Numerical Investigation of Vortex Breakdown in Compressible, Swirling Nozzle-Jet Flows

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Instantaneous ($\rho_w = 0.2$)-iso-surface plot for the rotating nozzle setup at time $t = 400$ intersected by a $(r, z)$-plane (DNS result). Iso-surface coloured by distance perpendicular to the intersection plane (front-to-back: yellow-to-white-to-black), see Fig. 5.10.

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NUMERICAL INVESTIGATION OF VORTEX BREAKDOWN IN COMPRESSIBLE, SWIRLING NOZZLE-JET FLOWS

A dissertation submitted to

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presented by

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Abstract

Vortex breakdown is a phenomenon which is of importance in a large variety of swirling flows. In environmental (tornadoes, hurricanes) as well as in internal (turbines, combustion chambers) and external (delta wing aircraft) flows of technical relevance it is of great interest to understand the underlying physical mechanism responsible for the occurrence of vortex breakdown and to know the parameter set at which vortex breakdown takes place. Despite decades of intense research no consensus has been found for the explanation of the instability mechanism and a widely accepted theoretical framework is lacking.

The present work deals with the numerical investigation of swirling jet flows as a model problem. The numerical code PARACONCYL is utilised to perform Direct- and Large-Eddy Simulations (DNS/LES) on massively parallel computing architectures. The compressible Navier-Stokes equations are solved on a cylindrical grid using high-order spatial and temporal numerical schemes. For the subgrid closure the Approximate Deconvolution Model (ADM) is applied. To account for a more realistic setup a nozzle is included in the computational domain modelled as an isothermal wall, which either rotates with the mean flow or is kept at rest. The effects of boundary conditions imposed at the inflow and outflow of the computational domain on the breakdown behaviour of the swirling jet are assessed in detail.

The setups with a rotating nozzle and a nozzle kept at rest are compared for identical initial integral swirl numbers. Main differences in the flow field results as well as in the vortex breakdown configuration are reported. The instability mechanism for the two setups is inherently different leading to significant differences in the mode and frequency selection. A single-helix type instability dominating the vortex breakdown configuration is found for both setups.

The Mach number is varied for the setup with a rotating nozzle wall to investigate its impact on the vortex breakdown behaviour of the swirling jet flow. The effect of the swirl intensity on the stability of the swirling jet is studied for both the rotating nozzle and the nozzle kept at rest.

Although the swirling jet is highly sensitive to the parameters investigated, the instability mechanism leading to vortex breakdown is remarkably robust.
Kurzfassung


Ergebnisse für die rotierende und die stehende Düse werden bei identischer initialer integraler Drallzahl verglichen. Grundsätzliche Unterschiede der resultierenden Strömungsfelder und der Konfiguration des Wirbelplatzens werden beschrieben. Der Instabilitätsmechanismus der beiden Aufbauten unterscheidet sich inhärent, was zu bedeutenden Unterschieden in der Moden- und Frequenzselektion führt. Eine Spiralmode mit der azimutalen Wellenzahl 1 dominiert das Strömungsfeld im Bereich des Wirbelplatzens für die beiden untersuchten Aufbauten.

Die Machzahl wird für den Aufbau mit rotierender Düse variiert, um den Einfluss dieser auf das Wirbelplatzen des Freistrahls zu untersuchen. Der Effekt der Drallintensität auf die Stabilität des drallbehafteten Freistrahls wird sowohl für die stehende als auch die rotierende Düsenwand untersucht.
Obwohl der Drallstrahl besonders empfindlich gegenüber Parameterveränderungen ist, ist der Instabilitätsmechanismus, der zum Wirbelplatzen führt, bemerkenswert robust.
Acknowledgments

I wish to thank Prof. Dr. Leonhard Kleiser for giving me the opportunity to work on the research project which led to the present thesis. I greatly acknowledge the freedom he gave me in conducting research and in digging into topics related but not closely connected to the main project. The fruitful discussions with him and the co-examiner of the present thesis, Prof. François Gallaire, are kindly acknowledged.

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Zürich, December 2013

Tobias Luginsland
## Contents

Nomenclature \hspace{1cm} V

1 Introduction \hspace{1cm} 1
  1.1 Motivation of current work \hspace{1cm} 1
  1.2 Overview of the thesis \hspace{1cm} 4

2 Literature survey \hspace{1cm} 7
  2.1 Experiments \hspace{1cm} 7
    2.1.1 Devices utilising a nozzle at rest \hspace{1cm} 8
    2.1.2 Rotating nozzle devices \hspace{1cm} 12
    2.1.3 Coaxial jets and combustors \hspace{1cm} 13
  2.2 Numerical Simulations \hspace{1cm} 14
    2.2.1 Low to moderate Reynolds number incompressible flows \hspace{1cm} 15
    2.2.2 High Reynolds number incompressible flows \hspace{1cm} 15
    2.2.3 Compressible flows \hspace{1cm} 16
    2.2.4 Coaxial jets, combustors and turbines \hspace{1cm} 17
  2.3 (Semi-)Analytical Studies \hspace{1cm} 18
  2.4 Stability Analysis \hspace{1cm} 19
    2.4.1 Model vortices \hspace{1cm} 19
    2.4.2 Jets and wakes \hspace{1cm} 20
    2.4.3 Swirling pipe flow \hspace{1cm} 22
  2.5 Control of Vortex Breakdown \hspace{1cm} 23
    2.5.1 Axial forcing \hspace{1cm} 23
    2.5.2 Helical forcing \hspace{1cm} 24
    2.5.3 Other forcing approaches \hspace{1cm} 26
  2.6 Swirl number definition \hspace{1cm} 28

3 Computational framework \hspace{1cm} 33
  3.1 Basic equations and numerical schemes \hspace{1cm} 33
  3.2 Code parallelisation strategy \hspace{1cm} 37

4 Assessment of boundary conditions \hspace{1cm} 39
  4.1 Introduction \hspace{1cm} 40
  4.2 Numerical framework \hspace{1cm} 43
  4.3 Boundary condition setup \hspace{1cm} 44
<table>
<thead>
<tr>
<th>Contents</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Publications</td>
<td>201</td>
</tr>
<tr>
<td>Curriculum vitae</td>
<td>203</td>
</tr>
</tbody>
</table>
Nomenclature

Roman symbols

- **A**: amplitude
- **a**: swirl parameter, speed of sound
- **a, b, c**: coefficients of numerical scheme for spatial derivatives
- **c_p**: heat capacity at constant pressure
- **C**: characteristic wave
- **d**: nozzle wall thickness
- **E**: total energy
- **f**: function value
- **F, G, H**: flux vectors
- **i**: imaginary unit
- **J**: Jacobian
- **L**: nozzle length
- **Ma**: Mach number \( (= \frac{w^\circ}{\sqrt{\gamma R_{\text{air}}^\circ T_c^\circ}} = \frac{w^\circ}{a_c^\circ}) \)
- **N**: number of grid points
- **n**: azimuthal wave number
- **Pr**: Prandtl number \( (= \frac{c_p^\circ \kappa^\circ}{\mu_c^\circ} = 0.7) \)
- **R, R_o**: inner/outer radius of the nozzle
- **R_{\text{air}}**: gas constant of air \( (= 287.15 J/kg/K) \)
- **r**: radial co-ordinate
- **Re**: Reynolds number \( (= \frac{\rho_c^\circ w_c^\circ R^\circ}{\mu_c^\circ}) \)
- **S**: swirl number
- **St**: Strouhal number \( (= f^\circ \cdot R^\circ / w_c^\circ) \)
- **Su**: Sutherland constant \( (= 0.404) \)
- **T**: temperature
- **t**: time
- **u, v, w**: radial, azimuthal and streamwise velocity components
- **U**: vector of conservative variables
- **U, V, W**: Cartesian velocity components
- **X, Y, Z**: Cartesian co-ordinates
- **z**: streamwise co-ordinate
Greek symbols

$\alpha, \beta$ coefficients of numerical scheme for spatial derivatives
$\gamma$ (ratio of specific heats or) isentropic exponent ($= 1.4$)
$\Delta$ increment
$\theta$ azimuthal co-ordinate
$\kappa$ heat conductivity
$\mu$ dynamic viscosity
$\pi$ mathematical constant, ratio of a circle’s circumference to its diameter
$\rho$ density
$\sigma$ sponge geometry function
$\tau$ stress tensor
$\chi$ subgrid-scale model parameter
$\xi, \eta, \zeta$ generalized co-ordinates

Subscripts

$(\cdot)_{bc}$ boundary condition quantity
$(\cdot)_{c}$ centreline quantity, critical quantity
$(\cdot)_{dom}$ dominant quantity
$(\cdot)_{E}$ Euler flux
$(\cdot)_{hp}$ high-pass filtered quantity
$(\cdot)_{i}$ imaginary quantity
$(\cdot)_{lip}$ nozzle lip quantity
$(\cdot)_{max}$ maximum quantity
$(\cdot)_{min}$ minimum quantity
$(\cdot)_{r}$ radial quantity, real quantity
$(\cdot)_{ref}$ reference quantity
$(\cdot)_{S}$ Stokes flux
$(\cdot)_{sat}$ saturation quantity
$(\cdot)_{w}$ wall quantity
$(\cdot)_{z}$ streamwise quantity
$(\cdot)_{\theta}$ azimuthal quantity

Superscripts

$(\cdot)'$ fluctuation quantity, derivative
$(\cdot)$ Fourier-transformed quantity
$(\cdot)^{\circ}$ dimensional quantity
$(\cdot)_{\infty}$ free-stream or far-field quantity
Other symbols and operators

\( \cdot \) (inner or) scalar product, e.g. \( \phi = \chi \cdot \psi \)

\( | \cdot | \) absolute value

\( \langle \cdot \rangle \) averaged quantity

\( (\cdot) \) vector quantity

\( [\cdot] \) matrix quantity

Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADM</td>
<td>Approximate Deconvolution Model</td>
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<tr>
<td>CFL</td>
<td>Courant-Friedrichs-Lewy (condition or number)</td>
</tr>
<tr>
<td>DNS</td>
<td>Direct Numerical Simulation</td>
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<tr>
<td>LES</td>
<td>Large-Eddy Simulation</td>
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<tr>
<td>NBL</td>
<td>Nozzle Boundary Layer</td>
</tr>
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<td>OSL</td>
<td>Outer Shear Layer of the swirling jet</td>
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<td>PIV</td>
<td>Particle Image Velocimetry</td>
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<td>PVC</td>
<td>Precessing Vortex Core</td>
</tr>
<tr>
<td>SGS</td>
<td>Subgrid Scale</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

The physical phenomenon of vortex breakdown occurs in many technical applications, e.g. on delta wing aircraft (Peckham & Atkinson, 1957) and in vortex burners (Chigier & Chervinsky, 1967), and can also be observed in nature, e.g. in dust devils, tornadoes and hurricanes (Burggraf & Foster, 1977). It is therefore of great interest to understand the fundamental features of vortex breakdown, to know the parameter set at which vortex breakdown occurs, and to get insight into possible control mechanisms of this special flow configuration. Although in more than five decades of intense research many attempts were made to explain this flow feature, a commonly accepted theory of the underlying mechanism is still missing (see Chap. 2).


1.1 Motivation of current work

Starting point of the present study are the previous investigations by Liang (2003), Liang & Maxworthy (2005), Müller (2007) and Müller & Kleiser (2008a). Liang & Maxworthy (2005) investigated incompressible swirling jets emanating from a rotating pipe similar to Faccioilo et al. (2007). They found that vortex breakdown is likely related to a change from convectively to absolutely unstable helical modes (see also Sec. 2.1). For sufficiently high swirl they found a strong evidence of a bifurcation of super-critical Hopf-type, in agreement with recently published results.
(Oberleithner et al., 2012, 2011b). Müller & Kleiser (2008a) investigated swirling jets in the compressible regime and reproduced qualitatively the results by Liang & Maxworthy (2005). They were able to capture the fundamental physics of the underlying swirling flow undergoing vortex breakdown using their numerical framework. The authors reported an interaction between the upstream-shifting recirculation region and the domain inflow boundary treatment by means of a numerical sponge layer technique which prevented helical instabilities to grow naturally. Müller & Kleiser (2008a) therefore suggested to include a nozzle into the computational domain to avoid spurious boundary-condition effects on the development of vortex breakdown.

In the present investigation we follow this suggestion. A nozzle has been included into the computational domain (Bühler et al., 2010; Luginsland et al., 2010; Luginsland & Kleiser, 2011b) in order to account for more realistic inflow boundary conditions and to avoid the interaction of the swirling jet with the sponge layer at the inflow. With the improved treatment of the boundary condition at the inflow of the numerical domain we are now able to investigate swirling jet flows undergoing vortex breakdown with inclusion of the physics of the entire nozzle flow. To the best of our knowledge this is a new approach in the field of numerical investigations of swirling jet flows. In experiments it may be challenging to track the flow within the nozzle either because the nozzle wall is non-transparent or due to reflections at the nozzle wall, making it difficult to use imaging techniques such as PIV (Raffel et al., 2007). Accordingly, the number of references in literature dealing with swirling pipe/nozzle and jet flows at the same time is very limited (Facciolo et al., 2007). Numerical simulations take into account either no nozzle at all or rarely use indirect methods such as the immersed-boundary technique, cf. Freitag & Klein (2005). Only three studies are known to the author dealing with an included nozzle wall in the context of freely evolving (non-swirling) round jets: The axisymmetric investigation by DeBonis & Scott (2002) and later the fully three-dimensional study by Sandberg et al. (2011) and Sandberg et al. (2012) based on a study of flow past a trailing edge (Sandberg & Sandham, 2008; Sandberg et al., 2007), where the wall is modelled as rigid and isothermal. Bühler & Kleiser (2011), Bühler et al. (2010), Bühler (2013) and Bühler & Kleiser (2012) investigated computational aero-acoustics of nozzle-jet flow configurations parallel to the present investigation using the same basic numerical setup. Besides the fact that we consider compressible flows of swirling jets, the inclusion of the nozzle into our computational
domain is therefore a unique feature in numerical swirling jet flow research and opens up a wide field for investigating new aspects related to it.

With the present investigation, we intend to contribute to the clarification of significant questions raised in the literature including the following ones:

- Where is the 'wave-maker' of the flow exciting the single-helix type instability, which is believed to be globally unstable? Is it located in front of the recirculation zone, in the lee of the vortex breakdown bubble, in the inner or outer shear-layer of the swirling jet (Liang & Maxworthy, 2005)?

- Which type of instabilities dominate the flow field? Where and when are those instabilities initiated? What are the dominating frequencies and azimuthal wave numbers?

- Where do the helical instabilities grow and which mechanisms lead to the growth?

- Is vortex breakdown to be understood as a 'core-mechanism' (Gallaire et al., 2004b)? Is vortex breakdown induced by a vortex core gyrating around the centreline of the swirling jet?

- Is there a significant influence of the Mach number on vortex breakdown?

In addition, we raise the following questions related to the nozzle flow itself:

- How do the boundary layers at the nozzle wall impact the vortex breakdown behaviour (Müller, 2007)?

- Which type of instabilities grow in the nozzle flow regime for the cases of a rotating nozzle and a nozzle kept at rest?

- Are there significant upstream effects of vortex breakdown observed within the nozzle?

In the following Sec. 1.2 we give an overview of the thesis and point out the specific objectives of the present work.
1.2 Overview of the thesis

In Chap. 2 we introduce the reader into the current state-of-the-art in vortex breakdown research and discuss the choice of the swirl number in the present investigation. In Chap. 3 we describe the numerical framework used and give some details of the computational code PARACONCYL. The results are split up into three main parts (Chaps. 4 - 6), which are independent investigations:

- Chap. 4: Assessment of boundary conditions

  In this section we investigate the effects of changes in boundary conditions at the inflow and outflow of the computational domain. The comparison includes six different setups, for which differences in instantaneous as well as mean flow properties are reported. The investigation is comparable to the study of Ruith et al. (2004) for incompressible swirling jet flows, but differs in accounting for compressibility and in extending to a larger set of inflow and outflow conditions and their combination. We give a recommendation for the combination of boundary conditions we consider is most appropriate in the present context and choose a corresponding setup for the numerical simulations.

- Chap. 5: Comparison of the configuration with a rotating nozzle and a nozzle kept at rest

  We compare results of two basic configurations with either a rotating nozzle or a nozzle kept at rest (Chap. 5). We report on differences in the results which are inherently connected to changes in the setup and discuss the different instability mechanisms observed for the nozzle flow regime upstream of vortex breakdown. A description of the helical instabilities dominating the flow is given. We show that the flow state upstream of vortex breakdown significantly influences the breakdown of the swirling jet. The comparison serves as a basis for the subsequent parametric studies.

- Chap. 6: Parametric studies

  In Chap. 6 the results of the parametric studies are presented. Swirl number variations for the rotating nozzle and the nozzle
kept at rest are investigated (Sec. 6.1) and the critical amount of swirl for the onset of vortex breakdown is determined. The mode selection mechanism depending on the amount of swirl is discussed for both setups and the variation in size and shape of the recirculation zone is described. Wherever possible we compare our findings to results reported in the literature and comment on the ongoing discussion of the possible onset of a globally unstable single-helix type instability. In addition, the impact of Mach number variations on the vortex breakdown behaviour is studied (Sec. 6.2).

We conclude our study in Chap. 7 and identify possible future investigations. Supplementary results of a grid resolution study are given in the appendix App. A.
Chapter 2

Literature survey

In this chapter we review the current state-of-the-art in vortex breakdown research mainly focusing on swirling jet flows. An overview of the literature is given including recent experimental, numerical and analytical investigations (Secs. 2.1 - 2.3). Furthermore, main findings of recent studies concerning the hydrodynamic stability of swirling jet flows and model vortices are summarized (Sec. 2.4). The literature survey is completed by a section on vortex breakdown control (Sec. 2.5).

Several definitions of the swirl number—the critical measure in studies on vortex breakdown in swirling jet flows—introduced in literature are summarized in Sec. 2.6. We evaluate and compare the different swirl numbers for the specific setup used in the present investigation and make a choice which definition to use in the remainder of the present study.

2.1 Experiments

Since the first observation of vortex breakdown by Nuttal (1953) and later by Peckham & Atkinson (1957), Lambourne & Bryer (1961) and Harvey (1962) a large number of experimental studies were undertaken concerning this physical phenomenon. Chigier & Chervinsky (1967) provided time-averaged velocity profiles of a confined swirling jet and observed a recirculation zone for sufficiently high swirl. Cassidy & Falvey (1970) focused on the unsteady nature of the flow after occurrence of vortex breakdown. Sarpkaya (1971), Sarpkaya (1995), Faler & Leibovich (1977), Faler & Leibovich (1978), Escudier & Zehnder (1982), Brücker & Althaus (1992), Brücker & Althaus (1995), Brücker (1993) and Brücker (2002), among others, conducted experiments of confined swirling jet flow in the incompressible, laminar and weakly turbulent regime. Parameter sets were identified for the onset of vortex breakdown, up to seven different vortex breakdown types (helix, double-helix, bubble, e.g.) were described.
2.1.1 Devices utilising a nozzle at rest

Panda & McLaughlin (1994) performed experiments of swirling turbulent jets and focused on axisymmetric and helical instabilities at swirl number $S_5 = 0.5$. The jet did not show any organized structures in the shear-layer but featured weak and irregular coarse structures. Vortex pairing was prevented due to swirl being present. For the swirl-free jet the spreading and the mass entrainment (Townsend, 1966) were lower than in the case of a swirling jet. For the $Re = 22'000$ jet the azimuthal mode $n = \pm 1$ grew much faster than the axisymmetric mode $n = 0$, whereas at $Re = 57'000$ the growth rates of both modes were nearly identical. Oberleithner & Paschereit (2009) studied a turbulent air jet at low swirl numbers ($S_5 \leq 0.6$) in the pre-breakdown regime. Helical modes were observed to be linearly amplified with an increased rate of amplification for increased swirl number. Due to shear-layer instabilities energy was shifted to lower frequencies for rising swirl number and the maximum amplification was reduced. Mode $n = +2$ was identified as the most unstable for swirl numbers $S_5 \geq 0.4$. For low-swirl jet flows modes $n = 0, +1$ were most unstable. The helices were of co-rotating, counter-winding type, cf. Billant et al. (1998) and Liang & Maxworthy (2005). Oberleithner et al. (2010) investigated the instability characteristics of moderately swirling jets by means of hot-wire measurements and linear spatial stability analysis of time-averaged data. The authors found that increased swirl reduced the amplification rate of the instabilities due to enhanced shear-layer growth. Mode $n = +1$ was most unstable, a counter-winding, co-rotating helical instability. The velocity overshoot at the centreline of the jet due to a contracting nozzle used in the experiments did not influence the results of the stability analysis, in agreement with results found in literature, cf. Gallaire & Chomaz (2003b). The axial phase velocity of instabilities depended on the amount of swirl and the azimuthal wave number, inhibiting the interaction between two azimuthal modes forced simultaneously at the same frequency. Waves in swirling jets were found to be dispersive for the entire range of unstable frequencies, which was assumed to be the reason for the nonexistence of subharmonic resonance in swirling jets, cf. Panda & McLaughlin (1994). Experimental and theoretical results of the stability analysis were in good agreement, despite a clear selection mechanism missing in the latter. Billant et al. (1998) conducted experiments in a huge water tank at moderate Reynolds numbers ($Re \leq 1'690$) and studied the influence of the swirl intensity. The authors
found the helical modes $n = +2, +3$ (counter-winding, co-rotating) to be dominant in the pre-breakdown stage. In the post-breakdown stage the authors observed a competition of helical modes of similar type. They distinguished between different vortex breakdown types: a bubble and a cone type, both either symmetric or asymmetric. The authors reported a strong dependency of the vortex breakdown type selection mechanism on temperature differences. Loiseleux & Chomaz (2003) investigated laminar swirling water jets in the pre-breakdown stage and focused on instabilities breaking the symmetry of the jet. The authors distinguished between three dynamical regimes depending on the swirl number $S_2$. For low swirl $S_2 \leq 0.6$ the instability mechanism was similar to that of a non-swirling jet, Kelvin-Helmholtz like disturbances dominated (counter-rotating vortex pairs), helical modes started growing with increasing swirl. At the transitional swirl level at $S_2 \approx 0.6$ the jet behaved like a non-swirling jet, azimuthal modes were nearly non-detectable. In the intermediate swirl range $0.6 < S_2 \leq 1.0$ vortex rings competed with growing helical modes. For high swirl $1.0 < S_2 \leq 1.3$ an interaction of vortex rings and helical modes was observed and a bending mode with wave number $n = +1$ developed which was of co-rotating, counter-winding type. Seele et al. (2008) investigated the mean properties of an incompressible, fully turbulent swirling jet at moderate Reynolds number in water. The high turbulence level was generated by a tripping device mounted inside the contraction of the nozzle. The authors found a strong influence of the nozzle contraction on the jet development (Leclaire & Jacquin, 2011), namely an overshoot in the streamwise velocity at the jet centreline leading to difficulties in deriving scaling laws for the swirling jet. Nevertheless, scaling laws were found for large swirl and locations sufficiently far downstream of the nozzle exit plane. For increasing swirl the spreading rate and also the turbulence level increased and the stagnation point moved upstream. The location of vortex breakdown was found to be highly fluctuating. Oberleithner et al. (2012) studied swirling water jets at moderate Reynolds number. The authors found that vortex breakdown occurred intermittently at a wide range of swirl numbers before it was detected in the mean flow field. The intermittent vortex breakdown state was dominated by strong oscillations of the location of the vortex breakdown bubble. An axisymmetric flow state developed for increased swirl accompanied by a super-critical to sub-critical transition of the inflowing vortex core. For further increased swirl a super-critical Hopf-bifurcation to a globally unstable single-helix type instability was observed leading to a spiral shape
of vortex breakdown.

In their study of incompressible jets at low swirl numbers, Gilchrist & Naughton (2005) stated that the effect of different initial azimuthal velocity distributions on the growth rate enhancement of the jet was very small. In contrast, the swirl number was found to be critical for the enhancement if it exceeded a certain value. Their findings were in contradiction to the results by Chigier & Chervinsky (1967) showing a linear dependency of growth rate on swirl number. The authors distinguished between three different regions of swirl-enhanced growth rate, namely: (i) a region of low swirl where no enhancement effect existed, (ii) a medium swirl region where enhancement scaled with swirl and (iii) a region where vortex breakdown occurred due to sufficiently high swirl numbers and the whole process was dominated by the bursting of the jet. Surprisingly, the time-averaged results of the study agreed well with experimental results found in literature where artificial effects of the swirl generation and non-typical nozzle exit conditions were likely present, which were avoided in the present facility. The authors suggested that the dynamics created by the presence of swirl dominated the mixing process and the exit conditions only played a secondary role, see Leclaire & Jacquin (2011) for a discussion of nozzle exit conditions in the context of highly swirling jets. Comparably, Farokhi et al. (1989) investigated the effect of initial swirl distribution on the evolution of a turbulent jet and found out that the swirl number on its own was inadequate in describing the mean swirling jet behaviour in the near field. In the work of Toh et al. (2010) a swirling mixing layer undergoing vortex breakdown was considered and different swirl number definitions were discussed in detail. For different definitions the critical swirl number at which vortex breakdown occurred naturally had a different value and the conclusion was drawn that defining a certain swirl number is meaningful only in combination with precisely defined inflow velocity profiles. These findings were a confirmation of results by Farokhi et al. (1989). For increased swirl number the velocity decay at the jet axis was observed to be stronger, the jet spreading was larger and the turbulence level was higher, cf. Maciel et al. (2008).

Park & Shin (1993) concentrated on the entrainment characteristics of swirling jets at high Reynolds number. The authors found that the swirl intensity promoted entrainment. For strong swirl a precessing vortex core was observed enhancing the spreading of the jet further, leading to even stronger entrainment. Continuing the work of Sarpkaya (1995), Novak & Sarpkaya (2000) observed a bypass-mechanism in high Reynolds
number flows in pipes ($Re = 3 \times 10^5$). After a helical mode formed the flow directly broke down into turbulence circumventing the vortex breakdown stage observable at lower Reynolds numbers. The authors emphasised that vortex breakdown at sufficiently high Reynolds numbers occurred in a dramatically different fashion compared to low Reynolds number flows. The authors therefore concluded that the applicability of the information deduced from laminar flow studies to the prediction of breakdowns in turbulent flows is highly questionable. In their study of an incompressible swirling jet at high Reynolds number in the medium to high swirl regime, Martinelli et al. (2007) found that the true turbulence intensity, associated with the small-scale flow turbulence, was independent of the swirl parameter. Furthermore it was possible to separate the periodic fluctuations due to the vortex core from the small scales, leading to the result that the amplitude of the fluctuations corresponding to the former were dependent on Reynolds number and swirl parameter. In contrast, the characteristic size of the precession region was nearly independent of the Reynolds number. Oberleithner et al. (2011b) conducted experiments in air at a Reynolds number of $Re_D = 20'000$ of a swirling jet over a range of swirl numbers $S_5$. The authors focused on the description of three-dimensional coherent structures by means of POD and compared the empirical results to results from a special spatial linear stability analysis. In this stability analysis the Reynolds number, $Re$, and the complex axial wave number, $\alpha$, were assumed to vary in the streamwise direction. The analysis was carried out on subsequent downstream positions to account for the non-parallel base flow assuming the instabilities in the outer shear-layer to be of convective type. A frequency was measured which dominates the whole flow. The stability investigation was restricted to the positive frequency regime, especially the global dominant frequency was used as input parameter for the stability investigation. As earlier by Liang & Maxworthy (2005), evidence was found for the existence of a super-critical Hopf bifurcation on which a global mode could establish. The global mode was identified to be a co-rotating, counter-winding single-helix which was triggered by the precessing of the vortex core in the inner region of the jet, cf. Liang & Maxworthy (2005).

Leclaire & Jacquin (2011) investigated the generation of swirling jets in the context of rotating pipe flows with a final contraction in the high Reynolds number regime. For swirl numbers higher than the critical level the flow did not undergo vortex breakdown in the pipe section independent of the contraction ratio, in contradiction to results found in literature, see
the model study of Leclaire & Sipp (2010). The authors studied the effect of the contraction ratio on the turbulence level in the exit plane of the pipe and found the turbulence originated in the pipe flow upstream of the exit rather than resulting from upstream-travelling disturbances initiated in the jet region. A single-helical mode was observed in the pipe exit plane at a frequency of the rotating honeycomb used for swirl generation. The hypothesis was written that this instability was the resonant response of the pipe flow to the harmonic excitation caused by the honeycomb. The authors concluded that a convergent nozzle may lead to artificially high turbulence levels being therefore no longer beneficial for the generation of high Reynolds, high swirl number flows (as opposed to non-swirling flows). They found that the use of a contracting nozzle should be avoided in any study concerning vortex breakdown to circumvent a transition to a sub-critical flow already in the pipe upstream of the nozzle exit.

Mourtazin & Cohen (2007) conducted experiments in the laminar in-compressible regime concerning the effect of buoyancy on vortex breakdown, continuing the work of Billant et al. (1998) who first reported the sensitivity of vortex breakdown to small temperature differences. Vortex breakdown was suppressed (enhanced) by a negative (positive) temperature difference between the jet core and the surrounding fluid. Vortex breakdown occurred at lower swirl number for heated jet flows, see Lim & Redekopp (1998) who reported a promotion of absolute instability for positive pressure gradients. The critical swirl number at which vortex breakdown occurred was in good agreement with the criterion introduced by Billant et al. (1998) and extended to account for buoyancy effects. The authors additionally pointed out that differences in the velocity profiles had a significant effect on the breakdown type to occur (bubble or cone).

2.1.2 Rotating nozzle devices

Liang (2003), Liang & Maxworthy (2005) and Liang & Maxworthy (2008) extended the work of Billant et al. (1998) using a long rotating pipe attached to a large water tank to generate swirling jets. The authors conducted experiments in either stationary or rotating surroundings (the entire tank was rotated either with or against the mean flow) at Reynolds numbers up to $Re = 2'000$. Helical modes $n = +2, +3$ (co-rotating, counter-winding) were found to be dominant before vortex breakdown (cf. Billant et al. (1998)), while after vortex breakdown modes $n = +1, +2$ competed with each other, with mode $n = +1$ most unstable. In the
post-breakdown stage the dominant modes were suggested to be self-excited/globally unstable, a behaviour identified as a super-critical Hopf-bifurcation: the saturation amplitude of modes $n = +1, +2$ depended linearly on the critical swirl parameter, cf. Huerre & Monkewitz (1990). Co-rotating the surrounding tank decreased the critical amount of swirl leading to vortex breakdown while it maintained approximately unchanged for counter-rotating the tank. The flow criticality and shear-layer morphology remained unchanged with Reynolds number. The authors concluded that the swirl difference of the jet to the ambient fluid had only a minor effect on the flow criticality, which depended mainly on the velocity distribution of the vortex core.

