Master Thesis

Active Control of the Pantograph-Catenary Interaction in a Finite Element Model

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Raphael Schär

Active Control of the Pantograph-Catenary Interaction in a Finite Element Model

Master Thesis

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And
Institute for Dynamic Systems and Control
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Preface

In late 2011, I decided to write my master thesis abroad and started looking for possible research groups. From the very beginning, it was clear that I wanted to write my thesis in Sweden to be able discover this country that I knew from previous short travels. At KTH Stockholm, I found the Rail Vehicle Division and I am very thankful that Professor Sebastian Stichel accepted my application.

The continuous support from Dr. Per-Anders Jönsson and Professor Sebastian Stichel during my master thesis was very much appreciated and I learned a lot during these six months I was working in their research group. I also want to thank Professor Lino Guzzella and Dr. Christopher Onder for their support from ETH Zurich and that they made it possible that I could write my thesis abroad.

From the Rail Vehicle Division, I additionally thank Saeed Hosseinnia, who shared the office with me for most of the time at KTH, for the helpful answers to my questions, teaching my some basic Farsi language and sharing many coffee and ice cream breaks with me.

A special thanks goes to my girlfriend Bettina Sommer who supported my wish to study abroad and came four times to visit me which was always highly appreciated. I also like to thank my parents for their continuous support during my studies.

Besides studying, I had the change to discover the beautiful nature around Stockholm and some very nice places in Sweden. The good access to the Baltic sea made it possible to additionally discover cities in three other countries. In winter, I learned long distance ice skating thanks to my colleague Dirk Thomas. And in Summer, I could do what I like the most which is being outside with a tent and friends. A big thanks goes to Lukas Bühler, Ralf Schütte, Julio Cézar and Lyn Bentschik for making my stay in Stockholm such a great experience. I am also very happy that some friends from Switzerland came for holiday to Stockholm and discovered this great city together with me.

Thank you all. These were seven wonderful months that I will never forget.
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Abstract

This work analyzes the implementation of an active train pantograph in a full finite element model in the program Ansys. As controller design method, the $\mathcal{H}_\infty$ method was taken in order to cope with the different uncertainties in the given system. The focus lies on the contact force between the pantograph and the catenary. The goal was the reduction of the contact force standard deviation in order to allow higher train speeds on existing lines. An additional goal is the use of multi train configurations. This means that two coupled trains with a distance between the two pantographs of 100 meters can run with high speed on existing lines. Current regulations limit the distance to 200 meters. In addition to the active solutions, different modifications of the given pantograph were investigated.

The simulations showed that the desired speed of 280 km/h is achieved on existing lines in multi train configuration. For only one train, a speed of up to 300 km/h can be reached.

More important, by using an estimator, the standard deviation values for these speeds were still below the limitations and hence, it is possible to implement this solution in a real system.
## Nomenclature

### Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
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<tr>
<td>$A$</td>
<td>Area of the beam element</td>
<td>$[m^2]$</td>
</tr>
<tr>
<td>$A$</td>
<td>System matrix of the continuous transfer function</td>
<td>[-]</td>
</tr>
<tr>
<td>$B$</td>
<td>Input matrix of the continuous transfer function</td>
<td>[-]</td>
</tr>
<tr>
<td>$C$</td>
<td>Output matrix of the continuous transfer function</td>
<td>[-]</td>
</tr>
<tr>
<td>$c_1$</td>
<td>Damping value in the pantograph model</td>
<td>$[Ns/m]$</td>
</tr>
<tr>
<td>$c_2$</td>
<td>Damping value in the pantograph model</td>
<td>$[Ns/m]$</td>
</tr>
<tr>
<td>$D$</td>
<td>Feedthrough matrix of the continuous transfer function</td>
<td>[-]</td>
</tr>
<tr>
<td>$d$</td>
<td>Disturbance signal</td>
<td>[N]</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance</td>
<td>[m]</td>
</tr>
<tr>
<td>$d_{2pan}$</td>
<td>Distance between two pantographs</td>
<td>[m]</td>
</tr>
<tr>
<td>$\tilde{e}$, $e_1$, $e_2$, $e_3$</td>
<td>Performance signals that are to be minimized to meet the control objectives</td>
<td>[N]</td>
</tr>
<tr>
<td>$e$</td>
<td>Error signal</td>
<td>[N]</td>
</tr>
<tr>
<td>$F$</td>
<td>System matrix of the discrete transfer function</td>
<td>[-]</td>
</tr>
<tr>
<td>$F_{\text{aero}}$</td>
<td>Aerodynamic uplift force</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{\text{aero,closed}}$</td>
<td>Aerodynamic uplift force in closed driving direction</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{\text{aero,open}}$</td>
<td>Aerodynamic uplift force in open driving direction</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_c$</td>
<td>Contact Force</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{c1}$</td>
<td>Force related to the damping coefficient $c_1</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{c2}$</td>
<td>Force related to the damping coefficient $c_1</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{\text{fric1}}$</td>
<td>Friction in the pantograph model</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{\text{fric2}}$</td>
<td>Friction in the pantograph model</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{k1}$</td>
<td>Force related to the spring constant $k_1</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{k2}$</td>
<td>Force related to the spring constant $k_2</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{\text{pre}}$</td>
<td>Preforce in the pantograph model</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_1$</td>
<td>Force on the upper mass in the pantograph model</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_2$</td>
<td>Force on the lower mass in the pantograph model</td>
<td>[N]</td>
</tr>
<tr>
<td>$f_0$</td>
<td>Frequency of the movement of the masses</td>
<td>[Hz]</td>
</tr>
<tr>
<td>$G$</td>
<td>Input matrix of the discrete transfer function</td>
<td>[-]</td>
</tr>
<tr>
<td>$G(s)$</td>
<td>Closed loop transfer function</td>
<td>[-]</td>
</tr>
<tr>
<td>$G_p(s)$</td>
<td>Perturbed closed loop transfer function</td>
<td>[-]</td>
</tr>
</tbody>
</table>
**Output matrix of the discrete transfer function** \( H \) [-]

**Moment of inertia** \( J \) [kgm²]

**Controller** \( K(s) \) [-]

**Stiffness of the catenary** \( k_c \) [N/m]

**Spring constant in the pantograph model** \( k_1 \) [N/m], \( k_2 \) [N/m]

**Defines the combination of what is desired to be estimated** \( L \) [-]

**Open loop transfer function** \( L(s) \) [-]

**Perturbed open loop transfer function** \( L_p(s) \) [-]

**Length** \( l \) [m]

**Upper mass in the pantograph model** \( m_1 \) [kg]

**Lower mass in the pantograph model** \( m_2 \) [kg]

**Additional point masses in the pantograph model** \( m_{add} \) [kg]

**Total mass** \( m_{tot} \) [kg]

**Mass of the beam in the Ansys model** \( m_{11b} \) [kg]

**Point masses in the Ansys model** \( m_{11p} \) [kg]

**Noise signal** \( n \) [N]

**Transfer function for \( H_\infty \) control** \( P(s) \) [-]

**Actuator model** \( P_{act} \) [-]

**Disturbance Model** \( P_d(s) \) [-]

**Nominal model of the pantograph** \( P_{nom} \) [-]

**Weight of the disturbance influence on each state** \( Q \) [-]

**Weight for the sensor noise** \( R \) [-]

**Reference signal** \( r \) [N]

**Weight of the importance of each state** \( S \) [-]

**Sensitivity transfer function** \( S(s) \) [-]

**Complementary sensitivity transfer function** \( T(s) \) [-]

**Sampling time** \( T_s \) [s]

**Width of the beam element** \( t_{11} \) [m]

**Time until 90 % of the demanded value is reached** \( t_{90} \) [s]

**Control variables, controller signal** \( u \) [N]

**Train speed** \( v \) [km/h]

**Critical train speed** \( v_{crit} \) [km/h]

**Output value of the disturbance scale \( \Delta \)** \( v, v_1, v_2, v_3 \) [-]

**Actuator performance function** \( W_{aP} \) [-]

**Disturbance weight** \( W_d \) [-]

**Error performance function** \( W_{eP} \) [-]

**Input uncertainty** \( W_i \) [-]

**Output uncertainty** \( W_o \) [-]

**System uncertainty** \( W_s \) [-]

**Output performance function** \( W_{yP} \) [-]
\( w \) Exogenous signals (disturbances and commands) \([N]\)

\( x_0 \) Amplitude of the movement of the masses \([m]\)

\( y \) Output signal \([N]\)

\( \hat{y} \) Input value for the controller \( r - y \) \([N]\)

\( z, z_1, z_2, z_3 \) Input value to the disturbance scale \( \Delta \) \([-\text{-}]\)

\( \Delta \) Uncertainty weighting block or disturbance scale \([-\text{-}]\)

\( \theta \) Inverse of an upper bound for the estimation error \([-\text{-}]\)

\( \mu \) Structured singular value \([-\text{-}]\)

\( \rho \) Spectral radius \([-\text{-}]\)

\( \sigma \) Singular value \([-\text{-}]\)

**Indices**

aero Aerodynamic

a, act Actuator

add additional

crit critical

d Disturbance

e Error

fric friction

i Input

nom Nominal

n Noise

o Output

p perturbed

P Performance

pre Preforce

s System

tot Total

y Output

\( \infty \) Infinity
**Acronyms and Abbreviations**

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>COM</td>
<td>Control oriented Model</td>
</tr>
<tr>
<td>DOF</td>
<td>Degree of Freedom</td>
</tr>
<tr>
<td>ETH</td>
<td>Eidgenössische Technische Hochschule</td>
</tr>
<tr>
<td>ILC</td>
<td>Iterative Learning Control</td>
</tr>
<tr>
<td>KTH</td>
<td>The Royal Institute of Technology</td>
</tr>
<tr>
<td>NP</td>
<td>Nominal Performance</td>
</tr>
<tr>
<td>RS</td>
<td>Robust Stability</td>
</tr>
<tr>
<td>RP</td>
<td>Robust Performance</td>
</tr>
<tr>
<td>SSS400</td>
<td>A panhead for the WBL88 Pantograph that is modified for higher speeds</td>
</tr>
<tr>
<td>SYT</td>
<td>A type of catenary system with stich wire</td>
</tr>
<tr>
<td>WBL88</td>
<td>A Pantograph Type</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Pantograph Catenary Interaction

Figure 1.1 shows an electric train while driving. The stationary system that consists of poles and the wires with the electric power supply is called catenary. The train is connected to the catenary system via a pantograph that is mounted on the roof of the train. The catenary has two wires, the contact wire which is connected to the pantograph and the messenger wire above the contact wire which are linked together over the droppers. In the analyzed catenary system, the droppers have a distance of about 9 meters and the poles have a distance of 60 meters. The pantograph catenary interaction is discussed in more detail in Chapter 2.

Figure 1.1: A system overview with the train, the pantograph and the catenary system.

[Image of a train with labels for pantograph, messenger wire, contact wire, dropper, pole, and train]
1.2 Goal

The aim of the present work is to allow higher train speed on existing lines in multi train configurations for the Gröna Täget (Green Train) programme in Sweden. The speed shall be increased from 200 km/h to around 280 km/h. Multi train configurations mean that two or more trains are coupled and they are both connected with a pantograph to the catenary. According to current specifications, the minimal distance between two pantographs for high speed trains is 200 meters. In this thesis, the aim is to reduce this distance to 100 m. Both, the higher speed and the closer distance between two pantographs introduce higher variations of the contact force between the pantograph and the catenary. These variations should be reduced by implementing active control strategies. These control strategies will be verified in a full finite element model in Ansys. The specifications for the mean value and the standard deviation of the contact force are given by [2].

1.3 Background

Some authors proposed passive solutions such as [3], where the pre-load was lower to reduce the contact force variations in multi train configurations on higher speeds.

Other authors focused on active pantographs where the pantograph mostly was represented with a two mass model, where the upper mass represents the head of the pantograph and the lower the frame of the pantograph (see also Figure 1.2). Interest in active pantograph started around 20 years ago with the first high speed trains and [4] gives a very interesting summary of different control strategies for active pantographs. One point that limits the speed up is the wave propagation. If the train speed is higher than 80 % of the wave propagation speed, the stiffness of the catenary changes enormously. This speed varies for different types of catenarys and is, for the system investigated in this thesis, 320 km/h. As a speed increase up to 280 km/h is desired in this thesis, this is no particular concern here.

Other papers such as [5] could achieve good reduction of the contact force vibration. [6] used a wire between the train roof and the upper mass as an actuator. Therefore a faster controller could be designed without the drawback of additional mass on the upper mass. [7] used $H_\infty$ control and achieved good results and in [8], a new pantograph for high speed trains has been developed and real measurements were performed. Except for the last work, most authors used a simplified model of the catenary for their analysis. These models have, for example, no wave propagation included. Additionally, the catenary is represented as a varying stiffness where the effects of the droppers are not well represented.
Chapter 1. Introduction

Figure 1.2: A pantograph with the main parts, the pantograph head and the pantograph frame.

1.4 Organization

In this thesis, the work of [9] is continued. In [9] the two mass model of the pantograph was represented with a multi body model in Gensys, a software developed in Sweden for such analyses [10]. Now, the controller is implemented in the full finite element model that is available for the software Ansys. This program supports the wave propagation in the contact wire and consists of a full model of the catenary system. Hence, the analysis is closer to reality.

In Chapter 2, the model of the pantograph and the catenary is shown. Chapter 3 discusses the theoretical background of $\mathcal{H}_\infty$ control design and the controller design. In Chapter 4, the results with the controller from Chapter 3 are summarized. Chapter 5 draws a conclusion of the thesis and Chapter 6 gives some ideas for future work on this topic.
Chapter 2

Pantograph Catenary Interaction

In this chapter, the different models that were used in Ansys and Matlab are introduced. In Ansys a two mass oscillator with three degrees of freedom (DOF) represents the pantograph whereas the control oriented model (COM) is reduced to two DOF. In [9], a three mass model was introduced and this is also shortly discussed in this chapter. In addition, different changes on the original configuration are shown and they are analyzed on their performance in the next chapters.

2.1 Modeling in Ansys

In this section, the modeling of the pantograph and catenary in the finite element program Ansys is discussed.

2.1.1 Two Mass Model in Ansys

A lumped mass model represents the pantograph. The parametrization is given by earlier projects such as [3] an [9]. The identification has been carried out by Trafikverket. Figure 2.1 shows the two mass oscillator. The pantograph has three degree of freedom which are the movement in y-direction of both masses, $x_1$ and $x_2$ and the roll of the upper mass $x_3$.

