k \in [t+1, t+N+m-2]$, are the predicted values of the desired output, and ξ_T and ξ_C are positive definite weighting matrices chosen by the control designer. These two matrices can be tuned to achieve a trade-off between penalizing the tracking error of the nominal model and the control effort. The prediction error term $\hat{d}(t)$ in the cost function is used to introduce feedback and guarantee offset free reference tracking. In fact, with this addition the proposed control algorithm exhibits integral action.

Satisfaction of input constraints can be enforced by the following set of inequalities:

$$\begin{matrix} Cu(k|t) \leq & d \\ E\Delta u(k|t) \leq & f \end{matrix} \ \forall k \in [t, t+N+m-2]. \quad (21)$$

And the robust satisfaction of the output constraints can be enforced by:

$$\overline{y}(q_l, k+1|t) \leq p_l,\ \forall l = 1, \ldots n_o,\ \forall k \in [t, t+N+m-2], \quad (22)$$

where q_l and p_l stand for the l^{th} rows of the matrix Q and vector p, and $\overline{y}(q_l, k+1|t)$ denote the predicted local bounds (14), based on the past input sequence applied up to the time step t and on the predicted control moves up to time step k, and are given by:

$$\overline{y}(q_l, k+1|t) = \max_{H_j \in \mathcal{F}_j(\mathcal{M}_0^t)} \sum_{j=1}^{n} q_{lj} \left(V_j(k|t) H_j + \epsilon_{d_j} \right). \quad (23)$$

For fixed values of N, ξ_T and ξ_C we can now define the following finite horizon optimal control problem (FHOCP) at time t:

$$\begin{aligned} &\min_U J(U, \tilde{y}(t), \tilde{\mathcal{U}}_{t-m}^{t-1}) \\ &\quad \text{subject to } (21),\ (22). \end{aligned} \quad (24)$$

The resulting FHOCP can be recast into a quadratic program (QP) by using the ideas of robust linear programming, see e.g. Ben-Tal et al. [2009]. We embed (24) in the following receding horizon scheme:

Algorithm 4.1. (Adaptive MPC)

1) At time step t, compute the polytopes $\mathcal{F}_j(\mathcal{M}_0^t), j = 1, \ldots, n$ based on $\tilde{\mathcal{U}}_{t-m}^{t-1}$ and $\tilde{y}(t)$, by using the bounded complexity update algorithm;
2) Find $H_{c_j}(t), j = 1 \ldots, n$ by solving the linear programs (13);
3) Solve the problem (24), let $u(k|t)^*, k \in [t, t+N+m-2]$ be the computed control sequence;
4) Apply $u(t) = u^*(t|t)$, set $t = t+1$, go to 1).

■

The proposed control algorithm is an indirect adaptive controller, as it involves the estimation of the plant model on which the control computation is based. The algorithm guarantees robust satisfaction of both input and output constraints, as shown by the following result, for which we omit the proof.

Theorem 4.1. Let the Assumptions 1–2 hold, and assume that the problem (24) is feasible at time $t = 0$. Then the problem (24) remains feasible and closed-loop system obtained by applying the Algorithm 4.1 is guaranteed to satisfy input and output constraints $\forall t \geq 0$. ■

In practice, the condition that the problem is feasible for $t = 0$ means that the initial assumptions are selected such that, if the system is initially at rest, there exists a nonzero input sequence that does not violate the input and output constraints for all the plants in the initial polytopes $\mathcal{F}_j(\mathcal{M}_0^t)$, which is a reasonable condition. This condition, together with the fact that the initial nominal model is constructed from nonzero points inside the sets $\mathcal{K}_{ji}(L_{ji}, \rho_{ji}, \eta_{ji})$, ensures that the Algorithm 4.1 does not result in a trivial control law of always applying zero control input.

