Acoustically driven particle transport in fluid chambers

Author(s):
Möller, Dirk Björn

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Acoustically Driven Particle Transport in Fluid Chambers

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DIRK BJÖRN MÖLLER
MSc ETH Masch.-Ing.
born July 26, 1982
citizen of Zurich (Switzerland) and Germany

accepted on the recommendation of
Prof. Dr. J. Dual, examiner
Prof. Dr. T. Rösgen, co-examiner

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Abstract

This work models and experimentally investigates the transport of suspended micro-
particles with two acoustic methods. One method is continuous frequency sweeping based
on acoustic radiation forces. The other method uses fluid flow generated by acoustic
streaming. Furthermore, particle traps are presented which are based on single frequency
acoustic radiation forces and can be combined with the presented acoustic streaming de-
vices. Continuous frequency sweeping makes use of the shift of pressure nodes of standing
waves when the frequency is changed. Acoustic radiation forces arise as non-linear ef-
fects when sound waves are scattered due to the presence of a particle and hence the
method is ‘contactless’. Acoustic streaming is a net mean fluid flow generated by acoustic
oscillations. This non-linear effect can be divided into bulk attenuation driven acoustic
streaming due to the spatial attenuation of acoustic oscillations in free space, and bound-
ary layer driven acoustic streaming due to the friction between the oscillating medium
and a solid boundary.

Particles or beads with a specific surface affinity to a target substance are often used for
purification. This is of a particular interest for point of care or lab on a chip devices
which aim at reducing the size of the analysis apparatus, the analysis chip, or both. One
application followed here is nucleic acid concentration. The requirements for this appli-
cation are a closed system, disposable devices and sample volumes in the mL range. The
investigated ultrasonic devices are mm-sized and mainly fabricated from plastics.

Particle transport with continuous frequency sweeping has been described experimen-
tally in literature, but without providing a physical model. In this work continuous
frequency sweeping is described with a one dimensional analytical model. Two device
designs are proposed to employ the method. The first is a planar square device geometry
with a transducer placed in line with the particle transport direction, and the second is
a planar device where the transducer is placed perpendicular to it. The latter is used
to investigate wave coupling mechanisms. The devices are investigated numerically with
a simulation of the pressure field. In addition, a particle tracing simulation with time
dependent frequency is developed. Particle experiments are combined with biochemical
experiments and a simple schlieren visualization setup is designed which can visualize the
full pressure field in real time. In this work, it is demonstrated that continuous frequency
sweeping relies on attenuated standing waves which are present at all frequencies in a
cavity, the shift of pressure nodes of any off-resonance standing wave depends on the
position of the excitation and particles can be moved from one side of a cavity to another
and back. Moreover, averaging effects are important for particle transport with a high
particle collection yield. Some of the most critical parameters are the total attenuation of the system, the used frequency ranges, and the frequency sweep rate. Furthermore, particle properties are important and it is advantageous to avoid sedimentation. For good performance, a rigid wave reflector plate and direct coupling of waves are more important than plate vibrations. The presented model, simulations and experiments are in agreement and proof is given, that the method can be used to concentrate DNA.

Bulk attenuation and boundary layer driven acoustic streaming are investigated with numerical simulations optimized for each type. Particle image velocimetry is used to measure the velocity field of attenuation driven acoustic streaming devices experimentally. For this type of streaming and rectangular devices, velocity fields with a flow away from the transducer, as known from literature, are reported as well as less intuitive fields with a flow towards the transducer. These flows can be explained with multiple wave reflections which appear with a long characteristic attenuation length. For the same type of streaming ring type chambers are proposed and developed which have a side channel with a constant fluid flow decoupled from the acoustic field. Geometry changes can be used to guide wave reflections and efficiently produce bulk flow with acoustic streaming. The presented numerical simulation of boundary driven acoustic streaming with an oscillating wall has a high convergence and is in agreement with an analytical solution given in literature.


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Introduction

This first chapter starts with a motivation and background information for the thesis. The aim and scope of the thesis is presented in Section 1.2 and an outline of the chapters of this thesis is given in Section 1.3.

1.1 Background and Motivation

Particle manipulation techniques for suspended micro-particles are applied in life sciences and have a large number of emerging applications. These include applications in sorting, concentration, separation, rotation, analysis, automatization and cell handling amongst others [97, 151, 123, 34]. Beads are often used in molecular biology [163]. Examples are glass particles for nucleic acid (DNA) purification or any sort of particles with functionalized surfaces. A specific particle surface affinity to a target substance can be used for DNA, RNA and protein extraction, purification or isolation and concentration of the target. These bio-molecular applications often require a closed system to avoid sample contamination.

Manipulation techniques can be divided into three categories. One category is large scale techniques which rely on gravity dependent characteristics or mechanical contact. Examples are sedimentation, flocculation, or centrifugation and filtration. These are the classic manipulation techniques. Another category is particle manipulation with external fields. These techniques are suitable for single particle manipulation and do no rely on fluid flow. Examples are optical, electrical, magnetic and acoustic principles. A third category is associated to hydrodynamic mechanisms. Examples are cross stream migration and hydrodynamic filtration, or vorticity induced trapping which includes acoustic streaming.

Classic Particle Manipulation

Classic methods such as sedimentation, centrifugation and filtration are particularly suited for large scale applications, but typically give less control over the manipulation of single particles. 
Sedimentation has the advantage of simplicity and is inexpensive to set up. However, the
method can be slow, or even rendered useless if drag forces or Brownian motion dominate over buoyancy or gravitational forces, particularly so, if turbulence is present in the suspending medium.

Centrifugation makes use of centrifugal forces and is able to increase the rate of sedimentation by orders of magnitudes. While most centrifuges are bulky, even for microfluidics applications [53], there are also attempts to miniaturize them [105]. Main limitations remain either with moving parts, or with external driven fluid flow.

Filtration is a mechanical or physical separation method. It builds up on fluid flow or moving parts. Advantages are simplicity and potential high contact between suspension and filter medium. This makes filtration particularly useful for biomedical or chemical applications. Limitations are immobilization of the filtrate, reduced efficiency when particles start to build up and form a filter cake, or potential damage to the filtrate due to the physical nature of the method.

Particle Manipulation with External Fields

Non-contact techniques to manipulate particles include optical, electrical, magnetic and acoustic principles. These techniques can be operated with zero or near zero net fluid flow.

Optical tweezers are a well establish method to trap and manipulate micro-particles with highly focused beams of light [3]. Particles trapped in the beam are in balance between attractive optical gradient forces and repulsive scattering or radiation pressure forces which push the particles along the beam. The attractive forces depend on the particle properties such as refractive index and the profile or incident angle of the trapping light beam. Optical tweezers have been used to trap dielectric spheres, cells and metal particles amongst others [56]. They offer to measure with high precision and are ideal to exert forces on biological samples and measure their response. On the downside optical tweezers can potentially cause damage to the sample, for example by heating, and rather complex and expensive setups are required which are hard to miniaturize.

Dielectrophoresis (DEP) is a phenomenon in which an electrical field gradient is used to induce a dipole moment in a suspended particle [132]. A particle polarization which is higher than that of the suspending medium leads to attractive forces towards higher field strength. This is called positive DEP. Negative DEP refers to the case of lower polarization of the particle and repulsive forces. DEP is well suited for size fractionation since the force scales with the volume of the particle. Potential drawbacks are dependencies on medium conductivity and particle polarizability or current induced heating.

Optical tweezers and dielectrophoresis can be combined to build optoelectronic tweezers. This significantly reduces power consumption of optical tweezers and enables massively parallel manipulation [20].

Magnetic forces act on particles in a magnetic field gradient. These can be ferromagnetic, paramagnetic or diamagnetic particles. Typically superparamagnetic nanometer sized particles are used which have no magnetic memory [127]. Particles in the micrometer range usually show hysteretic magnetization behavior. Paramagnetic particles experience an attractive force towards the magnetic field maxima. Advantages are relatively high
forces in a short range and the manipulation of a large number of particles. On the other hand the force field is typically highly inhomogeneous.

Acoustic or ultrasonic particle trapping, also known as acoustophoresis, makes use of acoustic radiation forces which act on particles in an acoustic field. This field typically is associated to a standing wave field which exhibits large pressure amplitudes. Particles with a higher density and speed of sound than the suspending fluid typically collect in the pressure nodes, while particles with lower density and speed of sound typically collect in the pressure antinodes [59, 16]. The method has some unique advantages and bypasses some of the drawbacks inherent to the other methods. It is contactless and allows simultaneous and synchronous manipulation of a large number of particles with periodic force fields. With bulk waves the method is relatively inexpensive and simple in setup and operation. It is physically non-destructive, chemically inert and can be applied to both microfluidic systems and large scale systems [71, 121]. Acoustophoresis has a high cell viability which allows to manipulate cells for hours or days without [185]. The method works for most particles regardless of their optical, electrical or magnetic properties [121, 30]. The method can be combined with other methods [50].

Hydrodynamic Mechanisms

Different hydrodynamic mechanisms have been used for particle manipulation. These include pinched flow fractionation, hydrodynamic filtration and methods which can be summarized in two families, the cross stream migration based methods and vortical flow methods. A detailed review of these techniques can be found in recent reviews [77, 96] and are summarized here.

Pinched flow fractionation and hydrodynamic filtration have been used for size fractionation [96]. The former is based on a system with particle suspension flow and pure liquid combined in a pinched segment. The latter makes use of gradual outflow through side capillaries or channels.

Cross stream migration of suspended particles in confined flow refers to the lift force that particles experience when exposed to a shear flow. The force is perpendicular to the flow direction and dependent on a carrier fluid and particles properties as well as on the flow field. This includes inertial effects, viscoelastic focusing, deformability-selective cell separation and dean flow [77]. Inertial effects are utilized in microfluidic channels with focusing, pinching and extraction regions. Amongst others, this allows size fractionation, or to concentrate particles in a specific region of the channel. Viscoelastic focusing has been used to move particles towards the walls in the flow of shear-thinning liquid and towards the centerline with elastic fluid. Dean flow uses the mismatch of velocities in curved channels to create symmetrical vortices. Combined with inertial and secondary flow effects this has been used for size fractionation or to concentrate particles in the flow. Vorticity can be used to collect particles from a large range and trap them with short range forces. This allows to trap, release and sort particles. Techniques used include confined vortices where particles are pushed towards the center of the vortex in a confinement region, stagnation points between vortices, or the combination with secondary force fields [103]. Methods to produce vortices include geometrical channel modifications, electro-
osmosis, dielectrophoresis, and acoustic streaming [77]. Applications include trapping of DNA with AC electro-osmosis [32], or with a stagnation region [88]. The methods can be of particular interest for the detection and analysis of samples with low diffusivity, such as of DNA. The Brownian transport to a localized sensor surface can be enhanced with vorticity induced trapping [103].

Acoustic streaming is a net mean fluid flow generated by sound. The method allows to produce vortices with high velocities close to a boundary or wall. This is mainly achieved with boundary layer driven acoustic streaming. In addition a flow system can be driven with attenuation driven acoustic streaming [99, 38]. The method is well suited to be combined with other methods and can drive fluid flow with long and short range forces. It can be used for microscale and large scale systems and is bio-compatible even though shear stresses from a fluid flow can cause membrane rupture and cell lysis in high velocity gradients [185, 42]. One of the main fields of applications is enhancement of convective heat transfer [176]. Other fields are micro mixing [173], ultrasonic cleaning, micro pumps [140, 144] or measuring absorption coefficients [131]. These areas of applications arise from the ability of acoustic streaming to overcome the smooth or laminar flow which dominates in microfluidics. In addition when handling small volumes mechanically moving parts can be inapplicable, this is where the contactless driving mechanism comes into play. A main disadvantage is the rather low efficiency which is also due to low efficiencies of ultrasonic transducers. For a more detailed overview of applications the reader is referred to [171, 186].

Nucleic Acid Concentration

The direct detection of genetic material of a virus or bacterium can shorten the window period between first infection and reliable detection of the infection. Nucleic acid testing (NAT) includes any of various techniques in molecular biology which allow the direct and sequence-specific detection of nucleic acid. NAT is of interest for many infectious diseases. An often targeted example is tuberculosis. A more detailed list with examples and technical considerations of NAT can be found in [21]. NAT provides potentially rapid methods for diagnosis, speciation and drug-susceptibility assessment [76]. It has a high analytical sensitivity compared to other tests based on immune reactions, virus isolation, or cell culture [29]. Today, NAT is mainly performed by skilled personnel using bulky instrumentation in centralized laboratories [115].

On the other hand, point of care (POC) testing which is defined as medical testing at or near the site of a patient is advantageous when a rapid answer is required. POC devices can thus aim at reducing the need for disposable laboratory supplies, and trained personnel. This is particularly of interest for low resource settings such as in the developing world where NAT shows a large potential, and might be a crucial element for complex situations like the diagnosis of tuberculosis of HIV-positive individuals [76]. The flow process towards NA analysis includes many process steps which can strongly vary in order and number. They can be summarized as sample preparation, amplification, and detection. It starts with the collection of samples, e.g. whole blood or urine. Further steps include extraction where the DNA is released from the cell, separation,
isolation and concentration as well as quantitation and amplification. Final steps are analysis and interpretation. The integration of sample preparation up to separation and concentration in a small in-field device or POC-NAT is a weak link [107, 29]. Examples of sample preparation techniques in POC-NAT systems are affinity based target capture and sonication, magnetic silica particle-based extraction, solid-phase nucleic acid extraction, and column- or filter-based methods [29, 115, 9]. The extraction or concentration of nucleic acids for nucleic acid-based assays is a crucial step in the sample preparation. In view of thermal cycling as used for polymerase chain reaction (PCR) based methods small volumes containing as much of the desired genetic material as possible is often the goal. The most consistent and efficient procedures have a solid-phase extraction with silica or glass in the presence of a chaotropic reagent. Automated POC-NAT systems are mainly based on silica coated magnetic beads. These modern systems have total analysis times of less than \(45\) min [115].

The concentration or availability of biomarkers in the collected sample can vary significantly. This also depends on the sample type, where e.g. urine typically is a large volume sample with low concentration while blood has much higher concentrations of biomarkers. The final concentrated sample volume can be in the \(\mu\)L range, whereas the original sample can have several mL. This can be a limiting factor for microfluidic systems [115, 11].

1.2 Aim and Scope

The aim of this thesis is to investigate and realize acoustic particle transport methods which are suitable for applications like DNA concentration. The focus is on two methods. The first is continuous frequency sweeping which is an acoustic radiation force based method. The second method is acoustic streaming which is used to transport particles with drag forces. The goal of DNA concentration builds up on the idea of beads as target for biomarker adsorption and the transport of these beads. The advantages of acoustic methods shall be combined with the requirements towards an ideal nucleic acid extraction technology for a low resource setting or a point of care device. These requirements include minimal processing steps, being self-contained to avoid sample contamination during processing, and to enable the use of large samples volumes of at least 1 mL. The idea of a disposable low cost device makes plastics particularly interesting as a device target material. The majority of ultrasonic particle manipulation devices can be found in the macro-[71] and micro-scale range [119, 97, 47, 40]. Here the focus is on the intermediate mm-scale range [62, 23]. The literature on ultrasonic particle manipulation devices which use plastics is rather limited. Examples are large scale devices [164] and smaller devices which are either only partly made out of plastics [51] or have channels worked into bulk material [52]. Here the focus is on thin-walled, disposable devices. Typically ultrasonic particle concentration devices use a continuous flow system with an externally driven flow [97, 40]. The focus is on closed systems here in contrast. Both effects, acoustic radiation forces and acoustic streaming cope with this feature.
Acoustic Radiation Force Driven Particle Transport

This work follows up the work on acoustic radiation forces in Prof. Dual’s group. This includes the work on mm-sized half open glass-metal devices by Haake [62] and silicon micro-machined devices by Oberti [119].

The goal is to find and investigate an acoustic radiation force based particle transport method which fulfills the previously mentioned requirements, such as a closed system and a disposable device in the mm-scale. A well suited method is continuous frequency sweeping which is investigated in more detail. Literature shows several reports of systems which operate with, or methods similar to continuous frequency sweeping [64, 164, 184, 153, 106]. However, these reports partly address the observations of other mechanisms like the use of beat frequencies or resonant mode switching and or lack a detailed explanation. Lipkens et.al. [101] gives a two-dimensional theoretical model for particle trajectories and provides basic investigations of continuous frequency sweeping. Here a goal is to provide an analytical discussion of the mechanism which provides detailed particle paths. Furthermore, numerical models are presented which can describe systems operating with continuous frequency sweeping. This includes 1D, 2D, and 3D modeling of acoustic fields, and the establishment of a method suitable to describe the system behavior over a large frequency range. Most numerical models of acoustophoretic devices found in literature focus on microfluidics systems which operate with a few wavelengths and without utilizing time dependent frequency changes [121, 151, 112, 55]. In contrast, here the focus is on multi-wavelength systems with time dependent frequency changes. This includes the implementation of a suitable particle tracing simulation.

A further goal is to propose and test continuous frequency sweeping systems in experiment. An aim of these experiments is a verification of the analytical and numerical models. This includes common particle experiments as well as the aim to show whether such a proposed system can be used to concentrate DNA. The experimental work also includes the goal to find and realize methods suitable to investigate the time dependent pressure field which is given with schlieren visualization.

The goal to propose and realize an acoustophoretic DNA concentration device includes the investigation towards general device design criteria. These design criteria include a variation of device materials, geometrical changes, and an understanding of device vibrations and wave coupling.

Acoustic Streaming Driven Particle Transport

The goal is to investigate acoustic streaming in closed, mm-sized plastic chambers. An aim is to utilize attenuation driven acoustic streaming with low power consumption compared to conventional, high power ultrasonic horn transducer configurations. With this background the goal is to propose and investigate acoustic streaming devices which can be combined with a particle trap to concentrate particles. This includes an experimental investigation as well as numerical models. The focus of the numerical models is an efficient implementation which is able to simulate multi-wavelength attenuation driven acoustic streaming in 2D and includes the acoustic interaction with the structure. Another goal is to present and compare a boundary layer driven acoustic streaming simulation, including
1.3. Outline

Oscillating boundaries, based on well established theory [117, 136, 99, 142, 175].
The investigations of acoustic streaming includes different flow patterns in rectangular chambers and a variation of different geometries with a focus on ring type chambers. The goal of particle concentration with acoustic streaming in mm-sized chambers includes the investigation of particle traps suitable for this task. In literature, particle traps are typically in the micro- or macro-scale and are realized with materials such as glass or silica [40] while the focus is on plastics here.

1.3 Outline

In Chapter 2, the theoretical background is presented. This covers primary and secondary acoustic radiation forces. In addition acoustic streaming is introduced with an overview of the topic and a discussion of the governing equations, reference systems and boundary conditions. A section about attenuation and damping sets different attenuation parameters and mechanisms into context. Finally the acoustic impedance and impedance matching is briefly covered.

In Chapter 3, experimental and numerical methods are described. It starts with a description of the numerical simulation which includes a linear acoustics and structural system model, a part describing a particle tracing model and concludes with a numerical model of acoustic streaming. In the second part the basic experimental setup is described which is used for different acoustic devices. This part also covers the fabrication of the acoustic devices. In a further section the schlieren visualization method is described and a vertical schlieren setup is presented. The chapter concludes with a description of a particle image velocimetry setup.

In Chapter 4, first particle transport methods which rely on acoustic radiation forces are summarized. Out of these, the focus is on continuous frequency sweeping which is discussed next with a 1D analytical model. The next section is a core topic of the thesis and covers a square mm-sized acoustic chamber which is used in combination with continuous frequency sweeping. This section starts with a system description and is followed with numerical models such as an isoline description, a discussion of pressure fields and particle tracing simulations. In a final part of this section particle experiments, schlieren visualizations and DNA concentration experiments are presented. The chapter concludes with a section about wave coupling mechanisms which are discussed with a planar device with numerical simulations as well as with experimental work.

In Chapter 5, aspects of acoustic streaming driven particle transport are presented. This starts with a discussion of bulk attenuation driven acoustic streaming in mm-sized plastic chambers. The streaming in different device geometries is covered. Geometries are rectangular chambers, the square chamber introduced in the previous chapter and a variation of more complex geometries. In addition, ring type chambers are presented which can be combined with ultrasonic particle traps. The chapter concludes with a discussion of boundary layer driven acoustic streaming between two concentric cylinders.

In Chapter 6, conclusions are presented and the main contributions of this thesis are summarized. Finally, some of the remaining open questions are discussed in an outlook.
Theoretical Background

In this chapter the theoretical background is given for the treatment of acoustic fields and the non-linear acoustic effects of acoustic radiation force and acoustic streaming. This is mainly a collection of well established textbook material, supplemented with a short literature survey. In Section 2.1 the governing equations of hydrodynamics are presented. This includes scaling and dimensionless numbers and the division of the problem into a first and second order problem which is particularly of interest for acoustic streaming. The different boundary conditions and a discussion of the reference system are presented in Section 2.2. The effect of the acoustic radiation force is introduced in more detail in Section 2.3. The effects of acoustic streaming is introduced in Section 2.4. In Section 2.5 different concepts of attenuation and damping and the most recent literature of attenuation in water and some plastics is given. The above mentioned theory is important for the numerical simulations presented in this thesis. The specific acoustic impedance is a continuum concept presented in Section 2.6, and mainly serves as basis for the discussion of the acoustic device design.

2.1 Governing Equations

The equations presented here start with the general equations of hydrodynamics and follow the essential idea of Rayleigh’s treatment of the problem which is dividing the problem into a first and second order problem. This division is an established procedure and is well presented by e.g. Eckart [38], Nyborg [118], or Riley [142] and is followed here.

The fluid is assumed to be isotropic and Newtonian and the density, pressure and velocity are given in Eulerian description $\rho(x,t)$, $p(x,t)$, $v(x,t)$. The fluid motion is generally governed by the equation of continuity and the Navier-Stokes (NS) equations, see e.g. [89]. They are presented here in vector form with expanded material derivatives and without external body forces as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0$$

(2.1)
\[
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \mu \nabla^2 \mathbf{v} + \left( \frac{\mu}{3} + \mu_B \right) \nabla \nabla \cdot \mathbf{v},
\]

(2.2)

where \( \mu \) and \( \mu_B \) are the dynamic and bulk viscosity, respectively. Physically the terms on the left hand side of the NS equations represent inertia with unsteady and convective acceleration. The effect of stress in the fluid is given on the right hand side with pressure gradient and viscous forces. The latter are divided into a term describing shear effects and one describing the rate of expansion in a compressible fluid. For temperature effects this set of equations can be supplemented with the energy equation, however, all processes are assumed to be isothermal here.

Scaling and Dimensionless Numbers

The general equations of fluid dynamics describe a variety of phenomena. Their individual contribution for a given problem can vary significantly and must be judged against each other. Dimensionless numbers allow to compare the individual contributions.

For viscous fluids a difference in oscillations between a wall and the fluid generates time harmonic vorticity due to the no-slip boundary condition. The time harmonic part of the vorticity diffuses away from the boundary confining wall effects to a boundary layer. This boundary layer is called acoustic or Stokes boundary layer and is of size

\[
\delta \sim \sqrt{\frac{2\nu}{\omega}},
\]

(2.3)

where \( \nu = \mu/\rho \) is the kinematic viscosity. Outside of this layer the unsteady vorticity decays exponentially which basically leaves an irrotational outer region.

The Reynolds number is a dimensionless number that gives a measure of the ratio of inertial forces to viscous forces and is given by

\[
Re = \frac{vR}{\nu},
\]

(2.4)

where \( v \) and \( R \) are characteristic flow velocity and length, respectively. For boundary layer driven streaming and curved boundaries the order of \( R \) is given by the wall curvature radius and for attenuation driven streaming \( R \sim 1/\alpha_s \), where \( \alpha_s \) is the spatial attenuation coefficient discussed in more detail in Section 2.5. The steady inertial force and viscous forces are in balance which results in a velocity scale for the steady streaming \( v \sim U_0^2/\omega R \) [158] and a streaming Reynolds number [159] can be formulated

\[
Re_s = \frac{U_0^2}{\omega \nu},
\]

(2.5)

where \( U_0 \) is the amplitude of the velocity oscillations or in other words the characteristic particle velocity amplitude of the sound field. In the micro scale and for high frequencies \( Re_s \ll 1 \) typically holds true resulting in Stokes flow or the RNW (Rayleigh, Nyborg and Westervelt) streaming regime. For high power acoustic beams, in contrast, the streaming is at higher Reynolds numbers and the convective terms must be included. The latter
streaming is sometimes called Stuart streaming. A more detailed discussion of acoustic streaming regimes is given in Section 2.4.

The Strouhal Number $Sr$ characterizes the frequency and is given by

$$Sr = \frac{R \omega}{U_0} = \frac{1}{\varepsilon},$$

(2.6)

where the inverse of $Sr$ is the parameter $\varepsilon$, used as perturbation parameter. Thus $\varepsilon \ll 1$ when amplitudes of oscillations are small compared with $R$.

The Womersley number $M$ is a large frequency parameter defined by the relation

$$M^2 = \frac{\omega R^2}{\nu} = Re \cdot Sr$$

(2.7)

and expresses the ratio of the transient or oscillatory inertia force to the shear force. The vorticity is confined to the Stokes boundary layer (2.3) for $M \gg 1$. In contrast when $M$ is small or $M \ll 1$ the vorticity is no longer confined to a thin shear wave layer. The latter is an uncommon case which also leads to a more elaborate theoretical treatment [141]. As a good approximation the streaming velocity at the outer edge of the Stokes layer can be used as slip velocity for the outer streaming for $M \gg 1$ [146].

The Mach number given by

$$Ma = \frac{U_0}{c_0}$$

(2.8)

is a measure for the relative density variation and is proportional to a dimensionless wave number $kR$ over $Sr$. A more detailed discussion of the dimensionless parameters can be found in the pioneering work of Stuart [159] or in [177, 142, 158].

For the discussion of the governing equations thermal effects are usually neglected. These can include the non-adiabatic case and convection as well as thermo-viscous effects [139, 143]. The thermal boundary layer and the Prandtl ($Pr$) number defined as the ratio between viscous and thermal diffusion rate can give some insight into the thermal contributions. Rednikov and Sadhal [137] theoretically investigated the influence of thermo-viscous effects in the non-adiabatic case for streaming near a motionless boundary and Muller et.al. [112] recently presented a numerical study taking thermo-viscous effects into account.

**Perturbation**

Between the time scale of the sound field and that of the time averaged effect acoustic streaming is a large gap. For ultrasound this is typically micro-seconds versus seconds. Solutions for both cases can be obtained with the perturbation technique. A more detailed account of this technique and the NS equations is given in [15, 146]. This technique is similar to the older method of successive approximations (Stokes perturbation) often used in the literature e.g. by Nyborg [118], but in addition provides means for suppressing resonant forcing. With a perturbation parameter $\varepsilon \ll 1$ the flow parameters can be
expanded up to second order into

\[
\begin{align*}
\rho &= \rho_0 + \varepsilon^1 \rho_1 + \varepsilon^2 \rho_2 \\
p &= p_0 + \varepsilon^1 p_1 + \varepsilon^2 p_2 \\
v &= v_0 + \varepsilon^1 v_1 + \varepsilon^2 v_2
\end{align*}
\]  

(2.9)

where the free or unperturbed solution is indicated with the subscript ‘0’. Suitable values for the perturbation parameter \(\varepsilon\) are the acoustic Mach number (2.8) or the inverse Strouhal number (2.6). The system is initially at rest and therefore the fluid particle velocity \(v_0 = 0\). Variables with subscript ‘1’ denote the first order approximations and represent the sound field whereas ‘2’ denotes the second order terms. Here generally the time-averaged second order field is regarded in which case \(v_2\) is the streaming velocity. It can also be convenient to write the field variable expansion in terms of a steady ‘dc’ part and second harmonic fields, as an example for the velocity by \(v = v_0 + \varepsilon^1 v_1 + \varepsilon^2 (v_{dc} + v_2)\) [12] but is not followed here.

**First Order System**

For small wave amplitudes and MHz frequencies the generation of sound waves can be assumed to be an isentropic process and the equation of state is linear. For ideal conditions it is given by

\[
p_1 = \rho_0^2 \rho_1.
\]  

(2.10)

An example with a constitutive equation including entropy is given by Bradley [12].

The continuity (2.1) and NS (2.2) equations for the first order \((\varepsilon^1)\) variables are then

\[
\frac{\partial \rho_1}{\partial t} + \rho_0 \nabla \cdot v_1 = 0, \\
\rho_0 \frac{\partial v_1}{\partial t} = -\nabla p_1 + \mu \nabla^2 v_1 + \left( \frac{\mu}{3} + \mu_B \right) \nabla \nabla \cdot v_1.
\]  

(2.11, 2.12)

The nonlinear advective term \(v \cdot \nabla v\) is neglected in eq. (2.12) due to the assumption of disturbances of small amplitude in the sound field.

**Second Order System**

The equations for the steady acoustic streaming are obtained by taking the time average of eqs. (2.1) and (2.2) up to the second order \((\varepsilon^2)\) which gives

\[
\rho_0 \nabla \cdot v_2 = -\nabla \cdot (\rho_1 v_1),
\]  

(2.13)

\[
-\nabla p_2 + \mu \nabla^2 v_2 + \left( \frac{\mu}{3} + \mu_B \right) \nabla \nabla \cdot v_2 = F_R,
\]  

(2.14)

where time-averaging is indicated with pointy brackets \(\langle \rangle\). The source term

\[
F_R = \left\langle \rho_0 (v_1 \cdot \nabla v_1 + \rho_1 \frac{\partial v_1}{\partial t}) \right\rangle = \rho_0 \langle v_1 \cdot \nabla v_1 + v_1 \nabla \cdot v_1 \rangle
\]  

(2.15)
is the spatial variation of the Reynolds stress where the last term addresses compressibility. With the chosen perturbation expansion the convection term in the NS equations (2.2) ends up as $\rho_0 \mathbf{v}_2 \cdot \nabla \mathbf{v}_2$ which is considered a term of fourth order and therefore not included in the second order equation (2.14). The right hand side of the second order continuity equation (2.13) represents a mass source term $\dot{m} = -\nabla \cdot \langle \rho_1 \mathbf{v}_1 \rangle$. This term is assumed to be zero here, but can be of interest for e.g. heat transfer problems [12, 176]. The resulting flow with viscous forces dominating over advective inertial forces is called Stokes flow or creeping flow. The given eq. (2.14) holds for $Re_s \ll 1$ where $Re_s$ is the streaming Reynolds number (2.5).

### Acoustic Wave Equation

In this section practical and often used limits of the first order equations (2.10), (2.11), and (2.12) are presented. In the NS equations (2.2) dissipation is described with the viscous terms. These terms can be combined and the linearized continuity (2.11) and NS equations (2.12) can be reduced to a lossy wave equation

$$\frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} = \nabla^2 p + \tau_s \frac{\partial^2 p}{\partial t} , \quad \tau_s = \frac{4}{3} \mu + \mu_B \rho_0 c_0^2$$

(2.16)

if expressed as a function of the pressure and by use of the linear equation of state (2.10). The viscous relaxation time is given by $\tau_s$. In the inviscid limit the lossy wave equation (2.16) reduces to an ordinary wave equation. Assuming time harmonic oscillations with $e^{-i\omega t}$ the lossy wave equation (2.16) can be simplified and reduced to a lossy Helmholtz equation

$$\nabla^2 p + k_c^2 p = 0$$

(2.17)

with a complex wave number $k_c = k + i\alpha_s$, where $\alpha_s$ is the spatial attenuation coefficient. This coefficient is commonly used to describe acoustic attenuation and is discussed in more detail in Section 2.5. In the inviscid limit the velocity directly follows from eq. (2.12) as

$$\mathbf{v}_1 = \frac{-i}{\omega \rho_0} \nabla p_1 .$$

(2.18)

For the first order velocity $\mathbf{v}_1$ this limit is typically valid for applications where viscous shear effects are small. These are e.g. attenuation driven acoustic streaming or acoustic radiation force effects.

### 2.2 Reference System and Boundary Conditions

The description of equations and boundary conditions in different reference systems, or the conversion from Eulerian into Lagrangian description and back, is important for the formulation of the boundary conditions of an oscillating boundary and boundary layer driven streaming. This is discussed here and a set of boundary conditions which apply to both, linear acoustics and flow problems are given.
Eulerian and Lagrangian Description

The given governing equations are developed within the Eulerian system, since the Lagrangian equations of motion tend to be more difficult particularly regarding non-linearities [133]. Nonetheless the conservation laws are inherently Lagrangian and for the linear theory of sound no such distinction exists as shown with eq. (2.19). While the theory is developed in Eulerian description, many fluid velocity measurement (velocimetry) and visualization techniques like particle image velocimetry (PIV) or particle tracking velocimetry (PTV) are based on Lagrangian data obtained from seeding particles. In the case of PIV this data is then used to generate an Eulerian velocity field description.

The Lagrangian velocity field can be obtained utilizing the material derivative which expanded to first and second order is

\[
\frac{Dv_{1L}}{Dt} = \frac{\partial v_1}{\partial t}, \\
\frac{Dv_{2L}}{Dt} = \frac{\partial v_2}{\partial t} + v_{1F} \cdot \nabla v_1,
\]

respectively, where the subscript ‘L’ denotes the Lagrangian velocity and ‘F’ the fluid particle velocity which have been introduced for clarity. The final Lagrangian velocity of the steady streaming is obtained by time-averaging

\[
v_{2L} = v_2 + \left\langle \int v_1 dt \cdot \nabla v_{1F} \right\rangle = v_2 + v_S
\]

(2.20)

where the last term is known as Stokes drift velocity often loosely referred to as Stokes drift. With a complex time harmonic assumption \(e^{-i\omega t}\) for the first order velocity and with \(v_{1F} = v_1\) the Stokes drift can be written as

\[
v_S = \frac{i}{\omega} \left\langle v_1 \cdot \nabla v_1 \right\rangle.
\]

(2.21)

Here \(\omega = 2\pi f\) is the angular frequency and \(f\) the ordinary frequency. In words, the Stokes drift describes the difference between the Lagrangian and the Eulerian velocities that arise from filtering out the high frequency motion when time-averaging Eulerian velocities which most observation techniques necessarily do. A more detailed discussion of the Stokes drift can be found in [104].

Boundary Conditions

The time harmonic and stationary solutions of the first and second order governing equations can be obtained in combination with the appropriate boundary conditions. The bounding surface is denoted with \(B\).

For the first order fields (2.11) and (2.12) the most common boundary conditions can be expressed as Dirichlet boundary conditions which are

\[
p_1|_B = 0, \\
\mathbf{n} \cdot \mathbf{v}_1|_B = 0
\]

(2.22)  (2.23)
2.3. Acoustic Radiation Force

for a soft wall or open boundary and a hard wall boundary condition, respectively. For the latter \( n \) denotes the unit normal vector of the boundary. The inviscid case is often directly solved for the pressure field \( p_1 \). In this case the hard wall boundary condition can be given as a Neumann boundary condition by

\[
\mathbf{n} \cdot \nabla p_1 \big|_B = 0
\]  
(2.24)

and an excitation boundary conditions can be given with an acceleration

\[
\mathbf{n} \cdot \frac{\partial \mathbf{v}_1}{\partial t} \big|_B = \frac{1}{\rho_0} \mathbf{n} \cdot \nabla p_1 \big|_B
\]  
(2.25)

in normal direction. Another type of boundary condition is the lossy or non-reflective boundary condition. Examples of theories treating those boundary conditions are the matched boundary or radiation boundary conditions [15, 48].