This thesis analyses two different pantograph types. The standard WBL88 from Schunk and the adapted WBL88 with SSS400 panhead, a pantograph designed for higher trainspeeds. The main focus is on the pantograph with the SSS400 panhead. In addition, the pantograph with SSS400 panhead is slightly modified for an analysis of the influence of changes in the mass, spring and damping coefficients. These values are shown in Section 2.2.5.
The parametrization is given by Table 2.1 and the aerodynamic force on the upper mass as a function of the speed by Figure 2.2 and Eq. (2.1).

\[
F_{\text{aero, closed}} = -0.0037 \cdot v + 0.00025 \cdot v^2 \\
F_{\text{aero, open}} = 0.0094 \cdot v + 0.00027 \cdot v^2
\]  

(2.1)

where \( v \) is the train speed in km/h. This equations have been obtained by [11].

2.1.2 Catenary Model in Ansys

In this thesis, the SYT 7.0/9.8 catenary is analyzed. Y indicates the stitch wire which can be seen in Figure 2.4 as the wire connecting the messenger (upper) wire horizontally close to the pole. The two numbers stay for the force applied at the end of the wire. In this case, 9.8 kN and 7.0 kN are applied to the contact (lower) and messenger (upper) wire, respectively. The Ansys model includes presag which is a method to reduce the contact force variations. Presag is a static shape of the contact wire to achieve an additive stiffness in the middle of the span [9]. Additionally, a zig-zag configuration of \( \pm 300 \text{mm} \) is considered which is important that the wire and the pantograph are not always connected at the same point in order to reduce wear and tear.

The distance between two poles is set to 60 m, whereas in reality this distance can vary between 55 and 65 m. The two wires are connected with droppers every 9 meters. More information on the catenary can be found in [9], where also the first eigenmode of 0.79 Hz of the coupled pantograph catenary system was derived. The first eigenmode for the uncoupled catenary system is 0.84 Hz.

![Figure 2.4: The catenary between two poles with a distance of 60 meters. The upper wire is the messenger wire and the lower wire is the contact wire where the contact with the pantograph takes place. These two wires are connected with droppers every 9 meters. The black points indicate where the wires are connected to the poles. The stitch wire is not connected to the poles.](image-url)
Figure 2.1: The pantograph model that is used in Ansys. It consists of different spring, damper and friction elements.

Table 2.1: Parametrization of the WBL88 pantograph (left column) and the WBL88 pantograph with SSS400 panhead (right column). These data can not be found in this public version of the thesis due to confidentiality reasons.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>WBL 88</th>
<th>SSS-400</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper mass $m_1$ [kg]</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Additional mass $m_{add}$ [kg]</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Lower mass $m_2$ [kg]</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Spring constant $k_1$ [N/m]</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Damping value $c_1$ [Ns/m]</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Friction $F_{fric1}$ [N]</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Spring constant $k_2$ [N/m]</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Damping value $c_2$ [Ns/m]</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Friction $F_{fric2}$ [N]</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Preforce $F_{pre}$ [N]</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Force $F_1$ [N]</td>
<td>$F_{aero} + (m_1 + 2 \cdot m_{add}) \cdot 9.81$</td>
<td></td>
</tr>
<tr>
<td>Force $F_2$ [N]</td>
<td>$F_{pre} + m_2 \cdot g$</td>
<td></td>
</tr>
</tbody>
</table>
Figure 2.2: Aerodynamic force on the pantograph depending on the driving direction according to Eq. (2.1). The gray and black line indicate the open and closed driving direction, respectively.

Figure 2.3: The gray and the black arrow indicate open knee and closed knee driving direction, respectively.


\[ \begin{align*}
F_1 & = k_1 x_1 \\
F_2 & = k_2 x_2 \\
F_c & = m_2 \ddot{x}_2 \\
m_1 \ddot{x}_1 & = F_1 - k_1 x_1 - c_1 \dot{x}_1 \\
m_2 \ddot{x}_2 & = F_2 - k_2 x_2 - c_2 \dot{x}_2 + F_c
\end{align*} \]

Figure 2.5: The control oriented model of the pantograph. The parameters for this model can be found in Table 2.2.

2.2 Control Oriented Model

In order to have a control oriented model (COM), the model from section 2.1.1 has been adapted. The following assumptions were made:

- No roll of the upper mass.
- No zig-zag and presag.
- No friction is considered. Instead an additional damping coefficient is introduced.
- The damping and spring coefficients \( c_1 \) and \( k_1 \) between the lower and upper masses are summarized to only one of each element.
- The contact force is represented by a constant stiffness value.

These assumptions are necessary to have a linear and time invariant model and a low number of states.

The model and the parametrization is shown in Figure 2.5 and in Table 2.2 respectively. Section 2.2.1 discusses the different sensor and actuator possibilities, Section 2.2.2 derives the mathematical representation that is used later for the controller design, Section 2.2.3 shows the linearization of the friction elements and Section 2.3 compares the different configurations from Section 2.2.1. In Section 2.2.4, a three mass model is discussed shortly. Section 2.2.5 shows possible changes in the parametrization of the pantograph.
Table 2.2: Parametrization of the COM of the WBL88 Pantograph with SSS400 panhead including the additional damping from Section 2.2.3. These data cannot be found in this public version of the thesis due to confidentiality reasons.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SSS400</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper mass (m_1) [kg]</td>
<td>?</td>
</tr>
<tr>
<td>Lower mass (m_2) [kg]</td>
<td>?</td>
</tr>
<tr>
<td>Spring constant (k_1) [N/m]</td>
<td>?</td>
</tr>
<tr>
<td>Damping value (c_1) [Ns/m]</td>
<td>?</td>
</tr>
<tr>
<td>Spring constant (k_2) [N/m]</td>
<td>?</td>
</tr>
<tr>
<td>Damping value (c_2) [Ns/m]</td>
<td>?</td>
</tr>
<tr>
<td>Average stiffness of the catenary (k_c) [N/m]</td>
<td>?</td>
</tr>
</tbody>
</table>

2.2.1 Sensor and Actuator Configurations

In this thesis, four different configurations are analyzed:

- Configuration 1: Measurement of the contact force \(F_c\) and adding the control force to \(F_2\).
- Configuration 2: Measurement of the contact force \(F_c\) and adding the control force to \(F_1\).
- Configuration 3: Measurement of the acceleration \(\ddot{x}_1\) or position \(x_1\) and adding the control force to \(F_2\).
- Configuration 4: Measurement of the velocity \(\dot{x}_1\) and adding the control force to \(F_1\) by using a controller with a constant gain (Sky-hook principle).

For configurations 1 and 2, the contact force \(F_c\) is represented by an average stiffness of the catenary \(k_c\). In configuration 3, an estimator is needed to derive the contact force. This will be discussed in Section 3.2. The focus lies on configuration 1, as an actuator on the lower mass has several advantages compared to an actuator on the upper mass and the estimation of the contact force is not the main topic of this thesis. However, an estimator is a key point for the later implementation because the contact force cannot be measured in reality. Therefore, the potential of the other configurations is also explored.

Adding the control force to \(F_1\) has the disadvantage of adding more mass to \(m_1\) and the space around the panhead is limited. Therefore, from a practical point of view, adding the control force to \(F_2\) may be more acceptable [4] as the actuator could be placed right next to the air spring that provides the constant force in current state of the art pantographs. This method would also guarantee fail-safety, because the air spring can still provide the constant preforce if the actuator fails. Some other possible configurations were introduced in Section 1.3.

Controlling a constant position has been analyzed shortly, but the problem is the varying stiffness of the catenary and therefore it is more reasonable to control the force which is influenced both from the stiffness and the varying position.
2.2.2 Mathematical Representation

Newtons second law applied on the upper and lower mass results in

\begin{align}
    m_1 \ddot{x}_1 &= F_1 - F_{c1} - F_{k1} - m_1 \cdot g \\
    m_2 \ddot{x}_2 &= F_2 + F_{k1} + F_{c1} - F_{k2} - F_{c2} - m_2 \cdot g
\end{align}

and the forces can be written as

\begin{align}
    F_c &= k_c \cdot x_1 + d \\
    F_{k1} &= k_1 \cdot (x_1 - x_2) \\
    F_{k2} &= k_2 \cdot x_2 \\
    F_{c1} &= c_1 \cdot (\dot{x}_1 - \dot{x}_2) \\
    F_{c2} &= c_2 \cdot \dot{x}_2.
\end{align}

The parameters are set according to the values from Table 2.2 and \(d\) stands for the disturbance input in the model.

In order to get a representation of first order systems, additional states \(x_3\) and \(x_4\) were introduced

\begin{align}
    x_3 &= \dot{x}_1 \\
    x_4 &= \dot{x}_2 \\
    \dot{x}_3 &= x_1, \\
    \dot{x}_4 &= x_2.
\end{align}

Hence the following linear and time-invariant (LTI) state space representation was derived to

\[
\begin{bmatrix}
    \dot{x}_1 \\
    \dot{x}_2 \\
    \dot{x}_3 \\
    \dot{x}_4
\end{bmatrix} = \begin{bmatrix}
    0 & 0 & 1 & 0 \\
    -\frac{\left( k_1 + k_c \right) + \frac{k_1}{m_1}}{m_2} & 0 & 0 & 1 \\
    -\frac{c_1}{m_1} & -\frac{c_1}{m_2} & -\frac{(c_1 + c_2)}{m_2} & 0 \\
\end{bmatrix} \cdot \begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
    x_4
\end{bmatrix} + \begin{bmatrix}
    0 \\
    0 \\
    -\frac{d}{m_1} \cdot \frac{F_2}{m_2} \\
    \end{bmatrix},
\]

with

\[
y = [k_c \ 0 \ 0] \cdot \vec{x},
\]

where the output vector is according to configurations 1 and 2. \(F_2\) and \(F_1\) represent the control force for configuration 1 and 2, respectively. From the linearization a value \(y_{nom}\) of 72.69 N results. This force origins from the sum of the preforce \(F_{pre}\) of 60 N and the aerodynamic uplift at 200 km/h in open direction.
2.2.3 Linearization of the Friction

The friction elements in Ansys are nonlinear. For the control oriented model, a linear representation is needed. The friction $F_{\text{fric}}$ is replaced by an additional damping coefficient $c$ such that the energy dissipation is similar. The following procedure results in a suitable $c$:

- Perform a simulation with the full nonlinear model in Ansys.
- Find the frequency $f_0$ and amplitude $x_0$ of the movement of the lower and upper mass.
- Calculate the damping coefficient over the energy dissipation of the viscous damping in, $A = \pi \cdot 2\pi \cdot f_0 \cdot c \cdot x_0^2$, and the friction damping, $A = 4 \cdot F_{\text{fric}} \cdot x_0$.

The values are shown in Table 2.3. The resulting damping coefficients $c_1$ and $c_2$ are already included in the parametrization in Table 2.2.

<table>
<thead>
<tr>
<th>Mass</th>
<th>$F_{\text{fric}}$ [N]</th>
<th>$x_0$ [m]</th>
<th>$f_0$ [Hz]</th>
<th>$c$ [Ns/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper mass</td>
<td>4</td>
<td>0.004</td>
<td>7</td>
<td>29</td>
</tr>
<tr>
<td>Lower mass</td>
<td>3</td>
<td>0.010</td>
<td>1</td>
<td>61</td>
</tr>
</tbody>
</table>

2.2.4 Three Mass Model

In [9], a three mass model was derived in order to represent the first eigenmode of the combined system which is 0.79 Hz. By applying the same procedure, the resonance frequencies of the two pantograph eigenmodes were changed in a way, that the original representation got lost. Therefore, this approach was not investigated in more detail. The interested reader is referred to [12] for the derivation of the relative damping of the two mass model and to the Rayleigh parametrization for the relative damping coefficients of the three mass model.
2.2.5 Modifications of the Pantograph with SSS400 Panhead

The SSS400 panhead has been designed for higher speeds. However, it might be helpful to modify the configuration for an implementation of an active controller in the system. Therefore, different configurations are analyzed:

- **Modification 1:** $m_{1,new} = \frac{1}{2} \cdot m_1$, $m_{2,new} = \frac{1}{2} \cdot m_2$, $k_{1,new} = 2 \cdot k_1$.
- **Modification 2:** $m_{1,new} = \frac{1}{2} \cdot m_1$, $m_{2,new} = \frac{1}{2} \cdot m_2$, $k_{1,new} = 2 \cdot k_1$, $c_1 = 100 \text{Ns/m}$.
- **Modification 3:** $m_{1,new} = 2 \cdot m_1$, $m_{2,new} = 2 \cdot m_2$, $k_{1,new} = \frac{1}{2} \cdot k_1$.
- **Modification 4:** $m_{1,new} = 2 \cdot m_1$, $m_{2,new} = 2 \cdot m_2$, $k_{1,new} = \frac{1}{2} \cdot k_1$, $c_1 = 100 \text{Ns/m}$.
- **Modification 5:** $c_1 = 100 \text{Ns/m}$.

These modifications try to simulate the behaviour of a higher and lower pantograph with or without included damping. The next section shows bodeplots of these modifications to get a better overview of the changes in the frequency domain.
2.3 Model Analysis

Figure 2.6 shows a step response of 50 N applied on the lower and upper mass of the nonlinear model in Ansys. The response is the force that results out of this analysis minus the force that results without a step applied. The step was applied at a speed of 200 km/h in open driving direction. A step response analysis is a typical tool used to get an overview of the system to analyze. Table 2.4 shows an overview of the delay time and the $t_{90}$ time, which is the time where 90% of the reference value is reached. As the stiffness varies over the step, both the delay and the $t_{90}$ time are approximations, but they indicate the behaviour when a force is applied. The reaction is faster on the upper mass, but the final value is reached at the same time for both configurations.

Table 2.4: Overview of the time delay and the $t_{90}$ time for configuration 1 and 3 without any modification.

<table>
<thead>
<tr>
<th>Step on the upper mass</th>
<th>Step on the lower mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{90}$ Time, First Step</td>
<td>0.3 s</td>
</tr>
<tr>
<td>$t_{90}$ Time, Second Step</td>
<td>0.11 s</td>
</tr>
<tr>
<td>Delay, First Step</td>
<td>0.04 s</td>
</tr>
<tr>
<td>Delay, Second Step</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 2.7 shows a bode plot of configuration 1 and in Figure 2.8 the bode plot is compared to configuration 2. The resonance frequency is 0.8 Hz and 5.9 Hz for both configurations. The relative damping of the first and second frequency is 0.407 and 0.099, respectively. The system obtained in Section 2.2.2 is stable, observable and controllable in case of all modifications. The eigenvalues and eigenfrequencies for all configurations and modifications are shown in Table 2.5. Modifications 1 and 2 are have higher eigenfrequencies and modification 3 and 4 have lower eigenfrequencies than the original system. Figure 2.9, 2.10, 2.11, 2.12 and 2.13 show modifications 1, 2, 3, 4 and 5. Configuration 4 is a P-controller that takes the speed of the upper mass as an input and the resulting theoretical damping force is the control force and therefore the plant model is the same as for configuration 2. Figure 2.14 shows the bode plot of the standard WBL 88 pantograph.

