Influence of varying material properties on the load-bearing capacity of glued laminated timber

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INFLUENCE OF VARYING MATERIAL PROPERTIES ON THE LOAD-BEARING CAPACITY OF GLUED LAMINATED TIMBER

A thesis submitted to attain the degree of
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Gerhard Fink
Zurich, January 2014
Glued laminated timber (GLT) is a structural product composed of several layers of timber boards glued together. GLT components have many advantages, such as the larger range of available component dimensions to choose from, the environmental sustainability or the efficient ratio between weight and load-bearing capacity. Because of that, GLT beams have been established as one of the most important products in timber engineering in the last decades.

As a natural grown material, timber properties exhibit higher variability, compared with other building materials. The variability is pronounced not only between different structural elements but also within single elements, the latter being highly related to the occurrence of knot clusters. Due to the highly inhomogeneous structure of timber, the prediction of the material properties of GLT beams is affected by large uncertainties. In the presented thesis, the influence of varying material properties on the load-bearing capacity of GLT beams was investigated. Thus the thesis contributes to develop the quality of GLT beams, in terms of reliability and efficiency.

Detailed, non-destructive investigations of altogether 400 timber boards were performed. Thereby, different strength and stiffness related indicators, such as the position and characteristic of knots, or the eigenfrequency, were assessed. Furthermore, non-destructive tensile test were performed to estimate the stiffness properties of knot clusters. Out of the investigated timber boards, GLT beams having a precisely-known beam setup were fabricated. As a result, the exact position of each particular timber board (and each particular knot cluster) within the GLT beams was known. Afterwards, bending tests were performed to estimate the load-bearing capacity of these GLT beams. Thereby, the influence of knot clusters and finger joint connections on the deformation and failure behaviour was investigated.

In addition to the experimental investigations, a probabilistic approach for modelling GLT beams (referred to as GLT model) was developed. Thereby, at first, timber boards are simulated according to their natural growth characteristics. Afterwards, out of the simulated timber boards, virtual GLT beams are fabricated. Finally, the load-bearing behaviour of these GLT beams is estimated by using a numerical model. To assure the quality of the numerical model, it was validated with the test results. Using the GLT model, the influence of different parameters, such as the position and characteristics of knots, or the quality of finger joint connections, on the load-bearing capacity of GLT beams was investigated.

One further goal of this thesis was the investigation of machine-grading indicators, that are measured during the grading process. Therefore, all the investigations presented in this thesis are conducted for indicators measured in laboratory and machine-grading indicators. The same applies for the GLT model, which was also developed for both types of indicators.
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Zusammenfassung

Brettschichtholz (BSH) ist ein aus mehreren Holzbrettern zusammengesetzter Baustoff, welcher sich durch zahlreiche Vorteile, wie die freie Wahl der Abmessungen, die ökologische Verträglichkeit oder das günstige Verhältnis zwischen Eigengewicht und Tragfähigkeit auszeichnet. Auf Grund dieser Vorteile zählt BSH zu einem der meistverwendeten Bauprodukte im Ingenieurholzbau.


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Chapter 1

Introduction

Timber is a natural grown material and, therefore, compared with other building materials, timber properties exhibit higher variability. Due to the highly inhomogeneous structure of timber, its variability is pronounced not only between different structural elements but also within single elements. The variability between structural elements results from different growth conditions and cutting processes whereas the within-member variability is highly related to the occurrence of knot clusters.

Glued laminated timber (GLT) is a structural product composed of several layers of timber boards glued together. Within GLT beams, the variability of the material properties is slightly reduced through homogenisation. Compared to solid wooden members, GLT beams have many advantages, such as a lower variability of the material properties, or the larger range of available component dimensions to choose from. Because of that, GLT beams have been established as one of the most important products in timber engineering within the last decades.

The material properties of the timber boards used for GLT fabrication are contributing substantially to the load-bearing capacity of the GLT beams. Therefore, before the fabrication process, timber boards are graded into the so-called strength grades; using either visual or machine grading techniques. Afterwards, timber boards of one (or more) strength grades, are used to fabricate GLT beams of different strength classes.

For the application of GLT beam as structural members, the load-bearing capacity of each strength class has to be estimated. In this regard, there are two different approaches: (a) experimental investigation of an adequate number of GLT beams and (b) simulation methods (At first GLT beams are simulated so that the beam setup is represented in a most realistic manner. Afterwards the material properties of the simulated beams are estimated using mechanical models.) Because of the complexity of timber and, therefore, the large number of influencing parameters the second approach has to be established as the more efficient within the last decades.

The load-bearing capacity of GLT beams, or more precise the characteristic value of the bending strength has been investigated for more than 30 years. However, even though the load-bearing behaviour has been investigated over such a long period, it is not fully understood yet. As a result the load-bearing capacity of GLT beams, is subjected to large uncertainties. As a
consequence, in order to compensate these uncertainties, large partial safety factors are required for the structural design of GLT constructions.

A better understanding about the influence of varying material properties on the load-bearing capacity of GLT might facilitate the understanding of the mechanical behaviour of GLT. This could lead to optimised grading criteria for GLT lamellas, with the objective to reduce the variability of GLT beams. As a result the application of GLT as a structural building material would be optimised.

1.1 Aim of the work

The main objective of the presented thesis is the investigation of the influence of varying material properties on the load-bearing capacity of GLT beams. Therefore, (a) experimental investigations on GLT beams with well known local material properties were performed, and (b) a probabilistic approach for modelling GLT beams (referred to as $GLT\ model$) was developed.

*Experimental investigations:* Detailed, non-destructive investigations of timber boards were performed. Thereby, different strength and stiffness related indicators, such as the position and characteristic of knots, or the eigenfrequency, were assessed. Furthermore, non-destructive tensile test were performed to estimate the stiffness properties of knot clusters. From the studied timber boards, GLT beams were fabricated. As a result, the exact position of each particular timber board (and each particular knot cluster) within the GLT beams is known – GLT beams having well-known local material properties were fabricated. Afterwards, bending tests were performed to estimate the load-bearing capacity of these beams. Thereby, the influence of knot clusters and finger joint connections on the deformation and failure behaviour was investigated.

*GLT model:* Based on the results of the experimental investigations on timber boards, probabilistic models and material models were developed. The probabilistic models are essential for modelling the variability of structural timber, and the material models are essential for the prediction of the strength and stiffness properties of timber board sections. Furthermore, a numerical model for the estimation of the load-bearing capacity of GLT beams was developed. The numerical model was validated with the investigated GLT beams. Taking into account the three models, the GLT model was developed. The GLT model gives the opportunity to investigate the influence of different parameters, such as the position and characteristics of knots, or the quality of finger joint connections, on the load-bearing capacity.

One further goal of this thesis was to support the enhancement of grading criteria for both, visual and machine graded timber. Therefore, the GLT model is applicable for visual and machine measurable indicators.

1.2 Outline and overview

The overview of the thesis is illustrated in Fig. [1.1](#). In Chapter [2](#), a brief introduction of all relevant topics of the presented work is given, which includes three main parts: timber as a
1.2. Outline and overview

structural material, the load-bearing behaviour of GLT and aspects of structural reliability. Chapter 3 gives an overview on the conducted experimental investigations on timber boards and GLT beams. Afterwards, the main part of the thesis, the development of the GLT model, is described. Therefore, initially three sub-models are developed. Chapter 4 model for the probabilistic representation of strength and stiffness related indicators, Chapter 5 material model to predict the material properties based on strength and stiffness related indicators, and Chapter 6 numerical model to estimate the load-bearing capacity of GLT beams having well-known local material properties. Taking into account all three sub-models, a probabilistic approach for modelling GLT beams is presented (Chapter 7). In Chapter 8 the approach is extended to machine-grading indicators. The thesis concludes with Chapter 9 where the advantages, possibilities and limitations of the presented approach are summarised.

![Schematic overview of the thesis](image-url)
Chapter 1. Introduction
Chapter 2

State of the art

The intention of this chapter is to give an overview about the state of the art of all the relevant topics concerning this thesis. This includes three main issues: timber as a structural building material (Chapter 2.1), the load-bearing capacity of GLT (Chapter 2.2), and aspects of structural reliability (Chapter 2.3).

2.1 Timber as a structural material

Timber is a widely available natural resource but highly complex due to its material anisotropy and inhomogeneity. In the following paragraphs, a brief introduction into the mechanical performance of timber as a structural building material is given. It is particularly focused on the load-bearing behaviour under tensile load, which is the most relevant material behaviour concerning the load-bearing capacity of GLT beams.

2.1.1 Mechanical performance of structural timber

"The two products - wood, in the sense of clear defect-free wood and timber, in the sense of commercial timber - have to be considered as two different materials and that must be respected when strength properties are developed for engineering purposes" – Madsen et al. (1992)

Structural timber components, such as squared timber or timber boards, are components having structural dimensions, which are sawn out from the trunk of a tree. Nowadays, in Norway spruce (Picea abies) timber, typical dimensions up to a cross-sectional area $b \cdot t = 225 \cdot 75 \text{ mm}$ and a length $l = 5'000 \text{ mm}$ are common (Steer 1995). To describe the mechanical performance of structural timber components both; (a) the material properties of clear defect-free timber, and (b) the influence of growth irregularities have to be considered.

Clear defect-free timber can be described using an orthotropic material behaviour having three main axis: longitudinal, radial and tangential (Niemz 2005). The material orthography is a result of the orientation of the micro fibrils inside the cell walls; for a detailed description see e.g. Shigo (1989). The material properties of clear defect-free timber have been investigated within numerous studies (e.g. Kollmann et al. 1968). The results show that the strength and
stiffness properties in longitudinal direction are significantly larger than in radial and tangential directions, which are relatively similar.

When sawing structural timber components out of a tree, in general no consideration is taken on the position within the trunk; i.e. the orientation of the growth rings (annual rings) within one particular cross-section is more or less random. However, the components are cut out rather parallel to the trunk axis, thus the grain orientation can be assumed parallel to the longitudinal axis of the timber board. Therefore, for engineering applications the material properties are usually described using a transversal-isotropic constitutive equation; i.e. material properties parallel to the grain and perpendicular to the grain.

Timber that is loaded in tension transmits the load by its tensioned fibres in longitudinal direction. In a hypothetical defect-free specimen, the grains would be located perfectly parallel to each other in the longitudinal direction and the load-bearing capacity would be maximized. Based on the fact that timber is a natural grown material, the grain orientation of commercial timber boards might deviate from being exactly parallel to the board’s longitudinal axis. Two reasons for this might be distinguished; global deviation due to spiral grain and local deviation due to knots and knot clusters. The spiral orientation of the fibres in the tree trunk is described by the so-called spiral grain angle \cite{Harris1989}. For Norway spruce specimens the magnitude of the spiral grain angles has been found to vary in general between zero and five degrees. The strength and stiffness properties are about 15 – 20% lower for a timber board with a spiral grain angle of four degrees compared to a timber board with no spiral grain angle \cite{Gerhards1988, Ormarsson1998, Pope2005}. In addition to the spiral grain the occurrence of knots or knot clusters influence the grain orientation. Knots lead to local changes in the grain angle, that is combined with a significant reduction of the load-bearing capacity. An overview of different models that describe the distribution of the grain orientation around knots is given in \cite{Foley2001, Foley2003}.

In additional to spiral grain and the occurrence of knots, the material properties of structural timber components depend on physio-morphological parameters such as the annual ring width, the density or the distance to the pith, and other growth irregularities such as wane, reaction wood, cracks or resin pockets.

Due to the dimensions of structural timber components, the above mentioned growth irregularities have to be considered when describing the material performance. Growth irregularities lead to a change of the mechanical performance; in general to a reduction of the strength and stiffness properties (within structural timber components, stiffness properties are defined as the mean stiffness of the entire cross-section, and strength properties are defined as the load-bearing capacity in relation to the cross-section). The strength and stiffness reduction can be global (reduction over the entire length) and local (reduction of board sections).

Through the occurrence of growth irregularities, the material properties of structural timber components are significantly lower, compared to the material properties of clear defect-free timber. In structural timber components produced from Norway spruce especially the occurrence of knots and knot clusters are of particular importance. This is explained in more detail within the following chapters.
2.1. Timber as a structural material

Fig. 2.1: (left) knot arrangement within the cross-section of a tree trunk, (middle) influence of the sawing pattern on the knot distribution within sawn timber boards, and (right) resulting knot area within the cross-section of one timber board (Fink et al. 2012).

2.1.2 Variability of material properties within structural timber

Timber is a natural grown material that has, compared to other building materials, a large variability in its load-bearing behaviour. This variability can be observed between different growth regions, between different timber boards within the same growth region and even within one particular timber board (Sandomeer et al. 2008). However, timber boards are graded into strength grades, thus for engineering purposes a subdivision into (a) the variability between timber boards of the same strength grade, and (b) the variability within timber boards is sufficient.

The variability between timber boards or rather the variability of the undisturbed timber (knot free timber – referred to as clear wood), is related to different growth and sawing characteristics; e.g. growth region, sapwood-heartwood, annual ring width, density or distance to the pith.

The variability of the strength and stiffness properties within structural timber boards is highly dependent on morphological characteristics of the tree, especially on knots and their arrangement. Nordic spruce timber components are commonly characterised by a sequence of knot clusters divided by sections without knots. Knot clusters are distributed over the length of the board with rather regular longitudinal distances. Considering the trunk of a tree, the average distance between the clusters is directly related to the yearly growth of the tree. Within one knot cluster, knots are growing almost horizontally in radial direction. Every knot has its origin in the pith. The change of the grain orientation appears in the area around the knots. In Fig. 2.1 (left) the knots (black area) and the ambient area with deviated grain orientation (grey area) within one cross-section of the tree trunk are illustrated. Since the individual boards are cut out of the timber trunk, during the sawing process, the well-structured natural arrangement of the knots becomes decomposed due to different sawing patterns. As a result, numerous different knot arrangements appear in sawn timber (Fig. 2.1).

As mentioned above, the occurrence of knots and knot clusters leads to a significant local reduction of the strength and stiffness properties. A possible distribution of the material properties over the length of a timber board is introduced in Riberholt & Madsen (1979); see Fig. 2.2.
Chapter 2. State of the art

Models to describe the variability of structural timber

In the past, numerous models have been developed to describe the variability of the material properties of structural timber. Well-known models for the stiffness variability are Kline et al. (1986) and Taylor & Bender (1991). In both models, timber boards are subdivided into sections with equal length (762 mm and 610 mm, respectively), without consideration of the natural growth characteristics; i.e. without considering the occurrence of knots and knot clusters. For each of these sections the stiffness is measured and the lengthwise stiffness variation of the timber board sections is described.

To describe the variability of the strength properties the following models are known: Taylor & Bender (1991), Lam & Varoglu (1991a,b), Czmoch et al. (1991), and Isaksson (1999). The latter two consider the natural growth characteristics within structural timber components; i.e. the timber boards are subdivided into sections containing major knots and/or knot clusters and sections without knots.

An alternative procedure, to describe the variability of structural timber can be made via a description of strength and stiffness related indicators; e.g. density (for the between-member variability) and tKAR (for the within-member variability) – see Tab. 2.1 for definition of tKAR-value. This approach is chosen by Ehlbeck et al. (1985a), Blaß et al. (2008). In both studies, the within-member variability is described based on the investigation of Colling & Dinort (1987). There, the timber boards are subdivided into sections with equal length of 150 mm.

2.1.3 Prediction of material properties

The variability between timber boards or rather the variability of clear wood, is related to different growth and sawing characteristics. For the predictions of the mean material properties different non-destructive tests methods were developed in the last decades. The most common methods are the eigenfrequency measurement (Kollmann & Krech 1960, Görlacher 1984, 1990b), the ultrasonic runtime measurement (Steiger 1995, 1996) and the density measurement. In several studies correlations between those parameters and the material properties are analysed (e.g. Görlacher 1984, Steiger 1996, Denzler 2007). In particular the first two methods, which
2.1. Timber as a structural material

are eigenfrequency and ultrasonic runtime, show an exceptionally good correlation to the mean material properties.

The variability of the load-bearing behaviour within timber boards, is highly dependent on morphological characteristics of the tree, especially on knots and their arrangement. Accordingly, numerous studies have been conducted to identify knot related indicators that are capable of describing the influence of knots on the load-bearing behaviour of timber boards, relevant for the design of timber structures. In general, the developed indicators can be categorised into two groups: group 1 represents knot indicators that are assessed based on visible knot pattern (measurable at the surface of the timber board), and group 2 represents knot indicators, which are assessed based on cross-section area of the knots. In Tab. 2.1 the most relevant indicators of group 2 are summarised.

The interrelation between the knot indicators described in Tab. 2.1 and the load-bearing behaviour is assumed to be known. In Denzler (2007), Isaksson (1999), and Boatright & Garrett (1979a,b) the interrelation between the ultimate bending capacity and different knot indicators is analysed. In the study of Isaksson (1999) the correlation coefficient between the bending capacity and the two knot indicators tKAR and mKAR is addressed and a correlation coefficient of $\rho = 0.40$ has been identified. Additionally, Isaksson (1999) has developed knot indicators for the prediction of component ultimate bending capacity that are based on the visible knot pattern with similar correlation. Denzler (2007) has identified significantly larger correlation coefficients between the ultimate bending capacity and the knot indicators ($\rho = 0.59$ for tKAR and $\rho = 0.63$ for mKAR). Furthermore, Denzler (2007) has developed alternative knot indicators containing the perpendicular distance of a knot to the neutral axis. However, the implementation of this feature has not yielded any increase of the correlation ($0.34 < \rho < 0.43$). Boatright & Garrett (1979a,b) have analysed the influence of the knot ratio on the percentage reduction of the bending capacity of clear specimens ($\rho = 0.39$). Courchene et al. (1996) have analysed the interrelation between tKAR and bending capacity as well as between tKAR and ultimate tensile capacity. In both cases, the relationship is illustrated qualitatively and no significant correlation can be observed. Mitsuhashi et al. (2008) have compared two knot indicators (ARF and CWAR) with the ultimate tensile capacity. For two different samples the knot indicator CWAR leads to a correlation coefficient of $\rho = 0.33$ and $\rho = 0.14$. The knot indicator ARF shows a larger correlation $\rho = 0.44$ and $\rho = 0.26$, respectively.

The interrelation between the stiffness properties and the knot area are analysed in Samson & Blanchet (1992). There the influence of single centre knots on the bending stiffness is analysed. The results show that the influence is quite small; e.g. a knot with a projected area of 1/3 of the cross-section leads to a reduction of the bending stiffness of 10%. Furthermore, Samson & Blanchet (1992) detected no significant differences between intergrown and dead knots.

Regarding the between and within-member variability of the material properties it is obvious that an efficient model for the prediction of the local strength and stiffness properties should include at least two indicators: (a) one that describes the mean material properties of the entire timber board, to consider the between-member variability of the mean material properties, and
<table>
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<tr>
<th>Abbr.</th>
<th>Name</th>
<th>Description</th>
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| tKAR  | **total knot area ratio** (Isaksson 1999) | - Ratio between the projected knot area within a length of 150 mm and the cross-section area  
- Overlapping knots are counted only once |
| mKAR  | **marginal knot area ratio** (Isaksson 1999) | - Ratio between the projected knot area within a length of 150 mm and the cross-section area  
- Calculated at the outer quarter of the cross-section area  
- Overlapping knots are counted only once  
- Developed for bending |
| CWAR  | **clear wood area ratio** (Mitsuhashi et al. 2008) | - The CWAR is the complement of the knot area ratio (for a length of 100 mm) |
| ARF   | **area reduction factor** (Mitsuhashi et al. 2008) | - The ARF is the complement of knot area ratio (for a length of 100 mm), including a local area reduction factor based on Hankinson’s formula:  
\[ f_{t,\theta} = \frac{0.1}{\sin^{1.4}\theta + 0.1 \cdot \cos^{1.4}\theta} \cdot f_{t,0} \]  
(b) one that describe the local strength and stiffness reduction through the occurrence of knots and knot cluster, to consider the within-member variability.
2.2 Load-bearing behaviour of GLT

GLT is a structural timber product composed of several layers of timber boards glued together. Structures out of GLT have many advantages compared to solid wooden structures, such as the lower variability of the material properties, or the range of component dimensions to choose from. Through this, GLT has become one of the most important timber products in timber engineering within the last decades.

2.2.1 Fabrication of GLT

In the following paragraphs, a brief introduction into the fabrication process of GLT beams is given; for a more detailed description see e.g. Thelandersson et al. (2003).

GLT beams are produced mostly out of timber boards having a thickness $t_1 = 30 - 50 \text{ mm}$. In a first fabrication step the timber boards have to be finger jointed. Hereby it is important that no finger joint is placed in areas of knots or knot clusters. To fulfil the requirements of EN 14080 (2009), the distance between a knot and the finger joint shall not be less than three times the knot diameter. Following, the timber boards are glued together (finger joint connection – FJ) to produce endless lamellas. The endless lamellas are cut into single lamellas having the length of the GLT beam. Afterwards, the single lamellas are planed (on the top and the bottom surface) and glued together. For practical and optical reasons the surface of the GLT beams are planed to the final dimensions. In Fig. 2.3 the principle of the GLT fabrication is illustrated.

For the fabrication of GLT beams usually Phenol-Resorcinol-Formaldehyde (PRF), Melamine Urea Formaldehyde (MUF) or polyurethane (PUR) are used as adhesives. Theoretically GLT beams can be produced in any size. However, for practical reasons (factory size, transport, etc.) GLT having a length up to 30 m are common (Steer 1995).

2.2.2 Mechanical performance of GLT

The mechanical performance of GLT beams can be described as a combined performance of its single components. That includes the mechanical performance of timber boards, FJ and glue-lines between the timber boards. Further, the arrangement of timber boards and FJ within the GLT beams has to be considered. According to the large variability within and between
timber boards, it is obvious that the mechanical performance of GLT beams cannot be described explicit using a simplified model. In the following paragraphs, information that is useful for understanding the load-bearing behaviour of GLT beams under bending is described.

**Tensile capacity of timber boards – bending capacity of GLT**

As mentioned above the mechanical performance and thus the bending capacity of GLT beams is a combination of several independent parameters. However, assuming the Euler-Bernoulli bending theory, the bending stresses over the cross-section can be described using Eq. (2.1);

\[
\sigma_m = \frac{M}{I} \cdot z \tag{2.1}
\]

where \( M \) is the bending moment, \( I \) is the second moment of area, and \( z \) is the vertical distance to the beam axis. Within each lamella the bending stresses can be subdivided into normal stresses and bending stresses. The bending stresses are constant for each lamella and relatively small compared to the normal stresses; in particular within the outmost lamellas. Thus within a GLT beam having numerous lamellas, the bending stresses can be approximated with the normal stresses within the lamellas (Fig. 2.4). However, timber under compression is quite ductile, thus the bending failure is related to the tensile capacity of the lowest lamella. The origin of the failure is often a weak section located in the lowest lamella. This could be a major knot, a knot cluster or a FJ. Accordingly, several models are developed to predict the bending capacity of GLT based on the tensile capacity of the source material. In Chapter 2.2.3 a few selected models are introduced.

**Homogenisation (Lamination effect)**

The material properties of GLT beams are analysed in numerous of studies. Those studies show that the the variability of the resistance is smaller than that of solid timber. This is a result of the homogenisation; i.e. local weak sections, such as knot clusters are distributed more homogeneously than in solid timber (e.g. Colling [1990], Schickhofer et al. [1995]).

In general the bending capacity of GLT beams exceed the tensile capacity of the lamellas; i.e. the most loaded lamella of the GLT (outmost lamella) can withstand higher tensile stresses than the individual lamella. This is mostly due to the lamination effect. To quantify the lamination
2.2. Load-bearing behaviour of GLT

effect the lamination factor $k_{\text{lam}}$ is introduced Eq. (2.2); where $f_{m,g}$ is the bending strength of the GLT beam and $f_{t,l}$ is the tensile strength of the lamination.

$$k_{\text{lam}} = \frac{f_{m,g}}{f_{t,l}} \quad (2.2)$$

The lamination effect is explained in numerous of publications (e.g. Falk & Colling 1995, Serrano & Larsen 1999). Summarised, it can be explained with the following effects:

- **Dispersion effect**: The material properties of GLT are more homogeneous than those of structural timber. As a result the probability that a single defect has an influence on the load-bearing capacity is highly reduced compared to solid timber.

- **Reinforcing effect**: Local weak sections, such as knot clusters or FJ, are reinforced by the adjacent lamellas.

- **Effect of test procedure**: According to standard test methods for the estimation of the tensile strength, such as EN 408 (2003), specimens are loaded centric without any lateral restraints. Through unsymmetrical defects, such as edge knots (Fig. 5.3) lateral bending stresses are induced, which are reducing the tensile capacity. In GLT lateral bending is prevented due to the adjacent lamellas.

In addition to the above mentioned effects, also the size effect and the load configuration effect may have an influence on the difference between $f_{m,g}$ and $f_{t,l}$. Both effects are introduced in the following.

**Size effect**

The strength of a structural component depends on its dimensions, mainly for materials that show brittle failures. This can be explained by the weakest link theory (Weibull theory, according to Weibull 1939), which states that the load-bearing capacity of a structural component corresponds to load-bearing capacity of its weakest link. The probability of occurrence of a weak zone within a component increases with increasing volume. Thus the load-bearing capacity decreases with increasing volume.

Assuming that the probability of failure of a single element can be described with a Weibull distribution (Tab. 2.2) the probability that a series of $n$ elements (total volume $V$) will fail under constant tensile stresses $\sigma$ can be described as following (Madsen & Buchanan 1986, Colling 1986a,b, Barrett et al. 1995, Isaksson 1999):

$$P_f(\sigma) = 1 - \exp\left(-n\left(\frac{\sigma}{b}\right)^p\right) = 1 - \exp\left(-V\left(\frac{\sigma}{b}\right)^p\right) \quad (2.3)$$

Keeping the probability of failure constant for two different volumes $V_1$ and $V_2$, a relation between the load-bearing capacities can be calculated; Eq. (2.4). Based on Eq. (2.4) the size effect $k_{\text{size}}$ is derived; Eq. (2.5). Here $V$ denotes the volume of the specimen and $V_0$ denotes the reference volume.
\[ P_f(\sigma_1) = P_f(\sigma_2) \rightarrow \frac{\sigma_2}{\sigma_1} = \left( \frac{V_1}{V_2} \right)^{1/p} \quad (2.4) \]

\[ k_{\text{size}} = \left( \frac{V}{V_0} \right)^{\eta}, \quad \eta = -\frac{1}{p} < 0 \quad (2.5) \]

As mentioned above, the Weibull theory can be used to describe the load-bearing capacity of materials exhibiting brittle failure; e.g. timber under pure tension. However, a bending failure is different from a tensile failure. There, both tension and compression are involved; failure under compression is quite ductile, thus the effect might be reduced. According to Thelandersson et al. (2003) typical values for \( \eta \) are between \(-0.1\) and \(-0.4\).

**Load configuration effect**

Another reason for the higher bending capacity (compared to the tensile capacity of a single lamella) is the load configuration effect. This effect is explained with the following example. For simplification it is be assumed that a specimen fails when a single section is stressed up to its resistance, and a timber board loaded under pure tension is assumed to have uniform tensile stresses over its entire length; i.e. the tensile stresses are identical within each cross-section. Hence the tensile capacity of the timber board corresponds to the tensile strength of its weakest section. In contrast, the stresses within a GLT beam loaded under four-point bending are different over the length of the beam. The failure will occur within the area having the lowest resistance in relation to the applied stresses, which is not always the weakest section. The load configuration effect is analysed by Colling (1986a,b), Isaksson (1999) for different load configurations. Comparing a GLT beam having a constant bending moment with one under four-point bending the bending capacity of the latter one is about 18% higher (Isaksson 1999).

**Finger joint connections**

Similarly to knot clusters, also finger joint connections can be considered as local weak sections within GLT beams. In general the load-bearing capacity of FJ is significantly smaller than that of the adjacent clear wood, whereas the stiffness is comparable (Ehlbeck et al. 1985a, Heimeshoff & Glos 1980). According to Colling (1990) the tensile strength of FJ can be assumed to be similar to the tensile strength of a knot cluster with \( t_{\text{KAR}} = 0.25 - 0.30 \). Due to the relatively large stiffness FJ attract higher stresses, compared to knot clusters.

The influence of FJ has been investigated in numerous studies. In these studies always a certain amount of the tested GLT beam failed in areas of FJ. However, the amount of the failures where FJ are involved varies between the investigations. Colling (1990) analysed the influence of FJ on a compilation of numerous studies; altogether the compilation included \( 1'767 \) GLT beams. The investigation showed that about 79% of the investigated GLT beam, having a FJ located in the lowest lamella within the area of the maximal bending moment, failed through the FJ. In addition Colling (1990) tested 42 GLT beams himself. The results show that the influence of FJ on the type of failure is directly related to the \( t_{\text{KAR}} \)-value. GLT beams produced out
of timber boards where tKAR $\geq 0.35$ failed through the knot cluster (all tested specimens), whereas about 2/3 of the GLT beams with tKAR $\leq 0.35$ failed in the area of a FJ. In the study of Johansson (1990) only 31% failed through FJ. Blaß et al. (2008) presented the failure within the lowest lamella of altogether 50 GLT beams, of the strength classes GL32c and GL36c. 6 FJ-failure, 37 timber failure (knot cluster or clear wood) and 7 combined failure (FJ and timber) are documented. Thus only 13 of the GLT beam failed connected to a FJ (26%). Conspicuous is that within the lower strength grade significantly more FJ-failures (9) occur than in the upper strength glass (4), which is contradictory. Schickhofer (1996) investigated 115 GLT beams. The investigation showed that the amount of failures related to FJ increases with increasing timber quality. GLT fabricated out of timber boards MS10 failed in 5-9% trough FJ, MS13 in 11%, and MS17 in 24-39%. Further, it seems that the probability of a FJ-failure decreases with increasing GLT dimensions. Falk et al. (1992) have investigated altogether 312 GLT beam produced out of Norwegian spruce of three different strength classes, that are comparable to GL28h, GL32c, and GL32h according to EN 1194 (1999). In 23%, 34% and 44% of the GLT beams a FJ-failure was detected.