For the second order fields (2.13) and (2.14) the no-slip boundary condition is applied on a wall. If the wall is rigid or non-oscillating, the no-slip condition is

\[
\mathbf{v}_2 \big|_{\mathcal{R}B} = 0.
\]  
(2.26)

At an oscillating surface the no-slip boundary condition must be applied at the displaced surface, thus the Lagrangian velocity at the boundary is \( \mathbf{v}_2 \big|_{\mathcal{B}} = 0 \). With the aid of eq. (2.20) the time-averaged surface velocity on an oscillating boundary is obtained as

\[
\mathbf{v}_2 \big|_{\mathcal{OB}} = -\mathbf{v}_S
\]  
(2.27)

in Eulerian description. An open or flow boundary condition can be defined by assigning a value to the stress given by the left hand side of eq. (2.14) in normal direction. As an example a fully open boundary is defined by setting the stress equal to zero. An open normal flow is additionally defined by \( \mathbf{t} \cdot \mathbf{v}_2 \big|_{\mathcal{B}} = 0 \), where \( \mathbf{t} \) is the tangential direction.

2.3 Acoustic Radiation Force

The acoustic radiation force is a result of a change in wave momentum due to scattering at an inhomogeneity such as bubbles or solid particles. The forces arise as a time averaged effect and would vanish in a linear approximation but can be derived taking second order terms into account in theory. Forces exerted on a single particle are called primary acoustic radiation forces and any interaction between two or more surfaces is termed secondary acoustic radiation force.

Primary Force

The force on a particle can be calculated by integrating the time averaged hydrodynamic traction up to the second order given with eqs. (2.13) and (2.14) over the particle surface. However in most cases this is not very practical and a number of simplified analytical expressions have been derived using different sets of equations and boundary conditions.
The analysis of the problem goes back to King [80] who provided a force expression for a rigid and movable sphere in an inviscid fluid. Yosioka and Kawasima [190] extended the work further. They treat small particle deformations with a general force integration for a compressible particle on the equilibrium particle surface. In addition they treat large deformations of a sphere in an inviscid fluid. Their formulation is very convenient for modeling of arbitrary shaped particles. Gor’kov [54] summarized and generalized their work for a small sphere in a standing wave field and expressed the acoustic radiation force

\[ F_{\text{rad}} = -\nabla U \]  

as the gradient of a potential \( U \), also referred to as Gor’kov potential. The potential expressed as a function of the compressibility \( \kappa \) as in [16] is

\[ U = \frac{4}{3} \pi r^3 \left( \frac{1}{2} f_1 \kappa_0 \langle p^2 \rangle - \frac{3}{4} f_2 \rho_0 \langle v^2 \rangle \right), \]  

where \( r \) is the particle radius and \( \langle p^2 \rangle \) and \( \langle v^2 \rangle \) are the mean square fluctuations of the fluid pressure and velocity, respectively. These quantities are of the incident wave at the particle center of the pressure and velocity field when the particle is absent. The coefficients \( f_1 \) and \( f_2 \) are physically related to the monopole and dipole scattering and are expressed by

\[ f_1 = 1 - \frac{\kappa_s}{\kappa_0} \quad \text{and} \quad f_2 = \frac{2(\rho_s - \rho_0)}{2\rho_s + \rho_0}, \]  

where \( \rho \) is the density and the subscripts ‘s’ and ‘0’ denote particle and fluid quantities, respectively. The material compressibility of the fluid is given with \( \kappa_0 = 1/(\rho_0 c_0^2) \), where \( c_0 \) is the speed of sound. The solid compressibility \( \kappa_s \) is given as inverse of the bulk modulus. Gor’kov’s formula is a good approximation for a large number of ultrasonic particle manipulation devices and has been applied widely for their design [16].

Doinikov [31] extended the work in a paper series to include viscous and thermal effects for a sphere. Different authors have since extended the work [109, 26, 179]. Viscous effects are mainly of interest for traveling wave applications or for smaller particles. A more detailed discussion of the influence of viscosity for typical ultrasonic particle manipulation devices is included in the work of Settnes and Bruss [156]. The most recent work is given by Wang [178] with an expression for a small particle in a viscous fluid and an extension of Gor’kov’s theory where potential and non potential forces are indicated.

**Secondary Force**

In addition to the radiation forces arising from the primary acoustic field, the scattered fields between particles lead to secondary forces. For the interaction between two particles these attractive and repulsive secondary forces are commonly called Bjerknes forces [25]. Particles trapped at pressure nodes experience an attractive force if the interconnection line between particles is perpendicular to the primary acoustic field. This leads to the formation of particle clumps. The forces scale with the distance squared and to the fourth power depending whether the rigidity or the compressibility dominates, respectively. A
more detailed discussion of the forces for typical particle sizes is given in [119]. An example utilizing these forces to trap particles in a continuous flow system can be found in [67]. There is a larger number of publications related to the interaction between bubbles where these forces are typically stronger [128]. A review of secondary forces for ultrasonic devices can be found in [59]. A discussion of secondary forces between a wall and a rigid cylinder is given in [180].

2.4 Acoustic Streaming

Acoustic streaming is a net mean flow driven by acoustic oscillations. Similar to acoustic radiation forces, acoustic streaming is a non-linear, time-averaged effect which typically becomes significant with periodic oscillations of high frequency and or amplitude. The phenomenon has been known for more than a century, Faraday reporting streams of air above vibrating plates in 1831 and a deeper analysis was given by Rayleigh in 1884 [136].

From a phenomenological point of view acoustic streaming can be divided into two types of streaming, bulk attenuation driven and boundary layer driven acoustic streaming. Physically both types of streaming can be interpreted as a time-averaged momentum transfer from the oscillations to the medium [99]. In the case of bulk attenuation driven streaming the energy transferred comes from the spatial attenuation of acoustic oscillations in free space. Thus the linear acoustic part is dominated by the compressibility of the host fluid and both the flow field and sound field can be irrotational. Following the nature of this mechanism bulk attenuation streaming is also called traveling wave streaming.

On the other hand, in the case of boundary layer driven streaming the energy transferred comes from the friction between the oscillating medium and a solid boundary. Thus viscosity comes also into play. The term ‘boundary layer’ generally refers to the acoustic or Stokes boundary layer. The no-slip boundary condition impresses a mainly rotational character on the viscous medium in the boundary layer. The streaming in the boundary layer is referred to as inner streaming and the one in the bulk as outer streaming. The outer streaming is driven by the inner streaming.

In literature a number of different terms are commonly used for the two types of acoustic streaming mentioned above which may lead to some confusion since some of them are also known for specific solutions. Bulk attenuation streaming or traveling wave streaming is also called Eckart streaming or quartz wind and in scarce cases sonic wind. Other common terms for boundary layer driven streaming are Rayleigh or Schlichting streaming. Eckart streaming also refers to the solution of a plane wave traveling along a circular tube with rigid walls treated by Eckart [38]. Quartz wind may also refer to an ultrasonic beam propagating into an infinite half space and originates from observations with quartz oscillators in water [17]. Rayleigh streaming may also refer to the inner and outer streaming around a vibrating, flat wall [136]. Schlichting streaming may also refer to the acoustic streaming around a vibrating, curved wall or refer to inner streaming. An example is the streaming around a stationary cylinder due to an oscillating irrotational fluid at infinity [150]. Both, Schlichting and Rayleigh investigated this streaming using.
the Stokes boundary layer theory.

Sometimes acoustic streaming is also classified by the regime of streaming. The term RNW streaming, after Rayleigh, Nyborg and Westervelt, is used for streaming which can be described with Stokes flow [99]. This is the most widely treated regime. On the other hand, acoustic streaming with a significant convective contribution is called Stuart streaming [159, 169, 99]. The convective term appears as fourth order term in the time averaged equations. Stuart included this term in the equations for very high frequencies and boundary layer driven streaming. In this case a double boundary layer is formed. The convective term is also important for high power acoustic beams [169].

There are a number of publications extending and reviewing the theories and solutions exhibited above. Nyborg investigated streaming measurements and suggests that these measurements cannot be used to distinguish between absorption mechanisms such as scattering and thermal relaxation [116]. In addition Nyborg extended the application of the boundary layer theory for streaming effects from cylinder surfaces to arbitrary smooth surfaces [117]. Lighthill [99] set the wave attenuation in respect to an acoustic energy flux. Riley [142] gives a more recent review of acoustic streaming including heat transfer. He also generalized Schlichting’s method and proposed dimensionless governing equations [141]. Lee and Wang [91, 92] extend Nyborg’s solution of a cylinder in a standing wave for an arbitrary position of the cylinder and they showed that a viscous torque can be generated on a cylinder by two orthogonally standing waves. Nyborg [118] discussed mass transport and Bradley [12] extended Nyborg’s theory to non-rectilinear fluid motion cases. Rednikov and Sadhal [137] recently summarized and generalized Nyborg’s theory including non-adiabatic effects in the Stokes layer and provided a detailed theoretical framework and examples.

2.5 Attenuation and Damping

The terms attenuation and damping are often loosely interchanged and can be described as the reduction in the magnitude of oscillations by dissipation of energy. This effect can be described mathematically in a variety of ways using different parameters. For ultrasound attenuation typically refers to a spatial change and damping to a temporal change in amplitude. Another parameter often used in ultrasonics is the quality factor or $Q$-factor which can be referred to the idea of energy dissipation directly. There are a large number of other parameters, such as loss angle, isotropic loss factor, damping parameter, damping ratio, logarithmic decrement and characteristic time. Attenuation is a fundamental part of the modeling of acoustic systems. In the first section the attenuation coefficient is discussed. The basic damping concepts can be found in literature, e.g. by Kinsler [81] and are followed here. This is supplemented with literature values for different materials used in this thesis and brief calculations showing the limit of given approximations and the link towards a complex speed of sound. The $Q$-factor and loss factor as well as different complex damping variables are used in the numerical models described in Section 3.1.
2.5. Attenuation and Damping

Attenuation Coefficient

The spatial attenuation coefficient $\alpha_s$ is related to viscous attenuation. The coefficient can be related to the first order equations (2.10), (2.11), and (2.12), if these equations are expressed as a function of the pressure and assuming complex time harmonic oscillations, as shown in Section 2.1. This leads to a complex wave number

$$k_c = k + i\alpha_s = \frac{\omega}{c_0} \sqrt{\frac{1 + i\omega \tau_s}{1 + \omega^2 \tau_s^2}}. \quad (2.31)$$

For most viscous liquids except highly viscous ones the viscous relaxation time $\tau_s$ is of the order of $10^{-12}$ s [37]. Unless frequencies are in the upper GHz range $\omega \tau_s \ll 1$ holds and by further simplification, $\alpha_s$ can be approximated with

$$\alpha_s \approx \frac{1}{2} \omega \tau_s = \frac{\omega^2}{2 \rho_0 c_0^2} \left( \frac{4}{3} \mu + \mu_B \right) \quad (2.32)$$

and the real-valued wavenumber is $k = \omega/c_0$. The solution to the Helmholtz equation (2.17) for a plane wave traveling in positive $x$-direction is

$$p = \hat{p} e^{i(kx - \omega t)} = \hat{p} e^{-\alpha_s x} e^{i(kx - \omega t)} \quad (2.33)$$

which represents an exponential decay in space of the wave with $\alpha_s$. Note that the time harmonic factor used has a negative exponent in time. With a positive factor the complex wave number has to be in the form of $k_c = k - i\alpha_s$ to fulfill an exponential decay of the wave. In literature $\alpha_s$ is sometimes calculated with the extended Stokes’ law which was originally derived neglecting bulk viscosity and in comparison overestimates its contribution by $1/3$.

As shown in eq. (2.32) an increase with the frequency squared is a good approximation for viscous low loss materials such as water. Similarly the attenuation of many other materials can be described with a simple frequency power law with constants $\alpha_0$ and $\alpha_1$,

$$\alpha_s(f) = \alpha_0 + \alpha_1 f^n, \quad \eta \in [0, 2], \quad (2.34)$$

where $f$ is the frequency and $\alpha_0$ generally is set to zero [161]. The energy dissipation of structural metals is approximately independent of the frequency [81] and thus $\eta = 1$. In the ultrasonic regime many plastics or polymers follow a power law with $\eta$ around 1 [82, 161, 152]. Polymers also have a strong temperature dependence of the attenuation [182]. Besides water single crystalline materials follow a quadratic power law [161].

A summary of basic material coefficients for water, aluminum and some plastics is shown in Table 2.1. The values are normalized with $f^n$ for better comparison. Acoustic attenuation properties of plastics are scarce in literature and are more of an approximate guidance level. Here they are shown for $\eta = 1$. The values given for PMMA (poly-methylmethacrylate) or acrylic glass, PS (polystyrene) and PE (polyethylene) are from Selfridge [152] and Al from [78], other references for plastics are e.g. [18, 4, 82].

Some more precise data for the bulk viscosity of water has been obtained with experimental measurements only recently [37, 74]. As Dukhin and Goetz [37] pointed out many
### Table 2.1: Material properties where ‘r.t.’ stands for room temperature. Bulk wave speed \(c_0\) is given for water and longitudinal wave speed \(c_L\) for the solids. Values for water at 20 °C are interpolated. The exponent \(\eta\) is 2 for water and 1 for the solids. The last column shows the frequency values at which the attenuation values are recorded.

<table>
<thead>
<tr>
<th>Material</th>
<th>Temp. °C</th>
<th>Density (\rho) kg m(^{-3})</th>
<th>Speed of sound (c_0, c_L) m s(^{-1})</th>
<th>Dyn. visc. (\mu) mPa s</th>
<th>Bulk visc. (\mu_B) mPa s</th>
<th>Attenuation coefficient (\alpha_s) dB m(^{-1}) MHz(^{-\eta})</th>
<th>(\alpha_s/\eta) MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>20</td>
<td>998</td>
<td>1481</td>
<td>1.01</td>
<td>2.93</td>
<td>0.23</td>
<td>@</td>
</tr>
<tr>
<td>Water</td>
<td>25</td>
<td>997</td>
<td>1497</td>
<td>0.89</td>
<td>2.47</td>
<td>0.19</td>
<td>@</td>
</tr>
<tr>
<td>PMMA</td>
<td>r.t.</td>
<td>1190</td>
<td>2750</td>
<td>-</td>
<td>-</td>
<td>128</td>
<td>5</td>
</tr>
<tr>
<td>PS</td>
<td>r.t.</td>
<td>1040</td>
<td>2320</td>
<td>-</td>
<td>-</td>
<td>72</td>
<td>5</td>
</tr>
<tr>
<td>PE</td>
<td>r.t.</td>
<td>920</td>
<td>1950</td>
<td>-</td>
<td>-</td>
<td>48</td>
<td>5</td>
</tr>
<tr>
<td>Al</td>
<td>r.t.</td>
<td>2698</td>
<td>6374</td>
<td>-</td>
<td>-</td>
<td>0.35</td>
<td>10</td>
</tr>
</tbody>
</table>

textbooks assume liquids to be incompressible and bulk viscosity therefore plays no role. Nonetheless its contribution to the attenuation of plane waves is more important than that of the dynamic viscosity.

In literature the attenuation coefficient is typically given in a logarithmic scale like decibel (dB) and normalized by the frequency squared calculated by

\[
\alpha_{sdB} = \frac{10}{f^2 \Delta x \log_{10} \left( \frac{I}{I_d} \right)}, \tag{2.35}
\]

where \(\Delta x\) is the attenuation distance and \(I\) and \(I_d\) are the acoustic intensities at reference and attenuated state. Sometimes Neper (Np) defined by the natural logarithm is used instead of dB. The acoustic intensity \(I = pv_1\) is proportional to the amplitude squared given by eq. (2.33) and therefore the logarithmic and non-logarithmic attenuation coefficients can be converted by

\[
\alpha_s = \frac{\ln (10)}{20} \alpha_{sdB}. \tag{2.36}
\]

The theory summarized here provides a link between energy dissipation in the NS equations (2.2) and a simplified model with attenuation in space as given with \(\alpha_s\). This helps to compare damping in boundary layer driven streaming with that of attenuation driven streaming. An alternative concept which can build a link between different damping concepts is the damping ratio. The lossy wave equation (2.16) and separation of variables with respect to time and space leads to the Helmholtz equation as discussed above and to an equation of motion for a harmonic oscillator. The solution of the equation of motion leads to the damping ratio which characterizes the frequency response and is not discussed in more detail here.
2.5. Attenuation and Damping

**Q-factor and Loss Factor**

The quality factor or $Q$-factor is a measure for how much under-damped an oscillator is. It describes the energy stored in a system in relation to the rate of energy dissipation or

$$Q = 2\pi \frac{\text{Energy Stored}}{\text{Energy dissipated per cycle}}.$$  \hspace{1cm} (2.37)

For a harmonic oscillator the loss or dissipation factor $\eta$ which is equal to loss tangent is the inverse of the $Q$-factor

$$\eta = \frac{1}{Q} = \sum_i \frac{1}{Q_i}$$ \hspace{1cm} (2.38)

where the reciprocal of the total $Q$-factor is equivalent to the sum of the reciprocals of the individual $Q_i$. In an ultrasonic device these $Q_i$ can be for the fluid, structure, glue, support and so forth. A discussion of the measurement of the $Q$-factor in ultrasonic systems can be found in Acoustofludics 6 [33].

While the attenuation coefficient $\alpha_s$ is often used as a material coefficient independent of any geometrical dimensions, the $Q$-factor in contrast is particularly suited to describe systems with different parts and geometrical constraints. Both terms can be related to each other by using eq. (2.38) where the energy is proportional to the pressure (2.33) squared. The distance a plane wave travels during one wave cycle is $2\pi/k$ which then gives

$$Q = \frac{2\pi}{1 - e^{-4\pi\alpha_s/k}} \approx \frac{k}{2\alpha_s}.$$ \hspace{1cm} (2.39)

The inverse proportional relation to the attenuation coefficient $\alpha_s$ on the right hand side of eq. (2.39) is a good approximation and is also known as Kolsky [83] model.

**Complex Damping Variables**

If restricted to a harmonic motion, complex damping variables similar in form to the complex wave number (2.31) can be formulated. They can simplify calculations significantly. For the complex variables, a conventional negative exponent in time is assumed here. In other fields of physics a positive exponent in time is usually adopted which leads to a change in sign of the complex contribution.

For ultrasonic particle manipulation devices a complex speed of sound $c_c$ is common [59]. Inserting eq. (2.39) into the complex wave number $k_c = k + i\alpha_s$ (2.31) and assuming $Q \gg 1$ gives

$$c_c = c_0 \frac{1 - i\frac{\sqrt{Q}}{2\pi}}{1 + i\frac{\sqrt{Q}}{2\pi}} \approx c_0 \left(1 - \frac{i}{2Q}\right).$$ \hspace{1cm} (2.40)

While the complex wave number can be directly interpreted as an exponential decay in space, the complex sound speed is a more abstract concept which is also related to the concept of a complex Young’s modulus $E_c$ [24]. The latter is used to model the damping of the piezoelectric transducer. According to the theory of linear viscoelastic materials
and for small damping $E_c$ can be written as

$$E_c = E_0 \left(1 - \frac{i}{Q}\right), \quad (2.41)$$

where $E_0$ is the storage modulus which is a measure for the stored energy and represents elasticity. The $Q$-factor is given by $Q = E_0/E_L$, where $E_L$ is the loss modulus which is a measure for energy dissipation and represents viscous properties. For a viscoelastic solid the phase between applied force and deformation can be expressed by the loss tangent $\tan \phi$ which is inversely proportional to $Q$.

### 2.6 Impedance Matching

The impedance $Z$ gives a measure of some system behaviors. In electrical engineering this is the extent to which a circuit opposes the flow of electricity. The acoustic impedance defined as the ratio of the sound pressure to the particle velocity is an indication for the pressure amplitudes in a resonating system. The impedance is frequency dependent and can be used to determine the frequency response of a system. Acoustic impedance is often treated in analogy to electrical impedance.

The specific acoustic impedance of a plane wave with phase velocity $c_0$ is an intrinsic measure defined as

$$Z_0 = \rho_0 c_0. \quad (2.42)$$

In the dispersion free case $Z_0$ is not a function of the frequency. Other phrases used for $Z_0$ are acoustic, or characteristic impedance, where the latter is also defined per unit area.

The energy or intensity of a plane wave reflected or transmitted at an interface between two materials can be estimated with

$$e_r = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2 \quad \text{and} \quad e_t = 1 - e_r, \quad (2.43)$$

where $e_r$ and $e_t$ are the energy reflection and transmission coefficients, respectively. Table 2.2 lists $e_r$ between water and a number of materials as well as their specific acoustic impedance. The material data values are from Table 2.1 and for Pz26 given by the manufacturer Ferroperm Piezoceramics.

<table>
<thead>
<tr>
<th></th>
<th>Air</th>
<th>Water</th>
<th>PMMA</th>
<th>PS</th>
<th>PE</th>
<th>Al</th>
<th>Pz26</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_0 \left[\text{MPa s m}^{-1}\right]$</td>
<td>$4 \cdot 10^{-4}$</td>
<td>1.48</td>
<td>3.27</td>
<td>2.41</td>
<td>1.79</td>
<td>17.2</td>
<td>15.7</td>
</tr>
<tr>
<td>$e_r \left(\text{Water - }'\right)$</td>
<td>0.99</td>
<td>0</td>
<td>0.14</td>
<td>0.06</td>
<td>0.01</td>
<td>0.71</td>
<td>0.69</td>
</tr>
</tbody>
</table>

Table 2.2: Specific acoustic impedance $Z_0$ and energy reflection coefficient $e_r$ between water and the given materials.

Piezoelectric elements or metals usually have a much larger impedance than water or
plastics. The energy transmission from the transducer to the fluid can be optimized with an impedance matching layer which usually has a width of $\lambda/4$ or odd multiples of it. Like this reflections in the layer superimpose in phase and the waves transmitted back into the transducer cancel out with the reflected incident wave. This gives a better pulse compactness which is mainly of importance for systems with a backing layer like used in ultrasonic imaging. The specific impedance $Z_2$ of such a layer can be determined with the Desilets [28] model, or the similar Collin [22] model

$$Z_2 = \sqrt{Z_1 Z_3}.$$  \hspace{1cm} (2.44)

The energy transmission efficiency can be further improved with multisection constructions. A more detailed description of transducer design can be found in [174]. Additional discussion of the acoustic impedance can be found in [81, 5, 60].
Chapter 3

Numerical and Experimental Methods

In this chapter the numerical and experimental framework is presented. The numerical simulation is presented in three main parts. First the linear acoustics and structural system model is presented in Section 3.1.1. It is used to calculate the acoustic pressure field and serves as a starting point for more detailed simulation models. Particle tracing simulation models are presented in Section 3.1.2. They are used to simulate particle transport with continuous frequency sweeping and acoustic particle traps. Finally the numerical model of attenuation and boundary layer driven acoustic streaming is presented in Section 3.1.3. The experimental setup used to operate the acoustic devices and the fabrication of the acoustic devices is presented in Section 3.2. A schlieren visualization setup is presented in Section 3.3. In Section 3.4 the setup used for particle image velocity measurements is presented.

3.1 Numerical Simulation

Modeling of acoustic systems helps to gain a better understanding of their mode of operation and is a valuable tool to improve their functionality. For resonating systems the time harmonic pressure and velocity fields in the fluid as well as the displacement fields in the solid structure are of particular interest. They can be used to derive a number of other quantities which are of interest like the acoustic radiation force or momentum. An analytical model allows a rather direct insight in the relation between model parameters and the underlying differential equations describing the problem. However, an analytical derivation is often too complex, particularly for problems beyond a 1D model. Another attempt to solve the problem is using numerical methods. There is a large number of different numerical methods which can also be combined and suit different problem sets best. An example of an indirect domain discretization method is the boundary element method (BEM). Among the direct domain discretization methods there are the finite element method (FEM), the finite difference method (FDM), the finite volume method (FVM) or spectral methods. The advantage of the latter is accuracy while FVM and BEM can be computational powerful for special problems. FDM is among the simplest to implement. One of the most common methods is FEM due to its great flexibility with
complex geometries. More information about numerical methods can be found in [162].

The FEM software package COMSOL Multiphysics is used here. One of its strength is the ability to easily combine different physical problem descriptions in a single model. COMSOL introduced a number of add-on releases over recent years which find application here. These are e.g. LiveLink for MATLAB with version 4.0 which allows scripting or the more recent particle tracing module which includes tools for the calculation of particle trajectories in a fluid including particle-field interactions. There are also a number of changes regarding predefined boundary conditions in the graphical user interface or application module content. The model descriptions given here are based on COMSOL Multiphysics 4.3a.

3.1.1 Linear Acoustics and Structural System Model

A typical model consists of a fluid domain, a surrounding mechanical structure and a piezoelectric transducer. The linear acoustics model including fluid-structure interaction presented here, follows the first models by Neild et.al. [113] which have been expanded by Oberti [119] and Möller [110]. This is supplemented with structural damping and a model for glue layers and clamping. The numerical models of resonant ultrasonic devices in literature typically have a fluid cavity which extends over a small number of wavelengths [121, 123, 112, 15, 49]. Devices with a large number of wavelengths (> 10) are presented here. This increases the computational cost significantly and therefore meshing and solver options are discussed in more detail.

Application modes

In the case of an inviscid fluid, the fluid domain is modeled with the Pressure Acoustics, frequency domain application mode to solve eq. (2.17). The pressure is the dependent field variable which is used to calculate the velocity with eq. (2.18). The mechanical structure is either modeled with the Piezoelectric Devices module and the incorporated Linear Elastic Material Model or equivalently with the Structural Mechanics, Solid Mechanics module. With the previous one there is the advantage that the internal boundary conditions are preset. The piezoelectric transducers are modeled with the Piezoelectric Material Model feature. A review of continuum mechanics and piezoelectricity for ultrasonic particle manipulation can be found in [36, 35]. For the 2D models it is assumed that the normal extent is large compared to the in plane one and any solid parts thus are approximated with a plane strain assumption. Dependent field variables are the displacement field and the voltage.

Glue Layers and Clamping

Glue layers have an expected thickness $d$ below 100 $\mu$m. This evokes mesh discretization problems with an explicit glue layer model. For small $d$ a thin elastic layer model is a good approximation. In COMSOL this model can be used in any structural module as interior boundary. The model is used with a lossy spring constant per unit area. Assuming
isotropic material behavior and a rather large Poisson’s ratio the spring constant in normal and tangential directions approximately are

\[ k_n = \frac{K}{d} \quad \text{and} \quad k_t = \frac{G}{d}, \]  

(3.1)

respectively, where \( K \) is the bulk modulus and \( G \) the shear modulus.

A foundation or clamping of the whole structure is modeled with a spring foundation model which has the same physics as the thin elastic layer model.

Material Properties

The fluid is modeled as water with a density \( \rho = 998 \text{kg m}^{-3} \) and a speed of sound of \( c_0 = 1481 \text{m s}^{-1} \) as given in Table 2.1. In the inviscid case, the attenuation is modeled with a complex speed of sound (2.40) or wave number (2.31) and \( Q(f) = 2000 f^{-1} \text{MHz} \). For the surrounding structure glass, aluminum and PMMA (acrylic glass) are used. The piezoelectric material is modeled as Pz26 with the parameters given by the manufacturer, Ferroperm Piezoceramics and complex damping parameters which have been introduced in Section 2.5. The transverse isotropic elasticity or stiffness matrix \( C \) has the components

\[
C_{11} = C_{22} = 168 \text{GPa}, \quad C_{33} = 123 \text{GPa}, \quad C_{44} = C_{55} = 30.1 \text{GPa}, \quad C_{66} = 28.8 \text{GPa}, \\
C_{12} = C_{21} = 110 \text{GPa} \quad \text{and} \quad C_{13} = C_{23} = C_{31} = C_{32} = 99.9 \text{GPa},
\]

and the damping is given with \( Q = 180 \). The coupling matrix \( K \) connects the mechanical and electrical properties and the components are

\[
K_{15} = K_{24} = 9.86 \text{C m}^{-2}, \quad K_{31} = K_{32} = -2.8 \text{C m}^{-2}, \quad \text{and} \quad K_{33} = 14.7 \text{C m}^{-2}.
\]

The complex relative permittivity \( \varepsilon \) has the components \( \varepsilon_{11} = \varepsilon_{22} = 828 \) and \( \varepsilon_{33} = 700 \) and damping is given with an isotropic loss factor \( \eta = 0.003 \). The material density is \( \rho = 7700 \text{kg m}^{-3} \).

PMMA is modeled with a Young’s modulus \( E_0 = 3300 \text{MPa} \), a Poisson’s ratio \( \nu = 0.37 \), a density \( \rho = 1190 \text{kg m}^{-3} \), and an isotropic loss factor of \( \eta = 0.003 \) according to the manufacturer Evonik Röhm GmbH. Aluminium is modeled with the material properties from the COMSOL Multiphysics Material Library and a loss factor \( \eta = 2 \cdot 10^{-5} \) [78]. The attenuation and damping parameters are varied for certain models to account for a change in the total damping of the system.

The particles are modeled as copolymer, silica or glass spheres. The copolymer particles are from Duke Scientific (now Thermo Scientific) with a density of \( 1050 \text{kg m}^{-3} \) and an assumed speed of sound of \( 3000 \text{m s}^{-1} \). The silica particles are from Kisker Biotech and they are porous with a density of \( 2000 \text{kg m}^{-3} \) and an assumed speed of sound of \( 5500 \text{m s}^{-1} \). The glass particles have a density of \( 2500 \text{kg m}^{-3} \) and an assumed speed of sound of \( 5600 \text{m s}^{-1} \).

Temperature Influences

For ultrasonic particle manipulation, temperature often plays a crucial role when operated for a long time. This is mainly due to the heat produced by the transducer which in
turn changes the behavior of the device as all material properties are a function of the
temperature. Most liquids have a strong temperature dependence for the viscosity and
for the speed of sound. For an unaltered device the latter leads to a change in resonance
frequency. Viscosity comes into play with drag forces and for acoustic streaming. For solid
parts the most important effect might be the temperature increase in glue layers which
will change the damping of the system. Furthermore, the ceramic transducers will change
their properties with very excessive heat up to loosing the polarization when heated above
the Curie temperature. It is feasible to assume isothermal conditions for the numerical
simulations since most results are of qualitative character. The given values are for a
reference temperature of 20°C.

**Boundary Conditions**

The basic boundary conditions for a model setup with the Piezoelectric Devices module
is presented in Table 3.1. The BCs of the Linear Elastic Material mode also apply for the
Piezoelectric Material mode. Both are solved for the displacement field \( \mathbf{d} \) and therefore
interior boundaries between those two are not listed. The value of the electric potential

<table>
<thead>
<tr>
<th>Boundary Mode</th>
<th>Boundary</th>
<th>Boundary Condition</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acoustics</td>
<td>Structure</td>
<td>Normal acceleration</td>
<td>(2.25)</td>
</tr>
<tr>
<td></td>
<td>Air</td>
<td>Soft wall</td>
<td>(2.22)</td>
</tr>
<tr>
<td></td>
<td>Symmetry</td>
<td>Hard wall</td>
<td>(2.24)</td>
</tr>
<tr>
<td>Linear Elastic Material</td>
<td>Fluid</td>
<td>Boundary load</td>
<td>( \mathbf{F} = -\mathbf{n} \cdot \mathbf{p} )</td>
</tr>
<tr>
<td></td>
<td>Air</td>
<td>Free</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Symmetry</td>
<td>Symmetry</td>
<td>( \mathbf{n} \cdot \mathbf{d} = 0 )</td>
</tr>
<tr>
<td>Piezoelectric Material</td>
<td>Electrode 1</td>
<td>Ground</td>
<td>( V_0 = 0 ) V</td>
</tr>
<tr>
<td></td>
<td>Electrode 2</td>
<td>Electric potential</td>
<td>( V_0 = 20 ) V</td>
</tr>
<tr>
<td></td>
<td>Neutral</td>
<td>Zero charge</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1: Boundary conditions (\( \mathbf{d} \) displacement, \( \mathbf{n} \) domain normal vector).

\( V_0 \) may vary and takes the given value if not mentioned otherwise.
Symmetries can be used if the model is fully symmetric with respect to all geometry,
material properties, and boundary conditions. Here the harmonic frequency response is
investigated which relies on an excitation boundary condition and thus the solution is
symmetric if the problem is symmetric as described before. This is in contrast to an
eigenfrequency analysis which has asymmetric solutions as well.

**2D Meshing and Solver**

In a mesh convergence study mapped or quad mesh elements (ME) and triangular un-
structured ME of different discretization order \( \mathcal{O}(h) \) are investigated. The following study
is for the square chamber presented in Chapter 4.3. However, similar results are obtained for the other geometries presented in this work. Figure 3.1 shows the normalized root mean square error (NRMSE) also known as coefficient of variation in a logarithmic scale over the ratio of wavelength $\lambda$ and maximum element size $d$. This ratio approximately gives the number of ME per wavelength. The NRMSE is evaluated with the absolute pressure along the symmetry line in 2D of the top view at the peak resonance of 2.0795 MHz as shown in Figure 4.17. The reference values are obtained with mapped ME of $O(h^2)$ and $\lambda/d = 70$. For a coarse discretization free triangular ME show a better convergence,

![Figure 3.1: Mesh convergence study for 2D meshes, where $\lambda/d$ is the ratio of wavelength to maximum element size.](image)

however, with the cost of a larger DOF due to the larger number of ME. For $\lambda/d = 5$ the increase is about 20%. The larger error with mapped elements compared to triangular elements is mainly an amplitude scaling. Thus qualitative information can also be obtained with mapped ME of $O(h^2)$ and $\lambda/d = 5$. In this case the NRMSE is 0.40 and the amplitude corrected NRMSE is about 0.1. Elements of third order converge much faster but come with an increase of the DOF of more than a factor 2.5.

All structured or rectangular geometries are meshed with mapped ME and other geometries with free triangular ME. The used ME typically have quadratic element order and at least 15 ME per wavelength in 2D simulations. Simulations over a large frequency range are calculated with frequency dependent meshing. This is done with the Parametric Sweep feature. The mesh is re-meshed with steps of about 20%.

The degree of freedom (DOF) is a good measure for the memory requirements and computational time. The DOF equals to the number of nodes times dependent variables. As an estimate, the number of nodes of quad elements compared to triangular elements are doubled with 2D meshes and increased by factor 6 with 3D meshes. A change of the discretization order from quadratic to cubic order typically results in an increase of the DOF of at least a factor 2.5. More details and examples can be found in the COMSOL Knowledge base.

The stationary problems are solved in the frequency domain with a direct solver and fully coupled. The solver is a MUMPS (MULTifrontal Massively Parallel sparse direct Solver) with standard settings proposed by COMSOL.
3D Meshing and Solver

3D simulations have significantly larger memory requirements and are computationally more expensive compared to 2D simulations. For systems with a large wavelength to device dimension ratio this is particularly critical.

In general brick mesh elements are used. They are defined with a Mapped mesh extruded in the third dimension with the Swept feature. Like this a high mesh quality is obtained. At least 6.6 ME per wavelength are used in 3D simulations. In selected cases tetrahedral elements are used. They are used in combination with the General Projection feature which is only implemented for this type of elements. The feature allows to integrate along a line or curve. An example is to map a volume to a surface.

Similar to the 2D case, the problems are solved with a direct MUMPS with full coupling. The shape functions have a quadratic discretization order.