Facciolo & Alfredsson (2004), Facciolo et al. (2007) and Maciel et al. (2008) conducted swirling pipe and jet experiments at Reynolds numbers up to $Re = 33'500$ in air. The authors found a counter-rotating jet core near the pipe exit and dominant helical structures of low mode number. They linked the counter-rotating vortex core to the Reynolds stress distribution of the pipe flow. Maciel et al. (2008) noticed an increase in turbulence level with increasing swirl number. Örlü & Alfredsson (2008) reported on the mixing characteristics in the near field of a swirling jet and found that swirl enhanced mixing in agreement with Alekseenko et al. (2008) who found counter-rotating large scale structures being responsible for this trend.

2.1.3 Coaxial jets and combustors

Champagne & Kromat (2000) performed experiments of co-axial jets with and without swirl, continuing the work of Ribeiro & Whitelaw (2006). The authors observed the development of an internal recirculation zone whose appearance depended on the amount of swirl in the outer jet. The critical amount of swirl necessary depended on the mass flow ratio of the jets. The formation of the recirculation zone seemed to be independent of the growth of convectively unstable modes.

Adzlan & Gotoda (2012) investigated vortex breakdown in a coaxial swirling jet with a density difference at low Reynolds numbers. For an increasing bulk velocity of the outer jet the stagnation point in front of the recirculation region was shifted in the downstream direction due to an increase in mean axial velocity along the jet centreline. Due to the larger centrifugal forces acting on the outer swirling jet its spreading angle was
larger compared to the spreading angle of the inner jet for all Reynolds numbers investigated.

In their experimental study of swirling jets in a model burner, Cala et al. (2006) observed three different vortical structures interacting with each other. The experiments were performed at high Reynolds number $Re \approx 15'000$ with swirl number $S = 1.01$. In the jet breakdown zone co-rotating co-winding helices were detected in the inner region, whereas counter-winding spirals were visible in the outer azimuthal shear-layer.

Umeh et al. (2010) performed experiments and numerical simulations of non-reacting vortex breakdown in a more applied context of a swirl-stabilized combustor. The authors found good agreement between experiments and Reynolds-averaged Navier-Stokes (RANS) simulations. In addition the authors compared the theoretically predicted critical amount of swirl according to the $Q$-vortex model of Rusak et al. (1998) and the Rankine vortex model of Keller et al. (1985) for the first appearance of vortex breakdown in the pipe expansion plane as well as in the inlet pipe section. The authors concluded that the theoretical model predicted the critical swirl amount well. Continuing their previous study, Umeh et al. (2012) investigated experimentally the effects of vortex breakdown in a swirl-stabilized combustor by means of PIV. The vortex breakdown region was shifted in the downstream direction for a preheated flow case compared to a cold flow case with upstream pressure and mass flow held fixed. The effect of the vortex breakdown onto combustion dynamics was discussed.

2.2 Numerical Simulations

In recent years the phenomenon of vortex breakdown was investigated using computational tools. One of the first researchers simulating vortex breakdown of swirling flows was Bossel (1969). Mainly Direct Numerical and Large-Eddy Simulations (DNS, LES) were the methods of choice to get insight into the mechanisms underlying this flow. While for the direct approach no modelling of the subgrid-scales is necessary (Moin & Mahesh, 1998), LES are in need of modelling the fine scales to circumvent the turbulence closure problem (Lesieur & Metais, 1996). Rogallo & Moin (1984) reviewed the numerical simulation of turbulent flows. For an introduction to LES in the incompressible and the compressible regime see Sagaut (2006) and Garnier et al. (2009), respectively.
2.2 Numerical Simulations

2.2.1 Low to moderate Reynolds number incompressible flows

Spall & Gatski (1991) studied the topological structure of vortex breakdown in the incompressible regime for a laminar flow at low Reynolds number. The authors found four different types of vortex breakdown, namely two helices, a spiral and a bubble-type. The axial velocity of the co-flow was found to have a significant effect on the position and type of vortex breakdown.

Kollmann et al. (2001) investigated a low Reynolds number incompressible jet at swirl number $S_2 = 1.32$ numerically reproducing two experiments by Billant et al. (1998) and found qualitative agreement.

Ruith & Meiburg (2002) and Ruith et al. (2003) studied swirling jets and wakes using direct numerical simulations and linear local and global stability analyses. The authors restricted their study to nominally axisymmetric, incompressible jets at low to medium Reynolds numbers and concluded that the basic form of vortex breakdown was axisymmetric. Helical modes in the post-breakdown stage were caused by a pocket of absolute instability in the wake of the recirculation bubble leading to a self-excited global mode.

Kollmann (2011) simulated axisymmetric and fully 3D swirling jets at low Reynolds numbers $Re = 400$ and $Re = 2000$ and a high swirl number $S_2 = 1.6$. For the low Reynolds number the flow approached an asymptotic state which was dominated by a bubble and multiple recirculation zones. This state was insensitive to disturbances superimposed at the inflow. The higher Reynolds number jet did not approach a steady-state, neither for axisymmetric nor for fully 3D simulations. Symmetry breaking disturbances superimposed at the inflow led to a fast development of a 3D flow with up to four recirculation zones.

2.2.2 High Reynolds number incompressible flows

Using LES García-Villalba et al. (2006) investigated an incompressible turbulent swirling jet at high Reynolds number and moderate swirl level. The simulation resembled experiments done using a movable block swirl generator. No-slip walls were included into the computational domain to model the nozzle and a slight co-flow was added to the swirling jet. Two dominant families of large-scale structures were identified as a results of Kelvin-Helmholtz instabilities. Freitag & Klein (2005) performed DNS in a context similar to García-Villalba et al. (2006) at lower Reynolds number. The nozzle included in the computational domain was modeled
using the immersed boundary technique. Good qualitative as well as quantitative agreement was found with experimental results in literature. The precessing vortex core was characterized as a single-helical structure directly at the nozzle lip, which possibly changed its type to an instability with higher azimuthal wave number further downstream.

Maciel et al. (2008) conducted LES of a $Re = 24'000$ incompressible water jet at swirl numbers $S_9 = 0.0$ and $S_9 = 0.5$. At the inflow results of a fully turbulent pipe flow precursor simulation were applied. The authors stated that the non-resolved small scale structures in the pipe did not substantially influence the jet development and the breakdown to turbulence. Modes $n = +2$ and $n = +3$ were found to be dominant, counter-rotating against the mean flow.

Moet et al. (2005) presented results of DNS and LES of propagating pressure waves in a Lamb-Oseen vortex. Due to an increase in axial velocity behind the travelling wave helical instabilities were triggered for sufficiently high swirl. Vortex bursting occurred for two pressure waves intersecting each other.

For a numerical simulation of vortex breakdown over a high angle of attack delta wing aircraft, see Mary (2002).

### 2.2.3 Compressible flows

Melville (1996) solved the compressible Euler equations to study vortex breakdown of a subsonic free vortex of Burger’s type. For a corrected pressure coefficient the effect of Mach number on the vortex breakdown was reversed (Herrada et al., 2003) leading to a promotion and upstream shift of the recirculation region. The vortex breakdown configuration was governed by a double-helix type structure, co-rotating with the mean flow and winding in the opposite direction, but different to the structure observed by Sarpkaya (1971). Herrada et al. (2003) investigated the effects of compressibility on the critical swirl number leading to vortex breakdown for pipe flow at low Reynolds numbers. For high Mach numbers in the subsonic regime a comparably higher amount of swirl was necessary for vortex breakdown to occur. It was shown that more than one steady-state solution was found at the same parameter set (bifurcation, bi-stability), a feature of the underlying flow well known for the incompressible regime, cf. Vyazmina et al. (2009). Müller (2007) and Müller & Kleiser (2008a) used Direct and Large-Eddy Simulations (DNS/LES) to investigate the breakdown of compressible swirling mixing layers and jets. Natural and forced
swirling jets were considered and linear stability analysis was applied to identify unstable modes. The results showed good qualitative agreement with the experimental observations by Liang & Maxworthy (2005). At sufficiently high swirl the jet broke down and a conical breakdown state established with a pronounced recirculation zone around the jet axis. The more rapid breakdown of the jet and the stronger deceleration of stream-wise velocity at the jet axis was suggested to be linked to compressibility effects.

To our knowledge, swirling jet flows undergoing vortex breakdown were not investigated extensively in the compressible, subsonic regime so far with the exception of a previous project in our research group (Müller, 2007; Müller & Kleiser, 2008) and the study by Melville (1996) in the inviscid regime.

2.2.4 Coaxial jets, combustors and turbines

In the context of a gas turbine combustor, Jochmann et al. (2006) performed unsteady Reynolds-averaged Navier Stokes (URANS) simulations focusing on the time-dependent properties of vortex breakdown. A spiral-type vortex breakdown was found precessing in time leading to an asymmetric flow field. The spiral, which was of single-helix type, gyrated around a zone of strong negative streamwise velocity. Wang et al. (2007) used Large-Eddy Simulations to investigate swirling flows in an operational gas-turbine injector. In this study the results were compared with experiments conducted simultaneously. A central recirculation zone at the axis was found, the flow being dominated by helical instabilities. The winding sense of the instabilities, which were of double-helix type, depended on the gradient of azimuthal velocity around the stagnation point. Comparing co- and counter-rotating inflow configurations the authors found that the latter led to a more efficient and stable combustion due to a shorter recirculation zone.

Duwig & Fuchs (2007) investigated the interaction of vortex breakdown with a swirl-stabilized premixed flame using LES and a filtered flamelet model. Results showed that for increased swirl the stabilized flame and especially the precessing vortex core did not change substantially, while for decreasing swirl the flame shape changed and the precessing vortex core diminished. POD analysis shed light on the structure of the precessing vortex core which was found to be a helical instability consisting of two counter-rotating helices.
Dinesh & Kirkpatrick (2009) used Large-Eddy Simulations to investigate low-to-high swirling jets in a model Sydney swirl burner at high Reynolds numbers. The results were compared with experiments by Al-Abdeli & Masri (2004) and good quantitative agreement was found in dominant Strouhal numbers at different swirl intensities. The frequency dominating the flow was due to the precession of the vortex core at the jet centreline, cf. Oberleithner et al. (2011b).

Susan-Resiga et al. (2010) performed numerical simulations of a swirling flow in the context of a model Francis turbine. The axisymmetric flow underwent vortex breakdown leading to a loss in performance of the diffuser due to a discharge decrease of the turbine. Using an injection device in the streamwise direction at the runner crown tip vortex breakdown was prevented and losses in performance were minimized.

2.3 (Semi-)Analytical Studies

In his analytical work, Benjamin (1962) proposed an axisymmetric theory analogous to hydraulic jumps, which explains vortex breakdown as a finite transition between conjugate flow states. Vortex breakdown was understood as the transition from a super-critical to a sub-critical flow, the latter supporting upstream-travelling waves. In Benjamin (1967) the author reinforced his theory of conjugate flows and extended it by an analysis of finite-amplitude stationary waves.

Shtern & Hussain (1993) and Shtern & Hussain (1996) investigated the hysteresis behaviour of swirling jets by introducing analytical models. The authors related the hysteresis loops found to hydraulic jump transitions between various flow regimes. Up to four kinds of jumps were found, one of them identified as vortex breakdown. A review on theoretical studies of swirling jet flows is also available, cf. Shtern & Hussain (1999). In Shtern et al. (2000) the authors extended their theoretical investigation of swirling jets to a model with an exponentially decaying streamwise velocity. The vortex breakdown model proposed in Shtern et al. (1997) extended previous work to account for turbulence.

Blackmore et al. (2008) introduced a model for bubble-type vortex breakdown based on a pair of slender coaxial vortex rings immersed into a swirling flow. The model captured a large number of characteristic phenomena observed in studies of vortex breakdown. Numerical results were given illustrating the features of the model.

Theoretical studies provide a proper insight into the physical mech-
2.4 Stability Analysis


2.4.1 Model vortices

Lessen et al. (1974) and Lessen & Paillet (1974) were the first to work on the stability analysis of swirling flows. The authors investigated the linear temporal stability of an incompressible trailing line vortex, which was defined similar to that of Batchelor (1964). Duck & Foster (1980) extended this study using a efficient finite-difference method (instead of a shooting method) being therefore able to determine less unstable modes, in addition to the modes found in the previous study which were verified. Khorrami (1991) and Mayer & Powell (1992), besides others, contributed results for the viscous case. Duck (1986) investigated the stability of the Batchelor vortex (Batchelor, 1964) in the limit of vanishing viscosity. Duck & Khorrami (1992) extended these studies to account for viscous effects and found good agreement between results of asymptotic theory and numerical investigations. The effect of compressibility on the stability of the viscous Batchelor vortex was analysed by Stott & Duck (1995). The linear impulse response of the viscous Batchelor’s vortex was studied by Delbende et al. (1998) to determine the absolute/convective instability characteristics of this flow type. Swirl was found to strongly promote the onset of absolute instability for wakes and jets, counter-rotating helical modes were determined which switched over first from convective to absolute instability. For wakes mode $n = -1$ and for jets
modes $n \leq -2$ changed their type first from convectively to absolutely unstable. Olendraru et al. (1999) confirmed the results of Delbende et al. (1998) in the inviscid limit and extended them to a larger regime of Reynolds numbers and base flows in their work Olendraru & Sellier (2002). Fabre & Jacquin (2004) extended the work of Olendraru & Sellier (2002) by investigating the temporal counterparts of the spatial instabilities previously found. The viscous temporal instabilities found existed in a parameter range of swirl and Reynolds number which was thought to be a stable flow regime. The instabilities were similar to the centre modes described by Stewartson et al. (1988).

The stability of Long’s vortex, which is believed to be a model for tornado-like flows, was analysed by Foster & Duck (1982) and an instability was found to short-wave, helical modes, whereas the vortex was stable to the axisymmetric mode.

Gallaire & Chomaz (2003) studied the screened Rankine vortex with added plug flow as a model problem for swirling jets by means of temporal stability analysis in the inviscid regime. The authors found that azimuthal shear destabilized helical modes $|n| \geq 2$, whereas the bending modes $n = \pm 1$ were stabilized by Kelvin waves in the vortex core region. All modes were destabilized by the axial shear and centrifugal instability. Their spatio-temporal study, which is a continuation of the study by Loiseleux et al. (1998), led to the result that centrifugal instability and azimuthal shear influenced the absolute instability of helices with positive wave number in a promoting manner.

### 2.4.2 Jets and wakes

Loiseleux et al. (1998) investigated the effect of swirl on the stability of jets and wakes. As a model flow the authors analysed the inviscid Rankine vortex with added plug flow and found that swirl significantly enhanced absolute instability of negative modes. The transition from convective to absolute instability was determined depending on swirl number and favourable comparison was found with threshold values for vortex breakdown in literature. Loiseleux et al. (2000) continued their previous work by investigating the instability nature of swirling jet/wake shear-layers following Martin & Meiburg (1994) and Lim & Redekopp (1998). The authors found centrifugally (de-)stabilizing swirl differences to promote absolute instability. The authors determined the transitional helical modes
leading to the change from convectively to absolutely unstable flow. The type of transitional mode was defined by the sign of the swirl difference.

Gallaire & Chomaz (2003b) investigated the pre-breakdown stage of swirling jets by means of temporal and spatio-temporal linear stability analysis in the incompressible flow regime continuing the work of Delbende et al. (1998) and Olendraru et al. (1999). The temporal analysis did not lead to a clear mode selection mechanism and experimental observations of Billant et al. (1998) could not be reproduced. The flow was first absolutely unstable to a counter-rotating, co-winding mode \( n = -2 \) which was suggested to be a double-helix observed in experiments, cf. Ruith & Meiburg (2002). In a follow-up study by Gallaire et al. (2006) a non-linear global mode analysis gave rise to the interpretation of spiral vortex breakdown as a global mode developing on the axisymmetric breakdown state. The spiral was triggered by a region of local absolute instability in the wake of the breakdown bubble, supporting the assumptions of Liang & Maxworthy (2005) and Oberleithner et al. (2011b). Healey (2008) theoretically investigated the absolute instability of inviscid axisymmetric swirling jets. For a jet with uniform axial velocity and superimposed solid-body rotation issuing into still fluid he found a change from convective to absolute instability for confining the jet with a concentric cylinder independent of its radius. The author suggested that the transition from convective to absolute instability could be associated with the onset of an unsteady vortex breakdown. The swirl required to produce this transition could be either greater or less than the swirl required to produce the transition from super-critical to sub-critical flow, depending on the details of the basic velocity profiles. Healey (2008) confirmed findings by Gallaire & Chomaz (2003b) which show that centrifugal instability enhanced absolute instability.

Rukes et al. (2011) successfully applied spatio-temporal linear stability analysis to time-averaged experimental data of swirling jets undergoing vortex breakdown violating the parallel-flow assumption. By setting the azimuthal mode number to the naturally occurring single-helix type instability \( n = +1 \) the authors showed that two pockets of absolute instability were present: one region directly downstream of the nozzle lip, the other one in the region of the vortex breakdown bubble in agreement with findings of Gallaire et al. (2006). A similar approach was used by Qadri et al. (2011) combining local and global stability analyses of results of steady-state axisymmetric Navier-Stokes solutions following Ruith et al. (2003) in the definition of inflow boundary conditions. The authors found a wave-
maker situated in the recirculation bubble, in contrast to their expectation that the dominating frequency came from the wake of the jet. The local stability analysis predicted two regions of absolute instability, one located in the recirculation bubble, the other in the wake, cf. Rukes et al. (2011). The first location corresponded to the instability core found in the global analysis, supporting the conclusion that the linear global mode was being driven by a wave-maker located in the breakdown bubble. The authors remarked that the local analysis was valid only for slowly developing, weakly non-parallel flows and failed to conclusively identify the wave-maker location in flows with more than one region of absolute instability. Meliga et al. (2012) analysed the global linear and non-linear bifurcation characteristics of swirling jets for flow configurations along the lines of Ruith et al. (2003). The authors focused on nominally axisymmetric swirling jets and found bifurcations to a single- and a double-helix type instability co-rotating with the mean flow and winding in the opposite direction. At large swirl the double-helix was found to be the only stable solution while for low to moderate swirl two sub-critical subsequent bifurcations first to a single- and than to a double-helix were observed.

Khorrami (1995) studied the temporal instability of a compressible swirling axisymmetric jet. He found that a modest amount of swirl added to the flow led to instability growth rates that were increased substantially. Swirling jets were found to be unstable to helical modes of high wave number, to which non-swirling jets were stable. Additionally swirl diminished the stabilizing effect of high Mach numbers, cf. Herrada et al. (2003).

2.4.3 Swirling pipe flow

The temporal and spatial stability characteristics of swirling pipe flow were investigated first by Pedley (1969) and Mackrodt (1976). They found that a superimposed azimuthal velocity component (solid-body rotation) dramatically destabilizes the pipe flow. Salwen & Cotton (1980) and Cotton & Salwen (1981) continued their work and later Landman (1990a) and Landman (1990b) focused on the generation of helical waves in rotating pipe flow and their time-dependence. Wang & Rusak (1996) continued the studies on swirling pipe flow stability and Rusak & Lee (2002) and Rusak et al. (2007) extended the results to the compressible regime. A recent investigation of the stability of swirling pipe flow is the study by Rusak et al. (2012) which deals with the global non-linear stability of swirling flow in a long finite-length pipe explicitly discussing
the transition to vortex breakdown, cf. also Meliga et al. (2012).

2.5 Control of Vortex Breakdown

The control of vortex breakdown in swirling flows is a major research area with applications in the field of external aerodynamics such as delta wing aircraft (Akilli et al., 2003), swirling flow in cylinders (Jørgensen et al., 2010) and pipes (Meliga & Gallaire, 2011) as well as in combustion applications (Syred, 2006). For a review of flow control see Bewley (2001) and, especially for swirling flows undergoing vortex breakdown, Mitchell & Delery (2001). Theofilis & Colonius (2011) give an overview over control in globally unstable flows. The textbook by Gad-el Hak (2007) introduces to active and passive control strategies for a large variety of flows.

2.5.1 Axial forcing

Khalil et al. (2006) experimentally investigated the effect of axial pulsing on the location of vortex breakdown for unconfined highly swirling jets at low Reynolds number (cf. Lopez et al. (2008) for a similar approach in the context of a closed cylinder). The authors found that for an unforced jet the natural frequency of shear-layer vortex shedding was independent of the Reynolds and swirl number. For forcing with low frequencies up to twice the natural one the jet locked onto the forcing frequency. For higher forcing frequencies vortex breakdown was not receptive anymore and the jet shed at its natural frequency, even for high forcing amplitudes. The breakdown position was shifted substantially downstream for forcing with approximately the natural frequency at critical swirl number. Thereby the breakdown structure was destroyed and the axial velocity profile altered in such a way that breakdown was delayed. Oberleithner et al. (2007)—in some sense continuing the work of Khalil et al. (2006)—used axisymmetric forcing of mode \( n = 0 \) at low to high amplitudes to study the effect on a highly swirling water jet at moderate Reynolds numbers. For higher Reynolds numbers the authors found in their experiments that for increasing swirl intensity vortex breakdown did not occur abruptly but continuously. For a favoured parameter set (not further specified) vortex breakdown was shifted downstream accompanied with a decrease of intensity. Oberleithner et al. (2008) continued the work of Seele et al. (2008) by forcing the jet at \( St = 0.35 \) and \( St = 0.92 \) in a sinusoidal way with
high amplitudes. Without forcing the authors found modes $n = +2, +3$ being dominant confirming the results of Gallaire & Chomaz (2003b) and Liang & Maxworthy (2005). When applying axial forcing, mode $n = 0$ was initially dominant upstream of vortex breakdown and energy was transferred to mode $n = +2$ further downstream. Forcing at $St = 0.92$ was found to be more efficient in controlling the flow under investigation. A lock-in onto the forcing frequency was reported. The vortex breakdown was less pronounced and shifted downstream. The authors supposed that a connection existed between the shear-layer instability and the criticality of vortex breakdown.

Iudiciani & Duwig (2011) performed LES to study the flame/vortex interaction in the presence of a precessing vortex core in a burner geometry at a Reynolds number $Re = 81'000$ in the incompressible regime (temperature-induced density differences were accounted for only). The authors forced the flow axially at various frequencies and low-level amplitudes. They found that forcing with frequencies lower than the natural frequency led to an upstream shift of the recirculation region while higher frequencies did not affect the flow field substantially, concluding that the precessing vortex core is an effective damper of high frequency axial pulsations.

2.5.2 Helical forcing

As mentioned in Sec. 2.1, Panda & McLaughlin (1994) conducted experiments with axisymmetric and helical (co- and counter-rotating mode $n \pm 1$) low amplitude forcing. For the forced jet acoustic excitation at the nozzle was applied and the streamwise development of triggered disturbances was monitored. With increasing Reynolds number the forcing frequency increased for which the swirling jet is most receptive. The swirling jet was more insensitive to external excitation compared to a non-swirling jet and the overall growth of the instabilities was substantially smaller. The authors found that the overall growth of the forced instabilities was small and vortex pairing was suppressed, arguing that this was due to the rapid growth of momentum thickness due to the mean flow azimuthal velocity. Both helical modes had the same overall growth in spite of a preferred direction of rotation by the mean flow, in contradiction to Oberleithner et al. (2009).

Gallaire et al. (2004b) performed experiments in the laminar incompressible regime at high swirl numbers, extending the study of Billant et al.
(1998) to azimuthally forced jets, see also Sec. 2.1. At the nozzle exit the authors disturbed the flow with co- and counter-rotating helical modes. The authors found that the receptivity of the swirling jet was poor for forcing with natural azimuthal mode $n = +2$ (co-rotating double-helix) and natural frequency at $Re = 1490, S_2 = 0.84$, the second swirl regime according to Loiseleux & Chomaz (2003). On the contrary, forcing with azimuthal modes $n = \pm 2, \pm 3$ with an amplitude one order of magnitude higher than the natural one resulted in a strong response of the swirling jet. In general vortex breakdown was found very robust to any of the forcing parameter sets applied, which was argued to be due to the fact that the mechanism leading to vortex breakdown was located at the jet centreline (negative azimuthal vorticity at the jet axis), therefore being insensitive to any forcing at the outer shear-layer, cf. Brown & Lopez (1990). (This is in contradiction to Oberleithner et al. (2007), Oberleithner & Paschereit (2009), Oberleithner et al. (2009) and Oberleithner et al. (2011_a), who were able to invert the natural energy transfer in the breakdown bubble, see below.).

Besides naturally evolving swirling jets (see Sec. 2.1), Liang & Maxworthy (2005) also investigated the effects of azimuthal forcing using counter-winding modes $n = +1$ and $n = +2$. Below the critical swirl level, modes $n = +1$ and $n = +2$ depended proportionally on the forcing amplitude, which indicated the convectively unstable nature of the helical modes at this stage. For higher swirl numbers the flow response seemed insensitive to external forcing, which led to the conclusion that modes $n = +1$ and $n = +2$ saturated in amplitude, changing their type from convectively to globally unstable.

Continuing their earlier work (Oberleithner et al., 2007), Oberleithner & Paschereit (2009) studied moderately swirling jets at a Reynolds number of $Re = 60'000$ employing axial and azimuthal forcing ($n = +2$) with low amplitudes in the range of $St = 0.4, \ldots, 0.9$ observing co-rotating, counter-winding helical modes. For low swirl the growth of the large-scale amplitudes changed linearly with increased frequency. The shear-layer thickness was initially small leading to a linear response of the swirling jet to the applied forcing. For moderate swirl the response maintained its linear behaviour while amplitudes were much higher and decaying faster in the streamwise direction. A second region of amplification developed with dominant mode $n = +1$ downstream of the location where mode $n = +2$ was amplified. The shedding frequency was reduced for increased swirl due to the initially thicker shear-layer
and its enhanced growth in the streamwise direction. Oberleithner et al. (2009) azimuthally forced a high Reynolds number swirling jet undergoing vortex breakdown \((S_5 = 1.22)\) and found a preferred sense of winding of helical modes, in contradiction to Panda & McLaughlin (1994): Counter-winding modes \((n \geq 0)\) led to very low amplification while co-winding modes \(n = -4, \ldots, -2\) showed exponential growth in the outer and the inner shear-layer \((St = 0.54)\). For increased forcing amplitude the energy of the amplified wave responded linearly, cf. Oberleithner et al. (2011b). For forcing with a co-winding single-helix close to the natural frequency of the jet, a lock-in onto the forced frequency was observed. No exponential growth was detectable anymore. In conclusion, mode \(n = -1\) triggered the self-excited, global mode observed by Liang & Maxworthy (2005), while forcing of modes \(n \leq -1\), which were convectively unstable, damped the absolute unstable mode, shifting the breakdown location downstream. Oberleithner et al. (2011a) experimentally investigated the controllability of the globally unstable single-helix mode by means of open-loop control for a high Reynolds number jet at swirl number \(S_5 = 1\). A double-helix instability was used to force the swirling jet and was found to be convectively unstable, thus leading to the insight that the outer shear-layer acted as a linear amplifier of upstream perturbations. The global instability was finally partly suppressed by the forcing applied, the recirculation zone being shortened and shifted downstream depending on the forcing frequency. The control mechanism was explained as an energy transfer from the outer to the inner shear-layer downstream of the stagnation point at the beginning of the breakdown bubble. Forcing therefore reversed the natural mechanism of energy exchange from the inner to the outer shear-layer.

### 2.5.3 Other forcing approaches

Husain et al. (1997) and Husain et al. (2003) studied the influence of added near-axis swirl on vortex breakdown in a closed cylindrical container by means of experiments. The authors found that co-rotation at the axis suppressed vortex breakdown, while counter-rotation enhanced the breakdown leading to several recirculation bubbles. The authors argued that the co-rotation decreased the positive pressure gradient along the axis and the swirl number while counter-rotation induced centrifugal instability. Herrada & Shtern (2003b) and Herrada & Shtern (2003a) continued the work of Husain et al. (1997) and Husain et al. (2003) numerically and extended the previous study by applying a temperature gradient in
the axial direction. The authors found that a negative (positive) axial temperature gradient could enforce (diminish) the vortex breakdown enhancement by the counter-rotation. The underlying mechanism of the vortex breakdown control was centrifugal and gravitational convection. For higher $Ma$ and $Re$ numbers the effects of the centrifugal convection became more significant. Density variations induced by the temperature gradients had a larger effect on the flow than those induced by Mach number effects, cf. Billant et al. (1998) and Mourtazin & Cohen (2007). In their experiments Ismadi et al. (2011) and Meunier et al. (2011) investigated the control of vortex breakdown by density differences inside a cylinder with a rotating top-lid, continuing the work of Herrada & Shtern (2003$b$) and Herrada & Shtern (2003$a$). The authors stated that injecting a heavier fluid compared to the ambient one at the vortex axis enhanced bubble-type vortex breakdown while the injection of a lighter fluid had no preventing effect. Tan et al. (2009) numerically studied the ability to control vortex breakdown in a cylinder with a rotating end-wall by adding swirl using a small disk embedded in the non-rotating top-lid (cf. Husain et al. (1997) and Husain et al. (2003)). The authors showed that co-rotation of the disk promoted breakdown while counter-rotation had the opposite effect. The effect of the co-rotation on vortex breakdown was similar to the effect of increased Reynolds number.

Akilli et al. (2003) experimentally investigated the effect of a wire co-axially aligned with the leading-edge vortex of a delta wing and found that onset of vortex breakdown could be altered depending on the length of the wire. See also the review on control of vortex breakdown over delta wings by Mitchell & Delery (2001). Paschereit et al. (2006) experimentally and numerically studied the effect of an instability associated with vortex breakdown in a swirl-stabilized combustor. The recirculation within the breakdown region depended on the swirl intensity and on downstream pressure fluctuations in the combustor. Control of the instability was achieved by a passive method by means of a extended lance and fuel injection directly into the recirculation zone to actively stabilize the flame.