**Load transmission between lamellas**

As already mentioned, one advantage of GLT beams is that local weak sections, such as knot clusters or FJ are reinforced by the adjacent lamellas. In order to ensure this effect, the load transfer between lamellas has to be sufficient. This is of particular importance in areas of large stiffness differences between the lamellas, which leads to local shear stresses.

In addition, the load transmission between lamellas might have an influence on the load-bearing capacity in the case of a local failure. This has been investigated in Serrano & Larsen (1999). The strain energy released through the failure can lead to a failure of the entire GLT beam or have only a minor influence, if the adjacent lamellas are able to take the additional stresses. Furthermore, the investigation of Serrano & Larsen (1999) shows that a failure of the outmost lamella will most likely lead to the failure of the entire GLT beam. Only GLT beams having very thin lamellas up to 10 mm would have a delamination without a failure of the entire GLT beam.

### 2.2.3 Modelling of GLT

The load-bearing capacity of GLT beams, or rather the characteristic value of the bending strength $f_{m,g,k}$, has been investigated for more than 30 years in numerous different studies. The studies can be subdivided into two groups. Those where a model for $f_{m,g,k}$ is identified based on (a) experimental investigations, and (b) simulation models. Regarding the large number of influencing parameters, the second approach has been established as the more efficient within the last decades. Next, a few selected models are briefly introduced. For a detailed description of those models please see the literature mentioned in the corresponding paragraphs.

---

1 quoted in Thelandersson et al. (2003)
Chapter 2. State of the art

Models based on experimental investigations

In the 90s, numerous studies have been conducted to develop a model to predict the characteristic bending strength $f_{m,g,k}$, based on the results of experimental investigations. The outcome of the majority of the studies are empirical equations based on the characteristic tensile strength $f_{t,0,l,k}$ of the source material. Examples are the studies of Riberholt et al. (1990), Falk et al. (1992), Gehri (1992, 1995), Schickhofer (1996). In Brandner & Schickhofer (2008) a detailed compilation of the models is given.

Model of Foschi and Barrett

Foschi & Barrett (1980) presented an approach to model GLT. There, initially GLT beams are simulated where the beam setup should represent the natural variability of timber. Following the material properties of the simulated beams are estimated using a finite element model (FEM). The GLT beams are subdivided into elements having a constant length of 150 mm. The element height and width correspond to the cross-section dimensions of the lamellas. A specific density and a specific knot diameter are allocated to each element; FJ are not considered within the GLT simulation. To develop the model for the probabilistic representation of the specific knot diameter, timber boards are subdivided into sections having a constant length of 152 mm without taking into account the natural growth characteristics of timber. For those sections, a specific knot diameter was identified.

In the next step, the material properties of the elements are estimated based on information about the density and the specific knot diameter. Therefore, empirical material models are used. The material models are developed based on test results of large specimens (3'660 mm) and estimations of the variability of the specific knot diameter.

Subsequent to the GLT simulation, the load-bearing capacity of each simulated GLT beam is estimated using FEM. The material properties are assumed to be linear elastic. To estimate the load-bearing capacity, a brittle failure, so-called weakest link failure criterion, is chosen. For the calibration of the FEM model parameters, four-point bending tests were performed. Therefore, it is obvious that the simulations show a relatively good correlation to the measured values.

Prolam Model

Bender et al. (1985) and Hernandez et al. (1992) presented the so-called Prolam Model. In a first step, the assembly of GLT beams is simulated. Input parameters are (a) the length of the timber boards, (b) the strength and stiffness properties of 610 mm long timber board elements, and (c) the strength and stiffness properties of FJ. The strength and stiffness properties of the timber board elements are estimated based on the investigation of Taylor & Bender (1991).

Afterwards, the load-bearing capacity of the simulated GLT beams is estimated using a transformed section method. The load-bearing capacity of the GLT beam is defined by the capacity of its weakest cross-section. Local failure of a lamella cross-section, such as failed FJ, are allowed; i.e. they do not necessary lead to a failure of the GLT-beam.
2.2. Load-bearing behaviour of GLT

Karlsruher Rechenmodel

The most popular approach for modelling GLT is the so-called Karlsruher Rechenmodel; see e.g. [Ehlbeck et al. 1985b, a, c; Colling 1990], or more recent [Blaß et al. 2008]. The model is similar to the model of Foschi & Barrett (1980); i.e. it is a combination of two separate programs: one model to simulate the beam setup and a FEM to estimate the load-bearing capacity.

To model the beam setup, at first, an endless lamella composed of a series of 150 mm long elements is simulated. Afterwards, the endless lamella is cut to the specific beam length to create GLT beams. The lamella contains two different kinds of elements: timber and FJ. The position of the FJ is modelled in accordance with the fabrication process (using the length of timber boards). Following each timber section a specific dry density \( \rho_0 \) and a specific tKAR-value are allocated. \( \rho_0 \) is assumed to be constant within one timber board. For the simulation of the tKAR-value, at first, a tKAR\(_{\text{max}}\) within a timber board is simulated. Based on tKAR\(_{\text{max}}\), a specific tKAR-value is allocated to all other sections within the timber board, based on an investigation by Görlacher (1990a). \( 2/3 \) of the timber sections are free of knots, thus the tKAR-value is zero. As already mentioned in Chapter 2.1.2, the tKAR-value model is developed based on the investigation of Colling & Dinori (1987). Both parameters \( \rho_0 \) and tKAR\(_{\text{max}}\) are modelled using a beta distribution, having an upper and lower limit; i.e. low and high realisation of both parameters are prevented. The distribution functions are developed for timber boards having different grading criteria.

In the next step, the strength and stiffness properties of each particular timber board section are estimated based on \( \rho_0 \), tKAR-value and FJ. Therefore, material models developed by Glos (1978) and Ehlbeck et al. (1985a) are used, see also Heimeshoff & Glos (1980) for the test setup. The material models are developed based on material properties measured on small specimens having a testing length of 137.5 mm. This might have an influence on the estimated material properties; the influence of the specimen size is described in Chapter 5. However, the material model already consists of correlations between strength and stiffness properties and correlations between elements of the same timber board.

In the second program, the load-bearing capacity of the simulated GLT beams are estimated using FEM. The material behaviour is assumed as orthotropic; ideal elastic for tension and ideal elasto-plastic for compression. A failure within the lowest lamella is assumed as the failure criterion of the GLT beam. A detailed description about the FEM program is given in Frese (2006).

One of the outcomes of the Karlsruher Rechenmodel was an empirical equation to predict the characteristic value of the bending strength, based on characteristic values of the source material. A distinction was made between visual and machine graded timber (e.g. Frese & Blaß 2009, Frese et al. 2010).

Model summary

The quality of the simulation models has improved since the first approach developed by Foschi & Barrett (1980). Thereby in particular the development of the Karlsruher Rechenmodel has
to be mentioned. However, there are still some opportunities for improvement such as (a) the use of more efficient strength and stiffness related indicators, (b) an improvement of the probabilistic description of timber boards, or (c) an improvement of the material models. Detailed explanations about the improvements are given in the corresponding chapters. Furthermore, it has to be mentioned that until now none of the GLT models has ever been validated withGLT beams with well-known local material properties; i.e. GLT beams where the exact position of each particular knot cluster and each particular finger joint connection is known. The only exception is Ehlbeck & Colling (1987a,b), who tested altogether nine GLT beams, where the above-mentioned information of the lowest two lamellas is known. However, only in two GLT beams, a FJ was placed in the highest loaded area – both failed within the FJ. As a result, the quality of the numerical models, in terms of considering varying material properties and detecting the type of failure, is not completely proved yet.

Another more general disadvantage of all existing approaches is that they are based on strength and stiffness related indicators measured in the laboratory. Usually the measurement of those indicators is very time consuming and thus not really efficient for practical application. Nowadays, timber boards are often graded with measurement devices where global and local strength and stiffness related indicators are automatically measured and documented. For a practical application it would be more successful if the GLT models are based on such machine-grading indicators.

The final outcome of the majority of the investigations (experimental investigations and simulation models) is an empirical equation to predict the characteristic value of the bending strength $f_{m,g,k}$, based on characteristic values of the source material; e.g. characteristic value of the tensile capacity of the lamellas $f_{t,0,l,k}$ or the bending capacity of FJ $f_{m,j,k}$. From a scientific perspective, the origin of the empirical values within those equations is often un reproducible. One example is the equation given in the current version of the EN 14080 (2013), that contains altogether seven empirical values; Eq. (2.6). Furthermore, none of those models consider the variability of the material properties of the source material.

$$f_{m,g,k} = -2.2 + 2.5 f_{t,0,l,k}^{0.75} + 1.5 \left( \frac{f_{m,j,k}}{1.4 - f_{t,0,l,k}} + 6 \right)^{0.65}$$  (2.6)
2.3 Aspects of structural reliability

Typical problems in structural engineering such as design, assessment, inspection and maintenance planning are decision problems subject to a combination of inherent, modelling and statistical uncertainties. Those uncertainties have to be considered, especially within the field of timber engineering, where the material properties exhibit large variabilities and uncertainties. Structural reliability theory is concerned with the rational treatment of these uncertainties. In the following paragraphs, a few selected aspects concerning structural reliability are introduced. The introduction is based on standard literature concerning structural reliability, such as Thoft-Christensen & Baker (1982), Melchers (1999), Ditlevsen & Madsen (1996), Madsen et al. (2006), and Faber (2009).

2.3.1 Limit state principle

Failure of a structural component is defined as an unfulfilment of its associated requirements. These can be serviceability limit state requirements (e.g. excessive deformation, vibration) or ultimate limit state requirements (e.g. instability, rupture).

One elegant approach to describe failure is by using the limit state function \( g(x) \), according to Eq. (2.7). Here, \( x \) are realisations of the random variables \( X \), representing all uncertainties. For structural components the limit state function can be expressed through resistance \( R \) and load \( S \).

\[
F = \{ g(x) \leq 0 \} \quad \text{with} \quad g(x) = r - s 
\]

In the case of a bending failure, which is a typical ultimate limit state failure of GLT beams, the limit state function is defined as \( g(x) = f_m - \sigma_m \); where, \( f_m \) denotes the bending strength (resistance of the structural member), and \( \sigma_m \) denotes the bending stresses (as a function of the applied load). It is obvious that each realisation where \( f_m \leq \sigma_m \) leads to failure. Taking into account the entire range of the random variable \( X \), the probability of failure can be described using Eq. (2.8). Here \( f_X \) is the joint probability function of the variable \( X \).

\[
P_f = P(g(X) \leq 0) = \int_{g(x) \leq 0} f_X(x) \, dx \tag{2.8}
\]

In general, both the applied load \( S \) and the resistance \( R \) are functions of time. In many cases the applied load shows a large variability over time, depending on environmental conditions (snow, wind) and use. The resistance of a structural member is also a function of time; e.g. decreasing resistance over the time through deterioration processes. A typical realisation of \( R(t) \) and \( S(t) \) is illustrated in Fig. 2.5. It is obvious that the probability of failure \( P_f \) increases over time.

For practical application, it is often difficult and time consuming to consider the time dependency, thus in many applications it is not considered. For a more detailed description see e.g. Melchers (1999) and Faber (2009).
2.3.2 Reliability based code calibration

Modern design codes, such as the Eurocodes \cite{2002}, are based on the so-called load and resistance factor design (LRFD) format. Next, the principle of LRFD is explained for the case of two loads; one that is constant and one that is variable over time. The LRFD equation is given in Eq. \cite{2009}. Here $R_k$, $G_k$, and $Q_k$ are the characteristic values of the resistance $R$, the permanent load $G$, and the time variable load $Q$. $\gamma_m$, $\gamma_G$ and $\gamma_Q$ are the corresponding partial safety factors. $z$ is the so-called design variable, which is defined by the chosen dimensions of the structural component.

\[ z \frac{R_k}{\gamma_m} - \gamma_G G_k - \gamma_Q Q_k = 0 \] (2.9)

The characteristic values for both load and resistance are in general defined as fractile values of the corresponding probability distributions. In Eurocode 5 \cite{2004} the following characteristic values are defined: $R_k$ is the 5\% fractile value of a Lognormal distributed resistance, $G_k$ is the 50\% fractile value (mean value) of the Normal distributed load (constant in time), and $Q_k$ is the 98\% fractile value of the Gumbel distributed yearly maxima of the load (variable in time).

The corresponding partial safety factors can be calibrated to provide design solutions ($z$) with an acceptable failure probability $P_f$ (Eq. \ref{2.10}). Here $R$, $G$, and $Q$ are resistance and loads represented as random variables, $z^* = z(\gamma_m, \gamma_G, \gamma_Q)$ is the design solution identified with Eq. \ref{2.9} as a function of the selected partial safety factors, and $X$ is the model uncertainty.

\[ P_f = P\{g(X, R, G, Q) < 0\} \] with \[ g(X, R, G, Q) = z^* XR - G - Q = 0 \] (2.10)

Often the structural reliability is expressed with the so-called reliability index $\beta$ (Eq. \ref{2.11}). A common value for the target reliability index is $\beta \approx 4.2$ which corresponds to a probability of failure $P_f \approx 10^{-5}$ \cite{2001}.

\[ \beta = -\Phi^{-1}(P_f) \] (2.11)
2.3. Aspects of structural reliability

In general, different design situations are relevant; i.e. different ratios between $G$ and $Q$. This can be considered using a modification of Eq. (2.9)–(2.10) into Eq. (2.12)–(2.13). $\alpha_i$ might take values between 0 and 1, representing different ratios of $G$ and $Q$. $\hat{R}$, $\hat{G}$, and $\hat{Q}$ are normalized to a mean value of 1. For each $\alpha_i$ one design equations exists, thus altogether $n$ different design equations have to be considered.

$$z_i \frac{\hat{R}_k}{\gamma_m} - \gamma_G \alpha_i \hat{G}_k - \gamma_Q (1 - \alpha_i) \hat{Q}_k = 0 \quad (2.12)$$

$$g_i(X, \hat{R}, \hat{G}, \hat{Q}) = z_i^* X \hat{R} - \alpha_i \hat{G} - (1 - \alpha_i) \hat{Q} = 0 \quad (2.13)$$

Afterwards, the partial safety factors ($\gamma_m$, $\gamma_G$, and $\gamma_Q$) can be calibrated by solving the optimisation problem given in Eq. (2.14).

$$\min_{\gamma} \left[ \sum_{j=1}^{n} \left( \beta_{\text{target}} - \beta_j \right)^2 \right] \quad (2.14)$$

The reliability based code calibration is briefly introduced to illustrate the influence of uncertainties (load and resistance), in respect to codes. Please find more information in (e.g. JCSS 2001, Faber & Sørensen 2003). For examples of applications in the area of timber engineering see also Kohler et al. (2012).

2.3.3 Basic variables of the resistance

An elegant method to describe the variability of the material properties (resistance) is by using distribution functions. In the following paragraphs, distribution function that are relevant for this thesis are introduced (Tab. 2.2). For an overview of other distribution functions see e.g. Benjamin & Cornell (1970) and Hahn & Shapiro (1967).

**Normal distribution:** The Normal distribution is the most used distribution function. According to Benjamin & Cornell (1970), the sum of many independent random values are Normal distributed (central limit theorem). The range of the Normal distribution is $-\infty < x < \infty$, which gives always a certain probability of negative values. This is contradictory when modelling the resistance of materials, which can never be negative.

**Lognormal distribution:** A random variable is Lognormal distributed if its logarithm is Normal distributed. For practical applications the Lognormal distribution has the advantage that it precludes negative values.

**Exponential distribution:** The interval between two sequential events that follow a Poisson process is Exponential distributed. One example is the distance between two adjacent knot clusters within the trunk of a Norway spruce tree (see Chapter 4).

**Gamma distribution:** The Gamma distribution can be seen as a generalized version of the Exponential distribution. It describes the interval to the $n$th event of a Poisson process. The distribution is generalized when $k$ is not an integer.
**Weibull distribution:** The Weibull distribution can be used to describe extreme minima. In engineering applications it is common to use the Weibull distribution to describe the strength of a structural component, especially for brittle materials (see also Chapter 2.2.2).

**Estimation of distribution parameters**

There are several methods to estimate the parameters of the distribution function based on a data sample; e.g. methods of moments or maximum likelihood method (MLM). Here, only the MLM is introduced. The basic principle of the MLM is to find the parameters of the chosen distribution function which most likely reflect the data sample. The parameters of the distribution function are estimated by solving the optimisation problem given in Eq. (2.16). The likelihood $L(\theta|\hat{x})$ of the observed data is defined according to Eq. (2.15), where $\theta$ represents the parameters, $f_X$ is the density function of the random variable X, and $\hat{x}$ the measured values of the data sample.

$$L(\theta|\hat{x}) = \prod_{i=0}^{n} f_X(\hat{x}_i|\theta)$$  \hspace{1cm} (2.15)

$$\min_{\theta} (-L(\theta|\hat{x}))$$  \hspace{1cm} (2.16)

The uncertainties of the MLM estimators can be expressed with covariance matrix $C_{\Theta\Theta}$, where the diagonals are the variances of the estimated distribution parameters and the other elements are the covariances between the parameters. The covariance matrix $C_{\Theta\Theta}$ is defined as the inverse of the Fisher information matrix $H$. The components of $H$ are determined by the second order partial derivatives of the log-likelihood function; see e.g. Faber (2012).

$$C_{\Theta\Theta} = H^{-1}$$  \hspace{1cm} (2.17)

$$H_{ij} = -\frac{\partial^2 l(\theta|\hat{x})}{\partial \theta_i \partial \theta_j}|_{\theta = \theta^*}$$  \hspace{1cm} (2.18)

**2.3.4 Uncertainties in reliability assessment**

As mentioned above, decision problems are in general subjected to uncertainties. It is widespread to distinguish between aleatory uncertainties (inherent natural variability) and epistemic uncertainties (model and statistical uncertainties), see e.g. Melchers (1999), Faber (2009).

**Inherent natural variability:** Uncertainties according to the inherent natural variability results from the randomness of a phenomenon. An example is the realisation of the applied wind or snow load on a construction.

**Model uncertainties:** Uncertainties that are associated with the inaccuracy of our physical or mathematical models.

**Statistical uncertainties:** The statistical evaluation of test results is connected to statistical uncertainties. They can be reduced through an increased number of specimens.
2.3. Aspects of structural reliability

<table>
<thead>
<tr>
<th>Distribution function</th>
<th>Density function [ f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{1}{2} \left( \frac{x-\mu}{\sigma} \right)^2 \right) ]</th>
<th>Range [ -\infty &lt; x &lt; \infty ]</th>
<th>Mean value [ \mu ]</th>
<th>Standard deviation [ \sigma ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal distribution</td>
<td>[ f(x) = \frac{1}{\sigma} \phi \left( \frac{x-\mu}{\sigma} \right) ]</td>
<td>[ \sigma &gt; 0 ]</td>
<td>[ \sigma ]</td>
<td>[ \sigma &gt; 0 ]</td>
</tr>
<tr>
<td>Lognormal distribution (shifted)</td>
<td>[ f(x) = \frac{1}{(x-\epsilon)\zeta} \phi \left( \frac{\ln(x-\epsilon)-\lambda}{\zeta} \right) ]</td>
<td>[ \epsilon &lt; x &lt; \infty ]</td>
<td>[ \mu = \epsilon + \exp \left( \lambda + \frac{\zeta^2}{2} \right) ]</td>
<td>[ \zeta &gt; 0 ]</td>
</tr>
<tr>
<td>Exponential distribution (shifted)</td>
<td>[ f(x) = \lambda \exp(-\lambda(x-\epsilon)) ]</td>
<td>[ \epsilon \leq x &lt; \infty ]</td>
<td>[ \mu = \epsilon + \frac{1}{\lambda} ]</td>
<td>[ \sigma = \frac{1}{\lambda} ]</td>
</tr>
<tr>
<td>Gamma distribution</td>
<td>[ f(x) = \frac{\nu(x)^{k-1}}{\Gamma(k)} \exp(-\nu x) ]</td>
<td>[ 0 \leq x &lt; \infty ]</td>
<td>[ \mu = \frac{k}{\nu} ]</td>
<td>[ \sigma = \frac{\sqrt{k}}{\nu} ]</td>
</tr>
<tr>
<td>2p - Weibull distribution</td>
<td>[ f(x) = \frac{(\frac{x}{b})^{p-1}}{b} \exp \left( -\left( \frac{x}{b} \right)^p \right) ]</td>
<td>[ 0 &lt; x &lt; \infty ]</td>
<td>[ \mu = b \Gamma \left( 1 + \frac{1}{p} \right) ]</td>
<td>[ \sigma = b \cdot \sqrt{\Gamma \left( 1 + \frac{2}{p} \right) - \Gamma^2 \left( 1 + \frac{1}{p} \right)} ]</td>
</tr>
</tbody>
</table>
Often a model is subjected to all three types of uncertainties. E.g. in an empirical model to predict the tensile strength of timber boards ($f_t$) based on the tKAR-value: $f_t = \beta_0 + \beta_1 \cdot \text{tKAR}$. Here the model uncertainties are a result of the inappropriate model, they can be reduced through an improvement of the model; e.g. additional indicators. Furthermore, both empirical parameters $\beta_i$ might be connected with statistical uncertainties since they are estimated based on a limited number of data. However, even with a model that is physically and mathematically ‘perfect’ (i.e. no model and statistical uncertainties), some uncertainties will remain according to the inherent natural variability of timber.

### 2.3.5 Methods of structural reliability

There exists different approaches to calculate the probability of exceeding or being below a certain threshold, such as the probability of failure or the probability of being below a certain load-bearing capacity. They can be classified into two groups: approximation methods (e.g. FORM, SORM) and simulation methods. An overview of different methods of structural reliability is given in the corresponding literature (e.g. [Melchers 1999](#), Faber 2009).

In this thesis, only the straight-forward classical Monte Carlo simulation method (MC) is used. Assuming a random variable is represented through a set of independent random variables $X$. The outcome of the limit state function $g(x)$ can then be predicted for each realization of $X$. In the case of a limit state function the outcome can only be within the failure domain $g(x) \leq 0$ or within the safe domain $g(x) > 0$. The probability of failure $P_f$ can be predicted after an infinite number of realisations $n$. Here $n_f$ is the number of realisations that end up in the failure domain.

$$P_f = P(g(x) \leq 0) = \lim_{n \to \infty} \frac{n_f}{n} \quad (2.19)$$

It is obvious that with increasing number of realisations $n$, the outcomes become more precise. In the present thesis, the MC method is used to estimate the characteristic value of the load-bearing capacity (5% fractile). As a result, only a relatively small number of simulations ($n \approx 10^3 - 10^4$) is necessary.

However, often significantly smaller failure probabilities have to be estimated; in structural reliability analysis failure probabilities $P_f \leq 10^{-6}$ are common. Therefore, $n \geq 10^8$ simulations would be essential for a realistic estimation, concerning a specific decision problem. To optimize the calculation time for such applications, more advanced MC methods, such as important sampling, were developed. Please find more information in e.g. [Melchers 1999](#).
Chapter 3

Experimental investigation

Within the scope of this thesis, numerous experimental investigations were conducted. They can be subdivided in two main parts: (1) investigations of timber boards and (2) investigations of GLT beams. Within the first part, altogether 400 timber boards were investigated, mainly non-destructively. Afterwards, out of the investigated timber boards, 24 GLT beams having well-known local material properties were fabricated. In the second part, these GLT beams were investigated destructively. Thereby, it is particularly focused on the influence of knot clusters and finger joint connections (FJ) on the load-bearing behaviour of the GLT beams. The experimental investigations are described in detail in two test reports (Fink & Kohler 2012, Fink et al. 2013b). In this thesis only a short summary of the conducted tests is presented.

3.1 Structural timber

In the first part of the experimental investigation, the material properties of timber boards are analysed, mostly non-destructively. A detailed description is given in Fink & Kohler (2012). For further information, see also Fink & Kohler (2011), Fink et al. (2011, 2012).

3.1.1 Specimens

The investigation was performed on two sample sets, each comprising 200 specimens; the species is Norway spruce from southern Germany. The dimensions of the timber boards are 126·44·4'000 mm. All timber boards are graded into the strength grades L25 and L40. According to the European standard EN 14081-4 (2009), these strength grades require a minimum characteristic tensile capacity of 14.5 MPa and 26.0 MPa, respectively.

The timber boards were graded with the GoldenEye-706 grading device manufactured by MiCROTEC (Brixen, IT) (Giudiceandrea 2005). This is a grading device that combines the measurement of the dynamic modulus of elasticity, based on eigenfrequency, with an X-ray measurement, to detect knots. Through the significant larger density of knots, compared to the density of defect-free timber, knots are visible in a grey scale image. They can be detected in size and position, by using image processing. As a result, for all the 400 timber boards, the
machine-grading indicators, i.e. an estimation of the dynamic modulus of elasticity ($E_m$) and a knot indicator ($K_m$), are known.

### 3.1.2 Conducted tests

For all timber boards, the dimensions and the position of every knot with a diameter larger than 10mm were assessed. Furthermore, the following parameters were measured: ultrasonic runtime, eigenfrequency, dimensions, weight and moisture content. Based on the eigenfrequency and the ultrasonic runtime, the corresponding dynamic moduli of elasticity ($E_{dyn,F}$ and $E_{dyn,US}$) of the timber boards are calculated according to Eq. (3.1) (Görlacher 1984, Steiger 1996); where $f_0$ is the eigenfrequency, $\nu$ is the ultrasonic wave speed, $l$ is the board length and $\rho$ is the density. Both $E_{dyn,F}$ and $E_{dyn,US}$ have to be considered as average values over the entire length of the timber board. The assessed values are corrected to a reference moisture content according to EN 384 (2010).

$$E_{dyn,F} = (2lf_0^2)^2 \rho \quad E_{dyn,US} = \rho \nu^2$$

In addition to the knot measurement and the estimation of strength and stiffness related indicators, destructive and non-destructive tensile tests were performed to investigate the tensile stiffness of timber boards, as well as the deformation and failure behaviour of selected knot clusters. The investigations are explained in more detail in the following paragraphs. In order to ensure the comparability of the test results, all tensile tests are performed with standard moisture content according to EN 408 (2003); i.e. equilibrium moisture content of the specimen in standard climate: $(20 \pm 2) ^\circ C$ and $(65 \pm 5)$% relative humidity.

**Non-destructive tensile tests – investigation of local stiffness properties**

Non-destructive tensile tests are performed on half of the timber boards to investigate local stiffness properties. The specific characteristic of this part of the experimental investigation is that the timber boards were previously subdivided into (a) sections containing knot clusters or large single knots (referred to as knot sections, KS), and (b) sections between the knot sections (referred to as clear wood sections, CWS). For all sections the corresponding expansions are measured using an optical camera device Optotrak Certus (s-Type), Northern Digital Inc. (Waterloo, Canada). In order to do that, at the beginning and the end of each section and at the edge of the total measured area (optical range of the infrared camera), three high frequently infrared light emitting diodes (LEDs) are mounted (Fig. 3.1). The timber boards were loaded with an axial tensile force that represents 45% of the estimated maximum tensile capacity; the estimation is based on the measurements of the GoldenEye-706 grading device. During the tensile tests, the LEDs send light impulses with a constant frequency of 20 Hz. Using these light impulses, the infrared camera device measures the position of the LEDs. Based on these measurements, the strains of the board sections are estimated.

During the test phase it became evident that local strains are highly affected by the knot arrangement within the KS; e.g. if a knot cluster contains a splay knot or a narrow side knot
3.1. Structural timber

Fig. 3.1: (left) illustration of the experimental setup, (right) illustration of the LED-arrangement around a knot cluster

![Diagram of experimental setup and LED arrangement around a knot cluster]

Fig. 3.2: Example of the modulus of elasticity distribution

![Graph showing modulus of elasticity distribution with different sections denoted by different lines]

(Fig. 3.2), the measured strains on the top side of the timber board, are different compared to them on the bottom side. For that reason, each timber board is tested twice: one time on the top and one time on the bottom side. Both measurements are considered to estimate the strains at the boards’ axis. For the estimation of the mean stiffness of a timber board, the outmost LEDs are used.

The properties of KS depend on several parameters, such as size of knots, knot arrangement and/or knot orientation. Thus, the probabilistic description of their properties is difficult. Therefore, weak sections (WS) with an unit length $l_{WS} = 150$ mm are introduced. The stiffness of a WS ($E_{j,WS}$) is calculated utilizing the corresponding $E_{j,KS}$ and the stiffness of the two adjacent CWS ($E_{j-1,CWS}$ and $E_{j+1,CWS}$). $l_{j,KS}$ denotes the length of the KS (Fig. 3.2):

\[
\frac{1}{E_{j,WS}} = \frac{1}{l_{WS}} \left( l_{j,KS} \frac{E_{j,KS}}{E_{j,KS}} + \frac{l_{WS} - l_{j,KS}}{2E_{j-1,CWS}} + \frac{l_{WS} - l_{j,KS}}{2E_{j+1,CWS}} \right) \quad \text{for} \quad l_{j,KS} \leq 150 \text{ mm} \tag{3.2}
\]

\[
E_{j,WS} = E_{j,KS} \quad \text{for} \quad l_{j,KS} \geq 150 \text{ mm}
\]
Investigation of the deformation and failure behaviour of knot clusters

In this part of the experimental investigation the influence of knots, and their arrangements, on the deformation and the failure behaviour is analysed. Destructive and non-destructive tensile tests were performed. Previously to the tensile tests, the investigated knot clusters were prepared with a speckle pattern; Fig. 3.3 (right). During the tensile tests, pictures were taken with a constant frequency. Based on these pictures, the relative displacements within the investigated knot cluster and, thus, the strain distribution (at the surface) were calculated using the digital image correlation software VIC 2D, Correlated Solutions, Inc. (Columbia, USA).