The above mentioned solver and discretization provided the best results but other settings have been evaluated as well. For systems with high memory requirements iterative solvers can be an interesting option. Generally they do not require a matrix factorization and operate with preconditioning steps. Among other reasons this eases the memory requirements. However, the different iterative methods are typically specific to different physics and cannot be used for general purpose. In addition, the solvers are not as robust as direct solvers. An example of an iterative solver is a GMRS solver combined with a Geometric Multigrid. This reduces the required memory with the same mesh and discretization order to at least a third. Nevertheless at the same time the mesh quality has to be increased in order to attain that the solver converges. Hence no memory is saved and instead the computational time is significantly increased for a simulation which is close to convergence.

3.1.2 Particle Tracing

The time harmonic pressure field and with it the acoustic radiation force potential provide direct information about stationary particle positions. In addition any stationary force field such as gravitational forces can be included. However, any time dependent forces cannot be included. A possible approach to overcome this limitation is the mass transport equation [168] briefly presented in Section 4.3.4. The Newtonian particle tracing model is another way to achieve time dependence. For the latter the governing equation for a single particle of mass $m$, velocity $\mathbf{v}$, and position $\mathbf{x}$ is

$$m \frac{\partial \mathbf{v}}{\partial t} = \sum_i F_i (\mathbf{x}, \mathbf{v}) ,$$

(3.2)

where $F_i$ are all forces acting on the particle. Here these forces are acoustic radiation forces, Stokes drag forces, and in some simulations gravitational forces. A number of other forces can be of interest. For particles in the lower $\mu$m to sub $\mu$m range particle induced acoustic streaming forces gain importance. Particle-particle interaction can be of greater interest with large particles and or with a great number of particles. However, this increases the computational time significantly. Neither of these additional forces are
3.1. Numerical Simulation

included here. A model including pressure driven flow is presented by Townsend et al. [166] and one including boundary layer driven acoustic streaming is presented by Muller [112]. Those two models are limited to the case where the frequency is not a general function of time or in other words limited to single frequencies. Particle tracing simulations at a single frequency is of interest for particle traps presented in Section 5.2. These simulations balance drag forces arising from the flow field and acoustic radiation forces. Particle tracing at a single frequency can be simulated using the graphical user interface of COMSOL. Particle tracing simulations of frequency sweeping involve time dependent frequencies. For this purpose single frequency particle tracing simulations are extended over several frequencies using LiveLink for MATLAB.

**Basic Model**

For a single frequency in 2D, the linear acoustics and structural system model is extended with the Particle Tracing for Fluid Flow module to solve Newton’s law of motion. Forces on the particles are calculated with the Acoustophoretic Force and Drag Force feature. Particle trap simulations include fluid flow calculated with the Creeping Flow module and a shallow channel approximation. Some frequency sweeping models include gravity or buoyancy forces calculated with the Gravity Force node. Particle-particle interactions are neglected.

The fluid chamber walls are modeled with Bounce wall conditions and a second order wall accuracy model. The initial particle locations and velocities are set with the Release from Grid feature.

The models are solved in two study steps: First the linear acoustics and structural system model is solved in the frequency domain. For some models this includes the stationary solver of fluid flow. In a second step the time dependent particle tracing is solved. For the latter step the values of dependent variables or more specifically, the values of variables not solved for, are taken from the first study step. Models which include fluid flow have two dependencies. More than one dependency can either be linked externally or directly within the graphical user interface from COMSOL Multiphysics 4.3b on.

The mesh is similar to the acoustic model, but for higher stability of the time dependent solver the mesh size is increased to at least 20 to 40 mesh elements per wavelength. Gravitational forces break the symmetry present in most models and therefore the full model geometry is used in those cases. The time dependent solver is an iterative GMRES solver with Jacobi node attribute. The solver in addition has a Fully Coupled attribute and the time stepping method is Generalized alpha as proposed by COMSOL. For a single frequency Free time steps are good. Strict time steps give better results for the presented implementation over several frequencies in series.

**Multiple Frequencies**

The particle manipulation devices assume a time harmonic state and therefore the models are solved in the frequency domain. As a consequence also particle tracing over a frequency range is calculated with a discretization of the frequency. This is realized with a Matlab...
script which loads the COMSOL model in the server, introduces the required model features, and then solves the model for each frequency step while looping the individual solutions. Both, the particle position and velocity, are retained with each instance even though the velocity has hardly any influence due to the low mass of the particles. To do so a Release from Grid feature is created for every single particle and evaluation features for each of the four variable sets are created. The variables are the $x$ and $y$ values of the position and velocity.

The initial particle distributions are quasi-random with the goal to fill the space uniformly. Unlike random or pseudo-random distributions, quasi-random ones fail statistical tests for randomness. An initial particle distribution is shown in Figure 4.21 in Chapter 4.3. The distributions are produced with the Statistics Toolbox of Matlab generating a Halton 2D point set skipping the first 1000 sequence values, retaining every 101-st point and finally reverse radix scrambled.

The implemented model aims for a good balance between computational time on one hand and reasonable convergence and accuracy on the other hand. There are two main issues with the time dependent solver. One is particles diffusing into the side walls and the other issue is empty output variables of individual particles. Forces towards the side walls can cause particles to diffuse into the side walls due to a limited precision of the final particle position after a single frequency step. If such a position is directly assigned to the next instance of the loop, an invalid starting position of the particle is the result and the solver terminates the calculation of that particle. To avoid this problem the final particle positions are checked after every complete solver instance. If the particle center position is closer to the wall than half the particle diameter, the particle position for the next step is moved by half the particles diameter away from the wall. Terminated or not converged particle paths mainly arise with very small forces acting on the particles. Therefore the output variables are checked for empty entries after every loop instance. If this is the case, the particle position is repopulated with the position of the last increment and the particle velocity is set to zero. This problem often occurs close to a wall where forces tend to be small. For the described models the number of these corrections is the highest with gravitational forces. The corrections can occur at up to 20% of the total solving steps as particles move along the wall. For the simulation results presented here this number is at 2% to 5% of all steps. The occurrence of the two discussed problems can be lowered with a higher convergence and accuracy of the time dependent solver. This can be achieved with smaller time steps, smaller drop tolerances, more iterations, or smaller absolute tolerances. However, this can increase the computational time by orders of magnitude while the total simulation is also limited by the discretization of the frequency. The spatial discretization in comparison is not the main limiting factor. The introduction of boundary layer elements does not relieve the wall problem.

### 3.1.3 Acoustic Streaming

The numerical simulation of acoustic streaming is an ongoing topic for one reason because it is rather computationally expensive. Reports in literature include attenuation driven acoustic streaming in different regimes. These are e.g. streaming in micro-channels...
3.1. Numerical Simulation

[149], approximations with lattice Boltzmann simulations [72], or approximations for the streaming in an ultrasonic horn reactor [169]. In contrast to these simulations, the simulation of bulk attenuation driven acoustic streaming presented here includes coupling of the acoustic field with the structure, treats the general case, and is optimized to simulate problems with a large number of wavelengths. For this reason the simulation is in 2D.

Boundary layer driven acoustic streaming typically includes attenuation driven acoustic streaming but normally is simulated for a problem set where boundary layer driven acoustic streaming dominates. Examples are the streaming between motionless parallel plates [112, 2] or enhanced heat transfer between parallel beams, including plate vibrations given by Wan and Kuznetsov [175]. Other boundary layer driven acoustic streaming examples are the streaming produced with surface acoustic waves in droplets or micro cavities [84, 43]. The simulation presented here is similar to the implementation of Wan and Kuznetsov [175] however, here the final velocity field is expressed in Lagrangian representation.

The numerical simulation of acoustic streaming follows the idea to split the problem in a first and second order system as introduced in Section 2.4. Thus the numerical procedure consists of two main study steps. Both, the simulations of bulk attenuation driven and boundary layer driven acoustic streaming share this procedure. They differ in the assumptions made which results in different implementations. They are presented in more detail in the following subsections while the procedure is briefly given in general terms next.

First the acoustic or first order velocity field \( \mathbf{v}_1 \) given with eqs. (2.10), (2.11), and (2.12) is calculated. This is done with a fully coupled model between acoustic and structural domain as described in Section 3.1.1.

In the second step the streaming or second order velocity field \( \mathbf{v}_2 \) given with eqs. (2.13), (2.14), and (2.15) is calculated. The Reynolds stress related force \( \mathbf{F}_R \) (2.15) acts as volume force. The force is calculated from the first order field and the real valued time average is expressed as product of complex and complex conjugate. The time averaged product of two harmonic functions \( f \) and \( g \) with complex exponential is

\[
\langle fg \rangle = \frac{1}{2} \text{Re} \left( \langle fg \rangle \right),
\]

where the overline denotes complex conjugation. The force \( \mathbf{F}_R \) including compressibility hence in 2D and component notation is

\[
F_{Rx} = \frac{1}{2} \rho_0 \text{Re} \left( 2u_1 \frac{\partial u_1}{\partial x} + \frac{\partial u_1}{\partial y} + \frac{\partial v_1}{\partial y} \right),
\]

\[
F_{Ry} = \frac{1}{2} \rho_0 \text{Re} \left( 2v_1 \frac{\partial v_1}{\partial y} + \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial x} \right),
\]

where \( \mathbf{v}_1 = (u_1, v_1)^T \).

**Bulk Attenuation Driven Acoustic Streaming**

An important simplification for bulk attenuation driven acoustic streaming originates from the assumption that viscous shear effects can be neglected for the acoustic velocity
field. The attenuation is expressed as spatial attenuation of acoustic oscillations in free space with the pressure field as described in Section 2.1 and Section 2.5. The velocity field on the other hand is considered in the inviscid and irrotational case as a function of the pressure field. Zero viscous shear also leads to the assumption that the vibrating boundaries can be modeled as motionless boundaries for the streaming field. The simulation scheme including these simplifications is shown in Figure 3.2.

![Figure 3.2: Bulk attenuation driven acoustic streaming simulation scheme.](image)

In COMSOL Step 1 corresponds to the Pressure Acoustics application mode in the frequency domain. This step is described in more detail in Section 3.1.1 including the structural system model.

The second order velocity field is implemented as Stokes flow with the Creeping Flow module and solved for the stationary solution. To account for wall friction in the third dimension a shallow channel approximation is used. To create a well defined problem, a pressure point constraint is set somewhere on the boundary, typically with \( p_2 = 0 \) somewhere away from the transducer. The walls of the fluid chamber are set to the no-slip boundary condition. The fluid domain has to be closed that is no flow over a boundary is allowed to obtain a numerically stable solution.

The direct solvers MUMPS and PARADISO are used for the linear acoustics and creeping flow, respectively. Despite the simplifications made with this implementation, the simulations are computationally expensive. Compared to simulations of the pressure field as final parameter, the discretization order of the shape function is way more critical here. The driving volume force for the second simulation step is linked over two derivatives to the pressure field. Thus at least a cubic or better quartic discretization order of the pressure shape function is required for the problem to fully converge. The structural displacement field is calculated with a cubic discretization order. For the creeping flow best results are obtained with elements of cubic and quadratic order for the velocity \( \mathbf{v}_2 \) and pressure \( p_2 \) field, respectively. The solver also finds a solution with lower element orders which sometimes is confused with a valid solution. The element order is an important point which is a potential source of error for work found in literature. Unstructured or non rectangular geometries are modeled with free triangular mesh elements. The structured geometries are modeled with mapped quad elements. At least 10 mesh elements per wavelength are used. Edges of the structure into the fluid domain, such as of a structural cavity within the fluid domain, are modeled as round corners instead. The corner radius
3.1. Numerical Simulation

is chosen in the order of the largest mesh elements and the mesh is locally refined at the corners in order to prevent artifacts which arise at the discontinuity in the derivative of the pressure field used for the volume force $F_R$. Inner corners of the structure with an inclosing angle are left unaltered. Calculating with lower shape function orders but a finer mesh significantly increases computational time assuming the same degree of convergence towards the exact solution. Symmetries are used if applicable and modeled with symmetry boundary conditions in all domains.

Boundary Layer Driven Acoustic Streaming

Boundary layer driven acoustic streaming is dominated by the friction between the oscillating medium and a solid boundary and thus viscous shear effects have to be taken into account for the acoustic velocity field. Hence the acoustic field is described with the full linear equation set (2.11), (2.12), and (2.10) and no further assumptions apply. However, an implementation with coupled equations which are a function of the pressure and velocity field are computationally not beneficial. Setting up an uncoupled equation set and only solving for the velocity field has several advantages. It results in a reduced DOF and typically increases the convergence of the problem. In component notation the equations for the velocity field are given with

$$
\begin{align*}
- \frac{\omega^2}{c_0^2} u_1 &= (1 - i\omega \tau_s) \frac{\partial^2 u_1}{\partial x^2} - i\omega A \frac{\partial^2 u_1}{\partial y^2} + (1 - i\omega(\tau_s - A)) \frac{\partial^2 v_1}{\partial y \partial x}, \\
- \frac{\omega^2}{c_0^2} v_1 &= (1 - i\omega \tau_s) \frac{\partial^2 v_1}{\partial y^2} - i\omega A \frac{\partial^2 v_1}{\partial x^2} + (1 - i\omega(\tau_s - A)) \frac{\partial^2 u_1}{\partial x \partial y},
\end{align*}
$$

(3.5)

where $A = \mu/(\rho_0 c_0^2)$ and $\tau_s$ is the viscous relaxation time given with equation (2.16). This form provides an idea of the attenuation where the factor $\omega \tau_s$ can be directly related to the spatial attenuation coefficient $\alpha_s$ given with eq. (2.32) or related to the $Q$-factor with $\omega \tau_s \approx 1/Q$. The eq. (3.5) can be found in literature in different composition and assuming a positive harmonic ansatz [176]. Solving directly for the pressure field or a general potential does not provide the advantage given in the inviscid case because the velocity field can no longer be expressed as a simple gradient of the potential field. In addition such an approach increases the constraints for the discretization shape function order.

The no-slip boundary condition at an oscillating surface is crucial for boundary layer driven acoustic streaming and is given with eq. (2.27). With eq. (3.3) the Stokes drift velocity in component notation becomes

$$
\begin{align*}
\begin{aligned}
u_s &= -\frac{1}{2} \text{Re} \left( \frac{i}{\omega} \left( \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} \right) \right), \\
v_s &= -\frac{1}{2} \text{Re} \left( \frac{i}{\omega} \left( \frac{\partial v_1}{\partial y} + \frac{\partial u_1}{\partial x} \right) \right).
\end{aligned}
\end{align*}
$$

(3.6)

It should be noted that the sign of equations (3.5) and (3.6) amongst others are depending on the sign of the harmonic ansatz and therefore have to be used consistently. Here $e^{-i\omega t}$ is used.
The simulation scheme for boundary layer driven acoustic streaming is shown in Figure 3.3. Here the driving force $F_R$ is implemented including compressibility since this comes at hardly any additional computational cost. However, for boundary layer driven acoustic streaming the corresponding term can generally be omitted.

In COMSOL the linear acoustics eq. (3.5) in Step 1 are solved with the general form PDE interface in the frequency domain. The boundaries are modeled with Dirichlet boundary conditions where the wall velocity is set equal to the structural wall velocity. In Step 2 the Eulerian streaming velocity field is calculated with the Creeping Flow module where $F_R$ (3.4) acts as volume force. A pressure point constraint is introduced and a shallow channel approximation is used similar as for bulk attenuation driven streaming. The wall boundary condition is given with eq. (2.27). The final Lagrangian velocity is given by eq. (2.20).

The direct solvers MUMPS and PARADISO are used for the PDE mode and creeping flow, respectively. The discretization order is critical and higher orders provide better results similar to bulk attenuation driven streaming simulations. The PDE is solved with Lagrange shape functions of quartic order. Lower orders introduce artifacts at the boundaries. This problem is even larger when the fluid domain is not coupled to a structural mechanics domain, but modeled with a fixed boundary without additional freedom. The velocity field $v_2$ and pressure field $p_2$ are solved with cubic and quadratic elements, respectively.

The implementation of the first order velocity field can be potentially improved with a weak formulation implementation. The Thermoacoustics Model in the frequency domain is another alternative implementation in COMSOL. This pre-implemented model additionally includes energy equations. Such an implementation reduces numerical artifacts at the boundaries and solves directly for the velocity and the pressure field. However, even in the adiabatic case with thermal conduction equal to zero, the Thermoacoustics Model implementation is computationally more expensive due to increased DOF.
3.2 Experimental Setup and Device Fabrication

The experimental setup and the manufacturing of devices are very similar for all the devices presented in Chapter 4.3 and Chapter 5. They are presented here in more detail while the device dimensions and specific materials used for each individual device are presented in the according chapters.

Device Fabrication

The devices focus on plastic as manufacturing material. Plastic potentially allows cheap mass production even with more complex geometries. In addition transparent plastic provides means for visual inspection. The material properties of plastic significantly deviates from other common materials used in acoustics such as glass, metal, or silicon as shown in Table 2.1 and Table 2.2. However, some of the latter materials also find application in the presented devices. This mainly for comparison and performance tests.

There are a number of different plastics which fulfill the basic requirements such as transparency and there are different methods to machine them. A good compromise between these options is achieved with a three layer structure and PMMA (acrylic glass). These layers are a main frame consisting of the side walls and a top and bottom cover plate or foil. An illustrated example can be found in Chapter 4.3. The main frame is laser or waterjet cut from a 3mm thick PMMA sheet. The laser cutting is done with a VersaLaser VL3.50 laser engraver machine with a beam diameter of about 0.2mm. The advantage of laser cutting is a glossy, rather transparent cut surface. The disadvantages are a relatively large cutting angle tolerance of up to 2° and large residual stresses in the material. The method is better suited for large feature sizes and \( w_{\text{wall}} = 1 \) mm is a critical lower limit. For thinner walls waterjet cutting is better suited. This method does not suffer from heat problems and creates only very small residual stresses. In addition cutting angle tolerances are well below 1°. The cut surfaces are smoother but with a mat finish. The PMMA components are glued together with GBL (gamma-butyrolactone) which dissolves PMMA forming a perfect bond. It strongly simplifies the assembling due to its low viscosity and long curing times under ambient conditions. The top and bottom cover plates with a thickness of 1mm are laser cut as well. Plates or foils which are thinner than 1 mm are mechanically cut to size after assembling.

PMMA is ideal when it comes to cutting to size and glueing. However, compared to some other transparent plastics the acoustic properties are not ideal with relative large attenuation as shown in Table 2.1. Therefore PS (polystyrene) is also used. PS parts are mechanically machined or assembled from prefabricated boxes. Furthermore, Type 1 polyester (hydroxy carboxylic acid) film from 3M finds application as top and bottom cover foil. No acoustic properties of this polyester were available but the acoustic performance in experiment indicates low damping, maybe similar to PET which is a Type 2 polyester. To reduce material damping even further 0.5 mm thick untreated or anodized aluminum plates are used. Either only the bottom cover plate or both bottom and side walls are replaced.

For any material other than PMMA either standard two component epoxy or A+P
Cyana 50 from Angst+Pfister is used. The epoxy forms a mechanical bond and therefore adhesion can be increased by roughening up the surfaces before they are glued together. Cyanolit is an instant glue which requires perfect pre-alignment of the parts and is also rather aggressive on any surface it has contact to. An advantage is that any leakage can be filled with the glue by capillary forces.

**Transducer and Wave Guide**

The devices are excited to vibration with Pz26 piezoelectric plate elements from Ferroperm Piezoceramics. They are operated in thickness extension mode and have a thickness of 1 mm resulting in an idealized center frequency of 2.067 MHz. The elements are cut with a wafer saw providing high grade cut edges. The electrodes are contacted with 0.15 mm thick enameled copper wires which add only very little mass to the electrodes and are highly flexible. The wires are attached using a small drop of CircuitWorks conductive epoxy CW2400 from ITW Chemtronics.

The piezoelectric elements are either directly attached to a sidewall of the device or are operated with a waveguide in between. To contact the electrode facing towards the device two approaches are used. One solution is with elements which overlap a device edge and thus provide a small contact area on the transducer electrode. Another approach is to form a small film on the device with excess conductive epoxy or silver paint such that the wire can be placed beneath the transducer.

The maximal pressure amplitudes which can be achieved with directly attached transducers are relatively good. However, additional bending modes of the transducer can lead to undesirable inhomogeneities in the pressure amplitude distribution. In addition heat and reproducibility can be an issue. These problems can be addressed with a waveguide between device and transducer. The waveguides are made of aluminum and provide passive cooling and can also be armed with active cooling measures. Such waveguides have been used for the square chamber device and are described in more detail in Chapter 4.3.

Piezoelectric elements with varying thickness have been used for higher and lower center frequencies. These elements are 2 mm and 0.5 mm thick and have a center frequency of 1.018 MHz and 4.113 MHz, respectively. Transducer arrays are an approach to address undesirable issues arising from plate bending modes of the piezoelectric elements. The arrays are produced by gluing a plate element on an aluminum support and cutting the plate into pieces. The centers of the elements are 1 mm apart with a spacing of 200 µm in between. The back of the electrodes are contacted with a thin aluminum film to keep mass loading small. Compared to the simpler waveguide constructions these arrays did not prove advantages in experiment and therefore found only limited use. Commercial transducer arrays often find application in conventional ultrasonic imaging. However these transducers are designed for pulsed operation and therefore have an absorbing back layer which strongly reduces performance in resonant systems.

**Experimental Setup**

The devices are placed under an Olympus SZH microscope with zoom capabilities. The microscope has a relatively large working distance range of 39 mm to 198 mm. It is
equipped with a double optical path which allows simultaneous inspection by eye and recording with a camera. The used camera is a Lynx IPX-2M30-G from Imperx. It is a color progressive scan camera with 33 fps at 1600 px × 1200 px. The camera is connected to a computer for control, live streaming, and recording. The schematic setup is shown in Figure 3.4. The devices are illuminated with one or more flexibly placed light sources.

![Figure 3.4: Schematic of the experimental setup around a square transparent chamber with transducer.](image)

The piezoelectric element is driven with a function generator which is connected to an ENI 2100L RF power amplifier. The amplifier is not voltage stabilized and therefore the voltage varies with the frequency. These variations are not linear. Function generators used are either a SRS DS345, 30MHz synthesized function generator or an Tektronix AFG 3022B function generator.

Devices with a fixed transducer are loosely placed on a small, thin, and transparent film strip which is strained over two bars. Transducers which are not fixed to the device are clamped as shown in Figure 3.5. This mainly applies to the square chamber described in Section 4.3.1. The figure shows a configuration with an aluminum waveguide which

![Figure 3.5: Clamping of a square device from one or two sides. The waveguides have tubing for active cooling.](image)

has tubing for active cooling. A square chamber can be actuated from one side as shown in the left image or from two sides as shown in the right image. The configuration is
quite open such that the devices can be illuminated from many different angles including
top and bottom illumination. This eases access to the device to fill or refill the device.
Decoupling of the device from the surrounding greatly improves its performance. Good
results have been obtained with pairs of wooden spacers made of birch wood toothpicks
with a diameter of 2 mm. They are placed symmetrically on each side. The distance
between two spacers is typically half a device length. Other approaches like a metal grid
with a zig-zag pattern or different types of foam did not show as good results as obtained
with the wooden spacers. Softer materials allow more tolerances and therefore birch wood
has been chosen over beech tree toothpicks.
The influence of the clamping pressure has been found to be rather small as long as it is
small enough not to deform the device and high enough to provide a firm fit. Therefore
no precise measurement has been done and the clamping pressure has been set by hand.
Grease-less clinical ultrasound gel has been used to enhance the ultrasonic coupling
between transducer or waveguide and device. In some cases a simple water film has been
used instead. Drying or evaporation of these films is an issue with long term experiments,
particularly without actively cooled waveguides and high driving voltages.
The tubing of the actively cooled waveguides have been connected to a Paar Physica
Viscotherm VT2 cooling machine. A syringe pump Model 200 Series of KD Scientific has
been used for flow through experiments.
Most of the plastic devices presented have two small openings which allow to fill
the chamber with suspension or water. Air inclusions in the fluid chamber are easily
introduced. In order to avoid this the chambers have to be filled against gravity which
means that first the chamber is put up in an angle such that one opening is at the highest
point of the device. Next, the fluid is poured in the chamber from the second hole which
then typically is somewhere close to the lowest part of the device. This works particularly
good with hydrophobic devices. Only the first drops of liquid require some increased
pressure. Finally the liquid is sucked into the device against gravity due to the surface
tension and contact angle. This assumes that the opening is significantly smaller than
the fluid chamber cross section.

3.3 Schlieren Visualization
The schlieren method is a proven tool for the experimental characterization of optically
transparent liquids and for the visualization of ultrasonic fields [124, 114, 19]. This method
makes visible spatial variations in the refractive index of the liquid. Thus any quantity
related to the refractive index can be visualized, these are e.g. temperature variations, or
density variations caused by fluid flow or by acoustic waves. The method is similar to
shadowgraphy and first known reports of a schlieren setup are from R. Hooke in the year
1665 for the visualization of thermal disturbances in air. The term schlieren goes back to
A. Töpler who reported the first advanced schlieren setup in the year 1864. A more modern
account on the schlieren technique in air is given by Settles [155]. Reports suggest a large
variety of different setups based on mirrors and transparent lenses including bidirectional
setups with a mirror as background [157]. Specialized methods are e.g. the background
oriented schlieren method [138], the color schlieren method [155] or rainbow schlieren [75]. Recent reviews [39, 69] have successfully used these methods for quantitative analysis of thermal and flow applications. An example of the quantification of the pressure density with real-time pulsed schlieren imaging is given by [68]. Methods similar to schlieren visualization are e.g. phase contrast and interferometry methods such as the differential interference contrast method. There are also commercially available schlieren systems like the OptiSon ultrasonic beam analyzer from Onda Corporation.

Application fields of schlieren visualization are e.g. energy optimizations, safety test or material testing. A large area is the visualization of fluid flows or of sound shock waves [165, 155].

For ultrasonic particle manipulation devices, particle tracking [6] is also a very strong tool to characterize the pressure field or to visualize fluid flow. While ultimately both methods can be combined, the schlieren method offers some unique advantages. In contrast to particle tracking, its timescales are not limited by drag forces so it is much faster and, with the proper equipment, even able to measure traveling waves. Furthermore, it is a non invasive method what means no seeding particles are required which might influence the fluid properties or the acoustic field. Other advantages of an optical method consist in the fact that the complete area of interest can be imaged simultaneously and over a long time frame even for fast frequency changes in real time while resolving complete pressure waves of a standing wave field. The resolution and frequencies of the ultrasonic field which can be imaged are mainly limited by the optical wavelength and the resolution of the optical components used. Reports show schlieren images of ultrasound in water of over 100 MHz used in medical applications [191]. The structure containing the fluid, like any obstacle in the optical light path, causes interference fringes which can be a limiting factor for small devices or if one is interested in features close to a wall. The same problem can arise when using the schlieren method in combination with particles.

Method and Setup

A schlieren setup has been designed and built to investigate a mm-sized square chamber presented in Chapter 4.3. A first proposal and preliminary tests of the setup are presented in N. Degen’s bachelor thesis [27] and a refined setup is presented in Möller et.al. [111]. The setup is a conventional (Töpler) three lens schlieren system which has been mounted vertically as shown in Figure 3.6. The acoustic devices have a height of 3 mm which is the same as the assumed acoustic beam thickness $L$. They are operated in the lower MHz frequency at wavelength $\Lambda$. Assuming the optical axis to be parallel to the ultrasonic wave front and an optical wavelength $\lambda$ of at least 700 nm the Klein-Cook parameter $Q = \frac{2\pi\lambda L}{\Lambda^2} \ll 1$ and thus Raman-Nath diffraction [135] is expected. In other words, the acoustic beam thickness $L$ or length that the optical waves cross the acoustic waves, is sufficiently small for the effect of multiple diffraction to be neglected.

For Raman-Nath diffraction the diffraction angle $\theta$ is given by $\sin \theta = \frac{\lambda}{\Lambda}$ for the first intensity peak. In literature [124] the Bragg diffraction angle for one main peak is used interchangeably since both angles are approximately the same. The diffracted beam separation distance $\varepsilon = \theta f_2$ is a measure for the sensitivity of the schlieren setup.
Compared to higher ultrasound frequencies, the requirements here are reduced for camera and camera objective to resolve a single wavelength with the same resolution. However, with lower frequencies the requirements on the quality of the optical components and the filtering are increased due to a smaller $\theta$. For a large ratio of focal length $f_1$ to pinhole diameter $d_p$ the relation $\alpha \propto d_p / f_1$ holds, where $\alpha$ is the beam deviation angle at the collimation lens $L_1$ which should be as small as possible. For $d_p$ there is a lower limit in the order of the Airy disc and Rayleigh criterion $d_p = 2\sqrt{2} f_1 \lambda$.

For the work presented here the two field lenses ($L_1$, $L_2$) have a lens diameter of 50 mm where lens $L_1$ is a planoconvex lens with a focal length of 250 mm and lens $L_2$ is an achromatic lens doublet with a focal length of 400 mm which is also corrected for spherical aberration and coma. A modified high power white LED with a 300 $\mu$m pinhole has been used as light source. Since the acoustic devices are operated in quasi stationary mode relatively long camera exposure times can be used as compared to imaging of traveling waves. This eases the requirements for the light source and camera in terms of brightness. In the cut-off or Fourier plane either a knife edge filter which allows to emphasize directional effects dependent on its rotational position or a dark field filter has been used. The filters are placed on micrometer sliders for accurate positioning. The camera objective is a Minolta 135 mm and the camera is a Lynx IPX-2M30-G resulting in a spatial resolution in the order of 20 $\mu$m/pixel. The focal plane of the objective is at the position of the acoustic device resulting in an image where the spacing of the interference fringes is equal to the ultrasonic wavelength multiplied by the lateral magnification of real images [124]. The single components of the setup can easily be exchanged.

The quality of the imaging setup has the potential to be improved in order to record with higher sensitivity and homogeneity e.g. with a more sophisticated lens setup and light source. For high speed recordings laser light sources can be of interest. Additional improvements can be achieved by introducing color filters and light blocking filters of
different shape [154] or with more advanced spatial filtering [170].

3.4 Particle Image Velocimetry

Particle image velocimetry (PIV) is an optical technique which uses particles to determine the velocity field in fluid dynamics. The tracer or seeding particles should be small enough not to disturb the flow field and have a density similar to the suspension to minimize gravitational or inertial effects. The velocity is determined from the particle positions captured at two or more time instances. The particle positions are typically captured in 2D. PIV is covered broadly in literature and a more detailed description can be found in [134] as an example. A related technique is particle tracking velocimetry (PTV) where the positions of the particles are explicitly tracked e.g. in a 3D space. Recent reviews of microfluidic velocimetry techniques which find application in acoustofluidic devices are given by [188, 100].

The experimental setup used is similar to the one shown in Figure 3.4. The illumination is replaced with a light sheet as shown in Figure 3.7. This configuration requires transparent side walls which do not strongly distort the light beam. Thus waterjet cut transparent plastic cannot be used and even laser cut plastic is not optimal. The light sheet is generated with a solid state laser Ciel 300mW from Laser Quantum, equipped with a light sheet beam expander. Alternatively a green laser module with cylindrical rod lens as beam expander is used. Seeding particles are dedicated PSP-20 polyamid particles from Dantex Dynamic with a diameter of 20\(\mu\)m. However, these particles can be influenced significantly by acoustic radiation forces. Therefore also smaller particles like 9\(\mu\)m copolymer spheres have been used.

There are a number of free and commercial PIV software tools available. The open source software PIVlab 1.13 of Thielicke and Stammhuis is used here. This is a Matlab based tool with a graphical user interface. To enhance the results at least 30 time frames are used to calculate a velocity field.
Acoustic Radiation Force Driven Particle Transport

There are a number of different transport methods based on acoustic radiation forces. They are briefly introduced in Section 4.1. Here the focus is on the continuous frequency sweeping method which is discussed with a 1D analytical model in Section 4.2.1 and realized with a square mm-sized chamber presented in Section 4.3. Resonant mode switching can be realized with the same system as continuous frequency sweeping and is mentioned in the discussion as a comparison. The aim to build a system with a closed chamber arises for biomedical applications. The chamber is preferably disposable and easy to manufacture which can be realized with plastics. Finally, in Section 4.4 coupling of the waves into the fluid is discussed which is important for the design of continuous frequency sweeping based devices.

4.1 Transport Methods

Moving particles over distances in the order of wavelengths with acoustic radiation forces can be achieved with a number of different approaches. The basic requirement is a pressure gradient strong enough to trap particles and with a change slow enough to overcome drag forces. The position of the force gradients can be controlled by changing the frequency, amplitude, phase, and space function, or any suitable subset of these. Approaches can be loosely referred to as alteration of physical boundary conditions, using transducer arrays, setting up a beat frequency, amplitude modulation, phase control, and continuous frequency sweeping.

Alterations of physical boundary conditions are: The position change of a reflecting wall [130], or the displacement of an entire half open device [64], or the displacement of a transducer [65]. The latter two allow to transport particles trapped in nodal planes with a constant speed while all three have the advantage to operate the transducer at its resonance frequency. Such systems have mechanically moving parts and in the cited cases open or changing fluid volumes which can be a drawback.

Transducer arrays can be modulated to obtain a pressure maximum in a desired lo-
cation. Examples are planar [49] and linear concave transducers arrays [87] which can move particles in transverse direction to the sound beam axis. These devices can be very versatile but often suffer from smaller pressure gradients in transverse direction compared to the gradients in normal direction to the sound beam. A more complex example is a circular transducer array [57] which allows to position particle clumps locally.

Beat frequencies with a slow group velocity arise from two oppositely traveling waves with a small frequency difference. Such beat frequencies are also referred to as pseudo standing waves [130] or drifting fields [60]. Numerous mechanisms to create these pseudo standing waves are proposed [130, 183]. Examples are alteration of boundary conditions, two oppositely placed transducers with absorbing back layer end or changing the frequency very fast so that source and reflected wave superimpose with a sufficient frequency difference. Pseudo standing wave devices can suffer from a predominant true standing wave [129]. In addition the pressure gradients are lower compared to resonant devices.

Amplitude modulation can be used to create a slowly translating standing wave as a result from two superimposed modulated standing waves. This gives a perfectly homogeneous movement of pressure nodes but it also sets up restrictions on the boundary conditions which are hard to meet. Possible examples are ring shaped devices [65] or planar open devices [64].

Phase control can be used to control the position of the nodes of a standing wave generated by two counter propagating waves. The standing wave again has a homogeneous movement, similar to the amplitude modulation, however, the amplitudes are typically lower since the devices do not operate in resonant mode. One example is a cavity with matching and attenuation layers at two opposing transducers to absorb the oncoming waves [23]. Another example is the use of crossing sound beams to avoid reflections. The latter example can be used for three-dimensional manipulation [85].

Resonant mode switching is exciting resonant standing waves and moving particles by changing the mode number and with it the position of the pressure nodes. With a large number of wavelengths per cavity length particles can be moved in small incremental steps [148]. This technique is well suited to be combined with other techniques like continuous frequency sweeping or transducer arrays [86, 87].

Most of the previously mentioned examples are rather large scale. Surface acoustic wave (SAW) devices, driven with interdigital transducers, are a good alternative to bulk transducer driven devices for micro particle transport. Examples are devices which use the phase control method [126, 125], or resonant mode switching [30]. Large scale examples with particles in air, instead of fluid, are mode switching [167], or flexural plate vibrations combined with suitable geometrical boundary conditions [10].