Gallaire et al. (2004$a$) developed a closed-loop control model for the finite-length pipe model of Wang & Rusak (1996) in a more theoretical context. By means of an optimal control approach the authors showed that vortex breakdown was prevented up to a swirl level 13% higher than the swirl sufficient for the onset of the global instability in the non-controlled case. The control mechanism was proven to be robust to noise and uncertainties in the parameter settings. Gallaire & Meliga (2010)
and Meliga & Gallaire (2011) studied theoretically and numerically the axisymmetric flow in constricted finite-length pipes with different levels of confinement. To control the swirling flow, a low-flow rate annular jet directed radially inwards was positioned at the location of confinement on the pipe wall. Maximum effectiveness was achieved for control by means of the wall jet for placing the jet at the outlet of the pipe contraction. In addition the inflow axial velocity profile was modified to outdo the wall-jet technique, being even more effective in maintaining the columnar state of the flow. Yu & Meguid (2009) numerically studied the effect of wavy side-walls on vortex breakdown in a cylindrical chamber with a rotating end-wall and showed that, depending on the period, amplitude and orientation of the wall-shape vortex breakdown was either enhanced or suppressed (cf. Gallaire & Meliga (2010) and Meliga & Gallaire (2011) for a similar approach).

García-Villalba & Fröhlich (2006) performed LES of free annular jets at high Reynolds and swirl numbers and added a pilot jet near the axis with small axial and angular momentum. The authors found that the mean flow properties were nearly unaffected by the pilot jet, while instantaneous vortical structures were destroyed. The control mechanism was even more effective in the case of a swirling pilot jet.

O’Connor & Lieuwen (2012) forced a swirling annular jet with a transverse acoustic excitation. For the natural flow a single- and double-helix type instability dominated the flow co-rotating with the mean flow. The authors showed that the recirculation region is sensitive to high-amplitude acoustic excitation and that the flow response is asymmetric for asymmetric forcing as known from literature. Nevertheless, the mean flow properties changed slightly only in general, with an exception for a bifurcation case where the flow showed a sharp transition behaviour. The authors drew the conclusion that significantly different types of responses of the swirling annular jet exist, consistent with the known highly non-linear response of globally unstable flows on forcing. No lock-in of the natural frequency on the forcing frequency was observed, in contradiction to results reported in Oberleithner et al. (2011b).

2.6 Swirl number definition

In the following we discuss the definition of the swirl number, a parameter characteristic for swirling flows and widely used for their classification. We refer thereby to discussions by Toh et al. (2010) and Oberleithner et al.
(2012) and restrict ourselves to a comparison of differently defined swirl numbers and their applicability to the swirling jets in the specific configurations investigated here.

Since the first observation of vortex breakdown by Nuttal (1953) several different definitions for the swirl number $S$ have been introduced. The swirl number is the parameter describing the behaviour of swirling flow and is used to distinguish different flow regimes, see for example Oberleithner et al. (2012) besides many others. In the pre-breakdown regime ($S < S_{c1}$) no recirculation zone is observed and the streamwise centreline velocity is positive for all time. The transitional vortex breakdown regime ($S_{c1} \leq S < S_{c2}$) is characterized by intermittently occurring negative streamwise centreline velocity and a vortex breakdown configuration, but the mean flow field does not show a back-flow at the centreline. In the regime of stable vortex breakdown ($S > S_{c2}$) a recirculation region is observed constantly in the instantaneous flow field as well as in the mean flow.

Tab. 2.1 summarizes proposed definitions for the swirl number found in the literature. We distinguish between locally defined swirl numbers such as $S_1 - S_3$ and those who take at least one integral quantity into account ($S_4 - S_9$). The advantage of locally defined swirl numbers is the much easier computation, or measurement, of the quantities taken into account. A drawback of this approach is that the information contained in locally defined swirl numbers is limited to some extent and sometimes meaningful only in specific experimental/computational configurations. This might be a disadvantage especially for our purpose here, since we investigate swirling jet flows emanating from rotating nozzles and nozzles kept at rest. For example, definition $S_9$ fails for the nozzle kept at rest due to the zero azimuthal velocity at the nozzle wall. The effort in determining an integral swirl number is somewhat higher, but the resulting parameter might be more meaningful, especially when computed at downstream positions in the vortex breakdown region, see Sec. 5.4.

Fig. 2.1 shows several swirl numbers $S_i$ according to the definitions given in Tab. 2.1 for increasing swirl parameter $a$. The initial velocity profiles introduced in Sec. 4.3.1 and utilised for imposing the inflow boundary conditions are used for the calculation, see Eqs. 5.2 - 5.4. Depending on the initial distribution of the azimuthal velocity, the several definitions lead to either a smaller ($S_2, S_3, S_7, S_8$) or a larger ($S_1, S_9$) swirl number for the rotating nozzle compared to the nozzle kept at rest. Only definition $S_4$ provides approximately identical swirl number values for increasing swirl parameter $a$, making it theoretically possible to compare both nozzle se-
tups easily. As we will see in Sec. 5.4, swirl numbers $S_2, S_3, S_7, S_9$ tend to infinity in the region of vortex breakdown downstream of the nozzle while definition $S_8$ is approximately zero there. So the streamwise development of those swirl numbers makes their application doubtful in the present context. For swirl numbers $S_1, S_4, S_5$ we observe a large variation over downstream distance especially for the latter two in the rotating nozzle setup. The definition $S_6$, which requires most complex calculation, leads to the smoothest distribution of the swirl number in both setups investigated. We therefore decide to use swirl number $S_6$ for comparison.

![Swirl numbers computed for the velocity profiles Eqs. 5.1 - 5.4 according to different definitions (Tab. 2.1) for the rotating nozzle and the nozzle kept at rest. The grey-shaded area marks the swirl parameter range $a \geq a_{c2}$ where stable vortex breakdown occurs, see Sec. 6.1. $S_1$ (red), $S_2$ (green), $S_3$ (blue), $S_4$ (violet), $S_7$ (turquoise), $S_8$ (yellow), $S_9$ (black).](image)

We want to point out that in the context of compressible swirling flows it might be more intuitive to use Favre-averaged flow quantities instead of the Reynolds-averaged mean values, see Bilger (1975) and the references therein. For the sake of comparability to results reported in literature we stay with swirl number $S_6$ throughout the present investigation.
<table>
<thead>
<tr>
<th>Reference</th>
<th>Swirl number definition $S_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Escudier &amp; Keller (1985)</td>
<td>$S_1 = \frac{2 \cdot \max(v)}{\max(w)}$</td>
</tr>
<tr>
<td>Billant et al. (1998), Loiselleux &amp; Chomaz (2003), Gallaire &amp; Chomaz (2003$b$)</td>
<td>$S_2 = \frac{2 \cdot v(r=R/2)}{w(r=0)}$</td>
</tr>
<tr>
<td>F. Gallaire, private comm.</td>
<td>$S_3 = \frac{\partial}{\partial r} v(r=0)$</td>
</tr>
<tr>
<td>Chervinsky &amp; Chigier (1965)</td>
<td>$S_4 = \frac{\int_0^{r_\infty} (w \cdot v \cdot r^2) dr}{R \cdot \int_0^{r_\infty} (w^2 - \frac{v^2}{2}) \cdot r dr}$</td>
</tr>
<tr>
<td>Chervinsky &amp; Chigier (1965), Panda &amp; McLaughlin (1994), Lu &amp; Lele (1999), Oberleithner et al. (2012)</td>
<td>$S_5 = \frac{\int_0^{r_\infty} (w \cdot v \cdot r^2) dr}{R \cdot \int_0^{r_\infty} (w^2 - \frac{v^2}{2} + w'^2 - \frac{v'^2}{2}) \cdot r dr}$</td>
</tr>
<tr>
<td>Oberleithner et al. (2012)</td>
<td>$S_6 = \frac{\int_0^{r_\infty} (\left((w \cdot v + u' \cdot v') \cdot r^2\right) dr}{R \cdot \int_0^{r_\infty} \left(w^2 - \frac{v^2}{2} + w'^2 - \frac{v'^2}{2}\right) \cdot r + \left(w \frac{\partial u}{\partial z} + u \frac{\partial u}{\partial r} + \frac{\partial (w' u')}{\partial z}\right) \cdot \frac{r^2}{2} dr}$</td>
</tr>
<tr>
<td>Billant et al. (1998)</td>
<td>$S_7 = \sqrt{\int_0^{r_\infty} \frac{v^2}{w(r=0)} dr}$, $S_8 = \left(\frac{\max(v)}{\int_0^{r_\infty} \frac{w}{wdr}}\right)^{-1}$</td>
</tr>
<tr>
<td>Liang &amp; Maxworthy (2005), Facciolo et al. (2007), Örlü &amp; Alfredsson (2008), Leclaire &amp; Jacquin (2011)</td>
<td>$S_9 = \frac{v(r=R,z)}{r_\infty \int_0^{r_\infty} wdr}$</td>
</tr>
</tbody>
</table>

Table 2.1: Several swirl number definitions according to references in literature.
Chapter 3

Computational framework

In this chapter we introduce the numerical schemes utilised in the present investigation. The numerical framework has been used before in previous research projects and has been validated extensively, cf. Müller (2007), Müller & Kleiser (2007a), Keiderling et al. (2004) and Keiderling (2009) and the references therein. Therefore we describe the numerical framework used in the present investigation as briefly as possible, closely following Müller (2007, Chap. 3).

3.1 Basic equations and numerical schemes

Our simulation code PARACONCYL solves the compressible Navier-Stokes equations in a conservative formulation (Anderson et al., 1968) with generalized co-ordinates:

\[
\left( \frac{U}{J} \right)_t + \left( \frac{F_E}{J} \right)_\xi + \left( \frac{G_E}{J} \right)_Y + \left( \frac{H_E}{J} \right)_\zeta = \left( \frac{F_S}{J} \right)_\xi + \left( \frac{G_S}{J} \right)_Y + \left( \frac{H_S}{J} \right)_\zeta, \quad (3.1)
\]

where \( \frac{U}{J} \) is the vector of conservative variables and \( \frac{F_E}{J}, \frac{G_E}{J}, \frac{H_E}{J} \) are the inviscid and \( \frac{F_S}{J}, \frac{G_S}{J}, \frac{H_S}{J} \) are the viscous fluxes, respectively, cf. Müller (2007, Eq. 3.3 + 3.4, 3.11-3.14). The Jacobian \( J \) relates the generalized co-ordinates \( (\xi, \eta, \zeta) \) to the Cartesian co-ordinates \( (X, Y, Z) \), which can be related to the cylindrical co-ordinates \( (r, \theta, z) \) with velocity components \( (u, v, w) \), cf. Müller (2007, Eq. 3.6-3.9). The vector of conservative variables is given in the following form:

\[
\vec{U} = \begin{pmatrix} \rho \\ \rho U \\ \rho V \\ \rho W \\ E \end{pmatrix}, \quad (3.2)
\]
with $\rho$ the density, $U, V, W$ the Cartesian velocity components and $E$ the total energy:

$$E = \frac{p}{\gamma - 1} + \frac{\rho}{2} (U^2 + V^2 + W^2),$$  \hspace{1cm} (3.3)$$

where $p$ is the pressure and $\gamma = 1.4$ is the ratio of specific heats. Eq. 3.1 is solved on a cylindrical computational grid together with the equation of state for an ideal gas

$$\gamma Ma^2 p = \rho T,$$ \hspace{1cm} (3.4)$$

where $Ma$ is the Mach number and $T$ is the temperature. Spatial derivatives in the streamwise and the radial directions are discretised using high-order compact finite difference schemes (Lele, 1992) with up to 10th order accuracy in the interior of the domain:

$$\beta f'_{i+2} + \alpha f'_{i+1} + f'_i + \alpha f'_{i-1} + \beta f'_{i-2} = \frac{a f_{i+1} - f_{i-1}}{2h} + \frac{b f_{i+2} - f_{i-2}}{4h} + \frac{c f_{i+3} - f_{i-3}}{6h},$$ \hspace{1cm} (3.5)$$

where the derivatives $f'_i$ are approximated by a set of values $f_i$ on an equidistant numerical grid. The coefficients are given as follows according to Lele (1992):

$$\alpha = \frac{1}{2}, \beta = \frac{1}{20}, a = \frac{17}{12}, b = \frac{101}{150}, c = \frac{1}{100}.$$ \hspace{1cm} (3.6)$$

At the borders of the domain we employ shifted and one-sided compact schemes of 6th, 5th and 3rd order accuracy in the second-next-to-, next-to- and the boundary grid point leading to an overall accuracy of 6th order, cf. Müller (2007, Sec. 3.2.2). For the circumferential direction a pseudo-spectral method is implemented to calculate the spatial derivatives in spectral space:

$$\left. \frac{df}{d\theta} \right|_l = 2\pi \sum_{n=-N_\theta/2}^{N_\theta/2-1} (in) \hat{f}_n \exp \left( \frac{2\pi iln}{N_\theta} \right),$$ \hspace{1cm} (3.7)$$

where $N_\theta$ is the number of azimuthal grid points and the subscript $l$ denotes the $l$th grid point at which the derivative is calculated.

For time integration the LDDRK-scheme of Berland et al. (2006) and Berland et al. (2007) is used, a scheme employed widely in aero-acoustics
3.1 Basic equations and numerical schemes

research due to its superior dissipation and dispersion properties. The Runge-Kutta scheme is of explicit type with an accuracy of 4th order (Müller, 2007, Eq. 3.41):

\[
\begin{cases}
  t_h^{[i]} = \alpha_{RK} t_h^{i-1} + \Delta t \\
  t^{[i]} = t^{[i-1]} + \beta_{RK} t_h^{[i]} \\
  w^{[i]} = \alpha_{RK} w^{[i-1]} + \Delta t f(u^{[i-1]}, t^{[i-1]}) \\
  u^{[i]} = u^{[i-1]} + \beta_{RK} w^{[i]}
\end{cases}
\]  

(3.8)

where \( s = 6 \) is the number of stages and \( u^{[0]} = u^{n-1} \), \( u^{[s]} = u^n \), \( t^{[0]} = t^{n-1} \). The intermediate time level \( t^{[i]} \) and the intermediate flow field \( u^{[i]} \) are determined by the auxiliary variables \( t_h^{[i]} \) and \( w^{[i]} \). For \( \alpha_{RK}^{[1]} \equiv 0 \) the algorithm is self-starting. The coefficients \( \alpha_{RK}^{[i]}, \beta_{RK}^{[i]} \) are given in Berland et al. (2006) and Müller (2007, Tab. 3.1). The time-step is set constant throughout the present investigation according to a CFL-criterion.

For modelling the subgrid scales we use the approximate deconvolution model in the relaxation-term formulation (ADM-RT), see Stolz & Adams (1999), Stolz et al. (2001b), Stolz et al. (2001a) and Adams (2011) and Malaspinas & Sagaut (2011) for newest developments:

\[
\frac{\partial U(x, t)}{\partial t} = \text{RHS}(U(x, t)) - \chi \cdot (U(x, t))_{hp},
\]  

(3.9)

where \( \chi \) is the model parameter and the subscript \( hp \) denotes the high-pass filtered state vector, cf. Gräf (2012, Eq. 2.21). Müller & Kleiser (2008b) showed that this SGS model is applicable to swirling jet flow undergoing vortex breakdown leading to quite favorable results. The model parameter \( \chi = 1/\Delta t \) is set constant throughout the present study following Keiderling & Kleiser (2007), Keiderling (2009) and Gräf (2012). Müller (2007) compared DNS results with LES data and showed that the choice of a constant SGS model parameter is applicable to swirling jet flows leading to favourable results. Stolz (2000) and Schlatter (2005) showed for different flow types that the sensitivity of the results on parameter changes is low over a wide range of \( \chi \).

For a vanishing radial co-ordinate \( r \to 0 \) a singularity treatment is implemented according to Mohseni & Colonius (2000) shifting the innermost grid point slightly off the axis. For the eight inner circular grid lines mode-clipping (Freund et al., 2000b) is employed, leading to at least six modes being retained at the innermost circular grid line to lessen time-step restrictions (Keiderling, 2009).
At all domain boundaries non-reflecting conditions are implemented, see Thompson (1987), Poinsot & Lele (1992) and Kim & Lee (2000) and references therein. The boundary conditions are supplemented with sponge layers (Bodony, 2006; Israeli & Orszag, 1981) to avoid a mean pressure drift (outflow/far-field) and to damp upstream and downstream travelling waves (in-/outflow), see Colonius (2004) for a review. This is achieved by adding a forcing term to the right-hand side of the Navier–Stokes equations within a sponge region

\[
\frac{\partial \mathbf{U}(x,t)}{\partial t} = \text{RHS}(\mathbf{U}(x,t)) + \sigma(x) \cdot (\mathbf{U}_{\text{ref}}(x,t) - \mathbf{U}(x,t)),
\]

where the support of \(\sigma(x)\) defines the sponge geometry as specified below. The forcing is designed to drive the current solution \(\mathbf{U}(x,t)\) towards a reference state \(\mathbf{U}_{\text{ref}}(x)\).

Taking into account the computational domain definition \(-5 \leq z \leq 15\) and \(0 \leq r \leq 10\) used throughout the present investigation, the sponge regions are defined by

\[
\sigma(x) = \sigma_I(x) \cdot S_I + \sigma_O(x) \cdot S_O + \sigma_\infty(x) \cdot S_\infty,
\]

with the inflow, outflow and far-field sponges

\[
\begin{align*}
\sigma_I(x) &= \exp(-2.5 \cdot (z + 5)^3) \\
\sigma_O(x) &= 0.5 \cdot (1 - \text{erf}(1.05 \cdot (16.5 - (z + 5)))) \\
\sigma_\infty(x) &= [1 - (0.5 \cdot (1 - \text{erf}(1.05 \cdot (16.5 - (z + 5)))))] \\
&\quad - (\exp(-0.5 \cdot (z + 5)^3))] \\
&\quad \cdot [0.5 \cdot (1 - \text{erf}(1.05 \cdot (8.5 - r)))]
\end{align*}
\]

where \(\text{erf}\) denotes the error function. The diagonal matrices \(S_I, S_O, S_\infty\) define on which of the five variables the sponges act, and are defined by

\[
\begin{align*}
S_{I,ij} &= \begin{cases} 
1, & i = j \\ 
0, & i \neq j
\end{cases} \\
S_{O,ij} &= \begin{cases} 
\in [0, 1], & i = j \\ 
0, & i \neq j
\end{cases} \\
S_{\infty,ij} &= \begin{cases} 
1, & i = j = 1, \ i = j = 5 \\ 
0, & i \neq j
\end{cases}
\end{align*}
\]
3.2 Code parallelisation strategy

At the inflow a Dirichlet boundary condition can be employed instead of the non-reflecting condition. More details on the boundary conditions are given in Chap. 4.

The cylindrical grid used is refined around the nozzle lip in the streamwise direction and coarsened at the domain outflow boundary. For the radial direction the grid spacing is refined in the region of the nozzle wall to account for the viscous sublayer within the nozzle and the shear-layer developing downstream of the nozzle lip. The grid spacing is coarsened smoothly from $r = 2$ onwards. The overall grid generation follows Keiderling (2009) and Müller (2007). For a grid convergence study and details on the numerical grid we refer to App. A.

A nozzle is included in our computational domain to account for a more realistic setup (Bühler, 2013; Bühler et al., 2010), see previous publications by the author (Luginsland & Kleiser, 2011a, b). At the nozzle wall the spatial schemes are modified in the same manner as for the domain boundaries to decouple the inner wall grid points from the physical flow region, for details see Bühler (2013) and Bühler et al. (2010). Dirichlet boundary conditions are implemented at the nozzle wall to set three velocities and the temperature following Sandberg et al. (2011) and Bühler et al. (2010).

3.2 Code parallelisation strategy

To allow for the application on massively parallel computing architectures we parallelised our in-house simulation code CONCYL (COmpressible Navier-Stokes in CYLindrical geometries). The simulation code has been used extensively in previous projects on vector-based architectures (Keiderling, 2009; Keiderling & Kleiser, 2007, 2008; Keiderling et al., 2009, 2004; Müller, 2007; Müller & Kleiser, 2007a,b, 2008a,b). The parallelisation approach is of mixed type: for applying the subgrid-scale model we filter the flow field and use therefore the ghost-cell approach to have the necessary data available on all processors involved. For calculating the spatial derivatives we transpose the flow field from a ring-configuration to a beam-configuration and then to a double wedge-configuration and back to avoid solving a secondary system (Schur problem), see Bühler (2013) and the references therein for details. The weak scaling properties of the parallelized code PARACONCYL are therefore determined by the data transposition procedure, see Luginsland et al. (2010) for a communication cost estima-
tion and scaling properties. We highly optimized the simulation code for massively parallel architectures during the parallelisation procedure.
Chapter 4

Assessment of boundary conditions

We investigate the sensitivity of numerical simulation results for swirling jet flows undergoing vortex breakdown to inflow and outflow boundary conditions. The compressible regime at Mach number $Ma = 0.6$ and Reynolds number $Re = 5000$ is considered. The swirl velocity is approximately of the same magnitude as the streamwise centreline velocity at inflow. We perform Large–Eddy Simulations using high-order discretisation schemes in space and time. A rotating nozzle with isothermal wall is included in the computational domain. Six different combinations of inflow and outflow boundary conditions are investigated. These use a Dirichlet condition at the inflow supplemented with a sponge layer imposing up to five variables and a sponge layer at the outflow acting on several combinations of variables, applied together with non-reflecting boundary conditions. The advantages and drawbacks of each setup are investigated. The qualitative features of the swirling jet undergoing vortex breakdown are robust to changes in the inflow and outflow boundary conditions, i.e., conical shear–layers, a recirculation bubble, the existence of a single-helix type instability, and the occurrence of a dominant frequency, are all captured by combinations of the boundary conditions investigated. However, significant quantitative differences are observed depending on the conditions set at inflow and outflow. In particular, the locations of the stagnation points and the spreading angle of the swirling jet are strongly influenced. The size and shape of the recirculation bubble change as well, as does the intensity of the recirculation flow and of the counter-rotating motion observed at the jet centreline. The dominant frequency in the breakdown region also depends on the setup. As a result of this study, we recommend setting the three velocity components, density, and pressure at the inflow and outflow using sponge layers supplementing non-reflecting boundary conditions as the most suitable choice.
4.1 Introduction

The physical phenomenon of vortex breakdown was first observed by Nuttal (1953) in the context of swirling flow in a cylindrical container and later by Lambourne & Bryer (1961) for a delta wing aircraft at high angle of attack. Vortex breakdown occurs in many technical applications (e.g., delta-wing aircraft (Peckham & Atkinson, 1957), vortex burners (Chigier & Chervinsky, 1967)) and can also be observed in nature (dust devils, tornadoes, hurricanes (Burggraf & Foster, 1977)). A field of ongoing research are swirling jet flows undergoing vortex breakdown. For a sufficiently high circumferential velocity relative to the streamwise velocity, vortex breakdown occurs. The flow state of a vortex breakdown is characterised by a strong recirculation in the centreline region of the swirling flow and a high radial spreading rate (Billant et al., 1998). It is of great interest to understand the fundamental features of vortex breakdown, to know the parameters at which it occurs, and to get insight into possible control mechanisms of this special flow configuration. Although in more than five decades of intense research many attempts were made to explain vortex breakdown, a widely accepted theory is still missing. For reviews of the vortex breakdown phenomenon, we refer to Hall (1966), Hall (1972), Leibovich (1978), Escudier (1988), Delery (1994), Shtern & Hussain (1999), Lucca-Negro & O’Doherty (2001) and especially focusing on control, Mitchell & Delery (2001). The review by Krause (2003) focuses on three examples of vortical flows including vortex breakdown.

Recent experimental studies on swirling jet flows in the incompressible regime (Gallaire et al., 2004b; Liang & Maxworthy, 2005, 2008; Oberleithner et al., 2011b) revealed the presence of a globally unstable mode. The global mode overwhelms the entire flow, acting as the wave-maker for the helical shear-layer instabilities of the conical vortex breakdown. These results are supported by linear stability analysis (Gallaire et al., 2006), leading to a maximum of two absolutely unstable flow regions: the first one located directly downstream of the nozzle, and the other one located in the leeward region of the breakdown bubble.

Published results of numerical investigations are mainly based on solutions of the incompressible Navier–Stokes equations in the low Reynolds number regime ($Re \leq 1000$) for swirling jet flows (Ruith et al., 2003), and for moderate to high Reynolds numbers in the context of swirl burners (Dinesh & Kirkpatrick, 2009) and turbines (Susan-Resiga et al., 2010). To the best of our knowledge, the boundary conditions in these
studies were chosen without much further discussion, and only Ruith et al. (2004) reported an assessment of the influence of far-field conditions on the flow characteristics. They recommended radiation conditions at the far-field boundary of the computational domain while using Dirichlet boundary conditions at inflow and a convective outflow condition. The flows under investigation were laminar, incompressible low Reynolds number swirling jets and wakes. The results revealed a high sensitivity of the vortex breakdown structure and the entrainment streamlines to the choice of the far-field boundary conditions.

García-Villalba et al. (2004) investigated the influence of the inflow boundary location in the context of swirl burners. They found that for certain inflow boundary locations, highly unsteady large-scale coherent structures found in corresponding experiments were not present at all in their simulations. The types of boundary conditions were held fixed for all three simulations performed: a Dirichlet condition at the inflow, a convective condition at the outflow and free-slip conditions in the far-field.

Gallaire & Chomaz (2004) discussed the role of boundary conditions in the context of local/global stability analysis of inviscid swirling pipe flow following Wang & Rusak (1996). They extended the study of Wang & Rusak (1996) by investigating three additional sets of boundary conditions at the in-/outlet, namely a vanishing radial/streamwise velocity perturbation and a combination of both. Changing the boundary conditions independently led to a weakly stabilized flow, while changing both at a time led to an unconditionally stable flow independent of the swirl considered. The authors concluded that suitable in- and outlet conditions were sufficient for the flow to become absolutely unstable in the absence of any local instability mechanism.

Leclaire & Sipp (2010) theoretically investigated the influence of the upstream boundary conditions on the bifurcation structure leading to vortex breakdown following Wang & Rusak (1997). They varied the streamwise and azimuthal velocity profiles at the inflow in combination with a third condition chosen either as a fixed azimuthal vorticity or as a vanishing radial velocity. At the pipe wall, free-slip conditions were applied. The authors restricted their study to an incompressible inviscid flow in a finite-length pipe of constant cross-section, and found up to six different bifurcation scenarios. Flows with a large rotational core were particularly sensitive to an accurate modelling of the upstream boundary conditions.

Melville (1996) studied the breakdown behaviour of an isolated, unconfined Burgers-type vortex in the inviscid, compressible, subsonic regime.
He solved the compressible Euler equations under the following boundary conditions: Three velocities were set at the inflow together with the pressure extrapolated from the interior of the domain. At the outflow, all five variables were set, leading to a formal ill-posedness. The upstream effect of this ill-posedness was minimized by choosing the domain size to be sufficiently large.

Herrada et al. (2003) investigated the effects of compressibility at $Ma < 1$ on vortex breakdown in pipes solving the axisymmetric Navier–Stokes equations. They set all five variables at the inflow assuming a uniform temperature distribution. At the outflow, zero gradient conditions were imposed and compared to non-reflecting conditions, leading to indistinguishable results.

Liu et al. (1993) solved the full compressible Navier–Stokes equations to investigate vortex breakdown of swirling jets in the supersonic regime. The authors found only very little effect of the outflow conditions on the vortex breakdown configuration in bounded and unbounded domains for two reasons: first, the flow field was mainly supersonic at the outflow (due to a bypass flow), and second, the numerical domain was large enough to prevent interactions between the outflow boundary conditions and the vortex breakdown region.

The contributions by Müller (2007), Müller & Kleiser (2007b) and Müller & Kleiser (2008a) concern swirling jets undergoing vortex breakdown in the compressible subsonic regime. A Dirichlet condition at the inflow was used in combination with a sponge layer (Bodony, 2006) for all five conservative variables. The advantages of this choice are the possibility of imposing precise disturbances at the inflow to trigger the swirling jet flow and the ability to damp upstream-travelling waves. At the outflow and far-field boundary, non-reflecting conditions (Poinsot & Lele, 1992) were used in combination with sponge layers for three velocities, density and pressure and for density and pressure, respectively.

Since it is well known that swirling flows undergoing vortex breakdown are highly sensitive to upstream and downstream conditions (Escudier & Keller, 1985) and especially to any physical or artificial perturbations, it is of great interest to assess the influence of the boundary conditions on computational results for a swirling jet flow. The aim of the present study is to identify the most appropriate combination of inflow and outflow boundary conditions for studying vortex breakdown of swirling jet flows including nozzle modelling. We restrict our investigation to the subsonic, compressible regime at moderate Reynolds number $Re = 5000$. 
This chapter is organized as follows. In Sec. 4.2 the numerical framework is introduced. In Sec. 4.3.1 the basic simulation setup is presented, followed by the setup variations described in Sec. 4.3.2. In Sec. 4.4 the results on the influence of the boundary conditions on flow characteristics are presented. In Sec. 4.5 we summarize and discuss our findings. We conclude our study in Sec. 4.6 and give a recommendation for the most appropriate setup for simulations of swirling jets undergoing vortex breakdown in the compressible regime.

4.2 Numerical framework

In this section, we summarize the basic approach and the numerical methods used in the present investigation. An extensive documentation is given in Müller (2007). We solve the compressible Navier–Stokes equations in a conservative formulation on a cylindrical grid, see Fig. 4.1 for a sketch of the setup. The governing equations are non-dimensionalized using the nozzle inner radius $R^\circ$ and centreline quantities, such as streamwise velocity $w^\circ_c$, density $\rho^\circ_c$, dynamic viscosity $\mu^\circ_c$ and temperature $T^\circ_c$ ($^\circ$ indicates dimensional quantities). The Reynolds number is set to $Re =$

![Figure 4.1](image)

Figure 4.1: Computational domain for simulations. Dirichlet conditions marked in red, non-reflecting boundary conditions marked in green. Sponge layers are shaded in grey or hatched. Nozzle wall marked in blue. Dashed boundary conditions at inflow and the hatched sponge layer at outflow are used optionally depending on the specific setup tested. Quantities given are imposed for all setups without exception (inflow/far-field sponge and nozzle wall).
ρcw = 5000 and the Mach number is $Ma = \frac{w_c}{\sqrt{\gamma R_{\text{air}} T_c}} = 0.6$, with $\gamma = 1.4$ the ratio of specific heats and $R_{\text{air}} = 287.15 J/kg/K$ the gas constant of air. We define the integral swirl number $S_6$ according to Tab. 2.1 (Oberleithner et al., 2012), which leads to an initial swirl number of $S_6 = 0.75$, lying above the threshold for vortex breakdown.