The deformation behaviour was investigated on altogether 40 knot clusters. To get an optimal understanding about the strain distribution within the knot cluster, each knot cluster is measured twice (on the top and bottom faces). Fig. 3.3 (left) illustrates the tensile testing machine in the laboratory at ETH Zurich. In addition to the non-destructive tensile tests, selected knot clusters are tested until failure. Therefore, the same tensile testing machine as for the non-destructive tests are used.

Typical results are presented in Fig. 3.4 for a knot cluster containing two knots. The illustration shows the longitudinal strains (strains in board/load direction), the transversal strains (strains perpendicular to the board’s axis), and the shear strains, from both sides of the investigated knot cluster. On top, the strains on one side of the knot cluster, measured during the non-destructive tensile test (tensile load 55 kN), and below, the strains of the opposite side, measured during the destructive tensile test. The failure occurred under a tensile load of 141 kN that corresponds to mean tensile stresses of 25.4 MPa in the cross-section. The dashed...
3.1. Structural timber

Fig. 3.4: Strain distribution around a knot cluster containing two knots and the associated fracture pattern. Top: upper side, tensile load: 55 kN (9.92 MPa). Bottom: lower side, tensile load: 141 kN (25.4 MPa). The dashed line illustrates the knot located on the opposite side of the board. The dash-dotted line shows the fracture pattern.

line illustrates the knots located on the corresponding opposite side of the timber board. The dash-dotted line illustrates the fracture pattern.

Additional destructive tensile tests - investigation of the tensile capacity

In addition to the investigations on the 400 timber boards described above, the results of a parallel running study were also used in this research. There, altogether 450 timber boards were investigated. The investigation was performed on three sample sets, each comprising 150 randomly selected Swiss grown Norway spruce specimens. The dimensions of the specimens in the reference samples were $90 \cdot 45 \cdot 4'000$ mm, $110 \cdot 45 \cdot 4'000$ mm, and $230 \cdot 45 \cdot 4'000$ mm, respectively. The timber boards were randomly selected and not graded, thus their material properties should represent the basic population of Swiss grown Norway spruce.

From the 450 timber boards, the same strength and stiffness related indicators were identified; i.e. knot measurements, ultrasonic runtime, eigenfrequency, dimensions, weight and moisture content. Furthermore, the timber boards were also investigated by the GoldenEye-706 grading device, thus the machine-grading indicators were also known. In addition, destructive tensile
tests were performed to estimate the tensile capacity. The tests have been performed using the same tensile testing machine, in accordance with EN 408 (2003); i.e. the required testing length is at least nine times the width of the timber boards. In order to collect as much information as possible about the individual timber boards, the test length is maximized over the whole testable length of the timber boards, being just limited by the clamping jaws. The resulting testing range is 3’360 mm.

### 3.1.3 Summary of the test results

In the first part of the experimental investigations, different, mainly non-destructive, measurements on timber boards were performed. Thereby, a large database was produced, on which the influence of different parameters (knots, \( E_{\text{dyn,F,}} \) etc.) on the tensile strength and stiffness properties of timber board (sections) can be investigated (this is explained in detail in Chapter 5). However, the analysis shows that global parameters (\( \rho, E_{\text{dyn,F}} \) and \( E_{\text{dyn,US}} \)) are strongly correlated to the mean material properties, whereas the local strength and stiffness reduction within timber boards depends on the occurrence of knots and knot clusters. Therefore, the first group is of particular importance for the estimation of the between-member variability, whereas the latter one influences the within-member variability. To show the influence of knots on the within-member variability, the estimated tensile stiffness and the corresponding tKAR-value of one specimen are illustrated in Fig. 3.5. It is obvious that within the areas of knots, the tensile stiffness is significantly reduced.

**Fig. 3.5:** Illustration of the measured tensile stiffness and the corresponding tKAR-value of one timber board
3.2 GLT beams

The second part of the experimental investigation is concerned with the investigation of GLT beams. There, the focus lies on the investigation of global phenomena (load-bearing capacity and bending stiffness) and local phenomena (deformation behaviour of selected areas and type of failure). A detailed description of the conducted tests is given in Fink et al. (2013b). For further information see also Fink et al. (2013a).

3.2.1 Specimens

Out of the investigated timber boards, GLT beams having well-known local material properties were fabricated. Each GLT beam contains eight layers of lamellas. The dimensions of the beams are \( b \cdot h \cdot l = 115 \cdot 320 \cdot 6'000 \text{ mm} \); hence the ratio \( l/h \approx 19 \) (see EN 408 (2003)). According to EN 14080 (2009), timber boards of the strength grades L25 and L40 fulfil the requirements to produce GLT beams of the strength class GL24h and GL36h, respectively. At this point it has to be mentioned that the strength grade GL36h no longer exists in the current version of the EN 14080 (2013).

For the fabrication of the GLT beams, the position of each timber board and each FJ are defined before the GLT fabrication. Thus GLT beams are fabricated, where the exact position of each particular timber board is precisely-known. The timber boards for which the local stiffness properties were measured are built in the lower three lamellas of the GLT beams. Accordingly, for all 24 GLT beams the following information is known: (1) position of each FJ, (2) position of each knot with a diameter larger than 10 mm, (3) density of each timber board, (4) estimated mean stiffness properties of each timber board \( (E_{\text{dyn,F}} \text{ and } E_{\text{dyn,US}}) \), and (5) measured stiffness properties of each KS and each CWS located in the tensile loaded area of the GLT beams. In addition to the parameters measured in the laboratory, also the machine-grading indicators from the GoldenEye-706 grading device are known: (6) estimation of the dynamic modulus of elasticity \( E_m \), and (7) knot parameter \( K_m \). In Fig. 3.10 the material properties within one GLT beam are illustrated: (a) tKAR-value, (b) \( E_{\text{dyn,F}} \), and (c) measured tensile stiffness. The black lines illustrate the position of FJ.

Altogether 24 GLT beams having well-known local material properties were fabricated and investigated. Half of them were fabricated with timber boards of the strength grades L25 and the other half with L40. For both strength grades, three different types of GLT beams were produced, each four beams. The first type of GLT beams was produced out of randomly selected timber boards. The second and third types are produced with so-called 'homogeneous' or 'inhomogeneous' lamellas in the tensile loaded area of the GLT beams. Here 'homogeneous' lamellas show relatively low mean stiffness properties combined with comparative small within-member variability (small knot clusters). On the other hand 'inhomogeneous' lamellas have a relatively large variability of the material properties within the member; i.e. large knot clusters. The timber boards were subdivided based on the measurements of the GoldenEye-706 grading device (see Fink & Kohler (2012) for a detailed description).
The idea behind this subdivision is that GLT beams having 'homogeneous' lamellas in the tensile loaded area can allow larger deformation (low mean stiffness), without large stress peaks within the area of knot clusters (small variability of the material properties). This might lead to an increase on the stress redistribution from the lowest lamella to the lamella located above and thus to an increase of the load-bearing capacity. In contrast, GLT beams having 'inhomogeneous' lamellas in the tensile loaded area should lead to GLT beams where the load-bearing capacity is highly related to the tensile capacity of the weakest section within the lowest lamella.

The arrangement of the timber boards within the GLT beams were randomly, except the lower three lamellas of the 'homogeneous' and the 'inhomogeneous' beams. There, the positions of the timber boards were chosen so that specified arrangements of weak sections occurred. Attempts were made to produce GLT beams where (a) knot clusters lie above each other, and (b) knot clusters are diagonally shifted. Thereby it was particular focused on the main stressed area. However, timber is a natural grown material and thus the arrangement of knots is not completely regular. Consequently, the majority of the GLT beams included both knot clusters located above each other and knot clusters that were diagonally shifted.

3.2.2 Conducted tests

The experimental investigation was performed on the 24 GLT beams described before. From all of them the load-bearing capacity and the bending stiffness were identified and the type of failure was investigated. Additionally, local strains within the GLT beams were analysed using the same optical camera device used for the investigation of timber boards (Chapter 3.1). On four GLT beams the strains were analysed over the entire main stressed area (area where the bending moment is maximum). For the other 20 GLT beams, the strains were measured on two selected local areas (in general, areas containing knot clusters or FJ). To ensure an optimal comparability between the test results, all beams were tested with the same moisture content $u = 10 - 12\%$.

Investigation of the load-bearing capacity and the bending stiffness

During the four-point bending tests, each GLT beam was loaded up to failure. The failure is defined as the first explicit crack in the GLT beam that leads to an abrupt, significant deformation of the specimen. The ultimate load $F_u$ is the applied load at the moment of failure. Given $F_u$, the bending strength $f_{m,g}$ is calculated with Eq. (3.3), according to EN 408 (2003); here $a$ is the distance between the point of load application and the nearest support, $b$ and $h$ are the width and height of the cross-section.

$$f_{m,g} = \frac{3F_u \cdot a}{bh^2} \quad (3.3)$$

The bending strength $f_{m,g}$ was estimated on specimens with a cross-section $b \cdot h = 115 \cdot 320$ mm. According to EN 1194 (1999), $f_{m,g}$ should be estimated based on specimens with a cross-section of at least $b \cdot h = 150 \cdot 600$ mm. In the case of smaller tested specimens, the estimated bending
3.2. GLT beams

strength $f_{m,g}$ has to be reduced with the size factor $k_{\text{size}}$ according to Eq. (3.4) to consider the size effect (denoted $f_{m,g,\text{size}}$).

$$f_{m,g,\text{size}} = f_{m,g} \cdot k_{\text{size}}$$

$$k_{\text{size}} = \left( \frac{b}{150} \right)^{0.05} \left( \frac{h}{600} \right)^{0.1} = 0.927$$

(3.4)

The global bending stiffness $E_{m,g}$ is estimated based on the measured vertical displacement of the middle point of the GLT beam. The vertical displacement is measured with two different systems; (1) linear variable differential transformer (LVDT) located in the middle lower side of the beam, and (2) the optical camera device described before (using the LED located in the middle of the lowest lamella). Using both measurement systems, similar displacements were identified.

Taking into account the measured force and the associated vertical displacement, the global bending stiffness $E_{m,g}$ is estimated using Eq. (3.5), according to EN 408 (2003), using a linear regression analysis with all data within the interval $0.1 - 0.4 F_u$. For all specimens the coefficient of correlation was $\rho(F, w) > 0.99$. $F_2 - F_1$ and $w_2 - w_1$ denote the rates of loading and deformation within the load interval $0.1 - 0.4 F_u$. The shear modulus $G$ is assumed as infinite according to EN 408 (2003).

$$E_{m,g} = \frac{3a l^2 - 4a^3}{2bh^3} \left( \frac{w_2 - w_1}{F_2 - F_1} - \frac{6a}{5Gbh} \right)$$

(3.5)

The results are summarised in Fig. 3.6–3.7. Here the black lines corresponds to the required values of the material properties given in EN 1194 (1999), combined with the recommended COV given in JCSS (2006): $f_{m,g,k} = 24 \text{ MPa (COV = 0.15)}$ and $E_{m,g,\text{mean}} = 11'600 \text{ MPa (COV = 0.13)}$ for GL24h, and $f_{m,g,k} = 36 \text{ MPa (COV = 0.15)}$ and $E_{m,g,\text{mean}} = 14'700 \text{ MPa (COV = 0.13)}$ for GL36h ($f_{m,g}$ and $E_{m,g}$ are assumed to be Lognormal distributed). It seems that the strength properties of both strength classes corresponds to the proposed values. The same applies for $E_{m,g,\text{mean}}$. However, the COV for the bending stiffness recommended in JCSS (2006) seems to be significantly overestimated.

As mentioned at the beginning of this chapter, for both strength classes, three kinds of GLT beams were produced: random, homogeneous and inhomogeneous GLT beams. Between the three types, no unambiguous differences could be detected; neither for $f_{m,g}$ nor for $E_{m,g}$. This might be the result of (a) the relatively low variability of the material properties within one strength grade or (b) the small sample size.

Investigation of the type of failure

All investigated GLT beams failed under bending; i.e. the lower lamellas show tensile failures within the area of high bending moment (e.g. Fig. 3.8). The tensile failure of the failed lamellas were inspected in detail. A distinction between the following three types of failure is made: (a) tensile failure of the lamella within the area of a knot cluster – a knot cluster is defined as a
lamella cross-section with tKAR ≥ 0.1, (b) tensile failure of the lamella within the area of a FJ, and (c) tensile failure within clear wood (defect-free timber). In the case that a lamella failed because of a combination of more than one failure type, only the leading failure is documented.

The identified types of failure are different for each strength class. In the lower strength class GL24h: 9 knot clusters, 3 clear wood and no FJ-failure were detected in the lowest lamella. In contrast, the upper strength class GL36h shows 5 knot clusters, 2 clear wood and 5 FJ-failure. The results corresponds to the investigations of former studies (see Chapter 2) and show that the influence of FJ is more important for higher strength classes.

Investigation of the deformation behaviour of the entire GLT beam

On four GLT beams the strains were analysed over the entire main stressed area (area where the bending moment is maximum). For the investigation a regular LED setup was chosen (Fig. 3.9). The vertical distance between the LEDs corresponded to the thickness of the lamella \( t_l \approx 40 \text{ mm} \).
3.2. GLT beams

Fig. 3.8: Failed GLT beam L25-IH-2

and the horizontal distance between the LEDs was 50 mm. Using this LED setup the entire beam height over a length of 3’300 mm is investigated; i.e. only the outmost 1’350 mm (on both sides) were outside the optical range of the measurement device.

A total amount of 408 LEDs were glued on each of these GLT beams. As a result of the large number of LEDs, a relatively small measurement frequency of about 2 Hz was chosen. When attaching the LEDs to the GLT beams, no effort was taken on the beam surface pattern; i.e. the LEDs were glued on the identified position, independently of knots, pitch pockets and other defects. Unfortunately, LEDs that are glued on knots or other irregularities are often unusable for a local strain estimation. As a result, local erroneous measurements occurred; see [Fink et al. (2013b)] for a detailed description.

The estimated strains (parallel and perpendicular to the beam axis) are illustrated in Fig. 3.11. Here, the black areas illustrate erroneous measurements. The estimated strains clearly indicating areas under tension (lower part) and compression (upper part). However, the results are
not suitable for an unambiguous description of the local strains. Nevertheless, the inspection shows that the measured strains on the surface were significantly larger when knots were located near the investigated surface. This indicates a non-linear strain distribution within the lamella’s cross-section.

Investigation of the deformation behaviour of selected sections

On the other 20 GLT beams, the strains were measured on two selected local areas. Thereby, the analysis was focused on the interaction between adjacent lamellas with different material properties; e.g. knot clusters or FJ, that are located in one of the two lowest lamellas. Attention was paid that the surface around the investigated knots or FJ was free of disturbances, to avoid erroneous measurements, as described before. Different constellations of knot clusters and FJ were identified and inspected.

Fig. 3.12 shows the LED arrangement in one inspected area, containing a knot cluster located in the second lowest lamella (marked with a cross). The corresponding axial strains are illustrated in Fig. 3.13. It is obvious that, within the area of the knot, significantly larger deformations occurred.

Consideration of all the investigated knot clusters and FJ, the following outcomes can be stated: (a) knot clusters are significantly weaker than FJ, (b) FJ are only marginally influencing
3.2. GLT beams

![Graphs of material properties](image1)

**Fig. 3.10:** Material properties of L25-R-1: (a) tKAR-value, (b) dynamic modulus of elasticity based on Eigenfrequency measurement, (c) measured modulus of elasticity (estimated with non-destructive tensile tests); the black lines illustrate FJ, the crosses illustrate the position of failure of the lowest two lamellas

![Graphs of strains](image2)

**Fig. 3.11:** Estimated strains of L25-R-1: (a) parallel to the beam axis, (b) strain perpendicular to the beam axis
the strain distribution, and (c) the strain distribution within the lamella cross-section of knot clusters seems to be non-linear.

3.2.3 Summary of the test results

In the second part of the experimental investigation the load-bearing capacity and the bending stiffness of altogether 24 GLT beams having well-known local material properties, were investigated. As a result, a database was produced, that can be used to validate numerical models to estimate the load-bearing capacity of GLT beams. This is applied on the numerical model developed within this thesis (see Chapter 6). However, the results can also be used for the validation of existing models.

Furthermore, the deformation behaviour of local weak sections (knot clusters and finger joint connections) in GLT beams in bending, as well as their influence on the bending failure was investigated. Such information is essential to get a better understanding about the load-bearing behaviour of GLT beams.
Chapter 4

Probabilistic representation of the variability of timber

Numerous models have been developed to describe the variability of the material properties in structural timber (Chapter 2). In the majority of those models, first the strength and/or stiffness properties of timber board sections are measured. Afterwards, probabilistic models are developed to describe those measured material properties. The drawback of such an approach is that the experimental investigations are very time consuming. Therefore, the amount of tested specimens is relatively small and thus the probabilistic models are affected to large statistical uncertainties. Furthermore, the developed models can only be used to describe the material properties of the investigated strength grades, dimensions, growing regions, and so on.

In this thesis, a model is developed where, instead of strength and stiffness properties, the so-called strength and stiffness related indicators (here $E_{\text{dyn,F}}$ and tKAR-value) are described probabilistically. Afterwards, the material properties can be estimated with the material model introduced in Chapter 5. One advantage of this approach is that strength and stiffness related indicators are easier to measure. Thus, the model can be extended for different strength grades or different growth regions with a comparable small amount of additional effort. A similar approach is presented in Blaß et al. (2008) to model the material properties of the lamellas within GLT. There, the dry density and a knot ratio are used as strength and stiffness related indicators.

The presented model, describes the growth characteristic of timber boards for two different strength grades (L25 and L40). It includes (a) the geometrical setup of timber boards – position of knot clusters, and (b) a hierarchical representation of two strength and stiffness related indicators – $E_{\text{dyn,F}}$ and tKAR. The model is developed based on the investigations described in Chapter 3.
Chapter 4. Probabilistic representation of the variability of timber

4.1 Geometrical setup

4.1.1 Definition of weak section

The positions of WS are extracted from the knot measurements. In order to do that, a definition of a WS is established. In the presented work, a threshold \( t_{\text{KAR}} = 0.1 \) is introduced; i.e. knot clusters with \( t_{\text{KAR}} \geq 0.1 \) are defined as WS, whereas knot clusters with \( t_{\text{KAR}} < 0.1 \) are neglected. Using this threshold, the majority of the knot clusters are identified as WS, especially knot clusters having a large \( t_{\text{KAR}} \)-value. In Fig. 4.1 the principle is illustrated on two timber boards. Choosing a threshold \( t_{\text{KAR}} = 0.3 \) (dashed line) only 1 WS / 2 WS are identified. It is obvious that if a smaller threshold is used, more WS are identified; e.g. \( t_{\text{KAR}} = 0.2 \) (dashed-dotted line) leads to 3 WS / 4 WS, \( t_{\text{KAR}} = 0.1 \) (solid line) to 7 WS / 6 WS, and \( t_{\text{KAR}} = 0.05 \) (dotted line) to 7 WS / 7 WS.

In the presented work (\( t_{\text{KAR}} = 0.1 \)), altogether 2'870 WS (L25: 1'416 WS, L40: 1'454 WS) are identified. As already described in Chapter 3, the length of a WS is assumed to be constant \( l_{\text{WS}} = 150 \text{ mm} \).

4.1.2 Distance between weak sections

The distance between WS (denoted \( d \)) is defined as the distance between the mid-points of two adjacent WS. In a growing tree, the appearance of knot clusters in the longitudinal direction of the trunk might be represented by a Poisson process and therefore the distances between knot clusters would be Exponential distributed. When sawing out timber boards from a tree not every knot cluster might appear in every particular timber board. The distance between knot clusters that appears in the boards might be best represented by the Gamma distribution, which corresponds to the distribution of the distance between the \( i^{\text{th}} \) and \( i + k^{\text{th}} \) occurrence of a Poisson process. The distribution is generalized when \( k \) is not an integer.
4.1 Geometrical setup

According to the definition, a WS has a constant length \( l_{WS} = 150 \text{ mm} \). Thus the minimal distance between two adjacent WS is \( d_{\text{min}} = 150 \text{ mm} \). As a result, a shifted Gamma distribution is used to describe \( d \) (Fig. 4.2):

\[
f(d) = \frac{\nu(d - l_{WS})^{k-1}}{\Gamma(k)} e^{-\nu(d - l_{WS})} + l_{WS} \quad \text{for} \quad l_{WS} \leq d \leq \infty
\] (4.1)

The estimated parameters and their COV are summarised in Tab. 4.1. The parameters correspond to expected values \( E(d_{L_{25}}) = 537 \text{ mm} \) and \( E(d_{L_{40}}) = 521 \text{ mm} \), and standard deviations \( \sigma(d_{L_{25}}) = 253 \text{ mm} \) and \( \sigma(d_{L_{40}}) = 240 \text{ mm} \). Thus, only a marginal difference between the two strength grades is identified: the difference between the expected values is about 16 mm. Consequently it will not be distinguished between the two strength grades when modelling \( d \): Taking into account both strength grades: \( E(d) = 529 \text{ mm} \) and \( \sigma(d) = 246 \text{ mm} \).

The small coefficients of variation of the two parameters \( k \) and \( \nu \) indicate that the number of specimens is apparently large enough for an efficient parameter estimation. However, the estimated parameters are highly related to the definition of WS (see e.g. Isaksson 1999, Fink & Kohler 2012). As mentioned above, a WS is defined as a section with \( t_{\text{KAR}} \geq 0.1 \). When choosing a lower \( t_{\text{KAR}} \)-value, more WS would be detected and thus the distance between them would decrease; e.g. \( t_{\text{KAR}} = 0.05 \) leads to \( E(d) = 485 \text{ mm} \) and \( \sigma(d) = 215 \text{ mm} \).

The estimated values can be compared with former studies; e.g. Isaksson (1999) and Colling & Dinort (1987). Isaksson also described \( d \) with the Gamma distribution and identified an expected value of 494 mm with a standard deviation of 310 mm. The results of this study are similar, although the timber has different origins (the timber of the present study is grown in the southern part of Germany, whereby the timber in the study of Isaksson is grown in southern Sweden), and a different definition of the WS (Isaksson defined a WS as a section with \( t_{\text{KAR}} \geq 0.5 \cdot t_{\text{KAR}}_{\text{max}} \)). Colling & Dinort investigated \( d \) on timber boards from Germany, Austria and Scandinavia. There, a mean distance between the WS of 450–500 mm was found. Furthermore, Colling & Dinort also did not identify significant differences of \( d \) between different strength grades.
As mentioned above, the distance between knot clusters within one particular tree is relatively regular. Thus it seems likely to model $d$ hierarchically; i.e. with a mean $d$ of a timber board and a variability of $d$ within the timber board. However, the investigation shows that the mean $d$ of a timber board is not regular enough for this approach. This might be a result of the sawing process combined with the definition of a WS (not every knot cluster within the tree might appear as a WS in every particular timber board). A further explanation might be the limited amount of data.

### 4.2 Strength and stiffness related indicators

#### 4.2.1 tKAR-value

According to definition, the tKAR-value of every timber board section has to be within the interval $[0, 1]$. Through the grading process, an upper limit $t\text{KAR}_{\text{limit}}$ could be introduced; e.g. visual graded timber is usually regulated through upper limits for permissible knot dimensions (e.g. DIN 4074-1 (2008), SIA 265-1 (2009)). It is obvious that $t\text{KAR}_{\text{limit}}$ is smaller for higher strength grades. However, it has to be considered that in reality the grading process is not perfect; i.e. there is a certain probability that the tKAR-value exceeds the defined threshold. Therefore, in the present study no upper limit through the grading process is introduced: $t\text{KAR}_{\text{limit}} = 1$.

To describe the tKAR-value of a WS, the Lognormal distribution with a truncated upper tail ($t\text{KAR}_{\text{limit}} = 1$) is used: $f^\ast_X(x)$ in Eq. (4.2). Here, $f_X(x)$ is the probability density function of the Lognormal distribution, and $F_X(t\text{KAR}_{\text{limit}})$ is the cumulative distribution function up to the upper limit, which is the area of the probability density function for the interval $[0, t\text{KAR}_{\text{limit}}]$. The truncated Lognormal distribution seems to be suitable, regarding to (a) the flat tail within the area of small tKAR-values (knot clusters with $t\text{KAR} < 0.1$ are not considered as WS), (b) the flat tail within the area of large tKAR-values (according to the grading criteria the probability of large tKAR-values is rather small), and (c) the truncated upper tail $t\text{KAR}_{\text{limit}}$ (Fig. 4.3). At this point it has to be mentioned that the area of the truncated tail of the density function for the strength grades L25 and L40 are $f_X(x|X \geq 1) = 1 - F_X(t\text{KAR}_{\text{limit}} = 1) = 4 \cdot 10^{-5}$ and $8 \cdot 10^{-8}$, respectively. Thus, its influence is rather small. However, it is a threshold that can not exceed by definition.

$$f^\ast_X(x) = \frac{f_X(x)}{F_X(t\text{KAR}_{\text{limit}})}$$  \hspace{1cm} (4.2)
4.2. Strength and stiffness related indicators

Assuming a truncated Lognormal distribution with \( t_{\text{KAR}}_{\text{limit}} = 1 \), the expected values of \( t_{\text{KAR}} \) are \( E(t_{\text{KAR}}_{L25}) = 0.240 \) and \( E(t_{\text{KAR}}_{L40}) = 0.192 \), with standard deviations \( \sigma(t_{\text{KAR}}_{L25}) = 0.095 \) and \( \sigma(t_{\text{KAR}}_{L40}) = 0.064 \). Thus the \( t_{\text{KAR}} \)-values of WS within timber boards of the lower strength grade are significantly larger (as it was foreseeable) and show a larger variation. Compared to former studies, here the mean \( t_{\text{KAR}} \) is relatively large; e.g. Colling & Dinort (1987) identified a mean \( t_{\text{KAR}} \) of 0.15–0.20 (considering only knot clusters with \( t_{\text{KAR}} \geq 0.1 \)). The reason for the increase might be the different grading criteria of the two studies. Further it has to be considered that the time between the studies is more than 25 years and thus the requirements to the forest and the sawmill industry have changed. This might have an influence on the quality of sawn timber (Blaß et al. 2008).

The \( t_{\text{KAR}} \)-value is described by a hierarchical model with two hierarchical levels (Kersken-Bradley & Rackwitz 1991; Köhler & Faber 2004); namely the meso- and micro-scale. The meso-scale describes the variability of a single timber board within a sample of timber boards. The micro-scale describes the variability within one timber board. The hierarchical model is given in Eq. (4.3). Here, \( \tau \) is introduced to describe the meso-scale variability and \( \epsilon \) to describe the micro-scale variability. To consider \( t_{\text{KAR}}_{\text{limit}} \), the combined realization of large \( \tau \) and large \( \epsilon \) is prevented.

The parameters of Eq. (4.3) and their uncertainties, expressed through the COV, are estimated using maximum likelihood; Eq. (2.15)–(2.18). The results are summarised in Tab. 4.2. According to the large amount of WS (2'870), the statistical uncertainties of the estimated parameters are rather small, especially for \( \mu \) and \( \epsilon \). The parameter \( \tau \) is estimated on a smaller number of data (200 specimens per strength grade), thus the uncertainties are larger.

For the estimation of the \( t_{\text{KAR}} \)-value, the natural growth characteristic is considered; i.e. the \( t_{\text{KAR}} \)-value of each particular knot cluster is measured. To clarify the importance of considering the natural growth characteristic, the results are compared with those of an alternative approach: timber boards are subdivided into sections with constant length of \( \Delta l = 150 \text{ mm} \). Afterwards, for each particular section the \( t_{\text{KAR}} \)-value is measured. The differences between the two approaches

![Fig. 4.3: \( t_{\text{KAR}} \)-value: (left) strength grade L25, (right) strength grade L40](image-url)
\[ t\text{KAR}_{ij} = \exp(\mu + \tau_i + \epsilon_{ij}) \]

with \( \tau_i + \epsilon_{ij} \leq \ln(t\text{KAR}_{\text{limit}}) - \mu \) \hspace{1cm} (4.3)

where

- \( t\text{KAR}_{ij} \): the tKAR of the WS \( j \) in a board \( i \).
- \( \mu \): the logarithmic mean tKAR of all WS within a sample of boards. \( \mu \) is considered to be deterministic.
- \( \tau_i \): the difference between the logarithmic mean tKAR of all WS within one board \( i \) and \( \mu \). \( \tau_i \) is represented by a Normal distributed random variable \( \tau_i \sim N(0, \sigma_\tau) \).
- \( \epsilon_{ij} \): the difference between WS \( j \) in a board \( i \) and the logarithmic mean tKAR of all WS within one board \( i \) (\( \mu + \tau_i \)). \( \epsilon_{ij} \) is represented by a Normal distributed random variable \( \epsilon_{ij} \sim N(0, \sigma_\epsilon) \).

**Tab. 4.2:** Estimated parameters to predict tKAR: Expected value, COV in brackets

<table>
<thead>
<tr>
<th></th>
<th>L25</th>
<th>L40</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu ) (COV)</td>
<td>-1.50 (0.0067)</td>
<td>-1.70 (0.0050)</td>
</tr>
<tr>
<td>( \sigma_\tau ) (COV)</td>
<td>0.184 (0.050)</td>
<td>0.171 (0.050)</td>
</tr>
<tr>
<td>( \sigma_\epsilon ) (COV)</td>
<td>0.335 (0.019)</td>
<td>0.281 (0.019)</td>
</tr>
</tbody>
</table>

are illustrated in Fig. 4.4. Through the subdivision of timber boards into sections with constant length, knot clusters which are located between two sections are subdivided. Accordingly, both sections contain one part of the knot cluster, instead of one section containing the entire knot cluster; e.g. the 2\textsuperscript{nd} and 3\textsuperscript{rd} knot cluster in Fig. 4.4. This is of particular relevance in the case of large knot clusters, where the probability that the knot clusters are located not only in a single timber board section is relatively large. Through the subdivision into elements with constant length, the following irregularities occur: (a) the number of sections containing knots increases, (b) the mean tKAR decreases, and (c) the upper part of the estimated probability density function is significantly underestimated.