5. NUMERICAL EXAMPLE

A numerical example is used to illustrate the performance of the proposed adaptive control algorithm. The system that we consider consists of two masses connected in series to a wall by springs with dampers as shown in Fig. 2. The

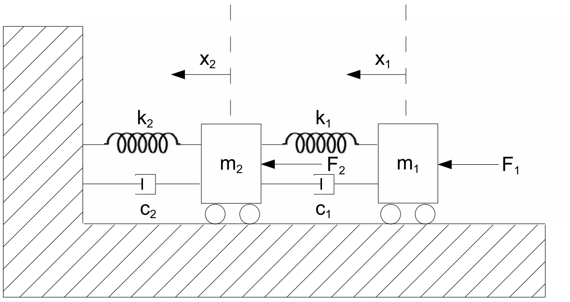

Fig. 2. Numerical example: system layout.

motion equations for the two masses are:

$$\begin{aligned} m_1\ddot{x}_1 &= F_1 - k_1(x_1-x_2) - c_1(\dot{x}_1-\dot{x}_2) \\ m_2\ddot{x}_2 &= F_2 + k_1(x_1-x_2) + c_1(\dot{x}_1-\dot{x}_2) - k_2x_2 - c_2\dot{x}_2 \end{aligned} \tag{25}$$

Taking the state vector $x = [x_1 \ \dot{x}_1 \ x_2 \ \dot{x}_2]^T$, the plant input vector $u = [F_1 \, F_2]^T$ and the plant output vector $y = [x_1 \, x_2]^T$, the following representation of the model is obtained:

$$\begin{aligned} \dot{x} &= \quad Ax + Bu \\ y &= \quad Cx + Du, \end{aligned} \tag{26}$$

where:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{k_1}{m_1} & -\dfrac{c_1}{m_1} & \dfrac{k_1}{m_1} & \dfrac{c_1}{m_1} \\ 0 & 0 & 0 & 1 \\ \dfrac{k_1}{m_2} & \dfrac{c_1}{m_2} & -\dfrac{k_1+k_2}{m_2} & -\dfrac{c_1+c_2}{m_2} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ \dfrac{1}{m_1} & 0 \\ 0 & 0 \\ 0 & \dfrac{1}{m_2} \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \qquad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The numerical values of Table 1 have been used in the simulations. Impulse responses from each of the inputs to each of the system outputs, obtained by discretizing the system model with sampling time $T_s = 0.6$ s and the impulse response preserving method, are shown in Fig. 3. It is assumed that the forces acting on the masses and

Table 1. Computational example: parameters for the two-mass-spring system.

k_1	k_2	c_1	c_2	m_1	m_2
1 N/m	4 N/m	0.8 Ns/m	0.99 Ns/m	0.1 kg	0.8 kg

their rates of change are limited as:

$$\begin{aligned} \begin{bmatrix} -\overline{u} \\ -\overline{u} \end{bmatrix} &\le u(t) \le \begin{bmatrix} \overline{u} \\ \overline{u} \end{bmatrix} \\ \begin{bmatrix} -\overline{\Delta u} \\ -\overline{\Delta u} \end{bmatrix} &\le \Delta u(t) \le \begin{bmatrix} \overline{\Delta u} \\ \overline{\Delta u} \end{bmatrix} \end{aligned}, \ \forall t.$$

In addition, the following box constraints on the system outputs are required to be satisfied:

$$\begin{bmatrix} -\overline{y}_1 \\ -\overline{y}_2 \end{bmatrix} \le y(t) \le \begin{bmatrix} \overline{y}_1 \\ \overline{y}_2 \end{bmatrix}, \ \forall t,$$

Values of the parameters defining the input and output constraints are listed in Table 2. Initial bounds on the impulse response coefficients are selected such that they are equal for all the input-output pairs. In addition, bounds on the measurement noise are selected to be equal for both

Table 2. Computational example:system constraints.

$\overline{u}$	$\overline{\Delta u}$	$\overline{y}_1$	$\overline{y}_2$
1	0.4	1.5	0.7

outputs. The lengths of the FIR models to be used are also selected to be the same for all the input-output pairs. Table 3 lists the values of the used control design parameters. Values that are identical for all the input-output pairs are denoted with no indexes. The weighting matrices ξ_T and ξ_C are selected as identity matrices. In the simulations a stochastic measurement noise, uniformly distributed on the interval $[-\epsilon_v, \epsilon_v]$ was used. The initial plant model was formed by random nonzero points inside the sets $\mathcal{K}_{ji}(L, \rho, \mu)$. In Fig. 3 we show that the initial bounds (6)

Table 3. Computational example: design parameters of the control system.