The nozzle length is $L = 5$ and the nozzle wall thickness is $d = (R^o_o - R^o) / R^o = 0.1$ where $R^o_o$ denotes the outer nozzle radius.

At all domain boundaries, non-reflecting conditions (Poinsot & Lele (1992) and references therein) are implemented. The boundary conditions are optionally supplemented with sponge layers (Bodony, 2006) to avoid a mean density drift and to damp reflections of upstream- and downstream-travelling waves at the inflow and outflow. At the inflow, a Dirichlet boundary condition is employed instead of the non-reflecting condition for some cases investigated. Dirichlet boundary conditions are implemented at the nozzle wall prescribing three no-slip velocities and the temperature as in Bühler et al. (2010). A detailed discussion of the employed boundary conditions and the different cases is given in Sec. 4.3.

The domain size is $L_r \times L_\theta \times L_z = 10R^o \times 2\pi \times 20R^o$ with the nozzle located at $-5 \leq z \leq 0$, see Fig. 4.1. The Large–Eddy Simulations are performed with a resolution of $N_r \times N_\theta \times N_z = 288 \times 128 \times 288$ grid points. The time-step $\Delta t = 0.004$ is kept fixed throughout the present investigation.

The data is sampled after 100 dimensionless time units (a transition to a quasi-steady flow field) for a time interval of 300 time units. Every fifth time-step is used for calculating mean flow properties (15’000 samples) and every 25th time-step is sampled for all other quantities (3’000 samples).

### 4.3 Boundary condition setup

Poinsot & Lele (1992) developed non-reflecting boundary conditions for direct numerical simulations of compressible viscous flows based on the characteristic analysis of Thompson (1987). The authors derived boundary conditions for the Euler equations and extended the results to the full Navier–Stokes equations. The entire analysis is based on local one-dimensional inviscid (LODI) relations for time variations of the amplitude of characteristic waves crossing the boundary of the computational domain, see Fig. 4.2. At the inflow boundary four characteristic waves ($C_2 - C_5$) enter and one wave ($C_1$) leaves the domain, while it is vice-versa
at the outflow boundary. The number of characteristic waves entering the domain determines the number of boundary conditions to be set. The estimated wave amplitude variations were then used to calculate the missing information, which was not set at the domain boundary, by solving the corresponding conservation equations and guaranteeing that the boundary conditions are non-reflective.

Based on their analysis Poinsot & Lele (1992) proposed the following choice of boundary conditions for solving the viscous compressible three-dimensional Navier–Stokes equations in the subsonic regime: at the inflow, three velocities and the temperature are set, while the density is determined by the flow state in the interior of the domain solving the continuity equation at the domain boundary. At the outflow, the pressure is set indirectly to the far-field pressure by properly adjusting the variation of the amplitude of the entering characteristic wave and by solving the energy equation. The wave amplitude variations for the four waves leaving the domain are determined by the interior flow field solving the three momentum equations and the continuity equation. No recommendation was given for the far-field boundary of the computational domain.

The experience gained in the present investigation showed that the combination of boundary conditions in accordance with the above recommendations (Poinsot & Lele, 1992) leads to numerical instabilities due to a mean density drift at the outflow boundary, a feature reported also in Colonius (2004). Therefore all setups investigated treat the boundary conditions differently than Poinsot & Lele (1992). Besides the references quoted in Sec. 4.1, there are several publications concerning non-swirling jet flows which also do not strictly comply with the suggestions.
of Poinset & Lele (1992). Bogey & Bailly (2006) applied non-reflecting boundary conditions and supplementary sponge layers setting density and pressure at the inflow, outflow and far-field, while at the inflow the three velocities were set additionally. Sandberg et al. (2011) used non-reflecting boundary conditions supplemented with sponge layers for density and pressure at the outflow and far-field, while imposing turbulent fluctuations at the inflow where all conservative variables were set. Bühler & Kleiser (2011) applied non-reflecting boundary conditions supplemented with sponge layers at the inflow and outflow for all conservative variables. A precursor simulation was performed to calculate a reference solution used at the outflow boundary. Setting the density at the inflow as well as the outflow thereby assured avoiding any drift (Colonius, 2004). Imposing the three velocity components at the outflow guaranteed a strictly positive streamwise velocity and therefore properly working non-reflecting boundary conditions. At the far-field boundary of the computational domain, the density and the pressure were set using sponge layers supplementing non-reflecting boundary conditions in accordance with Keiderling et al. (2009).

These publications, together with Herrada et al. (2003), Melville (1996) and Müller & Kleiser (2008a), indicate that there may be reasons for applying boundary conditions in practice that do not strictly follow the recommendations made by Poinset & Lele (1992). The choice of boundary conditions depends naturally on the flow to be investigated, and for compressible flows, depends further on the Mach number, which determines the propagation velocity of the waves and therefore of the information. It follows that the most suitable combination of boundary conditions has to be found separately for different classes of flows, and the setups reported in the literature are not necessarily applicable directly. We proceed by introducing the boundary condition setups analysed in the present study.

4.3.1 Basic Setup

In this section, we introduce the simulation setup that provides the reference case for our investigation (setup V1). The choice of boundary conditions for the basic setup (V1) is in accordance with Müller & Kleiser (2008a). We use a Dirichlet boundary condition at the inflow for all five variables denoted as $D[\rho, p, u]$ and apply non-reflecting conditions at the outflow and in the far-field of the computational domain. The analytical expressions and reference solutions for setting physical properties at the boundaries are introduced below.
4.3 Boundary condition setup

All domain boundaries are supplemented with sponge layers to avoid a drift of mean flow properties (ambient and far-field) and to reduce the reflection of upstream travelling waves at the inflow, see Chap. 3.

At the inflow, analytical profiles are used to set Dirichlet boundary conditions for the three velocities

\[
\begin{align*}
    u(r) &= 0 \\
    v(r) &= \begin{cases} 
    r, & r \leq 1.1 \\
    0, & r > 1.1
    \end{cases} \\
    w(r) &= \begin{cases} 
    1 - r^7, & r \leq 1 \\
    0, & r > 1
    \end{cases}
\end{align*}
\]

We emphasise that all three velocity components at the inflow plane \(z = -5\) outside of the nozzle \(r \geq 1.1\) are set identically to zero. The inflow plane outside of the nozzle acts therefore as a solid wall in accordance with the conditions in the investigations of Liang & Maxworthy (2005) and Billant et al. (1998), who performed experiments in a large water tank, as well as with those of Facciolo et al. (2007) and Oberleithner et al. (2011b), who attached a plate to the nozzle end. In addition, we set Dirichlet boundary conditions for the pressure and density, for which analytical expressions are obtained following (Herrada et al., 2003; Müller & Kleiser, 2008a):

\[
p(r) = \frac{1}{\gamma \cdot Ma^2} \cdot \exp \left( \int_{r'}^{r} \frac{\gamma \cdot Ma^2 \cdot v^2(r')}{T \cdot r'} dr' \right),
\]

assuming a uniform temperature distribution \(T(r) = 1\). For the non-swirling case, the pressure is constant across the inflow plane according to Eq. (4.4), in agreement with boundary conditions found in the literature, cf. Keiderling et al. (2009). Using the equation of state \(\gamma Ma^2 p = \rho T\) leads to

\[
\rho(r) = \frac{p(r) \cdot \gamma \cdot Ma^2}{T},
\]

which we use for setting the density. The resulting density and pressure distribution is in qualitative agreement with experimental results reported in Liu et al. (1993). For the inflow sponge layer, the reference state \(u_{\text{ref}}(x)\) is defined by the analytical inflow profiles (Eqs. 4.1-4.5).
At the outflow boundary, a sponge layer is used for all five variables (denoted as \( S[\rho, p, \mathbf{u}] \)), supplementing the non-reflecting boundary conditions. The reference state at the outflow is set according to the results of a precursor simulation applying non-reflecting boundary conditions at the outflow without any sponge layer. The results of the precursor simulation at the outflow boundary lead to the following reference state:

\[
\begin{align*}
\rho_{\text{ref}}(r, \theta, z = 15) &= \rho_{\text{pre}} = \rho_{\infty} \\
p_{\text{ref}}(r, \theta, z = 15) &= p_{\text{pre}} = p_{\infty} \\
u_{\text{ref}}(r, \theta, z = 15) &= u_{\text{pre}} = 0 \\
v_{\text{ref}}(r, \theta, z = 15) &= v_{\text{pre}} = 0 \\
w_{\text{ref}}(r, \theta, z = 15) &= w_{\text{pre}} = \frac{ar + b}{r^3 + cr^2 + dr + e},
\end{align*}
\]

where an analytical fitting function is used to match the streamwise velocity component \((a = -1.66, b = 9.26, c = -0.92, d = -9.34, e = 61.98)\).

At the far-field boundary, we apply non-reflecting boundary conditions similar to the radiation condition suggested by Ruith et al. (2004), supplemented with a sponge layer. In the far-field sponge, the pressure and density are set to analytical far-field values evaluated at \( r = r_{\infty} = 10 \) using Eqs. (4.4) and (4.5).

At the beginning of the simulation we initialise the flow field \( \mathbf{u}(r, \theta, z, t = 0) \) within the entire computational domain using the flow state at the inflow. The analytical inflow profiles introduced above (Eqs. 4.2 and 4.3) are slightly changed, leading to a smoother initialization of the flow field in the region \( \mathcal{R} = (1 \leq r \leq 1.1, \theta, z) \), see Fig. 4.3 for details. The family of analytical profiles used in the present investigation for the streamwise and azimuthal velocity were introduced by Gallaire & Chomaz (2003b) and applied also in Müller & Kleiser (2008a). The slight overshoot in the pressure and density distribution due to the changed initial azimuthal velocity profile does not influence the results of the present investigation. The flow field is disturbed with random noise with amplitude on the order of \( 10^{-4} \) at the initial time-step \( t = 0 \) to accelerate the transition of the swirling jet.

### 4.3.2 Setup variations

We vary the boundary conditions at the inflow and outflow to get insight into their effects on the computational results. The inflow sponge layer
4.3 Boundary condition setup

as well as the far-field sponge layer described in Sec. 4.3.1 are used for all simulations without modification. Besides the basic setup (V1) introduced above, five different combinations of inflow and outflow conditions were investigated:

- **V1**: $D[\rho, p, u], S[\rho, p, u]$

  A Dirichlet boundary condition is used for all five variables at the inflow of the computational domain. The sponge layer applied at the outflow boundary acts on all five variables as well. This setup corresponds to Müller & Kleiser (2008a) and serves as the basic setup in the present investigation.

- **V2**: $D[\rho, p, u], S[\ldots]$

Figure 4.3: Analytical profiles used for the inflow boundary conditions (solid lines —). Profiles for initialization printed in dashed lines ---. Temperature $T(r) = 1$ and radial velocity $u(r) = 0$ not plotted for brevity.
While a Dirichlet boundary condition is used for all five variables at the inflow, no sponge layer is used at the outflow of the domain.

- **V3**: $D[p, u], S[-]

  The Dirichlet condition at the inflow boundary is applied for the pressure and the three velocities. No outflow sponge layer is used. The non-reflecting boundary condition at the inflow therefore affects the density only.

- **V4**: $D[p, u], S[\rho, p]

  An outflow sponge layer accounting for the pressure and the density is used. At the inflow a Dirichlet boundary condition is set for the pressure and the three velocities.

- **V5**: $D[-], S[\rho, p]

  No Dirichlet boundary condition is applied at the inflow, the non-reflecting boundary conditions affect all five variables at the inflow boundary. At the outflow boundary the sponge layer acts on the density and the pressure.

- **V6**: $D[-], S[\rho, p, u]

  No Dirichlet boundary condition is applied at the inflow. At the outflow boundary the sponge layer acts on all five variables.

The different setups V1–V6 are summarized in Tab. 4.1 for comparison and also schematically displayed in Fig. 4.4. Results for the different setups are presented in the following section with a focus on a pairwise comparison to allow to precisely determine the effects of one change of a boundary conditions at a time. Comparisons made are V1 $\leftrightarrow$ V2 to investigate the effect of the full outflow sponge $S[\rho, p, u]$, V3 $\leftrightarrow$ V4 for determining the influence of the outflow sponge for pressure and density $S[\rho, p]$, V5 $\leftrightarrow$ V6 to clarify the role of the outflow sponge for the three velocities $S[u]$ and V1 $\leftrightarrow$ V6 to investigate the effect of the full Dirichlet boundary condition $D[\rho, p, u]$ at the inflow.
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Table 4.1: Conditions imposed at the inflow and outflow boundaries for the six different setups. The sponge layer at the inflow acts on all five variables for all setups V1–V6 investigated and is therefore not included in the table.
Assessment of boundary conditions

(a) Setup V1
(b) Setup V2
(c) Setup V3
(d) Setup V4
(e) Setup V5
(f) Setup V6

Figure 4.4: Comparison of boundary conditions investigated. Dirichlet boundary conditions marked in red, non-reflecting boundary conditions marked in green. Nozzle wall marked in blue. Sponge layers are shaded in grey at the inflow and far-field boundary, the outflow sponge is marked in orange. Quantities imposed within the inflow and far-field sponge layer and at the nozzle wall are not shown for clarity, see Fig. 4.1.

4.4 Numerical results

In this section, the numerical results for the different setups are presented. The results for the mean flow field are compared first (Sec. 4.4.1), followed by an analysis of the coherent structures dominating the flow field (Sec. 4.4.2) and a frequency analysis (Sec. 4.4.3). Since it is known from the literature that vortex breakdown is especially sensitive to physical or artificial fluctuations in the flow field (Ruith et al., 2003), the instantaneous pressure field is further studied (Sec. 4.4.4). The results are compared qual-
4.4 Numerical results

Itatively to results reported in the literature wherever reasonable. Since all setups considered here reproduce the flow features assumed to be critical for the occurrence of vortex breakdown, we focus on how the different combinations of inflow and outflow boundary conditions alter the flow field.

4.4.1 Mean flow properties

Fig. 4.5 displays streamlines of the \( \langle t, \theta \rangle \)-averaged flow field for the six setups under investigation. The pronounced recirculation zone deflects the streamlines radially outwards, influencing the entrainment of ambient fluid. Behind the recirculation zone, the streamlines are directed radially inwards until changing over to a quasi-parallel flow. The location of the breakdown bubble and also its radial extent depend significantly on the combination of boundary conditions chosen. For setups V1, V2 and V6, a third and fourth stagnation point appears on the jet centreline within the recirculation zone, due to a region of positive streamwise velocity. A second recirculation region exists, rotating in the opposite direction to the main recirculation, leading to the generation of a third shear–layer, cf. Faler & Leibovich (1978). The results for setups V3–V5 do not show this second vortical structure. Especially for setups V3 and V4, the recirculation region is substantially smaller than for setups V1, V2 and V6.

The qualitative differences in the flow field results are shown in Fig. 4.6, which shows the \( \langle t, \theta \rangle \)-averaged velocity profiles at a sequence of downstream positions. The overall picture shows that the results for setups V1, V2, and V6 are close together; as do those for V3 and V4, while setup V5 leads to velocity profiles slightly different from the results of setups V1, V2 and V6. Within the nozzle, the streamwise velocity maintains its initial flat shape for setups V3 and V4, while for the other setups a slight deviation is visible. In the nozzle end section at \( z = 0 \), the flow encounters a velocity deficit at the jet centreline, which is much stronger for setups V1, V2, V5 and V6 than for setups V3 and V4. This velocity decay is even more pronounced downstream of the nozzle, where the recirculation zone develops and the streamwise velocity is negative. Behind the nozzle lip, the maximum streamwise velocity is approximately the same for all setups, but the maximum is located more radially inwards for setups V3 and V4. In general, the streamwise shear–layer is shifted outwards in the radial direction due to the developing recirculation bubble. Accompanying the decrease in the streamwise velocity, the azimuthal velocity deviates from solid–body rotation, indicating the change from
Assessment of boundary conditions

Figure 4.5: Streamlines of the $\langle t, \theta \rangle$-averaged flow field. The grey bar indicates the nozzle wall and the circles the stagnation points at the jet centreline.

laminar to transitional flow (Facciolo et al., 2007). The deviation is less pronounced for setups V3 and V4. The radial velocity component grows significantly downstream of the nozzle lip. It is larger for setups V1, V2, V5 and V6 than for V3 and V4: the jet spreads rapidly and a conical shear–layer develops. Three radii downstream from the nozzle end section, the radial velocity is slightly negative and the spreading angle of the jet decreases again behind the recirculation bubble. For $z \geq 4$, the velocity profiles collapse: they are nearly identical for all setups.

Comparing the $\langle t, \theta \rangle$-averaged streamwise velocity at the jet centreline for the six setups (Fig. 4.7), we find that, upstream of the vortex break-
4.4 Numerical results

Figure 4.6: Axial development of $\langle t, \theta \rangle$-averaged streamwise, azimuthal and radial velocity (top to bottom). The grey bar indicates the position of the nozzle wall. V1 — (+), V2 —— (×), V3 ····· (⋆), V4 ———— (□), V5 —— (■), V6 ——— (○).

down, the results for V1, V2, V5, and V6 collapse, as do those of V3 and V4, as described in the last paragraph. The streamwise centreline velocity decreases (more strongly for setups V1, V2, V5 and V6) and a slight acceleration within the nozzle is observed. In the region of the nozzle end, a pronounced decay of the streamwise centreline velocity is observed. It is negative over the complete extent of the recirculation bubble for setups V3–V5, while it is slightly positive in its front part for setups V1, V2 and V6 associated with the secondary vortical structure visible in Fig. 4.5. Behind the recirculation region, the velocity is approximately constant for all setups, differing in absolute value. For $z \geq 10$, the outflow sponge
layer (if present) acts on the flow field, leading to an acceleration of the flow in the cases V1, V4, V5 and V6. The flow field in this region can no longer be assumed to be physical. Nevertheless, a possible upstream effect of this downstream flow region should be kept in mind.

Fig. 4.7: $\langle t, \theta \rangle$-averaged streamwise velocity at centreline. V1 --- (+), V2 - - - (x), V3 ...... (★), V4 ------- (□), V5 - - - (■), V6 ----- (○). The regions where the sponges act on the flow field are shaded in grey and hatched.

Fig. 4.8 displays the streamwise position of the recirculation region and the location of the minimum streamwise velocity at the centreline. For setups V1–V3, the location of the stagnation point in front of the recirculation zone is approximately the same, while for setups V4–V6 it is shifted upstream towards the nozzle end. Setups V1 and V2 have nearly identical locations for their minimum streamwise velocities, as do V3–V6. The second stagnation point on the jet centreline, indicating the end of the recirculation zone, is located about one radius further downstream for setups V1 and V2 than it is for setups V3–V5. The recirculation zone is therefore larger. For setup V6, the second stagnation point is situated approximately 0.5 radii further downstream than it is in setups V3–V5.

We define the jet half-width as the radial position at which $\langle w(r, z) \rangle / \langle w_c(z = -5) \rangle = 0.5$ (Liang & Maxworthy, 2005). $\langle w_c \rangle$ is extracted at the inflow plane. The results for the jet half-width collapse for setups V1, V2 and V6, while they differ for all other setups (Fig. 4.9). The maximum half-width is the largest for setups V1, V2 and V6, decreasing by about 0.5 radii for setup V3. For V4, the maximum half-width is
4.4 Numerical results

Figure 4.8: Streamwise extent of the $\langle t, \theta \rangle$-averaged recirculation region between first and last stagnation point on the jet centreline. The internal stagnation points found for setups V1, V2 and V6 located within the recirculation bubble are not considered here. The shaded area indicates the zone of recirculation ($\langle w_c \rangle \leq 0$).

slightly larger than for V3, while for V5 it is slightly smaller than for V1, V2 and V6. For $z \geq 1.5$, the streamwise velocity is $\langle w(r, z) \rangle < 0.5$ for all radial positions, therefore the jet half-width is no longer defined.

4.4.2 Azimuthal mode analysis

We define the amplitude of the azimuthal modes $n = 0, \ldots, 10$ as the time-averaged azimuthally Fourier-transformed instantaneous azimuthal velocity fluctuations:

$$A_\theta(r, z, n) = \langle |\hat{v}'(r, z, t, n)| \rangle_t,$$

(4.11)

where $\hat{()}$ denotes the Fourier transform and $q' = q - \langle q \rangle_t$ a fluctuating quantity. Fig. 4.10 shows the plots of the radially integrated amplitude $A_\theta$ to give an impression of the azimuthal modes observed in the flow field. In general, the amplitudes of all non-axisymmetric modes increase in the nozzle end section, and there is a period of exponential growth in the downstream direction is. For $z \geq 3$, the amplitude level decays down to the position $z \approx 6$, remaining approximately constant until the outflow sponge layer starts to act on the flow. For setups V1 and V6 all modes are damped due to the outflow sponge layers acting on all five variables.
Figure 4.9: $\langle t, \theta \rangle$-averaged jet half-width. In the region of the conical vortex breakdown, the radial position is evaluated in the outer shear–layer. The grey bar indicates the nozzle wall. V1 — (+), V2 — — (×), V3 — — — (★), V4 — — — (□), V5 — — (■), V6 — — — (○).

The results for the setups V2 and V3, which do not apply any sponge layer at the outflow, show a monotonic amplitude decrease for all azimuthal modes. For the outflow sponge layer acting on $p, \rho$, the amplitudes of all azimuthal modes increase artificially in the downstream direction, accompanied by an acceleration (see Fig. 4.7).

In the nozzle region, three behaviours are seen: for setups V1, V2 and V6 modes $n = 0, 1$ show a behaviour similar to the azimuthal modes of higher mode number, growing in the downstream direction with either $n = 0$ (V1 and V6) or $n = 1$ (V2) being dominant. Setups V3 and V4 lead to a dominant axisymmetric mode $n = 0$, whose amplitude decreases again already in the nozzle region at $z \approx -1.5$. This mode rises again in the region between the nozzle end and the recirculation zone for setups V3 and V4, and more weakly for setup V5. For setup V3, the axisymmetric mode is overwhelmed by mode $n = 1$ for the nozzle flow for some short spatial extent, while for setup V4 the single-helix type instability ($n = 1$) is only co-dominant at $z \approx -1.5$. For case V5, the axisymmetric mode $n = 0$ is dominant within the nozzle for $z \geq -3$, starting its decay at $z \approx 0.5$.

The integral of the azimuthal amplitude $A_\theta$ is shown in Fig. 4.11. Common to all results is that the amplitudes of azimuthal modes of higher mode number show an approximately exponential decay with increasing mode

$(\rho, p, u, v, w)$. The results for the setups V2 and V3, which do not apply any sponge layer at the outflow, show a monotonic amplitude decrease for all azimuthal modes. For the outflow sponge layer acting on $p, \rho$, the amplitudes of all azimuthal modes increase artificially in the downstream direction, accompanied by an acceleration (see Fig. 4.7).

In the nozzle region, three behaviours are seen: for setups V1, V2 and V6 modes $n = 0, 1$ show a behaviour similar to the azimuthal modes of higher mode number, growing in the downstream direction with either $n = 0$ (V1 and V6) or $n = 1$ (V2) being dominant. Setups V3 and V4 lead to a dominant axisymmetric mode $n = 0$, whose amplitude decreases again already in the nozzle region at $z \approx -1.5$. This mode rises again in the region between the nozzle end and the recirculation zone for setups V3 and V4, and more weakly for setup V5. For setup V3, the axisymmetric mode is overwhelmed by mode $n = 1$ for the nozzle flow for some short spatial extent, while for setup V4 the single-helix type instability ($n = 1$) is only co-dominant at $z \approx -1.5$. For case V5, the axisymmetric mode $n = 0$ is dominant within the nozzle for $z \geq -3$, starting its decay at $z \approx 0.5$.

The integral of the azimuthal amplitude $A_\theta$ is shown in Fig. 4.11. Common to all results is that the amplitudes of azimuthal modes of higher mode number show an approximately exponential decay with increasing mode
4.4 Numerical results

Figure 4.10: Radially integrated amplitude $A_\theta$. Regions where sponges act on the flow field are shaded in grey. $n = 0$ —— (blue), $n = 1$ - - - (red), $n = 2$ ···· (green), azimuthal modes of higher mode number in hierarchical order.

number $n_i$. The amplitudes of all modes increase from setup V1 to V3 and
decrease again for setups V4 and V5. For setup V6 this trend holds only for the azimuthal mode \( n = 0 \). Another exception is mode \( n = 3 \), which has a lower amplitude for setup V3 than for V4. While the single-helix mode \( n = 1 \) is dominant (accompanied by a co-dominant axisymmetric mode \( n = 0 \)) for setups V1–V3 and V6, the dominance is opposite for setups V4 and V5. Thereby, the dominance of the single-helix type mode \( n = 1 \) is much stronger for cases V2, V3 and V6 than for case V1.

\[
\int_{z=-5}^{z=10} \int_{r=0}^{\infty} A_{\theta} \, dr \, dz
\]

Figure 4.11: Integral amplitude of modes \( n = 0, \ldots, 10 \) scaled by the maximum global amplitude. Results for different setups ordered from left to right (V1–V6).

Figs. 4.12 and 4.13 display colour maps of the maximum amplitude over all azimuthal modes together with contour lines of the associated azimuthal mode number \( n_i \) at each spatial location. The results for all setups show local maxima at the centreline of the jet in the leeward part of the recirculation region. A second local maximum is situated at the outer side of the nozzle wall, where the flow is governed by a thin azimuthal boundary layer (see Fig. 4.6). The results for the setups V3 and V4 show a third region of remarkably high amplitude downstream of the nozzle end shifted inwards by approximately 0.25 radii. This region clearly dominates the flow globally due to its size and amplitude.

The regions where local maxima are observed are associated with corresponding azimuthal modes \( n_i \). The region at the outer side of the nozzle wall is dominated by the axisymmetric and single-helix type instabilities \( n = 0, 1 \). The centreline region is governed by a single-helix type instability. The globally dominant region found for setups V3 and
V4 is dominated by the axisymmetric mode $n = 0$. Compared to the other cases, this region is much larger in radial and streamwise extent. Directly at the nozzle lip, the flow is governed by azimuthal modes of higher mode number for all setups. The mode selection in the outer shear–layer varies for the different setups: this region is generally dominated by azimuthal modes of higher mode number for all setups. For setups V1–V3 and V6, a co-dominating single-helix appears, accompanied by a double-helix (case V3 only). For the cases V4 and V5, no single-helix dominance is found: instead, a double-helix instability governs the outer shear–layer together with azimuthal modes of higher mode number (case V5 only).
Figure 4.12: Maximum of $A_\theta(r, z)$ over all azimuthal modes $n_i$ and associated mode number (contour lines). **Violet-blue** contour lines indicate change from $(n = 0)$-dominance to $(n = 1)$-dominance, **Green-red** contour lines the change from $(n = 1)$-dominance to $(n = 2)$-dominance. Sponge layer regions are hatched in black, nozzle wall indicated in grey.
Figure 4.13: For caption see Fig. 4.12.
4.4.3 Frequency analysis

Figs. 4.14 and 4.15 display the maximum amplitude of the Fourier transformed time series of streamwise velocity fluctuations according to

\[ A(St_{\text{dom}}) = \max_{St} |\hat{w}'(r, \theta, z, St)|, \]  

where \( St = f \cdot R/w_c \) is a Strouhal number. The picture is in general twofold: For setups V1, V2, V5 and V6, the maximum amplitude is found in the inner shear–layer of the vortex breakdown cone with a second local maximum in the outer shear–layer (V2 and V6 only). For setup V6, a third maximum amplitude is visible within the recirculation zone at the jet centreline. For setup V5, the maximum in the inner shear–layer reaches into the front region of the recirculation bubble. The amplitudes observed in the recirculation bubble are much smaller than those of overall maximum. Setups V3 and V4 show a strong maximum in front of the recirculation region, which extends along the inner shear–layer in the radial-streamwise direction to the point where the maximum was found in the analysis of the azimuthal modes (see Fig. 4.12). A second local maximum is found for both setups in the outer shear–layer of the swirling jet. The maximum amplitudes for setups V3 and V4 are up to five times higher than for the other setups.