A direct comparison between the two approaches is illustrated in Fig. 4.5. Using the approach with the constant section length, the number of sections containing knots is significantly larger: 3’970 instead of 2’870. This is a result of (a) the subdivision of knot clusters into two sections containing knots, and (b) that small knot clusters with tKAR ≤ 0.1 are also considered.
4.2. STRENGTH AND STIFFNESS RELATED INDICATORS

Fig. 4.4: Schematic illustration of the differences between two approaches for the tKAR calculation

Fig. 4.5: Comparison between two approaches for the tKAR calculation: (left) strength grade L25, (right) strength grade L40

4.2.2 Dynamic modulus of elasticity

The dynamic modulus of elasticity based on eigenfrequency \( E_{\text{dyn,F}} \) is also described by a hierarchical model. \( E_{\text{dyn,F}} \) represents a mean value over the entire timber board. Thereby, only the first hierarchical level (meso-scale), which describes the variability of a single timber board within a sample of timber boards, is used (Eq. 4.4). \( E_{\text{dyn,F}} \) is assumed to be Lognormal distributed (e.g. [JCSE 2006] Köhler et al. 2007).

The results are summarised in Tab. 4.3. The estimated parameters correspond to expected values \( E(\text{dyn,F,L25}) = 11'630 \text{ MPa} \) and \( E(\text{dyn,F,L40}) = 15'980 \text{ MPa} \), and standard deviations \( \sigma(\text{dyn,F,L25}) = 1'450 \text{ MPa} \) and \( \sigma(\text{dyn,F,L40}) = 1'400 \text{ MPa} \). Even when the expected value of the upper strength grade is significantly larger (~35%), it seems that the standard deviation is almost unaffected.
\[ E_{\text{dyn,F},i} = \exp(\mu + \tau_i) \]  

where

- \( E_{\text{dyn,F},i} \) is the \( E_{\text{dyn,F}} \) of the board \( i \).
- \( E_{\text{dyn,F},i} \) is a Lognormal random variable
- \( \mu \) is the logarithmic mean of all \( E_{\text{dyn,F}} \) within a sample of boards. \( \mu \) is considered to be deterministic
- \( \tau_i \) is the difference between the logarithmic mean of \( E_{\text{dyn,F}} \) in the board \( i \) and \( \mu \). \( \tau_i \) is represented by a Normal distributed random variable \( \tau_i \sim N(0, \sigma_\tau) \)

![Fig. 4.6: Dynamic modulus of elasticity \( E_{\text{dyn,F}} \): (left) strength grade L25, (right) strength grade L40](image)

**Tab. 4.3: Estimated parameters to predict \( E_{\text{dyn,F}} \): Expected value, COV in brackets**

<table>
<thead>
<tr>
<th></th>
<th>L25</th>
<th>L40</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu ) (COV)</td>
<td>9.35 (0.0009)</td>
<td>9.68 (0.0006)</td>
</tr>
<tr>
<td>( \sigma_\tau ) (COV)</td>
<td>0.124 (0.050)</td>
<td>0.0877 (0.050)</td>
</tr>
</tbody>
</table>

### 4.2.3 Correlation between the parameters

In addition to the three parameters \((d, E_{\text{dyn,F}} \text{ and } \text{tKAR})\), their correlations are analysed with particular focus on the correlations between timber boards. That includes correlation between the following three parameters:

- mean tKAR in a timber board (referred to as tKAR\(_i\))
- dynamic modulus of elasticity based on eigenfrequency measurement \( E_{\text{dyn,F}} \)
- mean distance between the WS within one board (referred to as \( d_i \))

The estimated correlations are summarised in Tab. 4.4. All identified correlations are rather small, except \( \rho(\text{tKAR}_i, E_{\text{dyn,F}}) \) – considering both strength grades. This is plausible through the
4.3 Alternative approach – direct representation of the stiffness properties

fact that timber boards of higher strength grades have a larger mean stiffness and less and/or smaller knots. However, within one strength grade the correlations are small $|\rho| \leq 0.182$.

The low correlations between $d_i$ and the other two parameters ($t\text{KAR}_i$ and $E_{\text{dy}n,F}$) are also plausible, as a result that the distance between WS is only marginal influenced by the strength grade (Chapter 4.1.2). At this point it has to be mentioned that the low correlations might be a result of the sample size (200 timber boards per strength grade). However, due to the low correlations they are not considered within the model.

**Tab. 4.4: Correlation between the estimated parameters**

<table>
<thead>
<tr>
<th>Correlations</th>
<th>L25</th>
<th>L40</th>
<th>L25 &amp; L40</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho(t\text{KAR}<em>i, E</em>{\text{dy}n,F})$</td>
<td>-0.182</td>
<td>-0.085</td>
<td>-0.529</td>
</tr>
<tr>
<td>$\rho(t\text{KAR}_i, d_i)$</td>
<td>0.221</td>
<td>-0.159</td>
<td>-0.029</td>
</tr>
<tr>
<td>$\rho(E_{\text{dy}n,F}, d_i)$</td>
<td>-0.093</td>
<td>-0.062</td>
<td>-0.019</td>
</tr>
</tbody>
</table>

4.3 Alternative approach – direct representation of the stiffness properties

As an alternative to the approach introduced above, the material properties can be directly described. The variability of the tensile stiffness is investigated on a small sub-sample of 30 randomly selected timber boards of strength grade L25 (Fink & Kohler 2011). The approach is similar to the one used for the strength and stiffness related indicators. Instead of $E_{\text{dy}n,F}$ and $t\text{KAR}$ the measured stiffness properties of WS and CWS are described. For both, WS and CWS, two hierarchical levels are used to consider the within and the between-member variabilities. In addition, the correlation between the stiffness properties of WS and CWS is investigated; expressed through the correlation between the logarithmic mean of all WS within one board $i$ and logarithmic mean of all CWS within the same board (Fig. 4.7): $\rho(\tau_i,_{WS}, \tau_i,_{CWS}) = 0.787$.

The probabilistic model is given in Eq. (4.5), where the sections $j = \{1, 3, 5, \cdots, n\}$ are considered to be WS and the sections $j = \{2, 4, 6, \cdots, n - 1\}$ are considered to be CWS ($n$ is the number of sections within one timber board). The strong correlation between the mean stiffness properties of WS and CWS within one board is considered by the correlation of the variables $\tau_i,_{WS}$ and $\tau_i,_{CWS}$. The parameters are estimated based on the measured stiffness properties of 149 WS and 119 CWS, and are summarised in Tab. 4.5.
\[ E_{ij} = \begin{cases} \exp(\mu_{WS} + \tau_{i,WS} + \epsilon_{ij,WS}) & \text{for } j = \{1, 3, 5, \ldots, n\} \\ \exp(\mu_{CWS} + \tau_{i,CWS} + \epsilon_{ij,CWS}) & \text{for } j = \{2, 4, 6, \ldots, n - 1\} \end{cases} \] (4.5)

where

- \( E_{ij} \) is the stiffness of the WS \( j \) in a board \( i \). \( E_{ij} \) is a Lognormal distributed random variable.
- \( \mu_{WS}, \mu_{CWS} \) is the logarithmic mean of all WS/CWS within a sample of boards. \( \mu \) is considered to be deterministic.
- \( \tau_{i,WS}, \tau_{i,CWS} \) is the difference between the logarithmic mean of all WS/CWS within one board \( i \) and \( \mu \). \( \tau_{i} \) is represented by a Normal distributed random variable \( \tau_{i} \sim N(0, \sigma_{\tau}) \).
- \( \epsilon_{ij,WS}, \epsilon_{ij,CWS} \) is the difference between WS/CWS \( j \) in a board \( i \) and the logarithmic mean of all WS/CWS within one board \( i \) (\( \mu + \tau_{i} \)). \( \epsilon_{ij} \) is represented by a normal distributed random variable \( \epsilon_{ij} \sim N(0, \sigma_{\epsilon}) \).

<table>
<thead>
<tr>
<th></th>
<th>WS</th>
<th>CWS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu ) (COV)</td>
<td>9.15 (0.0017)</td>
<td>9.41 (0.0016)</td>
</tr>
<tr>
<td>( \sigma_{\tau} ) (COV)</td>
<td>0.138 (0.131)</td>
<td>0.144 (0.134)</td>
</tr>
<tr>
<td>( \sigma_{\epsilon} ) (COV)</td>
<td>0.120 (0.058)</td>
<td>0.0648 (0.068)</td>
</tr>
<tr>
<td>( \rho(\tau_{i,WS}, \tau_{i,CWS}) )</td>
<td>0.787</td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 4.7:** Correlation between \( \tau_{i,WS} \) and \( \tau_{i,CWS} \)
4.4 Model summary

In this chapter, a probabilistic model for the representation of strength and stiffness related indicators of two strength grades (L25 and L40) was developed. The specific characteristic of this model is that the natural growth characteristic of timber is considered; i.e. the position and the characteristics of knot clusters can be simulated.

Three parameters were considered \( (d, \text{tKAR} \text{ and } E_{\text{dyn,F}}) \): \( d \) describes the distance between knot clusters, tKAR describes the characteristics of knot clusters, and \( E_{\text{dyn,F}} \) describes the mean material properties of the timber boards. For all three parameters the most suitable distribution functions were selected, in a way that the basic population of each indicator is best represented. To consider the within-member correlation, the tKAR-value is modelled hierarchically. The model is summarised in Tab. 4.6. No unambiguous correlation could be detected between the three parameters.

At this point it has to be mentioned that in this thesis only probabilistic models of two strength grades (L25 and L40) are developed. All timber boards were (a) graded with the same grading device, (b) grown in the same region, and (c) have the same dimensions. As a result, the probabilistic description might be different for timber boards that are (a) visually graded or graded with another device, (b) from different growth regions, or (c) have different dimensions.

**Tab. 4.6:** Compilation of the model for the probabilistic representation of timber: Expected value, COV in brackets

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>Strength grade</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>L25</td>
</tr>
<tr>
<td>( d )</td>
<td>Eq.(4.1)</td>
<td>( k ) (COV)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \nu ) (COV)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \rho(k, \nu) )</td>
</tr>
<tr>
<td>tKAR</td>
<td>Eq.(4.3)</td>
<td>( \mu ) (COV)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \sigma_{\tau} ) (COV)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \sigma_e ) (COV)</td>
</tr>
<tr>
<td>( E_{\text{dyn,F}} )</td>
<td>Eq.(4.4)</td>
<td>( \mu ) (COV)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \sigma_{\tau} ) (COV)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Chapter 4. Probabilistic representation of the variability of timber
Chapter 5

Prediction of material properties

In this chapter a model to predict the strength and stiffness properties of timber boards, based on strength and stiffness related indicators, is developed. First, the interaction between different indicators and the measured material properties is investigated. Afterwards, the most efficient indicators are considered for the development of the material model. Finally, these material models are validated. This study is based on the timber boards described in Chapter 3.

5.1 Investigation of strength & stiffness related indicators

The interaction between different strength and stiffness related indicators and the measured material properties is investigated. This includes indicators for predicting the mean properties of a timber board (referred to as global indicators) as well as indicators to predict the within-member variability (referred to as knot indicators).

5.1.1 Global indicators

The variability between timber boards or, rather, the variability between the defect-free timber is related to different growth and sawing characteristics. Typical non-destructive indicators for the mean material properties are the eigenfrequency (Görlacher 1984, 1990b), the ultrasonic runtime (Steiger 1996), and the density.

In the presented work, these three indicators were measured on all investigated timber boards. In addition, the influence of the position of the timber board within the trunk of the tree is investigated. Therefore, two additional indicators are identified: distance to the pith $d_p$, and angle of the annual rings $\alpha$. For timber boards for which the pith is within the cross-sectional area of the timber board, its position is measured. Otherwise, the position is estimated based on the knot measurement. The position of the pith is assumed to be the intersection of all knot-axes in a timber board; Fig. 5.2 (d, e) – see Fink & Kohler (2012) for a detailed description. Based on the position of the pith, both indicators ($d_p$ and $\alpha$) are calculated. Indicator $\alpha$ is described in degrees within the interval $[0, 90]$. 
The interrelation between the five measured parameters to the measured mean stiffness $\overline{E}$ and the measured tensile strength $f_t$ are estimated based on 200 and 443 timber boards, respectively. The results are summarised in Tab. 5.1. It seems that the three common indicators ($E_{\text{dyn,F}}$, $E_{\text{dyn,US}}$ and $\rho$) are quite efficient to predict the material properties, in particular $E_{\text{dyn,US}}$, and $E_{\text{dyn,F}}$ to predict $E$. Whereas, both indicators describing the position of the timber board within the trunk of the tree have no or only minor correlation to $E$ and $f_t$.

A direct comparison between the two most efficient indicators shows that $E_{\text{dyn,F}}$ is slightly more appropriate for the prediction of the material properties; see also Fig. 5.1. Further the analysis shows that $E_{\text{dyn,US}}$ is about 2'700 MPa larger than $E_{\text{dyn,F}}$.

The correlation between the five global indicators is analysed on both samples (643 timber boards). The results are summarised in Tab. 5.2. Between the three common indicators ($E_{\text{dyn,F}}$, $E_{\text{dyn,US}}$ and $\rho$) strong correlations are detected. In contrast, only minor correlations to and between the other two indicators ($d_p$ and $\alpha$) are identified. Because of the large correlation between $E_{\text{dyn,F}}$, $E_{\text{dyn,US}}$ and $\rho$, only one of them is essential for a model to predict the material properties. A combination of more than one indicator can only led to a minor improvement.

5.1.2 Knot indicators

The within-member variability of the stiffness properties is highly related to knots and their arrangement. In the following paragraph, the influence of knot indicators on the strength and stiffness properties is analysed. In order to do that, at first, different knot indicators are identified. All of them are analysed for a section length of 150 mm. Every knot in the timber board is assumed to have cylindrical shape. Next, eight selected knot indicators are introduced; for other
5.1. Investigation of strength & stiffness related indicators

Fig. 5.1: Correlation between $E_{\text{dyn,US}}$ respective $E_{\text{dyn,F}}$ and mean tensile stiffness $\overline{E}$ (left), and tensile capacity $f_t$ (right).

In addition to the knot indicators described above, also the type and the position of the knots are investigated. In order to investigate the influence of the type of knots, they were categorized into four groups: side knots, edge knots, splay knots and narrow side knots (Fig. 5.3). The projected knot area is calculated separately for each type of knot. Afterwards, the optimal combination of the four groups was investigated using linear regression. As a result, only a minor improvement is detected compared to the tKAR-value. This indicates that the influence of the type of knot on the strength and stiffness properties of WS is only marginal.
Chapter 5. Prediction of material properties

Fig. 5.2: (a) projected knot area (b) projected area with local deviation of the grain orientation (c) knot diameter (d) distance to the pith (e) angle of the annual rings

Fig. 5.3: Type of knots, in accordance with DIN 4047-1 (2008), (adapted from Glos & Richter 2002)

Following the same principle, also the position of the knots was investigated. For this purpose, the cross-section of the timber board is subdivided into three parts: inner, middle, and outer (Fig. 5.4). The projected knot areas are calculated separately and their influence was analysed. Again, only a minor improvement is detected, compared to the tKAR-value. This indicates that also the knot position is only marginal influencing the tensile strength and stiffness properties of WS.

Correlation between knot indicators and tensile stiffness

In the following paragraph, the influence of the introduced knot indicators on the tensile stiffness of WS ($E_{WS}$) as well as on the local stiffness reduction is analysed. The differences to the mean stiffness of the timber board ($\bar{E} - E_{WS}$) and the differences to the mean stiffness of the CWS within the timber board ($E_{CWS} - E_{WS}$) are investigated. This analysis used the measured stiffness properties of altogether 864 WS. The results are summarised in Tab. 5.3. The indicators describing the knot area projected on the cross-sectional area (B)–(D) are the most efficient for the prediction of the tensile stiffness. Nevertheless, also the indicators that are based on the sum of all visual knot diameters (G)–(H) show large correlations. On the other hand the maximal knot diameter (E)–(F) has a significantly lower influence on the stiffness properties, similarly

Fig. 5.4: Projected knot area subdivided into three parts
5.1. Investigation of strength & stiffness related indicators

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{WS}$</td>
<td>-0.229</td>
<td>-0.492</td>
<td>-0.488</td>
<td>-0.462</td>
<td>-0.352</td>
<td>-0.318</td>
<td>-0.451</td>
<td>-0.430</td>
</tr>
<tr>
<td>$\overline{E} - E_{WS}$</td>
<td>0.288</td>
<td>0.549</td>
<td>0.539</td>
<td>0.547</td>
<td>0.338</td>
<td>0.335</td>
<td>0.478</td>
<td>0.487</td>
</tr>
<tr>
<td>$E_{CWS} - E_{WS}$</td>
<td>0.253</td>
<td>0.551</td>
<td>0.540</td>
<td>0.550</td>
<td>0.361</td>
<td>0.366</td>
<td>0.467</td>
<td>0.472</td>
</tr>
<tr>
<td>$f_t$</td>
<td>-0.295</td>
<td>-0.585</td>
<td>-0.574</td>
<td>-0.558</td>
<td>-0.152</td>
<td>-0.383</td>
<td>-0.423</td>
<td>-0.538</td>
</tr>
</tbody>
</table>

Tab. 5.3: Correlation between $E_{WS}$, $\overline{E} - E_{WS}$, $E_{CWS} - E_{WS}$, $f_t$ and knot indicators

To the number of knots within a WS. Another outcome of the investigation is that, in general, knot parameters are more efficient for the prediction of the local stiffness reduction than for the direct prediction of the local stiffness properties of WS.

Correlation between knot indicators and tensile capacity

Just as for the variation of stiffness properties, the variation of strength properties within one timber board is highly related to knots and their arrangement. The influence of the same knot indicators, as introduced above, is analysed. All knot indicators are calculated for every WS ($t_{KAR} \geq 0.1$) within the testing range of the boards (altogether 2'577 WS). For the analysis it is assumed that the tensile capacity of the weakest section (here the knot cluster with the largest tKAR-value) corresponds to the tensile capacity of the entire timber board.

The calculated correlations, summarised in Tab. 5.3, are similar to the correlations identified for $E_{WS}$, even slightly better. That indicates a stronger influence of knots on the local strength reduction compared to the stiffness reduction.

Correlation between different knot indicators

In addition to the correlation between the knot indicators and the material properties, also their correlation is investigated. The knot indicators of both board samples are taken into account (altogether 3’441 WS). The results, summarised in Tab. 5.4, show that the most efficient indicators (B)–(D) and (G)–(H) are strongly correlated, whereas the correlation to the other three indicators is significantly lower. As a result, a combination of different knot indicators only leads to a minor improvement. This is explained in detail in Fink et al. (2011).

An additional outcome is that indicators (B) and (C) are correlated almost perfectly; i.e. $\rho(A, B) \approx 1$. This is because only a very small proportion of the projected knot areas over-lap each other. Consequently, it might be reasonable to neglect the reduction of the over-lapping part of the knot areas in the definition of the tKAR-value.
5.1.3 Summary

The investigation of the strength and stiffness related indicators clearly shows that for an efficient prediction of the material properties of knot clusters at least two indicators are necessary: one to describe the mean material properties (respectively the material properties of defect-free timber) and one to describe the local strength and stiffness reduction through knots. Due to the large correlation between the global indicators, a combination of more than one global indicator would lead only to a minor improvement. The same applies for the knot indicators. As a result of the wide agreement, it is suggested to use $E_{\text{dyn,F}}$ as the global indicator and tKAR-value as the knot indicator.
5.2 Material model

With regard to the between and within-member variability of the material properties, it is obvious that an efficient model for the prediction of the local strength and stiffness properties should include at least two indicators: (a) one that describes the mean material properties of the entire timber board, in order to consider the between-member variability of the mean material properties and (b) one that describes the local strength and stiffness reduction through the occurrence of knots and knot clusters, in order to consider the within-member variability.

In the presented material model, $E_{\text{dyn,F}}$ and $t_{\text{KAR}}$ are chosen to consider the between and within-member variability, respectively.

The material model is developed based on the experimental investigations described in Chapter 3, i.e. the material properties were measured on full-scale timber boards having a testing length $> 3'300$ mm. The main focus lies on the prediction of the tensile strength and the tensile stiffness of knot clusters. However, the model can also be used to predict the material properties of the defect-free timber board sections between knot clusters.

The material properties are described by linear regression models (Eq. 5.1), where $Y$ is the predicted strength/stiffness, $\beta_i$ are the regression coefficients, $X_i$ are the input variables, and $\varepsilon$ is the error term. The input variable $X_1$ stands for $E_{\text{dyn,F}}$ and the input variable $X_2$ stands for $t_{\text{KAR}}$.

\[
\ln(Y) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon \tag{5.1}
\]

The parameters of the regression model and their uncertainties are estimated using the maximum likelihood method. Based on the assumption of a Normal distributed error term $\varepsilon$, the parameter of the regression model $\beta_i$ and the standard deviation of the error term $\sigma_\varepsilon$ can be calculated as follows, where $n$ denotes the number of data and $k$ denotes the number of regression coefficients:

\[
\beta = (\hat{X}^T \hat{X})^{-1} \hat{X}^T \hat{y} \tag{5.2}
\]

\[
\sigma_\varepsilon^2 = \sqrt{\frac{\sum_{i=1}^{n} \varepsilon_i^2}{n - k}} \tag{5.3}
\]

5.2.1 Model to predict the tensile stiffness

In this chapter, two different stiffness models are developed. The first one can be used to predict the stiffness of defect-free timber, whereas the second one can be used to predict the local stiffness properties of a knot cluster.

Model to predict the stiffness of defect-free timber

The first stiffness model is developed in order to predict the mean stiffness properties of defect-free timber within one board $E_{\text{CWS}}$. Here $E_{\text{CWS}}$ is calculated with the stiffness of all measured

\footnote{the major part of the material model is already published in Fink & Kohler (2014)}
Chapter 5. Prediction of material properties

Tab. 5.5: Parameters for the model to predict $E_{\text{CWS}}, \overline{E}$ and $E_{\text{WS}}$

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>Expected value</th>
<th>COV</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta_0$</td>
<td>8.52</td>
<td>0.0026</td>
<td>$\rho(\beta_0, \beta_1) = -0.954$</td>
</tr>
<tr>
<td>$E_{\text{CWS}}$</td>
<td>$\beta_1$</td>
<td>$7.12 \cdot 10^{-5}$</td>
<td>0.023</td>
<td>$\rho(\beta_0, \sigma_\varepsilon) \approx 0$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_\varepsilon$</td>
<td>$5.47 \cdot 10^{-2}$</td>
<td>0.052</td>
<td>$\rho(\beta_1, \sigma_\varepsilon) \approx 0$</td>
</tr>
<tr>
<td></td>
<td>$\beta_0$</td>
<td>8.42</td>
<td>0.0021</td>
<td>$\rho(\beta_0, \beta_1) = -0.968$</td>
</tr>
<tr>
<td>$\overline{E}$</td>
<td>$\beta_1$</td>
<td>$7.41 \cdot 10^{-5}$</td>
<td>0.017</td>
<td>$\rho(\beta_0, \sigma_\varepsilon) \approx 0$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_\varepsilon$</td>
<td>$4.40 \cdot 10^{-2}$</td>
<td>0.052</td>
<td>$\rho(\beta_1, \sigma_\varepsilon) \approx 0$</td>
</tr>
<tr>
<td></td>
<td>$\beta_0$</td>
<td>8.41</td>
<td>0.0027</td>
<td>$\rho(\beta_0, \beta_1) = -0.922$</td>
</tr>
<tr>
<td>$E_{\text{WS}}$</td>
<td>$\beta_1$</td>
<td>$7.69 \cdot 10^{-5}$</td>
<td>0.019</td>
<td>$\rho(\beta_0, \beta_2) = -0.564$</td>
</tr>
<tr>
<td></td>
<td>$\beta_2$</td>
<td>$-9.02 \cdot 10^{-1}$</td>
<td>0.040</td>
<td>$\rho(\beta_1, \beta_2) = 0.234$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_\varepsilon$</td>
<td>$1.00 \cdot 10^{-1}$</td>
<td>0.024</td>
<td>$\rho(\beta_1, \sigma_\varepsilon) \approx 0$</td>
</tr>
</tbody>
</table>

Fig. 5.5: Model to predict (left) $E_{\text{CWS}}$, (right) $\overline{E}$

CWS within one board, in accordance with Hook’s law for serial springs (Eq. 5.4). In addition, a model for the prediction of the mean tensile stiffness of the entire timber board $\overline{E}$ is developed. For both models, only the first indicator $E_{\text{dyn,F}}$ is taken into account.

$$\frac{\sum_{i=1}^{n} l_i}{E_{\text{CWS}}} = \frac{1}{\sum_{i=1}^{n} E_{i,\text{CWS}}}$$  \hspace{1cm} (5.4)

With both models, large correlations $\rho \approx 0.96$ between the predicted and the measured stiffness properties are identified (Fig. 5.5). The estimated regression coefficients, the standard deviation of the error term, and their coefficients of variation are summarised in Tab. 5.5. A comparison between the two regression models shows that $E_{\text{CWS}}$ is in general about 700 MPa larger than $\overline{E}$. The differences between $E_{\text{CWS}}$ and $\overline{E}$ are almost constant for all timber boards.
Model to predict the stiffness of knot clusters

In the following paragraphs, a model is developed in order to predict the tensile stiffness of each particular WS ($E_{WS}$). Therefore, the measured stiffness properties of altogether 864 WS are taken into account. As described above, the model contains two parameters ($E_{dyn,F}$ and tKAR). The estimated regression coefficients, the standard deviation of the error term and their coefficients of variations are summarised in Tab. 5.5. Using this model, a rather large correlation $\rho = 0.912$ between the measured and the predicted stiffness can be identified (Fig. 5.6).

In order to control the outcome, a cross validation with four randomly selected sub-samples is performed [Hastie et al. 2001]. Thereby, the model is developed based on three sub-samples and the results are validated using the fourth sub-sample. This is done four times for every sub-sample. The estimated model parameters are only slightly different and the correlation between the measured and the estimated stiffness properties are $0.90 < \rho < 0.92$.

The presented model is developed in order to predict the tensile strength of WS. If the model is used to predict the stiffness of the CWS (using tKAR = 0), they are slightly underestimated. The difference of the measured and the predicted $E_{CWS}$ is about 3%. However, the measured and the predicted stiffness still show a large correlation $\rho = 0.953$. Thus, the model can also be applied to the prediction of the $E_{CWS}$, when considering the underestimation.

In all the here described stiffness models, a very high correlation coefficient has been identified. This might be partly influenced by the investigated timber boards, which are two sub-samples of two different strength grades. However, the influence of the two sub-samples on the model parameters and the error term should be rather small as a result of the large variability of the measured stiffness properties within both strength grades.

5.2.2 Model to predict the tensile strength

Two models for the prediction of the tensile strength are developed. The first one can be used for the prediction of the tensile capacity of the entire timber board $f_t$, whereas the second model can
be used to predict the tensile strength of each particular knot cluster $f_{t,WS}$. For the estimation of both models the experimental results of the destructive tensile tests are used.

Model to predict the tensile strength of timber boards

The following model is developed in order to predict the tensile strength of the entire timber board. It is assumed that the tensile strength of the timber board corresponds to the tensile strength of the weakest section within the measured length. The weakest section is assumed to be the knot cluster having the largest tKAR-value. In order to ensure an optimal comparability to the stiffness model described above, the same parameters are chosen that are $E_{dyn,F}$ and tKAR. In Tab. 5.6 the estimated regression coefficients, the standard deviation of the error term and their coefficients of variations are summarised. Applying the model, a wide agreement ($\rho = 0.782$) between the estimated and the measured tensile capacities is identified (Fig. 5.7).

As known from several studies (e.g. Riberholt & Madsen 1979, Taylor & Bender 1991, Courchene et al. 1996, Isaksson 1999, Köhler 2006), the characteristics of the weakest section within a timber member is related to its length. With increasing length, the largest tKAR-value within a timber board ($t_{KAR_{max}}$) increases and thus the tensile capacity decreases (size effect). Accordingly, the developed model can (without considering the size effect) only be used for the prediction of the tensile capacity of specimens having similar dimensions.

Model to predict the tensile strength of knot clusters

The second strength model is developed to predict the tensile strength of knot clusters or rather the tensile strength of WS. For the calculation, all WS within the measured area are considered; that includes a total number of 2'577 WS. As described above, a WS is defined as a section with tKAR $\geq 0.1$. Based on the results of the destructive tensile tests, the tensile capacity of the timber boards, and thus the tensile strength of the weakest section within each board, are known. Further, it is known that the tensile strength of all other WS is at least the tensile capacity of the corresponding timber board. In case of a timber board (three WS, tensile capacity of the

### Tab. 5.6: Parameters for the model to predict $f_t$ and $f_{t,WS}$ [MPa]

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>Expected value</th>
<th>COV</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_t$</td>
<td>$\beta_0$</td>
<td>2.14</td>
<td>0.047</td>
<td>$\rho(\beta_0, \beta_1) = -0.944$</td>
</tr>
<tr>
<td></td>
<td>$\beta_1$</td>
<td>1.13 $\cdot 10^{-4}$</td>
<td>0.059</td>
<td>$\rho(\beta_0, \beta_2) = -0.751$</td>
</tr>
<tr>
<td></td>
<td>$\beta_2$</td>
<td>-1.08</td>
<td>0.120</td>
<td>$\rho(\beta_1, \beta_2) = 0.520$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_e$</td>
<td>2.77 $\cdot 10^{-1}$</td>
<td>0.034</td>
<td>$\rho(\beta_1, \sigma_e) \approx 0$</td>
</tr>
<tr>
<td>$f_{t,WS}$</td>
<td>$\beta_0$</td>
<td>2.96</td>
<td>0.0067</td>
<td>$\rho(\beta_0, \beta_1) = -0.922$</td>
</tr>
<tr>
<td></td>
<td>$\beta_1$</td>
<td>8.50 $\cdot 10^{-5}$</td>
<td>0.017</td>
<td>$\rho(\beta_0, \beta_2) = -0.596$</td>
</tr>
<tr>
<td></td>
<td>$\beta_2$</td>
<td>-2.22</td>
<td>0.016</td>
<td>$\rho(\beta_1, \beta_2) = 0.274$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_e$</td>
<td>1.50 $\cdot 10^{-1}$</td>
<td>0.014</td>
<td>$\rho(\beta_1, \sigma_e) \approx 0$</td>
</tr>
</tbody>
</table>
5.2. Material model

entire board $f_t = 30 \text{ MPa}$) the following information about the tensile strength of the WS can be obtained: the tensile strength of the weakest section is $f_{t,WS} = 30 \text{ MPa}$ and the tensile strength of the other two WS is $f_{t,WS} \geq 30 \text{ MPa}$.