**Continuous Frequency Sweeping**

The method of a continuous frequency change or frequency sweeping makes use of the pressure field between resonances to alter the particle path. Apart from this, continuous frequency sweeping is similar to resonant mode switching. Figure 4.1 illustrates the principle. A particle trapped in a pressure node at $x_1$ moves in positive $x$-direction when the frequency is increased as indicated with a black solid line. In contrast, with resonant
mode switching a particle moves to the nearest pressure node. Increasing the frequency continuously from mode number 3 to 6 moves the particles located at $x_1$, $x_3$, and $x_5$ to $x_2$, $x_4$, and $x_6$, respectively. Continuous frequency sweeping has been realized in various devices, e.g. in a planar device [64], or in tubular devices [164, 184], or in small [153] and micro [106] scale devices. The method is very close to mode switching and can have similarities with the pseudo standing wave method since a changing frequency is used in both cases. Both, resonant mode switching and continuous frequency sweeping can be operated with repeating frequencies to transport particles over larger distances than what a single frequency range gives. The condition is that the last position of a particle is closer to a new node than to the last. As an example the particle at $x_3$ cannot move to $x_5$ if the frequency is only increased from mode number 3 to 4.

Figure 4.1: Principle of continuous frequency sweeping. The pressure field of resonant standing waves are indicated with light gray shades and the arrows indicate a particle travel path. Sweeping the frequency three times moves a particle from $x_1$ to $x_7$.

4.2 Continuous Frequency Sweeping Model

The basic principle of continuous frequency sweeping is described in Section 4.1 together with a brief introduction of similar methods. Continuous frequency sweeping and mode switching are proven tools for particle transport and have been applied in a variety of devices [64, 164, 184, 153, 106, 102]. However, most publications are of purely experimental nature and lack a proper explanation. Lipkens et.al. [101] gives a two-dimensional theoretical model for particle trajectory calculations and provides some basic explanations. Here, an analytical model discussion is presented which includes detailed particle paths. The difference between boundary conditions and the expected particle paths is studied in a one dimensional wave model for both techniques. Solutions to the wave equation subject to different boundary conditions can be found in fundamental wave motion text books such as Fahy [41] or in the context of ultrasonic devices by Bruus [15, 13]. These solutions typically lack a discussion of the lossy wave equation (2.16) over a varying frequency range. However, as will be shown here, continuous frequency sweeping strongly relies on attenuation which necessitates such a discussion.
4.2.1 Analytical Model

The one dimensional wave equation is a special case which is directly satisfied by displacement or velocity field variables compared to the three dimensional case which is only satisfied by pressure and velocity potential. While the displacement may serve as a more intuitive quantity, the pressure can be better related to the acoustic radiation potential (2.29). For a plane standing wave \( p(x,t) = \hat{p} \cos(k_c x)e^{-i\omega t} \) the potential simplifies to

\[
U = \pi r^3 \kappa_0 \hat{p}^2 \left( \frac{1}{3} f_1 \left| \cos^2(k_c x) \right| - \frac{1}{2} f_2 \left| \sin^2(k_c x) \right| \right).
\]  

(4.1)

Thus \( U \) is proportional to the absolute value of the pressure squared for particles which collect at the pressure nodes and with an additional phase shift of \( \pi/2 \) for particles which collect at the pressure antinodes. For simplicity the following descriptions assume the former type of particles and can be applied in analogy to the latter type by incorporating the phase shift. Further assumed is the presence of a number of particles at a time and distributed over the pressure field. The situation is different for single particles where more frequency modulation options appear.

For ease of use the solution of an ordinary wave equation with a complex wave number, as presented in Section 2.1, is used here. In a one-dimensional system the solution of a standing wave can be written as linear superposition of a right moving and a left moving wave. The ansatz needs to satisfy a complex time harmonic function as shown in Section 2.5. It proves convenient to write the time harmonic solution in the form

\[
p(x,t) = \hat{p} \cos(k_c x + \varphi)e^{-i\omega t},
\]  

(4.2)

where \( \varphi \) is the phase angle.

The solution within a cavity ranging from \( \ell_1 \) to \( \ell_2 \) is discussed subject to three different sets of boundary conditions. All three cases have an acceleration boundary condition with amplitude \( \hat{a}_n \) resulting in

\[
\frac{\partial p}{\partial x}(x) = \rho_0 \hat{a}_n e^{-i(\omega t + \varphi_t)}
\]  

(4.3)

at \( x = \ell_1 \) and with \( \varphi_t = 0 \) in common, where \( \varphi_t \) is the phase difference in time. For \( \omega \) it has been anticipated that the system response frequency equals the excitation frequency. The second boundary at \( \ell_2 \) is either a soft wall, a hard wall, or a second excitation boundary condition. First these three cases are presented, followed by a general discussion of the solutions.

The following discussion includes illustrations of attenuated pressure fields indicated with the absolute value of the complex pressure \( |p| \). Unlike \( |\text{Re} (p)| \) which shows the pressure field for a time instance and thus also with phase, the complex modulus or magnitude of the pressure \( |p| \) shows the time independent pressure amplitude.

**Excitation – Soft Wall Boundary Condition**

Solving the equation for a standing pressure wave (4.2) subject to the excitation boundary condition eq. (4.3) at \( x = \ell_1 \) and the soft wall boundary condition eq. (2.22) at \( x = \ell_2 \)
results in

\[ p = \rho_0 \hat{a}_n \frac{\sin (k_c (x - \ell_2))}{k_c \cos (k_c (\ell_1 - \ell_2))} e^{-i\omega t}. \]  \hspace{1cm} (4.4)

Resonances are identified in the unattenuated case where \( k_c = k \) and when the denominator equals zero. Thus the resonance condition is

\[ \ell = \frac{\lambda}{4} (2n - 1), \quad n \in \mathbb{N}, \]  \hspace{1cm} (4.5)

where \( \ell = \ell_2 - \ell_1 \) is the cavity length.

Figure 4.2 shows the absolute value \(|p|\) of the complex pressure field (4.4) as color plot for frequency \( f \) over distance \( x \). The excitation boundary is at \( \ell_1 = 0 \) and the open boundary at \( \ell_2 = 1 \). The plot is shown for a constant spatial attenuation coefficient with a value which equals to \( Q = 10f \) and with \( \hat{a}_n \sim f \) and the wavelength equals unity at \( f = 1 \). Four consecutive resonances are shown where the lowest is the third cavity resonance mode. The pressure field shows a significant contribution between resonances due to attenuation. These intermediate fields lead to different travel paths of particles moved by acoustic radiation forces with continuous frequency sweeping when compared to mode switching. This is illustrated with a further idealized model in the second plot of Figure 4.2. The figure shows isolines for \( p = 0 \) (gray lines) and linear connection lines between the closest resonant pressure nodes with solid black lines. The isolines are obtained from the weakly damped and real valued pressure field with \( \omega t = n\pi \) for all \( n \in \mathbb{N} \) and a constant amplitude. In such an illustration of the pressure field horizontal isolines with \( p = 0 \) appear at the resonances due to the frequency dependent phase shift. Here the focus is on the phase independent pressure field where no horizontal isolines are present. The horizontal light gray lines indicate the resonances in the figure.

Particles trapped in pressure nodes follow the isolines when the frequency is changed continuously with the given assumptions. Particles move towards or away from the excitation boundary when the frequency is decreased or increased, respectively. With mode
switching where only the resonances are used, particles will move towards the closest pressure nodes as indicated with the black lines. Thus, in contrast to a continuous frequency sweeping, particles will move either towards the center, or towards the boundaries. This happens in an asymmetric way where the black lines indicate particle paths for increasing frequency only. For decreasing frequency the paths are the same except for nodes adjacent to the center of the cavity at \( x = 0.5 \). Hence, particles can be moved from one cavity boundary to the other, even with mode switching, if the frequencies are chosen accordingly. That is, first all particles are moved towards the center by decreasing the frequency and then towards the excitation boundary if the frequency change is reversed at an even mode number, or towards the open boundary if changed at an odd mode number. However, for large particle clumps this is not practicable since then not all particles move to one side selectively.

**Excitation – Hard Wall Boundary Condition**

The solution of a standing pressure field with an excitation boundary condition eq. (4.3) at \( x = \ell_1 \) and a hard wall boundary condition eq. (2.24) at \( x = \ell_2 \) is similar to the excitation and soft wall boundary condition case, but with a spatial phase shift of \( \varphi = \pi/2 \). The solution is

\[
p = -\rho_0 \hat{a}_n \frac{\cos (k_c (x - \ell_2))}{k_c} \sin (k_c (\ell_1 - \ell_2)) e^{-i \omega t}
\]

and the resonance condition thus is given with

\[
\ell = \frac{\lambda}{2n}, \quad n \in \mathbb{N}.
\]

Figure 4.3 shows the same plot types as Figure 4.2 in the previous case but for equation (4.6). The resonances now show a symmetrical pressure field and their frequencies are

![Figure 4.3: Case described with eq. (4.6) which has an excitation at \( x = 0 \), and a hard wall boundary at \( x = 1 \). Dark blue denotes minima of |\( p \)|, and dark red maxima of |\( p \)|. Isolines are shown as gray lines, and particle paths for mode switching as black lines. These linear black lines are introduced between the position of the closest pressure nodes of consecutive resonances.](image)

shifted by \( c_0/4\ell \) according to the change in phase compared to the soft boundary case.
The pressure field between the resonances however, still is strongly asymmetric. Unlike for the open boundary case particles cannot be moved to the boundary but only close to it. Nonetheless, the basic behavior is very similar in both cases. For a continuous frequency change particles are again moved towards the excitation with a frequency increase and towards the hard boundary when decreasing the frequency. For mode switching the pressure node distribution is fully symmetric and particles can only be moved either to the center of the cavity or equally towards the left and right boundaries as illustrated in the isolines plot.

**Excitation – Excitation Boundary Condition**

The third case is with two excitation boundary conditions given with eq. (4.3). One boundary is at $x = \ell_1$, and the other at $x = \ell_2$. The standing pressure field for the fully symmetric case with a phase difference between the two excitations of $\varphi_t = \pi$ is

$$p = \rho_0 \hat{a}_n \frac{\cos \left( k_c \left( x - \frac{1}{2} (\ell_1 + \ell_2) \right) \right)}{\sin \left( \frac{k_c}{2} (\ell_1 - \ell_2) \right)} e^{-i\omega t}. \quad (4.8)$$

The solution for the case with equal excitations at both boundaries ($\varphi_t = 0$) has a phase difference of $\varphi = \pi/2$ compared with the fully symmetric case (4.8). The corresponding resonance conditions are

$$\ell = \lambda n \quad \text{and} \quad \ell = \lambda \left( n - \frac{1}{2} \right), \quad n \in \mathbb{N} \quad (4.9)$$

which have a factor of $\lambda$ compared to $\lambda/2$ for the two former cases.

Figure 4.4 illustrates the pressure field (4.8) for the same parameters as the previous two cases, again with normalized $|p|$ as color plot and isolines as gray lines. The center of the cavity forms the symmetry axis and thus the derivative of the pressure along it...
equals zero which is the condition of a hard wall boundary. Thus all resonances have a pressure antinode in the center and particles can never reach the very center of the cavity. Nonetheless, the particles move towards the center when continuously increasing the frequency, or they move from the center towards the boundaries when decreasing the frequency.

With mode switching and increasing frequency particles are moved towards the center and the boundaries at the same time and in equal parts. When decreasing the frequency the particles move towards the pressure nodes of the first resonance which are at $\ell/4$ and $3\ell/4$, respectively.

Two more cases are illustrated in Figure 4.5. One case is with $\varphi_t = 0$ between both excitations. This case can be regarded equivalent to the $\varphi_t = \pi$ case, but with a soft wall condition as symmetry plane and the according phase shifts of resonances and pressure nodes. The second plot shows the double excitation case for $\varphi_t = \pi/2$. The pressure field can be described with the superposition of two hard wall boundary condition field solutions (4.6) with exchanged excitation boundaries where one has a phase shift $\varphi_t = \pi/2$. The isolines are similar to the hard wall boundary condition case but with alternating sign accentuation between the resonance modes.

![Figure 4.5: Cases which have two excitation boundaries ($x = 0$ and $x = 1$), with phase shifts of $\varphi_t$. Dark blue denotes minima of $|p|$, and dark red maxima of $|p|$.

For increasing mode numbers and between odd and even modes, the field is similar as with an excitation at $\ell_1$ and between the other modes as if the excitation is at $\ell_2$. For a negative phase shift $-\pi/2$ this order is reversed.

Double excitation can be particularly interesting for mode switching since the pressure amplitudes are large compared to the single excitation cases. In simplified terms the amplitudes are doubled and increased by the factor $\sqrt{2}$ in the case of a resonance with zero or $\pi$ phase shift and in the case with $\pm\pi/2$ phase shift, respectively.

### 4.2.2 System Design Criteria

Continuous frequency sweeping relies on frequencies between the resonance frequencies to transport particles. However, not the whole frequency spectrum is required. Furthermore, the particle transport velocity depends on several parameters. In this section these
parameters are discussed and guidelines to design a system are given. The presented discussion is based on the analytical solutions given in Section 4.2.1.

**Resonances and Particle Velocity**

From the resonance conditions for the soft (eq. (4.5)), and hard wall (eq. (4.7)) boundary case resonances appear with a factor of $\lambda/2$ between resonances and for the symmetrical boundary case (4.9) this factor is $\lambda$. Thus for the former case the frequency difference $\Delta f_{\text{res}}$ between two consecutive resonances is

$$\Delta f_{\text{res}} = f_{\text{res}2} - f_{\text{res}1} = \frac{c_0}{2\ell},$$

where $f_{\text{res}1}$ and $f_{\text{res}2}$ are the resonance frequencies of the lower and higher resonance mode, respectively and $\ell$ is the cavity length. With the spatial attenuation $\alpha_s \propto f^2$ this difference $\Delta f_{\text{res}}$ then results in a change in attenuation between two consecutive resonances of

$$\frac{\alpha_{\text{res}2}}{\alpha_{\text{res}1}} = \left(\frac{c_0}{2\ell} + f_1\right)^2 f_2^2.$$

Hence, the assumption previously made for Figures 4.2 to 4.5 of a constant attenuation coefficient is feasible for $f \gg c_0/2\ell$. For a system with water at 1 MHz and a cavity length of 1 cm this change (eq. (4.11)) is in the order of 1%. Assuming that particles follow the isolines, the change in position per frequency is

$$\frac{\partial x}{\partial f} = \frac{\ell_2 - x}{f},$$

for the soft, and hard wall boundary case. The gradient is the largest for $x \to \ell_1$ and a large ratio of $\ell$ to $f$. Gradient times frequency sweep rate gives the local particle velocity $v_p$. The sweep rate $f_{\text{rate}}$ is defined as the number of sweeps per second. Assuming a constant $f_{\text{rate}}$ the velocity can be expressed as

$$v_p = f_{\text{rate}}f_{\text{span}}\frac{\ell_2 - x}{f},$$

where $f_{\text{span}}$ is the sweep span defined as end frequency minus the starting frequency of a single sweep.

**Attenuation**

The conditions for a purely standing wave are only fulfilled where a reflection condition is given in a system with attenuation in the fluid and no reflection losses. In the soft, and hard wall boundary case this is at $x = \ell_2$ and in the symmetric excitation case at $x = 0.5$. In Figure 4.2, for example, the pressure $|p|$ is only zero at $x = 1$ where the wave is reflected. In contrast, at the excitation or $x = 0$ the amplitude ratio between excited wave and reflected wave is the largest and thus the traveling wave field contribution is the
largest there. For systems where the main attenuation originates from the fluid properties, the space dependent attenuation builds a limitation for continuous frequency sweeping when the traveling wave starts to become significant in comparison to the standing wave. However, in most cases it is feasible to assume that the standing wave is homogenous in space, given a system with water which has a rather low attenuation (Table 2.1). In other words, the maximal pressure amplitudes do not significantly change over distance.

Critical Limit of Attenuation Influence

For the discussion of isolines such as in Section 4.2.1 a constant pressure amplitude is assumed. In real applications the pressure field can often be expected to be similar to a case as shown with the color plots in Figures 4.2, 4.3 and 4.4. This rises the question how far off-resonance a pressure field has to be established. For the soft wall and hard wall boundary condition the situation is most critical for the pressure node closest to the excitation. As an example, the zero pressure isolines for the hard wall boundary condition case are

\[ x_{\text{iso}} = \frac{\lambda}{4} (1 - 2n) + \ell_2, \quad n \in \mathbb{N}, \quad (4.14) \]

where the argument is chosen such that the isolines include the critical pressure node when \( n \) equals the corresponding mode number. A frequency range \( f_{\text{var}} \) expanding from the resonance is defined. Within this range it is assumed, that particles will follow the isolines, as indicated in Figure 4.6. The range \( f_{\text{var}} \) is defined to be constant between two resonances. The inequality

\[ x_{\text{iso}} (n, f (n) + f_{\text{var}}) > \frac{1}{2} \left( x_{\text{iso}} (n + 1, f (n + 1) - f_{\text{var}}) + x_{\text{iso}} (n, f (n + 1) - f_{\text{var}}) \right) \quad (4.15) \]

defines \( f_{\text{var}} \) such that the particles follow the isolines rather than the nearest pressure nodes of the resonances. For the hard wall boundary case this results in the condition

\[ f_{\text{var}} > c_0 \frac{n - 1}{2 \ell 4n - 1}, \quad n \in \mathbb{N}. \quad (4.16) \]

Figure 4.6 illustrates the particle paths when the influence of the pressure field in the region \( f_{\text{var}} \) is taken into account. Curved gray lines show the isolines around the resonances and solid black lines the link in between.

For the soft wall boundary case the zero pressure isolines are

\[ x_{\text{iso}} = \frac{\lambda}{2} (1 - n) + \ell_2, \quad n \in \mathbb{N} \quad (4.17) \]

and \( f_{\text{var}} \) can be formulated similarly to the hard wall boundary case as

\[ f_{\text{var}} > c_0 \frac{2n - 3}{4 \ell 4n - 3}, \quad n > 1, \quad n \in \mathbb{N}. \quad (4.18) \]

Assuming a device with only one excitation site, systems operating with high mode numbers can be designed to fulfill at least

\[ \lim_{n \to \infty} f_{\text{var}} (n) = \frac{\Delta f_{\text{res}}}{4}. \quad (4.19) \]
4.3 Square Chamber

The square chamber is introduced as a device operated with continuous frequency sweeping and is the main example for the method in this work. First, a systems description is given in Section 4.3.1. Next, numerical simulations are presented in Sections 4.3.2, Section 4.3.3, and Section 4.3.4. Finally, experimental work is presented in Section 4.3.5, Section 4.3.6, and Section 4.3.7.

4.3.1 System Description

An illustration of the square device is shown in Figure 4.7, including particles suspended in blue water and a single transducer element. This configuration allows to move particles back and forth in $x$-direction. Particles can also be moved simultaneously in $x-$ and $y$-direction with the help of a second transducer placed orthogonally to the first. Thus, particles can be agglomerated in a corner. The device has been designed for a possible application of nucleic acid concentration in a POC device. Therefore, the size of the

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Figure 4.6: Case which has an excitation at $x = 0$, and a hard wall boundary at $x = 1$. Illustrated are idealized particle paths if only frequencies with $f_{\text{var}}$ around the resonance frequencies are excited. Isolines are shown as gray curved lines, and the particle path in between as solid black lines.

Figure 4.7: Square chamber with transparent plastic walls, filled with particle suspension and a transducer attached to a side wall.
fluid chamber is chosen such that it can contain a volume of at least 1 ml. Furthermore, the device is closed and a focus is on plastic as device material. Details of the device manufacturing and experimental setup are given in Section 3.2. The following description focuses on the materials, dimensions, and configurations used here.

**Device**

A typical device is constructed with a three layer structure and made of PMMA. The layers are a 3 mm high main frame consisting of the four side walls, and a top and bottom cover as shown in Figure 4.8. The dimensions of a reference design are given here. They are an outer length of \( \ell = 22 \) mm, a frame wall thickness \( w_{wall} = 1 \) mm, and a top and bottom cover layer with \( w_{cover} = 0.25 \) mm. Furthermore, PMMA devices have been used which have 0.5 mm thick frame walls and are combined with cover thicknesses of \( w_{cover} = 0.5 \) mm and 0.175 mm. Both, laser and waterjet cutting have been used to cut the main frame. The top cover has two holes with a diameter of 1 mm to 1.5 mm placed in two opposite corners, as indicated in Figure 4.7.

In order to reduce damping PS (polystyrene) devices have been used which have been manufactured from prefabricated boxes with \( \ell = 23 \) mm and \( w_{wall} = 1 \) mm where the bottom cover and the side walls build a single piece. The top cover is either a 1 mm PS plate or a 100 µm thick polyester film. These polyester films have also been used in combination with PMMA frames. To reduce material damping even further, 0.5 mm thick untreated or anodized aluminum plates have been used. Either only the bottom cover plate or both, bottom and side walls have been replaced.

**Transducer and Wave Guide**

The piezoelectric elements are either directly attached to a sidewall of the device, or are operated with a waveguide in between. A typical waveguide cross section with dimensions is shown in Figure 4.9 where the position of the transducer is indicated as hatched rectangle. The waveguide is made of aluminum which provides passive cooling. Addition-
ally active cooling is introduced. This is achieved with 4 mm diameter aluminum tubing and coolant flowing through. Waveguide and transducer are either glued together with conductive epoxy or standard epoxy (Araldit Rapid). The relatively large contact area between transducer and waveguide requires special attention in order to avoid air inclusions in the glue layer while keeping the layer as thin as possible. This is because the glue layer has a particularly high damping and can thus influence the overall performance. A good glue layer can be obtained by adding a glue drop in the center between transducer and waveguide and remove any excessive glue after both have been put together under load. Curing is done under load.

### 4.3.2 Frequency Response Simulations

Isolines of zero pressure level abstract the full pressure field to the locations where particles with a density and compressibility larger than the surrounding medium will collect. They are useful to illustrate data sets with large pressure amplitude variations. The isolines are shown in plots of frequency over cavity length, similar to the 1D analytical discussion in Section 4.2.1.

First a 1D reference simulation is presented which serves as reference for experimental work. Furthermore, the 1D simulations are used to discuss the influence of the device wall thickness, fluid resonances, and amplitude distributions. A detailed description of the numerical model is given in Section 3.1.1.

#### 1D Reference Simulation

Figure 4.10 shows isolines of the fluid cavity in gray. The isolines are obtained from a 1D simulation of the pressure field with $\omega t = \pi$. Hence a phase shift of the solution at a resonance results in an additional contribution to the isolines as discussed in Section 4.2.1. In an unattenuated case these additional isolines are perfectly horizontal lines in the figure. Here the attenuated case is treated where isolines are generally not constant over time. The effect of attenuation is the largest at small $x$ and large $f$ where the distortion of isolines is the largest. Locally, this change is the largest close to the resonance frequencies as discussed in Section 4.2.1. However, the non-horizontal isolines shown here are still a reasonable approximation of the acoustic radiation force potential minima.
In Figure 4.10, the black arrows indicate the travel path for an increasing frequency and a selected particle which is trapped in a potential minimum. A particle positioned at the wall close to the transducer at $x=0$ mm moves up to $x_1=8.3$ mm, when the frequency is continuously increased from 1.5 MHz to 2.5 MHz. As indicated, repeating this sweep 6 times moves the particle up to $x_6$, close to the opposing wall. With an additional 7-th sweep the particle can be move to the very wall at $x=20$ mm.

The acoustic field is dominated by the outer boundary between PMMA wall and air rather than by the interior boundary with the water, due to a larger impedance step there. Thus, the pressure node position in average over the frequency range, at both water cavity boundaries, is closer to the PMMA boundary than in a case where the wave reflection is dominant at the water cavity boundaries. Examples of the latter case are the cases treated in the analytical model Section 4.2.1. A good estimate of the impact from each boundary can be obtained by comparing the specific acoustic impedances given in Table 2.2. The energy reflection coefficient (2.43) between water and PMMA is 0.14 while it is approximately 1 between air and PMMA. Hence, changing the wall thickness alters the pressure field in the fluid cavity significantly.

**1D Simulation of Wall Thickness**

The influence of different wall thicknesses is as expected from the analytical solution of a single cavity given in Section 4.2.1. Increasing the wall thickness results in a longer cavity and thus more resonances within the given frequency range. Changing the wall thickness close to the excitation has only very little additional effect on the zero pressure isolines in the fluid cavity. This is in agreement with what is expected from eqs. (4.14) and (4.17). The solution is shifted with the cavity length and thus the isolines mainly change in the wall. In contrast, a change of the thickness of the reflecting wall at $x=20$ mm has a
significant influence on the pressure field.
A wall thickness of 2 mm and 4 mm are shown in Figure 4.11 as an example. The dimensions of the water cavity and the wall close to the transducer are unaltered. In this situation the change of the pressure field close to the transducer is rather small apart from the additional resonances. The change increases with closing distance to where the boundary is changed. For this reason only the regions around the changed boundaries are shown in Figure 4.11. The zero pressure isolines in the PMMA wall are shown with light gray lines, in addition to the isolines in the fluid cavity. Doubling the wall thickness

![Diagram](image)

Figure 4.11: 1D simulation. Zero pressure isolines in the fluid and in the PMMA wall at the right boundary with a 2 mm and a 4 mm PMMA wall. The thickness of the left PMMA wall and the length of the fluid cavity remain constant at 1 mm, and 20 mm, respectively. The black arrows indicate the particle path of a particle started at $x = 0$ mm.

from 1 mm to 2 mm increases the node shift rate, such that a particle traverses the cavity in 5 full steps and one partial step compared to 6 steps plus one partial step before. In the 1 mm wall case the final location of the tracked particle after every step, and at 2.5 MHz, coincides well with a pressure node of the next step at 1.5 MHz. In other words, shown with the schematic in Figure 4.1, the distance between the pair $x_2$ and $x_3$, and the pair $x_4$ and $x_5$ is very small. However, this good coincidence is not given for all possible particle paths. In the presented 2 mm wall case this is shifted for worse, therefore an average particle would move even further. In the second shown case, with a 4 mm wall, an average particle moves slightly less far. Here, the tracked particle requires only 4 full steps plus one partial step. The path is shown for sweeps with increasing frequencies in all examples. For decreasing frequencies and particles moving in the opposite direction, the path can deviate from the shown path due to the described possible mismatch of the particle position at 1.5 MHz and 2.5 MHz.

The case with a 1 mm wall, shown in Figure 4.10, has 26 resonances. With a 2 mm wall this is increased to 31, and with a 4 mm wall to 33 resonances. The reflection of waves
at the interface between wall and PMMA introduces a modulation of the isoline function. In the ideal case, without inner reflections, this function is inversely proportional to the frequency. With other plastics like PS or PE, these effects caused by the inner boundary, can be reduced since their specific acoustic impedance is even closer to water than PMMA, as shown in Table 2.2. Furthermore, their attenuation coefficient is smaller (Table 2.1), which is beneficial.

In summary, increasing the wall thickness close to the transducer reduces the particle transport velocity and peak pressure amplitudes. Increasing the thickness of the second wall with an air interface increases the particle transport velocity close to this wall, but has a limited influence on the isolines close to the transducer. Thus, this positive effect has to be weighted against increased attenuation.

1D Simulation of Resonance Frequencies

The shape of the isolines is related to the frequency difference $\Delta f$ between two resonances. Three different 1D examples are shown in Figure 4.12. The frequency differences $\Delta f$ are plotted at the mid frequency between two resonances. ‘Water’ refers to the analytical case.

![Figure 4.12: 1D simulation. Frequency difference between two resonances for the case of fluid cavity only (Water), including both side walls (Water + PMMA), and with a piezoelectric element at one side (Water + PMMA + Pz26). The horizontal axis shows the average frequency of two resonances, which are $\Delta f$ apart.](image)

with an open boundary and results in 26 resonances. The other two cases are numerical simulations with a resolution of 100 Hz and refer to the reference case with 1 mm thick walls. Adding the walls increases the total length and thus the number of resonances to 28 and introduces a slight modulation which depends on the number of wavelengths in the walls. Adding a transducer increases the length further and introduces an additional resonance in the cavity around the fundamental resonance frequency of the transducer at around 2 MHz. The resonance frequency modulations shown in Figure 4.12 have to be taken into account with single particle applications which aim at moving the particle a well defined distance. On the other hand, these modulations play a minor role for particle concentration applications with a large number of particles where the average movement
is of interest. Furthermore, the non-constant $\Delta f$ between resonances has to be taken into account with resonant mode switching applications. Continuous frequency sweeping in contrast is less sensitive to changes in the resonance frequency since the average effect of multiple frequencies is used.

2D Isoline Simulations

Figure 4.13 shows the zero pressure isolines in gray which appear along the symmetry line of the top view 2D simulation in the fluid cavity. The shown black arrows indicate the travel path of a single particle with repeated frequency sweeping and are the ones obtained from the 1D simulation shown in Figure 4.10. This top view has an aspect ratio of fluid cavity length to side wall thickness of 10 to 1 and the transducer has an aspect ratio of 22 to 1. Therefore, similarities with the 1D solution are expected. The 2D simulation shows strong local distortions or jitter of the isolines. These distortions are particularly distinct around 1.5 MHz, but appear also at other frequencies.

Figure 4.13: 2D simulation of top view. Zero pressure isolines (gray) along the symmetry line of the square chamber. The black arrows indicate an idealized (1D simulation) particle path.

Figure 4.14 shows the isolines extracted from the side view symmetry line. Again, jitter is shown around 1.5 MHz, thus chances are increased that these frequencies perform rather bad in transporting particles. In contrast, in the top view simulation jitter is shown around 2.5 MHz which does not appear in the side view. At some locations and over some frequencies particles might even be transported crosswise to the rest. An example for this is indicated with the area denoted with A1. Within the frequency range covered by A1 all isolines are strongly distorted over the full fluid cavity length. This is not necessarily the case for all frequencies as shown with the areas B1 and B2. While the isolines within the 3 mm of B1 are strongly distorted, the ones in B2 are closer to the 1D case. The local isoline or pressure field distortions are the main reason why an increased frequency
sweeping range is advantageous. The quality of the pressure field is discontinuous over all length dimensions as well as over the frequency and thus averaging is important. In contrast, a 1D or simplified analytical model could lead to the assumption that a device is best operated exclusively with frequencies very close to the fundamental transducer resonance at $2\,\text{MHz}$ where the pressure amplitudes are the highest.

**Amplitude Distribution**

Figure 4.15 shows the averaged absolute value of the normalized acoustic radiation force (eq. (2.28)) in a semi logarithmic scale. The values are obtained from the same 1D simulation as presented in Figure 4.10 and show the average force directed along the fluid cavity length. The plot is shown as normalized plot since 1D simulations over-estimate the pressure amplitudes. The force is calculated from the complex valued pressure and thus an additional term from traveling waves is included. A symmetric frequency range around the fundamental transducer resonance frequency of $2.067\,\text{MHz}$ starts at $1.634\,\text{MHz}$ if the upper limit of $2.5\,\text{MHz}$ is fixed. This fits with the force amplitude plot. The resonance force amplitudes at the lower frequency range end are larger than at the upper end at $2.5\,\text{MHz}$. Yet, the amplitudes of the damped frequencies are lower in the former case compared to the latter. Therefore, the lower frequencies appear to be more critical for frequency sweeping.

Whether the shape of the isolines for a certain frequency is important for the particle transport also depends on the local force or pressure amplitudes. Figure 4.16 shows the averaged and then smoothed values of the complex pressure field where the averaging is along the fluid cavity length in $x$-direction. The smoothing is over the frequency and is obtained with an even line resolution of $10^3$ points and a smoothness parameter of $5 \cdot 10^5$, with the algorithm described in [45]. The three solid lines represent the previously

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Figure 4.14: 2D simulation of side view. Zero pressure isolines along symmetry line of device (gray lines). The plot areas A1, B1, and B2 indicate isolines which result in negative, ineffective, and good device performance, respectively.
4.3. Square Chamber

Figure 4.15: 1D simulation. Absolute value of the normalized acoustic radiation force. Values are averaged over the fluid cavity length. The fundamental transducer resonance is at 2.067 MHz. The individual peaks correspond to the fluid resonances.

discussed cases along the symmetry line of the device, of a 1D solution, 2D top view, and 2D side view as shown in Figures 4.10, 4.13, and 4.14, respectively. The amplitudes are normalized with the maximal amplitude of the 1D solution. The amplitude response of the top view simulation qualitatively follows the 1D solution. In both cases, 2D top, and 2D side view, the amplitude response remains qualitatively the same if evaluated at a different $y$-position. However, the average amplitudes are lower. This behavior of largest amplitudes in the center and smaller ones towards the boundaries is illustrated in Figure 4.16 with the side view case where in addition to the symmetry case, the boundary case is shown with a dashed line. In the side view case the distribution shows a minimum at the fundamental transducer resonance rather than a maximum as expected from the 1D and 2D top view case. This is also reflected in the values of the resonance frequencies. The first resonance in the 1D case is at 1.50925 MHz and in the 2D top view case it is at 1.51000 MHz. They are relatively close considering a typical $\Delta f$ in the order of 34 kHz (Figure 4.12). In contrast, the resonances of the 2D side view are considerably shifted where the first resonance is at 1.52400 MHz. The resonance frequencies are evaluated with a frequency resolution of 500 Hz. Resonances evaluated as an argument of maximum mean pressure amplitude or maximum peak amplitude results in the same resonant frequencies.

The discussion of acoustic radiation force response shows that both, lower and upper frequencies around the resonance frequency, are well suited for continuous frequency sweeping, regardless of different peak amplitudes. The discussion of the pressure amplitudes shows the limits of 2D simulations which provide considerably different values for top and side view geometry. Furthermore, a 1D model is a reasonable approximation for the top view geometry averaged amplitude distribution and the amplitude distributions show a good consistency throughout the device.
Figure 4.16: Average absolute values of the pressure field. The values are normalized with the peak pressure amplitude of the 1D simulation (1D). The values from the 2D simulations are evaluated along the symmetry line of the top view (2D top sym.) and along the symmetry line of the side view (2D side sym.) as well as along the boundary parallel to the symmetry line (2D side boundary).

### 4.3.3 Pressure Field Simulations

In this section the whole pressure field within the device at selected frequencies is discussed. This is important to estimate the particle transport capabilities of continuous frequency sweeping in a full fluid domain. The simulations all have one or more symmetries, but for illustrative purposes the full fields are shown. The figures presented in this section share the same figure type. The absolute values of the complex pressure fields $|p|$ in the fluid domains are shown as grayscale images. The PMMA frame is indicated as white boundary and the Pz26 element as gray block. The plots are typically shown at a resonance frequency. In a multidimensional system resonance frequencies can be determined differently. One way is to evaluate the pressure amplitude along a line in direction of the extension of the transducer, e.g. along the symmetry line. This approach is used here. For a 2D field another reasonable way to determine resonances is to integrate the field quantity over the whole domain. Evaluating along a line, or the whole domain, provides approximately the same resonance frequencies in the examples presented here.