The Strouhal numbers associated with the overall maximum Fourier amplitudes are given in Tab. 4.2. The highest Strouhal numbers are observed at the nozzle lip in the outer shear–layer of the vortex breakdown cone where the amplitudes are low. The variation of the Strouhal numbers found at the nozzle lip is high. The dominant Strouhal numbers are low and identical for all setups except for setups V1 and V6. The associated Fourier amplitudes vary by a factor of up to approximately 4. As mentioned in the previous paragraph, the locations where the maximum Fourier amplitudes are found differ substantially for the setups under investigation. This can be understood by comparing these locations with the mean flow fields plotted in Fig. 4.5: depending on the length and the radial extent of the recirculation zone, the position of the inner shear–layer is shifted in the streamwise and the radial direction. Therefore, the location where the maximum amplitude is found is shifted in the same manner since it is associated with the inner shear–layer.
### 4.4 Numerical results

<table>
<thead>
<tr>
<th>Setup</th>
<th>$St_{\text{dom}}$</th>
<th>$A(St_{\text{dom}})$</th>
<th>$(r, \theta, z)(St_{\text{dom}})$</th>
<th>$St_{\text{lip}}$</th>
<th>$A(St_{\text{lip}})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>0.0073</td>
<td>0.5579</td>
<td>(1.2545, 0, 2.4593)</td>
<td>1.4502</td>
<td>0.0218</td>
</tr>
<tr>
<td>V2</td>
<td>0.0024</td>
<td>0.4987</td>
<td>(1.2545, 0, 1.7445)</td>
<td>0.7104</td>
<td>0.0336</td>
</tr>
<tr>
<td>V3</td>
<td>0.0024</td>
<td>2.2591</td>
<td>(0.5458, 0, 1.3596)</td>
<td>0.5175</td>
<td>0.0621</td>
</tr>
<tr>
<td>V4</td>
<td>0.0024</td>
<td>1.3221</td>
<td>(0.6710, 0, 1.4696)</td>
<td>2.1191</td>
<td>0.0037</td>
</tr>
<tr>
<td>V5</td>
<td>0.0024</td>
<td>1.0333</td>
<td>(1.1115, 0, 1.7445)</td>
<td>1.3940</td>
<td>0.0075</td>
</tr>
<tr>
<td>V6</td>
<td>0.078</td>
<td>0.403</td>
<td>(1.101, 0, 2.734)</td>
<td>0.903</td>
<td>0.026</td>
</tr>
</tbody>
</table>

Table 4.2: Overall dominant Strouhal number ($St = f \cdot R/w_c$) with amplitude and location and Strouhal number at nozzle lip with amplitude.
Figure 4.14: Maximum amplitude of the Fourier-transformed instantaneous streamwise velocity fluctuations. Sponge layer regions are hatched in black, nozzle wall indicated in grey.
Figure 4.15: For caption see Fig. 4.14.
### 4.4.4 Pressure fluctuations

It is known from the literature (Ruith et al., 2003) that the vortex breakdown phenomenon is highly sensitive to any physical or artificial disturbance. We monitor the development of pressure fluctuations on the jet centreline. The results are summarized in Tab. 4.3, showing the spatially-averaged maximum value of the absolute pressure fluctuations and the maximum value of the absolute pressure fluctuations together with the streamwise position of its occurrence. As it is reported in Martinelli et al. (2012), the fluctuations induced by the precessing vortex core, which gives rise to a single-helix type instability, are substantially larger than the background turbulence. Therefore we expect the largest pressure fluctuations to occur at the jet centreline, where the single-helix type mode is located. For setups V1, V3 and V6, we find the maximum pressure fluctuations in the region of the recirculation zone and slightly leeward of it as expected (see Fig. 4.8), where the single-helix mode \( (n = 1) \) is dominant. The spatially averaged pressure fluctuations are approximately two times higher for setup V3 than for V1 and V6. Setup V2 leads to a maximum pressure fluctuation located in the end section of the nozzle. The maximum fluctuations observed within the nozzle are significantly higher for setup V2 than for all other setups. For setups V4 and V5 the maximum is found at the outflow boundary due to the specifics of the action of the sponge layer. The maximum of the pressure fluctuation is artificially high for these two cases, induced by the increase in the azimuthal amplitude \( A_\theta \), see Fig. 4.10.

| Setup | \( \langle \text{max} |p'| \rangle_{z,r=0} \) | \( \text{max}(\text{max} |p'|) \) | \( z(\text{max}(\text{max} |p'|)) \) |
|-------|-----------------|-----------------|-----------------|
| V1    | 0.034           | 0.135           | 3.01            |
| V2    | 0.053           | 0.127           | -0.07           |
| V3    | 0.057           | 0.225           | 2.68            |
| V4    | 0.084           | 0.578           | 13.31           |
| V5    | 0.069           | 0.593           | 14.23           |
| V6    | 0.034           | 0.113           | 4.44            |

Table 4.3: Maximum instantaneous and spatially-averaged pressure fluctuations on the jet centreline and the corresponding downstream position.
4.5 Summary and discussion

We performed Large–Eddy Simulations of swirling jet flows undergoing vortex breakdown in the compressible regime \((Ma = 0.6, Re = 5000, S_{\gamma} = 0.75)\). Our setup included a nozzle modelled as an isothermal rotating wall. High-order schemes were used for the spatial derivatives (Lele, 1992) in combination with a Runge–Kutta time integration scheme with optimized dissipation and dispersion properties (Berland et al., 2006). The sub-grid scales were treated with the approximate deconvolution model (ADM-RT) (Schlatter et al., 2006).

Six different combinations of inflow and outflow boundary conditions (V1–V6) were studied, in order to get more insight into their effect on the flow field. At the inflow, three different boundary conditions were investigated:

- \(D[\rho, p, u]\): a Dirichlet boundary condition for all five variables \((\rho, p, u, v, w)\) used for setups V1 and V2,
- \(D[p, u]\): a Dirichlet boundary condition for the pressure and three velocities applied for setups V3 and V4 and
- \(D[-]\): no Dirichlet boundary condition at all (setups V5 and V6).

All three cases were supplemented with a sponge layer at the inflow for all five variables. Non-reflecting conditions were applied to all variables at the inflow if no Dirichlet condition was imposed. At the outflow boundary, we investigated the following conditions:

- \(S[\rho, p, u]\): a sponge layer for all five variables (setup V1 and V6),
- \(S[\rho, p]\): a sponge layer for density and pressure only (setups V4 and V5) and
- \(S[-]\): no sponge layer at all (setups V2 and V3).

In addition to the sponge layer, non-reflecting boundary conditions were used at the outflow boundary. At the far-field boundary, non-reflecting conditions were implemented, supplemented with a sponge layer for pressure and density for all setups investigated.

The results for all six setups are generally in good qualitative agreement with experimental and numerical results found in the literature (Facciolo et al., 2007; Liang & Maxworthy, 2005). Qualitatively, the
overall flow behaviour is robust with respect to the changes made in the inflow and outflow boundary conditions, i.e., a recirculation zone, a conical vortex breakdown configuration, competing azimuthal modes, and a dominant frequency are observed for all setups. Moreover, the results provide evidence that the effects of the nozzle lip and the azimuthal boundary layer at the outer side of the nozzle substantially influence the development of the swirling jet flow.

Nevertheless, depending on the combination of inflow and outflow boundary conditions, the flow field changes quantitatively. The dominant frequency, the location and shape of the recirculation bubble, the spreading angle, and the selection of azimuthal modes is altered. Additionally, the intensity of the recirculation and of the counter-rotating motion observed in the front part of the breakdown bubble is changed.

Simulation setups V1 and V2 applying $D[\rho, p, u]$ at the inflow and setups V3 and V4 using $D[p, u]$ lead to very similar overall results, independently of the type of outflow sponge layer. The same is true for setups V5 and V6, which do not apply a Dirichlet boundary condition $D[-]$, but to a lesser extent. One observation is therefore that modifications in the outflow boundary treatment lead to less pronounced changes in flow properties than modifications at the inflow, see also Darmofal (1996). This result is to be expected for sufficiently large computational domains. Nevertheless possible upstream effects of the outflow boundary treatment, especially on instantaneous flow properties, should be kept in mind as we will discuss in the remainder of this section.

The comparison of the results for simulation setups V2 and V3 ($D[\rho, p, u] \leftrightarrow D[p, u]$) and V4 and V5 ($D[p, u] \leftrightarrow D[-]$) reveals that modifications in the inflow boundary conditions lead to significant changes in the mean flow field properties as well as in the selection of dominant azimuthal modes and frequencies. The present study shows that a Dirichlet boundary condition for the pressure and three velocities $D[p, u]$ leads to a dominance of the axisymmetric mode in the front part of the recirculation bubble, which is observed solely for this boundary condition. Besides this most probably artificial effect, the full Dirichlet boundary condition $D[\rho, p, u]$ as applied in Müller & Kleiser (2008a) leads to high pressure fluctuations within the nozzle, at least in combination with certain outflow boundary conditions. As it is well known from the literature that time-independent flow properties are imposed properly using sponge layers (cf. Colonius (2004)), we suggest omitting the Dirichlet condition completely $D[-]$, at least in the present context of naturally evolving
swirling jets. This is in accordance with simulations performed in the aero-acoustics field, cf. Bühler & Kleiser (2011); Bühler et al. (2014), where the results are even more sensitive to any disturbances, and the boundary conditions are formulated correspondingly. A comparison of the results for setups V1 and V6 \( D[\rho, p, u] \leftrightarrow D[-] \) in combination with \( S[\rho, p, u] \) shows that besides the beneficial effect of avoiding high pressure fluctuations within the nozzle, the flow field changes only slightly.

The outflow sponge layer acting on density and pressure \( S[\rho, p] \) introduces an artificial increase of Fourier amplitudes of the helical modes along with an artificially high level of pressure fluctuations. The helical modes are excited by this outflow boundary treatment. This artificial excitation may influence the upstream development of the flow. The breakdown behaviour may change due to the outflow boundary treatment, particularly due to the single-helix mode that is believed to be absolutely unstable (Gallaire et al., 2006).

In contrast to the sponge layer’s imposing density and pressure at the outflow only, the sponge layer additionally accounting for the three velocities \( S[\rho, p, u] \) damps all helical modes at the downstream boundary of the domain, thereby ensuring a low level of pressure fluctuations, which is preferable in the context of non-reflecting boundary conditions. We prefer the sponge layer for all five variables, in order to avoid a mean density drift at the downstream end of the domain as reported in Colonius (2004) and to guarantee a strictly positive streamwise velocity at the outflow. A drawback of setting velocities \( u \) at the outflow is the need for physical data at a downstream location where the flow field is not known a priori. Using a precursor simulation (performed without any sponge layer acting at the outflow boundary) is a feasible way to get statistical data to be imposed at the outflow, which introduces a certain effort.

### 4.6 Conclusions

Concluding the present study, we recommend setting five variables at the inflow and outflow boundary of the computational domain using sponge layers in combination with non-reflecting conditions. This combination of boundary conditions \( D[-], S[\rho, p, u] \), referred to herein as setup V6, allows simulations of swirling jets that ensure a minimum of artificial disturbances of the flow field.
Chapter 5

Comparison of the configuration with a rotating nozzle and a nozzle kept at rest

Vortex breakdown of swirling jet flows is investigated in the compressible, subsonic regime by means of Direct Numerical Simulation (DNS). The Reynolds number is $Re = 5000$ and the flow is moderately compressible with Mach number $Ma = 0.6$. The integral swirl number is $S_6 = 0.85$. The present investigation aims to clarify the role played by the nozzle wall motion for the vortex breakdown of the swirling jet. We study the nozzle flow as well as the swirling jet flow simultaneously, a novel approach in the field of vortex breakdown investigations. Depending on the nozzle wall motion, the flow significantly differs upstream of the vortex breakdown: for the rotating nozzle, the flow inside the nozzle is purely laminar and the azimuthal boundary layer at the outer nozzle wall gives rise to the axisymmetric mode $n = 0$ and the single-helix type instability with azimuthal wave number $n = 1$. With the nozzle at rest, a transitional flow is observed within the nozzle, where a helical instability with azimuthal wave number $n = 12$ dominates the flow, growing in the boundary layer at the nozzle wall. The helical instabilities observed for the nozzle flow interact with the developing vortex breakdown and the conical shear-layer downstream of the nozzle. This interaction results in a vortex breakdown configuration which is shifted in the upstream direction and which has a smaller radial and streamwise extent for the nozzle at rest compared to the rotating nozzle. The recirculation intensity is higher than for the nozzle in rotation. The dominant frequency is highly influenced by the flow upstream of the vortex breakdown and is substantially higher for the nozzle at rest. Although the nozzle flow field differs for the two nozzle configurations and therefore alters the vortex breakdown downstream of the nozzle, a single-helix type instability $n = 1$ governing the vortex breakdown is found for both cases. This gives strong evidence of the robustness of the instability mechanisms leading to vortex breakdown.
5.1 Introduction

The physical phenomenon of vortex breakdown occurs in many technical applications, e.g. delta wing aircraft (Peckham & Atkinson, 1957) and vortex burners (Chigier & Chervinsky, 1967), and can also be observed in nature (Burggraf & Foster, 1977). A field of ongoing research are swirling jet flows undergoing vortex breakdown (Oberleithner et al., 2011b). For a sufficiently high circumferential velocity relative to the streamwise velocity vortex breakdown occurs. The flow state of vortex breakdown is characterised by a strong recirculation in the centreline region of the swirling flow and a high radial spreading rate (Billant et al., 1998). Helical instabilities of co- and counter-rotating type, winding against or in the mean flow direction, dominate the flow field (Gallaire & Chomaz, 2003b). Recently, experimental observations gave strong evidence of the existence of a super-critical Hopf bifurcation to a globally unstable single-helix type mode (Liang & Maxworthy, 2005; Oberleithner et al., 2011b). It is of great interest to understand the fundamental features of vortex breakdown, to know the parameters at which vortex breakdown occurs, and to get insight into possible control mechanisms for this special flow configuration. Although in more than five decades of intense research many attempts were made to explain vortex breakdown, a commonly accepted theory of the underlying mechanisms is still missing. For reviews of the vortex breakdown phenomenon, we refer to Lucca-Negro & O’Doherty (2001) and references therein.

Experimental investigations reported in literature vary in the design of the devices used for generating the swirling jet flow as well as in whether the nozzle itself is kept at rest or not. The studies mainly split up in two groups either utilising a long rotating pipe with a straight or contracting nozzle attached, e.g. Liang & Maxworthy (2005) and Facciolo et al. (2007), or a pipe and a nozzle at rest in combination with a rotating honeycomb or guiding vanes to generate swirl, e.g. Billant et al. (1998), Oberleithner et al. (2011b) and Leclaire & Jacquin (2011). Following from the variations in the experimental setups, the flow upstream of the swirling jet differs as well. For the first group of experiments, the azimuthal velocity component at the wall is identical to the rotation speed of the nozzle, while for the latter the fluid is at rest at the nozzle wall. The differences in the experimental setups make it difficult to compare results of various studies, especially because the flow state upstream of vortex breakdown within the nozzle is seldom known and a discussion is
found only in Facciolo et al. (2007) and Leclaire & Jacquin (2011), but it is restricted to flows before the onset of vortex breakdown.

Facciolo et al. (2007) investigated experimentally and numerically turbulent swirling pipe and jet flows and focused on turbulent properties of the flows. Both flow regimes were studied separately and an interaction of the pipe flow and the swirling jet flow was neglected. Since the swirl intensity was below the threshold for vortex breakdown, no upstream effects of the swirling jet on the pipe flow were considered. The authors found that the fully developed flow within the pipe was not in solid-body rotation due to the Reynolds shear stresses acting, which is in agreement with findings reported in Orlandi & Fatica (1997). A counter-rotating core was found in the jet flow regime at the centreline of the swirling jet in the front part of the recirculation zone.

Leclaire & Jacquin (2011) reported on high-Reynolds number rotating flows in a pipe with a final contraction. The authors found that vortex breakdown within the pipe is suppressed by the contraction at the pipe end and observed instead standing axisymmetric Kelvin waves to be present. The observations made were independent of the contraction ratio of the pipe. The authors observed a high fluctuation level in the pipe exit plane and a spiralling motion, both connected to the flow upstream of the pipe exit plane and its change in criticality from super-critical to sub-critical (Benjamin, 1962). To avoid this change in criticality already within the pipe and to guarantee for smooth flow conditions in the pipe exit, especially at high swirl, Leclaire & Jacquin (2011) suggested to exclude a final contraction in swirling jet flow experiments. Although the swirling jet flow was not studied in this investigation, it gives strong evidence of the importance of the upstream flow conditions within the pipe on the vortex breakdown configuration.

In the present investigation, we study swirling nozzle-jet flows to bridge the gap in the experimental investigations reported in literature, where either swirling pipe or jet flows are studied separately without allowing for an interaction of the two. The nozzle flow and the swirling jet flow are investigated simultaneously, a novel approach in the field of vortex breakdown research. At the same time, we compare swirling nozzle-jet flows emanating from both a rotating nozzle and a nozzle kept at rest. Our approach makes it possible to study the two setups and provides the link between the two groups of experimental investigations found in literature. The present study is thus a complement to the studies reported in literature, where either a rotating or a nozzle at rest
Comparison of the two nozzle configurations

Figure 5.1: Computational domain for simulations. Non-reflecting boundary conditions marked in green. Sponge layers are shaded in grey. The nozzle wall is indicated in blue.

is considered. We use a numerical framework introduced in Chap. 3 (Bühler et al., 2010; Luginsland & Kleiser, 2011b, 2013a, b), which allows for a minimum of setup differences and a precise study of the effects of the nozzle configuration on the flow field. Our aim is to provide insight into the instability mechanisms inherently connected to the nozzle either rotating with the mean flow or kept at rest. Furthermore, we report on the effects of the flow within the nozzle on the vortex breakdown behaviour of the swirling jet for the two nozzle configurations.

This chapter is organised as follows: in Sec. 5.2 we introduce the numerical framework and give details on computational aspects of the present study. We discuss the setup differences for the rotating nozzle and the nozzle at rest in Sec. 5.3. In Sec. 5.4 numerical results are presented and compared to results reported in literature. We summarize our study, discuss the main findings and conclude the investigation in Sec. 5.5.

5.2 Numerical framework

In this section, we summarize the basic approach and the numerical methods used in the present investigation. An extensive documentation is given in Chap. 3 and in Müller (2007). The compressible, three-dimensional Navier-Stokes equations are solved numerically on a cylindrical grid, see Fig. 5.1 for a schematic sketch of the setup. The Reynolds number is set
5.3 Setup differences for the two nozzle configurations

The nozzle length is $L = 5$ and the nozzle wall thickness is $d = (R^o_o - R^o) / R^o = 0.1$ where $R^o_o$ is the outer nozzle radius. The nozzle is straight in accordance with suggestions given by Leclaire & Jacquin (2011). At the domain boundaries, non-reflecting conditions (Poinsot & Lele, 1992) are implemented, see Fig. 5.1. The non-reflecting conditions are supplemented with sponge layers (Bodony, 2006) for five variables ($\rho, p, u, v, w$) at the inflow and outflow and for pressure and density ($p, \rho$) in the far-field, respectively. The imposed reference solution at the outflow is obtained by performing a precursor simulation. At the nozzle wall three velocities and the temperature are prescribed. The setup of boundary conditions as well as the initialization is in accordance with the recommendations given in Chap. 4.

To obtain the results presented in Sec. 5.4 the simulations were run with a grid resolution of $N_r \times N_\theta \times N_z = 480 \times 288 \times 480$ on a domain with size $L_r \times L_\theta \times L_z = 10R^o \times 2\pi \times 20R^o$. For a grid resolution study we refer to Luginsland & Kleiser (2013a) and to App. A. The time-step is chosen to $\Delta t = 0.002$ according to a CFL criterion (Müller, 2007). The simulations were performed as Direct Numerical Simulations (DNS) with filtering of the flow field applied every second time-step to stabilize the simulation, cf. Müller (2007). The simulations were run on 768 cores in parallel on a CRAY XE6 super-computer (Monte Rosa) at the Swiss National Supercomputing Centre (CSCS), Lugano.

5.3 Setup differences for the rotating nozzle and the nozzle at rest

We perform numerical simulations for compressible swirling jet flows undergoing vortex breakdown with either a rotating nozzle or a nozzle at rest included in the computational domain. The rotating nozzle setup follows Liang & Maxworthy (2005) and Facciolo et al. (2007), who utilised a rotating pipe to generate swirl. The nozzle at rest follows Billant et al. (1998) and Oberleithner et al. (2011b), who used either guiding vanes or a rotating honeycomb for swirl generation. For both
Comparison of the two nozzle configurations

configurations, we follow Müller & Kleiser (2008a), Luginsland & Kleiser (2013a) and the recommendations given in Chap. 4 to define inflow boundary conditions for pressure and density by integrating the radial momentum equation and using the equation of state ($\gamma Ma^2p = \rho T$), assuming a uniform temperature distribution $T(r) = 1$. The radial and streamwise velocity components at the inflow are defined as follows:

$$u(r) = 0, \quad (5.1)$$

$$w(r) = \begin{cases} 1 - r^7, & r \leq 1 \\ 0, & r > 1 \end{cases}, \quad (5.2)$$

We choose a flat-top profile for the streamwise velocity component $w$ to match approximately the conditions downstream of the nozzle reported in literature, cf. Liang & Maxworthy (2005). At the in- and outflow boundary of the numerical domain we choose conditions according to the findings documented in Chap. 4, the far-field boundary conditions follow Ruith et al. (2004). At all domain boundaries we apply non-reflecting boundary conditions to set $\rho, p, u, v, w$ at the in- and outflow and $\rho, p$ in the far-field. We want to emphasise here that all three velocity components at the inflow outside of the nozzle are set identically zero. The inflow plane outside of the nozzle acts therefore as a solid wall in accordance with the investigation of Liang & Maxworthy (2005) and Billant et al. (1998), who performed experiments in a large water tank, as well as to Facciolo et al. (2007) and Oberleithner et al. (2011b), who attached a plate to the nozzle end. The water tank was large enough to avoid far-field effects on the development and breakdown of the jet. The nozzle extended into the tank. In contrast, Oberleithner et al. (2011b) and Toh et al. (2010) besides others use a facility for their investigation for which the swirling jet does not emanate into a container so that the swirling jet develops comparably freely. Nevertheless an actuation device with approximately $11R$ radius (Oberleithner, 2012) was attached to the contracting nozzle acting locally in a similar fashion as the inlet wall of the water tank of Liang & Maxworthy (2005). In Toh et al. (2010) the nozzle is positioned in the middle of a $62R \times 62R$ platform, a setup which corresponds approximately to our numerical setup with open lateral boundaries. Effects of the actuation device or the supporting platform, respectively, on the breakdown of the swirling jet were not reported by the authors.

For the rotating nozzle setup, the nozzle wall is in rotation with the
mean flow. The azimuthal velocity component is defined as

\[ v(r) = \begin{cases} r, & r \leq 1.1 \\ 0, & r > 1.1. \end{cases} \] (5.3)

Due to the rotation of the nozzle, a boundary layer develops at the outer side of the nozzle wall. At the inner side of the nozzle wall a streamwise boundary layer is present due to the non-slip condition for the streamwise velocity component. The vortex core of the swirling jet (Green, 1995; Saffman, 1995; Wu et al., 2005) is comparably large due to the nozzle wall in rotation.

The fluid at the outer side of the nozzle wall is still for the nozzle at rest and no boundary layer develops. For the nozzle at rest, the azimuthal velocity component is defined analytically, similar to the expression used by Müller & Kleiser (2008a):

\[ v(r) = \begin{cases} 1.38 \cdot r \cdot \exp\left(-\left(\frac{r}{0.9}\right)^{18}\right), & r \leq 1 \\ 0, & r > 1. \end{cases} \] (5.4)

The azimuthal velocity profile is defined in such a way that the maximum azimuthal velocity within the nozzle is identical for both setups as well as the initial integral swirl number \( S_6 = 0.85 \) at \( t = 0 \). At the inner side of the nozzle wall, a streamwise-azimuthal boundary layer develops due to the nozzle wall at rest.

Fig. 5.2 shows schematic sketches of the two setups including the boundary layers inherently related to the specific conditions imposed at the nozzle wall. We want to point out that the instability mechanism in the nozzle flow regime is significantly different for both setups due to the different boundary layers. For the nozzle at rest, helical instabilities (Del Pino et al., 2003; Fernandez-Feria & Del Pino, 2002; Vaidya et al., 2011) are expected to grow within the boundary layer at the inner side of the nozzle wall due to shear and centrifugal instability mechanisms (Herrada et al., 2004) and a transition to turbulence takes place for relatively short nozzles.

For the rotating nozzle setup, the boundary layer at the inner side of the nozzle is comparably stable and instabilities grow substantially slower in the downstream direction (Fernandez-Feria & Del Pino, 2002). The nozzle flow is expected to stay laminar due to its short length, cf. Imao et al. (1992). Imao et al. (1996) and later Yang (2000) observed
Comparison of the two nozzle configurations

Figure 5.2: Schematic overview over the different setups investigated with developing boundary and shear-layers for the typical breakdown configuration for the rotating nozzle and the nozzle at rest. Yellow: azimuthal boundary layer, orange: streamwise boundary layer, green: inner jet shear-layer, violet: outer jet shear-layer, turquoise: shear-layer due to the secondary vortical structure within the breakdown bubble. The nozzle wall is indicated in blue, (numbered) stagnation points are marked in red, the flow direction is shown with arrows.

that for turbulent rotating pipe flow flow swirl stabilizes the flow due to the suppression of turbulence by the centrifugal force. Pedley (1969) and Mackrodt (1976) showed that for laminar swirling pipe flow the opposite is the case (destabilization). The boundary layer at the outer side of the nozzle is centrifugally unstable and azimuthal modes are expected to grow in the downstream direction.

Due to the differences in the instability mechanisms for the two setups investigated, the flow conditions downstream of the nozzle, where vortex breakdown takes place, differ as well. As we will report in Sec. 5.4, helical instabilities of different azimuthal mode number grow and saturate at unequal amplitude levels. These modes interact with the developing conical shear-layer in the region of vortex breakdown and additionally with the growing single-helix type instability (Garg & Leibovich, 1979) believed to dominate the recirculation zone and the inner shear-layer surrounding the vortex breakdown bubble (Liang & Maxworthy, 2005; Oberleithner et al., 2011b).
5.4 Comparison of DNS results

5.4.1 Mean flow properties

Fig. 5.3 displays the \( \langle t, \theta \rangle \)-averaged distribution of the three velocity components at sequent positions downstream of the domain inlet. For the rotating nozzle, the streamwise velocity component decreases in the streamwise direction due to the upstream effect of vortex breakdown, while maintaining its flat-top shape within the nozzle. The azimuthal velocity component remains unaltered for the entire nozzle indicating a purely laminar flow. As reported by Orlandi & Fatica (1997) and Imao et al. (1992) and referring to Reich & Beer (1989) and White (1964), for a turbulent rotating pipe flow the azimuthal velocity component deviates strongly from an initial solid-body rotation and the streamwise velocity component tends to a parabolic shape with increasing swirl being stabilized. Both effects are not observed in the present investigation. The radial velocity is identically zero for the entire nozzle flow. In the nozzle end section the jet starts to spread radially. At the outer side of the rotating nozzle, a boundary layer develops in the downstream direction increasing in thickness.

For the nozzle at rest, the streamwise velocity component deviates from the initial flat-top profile directly downstream of the inflow plane indicating a transitional flow within the nozzle. The streamwise and azimuthal velocity profiles within the nozzle compare qualitatively well with findings reported in Vaidya et al. (2011) and Kitoh (1991). The velocity deficit at the centreline is stronger for this setup. The azimuthal velocity component is altered, the initial maximum decreases and radially smears out due to the development of the boundary layer at the inner side of the nozzle wall. The radial velocity component is approximately zero and increases in the end section of the nozzle being smaller compared to the rotating nozzle setup.

Downstream of the nozzle, the maxima for all three velocity components are situated further radially outwards for the rotating nozzle compared to the nozzle at rest. The spreading angle is larger for the rotating nozzle wall and the maximum radial velocity is higher, indicating a stronger vortex breakdown. The entrainment of ambient fluid is stronger directly downstream of the nozzle for the rotating nozzle compared to the nozzle kept at rest, while it is vice-versa further downstream \((z \geq 3)\). The radial and azimuthal velocity components approximately coincide far downstream for both setups, while the streamwise velocity shows still significant differences.
The present results compare qualitatively well with findings reported in literature, cf. Liang & Maxworthy (2005) and Facciolo et al. (2007) for the rotating nozzle and Martinelli et al. (2007), Oberleithner et al. (2011b), Oberleithner (2012) and Toh et al. (2010), respectively, for the nozzle at rest, respectively. The comparison is restricted to a qualitative analysis, because the initial conditions for the various experiments, the inflow conditions upstream of vortex breakdown as well as the parameters Reynolds, Mach and swirl number, \((Re, Ma, S)\), significantly differ. While without exception all referred investigations concern the incompressible regime, our investigation focuses on moderately compressible swirling jets at \(Ma = 0.6\) following the numerical investigations by Müller (2007) and Müller & Kleiser (2008a). Comparing our mean flow results with Müller & Kleiser (2008a), we find qualitative agreement as well, but the comparison cannot be conclusive—as for the other references—because of differences in the setups mentioned above, namely in this case the instability mechanisms introduced by modelling the nozzle wall.

The \((t, \theta)\)-averaged temperature, pressure and density are shown in Fig. 5.4. For the nozzle at rest, there is a stronger increase of temperature observed within the nozzle. This effect is due to the stronger deceleration of fluid discussed above. At the inner side of the nozzle wall the temperature increases due to the developing boundary layer. A similar observation, albeit less pronounced, is made for the rotating nozzle within the boundary layer at the outer nozzle side. The temperature increase within the boundary layers can be explained by dissipative heating, cf. Lele (1994). Downstream of the nozzle, the temperature maximum is located in the inner shear-layer of the swirling jet for the rotating nozzle and in the core region of the jet for the nozzle at rest. Far downstream, the temperature converges to a smooth distribution for both setups.

Pressure and density increase more strongly in radial direction according to Eqs. 4.4 and 4.5 for the nozzle at rest due to the differences in the initial azimuthal velocity distributions (cf. Eqs. 5.3 and 5.4), leading to higher far-field values (Luginsland & Kleiser, 2013a; Müller & Kleiser, 2008a). Within the boundary layer at the inner side of the nozzle wall, pressure and density grow more strongly in the downstream direction for the nozzle at rest. At the centreline, a similar observation is made due to upstream effects of vortex breakdown: the local positive pressure and density gradient in the downstream direction is stronger. The pressure distribution for the nozzle at rest is qualitatively comparable to results reported in Kitoh (1991). At the centreline of the swirling jet downstream
5.4 Comparison of DNS results

Figure 5.3: Axial development of $\langle t, \theta \rangle$-averaged streamwise, azimuthal and radial velocity (top to bottom). The grey bar indicates the position of the nozzle wall. Rotating nozzle $- - - (\times)$, nozzle at rest $- - - (\times)$.

of the nozzle, pressure and density are nearly identical for both setups, while differences in the far-field are still observed due to the far-field sponge imposing different reference solutions for the two setups.

The $\langle t, \theta \rangle$-averaged streamwise centreline velocity is depicted in Fig. 5.5. For the rotating nozzle, the flow within the nozzle is characterized by an initial decline in the front part of the nozzle followed by a slight rise down to the location $z = -1$ where the dramatic drop sets in due to the developing vortex breakdown. The nozzle at rest leads to a much stronger initial decline of the centreline streamwise velocity. The velocity drop due to vortex breakdown sets in at approximately the same downstream position as for the rotating nozzle and the slope is identical. A distinct breakdown
Figure 5.4: Axial development of $\langle t, \theta \rangle$-averaged temperature, pressure and density (top to bottom). The grey bar indicates the position of the nozzle wall. Rotating nozzle $+$, nozzle at rest $\times$.

region develops for both setups. The minimum centreline streamwise velocity is slightly smaller for the nozzle at rest compared to the setup with a rotating nozzle. The flow recovers at a position further upstream compared to the rotating nozzle setup, to about 35% of the initial centreline velocity. Downstream of the increase, the centreline velocity decreases in the streamwise direction to the location, where the sponge starts to act on the flow field. For the rotating nozzle, the flow downstream of the recirculation region increases slightly, no pronounced recovery is observed.