With the given information it is possible to estimate the regression parameters by using linear regression analysis for censored data (see e.g. Buckley & James 1979, Chatterjee & McLeich 1981). In Fig. 5.8 the principle of the regression analysis for censored data is illustrated. Fig. 5.8 (a) illustrates the measured tensile capacity $f_{t,i}$ of three timber boards (each timber board has two WS) and the corresponding parameter of each WS (e.g. tKAR-value). The weakest knot cluster within each member WS+ is represented by '+' and the other knot cluster WSo are represented by 'o'. Further, the illustration shows a linear regression line that describes the relation between the parameter of the weakest section and the corresponding tensile capacity. The regression curve and its corresponding $\sigma_\varepsilon$ are calculated with Eq. (5.2)–(5.3). They are used as the start value for the following calculations.

According to the principle of a linear regression model, it is assumed that the error term $\varepsilon$ is Normal distributed around the regression model $\varepsilon \sim N(0, \sigma_\varepsilon)$. Based on this, the strength of WSo ($f_{t,WS,o}$) can be estimated by the expected value of the truncated Normal distribution (grey area) according to Eq. (5.5). Here, $f_{t,reg}$ denotes the expected tensile strength according to the regression model.

$$ f_{t,WS,o} = \frac{\int_{f_{t,i}}^{\infty} x \cdot f(x) dx}{\int_{f_{t,i}}^{\infty} f(x) dx} \quad \text{with} \quad f(x) = \frac{1}{\sigma_\varepsilon \sqrt{2\pi}} \exp \left( -\frac{1}{2} \left( \frac{x - f_{t,reg}}{\sigma_\varepsilon} \right)^2 \right) \quad (5.5) $$

The estimated strength is illustrated with a 'x' in Fig. 5.8 (b). Using the estimated strength of WSo and the measured strength of WS+, a new regression model with a corresponding $\sigma_\varepsilon$ can be calculated. The new regression line is illustrated as dashed line. With the new regression line and the corresponding $\sigma_\varepsilon$ the strength of WSo can be estimated again, following...
the principle described above. The new estimated strengths are illustrated as black dots in Fig. 5.8 (c). With the new estimated strength of WS, and the measured strength of WS+, a new regression model (dashed-dotted line) with a corresponding $\sigma_e$ can be estimated. This iteration has to be repeated up to the convergence criterion. In this study, a change between the estimated regression parameters in iteration step $i$ and iteration step $i + 1$ of 0.005% is chosen. The estimated regression parameters of the strength model are summarised in Tab. 5.6 Furthermore, Fig. 5.9 illustrates the correlation between the measured/estimated tensile strength with the predicted tensile strength of all WS. It is obvious that the estimated tensile strength of the WS is located within the area near right of the regression model.

At this point, it has to be mentioned that this model is developed for the prediction of the tensile strength of WS. If the model is used for the prediction of the tensile strength of CWS (using tKAR = 0), it will be slightly underestimated. Further it has to be considered that the standard deviation of the estimated error term $\sigma_e$ will be underestimated using censored regression analysis. That results from the fact that the regression model is developed by using measured and estimated material properties and the latter ones are in general located nearby the regression line.

Another area of application of this model is the prediction of the tensile capacity of the entire timber board $f_t$. Therefore, the tensile capacity of one timber board $f_{t,i}$ is defined as the tensile strength of the weakest section within the board ($j$ denotes the number of WS within the board $i$):

$$f_{t,i} = \min_j(f_{t,WS,ij}) \tag{5.6}$$
5.2. Material model

A comparison between the measured and the predicted tensile capacity shows a rather large correlation $\rho = 0.751$ (see the ‘+’ in Fig. 5.9). An advantage of this method is that the tensile capacity of timber boards can be predicted independently of the board length.

Comparisons of the models

In the following paragraphs, the two strength models described above are compared with each other (both are summarised in Tab. 5.6). The significantly higher value of parameter $\beta_0$ in the second model indicates a higher predicted tensile strength within areas with small tKAR. On the other hand, the higher absolute value of parameter $\beta_2$ explains larger local strength reduction due to knots.

In Fig. 5.10, the estimated tensile strength of all WS (calculated with both models) are illustrated. Here ‘+’ denotes the predicted tensile strength of the weakest section WS and ‘o’ denotes the predicted strength of all other WSs. It is obvious that the first strength model (model to predict $f_t$) underestimates the tensile strength of the majority of the WS. The difference between the two models is on average $\Delta f_{t,WS} = 7.70$ MPa. However, the prediction of the tensile capacity of the timber boards $f_t$ (which is assumed to be the tensile strength of the section with tKAR$_{\text{max}}$) is relatively similar, especially for timber boards having a small tensile strength.

The difference between the two models can be explained by the following example. Let us assume that a timber board with a tensile capacity $f_t = 30$ MPa contains a number of knot clusters. Let us assume further that the two largest knot clusters have the same tKAR-value (tKAR = 0.25). The first strength model is developed with the information about the tensile strength of one knot cluster with the specific tKAR-value; i.e. tKAR = 0.25 $\rightarrow f_{t,WS} = 30$ MPa. The second model is calibrated with the information that only one knot clusters with the specific tKAR-value will fail tKAR = 0.25 $\rightarrow f_{t,WS,1} = 30$ MPa, whereas the second knot cluster shows a higher tensile strength tKAR = 0.25 $\rightarrow f_{t,WS,2} \geq 30$ MPa.
The consideration of all WS within a timber board (not only the weakest) is of particular importance, since the differences between tKAR-values of WS within the same timber board are relatively small. In more than 75% of the investigated timber boards with more than one WS, the difference between the two largest tKAR-values is smaller than 5%.

5.2.3 Model to predict the tensile strength & stiffness of FJ

Using the presented models, it is also possible to estimate the tensile strength and stiffness of finger joint connections (FJ). Therefore, the corresponding literature has to be taken into account. The stiffness properties of FJ are analysed in Samson (1985), Heimeshoff & Glos (1980) for bending and Ehlbeck et al. (1985a) for tension. All studies showed no significant difference to the stiffness properties of CWS. Thus, $E_{t,j}$ is assumed to be the mean of the two adjacent CWS; Eq. (5.7).

To model the tensile strength of finger joints $f_{t,j}$, which is one of the most important parameters to model the mechanical performance of GLT, a very simple and plausible approach is chosen: it is assumed that $f_{t,j}$ is equal to $f_{t,WS}$ having a specific tKAR-value (Eq. (5.7)). This approach was already mentioned in other studies (e.g. Pellicane et al. 1987, Colling 1990). Based on the mentioned literature and the experimental experience of the research group at ETH Zurich, $0.2 \leq t\text{KAR} \leq 0.3$ seems to be realistic.

$$E_{t,j} = \frac{1}{2} \sum_{i=1}^{2} E_{t,CWS,i} \quad f_{t,j} = \min_{i=1,2} \{f_{t,WS,i}|t\text{KAR}\}$$  \hspace{1cm} (5.7)

5.2.4 Model to predict the compressive strength & stiffness properties

For the estimation of the compressive stiffness, the results of former investigation are taken into account (Heimeshoff & Glos 1980, Ehlbeck et al. 1985a, Sell 1997, Blaß et al. 2008). The analysis shows that WS under compression load are slightly stiffer (~5%) than under tensile...
load, whereas CWS are slightly weaker (∼4%), for timber boards of typical strength grades. Only for FJ, significantly reduced stiffness properties (10–15%) are identified.

Comparison of compressive strength versus tensile strength shows that defect-free wood specimens have a significant smaller compressive strength (e.g. Sell 1997). On the other hand, the characteristic values of timber boards given in the EN 338 (2010) shows the opposite. One reason therefore is the size or length effect (the characteristic values are identified on specimens with different lengths – see EN 408 (2003)). Nevertheless, it indicates that strength reduction through the occurrence of knots in compression is smaller than in tension. The same results can be identified through an investigation of existing material models (e.g. Blaß et al. 2008).

However, the main objective of the present work is the investigation of the bending capacity of GLT beams ($f_{m,g}$). It is well established that $f_{m,g}$ is highly related to the tensile strength of its weak zones (knot clusters and FJ) located in the tensile loaded area of the GLT beam. Therefore, it is of particular importance that the material model shows a good accuracy within the tensile related material properties, whereas the accuracy of the compressive properties is of minor importance. As a result of the relatively small stiffness differences, it is not distinguished between tensile and compressive stiffness in the GLT model (introduced in Chapter 7); i.e. the compressive stiffness is assumed to be equal to the tensile stiffness. Furthermore, in the GLT model a tensile failure is assumed as the failure criterion, and the compressive strength is not considered.

5.2.5 Model uncertainties

It is obvious that the prediction of material properties is associated to model uncertainties. The model uncertainties, expressed through the error term $\varepsilon$, are identified for the tensile strength and stiffness model. However, using censored regression analysis to estimate the parameter, underestimates the model uncertainties. To compensate that, a larger $\sigma_{\varepsilon} = 0.2$ is assumed.

To consider the correlation of the material properties in each particular member, the error term $\varepsilon$ is separated into two parts: one part for the uncertainty of the mean material properties (constant within one timber board – denoted $\varepsilon_1$) and the other part for the uncertainty of the strength/stiffness reduction of each particular WS (denoted $\varepsilon_2$). A ratio between the two parts of $\varepsilon_1 : \varepsilon_2 = 2 : 1$ is chosen, in accordance with the investigations of Colling (1990).

Furthermore, the correlation between the strength and stiffness properties has to be considered. Therefore, a correlation between strength and stiffness related error terms $\rho = 0.8$ is assumed. To model FJ, the same model uncertainties are assumed as for WS.

5.2.6 Comparison with existing models

In order to verify the introduced material model, it is compared with existing models. Therefore, the above mentioned model presented in Blaß et al. (2008) is used as a reference model. Between the two model approaches, fundamental differences exist:

- Test setup: the test configuration is different in the two studies.
Specimen size / testing length: the reference model was developed based on experimental investigations with specimens having a testing length of 137.5 mm; in the present study the testing length is greater than 3’300 mm.

Measurement length: in the present study, the measurement length to measure the stiffness properties varies depending on the natural growth characteristics of the investigated specimen.

Sample selection: the reference model was developed based on randomly selected timber board segments; here a model is developed to predict $E_{WS}$ and $f_{t,WS}$ – therefore only sections containing knot clusters are considered.

Input parameters: the global indicator for the description of the material properties of the defect-free timber is different; here $E_{\text{dyn},F}$ is used instead of $\rho_0$.

In the following paragraphs, the stiffness properties $E_{\text{CW}S}$ and $E_{WS}$, as well as the tensile capacity of the timber boards $f_t$ (tensile strength of the WS with tKAR$_{\text{max}}$) are estimated using both approaches. The results are illustrated in Fig. 5.11–5.12. In all illustrations there is an accordance between the estimated material properties (for both approaches) and the measured material properties within the upper part; i.e. for timber boards or WS having relatively high strength and stiffness properties. However, in the lower part, significant differences are detected. It is obvious that the here developed models show good accordance – as they have been developed on the data-set itself. Nevertheless, all three illustrations indicate that the predictions using the reference model significantly overestimate the material properties, especially for low values of stiffness and strength. The overestimation is on average $\varepsilon(E_{\text{CW}S}) = 886$ MPa, $\varepsilon(E_{WS}) = 374$ MPa, and $\varepsilon(f_t) = 9.80$ MPa.

The differences in Fig. 5.11 (left) might be a result of the measurement length. In the present study, the tensile stiffness of the clear wood $E_{\text{CW}S}$ is measured on the entire length between two adjacent knot clusters. Small defects between the knot clusters are not explicitly considered. Thus the measured stiffness might be lower than the stiffness of a defect-free specimen. However, it still represents the mean stiffness properties of the clear wood sections.

It seems likely that the differences in Fig. 5.11 (right) and Fig. 5.12 (left) come from the different dimensions of the test specimens. A drawback of small test specimens is that effects that reduce the strength and stiffness properties, such as the influence of local grain deviation before and after the knot clusters are not considered. Further, lateral bending due to knots might be prevented. This influence is of particular importance for knot clusters having numerous knots and thus a large tKAR. Another reason for the differences might be the sample selection. As mentioned above, the emphasis of the present study is the investigation of the material properties of knot clusters and not of board sections. Thus, in particular sections containing knot clusters are considered.

For comparison of the tensile capacities, the different test setups have to be considered. When measuring the tensile strength using the test configuration described in Heimeshoff & Glos (1980), the investigated timber boards are supported through glued timber boards within the transition area. Thus, knot clusters, where parts of the fracture are outside the testing length (137.5 mm),
are reinforced. The influence might be significant because in the majority of the investigated timber boards the area of the fracture was above 137.5 mm.

In addition to the material model, also the strength model for FJ is verified. The tensile strength of the FJ is assumed to be the tensile strength of a WS with \( t_{KAR} = 0 \). As before, the model is compared to the reference model presented in [Blaß et al. (2008)]. There \( f_{t,j} \) is modelled as a function of the dry density \( \rho_0 \), here it is a function of \( E_{\text{dyn,F}} \). In both models \( f_{t,j} \) is estimated based on the global indicator (\( \rho_0 \) or \( E_{\text{dyn,F}} \)) from the weaker of the two associated timber boards. To describe the interaction between the two indicators, a linear regression model, based on all 950 investigated timber boards, is used: \( E_{\text{dyn,F}} = -5.6 \cdot 10^3 + 43\rho_0 \). In Fig. 5.12 (right) the estimated \( f_{t,j} \) for typical dry densities \( 350 \leq \rho_0 \leq 500 \) are illustrated. In the presented model, the influence of the stiffness related indicator is larger; i.e. the curve is steeper. However, it seems that is \( t_{KAR} = 0.20 \) is used, the present model is similar to the reference model, in particular for timber boards with a small \( \rho_0 \), which are the most relevant for modelling GLT.

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**Fig. 5.11:** Verification: (left) model to predict \( E_{\text{WS}} \), (right) model to predict \( E_{\text{WS}} \)

**Fig. 5.12:** Verification: (left) model to predict \( f_t \), (right) model to predict \( f_{t,j} \)
5.3 Application – modelling of timber boards & FJ

5.3.1 Timber boards

Applying the material model described before and the probabilistic model introduced in Chapter 4, it is possible to model timber boards, or rather the tensile strength and stiffness properties within timber boards. The procedure is as follows:

1. Modelling the parameters $E_{\text{dyn,F}}$, $d$, and tKAR:
   - Modelling timber boards with a specific $E_{\text{dyn,F}}$ – Eq. (4.4)
   - Modelling the position and the tKAR-value of each particular WS – Eq. (4.1), (4.3)

2. Allocate the strength and stiffness properties:
   - Estimation of the tensile strength and stiffness of CWS and WS – Eq. (5.1)
   - Modelling the uncertainties $\varepsilon_1$ and $\varepsilon_2$

Within one particular timber board, the global indicator $E_{\text{dyn,F}}$ is constant. Thus, and because of the hierarchical model of tKAR, the within-member correlation is considered; i.e. the correlation of board sections within one timber board. Both material properties (tensile strength and tensile stiffness) of one particular timber board section, are calculated with the same input parameters ($E_{\text{dyn,F}}$ and tKAR). Thus, the correlation between tensile strength and tensile stiffness is automatically considered.

In the following paragraphs, the tensile strength and stiffness properties of the simulated timber boards are estimated. For this purpose, timber boards with a length $l = 1'620$ mm are simulated. The chosen length corresponds to the required testing length for typical timber boards $b \cdot t_l = 180 \cdot 40$ mm, according to EN 408 (2003).

Based on a sufficient amount of simulated timber boards, several parameters of the material properties, such as the characteristic value of the tensile strength $f_{t,k}$ or the mean stiffness $E_{t,\text{mean}}$ can be calculated. In the present example $n = 10^4$ timber boards are simulated.

Tensile strength

To estimate the tensile strength $f_t$ of the simulated timber boards, the following failure criterion is chosen: the tensile strength of a timber board is assumed to be the tensile strength of the weakest section of the timber board. Taking into account $f_t$ of all simulated timber boards, the characteristic value of the tensile strength $f_{t,k}$ is calculated. Furthermore, the mean value $f_{t,\text{mean}}$ and the COV are estimated assuming a Lognormal distribution; the results are summarised in Tab. 5.7. The estimated tensile strength of the simulated timber boards and the fitted Lognormal distribution are illustrated in Fig. 5.13 (left).

Tensile stiffness

The tensile stiffness $E_{t,\text{mean}}$ of the timber boards is calculated with the tensile stiffness of the single timber board sections, in accordance with Hook’s law for serial springs. Based on the
5.3. Application – modelling of timber boards & FJ

Tab. 5.7: Compilation of the simulated material properties [MPa] (compared with the required/recommended values given in literature)

<table>
<thead>
<tr>
<th></th>
<th>Strength grade L25</th>
<th>Strength grade L40</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Simulation</td>
<td>Literature</td>
</tr>
<tr>
<td>( f_{t,k} )</td>
<td>14.8</td>
<td>14.5(^a)</td>
</tr>
<tr>
<td>( f_t )</td>
<td>26.7</td>
<td>-</td>
</tr>
<tr>
<td>( \text{COV}(f_t) )</td>
<td>0.33</td>
<td>0.30(^b)</td>
</tr>
<tr>
<td>( E_t )</td>
<td>10'400</td>
<td>11'000(^a)</td>
</tr>
<tr>
<td>( \text{COV}(E_t) )</td>
<td>0.15</td>
<td>0.13(^b)</td>
</tr>
<tr>
<td>( f_{t,j,k} )</td>
<td>21.2</td>
<td>19.5(^c)</td>
</tr>
<tr>
<td>( f_{t,j} )</td>
<td>30.1</td>
<td>-</td>
</tr>
<tr>
<td>( \text{COV}(f_{t,j}) )</td>
<td>0.20</td>
<td>-</td>
</tr>
</tbody>
</table>

\(^a\) according to EN 14081-4 (2009)
\(^b\) according to JCSS (2006)
\(^c\) according to EN 1194 (1999): \( f_{t,j,k} \geq 5 + f_{t,k} \)

Simulations, the expected value of \( E_{t,\text{mean}} \) and the COV are estimated, assuming a Lognormal distribution (Tab. 5.7).

Size effect

A further application of the model is the investigation of the size effect. Timber boards having different lengths are simulated. From the simulated timber boards the tensile strength and stiffness properties \( (f_t \text{ and } E_t) \) are estimated and the corresponding characteristic values \( (f_{t,k} \text{ and } E_{t,\text{mean}}) \) are calculated. The results are illustrated in Fig. 5.13 (right). It is obvious, that the tensile strength decreases with increasing board length, whereas no influence on the mean tensile stiffness can be detected.

Next, a two dimensional Weibull distribution is used to describe the size effect; Eq. (2.3)–(2.5) – reference length \( l = 1'620 \text{ mm} \). With a chosen parameter \( \eta = -0.15 \) for strength grade L25, and \( \eta = -0.10 \) for strength grade L40 a good accuracy to the simulated data is identified; Fig. 5.13 (right). The identified parameters correspond to those found in the literature (see e.g. the literature compilation in Isaksson 1999). It seems that the influence of the size effect decreases with increasing quality of the timber boards. That can be explained with the lower variability of the material properties of higher strength grades.

tKAR\(_{\text{limit}}\)

To model the tKAR-value of WS it is assumed that the grading process is not perfect, thus \( tKAR_{\text{limit}} = 1 \) (Chapter 4.2.1). However, to simulate a perfect grading process, any \( tKAR_{\text{limit}} \) can be considered in the presented approach. The strength grade L25 (EN 14081-4 (2009)), is
comparable to the strength grade C24 (EN 338 (2010)): \( f_{t,k} = 14.5 \text{ MPa} \) and \( f_{t,k} = 14.0 \text{ MPa} \), respectively. According to EN 1912 (2012), the strength grade C24, corresponds to visually graded timber S10. For S10, a knot cluster tKAR \( \leq 0.50 \) is acceptable, according to DIN 4074-1 (2008). Thus tKAR\(_{\text{limit}} = 0.50 \) seems to be an accurate grading criteria for strength grade L25. Using tKAR\(_{\text{limit}} = 0.50 \), the tensile strength would slightly increase: \( f_{t,k} \approx 8\% \). However, it has to be mentioned that for timber graded with GoldenEye-706 grading device, no upper limit for knots is defined.

For the upper strength grade L40, a direct comparison to a visual grading criteria is not possible, as visual grading of Norway spruce is only allowed up to strength grade C30 (see EN 1912 (2012) and DIN 4074-1 (2008)). However, a limitation tKAR\(_{\text{limit}} = 0.40 \) might be accurate. In that case \( f_{t,k} \) increases \( \approx 3\% \).

**Model uncertainties – material model**

The material model (strength and stiffness model) is subjected to model uncertainties. They are expressed through the error term \( \epsilon \sim N(0, \sigma_\epsilon) \) – Eq. (5.1). The sensibility of the error term \( \epsilon \), on the characteristic value of the tensile strength \( f_{t,k} \), is investigated. It is obvious that with an increase of \( \sigma_\epsilon \), the estimation of \( f_{t,k} \) will decrease; Fig. 5.14 (left). The illustration indicates that the model is sensitive to the chosen \( \sigma_\epsilon \), in particular for higher strength grades.

To consider the within-member correlation, \( \epsilon \) is separated into two parts: \( \epsilon_1 \) for the uncertainty of the mean material properties (constant within one timber board), and \( \epsilon_2 \) for the uncertainty of the strength/stiffness reduction of each particular WS. A ratio between these two parts of \( \epsilon_1 : \epsilon_2 = 2 : 1 \) is chosen in accordance with the investigations of Colling (1990). However, for the presented material model this assumption is investigated. The influence of \( \epsilon_2 \) is larger, because in average more than one WS occurs within one timber board: \( f_{t,k}(\epsilon_1, \epsilon_2 | \epsilon_1 < \epsilon_2) < f_{t,k}(\epsilon_1, \epsilon_2 | \epsilon_1 > \epsilon_2) \). However, the influence is rather small; e.g. \( \leq \pm 2\% \) for the characteristic value of the tensile strength.
5.3. Application – modelling of timber boards & FJ

Statistical uncertainties

To predict the characteristic value of the tensile strength $f_{t,k}$ of timber boards, four models ($d$, $t_{KAR}$, $E_{dyn,F}$, and $f_{t,WS}$) with altogether 11 parameters have to be taken into account. The four models are not or only marginally correlated, thus their correlation is not considered (see Chapter 4.2.3). However, for all parameters the expected values and their statistical uncertainties, as well as their correlations (correlations between parameters of the same model), are identified. Assuming that the statistical uncertainties of the parameters are Normal distributed, the statistical uncertainties of $f_{t,k}$ are calculated.

For this purpose, the characteristic value of the tensile strength $f_{t,k}$ is calculated 100 times, each time based on $10^4$ simulated timber boards. The variability of $f_{t,k}$ is $\text{COV}(f_{t,k}) = 0.019$. Thus, it seems that the influence of the statistical uncertainties is rather small.

5.3.2 Finger joint connections

The introduced approach can also be used to model the tensile strength of FJ ($f_{t,j}$). Therefore, two independent timber boards (clear wood) are simulated. $f_{t,j}$ is calculated just as a WS, within the weaker timber board, having a specific $t_{KAR}$-value; here $t_{KAR} = 0.2$. Based on a sufficient amount of simulated FJ, the mean value and the characteristic value of the tensile strength ($f_{t,j,\text{mean}}$ and $f_{t,j,k}$) are calculated. The results are summarised in Tab. 5.7.

In the present model it is assumed that the tensile strength of a FJ corresponds to the tensile strength of a WS having specific $t_{KAR}$-value. It is obvious that a larger $t_{KAR}$-value leads to a decrease of $f_{t,j}$. This influence is illustrated in Fig. 5.14 (right). Considering that the required values are 19.5 MPa and 31.0 MPa, respectively, it seems that the chosen value $t_{KAR} = 0.2$ is quite realistic. Using a higher $t_{KAR}$-value, the required values of the characteristic tensile strength are significantly underestimated; e.g. $t_{KAR} = 0.25$ led to $f_{t,j,k} = 17.0$ MPa and $f_{t,j,k} = 24.5$ MPa, respectively.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5_14}
\caption{(left) influence of $\sigma_c$ on the characteristic value of the tensile capacity of timber boards, (right) influence of $t_{KAR}$ on the characteristic value of the tensile capacity of finger joint connections}
\end{figure}
5.4 Model summary

In this chapter a model for the prediction of the material properties based on strength and stiffness related indicators was developed. First, the efficiency of numerous indicators and their correlation were investigated. As a result of wide agreement, two indicators were selected: \( E_{\text{dyn,F}} \) to consider the between-member variability and tKAR to consider the within-member variability.

Taking into account the two indicators (\( E_{\text{dyn,F}} \) and tKAR), material models were developed. The model are particularly focused on the prediction of the tensile stiffness and the tensile strength of knot clusters. For the latter, the censored regression analysis was used; i.e. a method where both, equality type and inequality type information are considered. Compared to other material models, the approach presented here, shows a significant larger influence of knots on the local strength reduction. If the model is applied for the prediction of the tensile strength and stiffness properties of clear wood (using tKAR = 0), these are slightly underestimated (\( \approx 3\% \)). The model parameters are summarised in Tab. 5.8.

In the last part of the chapter, an application of the material model is presented. Timber boards and finger joint connections are simulated using the probabilistic model described in Chapter 4. Afterwards, the material properties are allocated using the presented material model. From the simulated timber boards, the tensile capacity and the tensile stiffness are estimated. A comparison between the simulated and the required/recommended values indicates that the present approach can be used for modelling the tensile related material properties of timber boards and finger joint connections.

Tab. 5.8: Compilation of the material model to predict \( E_{\text{WS}} \) and \( f_{t,\text{WS}} \) [MPa]

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>Expected value</th>
<th>COV</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_{\text{WS}} )</td>
<td>( \beta_0 )</td>
<td>8.41</td>
<td>0.0027</td>
<td>( \rho(\beta_0, \beta_1) = -0.922 )</td>
</tr>
<tr>
<td></td>
<td>( \beta_1 )</td>
<td>( 7.69 \cdot 10^{-5} )</td>
<td>0.019</td>
<td>( \rho(\beta_0, \beta_2) = -0.564 )</td>
</tr>
<tr>
<td></td>
<td>( \beta_2 )</td>
<td>( -9.02 \cdot 10^{-1} )</td>
<td>0.040</td>
<td>( \rho(\beta_1, \beta_2) = 0.234 )</td>
</tr>
<tr>
<td></td>
<td>( \sigma_{\varepsilon} )</td>
<td>( 1.00 \cdot 10^{-1} )</td>
<td>0.024</td>
<td>( \rho(\beta_1, \sigma_{\varepsilon}) \approx 0 )</td>
</tr>
<tr>
<td>( f_{t,\text{WS}} )</td>
<td>( \beta_0 )</td>
<td>2.96</td>
<td>0.0067</td>
<td>( \rho(\beta_0, \beta_1) = -0.922 )</td>
</tr>
<tr>
<td></td>
<td>( \beta_1 )</td>
<td>( 8.50 \cdot 10^{-5} )</td>
<td>0.017</td>
<td>( \rho(\beta_0, \beta_2) = -0.596 )</td>
</tr>
<tr>
<td></td>
<td>( \beta_2 )</td>
<td>( -2.22 )</td>
<td>0.016</td>
<td>( \rho(\beta_1, \beta_2) = 0.274 )</td>
</tr>
<tr>
<td></td>
<td>( \sigma_{\varepsilon} )</td>
<td>( 1.50 \cdot 10^{-1} )</td>
<td>0.014</td>
<td>( \rho(\beta_1, \sigma_{\varepsilon}) \approx 0 )</td>
</tr>
</tbody>
</table>
Chapter 6

Estimation of the load-bearing capacity of GLT

Applying the probabilistic model (Chapter 4) and the material model (Chapter 5), timber boards and thus GLT beams with well-known local material properties can be simulated. In a next step the load-bearing capacity \( f_{m,g} \) and the bending stiffness \( E_{m,g} \) of simulated GLT beams are estimated. In order to do that, a numerical strain-based model (FEM) is developed that takes into account the local material properties of the entire GLT beam. In the current chapter this numerical model is introduced and, subsequently validated with the 24 tested GLT beams with well-known local material properties.

6.1 Model description

In this chapter, a numerical strain-based model for the estimation of the load-bearing capacity of GLT beams is introduced\(^1\). The characteristic of the presented model is that the bending failure of the GLT beam is defined by a tensile failure of an entire lamella cross-section. Local failure mechanism through, e.g. local growth irregularities within a knot cluster, are not explicitly considered.

The intention behind this simplified failure mechanism is the following. The failure mechanism of timber boards and thus of GLT beams is highly related to local growth irregularities, such as the local grain deviation around knots. Neither the probabilistic representation of the local growth irregularities nor the resulting failure mechanism due to those growth irregularities are fully understood yet. However, both are indirectly considered in the material properties (in structural timber components, stiffness properties are defined as the mean stiffness of the entire cross-section and strength properties are defined as the load-bearing capacity in relation to the cross-section). Using the probabilistic model and the material model described in the previous chapters, these ’mean material properties of the timber board cross-section’ are modelled. Hence,

\(^1\)The numerical model is developed based on Bathe (1996) and Betten (2003)
it seems to be most correct that a failure is defined through the exceedance of the acceptable mean axial strains (mean axial stresses) of an entire lamella cross-section.