---

1 A bode plot shows the amplitude and the phase of a transfer function over a specific frequency range. It is a useful tool to see the gain and phase shift applied to a signal with a specific frequency. These bodeplots result out of the linearized model in Matlab.
Figure 2.6: A step of 50 N in dotted is applied on the lower mass \( (F_2) \) and upper mass \( (F_1) \). The response is shown in black full (Configuration 1, \( F_2 \)) and gray (Configuration 2, \( F_1 \)).

Table 2.5: Overview of the eigenvalues and eigenfrequencies of all pantograph types.

<table>
<thead>
<tr>
<th>Pantograph Type</th>
<th>First Eigenvalue</th>
<th>Second Eigenvalue</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First Eigenfrequency</td>
<td>Second Eigenfrequency</td>
</tr>
<tr>
<td>With SSS400 Panhead</td>
<td>-3.709 ± 37.340i</td>
<td>-2.604 ± 5.848i</td>
</tr>
<tr>
<td></td>
<td>5.972 Hz</td>
<td>1.019 Hz</td>
</tr>
<tr>
<td>WBL 88 Pantograph</td>
<td>-11.060 ± 29.802i</td>
<td>-2.679 ± 5.675i</td>
</tr>
<tr>
<td></td>
<td>5.059 Hz</td>
<td>0.999 Hz</td>
</tr>
<tr>
<td>Modification 1</td>
<td>-7.580 ± 72.535i</td>
<td>-5.045 ± 7.832i</td>
</tr>
<tr>
<td></td>
<td>11.607 Hz</td>
<td>1.483 Hz</td>
</tr>
<tr>
<td>Modification 2</td>
<td>-29.285 ± 66.631i</td>
<td>-5.084 ± 7.830i</td>
</tr>
<tr>
<td></td>
<td>11.584 Hz</td>
<td>1.486 Hz</td>
</tr>
<tr>
<td>Modification 3</td>
<td>-1.781 ± 19.738i</td>
<td>-1.375 ± 4.059i</td>
</tr>
<tr>
<td></td>
<td>3.154 Hz</td>
<td>0.682 Hz</td>
</tr>
<tr>
<td>Modification 4</td>
<td>-7.134 ± 18.333i</td>
<td>-1.458 ± 4.064i</td>
</tr>
<tr>
<td></td>
<td>3.131 Hz</td>
<td>0.687 Hz</td>
</tr>
<tr>
<td>Modification 5</td>
<td>-14.525 ± 34.4410i</td>
<td>-2.660 ± 5.851i</td>
</tr>
<tr>
<td></td>
<td>5.949 Hz</td>
<td>1.023 Hz</td>
</tr>
</tbody>
</table>
Figure 2.7: Bode plot of the WBL 88 pantograph with SSS400 panhead in configuration 1.

Figure 2.8: Bode plot of the WBL 88 pantograph with SSS400 panhead in configuration 2. The dotted lines indicate configuration 1 as a comparison.

Figure 2.9: Bode plot of the WBL 88 pantograph with SSS400 panhead in configuration 1 with modification 1. The dotted lines indicate configuration 1 as a comparison.
Figure 2.10: Bode plot of the WBL 88 pantograph with SSS400 panhead in configuration 1 with modification 2. The dotted lines indicate configuration 1 as a comparison.

Figure 2.11: Bode plot of the WBL 88 pantograph with SSS400 panhead in configuration 1 with modification 3. The dotted lines indicate configuration 1 as a comparison.
Figure 2.12: Bode plot of the WBL 88 pantograph with SSS400 panhead in configuration 1 with modification 4. The dotted lines indicate configuration 1 as a comparison.

Figure 2.13: Bode plot of the WBL 88 pantograph with SSS400 panhead in configuration 1 with modification 5. The dotted lines indicate configuration 1 as a comparison.

Figure 2.14: Bode plot of the WBL 88 pantograph without SSS400 panhead. The dotted lines indicate configuration 1 as a comparison.
2.4 Multi Pantograph Configuration

As introduced in Chapter 1, multi train configurations are of particular interest. Therefore, the Ansys model was changed such that a second pantograph is considered. This second pantograph is an exact copy of the first one and the investigated distance between these two pantographs is 100 meters. Current regulations limit the distance between two pantographs to 200 m for high speed trains. The Gröna Tåget (Green Train) programme in Sweden intends to run short trains and couple them between bigger cities. Therefore, a distance of 100 m is investigated.

2.4.1 Critical Speed

If more than one pantograph is in use, the critical speed can be calculated as a function of the distance between the pantographs and the first eigenfrequency of the combined system. In [9], the first eigenfrequency was found to be 0.79 Hz. From the following relationship

\[ f = \frac{\text{speed}}{\text{distance}} \text{[Hz]} \]  

(2.14)

the critical speed for a distance of 100 m between two pantographs is

\[ v_{\text{crit}} = 0.79 \cdot d_{\text{pan}} = 0.79 \cdot 100 = 79\text{[m/s]} = 284.4\text{[km/h]} \]  

(2.15)
2.4. Multi Pantograph Configuration
Chapter 3

Active Pantograph

In this chapter, the controller for the active pantograph is designed. First the $\mathcal{H}_\infty$ method that is used to derive the controller is introduced and then applied. Later in this chapter, a contact force estimator is derived and the last section shows the implementation of the active pantograph into the finite element program Ansys which is used to analyze the controller behaviour.

3.1 $\mathcal{H}_\infty$ Control of the Contact Force

In this section, first the control problem formulation is stated, then the $\mathcal{H}_\infty$ Control method is introduced and applied.

3.1.1 Control Problem Formulation

The goal of the implementation of an active control is the reduction of the standard deviation of the contact force, hence reducing the contact force variation and holding the contact force as constant as possible. The disturbances are mainly caused by the droppers and the poles. The following frequencies are important for a train speed between 200 and 300 km/h:

- First eigenfrequency of the pantograph: 1.02 Hz
- Second eigenfrequency of the pantograph: 5.97 Hz
- Disturbances by the poles every 60 meters: 0.9 to 1.4 Hz
- Disturbances by the droppers every 9 meters: 6.2 to 9.3 Hz
- Disturbances by several harmonics of the poles and droppers.

A rule of thumb says, that the disturbances should be 10 times slower than the crossover frequency of the system. This is not the case in this problem formulation. Therefore, a total rejection of the disturbances is not possible. However, a certain reduction can be achieved.

As a control strategy, $\mathcal{H}_\infty$ Control has been chosen because it is a method that shapes the transfer function in the frequency domain whereas, for example, the LQG method penalizes the states and hence, the resulting transfer function cannot be tuned in the same way as with the $\mathcal{H}_\infty$ method.
In control theory, the disturbance is normally considered in the way as shown in Figure 3.1. However, as the exact input of the disturbance into the model is known from section 2.2.2 we can assume the structure in Figure 3.2. Hence, the closed loop transfer function from the disturbance $d$ to the output $y$, the sensitivity transfer function $S(s)$, is:

$$S(s) = \frac{Y(s)}{D(s)} = \frac{P_d(s)}{1 + K(s) \cdot P(s)} \quad (3.1)$$

where $P_d$ represents the disturbance model.

Figure 3.1: Standard definition of a feedback system with the disturbance added to the output signal.

Figure 3.2: Adapted feedback system with the disturbance as a second model input.
### 3.1.2 Introduction to $\mathcal{H}_\infty$ Control

This section explains the uncertainties, the nominal and robust stability, nominal and robust performance in order to better understand the robust control problem formulation in the next section. $\mathcal{H}_\infty$ control designs a controller $K(s)$ such that the peak of the upper singular value of the closed loop transfer function from $w$ to $\tilde{e}$ as shown in Figure 3.3 is minimized. In a second step, the $\mu$-synthesis, the upper singular value including the uncertainties $\Delta$ is minimized such that robustness is guaranteed. And by robustness, robustness with respect to model uncertainties is meant.

![Figure 3.3: The system P that is to control with a controller K in order to cope with the uncertainties $\Delta$.](image)

Figure 3.3: The system $P$ that is to control with a controller $K$ in order to cope with the uncertainties $\Delta$. 

3.1. $H_\infty$ Control of the Contact Force

Uncertainties
The design of feedback controllers in the presence of non-parametric and unstructured uncertainty is the raison d’être for $H_\infty$ feedback optimization. Uncertainty in the plant model may have several origins [13]:

- Parameters that are only known approximately or have an error.
- Parameters in the linear model that change due to nonlinearities or operating conditions.
- Measurement devices have imperfections including uncertainties in the manipulated inputs.
- Uncertainties may represent neglected dynamics in order to have a lower-order model.
- Difference of the implemented controller to the obtained due to order reduction.
- The uncertainty may exceed 100 % at higher frequencies as neither the structure nor the model order is known anymore.

One method to represent uncertainties is the multiplicative type which is shown in Eq. (3.2) where $\Delta_I(s)$ being any stable transfer function which at each frequency is less than or equal to one in magnitude. Additionally, the uncertainties $\omega_I(s)$ must be stable and have minimum phase.

$$G_p(s) = G(s) \cdot (1 + \omega_I(s)\Delta_I(s)) \text{ with } |\Delta_I(j\omega)| \leq 1 \forall \omega \quad (3.2)$$

Robust Stability (RS)
Nominal stability is guaranteed by the the $H_\infty$ method which means that the real parts of all eigenvalues have a value equal or smaller than zero. Robust stability on the other hand guarantees stability including the uncertainties. For a single input/single output (SISO) system with multiplicative uncertainties as introduced in Eq. (3.2), we can prove robust stability graphically by requiring that the loop transfer function

$$L_p = G_pK = G(s)K \cdot (1 + \omega_I\Delta_I) \quad (3.3)$$

does not encircle the point -1 for any $L_p$. Hence we say:

$$|\omega_I L| < |1 + L|, \forall \omega \quad (3.4)$$

$$\frac{|\omega_I L|}{1 + L} < 1, \forall \omega \quad (3.5)$$

$$RS \Leftrightarrow |\omega_I T| < 1, \forall \omega \quad (3.6)$$

Where the last equation indicates that we have to detune the system (i.e. make $T$ small) at frequencies where the relative uncertainty exceeds 1 in magnitude. See [13] for more details.
Robust Performance (RP)

Nominal performance (NP) is guaranteed when the transfer function \( L(j\omega) \) does not cross a circle with radius \( |\omega_p(j\omega)| \) around the point -1. This means mathematically that

\[

\text{NP} \iff |\omega_p| < |1 + L|, \forall \omega
\]  

(3.7)

Robust performance on the other side is a combination of robust stability and nominal performance. This means that in Eq. (3.7) \( L \) is replaced by \( L_p \) from Eq. (3.3). [13] shows that the following condition has to hold:

\[

\text{RP} \iff \max_{\omega} (|\omega_p S| + |\omega_l T|) < 1
\]  

(3.8)

3.1.3 Robust Control Problem

The applied \( \mathcal{H}_\infty \) control method in this section is based on the lecture advanced topics in control taught at ETH Zurich by Roy Smith [14] and a textbook on this topic [13]. The interested reader is also referred to [15] for an engineering aspect of \( \mathcal{H}_\infty \) control and some insights into possible performance functions. Additionally, [16] discusses the method that is used in Matlab. Later in section 3.1.3 the control problem is solved.

To design an \( \mathcal{H}_\infty \)-Controller, the structure of Figure 3.3 is considered where \( K \) is a controller to stabilize the system \( P \) including uncertainties \( \Delta \). In a first step, the system including the uncertainties and the weighting functions shown in Figure 3.4 is derived. Later, the controller \( K \) is designed based on this weighting functions.

\( M\Delta \)-Form

In the \( M\Delta \) structure, the scaling of the uncertainties is considered in a separate block, the \( \Delta \) block. Robust stability is guaranteed if and only if the loop transfer function \( M\Delta \) does not encircle -1 for all uncertainties \( \Delta \). Thus,

\[

\text{RS} \iff |1 + M\Delta| > 0, \forall \omega, \forall |\Delta| \leq 1
\]  

(3.9)

Such a \( M\Delta \) structure is shown in Figure 3.5 where \( M \) is represented by the closed loop transfer function \( G(s) \). To derive such a \( M\Delta \)-structure, a system overview as in Figure 3.4 is created.

In Figure 3.4 the actuator model \( P_{\text{act}} \), the input uncertainty \( W_i \), the nominal system \( P_{\text{nom}} \), the system uncertainty \( W_s \) and the output uncertainty \( W_o \) describe the input/output behaviour. In addition, weighting functions on the input \( u \) (\( W_{uP} \)), the output \( y \) (\( W_{yP} \)), the error \( r - y \) (\( W_{eP} \)), the noise signal \( n \) (\( W_n \)) and the disturbance signal \( d \) (\( W_d \)) were introduced.
3.1. $\mathcal{H}_\infty$ Control of the Contact Force

Figure 3.4: System Overview including the Uncertainties and the Weighting Functions. This is the problem setup as it is used in the Matlab/Simulink environment. It is a more detailed sketch of the overview given in Figure 3.3.

Out of Figure 3.4, the following equation is derived to get the M-\(\Delta\)-form of figure 3.5:

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 & W_d P_{\text{nom}} & 0 & W_d P_{\text{act}} & W_d P_{\text{nom}} W_d & 0 & 0 \\ 0 & W_d & 0 & W_d P_{\text{act}} & W_d P_{\text{nom}} W_d & 0 & 0 \\ 1 & -W_{d, P} & -W_{d, P} P_{\text{act}} & -W_{d, P} P_{\text{nom}} P_{\text{act}} & -W_{d, P} P_{\text{nom}} W_d & -W_{d, P} W_n & W_{d, P} \\ 0 & -1 & W_{d, P} P_{\text{nom}} -P_{\text{act}} & W_{d, P} P_{\text{nom}} P_{\text{act}} & -P_{\text{nom}} P_{\text{act}} & W_{d, P} W_n & 0 \\ 0 & 0 & W_{d, P} & -P_{\text{nom}} W_d & -W_{d, P} W_n & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

where the following notation is used for the signals [14]:

- $u$: control variables: $u$.
- $w$: exogenous signals (disturbances and commands): $d$, $n$ and $r$.
- $\tilde{e}$: performance signals that are to be minimized to meet the control objectives: $e_1$, $e_2$ and $e_3$.
- $\tilde{y}$: the input value for the controller: $r - y$.
- $z$: input value to the disturbance scale $\Delta$: $z_1$, $z_2$ and $z_3$.
- $v$: output value of the disturbance scale $\Delta$: $v_1$, $v_2$ and $v_3$. 