Fig. 5.6 shows the streamwise development of the $\langle t, \theta \rangle$-averaged integral swirl number $S_6$ according to Oberleithner et al. (2012) and introduced in Sec. 2.6. The swirl number is identical for both setups at
5.4 Comparison of DNS results

Figure 5.5: \(\langle t, \theta \rangle\)-averaged streamwise velocity at centreline. Rotating nozzle — (+), nozzle at rest — - - (×). Regions where sponges act on the flow field are shaded in grey.

The inflow plane of the domain \((S_6 = 0.85)\) and increases linearly in the front section of the nozzle to a maximum value of \(S_6 \approx 1.05\) for the nozzle at rest and \(S_6 \approx 1.26\) for the rotating nozzle, respectively. For the nozzle at rest, the swirl number decreases in the downstream section of the nozzle to an approximately constant level downstream of it \((S_6 \approx 0.65)\). The integral swirl number decreases in the downstream part of the nozzle for the rotating nozzle. It increases substantially in the nozzle end section due to a strong decrease in the second term in the denominator in the definition of the integral swirl number \(S_6\) (see Tab. 2.1), \((-\langle v \rangle^2/2)\), approaching a quasi-constant level further downstream at a much higher level \((S_6 \approx 0.975)\) compared to the nozzle at rest. Spatially averaging the integral swirl number in the vortex breakdown region leads to \(S_6 \approx 1.05\). We observe that the integral swirl number \(S_6\) varies in the downstream direction for both setups investigated, as expected, and that even for an identical initial integral swirl number \(S_6 = 0.85\) the \(\langle t, \theta \rangle\)-averaged swirl number differs at identical streamwise locations.

5.4.2 Vortex breakdown structure

The recirculation region due to vortex breakdown is significantly smaller for the nozzle at rest, see Fig. 5.7. The location, where the minimum centreline streamwise velocity is observed, is found in the leeward part of
Comparison of the two nozzle configurations

Figure 5.6: $\langle t, \theta \rangle$-averaged integral swirl number according to Tab. 2.1 according to Oberleithner et al. (2012). Rotating nozzle — (+), nozzle at rest - - - (×). Regions where sponges act on the flow field are shaded in grey.

the recirculation zone, which is situated $0.5z$ downstream of the nozzle with a length of $1.75z$. In the case of a rotating nozzle wall, the first stagnation point is located $0.8z$ downstream of the nozzle, the length of the recirculation region is approximately $3z$. The location of the minimum centreline streamwise velocity is shifted by approximately $0.35z$ off the centre of the recirculation bubble in the downstream direction due to the secondary vortical structure in the front part of the recirculation region.

Streamlines of the $\langle t, \theta \rangle$-averaged velocity field (Fig. 5.8) confirm the previously reported observations. The recirculation bubble is larger in streamwise as well as in radial extent for the rotating nozzle compared to the nozzle at rest. Additionally, the internal structure is more complex (see also Fig. 5.2). A secondary vortical structure is present, introducing two additional stagnation points at the centreline of the swirling jet and an internal shear-layer in the front region of the recirculation zone, in accordance with findings by Faler & Leibovich (1978). It is clearly visible that the radial spreading is much higher for the setup with a rotating nozzle compared to the nozzle at rest. The outer jet shear-layer is shifted substantially radially outwards due to the large recirculation bubble, while for the nozzle at rest it is deflected moderately.

Fig. 5.9 displays the time-averaged three-dimensional flow field for the two setups under investigation. The recirculation bubble for the rotating
5.4 Comparison of DNS results

Figure 5.7: Streamwise extent of the \(\langle t, \theta \rangle\)-averaged recirculation region between first and last stagnation point on the jet centreline. Internal stagnation points located within the recirculation bubble found for the setup with a rotating nozzle are not considered here. Shaded area indicates the zone of recirculation \(\langle w_c \rangle \leq 0\).

Figure 5.8: Streamlines of the \(\langle t, \theta \rangle\)-averaged flow field. The grey bar indicates the nozzle wall and the circles the stagnation points at the jet centreline.

The nozzle is of approximate spherical shape, while for the nozzle kept at rest the bubble is drop-like shaped. The streamwise velocity is negative in the leeward part of the recirculation zone for both nozzle configurations, while it is approximately zero (nozzle at rest) and slightly positive (rotating nozzle) in its front part, cf. Fig. 5.5. At the outer side of the rotating nozzle the streamwise velocity is marginally negative, see also Fig. 5.3. For both setups, the fluid counter-rotates weakly against the mean flow in the front part of the recirculation zone, as reported in Faccio & Alfredsson (2004).
Comparison of the two nozzle configurations

Figure 5.9: Time-averaged streamwise, $w$ (red-blue), and azimuthal velocity, $v$ (green-yellow), with streamlines. The nozzle wall is indicated in grey.

In the wake of the vortex breakdown bubble, a time-independent single-helix type instability is observed for both setups, co-winding with the mean flow direction in agreement with observations reported in Sarpkaya (1971). Fig. 5.10 additionally illustrates the differences in shape and size of the vortex breakdown bubble and the tail in the wake of the recirculation zone.

The instantaneous mass flux $\rho w$ shows a significant stronger recirculation for the setup with rotating nozzle, see Fig. 5.11. Strong fluctuations are observed at the outer side of the rotating nozzle wall where instabilities grow in the downstream direction. For the nozzle at rest, azimuthal modes of high wave number grow in the boundary layer.
Figure 5.10: Instantaneous ($\rho w = 0.2$)-iso-surface plot for the rotating nozzle and the nozzle at rest at time $t = 400$ intersected by a $(r, z)$-plane. Iso-surface coloured by distance perpendicular to the intersection plane (front-to-back: yellow-to-white-to-black).
Comparison of the two nozzle configurations at the inner side of the nozzle wall, see also Figs. 5.13 and 5.14. The maximum value for $\rho w$ is larger for the nozzle at rest.

Figure 5.11: Instantaneous mass flux $\rho w$ for the rotating nozzle and the nozzle at rest at time $t = 400$ at the end of the simulation.
5.4 Comparison of DNS results

Fig. 5.12 visualizes an azimuthal mode with wave number $n = 6$ which is of co-rotating co-winding type for both setups similar to helical instabilities observed by Cala et al. (2006). It is located in the outer shear-layer of the swirling jet downstream of the nozzle. This helical instability exists together with azimuthal modes of smaller wave number, see below for a detailed discussion of the dominating azimuthal modes.

![Instantaneous mass flux](image)

Figure 5.12: Instantaneous mass flux ($\rho w = 0.25$)-iso-surface plot for the rotating nozzle and the nozzle at rest at a sequence of time-steps ($398t$, $399t$, $400t$) (left-to-right). Iso-surfaces colored with radial co-ordinate.

5.4.3 Instantaneous flow field

Figs. 5.13 and 5.14 show the instantaneous mass flux field $\rho w$ at subsequent downstream positions on $(r, \theta)$-intersection planes at the end of the simulation runs. The instantaneous mass flux $\rho w$ shows a significantly stronger recirculation for the rotating nozzle compared to the nozzle kept at rest, while it is vice-versa for the maximum value of $\rho w$ (Figs. 5.13g and
5.14g). Strong fluctuations are observed at the outer side of the rotating nozzle wall due to the azimuthal boundary layer present (Fig. 5.13a-d). In this region, an axisymmetric mode and a single-helix type mode is observed, which co-rotates with the mean flow and which is of counter-winding type, see Sec. 5.4.4 for further discussion. For the nozzle kept at rest, a helical instability with high azimuthal wave number is located at the inner side of the nozzle wall within the streamwise-azimuthal boundary layer (Fig. 5.14b), which already decays again upstream of the nozzle end section. This helical mode co-rotates with the mean flow with a winding sense opposite to it. Downstream of the nozzle, the swirling jet breaks down more dramatically for the rotating nozzle compared to the nozzle at rest (Figs. 5.13f-i and 5.14f-i). The region of negative streamwise mass flux is much larger for the rotating nozzle indicating a larger spatial extent of the vortex breakdown structure.

5.4.4 Azimuthal mode analysis

The amplitude of the azimuthal modes $n = 0, \ldots, 10$ according to Eq. 4.11 is shown in Fig. 5.15. The flow downstream of the nozzle is dominated by the single-helix type instability $n = 1$ for both setups. All modes grow exponentially downstream of the nozzle up to comparable amplitude levels. A local dominance of the axisymmetric mode $n = 0$ directly downstream of the nozzle is observed for the rotating nozzle, while for the nozzle at rest the axisymmetric mode is only co-dominant to the single-helix type instability $n = 1$.

For the nozzle flow the picture is different (rotating nozzle): helical modes $n = 0, \ldots, 20$ grow at the outer side of the nozzle wall saturating at amplitude levels hierarchically ordered by the mode number. The axisymmetric mode $n = 0$ dominates the flow together with a co-dominant single-helix type instability. The azimuthal amplitudes of all modes decrease already again towards the nozzle end section in the region $-2.5 \leq z \leq 0$. For the nozzle at rest, the amplitudes of the helical modes are in general up to one order of magnitude smaller compared to the rotating nozzle. The nozzle flow is strongly dominated by a $(n = 12)$-mode together with a co-dominant single-helix mode $n = 1$ and mode $n = 13$. The azimuthal wave number of the dominating mode is thereby grid resolution dependent due to a variation of the cutoff wave length of the filter, applied for stabilizing the simulation (see Müller (2007) for details). This is in accordance with results reported in Keiderling & Kleiser (2008).
5.4 Comparison of DNS results

Figure 5.13: Instantaneous mass flux $\rho w$ colour plots at different downstream positions ($r, \theta$-planes) for the rotating nozzle at the end of the simulation. The base flow is directed into the drawing plane and rotates with counter-clockwise orientation. The nozzle wall is indicated in blue.

The authors investigated the effects of the azimuthal resolution on the selection of helical modes and concluded that consistent results with high predictive quality were obtained, but emphasised that grid-independent
Comparison of the two nozzle configurations

Figure 5.14: Instantaneous mass flux \( \rho w \) colour plots at different downstream positions \((r, \theta)-planes\) for the nozzle at rest at the end of the simulation. The base flow is directed into the drawing plane and rotates with counter-clockwise orientation. The nozzle wall is indicated in blue.
results were not to be expected. Depending on the azimuthal grid resolution, the azimuthal mode number of the dominating mode corresponds to $n = \sqrt{N_\theta}$ for $N_\theta = 36$ and $n = \sqrt{N_\theta/2}$ for $N_\theta = 128$ and $N_\theta = 288$ in the present investigation. The existence of a helical mode of high azimuthal mode number is confirmed by observations reported in Hof et al. (2004) and Vaidya et al. (2011). However, the azimuthal mode number ($n = 12$) determined in the present investigation might not be conclusive. As reported in Sec. 6.1, the wave number of the dominating azimuthal mode depends additionally on the swirl intensity.

The integrated azimuthal amplitude is shown in Fig. 5.16. As mentioned above, the amplitudes of the azimuthal modes $n_i$ are higher for the rotating nozzle due to their strong growth at its outer side. Only the amplitude of mode $n = 12$ is comparably high for the nozzle at rest, because of its growth in the boundary layer at the inner side of the nozzle wall. While the flow for the nozzle at rest is dominated by the single-helix type instability $n = 1$, the flow for the rotating nozzle is governed by the axisymmetric mode $n = 0$ due to its dominance in the nozzle flow. The total azimuthal amplitude contained in the flow with the rotating nozzle is higher than for the nozzle at rest, see Tab. 5.1.

Figs. 5.17 and 5.18 show the maximum amplitude of the azimuthal modes $n = 0, \ldots, 20$ at each location in space and the associated mode numbers $n_i$. For the rotating nozzle flow the overall maximum amplitude is observed at the outer side of the nozzle wall within the boundary layer,
Comparison of the two nozzle configurations

![Graph showing comparison of nozzle configurations](image)

**Figure 5.16:** Integral amplitude of azimuthal modes $n = 0, \ldots, 20$ scaled by the maximum global amplitude. Black: rotating nozzle, grey: nozzle at rest.

<table>
<thead>
<tr>
<th>Setup</th>
<th>$\sum_n \left( \int_{z=15}^{r=\infty} \int_{r=0}^{\infty} A_\theta , dr , dz \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotating</td>
<td>1</td>
</tr>
<tr>
<td>At rest</td>
<td>0.783</td>
</tr>
</tbody>
</table>

**Table 5.1:** Total amplitude of azimuthal modes $n = 0, \ldots, 20$ scaled by the global maximum total amplitude.

which is centrifugally unstable. Local maxima are located within the inner and outer shear-layer of the swirling jet and in the leeward part of the recirculation zone, where the single-helix type instability is found. At the outer side of the nozzle, the flow is dominated by an axisymmetric and single-helix type mode. In the centreline region, the single-helix mode shows the highest amplitude. The breakdown region and the outer shear-layer is governed by azimuthal modes $n = 1, \ldots, 3$, the inner shear-layer by the axisymmetric mode. At the nozzle lip, azimuthal modes of higher wave number grow and interact with the outer shear-layer of the swirling jet.

The picture for the nozzle at rest is different: high amplitudes are observed in the boundary layer at the inner side of the nozzle wall, where the $(n = 12)$-mode dominates the flow field. As for the rotating nozzle, the centreline region is dominated by the single-helix type instability. Local maxima are located in the inner and outer shear-layer of the
swirling jet and in the leeward part of the recirculation zone, comparable to the rotating nozzle, but at higher amplitudes. A local maximum situated downstream of the vortex breakdown bubble is observed for the nozzle at rest only. In the breakdown region, modes $n = 1$ and $n = 2$ are observed, while the inner shear-layer is dominated by the axisymmetric mode $n = 0$. The outer shear-layer is governed by the ($n = 13$)-mode already observed in the nozzle boundary layer together with modes $n = 1$ and $n = 2$ and other modes of high azimuthal wave number. Further downstream, modes $n = 1$ and $n = 2$ are observed in the outer shear-layer of the swirling jet. The nozzle lip does not seem to have a strong influence on the mode selection for the nozzle at rest.

5.4.5 Frequency analysis

The maximum Fourier-amplitudes of the streamwise velocity fluctuations transformed in time according to Eq. 4.12 are shown at each location in space for both setups in Fig. 5.19. For the nozzle at rest, maxima are
Comparison of the two nozzle configurations

located within the boundary layer at the inner side of the nozzle wall and in the inner shear-layer of the swirling jet (overall maximum). For the rotating nozzle, the maximum amplitude is about three times smaller compared to the nozzle at rest. Local maxima are observed at the outer side of the nozzle, in the jet shear-layers and at the centreline within the recirculation zone. The overall maximum is found in the inner shear-layer of the swirling jet in accordance with results for the nozzle at rest.

The Strouhal numbers corresponding to the maximum amplitudes are substantially different for both setups, see Fig. 5.20 and Tab. 5.2. This is not surprising because of the different locations where the maximum amplitudes are observed and the different underlying instability mechanisms. The Strouhal number varies significantly depending on the spatial location (cf. Fig. 5.20) and the azimuthal mode associated with, see Fig. 5.18.
Figure 5.19: Maximum amplitude of the Fourier-transformed instantaneous stream-wise velocity fluctuations. Sponge regions are hatched in black, the nozzle wall is indicated in grey.

<table>
<thead>
<tr>
<th>Setup</th>
<th>$St_{dom}$</th>
<th>$A(St_{dom})$</th>
<th>$(r, \theta, z)(St_{dom})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotating</td>
<td>0.005</td>
<td>0.428</td>
<td>(1.223, 0, 2.519)</td>
</tr>
<tr>
<td>At rest</td>
<td>0.103</td>
<td>1.331</td>
<td>(0.695, 0, 1.613)</td>
</tr>
</tbody>
</table>

Table 5.2: Overall dominant Strouhal number ($St = \frac{f \cdot R}{\omega_c}$) with amplitude and location.
Figure 5.20: Strouhal numbers associated with the maximum amplitude of the Fourier-transformed instantaneous streamwise velocity fluctuations. Sponge regions are hatched in black, the nozzle wall is indicated in grey.
5.5 Summary, discussion and conclusions

Vortex breakdown of swirling jet flows in the compressible regime at Reynolds number $Re = 5000$ and Mach number $(Ma = 0.6)$ was investigated by means of Direct Numerical Simulation (DNS). A nozzle was included in our computational domain, modelled as an isothermal wall. The nozzle was either in rotation with the mean flow or kept at rest. We discussed the main differences in the flow field and especially in the instability mechanisms associated with the different nozzle configurations. We compared numerical results for the setup with either a rotating nozzle wall or a nozzle wall at rest. The initial integral swirl number $S_6 = 0.85$ was identical for both setups investigated.

For a rotating nozzle wall, the swirling jet breaks down more intensively. The recirculation zone is longer in streamwise and radial extent, the spreading angle is larger. The first stagnation point on the jet centreline is located further downstream compared to the nozzle at rest, as is the second stagnation point. A secondary vortical structure is observed in the front part of the recirculation zone for the rotating nozzle, introducing an additional shear-layer and two stagnation points. The minimum streamwise centreline velocity is slightly larger for the rotating nozzle. The differences described are linked to the higher integral swirl number observed in the vortex breakdown region for the rotating nozzle. However, the changes in the flow configuration cannot be explained exclusively by swirl number effects: independent of the integral swirl number, differences are observed due to the different underlying instability mechanisms observed in the nozzle flow.

The mode selection in the nozzle flow differs significantly for both setups, in the type of helical instabilities selected as well as in the amplitude level. For the rotating nozzle wall, a boundary layer develops at the outer side of the nozzle, which is centrifugally unstable to the low wave number azimuthal modes $n = 0$ and $n = 1$. The nozzle flow is laminar except for upstream effects of the vortex breakdown. The nozzle at rest leads to an unstable boundary layer at the inner side of the nozzle wall, where a $(n = 12)$-azimuthal mode grows together with a single-helix $n = 1$ and a mode of high azimuthal wave number $n = 13$. The azimuthal wave number of the dominant mode is thereby grid resolution dependent in agreement with Keiderling & Kleiser (2008) due to the filter operation applied for stabilizing the direct numerical simulation. The amplitudes of the azimuthal modes are generally higher for the rotating nozzle. The maximum amplitude is observed at the outer side of the nozzle wall within the boundary.
Comparison of the two nozzle configurations

layer for the rotating nozzle, while it is located in the boundary layer at the inner side of the nozzle wall for the nozzle at rest. Downstream of the nozzle, the amplitudes are comparably high for both setups investigated. For both setups, the jet flow is governed by a single-helix type instability accompanied by an axisymmetric mode together with a double-helix mode.

A frequency analysis reveals that the maximum Fourier-amplitudes are observed in the nozzle boundary layer and in the inner shear-layer of the swirling jet for the nozzle at rest. The rotating nozzle case shows three maxima at the jet centreline and in the inner and outer shear-layer of the swirling jet. The dominant Strouhal number and the associated Fourier-amplitude is higher for the nozzle kept at rest.

The present study bridges the gap between investigations of swirling jet flows reported in literature utilising rotating nozzle devices (Facciolo et al., 2007; Liang & Maxworthy, 2005) and nozzles kept at rest (Billant et al., 1998; Leclaire & Jacquin, 2011; Oberleithner et al., 2011b). A numerical framework is used that allows to study the two nozzle configurations simultaneously with a minimum of setup differences to guarantee a reasonable comparison. Furthermore, the present study goes beyond recent experimental and numerical investigations of swirling jet flows undergoing vortex breakdown in studying the nozzle flow and the swirling jet flow at the same time allowing for an interaction of the two. The present results give strong evidence of the significant influence of the nozzle configuration, whether the nozzle rotates with the mean flow or is kept at rest, on the vortex breakdown behaviour of the swirling jet. The boundary layers developing at the inner (nozzle at rest) and outer side (rotating nozzle) of the nozzle strongly influence the vortex breakdown configuration. Swirling jets with an initially identical swirl number show significant variations in the size and shape as well as in the intensity of vortex breakdown. Consequently, the present observations re-emphasise that comparisons of results reported in literature should be made with care, especially when the flow upstream of vortex breakdown is not reported on and therefore unknown.

The conclusion is drawn that the changes observed in the nozzle (laminar flow ↔ transitional flow), the different types of boundary layers developing in downstream direction and following from this the variation in the mode selection upstream of vortex breakdown are responsible for the substantially different flow fields observed for the swirling jet. We want to point out that, besides the differences observed and described in the last section, the overall instability mechanism leading to vortex breakdown as well as the mode selection (single-helix type instability) in the recirculation
region is identical for both setups investigated. This gives strong evidence of the robustness of the mechanisms leading to vortex breakdown.

The present study clarifies the important role of the nozzle flow for the vortex breakdown of the swirling jet. It therefore strongly encourages to include the observation of the nozzle flow into studies of swirling jet flows to provide a more complete picture of the entire flow field and to allow for a more precise comparison of different studies.
We investigate the influence of the governing parameters on the flow field. We vary one parameter at a time to determine the effects on the flow field due to the changes made. The physical parameters defining the flow field such as the swirl parameter $a$ and the Mach number $Ma$ are varied. While the Mach number $Ma$ is varied only for the case of a rotating nozzle wall, the swirl parameter $a$ is studied additionally for the nozzle wall kept at rest. Tab. 6.1 gives an overview of the parameters varied throughout the investigation. For all numerical investigations performed (LES), the Reynolds number is $Re = 5000$, the nozzle length is $L = 5$, the nozzle wall thickness is $d = 0.1$ and the nozzle wall temperature is $T_w = 1$. The simulations were performed and data was sampled as described in Sec. 4.3.1. For grid resolution details see App. A.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range of investigation</th>
</tr>
</thead>
</table>
| Rotating  | $a$ 0.75, 0.85, 0.95, 0.975, 1.0, 1.125, ...  
|           | ... 1.25, 1.375, 1.5    |
|           | $Ma$ 0.4, 0.45, 0.5, 0.6, 0.7, 0.8 |
| At rest   | $a$ 0.75, 0.85, 0.95, 1.0, 1.05, 1.1, ...  
|           | ... 1.15, 1.2, 1.25    |

Table 6.1: Variation of physical parameters. The parameters of the reference case are marked in red.
6.1 Swirl parameter variation

Vortex breakdown of compressible swirling nozzle-jet flows is investigated by means of Large-Eddy Simulation (LES). The Reynolds number is $Re = 5000$ and the swirling jet is moderately compressible ($Ma = 0.6$). The swirl intensity is varied according to Tab. 6.1 to study the effects on the nozzle-jet flow field for the two nozzle configurations. Since the nozzle flow field upstream of vortex breakdown is significantly different for the two nozzle configurations (cf. Chap. 5), we observe that the nozzle flow field is altered generally in a different fashion by varying the swirl intensity. While for the rotating nozzle the axisymmetric and single-helix type instability is excited for increasing swirl, helical modes of high wave number ($n = 6, \ldots, 9$) grow within the nozzle kept at rest. For a comparably high maximum initial swirl velocity, the integral swirl number is substantially larger in the vortex breakdown region for the rotating nozzle than for the nozzle kept at rest. Despite the differences observed, remarkable similarities are found for the two nozzle configurations investigated. At the stage of vortex breakdown, the recirculation region and the conical shear-layer is dominated by a single-helix type mode together with an axisymmetric and a double-helix type instability for both setups. Irrespective of whether the nozzle rotating with the mean flow or kept at rest, there is a strong indication of the existence of a bifurcation of super-critical Hopf type to this single-helix mode, which is believed to be absolutely unstable, in agreement with findings reported in literature. Nevertheless, the amount of swirl sufficient for the occurrence of vortex breakdown and to let the bifurcation take place differs substantially for the two different setups.

We report on the results for both cases separately (Secs. 6.1.1 and 6.1.2) because the underlying instability mechanism differs significantly in the nozzle flow for the two setups chosen, cf. Chap. 5. We discuss and summarize our findings in Sec. 6.1.3 and point out the main differences between the rotating nozzle wall and the nozzle wall kept at rest for a changing swirl parameter $a$. In Sec. 6.1.4 we conclude the present study.

6.1.1 Rotating nozzle

Fig. 6.1 displays the streamwise, azimuthal and radial $(t, \theta)$-averaged velocities for an increasing swirl intensity at a sequence of downstream positions. The decline in the streamwise velocity along the jet centreline is stronger for an increasing swirl intensity as expected, cf. Liang & Maxworthy.
While for $a < 1$ the velocity decline is negligibly small for the flow within the nozzle, the deviation from the initial flat top profile increases for $a \geq 1$. Downstream of the nozzle at $z = 1$, the local maximum streamwise velocity is located within the conical shear-layer for $a \geq 1$, while for $a < 1$ the flow maintains its flat top shaped velocity profile approximately. The local maximum streamwise velocity within the conical shear-layer increases for $a = 1$ to $a = 1.125$ and decreases again for higher swirl parameters $a \geq 1.25$. We will clarify this observation in the discussion of the streamwise centreline velocity in the remainder of this section.

The larger the swirl the more pronounced is the deviation of the azimuthal velocity from solid-body rotation, indicating the transition to turbulence (Facciolo et al., 2007). At $z = 4$, the velocity profiles collapse for all swirl intensities investigated. At the outer side of the nozzle wall, an azimuthal boundary layer develops in the downstream direction. As reported in Chap. 5, this boundary layer is highly unstable to azimuthal modes of low wave number, which interact with the conical shear-layer downstream of the nozzle lip.

The radial velocity shows a monotonic trend in the nozzle flow regime: the higher the swirl intensity, the larger the radial velocity. An exception is observed for $a = 1.5$ at $z = -3$, where the radial velocity component is negative. The same trend is observed in the nozzle end section with a negative radial velocity at the outer nozzle wall due to entrainment of ambient fluid. The entrainment velocity increases with the swirl, cf. Komori & Ueda (1985) and McIlwain & Pollard (2002). At $z = 1$, the radial velocity increases to a maximum for $a = 1$ and then decreases again for a higher swirl intensity. The local maximum is shifted radially outwards. The radial velocity is much higher for $a = 1$ compared to the results of all other cases. As we will explain in the next paragraph, this is due to the change in the vortex breakdown configuration for a higher swirl intensity, see Fig. 6.2. Further downstream at $z = 2$, the radial velocity is negative for $a > 1$ indicating the entrainment of ambient fluid and the downstream end of the recirculation region.

Fig. 6.2 displays the $\langle t, \theta \rangle$-averaged streamwise velocity at the jet centreline. In general, a velocity decrease is observed within the nozzle for all values of $a$, with an increase in the streamwise centreline velocity above the initial maximum for $a \leq 1$ only. We define stable vortex breakdown to occur for $\langle w_c \rangle \leq 0$ in contrast to intermittent vortex breakdown ($w_c(t) \leq 0$) in agreement with Oberleithner et al. (2012). According to this definition, stable vortex breakdown occurs for $a \geq 0.975$, while for smaller swirl inten-
Figure 6.1: Axial development of $\langle t, \theta \rangle$-averaged streamwise, azimuthal and radial velocity (top to bottom). The grey bar indicates the position of the nozzle wall.

sities no recirculation zone is observed. For $a \geq 0.975$, the streamwise extent of the recirculation zone increases with increasing swirl. For the cases undergoing stable vortex breakdown the picture is threefold: for $a = 0.975$, one recirculation zone exists with two stagnation points at the jet axis. The decrease of the streamwise centreline velocity to the global minimum is monotonic. For $1 \leq a \leq 1.125$, the centreline velocity decreases to a first local minimum, increases again to a positive value and decreases to the global minimum finally. The flow displays four stagnation points on the jet axis. For $a \geq 1.25$, a first quasi-breakdown occurs within the nozzle, which is weaker than the second breakdown developing further downstream. The streamwise centreline velocity is generally positive within the nozzle, only $a = 1.5$ leads to $\langle w_c \rangle \leq 0$ and to a small vortical structure within the
nozzle. The streamwise centreline velocity shows four stagnation points (two for $a = 1.375$). For $z \geq 10$, the outflow boundary treatment acts on the flow field and the centreline velocity is shifted to a reference value.

The overall minimum streamwise centreline velocity decreases linearly in the swirl parameter regime before the onset of stable vortex breakdown in agreement with findings reported in Oberleithner et al. (2012), see Fig. 6.3. Stable vortex breakdown occurs for $a_{c2} \geq 0.97$ (linearly interpolated). In the breakdown regime, the minimum streamwise centreline velocity decreases further to a minimum for $a = 1$. A second approximately linear regime is observed for $a \geq 1.125$. This second linear regime corresponds to the flow cases described in the previous paragraph, for which a first velocity decrease is observed within the nozzle, including the case $a = 1.125$, which links the second and the third vortex breakdown regime (I: no breakdown, II: stable breakdown, III: first velocity deficit + stable breakdown).

Fig. 6.4 displays the length of the recirculation zone defined by the most up- and downstream stagnation points at the jet centreline, together with the location where the minimum streamwise centreline velocity is found. For $a \leq 0.975$ before the onset of stable vortex breakdown, the streamwise centreline velocity is smallest at the outflow boundary of the computational domain and therefore the positions are plotted

![Figure 6.2: $\langle t, \theta \rangle$-averaged streamwise velocity at centreline. Regions where sponges act on the flow field are shaded in grey.](image-url)
for completeness only. In the breakdown regime, the length of the recirculation zone increases with increasing swirl, with the exception of case $a = 1.125$, for which the length of the recirculation zone decreases due to a strong secondary vortical structure, see also Fig. 6.7. Due to this secondary vortical structure, the minimum velocity is located nearly at the position of the second stagnation point. For a higher swirl intensity $a \geq 1.25$, the first stagnation point is located within the nozzle. The location for the minimum streamwise centreline velocity is situated approximately in the middle of the recirculation zone for $a = 1$ and $a = 1.25$. For $a \geq 1.375$, this location is shifted in upstream direction towards the location of the first stagnation point. In general, the location where the minimum streamwise centreline velocity is found depends on the existence and the strength of the secondary vortical structure in the front part of the recirculation region as well as on the swirl intensity.