6.1.1 Load configuration

The two-dimensional model described here is developed in order to estimate the load-bearing behaviour of GLT beams under four-point bending, in accordance with EN 408 [2003]; see Fig. 6.1. The GLT beam is supported, on the outer parts of the beam and the load $F = 1$ kN is applied in the vertical direction, at a distance of $l/3$ from the supports.

6.1.2 Element mesh

The GLT beam is subdivided into elements with constant dimensions: element length $l_e = 50$ mm and element height $h_e = t_l$ (lamella thickness, here $h_e \approx 40$ mm). The resulting element mesh is illustrated in Fig. 6.2. The strength and stiffness properties within the entire element are assumed to be constant, having a specific $f_i, E_i$. The elements are modelled as iso-parametric four-node elements having isotropic, linear elastic material properties. The corresponding strain-displacement matrix $B$, the material matrix $C$ and the determinant of the Jacobian matrix $J$ are given in Eq. (6.1–6.2). Here, $r$ and $s$ are the local element coordinates, $E$ is the element stiffness, $b$ is the element width (width of the GLT beam) and $\nu$ is the Poisson’s ratio.

$$
B = \frac{1}{2l_e h_e} \begin{bmatrix}
-h_e(1-s) & 0 & h_e(1-s) & 0 & \ldots \\
0 & -l_e(1-r) & 0 & -l_e(1+r) & \ldots \\
-l_e(1-r) & -h_e(1-s) & -l_e(1+r) & h_e(1-s) & \ldots \\
h_e(1+s) & 0 & -h_e(1+s) & 0 & \ldots \\
0 & l_e(1+r) & 0 & l_e(1-r) & \ldots \\
l_e(1+r) & h_e(1+s) & l_e(1-r) & -h_e(1+s) & \ldots \\
\end{bmatrix}
$$

$$
C = \frac{E \cdot b}{1-\nu^2} \begin{bmatrix}
1 & \nu & 0 \\
\nu & 1 & 0 \\
0 & 0 & \frac{1-\nu}{2} \\
\end{bmatrix} \quad J = \begin{bmatrix}
\frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\
\frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \\
\end{bmatrix} \quad \det J = \frac{l_e h_e}{4}
$$

Fig. 6.1: Load configuration
Fig. 6.2: (top) Element mesh, (bottom) iso-parametric 4-node elements
6.1.3 Local stiffness matrix

Under consideration of \( B \), \( C \), and \( J \) the local stiffness matrix \( K^e \) can be calculated using Eq. (6.3). For the case of an iso-parametric 4-node element, having isotropic material properties an analytical solution for \( K^e \) exists; Eq. (6.4).

\[
K^e = \int_{-1}^{1} \int_{-1}^{1} B^T C B \cdot \det J \cdot \text{dr} \cdot \text{ds} \quad (6.3)
\]

\[
K^e = D \cdot \begin{bmatrix}
k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} & k_{17} & k_{18} \\
k_{22} & k_{23} & k_{24} & k_{25} & k_{26} & k_{27} & k_{28} & k_{33} \\
k_{33} & k_{34} & k_{35} & k_{36} & k_{37} & k_{38} & k_{44} & k_{45} \\
k_{44} & k_{46} & k_{47} & k_{48} & k_{55} & k_{56} & k_{57} & k_{58} \\
k_{55} & k_{56} & k_{57} & k_{58} & k_{66} & k_{67} & k_{68} & k_{77} & k_{78} & k_{88} \\
\end{bmatrix}
(6.4)
\]

\[
k_{11} = 8\beta + 4\alpha (1 - \nu) \quad k_{23} = 3 - 9\nu \quad k_{36} = k_{14} \quad k_{56} = k_{12}
\]
\[
k_{12} = 3 + 3\nu \quad k_{24} = 4\alpha - 4\beta (1 - \nu) \quad k_{37} = k_{15} \quad k_{57} = k_{13}
\]
\[
k_{13} = -8\beta + 2\alpha (1 - \nu) \quad k_{25} = k_{16} \quad k_{38} = k_{12} \quad k_{58} = k_{14}
\]
\[
k_{14} = -3 + 9\nu \quad k_{26} = -4\alpha - 2\beta (1 - \nu) \quad k_{44} = k_{22} \quad k_{66} = k_{22}
\]
\[
k_{15} = -4\beta - 2\alpha (1 - \nu) \quad k_{27} = k_{14} \quad k_{45} = k_{23} \quad k_{67} = k_{23}
\]
\[
k_{16} = -3 + 9\nu \quad k_{28} = -8\alpha + 2\beta (1 - \nu) \quad k_{46} = k_{28} \quad k_{68} = k_{24}
\]
\[
k_{17} = 4\beta - 4\alpha (1 - \nu) \quad k_{33} = k_{11} \quad k_{47} = k_{12} \quad k_{77} = k_{11}
\]
\[
k_{18} = 3 - 9\nu \quad k_{34} = k_{16} \quad k_{48} = k_{26} \quad k_{78} = k_{16}
\]
\[
k_{22} = 8\alpha + 4\beta (1 - \nu) \quad k_{35} = k_{17} \quad k_{55} = k_{11} \quad k_{88} = k_{22}
\]

\[
D = \frac{E \cdot b}{24(1 - \nu^2)} \quad \alpha = \frac{h_e}{l_e} \quad \beta = \frac{l_e}{h_e}
\]

6.1.4 Assembling global stiffness matrix

As mentioned above the strength and stiffness properties are different for each element; i.e. there are \( n \cdot m \) different \( K^e \) matrices: For instance, in the case of a GLT beam \( l \cdot h = 10'800 \cdot 600 \text{ mm} \) (recommended dimension according to EN 1194 (1999)) this implies 3'240 elements, which have to be assembled in one global stiffness matrix \( K \). Therefore, the orientation and position of each local element within the global element mesh have to be considered. In Eq. (6.5) the local and global orientations of the 1st element are shown (Fig. 6.2).

\[
o_{\text{local}} = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \quad o_{\text{global}} = \begin{bmatrix} 2 & n + 3 & n + 2 & 1 \end{bmatrix} \quad (6.5)
\]
In order to illustrate the principle of assembling the local stiffness matrices $K^e$ to the global stiffness matrix $K$, the location of the 1st element within $K$ is illustrated (denoted $K^1$):

$$
K^1 = \begin{bmatrix}
    u_1 & v_1 & u_2 & v_2 & \ldots & u_{n+2} & v_{n+2} & u_{n+3} & v_{n+3} & \ldots \\
    u_1 & k_{17}^1 & k_{18}^1 & k_{21}^1 & k_{22}^1 & \ldots & k_{75}^1 & k_{76}^1 & k_{73}^1 & k_{74}^1 & \ldots \\
    v_1 & k_{57}^1 & k_{88}^1 & k_{81}^1 & k_{82}^1 & \ldots & k_{85}^1 & k_{86}^1 & k_{83}^1 & k_{84}^1 & \ldots \\
    u_2 & k_{17}^1 & k_{18}^1 & k_{11}^1 & k_{12}^1 & \ldots & k_{15}^1 & k_{16}^1 & k_{13}^1 & k_{14}^1 & \ldots \\
    v_2 & k_{27}^1 & k_{28}^1 & k_{21}^1 & k_{22}^1 & \ldots & k_{25}^1 & k_{26}^1 & k_{23}^1 & k_{24}^1 & \ldots \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \ldots & \vdots & \vdots & \vdots & \vdots & \ldots \\
    u_{n+2} & k_{57}^1 & k_{58}^1 & k_{51}^1 & k_{52}^1 & \ldots & k_{55}^1 & k_{56}^1 & k_{53}^1 & k_{54}^1 & \ldots \\
    v_{n+2} & k_{57}^1 & k_{68}^1 & k_{61}^1 & k_{62}^1 & \ldots & k_{65}^1 & k_{66}^1 & k_{63}^1 & k_{64}^1 & \ldots \\
    u_{n+3} & k_{37}^1 & k_{38}^1 & k_{31}^1 & k_{32}^1 & \ldots & k_{35}^1 & k_{36}^1 & k_{33}^1 & k_{34}^1 & \ldots \\
    v_{n+3} & k_{37}^1 & k_{48}^1 & k_{41}^1 & k_{42}^1 & \ldots & k_{45}^1 & k_{46}^1 & k_{43}^1 & k_{44}^1 & \ldots \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots 
\end{bmatrix} \quad (6.6)
$$

By following the same procedure, all the other elements can be assembled in $K$. For the GLT beam mentioned before ($l \cdot h = 10'800 \cdot 600$ mm), there are $m \times n = 3'240$ elements, and $(m + 1) \times (n + 1) = 3'742$ element-nodes. Thus the size of the global stiffness matrix $K$ would be $6'944 \times 6'944$. In Eq. (6.7), the first three element-nodes of $K$ are illustrated.

$$
K = \begin{bmatrix}
    u_1 & v_1 & u_2 & v_2 & u_3 & v_3 & \ldots \\
    u_1 & k_{17}^1 & k_{18}^1 & k_{11}^1 & k_{12}^1 & 0 & 0 & \ldots \\
    v_1 & k_{57}^1 & k_{88}^1 & k_{81}^1 & k_{82}^1 & 0 & 0 & \ldots \\
    u_2 & k_{17}^1 & k_{18}^1 & k_{11}^1 + k_{77}^1 & k_{12}^1 + k_{78}^1 & k_{21}^1 & k_{22}^1 & \ldots \\
    v_2 & k_{27}^1 & k_{28}^1 & k_{21}^1 + k_{87}^1 & k_{22}^1 + k_{88}^1 & k_{31}^1 & k_{32}^1 & \ldots \\
    u_3 & 0 & 0 & k_{17}^2 & k_{18}^2 & k_{11}^2 + k_{77}^2 & k_{12}^2 + k_{78}^2 & \ldots \\
    v_3 & 0 & 0 & k_{27}^2 & k_{28}^2 & k_{21}^2 + k_{87}^2 & k_{22}^2 + k_{88}^2 & \ldots \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ldots 
\end{bmatrix} \quad (6.7)
$$

### 6.1.5 Calculation of the axial stresses

Taking into account the global stiffness matrix $K$ of the entire GLT beam, and the load vector $f$, the deformed element mesh under a unit load $F = 1$ kN is calculated by solving:

$$
u = K^{-1}f \quad (6.8)$$
Based on the position of the element-nodes, the mean change of length in axial direction $\Delta l_i$ of each element $i$ is calculated. Afterwards, the mean axial strains $\varepsilon_i$ and the mean axial stresses $\sigma_{t,i}$ are calculated:

$$\varepsilon_i = \frac{\Delta l_i}{l_0}$$
$$\sigma_{t,i} = \varepsilon_i \cdot E_i$$  \hspace{1cm} (6.9)

### 6.1.6 Failure criterion

Through a comparison of the mean axial stresses $\sigma_{t,i}$ with the corresponding tensile strength $f_{t,i}$, the load $F$ at which the first element fails under tension is identified (denoted $F_1$, for the first load cycle). Taking into account the load $F_1$ and the deformation of the GLT beam, the corresponding load-bearing capacity $f_{m,g,1}$ and the corresponding bending stiffness $E_{m,g,1}$ are calculated. Thereafter, the stiffness of the specific element is assumed to be zero $E_{t,i} \approx 0$ and the calculation is repeated. This time another element fails under a load $F$ (denoted $F_2$, for the second load cycle). Again the corresponding load-bearing capacity $f_{m,g,2}$ and the corresponding bending stiffness $E_{m,g,2}$ are calculated. This calculation is repeated up to a significant stiffness reduction (here a reduction of 1%, compared to the initial stiffness $E_{m,g,1}$, is assumed as the threshold). Finally, the material properties of the GLT beam are:

$$f_{m,g} = \max_j \{f_{m,g,j}\}$$
$$E_{m,g} = E_{m,g,1}$$  \hspace{1cm} (6.10)

For short GLT beams, such as the investigated in this thesis (Chapter 3), a significant stiffness reduction usually occurs after the first load cycle. Thus the load-bearing capacity of the GLT beam corresponds to the load-bearing capacity identified in the first load cycle: $f_{m,g} = f_{m,g,1}$. However, the introduced failure criterion is important to cover low realisations of $f_{t,i}$, which would not lead to a failure of the GLT beam; e.g. an element located in the middle of the beam. This is more important for longer and higher beams, where the probability of low realisations is larger. For clarification of the principle, the failure criterion is explained with two examples:

**Example 1:**

Assuming a randomly simulated GLT beam with eight lamellas, illustrated in Fig. 6.3. The top of the illustration shows the strength and stiffness properties. The resulting mean axial strain/stresses under a load $F = 1$ kN are illustrated under the stiffness properties. A comparison between the mean axial stresses and the tensile strength indicates a knot cluster (tKAR = 0.33) located in the lowest lamella as the most critical element. This element fails under a load $F_1 = 52.3$ kN, which corresponds to $f_{m,g,1} = 26.6$ MPa and $E_{m,g,1} = 10'720$ MPa. Afterwards, the stiffness of the failed element is assumed to be zero $E_{t,i} \approx 0$ and the calculation is repeated. This time the bending stiffness is significantly lower $E_{m,g,2} = 10'570$ MPa and the calculation ends. The final outcomes are the results from the first load cycle: $f_{m,g} = 26.6$ MPa, $E_{m,g} = 10'720$ MPa, and type of failure – knot cluster tKAR = 0.33.
Example 2:
Let us assume, the same GLT beam having a very low realisation of the strength properties of a finger joint connection \( f_{t.i} = 10 \text{ MPa} \) located in the third lowest lamella; illustrated with a circle in Fig. 6.4. It is obvious that this particular element will fail first: \( F_1 = 47.9 \text{ kN} \), \( f_{m.g,1} = 24.4 \text{ MPa} \), and \( E_{m.g,1} = 10'720 \text{ MPa} \). Following the same procedure as in Example 1, the specific element stiffness is assumed to be zero \( E_{t.i} \approx 0 \) and the calculation is repeated.

The new bending stiffness \( E_{m.g,2} = 10'700 \text{ MPa} \); i.e. the stiffness reduction is only marginal \( E_{m.g,2} \approx E_{m.g,1} \). This time the knot cluster located in the lowest lamella is detected as the weakest section: \( F_2 = 52.3 \text{ kN} \) and \( f_{m.g,2} = 26.6 \text{ MPa} \). Again the specific element is assumed to be zero \( E_{t.i} \approx 0 \) and the calculation is repeated. In the third load cycle the bending stiffness is significantly lower \( E_{m.g,3} = 10'550 \text{ MPa} \), the calculation ends. According to Eq. (6.10) the final outcome is: \( f_{m.g} = f_{m.g,2} = 26.6 \text{ MPa} \), \( E_{m.g} = E_{m.g,1} = 10'720 \text{ MPa} \), and type of failure – knot cluster \( tKAR = 0.33 \).

In both examples, the identified load-bearing capacity is about the same. This is reasonable as a result that a local weak zone, such as an inadequate finger joint connection (FJ), located in the middle of the GLT beam does not significantly influences the load-bearing capacity.

### 6.2 Verification of the numerical model

In this study, 24 GLT beams with well-known beam setup were produced and tested. From the GLT beams (a) the position and \( E_{dyn,F} \) of each timber board, (b) the position of each FJ, and
Chapter 6. Estimation of the load-bearing capacity of GLT

Stiffness properties

\[ F = 52.3 \text{kN} \]
\[ E = 10,720 \text{MPa} \]

Axial strains

Axial stresses

factor = 1.0

\[ E \approx 0 \]

New stiffness properties

\[ E_{n,g,1} = 10,700 \text{MPa} \]

Axial strains

Axial stresses

factor = 1.0

\[ E \approx 0 \]

New stiffness properties

\[ E_{n,g,3} = 10,550 \text{MPa} \]

Fig. 6.4: Example 2
6.2. Verification of the numerical model

Fig. 6.5: Estimated bending stiffness (left) and estimated load-bearing capacity (right) of one beam - GL36h, 100 realisations

(c) the position and tKAR-value of each knot cluster are precisely-known: Chapter 3. Taking into account information about \( E_{\text{dyn,F}}, t\text{KAR} \) and FJ, the strength and stiffness properties of timber board sections can be estimated using the material models introduced in Chapter 5. In this example a tensile strength of the FJ \((f_{t,j} = f_{t,\text{WS}|t\text{KAR}=0.2})\) is assumed. The Poisson’s ratio is assumed to be \( \nu = 0.5 \). As already mentioned, the material model is subjected to model uncertainties, expressed through the error term \( \epsilon \). As a result, different realisations of the GLT beams are possible. In this study, 100 possible realisations of each GLT beam are simulated and their material properties are estimated. It is obvious that different realisations lead to different material properties. In Fig. 6.5, the estimated load-bearing capacities and the estimated bending stiffness of all 100 realisations of one GLT beam (strength class GL36h) are illustrated. The majority of the realisations are located near the test results. However, there are also realisations significantly above or below. For this particular beam the uncertainty of (a) the estimated load-bearing capacity is \( \text{COV} = 0.11 \), and (b) the estimated bending stiffness is \( \text{COV} = 0.03 \). The other GLT beams show similar results.

6.2.1 Load carrying capacity & global modulus of elasticity

Taking into account all the 100 realisations, an expected value of the load-bearing capacity and the bending stiffness are estimated. In Fig. 6.6, the measured and the estimated values of all 24 GLT beams are illustrated. Overall a good agreement between the measured and the estimated material properties could be observed.

In average, the load-bearing capacity \( f_{m,g} \) is slightly underestimated by 2.5 MPa (~6%). The maximum underestimation is 10.0 MPa and the maximum overestimation is 7.3 MPa. For the bending stiffness \( E_{m,g} \), the mean underestimation is 120 MPa (~1%), the maximum underestimation and the maximum overestimation are 980 MPa.
6.2.2 Type of failure

In addition to the load-bearing capacity and the bending stiffness, also the type of failure is investigated. Only in 5 GLT beams (GL24h: 0 and GL36h: 5), a FJ-failure in the lowest lamella is observed in the experimental investigation. The numerical analysis shows a comparable result: in 10 GLT beams (GL24h: 2 and GL36h: 8) a FJ-failure in the lowest lamella is observed. All 5 'real' FJ-failure are detected.

6.3 Model summary

In this chapter a numerical strain-based model for the estimation of the load-bearing capacity, the bending stiffness and the type of failure is introduced. The model takes into account the local material properties of the entire GLT beam; i.e. the strength and stiffness properties of timber board sections with a length of 50 mm. The characteristic of the model is that the bending failure of the GLT beam is defined by a tensile failure of an entire lamella cross-section. Local failure mechanism through, e.g. local growth irregularities within a knot cluster, are not explicitly considered.

The GLT model is validated with 24 GLT beams having well-known local material properties. Between the estimated and the measured material properties a wide agreement is observed. As a result, it seems likely to accurately estimate the material properties ($f_{m,g}$ and $E_{m,g}$) of GLT beams, having well-known information about $E_{\text{dyn},F}$, tKAR-value, and FJ.
Chapter 7

GLT model

In this chapter a probabilistic approach for modelling the material properties of GLT (referred to as GLT model) is presented. The approach contains the probabilistic, the material and the numerical models described in the previous Chapters 4, 5, and 6.

At first the structure of the GLT model is introduced and the main differences to existing approaches are summarised. Afterwards, its application is illustrated on selected examples. Finally, the influence of different input parameters, such as the beam’s dimensions or the quality of finger joint connections (FJ), on the load-bearing capacity of GLT beams is investigated.

7.1 GLT model – structure

The structure of GLT model is illustrated in Fig. 7.1 and Fig. 7.2; it contains four independent sub-models: (1) simulation of timber boards, (2) fabrication of GLT beams, (3) allocation of material properties, and (4) a numerical model for the estimation of the load-bearing capacity. In the following chapters, these sub-models are introduced.

![Fig. 7.1: Framework for modelling GLT](image-url)
Chapter 7. GLT model

7.1.1 Simulation of timber boards

Timber boards are simulated using the probabilistic model introduced in Chapter 4. The specific characteristic of this model is that the natural growth characteristics of timber is considered; i.e. the position and the characteristics of knot clusters are simulated. The model includes a representation of the geometrical setup (distance between WS $d$), as well as a hierarchical representation of both strength and stiffness related indicators ($E_{\text{dyn,F}}$ and tKAR). The simulation of timber boards is as follows: (1) modelling of timber boards having a specific $E_{\text{dyn,F}}$, (2) simulating the position of all knot clusters within the timber board, and (3) allocating a specific tKAR-value to each knot cluster.

With the presented model, timber boards of any board length can be simulated. Through the hierarchical modelling of tKAR, the weakest knot cluster within a timber board (tKAR$_{\text{max}}$) depends on the board length. In this part of the simulation process, overlength timber boards are simulated. They are sawn to a specific length in the second sub-model (Fabrication of GLT).
7.1.2 Fabrication of GLT

In the second step, the simulated timber boards are virtually finger jointed to form endless lamellas, which are then cut to the specific beam length, and glued together to GLT beams. Thereby, in principle every kind of fabrication procedure can be recreated. In the present study, the timber boards are simulated in accordance with the investigations of Larsen (1980)\(^1\) and Ehlbeck & Colling (1987a)\(^1\); i.e. shorted boards \(L \sim N(2.15, 0.50)\) or non-shorted boards \(L \sim N(4.3, 0.71)\). Further, a minimal and a maximal board length is introduced \(l = [1, 6]\). The distance from the board edges to the outermost WS is assumed to be \(d - 100\) mm, to simulate the fabrication process most realistically (Fig. 7.2).

After this part of the simulation process, GLT beams are virtually fabricated, where (a) the position and \(E_{\text{dyn,F}}\) of each timber board, (b) the position and tKAR-value of each knot cluster, and (c) the position of all FJ are precisely-known.

7.1.3 Allocation of material properties

After the GLT fabrication, the material properties are allocated using the material models described in Chapter 5. The strength and stiffness properties of each timber board section and FJ are calculated based on the associated indicators \(E_{\text{dyn,F}}\) and tKAR-value (for FJ a specific tKAR-value has to be assumed). Both the strength and stiffness properties are calculated with the same indicators (\(E_{\text{dyn,F}}\) and tKAR), and therefore, the correlation between them is automatically considered; e.g. timber board sections with a small \(E_{\text{dyn,F}}\) and a large tKAR-value have low strength and low stiffness properties.

Within one particular timber board, the global indicator \(E_{\text{dyn,F}}\) is constant. Thus, and because of the hierarchical model of the tKAR-value, sections within the same timber board are correlated (within-member correlation).

7.1.4 Numerical model – Estimation of the load-bearing capacity

In the last sub-model, the load-bearing capacity \(f_{\text{m,g}}\), the bending stiffness \(E_{\text{m,g}}\), and the type of failure of the simulated GLT beams are estimated. The analysis is performed with the numerical strain-based model described in Chapter 6. The specific characteristic of this model is that the bending failure of the GLT beam is defined by a tensile failure of a lamella’s entire cross-section. Local failure mechanism through, e.g. local growth irregularities within knot clusters, are not explicitly considered.

At this point, it has to be mentioned that the numerical model presented here is clearly simplified. However, theoretically, the numerical model can be replaced by a more advanced model that considers anisotropy, plasticity, shear and so on.

\(^1\) cited in Wiegand (2003)
7.1.5 Monte Carlo simulation

Through the application of all four sub-models, GLT beams are simulated and their material properties are estimated. Based on a sufficient amount of simulations, values of interest, such as the characteristic value of the load-bearing capacity \( f_{m,g,k} \), or the mean value of the bending stiffness \( E_{m,g,\text{mean}} \), can be calculated. Furthermore, the most suitable distribution functions of \( f_{m,g} \) and \( E_{m,g} \) can be fitted. Examples are illustrated in Fig. 7.3.

The presented approach is developed to investigate the influence of different parameters, such as the position and characteristics of knot clusters, or the quality of FJ, on the load-bearing capacity of GLT beams. To perform a probabilistic investigation of the parameters, it is necessary that (a) the calculation time is within an acceptable range, and (b) the simulations can be performed in parallel. To fulfil both requirements, the GLT model is programmed in Matlab©. The probabilistic investigation is performed on Central HPC Cluster Brutus. There, the Matlab©-code is compiled into standalone executable programs by using the Matlab© Compiler mcc. The compiled program does not check out a Matlab© license, thus the model can run in parallel, independently of program licences. As a result, probabilistic investigations can be performed within a few minutes/hours, depending on the number of simulations and beam dimensions (number of elements).

7.1.6 Differences to existing approaches

In this chapter the presented GLT model is compared with existing approaches for modelling GLT beams; e.g. Foschi & Barrett (1980), Ehlbeck et al. (1985b,a,c), Colling (1990), Blaß et al. (2008). It is obvious that the structure is comparable; i.e. GLT beams having a specific beam setup are simulated and their load-bearing capacities are estimated. However, in detail significant differences exist. The most important of them are summarised:

- **Strength and stiffness related indicators:** All above mentioned models contain two strength and stiffness related indicators: one to describe the material properties of the defect-free timber (density \( \rho \)), and one that describes the local strength and stiffness reduction due to knots (usually tKAR). In the presented study, attention was paid to find the most efficient indicators (Tab. 5.1 and 5.3). The analyses show that \( E_{\text{dyn,F}} \) seems to be significantly better for modelling defect-free timber, whereas the efficiency of tKAR could be confirmed. Thus \( E_{\text{dyn,F}} \) and tKAR are chosen as strength and stiffness related indicators.

- **Probabilistic description of timber boards:** The probabilistic model takes into account the natural growth characteristics of timber\(^2\) i.e. the position and the characteristic of knot clusters are simulated. For the probabilistic description of all three indicators of the probabilistic model (\( E_{\text{dyn,F}} \), tKAR and \( d \)), the most applicable distribution functions are selected, in such a way that the basic population of each indicator is best represented.

- **Length independent timber boards:** In the presented approach, the tKAR-value is modelled hierarchically. As a result, the weakest knot cluster within a timber board (tKAR\(_{\text{max}}\)) depends

\(^2\)In the model presented by Colling (1990), the natural growth characteristics are considered too.
on the timber board length; i.e. $t\text{KAR}_{\text{max}}$ is larger for long timber boards than for short ones. Thus timber boards of each length can be simulated. In contrast, Blaß et al. (2008) describe $t\text{KAR}_{\text{max}}$ by a distribution function (the $t\text{KAR}$-values of the other WS are modelled based on $t\text{KAR}_{\text{max}}$). However, the estimated distribution function of $t\text{KAR}_{\text{max}}$ is investigated on a sample of timber boards, having a specific length. As a result, $t\text{KAR}_{\text{max}}$ would be overestimated when simulating shorter timber boards, and vice versa. The length independence of timber boards is of particular importance for the investigation of the influence of the timber board length on the load-bearing capacity.

- **Estimation of the material properties:** The tensile related material properties are estimated on full scale tensile tests; testing length $>3'300$ mm. Compared to former studies where the testing length was $137.5$ mm; see Heimeshoff & Glos (1980) for the test setup. A drawback of using small test specimens is that effects that are reducing the strength and stiffness properties, such as the influence of local grain deviation before and after the knot clusters, are not considered. The influence is of particular importance for large knot clusters having numerous knots, where the knot cluster itself is larger than the testing length.

- **Material model:** The load-bearing capacity of GLT is highly dependent on the tensile related material properties of knot clusters. Therefore, the material models are developed to predict the tensile stiffness and the tensile strength of knot clusters ($t\text{KAR} \geq 0.1$), instead of timber board sections. Compared to other material models, the approach presented here shows a significant larger influence of knots on the local strength reduction.

- **Numerical model:** For the estimation of the load-bearing capacity of the simulated GLT beams, a numerical strain-based model is developed. The characteristic of this model is that the bending failure of the GLT beam is defined by a tensile failure of a lamella’s entire cross-section. Thereby, only the mean axial strains (mean axial stresses) are considered. Compared to the numerical model of other approaches, the presented one has a very simple failure criterion.

- **Validation:** Until now, none of the numerical models has ever been validated with GLT beams having well-known local material properties; i.e. GLT beams where the exact position of each particular knot cluster and each particular FJ is known. The only exception is Ehlsbeck & Colling (1987a,b), who tested altogether nine GLT beams, where the above-mentioned information of the lowest two lamellas were known. However, only in two GLT beams, a FJ was placed in the highest loaded area – both failed within the FJ. As a result, the quality of the numerical models, in terms of considering varying material properties and detecting the type of failure, is not completely proved yet. The presented approach is validated on all together 24 GLT beams having well-known local material properties – in all 24 GLT beams a FJ is located in the high loaded area of the lowest lamella.

- **Independent sub-models:** The approach contains four independent sub-models; i.e. each of them can be exchanged separately. Therefore, the approach can be easily upgraded in the case that new probabilistic models, better material models or more advanced numerical models are developed.
Machine-grading indicators: As already mentioned in the introduction, one of the major goals of this thesis is to support the enhancement of machine graded timber, or more precisely machine-grading indicators. Thus, the entire GLT model (including the probabilistic model and the material model) is developed in such a way that the strength and stiffness related indicators \( E_{\text{dyn},F} \) and \( t\text{KAR} \) can be exchanged with the machine-grading indicators \( E_m \) and \( K_m \). The application is introduced in Chapter 8.

In conclusion, even though the GLT model has a similar structure to existing approaches, significant differences can be found within each particular part of the model. In addition to the differences, it has to be mentioned that the entire approach, including all the experimental investigations and all the developed models, is developed within one research project. Thus, points of conflict, such as missing information through knowledge transfer or misinterpretations, could be avoided. In comparison to other models this is different, there the experimental investigations of timber boards, the probabilistic model, the material model and the numerical model are developed within more than one research projects.