Figure 3.5: The M-\(\Delta\) form, here with $G(s)$ as the closed loop system and for the analysis called $M$. 

Hence, the matrix $M_{11}$ ($z = M_{11} \cdot v$) follows
\[
M_{11} = \begin{bmatrix}
0 & W_s \cdot P_{nom} & 0 \\
0 & 0 & 0 \\
W_o & W_o \cdot P_{nom} & 0 
\end{bmatrix},
\]
And
\[
\Delta = \begin{bmatrix}
\Delta_1 & 0 & 0 \\
0 & \Delta_2 & 0 \\
0 & 0 & \Delta_3 
\end{bmatrix} = \begin{bmatrix}
\delta_1 & 0 & 0 \\
0 & \delta_2 & 0 \\
0 & 0 & \delta_3 
\end{bmatrix} \in \mathbb{C}^{3 \times 3}.
\]

The $\Delta$ matrix has diagonal form as all the signals are scalars due to the fact that the system is a SISO one. This structure is of importance for the D-K iteration.

**Nominal Model and Uncertainties**

The following uncertainties are considered:

- Input Uncertainty: Constantly 5% error due to possible errors in the actuator etc.
- System Uncertainty: Changing stiffness $k_c$. See Eq. (3.13).
- Output Uncertainty: Constantly 20% error due to possible sensor errors and estimator errors in a later stage of the project.

According to [13], it is possible to combine the input and output uncertainties for SISO systems. However, not the same values can be taken and the understanding of the uncertainties can get lost. Therefore, these two uncertainties have not been combined in this thesis. The resulting transfer functions are

\[
P_{nom} = \frac{288.5s + 5.771e4}{s^4 + 12.63s^3 + 1488s^2 + 7636s + 5.771e4}, \quad (3.10)
\]
\[
P_{act} = 1, \quad (3.11)
\]
\[
W_i = 0.05, \quad (3.12)
\]
\[
W_s = \frac{0.1s + 0.001}{0.2857s + 1}, \quad (3.13)
\]
\[
W_o = 0.2. \quad (3.14)
\]
Weighting Functions

Different strategies to choose the weighting functions such as S/KS, S/T etc. were introduced in [13] and [15]. For this specific problem formulation with the desire of disturbance rejection with respect to uncertainties, the S/KS method is most suitable. By choosing this method, a bound for the sensitivity transfer function $S(s)$ and the controller $K(s)$ is chosen as indicated in Figure 3.7. The error transfer function $W_eP$ is designed such that it has a value higher than 1 for low frequencies and values lower than 1 for high frequencies were no disturbance rejection is possible. The inverse of this transfer function is then the bound indicated in Figure 3.7. In other words, low frequency terms get a higher weight such that these terms are then lowered by the optimization in order to have a singular value below 1 from the inputs, e.g. the disturbance input, to the output, e.g. the error performance output. Additionally, the error transfer function is also needed so that the controller actually starts tracking the reference value.

The actuator performance function $W_aP$ has a high value for high frequencies such that the actuator power gets reduced for these high frequencies. Hence the bound in Figure 3.7 for the controller $K(s)$ is the inverse of the performance function $W_aP$. The upper frequency for the controller was bound at one decade faster than the system.

For this method, the output performance function $W_yP$ can be zero, because the other performance functions are sufficient to design the controller. Additionally, the weighting function for noise has not been considered in this problem, because no noise description is available and it is an additional method to shape the high frequency terms which is already done by $W_aP$. For the same reason, the actuator transfer function $P_{act}$ is 1.

Figure 3.8 and Eq. (3.15) show all weighting and performance functions. It is important that all of these functions are proper and stable.

\[
W_eP = \frac{0.04082s^2 + 0.5714s + 2}{0.1623s^2 + 0.8058s + 1} \quad W_n = 0
\]
\[
W_aP = \frac{0.00175s + 0.07}{0.00025s + 1} \quad W_d = \frac{1}{0.2015s + 1}
\]
\[
W_yP = 0
\]
Figure 3.6: An overview of the uncertainties (input (red), system (blue), output (magenta)) and the nominal system (black) over the frequency range of interest.

Figure 3.7: Schematic overview of the bounds for the sensitivity transfer function $S(s)$ and the controller $K(s)$ with the $S/KS$ weighting method as chosen in this thesis.

Figure 3.8: Overview of the weighting functions (blue: error performance, green: actuator performance, cyan: disturbance weight) over the frequency range of interest.
3.1.4 Nominal $H_\infty$-Controller

A controller $K$ is to find for the system in Figure 3.9 in order to be optimal for the introduced weighting functions. In this nominal approach, the uncertainties were neglected.

![System overview: Nominal system $P(s)$ with a controller $K(s)$](image)

Figure 3.9: System overview: Nominal system $P(s)$ with a controller $K(s)$.

Nominal Controller Design

The structure used to find an optimal controller was introduced previously in Figure 3.4 including all the weighting functions. Out of this figure and the information given in [14], one can derive a nominal system and then use the Matlab command $\text{hinfsyn}$ (see [16] for a detailed explanation) to derive a nominal $H_\infty$ controller. The weighting functions are then omitted and with the resulting controller, the loop is closed. For the analysis of the nominal controller, the uncertainties are still neglected.

In Figure 3.10 a disturbance step of 20 N is applied to the nominal closed loop system. It can be seen that a certain rejection is achieved. A detailed analysis of the results can be found in Chapter 4. Figure 3.11 shows the controller $K(s)$, the open loop transfer function $L(s)$ from the input to the output, the closed loop transfer function $T(s)$ from the input to the output and the closed loop transfer function $S(s)$ from the disturbance to the output.

![Response of a disturbance step of 20 N on the system](image)

Figure 3.10: Response of a disturbance step of 20 N on the system in blue. Control action in green.
Figure 3.11: An overview of the controller $K(s)$ (red), the open loop transfer function $L(s)$ from the input to the output (blue), the closed loop transfer function $T(s)$ from the input to the output (black) and the closed loop transfer function $S(s)$ from the disturbance to the output (gray).

**Robustness Analysis**

[13] defines the structured singular value, $\mu$, as a function which provides a generalization of the singular value, $\sigma$, and the spectral radius, $\rho$. $\mu$ is used to get necessary and sufficient conditions for RS and RP. The definition is:

$$\text{Find the smallest structured } \Delta \text{ (measured in terms of } \sigma(\Delta)) \text{ which makes } \det(I - M\Delta) = 0; \text{ then } \mu(M) = \frac{1}{\sigma(\Delta)}.$$  

Where $M$ and $\Delta$ are the same as for the $M\Delta$ structure previously introduced. [13] goes into more detail on this topic. Most important is, that three different $\mu$ tests are done in the procedure in [16] and the final results is a plot of $\mu$ for RS, NP and RP over a defined frequency range. The goal is to have the peak value of all three $\mu$-tests below 1.

As seen in Figure 3.12, the nominal controller satisfies all criteria such as nominal performance as well as robust performance and stability because the maximal value of all singular value plots is below 1.
3.1. \( H_{\infty} \) Control of the Contact Force

### \( \mu \)-Synthesis and D-K Iteration

The structured singular value \( \mu \) is a very powerful tool for the analysis of robust performance with a given controller. However, one may also seek to find the controller that minimizes a given \( \mu \)-condition: this is the \( \mu \)-synthesis problem \cite{13}. There is no direct method to synthesize a \( \mu \)-optimal controller available. However, a method known as DK-iteration can be applied. The method starts with the upper bound on \( \mu \) in terms of the scaled singular value:

\[
\mu(N) \leq \min_D \bar{\sigma}(DMD^{-1})
\]

(3.16)

where \( D \) is any matrix which commutes with \( \Delta \), that is \( \Delta D = D\Delta \).

The idea is to find the controller that minimizes the peak value of this upper bound, namely

\[
\min_K (\min_D \|\bar{\sigma}(DMD^{-1})\|_{\infty})
\]

(3.17)

by altering between minimizing \( \|\bar{\sigma}(DMD^{-1})\|_{\infty} \) with respect to either \( K \) or \( D \) while holding the other constant. This method is explained in more details in \cite{13}. It is important to mention that the order of the controller is increased by using this approach. Additionally, controller that are synthesized with this method are from experience slower and more conservative than the nominal controller resulting from the nominal synthesis.

As seen in the robustness analysis, the controller satisfies the criteria and, therefore, this method is not applied in this work.

### 3.1.5 Other Configurations and Modifications

In order to compare the different pantograph modifications and controller configurations, the same weighting functions as specified in Section 3.1.3 are used to derive all controllers used in the next chapter.
3.1.6 Controller Discretization

In Matlab, a continuous controller is derived. Unfortunately, it is not possible to apply such a controller direct in ANSYS. Therefore, the controller needs to be discretized with an appropriate sampling time. The controller was discretized in Matlab with the command \texttt{c2d} and a sampling time $T_s$ of 0.002 s. Tustin emulation was used as the discretization method. This emulation method replaces the Laplace variable $s$ by:

$$s = \frac{2 \cdot (z - 1)}{T_s \cdot (z + 1)}$$ (3.18)

where $z$ represents the unit time shift operator and $T_s$ the sampling time.
3.2 Contact Force Estimation

This section discusses the estimation of the contact force between the pantograph and the catenary. An estimation of the force is needed, because the force cannot be measured directly. First, a justification for the choice of $\mathcal{H}_\infty$ estimation is given. Later, estimators with two different sensor configurations are derived and compared. Up to date, a pantograph does not have a sensor that could be used for an estimation. Hence, this is also a study on which sensor is more interesting to use for an estimator.

3.2.1 Comparison between Kalman Filtering and $\mathcal{H}_\infty$ Estimation

Kalman filtering assumes that the message generating process has a known dynamics and that the exogenous inputs have known statistical properties. Unfortunately, these assumptions limit the utility of minimum variance estimators in situations where the message model and/or the noise description are unknown [17].

In this application, the noise description is indeed unknown and a very simplified model is used. Hence, a minimization of the estimation error is a good approach [18].

3.2.2 Theoretical Background to $\mathcal{H}_\infty$ Estimation

This is a short summary on discrete steady-state $\mathcal{H}_\infty$ estimation by [18]. Suppose we have a discrete system

$$x_{k+1} = F \cdot x_k + G \cdot u_k + w_k,$$

$$y_k = H \cdot x_k + v_k,$$

$$z_k = L \cdot x_k,$$

where $F$, $G$, $H$ represent the discretized system matrix with the zero order hold (ZOH) method. $w_k$ and $v_k$ are noise terms. The goal is to estimate $z_k$ such that

$$\lim_{N \to \infty} \frac{\sum_{k=0}^{N-1} \| z_k - \hat{z}_k \|^2_S}{\sum_{k=0}^{N-1} (\| w_k \|^2_Q + \| v_k \|^2_R)} < \frac{1}{\theta}$$

(3.22)

where $Q$, $R$ and $S$ are symmetric positive definite matrices and $\theta$ is a scaler that must be chosen based on the problem. The steady-state filter is defined as

$$\tilde{S} = L^T S L,$$

$$K = P[I - \theta \tilde{S} P + H^T R^{-1} H P]^{-1} H^T R^{-1},$$

$$\hat{x}_{k+1} = F \hat{x}_k + G u_k + F K (y_k - H \hat{x}_k),$$

$$P = FP[I - \theta \tilde{S} P + H^T R^{-1} H P]^{-1} F^T + Q.$$

(3.23)

(3.24)

(3.25)

(3.26)

The last equation can be rearranged with the matrix inversion lemma and then be solved in Matlab with the command DARE.
Chapter 3. Active Pantograph

Compared to the time variant filter, all matrices can be precalculated and the gain K can be directly implemented. In order to apply the method, the following variables need to be defined:

- $S$ is the weight of the importance of each state.
- $Q$ is a weight of the disturbance influence on each state.
- $R$ is the weight for the sensor noise.
- $L$ defines the combination of what is desired to be estimated. The unity matrix can be chosen if all states shall be estimated.
- $\theta^{-1}$ is the upper bound for the error.

### 3.2.3 Design of a $H_\infty$ Estimator

According to the specification from the last section, the following matrices were defined:

\[
S = \begin{bmatrix}
50 & 0 & 0 & 0 \\
0 & 10 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix} \quad (3.27)
\]

\[
Q = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix} \quad (3.28)
\]

\[
R = 1 \quad (3.29)
\]

\[
L = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix} \quad (3.30)
\]

\[
\theta = 10 \quad (3.31)
\]

where the higher weight on the position states have been derived by analysis in Matlab. 50 and 10 is the weight of the state $x_1$ and $x_2$, respectively. As there is no information on noise and disturbance influence, these values were kept at a value of 1.
3.2.4 Comparison and Conclusion

Figure 3.13 shows the acceleration $\ddot{x}_1$, position $x_1$ and contact force over one span. It is clearly visible that the acceleration provides more information about the contact force and hence this sensor is used to estimate the contact force in this configuration.

Figure 3.14 show the estimators based on the position and acceleration measurement. There is hardly any change in the estimation based on the position sensor. Figure 3.15 shows an other approach with an additional direct mathematical interpretation of the acceleration sensor:

$$ F_{\text{contact}} = F_{\text{estimator,acc}} + a \cdot m_1 $$

where $F_{\text{contact}}$ is the final contact force estimation, $F_{\text{estimator,acc}}$ the estimated force based on the $H_\infty$ estimation method with an acceleration sensor and $a$ the acceleration measurement. This estimation provides more information than the other two approaches but a further tuning might be necessary. Both methods that are shown in Figure 3.15 are analyzed in the next chapter.

Figure 3.13: Measurement of the acceleration in the upper figure, the position in the middle figure and the force in the lower figure with a train speed of 280 km/h.
Figure 3.14: Comparison of the estimator based on the position sensor (gray) and on the acceleration sensor (black). The real contact force is shown in blue. The train speed is 200 km/h.