#### 2D Pressure Field

Figure 4.17 shows the absolute pressure $|p|$ of the top and side view 2D simulation at resonance frequencies of the top view geometry. The frequencies 1.5100 MHz and 2.0795 MHz are the first resonance frequency in the range from 1.5 MHz to 2.5 MHz and the frequency with highest total amplitude, respectively. Resonance frequencies which are below the fundamental transducer resonance, thus of about up to 1.9 MHz, all show a rather smooth pressure field in the top view geometry as shown for 1.5100 MHz. The pressure nodal lines are almost straight. The top view pressure field at 2.0795 MHz shows a characteristic pattern for the frequency range around the fundamental transducer resonance. For these frequencies, standing waves in directions deviating from the main resonance in $x$-
4.3. Square Chamber

Figure 4.17: Absolute pressure field $|p|$ of the top and side view 2D simulation at 1.5100 MHz and 2.0795 MHz. These two frequencies are resonance frequencies obtained from the top view 2D simulation and are well below, and around the fundamental transducer resonance.

Direction are significant. Thus, hardly any pressure nodal lines appear as straight lines, but rather as wavy lines. Frequencies above the peak resonances typically show rather straight pressure nodal lines, but less straight lines than the frequencies below 1.9 MHz. Furthermore, the frequencies around 2.5 MHz show a similar behavior than off-resonances and are discussed next.

Figure 4.18 shows $|p|$ of resonance frequencies of the side view. In the frequency range from 1.5 MHz to 2.5 MHz, the frequencies 1.524 MHz and 2.496 MHz are the first and last resonance frequency, respectively. The top views show off-resonance pressure fields. These fields have a large variety of different patterns with changing frequency. They have bands or areas along the $x$-axis in common which can be large in number and can have high pressure amplitudes. The elongation of these bands in $y$-direction ranges from less than 1 mm up to several millimeters. They are of particular interest since their position along the $y$-axis can change relatively fast within kHz and their pressure gradient in $y$-direction can be large enough to trap particles. Thus, these bands might serve as a faster mean to transport particles locally. The top view at 2.496 MHz shows larger amplitudes at the side walls ($y = 0$ mm and 20 mm) close to the transducer. This is also the case with other
off-resonance frequencies, releasing the low pressure amplitudes problem in these areas. The off-resonances shown in Figure 4.17 and Figure 4.18 have significantly lower pressure amplitudes. This is as expected from the shift of resonance frequencies between top and side view described in Section 4.3.2.

Pressure field resonances of the side view geometry do not show as characteristic patterns for the frequency range as the resonances of the top view simulation. The side view patterns can take forms similar to the off-resonance patterns shown previously in Figure 4.17.

Common to all frequencies is the averaged pressure distribution in the top view with largest or large amplitudes around the center ($y = 10 \text{ mm}$) and very low amplitudes at the side walls ($y = 0 \text{ mm}$ and $20 \text{ mm}$). As previously discussed and shown in Figure 4.16, the side view has this tendency of low amplitudes at the side walls as well. However, not as pronounced as in the top view. Openings in the top cover of a real device probably amplify the effect. On the other hand, the influence of openings might not be that strong when they are introduced in an area where the pressure is low anyway.

In summary, the pressure field close to the side walls is critical with low pressure am-
4.3. Square Chamber

Amplitudes throughout the frequency range. For applications which aim at aligning particles in a regular grid the lower frequencies are best suited. The frequencies around the transducer resonance provide pressure fields which lead to particle clumps. Higher frequencies tend to have uneven pressure distributions with low and high performance areas. This can be critical for single frequency applications like resonance mode switching and shows the advantage of averaging with continuous frequency sweeping.

2D Pressure Field with Double Sided Excitation

Excitation from two sides can be used to agglomerate particles in one corner of the chamber rather than along one of the boundaries. Furthermore, a second transducer can be used to move particles to the center of the chamber where pressure amplitudes are highest. This helps to avoid particles stuck to the side walls and can be achieved with resonant mode switching.

Figure 4.19 shows the absolute pressure $|p|$ for two resonance frequencies. The pressure fields are very similar to the superposition of two perpendicular single transducer solutions. The two transducers are almost decoupled and an additional transducer does not significantly disturb the field caused by a single transducer. This is mainly true for all frequencies where a single transducer does not cause a strong resonance in the fluid cavity and perpendicular to the main extension direction of the transducer. This is the case for most frequencies in the top view simulation. The shown pressure fields are similar to a superposition of two standing waves with same amplitude and frequency. In this case particles which collect in the pressure nodes will form oval particle clumps rather than

Figure 4.19: Absolute pressure field $|p|$ at resonance frequencies of the top view 2D simulation well below and above the fundamental transducer resonance.
circular clumps. A small frequency difference in the order of 25 Hz between both standing waves as presented by Oberti et.al. [122] is a simple way to obtain circular clumps.

3D Pressure Field

A large ratio of wavelength to device dimensions makes 3D numerical simulations computationally expensive and it demands large amounts of memory. Moreover, a large parameter set increases the amount of data. This is where 1D and 2D simulations are particularly strong due to the significantly reduced amount of data. However, 2D simulations are limited in general and here the reported mismatch of resonances and pressure patterns between top view and side view 2D simulation clearly shows the limits. Furthermore, a 2D simulation does not allow to simulate the $yz$-cross section. A 3D simulation of a constricted frequency range of 1.5 MHz to 1.6 MHz is introduced to evaluate these limits. The symmetries are used and only a quarter of the actual domain builds the simulation domain. With a maximum element size $d = 1.4 \cdot 10^{-4}$ mm at least 6.6 elements per wavelength are obtained in the fundamental directions and the total DOF is over $2 \cdot 10^{6}$. With the limited element number the simulation is in a critical convergence range where an error in the form of an amplitude factor can be expected. However, the qualitative pressure fields are considered reasonably converged.

Figure 4.20 shows $|p|$ as grayscale images of the 3D simulation at a frequency of 1.528 MHz. This is the first resonance frequency, followed by 1.554 MHz and 1.585 MHz. The resonance frequencies are evaluated as frequencies with maximum average pressure amplitude in the whole fluid domain. Evaluating the central symmetry line at $y = 10$ mm and $z = 1.5$ mm as in the 2D cases results in approximately the same frequencies. These resonances in 3D are closer to the 2D side view than top view resonances. The shown top view and side view are the cross sections of the symmetry planes and the two $xz$-cross sections are taken from $x_1$, and $x_2$, respectively, as indicated. The pressure field distributions in the $xz$-cross sections are as expected from a rectangular chamber. They appear close to an interpolation of the top and side view cross sections. The top view cross section shows an amplitude modulation indicated with white arrows. These modulations appear with most frequencies in the investigated range. The modulations are also shown in $xz$-cross sections, for which they are rather similar in 3D and 2D. In 3D, the modulations are far less pronounced in an average field over the $z$-direction while averaging in $y$-directions has no significant influence on the field patterns.

In summary, the side view shows a close match of resonance frequencies and pressure field patterns between 3D and 2D. The main features of the top view 2D simulation, such as low pressure values at the side walls and pressure ‘bands’ are shown in both simulations. The results of the comparison between 2D and 3D simulations are as expected from the different aspect ratios between device height and width. This implies a larger effect for the 2D top view simulations considering influences in $z$-direction than taking influences in $y$-direction into account for the 2D side view simulations.
4.3.4 Particle Tracing Simulations

For frequency sweeping the main interest is not in the particle paths or equilibrium positions at a single frequency, but rather in the paths over a frequency range. This time dependence has been realized with a particle tracing model which has been extended to resolve frequency changes as described in Section 3.1.2.

Side View Simulations without Gravity

Figure 4.21 shows a particle tracing simulation of the 2D side view model. The 100 initial pseudo-random particle positions are indicated with gray circles and the final particle positions with black circles. The particles are 26\(\mu\)m glass spheres and for better visibility their diameter is enlarged in the figure. The particle paths are shown with gray lines and are for a linear frequency sweep from 1.5 MHz to 2.5 MHz in 20 s. The simulation is performed with a frequency resolution of 1 kHz and without gravitational forces. The particles move away from the transducer as expected. Most of them agglomerate in the center around \(z = 1.5\) mm and some particles move close to the side walls at \(z = 0\) mm and 3 mm but hardly any particles remain in the intermediate areas. The particles tend...
Figure 4.21: Particle tracing simulation with 2D side view geometry. The 100 initial particle positions are indicated with gray circles and final positions with black circles where their size is not to scale. The gray lines indicate the particle paths for a linear frequency sweep from 1.5 MHz to 2.5 MHz in 20 s. Particles are 26 µm glass spheres.

to form clumps even though no secondary acoustic radiation forces are included in the simulation. This clump forming behavior is also reflected in the symmetric appearance of the final particle locations. The pressure field is also symmetric, but the initial particle locations are not.

Figure 4.22 shows the particle location probability as grayscale plot with frequency over distance \( x \). The results are from the same simulation as shown in Figure 4.21. The probability is calculated with a \( \Delta x \) below the shown line width and thus the lines effectively denote the particle paths. Light gray denotes a single particle and black 10 particles. Hence, the particle clump shown at \( x = 16 \) mm, and 2.5 MHz, in Figure 4.21 as an example consists of 10 particles. A comparison with the isoline plot in Figure 4.14 shows good agreement. The particle paths are very close to the isolines with stronger disturbances close to 1.5 MHz and around 2 MHz which is as predicted by the isoline plot. However, with particle tracing these disturbances are stronger close to the transducer since there the frequency gradients of the isolines are the largest and thus also the influence of the Stokes drag is the largest.
Frequency Sweeping Rate

The particle transport performance has been investigated for different frequency sweeping
rates. The simulations have the same geometry and parameters as the simulations of the
side view geometry without gravity presented in Figures 4.21, and 4.22, except for different
frequency ramp times.

Figure 4.23 shows the averaged travel distances of all 100 particles as a function of
the frequency. The previously shown simulation with a ramp time of 20 s is already close
to the maximum travel distance which is slightly above 4 mm. The characteristics of
the travel distance curve of 1.25 s reflect those of the pressure amplitude curves of the
2D side view simulations, shown in Figure 4.16. This is, the total travel distance is the
largest where the amplitudes are the largest, around 1.9 MHz and 2.2 MHz. For the shown
times it holds, the shorter the ramp time, the larger is the total travel distance per ratio
ramp time to fixed reference time. However, this does only apply for a limited number of
frequency sweep cycles. The problem is that for faster ramp times the particles close to
the transducer are no longer transported while the transport at the other end, towards
$x = 20$ mm, is much less ramp time critical in comparison. Or in other words, the particle
collection yield decreases with shorter ramp times. This is also indicated in Figure 4.22.
Thus, up to a certain limit it can be advantageous to speed up the sweep rate, both,
within a single sweep cycle and over many sweep cycles. Moreover, with ramp times of
10 s and less, the particles no longer collect in the center at $z = 1.5$ mm, as they mostly
do with 20 s. However, this is not of significance for the performance.

Particle tracing resolves the influence of damping. The maximum force amplitudes are
higher for the lower frequencies around 1.5 MHz compared to the frequencies towards
2.5 MHz, as shown in Figure 4.15. Nonetheless, it is with the lower frequencies where
the transport of particles first fails with decreasing ramp times. This is because the minimum or off-resonance frequencies are lower. Changing the acoustic radiation force on the particles has a similar effect on the system as changing the ramp time.

Side View Simulations with Gravity

Gravity can influence the particles positions significantly and sedimentation is a known issue in experiments. High particle material densities typically result in high acoustic radiation forces. This is the case comparing glass and silica with copolymer particles.

Figure 4.24 shows the same configuration as presented with Figure 4.21, but including a gravitational force in negative $z$-direction. The majority of all particles reaches the bottom wall at $x = 0$ mm after 5 s out of 20 s, and some few remain levitated for about 15 s around $x = 16$ mm. The particles bounce or slide along the wall and no Coulomb

![Figure 4.24: Particle tracing simulation with 2D side view geometry including gravitational forces. Initial 100 particle positions are indicated with gray circles and final positions with black circles where their size is not to scale. The gray lines indicate the particle paths for a linear frequency sweep from 1.5 MHz to 2.5 MHz in 20 s. Particles are 26 µm glass spheres.](image)

or similar friction model is included in the simulation. Furthermore, the Gor’kov model (eq. (2.29)) used to calculate the primary acoustic radiation forces is no longer valid in the proximity of a wall and here the particles are always treated as being in an infinite volume. The frequency dependent behavior of the system is shown in Figure 4.25 which shows the particle location probability along the $x$-axis. Most particles take 5 s to settle to the bottom wall. This corresponds to a frequency of 1.75 MHz. The particles which are close to the wall tend to form clumps. Here no particle-particle interaction is included and the clumps are reduced to a diameter of a single particle. The performance over one frequency sweep cycle is reduced. The particles in average are only transported 3.0 mm compared to 3.8 mm without gravity. However, large particle clump diameters and particles stacking up on each other can increase the force on the particles which in turn increases the performance of a real system.

The simulations including gravitational forces show that particle sedimentation is an issue since the levitating forces are generally not strong enough with glass or silica particles.
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Figure 4.25: Particle location probability calculated from the 2D side view particle tracing simulation shown in Figure 4.24 which includes gravitational forces. Light gray denotes a single particle and black 10 particles.

**Side View Resonant Mode Switching**

Frequency sweeping relies on damped off-resonance frequencies. To illustrate their importance the simulation is directly compared to frequency mode switching, shown in Figure 4.26. The figure illustrates results from the 2D side view simulation without gravity and the grayscale plot shows the particle location probability along the $x$-axis. For frequency mode switching only the resonance frequencies are excited. The pressure along the center or symmetry line of the simulation, illustrated in Figure 4.14, is taken as argument to evaluate the resonance frequencies. The particle tracing simulation is performed with the same total time of 20 s as the previous sweeping simulations for better comparison. The $\Delta f$ between two resonances is not constant and the excitation frequencies are mapped onto a linear frequency range. Thus, the chosen time intervals between resonances are not constant. The shown simulation is calculated with a fixed 2 kHz resolution. Instead of

Figure 4.26: Particle location probability of a 2D side view particle tracing simulation with frequency mode switching. Particles are 26 $\mu$m glass spheres and the simulated time span is 20 s. Light gray denotes a single particle and black 6 particles.
the frequency mapping, which is restricted to the 2 kHz resolution, the resonances can be used directly. Here, the 33 resonances have been determined with a resolution of 100 Hz. If the simulations are fully converged, both simulations should provide the same results. A comparison of both simulations shows comparable results where a few particle paths deviate between both simulations. This is an indication that the frequency discretization of at least 2 kHz is reasonable, but does not result in fully converged simulations.

The simulation results with resonant mode switching show particle paths in \( x \)-direction of particles moving towards both side walls instead of moving from one side to the other, as shown previously with the continuous frequency sweep simulations. This is in agreement with the 1D analytical model discussion in Section 4.2.1. In \( y \)-direction most particles agglomerate along the symmetry line and some along the side walls, similar as observed with frequency sweeping.

The particles move rather fast from pressure node to pressure node, due to the high amplitudes of the resonances. With the exception of a few frequencies and locations most particles move in less than 0.05 s between the pressure nodes of two consecutive resonance frequencies. Hence, the overall performance is similar for a total frequency mode switching cycle time of less than 1 s. This can be used to improve the overall performance of a frequency sweeping device once all particles are on one half of the device. However, in real applications bubbles or evaporating fluid can change the resonance frequencies and thus alter the performance for worse. In addition, a resonator frequency shift can occur due to particle migration [90].

**Top View Simulations**

Particle tracing simulations of the top view geometry show a better particle transport performance compared to the 2D side view simulations. The total displacement of all particles of a 20 s frequency sweep in the top view corresponds to about a sweep time of 40 s in the side view. The areas of lower average pressure amplitude close to the side walls at \( y = 0 \) mm and \( y = 20 \) mm do not show significant problems in the particle tracing simulations. Only particles which are closer than about 0.5 mm to the side walls tend to be stuck in the low pressure field. Most particles in the low pressure field regions move towards the center of the fluid chamber, in addition to the movement in the desired \( x \)-direction. For a 20 s sweep time this movement is in the order of 2 mm and generally is most pronounced for frequencies in the range of 1.9 MHz to 2.2 MHz. Fast changes of the pressure field cause particles at any position to move back and forth in \( y \)-direction, but not as pronounced as close to the side walls.

Similar as in the side view simulations, the particles tend to form clumps. However, in the top view, these are not mainly located along the symmetry line of the device, but at different locations within the fluid domain.

**Damping**

The presented models are based on a relatively low \( Q \)-factor of 2000 at 1 MHz for water. This value has been chosen because it provides reasonable overall damping in the simulations and is common in literature [59]. However, actual attenuation values of water are
much lower as shown in Table 2.1 which corresponds to $Q = 80,000$ at 1 MHz. In the side view particle tracing simulation values of up to this $Q$-factor value have only a very small influence on the simulation result.

**Particle Size**

The particle tracing simulation is well suited to investigate the influence of different particles. An example is 9.6 $\mu$m copolymer particles with a density of 1050 kg m$^{-3}$. These particles are smaller and have a lower density compared to the previously presented 26 $\mu$m glass spheres. Hence both, acoustic radiation forces and gravitational forces are much smaller. A 2D particle tracing simulation of the side view with a frequency ramp time of 20 s confirms the expected results. The simulation is similar to the one presented in Figure 4.24. The particles hardly sediment with a total integrated travel distance of 0.04 mm in $z$-direction. The particles do not form particle clumps in the given time and travel around 1.49 mm in $x$-direction. This is similar to the performance with the glass spheres and a ramp time of 2 s.

Another example is silica particles with a diameter of 10 $\mu$m and a density of 2000 kg m$^{-3}$. The diameter is comparable to the copolymer particles but the density is significantly higher. The total travel distance in $z$-direction is 0.68 mm. Thus, sedimentation is noticeable and about one forth of the particles end up close to the wall while most the remaining particles end up close to the symmetry line. The total travel distance in $x$-direction is 2.17 mm. This is a typical trade-off where particles with a density close to that of water also experience low acoustic radiation forces as shown with eq. (2.29).

**Mass Transport Equation**

For models with a large number of particles, typically the change of particle concentration rather than single particle paths are of interest. The time dependent changes can be described with a diffusion model. Trujillo et.al. [168] modified the mass transport equation to account for acoustic radiation forces, included the model in a 1D simulation of the square device presented here and compared the numerical results with experimental results. The final mass balance of particles included in the model can be described with the three general terms accumulation, diffusion, and convection. To account for high particle concentrations the diffusivity is calculated with a correction proposed by [46], instead of the Einstein-Stokes equation. The mass fraction of particles is $\chi_p = 0.01$. The experimental results used for the comparison with the numerical results are presented in Section 4.3.5 and shown in Figure 4.28. The mass fraction distribution of particles after 1, 2, and 6 frequency sweep cycles of 20 s each are in agreement with experimental results. The 1D simulation shows a partial reversed displacement of particles which cannot follow the force potential minima fast enough. The pressure node displacement is the fastest close to the transducer and thus this effect is the strongest there. This is also observed in experiments.

Both methods, particle tracing, as well as the mass transport equation approach, suffer from numerical problems. Due to numerical instabilities the latter is mainly limited to
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low excitation voltages, that is low acoustic radiation forces. In contrast, particle tracing works best with large acoustic forces.

4.3.5 Particle Experiments

The presented continuous frequency sweeping experiments focus on a frequency range of 1.5 MHz to 2.5 MHz, excited with 1 mm thick piezoelectric elements. This range is continuously repeated or in other words, modulated with a sawtooth frequency modulation where the modulation frequency is given as ramp rate. Individual observations and operation of the devices have been performed over larger frequency ranges and with different modulations. The main conclusions are also valid for the frequency range presented here.

Continuous Frequency Sweeping Experiment

In this section two typical particle experiments are introduced. Figure 4.27 shows a particle experiment with 26 µm glass particles suspended in water. The device has the same dimensions as the reference device which is the same as the one presented in the numerical simulations. That is a PMMA device with 1 mm thick walls and 250 µm thick top and bottom covers. The device is excited at the left boundary and the transducer voltages range from 2 V to 10 V. The repetition rate is 0.1 Hz, and the sweep range is 1.5 MHz to 2.5 MHz. There are two 1 mm openings used to fill the chamber in the top cover. They are shown in the upper left and lower right corner. The figure shows the initial particle distribution at t = 0 s and the distributions after 40 s and 120 s at an instance with active acoustics at 2.5 MHz. At t = 0 s the particles are distributed throughout the device with a strongly reduced concentration below the opening in the lower right corner and an increased number of particles in the first 5 mm close to the transducer.

![Figure 4.27: Continuous frequency sweeping experiment with 26 µm glass particles suspended in water. The device is a PMMA device with 1 mm side walls and the transducer and Al waveguide are placed on the left. Shown are initial particle distribution at 0 s, and the particle distributions after 4 and 12 sweep cycles of 1.5 MHz to 2.5 MHz.](image)

Particles very close to the side walls at y = 0 mm and 20 mm are not, or hardly transported while the particles which are in proximity to the wall are moved towards the center of the fluid chamber. This behavior is in good agreement with the particle tracing simulation
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given in Section 4.3.4.

An experiment with 9.6 µm copolymer particles is shown in Figure 4.28. These particles sediment much slower compared to the 26 µm glass particles. The device has a PS base and a 100 µm thick polyester cover foil. PS has a lower acoustic attenuation (Table 2.1) and a lower acoustic impedance compared to PMMA which results in an increased performance. The openings in the top cover have a diameter of 1.5 mm. The excitation is similar to the previously presented experiment but with a repetition rate of 0.05 Hz. The figure shows the initial undisturbed particle distribution at $t = 0$ s, after two cycles or 40 s, and after 120 s. The latter two particle distributions are at a frequency of 2.5 MHz. This combination of a high concentration of small particles in a PS device still gives reasonable results compared to the experiment with larger particles and PMMA device presented in Figure 4.27.

The number of particles in the device influences the performance. A large number of particles increases the performance at the beginning where particle clumps decrease the probability of particles sticking to the ground and in addition increases the acoustic radiation forces on the particles which is the case as long as the clumps are smaller than about a quarter wavelength. This condition is typically violated after several frequency sweeping cycles when particles start to agglomerate close to the target wall. These agglomerates alter the pressure field in the whole device and thus decrease the total particle transport performance. This is the case after 40 s in the experiment shown in Figure 4.27, as an example.

**Evaluation of Particle Concentration Yield**

Multiple particles stack up on each other when they are confined to a pressure nodal plane. When exciting at a resonance frequency, the acoustic radiation forces are often large enough to levitate the particles to the first pressure node in $z$-direction. During frequency sweeping particles start to sediment between resonances. This stacking of particles makes it hard to evaluate the local particle concentration through color intensity methods. A
The direct mapping of the change of intensity underestimates the number of moved particles. The change of color intensity is defined as the intensity at the time normalized by the intensity of the undisturbed particle distribution at $t = 0$ s. The change of intensity is presented here for the experiment shown in Figure 4.27 as an example. After 120 s 80% of all particles are located in the last 6.1 mm along the wall to the right with the assumption that the darkest value corresponds to a state with zero particles. This value is reduced to about 3.8 mm when the acoustics are turned off and the particles start to sediment. The effective concentration is closer to a confinement length of 2 mm based on pipette concentration samples which also only provide an estimate. Running the experiment for a total of 4 min improves the length of the band with a high particle concentration to 1.7 mm. However, still not all particles are moved in this case. Mainly particles sticking to the ground and the particles at the side walls remain unmoved. Changing the thickness of the device side walls as discussed with the numerical simulation in Figure 4.11 did not show a higher concentration yield. Side wall thicknesses of 10 mm and 20 mm both showed too large damping of the acoustic pressure field and thus even poorer device performance than with a 1 mm thick side wall.

**System Time Response**

Figure 4.29 shows the image color intensity over time for the same experiment as shown in Figure 4.28. The intensities are averaged over the $y$-direction for each frame and normalized with the intensities at $t = 0$ s. Black corresponds to the highest intensity or largest particle concentration. One sweep cycle corresponds to 20 s. In an ideal case 6 to 7 sweep cycles suffice to move a particle from $x = 0$ mm to 20 mm, as shown with the 1D model and Figure 4.10. After about 80 s, thus 4 cycles, large particle clumps reduce the overall performance significantly and the intensity distribution in the figure only changes slightly for increasing time.

In general an aluminum base layer increases the performance of the device. Figure 4.30 shows the color intensity of an example with 1 mm thick PMMA walls and a 0.5 mm anodized aluminum plate. A somewhat lower particle concentration than in the previous two examples has been used and the particles are 10 µm silica spheres. The excitation is with the same frequency range and voltage as for the previous two cases and the repetition rate is 0.05 Hz. This configuration results in a small number of particles sticking to the ground plate and also in a low particle concentration at the side walls. Therefore, this experiment provides an intensity field averaged in $y$-direction with only small disturbances of the effective acoustics. The intensities are normalized with the values at $t = 0$ s and are plotted with a logarithmic function for a better comparison of the low intensity values. The intensities indicate the particle travel paths which show a very high reproducibility between the sweeping cycles of 20 s each. Even small changes of the particle paths are reproduced.

For the sweep cycle from 1.5 MHz to 2.5 MHz starting at $t = 40$ s, the particle travel paths obtained from the 1D numerical simulation are indicated with black arrows. The arrows are the same as presented in the isolines Figure 4.10. The good match of simulation and experiment and the high reproducibility of the particles paths within an experiment
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Figure 4.29: Averaged (y-direction) image color intensities for the same experiment as shown in Figure 4.28. The values are normalized with the intensities at $t = 0 \text{s}$. Black corresponds to the highest intensity or largest particle concentration. One sweep cycle of 1.5 MHz to 2.5 MHz corresponds to 20 s.

shows that for most cases the presented 1D simulation provides a reasonable model for the particle paths. The 2D simulations and isoline plots as well as the particle tracing simulations predicted a lower performance around 1.5 MHz and for smaller frequency ranges around other frequencies. This is in good agreement with the experiments and is also shown in Figure 4.30. This plot in addition shows the influence of particle clumps. For the first second at $t = 0 \text{s}$ the almost evenly distributed particles are hardly influenced by the acoustic field. Once the particles are concentrated in the pressure nodes they are also moved by the low acoustic field, as shown at the frequency step from 2.5 MHz to 1.5 MHz at $t = 20 \text{s}$ and $t = 40 \text{s}$.

All three types of simulations presented in Section 4.3.2, Section 4.3.3, and Section 4.3.4 show that it is advantageous to use a larger frequency range because the particle transport performance is not constant over $x$ for all frequencies. This is also the case in the experiments. An example is the frequency range over distance marked as plot area A2 where the intensities show a poor or even counterproductive particle transport performance. However, the performance is good for the same frequency range but other locations in the chamber. An example is marked with the plot area A1. For a frequency range around another center frequency this can almost be interchanged as indicated with B1 and B2.

**Back and Forward Particle Transport**

Particles are moved back towards the transducer when the frequency is decreased. The operation with increasing frequency typically is preferable because the particle velocities
Figure 4.30: Averaged \((y\text{-direction})\) image color intensities for an experiment with aluminum base layer. The values are normalized with the intensities at \(t = 0\) s. The intensities are plotted with a logarithmic scale where black corresponds to the highest intensity or largest particle concentration. One sweep cycle of 1.5 MHz to 2.5 MHz corresponds to 20 s. The black arrows indicate isolines obtained from a 1D numerical simulation. The plot areas marked with dashed boxes indicate good (A1 and B2) and bad (B1 and A2) particle transport performance.

are the largest close to the transducer. Therefore the risk of particles lying behind is reduced in the operation with increasing frequencies. If a particle does hardly move during one sweep cycle, it can catch up faster than in the inverse operation. Apart from this the performance with initially even distributed particles is almost the same in both cases. This underlines that the effect of frequency sweeping is not due to an acoustic energy gradient between transducer and target wall. Instead, it is the attenuation between resonances. However, the performance can be reduced with a larger number of particles when the goal is to move the particles back and forth from one wall to the other. This is
because a larger particle agglomeration disturbs the acoustic field.

**Particle Bands**

The formation of particle bands over several wavelengths which move in $y$-direction is often but not consistently observed. Figure 4.31 shows two particle bands indicated with white arrows. The shown device is a PS device with the same configuration and experimental parameters as the experiment shown in Figure 4.28 but with a higher excitation voltage range of 5 V to 20 V. The first image is at a frequency of 2.200 MHz and the second image is taken 0.35 s latter at a frequency of 2.175 MHz. Mainly particles in the upper of the two bands are moved in $y$-direction between the two frames. Typically particles hardly move sideways if they are close to, but not within these bands. Furthermore, the direction of the movement can change its sign rapidly. This shows that this type of movement is mainly caused by focused ultrasound and not fluid flow. Experiments with additional small seeding particles which are less influenced by the ultrasound confirms this observation.

![Figure 4.31: Two images of a continuous frequency sweeping experiment with 9.6 µm copolymer particles suspended in water in a PS device excited from the left. The images are at 2.2 MHz and 2.2175 MHz, and the time span between them is 0.35 s. The white arrows indicate two bands with a larger number of particles.](image)

The particle bands mainly appear with high transducer driving voltages as shown in Figure 4.31, or flawed device geometries, or with devices where the transducer has been glued to the device. The previously presented experiments do not fall in any of these categories and only show movement in $y$-direction with small velocities in the order of the velocities in $x$-direction. The numerical simulation also shows these bands and is closer to the latter case with less pronounced pressure amplitudes. In sporadic cases, experiments performed with a repetition rate of 0.05 Hz show particle band movements in $y$-direction of more than 2 mm in 0.1 s where the intended particle movement in $x$-direction is less than 0.1 mm in the same time.

**Fluid Flow**

Fluid flow is mainly due to Stokes drag, thermal convection, or acoustic streaming. Fluid flow in combination with Stokes drag can influence the desired particle paths significantly. Sources of fluid flow are acoustic streaming, thermal effects, or fluid dragged with moving
particles. The latter is mainly an issue with a large number of particles which, distributed in a pressure nodal plane, form sort of a surface or wall. An example often observed in experiments is particle clumps in the center of the device which drag fluid with them. This flow causes the fluid at the side walls to flow back, because there the number of particles and the pressure amplitudes are lower. This flow is also observed in experiments where the square chamber has been placed upright such that the gravitational field is oriented in $x$ or in-plane direction. With active acoustics the particles are held against gravity and as soon as the excitation is turned off the particles sediment and induce a flow. This method is also known as ultrasonically enhanced sedimentation [71].

Thermal convection can lead to a flow which is circular in the $xz$-plane. In the upper part of the chamber, the flow direction is the same as the particle transport direction if the frequency is increased. In the lower part of the chamber, the flow direction points against the desired particle movement direction. This is where most particles with a higher density than the fluid are located. Thus the effect on the particle transport is less significant when the particles are moved with decreasing frequency. Further thermal effects are discussed in the schlieren visualization Section 4.3.6.

The most significant effect is acoustic streaming. Fluid flow due to thermal effects or due to moving particles is typically small, smaller than the intended velocity of the particles. Acoustic streaming in contrast can reach very high velocities. In extreme cases the peak velocities can reach over $50 \text{ mms}^{-1}$ with an excitation from two sides and the same excitation frequencies and amplitudes as with the presented particle experiments. Acoustic streaming is relatively hard to control and in most experiments no significant acoustic streaming was observed. These velocities give rise to use acoustic streaming on purpose and streaming is discussed in more detail in Chapter 5 where acoustic streaming in the square chamber is discussed in Section 5.1.2.

Two Sided Excitation

Experiments with an excitation on two orthogonal side walls and with the same frequency, mainly showed smaller particle clumps instead of longitudinal stretched clumps or lines observed with single sided experiments. Figure 4.32 shows an experiment with a PS device and the same parameters as the experiment shown in Figure 4.28, but with an additional transducer along the wall at $x = 0 \text{ mm}$. Both transducers drive the particles towards a wall where the transport performance alongside the wall is reduced. Therefore, the final particle distribution is in a L-shape in the corner of the device. This is in good agreement with the numerical simulation which shows this behavior at the walls. Once most particles are moved towards the walls, simply reversing the frequency sweep direction for a limited time reduces the wall problem close to the transducers. This is the case because the particle moving velocities are not constant over the distance and are the smallest at the target wall away from the transducer. This velocity difference is indicated with $v_x$ and $v_y$ in the figure for an arbitrary chosen point. The difference is only used beneficially when the particles are moved towards the target walls which gives that approach a limited performance. This can be improved when four transducers are used, one at each side. In this case the mentioned velocity difference can also be used beneficially in the stage where
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Figure 4.32: Continuous frequency change experiment with 9.6 µm copolymer particles suspended in water. The device is a PS device excited from the left and lower boundary. Shown are initial particle distribution at 0s and the particle distributions after 2 and 6 sweep cycles of 1.5 MHz to 2.5 MHz.

the particles are moved away from the wall.

Most of the two sided experiments have been performed with a symmetrical setup and equal frequencies and amplitude on both excitations. Nonetheless, oval shaped particle clumps have not been a major problem as can be expected from other systems with two sided excitation \[122\]. Reasons for this fact can be that the excitation frequency is continuously changing and the averaged force on the particles is of interest. An unbalanced performance of both orthogonal particle transport directions has been an issue. Over time one or the other side is dominating and the particles are not always moving diagonal through the chamber. One reason for this behavior might be the pressure field which is more disturbed in one direction than the other due to particle agglomerates.

4.3.6 Schlieren Visualization Experiments

The strength of schlieren visualization is its ability to image very fast frequency changes and effects such as thermal convection or acoustic streaming. As an optical non invasive method it is a well suited extension to particle experiments.

The schlieren images are shown as grain extract images. For images with a linear intensity range from 0 (black) to 1 (white) a grain extract image $G$ is obtained with $G = \min(1, \max(0, B - F + 0.5))$, where $B$ is the background image and $F$ the front image. The front image is the schlieren image of interest. The resulting grain extract images have a pressure node where the local intensity field is the darkest, for both knife edge filter and dark field filter. Areas within the fluid chamber without a standing pressure wave present or with one of low amplitude show no local variation in the schlieren image.

Knife Edge Filter Pressure Fields

Figure 4.33 shows grain extract images where $B$ is a schlieren image of the device without ultrasound and $F$ a snap-shot image during sweeping. The images are obtained with a knife edge filter in $x$-direction. The devices are excited with a 1 mm thick piezoelectric element which is directly clamped to the device and placed on the left side of the images.
The excitation voltage range is 5 V to 20 V. The PMMA device has a 0.5 mm thick wall and the top and bottom cover are 0.5 mm glass slides. The glass is introduced to minimize disturbances in the optical path. A comparison with devices with PMMA cover did not show significant differences in the qualitative image of the acoustics. Glue between the glass and the PMMA forms a chamfer in the corners. Therefore, the walls appear thicker in the schlieren images. The excitation signal is a frequency sweep of 1.5 MHz to 2.5 MHz modulated with a saw-tooth with a frequency of 0.05 Hz.

![Figure 4.33: Grain extract schlieren images obtained with a knife edge filter. Excitation is from the left side.](image)

The first image is at 1.523 MHz which is the first resonance frequency within the excited range. The second frequency at 2.080 MHz is a resonance close to the fundamental transducer resonance frequency. The visualized nodal pressure planes show to be almost parallel with some disturbances around the four boundaries and in the center. The bright white stripe at the boundary along $x = 0$ mm close to the piezo element, as well as the other disturbances close to it are due to thermal gradients and convection caused by heat dissipation of the transducer. For prolonged experiments with high driving voltages and without cooling these thermal effects appear very clearly on the schlieren images. The disturbances are particularly strong with higher frequencies. Some disturbances can also be caused by convection in the air, even though they should be small in a vertical setup where the airflow of rising air is parallel to the light path.