7.2 Examples of application

In the following chapter, the results of selected investigations are illustrated and described. The simulated GLT beams have a height of \( h = 600 \text{ mm} \) and a span of \( l = 18 \cdot h = 10'800 \text{ mm} \), to assure an optimal comparability to the values given in EN 1194 (1999). GLT beams of strength grades GL24h and GL36h are simulated, both with shorted timber boards \( L \sim N(2.15, 0.50) \) and with non-shorted timber boards \( L \sim N(4.3, 0.71) \). Further, a minimal and a maximal timber board length is introduced \( l = [1, 6] \). The tensile capacity of FJ is assumed to be \( f_{t,j} = f_{t,\text{WS}}|_{\text{tKAR}=0.2} \). For each type of GLT beam, \( n = 10^3 \) simulations are conducted to estimate the material properties. Fig. 7.3 illustrates the bending stiffness and the load-bearing capacity of all \( n = 10^3 \) simulations for the lower strength class GL24h, non-shorted timber boards. The results of the simulations are summarised in Tab. 7.1. They show that the load-bearing capacity of both strength classes are slightly underestimated compared to the values given in EN 1194 (1999): \( f_{m,g,k} = 24 \text{ MPa} \) and \( f_{m,g,k} = 36 \text{ MPa} \). For both strength classes, the GLT beams fabricated out of longer boards have a slightly higher load-bearing capacity. This might be a result of the lower number of FJ. In addition to the absolute values, also their variability seems quite realistic, as JCSS (2006) recommends a COV = 0.15 for \( f_{m,g} \).

The results of the simulations are summarised in Tab. 7.1. They show that the load-bearing capacity of both strength classes are slightly underestimated compared to the values given in EN 1194 (1999): \( f_{m,g,k} = 24 \text{ MPa} \) and \( f_{m,g,k} = 36 \text{ MPa} \). For both strength classes, the GLT beams fabricated out of longer boards have a slightly higher load-bearing capacity. This might be a result of the lower number of FJ. In addition to the absolute values, also their variability seems quite realistic, as JCSS (2006) recommends a COV = 0.15 for \( f_{m,g} \).

The comparison between the estimated stiffness properties and the values given in EN 1194 (1999), \( E_{m,g,\text{mean}} = 11'600 \text{ MPa} \) and \( E_{m,g,\text{mean}} = 14'700 \text{ MPa} \), shows a wide agreement; i.e. small underestimation for the lower strength class (\( \sim 8\% \)) and a small overestimated for the upper strength class (\( \sim 3\% \)). On the other hand, the variability is significantly below the value given in JCSS (2006): COV = 0.13. However, as already mentioned in Chapter 3, the value recommended in JCSS (2006) seems to be too large. Compared to experimental investigations, the estimated variability is only slightly underestimated, e.g. Schickhofer (1995) with COV = 0.04 – 0.06, and Falk et al. (1992) with COV = 0.06 – 0.075.
Fig. 7.3: Estimated bending stiffness (left) and estimated load-bearing capacity (right) of GL24h, non-shorted timber boards. Number of simulations \( n = 10^3 \)

<table>
<thead>
<tr>
<th>Strength class</th>
<th>Board length</th>
<th>( f_{m,g} )</th>
<th>( f_{m,k} )</th>
<th>COV</th>
<th>( E_{m,g} )</th>
<th>COV</th>
<th>Type of failurea</th>
</tr>
</thead>
<tbody>
<tr>
<td>GL24h</td>
<td>shorted</td>
<td>27.1</td>
<td>21.1</td>
<td>0.14</td>
<td>10'700</td>
<td>0.03</td>
<td>26 62 12</td>
</tr>
<tr>
<td></td>
<td>non-shorted</td>
<td>27.7</td>
<td>21.4</td>
<td>0.15</td>
<td>10’600</td>
<td>0.04</td>
<td>14 78 8</td>
</tr>
<tr>
<td>GL36h</td>
<td>shorted</td>
<td>42.2</td>
<td>33.9</td>
<td>0.13</td>
<td>15’200</td>
<td>0.03</td>
<td>43 48 9</td>
</tr>
<tr>
<td></td>
<td>non-shorted</td>
<td>44.6</td>
<td>35.0</td>
<td>0.13</td>
<td>15’100</td>
<td>0.04</td>
<td>26 66 8</td>
</tr>
</tbody>
</table>

aType of failure in the lowest lamella

In addition to the strength and stiffness properties, also the type of failure is investigated. It is obvious that the amount of FJ-failure increases with increasing strength grade (timber boards of higher strength grades have less and smaller knots) and with decreasing board length (more FJ). In average, the amount of FJ-failure is about 20% for the lower and 35% for the upper strength class. This corresponds to different studies presented in the literature (Chapter 2). Compared to the results of other simulation models, the amount of FJ is relative small; e.g. Ehlbeck & Colling (1987b) estimated a number of FJ-failures of about 70%. The differences might be the result of a change of the FJ-quality since 1987, or the stronger influence of knots within the present material model. However, Ehlbeck & Colling (1987b) also detected a negative interrelation between the board length and the number of FJ-failure.

In conclusion, the results of the presented examples of application seem to be reasonable. All investigated parameters, that are (a) the characteristic value of the load-bearing capacity, (b) the variability of the load-bearing capacity, (c) the mean value of the bending stiffness, (d) the variability of the bending stiffness, and (e) the number of FJ-failure, correspond to required/recommended values or to the results of experimental investigations. This indicates that the presented GLT model can be applied for the investigation of GLT beams.
7.3 Parameter study

One of the main advantages of the presented GLT model is that the influence of different parameters on the load-bearing capacity, bending stiffness, and the type of failure can be investigated. In the following chapters, the influence of the beam size, the mean distance between WS, both strength and stiffness related indicators, and so on is investigated and discussed. The investigation is performed on GLT beams under four-point bending in accordance to EN 1194 (1999).

Unless otherwise mentioned, the following input parameters are chosen: the simulated GLT beams have a height of \( h = 600 \) mm and a span of \( l = 18 \cdot h = 10'800 \) mm, to assure an optimal comparability to the values given in EN 1194 (1999). GLT beams of strength grades GL24h and GL36h are simulated, with non-shorted timber boards \( L \sim N(4.3, 0.71) \). In addition, a minimal and a maximal timber board length is introduced \( l = [1, 6] \). The tensile strength of FJ is assumed to be \( f_{t,j} = f_{t,WS|tKAR=0.2} \). For each type of GLT beam, \( n = 10^3 \) simulations are conducted.

7.3.1 Size effect

One of the most important applications of the GLT model is the investigation of the size effect. As already mentioned in Chapter 2, the load-bearing capacity of a GLT beam is highly related to the load-bearing capacity of its weakest link(s). The probability of the occurrence of a weak zone within a GLT beam is increasing with increasing beam dimensions; i.e. the bending strength decreases with increasing beam dimensions. On the other hand it is established that the influence of local weak sections is decreasing with increasing beam dimensions, as a result of homogenisation (e.g. Schickhofer et al. 1995). Thus the variability of the material properties is decreasing.

To investigate the size effect, GLT beams with six different dimensions are simulated: \( h = 320, 400, 480, 600, 800, \) and \( 1'200 \) mm (8, 10, 12, 15, 20, and 30 lamellas, each 40 mm thick). To assure comparability with EN 408 (2003), a constant ratio \( l = 18 \cdot h \) is assumed. The spans of the beams are \( l = 5'760, 7'200, 8'640, 10'800, 14'400, \) and \( 21'600 \) mm. The results are illustrated in Fig. 7.4.

The illustration of the estimated load-bearing capacity, Fig. 7.4 (left), shows that both assumptions can be confirmed: (a) the mean value of the bending strength \( f_{m,g,mean} \) is decreasing with increasing beam dimensions, and (b) the variability of the bending strength is decreasing. Both effects are influencing the characteristic value \( f_{m,g,k} \) in opposite directions. It seems that the magnitude of both influences is similar and thus \( f_{m,g,k} \) is not, or only marginally influenced by the beam dimensions.

Fig. 7.4 (middle) illustrates the estimated bending stiffness \( E_{m,g} \). It is obvious that \( E_{m,g,mean} \) in not influenced by the beam’s dimensions. However, the effect of homogenisation is significant; i.e. the variability of \( E_{m,g} \) is decreasing with increasing beam dimensions.

The amount of FJ-failure is illustrated in Fig. 7.4 (right). The percentage of FJ-failure is decreasing with increasing beam dimensions. This is reasonable, given that the variability of the tensile strength is larger for WS than for FJ. The lower percentage of FJ-failures for very short GLT beams may seem to be inconsistent. However, it can be explained with the small
7.3. Parameter study

Fig. 7.4: Parameter study: size effect

Fig. 7.5: Parameter study: mean d
beam length. The span of the shortest beams is 5’760 mm, and thus the length of the maximal bending moment is only 1’920 mm. As a result, less than 50% of the simulated beams have a FJ located in the lowest lamella, within the area of the maximal bending moment.

### 7.3.2 Distance between WS

The geometrical setup of timber boards is described by the distance between WS ($d$). Four different types of GLT beams are simulated: mean $d = 400, 500, 600$, and $700$ mm (the standard deviation $\sigma = 246$ mm is assumed to be constant). The results are illustrated in Fig. [7.5]. It is obvious that both $f_{m,g}$ and $E_{m,g}$ are increasing with increasing $d$, as a result of the smaller number of WS. However, even though the number of WS (compared with the number FJ) is decreasing, no unambiguous influence of $d$, on the number of FJ-failure could be detected.

### 7.3.3 Dynamic modulus of elasticity

The probabilistic model of $E_{dyn,F}$ contains two parameters $\mu$ and $\tau$. $\mu$ represents the logarithmic mean and is considered to be deterministic. $\tau$ represents the between-member variability and is represented by a Normal distributed random variable $\tau \sim N(0, \sigma_{\tau})$.

In Fig. [7.6], the influence of the mean $E_{dyn,F}$ is illustrated. $\mu = 9.00$ corresponds to $E_{dyn,F} = 8’100$ MPa, $\mu = 9.25$ to $E_{dyn,F} = 10’400$ MPa, $\mu = 9.50$ to $E_{dyn,F} = 13’400$ MPa, and $\mu = 9.75$ to $E_{dyn,F} = 17’200$ MPa. It is obvious that an increase of $\mu$ leads to an increase of both, load-bearing capacity and bending stiffness.

The influence of the between-member variability (expressed through the parameter $\tau$) is illustrated in Fig. [7.7]. An increase of $\tau$ leads to a marginal decrease of the load-bearing capacity, combined with an increase of its variability. Both effects have a negative influence on the characteristic value. In addition, $\tau$ influences the load-bearing capacity of FJ $f_{t,j}$. Per definition, $f_{t,j}$ depends to the load-bearing capacity of the weaker of the two associated timber boards (Eq. [5.7]). It is obvious that $f_{t,j}$ is decreasing with increasing variability of $E_{dyn,F}$ and thus the number of FJ-failure increases. This effect is more significant for the upper strength class.

### 7.3.4 tKAR-value

The local strength and stiffness reduction through knot clusters is modelled with the tKAR-value. tKAR is described by a hierarchical model that contains three parameters: $\mu$ represents the logarithmic mean tKAR, $\tau$ represents the between-member variability, and $\epsilon$ represents the within-member variability. $\tau$ and $\epsilon$ are represented by Normal distributed random variables $\tau \sim N(0, \sigma_{\tau})$ and $\epsilon \sim N(0, \sigma_{\epsilon})$.

The influence of the mean tKAR ($\mu$) is illustrated in Fig. [7.8]. $\mu = -2.5$ corresponds to tKAR = 0.08, $\mu = -2.0$ to tKAR = 0.14, $\mu = -1.5$ to tKAR = 0.22, $\mu = -1.0$ to tKAR = 0.37, and $\mu = -0.5$ to tKAR = 0.61. The load-bearing capacity and the number of FJ-failure are decreasing significantly with increasing tKAR. In addition, also the bending stiffness is influenced.
7.3. Parameter study

- Parameter $m$
- Parameter $E_{dyn,F}$
- Parameter $t$

Fig. 7.6: Parameter study: $E_{dyn,F}$

Fig. 7.7: Parameter study: $E_{max}$
Chapter 7. GLT model

Fig. 7.8: Parameter study: tKAR

Fig. 7.9: Parameter study: tKAR
7.3. Parameter study

Fig. 7.10: Parameter study: $t_kAR - \varepsilon$

Fig. 7.11: Parameter study: $t_kAR$ limit
In addition to the mean tKAR, also the influence of its variability is investigated. Both the between-member variability ($\tau$) and the within-member variability ($\epsilon$) are analysed. In both cases, an increase of the variability is connected to a decrease of the load-bearing capacity and the number FJ-failures, whereas the bending stiffness seems to be unaffected (Fig. 7.9, 7.10). The decrease of the characteristic value of the load-bearing capacity is similar for $\tau$ and $\epsilon$. However, they have different causes: The decrease through the within-member variability ($\epsilon$) is mainly influenced by a decrease of the mean value $f_{m,g,\text{mean}}$, the variability keeps almost constant. On the other hand the decrease though the between-member variability ($\tau$) results from a combination of both effects; i.e. a decrease of the mean value $f_{m,g,\text{mean}}$ and an increase of the variability.

According to its definition, the tKAR-value of every section has to be within the interval $[0, 1]$. To consider the upper limit, a combined realization of large $\tau$ and large $\epsilon$ is prevented: $\text{tKAR}_{\text{limit}} = 1$. However, through the grading process, an upper limit $\text{tKAR}_{\text{limit}}$ might be introduced. In Fig. 7.11 the influence of different $\text{tKAR}_{\text{limit}}$ is illustrated. It seems that up to $\text{tKAR}_{\text{limit}}$,$\text{L25}$ $\approx$ 0.5 and $\text{tKAR}_{\text{limit}}$,$\text{L40}$ $\approx$ 0.4, which are realistic values for the two strength grades, the influence is rather small.

### 7.3.5 Finger joints

In the presented approach, the tensile strength of FJ is assumed to be equal to the tensile strength of a knot cluster having a specific tKAR ($f_{t,j} = f_{t,WS|\text{tKAR}}$). In Fig. 7.12 the outcomes of the GLT model using different tKAR-values ($0.2 \leq \text{tKAR} \leq 0.3$) are illustrated. It is clear that a larger tKAR-value leads to a lower tensile capacity $f_{t,j}$. As a result, the number of FJ-failures increase and thus the load-bearing capacity $f_{m,g}$ decreases.

The load-bearing capacity of a FJ depends on the material properties of the two adjacent timber boards and the fabrication quality. Using the presented model, the effects of a theoretical quality improvement can be investigated. An example is illustrated in Fig. 7.13. Logically increasing FJ-quality leads to an overall quality increase of the GLT beam, and the reverse is also true. For the upper strength class, the influence is significantly larger; e.g. a quality change of $\pm 20\%$ has the following influence on the characteristic value of the load-bearing capacity: +0.3 MPa and $-1.0$ MPa for GL24h, and +1.0 MPa and $-3.0$ MPa for GL36h. The results indicate that even a significant improvement of the FJ-quality of about $20\%$ would lead only to marginal improvement of the GLT beam. On the other hand, a reduction of the FJ-quality would influence the load-bearing capacity of GLT beams more clear. The influence of an increase of the FJ-quality was already investigated by Ehlbeck & Colling (1987b), who also reported a larger influence on the load-bearing capacity of GLT beams of higher strength classes.

### 7.3.6 Model uncertainties – material model

The material models (strength and stiffness models) described in Chapter 5 are subjected to model uncertainties. They are expressed through the error term $\epsilon \sim N(0, \sigma_\epsilon)$ – Eq. 5.1. As already mentioned in Chapter 5, the model uncertainties are underestimated if censored regres-
7.3. Parameter study

Fig. 7.12: Parameter study: $f_{t,j} = f_{w,s} | t_{KAR}$

Fig. 7.13: Parameter study: $f_{t,j}$

Fig. 7.14: Parameter study: $f_{w,s}$
sion analysis is used for the parameter estimation, which was done for the strength model. To compensate that, a larger $\sigma_e = 0.20$ was assumed (instead of $\sigma_e = 0.15$).

Fig. 7.14 illustrates the influence of $\sigma_e$. It is obvious that an increase of $\sigma_e$ leads to a reduction of the load-bearing capacity, whereas the bending stiffness is unaffected. In addition to the reduction of the load-bearing capacity, its variability is increasing. Both effects are negatively influencing the characteristic value of the load-bearing capacity $f_{m,g,k}$. However, the influence is still within an acceptable range; e.g. an increase from $\sigma_e = 0.15$ to $\sigma_e = 0.20$ leads to a reduction $\Delta f_{m,g,k} \approx 1$ MPa for the lower strength class, and $\Delta f_{m,g,k} \approx 2$ MPa for the upper strength class.

7.3.7 Statistical uncertainties

The presented approach contains three models for the probabilistic representation of timber boards ($d$, tKAR and $E_{dyn,F}$) and two material models ($f_{t,WS}$ and $E_{t,WS}$); with 15 parameters altogether. For all parameters, the expected values and their statistical uncertainties are identified. Furthermore, the correlations between the parameters of the same model are identified. The statistical uncertainties of the GLT model are calculated assuming that (a) the five models are uncorrelated (except the correlation between the error terms), and (b) the statistical uncertainties of the parameters are normally distributed.

In Fig. 7.15, the results of 100 realisation of $f_{m,g,k}$ and $E_{m,g,mean}$ are illustrated; strength class GL24h. Each realisation contains $10^3$ simulated GLT beams. It seems that the influence of the statistical uncertainties is rather small: $\text{COV}(f_{m,g,k}) = 0.016$, $\text{COV}(f_{m,g,mean}) = 0.012$, $\text{COV}(E_{m,g,mean}) = 0.011$ and $\text{COV}(FJ) = 0.082$.

7.4 Model summary

A probabilistic approach for modelling GLT beams is introduced. The GLT model contains four independent sub-models. At first, timber boards (or, more precisely, the strength and stiffness related indicators of timber boards) are simulated. Afterwards, GLT beams are fabricated (out of the simulated timber boards). Thereby every kind of fabrication procedure can be recreated; e.g. length of the timber boards, cutting criteria or beam dimensions. Subsequently to the GLT fabrication, the strength and stiffness properties of each timber board section are allocated, based on information about $E_{dyn,F}$, tKAR and FJ. In the last sub-model, the load-bearing capacity, the bending stiffness and the type of failure of the simulated GLT beams are estimated using a numerical strain-based model. For a probabilistic investigation of the input parameters, a Monte Carlo simulation is performed.

The application of the GLT model is illustrated on selected examples, and the influence of the following input parameters on the load-bearing capacity of GLT beams is investigated: (a) beam dimension, (b) distance between WS, (c) absolute value and variability of both strength and stiffness related indicators, (d) grading criteria (upper limits for tKAR-value), (e) quality of FJ, and (f) timber board length (distance between FJ). As an example, the investigation of
7.4. Model summary

Fig. 7.14: Parameter study: $\sigma(f)$
the beam dimensions indicates that an increase of the beam dimensions leads to a decrease of the mean value of the load-bearing capacity $f_{m.g,k}$, while the variability of $f_{m.g}$ is decreasing. Both effects are influencing the characteristic value $f_{m.g,k}$ in opposite directions. It seems that the magnitude of both influences are similar and thus $f_{m.g,k}$ is only marginally affected by the beam dimensions.

In conclusion, the results of all the presented examples of application seem to be reasonable. This clearly shows that the presented GLT model can be applied for the investigation of influencing parameters.

Fig. 7.15: Statistical uncertainties: 100 realisations, each $10^3$ simulations, (left) $f_{m.g,k}$ (right) $E_{m.g,mean}$ – strength class GL24h
Chapter 8

GLT model – using machine-grading indicators

The GLT model presented in Chapter 7 is based on two strength and stiffness related indicators measured in laboratory ($E_{\text{dyn,F}}$ and $t\text{KAR}$). In particular the measurement of $t\text{KAR}$ is time consuming and thus not really efficient for practical application. However, as mentioned before, all the investigated timber boards were machine graded using the GoldenEye-706 grading device. Therefore, in addition to the laboratory measured indicators, also the indicators measured by the grading device are known; i.e. an estimation of the dynamic modulus of elasticity ($E_m$) and a knot indicator ($K_m$). In the presented chapter, the GLT model will be extended for machine-grading indicators; i.e. the laboratory measured indicators ($E_{\text{dyn,F}}$ and $t\text{KAR}$) are exchanged by the indicators from the grading device ($E_m$ and $K_m$). Both the probabilistic and the material model are developed for the new indicators, following the same principles described in Chapters 4 and 5, respectively. Afterwards, the new material model is validated with the 24 GLT beams having well-known local material properties (Chapter 3).

The major advantage of using machine-grading indicators is that they are measured automatically during the grading process; i.e. $E_m$ and $K_m$ are measured for each particular timber board that is graded by a device that combines measurement of the dynamic modules of elasticity and X-ray, such as GoldenEye-706. As a result, both indicators can be collect automatically and thus new probabilistic models to describe the characteristics of timber boards of different strength grades, growing regions, cross-sections, and so on can be easily developed. A further advantage of machine-grading indicators is that they are reproducible and objective. To illustrate this advantage, the number of investigated knots has to be considered. In this study more than 7’000 knots (in 400 timber boards) are measured. It is obvious that through the huge amount of knots, typing errors are not preventable. Further it has to be considered that the geometrical shape of knots are often difficult to measure, in particular for intergrown knots. Thus even with clear definitions, the measurements would be varying between the individual persons, performing the knot measurements.
Fig. 8.1: Correlation between the laboratory measured and the machine-grading indicators: (left) global indicator $E_{\text{dyn,F}}$ and $E_m$, (right) knot indicator tKAR and $K_m$

8.1 Comparison between indicators

In this chapter the laboratory measured indicators ($E_{\text{dyn,F}}$ and tKAR) are exchanged by machine-grading indicators ($E_m$ and $K_m$). At first, the correlations between the indicators are investigated. The instigation is performed on the 200 timber boards (including 864 knot clusters), on which the stiffness properties are measured. The results are illustrated in Fig. 8.1. Both indicators measured by the grading device are strongly correlated to those measured in the laboratory: $\rho(E_m, E_{\text{dyn,F}}) = 0.98$ and $\rho(K_m, \text{tKAR}) = 0.77$. Thus, it seems to be adequate to extend the GLT model, introduced in Chapter 7, for machine-grading indicators.

For developing the probabilistic model and the material model, the positions and the characteristics of WS have to be identified. As before, they are extracted from the knot-profile, this time from the knot-profile measured with the grading device (grey line illustrated in Fig. 8.2). Therefore, a definition of a WS must be established. In the present work, the threshold for $K_m$ is defined in a way that the number of identified WS is similar as identified in Chapter 4. As a result, a threshold $K_m = 700$ (instead of tKAR = 0.1) is chosen; i.e. knot clusters with $K_m \geq 700$ are defined as WS, whereas knot clusters with $K_m < 700$ are neglected. Using this threshold, a total number of 2'824 WS (L25: 1'578 WS and L40: 1'246 WS) are identified, compared to 2'870 WS using tKAR=0.1.

8.2 Probabilistic model

Following the same principle as in Chapter 4, a probabilistic model for the representation of strength and stiffness related indicators is developed. The model contains one parameter to describe the distance between WS ($d$), and two strength and stiffness related indicators ($K_m$ and $E_m$). The probabilistic model is developed for two strength grades (L25 and L40), based on 200 timber boards per strength grade. As already mentioned, the positions and the characteristics
8.2. Probabilistic model

Fig. 8.2: Knot parameter distribution within one timber board

of the WS are extracted from the knot-profile. Based on the data, the model parameters are estimated. They are summarised in Tab. 8.1.

According to the growth characteristic of Norway spruce and the sawing process, a shifted Gamma distribution is selected to model \( d \) (Chapter 4). The estimated parameters, summarised in Tab. 8.1, correspond to expected values \( E(d_{L25}) = 487 \text{ mm} \) and \( E(d_{L40}) = 577 \text{ mm} \), and to standard deviations \( \sigma(d_{L25}) = 224 \text{ mm} \) and \( \sigma(d_{L40}) = 311 \text{ mm} \). In contrast with the model developed with the laboratory measured indicator tKAR, here a significant difference between the two strength grades is identified. This indicates that the knot parameter \( K_m \) might be more sensitive for the identification of WS.

The characteristics of WS are described with the knot indicator \( K_m \), which is assumed to be Lognormal distributed. \( K_m \) is modelled hierarchically, to consider the within-member correlation. As for tKAR, an upper limit exists per definition; the value of \( K_m \) for an black X-ray image (see Chapter 3). However, as the density of knots is not infinite, this value cannot be theoretically reached. Therefore, in the present thesis, an upper limit \( K_{m,\text{limit}} = 10'000 \) is chosen. According to Fig. 8.1 (right), this might be comparable to very large tKAR-values. Nevertheless, to model timber boards it is still possible to use a smaller \( K_{m,\text{limit}} \) to introduce a grading criterion. However, a comparison of the two probabilistic models (tKAR and \( K_m \)) shows that in both, the within-member variability (\( \epsilon \)) is significantly larger than the between-member variability (\( \tau \)). Furthermore, both models indicate a lower within-member variability (\( \epsilon \)) for the upper strength class.

For modelling the material properties of the clear wood, the global parameter \( E_m \) is used. As before, the global indicator is assumed to be Lognormal distributed. The estimated parameters correspond to expected values \( E(E_{m,L25}) = 12'020 \text{ MPa} \) and \( E(E_{m,L40}) = 16'300 \text{ MPa} \), and to standard deviations \( \sigma(E_{m,L25}) = 1'410 \text{ MPa} \) and \( \sigma(E_{m,L40}) = 1'420 \text{ MPa} \). The expected value of the upper strength grade is significantly larger (~35%), and the standard deviation seems almost unaffected by the strength grade. Compared to the global indicator measured in the laboratory \( E_{\text{dyn,F}} \), the expected values are slightly larger (2 − 3%).
Table 8.1: Compilation of model for the probabilistic representation of timber – based on machine-grading indicators

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>Strength grade</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>L25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.27 (0.021)</td>
</tr>
<tr>
<td>$d$</td>
<td>$k$ (COV)</td>
<td>0.0067 (0.017)</td>
</tr>
<tr>
<td></td>
<td>$\nu$ (COV)</td>
<td>0.650</td>
</tr>
<tr>
<td>$K_m$</td>
<td>$\mu$ (COV)</td>
<td>7.37 (0.0014)</td>
</tr>
<tr>
<td></td>
<td>$\sigma_\tau$ (COV)</td>
<td>0.194 (0.050)</td>
</tr>
<tr>
<td></td>
<td>$\sigma_\epsilon$ (COV)</td>
<td>0.376 (0.018)</td>
</tr>
<tr>
<td>$E_m$</td>
<td>$\mu$ (COV)</td>
<td>9.40 (0.0009)</td>
</tr>
<tr>
<td></td>
<td>$\sigma_\tau$ (COV)</td>
<td>0.114 (0.050)</td>
</tr>
</tbody>
</table>

Correlations between the three sub-models are investigated, mainly the correlation between timber boards of the same strength grade. That includes correlation between the mean $K_m$ with one timber board, $E_m$, and the mean distance between the WS within one board. As for the laboratory measured indicators, no unambiguous correlations are identified. Due to the low correlations, they are not considered within the model.

8.3 Material model

The material model is developed as described in Chapter 5, using $E_m$ and $K_m$ instead of $E_{dyn,F}$ and tKAR. As before, one indicator is used to describe the mean material properties of the timber boards ($E_m$), and the other one is used to describe the local strength and stiffness reduction due to knots ($K_m$).

8.3.1 Stiffness model

Taking into account both machine-grading indicators, a model for the prediction of the tensile stiffness of WS ($E_{WS}$) is developed. For the parameter estimation, the measured stiffness properties of altogether 846 WS are considered. The parameters are summarised in Tab. 8.2. A comparison with the model parameters identified in Chapter 5 shows large similarities; i.e. the parameters, their uncertainties and their correlations are similar. By applying the stiffness model to predict $E_{WS}$, a strong correlation $\rho = 0.920$ to the measured values is identified; Fig. 8.3 (left). The correlation is similar to the obtained with the laboratory measured indicators, even slightly better.

The presented stiffness model is developed in order to predict the tensile strength of WS. However, if the model is used for the prediction of the stiffness of the CWS (using tKAR = 0)
these are slightly underestimated. The difference of the measured and the predicted $E_{\text{CWS}}$ is about 4%.

8.3.2 Strength model

The model for the prediction of the tensile strength of WS ($f_{t,\text{WS}}$) is developed using censored regression analysis. This is a method that takes into account all WS within the testing range of the timber board (Chapter 5). The characteristics of the WS ($K_m$), are extracted from the knot-profile. Using the introduced threshold $K_m = 700$, altogether 2'987 WS are identified, compared to 2'577 WS when using $t\text{KAR} = 0.1$. Taking into account all identified WS, the parameters of the strength model can be estimated; they are summarised in Tab. 8.2. The model uncertainties, expressed through the error term $\varepsilon$, are about 20% smaller than when the laboratory measured indicators are used. However, as already mentioned, the model uncertainties are underestimated when using censored regression analysis to estimate the parameters. To compensate that, a larger $\sigma_{\varepsilon} = 0.16$ (corresponds to $\sigma_{\varepsilon,\text{lab}} = 0.2$ minus 20%) is assumed.

One possibility to validate the strength model is a prediction of the tensile capacity of the entire timber board $f_t$. Therefore, the tensile capacity of a timber board is defined as the tensile strength of the weakest section within the timber board (largest value of $K_m$). A comparison between the measured and the predicted tensile capacity show a rather high correlation $\rho = 0.818$, Fig. 8.3 (right), even better than when laboratory measured indicators are used.