Figure 3.15: Comparison of the estimator based on the acceleration sensor combined with a direct calculation out of the acceleration (gray) and on the acceleration sensor (black). The real contact force is shown in blue. The train speed is 200 km/h.
3.2. Contact Force Estimation
Chapter 4

Results and Discussion

This chapter consists of the results of the controller implementation in Ansys. These results are compared to the passive system. A focus lies on the WBL 88 pantograph with SSS400 panhead in single and double pantograph configuration in Section 4.1. The different controller configurations are also investigated in this section. In Section 4.2, the different system modifications are shown. The resulting simulation time is discussed in Section 4.3.

4.1 WBL 88 Pantograph with SSS400 Panhead

This section discusses the results of the WBL 88 pantograph with SSS400 panhead which has been designed for higher speed. These results are then compared to the results obtained in [9]. Multi train configurations and other controller configurations are also part of this section. The demanded value from the specification [2] is only considered in configuration 1.
4.1.1 One Pantograph

Passive Pantograph

[2] defines the demanded mean value of the contact force with the following equation:

\[ F_{\text{mean}} = F_{\text{stat}} + 0.00097 \cdot v^2 \]  

(4.1)

where \( v \) is the train speed in \( \text{km/h} \). The goal is to reach the specifications for \( 200 \text{ km/h} \) for higher speeds. Hence a mean value of 98.8 N is to reach. In addition, [2] demands a standard deviation of the contact force of 30 % of the mean value which is 29.6 N in this case and the mean value has to be within a bound of 10 %, hence between 88.92 N and 108.68 N.

Figure 4.1 provides a time domain overview of the signals of the passive system. It indicates the force over one span of 60 meters, the mean value and the standard deviation of the contact force.

In Figure 4.2 the mean value and the standard deviation of the contact force for different train speeds in open driving direction are shown. The increase in the mean value is caused by the increase in the aerodynamic uplift force. Table 4.1 shows the values for both driving directions. It can be seen, that the values do not differ significantly. As the values for the open driving directions are slightly higher, only these values are considered in the rest of the text.

Table 4.1: Comparison of the mean, standard deviation and RMS values without control in different driving direction (closed and open).

<table>
<thead>
<tr>
<th>Train Speed [\text{km/h}]</th>
<th>Driving Direction</th>
<th>Mean Contact Force [N]</th>
<th>Standard Deviation [N]</th>
<th>RMS value 0-2 Hz [\text{Hz}]</th>
<th>RMS value 0-5 Hz [\text{Hz}]</th>
<th>RMS value 5-20 Hz [\text{Hz}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 open</td>
<td>72.67</td>
<td>13.68</td>
<td>7.28</td>
<td>11.34</td>
<td>7.41</td>
<td></td>
</tr>
<tr>
<td>closed</td>
<td>69.22</td>
<td>13.49</td>
<td>7.16</td>
<td>11.23</td>
<td>7.28</td>
<td></td>
</tr>
<tr>
<td>220 open</td>
<td>74.7</td>
<td>16.08</td>
<td>8.01</td>
<td>12.86</td>
<td>10.02</td>
<td></td>
</tr>
<tr>
<td>closed</td>
<td>70.79</td>
<td>15.63</td>
<td>7.83</td>
<td>12.57</td>
<td>9.61</td>
<td></td>
</tr>
<tr>
<td>240 open</td>
<td>77.27</td>
<td>19.06</td>
<td>10.56</td>
<td>14.56</td>
<td>13.43</td>
<td></td>
</tr>
<tr>
<td>closed</td>
<td>72.9</td>
<td>18.62</td>
<td>10.3</td>
<td>14.37</td>
<td>12.88</td>
<td></td>
</tr>
<tr>
<td>260 open</td>
<td>80.07</td>
<td>22.27</td>
<td>12.07</td>
<td>16.03</td>
<td>17.19</td>
<td></td>
</tr>
<tr>
<td>closed</td>
<td>75.24</td>
<td>21.9</td>
<td>11.79</td>
<td>16.01</td>
<td>16.6</td>
<td></td>
</tr>
<tr>
<td>280 open</td>
<td>83.62</td>
<td>24.3</td>
<td>12.76</td>
<td>15.54</td>
<td>20.69</td>
<td></td>
</tr>
<tr>
<td>closed</td>
<td>78.29</td>
<td>24.19</td>
<td>12.39</td>
<td>15.53</td>
<td>20.56</td>
<td></td>
</tr>
<tr>
<td>300 open</td>
<td>87.4</td>
<td>30.43</td>
<td>16.83</td>
<td>19.34</td>
<td>25.89</td>
<td></td>
</tr>
<tr>
<td>closed</td>
<td>81.56</td>
<td>29.93</td>
<td>16.19</td>
<td>18.89</td>
<td>25.64</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.1: The contact force over one span in black. The mean value and the mean value ± the standard deviation are in blue and red, respectively. The dashed line indicate the location of the droppers. The train speed is 280 km/h.

Figure 4.2: The mean and standard deviation values over the train speed of interest in blue and red, respectively. The blue dashed line indicates the demanded mean value and the blue dash-dotted lines the bound for this mean value. The red dashed line indicates the upper bound for the standard deviation.
Active Pantograph, Configuration 1

In this section, the results of configuration 1 is shown. In configuration 1, the contact force is measured and the controller acts on the lower mass of the pantograph. Figure 4.3 shows the time domain behaviour with a controller. The upper plot shows the contact force for the active and passive case and the lower plot the control force over one span.

Figure 4.3 and tables 4.2 and 4.3 summarize the result with the same mean value as without controller. Figure 4.4 and tables 4.4 and 4.5 summarize the results with the mean value set to the demanded value according to [2]. A clear reduction of the standard deviation was achieved for the same mean value as in the passive case. However, this reduction is lost for lower speeds by demanding a higher mean value for the contact force.

Figures 4.6 and 4.7 show the distribution of the contact force and a Gaussian distribution overlying it.

Figure 4.3: The contact force of the passive system (gray) and the active system (black) is shown in the upper figure. The mean value and the mean value ± the standard deviation are in blue and red, respectively. The lower figure shows the control force. The train speed is 280 km/h.
Figure 4.4: The mean and standard deviation values over the train speed of interest in blue and red, respectively. The blue dashed line indicates the demanded mean value and the blue dash-dotted lines the bound for this mean value. The red dashed line indicates the upper bound for the standard deviation. The dotted blue and red lines indicate the passive case. A controller with the same reference value as the passive solution was applied.

Figure 4.5: The mean and standard deviation values over the train speed of interest in blue and red, respectively. The blue dashed line indicates the demanded mean value and the blue dash-dotted lines the bound for this mean value. The red dashed line indicates the upper bound for the standard deviation. The dotted blue and red lines indicate the passive case. A controller with the demanded reference value was applied.
Table 4.2: Overview of the reduction of the standard deviation of the contact force for the active solution with the reference value according to the passive solution.

<table>
<thead>
<tr>
<th>Train Speed [km/h]</th>
<th>Control Strategy</th>
<th>Mean Value [N]</th>
<th>Standard Deviation (STD) [N]</th>
<th>Reduction in STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>Passive</td>
<td>72.67</td>
<td>13.68</td>
<td>-17.8 %</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>72.65</td>
<td>11.24</td>
<td></td>
</tr>
<tr>
<td>220</td>
<td>Passive</td>
<td>74.7</td>
<td>16.08</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>75.17</td>
<td>13.57</td>
<td>-15.6 %</td>
</tr>
<tr>
<td>240</td>
<td>Passive</td>
<td>77.27</td>
<td>19.06</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>77.85</td>
<td>17</td>
<td>-10.8 %</td>
</tr>
<tr>
<td>260</td>
<td>Passive</td>
<td>80.07</td>
<td>22.27</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>80.82</td>
<td>20.58</td>
<td>-7.6 %</td>
</tr>
<tr>
<td>280</td>
<td>Passive</td>
<td>83.62</td>
<td>24.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>83.84</td>
<td>22.3</td>
<td>-8.2 %</td>
</tr>
<tr>
<td>300</td>
<td>Passive</td>
<td>87.4</td>
<td>30.43</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>86.98</td>
<td>26.13</td>
<td>-14.1 %</td>
</tr>
</tbody>
</table>

Table 4.3: Overview of the reduction of the RMS values of the contact force for the active solution with the reference value according to the passive solution.

<table>
<thead>
<tr>
<th>Train Speed [km/h]</th>
<th>Control Strategy</th>
<th>RMS 0-2 Hz</th>
<th>RMS 0-5 Hz</th>
<th>RMS 5-20 Hz</th>
<th>Change 0-2 Hz</th>
<th>Change 0-5 Hz</th>
<th>Change 5-20 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>Passive</td>
<td>7.28</td>
<td>11.34</td>
<td>7.41</td>
<td>-32.6 %</td>
<td>-29.5 %</td>
<td>5.0 %</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>4.91</td>
<td>8</td>
<td>7.78</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>220</td>
<td>Passive</td>
<td>8.01</td>
<td>12.86</td>
<td>10.02</td>
<td>-34.5 %</td>
<td>-27.1 %</td>
<td>0.8 %</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>5.25</td>
<td>9.37</td>
<td>10.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>Passive</td>
<td>10.56</td>
<td>14.56</td>
<td>13.43</td>
<td>-34.8 %</td>
<td>-23.8 %</td>
<td>3.7 %</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>6.88</td>
<td>11.09</td>
<td>13.93</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>260</td>
<td>Passive</td>
<td>12.07</td>
<td>16.03</td>
<td>17.19</td>
<td>-33.5 %</td>
<td>-25.3 %</td>
<td>8.1 %</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>8.03</td>
<td>11.98</td>
<td>18.59</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>280</td>
<td>Passive</td>
<td>12.76</td>
<td>15.54</td>
<td>20.69</td>
<td>-32.7 %</td>
<td>-24.3 %</td>
<td>1.1 %</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>8.59</td>
<td>11.76</td>
<td>20.91</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>Passive</td>
<td>16.83</td>
<td>19.34</td>
<td>25.89</td>
<td>-36.8 %</td>
<td>-27.7 %</td>
<td>-6.8 %</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>10.63</td>
<td>13.99</td>
<td>24.12</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4.4: Overview of the reduction of the standard deviation of the contact force for the active solution with the reference value according to the specifications.

<table>
<thead>
<tr>
<th>Train Speed [km/h]</th>
<th>Control Strategy</th>
<th>Mean Value [N]</th>
<th>Standard Deviation (STD) [N]</th>
<th>Change in STD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Passive</td>
<td>72.67</td>
<td>13.68</td>
<td>-1%</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>94.74</td>
<td>13.55</td>
<td></td>
</tr>
<tr>
<td>220</td>
<td>Passive</td>
<td>74.7</td>
<td>16.08</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>95.16</td>
<td>16.43</td>
<td>+2.2%</td>
</tr>
<tr>
<td>240</td>
<td>Passive</td>
<td>77.27</td>
<td>19.06</td>
<td>+2.4%</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>95.59</td>
<td>19.51</td>
<td></td>
</tr>
<tr>
<td>260</td>
<td>Passive</td>
<td>80.07</td>
<td>22.27</td>
<td>-5.2%</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>96.14</td>
<td>21.12</td>
<td></td>
</tr>
<tr>
<td>280</td>
<td>Passive</td>
<td>83.62</td>
<td>24.3</td>
<td>-8.9%</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>96.49</td>
<td>22.13</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>Passive</td>
<td>87.4</td>
<td>30.43</td>
<td>-13.1%</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>96.83</td>
<td>26.43</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.5: Overview of the reduction of the RMS values of the contact force for the active solution with the reference value according to the specifications.

<table>
<thead>
<tr>
<th>Train Speed [km/h]</th>
<th>Control Strategy</th>
<th>RMS 0-2 Hz</th>
<th>RMS 0-5 Hz</th>
<th>RMS 5-20 Hz</th>
<th>Change 0-2 Hz</th>
<th>Change 0-5 Hz</th>
<th>Change 5-20 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Passive</td>
<td>7.28</td>
<td>11.34</td>
<td>7.41</td>
<td>-23.6%</td>
<td>-23.0%</td>
<td>38.1%</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>5.56</td>
<td>8.73</td>
<td>10.23</td>
<td>-23.0%</td>
<td>-20.0%</td>
<td>33.1%</td>
</tr>
<tr>
<td>220</td>
<td>Passive</td>
<td>8.01</td>
<td>12.86</td>
<td>10.02</td>
<td>-25.8%</td>
<td>-19.9%</td>
<td>33.9%</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>5.94</td>
<td>10.3</td>
<td>13.42</td>
<td>-25.8%</td>
<td>-19.9%</td>
<td>33.9%</td>
</tr>
<tr>
<td>240</td>
<td>Passive</td>
<td>10.56</td>
<td>14.56</td>
<td>13.43</td>
<td>-26.1%</td>
<td>-21.7%</td>
<td>29.1%</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>7.8</td>
<td>11.4</td>
<td>17.34</td>
<td>-26.1%</td>
<td>-21.7%</td>
<td>29.1%</td>
</tr>
<tr>
<td>260</td>
<td>Passive</td>
<td>12.07</td>
<td>16.03</td>
<td>17.19</td>
<td>-26.7%</td>
<td>-26.9%</td>
<td>13.0%</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>8.85</td>
<td>11.72</td>
<td>19.42</td>
<td>-26.7%</td>
<td>-26.9%</td>
<td>13.0%</td>
</tr>
<tr>
<td>280</td>
<td>Passive</td>
<td>12.76</td>
<td>15.54</td>
<td>20.69</td>
<td>-26.8%</td>
<td>-25.2%</td>
<td>-0.6%</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>9.34</td>
<td>11.63</td>
<td>20.56</td>
<td>-26.8%</td>
<td>-25.2%</td>
<td>-0.6%</td>
</tr>
<tr>
<td>300</td>
<td>Passive</td>
<td>16.83</td>
<td>19.34</td>
<td>25.89</td>
<td>-32.3%</td>
<td>-25.3%</td>
<td>-6.8%</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>11.4</td>
<td>14.45</td>
<td>24.13</td>
<td>-32.3%</td>
<td>-25.3%</td>
<td>-6.8%</td>
</tr>
</tbody>
</table>
Figure 4.6: Distribution of the contact force for 200 km/h with control (black) and without control (gray). The standard deviation with and without control action is 11.24 N and 13.68 N, respectively.