The jet half-width is defined by the radial position for which $\langle w \rangle / \langle w_c \rangle = 0.5$ according to Liang & Maxworthy (2005). $\langle w_c \rangle$ is thereby extracted at the inflow plane. In the region of the conical vortex breakdown, the radial position is evaluated in the outer shear-layer. For increasing swirl, the jet half-width decreases with downstream position in the regime before the onset of stable vortex breakdown, see Fig. 6.5. For $a \geq 0.975$, the jet half-width increases with increasing swirl intensity as expected (Liang & Maxworthy, 2005; Oberleithner et al., 2012; Toh et al.,
6.1 Swirl parameter variation

Figure 6.4: Streamwise extent of the \( \langle t, \theta \rangle \)-averaged recirculation region between first and last stagnation point on the jet centreline. The internal stagnation points located within the recirculation bubble are not considered here. The dash-dotted line indicates the position of the nozzle end section, the grey-shaded area the parameter regime of stable vortex breakdown \((a \geq a_{c2})\). The white-shaded area indicates the region of recirculation \((\langle w_c \rangle \leq 0)\).

The maximum jet half-width found at \( z = 1.5 \) is smaller for \( a = 1.125 \) than for \( a = 1 \). This is due to the secondary vortical structure found in the upstream part of the recirculation region, which is stronger for \( a = 1.125 \) (see also Fig. 6.7).

There is no clear trend for the spreading angle given in Tab. 6.2. While in the regime \((a < a_{c2})\) the spreading angle decreases as the jet half-width, in the breakdown regime no clear trend is visible. While the spreading angle is initially increasing for \( 0.975 \leq a \leq 1 \), it decreases again for \( a = 1.125 \). For \( a \geq 1.125 \) the growth of the spreading angle is linear. The overall maximum value of \( \alpha \) is found for \( a = 1.375 \). For \( a = 1.5 \) no data is available because the streamwise velocity is \( \langle w \rangle < 0.5 \) for all positions downstream of the nozzle end section.

Fig. 6.6 shows the streamwise development of the integral swirl number \( S_6 \) according to Tab. 2.1. For \( a \leq 1 \), the integral swirl number \( S_6 \) is approximately constant in the nozzle flow regime and in the jet flow regime, with a slightly higher value in the latter. The differences in the integral swirl number observed in the two flow regimes (nozzle ↔ jet) increases with increasing swirl parameter \( a \) and the distribution is less smooth. For
Figure 6.5: \( (t, \theta) \)-averaged jet half-width calculated according to Liang & Maxworthy (2005). The grey bar indicates the nozzle wall.

| \( a \) | \( \alpha[^{\circ}] \) | \( |\alpha|/\alpha_{\text{max}}[-] \) |
|------|------|------------------|
| 0.75 | -3.56 | 0.04             |
| 0.85 | -5.56 | 0.062            |
| 0.95 | -3.64 | 0.04             |
| 0.975| 10.16 | 0.11             |
| 1.0  | 19.19 | 0.21             |
| 1.125| 11.77 | 0.13             |
| 1.25 | 17.07 | 0.19             |
| 1.375| 23.45 | 0.26             |
| 1.5  | -     | -                |

Table 6.2: Spreading angle \( \alpha \) for different swirl parameter \( a \) calculated according to Liang & Maxworthy (2005), scaled by the geometrically defined maximum possible spreading angle \( \alpha_{\text{max}} = \pi/2 \).

For \( a = 1.125 \), the integral swirl number \( S_6 \) increases substantially in the nozzle flow regime. For an even higher swirl intensity, the integral swirl number is negative in the nozzle flow regime (not plotted here) and the integral swirl number definition fails. The critical integral swirl number for the occurrence of stable vortex breakdown is \( S_{c2} = 1.02 \), corresponding to the critical swirl parameter \( a_{c2} = 0.97 \), spatially averaged in the recirculation region. A critical integral swirl number \( S_{c1} \) for the occurrence of intermittent vortex breakdown, as discussed in Oberleithner et al. (2012), could not be determined, because no vortex breakdown is visible in the instantaneous flow field for \( a \leq 0.95 \). Nevertheless, it may be possible to observe intermittent vortex breakdown for \( 0.95 < a < 0.97 \).
6.1 Swirl parameter variation

Fig. 6.6 displays streamlines of the \( \langle t, \theta \rangle \)-averaged flow field. For \( S < S_{c2} \), the jet flow is less columnar for an increasing swirl intensity as expected, due to the enhanced mixing and entrainment. For increasing swirl in the range \( 0.975 \leq a \leq 1.125 \), a secondary vortical structure in the front part of the recirculation region develops. This secondary vortical structure, increasing in size and intensity for \( a = 1.125 \) and decreasing again for \( a = 1.25 \), leads to a more complex flow field due to the existence of a third shear-layer within the recirculation zone, as discussed in Chap. 5. The size of the recirculation zone decreases for a stronger secondary vortical structure and the recirculation intensity is weakened, see also Fig. 6.3. Faler & Leibovich (1978) reported on the internal structure of vortex breakdown bubbles and found a secondary vortex cell comparable to our finding here. For \( a \geq 1.375 \), the secondary vortical structure disappears and the size of the recirculation zone grows significantly, while the centre of the recirculation bubble is shifted radially outwards. The vortex breakdown dominates nearly the entire nozzle flow. Additionally, a small vortex is observed for \( a = 1.5 \) situated in front of the recirculation region inside the nozzle.

We compare the azimuthal amplitude distribution \( A_\theta \) of the azimuthal modes \( n_i \) for different swirl intensities according to Eq. 4.11, see Figs. 6.8 and 6.9. The amplitudes generally decrease for an increasing mode number as expected (Oberleithner et al., 2012).
(a) $a = 0.85$

(b) $a = 0.95$

(c) $a = 0.975$

(d) $a = 1.0$

(e) $a = 1.125$

(f) $a = 1.25$

(g) $a = 1.375$

(h) $a = 1.5$

Figure 6.7: Streamlines of the $\langle t, \theta \rangle$-averaged flow field. The grey bar indicates the nozzle wall and the circles the stagnation points at the jet centreline.

In general, the overall amplitude level increases with swirl in the nozzle flow regime. The nozzle flow is governed by azimuthal modes
6.1 Swirl parameter variation

developing and growing in the downstream direction at the outer side of the nozzle wall within the azimuthal boundary layer, see Chap. 5. For $a \leq 1.125$, the nozzle flow regime is dominated by the axisymmetric mode $n = 0$, for $a = 1$ we find a co-dominance of the mode $n = 1$ in the downstream part of the nozzle. For $a = 0.975$, a high amplitude is found for the axisymmetric mode $n = 0$ observed at $z = -3$. For $a \geq 1.25$, for which a first velocity deficit is visible within the nozzle (see Fig. 6.2), the flow is dominated by the single-helix type instability associated with the precessing vortex core (Martinelli et al., 2007) together with a co-dominant axisymmetric mode. For $a = 1.5$, a double-helix type instability is co-dominant in the front part of the nozzle for about one radius in agreement to results reported in Ruith et al. (2003).

Within the nozzle end section, the amplitudes of all modes $n_i$ increase substantially for $a \leq 1.125$. The azimuthal modes growing in the boundary layer at the outer side of the nozzle wall are entrained into the jet and additionally destabilize the developing conical shear-layer, see also Fig. 6.1. For $a = 1.25$ and $a = 1.375$, the amplitudes of the modes $n = 0$ and $n = 1$ decrease within the nozzle end section, while the amplitudes increase for all other azimuthal modes. An amplitude peak is observed within the nozzle end section for $a = 1.5$ for modes $n = 1$ and $n = 2$, which is stronger compared to the increase found for cases with a lower swirl intensity.

Downstream of the nozzle, the single-helix type instability together with the axisymmetric mode and the double-helix type instability dominate the flow independent of the level of swirl. The amplitude distributions differ nevertheless significantly depending on whether or not vortex breakdown occurs. While for $a \leq 0.95$ no local amplitude maximum is found, a local amplitude maximum is observed in the breakdown regime for $a \geq 0.975$. This local amplitude maximum, observed for all azimuthal modes $n_i$ for $0.975 \leq a \leq 1.25$ and for modes $n \neq 0$ and $n \neq 1$ for $a \geq 1.375$, is located within the recirculation zone slightly in front of the position of the minimum streamwise centreline velocity, see also Fig. 6.4.

Fig. 6.10 displays the spatially integrated amplitude of the azimuthal modes $n = 0, \ldots, 10$. The swirling jets, which do not break down ($a \leq 0.95$) together with case $a = 0.975$ undergoing a weak stable breakdown, show generally higher amplitudes of the azimuthal modes $n = 2, \ldots, 10$. The same is true for mode $n = 0$ up to $a = 1.25$ and mode $n = 1$ up to $a = 1.125$, respectively. The reason for this observation is that coherent structures do not break down in the streamwise direction as rapid as for the cases with stronger swirl. For a sufficiently strong swirl
Figure 6.8: Radially integrated amplitude $A_\theta$. Regions where sponges act on the flow field are shaded in grey. $n = 0$ —— (blue), $n = 1$ - - - (red), $n = 2$ ----- (green), other azimuthal modes in hierarchical order.

$(a \geq 1.375)$, the amplitude of the modes $n = 0$ and $n = 1$ increases remark-
ably, up to a level higher than for cases with \( a \leq 0.975 \) due to a growth of those modes in the nozzle flow regime. An additional amplitude growth of the double-helix mode \( n = 2 \) is observed, which is less pronounced. An explanation for the observation that the vortex breakdown case \( a = 0.975 \) behaves similar to the non-breakdown cases is found when comparing the jet half-width and the spreading angle, see Fig. 6.5 and Tab. 6.2: for \( a \geq 1 \) the jet half-width increases substantially. The columnar state of the swirling jet is disturbed and a conical vortex breakdown develops. The stability of the jet shear-layer is reduced and the jet breaks down to turbulence further upstream. For a sufficiently strong swirl, the nozzle lip affects the mode selection additionally, see Figs. 6.12 and 6.13. For \( a \geq 1 \), the flow field therefore changes remarkably, while case \( a = 0.975 \) does not show those changes. The vortex breakdown configuration for \( a = 0.975 \) is similar to a bubble-type breakdown instead of a cone-type breakdown. Tab. 6.3 shows the total amplitude of the azimuthal modes \( n_i \) and gives additional evidence to the observations made.

Figs. 6.12 and 6.13 show the spatial distribution of the maximum amplitude and the corresponding azimuthal modes \( n_i \). For low values of the swirl parameter \( a \leq 0.95 \), the flow inside the nozzle is of approximately zero azimuthal amplitude. At the outer side of the nozzle wall at \( z = -4 \), a local amplitude maximum is located due to the azimuthal boundary layer, which grows in the downstream direction. This amplitude maximum increases with swirl intensity independent of the occurrence of stable vortex breakdown indicating that the azimuthal boundary layer
Figure 6.10: Integral amplitude of azimuthal modes $n = 0, \ldots, 10$ scaled by the maximum global amplitude.

<table>
<thead>
<tr>
<th>$a$</th>
<th>$\sum_n \left( \int_{z=-5}^{z=15} \int_{r=0}^{r=\infty} A_\theta , dr , dz \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>0.952</td>
</tr>
<tr>
<td>0.85</td>
<td>1.0</td>
</tr>
<tr>
<td>0.95</td>
<td>0.953</td>
</tr>
<tr>
<td>0.975</td>
<td>0.978</td>
</tr>
<tr>
<td>1.0</td>
<td>0.836</td>
</tr>
<tr>
<td>1.125</td>
<td>0.828</td>
</tr>
<tr>
<td>1.25</td>
<td>0.887</td>
</tr>
<tr>
<td>1.375</td>
<td>0.921</td>
</tr>
<tr>
<td>1.5</td>
<td>0.947</td>
</tr>
</tbody>
</table>

Table 6.3: Total amplitude of azimuthal modes $n = 0, \ldots, 10$ scaled by the global maximum total amplitude.

is more and more centrifugally unstable. The boundary layer thickness at the outer side of the wall increases in the downstream direction (see Fig. 6.1), while the maximum amplitude decreases. The outer nozzle boundary layer is dominated by the axisymmetric and the single-helix type instability further downstream. In the region of the nozzle lip, the outer nozzle boundary layer is entrained into the jet shear-layer and convectively unstable modes (Loiseleux et al., 1998; Oberleithner, 2012)
grow in the downstream direction leading to local amplitude maximum. At the nozzle lip, azimuthal modes of higher wave number dominate the flow governing the flow for a certain spatial region whose extent decreases with increasing swirl: the single- and double-helix modes are dominant in the shear-layer further downstream together with the axisymmetric mode for $a = 0.95$. A third maximum is located at the centreline of the jet at $z \approx 4.5$, where the relatively large velocity fluctuations of the outer nozzle boundary layer are entrained into the jet shear-layer and grow into the vortex core of the swirling jet, see Fig. 6.11 for a schematic overview. This region is governed by the single-helix type instability.

For the breakdown case $a = 0.975$ the third local maximum located at the jet centreline is associated with the leeward region of the vortex breakdown bubble, see Fig. 6.7, an observation in agreement with findings reported in Liang & Maxworthy (2005) and interpreted as a region of local absolute instability. A fourth maximum in the inner shear-layer exists together with local maxima (I) and (II). While for maximum (I) and (III) the mode selection stays the same, maximum (II) is governed by azimuthal modes of higher wave number together with a double-helix type instability. The region where higher modes dominate the flow within the jet shear-layer decreases in size. Maximum (IV) is dominated by the axisymmetric mode.

![Figure 6.11: Schematic overview over the local amplitude maxima observed in Figs. 6.12 and 6.13.](image)

Yellow: azimuthal boundary layer, orange: streamwise boundary layer, green: inner jet shear-layer, violet: outer jet shear-layer, turquoise: shear-layer due to the secondary vortical structure within the breakdown bubble. Nozzle wall indicated in blue, stagnation points marked in red, flow direction shown with arrows.

For $1 \leq a \leq 1.125$ local maxima (I) and (II) are observed, together with maximum (III) in the lee of the vortex breakdown region, maximum (IV) found for $a = 0.975$ is missing. The jet shear-layer associated with maximum (II) is thinner compared to a smaller swirl intensity and this region is dominated by azimuthal modes of wave numbers up to $n = 10$.
developing into the dominance of the single-helix type instability. The maximum region at the centreline is governed by the azimuthal mode \( n = 1 \). In the inner shear-layer axisymmetric and single-helix type instabilities are observed together with a double- and triple-helix type instability for case \( a = 1.125 \).

For \( a \geq 1.25 \), the local maximum (I) at the outer side of the nozzle wall increases with increasing swirl intensity as expected. Maximum (II) in the outer shear-layer of the jet is more locally concentrated in the nozzle lip region with growing amplitude for increasing swirl. The breakdown region moves upstream, see Fig. 6.7, and therefore the outer shear-layer of the jet interacts more intensively with the nozzle lip. Maxima (III) and (IV) are comparably weak. A maximum (V) not found for a smaller swirl intensity is visible in front of the recirculation zone. For higher \( a \), maximum (II) reaches into the nozzle flow and a second maximum of type (V) appears in front of the first velocity decline occurring for \( a \geq 1.375 \).

The nozzle flow regime is dominated by azimuthal modes \( n = 1 \) and \( n = 2 \), in agreement with findings reported in Imao et al. (1992) for high swirl intensities. The outer shear-layer at the nozzle lip is governed by single- and double-helix type instabilities, as the nozzle flow is, together with the axisymmetric mode (vanishing for increasing \( a \)). The flow downstream of the nozzle is governed by the same instabilities with the axisymmetric mode being dominant in a region off the centreline of the jet. The jet centreline region is dominated by a single-helix instability as observed for the whole swirl parameter regime investigated.

Figs. 6.14 and 6.15 display the spatial distribution of the maximum amplitude of the Fourier-transformed instantaneous streamwise velocity fluctuations according to Eq. 4.12. For \( a \leq 0.95 \), large amplitudes are found in the outer shear-layer of the swirling jet as expected. For the breakdown case \( a = 0.975 \), which lacks the second vortical structure within the recirculation zone, see Fig. 6.7, the maximum amplitude is found in front of the recirculation zone in agreement with Liang & Maxworthy (2005) who found a single-helix type instability (see Figs. 6.12 and 6.13), first in the vortex core and for higher swirl in the inner shear-layer of the jet. The precessing vortex core is associated to the maximum found. The maximum located within the outer shear-layer of the swirling jet is still observed but comparably less pronounced. For \( a = 1 \), the amplitude levels within the outer shear-layer and the recirculation zone are comparably high. The local maximum in front of the recirculation zone for \( a = 0.975 \) is not observed anymore. For \( a \geq 1.125 \), the highest amplitudes are located
6.1 Swirl parameter variation

within the inner shear-layer of the swirling jet enveloping the recirculation zone. This shear-layer merges for increasing swirl with the outer shear-layer where the entrainment of ambient fluid takes place at the nozzle lip. When the recirculation zone moves upstream into the nozzle for \( a \geq 1.25 \), the highest amplitudes are found in the nozzle region in front of and within the recirculation zone in addition to the maximum at the nozzle lip. For swirling jets undergoing a first velocity decline (\( a \geq 1.25 \)), two maxima are found within the nozzle merging for a higher swirl intensity.

Following Liang & Maxworthy (2005) and Oberleithner et al. (2011b), we plot the squared saturation amplitude \( A_{\text{sat}} \) against the swirl parameter, see Fig. 6.16. According to Huerre & Monkewitz (1990), a proportionality of the squared saturation amplitude to the critical parameter for the flow under investigation—in the present study the swirl parameter \( a \)—is an indication of a bifurcation of super-critical Hopf type. We observe a linear dependency of the squared saturation amplitude on the swirl parameter in agreement with results reported in literature (Liang & Maxworthy, 2005; Oberleithner et al., 2011b). The super-critical Hopf bifurcation to a single-helix type mode, believed to be globally unstable, sets in for a swirl parameter \( a_H = 1.12 \), which is higher than the amount of swirl sufficient for the occurrence of stable vortex breakdown (\( a_{c2} = 0.97 \)).

Tab. 6.4 summarizes the highest Fourier amplitudes found and the associated Strouhal numbers and spatial locations, together with the amplitudes and associated Strouhal numbers observed at the nozzle lip. While no clear trend is visible in the Strouhal numbers associated with the saturation amplitudes for \( a \leq 1.125 \), the value for \( St_{\text{dom}} \) is approximately constant for \( a \geq 1.25 \). This is in contradiction to Loiseleux & Chomaz (2003), Oberleithner et al. (2012) and Dinesh & Kirkpatrick (2009), who reported a slight and strong decrease of the dominant frequency, respectively, for increasing swirl intensity. The location, where the saturation amplitude is found is shifted downstream and radially outwards for increasing swirl (\( a \leq 0.95 \)). The saturation amplitude is found in the outer shear-layer of the swirling jet. In the breakdown regime (\( a \geq 0.975 \)), the saturation amplitude is located in front of the recirculation zone (\( a = 0.975 \)) and within the inner shear-layer (\( 1 \leq a \leq 1.25 \)). For a higher swirl intensity, the location where the saturation amplitude is found is situated within the nozzle, see also Figs. 6.14 and 6.15.

At the nozzle lip, the selected frequency depends strongly on the thickness of the outer jet shear-layer (Müller, 2007) and whether or not the inner shear-layer enveloping the recirculation zone merges with
the first. In general, the Strouhal numbers found at the nozzle lip are up to two orders of magnitude higher than the Strouhal numbers associated with the saturation amplitudes, while for the amplitudes we observe the opposite trend. There is no evidence of a mechanism relating the dominant Strouhal number to the Strouhal number at the nozzle lip, independent of whether or not the inner and outer shear-layers merge. Additionally, we find no evidence for a wave-maker mechanism as described by Oberleithner et al. (2011b): we cannot confirm the existence of a precessing vortex core that would impose its frequency on the inner shear-layer of the swirling jet overwhelming the entire flow. As reported in one of the previous paragraphs, we find a core-mechanism (Gallaire et al., 2004b) situated at the centreline of the jet, which gives rise to a single-helix type instability. This instability is also found within the inner shear-layer together with a double-helix type instability (triple-helix for \( a = 1 \) and \( a = 1.125 \) only). Due to the fact that the frequency within the inner shear-layer is different from the Strouhal number found in the precessing vortex core region of the jet without any clear connecting mechanism, we cannot support the existence of a wave-maker region at the centreline of the swirling jet and an amplification region within the inner shear-layer.

<table>
<thead>
<tr>
<th>( a )</th>
<th>( St_{\text{dom}} )</th>
<th>( A(St_{\text{dom}}) )</th>
<th>( (r, \theta, z)(St_{\text{dom}}) )</th>
<th>( St_{\text{lip}} )</th>
<th>( A(St_{\text{lip}}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>0.029</td>
<td>0.696</td>
<td>(0.753, 0, 1.690)</td>
<td>0.583</td>
<td>0.027</td>
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<tr>
<td>0.85</td>
<td>0.010</td>
<td>0.865</td>
<td>(0.846, 0, 1.800)</td>
<td>0.505</td>
<td>0.033</td>
</tr>
<tr>
<td>0.95</td>
<td>0.063</td>
<td>0.544</td>
<td>(1.203, 0, 2.570)</td>
<td>0.696</td>
<td>0.046</td>
</tr>
<tr>
<td>0.975</td>
<td>0.005</td>
<td>1.550</td>
<td>(0.404, 0, 1.744)</td>
<td>0.918</td>
<td>0.004</td>
</tr>
<tr>
<td>1.0</td>
<td>0.078</td>
<td>0.403</td>
<td>(1.101, 0, 2.734)</td>
<td>0.903</td>
<td>0.026</td>
</tr>
<tr>
<td>1.125</td>
<td>0.005</td>
<td>0.705</td>
<td>(0.846, 0, 1.250)</td>
<td>1.970</td>
<td>0.005</td>
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<td>1.25</td>
<td>0.156</td>
<td>2.390</td>
<td>(0.948, 0, 0.755)</td>
<td>1.300</td>
<td>0.009</td>
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<tr>
<td>1.375</td>
<td>0.159</td>
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<td>(0.503, 0, −1.119)</td>
<td>2.249</td>
<td>0.012</td>
</tr>
<tr>
<td>1.5</td>
<td>0.168</td>
<td>4.151</td>
<td>(0.650, 0, −3.778)</td>
<td>2.297</td>
<td>0.095</td>
</tr>
</tbody>
</table>

Table 6.4: Overall dominant Strouhal number \( (St = f \cdot R/w_c) \) with amplitude and location and Strouhal number at nozzle lip with amplitude.
Figure 6.12: Maximum of $A_\theta(r,z)$ over all azimuthal modes $n_z$ and associated mode number (contour lines). Violet-blue contour lines indicate change from $(n = 0)$-dominance to $(n = 1)$-dominance, Green-red contour lines the change from $(n = 1)$-dominance to $(n = 2)$-dominance. Sponge layer regions are hatched in black, nozzle wall indicated in grey.
Figure 6.13: For caption see Fig. 6.12.
6.1 Swirl parameter variation

Figure 6.14: Maximum amplitude of the Fourier-transformed instantaneous streamwise velocity fluctuations. Regions where sponges act on the flow field are shaded in white.
Figure 6.15: For caption see Fig. 6.14.
Figure 6.16: Squared saturation amplitude against swirl parameter $a$. In the super-critical regime a linear proportionality is an indication of a bifurcation of super-critical Hopf type, cf. (Huerre & Monkewitz, 1990). The grey shaded area indicates the stable vortex breakdown regime.
6.1.2 Nozzle at rest

Fig. 6.17 displays the streamwise, azimuthal and radial \((t, \theta)\)-averaged velocities for a varying swirl intensity at a sequence of downstream positions. For \(a \leq 0.95\), the streamwise velocity deviates slightly only from the initial flat top profile encountering a velocity decline at the centreline of the swirling jet within the nozzle. For flows undergoing vortex breakdown \((a \geq 1)\) the picture is twofold: for \(a = 1\), the flow behaves similar to the cases without vortex breakdown showing a stronger decrease in streamwise velocity in the end section of the nozzle. For \(a \geq 1.1\), a first velocity decline is observed in the front section of the nozzle together with a strong velocity increase further downstream for \(a \geq 1.1\). The first velocity decline is shifted in upstream direction for an increasing swirl intensity and is strongest for \(a = 1.2\). Downstream of the nozzle, the swirling jet breaks down for \(a \geq 1\). The streamwise velocity is negative at the jet centreline and the local maximum streamwise velocity is found in the conical shear-layer of the swirling jet. For \(a \leq 0.85\), the streamwise velocity approximately maintains its initial shape decelerating in the downstream direction. For \(a = 0.95\), the streamwise velocity at the jet centreline decreases to a minimum value downstream of the nozzle comparable to the value observed for the breakdown cases.

The azimuthal velocity profile is of solid-body rotation type in the core of the jet and decreases towards the nozzle inner wall, which is kept at rest. For cases without breakdown \((a \leq 0.95)\), the azimuthal velocity deviates from the initial distribution only in the boundary layer at the inner side of the nozzle, while it is nearly unaltered within the core region of the jet. A similar trend is observed for \(a = 1\). For \(a \geq 1.1\), the azimuthal velocity deviates additionally in the jet core region. This deviation comes along with the first streamwise velocity decline described above. For \(a \leq 0.95\), the flow starts to deviate from the initial solid-body rotation downstream of the nozzle end section and the swirl intensity decreases. For flows undergoing a stable vortex breakdown, the maximum azimuthal velocity is found within the conical shear-layer of the swirling jet, with a more distinct local maximum for the cases without a first velocity decline in the nozzle flow regime. Even further downstream, the local differences found downstream of the nozzle end section disappear and the profiles approximately collapse for all cases independent of the swirl intensity. Qualitatively, the mean flow results are in good agreement with results reported in literature, cf. Billant et al. (1998), Toh et al.
6.1 Swirl parameter variation

(2010), Oberleithner et al. (2011b) and Oberleithner et al. (2012).

For $a \leq 1$, the radial velocity is approximately zero within the nozzle, while for flows which show a first velocity decline the radial velocity is as high as $\pm 10\%$ of the streamwise centreline velocity. In the nozzle end section at the outer side of the nozzle wall, the radial velocity is negative due to entrainment for all cases, with a stronger entrainment for jets with a high swirl intensity (Komori & Ueda, 1985; McIlwain & Pollard, 2002). The entrainment of ambient fluid into the swirling jet is strongest one radius downstream of the nozzle end section decreasing in strength further downstream. For $a \geq 1$, the strong spreading of the jet is reflected by the radial velocity in the region downstream of the nozzle with highest values for $a = 1$ due to the specific breakdown configuration, see also Fig. 6.23. At $z = 5$, the radial velocity is approximately identical for all flow cases investigated.

Fig. 6.18 displays the $\langle t, \theta \rangle$-averaged streamwise velocity at the jet centreline. For $a \leq 0.95$, the streamwise centreline velocity is positive everywhere and no stable breakdown occurs. Stable vortex breakdown is observed first for $a = 1$ leading to a small recirculation zone approximately 2.5 radii downstream of the nozzle. For $a \geq 1.1$, a pronounced decrease of the streamwise centreline velocity is observed within the first half of the nozzle. The decrease comes along with an velocity increase in the second half of the nozzle leading to an overshoot for $a \geq 1.2$, see also Fig. 6.17. For $a = 1.2$, the decline is stronger compared to $a = 1.25$ and the streamwise centreline velocity is slightly negative within the nozzle. The velocity overshoot increases with the swirl intensity. Downstream of the nozzle, the streamwise centreline velocity decreases for $a \leq 0.95$, while a recovery is observed downstream of the recirculation region for flow cases undergoing stable vortex breakdown. For $z \geq 10$, the outflow sponge acts on the flow shifting the streamwise centreline velocity to a reference value.

The overall minimum streamwise centreline velocity decreases linearly before the onset of stable vortex breakdown in agreement with findings reported in Oberleithner et al. (2012), see Fig. 6.19. For a swirl parameter $a_{c2} \geq 0.995$ (linearly interpolated), a recirculation at the centreline of the swirling jet is observed. The minimum streamwise centreline velocity decreases significantly in this parameter range ($a = \pm a_{c2}$) indicating the change to the stable vortex breakdown regime. The minimum streamwise centreline velocity decreases linearly with increasing swirl intensity in the breakdown regime with an increase for $a = 1.25$. The observed increase for a high swirl intensity is in accordance with results reported
in Oberleithner et al. (2012).

Fig. 6.20 displays the length of the recirculation zone, defined by the stagnation points at the jet centreline, together with the position where the minimum streamwise centreline velocity is found. For $a \leq 0.85$, the streamwise centreline velocity decreases monotonically in the downstream direction with smallest values at the position where the outflow sponge starts to act on the flow. For $a = 0.95$, the streamwise centreline velocity decreases to a minimum at approximately $z = 4.6$ indicating that the location where the minimum streamwise centreline velocity is observed moves upstream in a linear fashion for $0.85 \leq a \leq 1$. The length of the recirculation zone grows slightly for increasing swirl, with an approximately constant location of the first stagnation point at $z \approx 0.2$.
6.1 Swirl parameter variation

Figure 6.18: \( \langle t, \theta \rangle \)-averaged streamwise velocity at centreline. Regions where sponges act on the flow field are shaded in grey.

Figure 6.19: \( \langle t, \theta \rangle \)-averaged minimum streamwise velocity at centreline. Regime of stable vortex breakdown \( (a \geq a_{c2}) \) shaded in grey.

for \( a \geq 1.05 \). The second stagnation point is shifted downstream for a higher swirl intensity similar to the location of the minimum streamwise centreline velocity. For all swirl intensities under investigation, the recirculation zone is situated downstream of the nozzle end section.
For increasing swirl, the jet half-width increases as expected (Liang & Maxworthy, 2005; Toh et al., 2010), see Fig. 6.21. Before the onset of stable vortex breakdown, the jet half-width is nearly constant for all downstream positions. For a higher swirl intensity, the jet half-width increases downstream of the nozzle end section due to the spreading of the jet and decreases further downstream in the leeward part of the recirculation zone. The spreading angle reflects the same trend as the jet half-width for an increasing swirl parameter, see Tab. 6.5.