8.4 Numerical model

In addition to the probabilistic and the material models, the application of the numerical model using machine-grading indicators is investigated. A definition of the tensile strength of FJ is established, in accordance with Eq. (5.7). It is assumed that the tensile strength of a FJ ($f_{t,j}$) is equal to the tensile strength of a WS ($f_{t,\text{WS}}$) having a specific $K_m$; here $K_m = 1'200$ is assumed. Thus, the tensile strength of a FJ is $f_{t,j} = f_{t,\text{WS}|K_m=1200}$.
Chapter 8. GLT model – using machine-grading indicators

The numerical model is validated with the GLT beams described in Chapter 3. Between the measured and the estimated material properties, a wide agreement could be identified (Fig. 8.4). In average the load-bearing capacity \(f\) is slightly underestimated by 1.4 MPa (\(~3\%\)). The maximum underestimation is 10.1 MPa and the maximum overestimation is 6.9 MPa. For the bending stiffness \(E_{m,g}\), the mean underestimation is 384 MPa (\(~3\%\)), the maximum underestimation is 1'080 MPa and maximum overestimation is 670 MPa. As a result of the large correlation, it seems likely to estimate accurately \(f_{m,g}\) and \(E_{m,g}\) of beams having well-known information about \(E_m\), \(K_m\) and \(FJ\).
### Tab. 8.3: Estimated material properties [MPa] and type of failure [%] – using machine-grading indicators

<table>
<thead>
<tr>
<th>Strength class</th>
<th>Board length</th>
<th>( f_{\text{m,g}} )</th>
<th>COV</th>
<th>( E_{\text{m,g}} )</th>
<th>COV</th>
<th>Type of failure&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>GL24h</td>
<td>shorted</td>
<td>28.6</td>
<td>0.12</td>
<td>7000</td>
<td>0.03</td>
<td>24 71 5</td>
</tr>
<tr>
<td></td>
<td>non-shorted</td>
<td>29.0</td>
<td>0.13</td>
<td>7000</td>
<td>0.04</td>
<td>13 81 6</td>
</tr>
<tr>
<td>GL36h</td>
<td>shorted</td>
<td>44.4</td>
<td>0.11</td>
<td>0000</td>
<td>0.03</td>
<td>44 49 7</td>
</tr>
<tr>
<td></td>
<td>non-shorted</td>
<td>45.5</td>
<td>0.11</td>
<td>0000</td>
<td>0.04</td>
<td>38 60 2</td>
</tr>
</tbody>
</table>

<sup>a</sup>type of failure in the lowest lamella

### 8.5 Example of application

Following the principle introduced in Chapter 7, GLT beams can be simulated and their load-bearing capacity can be estimated. For an optimal comparability, the same GLT beams, as in Chapter 7.2 are simulated: two strength grades (GL24h and GL36h), shorted timber boards \( L \sim N(2.15, 0.50) \) and non-shorted timber boards \( L \sim N(4.3, 0.71) \), height \( h = 600 \) mm, span \( l = 18 \cdot h = 10'800 \) mm, and tensile capacity of FJ \( (f_{t,j} = f_{t,WS} | K_{m} = 1200) \). For each type of beams \( n = 10^3 \) simulations are conducted. The results are summarised in Tab. 8.3.

The results show a small underestimation for the lower strength class GL24h and a small overestimation for the upper strength class GL36h, compared to the values given in EN 1194 (1999); \( f_{\text{m,g,k}} = 24 \) MPa and \( E_{\text{m,g,mean}} = 11'600 \) MPa for GL24h, and \( f_{\text{m,g,k}} = 36 \) MPa and \( E_{\text{m,g,mean}} = 14'700 \) MPa for GL36h. In both strength classes, the GLT beams fabricated out of longer timber boards have a higher load-bearing capacity as a result of the lower number of FJ. For the upper strength class GL36h, the influence of the board’s length is significantly larger; here \( \Delta f_{\text{m,g,k}} = 1.3 \) MPa. In addition to the characteristic values of the load-bearing capacity, also their variabilities seem quite realistic, JCSS (2006) recommend \( \text{COV} = 0.15 \) for \( f_{\text{m,g}} \). A comparison to the results from the GLT model using laboratory measured indicators, shows that the estimated load-bearing capacity is slightly larger, whereas the bending stiffness seems to be similar.

### 8.6 Summary

In the presented chapter, the GLT model introduced in Chapter 7 is extended to machine-grading indicators; i.e. the laboratory graded indicators (\( E_{\text{dyn,F}} \) and \( t_{KAR} \)) are exchanged by the indicators from the grading device (\( E_{\text{m}} \) and \( K_{m} \)). The major advantage of using machine-grading indicators is that they are measured automatically during the grading process; i.e. \( E_{\text{m}} \) and \( K_{m} \) are measured for each particular timber board that is graded by a device, combining measurements of the dynamic modules of elasticity and X-ray. As a result, new probabilistic models can be easily developed. A further advantage of machine measured indicators is that
they are reproducible and objective; i.e. typing errors and measurement uncertainties can be reduced.

The indicators measured with the grading device are compared with the indicators measured in the laboratory. For both, the global indicator and the knot indicator a strong correlation $\rho(E_m, E_{\text{dyn}, F}) = 0.98$ and $\rho(K_m, tKAR) = 0.77$ is identified.

The probabilistic and the material model, developed for the new indicators, show a wide agreement to them developed with the indicators measured in the laboratory. The validation of the material model even indicates a slightly better agreement.

The application of the numerical model, using machine-grading indicators, is validated with the 24 GLT beams having well-known local material properties. The estimation of the load-bearing capacity and the bending stiffness are slightly below the measured values. On average, the underestimation is only 1.4 MPa (~3%) for the load-bearing capacity and 384 MPa (~3%) for the bending stiffness.

The chapter concludes with an example of application. Thereby, four different types of GLT beams were simulated and the results correspond to those recommended by the literature.
Chapter 9

Conclusions and outlook

9.1 Conclusions

In the present thesis, the influence of varying material properties on the load-bearing behaviour of GLT beams was investigated. Experimental investigations on GLT beams with well-known local material properties were performed, and a probabilistic approach for modelling GLT beams was developed. In the following paragraphs, the most important topics and outcomes are summarised.

Experimental investigations – structural timber

Two samples, with 200 timber boards each, of strength grades L25 and L40 were investigated. The dimensions and position of every knot with a diameter larger than 10 mm were assessed for all timber boards. Furthermore, different strength and stiffness related indicators were measured; i.e. the density and the dynamic modulus of elasticity based on eigenfrequency and based on ultrasonic runtime.

On half of the timber boards, non-destructive tensile test were performed to investigate local stiffness properties. The investigated timber boards, were subdivided into sections containing knot cluster and sections between knot clusters. From each section, the corresponding expansion (and thus the tensile stiffness) was measured, using an optical camera device. Within the scope of this thesis, the stiffness properties of altogether 864 knot clusters were investigated.

To get a better understanding about local failure mechanisms, the deformation and failure behaviour of selected knot clusters were investigated, by destructive and non-destructive tensile tests. During the tensile tests, pictures were taken with a constant frequency. Based on these pictures, the relative displacements within the investigated knot cluster and thus, the strain distribution were calculated using a digital image correlation software.

The grading of all investigated timber boards, was performed by the GoldenEye-706 grading device. Thus, in addition to the results of the experimental investigation, also machine-grading indicators are known for all timber boards.
Experimental investigations – GLT beams

Out of the investigated timber boards, altogether 24 GLT beams having well-known local material properties were fabricated. The exact position of each particular timber board within the GLT beams was documented, and so, the (a) position of each FJ, (b) position of each knot with a diameter larger than 10 mm, (c) density of each timber board, (d) dynamic modulus of elasticity based on eigenfrequency and based on ultrasonic runtime of each timber board, (e) measured stiffness properties of all knot clusters, located in the tensile loaded area of the GLT beams, and (f) all indicators measured by the grading device, are precisely-known.

On all 24 GLT beams, four-point bending tests were performed. Thereby, the load-bearing capacity and the bending stiffness were measured, and the type of failure was investigated. Furthermore, the deformation behaviour of local weak sections, such as knot clusters or finger joint connections, as well as their influence on the failure of the GLT beams were investigated.

Investigation of strength & stiffness related indicators

The interaction between different strength and stiffness related indicators and the measured material properties was investigated. This included the investigation of numerous indicators to predict (a) the mean material properties of a timber board – global indicators, and (b) the within-member variability – knot indicators. The correlation between the indicators and the material properties, as well as the correlation between indicators, were analysed.

The investigation of the global indicators shows that the dynamic moduli of elasticity (based on eigenfrequency and ultrasonic runtime), are very efficient to predict the mean material properties. Whereas indicators that describe the position of the timber board within the trunk of the tree, i.e. distance to the pith and angle of the annual rings, have no or only minor correlation to the strength and stiffness properties.

According to the position and dimensions of knots, different knot indicators are identified. A comparison with the measured material properties shows that indicators describing the knot area (projected on the timber board’s cross-section), such as the tKAR-value, are the most efficient ones. However, their correlation is still relatively small $\rho \leq 0.6$. In addition to the knot parameter, also the influence of the type and position of knots was investigated. Both have only a marginal influence on the strength and stiffness properties.

The investigation of the strength and stiffness related indicators clearly shows that to efficiently predict local material properties ($E_{WS}$ and $f_{t,WS}$) at least two indicators are necessary: one to describe the material properties of defect-free timber, and one to describe the local strength and stiffness reduction due to knots. Due to its large correlation, the dynamic modulus of elasticity based on eigenfrequency $E_{\text{dyn,F}}$ is used as the global indicator, and the total knot area ratio (tKAR) is used as the knot indicator.
9.1. Conclusions

Probabilistic representation of the variability of timber

Based on the results of the experimental investigations, a probabilistic model for the representation of strength and stiffness related indicators was developed. The specific characteristic of this model is that the natural growth characteristics of timber are considered; i.e. the position and the characteristics of knot clusters can be simulated.

The probabilistic model contains one parameter to describe the geometrical setup of timber boards, distance between WS ($d$), and two strength and stiffness related indicators ($t_{\text{KAR}}$ and $E_{\text{dyn},F}$). For all three model parameters, the most suited distribution functions were selected, in a way that the basic population is best represented: $d$ – shifted Gamma distribution, $t_{\text{KAR}}$ – truncated Lognormal distribution, and $E_{\text{dyn},F}$ – Lognormal distribution. To consider the within-member correlation, the $t_{\text{KAR}}$-value is described by a hierarchical model having two hierarchical levels: (a) the meso-scale to describe the variability of a single board within a sample of boards and (b) the micro-scale to describe the variability within one board. Between the three parameters no unambiguous correlation could be detected.

Prediction of the material properties

Taking into account the two identified strength and stiffness related indicators ($E_{\text{dyn},F}$ and $t_{\text{KAR}}$), a material model was developed, which is particularly focused on the prediction of the tensile stiffness and tensile strength of knot clusters. The characteristic of this material model is that it is based on material properties measured on four meter long timber boards, tested over their entire length.

The stiffness model was developed based on the measured stiffness properties of altogether 864 knot clusters, using linear regression. When applying the model to predict stiffness properties, a large correlation ($\rho = 0.912$) between the measured and the predicted stiffness properties was identified.

To develop the strength model, the failure mechanism of timber boards had to be considered. In a destructive tensile test, a timber board fails only in one section, which is usually the weakest knot cluster. Therefore, the information obtained from one destructive tensile test is the tensile capacity of the weakest knot cluster and the minimal tensile capacity of all non-failed knot clusters. To consider both the equality type and inequality type information, the censored regression analysis is chosen. To estimate the parameters, altogether 2,577 knot clusters were considered. Applying the strength model to predict the strength properties of the investigated timber boards (defined as the tensile strength of the knot cluster with the largest $t_{\text{KAR}}$-value), a large correlation $\rho = 0.751$ between the measured and the predicted strength properties was identified.

The presented material model was also extended for the estimation of the material properties of finger joint connections. Therefore, a very simple and plausible approach was chosen: (a) the tensile stiffness is assumed to be the mean stiffness of the two adjacent defect-free timber boards sections, and (b) the tensile capacity corresponds to the tensile capacity of a knot cluster having a specific $t_{\text{KAR}}$-value.
The developed material model was compared with models from the literature. It indicates a significant larger influence of knots on local strength reduction.

**Simulation of timber boards & finger joint connections**

The material and probabilistic models were used to simulate timber boards and finger joint connections. Strength and stiffness properties of the timber boards and finger joint connections were estimated using simple assumptions; i.e. weakest link theory for the tensile strength and Hooke’s law for the tensile stiffness. Based on a sufficient amount of simulations, values of interest, such as the characteristic values of the tensile strength of timber boards and finger joint connections, were identified – all show a good accordance with the required/recommended values given in the codes and standards.

In addition to the material properties, also the size effect was investigated. Timber boards of different lengths were simulated and their tensile strength was estimated. The identified reduction of the tensile strength corresponds to the values found in the related literature.

**Estimation of the load-bearing capacity of GLT**

A numerical strain-based model to estimate the load-bearing capacity of GLT beams was developed. The model takes into account the local material properties of the entire beam; i.e. the strength and stiffness properties of timber board sections having a length 50 mm. The characteristic of this model is that the bending failure of the GLT beam is defined by a tensile failure of an entire lamella cross-section. Local failure mechanism are not explicitly considered; e.g. local growth irregularities within a knot cluster. In addition to the load-bearing capacity, the numerical model can be used to estimate the bending stiffness and the type of failure (e.g. knot cluster or finger joint connection) of GLT beams.

The model was validated with 24 GLT beams having well-known local material properties. The analysis shows a very good agreement between the measured and the estimated material properties. Thus, the presented model can be used to estimate accurately the load-bearing capacity and the bending stiffness of GLT beams where information about the beam setup is known.

**GLT model**

A probabilistic approach for modelling GLT beams was developed, which contains four independent sub-models. (1) Timber boards (or more precisely the strength and stiffness related indicators of timber boards) are simulated using the probabilistic model. (2) The simulated timber boards are used to fabricate GLT beams. Thereby, in principle, every kind of fabrication procedure can be simulated; e.g. length of the timber boards, cutting criteria or beam dimensions. (3) The strength and stiffness properties of each timber board section are allocated based on information about the strength and stiffness related indicators ($E_{\text{dyn,F}}$ and tKAR), and the position of finger joint connections. (4) In the last sub-model the load-bearing capacity, the
bending stiffness and the type of failure of the simulated GLT beams are estimated, using a numerical strain-based model. Based on a sufficient amount of simulations, values of interest such as the characteristic value of the bending strength or the mean value of the bending stiffness can be estimated. Therefore, Monte Carlo simulations are performed.

The application of the GLT model was illustrated on selected examples and the influence of the different input parameters, such as both strength and stiffness related indicators, the beam dimensions or the quality of finger joint connections, on the load-bearing behaviour of GLT beams were investigated. The results of all the examples of application are reasonable, which clearly shows that the presented GLT model can be used for the investigation of influencing parameters.

**GLT model – using machine-grading indicators**

The GLT model was extended for machine-grading indicators; i.e. the laboratory measured indicators ($E_{\text{dyn,F}}$ and $t\text{KAR}$) are replaced by indicators from the grading device ($E_m$ and $K_m$). The major advantage of machine-grading indicators is that they are measured automatically during the grading process; i.e. $E_m$ and $K_m$ are measured for each particular timber board that is graded by grading device that combines the measurement of the dynamic modules of elasticity and X-ray. As a result, new probabilistic models can be easily developed. An additional advantage of using machine-grading indicators is that they are reproducible and objective, thus typing errors and measurement uncertainties are reduced.

Probabilistic and material models were developed for the new indicators. For both models, a wide agreement to the developed with the indicators measured in the laboratory was identified. The validation of the material model indicates an even slightly better agreement. The application of the numerical model, using machine measured indicators, was validated with the 24 GLT beams having well-known local material properties. Again, very good agreement was identified, between the measured and the estimated material properties. The estimation of the load-bearing capacity, as well as the estimation of the bending stiffness were slightly below the measured values.

An example of application of the GLT model, using machine-grading indicators, was illustrated. Thereby four different types of GLT beams were simulated. The results show a wide accordance with the required/recommended values given in the codes and standards.

### 9.2 Originality of the work

The investigation on the influence of varying material properties on the load-bearing behaviour of GLT beams was defined as the main objective of this thesis. The motivation, was that increased knowledge on the influence of the variability of material properties might lead to a better understanding about the load-bearing behaviour of GLT beams, and thus to an improvement of GLT as a structural building material, in terms of reliability and efficiency.
In addition to experimental investigations, a probabilistic approach for modelling GLT beams was developed. The entire approach (including a probabilistic, a material and a numerical model), is developed for (a) indicators measured in the laboratory and (b) indicators that are automatically measured during the grading process. In particular the latter one offers new possibilities for the development of GLT beams.

Within the scope of this thesis, different subjects concerning timber boards and GLT beams were investigated. Some of the outcomes show new perspectives for modelling knot clusters, timber boards or GLT beams, others might facilitate the improvement of grading criteria. The most important results and outcomes are:

- The stiffness properties of 864 knot clusters are measured. For the stiffness measurement the growth characteristics of knot clusters were directly considered; i.e. the measured length corresponds to the length of the investigated knot cluster.

- The deformation and failure behaviour of knot clusters was investigated. New aspects about the influence of knot arrangements and the associated local grain deviation were discovered.

- The interrelation between numerous strength and stiffness stiffness related indicators and the tensile related material properties were investigated.

- A probabilistic model was developed to describe strength and stiffness related indicators. The specific characteristic of the model is that the natural growth characteristic of timber is considered.

- Material models were developed to predict the tensile strength and stiffness properties of knot clusters, based on strength and stiffness related indicators. The investigation shows that the influence of knots on the local strength reduction is significantly larger than expected.

- GLT beams having well-known local material properties were tested. The test results can be used to validate numerical models.

- A numerical strain-based model for the estimation of the load-bearing capacity of GLT beams was developed. The model was validated with GLT beams having well-known local material properties. A large correlation between the estimated and the measured material properties was identified. Therefore, the numerical model can be used to estimate the load-bearing capacity of GLT beams having well-known local material properties.

9.3 Limitations

One field of application of the presented GLT model is the estimation of the load-bearing capacity of GLT beams fabricated out of timber boards of one (or more) specific strength grades. Therefore, a probabilistic description of the timber boards of the respective strength grade(s) is essential. Within this thesis, probabilistic models of only two strength grades (L25 and L40) were developed. Further, it has to be considered that the timber boards are (a) graded with
the same grading device, (b) grown in the same region, and (c) have the same dimensions. The probabilistic description might be different if the timber boards are (a) graded visually or with another grading device, (b) from different growth regions, or (c) have different dimensions. However, new probabilistic models can be easily developed, using the approach for machine-grading indicators.

There are similar limitations regarding the material model. Both the strength and stiffness models are developed based on timber boards grown in Switzerland and southern Germany, respectively. The correlation between the strength and stiffness related indicators and the material properties might be slightly different for timber boards from different growth regions.

The material properties of finger joint connections were not investigated within this research project. Both the strength and stiffness properties were estimated based on the material properties of the adjacent timber boards, combined with an imposed strength reduction on the finger joint connection. The magnitude of the strength reduction, depends on the quality of the finger joint fabrication. To consider quality differences between GLT producers, the strength reduction is considered as an input variable. However, if the model is used to estimate the load-bearing capacity of GLT beams, this parameter has to be further investigated.

9.4 Outlook

The outcomes of this thesis present opportunities for an improvement of GLT as a structural building material, in terms of reliability and efficiency. In addition to clear opportunities, such as the improvement of grading criteria or the improvement of the finger joint quality, the following two approaches could significantly improve the reliability of GLT beams.

GLT beams with well-known beam setup: GLT beams could be fabricated in a way that the exact position of each particular timber board within the GLT beam is precisely-known. For timber boards graded with a grading device, that combines the measurement of the dynamic modules of elasticity and X-ray, the strength and stiffness related indicators over the entire board’s length are known. Thus, GLT beam with well-known beam setup can be fabricated; i.e. GLT beams with information about the strength and stiffness related indicators of each particular beam section. Using material models, such as the one presented in this thesis, the strength and stiffness properties of each beam section can be estimated. Afterwards, it is possible to estimate the load-bearing capacity of the GLT beam by using numerical models. Eliminating GLT beams with very low estimated load-bearing capacities would reduce the probability of GLT beams with insufficient material properties.

Automatic GLT fabrication: The approach of GLT beams having a well-known beam setup can be applied in the fabrication process. Through a combination of the grading process and the GLT fabrication, GLT beams having a planned beam setup can be produced. As a result, the arrangement of timber boards having relatively low material properties in highly loaded areas of the GLT beams can be avoided. Furthermore, timber boards could be finger jointed in a way that insufficient configurations of local weak sections are avoided; e.g. very large knot clusters
located above each other in the outer lamellas. The minimisation of such strength reducing effects should lead to a significant reduction of the variability of the load-bearing capacity of GLT beams.
# Nomenclature

## Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>ARF</td>
<td>Area reduction factor</td>
</tr>
<tr>
<td>C24</td>
<td>Strength grade for timber boards according to EN 338 (2010) with a characteristic tensile strength of 14.0 MPa</td>
</tr>
<tr>
<td>C30</td>
<td>Strength grade for timber boards according to EN 338 (2010) with a characteristic tensile strength of 18.0 MPa</td>
</tr>
<tr>
<td>COV</td>
<td>Coefficient of variation</td>
</tr>
<tr>
<td>CWAR</td>
<td>Clear wood area ratio</td>
</tr>
<tr>
<td>CWS</td>
<td>Clear wood section</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite element model</td>
</tr>
<tr>
<td>FJ</td>
<td>Finger joint connection</td>
</tr>
<tr>
<td>GL24h</td>
<td>Strength class for GLT beams according to EN 14080 (2009) with a characteristic bending strength of 24 MPa</td>
</tr>
<tr>
<td>GL36h</td>
<td>Strength class for GLT beams according to EN 14080 (2009) with a characteristic bending strength of 36 MPa</td>
</tr>
<tr>
<td>GLT</td>
<td>Glued laminated timber</td>
</tr>
<tr>
<td>KS</td>
<td>Knot sections</td>
</tr>
<tr>
<td>L25</td>
<td>Strength grade for GLT lamellas according to EN 14081-4 (2009) with a characteristic tensile strength of 14.5 MPa</td>
</tr>
<tr>
<td>L40</td>
<td>Strength grade for GLT lamellas according to EN 14081-4 (2009) with a characteristic tensile strength of 26.0 MPa</td>
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<tr>
<td>LED</td>
<td>Light emitting diode</td>
</tr>
<tr>
<td>LRFD</td>
<td>Load and resistance factor design</td>
</tr>
<tr>
<td>LVDT</td>
<td>Linear variable differential transformer</td>
</tr>
<tr>
<td>MC</td>
<td>Monte Carlo simulation method</td>
</tr>
</tbody>
</table>
mKAR Marginal knot area ratio
MLM Maximum likelihood method
MUF Melamine Urea Formaldehyde
PRF Phenol-Resorcinol-Formaldehyde
PUR Polyurethane
S10 Strength grade for visual graded timber, according to EN 1912 (2012)
tKAR Total knot area ratio
tKAR\_limit Upper limit of the tKAR-value
tKAR\_max Largest tKAR-value within a timber board
WS Weak section

**Upper-case roman letters**

B Strain-displacement matrix
C Material matrix
\(C_\Theta\) Covariance matrix
\(\overline{E}\) Measured mean stiffness [MPa]
\(E_m\) Knot indicator – machine graded [-]
\(E_{\text{CWS}}\) Stiffness of a CWS [MPa]
\(E_{\text{CWS}}\) Mean stiffness properties of defect-free timber within one timber board [MPa]
\(E_{\text{dyn,F}}\) Dynamic modulus of elasticity based on Eigenfrequency [MPa]
\(E_{\text{dyn,US}}\) Dynamic modulus of elasticity based on ultrasonic runtime [MPa]
\(E_{\text{KS}}\) Stiffness of a KS [MPa]
\(E_{m,g,\text{mean}}\) Mean value of the bending stiffness – GLT [MPa]
\(E_{m,g}\) Bending stiffness – GLT [MPa]
\(E_{t,j}\) Tensile stiffness – FJ [MPa]
\(E_{t,\text{mean}}\) Mean value of the tensile stiffness [MPa]
\(E_{\text{WS}}\) Stiffness of a WS [MPa]
\(F\) Force or Load [kN]
\(F_u\) Ultimate load [kN]
\(F_2 - F_1\) Rate of loading within the load interval between 0.1\(F_u\) and 0.4\(F_u\) [kN]
\(G\) Shear modulus [MPa]
Nomenclature

\( G_k \) Characteristic value of the permanent load
\( H \) Fisher information matrix
\( I \) Second moment of area
\( J \) Jacobian matrix
\( K \) Global stiffness matrix
\( K_e \) Local stiffness matrix
\( K_m \) Global indicator - machine graded [MPa]
\( L(\theta|\hat{x}) \) Likelihood
\( M \) Bending moment
\( P_f \) Probability of failure [-]
\( Q_k \) Characteristic value of the variable load
\( R \) Resistance
\( R_k \) Characteristic value of the resistance
\( S \) Load
\( V \) Volume
\( V_0 \) Reference volume

Lower-case roman letters

\( a \) Distance between the point of load transmission and the nearer support [mm]
\( b \) Width [mm], Model parameter – Weibul distribution [-]
\( d \) Distance between WS [mm]
\( d_{\text{min}} \) Minimal distance between WS [mm]
\( d_p \) Distance to the pith [mm]
\( e \) Visual knot diameter perpendicular to the board axis [mm]
\( f \) Visual knot diameter [mm]
\( f_{m,g,k} \) Characteristic value of the bending strength – GLT [MPa]
\( E_{m,g,\text{mean}} \) Mean value of the bending strength – GLT [MPa]
\( f_{m,g,\text{size}} \) Bending strength with considering the size effect – GLT [MPa]
\( f_{m,g} \) Bending strength [MPa]
\( f_{m,i,k} \) Characteristic value of the bending strength – FJ [MPa]
\( f_{t,0,1,k} \) Characteristic value of the tensile strength of the lamiation [MPa]
\( f_{t,0} \)  
Tensile strength parallel to the grain [MPa]

\( f_{t,j,k} \)  
Characteristic value of the tensile strength – FJ [MPa]

\( f_{t,j,\text{mean}} \)  
Mean value of the tensile strength – FJ [MPa]

\( f_{t,j} \)  
Tensile strength – FJ [MPa]

\( f_{t,k} \)  
Characteristic value of the tensile strength – timber board [MPa]

\( f_{t,l} \)  
Tensile strength of the lamination [MPa]

\( f_{t,\text{mean}} \)  
Mean value of the tensile strength – timber board [MPa]

\( f_{t,\text{reg}} \)  
Estimated tensile strength of a WS according to the regression model [MPa]

\( f_{t,\text{WS}} \)  
Tensile strength of a WS [MPa]

\( f_{t} \)  
Tensile strength – timber board [MPa]

\( f_{t,\theta} \)  
Tensile strength at an angle of inclination \( \theta \) [MPa]

\( f_{0} \)  
Eigenfrequency [Hz]

\( g \)  
Limit state function

\( h \)  
Height [mm]

\( h_{e} \)  
Element height [mm]

\( k \)  
Number of regression coefficients [-], Model parameter – Gamma distribution [-]

\( k_{\text{lam}} \)  
Lamination factor [-]

\( k_{\text{size}} \)  
Size factor [-]

\( l \)  
Length or span length [mm]

\( l_{\text{KS}} \)  
Length of a KS

\( l_{\text{WS}} \)  
Length of a WS

\( l_{0} \)  
Initial length [mm]

\( l_{e} \)  
Element length [mm]

\( n_{f} \)  
Number of simulations that ends up in the failure domain [-]

\( o_{\text{global}} \)  
Global element orientation [-]

\( o_{\text{local}} \)  
Local element orientation [-]

\( p \)  
Model parameter – Weibul distribution [-]

\( t \)  
Time [sec.]

\( t_{i} \)  
Thickness of lamella [mm]

\( u \)  
Moisture content [%]

\( u \)  
Displacement vector
Nomenclature

\[ w_2 - w_1 \] Rate of deformation within the load interval between 0.1F_u and 0.4F_u [mm]
\[ \dot{x} \] Measured values
\[ z \] Design variable
\[ z^* \] Design solution

### Upper-case greek letters

\[ \Delta l \] Change of length
\[ \Gamma \] Gamma function
\[ \Phi \] Standard normal distribution function

### Lower-case greek letters

\[ \alpha \] Angle of the annual rings [°]
\[ \beta \] Reliability index [-]
\[ \beta_{\text{target}} \] Target reliability index [-]
\[ \beta_i \] Regression coefficients [-]
\[ \epsilon \] Parameter to model tKAR, Parameter in the shifted Lognormal distribution, Parameter in the shifted Exponential distribution
\[ \varepsilon \] Mean axial strain [-], Error term of the regression model
\[ \eta \] Parameter to describe the size effect
\[ \gamma_G \] Partial safety factor – permanent load
\[ \gamma_m \] Partial safety factor – material
\[ \gamma_Q \] Partial safety factor – variable load
\[ \lambda \] Parameter in the Exponential distribution
\[ \mu \] Mean value, Parameter to model tKAR, Parameter to model \( E_{\text{dyn,F}} \)
\[ \nu \] Poisson’s ratio, Parameter in the Gamma distribution
\[ \phi \] Standard normal density distribution function
\[ \varphi \] Aperture angle [°]
\[ \rho \] Density [kg/m³]
\[ \rho_0 \] Dry density [kg/m³]
\[ \sigma_\epsilon \] Standard deviation of the parameter \( \epsilon \)
\[ \sigma_\tau \] Standard deviation of the parameter \( \tau \)
\( \sigma_m \)  Bensing stress
\( \sigma_t \)  Tensile stress
\( \sigma_x \)  Standard deviation of the error term
\( \tau \)  Parameter to model tKAR, Parameter to model \( E_{\text{dyn,F}} \)
\( \theta \)  Parameter of the regression model \([\cdot]\), Angle of inclination \( [\degree]\)
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