Figure 4.7: Distribution of the contact force for 280 km/h with control (black) and without control (gray). The standard deviation with and without control action is 22.3 N and 24.3 N, respectively.
Results from previous Work

This section summarizes the results obtained in [9], where active pantograph solutions were analyzed in the multi body simulation tool Gensys. This work considers a simplified model of the catenary. Two different analyses were performed. The results obtained with the same model as used in this thesis are summarized in the first part of Table 4.6. The results obtained with the three mass model shortly introduced in Section 2.2.4 are shown in second part of Table 4.6. As a control strategy, optimal control was used. The actuator considered in [9] is neglected for this comparison.

Table 4.6: Results of the optimal control strategy derived in [9] by using a two and three mass multi body model in the software Gensys [10].

<table>
<thead>
<tr>
<th>Train Speed [km/h]</th>
<th>Control Strategy</th>
<th>Mean Value [N]</th>
<th>Standard Deviation [STD] [N]</th>
<th>Reduction in STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>288 Passive, Two Mass Model</td>
<td>106.22</td>
<td>22.47</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Active, Two Mass Model</td>
<td>106.43</td>
<td>19.14</td>
<td>-14 %</td>
<td></td>
</tr>
<tr>
<td>200 Passive, Three Mass Model</td>
<td>69.46</td>
<td>14.71</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Active, Three Mass Model</td>
<td>69.11</td>
<td>5.7</td>
<td>-61 %</td>
<td></td>
</tr>
<tr>
<td>250 Passive, Three Mass Model</td>
<td>88.22</td>
<td>14.67</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Active, Three Mass Model</td>
<td>88.43</td>
<td>7.05</td>
<td>-52 %</td>
<td></td>
</tr>
<tr>
<td>300 Passive, Three Mass Model</td>
<td>101.55</td>
<td>14.70</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Active, Three Mass Model</td>
<td>101.65</td>
<td>8.04</td>
<td>-45 %</td>
<td></td>
</tr>
</tbody>
</table>
Active Pantograph, Configuration 2

Configuration 2 measures the contact force and the controller acts on the upper mass. The results of this configuration are summarized in table 4.7 and the contact force over one span for a train speed of 280 km/h is shown in Figure 4.8. Even though a faster response is possible as discussed in Section 2.3, the standard deviation is only reduced for some speeds. Figure 4.8 shows that the control force has two peaks between two droppers whereas the control force in configuration 1 only had 1 peak. One interpretation is, that the controller already acts too fast and the performance function could be changed such that a slower controller is derived.

![Graph showing contact force over time](image-url)

**Figure 4.8:** The contact force of the passive system (gray) and the active system (black) is shown in the upper figure. The mean value and the mean value ± the standard deviation are in blue and red, respectively. The lower figure shows the control force. The train speed is 280 km/h.
Table 4.7: Overview of the change in the standard deviation of the contact force for the active solution with the reference value according to passive solution with configuration 2.

<table>
<thead>
<tr>
<th>Train Speed [km/h]</th>
<th>Control Strategy</th>
<th>Mean Value [N]</th>
<th>Standard Deviation (STD) [N]</th>
<th>Change in STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>Passive</td>
<td>72.67</td>
<td>13.68</td>
<td>-22.8%</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>72.67</td>
<td>10.56</td>
<td></td>
</tr>
<tr>
<td>220</td>
<td>Passive</td>
<td>74.7</td>
<td>16.08</td>
<td>-8.3%</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>75.12</td>
<td>14.74</td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>Passive</td>
<td>77.27</td>
<td>19.06</td>
<td>+7.2 %</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>77.72</td>
<td>20.44</td>
<td></td>
</tr>
<tr>
<td>260</td>
<td>Passive</td>
<td>80.07</td>
<td>22.27</td>
<td>+7.7 %</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>80.69</td>
<td>23.99</td>
<td></td>
</tr>
<tr>
<td>280</td>
<td>Passive</td>
<td>83.62</td>
<td>24.3</td>
<td>+4.7 %</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>83.69</td>
<td>25.44</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>Passive</td>
<td>87.4</td>
<td>30.43</td>
<td>-5.2 %</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>86.99</td>
<td>28.85</td>
<td></td>
</tr>
</tbody>
</table>
Active Pantograph, Configuration 3

Table 4.8 summarizes the results of configuration 3. In configuration 3, an estimator is used to derive the contact force and the actuator acts on the lower mass. Estimator 1 is a pure $H_\infty$ estimator that uses the acceleration sensor as an input and estimator 2 is the same estimator but slightly adapted by including a direct mathematical interpretation of the acceleration sensor:

$$F_{\text{contact}} = F_{\text{estimator,acc}} + a \cdot m_1$$  \hspace{1cm} (4.2)

where $F_{\text{contact}}$ is the final contact force estimation, $F_{\text{estimator,acc}}$ the estimated force based on estimator 1 and $a$ the measured acceleration.

The simulation with a train speed of 300 km/h is only performed with one estimator to confirm the maximum possible top speed from configuration 1.

Table 4.8: Overview of the change in the standard deviation of the contact force for the active solution with the reference value according to passive solution. Estimator 1 is the pure $H_\infty$ estimator and Estimator 2 adds an additional part to this estimator.

<table>
<thead>
<tr>
<th>Train Speed [km/h]</th>
<th>Control Strategy</th>
<th>Mean Value [N]</th>
<th>Standard Deviation (STD) [N]</th>
<th>Change in STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 Passive</td>
<td>Estimator 1</td>
<td>72.67</td>
<td>13.68</td>
<td>-9.8%</td>
</tr>
<tr>
<td></td>
<td>Estimator 2</td>
<td>72.94</td>
<td>12.34</td>
<td>-5%</td>
</tr>
<tr>
<td>240 Passive</td>
<td>Estimator 1</td>
<td>77.27</td>
<td>19.06</td>
<td>-9.7%</td>
</tr>
<tr>
<td></td>
<td>Estimator 2</td>
<td>82.13</td>
<td>17.22</td>
<td>-1.5%</td>
</tr>
<tr>
<td>280 Passive</td>
<td>Estimator 1</td>
<td>82.57</td>
<td>18.77</td>
<td>-1.5%</td>
</tr>
<tr>
<td></td>
<td>Estimator 2</td>
<td>93.67</td>
<td>22.3</td>
<td>-8.2%</td>
</tr>
<tr>
<td>300 Passive</td>
<td>Estimator 1</td>
<td>94.1</td>
<td>23.8</td>
<td>-2.1%</td>
</tr>
<tr>
<td></td>
<td>Estimator 2</td>
<td>99.33</td>
<td>28.44</td>
<td>-6.5%</td>
</tr>
</tbody>
</table>
Active Pantograph, Configuration 4

Table 4.9 summarizes the change in standard deviation when configuration 4 was applied. Configuration 4 uses the velocity of the upper mass and multiplies this value with a constant that represents a damping with respect to an upper fixed point which is also referred as the sky. The method is, hence, also called sky hook method.

Table 4.9: Overview of the change in the standard deviation of the contact force for the active solution with the reference value according to passive solution with configuration 4.

<table>
<thead>
<tr>
<th>Train Speed [km/h]</th>
<th>Control Strategy</th>
<th>Mean Value [N]</th>
<th>Standard Deviation (STD) [N]</th>
<th>Change in STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>Passive</td>
<td>72.67</td>
<td>13.68</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sky Hook, (c_1 = 100)</td>
<td>72.96</td>
<td>14.44</td>
<td>+5.6%</td>
</tr>
<tr>
<td></td>
<td>Sky Hook, (c_1 = 200)</td>
<td>73.14</td>
<td>15.43</td>
<td>+12.8%</td>
</tr>
<tr>
<td>240</td>
<td>Passive</td>
<td>77.27</td>
<td>19.06</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sky Hook, (c_1 = 100)</td>
<td>77.35</td>
<td>19.37</td>
<td>+1.6%</td>
</tr>
<tr>
<td></td>
<td>Sky Hook, (c_1 = 200)</td>
<td>77.42</td>
<td>20.14</td>
<td>+5.7%</td>
</tr>
<tr>
<td>280</td>
<td>Passive</td>
<td>83.62</td>
<td>24.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sky Hook, (c_1 = 100)</td>
<td>83.31</td>
<td>24.9</td>
<td>+2.5%</td>
</tr>
<tr>
<td></td>
<td>Sky Hook, (c_1 = 200)</td>
<td>83.42</td>
<td>25.71</td>
<td>+5.8%</td>
</tr>
</tbody>
</table>
4.1.2 Two Pantographs

Passive Pantograph

The mean value and the standard deviation of the contact force without a controller is shown in Figure 4.9. The exact values are noted in the next section in Table 4.7.

![Figure 4.9: The mean and standard deviation values over the train speed of interest in blue and red, respectively. The blue dashed line indicates the demanded mean value and the blue dash-dotted lines the bound for this mean value. The red dashed line indicates the upper bound for the standard deviation. The left figure shows the first pantograph and the right figure the second pantograph.](image)

Active Pantograph, Configuration 1

Figure 4.10 and table 4.10 summarize the results of the multi train configuration with the same reference value as in the passive case. Figure 4.11 and table 4.11 summarize the results with the demanded reference value according to [2]. Figure 4.12 shows the force and control signal of both pantographs over one span of 60 meters. Applying the estimator at the maximal desired train speed of 280 km/h results in a standard deviation of the contact force of 22.12 N and 25.93 N for the first and second pantograph, respectively.
Figure 4.10: The mean and standard deviation values over the train speed of interest in blue and red, respectively. The blue dashed line indicates the demanded mean value and the blue dash-dotted lines the bound for this mean value. The red dashed line indicates the upper bound for the standard deviation. The dotted blue and red line indicate the passive case. A controller with the same reference value as the passive solution was applied. The left figure shows the first pantograph and the right figure the second pantograph.

Figure 4.11: A controller with the reference value according to the specifications was applied. The left figure shows the first pantograph and the right figure the second pantograph. See the above figure for a detailed explanation.
Table 4.10: Change in the standard deviation of the contact force for two pantographs in open driving direction for the same reference value as in the passive case.

<table>
<thead>
<tr>
<th>Train Speed [km/h]</th>
<th>Control Strategy</th>
<th>Mean Value [N]</th>
<th>Standard Deviation (STD) [N]</th>
<th>Change in STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>Passive, First Pantograph</td>
<td>72.93</td>
<td>13.46</td>
<td>-16.4%</td>
</tr>
<tr>
<td></td>
<td>Active, First Pantograph</td>
<td>72.69</td>
<td>11.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Passive, Second Pantograph</td>
<td>73.07</td>
<td>14.04</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active, Second Pantograph</td>
<td>72.68</td>
<td>12.64</td>
<td></td>
</tr>
<tr>
<td>220</td>
<td>Passive, First Pantograph</td>
<td>74.68</td>
<td>16.46</td>
<td>-10%</td>
</tr>
<tr>
<td></td>
<td>Active, First Pantograph</td>
<td>75.12</td>
<td>13.49</td>
<td>-18%</td>
</tr>
<tr>
<td></td>
<td>Passive, Second Pantograph</td>
<td>75.5</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active, Second Pantograph</td>
<td>75.18</td>
<td>17.95</td>
<td>-5.5%</td>
</tr>
<tr>
<td>240</td>
<td>Passive, First Pantograph</td>
<td>77.31</td>
<td>18.56</td>
<td>-9.3%</td>
</tr>
<tr>
<td></td>
<td>Active, First Pantograph</td>
<td>77.83</td>
<td>16.83</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Passive, Second Pantograph</td>
<td>78.95</td>
<td>22.09</td>
<td>-7.5%</td>
</tr>
<tr>
<td></td>
<td>Active, Second Pantograph</td>
<td>77.99</td>
<td>20.44</td>
<td></td>
</tr>
<tr>
<td>260</td>
<td>Passive, First Pantograph</td>
<td>80.03</td>
<td>22.14</td>
<td>-7.4%</td>
</tr>
<tr>
<td></td>
<td>Active, First Pantograph</td>
<td>80.8</td>
<td>20.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Passive, Second Pantograph</td>
<td>81.31</td>
<td>23.48</td>
<td>-7.8%</td>
</tr>
<tr>
<td></td>
<td>Active, Second Pantograph</td>
<td>81.71</td>
<td>21.64</td>
<td></td>
</tr>
<tr>
<td>280</td>
<td>Passive, First Pantograph</td>
<td>83.63</td>
<td>24.37</td>
<td>-8.7%</td>
</tr>
<tr>
<td></td>
<td>Active, First Pantograph</td>
<td>83.97</td>
<td>22.26</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Passive, Second Pantograph</td>
<td>84.25</td>
<td>30.68</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active, Second Pantograph</td>
<td>83.97</td>
<td>25.82</td>
<td>-15.8%</td>
</tr>
<tr>
<td>300</td>
<td>Passive, First Pantograph</td>
<td>87.34</td>
<td>30.41</td>
<td>-14.5%</td>
</tr>
<tr>
<td></td>
<td>Active, First Pantograph</td>
<td>86.98</td>
<td>26</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Passive, Second Pantograph</td>
<td>87.48</td>
<td>44.63</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active, Second Pantograph</td>
<td>87.25</td>
<td>34.23</td>
<td>-23.3%</td>
</tr>
</tbody>
</table>
Table 4.11: Change in the standard deviation of the contact force for two pantographs in open driving direction for the reference value according to the specifications.