**Frequency Resolved Experiments**

For the transport of particles the evolution in time of the pressure field is of interest. Figure 4.34 shows the grain extract image along the $x$-direction for a line at $y = 14$ mm. The plot is similar along the center of the device except that the disturbances are much stronger. The experiment is the same as shown in Figure 4.33. Thus also with a knife-edge filter in $x$-direction. The shown frequency range corresponds to a time range of 20 s. The plot is shown for 20 mm without the parts close to the walls covered with glue. The black arrows show isolines according to the 2D numerical simulation shown in Figure 4.10. The outer dimensions of the simulation and the ones of the experiment are the same. Most resonances can be clearly identified between 1.5 MHz and 2 MHz. They appear with
4.3. Square Chamber

Figure 4.34: Grain extract schlieren image along the $x$-direction for a line at $y = 14$ mm. The arrows indicate isolines obtained with a 2D numerical simulation. Excitation is from the left side.

rather narrow banded pressure amplitude peaks. Nonetheless, the pressure amplitudes between the resonances are still high enough to be imaged with the schlieren setup. This pressure behavior is in good agreement with the numerical simulations which also show rather narrow banded pressure amplitude peaks, as shown in Figure 4.15. The amplitude distribution around 2.25 MHz stands in contrast to this. The amplitudes are almost the same for resonance and off resonance frequencies. This behavior is observed in different schlieren experiments but not predicted by the numerical simulation.

**Dark Field Filter Pressure Fields**

Figure 4.35 shows two grain extract images obtained with a dark field filter. The two images are 5 kHz apart and are chosen to fall in the frequency range around 2.25 MHz where there is only a small change of total pressure amplitudes over frequency.

Figure 4.35: Grain extract schlieren images obtained with a dark field filter. Excitation is from the left side.

Unlike the knife edge filter, the dark field filter has no directional emphasis. The images
show strong changes in the pressure field amplitudes in $y$-direction. These changes appear throughout the full frequency range from 1.5 MHz to 2.5 MHz and are significant even within smaller frequency changes, as shown in the figure. Most of the changes are too fast to be visible with the particle experiments presented in Section 4.3.5. However, the changes might influence the particle transport performance of continuous frequency sweeping if very low repetition rates are used, or they can be used to transport particles in the $y$-direction. These changes are in good agreement with the numerical simulations presented in Section 4.3.3.

**Density Variations in the Fluid**

Schlieren visualization shows any type of density variations as long as the gradients are large enough. Some acoustic streaming effects fulfill this requirement as well.

Figure 4.36 shows a wave front caused by acoustic streaming. The streaming itself is caused by some disturbance like a small air bubble at the boundary towards the transducer. With a changing excitation frequency the resonance frequency of the disturbance is only met for a small time window. The background image $B$ used for the three shown grain extract images is an image right before the streaming burst starts. Like this any static part or low velocity change, as e.g. the bubble, will be suppressed, emphasizing high velocity changes like caused by fluid flow. The time passed between two images is 50 ms. Streaming caused by air bubbles can appear if the frequency matches the resonance condition of the bubble. With continuous frequency sweeping the resonance condition is typically only met for a short time and thus bubble induced streaming is not a major problem.

With liquids of different density in the chamber even further phenomena can be visualized. One example is shown in Figure 4.37 where two different lower harmonics are shown together with the pressure field. The wavelengths of the harmonics are indicated with arrows. Compared to the thermal effects present in the previous examples, particularly distinct in Figure 4.33, here the duration of the experiment is much longer resulting in larger volumes of water with different temperatures. Without acoustics the fluid volumes of different densities arrange according to gravity resulting in gradients along the optical axis which are not visible with the schlieren setup. With active acoustics the fluid volumes slowly rearrange with gradients in the $xy$-plane which can be visualized with the schlieren setup. The transient time for the change of these gradients is in the order of seconds. Structural vibrations in comparison have a significantly lower decay time, typically two or
three orders of magnitude lower for the same frequency as these lower harmonics. Effects

Figure 4.37: Schlieren visualization of acoustic pressure field and lower harmonics. The arrows indicate one wavelength.

of acoustic radiation forces on fluids of different density can also be observed with salt solutions. If salt solution is diluted with pure water and not thoroughly mixed before it is filled in the device this becomes visible in the schlieren images. When the acoustics are turned on the fluids rearrange and magnify the impression of the acoustic pressure field. However, this is not a stable or controlled process and the physics behind these effects are not within the scope of this work.

4.3.7 DNA Concentration Experiments

Nucleic acid purification or isolation is an important step for sensitive molecular analysis [189, 163]. This includes an extraction step with mechanisms such as silica-based surface affinity, electrostatic interaction, nanoporous membrane filtration, and binding to functionalized microparticles [79]. DNA can be concentrated further by moving or agglomerating the particles. A commonly used method for this are magnetic beads [7]. Particles can also be moved with acoustic radiation forces. Here the focus is on the feasibility to use beads in combination with acoustic radiation forces for DNA concentration. The main experiments of this section have been performed together with Stefan Lakämper who also performed the test-tube experiments.

Acoustic Device

The experimental parameters for the acoustics are the same as the ones given for the particle experiments in Section 4.3.5. These are mainly a transducer with Al waveguide, an excitation frequency range of 1.5 MHz to 2.5 MHz, a repetition rate of 0.05 Hz, and excitation voltages of 2 V to 10 V.

The use of biological samples with such a chamber arises additional requirements or considerations for the chamber or the acoustics. These are e.g. fluid properties, thermal control, or particle size which are also important for the biochemistry and are briefly discussed in the following.
Any sort of biological sample typically comes with a buffer solution with properties differing from pure water. Here these are e.g. sodium iodide (NaI) and phosphate buffered saline (PBS) buffer solution. Experiments with silicon devices [120] showed increasingly good particle manipulation performance comparing high concentration water salt solution, pure water, and isopropanol. These results have not been quantified and may also arise as secondary effects, e.g. due to a change of particle adhesion to the chamber walls or due to a change in the Stokes drag rather than a change of the acoustic radiation force field. In addition, a non-homogenous mixture of buffer solution and pure water can influence the acoustic field as described in the schlieren visualization Section 4.3.6.

Thermal control of the acoustic devices can be of interest for specific purification or polymerase. Devices with an Al base layer, similar to e.g. the one described in Figure 4.27, have been equipped with a filament which allows fast heating of the device. One limit of such a device is the thermal stability of PMMA, particularly for higher temperatures towards the boiling temperature of water. Furthermore, stress cracks can be induced with compound devices. However, these problems could be solved with a different choice of material. Heating also leads to boiling and bubbles. Large bubbles strongly disturb the acoustic field. These bubbles are hard to avoid even with preliminary degassing. Therefore, the presented acoustic experiments are performed at room temperature.

Particle size can become an issue with very small particles where boundary layer driven streaming becomes increasingly important. This is mainly the case for particles which collect in the pressure nodes and form a closed loop in the Stokes layer as shown in Section 5.3. Within this layer the transport of particles is dominated by diffusion. This can be resolved with particles which collect in the pressure antinode which do not form this sort of closed loop [147].

Proof of Principle of DNA Concentration with Acoustics

One approach to investigate the feasibility of using these chambers for DNA purification is using fluorescence to visualize the binding of DNA samples to particles within the chamber. Fluorescence is the spontaneous emission of light when a previously electromagnetically excited material falls back into its ground state.

Figure 4.38 shows such an experiment with the same device as shown in Figure 4.30 that is a device with anodized Al base layer. The excitation is from the left side. High concentrations of up to 0.03 mg mL$^{-1}$ double stranded DNA (dsDNA) labeled with FITC-ULS dissolved in PBS buffer have been used. This means that the DNA sample comes with linked fluorophore, in this case PlatinumBright. Particles are 10 µm streptavidine coated silica particles with a concentration of about 0.25 mg mL$^{-1}$. The illumination is from above with a TECBL-15G-488 laser from WSTech. Advantages of this setup are that the illumination at 488 nm is not in the UV range and thus a PMMA cover can be used and that the device base layer can be non transparent. A top illumination is flexible, however the used setup does not provide a fully homogeneous illumination as shown at $t = 0$ s with initial particle distribution. The next two images at 40 s and 120 s show fluorescence images. The final image at 140 s shows the chamber with additional white light illumination and the majority of particles agglomerated along the right boundary.
4.3. Square Chamber

Figure 4.38: DNA concentration experiment in square chamber where the excitation is from the left side and the base layer is anodized Al. The first three images show fluorescence images with initial, intermediate and final particle positions. The image at $t = 140$ s is with additional white light illumination.

The fluorescence of the particles at the right side at 120 s is not visible in comparable experiments without DNA sample.

Experiments performed with 5 µm particles showed similar results. From these results it can be concluded that it is possible to use continuous frequency sweeping to concentrate DNA. However, the experimental results do not allow to make any further quantitative conclusions.

Biochemistry

A number of biochemical experiments which do not include acoustics, have been performed in view of a quantitative analysis. These experiments are partly motivated by acoustic experiments and are summarized here for purposes of presentation.

The following experiments have been performed with NaI buffer solution and with ethidium bromide (EtBr) as fluorophore. The EtBr intercalates that is it forms a reversible inclusion between the bases of DNA. As a result EtBr fluoresces when excited with UV-light where the fluorescent light intensity is proportional to the DNA concentration. The molarity of NaI is 6 mol L$^{-1}$ if not stated otherwise. The particles are plain silica particles from Kisker Biotech with a diameter of 10 µm if not stated otherwise.

Test tube experiments have been performed in combination with beads. The described experiments are following the given description here. In a first incubation step the beads, DNA sample, and NaI solution are mixed including 15 min shaking. In a second separation step centrifugation at 5000 rpm for about 10 min is used to separate beads from supernatant. In a final step supernatant and bead pellet are resuspended and analyzed using agarose gel electrophoresis. A standard example mix consists of 100 µL silica beads with a concentration of 10 µg µL$^{-1}$ suspended in 4.8 mol L$^{-1}$ NaI and 20 µL dsDNA sample with a length of 10 kb and a concentration of 1 µg µL$^{-1}$ suspended in water and 80 µL NaI solution with 6 mol L$^{-1}$. For a typical sample kit size, the length of the DNA sample is on the longer side, still much smaller than genomic DNA, but pointing in that direction. A fixed length has been chosen for the aim of quantification.

An experimentation series has been performed to determine the holding capacity or
concentration of DNA that can be collected with silica particles and serves as reference. A gel electrophoresis image of dsDNA concentration experiments is shown in Figure 4.39. The first lane marked with M shows a DNA molecular weight standard. The next three lanes show dsDNA solutions where the numbers are the concentration in $\mu g \mu L^{-1}$. The remaining lanes show experiments with a constant amount of 10 $\mu$m particles present in total in the initial volume. Shown are pairs of bead pellet (B) and supernatant (S), where the numbers indicate the dsDNA amount in $\mu g$. The white arrow indicates a signal in the supernatant of the 40 $\mu g$ particle probe. Hence, it is expected that the particles are saturated with bound DNA at this concentration. As shown in the image the signal of the bead pellets in the gel is washed out or blurred. This makes a standard quantification unreasonable. Repeated washing steps could help solve this problem but did not result in quantifiable results.

Further tests show that the fluorescence of the particle solution in the wells does not increase significantly with higher dsDNA concentrations while the signal in the supernatant is increasing. Furthermore, an experiment with beads only in the solution also shows a signal in the well but none in the lane. This is likely due to adsorption of EtBr molecules to the plain silica beads which leads to fluorescence as the intercalation of EtBr with DNA bases does. This is particularly important to note for experiments where the fluorescence is visualized directly in the acoustic chamber.

The acoustic radiation force potential (2.29) is proportional to $r^3$, where $r$ is the particle radius. Classic magnetic beads are typically smaller than 1 $\mu$m while for the acoustics larger particles are advantageous. Therefore, the question of the binding capacity proportional to bead size arises. A test-tube series at standard conditions with silica particles of 20 $\mu$m, 10 $\mu$m, 5 $\mu$m and 0.5 $\mu$m in diameter has been performed. The mass concentration of the particles is kept constant and a saturation is found at around 20 $\mu g$, 40 $\mu g$, 80 $\mu g$ and 160 $\mu g$, respectively. This is in agreement with ideal conditions which would predict an
4.3. Square Chamber

inversely proportional relation between bead diameter and binding capacity for a constant mass volume. An exception are the 0.5 µm particles which have a lower saturation limit than would be expected from a linear extrapolation of the results of the larger particles.

Modified binding conditions can lead to optimal DNA binding conditions and also provide potential modifications for the acoustic conditions. Standard tests with different molarity ranging from 1 mol L\(^{-1}\) to 5 mol L\(^{-1}\) of the NaI solution have been performed. The results show optimal binding at 2 mol L\(^{-1}\).

Incubation experiments with subsequent gel analysis have been performed to ensure that no DNA binding to the chamber walls of the acoustic devices takes place. Therefore, different concentrations of DNA have been placed on glass, PMMA, and anodized Al which themselves are placed in a humidity chamber for 1 h. The results show that no significant binding to these materials occurs. A similar conclusion can be made from an experiment where particles have been shaken in both, a square chamber, and as comparison in a test tube. The results are shown in Figure 4.39 with bead solution (B) and supernatant (S). The setup follows the test tube approach and the solution has been removed from the device. The results also underline the importance of agitation which is even more evident in comparison to an experiment where particles fall through a square chamber driven by gravitational forces. The results are shown in Figure 4.39 with three lanes. The first lane (I) is taken from the entry point, the second lane (B) are the pelleted beads after 30 min sedimentation, and the last lane is the supernatant of the same. The results show that the amount of DNA collected with the beads and simple sedimentation is rather low.

Combined Experiments with Acoustics

One approach to investigate the feasibility of DNA concentration with acoustics is the one described in the proof of principle section where the DNA to particle bond is directly visualized in the chamber. Another approach is to extract the particles from the acoustic chamber once they have been concentrated close to an opening in the top cover. Supernatant has been removed from the second opening in the chamber as a control measure. These particles and supernatant can then be analyzed with agarose gel electrophoresis. If the DNA on the particles is not washed off prior to analysis, electrophoretic forces are used to separate the DNA from the beads. Such experiments have been performed with the same square chamber as shown in Figure 4.38 and 10 µm and 5 µm particles. Binding experiments provided a similar signal in the gel electrophoresis with the initial concentration and both extractions from the acoustic chamber. The fluorescence of the bead sample in the gel well is low. This is mainly due to difficulties in practice to remove the particles from the chamber completely. Reasons for this are the small opening in the top cover, particles distributed over a larger area than the opening, and particle attachment, as well as gravity.

The influence of ultrasound on the bond between DNA and particles has been investigated with a cross check where pre-incubated 10 µm particles have been manipulated with the acoustic device. Particles with different DNA saturation levels according to dsDNA amounts of 70 µg, 40 µg and 10 µg did not show a significant loss after acoustic manipulation. Thus, it is concluded that the applied ultrasound does not destroy the bond between
particles and DNA.

Experiments similar to the one described in the proof of principle section have been performed using a GenoPlex UV-light chamber from VWR for illumination and image recording. The setup is similar to the standard setup described in Section 3.2 but with transmitted UV-light from below. The top and bottom cover of the square chamber have been replaced with borosilicate glass cut from 0.5 mm thick glass wafers. These glass covers serve to avoid UV-light adsorption and result in a better acoustic performance than with PMMA covers. The experiments have been performed with NaI buffer solution and with ethidium bromide (EtBr) as fluorophore. A number of control experiments confirm that this configuration has no significant background UV-light adsorption at 260 nm of the device or solution except for the side walls of the device. Figure 4.40 shows experimental results with and without DNA sample. The excitation

![Figure 4.40: Fluorescence images of two experiments in the square chamber. The white arrows indicate particles and the bright frame is the PMMA wall. Excitation is with two transducers, one placed to the left and one at the bottom. The images are the final state after the particles have been move into one corner, back, and forward again. The more DNA is bound to the particles, the brighter they appear.](image)

is from two sides, from the left and bottom as shown in Figure 3.5. The waveguides are actively cooled at room temperature and high driving voltages of up to 20 V have been used. The duration of a single experiment is around 9 min where the particles have been moved into a corner, back towards the transducers, and then to the final position as shown in the images. For market applications a focus would be to improve this time. Here the focus is on the feasibility to concentrate DNA rather than optimizing the time which could lead to the use of aluminum devices. The white arrows in the images indicate the bulk amount of particles. The chambers have been filled with a mix of 1.3 mL NaI and 0.5 µL EtBr where 100 µL of this solution have been replaced with a solution of equal volume containing 50 µg mL⁻¹ of 10 µm silica beads. The experiment with DNA in addition has 100 µL DNA solution with a concentration of 1 µg mL⁻¹. The experiments show that the color intensity of the particles increases in the experiment with DNA while the experiment without DNA shows a rather low EtBr fluorescence. The fluorescence is low enough to be distinguished from DNA, but large enough that the particles can be visualized in the UV-light chamber.
Particle sedimentation is a typical and well known problem in acoustic devices [14]. For the square device sedimentation to the base layer is discussed in the particle tracing simulation Section 4.3.4 and in the particle experiments Section 4.3.5. With 10 µm silica beads in water or water like solutions this leads to particles which mainly agglomerate and are moved in the lower third or less of the chamber. This behavior is also observed in the experiments presented in Figure 4.40 and is one possible reason for the limited DNA concentration or collection yield. The performed experiments did not show any visual sign of acoustic streaming which possibly could serve as additional agitation. While classic techniques like test tube experiments include a strong agitation of the beads in the solution, this is not automatically given with the used square chambers and actuation. The importance of agitation is underlined with the previously described results in the biochemistry section. The experiments shown in Figure 4.39 show that a high DNA collection yield on the particles can be obtained with classic shaking independent whether the container is a test tube or acoustic chamber. Furthermore, particles collected with gravitational forces only show a low yield. Thus, it is believed that the lack of agitation is a main limitation factor of the performance of the acoustic devices to purify DNA.

In summary, it has been shown that DNA can be concentrated with the method of continuous frequency sweeping and the used ultrasound does not significantly hinder or destroy the bond of DNA to particles. There is some freedom for the choice of material as well as buffer solution with different molarity. This can be of interest for density matched particles which might result in lower acoustic radiation forces but help against sedimentation problems [94]. Large particles seem to be well suited to concentrate DNA. As a main problem remains the lack of agitation of particles in the acoustic chamber.

4.4 Wave Coupling

The transport of particles with continuous frequency sweeping depends on the damped frequencies between the resonances but also on the position of the excitation. For 1D this is discussed in Section 4.2.1 where the position of the fluid excitation and the one of the transducer coincide. In a 2D or 3D situation this can be replaced with a complex wave coupling between transducer, device, and fluid cavity. The model of a specific acoustic impedance and impedance matching normally suffice to describe the behavior between two materials. Geometrical influences and structural vibrations add complexity to the acoustic coupling between materials. An example where this becomes important is the influence of wave coupling over vibrating plates. The coupling over vibrating plates gains further significance with particle sedimentation. The direct interaction between plate and fluid has been investigated for different systems [63] and is of particular interest for surface acoustic wave devices [47]. Coupling of waves in 1D systems has been discussed in more detail e.g. by Gröschl [58, 60]. Here the focus is on changes of the wave coupling from transducer to the fluid due to structural geometry changes in 2D. These couplings are investigated with planar mm-sized devices where the transducer is placed below the fluid cavity. Such a layout is common with silicon micro machined acoustofluidic devices [123].
Specific Acoustic Impedance

A main difference between plastic and metal or silicon devices is the interface of largest acoustic impedance. As an example microfluidic devices are often manufactured with silicon microfabrication where a channel is etched into silicon [121]. The channel walls act as main reflector in this case. If the device dimensions are much larger than the channel width then eventually the outer device dimensions can be neglected. In contrast, the acoustic impedance difference between water and plastic is relatively small as shown in Table 2.2. Thus, with a plastic device the main wave reflection takes place at the outer dimensions of the device rather than the inner cavity walls. Hence, setting the whole structure to vibration at a random position of the device can work in some cases with silicon devices but is less suited for plastic devices.

To obtain the best energy transfer from transducer to fluid, impedance matching layers can be of interest as described in Section 2.6. However, the layers are optimal for a single frequency and thus not suited for continuous frequency change based devices.

4.4.1 System Description

The investigated devices are planar and have a transducer directly attached to it, as shown in Figure 4.41. The experimental setup and device fabrication are described in Section 3.2. The main difference is that here the transducer is glued to the device and therefore the devices are freely supported under the microscope. The support is either on foam material or on thin stripes of taut foil.

Device

The fluid chamber is built up from three layers as shown in Figure 4.42. The middle layer forms the side walls of the fluid chamber. This layer has been laser cut from a 1 mm PMMA sheet. The top and bottom cover have been varied in both, thickness and material. Both layers include PMMA sheets which are 250 µm, 0.5 mm, or 1 mm thick. In addition a 1 mm thick glass plate has been used as cover plate (top layer) which allows visual inspection similar to the PMMA plate. An alternative for the base layer is a 0.5 mm thick aluminum base plate with or without a cut. The cut is placed parallel to the transducer.
4.4. Wave Coupling

Figure 4.42: Dimensions and structure of planar device. The transducer is indicated with a hatched rectangle in the side view.

and in 1 mm distance from its edge. The cut separates the base layer into two parts which are 100 µm apart from each other.

As transducers 1 mm thick piezoelectric elements with a width of 2 mm and a length $w_{pz} = w$ are used. The transducer is placed at a distance $a$ from the left device boundary. To fill the chamber with water and particles two 1 mm openings have been added in the cover plate in opposing corners as shown in Figure 4.41.

A reference device is presented here. Its outer length and fluid chamber length are $\ell = 40$ mm, and $\ell_{\text{fluid}} = 30$ mm, respectively. The widths are $w = 12$ mm, and $w_{\text{fluid}} = 10$ mm. The top layer is a 250 µm PMMA layer, the middle layer a 1 mm PMMA plate, and the base layer is a 0.5 mm aluminum plate. The transducer is placed with $a = \ell_s = 5$ mm.

Wave Coupling Configurations

There is a large number of different transducer to device configurations [64, 121, 187, 95]. Microfluidic devices often have a small fluid domain compared to the device dimensions. Here the focus is on mm-sized devices where the size of the fluid domain is more dominant and the transducer placement becomes increasingly important.

Figure 4.43 shows three configurations in the cross section of the devices. In the first configuration the transducer is placed on the left side of the device. The elongation direction of the transducer and the desired wave propagation direction are in line. This configuration is used with the square chamber in Section 4.3. The other two configurations shown in Figure 4.43 illustrate the planar device, once with, and once without a cut in the base layer. Plate vibrations of the base layer are expected to play a major role in the configuration without a cut. The cut is introduced to decouple the base layer into two parts. It leaves a free water surface which is kept in place by its surface tension. This separation heavily reduces plate vibrations and thus the effect of coupling waves into the fluid by them.
Chapter 4. Acoustic Radiation Force Driven Particle Transport

Figure 4.43: Device cross section schematic of three different types of wave coupling from transducer over structure to fluid.

4.4.2 Numerical Simulation

The numerical simulations concentrate on 2D simulations of the side view cross section. Several aspects of the numerical simulations of this device have been investigated by E. Furger and are presented in his bachelor thesis [44]. The dimensions of the presented simulations refer to the reference device described in Section 4.4.1 if not specified differently.

Device Configurations

Figure 4.44 shows the absolute pressure fields of the fluid domain for three different configurations. The upper plots show a 3/2λ resonance in thickness or z-direction of the fluid cavity for each configuration. The lower plots show the peak resonance in x-direction in the frequency range 1.5 MHz to 2.5 MHz. With an uncut Al base layer, the amplitude

Figure 4.44: Absolute pressure fields $|p|$ of three different configurations. The plot at the higher frequency of each configuration shows the field for a 3/2λ resonance in thickness direction. The second plots show the strongest resonance in x-direction in the frequency range 1.5 MHz to 2.5 MHz.
of the resonance in $z$-direction decays slowly with increasing $x$. With a cut in the base layer high amplitudes are confined within the region above the transducer. The amplitude distribution of the cut Al and a PMMA base layer are similar. However, in comparison the resonance in $z$-direction in the range $x = 2\text{ mm}$ to $30\text{ mm}$ is less pronounced with the PMMA layer.

The strongest resonances of both Al configurations are approximately the same and below the fundamental transducer resonance of $2\text{ MHz}$, whereas the main resonance of the PMMA configuration is slightly above it. Thus these configurations are quite different even though the resonances in $z$-direction show similar behavior with a PMMA base layer as with a cut Al base layer. This is also shown in Figure 4.45 which shows the absolute pressure field along a line at half fluid cavity height for varying frequency.

\begin{figure}[h]
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\begin{subfigure}{0.4\textwidth}
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  \includegraphics[width=\textwidth]{al_base_layer}
  \caption{Al base layer}
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  \includegraphics[width=\textwidth]{pmma_base_layer}
  \caption{PMMA base layer}
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\begin{subfigure}{0.4\textwidth}
  \centering
  \includegraphics[width=\textwidth]{pmma_base_layer}
  \caption{PMMA base layer}
\end{subfigure}
\end{figure}

Figure 4.45: Absolute pressure field along a line at half fluid cavity height of the planar device. The two shown configurations are with uncut Al and PMMA base layer of $0.5\text{ mm}$ thickness.

Both plots show an amplitude modulation in $x$-direction which ranges over several wavelengths similar to the profile of a beat frequency. The wavelength of this modulation is $7.5\text{ mm}$ at the strongest resonance as an example. The corresponding acoustic wavelength in water is $\lambda = 0.78\text{ mm}$. With a PMMA base the modulation wavelengths are smaller. These modulations originate from plate vibrations. The two shown plots are very similar if the average of $|p|$ over the $z$-direction is plotted instead of a single line per frequency. Good performance of the device for particle transport is expected with high pressure amplitudes evolving over the whole length of the chamber and a frequency range which covers a node shift of at least one wavelength. The Al device fulfills this requirement. The PMMA device in contrast shows a too small frequency band with high pressure amplitudes and the amplitudes significantly decay with increasing $x$.

The cut Al base layer device shows a similar pressure field over the frequency as the uncut Al devices, but with lower amplitudes. The amplitudes are about $30\%$ lower.
Device Dimensions and Transducer Position

There are a number of other parameters which potentially change the performance of the device, such as outer dimensions or position of the transducer. Figure 4.46 shows the absolute pressure field averaged over the fluid cavity height at 2 MHz for two different parameters, starting from the reference device. The length $\ell$ of the device and correspondingly also $\ell_{\text{fluid}}$ is varied in the left plot where $\ell_{s}$ is not varied and the transducer center is at $x = 15$ mm. The right plot shows the result of a simulation where the transducer position $a$ is varied from the very left boundary to the center of the device.

![Device length and Transducer position](image)

Figure 4.46: Absolute pressure field averaged over the fluid cavity height. Both plots are at 2 MHz which is close to the transducer resonance frequency, and of a planar device with Al base layer.

The pressure isolines can be split into two domains when $\ell$ is changed. The pressure nodes remain almost constant between $x = 0$ mm and 15 mm since the distance between transducer and left boundary is kept constant. On the other hand the isolines follow the change of $\ell$ or the right boundary between $x = 15$ mm and 40 mm. The imprint of the 2 mm wide transducer to the solution is visible and rather locally confined. This behavior is in good agreement with the analytical solution of a 1D resonating cavity. An excitation acts as a variable boundary for changes in both, frequency and length. A change of $\ell$ shows different periodic changes of the total pressure amplitude along $\ell$ and between $x = 15$ mm and 40 mm. The most dominant has a periodicity of 1.30 mm. This is close to $\lambda/2$ of the longitudinal wave in Al which is 1.25 mm. The resonance condition for the fluid domain is $\lambda/2 = 0.37$ mm. Thus the resonance in the Al are dominant. The periodicity is more harmonic when the transducer is placed at $x = 0$ mm. The plot shows that the damping is relatively large allowing to use almost all lengths in the given range efficiently even with a single frequency. The long wavelength periodicity in $x$-direction is independent of the device length and has a fixed pressure node at the transducer. This is also shown in the transducer position...
plot with a variation of $a$. In this plot the isolines are constant over the whole $x$-range which is expected since neither length nor frequency are changed. The pressure field is weaker between $x = 0$ mm and 5 mm which is the range where the middle layer is fully covered with PMMA. Thus direct wave coupling into the fluid provides a constructive contribution.

The previous discussion indicates a significant contribution of plate vibrations. Be it as broadened contribution to resonances in the thickness direction as shown in Figure 4.44, or as shift and spread of the pressure amplitudes over the frequency range as shown in Figure 4.45, or with dominant periodic amplitude modulations. Nonetheless, an Al layer can also be used as reflector. Simulation results with the transducer attached on top of the device show a comparable performance even though the transducer is not directly coupled to the Al layer. The pressure field shows larger amplitudes around 1.95 MHz and 2.05 MHz with magnitudes in the same order as the reference configuration device. Furthermore, the amplitudes are higher with thinner top layers. This indicates that a rather rigid reflector is more important for good performance than wave coupling over such a rigid plate, be it Al or glass.

Damping

Apart from the wave coupling damping is very important for the performance of the devices. A very small damping of the devices would render continuous frequency sweeping inapplicable and the devices would work like frequency mode switching. On the other hand, the larger the damping, the lower are the peak pressure amplitudes and with it the acoustic radiation forces. From a 2D side view simulation it is seen that the contribution of the structural damping is rather large so that the main goal is to reduce damping. The qualitative results do not significantly change when the $Q$-factor of the water is increased. Small $Q$-factors of 500 or lower distort the results noticeably and are only applicable with a small number of wavelengths in the fluid chamber. Damping of glue layers modeled as thin elastic layer model on interior boundaries increases the total damping but the pressure field is not altered significantly. Damping introduced with a spring foundation model for the clamping provides a similar conclusion. These additional damping models are not included in the presented simulations since 2D models can hardly be used for quantitative analysis.

4.4.3 Experimental Results

The experiments have been performed with a large number of different devices. This is due to relatively uncritical side ratios of the side walls formed with the middle layer which eases the manufacturing a lot and allows to fabricate many different devices. None of the side walls is thinner than high. However, all of these devices feature relatively small transducers and at the same time have rather high damping compared to thin walled devices with a transducer at the side.

An example of a well working layout is the reference layout (sec. 4.4.1). Figure 4.47 shows an experiment with such a device. The Al base layer is anodized black for better contrast.
The particles are 55 µm copolymer spheres. A frequency sweep is performed with a range of 1.2 MHz to 2.3 MHz and a ramp rate of 0.1 Hz. The transducer voltage range is 2 V to 10 V.

At $t = 0$ s the particles are distributed homogeneously with the exception of an increased particle concentration in the lower right corner where also a 1 mm opening is placed. After 40 s most of the particles are transported to the right half of the fluid chamber and after 80 s to the right boundary. Moving particles in the region over the transducer is critical. This is also shown at $t = 40$ s where some particles are still trapped close to the left boundary. In a narrow frequency band the particles can still be moved and therefore the particles can eventually escape this region after several sweep cycles. With an inverse frequency sweep the particles can be moved back towards the transducer.

**Material Configurations**

The different material configurations are in good agreement with the numerical simulations. That is devices with a PMMA base layer perform poorly whereas an Al base layer provides better results. A cut in the Al base layer is comparable in performance to a PMMA base layer. This indicates that the total damping of the device plays a major role which can exceed the influence of plate vibrations.

The thickness of a PMMA top layer shows only a very small influence on the performance as long as the base layer is Al. If both, top and base layer are made out of PMMA then at least one should act as stiff reflector. This means a PMMA thickness of at least 0.5 mm to 1 mm. The poorest performance is obtained with two 125 µm PMMA foils. Glass has a lower material damping than PMMA and therefore better results could be expected with a glass top layer. This is only the case to a limited extent. A possible reason is that glass is bonded to the PMMA with an additional glue layer which potentially has a large damping, whereas PMMA to PMMA can be bonded with an almost perfect chemical bond without additional damping.

**Device Length and Transducer Position**

Different lengths of the devices have been tested successfully. Shorter devices with a length of 20 mm showed less damping which increases performance. Devices with a length up to 60 mm have been tested, but the performance becomes increasingly poor.
A number of devices have been tested with a larger width $w = 25\text{mm}$ but the same dimensions of the transducer as in the reference configuration. These devices have an increased damping, but apart from that show similar behavior as the reference design.

Devices with two transducers below both fluid chamber ends have been used to agglomerate particles in the center of the device with increasing frequency and back towards the boundary with decreasing frequency. A critical length is the distance from the transducer. With two transducers this distance is reduced which allows to increase the overall length of the devices. With a transducer placed below the center of the fluid chamber the particle have been moved in a similar manner, however with a lower transport performance compared to the operation with two transducers.

Other non-symmetric transducer positions, e.g. at one quarter of the fluid chamber length, have also been successfully used. This confirms that the transducer position is similarly essential as other boundary conditions and the system behavior described in the numerical simulation and Figure 4.46 is confirmed. Transducers located within the range spanned with $\ell_s$, that is below the PMMA of the middle layer, showed poor performance. This is in good agreement with the numerical simulation.
Acoustic Streaming Driven Particle Transport

Acoustic streaming is a net mean flow driven by acoustic oscillations. It can be divided into two types of streaming, bulk attenuation driven and boundary layer driven acoustic streaming. A more detailed description of acoustic streaming is given in Section 3.1.3.

Particles can be transported by drag forces. For micro-particles inertia can be neglected in most cases and the particles basically follow the fluid flow. Advantages of such a method are e.g. contactless handling and high achievable particle transportation speed. The speeds are relatively high with possible acoustic streaming velocities of several \( \text{mm s}^{-1} \) in mm-sized chambers [70, 8].

There are a number of applications as described in Section 3.1.3. Here the focus is on particle agglomeration or concentration and mixing. Particles can be agglomerated if streaming is combined with a particle trap. The particle trap holds particles in a flowing fluid. Over time, all particles driven through the trap with acoustic streaming will agglomerate in the trap and thus particles can be separated from the fluid.

Here the focus is on producing acoustic streaming with high efficiency and in a controlled way. In contrast, when efficiency is not a primary goal, high power acoustic horns can be used to move almost every fluid. However, besides efficiency issues, acoustic horns are also likely to produce cavitation and excessive heat. Furthermore, the presented theory is typically not valid for the latter with high amplitude oscillations.

First, bulk attenuation driven streaming is discussed in Section 5.1. This includes a discussion of streaming in rectangular chambers given in Section 5.1.1. The square chamber is introduced in Section 4.3 and streaming in this chamber is given in Section 5.1.2. The ring type devices are presented in Section 5.1.3 which are designed to work with acoustic radiation force based particle traps presented in Section 5.2 of this chapter. In the last Section 5.3, aspects of boundary layer driven acoustic streaming are discussed.
5.1 Bulk Attenuation Driven Acoustic Streaming

Bulk attenuation driven streaming refers to streaming where a momentum transfer from oscillations to the medium comes from the spatial attenuation of acoustic oscillations in free space. The presented devices have low wall curvatures and device dimensions which are larger than a wavelength in all directions. In addition, the acoustic excitation is chosen such that the wall surface velocity is expected to be the largest in normal direction to the chamber walls. Under this assumption attenuation driven acoustic streaming dominates and viscous attenuation in the acoustic boundary layer is negligible.