ϵ_v	ϵ_{d_1}	ϵ_{d_2}	L	μ	ρ	m	α	N
0.01	0.13	0.07	1.4	5	0.65	15	0.01	18

for the impulse response coefficients are selected quite conservatively. Yet the simulation results of Fig. 4 show that good reference tracking and output disturbance rejection is obtained. The control inputs applied by the proposed adaptive controller during the simulation are shown in Fig 5. As expected, both the input snd the output constraints are satisfied, even during the adaptation transient.

In order to robustly satisfy the output constraints, the

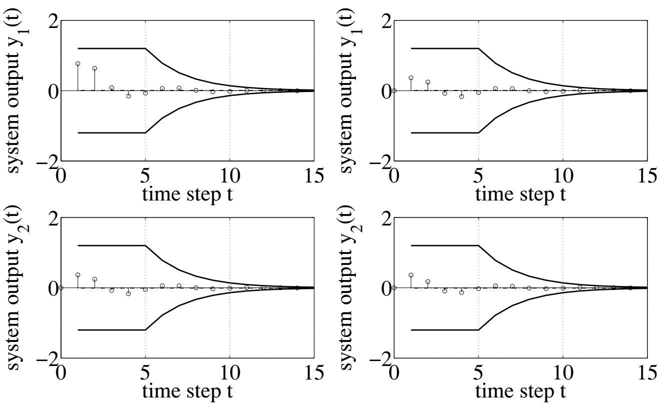

Fig. 3. Numerical example: discrete plant impulse responses (bars ending with o) compared with the used initial bounds on the impulse response coefficients (solid lines). Top left plot shows the transfer function from u_1 to y_1, top right plot from u_2 to y_1, bottom left from u_1 to y_2 and bottom right from u_2 to y_2.

newly proposed adaptive control algorithm introduces conservativeness during the adaptation transient which results in quite cautious control at the beginning. However, as the model uncertainty is reduced over time, the tracking performance of the controller improves, as it can be seen in Fig. 4.

6. CONCLUSION

We generalized a recently proposed adaptive control algorithm to the case of MIMO system and additive output

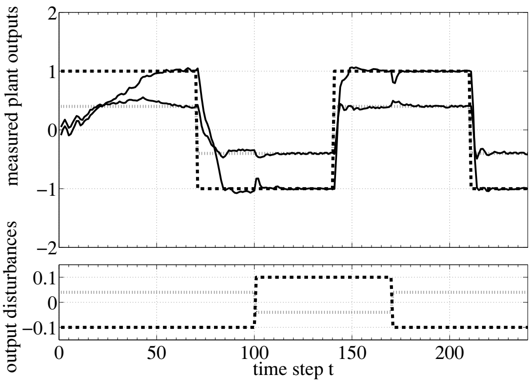

Fig. 4. Numerical example: simulation results obtained by the proposed adaptive controller. The upper plot compares the set reference $y_{des}(t)$ (dashed for y_1 and dotted for y_2) with the measured plant outputs $\tilde{y}(t)$ (solid lines). The lower plot shows the output disturbance pattern used in the simulation (dashed for d_1 and dotted for d_2).

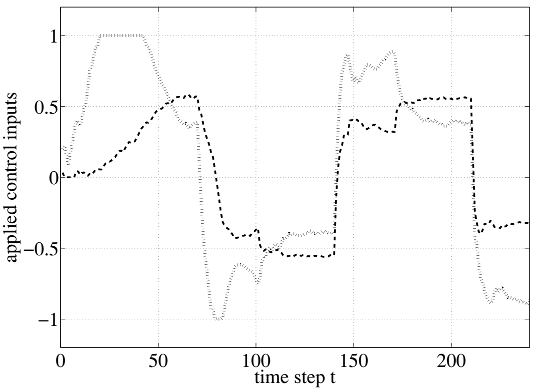

Fig. 5. Numerical example: Control inputs applied by the proposed adaptive controller (dashed for u_1 and dotted for u_2).

disturbance. In addition to the extension to the MIMO case, the proposed approach enjoys offset-free tracking when the additive disturbance is constant. The method relies on real-time SM identification to provide bounds on the predicted linear combination of system outputs. These bounds are used to design a receding horizon controller that is able to robustly satisfy output constraints. The algorithm is computationally tractable as it requires the solution of LPs and a QP only. In addition, recursive feasibility of all the optimization problems is guaranteed. Simulations on a numerical example show that the proposed controller results in good reference tracking and disturbance rejection.

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