<table>
<thead>
<tr>
<th>Train Speed [km/h]</th>
<th>Control Strategy</th>
<th>Mean Value [N]</th>
<th>Standard Deviation (STD) [N]</th>
<th>Change in STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>Passive, First Pantograph</td>
<td>72.93</td>
<td>13.46</td>
<td>+0.8%</td>
</tr>
<tr>
<td></td>
<td>Active, First Pantograph</td>
<td>94.76</td>
<td>13.57</td>
<td>+0.8%</td>
</tr>
<tr>
<td></td>
<td>Passive, Second Pantograph</td>
<td>73.07</td>
<td>14.04</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active, Second Pantograph</td>
<td>94.77</td>
<td>15.27</td>
<td>-8.8%</td>
</tr>
<tr>
<td>220</td>
<td>Passive, First Pantograph</td>
<td>74.68</td>
<td>16.46</td>
<td>-1.2%</td>
</tr>
<tr>
<td></td>
<td>Active, First Pantograph</td>
<td>95.12</td>
<td>16.26</td>
<td>+8.8%</td>
</tr>
<tr>
<td></td>
<td>Passive, Second Pantograph</td>
<td>75.6</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active, Second Pantograph</td>
<td>95.25</td>
<td>21.07</td>
<td>+10.9%</td>
</tr>
<tr>
<td>240</td>
<td>Passive, First Pantograph</td>
<td>77.31</td>
<td>18.56</td>
<td>+3.8%</td>
</tr>
<tr>
<td></td>
<td>Active, First Pantograph</td>
<td>95.57</td>
<td>19.27</td>
<td>-9.3%</td>
</tr>
<tr>
<td></td>
<td>Passive, Second Pantograph</td>
<td>78.95</td>
<td>22.09</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active, Second Pantograph</td>
<td>95.62</td>
<td>23.88</td>
<td>+8.1%</td>
</tr>
<tr>
<td>260</td>
<td>Passive, First Pantograph</td>
<td>80.03</td>
<td>22.14</td>
<td>+6.3%</td>
</tr>
<tr>
<td></td>
<td>Active, First Pantograph</td>
<td>96.11</td>
<td>21.1</td>
<td>-4.7%</td>
</tr>
<tr>
<td></td>
<td>Passive, Second Pantograph</td>
<td>81.31</td>
<td>23.48</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active, Second Pantograph</td>
<td>95.96</td>
<td>24.97</td>
<td>+6.3%</td>
</tr>
<tr>
<td>280</td>
<td>Passive, First Pantograph</td>
<td>83.63</td>
<td>24.37</td>
<td>-9.2%</td>
</tr>
<tr>
<td></td>
<td>Active, First Pantograph</td>
<td>96.46</td>
<td>22.12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Passive, Second Pantograph</td>
<td>84.25</td>
<td>30.68</td>
<td>+6.3%</td>
</tr>
<tr>
<td></td>
<td>Active, Second Pantograph</td>
<td>96.71</td>
<td>28.74</td>
<td>-6.3%</td>
</tr>
<tr>
<td>300</td>
<td>Passive, First Pantograph</td>
<td>87.34</td>
<td>30.41</td>
<td>-13.5%</td>
</tr>
<tr>
<td></td>
<td>Active, First Pantograph</td>
<td>96.85</td>
<td>26.29</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Passive, Second Pantograph</td>
<td>87.48</td>
<td>44.63</td>
<td>+17.8%</td>
</tr>
<tr>
<td></td>
<td>Active, Second Pantograph</td>
<td>97.19</td>
<td>36.68</td>
<td></td>
</tr>
</tbody>
</table>
Figure 4.12: The contact force over one span in the upper figure and the control force in the lower figure. The black and gray curves indicate the first and second pantograph, respectively. The train speed is 280 km/h.
4.1.3 Discussion

The previous section shows that the standard deviation (STD) can be reduced in the range of 7 to 17% when the same reference value as in the passive solution is taken and configuration 1 is used. The results from a previous study show similar STD reduction with the two mass model, whereas the three mass model shows STD reductions up to 60%. These results can not be achieved in the complex model used in this work.

Configuration 2 shows a STD reduction up to 22% for low speeds, but the STD is higher for higher speeds. The controller could be optimized for these higher speeds, but the solution from configuration 1 is preferred as the implementation is simpler in practice.

Due to the fact that it is not possible to measure the contact force, an estimator was designed and tested. The STD reduction is in the range of 9% which is less than with configuration 1 but this solution can be implemented in reality. The reduction with estimator 2 is smaller compared to estimator 1 even though this estimation was closer to the real contact force as shown in Section 3.2.4.

The sky hook method investigated as configuration 4 results in a higher STD than without a controller and will therefore not be further analyzed.

In the multi train configuration, the STD was reduced up to 23% for higher speeds on the second pantograph. According to the specifications, it is now possible to use such a configuration up to 280 km/h. A comparison of the controller signals shows that learning algorithm could be used. This means, that the second pantograph uses the same controller signal as the first and some further adjustments. This could be a solution to achieve a further STD reduction. A short analysis of an estimator used in the multi train configuration shows that the maximal train speed of 280 km/h can also be reached with this implementable solution.

For both cases, the single and double pantograph configurations, the STD reduction is lower if the demanded mean value \( \text{[2]} \) is reached. However, a speed of 280 km/h for two pantographs and 300 km/h for one pantograph is still possible. The root mean square (RMS) value in the range of 0-2 Hz and 0-5 Hz could be reduced up to 36%, whereas the RMS value in the range of 5-20 Hz is increased slightly for the same mean value as in the passive case. The increase can be explained by the controller action in this frequency region.
4.2 Other Pantographs

4.2.1 WBL 88 Pantograph

Table 4.12 summarizes the change in standard deviation for the WBL 88 pantograph with the standard panhead. Figure 4.13 shows the force over one span with a train speed of 200 km/h.

Figure 4.13: The contact force of the passive system (gray) and the active system (black) is shown in the upper figure. The lower force shows the control force. The train speed is 200 km/h. The mean value and the mean value ± the standard deviation are in blue and red, respectively.
Table 4.12: Overview of the change in the standard deviation of the contact force for the active solution with the WBL 88 pantograph in open driving direction.

<table>
<thead>
<tr>
<th>Train Speed [km/h]</th>
<th>Control Strategy</th>
<th>Mean Value [N]</th>
<th>Standard Deviation (STD) [N]</th>
<th>Change in STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>Passive</td>
<td>63.84</td>
<td>13.06</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>72.41</td>
<td>11.93</td>
<td>-8.7%</td>
</tr>
<tr>
<td>240</td>
<td>Passive</td>
<td>66.7</td>
<td>17.66</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>77.97</td>
<td>19.02</td>
<td>+7.7%</td>
</tr>
<tr>
<td>280</td>
<td>Passive</td>
<td>83.97</td>
<td>25.34</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>73.68</td>
<td>25.06</td>
<td>+1.1%</td>
</tr>
</tbody>
</table>
4.2.2 Pantograph Modifications

In Section 2.2.5, the following modifications of the WBL 88 pantograph with SSS400 panhead were defined:

- **Modification 1:** \( m_{1\text{,new}} = \frac{1}{2} \cdot m_1 \), \( m_{2\text{,new}} = \frac{1}{2} \cdot m_2 \), \( k_{1\text{,new}} = 2 \cdot k_1 \).
- **Modification 2:** \( m_{1\text{,new}} = \frac{1}{2} \cdot m_1 \), \( m_{2\text{,new}} = \frac{1}{2} \cdot m_2 \), \( k_{1\text{,new}} = 2 \cdot k_1 \), \( c_1 = 100\text{Ns/m} \).
- **Modification 3:** \( m_{1\text{,new}} = 2 \cdot m_1 \), \( m_{2\text{,new}} = 2 \cdot m_2 \), \( k_{1\text{,new}} = \frac{1}{2} \cdot k_1 \).
- **Modification 4:** \( m_{1\text{,new}} = 2 \cdot m_1 \), \( m_{2\text{,new}} = 2 \cdot m_2 \), \( k_{1\text{,new}} = \frac{1}{2} \cdot k_1 \), \( c_1 = 100\text{Ns/m} \).
- **Modification 5:** \( c_1 = 100\text{Ns/m} \).

Table 4.13, 4.14, 4.15, 4.16 and 4.17 show the results of modifications 1, 2, 3, 4 and 5, respectively. The reductions are always shown with respect to the solution without controller of this modification. As a comparison, the standard deviation in the passive case for the WBL 88 pantograph with SSS400 panhead is 13.68 N, 19.06N and 24.3 N for 200 km/h, 240 km/h and 280 km/h, respectively.

The passive solutions of modifications 1, 2 and 5 have a reduction in the standard deviation compared to the original configuration. Modifications 3 and 4, on the other hand, show an increase in the standard deviations for the passive and active case with respect to the original configuration.

Table 4.13: Overview of the change in the standard deviation of the contact force for the active solution with the reference value according to passive solution with modification 1.

<table>
<thead>
<tr>
<th>Train Speed [km/h]</th>
<th>Control Strategy</th>
<th>Mean Value [N]</th>
<th>Standard Deviation (STD) [N]</th>
<th>Change in STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>200 Passive</td>
<td>72.64</td>
<td>12.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Active</td>
<td>72.73</td>
<td>15.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>240 Passive</td>
<td>77.7</td>
<td>16.54</td>
<td></td>
<td>+25.2%</td>
</tr>
<tr>
<td>Active</td>
<td>77.9</td>
<td>40.77</td>
<td></td>
<td>+146.5%</td>
</tr>
<tr>
<td>280 Passive</td>
<td>83.7</td>
<td>20.06</td>
<td></td>
<td>+187%</td>
</tr>
<tr>
<td>Active</td>
<td>83.91</td>
<td>57.58</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4.14: Overview of the change in the standard deviation of the contact force for the active solution with the reference value according to passive solution with modification 2.

<table>
<thead>
<tr>
<th>Train Speed [km/h]</th>
<th>Control Strategy</th>
<th>Mean Value [N]</th>
<th>Standard Deviation (STD) [N]</th>
<th>Change in STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>Passive</td>
<td>72.58</td>
<td>11.83</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>72.73</td>
<td>12.88</td>
<td>+8.9%</td>
</tr>
<tr>
<td>240</td>
<td>Passive</td>
<td>77.45</td>
<td>16.23</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>77.91</td>
<td>24.71</td>
<td>+52.2%</td>
</tr>
<tr>
<td>280</td>
<td>Passive</td>
<td>83.61</td>
<td>20.06</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>83.91</td>
<td>38.74</td>
<td>+93.1%</td>
</tr>
</tbody>
</table>

Table 4.15: Overview of the change in the standard deviation of the contact force for the active solution with the reference value according to passive solution with modification 3.

<table>
<thead>
<tr>
<th>Train Speed [km/h]</th>
<th>Control Strategy</th>
<th>Mean Value [N]</th>
<th>Standard Deviation (STD) [N]</th>
<th>Change in STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>Passive</td>
<td>73.1</td>
<td>15.8</td>
<td>-15%</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>72.1</td>
<td>13.43</td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>Passive</td>
<td>77.84</td>
<td>24.03</td>
<td>-6.7%</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>78.07</td>
<td>22.43</td>
<td></td>
</tr>
<tr>
<td>280</td>
<td>Passive</td>
<td>84.73</td>
<td>33.08</td>
<td>-10.2%</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>83.8</td>
<td>29.69</td>
<td></td>
</tr>
</tbody>
</table>
Table 4.16: Overview of the change in the standard deviation of the contact force for the active solution with the reference value according to passive solution with modification 4.

<table>
<thead>
<tr>
<th>Train Speed [km/h]</th>
<th>Control Strategy</th>
<th>Mean Value [N]</th>
<th>Standard Deviation (STD) [N]</th>
<th>Change in STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>Passive</td>
<td>73.07</td>
<td>15.02</td>
<td>-4.9%</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>72.75</td>
<td>14.29</td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>Passive</td>
<td>78.02</td>
<td>22.24</td>
<td>-3.1%</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>78</td>
<td>21.55</td>
<td></td>
</tr>
<tr>
<td>280</td>
<td>Passive</td>
<td>84.66</td>
<td>30.73</td>
<td>-2.7%</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>83.79</td>
<td>29.91</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.17: Overview of the change in the standard deviation of the contact force for the active solution with the reference value according to passive solution with modification 5.

<table>
<thead>
<tr>
<th>Train Speed [km/h]</th>
<th>Control Strategy</th>
<th>Mean Value [N]</th>
<th>Standard Deviation (STD) [N]</th>
<th>Change in STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>Passive</td>
<td>72.59</td>
<td>13.02</td>
<td>-4.1%</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>72.74</td>
<td>12.48</td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>Passive</td>
<td>77.34</td>
<td>18.06</td>
<td>2.2%</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>77.89</td>
<td>18.46</td>
<td></td>
</tr>
<tr>
<td>280</td>
<td>Passive</td>
<td>83.79</td>
<td>23.27</td>
<td>+4.8%</td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>83.87</td>
<td>24.39</td>
<td></td>
</tr>
</tbody>
</table>
4.2.3 Discussion

Out of the five investigated modifications, no active solution results in a reduction of the standard deviation (STD) of the contact force. However, the passive solution of modifications 1 and 2 achieve more reduction than the active solution of the original configuration. The reason for the higher STD of some active solutions is the choice of weighting function. For all cases, the same weighting functions were used such that the solutions are comparable. But as the transfer function of the different modifications look different, the weighting functions should be chosen more wisely in a future analysis. The chosen weighting functions resulted in controllers that were not able to cope with the disturbances anymore and, therefore, started to oscillate.

A further investigation on these results and an optimization of these controllers may achieve a higher reduction of the STD.

4.3 Simulation Time

Without a controller, the simulation time for 10 spans varies between 2h 53min and 3h 5min. With a controller, the simulation time is between 3h 1min and 3h 40min. The simulations were performed on a HP Elitebook 8460p notebook with a 2.5-GHz Intel Core i5-2520M CPU. There was no significant difference whether one or two controllers are considered. One influence on the simulation time is the train speed as the track length is always the same. The CPU time per real time second is between 910 and 1400 seconds. The increase in simulation time from the passive to the active solution is between 4.6 and 18.9 %.
4.3. Simulation Time
Chapter 5

Conclusions

In this work, a controller was derived and implemented in a 3-dimensional pantograph catenary interaction model in the finite element program Ansys. Table 5.1 summarizes the most important results obtained in chapter 4.

The goal was the reduction of the standard deviation (STD) of the contact force between the pantograph and the catenary.

With a direct measurement of the contact force, the STD of the contact force was reduced up to 17.8 %. A possible solution that could be implemented in reality reaches a reduction of up to 9.8 %. This solution considers an estimator of the contact force as the contact force cannot be measured directly.

The reduction of the STD of the first and second pantograph in a multi train configuration for direct measurement is up to 16.4 % and 15.8 %, respectively. A top speed of 280 km/h and 300 km/h can be reached with a multi train and single train configuration, respectively. Most important, these top speeds are reached by using a contact force estimator. Hence, this is a solution that could be implemented. These top speeds can be reached by meeting the specifications for 200 km/h according to [2]. This means that it is possible to speed up the existing lines up to 280 km/h without violating these threshold values.

In most cases, other control configurations and modifications of the pantograph result in less reduction of the STD than the active solution. However, modifications 1 and 2 show a reduction of the STD without using a controller. These modifications consist of a lighter pantograph and a higher stiffness value between the masses \( k_1 \).

To sum up, two different approaches are possible to further investigate. Either the parametrization of the pantograph is changed as in modification 1 and 2 or a controller in combination with an estimator is applied to the WBL88 pantograph with SSS400 panhead.
Table 5.1: Overview of the reduction of the standard deviation of the contact force for the active solution with direct measurement of the contact force and with the contact force estimated. The reference value was chosen according to the passive solution.