First, rectangular chambers are presented in Section 5.1.1 with a focus on coupling mechanism and mixing. In Section 5.1.3 ring type devices are presented which are potentially suited to be combined with particle traps. Finally a number of different device layouts are presented in Section 5.1.4. A detailed description of the implementation of the numerical simulation is given in Section 3.1.3.

Regime and Assumptions

The driving mechanism is associated with traveling waves. For this reason compressibility is important and the full driving force \( F_R \) (2.15) has to be taken into account. On the other hand viscous driving mechanisms like in a boundary layer play a minor role which greatly simplifies the first order calculations as described in the theoretical Section 2.1. It is further assumed that the condition \( Re_s \ll 1 \) for the streaming Reynolds number (2.5) holds true and thus the streaming can be calculated as Stokes flow. With peak velocity amplitudes \( U_0 \) in the simulation, in the order of \( \text{m s}^{-1} \) at 2 MHz and with high transducer voltages, this assumption might not always be applicable. However, at most frequencies in the investigated frequency range, the velocity amplitude is below 1 m s\(^{-1}\) where the assumption holds. It is further assumed that the perturbation parameter \( \varepsilon \ll 1 \) and that the amplitude of oscillations \( U_0 \) is small compared with the attenuation length \( R \) which typically holds in the ultrasonic range and in water. Another assumption made is that \( Ma \ll 1 \) for the Mach number (2.8) and hence streaming is incompressible.

The absolute value of the pressure field is often a descriptive quantity for resonating systems. As an example, the acoustic radiation potential is typically closely related to the pressure field in multi-wavelength resonators as shown for a plane wave in Section 4.2.1. A similar relation can be formulated for the relation between the pressure field of a plane traveling wave \( p(x, t) = \hat{p} e^{-\alpha_s x} e^{i(kx-\omega t)} \) (eq. (2.33)) and the driving force of the attenuation driven streaming \( F_{Rx} \). Neglecting viscosity in the first order equations (2.12) and with eq. (2.15) the relation becomes

\[
F_{Rx} = \frac{1}{\rho_0 \omega^2} \left( k^2 + \alpha_s^2 \right) \hat{p}^2 \alpha_s e^{-2\alpha_s x}. \tag{5.1}
\]

With eq. (2.32) and with \( \omega \tau_s \ll 1 \), the volume force can be written in the form

\[
F_{Rx} = \frac{1}{\rho_0 c_0^2} \hat{p}^2 \alpha_s e^{-2\alpha_s x}. \tag{5.2}
\]
where $\alpha_s$ of water is proportional to the frequency squared. This 1D relation provides a more direct idea of the amplitude of the driving force but is no direct measure for the resulting streaming field. In the limit of a plane wave without spatial attenuation $p(x, t) = \hat{p}\cos(kx)e^{-i\omega t}$ the force can be derived similarly as for the attenuated case given with eq. (5.1) and is
\[
F_{Rx} = \frac{1}{\rho_0 c_0^2} \hat{p}^2 k \sin(2kx).
\] (5.3)
This relation has a similar frequency and wave amplitude dependence as the attenuated traveling wave case described with eq. (5.2). The constraints on the transducer are relieved since the driving force produced by attenuation driven acoustic streaming scales similarly with the acoustic field amplitude and frequency. The excitation frequency can be expressed as a function of the attenuation length $R$. For attenuation driven streaming $R \sim 1/\alpha_s$ and for water at $25^\circ C$ and $2\text{MHz}$ the attenuation length is $R = 11.4\text{m}$ with an attenuation coefficient given in Table 2.1. For the numerical simulations a $Q = 10000\text{ f^{-1} MHz}$ is used which results in $R = 2.4\text{m}$ at $2\text{MHz}$. These values of $R$ are significantly above the mm-scale and large $R$ can be avoided with frequencies in the upper MHz range. Frequencies in the lower MHz frequency range with frequencies not higher than around $20\text{MHz}$ are used here. Besides practical reasons in the experiments, the reason for this is the aim to have high power input and thus the possibility to obtain high bulk fluid velocities. Piezoelectric transducers have the highest efficiency around the fundamental transducer resonance. High frequencies can only be obtained with thin transducers since the resonance frequency scales inversely with the transducer thickness. However, the electrical loading capacity decreases with decreasing transducer size. Thus, the maximal mechanical power which can be obtained with a piezoelectric transducer decreases similarly assuming a constant effectiveness with which a piezoelectric material converts electrical energy into mechanical energy. A descriptive value for this conversion efficiency is given with the electromechanical coupling coefficient [35]. In reality this coupling is not constant and depends on a large number of parameters such as transducer size with, and without preloading, stacked or layered transducers, different piezoelectric materials, driving voltage, and frequency.

**Experimental Setup**

A number of experiments have been recorded with PIV where systematic problems appear. One problem is light scattering from the plastic side walls of the mm-sized chambers which introduces difficulties to track particles on a single plane only. This is particularly true with laser cut side walls which are not plane enough.

Furthermore, acoustic radiation forces trap or influence particle paths of the tracing particles used for PIV. This is mainly the case with single frequency excitation and large particles. The smaller the particles, the less of a problem these forces are. However, the particles are increasingly hard to resolve with digital image recording over large areas which can be resolved with local optical magnification. Another method to avoid trapping, than using small particles, is a fast change of the excitation frequency over a range of a few kHz. This works due to the different time scales in the order of $1\text{s}$ for acoustic
streaming and in the order of 1 ms for the acoustic pressure field to reach a steady state. This fast frequency change leads to a small net mean radiation force on the particles where fast means with respect to the time particles need to collect in an acoustic radiation force potential minimum. A net mean volume force on the fluid is still established causing acoustic streaming. This has been successfully used in experiment with ring type devices but the results presented here are for single frequencies or slow frequency changes only.

5.1.1 Rectangular Chamber

A rectangular device is one of the simplest planar layouts and very similar to a square chamber described in Section 4.3. The chamber is combined with a small transducer placed centered on a sidewall. The relatively simple layout makes such a device suitable to investigate the basic mechanism of attenuation driven acoustic streaming in a mm-sized planar device. In addition, mixing within a single chamber is of interest for DNA concentration as an example. The device configuration has similarities with the configuration for a plane wave traveling along a circular tube investigated by Eckart [38]. Several aspects of the rectangular chamber have been investigated by T. Hilsdorf and are presented in his master thesis [73].

Device

The rectangular devices have three layers. The 3 mm high middle layer forms the device side walls and is shown in Figure 5.1 in a top view together with the transducer. This layer consists of a PMMA main frame with 1 mm thick walls at three sides and an additional wall marked as ‘transducer wall’. The thickness of this wall is varied and in the case of a 1 mm thick transducer wall manufactured in one piece with the main frame. This middle layer is covered with a top and bottom layer made of polyester.

![Figure 5.1: Dimensions of the rectangular chamber design.](image)

In the following, the dimensions of a reference device are given which is used if not mentioned otherwise. The fluid chamber length and width are $\ell_f = 11.5$ mm and $w_f = 12.5$ mm, respectively. The transducer wall as well as the top and bottom cover are made of polyester with a width of 0.1 mm in experiment. For the simulations the transducer wall is modeled as PMMA. The 1 mm thick transducer has a width and length of 3 mm × 5 mm. The top cover has two 1 mm openings placed symmetrically in two corners close to the
5.1. Bulk Attenuation Driven Acoustic Streaming

wall at the far end from the transducer. More details of the device manufacturing are
given in Section 3.2 together with a description of the basic experimental setup.

**Numerical Simulation**

A numerical simulation has been used to investigate basic streaming patterns and the in-
fluence of different parameters. These are e.g. a change of the transducer wall thickness,
the length of the device, or the $Q$-factor. The simulations mainly cover the frequency
range around the fundamental transducer resonance frequency of 2 MHz. In addition,
frequencies around the second thickness mode of the transducer at 6 MHz have been
investigated. These are also frequencies where good results have been obtained in exper-
iment with transducer voltages of 27 V around 2 MHz and 14 V around 6 MHz. The same
driving voltages are used in the presented numerical simulations for better comparison.

The frequency dependent system behavior around the fundamental transducer res-
onance frequency is illustrated in Figure 5.2 which shows the spatial average values of
both, absolute acoustic pressure field, and absolute streaming velocity field for the refer-
dence device design. The spatial average is taken from the complete fluid domain. The
vertical, light gray lines indicate resonance frequencies of a 1D solution in $x$-direction and
are given for comparison. The pressure amplitude response shows a number of peaks with

![Figure 5.2: Spatial average values of the absolute acoustic pressure field and the abso-
lute streaming velocity. The values are from a 2D top view simulation of the rectangular
device.](image)

no clear trend for resonances in $x$-direction. This behavior is mainly due to the size of the
transducer which does not elongate over the total length of one side of the device. This
stands in contrast to the system behavior of the square chamber presented in Section 4.3.
The average streaming velocity response follows the pressure amplitudes qualitatively. In
contrast, a pressure field without a variation in lateral directions will create a quasi 1D
situation where no streaming can appear. Even though the amplitudes of these quantities
follow each other, the field patterns strongly deviate.

Figure 5.3 illustrates the deviation between acoustic pressure field and streaming veloc-
ity field patterns. The first frequency at 2.102 MHz is a low amplitude example and
the second frequency at 2.119 MHz is a high amplitude example. The plots show half
of the symmetric solution. The velocity field is shown as color plot together with black streamlines and with white arrows to indicate the flow direction.

Figure 5.3: Simulated absolute values of streaming velocity field and acoustic pressure field at 2.102 MHz and 2.119 MHz. Streamlines are shown as black lines and the flow field directions are indicated with white arrows.

The number of flow field patterns in the frequency range from 1.9 MHz to 2.2 MHz is large and they can change their shape significantly within frequency changes of 1 kHz which makes such a device potentially suited for mixing. Characteristic for these patterns are streamlines which expand either over the device length in $x$-direction or transverse to it in $y$-direction. Elongated vortices often align parallel to each other with different flow amplitudes as shown in the velocity field at 2.119 MHz and illustrated in Figure 5.3. These patterns also include inverse type flow fields with streams in the center of the device pointing towards the transducer as shown in Figure 5.4 for a frequency of 2.092 MHz. The term ‘inverse flow’ can also be understood in comparison to forward flow fields as the one shown in Figure 5.4 for a frequency of 6.671 MHz. This type of forward flow field can also be found around 2 MHz but with lower amplitudes. This is a typical pattern which has similarities with the streaming produced by a plane wave traveling along a circular tube as treated by Eckart [38]. The shown case around 6 MHz is discussed in more detail later.

There are a number of possible reasons for the large variety of velocity field patterns around 2 MHz which change fast with the frequency. These reasons are for example the excitation frequency, the PMMA structure, the transducer, or the attenuation length. Particular interest is on the inverse type flow since this stands in contrast to explanations which address the streaming patterns to acoustic energy gradients pointing away from
5.1. Bulk Attenuation Driven Acoustic Streaming

Figure 5.4: Acoustic streaming velocity fields with black streamlines and white arrows indicating the flow direction. The simulation at 2.092 MHz has an excitation voltage of 27 V and shows an inverse type flow field. The one at 6.671 MHz is with 14 V and shows a forward flow field.

The transducer in normal direction.

In the given model the attenuation length is much longer than the fluid chamber dimensions. To recap from eq. (2.39), the relation between attenuation length and $Q$-factor is given by

$$ R \sim \frac{1}{\alpha_s} = \frac{2c_0 Q}{\omega} \quad \text{(5.4)} $$

For the frequency range presented in Figure 5.2 and $Q = 10000 \text{ f}^{-1} \text{MHz}$ this results in $R > 2 \text{m}$ and thus an attenuation length which is over 100 times longer than the chamber length. No significant change in the field patterns is expected for a wide range of different $Q$-factors taking this difference in length scales as argument. A simulation with changing $Q$-factor leads to a similar conclusion. For $Q$-factors between 2000 and 20 000 the changes in the velocity field patterns are in average comparable to, or even smaller than those obtained with a 100 Hz change of the excitation frequency. The field amplitudes are almost proportional to the attenuation or for the velocity holds mean($|v|$) $\sim 1/Q$ which is in agreement with eq. (5.2). Simulations with $Q$-factors below $Q = 1000$ start to show significant changes in the flow patterns. However, such low $Q$-factors are no longer a valid assumption for water and multi-wavelength devices.

Flow fields similar to the ones described with the numerical simulation have been obtained in experiment and are discussed in more detail in the experimental results section. These results show velocities which are at least one order of magnitude lower than the sim-
ulation results. A reduction of the velocity amplitudes in the simulation with one order of magnitude could be obtained with a $Q$-factor for water which is one order of magnitude higher. However, such high $Q$-factor values lead to a discrepancy between experiment and simulation of the pressure fields. The experiments show rather flat resonance peaks which suggests larger damping. This can be due to the fact that the simulation is in 2D and thus perfectly rigid covers are assumed. Hence, acoustic attenuation in the third dimensions is neglected. Other discrepancies are e.g. losses into the surrounding, or wave scattering at particles, or reduced transducer efficiency due to heat. Apart from that also losses and inaccuracies in the device fabrication are likely to play a role. Furthermore, the 2D model with shallow channel approximation might not be accurate enough. Boundary layer driven acoustic streaming e.g. at the top and bottom cover can influence the results further and is not included in the simulation since its influence is expected to be small compared to the above mentioned effects. A more detailed discussion of boundary layer driven acoustic streaming is given in Section 5.3.

The PMMA structure and piezoelectric transducer build a main component of the system and therefore contribute to the flow field patterns. Their influence has been investigated with three simulations where the dimensions of the fluid domain are unaltered with respect to the reference design and are presented in the following.

First, a simulation without the PMMA structure except the part of the transducer wall between transducer and fluid show similar velocity field patterns as the simulations with the full reference design. Using a hard wall boundary condition, or a soft wall boundary condition on the free fluid boundaries changes the frequency specific field but provides similar quantitative behavior of the system over a frequency range. Both, flow field variety and significant pattern changes with small frequency changes are observed. The average amplitudes of pressure and flow field are higher without surrounding structure. Also inverse and forward type flow field patterns similar to the ones shown in Figure 5.4 are observed in this configuration.

Secondly, both PMMA structure and transducer have been removed. The excitation is with a normal acceleration boundary condition of constant amplitude and the same extent as the transducer of the reference configuration. This simulation again shows both inverse and forward type velocity field patterns. However, the variety of the flow patterns is not as high as in the previous simulations and forward flow patterns dominate.

As a third change, the thickness of the transducer wall has been changed. The investigated wall thicknesses are 0.1 mm, 0.5 mm, 1 mm and 2 mm. These simulations are also inspired by experimental results with a thin foil which tended to show better results compared to 1 mm walls. The simulations mainly show slightly smaller amplitudes due to the increased damping but no significant qualitative change. The reflection of the acoustic waves takes place at the outer wall interfaces rather than the boundary between fluid and PMMA since PMMA and water have similar specific acoustic impedances. Thus, a change of the outer length also changes the pressure field accordingly. Longer length means that the $\Delta f$ between resonances is smaller.

In summary, the structure and the transducer have a great influence on the frequency specific flow field but are not the main reason for the large variety of flow fields, nor for the fast changes in the flow field patterns around 2 MHz. In addition, the attenuation
around 2 MHz is not large enough to observe a direct link between the energy gradient in the fluid and the streaming flow field. Moreover, the attenuation length is one to two orders of magnitude larger than the device length which allows a large number of equally dominant but fundamentally different resonant modes in the fluid. This is believed to be the main reason for the described system behavior including inverse type flow patterns around 2 MHz.

An investigation of the fluid chamber length $\ell_f$ is motivated by two conditions. First the fluid chamber has to fulfill continuity of the flow field and hence a strong stream in the center of the device will also cause a back-flow at the side walls. Secondly, the ratio of attenuation length to device length is potentially important, comparable to a change of the fluid attenuation. For this reason simulations have been performed where the length $\ell_f$ of the reference design has been increased from 11.5 mm up to 40 mm. The simulation results with these long devices show streaming velocity patterns which vary with the frequency to a similar extent as the reference design simulations. Furthermore, the appearance of peak velocities in relative space is similar between short and long devices which means that some solutions also show peak velocities at the far end from the transducer. Devices with a length of up to 40 mm seem to be similarly suited for mixing. In addition, no clear trend can be given in the investigated length range with respect to whether longer devices are better suited to produce high streaming velocities in a closed rectangular volume.

In the previous discussion the influence of the frequency range around the fundamental transducer resonance has been discussed, e.g. with Figure 5.2, or otherwise this frequency range has been used for the investigation of other parameters. Next, the focus is on frequencies around the second fundamental transducer resonance frequency at 6.72 MHz. The investigation is motivated by good experimental results around this frequency as well as the attenuation which is strongly frequency dependent as shown with eq. (5.4). Figure 5.5 shows the spatial average of the absolute value of the acoustic pressure field and the streaming velocity field. Both field quantity responses follow each other qualitatively which is similar as around 2 MHz shown in Figure 5.2. However, at frequencies around

![Figure 5.5: Spatial average of the acoustic pressure $|p|$ and the streaming velocity $|v|$. The values are from a 2D top view simulation of the rectangular device.](image)
6.72 MHz the fluid resonances in $x$-direction are way more dominant. The pressure fields show a high amplitude beam in the device in $x$-direction for almost all frequencies. The beam is in extension of the transducer. The streaming velocity field patterns are all of the forward flow field type as a result and as shown at 6.671 MHz in Figure 5.4.

Neither increasing the $Q$-factor of water up to 100,000 nor removing the PMMA frame changes this behavior significantly. Some small vortices appear with the latter change at some frequencies and at the side walls or in the center but the main vortices are still similar to the previous described forward streaming type. Hence, a forward streaming field can be established with a configuration with 1 mm piezoelectric transducer excited at the second thickness mode. This configuration is neither very sensitive to small frequency changes nor to small changes of the structure.

**Experimental Results**

The rectangular chambers have been fabricated and tested as described in Section 3.2. The side walls of the devices presented here are all either fabricated from laser cut 3 mm PMMA sheets or are composed of cut 1 mm sheets which is to allow illumination with a laser light sheet through the side walls needed for PIV measurements. A description of the used PIV setup is given in Section 3.4. The illumination is at half device height. The devices have a permanently attached 1 mm Pz26 piezoelectric transducer and are loosely mounted on a transparent film strip strained over two bars.

A main focus is on the system behavior of the reference design device with different frequencies. These frequencies are mainly around the three first fundamental resonances at 2 MHz, 6.5 MHz and 10.8 MHz as determined with an electric impedance measurement. Figure 5.6 shows the measured streaming velocity fields at 2.00 MHz and 6.47 MHz and the transducer voltages are 27 V and 14 V, respectively. These are rather high voltages causing considerable heating. Therefore, the devices are not operated longer than 1 min at a time. The shown 2D flow fields do not satisfy continuity. Reasons for this are acoustic radiation forces on 20 µm polyamid particles and a flow field which is not fully planar. The flow field has vortices which can extend in the height or $z$-direction. Furthermore, light scattering from the laser cut side walls introduces difficulties to track particles on a single plane only. The velocity fields are not fully symmetric which is expected to be mainly due to tolerances in the fabrication of the device. Furthermore, small bubbles or particle agglomerations can influence the field. The shown velocity fields establish in less than a second. Similar experiments show times of several seconds until the maximum streaming velocities are reached. Possible reasons for the different times are changes in temperature which changes attenuation or the presence of particles and small bubbles which vary when a flow is established. The streaming patterns reach a steady state and frequency changes in the range of kHz have not produced significantly different streaming patterns. A frequency change mainly changes the velocity amplitude where streaming is only observed in a frequency range of a few kHz.

The flow field at 2 MHz shows an inverse type flow with a stream towards the transducer in the center of the device. The highest velocities are up to 0.7 mm s$^{-1}$. This type of flow has been reproduced with five experiments including experiments with a 0.5 mm thick
transducer wall and peak velocities of up to 1.1 mm s\(^{-1}\). The latter experiment shows a more distinct back flow in the center of the device but the 0.1 mm thick transducer wall example is shown for consistency.

With the numerical simulation an inverse type flow pattern is obtained at 2.092 MHz presented in Figure 5.4 which is similar to the one obtained at 2 MHz in the experiment shown in Figure 5.6. Simulation and experiment agree on the existence of inverse type flow patterns where one of the main differences between simulation and experiment is the fact that the simulation predicts a large variety of flow patterns which change relatively fast with frequency changes. Small amplitudes are a possible reason that a large number of streaming patterns do not show. The peak velocity in the 2D simulation is around 10 times higher which is likely due to a number of attenuation effects as discussed in the numerical simulation section.

The second flow pattern shown in Figure 5.6 is at 6.47 MHz and is a forward type flow. The peak velocity is 1.8 mm s\(^{-1}\). The shown pattern is in agreement with the numerical simulation which predicts such a forward type flow for most frequencies between 6 MHz and 7 MHz. An example is shown in Figure 5.4 at 6.671 MHz which is a peak resonance in the simulation. The frequency difference of experiment and simulation is mainly a shift of the transducer resonance. Additional mass load of the transducer due to wiring can be a cause which leads to the lower frequencies. Furthermore, there is a mismatch of fluid resonances which can be explained with a differing length of the experimental device due to tolerances in fabrication. Streaming was easier to obtain in experiment around the second transducer resonance than the first by trend. This behavior is in agreement with the numerical simulation and the previously mentioned broad appearance of similar flow patterns around this frequency.

Experiments at the third transducer resonance frequency around 10.8 MHz show a forward type streaming pattern similar as observed at 6.47 MHz. The flow field is slightly
more symmetrical compared to experiments performed with the same device but at lower frequencies. Peak velocities of \(2 \text{ mm s}^{-1}\) have been obtained with transducer driving voltages around \(5 \text{ V}\) and the reference device layout. Velocities of up to \(7 \text{ mm s}^{-1}\) have been obtained with a device with a longer transducer width and length of \(3 \text{ mm} \times 8 \text{ mm}\).

Experiments have been performed with modeling clay as damping material at the outer wall. The clay has been introduced with similar dimensions as the transducer at the opposite side of the device or \(x = 12.5 \text{ mm}\) and \(y = 0 \text{ mm}\). The motivation for the experiments is to test whether a traveling wave suffices to produce acoustic streaming in the chamber. It is assumed that the clay damps most of the waves and hence only a low amplitude wave is reflected. Such a setup is similar to the one used by Eckart [38] except that here the length in \(x\)-direction is much shorter and amplitudes probably lower. To increase the length experiments with 40 mm length have been performed as well. These experiments did not result in noticeable streaming. This is in agreement with the theory which leads to the assumption that only the superposition of traveling waves can provide large streaming amplitudes in such a small device filled with water.

In summary, it has been shown that attenuation driven acoustic streaming can be produced in mm-sized rectangular chambers around the fundamental transducer resonances. Both, forward and inverse type streaming have been produced and are shown to be in agreement with the numerical simulations. Furthermore, external acoustic absorption material prevents the generation of acoustic streaming.

### 5.1.2 Square Chamber

The square chamber is introduced in Chapter 4 where the focus is on acoustic radiation force driven particle transport or continuous frequency sweeping. Here the focus is on streaming effects which cause flow in the complete chamber volume. Discussed is the frequency range from \(1.5 \text{ MHz}\) to \(2.5 \text{ MHz}\) which is the same range as discussed in Section 4.3.

#### Numerical Simulation

The square chamber is a special case of the rectangular chamber discussed in Section 5.1.1. The latter includes a more detailed discussion of the influence of different parameters such as the PMMA structure, attenuation, or dimensions of rectangular devices. Several system behaviors are similar for both, rectangular and square chamber designs. One example is that the average of the acoustic pressure amplitude response and the streaming velocity response follow each other qualitatively. Thus, large streaming velocities are mainly obtained with fluid resonances of the pressure field. Furthermore, the streaming velocity field changes strongly with changing frequency. A change of \(1 \text{ kHz}\) around \(2 \text{ MHz}\) can lead to a totally different streaming pattern. However, typical streaming patterns vary between both chamber designs and are presented in the following.

Figure 5.7 shows the streaming velocity field at \(1.5100 \text{ MHz}\) and at \(1.9575 \text{ MHz}\) with black streamlines and white arrows which indicate the flow direction. The device is excited with a single transducer from the left side as indicated. The transducer voltage of \(20 \text{ V}\) is
the same as for the previous simulations of the square chamber. Both frequencies shown in the figure are resonance frequencies. The pressure field at a frequency of 1.5100 MHz and the same device configuration is shown in Figure 4.17. The velocity field at 1.5100 MHz is not a characteristic field but is motivated by experimental results presented in the Experimental Results subsection. The flow field at 1.9575 MHz shows a characteristic flow pattern with eight longitudinal vortices. Similar flow patterns with eight vortices appear at least five times between 1.9 MHz and 2.2 MHz with at least 18 kHz between two patterns. The peak velocities close to the fundamental transducer resonance frequency of 2 MHz are higher than the velocities around 1.5 MHz. They can reach over 0.2 m s⁻¹ even though a shallow channel approximation with a height of 3 mm has been used to account for the third dimension.

Figure 5.8 shows the streaming velocity field at 2.0320 MHz and at 1.9570 MHz of the square chamber with double sided excitation. The flow pattern at 2.0320 MHz is similar to experimental observations but is not a typical pattern observed with the numerical simulations. The second shown frequency at 1.9570 MHz shows a characteristic flow pattern for the investigated frequency range. The number and orientation of the individual vortices changes with the frequency. Often transitions from locally confined to longitudinal vortices are observed.

In summary, the shown flow patterns are well suited for mixing if operated at specific single frequencies. Acoustic streaming is generally undesired for particle transport with acoustic radiation forces and continuous frequency sweeping. With a changing frequency, strong streaming is generally unlikely which is due to the reason that the streaming patterns change fast and significantly with the frequency. In other words, there is no
central dominant stream as shown in Figure 5.4 which is present over a frequency range of several kHz.

**Experimental Results**

In experiments different types of fluid flow have been observed. Besides acoustic streaming, these are mainly Stokes drag induced streaming and thermal convection which are discussed in the fluid flow subsection of Section 4.3.5.

Acoustic streaming in square chambers has been observed roughly in every fifth or even less out of 100 frequency sweeping experiments in PMMA devices. With aluminum devices streaming has been observed roughly every tenth experiment out of 40 experiments. Producing acoustic streaming on purpose at a single frequency did not result in reproducible results. Reproducibility is one of the main issues, whether the goal is to obtain or to avoid acoustic streaming. These observations include both, experiments with permanently attached transducers as well as experiments where the transducer has been clamped to the device. Clamping is a potential source of reduced reproducibility.

The streaming patterns observed with continuous frequency sweeping experiments and one sided excitation are of similar type as the one shown at 1.5100 MHz in Figure 5.7 of the numerical simulation results. These patterns show flow towards the transducer close to the side walls at $x = 0$ mm and 20 mm. The flow in the center of the device is more homogeneous than the one shown in the numerical simulation. Peak velocities are around $1 \text{ mm s}^{-1}$ and below.

Experiments with continuous frequency sweeping and double sided excitation showed
patterns similar to the one shown at 2.0320 MHz obtained with the numerical simulation and shown in Figure 5.8. High streaming velocities of over \(50 \text{mm s}^{-1}\) have been observed. These high velocities have been observed with a single device made of PMMA with polyester foil cover and excitation voltages in the range 2 V to 10 V. The results have been reproduced in three different experiments where the device has been removed and put back in place in the configuration with clamped transducers. However, the results have not been reproduced with five similar devices with the same material parameters. These experiments have been performed with 9.6 µm copolymer particles which started to flocculate within a few seconds.

In summary, in most cases there is no or only low velocity acoustic streaming in the square chambers. The experiments showed the potential of acoustic streaming which can result in high streaming velocities even with relatively low power consumption.

### 5.1.3 Ring Type Devices

The ring type devices are designed towards an application of DNA concentration with functionalized beads. Acoustic streaming provides a non-contact method to drive fluid flow. This can be combined with a non-contact particle trap to concentrate particles. Ring type refers to the aim to produce fluid flow in a channel and flowing in a closed loop. In this way all particles in the chamber end up in the particle trap after some time. In addition, the fluid flows around trapped particles providing a potentially high contact between fluid particles or other material like DNA and the particles. More details about DNA concentration is given in Section 4.3.7 for a system where the particles are moved in a fluid at rest.

The ring type devices consist of two main parts, a fluid acceleration chamber and a side channel as illustrated in Figure 5.9. The aim of the acceleration chamber is to drive a fluid flow or produce a pressure difference with acoustic streaming. This pressure drives the flow in the side channel and closes the loop. Wave attenuation in the side channel can also contribute to the overall streaming. This functional splitting into a part which

![Figure 5.9: Transparent PMMA ring type device with openings in the top cover to fill the chamber.](image)

...
the aim is for particle agglomeration which can be achieved with a particle trap in the side channel. Suitable contactless particle traps based on acoustic radiation forces are described in Section 5.2.

This device design has similarities with proposed designs found in literature for ultrasonic pumps [131, 144, 140]. One proposal is based on an apparatus to measure sound absorption in liquids and consists of a main chamber with absorbing channel end and a return path [131, 144]. This design is mainly suited for large devices or frequencies in the upper MHz to GHz region. Another proposal from Rife et.al. [140] consists of a PMMA block mounted on a silicon wafer. Their device is operated around 50 MHz and they obtained mean streaming velocities of 1 mm s$^{-1}$. The device design proposed here aims for another scale and is fabricated fully in plastic. In addition, here the attempt is to avoid structural damping rather than utilizing it which leads to low performance by design. A main focus is to achieve high flow rates in the side channel. Another aim is to design devices with small dead volumes in the fluid chamber which is in contrast to the designs presented in literature. Dead volume means that there is no fluid exchange.

Device

The top view of a typical ring type device is shown in Figure 5.10. The schematic shows the main frame forming the 1 mm thick walls together with a 1 mm thick transducer at the left side. The 3 mm high PMMA frame is laser cut and covered by a top and bottom layer. These covers are either 100 µm thick polyester foils or 0.175 mm PMMA plates. A detailed description of the device manufacturing is given in Section 3.2.

![Figure 5.10: Dimensions of the ring type device with transducer shown hatched.](image)

A number of different device dimensions have been tested experimentally and are described in the experimental section. Most of these devices result in a volume of 1 ml to 2 ml. The three corners marked in Figure 5.10 have been changed individually to either a rectangular corner, a chamfered corner as shown with corner 1, or to a round transition between two corners.

The dimensions of a reference device are given here. The width $w$ times length $\ell$ are 20 mm $\times$ 40 mm and the 45° chamfer in corner 1 is placed with $w_{ch} = 6.5$ mm. The inlet or hole has a length of the straight part of $\ell_b = 20$ mm and an outer radius of $r = 2.5$ mm. The inlet is placed with $w_a = \ell_a = 7.5$ mm.
Numerical Simulation

The numerical simulations cover the frequency range around the fundamental transducer resonance frequency of 2 MHz. Goals are to find frequency dependent behavior or typical flow patterns, to optimize towards small dead volumes, and to investigate device designs which potentially increase the device performance towards high flow rates.

Figure 5.11 shows the streaming velocity field of a fully rectangular design. Streamlines are shown black and the flow direction is indicated with arrows. The excitation frequency is 2.022 MHz which is a frequency with a peak flow rate in the side channel. From general fluid mechanics it is expected that rectangular corners of both outer structure and hollow cavity are suboptimal however this device design serves as comparison for further changes. The shown flow field in the acceleration chamber is characteristic for this device design which means that multiple vortices appear with most excitation frequencies. The mean flow rate in the side channel shows strong variations with the frequency, but the flow does not change directions. Furthermore, the flow can be described as almost uniform potential flow. This flow comes with dead volumes in the two top corners number 2 and 3. The lower two corners have less dead volume with vortices expanding into the corners in comparison. This is in agreement with the volume force field $F_\text{R}$ which shows low amplitudes in the side channel. Hence, this device design fulfills the desired separation of fluid acceleration in the main chamber and pressure driven flow in the side channel.

Eckart’s investigation of acoustic streaming as well as quantitative measurements of attenuation in liquids aim at producing attenuation driven acoustic streaming with waves traveling along a cavity [38]. This is achieved with a transducer on one side and absorbing material on the other side of the cavity. Here this is achieved with a high damping material of the same extent as the transducer but introduced at the end of the acceleration chamber at $x = 39$ mm. This leads to lower flow rates in the side channel and frequency shifts of the flow rate response towards higher frequencies, due to the change of the boundary.

![Figure 5.11: Fully rectangular device design. Acoustic streaming velocity field with black streamlines and black and white arrows indicating the flow direction.](image)
conditions. These results are in agreement with the discussion of the attenuation length given in Section 5.1.1 which in summary state that the chamber length is one to two orders too small to be efficiently operated without focusing or without the use of reflections and with frequencies around 2 MHz.

The potential attenuation length of a traveling wave can be increased compared to the fully rectangular design with a chamfer in corner 1 as shown with the reference design in Figure 5.10. The reference design leaves the two top corners 2 and 3 unaltered. This is with the aim that the flow rate in the side channel is not influenced by the attenuation of waves in the side channel itself which in turn aims to the decoupling of acceleration chamber and side channel as achieved with the fully rectangular design. Figure 5.12 shows the streaming velocity of the reference device design with black streamlines and arrows which indicate the flow direction. The excitation frequency of 2.038 MHz is a frequency with peak flow rate in the side channel. The shown flow field with a single stream of high amplitude and low velocity vortices in the acceleration chamber is characteristic for this device design. The flow in the side channel is similarly homogeneous and unidirectional as the one of the fully rectangular device design presented in Figure 5.11.

The fully rectangular and the reference device design have similar cross sectional areas or volumes of 1.60 ml and 1.64 ml, respectively. Both have similar peak flow rates around 0.2 cm$^3$s$^{-1}$. However, the average flow rate is 0.046 cm$^3$s$^{-1}$ and 0.114 cm$^3$s$^{-1}$, respectively, between an excitation frequency of 2 MHz and 2.1 MHz, calculated with a resolution of 1 kHz. The average ratio between the flow rates is still around 2.6 with a range of 1.9 MHz to 2.2 MHz. Hence the reference device design performs better in average.

One of the drawbacks of the reference design is the large dead volume in the chamber. These volumes appear in both, the acceleration chamber as well as in the top corners of the side channel. A further device design proposed addresses these problems and is shown in Figure 5.13. The device has a chamfer in corner 2 which is expected to deflect traveling
waves towards the side channel and thus the attenuation of traveling waves can no longer be assumed to be mostly decoupled between side channel and acceleration chamber. This is also confirmed by the volume force field underlying this simulation. Shown are the

![Acoustic streaming velocity field with black streamlines and arrows indicating the flow direction.](image)

Figure 5.13: Acoustic streaming velocity field with black streamlines and arrows indicating the flow direction.

streaming velocity field with black streamlines and arrows indicating the flow direction. The excitation frequency of 2.091 MHz results in a peak flow rate in the side channel. The flow field shows hardly any vortices besides the main flow and the dead volume is small in comparison to the fully rectangular and the reference device design. This is also reflected in the flow rate frequency response which is more constant compared to the previous designs. The device design shown in Figure 5.13 has a fluid chamber volume of 1.45 ml. The transducer has the same thickness and thus resonance but is much shorter with a length of 6.5 mm compared to 12.5 mm of the reference device design. Hence also the energy input into the system is smaller. The resulting peak flow rates in the side channel are around 0.28 cm$^3$ s$^{-1}$. This is larger than the previously presented two designs. However, the according average flow rate over the frequency is range 0.08 cm$^3$ s$^{-1}$ and thus lies between the other two designs with flow rates of 0.046 cm$^3$ s$^{-1}$ and 0.114 cm$^3$ s$^{-1}$, respectively.