<table>
<thead>
<tr>
<th>Train Speed [km/h]</th>
<th>Control Strategy</th>
<th>Mean Value [N]</th>
<th>STD [N]</th>
<th>Reduction in STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>Passive</td>
<td>72.67</td>
<td>13.68</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>72.65</td>
<td>11.24</td>
<td>-17.8 %</td>
</tr>
<tr>
<td></td>
<td>Estimator 1</td>
<td>72.94</td>
<td>12.34</td>
<td>-9.8 %</td>
</tr>
<tr>
<td>240</td>
<td>Passive</td>
<td>77.27</td>
<td>19.06</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>77.85</td>
<td>17.42</td>
<td>-10.8 %</td>
</tr>
<tr>
<td></td>
<td>Estimator 1</td>
<td>82.13</td>
<td>17.22</td>
<td>-9.7 %</td>
</tr>
<tr>
<td>280</td>
<td>Passive</td>
<td>83.62</td>
<td>24.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>83.84</td>
<td>22.3</td>
<td>-8.2 %</td>
</tr>
<tr>
<td></td>
<td>Estimator 1</td>
<td>93.67</td>
<td>22.3</td>
<td>-8.2 %</td>
</tr>
<tr>
<td>300</td>
<td>Passive</td>
<td>87.4</td>
<td>30.43</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active</td>
<td>86.98</td>
<td>26.13</td>
<td>-14.1 %</td>
</tr>
<tr>
<td></td>
<td>Estimator 1</td>
<td>99.33</td>
<td>28.44</td>
<td>-6.5 %</td>
</tr>
</tbody>
</table>

Double Pantograph

<table>
<thead>
<tr>
<th>Train Speed [km/h]</th>
<th>Control Strategy</th>
<th>Mean Value [N]</th>
<th>STD [N]</th>
<th>Reduction in STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>Passive, First Pantograph</td>
<td>72.93</td>
<td>13.46</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active, First Pantograph</td>
<td>72.69</td>
<td>11.25</td>
<td>-16.4 %</td>
</tr>
<tr>
<td></td>
<td>Passive, Second Pantograph</td>
<td>73.07</td>
<td>14.04</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active, Second Pantograph</td>
<td>72.68</td>
<td>12.64</td>
<td>-10 %</td>
</tr>
<tr>
<td>240</td>
<td>Passive, First Pantograph</td>
<td>77.31</td>
<td>18.56</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active, First Pantograph</td>
<td>77.83</td>
<td>16.83</td>
<td>-9.3 %</td>
</tr>
<tr>
<td></td>
<td>Passive, Second Pantograph</td>
<td>78.95</td>
<td>22.09</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active, Second Pantograph</td>
<td>77.89</td>
<td>20.44</td>
<td>-7.5 %</td>
</tr>
<tr>
<td>280</td>
<td>Passive, First Pantograph</td>
<td>83.63</td>
<td>24.37</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active, First Pantograph</td>
<td>83.97</td>
<td>22.26</td>
<td>-8.7 %</td>
</tr>
<tr>
<td></td>
<td>Estimator 1, First Pantograph</td>
<td>93.49</td>
<td>22.21</td>
<td>-8.9 %</td>
</tr>
<tr>
<td></td>
<td>Passive, Second Pantograph</td>
<td>84.25</td>
<td>30.68</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Active, Second Pantograph</td>
<td>83.97</td>
<td>25.82</td>
<td>-15.8 %</td>
</tr>
<tr>
<td></td>
<td>Estimator 1, Second Pantograph</td>
<td>92.02</td>
<td>25.93</td>
<td>-15.5 %</td>
</tr>
</tbody>
</table>
Chapter 6

Outlook

The previous chapter has shown that it is possible to implement such a solution. However, until the implementation more work is necessary. This chapter is split in three main parts. One focuses on the model of the catenary and the pantograph, one on the control algorithm and one on other topics such as real measurements.

6.1 Optimizing the Control Oriented Model

The model used in Ansys has been validated before. For the control oriented model (COM) the same parametrization was taken. However, a detailed frequency analysis of a real pantograph might improve the transfer function of the pantograph and lead to a more exact linear model. Hence, also the approximation of the friction elements would not be necessary any more and a higher order model could be synthesized for a better estimator result. Additionally, the higher eigenmodes in the Ansys model could be investigated. only investigated the first eigenmode.

6.2 Possible Improvements of the Controller

A $\mathcal{H}_\infty$ controller offers very good tuning possibilities for such an application. However, other possibilities are available and a further analysis might achieve better results. Mainly two possibilities are interesting. On the one hand, a feedforward controller could be designed and on the other hand, iterative learning control could be further investigated. It is important to mention, that for both types of controllers, an estimation of the pole distance is the key point for the implementation in the real system, because in reality, the pole distance varies between 55 and 65 meters.

To apply a feedforward controller, the slow dynamics of the pantograph as shown in Section needs to be considered. This method could be useful, for example, to reduce the contact force peak after a pole. Iterative learning controller (ILC) has been analyzed shortly, but the simulation results were not satisfying and a further literature research on this topic needs to be done. ILC improves the controller signal over every iteration until the control error is zero.
In [19] a good overview about ILC can be found and different references were given for ILC with uncertainties in the system. For example, [20] shows a method where an existing controller is needed and an ILC is designed similar to the method used in this work. In [21] a method is proposed that uses current error feedback and, therefore, only an ILC is needed to control the system. Uncertainties were not discussed in this work. [22] applies a method that derives the controller and the ILC together including a $\mu$-synthesis and D-K-iteration. In [23], the author introduces a method where a pure memory is used as an ILC. A controller that guarantees robust performance in terms of $\mathcal{H}_\infty$ is required.

### 6.3 Further Steps

A further step in the analysis in Ansys is the implementation of irregularities such as curves, irregular wire geometry etc. Most of these irregularities are in a lower frequency region than the droppers. Hence, it is possible to react on such disturbances with the derived controller. Furthermore, the derived controller should be tested in a real application and if the gap to zero contact force is high enough, the demanded value could be lowered that a further STD reduction could be achieved.
Appendix A

ANSYS Implementation

In this appendix, the implementation of the control algorithm into the finite element program ANSYS is introduced. A more detailed description on the code can be found in appendix [B]. The model of the catenary and the pantograph has already been used in a previous work [3]. In this work, the additional implementation of a controller is carried out. The simulations are performed in ANSYS because of the catenary having many nonlinear elements that can not be represented in other simulation environments such as Matlab and Simulink.

The code is written in a way, that the controller design is done in Matlab and then the necessary code fragments are written into the Ansys file. There are methods available, where the controller is running in Simulink and Ansys is called in every time step. However, if several simulations of more than three hours should be performed together, it is more suitable to write the code into an Ansys file and start these files separately. Hence, the file could also be sent to a cluster and the user's computer would not be needed.

A.1 Important Commands

This section shows the important commands needed to implement a controller in ANSYS. See also [24] and [25] for a similar introduction about the implementation of a controller in ANsys.

A.1.1 Variable Definition

First, the correct pantograph has to be defined. It is important to say, that the mass definition taken in the ANSYS file does not affect the actual mass. The actual mass is calculated over the additional point mass $m_{11p}$ and the crossover area $A$ that is related to $m_{11b}$. From the equation for the moment of inertia and the sum of the masses

\begin{align}
J &= m_{11b} \cdot \frac{l^2}{12} + 2 \cdot m_{11p} \cdot d^2 \\
&\quad \text{(A.1)} \\
m_{\text{tot}} &= m_{11b} + 2 \cdot m_{11p} \\
&\quad \text{(A.2)}
\end{align}
Table A.1: Overview of the methods to define variables in Ansys

<table>
<thead>
<tr>
<th>Variable Type</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scalar</td>
<td>(a = 0)</td>
</tr>
<tr>
<td>Vector</td>
<td>*DIM,B,ARRAY,2,1</td>
</tr>
<tr>
<td></td>
<td>(B(1) = 1, 2)</td>
</tr>
<tr>
<td></td>
<td>*DIM,C,ARRAY,1,2</td>
</tr>
<tr>
<td></td>
<td>(C(1) = 1)</td>
</tr>
<tr>
<td></td>
<td>(C(2) = 2)</td>
</tr>
<tr>
<td>Matrix</td>
<td>*DIM,D,ARRAY,2,2</td>
</tr>
<tr>
<td></td>
<td>(D(1) = 1, 2)</td>
</tr>
<tr>
<td></td>
<td>(D(2) = 3, 4)</td>
</tr>
</tbody>
</table>

The following procedure is recommended to calculate all necessary parameters in the definition part of the ANSYS file:

\[
m_{11b} = \frac{-m_{tot} \cdot d^2 + J}{\frac{d^2}{12} - d^2} \quad (A.3)
\]

\[
m_{11p} = \frac{m_{tot} - m_{11b}}{2} \quad (A.4)
\]

\[
A = \frac{m_{11b}}{\rho \cdot l} \quad (A.5)
\]

\[
t_{11} = \sqrt{A} \quad (A.6)
\]

A value for the moment of inertia is needed for this calculations. From previous measurement campaigns, a value of 1.5 kgm\(^2\) was obtained. Additionally, the total length of the pantograph \(l = 1.6m\) and the distance to the point masses, \(m_{11p}, d = 0.6m\) as well as the density \(\rho = 7800\text{kg/m}^3\) of steel were given. However, if the moment of inertia is not known for a new parametrization it is more suitable to assume the same point masses and calculate a new moment of inertia. Otherwise the mass \(m_{11b}\) tends to zero.

Later, the vectors and the matrix needs to be initialized such that the program later knows what size this variable has. It is not necessary to predefine scalar variables but it can help the future users to get a better overview of the code. Table [A.1] shows the different methods to define a scalar, a vector and a matrices and Eq. (A.7) shows the result of these methods.

\[
a = 0
\]

\[
B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}
\]

\[
C = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}
\]

\[
D = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}
\]  

(A.7)
A.1.2 Save Data to a File

In order to analyze the results in Matlab, the obtained data can be stored in a csv-file. The file is initialized with the commands:
*CFOPEN,FILENAME,csv
*VWRITE var1, var2, var3

Where FILENAME is any name that can be chosen. This file must not exist before. Var1, var2 and var3 represent any variable that can be named here. This variable will be written on the first line of the file as the title. It is not necessary that it is the name of the variable that is actually stored in this line with the command:
*VWRITE,a,b,c
%0.0f,%0.0f,%0.0f

Where a,b,c are variables that have a specified value at this time in the code. The second line represents the way the number is stored. In this case it is a stored as a float variable. It is also possible to store a specific value of a vector, for example, B(1,1).

At the end of the program, the following line closes the csv file and makes it available for postprocessing:
*CFCLOSE

A.1.3 Matrix Calculations

Whereas scalar variables can be handled easily in the following way
\[ a = a + 5^a \]
the summation and multiplication need a special command:
Summation:  *VOPER,E(1),B(1),ADD,B(1)
Multiplication:  *MOPER,F(1),D(1),MULT,E(1)

Where B and D are the vector and matrix defined above ad E and F needs to be defined before this command is applied.

A.1.4 Do-Loop

Similar to the for-loop in Matlab and other programing languages, a DO-loop can be applied in Ansys for the same purpose. In this example, the time is increased by one time step with each new loop to guarantee the discrete behaviour:
*DO, t, Tstart, Tstop, Tstep
  Time,t
  definitions, calculations, etc.
  SOLVE
  *ENDDO

where t is the variable that will be increased by Tstep from Tstart to Tstop.
A.1.5 Reading Values of the Contact force, Acceleration and Position

In this application, the contact force, the acceleration and the position values were needed. The contact force of a contact element can be obtained with the command:

\*[GET,a,ELEM,11011,NMISC,44

The acceleration of a node can be obtained with the command:

\*[GET,b,NODE,10113,A,Y

And the velocity of a node can be obtained with the command:

\*[GET,c,NODE,10113,V,Y

And the position of a node can be obtained with the command:

\*[GET,d,NODE,10113,U,Y

Where a, b, c and d are the variable the value is stored in, 11011 is the element number of the contact element, 10113 is the node number of the node the position, velocity or acceleration should be taken from, Y indicates the direction of the acceleration, velocity or position.

In order to obtain the acceleration of the nodes, the following setting had to be changed:

\[\text{OUTRES, ALL, NONE}\]
to

\[\text{OUTRES, ALL, ALL}\]

The result file can be up to 15 GB with this option. Therefore, it is also possible to use the following code to only save all results of a specific node:

\[\text{nsel,s,,10113}\]
\[\text{CM,var1,NODE}\]
\[\text{OUTRES,A,ALL,var1}\]

where var1 is the name of this node group where the node 10113 is now stored in.

There are more contact elements defined that actually are necessary for the zick-zack of 300 mm. But it is important to consider them for a later implementation of disturbances such as crosswind, curves etc.

A.2 Communication between Matlab and Ansys

In order to write the matrices from Matlab into the Ansys file, they need to be converted to strings with the Matlab command num2str and then it is possible to write them into the Ansys file with the command fprintf and some additional settings. The program to do so was provided by [26].

When a matrix is converted to a string, it is important to remove the empty spaces and have only one comma between the entries. This is done in an input check before the actual program is called.

A.3 Postprocessing the Data from Ansys

The only drawback of Ansys is introduction of higher frequency terms due to the used contact elements between the pantograph and the catenary. Therefore, the force signal needs to be filtered with 20 Hz or averaged over the last 10 time steps which represents roughly one meter depending on the train speed.
Appendix B

Manual for the Interaction between Matlab and ANSYS

In this appendix, the main files for the interaction between Matlab and Ansys are explained.

B.1 Loading a Controller to Ansys

With the Matlab file ‘master.m’ all the necessary settings are written to an Ansys file. In this file, the following settings are possible:

- Train speed
- Length of the measurement
- Pantograph type
- Single or multi train configuration
- Active or passive solution
- Using the direct measurement of the contact force or an estimation for the controller
- Reference value for the controller
- Choose the working register
After these settings, the file can be executed and the following procedure is necessary to start the Ansys simulation:

- Start the program 'Ansyl Product Launcher'
- Choose 'Ansyl Batch'
- Choose the working register 'AnsysX' or 'Ansys_MultiX', where X is a number between 1 and 3
- Choose the input file 'AnsysX\LargerModel\V11_3D_analysis.dat' or 'Ansys_MultiX\LargerModel\V11_3D_analysis_multi.dat'
- Press Execute

B.2 Analyze the Results obtained with Ansys

The result files are stored in the working register and should be copied to the Matlab folder structure. There two different files are available for the analysis of the passive or active solution. In these files, similar settings as in the 'master.m' file can be set.