High amplitude standing waves are mainly present in the acceleration chamber of the fully rectangular and the reference device design. The pressure nodal lines are parallel to the transducer. Devices with three round or chamfered corners additionally have strong pressure amplitudes in the side channel. Those standing waves can be a problem with single frequency operation and the aim to transport particles. However, this can be circumvented with fast low range frequency changes as described in Section 5.1.

In summary, the acceleration chamber should be rather thin compared to its length to avoid vortices which produce dead fluid volume. The presented fully rectangular and reference device designs can be used to produce a rather homogeneous flow in the side channel. This flow is expected to be driven mostly by the acceleration chamber unless the corners of the device are changed to include the side channel in the fluid driving mechanism. A main goal has been reached by showing that the flow rate in the side
channel can be optimized with structural changes. This is achieved with devices where the attenuation length is much larger than the device dimensions and the acoustic wavelength in turn is much smaller than the device dimensions.

**Experimental Results**

In this section experimental results with the reference device design are presented and the influence of device design variations are discussed. Several aspects have been investigated by T. Hilsdorf and V. van Gemmeren and are presented in more detail in their bachelor thesis [172] and master thesis [73], respectively.

Figure 5.14 shows the velocity field of the reference design device with black arrows plotted over an image of the experiment. The field has been measured with PIV as described in Section 3.4. The shown device has a 2 mm thick transducer as indicated. The excitation frequency is 0.294 MHz which is below the fundamental transducer resonance of 1.027 MHz. The transducer excitation voltage is 5 V and the peak streaming velocities are around 15 mm s\(^{-1}\). The velocity field shows a relative homogenous flow in the side channel and large vortices in the acceleration chamber. The shown velocity field is distorted around the opening in the top cover close to the transducer due to problems with the PIV measurement. There the effective flow is of similar magnitude as in the side channel. The shown result is also influenced by particles trapped in pressure nodal planes. Sedimented particles trapped in lines parallel to the y-axis are shown as light gray in the image between \(x = 9.5\) mm and 28.5 mm.

Table 5.1 shows a list with different device layouts and some basic parameters together with the peak velocity \(v_{\text{max}}\) in the side channel. Further are given the excitation frequency \(f\), the transducer thickness \(d_{\text{Pz26}}\), and the excitation voltage \(V_{\text{Pz26}}\). In the table the ‘fully rectangular’ device refers to the geometry shown in Figure 5.11. The geometry ‘reference
### 5.1. Bulk Attenuation Driven Acoustic Streaming

<table>
<thead>
<tr>
<th>Device</th>
<th>$v_{\text{max}}$</th>
<th>$f$</th>
<th>$d_{P26}$</th>
<th>$V_{P26}$</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fully rectangular</td>
<td>$&lt; 1$</td>
<td>2.07</td>
<td>1</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Reference</td>
<td>1 - 2</td>
<td>2.07</td>
<td>1</td>
<td>5</td>
<td>5 experiments</td>
</tr>
<tr>
<td>Reference</td>
<td>10 - 30</td>
<td>0.294</td>
<td>2</td>
<td>5</td>
<td>10 experiments</td>
</tr>
<tr>
<td>Reference mod 1</td>
<td>34</td>
<td>0.294</td>
<td>2</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Reference mod 2</td>
<td>2 - 3</td>
<td>10.6</td>
<td>1</td>
<td>5</td>
<td>5 experiments</td>
</tr>
<tr>
<td>Long ring</td>
<td>$&lt; 1$</td>
<td>2.07</td>
<td>1</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Fully rect. short</td>
<td>3</td>
<td>10.85</td>
<td>1</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Fully rect. short</td>
<td>2</td>
<td>6.4</td>
<td>1</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>Fully rect. long</td>
<td>5</td>
<td>4.08</td>
<td>0.5</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Fully rect. long</td>
<td>5</td>
<td>20.5</td>
<td>0.5</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1: Summary of experimental streaming results with ring type devices where $v$ is the peak velocity in the side channel, $f$ the excitation frequency and $d_{P26}$ and $V_{P26}$ the transducer thickness and excitation voltage, respectively.

mod 1’ has a round corner 3 with a radius of 5 mm, a chamfer in corner 2 with chamfer width $w_{ch} = 5$ mm and the chamfer width in corner 1 is $w_{ch} = 10$ mm. The geometry ‘reference mod 2’ has a completely round end spanning over corners 1 and 2 with a radius of 10 mm and the inlet has a rectangular end towards the transducer with $\ell_a = 5$ mm and $\ell_b = 25$ mm. The ‘long ring’ refers to the geometry shown in Figure 5.13. The ‘fully rectangular short’ device has a length $\ell = 30$ mm and the ‘fully rectangular long’ device has a length of $\ell = 100$ mm.

All devices listed in Table 5.1 showed a homogeneous back flow in the side channel similar to the one shown in Figure 5.14 and a number of vortices in the acceleration chamber. Modified or reference design type devices showed several vortices which do not expand over the whole length of the devices. An exception is the long ring device which did not show vortices. This device has the narrowest acceleration chamber of all tested devices for the given length. Nonetheless the experiments with this device showed only rather slow streaming below $1 \text{mm} \text{s}^{-1}$ and vortices could appear at higher velocities. The observed flow character in the side channel and vortices in the acceleration chamber are in agreement with the numerical simulations which shows a similar behavior.

Length and size changes did not show significant changes in the device performance. The reference device has been reduced in size without a change of the geometrical constraints and showed similar results as an example. The small size has been designed to contain a fluid volume of 1 ml. Another example is the fully rectangular long device which mainly showed an increased number of vortices but did not increase the side channel device performance.

The acoustic streaming experiments generally proved to be rather sensitive to small
changes which could decide whether an experiment showed any streaming or not. This reproducibility issue can lead to devices which have the same nominal dimensions but show good, poor, or no streaming around the same frequencies in experiment. Besides manufacturing tolerances of the PMMA devices mainly transducer related issues led to device related problems. Transducers glued directly to the devices showed the best performance and reproducibility. However, this allows no adjustment to the transducer if a device does not work at all. This problem can be circumvented with clamped transducers which also allow to adjust the position individually or to replace the complete transducer if no streaming can be produced. However this approach lowers the reproducibility.

In summary, side channel velocities of up to $34 \text{mm s}^{-1}$ have been produced in experiment and the transducer thickness as well as the excitation frequencies showed to be a crucial element in the device design. The experiments are consistent with the numerical simulation and all show steady and homogeneous streaming in the side channel. Lower frequencies showed higher side channel velocities and higher frequencies steadier streaming patterns and by trend higher reproducibility where the latter remains a concern.

### 5.1.4 Complex Geometry Designs

In this section a W-shaped device is presented where the geometry dominates the flow pattern. In addition, a number of different device design concepts are briefly introduced. These designs include devices which aim at mixing only and devices which aim at producing a circular flow which can be used to concentrate particles in combination with a particle trap. The devices all have a PMMA frame and a fixed attached transducer. The layout is planar with device heights of $3 \text{mm}$. The experimental setup and device fabrication is given in Section 3.2.

The W-shape device has a symmetrical wall structure which aims at reflecting plane traveling waves in a way that counter-propagating waves are separated in space. Figure 5.15 shows an image of the device supplemented with the device outline and a $1 \text{mm}$ thick transducer. The black arrows show the velocity field measured with PIV at an

![Figure 5.15: Velocity field of W-shape device excited at 6.46 MHz. The transducer excitation voltage is 5 V and peak streaming velocities are around 4 mm s$^{-1}$. Two 1 mm openings in the top cover are shown in the corners close to the transducer.](image-url)
excitation frequency of 6.46 MHz and a voltage of 4.3 V which results in peak velocities of 3.2 mm s$^{-1}$. Doubling the voltage resulted in peak velocities of 8.5 mm s$^{-1}$. Increasing the voltage further up to 13 V resulted in peak velocities of around 9.1 mm s$^{-1}$. The strongly reduced gain of velocity with voltages above 10 V is expected to be mainly due to the transducer which reaches its strength limit due to its dynamic loading. The shown velocity field is rather symmetric compared to the results of the rectangular chamber as expected.

Experiments at 10.85 MHz and 4 V showed peak velocities of 11 mm s$^{-1}$. Experiments with a 2 mm thick transducer showed best results around 3.4 MHz. With a voltage of 5 V peak velocities of 4 mm s$^{-1}$ have been obtained. All these experiments showed the same shape of the velocity field which shows two vortices with backflow towards the transducer at the side walls ($y = \pm 10$ mm). The small changes in the flow field patterns stand in contrast to the experiments with the rectangular devices and the ring type devices presented in the previous sections which showed rather frequency dependent flow fields. The experiments with the W-shaped devices showed good reproducibility where the streaming fields have been reproduced with the same device in 10 different experiments performed with at least one day between experimentation. In summary, the W-shaped device is an example which shows that geometrical considerations can be used to build a mm-sized plastic chamber able to generate stable and reproducible attenuation driven acoustic streaming with attenuation lengths which exceed the device dimensions.

Generally attenuation driven acoustic streaming is rather hard to obtain and control with mm-sized planar plastic chambers excited with frequencies in the range 0.2 MHz to 20 MHz and small, 0.5 mm to 2 mm thick transducers where the smallest in-plane device dimension is at least two times the acoustic wavelength. Figure 5.16 shows a number of device concepts which fulfill these criteria and have a fluid volume in the range of 1 mL to 3 mL. The black arrows indicate the intended fluid flow and standing waves are indicated with parallel black lines. These devices showed no or hardly any streaming in experiment. The attenuation chamber has a damping structure and material at the opposite end of the transducer. With this design waves traveling from the transducer towards the absorption material are expected to dominate in the center of the device.

Ring corner devices aim at concentrating and trapping particles with a flow which moves particles trapped in a pressure nodal line along the line towards a wall in a corner. This trapping behavior has also been observed in other devices presented in Section 5.2.

Penta-hexa devices or ring type variations of it aim at coupling acoustic waves into a device which travel repeatedly in a circle. The shown device breaks symmetries which otherwise lead to standing waves as e.g. with two concentric rings.

The focusing device shows another method aimed at coupling traveling waves into a ring type device. The vortex device layout aims at producing streaming in a chamber where the pressure difference in the chamber is used to generate a flow in a side channel.
Figure 5.16: Different device design concepts for attenuation driven acoustic streaming. The arrows indicate the intended fluid flow and standing waves are indicated with parallel lines in the blue fluid domain.

5.2 Ultrasonic Trapping of Particles in Fluid Flow

Particles can be trapped with acoustic radiation forces which exceed the drag forces acting on the particles. This technique has been realized in a number of microfluidic devices such as in a borosilicate capillary [66] or in a silicon device with confocal ultrasonic cavity [160]. Further applications and devices can be found in a recent review of applications in acoustic trapping [40]. The devices reported in literature typically have a fluid flow in common which is driven with an external pump through small capillaries.

In this section ultrasonic particle traps are discussed which can be combined with acoustic streaming devices like the ring type devices presented in Section 5.1.3. The aim of such a combination between acoustic streaming device and particle trap is the agglomeration or concentration of particles. The combination results in a large contact between fluid and particles which is particularly well suited for an application such as the enrichment of low concentration samples like DNA. The aim here is to realize particle traps in mm-sized channels with low flow resistance compared to micrometer sizes capillaries. The channel height of 3 mm, the used materials, and the fabrication are the same as of the layered ring type devices presented in Section 5.1.3. The traps are designed in 2D which allows optical inspection or direct access to the particles in the third dimension.

Several aspects of particle traps have been investigated by I. Leibacher and T. Hilsdorf and are presented in their semester project [93] and master thesis [73], respectively.

Force Balance

A single particle can be trapped if the primary acoustic radiation force $F_{\text{rad}}$ (2.28) and the drag force are in equilibrium. Assuming Stokes drag the force from the flow on the
particle is

\[ F_{\text{Stokes}} = 6\pi\mu rv, \]

(5.5)

where \( v \) is the relative velocity and \( r \) the particle radius. With the acoustic radiation potential for a plane wave (4.1) the maximum velocity which still leads to particle trapping is

\[ v_{\text{max}} = \frac{2}{3}\mu k E_{\text{ac}} \phi r^2, \]

(5.6)

where

\[ E_{\text{ac}} = \frac{1}{4}\kappa_0 \beta^2 \quad \text{and} \quad \phi = \frac{1}{3}f_1 + \frac{1}{2}f_2 \]

(5.7)

are the acoustic energy density and acoustic contrast factor, respectively. This shows a quadratic relation between particle radius and flow velocity for a stationary particle. The acoustic radiation force field of a plane wave is of sinusoidal nature. Hence a stable equilibrium is assumed in addition to the assumption of a spherical particle with laminar flow and a plane acoustic wave.

For multiple particles secondary acoustic radiation forces come into play which can enhance the trapping capabilities. These forces can be utilized with large particles which increase the total acoustic force on small particles and thus increase the trapping capabilities.

### 5.2.1 Numerical Simulation

Two categories of particle traps have been investigated. One type of trap has a fluid channel inlet and an outlet which are in-line and without a discontinuity which would stop a plane traveling wave coming from the inlet. An example is the device shown in Figure 5.17. This type of device can be combined with ring type devices like the one shown in Figure 5.13 where traveling waves also drive acoustic streaming in the side channel or through the trap. A second type of particle trap works with no restriction of the position and the orientation for the channel inlet and outlet and has a discontinuity such that a plane traveling wave cannot pass through the device unreflected. These traps are denoted ‘shaped’ traps here. An example is the S-shape device shown in Figure 5.20. The simulations are in 2D and a detailed description of the model is given in Section 3.1. The particle tracing simulations are without gravitational forces and with 100 initial particles released at \( t = 0 \) s and distributed throughout the water domain proportional to the fluid velocity amplitude.

### In-Line Traps

An in-line trap design does not allow to establish a local standing wave with nodal lines perpendicular to the fluid flow. However, pressure gradients are typically the highest in the excitation direction.

Figure 5.17 shows the absolute value of the pressure field and the velocity amplitude of the flow field of a confocal trap design. The trap design is similar to the one of Svennebring et.al. [160] realized in silicon where the design here has a much wider fluid
channel and more direct excitation. The excitation frequency is at 2.144 MHz which leads to a resonance in the trap. The pressure field shows a focus in the center of the trap with strong gradients in both, excitation and fluid flow direction. The shown flow velocities result in a flow rate of 1.08 ml min$^{-1}$. Figure 5.18 shows a particle tracing simulation where the gray lines indicate particle paths and the black dots the particle positions. The simulation is performed with 26 µm glass particles which are not shown to scale. The forces are calculated based on the fields shown in Figure 5.17. The shown particle distribution is after 10 s. However, most particles reach their final equilibrium position after 6 s to 8 s. One black dot typically corresponds to multiple particles which take the same position. A simulation with the same parameters as before but with 10 µm particles shows that these particles are only trapped in the center of the chamber. For optimal performance the particles have to be concentrated along the center line of the inlet before they enter the trap. This can be achieved with frequency sweeping discussed in Chapter 4 as an example.

Another in-line design is with two standing waves which cross each other at an angle. The superposition of both results in a grid like pressure field. An example of such a device is shown in Figure 5.19 with a rhombohedral trap design. The figure shows the absolute pressure field at 1.976 MHz and the velocity amplitude of the flow field. The angle $\alpha$ between the two standing waves influences the pressure gradient along the channel in $x$-direction where 180° corresponds to a system with two opposing transducers. The smaller
the angle towards 90° the higher are the gradients in x-direction but the smaller is the intersection area between both standing waves assuming that the inlet and outlet channel are unaltered. The area can be expanded with a sawtooth like structure. However, this also increases the critical water volume with slow fluid velocities. Confocal device designs are better suited if the aim is to trap a small number of particles at one position only which can be achieved with pre-focusing the particles. Two crossing standing waves are better suited for devices with a large number of particles and they work without pre-focusing.

**Shaped Traps**

The shaped traps aim at establishing a single standing wave with nodal lines perpendicular to the streaming direction. An example is the S-shaped device shown in Figure 5.20. The standing wave excited at 2.031 MHz has pressure nodal planes perpendicular to the flow direction in the middle part of the device ranging from $y = 5$ mm to 10 mm. The vertical channel is chosen slightly wider to obtain a reduced flow velocity in the trapping region. Figure 5.21 shows a particle tracing simulation with 26 μm glass particles after 10 s. With these particles the acoustic radiation forces dominate and the particles are trapped in the complete vertical channel section. With 10 μm silica particles the acoustic radiation forces are weaker and particles above $y = 10$ mm are no longer trapped in the vertical channel. Particles which come from the inlet stay within the channel region of $y = 1$ mm to 4 mm.
In this region the particles experience drag forces in the direction of the pressure nodal lines. Hence particles are moved into low fluid velocity regions. The inlet channel can be extended around the transducer such that a pocket volume with almost resting fluid is generated. Large particle clumps are then moved into the pocket by particles which are still in the fluid flow.

### 5.2.2 Experimental Results

The particle traps have been investigated with an externally driven flow. The trap side walls have been laser cut from 3 mm thick PMMA sheets and are covered with 1 mm thick PMMA top and bottom covers. The devices have a length of 70 mm and have been excited with 1 mm thick Pz26 plates glued to the side walls. Single particle traps have been connected to a syringe pump with 4 mm silicone tubing. The excitation frequencies are around 2 MHz which is the transducer resonance frequency. A more detailed description of the experimental setup and device fabrication is given in Section 3.2.

The experiments suffer from the problem that larger particles like 26 µm glass particles sediment relatively fast. Sedimented particles are hard to move due to the no-slip boundary condition. Smaller particles with a density close to the one of water reduce this problem. Such particles experience much smaller acoustic radiation forces which makes trapping hard. The presented results have been obtained with copolymer particles which are introduced with a homogeneous distribution throughout the channel.

With the confocal device design shown in Figure 5.17 particles have been trapped with flow rates of 0.2 ml min$^{-1}$. However the particle trap yield is rather small and mainly small particle clumps have been trapped which can also make use of secondary acoustic radiation forces. Particles which have not been trapped were typically deflected and followed the pressure nodal lines through the trap. This behavior is in agreement with the particle tracing simulations. However, the simulation results show trapping of particles with more than five times higher flow rates.

With the S-shaped device shown in Figure 5.20 particles have been trapped with a flow rate of 0.1 ml min$^{-1}$. The particle trap yield is higher than with the presented in-line
traps but still only about half of the particles have been trapped. The multiple trapping sites along the fluid flow are advantageous for a small number of particles which increases the chance that particles which slip through the first pressure nodal plane are stopped at the next trapping site. However, this has the drawback that a particle clump can wipe through all pressure nodes and drag the already trapped particles with it.

In summary, the confocal design can be of interest for applications with a small number of particles trapped in a small region. In general shaped traps worked better than in-line traps.

5.3 Boundary Layer Driven Acoustic Streaming

Boundary layer driven acoustic streaming is a type of streaming where the momentum or energy transfer from oscillations to the medium comes from the friction between the oscillating medium and a solid boundary. The momentum transfer takes mainly place in a thin boundary layer. This makes this type of streaming particularly suited for microfluidic systems where viscous forces often dominate the fluid flow in the channels. In addition, the high velocity amplitudes in the boundary layer become increasingly important for the total flow with decreasing channel diameter. A more detailed description of boundary layer driven streaming is given in Section 2.4.

Advantages of boundary layer driven acoustic streaming compared with attenuation driven acoustic streaming are the small attenuation length and high velocities close to boundaries which can be helpful to move sedimented particles. However, excitation of, and implementation into a system which aims for a circular flow without additional vortices and thus dead fluid volume can produce further restrictions.

A possible implementation for fluid pumping with an oscillating air bubble is given by Ryu et.al. [145]. Boundary layer driven acoustic streaming has also been implemented for mixing in micro channels [1]. Different basic geometries such as cylinders, squares, and boundary inlets of these shapes can be used to excite streaming [98]. A more detailed review of applications of acoustic streaming in microfluidic devices can be found in [186].

Boundary layer driven acoustic streaming has been implemented in a numerical simulation which allows to calculate fully coupled systems with mechanical structure and piezoelectric excitation. However, here a reduced, dimensionless system model example is presented with the aim to compare the numerical simulation with an analytical solution given by J. Wang [181].

Regime and Assumptions

The driving mechanism of boundary layer driven acoustic streaming is dominated by viscous effects. Therefore, the full first order equations have to be taken into account as described in the theoretical Section 2.1. For the driving force \( \mathbf{F}_R \) (2.15) the viscous term is dominating. It is assumed that the condition \( Re_s \ll 1 \) holds and that thus the streaming can be calculated as Stokes flow. It is further assumed that the inertia parameter \( \varepsilon \ll 1 \) and that the amplitude of oscillations \( U_0 \) are small compared with the wall curvature radius \( R \). Only the cases where the vorticity is confined to the boundary
layer are regarded and thus it is assumed that $M \gg 1$. Further assumed is a large Mach number $Ma$ and hence streaming is incompressible.

**Dimensionless Model**

The presented example consists of two concentric cylinders as shown in Figure 5.22. The inner rigid cylinder is oscillating in $x$-direction and is located in a stationary container. A reference geometry has an inner radius $r_i = 1$ and an outer radius $r_o = 5$.

![Figure 5.22: Concentric cylinders](image)

Riley [141, 142] proposed the governing equations in dimensionless form. Here the dimensionless model given by Wang [181] has been implemented similarly as the dimensional model described in Section 3.1.3.

The first order ($\varepsilon^1$) equations are

$$M_p^2 \frac{\partial p_1}{\partial t} + \nabla \cdot \mathbf{v}_1 = 0,$$  \hspace{1cm} (5.8)

$$\frac{\partial \mathbf{v}_1}{\partial t} = -\nabla p_1 + \frac{1}{M^2} \nabla^2 \mathbf{v}_1,$$ \hspace{1cm} (5.9)

where $M_p = \omega^2 R^2/c_0^2$. A linear equation of state $p_1 = \rho_1$ is assumed. The second order ($\varepsilon^2$) time-averaged equations are

$$\nabla \cdot \mathbf{v}_2 = 0,$$ \hspace{1cm} (5.10)

$$-\nabla p_2 + \frac{1}{M^2} \nabla^2 \mathbf{v}_2 = \mathbf{F}_R.$$ \hspace{1cm} (5.11)

The system is regarded as incompressible and the compressible form of the continuity equation is used for numerical stability where the damping parameter $M_p^2 \rightarrow 0$ is approximated with small values. For $M = 10$ a value of $M_p^2 = 1 \cdot 10^{-6}$ is used.

**Mesh Convergence**

A mesh convergence study has been performed with the reference geometry and with $M = 10$. The full model geometry has been modeled to have a better measure of the numerical stability even though the problem is known to be symmetrical which would allow to model one quarter only. The mesh consists of free triangular mesh elements, which are refined close to the oscillating cylinder. The maximal element growth rate is 1.04.
The numerical results have been compared with an analytical reference solution given by Wang [181]. The analytical solution is valid uniformly within the entire regime and gives a more accurate model than the traditional boundary layer solutions. The normalized root mean square error (NRMSE) of the streaming velocity is shown in Figure 5.23 as a function of $\delta/d$. The ratio $\delta/d$ denotes the boundary layer thickness over the maximum element size $d$ which is the length between two nodes at the oscillating boundary. For the shown plot the radial velocities $v_r(r, \theta = 0)$ have been evaluated.

![Figure 5.23: Mesh convergence study of streaming velocity, where $\delta/d$ is the ratio of boundary layer thickness to maximum element size at the boundary.](image)

The velocities close to the oscillating cylinder are the most critical. An evaluation of the velocities over an area gives a lower weight of these velocities in comparison. Similar results are obtained from an evaluation of the azimuthal velocities $v_\theta(r, \theta = 45^\circ)$ and from the evaluation of the reference geometry with $r_o = 7$.

The convergence decreases slightly with more than 12 mesh elements per $\delta$ and keeps constant for a value of at least up to 35. The simulation has a high convergence stability even with a coarse mesh. A simulation where the boundary layer has been resolved with 2 mesh elements only results in a NRMSE of 0.02 which means that the basic flow field has been resolved and mainly an amplitude error remains.

**Results**

The focus here is on an example which resolves the boundary layer well. This can be obtained with the reference geometry and the dimensionless model. The results can be applied similarly to dimensional examples with water and ultrasound. However, in such examples the boundary layer thickness is in the µm-range and the cylinder diameters have to be chosen accordingly small.

Figure 5.24 shows the Lagrangian simulation results of the reference geometry with $M = 10$ and $U_0 = 1$. In the streamlines plot the full model geometry is shown where the arrows indicate the flow direction. The plots show the symmetrical nature of the solution. Furthermore, the streaming velocity field amplitude of one quarter of the domain is shown. The chosen Womersley number $M = 10$ is rather small resulting in a large boundary layer.
Characteristic for such a case is a velocity peak close to the cylinder. This changes with higher $M$ where the peak velocities shift towards the interface between the inner and outer vortices. This is illustrated in Figure 5.25 which shows the Lagrangian radial and azimuthal velocities $v_r$ and $v_\theta$, respectively.

Geometrical changes lead to a change of the flow field as expected from the analysis of the dimensionless numbers given in Section 2.1. Increasing the outer radius $r_o$ has a small influence on the velocities in the boundary layer but mainly increases the outer streaming velocities $v_r(r, \theta = 0)$. The inner radius $r_i$ is proportional to $M$ and thus directly affects the boundary layer region which dominates the total flow field.
Conclusions and Outlook

In this work two acoustic methods for the transport of suspended micro-particles have been covered. One method is continuous frequency sweeping which is based on acoustic radiation forces. The other method is acoustic streaming. The main achievements are summarized here.

A one dimensional analytical investigation of continuous frequency sweeping has been presented. The discussion shows that this technique makes use of damped resonances which appear at frequencies between resonance frequencies. It has further been shown that the variation of the boundary condition, such as a change of the excitation position or the number of excitations, can be used to create different particle transport paths without any change of the fluid cavity dimensions. Particles can be moved towards and away from the excitation. Different particle travel paths have been presented for frequency mode switching. Furthermore, continuous frequency sweeping has been investigated numerically and experimentally. Therefore a square mm-sized plastic chamber has been proposed which can be used to concentrate particles within a closed chamber. The numerical simulation includes a piezoelectric transducer, a mechanical structure, and a fluid domain model where damping has been modeled for all domains. The numerical calculations include 1D, 2D, and 3D simulations where it has been shown that there are significant differences between the dimensional order of the models, but still 1D and 2D give reasonable results for the system description. The results presented here focus on 2D simulations. The simulations show that frequency dependent pressure isoline plots are well suited to describe and compare acoustic systems which operate with time dependent frequencies. They have been used to show that continuous frequency sweeping works best if a frequency range is used which exceeds the minimal frequency range since the quality of the pressure field changes with frequency and in space. Therefore, averaging is required. It has been shown that the pressure amplitudes are the highest close to the fundamental transducer resonance but the pressure fields show strong variations at these frequencies which avoids good particle transport capabilities. The frequency difference between resonance frequencies is not constant and rather sensitive to the device layout. This is a point where the averaging property of continuous frequency sweeping is beneficial over frequency mode switching. The wall thickness of PMMA devices can be used to modulate
the acoustic field since the acoustic pressure field is dominated by the outer device dimensions. Attenuation is a crucial parameter which enables continuous frequency sweeping but can reach a critical limit such that acoustic radiation forces are too weak or that traveling waves are dominating over standing waves. A particle tracing simulation which resolves time dependent frequency changes has been presented. This simulation confirms the findings of the pressure field simulations and is in good agreement with the analytical model. Furthermore, the particle tracing simulation shows that particle sedimentation is an issue which can decrease particle transport performance significantly. It has further been shown that the particle tracing simulation can be used to find an optimum between particle properties, frequency range, and sweeping rate.

The same chamber geometry as for the numerical simulation has been investigated in experiment. Therefore plastic devices have been fabricated. Particles have been successfully moved back and forward through a device and concentrated in a corner. It has been shown that such a continuous frequency sweeping device can be used to concentrate DNA but that sedimentation and particle to fluid contact is an issue to resolve. For the experimental work clamping of a fluid chamber and transducer with waveguide has been established. It has been shown that a mm-sized square and planar chamber works best if for the ground plate material aluminum or a comparable low damping and rigid material is used, but that plastic foils still give good results. The particle experiments have been supplemented with schlieren visualization experiments which can continuously visualize the full pressure field in real time. Therefore, a suitable and simple schlieren setup has been designed. Good agreement of the imaged pressure field with the numerical simulation has been found. In addition, thermal effects, streaming, and lower harmonics have been visualized with schlieren imaging.

Continuous frequency sweeping depends on the location of the transducer and the wave coupling mechanism into the fluid. Wave coupling mechanisms such as in line coupling, plate vibrations, and fluid coupling have been investigated with planar layered devices. The numerical and experimental investigations show that different coupling mechanism can be used to build continuous frequency sweeping devices. It is the position of the excitation or transducer which foremost defines the operational capabilities of continuous frequency sweeping. Devices where the elongation direction of the transducer is perpendicular to the particle transport direction have been presented.

Attenuation driven acoustic streaming has been investigated numerically and in experiment for mm-sized devices. The presented numerical simulation includes coupling of acoustics and structure and is able to efficiently simulate multi-wavelength problems. Streaming patterns with fluid flow towards and away from the transducer have been shown in agreement with simulation and experiment. It has been shown that a large variety of streaming patterns, including the backflow pattern, arise in rectangular chambers when the attenuation length is one to two orders larger than the device dimensions. Experimental observations have shown streaming velocities of over 50 mm s$^{-1}$ in square mm-sized chambers. The flow rates are even higher in the numerical simulations. This proofs that high bulk flow velocities with relatively low power input can be generated with bulk attenuation driven acoustic streaming if the reflections or superposition of traveling waves are used for its benefit. Furthermore it has been shown numerically that attenuation driven
acoustic streaming is well suited for mixing in square chambers.
A ring type chamber concept has been proposed which allows to concentrate particles with attenuation driven acoustic streaming combined with particle traps. It has been shown numerically that these ring type devices can be designed with a fluid acceleration chamber and a side channel where the fluid flow is driven with a pressure difference. Changes of corner geometries can be used to increase flow rates and decrease dead volume. The qualitative findings of the simulation, such as passive and rather homogeneous flow in the side channel, have been confirmed in experiment. Acoustic radiation force based particle traps have been presented which can be fit into the side channel of the ring type devices. They have been investigated with a particle tracing simulation which combines fluid flow and acoustic radiation forces. The best results have been obtained with S-shaped devices where the flow direction is perpendicular to a standing wave.

Boundary layer driven acoustic streaming plays a minor role in the presented mm-sized chambers. However, it is a potent tool to create fluid flow. A presented numerical simulation of boundary layer driven acoustic streaming with oscillating boundaries has been compared with a dimensionless analytical solution from literature. It has been shown that the implementation has high convergence stability and shows high accuracy.

Both, continuous frequency sweeping and acoustic streaming are powerful tools for particle transport. Continuous frequency sweeping has the advantage to be relatively simple to realize and has the possibility to make use of averaging effects which makes the method rather insensitive to disturbances such as gas bubbles. However, the method is limited by the acoustic radiation force which decreases with particle size. Bulk flow generated with acoustic streaming in comparison is very well suited for the transport even of smallest particles. Attenuation driven acoustic streaming can generate high bulk flow rates, however it is more difficult to realize than continuous frequency sweeping. For an application like DNA concentration the potential of acoustic streaming compared to continuous frequency sweeping lies in the ability to provide means for mixing. Furthermore, a large fluid to particle interaction volume can be generated and the particle concentration times are within seconds when combined with a particle trap. Both methods can be realized with plastic devices. Plastic requires different design criteria compared to the common micromachined silicon devices. With plastic the focus is on the outer device boundaries rather than the inner boundaries. Plastic suffers from large attenuation but provides other advantages such as cheap manufacturing or a particularly good surface chemistry and therefore is well suited for ultrasonic devices.

**Outlook**

It has been shown that continuous frequency sweeping can be used to concentrate DNA. However, for the method to be competitive with magnetic beads in terms of processing time and efficiency further development is required. Particle to fluid contact is generally achieved with agitation and is a main issue which needs to be improved to increase efficiency. Acoustic streaming is a possible and potentially efficient method to resolve this, but a proof with DNA sample remains open.

Attenuation driven acoustic streaming has a high potential for mixing bulk fluid or driv-
ing a fluid flow efficiently. However, reproducibility and controllability of the fluid flow remain an issue. Computationally highly expensive simulations with full 3D problems or the combination of the simulations with optimization algorithms are examples which can be used to find and investigate geometries better suited than the ones presented here.

The presented devices focus on PMMA as device material. Detailed investigations of different plastics is of interest for cheap disposable ultrasonic particle manipulation devices. These materials can also serve as an alternative for the silicon generally used in microfabricated acoustophoretic devices, which is another open field of research.

A bottleneck of ultrasonic devices is a sufficiently high pressure amplitude, particularly for plastic devices. Improving transducer and waveguide design helps to increase the performance of ultrasonic devices. Advanced transducer designs, such as transducer arrays, can be mainly found for pulsed or traveling wave applications or for SAW devices. However, these transducers cannot provide high bulk wave amplitudes in continuous or resonant mode or even in multi-frequency mode.

The results presented in this thesis motivate further investigations which are not directly within the aim of a point of care DNA analysis device. An example is single particle transport with continuous frequency sweeping or mode switching. The possibilities to combine resonance modes multiply with single particle transport since only the pressure field at the location of the single particle has to be taken into account. This motivates optimization procedures of the frequency modulation which would be useless with particles distributed over the location of several pressure nodes, as investigated in this thesis.

Schlieren visualization is a strong tool to investigate acoustic micro manipulation devices. The method can be used to obtain a quantitative description of the pressure field, e.g. with a calibration of the presented setup. The experimental observations with the schlieren setup showed thermal effects and forces acting on fluid-fluid suspensions. An example is water with different concentrations of salt solution. These forces have potential applications in sensing or separation and are an open field for further investigations.

A simulation of boundary layer driven acoustic streaming with oscillating boundaries has been shown to be in good agreement with an analytical solution. However, experimental data found in literature is not detailed enough for a qualitative comparison of simulation and experiment. Missing data are e.g. excitation amplitudes and streaming velocity field. This thesis focused on closed devices. However, experiments with open devices or drops showed that continuous frequency sweeping can also be used to transport particles with open boundaries. Such devices could serve as interface to move particles from open fluid volumes into half closed systems or the other way round.

The above mentioned examples of research give an excerpt of the research fields which are motivated or have been touched by this work. The high potential of acoustic streaming to generate bulk flow and its large area of applications makes acoustic streaming a particularly interesting field for further research.
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Curriculum Vitae

Dirk Björn Möller
born July 26, 1982
Citizen of Zurich (Switzerland) and Germany

Education

2007 – 2013  Doctoral student at the Institute of Mechanical Systems, Center of Mechanics, ETH Zurich
1995 – 2002  Alexander von Humboldt Gymnasium Konstanz (D), Abitur
2002 – 2007  Studies on Mechanical Engineering at ETH Zurich with specialization in micro and nano-systems, graduation as MSc ETH Masch.-Ing.

Professional Experience

3/2007 – 12/2013  Research and teaching assistant at the Institute of Mechanical Systems, Center of Mechanics, ETH Zurich