Fig. 6.22 shows the streamwise development of the integral swirl number $S_6$ according to Tab. 2.1. While the distribution of the integral swirl number $S_6$ is approximately constant in the jet flow regime for all values of the swirl parameter, the integral swirl number increases substantially in the nozzle flow regime for increasing swirl. The critical swirl number for the occurrence of stable vortex breakdown is $S_{c2} = 0.65$ corresponding to the critical swirl parameter $a_{c2} = 0.995$, spatially averaged in the region of vortex breakdown. As for the setup with a rotating nozzle, we do not observe a vortex breakdown in the instantaneous flow field for $a \leq 0.95$ and therefore a critical integral swirl number $S_{c1}$ for the occurrence of intermittent vortex breakdown (Oberleithner et al., 2012) cannot be determined. Nevertheless, it may be possible to observe
6.1 Swirl parameter variation

Figure 6.21: \( \langle t, \theta \rangle \)-averaged jet half-width calculated according to Liang & Maxworthy (2005). The jet half-width is defined as the radial position for which \( \langle w \rangle / \langle w_c \rangle = 0.5 \) holds. \( \langle w_c \rangle \) is thereby extracted at the inflow plane. In the region of the conical vortex breakdown the radial position is evaluated in the outer shear-layer. The grey bar indicates the nozzle wall.

![Figure 6.21](image)

Table 6.5: Spreading angle \( \alpha \) for different swirl parameter \( a \) calculated according to Liang & Maxworthy (2005), scaled by the geometrically defined maximum possible spreading angle \( \alpha_{\text{max}} = \pi / 2 \).

| \( a \) | \( \alpha[^\circ] \) | \( |\alpha|/\alpha_{\text{max}}[-] \) |
|---|---|---|
| 0.75 | 0.56 | 0.006 |
| 0.85 | 0.59 | 0.006 |
| 0.95 | 0.86 | 0.009 |
| 1.0 | 4.57 | 0.051 |
| 1.1 | 3.97 | 0.044 |
| 1.2 | 9.33 | 0.104 |
| 1.25 | 9.75 | 0.108 |

intermittent vortex breakdown for \( 0.95 < a < 0.995 \).

Fig. 6.23 displays streamlines of the \( \langle t, \theta \rangle \)-averaged flow configuration to give an overall impression of the flow field. While for \( a \leq 0.95 \) the flow is quasi-parallel and no recirculation zone develops, the picture changes for \( a \geq 1 \). A recirculation zone develops and grows in the downstream direction for increasing swirl, see also Fig. 6.20. For \( a \geq 1.1 \), the location of the first stagnation point does not change, while the recirculation zone is elongated in the downstream direction. The internal structure of the recirculation zone is one large circulation with its centre shifting slightly for a changing swirl intensity. For \( a = 1.2 \), a small second recirculation zone develops in the front part of the nozzle (not shown here), locally
directing the streamlines radially outwards, see also Fig. 6.18.

We compare the amplitude distribution $A_\theta$ of the azimuthal modes $n_i$ for different swirl intensities in Figs. 6.24 and 6.25. The nozzle flow regime is governed by an axisymmetric mode $n = 0$ for $a = 0.75$. For a higher swirl intensity, azimuthal modes of higher wave number are dominant before the onset of stable vortex breakdown ($a < a_{c2}$): mode $n = 9$ is dominant for $a = 0.85$ and mode $n = 8$ with a co-dominant axisymmetric mode $n = 0$ for $a = 0.95$ and $a = 1$. For $1.1 \leq a \leq 1.2$, the wave number of the dominant azimuthal mode changes from $n = 8$ to $n = 7$, being overwhelmed by the axisymmetric and single-helix mode, especially in the downstream part of the nozzle. For $a \geq 1.25$, the wave number of the dominant mode reduces further to $n = 6$ and the single-helix type instability dominates the nozzle flow together with an axisymmetric and a double-helix mode. Azimuthal modes $n \geq 3$ decrease in amplitude in the downstream part of the nozzle for $a \geq 1.2$, after encountering a local maximum downstream of the location of the first velocity decline in the upstream part of the nozzle, see also Fig. 6.18. We have to keep in mind that the azimuthal wave number $n = 9, \ldots, 6$ of the instability dominating the nozzle flow is both dependent on the swirl intensity, as observed in the present investigation, and also on the grid resolution, see Chap. 5.

In the region of the nozzle end section, all modes increase in amplitude independent of the swirl intensity. Downstream of the nozzle, the flow
6.1 Swirl parameter variation

![Figure 6.23: Streamlines of the \(\langle t, \theta\rangle\)-averaged flow field. The grey bar indicates the nozzle wall and the circles the stagnation points at the jet centreline.](image)

is dominated by single- and double-helix type instabilities together with an axisymmetric mode \(n = 0\) independent of the swirl intensity. The single-helix type instability \(n = 1\) is amplified for increasing swirl. The
axisymmetric mode $n = 0$ dominates the flow downstream of the recirculation zone for $a = 1.1$. For $a = 1.1$, local maxima are visible within the front and the leeward part of the recirculation zone for $n = 1$, while for $a = 1$ a local maximum is found downstream of the recirculation zone for $n = 0$ and $n = 1$ together with a local maximum of the axisymmetric mode in the front part of the recirculation region, see also Fig. 6.23. For $a \geq 1.2$, the single-helix mode shows a local maximum in front of the recirculation zone. For $z \geq 10$, the outflow sponge acts on the flow field damping all helical modes.

Fig. 6.26 displays the spatially integrated amplitude of the azimuthal modes $n = 0, \ldots, 10$. For $a = 0.75$, the axisymmetric mode $n = 0$ is dominant. For higher modes $n$, the amplitudes decrease as expected for moderately swirling jets (Oberleithner et al., 2012). In general, the amplitude levels for all azimuthal modes increase for increasing swirl intensity. For $a \geq 0.85$, the amplitudes of the azimuthal modes with high wave number increase strongly, with the mode number decreasing for increasing swirl $(n = 9 \rightarrow 6)$ as discussed above. While for $a = 0.75$, $a = 1$ and $a = 1.1$ the amplitude is highest for the axisymmetric mode followed by the single-helix mode, it is vice-versa for the other values of the swirl parameter (with the double-helix mode co-dominant for $a = 0.85$). For $a \geq 1.1$, the amplitudes of all modes and especially of the modes $n = 0$ and $n = 1$ significantly increase and the dominance of the single-helix mode is more pronounced. The amplitude of the axisymmetric mode $n = 0$ decreases substantially for a change in the swirl parameter from $a = 1.2$ to $a = 1.25$, mainly due to an amplitude loss in the nozzle, see also Figs. 6.24 and 6.25.

Tab. 6.6 shows the total amplitude of the azimuthal modes $n_i$. The general trend of an increase in the amplitude for a higher swirl intensity described above is reflected. For $a = 1$ leading to a weak vortex breakdown, we find an exception of this general behaviour: the total amplitude decreases, which is probably due to the differences in the vortex breakdown configuration compared to a higher swirl intensity.

Figs. 6.27 and 6.28 show the spatial distribution of the maximum amplitude and the corresponding azimuthal modes $n_i$. Before the onset of stable vortex breakdown $a \leq 0.95$, the maximum azimuthal amplitude is found in the boundary layer on the inner side of the nozzle. The two-streak-structure is due to the streamwise and the azimuthal boundary layers, which do not coincide but show a radial offset to each other. For increasing swirl, the boundary layers coincide more and more due to the downstream development and the maximum smears
6.1 Swirl parameter variation

Figure 6.24: Radially integrated amplitude $A_\theta$. Regions where sponges act on the flow field are shaded in grey. $n = 0$ (blue), $n = 1$ (red), $n = 2$ (green), $n = 6$ (magenta), $n = 7$ (turquoise), $n = 8$ (orange), $n = 9$ (yellow), other azimuthal modes in hierarchical order.
out. Behind the nozzle, the maximum amplitude decreases relatively fast. Additional local maxima are visible for \( a = 1 \) in the leeward part (Liang & Maxworthy, 2005) of the recirculation bubble and in the inner shear-layer enveloping it. For \( a \geq 1.1 \), the amplitudes observed in the front part of the recirculation zone are increased together with the maximum in the inner shear-layer, which moves radially outwards due to the enhanced spreading of the swirling jet. For cases undergoing a first velocity decline within the nozzle, the maximum amplitude distribution
6.1 Swirl parameter variation

<table>
<thead>
<tr>
<th>$a$</th>
<th>$\Sigma_n \left( \int_{z=-5}^{z=10} \int_{r=0}^{r=\infty} A_\theta , dr , dz \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>0.7322</td>
</tr>
<tr>
<td>0.85</td>
<td>0.7958</td>
</tr>
<tr>
<td>0.95</td>
<td>0.81</td>
</tr>
<tr>
<td>1.0</td>
<td>0.7741</td>
</tr>
<tr>
<td>1.1</td>
<td>0.8872</td>
</tr>
<tr>
<td>1.2</td>
<td>1</td>
</tr>
<tr>
<td>1.25</td>
<td>0.9858</td>
</tr>
</tbody>
</table>

Table 6.6: Total amplitude of azimuthal modes $n = 0, \ldots, 10$ scaled by the global maximum total amplitude.

reflects this decline with a weak but visible local maximum in front of it.

For $a = 0.75$, the maxima in the nozzle boundary layer are associated with the axisymmetric mode. At the nozzle lip, azimuthal modes of high wave number rise while the centreline region of the swirling jet is governed by the single-helix mode further downstream. For $a = 0.85$, the nozzle boundary layer is dominated by the azimuthal mode $n = 9$ while the core region of the jet is dominated by the single-helix mode $n = 1$. Modes $n = 8, n = 1$ and $n = 0$ dominate the flow within and downstream the nozzle for $a = 0.95$. For $a = 1$, the swirling jet undergoes vortex breakdown and the recirculation region is governed by a single- and double-helix type instability. Within the nozzle, the maximum amplitude is identical to case $a = 0.95$. For $a \geq 1.1$, the wave number of the mode governing the nozzle boundary layer decreases and the whole flow is dominated by the single-helix type instability together with the axisymmetric mode.

Figs. 6.29 and 6.30 display the spatial distribution of the maximum amplitude of the Fourier-transformed instantaneous streamwise velocity fluctuations. For $a \leq 1.1$, we find the maximum Fourier-amplitudes in the boundary layer within the nozzle increasing in amplitude, see Fig. 6.27 for comparison. For the weak breakdown case $a = 1$, the maximum amplitude decreases and two additional local maxima in the inner and outer shear-layer in the breakdown region appear. The picture is even more complex for the swirl parameter $a = 1.1$: in addition to the local maxima observed in the inner and outer shear-layer in the breakdown region of the swirling jet, two maxima appear in the nozzle in front and behind the location of the first velocity decline. For $a \geq 1.2$, the whole
flow is dominated by the maxima observed in the inner shear-layer and in
the front region of the breakdown bubble.

Following Oberleithner et al. (2011b) we plot the squared saturation
amplitude $A_{\text{sat}}$ against the swirl parameter $a$, see Fig. 6.31. According to
Huerre & Monkewitz (1990) a proportionality of the squared saturation
amplitude to the critical parameter for the flow under investigation—in
the present investigation the swirl parameter $a$—is an indication of
a super-critical Hopf type bifurcation similar to findings reported in
Liang & Maxworthy (2005), Meliga et al. (2012) and Lopez (2012) and
in agreement with Oberleithner et al. (2011b), Oberleithner et al. (2012)
and Oberleithner (2012). The bifurcations takes place at a swirl level
$a_H = 1.065$, which is higher than the level sufficient for the occurrence of
stable vortex breakdown ($a_{c2} = 0.995$).

Tab. 6.7 summarizes the highest Fourier-amplitudes found and the as-
sociated Strouhal numbers and spatial locations. The Strouhal numbers
are in general low as reported by García-Villalba et al. (2006). While for
the regime before the onset of stable vortex breakdown no clear trend in
the dominant Strouhal number exists, the Strouhal numbers associated
with the maximum amplitudes decrease slightly for an increasing swirl
intensity above $a \geq 1$, in accordance with Loiseleux & Chomaz (2003),
Oberleithner et al. (2012) and Dinesh & Kirkpatrick (2009). The maxi-
mum amplitude increases significantly in the breakdown regime, see also
Fig. 6.31. The location where the maximum amplitude is found for $a \leq 1.1$
is situated within the boundary layer at the inner side of the nozzle wall,
while it is shifted downstream into the outer/inner shear-layer in the vortex
breakdown region for $a \geq 1.2$. Since the nozzle lip does not seem to have an
important influence on the swirling jet flow for the nozzle kept at rest (see
Chap. 5), the frequency selection at the nozzle lip is not discussed here.
<table>
<thead>
<tr>
<th>$a$</th>
<th>$St_{\text{dom}}$</th>
<th>$A(St_{\text{dom}})$</th>
<th>$(r, \theta, z)(St_{\text{dom}})$</th>
</tr>
</thead>
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<tr>
<td>0.75</td>
<td>0.004</td>
<td>0.977</td>
<td>(0.743, 0, −2.268)</td>
</tr>
<tr>
<td>0.85</td>
<td>0.527</td>
<td>2.633</td>
<td>(0.764, 0, −2.888)</td>
</tr>
<tr>
<td>0.95</td>
<td>0.076</td>
<td>3.115</td>
<td>(0.968, 0, −2.190)</td>
</tr>
<tr>
<td>1.0</td>
<td>0.33</td>
<td>1.221</td>
<td>(0.753, 0, −3.677)</td>
</tr>
<tr>
<td>1.1</td>
<td>0.203</td>
<td>2.026</td>
<td>(0.968, 0, −1.833)</td>
</tr>
<tr>
<td>1.2</td>
<td>0.186</td>
<td>6.087</td>
<td>(0.712, 0, 0.7)</td>
</tr>
<tr>
<td>1.25</td>
<td>0.186</td>
<td>5.287</td>
<td>(0.535, 0, 0.205)</td>
</tr>
</tbody>
</table>

Table 6.7: Overall dominant Strouhal number ($St = f \cdot R/w_c$) with amplitude and location.
Figure 6.27: Maximum of $A_\theta(r, z)$ over all azimuthal modes $n_i$ and associated mode number (contour lines). Violet-blue contour lines indicate change from $(n = 0)$-dominance to $(n = 1)$-dominance, Green-red contour lines the change from $(n = 1)$-dominance to $(n = 2)$-dominance. Sponge layer regions are hatched in black, nozzle wall indicated in grey.
6.1 Swirl parameter variation

(a) $a = 1.1$

(b) $a = 1.2$

(c) $a = 1.25$

Figure 6.28: For caption see Fig. 6.27.
Figure 6.29: Maximum amplitude of the Fourier-transformed instantaneous streamwise velocity fluctuations. Regions where sponges act on the flow field are shaded in white.
6.1 Swirl parameter variation

Figure 6.30: For caption see Fig. 6.29.
Figure 6.31: Squared saturation amplitude against swirl parameter $a$. In the super-critical regime a linear proportionality is an indication of a bifurcation of super-critical Hopf type, cf. (Huerre & Monkewitz, 1990). The grey shaded area indicates the stable vortex breakdown regime.
6.1.3 Comparison of results and discussion

We summarize our findings here and compare results for the two different setups (rotating nozzle and nozzle kept at rest). As discussed in Chap. 5, the setups with either a rotating nozzle or a nozzle kept at rest lead to different flow fields upstream of vortex breakdown. Consequently, the vortex breakdown configurations show some differences as well. In the following we focus on a comparison of the differences in the alteration of the flow field with respect to a swirl intensity variation. For general differences in the flow field due to the nozzle either in rotation or kept at rest we refer to Chap. 5.

The mean flow field changes in a different manner for an increasing swirl intensity for the two setups investigated. For the rotating nozzle, the recirculation region grows in radial as well as in upstream and downstream direction and the entire nozzle flow is affected by the vortex breakdown for high swirl intensities. The secondary vortical structure (Faler & Leibovich, 1978) observed solely for this nozzle setup is intensified for an increasing swirl until the breakdown configuration changes over to one large recirculation region. For the nozzle at rest, the recirculation zone is elongated in the downstream direction only. While the streamwise extent of the recirculation region is comparable to the rotating nozzle case for approximately identical integral swirl numbers, the radial extent is substantially smaller. The nozzle flow is only slightly affected by the breakdown of the swirling jet for the swirl intensities investigated.

The minimum streamwise centreline velocity decreases linearly for both setups before the onset of stable vortex breakdown in agreement with findings reported in Oberleithner et al. (2012). Both setups lead to a first velocity decline within the nozzle for sufficiently strong swirl intensities (smaller for the nozzle kept at rest compared to the rotating nozzle), comparable to observations reported in Leclaire & Jacquin (2011). While for both setups the streamwise centreline velocity increases downstream of the location of the first velocity decline within the nozzle, this increase is much stronger for the nozzle at rest leading to a streamwise centreline velocity overshoot.

The critical integral swirl number $S_c^2$ for the occurrence of stable vortex breakdown is significantly larger ($S_c^2 = 1.02$) for the setup with a rotating nozzle compared to the nozzle at rest ($S_c^2 = 0.65$). A critical integral swirl number $S_c^1$ for the occurrence of intermittent vortex breakdown (Oberleithner et al., 2012) is not found for both setups. For swirl param-
eters in the super-critical regime \((S \geq S_{c2})\), the (squared) saturation amplitude of the dominating single-helix type instability depends linearly on the swirl parameter for both setups, which is a strong indication for the existence of a super-critical Hopf-bifurcation (Huerre & Monkewitz, 1990; Liang & Maxworthy, 2005; Oberleithner et al., 2011). The super-critical Hopf-bifurcation is observed for a swirl parameter of \(a_H = 1.12\) for the rotating nozzle and for \(a_H = 1.065\) for the nozzle kept at rest. The flow is governed by a single-helix type instability in the breakdown regime with a co-dominant axisymmetric and double-helix mode.

In general, the azimuthal amplitudes observed for the coherent structures are smaller for the nozzle at rest compared to the setup with a rotating nozzle wall, independent of the swirl intensity. While for the rotating nozzle the maximum azimuthal amplitude is found at the outer side of the nozzle wall within the azimuthal boundary layer before the onset of vortex breakdown, the maximum is located inside the nozzle within the boundary layer for the nozzle at rest. For swirling jets undergoing stable vortex breakdown, the maximum is found eventually in the region of the breakdown bubble at the centreline and within the conical shear-layer enveloping the recirculation region for both setups. The maximum is associated with the single-helix type instability and increases for an increasing swirl level. The single-helix type instability is accompanied by an axisymmetric mode and a double-helix type instability for a high swirl intensity. For the nozzle at rest, the flow promotes additionally the growth of azimuthal modes with high wave numbers within the nozzle. The mode number of the azimuthal instability dominating the nozzle flow decreases from \(n = 9\) to \(n = 6\) for increasing swirl. In contrast, the rotating nozzle flow is governed by the single-helix type instability together with the axisymmetric and the double-helix type mode independent of the swirl intensity.

The underlying mechanism leading to vortex breakdown is identical for the rotating nozzle and the nozzle kept at rest. The vortex core, precessing around the centreline, dominates the flow together with the developing recirculation zone and the single-helix type instability, which is induced by the vortex core motion. Although a vortex core is found giving rise to the single-helix type instability growing in the inner shear-layer of the swirling jet (Liang & Maxworthy, 2005; Oberleithner et al., 2011), we do not observe a wave-maker region, which determines clearly a frequency globally dominating the entire flow field. The frequency, at which the vortex core precesses around the centreline, does not correspond to the frequency of the single-helix type instability found in the inner jet
shear-layer. The dominating frequency is higher for the nozzle kept at rest compared to the rotating nozzle.

6.1.4 Summary and conclusion

Vortex breakdown of compressible, swirling nozzle-jet flows was investigated by means of Large-Eddy Simulations (LES) at a Reynolds number of $Re = 5000$ and a Mach number of $Ma = 0.6$. A nozzle was included in the computational domain modelled as an isothermal wall either rotating with the mean flow direction or kept at rest. The present investigation focused on the effects of a variation of the swirl intensity on the breakdown behaviour of the swirling nozzle-jet flow. To the best of our knowledge the present swirl intensity study was the first in the compressible regime as well as the first including numerical nozzle modeling. Studying the nozzle flow and the swirling jet flow simultaneously and allowing for an interaction of the two led to new insights into their mutual influence especially at high swirl. We presented results for the two nozzle configurations and made a comparison to clarify how the differences introduced by the nozzle wall, either being in rotation or kept at rest, impact the modulation of the flow field for a varying swirl intensity.

The differences observed in the nozzle flow regime for the two nozzle configurations, namely the differences in the boundary layers as well as in the mode selection (see Chap. 5), play an important role for the development of the jet at a moderate swirl intensity below and slightly above the threshold for vortex breakdown. For a high swirl intensity above the threshold for the super-critical Hopf bifurcation to take place, the mechanism leading to vortex breakdown is enforced and the influence of the nozzle flow on the swirling jet flow is weakened. The globally unstable single-helix type mode induced by the precessing vortex core overwhelms the entire flow field and a remarkable upstream effect of vortex breakdown is observed within the nozzle flow.

We conclude from the present study that the inclusion of a nozzle into the computational domain is essential for the investigation of swirling jet flows, especially at a high swirl intensity. The mutual interaction of the nozzle flow and the swirling jet flow is captured by including a nozzle and allowing for a more precise study of the vortex breakdown phenomenon. The present results therefore reinforce the conclusions made in Chap. 5. Furthermore, the present study confirms the robustness of the super-critical Hopf-bifurcation to a globally unstable single-helix type
instability, which is observed for both nozzle configurations independent of the flow upstream of vortex breakdown. For the first time it has been shown that the mechanism leading to vortex breakdown in compressible swirling nozzle-jet flows is similar to the mechanism observed in the incompressible regime. Further studies should concern the impact of the nozzle geometry on the vortex breakdown of swirling nozzle-jet flows, namely the nozzle length and the nozzle wall thickness, for both a rotating nozzle and a nozzle kept at rest.
6.2 Mach number effects

We investigate the effect of a variation in the Mach number on the development and the vortex breakdown of swirling jets by means of LES, see Tab. 6.1 for an overview. While the Mach number is varied, all other parameters are held fixed to precisely carve out the effects of the changes made here. The investigation is performed for the setup with a rotating nozzle wall.

Results of the present study have been partially published in Luginsland & Kleiser (2013b).

6.2.1 Rotating nozzle

Fig. 6.32 displays the three $\langle t, \theta \rangle$-averaged velocity components at a sequence of downstream locations. For a small Mach number $Ma = 0.4$, vortex breakdown is suppressed, while for increasing Mach numbers the intensity of the vortex breakdown is enhanced. The streamwise as well as the azimuthal velocity components deviate from their initial distribution in the downstream part of the nozzle due to the vortex breakdown downstream of the nozzle end and its upstream effect on the nozzle flow. The larger the Mach number, the stronger the deviation. The nozzle flow is laminar, which is indicated by the solid-body rotation profile of the azimuthal velocity component. Downstream of the nozzle end, the velocity maxima are shifted radially outwards due to the vortex breakdown configuration and the developing conical shear-layers. The radial velocity component increases in the nozzle end section indicating high spreading rates of the swirling jets. It increases with Mach number for $Ma \leq 0.6$ and decreases again for higher Mach numbers. Further downstream at $z = 3$, the radial velocity component is negative as fluid is entrained into the swirling jet.

The $\langle t, \theta \rangle$-averaged temperature, pressure and density are shown in Fig. 6.33. For a higher Mach number the temperature within the nozzle increases in the downstream direction with a maximum in the end section of the nozzle at the centreline. The ratio of the far-field temperature to the centreline temperature $T_\infty / T_c$ decreases for increasing Mach number. The temperature increase is mainly due to the growth of the recirculation region (see Fig. 6.37) and the induced deceleration of the flow. Behind the nozzle end section the radial maximum is located within the inner shear-layer of the conical vortex breakdown (if observed). Further downstream the temperature converges slowly to a uniform profile. The
temperature distribution in the swirling jet flow regime is comparable to results reported in Elsner & Kurzak (1987) for moderate swirl intensities. By definition (Eq. 4.4) the pressure at the centreline is higher for small Mach numbers compared to large Mach numbers. For all cases under investigation the trend in the radial as well as in the downstream direction is comparable. Behind the nozzle the pressure at the centreline converges to the far-field pressure level.

For all values of the Mach number, the density at the centreline is identically 1 initially. It increases by definition more strongly in the radial direction for an increasing Mach number leading to a higher ratio of the far-field to centreline density $\rho_\infty / \rho_c$ in the nozzle flow regime (Eq. 4.5). At the centreline and the inner nozzle wall the density increases in the downstream direction, more strongly at the latter location, and especially stronger for higher Mach numbers. At the outer side of the nozzle wall within the azimuthal boundary layer, the density decreases in the downstream direction for high Mach numbers. The change in density at the inner and outer side of the nozzle wall and the development of a radial density gradient at the wall is due to dissipative heating and in agreement with Lele (1994). The effect of dissipative heating is stronger for higher Mach numbers. Downstream of the nozzle, the density converges to the far-field value in the downstream direction.

The $(t, \theta)$-centreline streamwise velocity is depicted in Fig. 6.34. The larger the Mach number is, the stronger is the velocity decrease in the downstream direction along the jet centreline. The deceleration in the nozzle end section is more pronounced for a higher Mach number. A vortex breakdown is observed for $Ma \geq 0.45$. The overall strongest backflow is found for $Ma = 0.5$ increasing for larger Mach numbers, a phenomenon we explain in the remainder of this section, see Fig. 6.37.

Fig. 6.35 shows the minimum streamwise centreline velocity for a varying Mach number. The minimum streamwise centreline velocity decreases linearly for increasing Mach number $Ma \leq 0.5$ and vortex breakdown occurs for $Ma \geq 0.44$ on (linearly interpolated). For $Ma > 0.5$, the minimum velocity increases approximately linearly which can be explained by the streamlines of the $(t, \theta)$-averaged flow field shown in Fig. 6.37, see below.

No recirculation along the centreline is observed at all for $Ma = 0.4$, see Fig. 6.36. For increasing Mach number, the first mean flow stagnation point at the centreline of the swirling jet moves upstream. The recirculation zone grows in radial and streamwise extent for $0.45 \leq Ma \leq 0.6$ and is smaller again for a higher Mach number due to the upstream-moving
second stagnation point, which marks the end of the recirculation zone. Its upstream motion can be explained by the growth of the secondary vortical structure within the recirculation region (Fig. 6.37), which grows for increasing Mach number, intensifying the positive streamwise centreline velocity induced. centreline streamwise velocity and a smaller recirculation zone. The position of the minimum centreline streamwise velocity $\langle w_c(z) \rangle = \min$ is shifted in the downstream direction to the leeward part of the breakdown zone for an increasing Mach number.

Fig. 6.37 displays streamlines of the $\langle t, \theta \rangle$-averaged flow field. For a sufficiently high Mach number $Ma \gtrsim 0.44$ the swirling jet undergoes vortex breakdown. While for $0.45 \leq Ma \leq 0.5$ the recirculation region consists of
Figure 6.33: Axial development of $\langle t, \theta \rangle$-averaged temperature, pressure and density (top to bottom). The grey bar indicates the position of the nozzle wall. $Ma = 0.4$ (red), $Ma = 0.45$ – – – (green), $Ma = 0.5$ …… (blue), $Ma = 0.6$ …… (violet), $Ma = 0.7$ – – – (turquoise), $Ma = 0.8$ …… (yellow).

one large vortical structure (Oberleithner et al., 2011b), a secondary vortical structure develops for $Ma \geq 0.6$, cf. Faler & Leibovich (1978). This secondary structure grows in extent for an increasing Mach number, intensifying the positive streamwise centreline velocity induced. The trend described is visible in the results discussed above, leading to a decreasing radial velocity component downstream of the nozzle, a larger minimum centreline streamwise velocity and a smaller recirculation zone.

The radially integrated azimuthal amplitude $A_\theta$ developing is plotted in Fig. 6.38. For $Ma = 0.4$, the amplitudes of all modes increase at first for the nozzle flow and decrease towards the nozzle end section due to the breakdown of helical structures. At the nozzle lip, where the outer
nozzle boundary layer is entrained into the swirling jet shear-layer, all modes grow in amplitude until a saturation sets in five radii downstream of the nozzle. The amplitudes of the azimuthal modes are highest for the single-helix type instability $n = 1$ followed by a double-helix instability and an axisymmetric mode. Higher modes have amplitudes decreas-
Figure 6.36: Streamwise extent of the \( \langle t, \theta \rangle \)-averaged recirculation region between first and last stagnation point on the jet centreline. Internal stagnation points located within the recirculation bubble are not considered here. The grey-shaded area indicates the parameter regime of stable vortex breakdown \( (Ma \geq Ma_c) \). The white-shaded area indicates the zone of recirculation \( (\langle w_c \rangle \leq 0) \).

In contrast to the case lacking vortex breakdown the amplitude distribution for \( Ma \geq 0.45 \) shows a local maximum in the leeward part of the recirculation zone, see also Fig. 6.37. The flow is governed by a single-helix type instability accompanied by either an axisymmetric or a double-helix type instability. For \( Ma = 0.45 \), the axisymmetric mode shows a global maximum in front of the recirculation zone.

The integral azimuthal amplitude distribution of the modes \( n = 0, \ldots, 10 \) is shown in Fig. 6.39. In general the amplitudes of all modes decrease significantly for an increasing Mach number. The only exception are modes \( n = 0, n = 2, n = 5 \) and \( n = 6 \) for \( Ma = 0.45 \), which show an amplitude increase (the latter three negligibly weak) and mode \( n = 0 \) for \( Ma = 0.7 \).
6.2 Mach number effects

Figure 6.37: Streamlines of the \(<t, \theta>\)-averaged flow field. The grey bar indicates the nozzle wall and the circles the stagnation points at the jet centreline.

Tab. 6.8 reflects the same over all trend: a linear decrease in the total azimuthal amplitude for increasing Mach number in accordance with Müller (2007) and Ray et al. (2009), who showed that an increase in the Mach number leads to a decrease in the spatial growth rate of the helical modes. A similar observation was reported by Freund et al. (2000a) for the growth rate of annular mixing layers.

Figs. 6.40 and 6.41 display colour maps of the maximum azimuthal amplitude at each location in space. The amplitude maximum located in the boundary layer at the outer side of the nozzle wall is nearly independent of the Mach number. A second maximum found in the leeward part of
Figure 6.38: Radially integrated amplitude $A_\theta$. Regions where sponges act on the flow field are shaded in grey. $n = 0$ —— (blue), $n = 1$ - - - (red), $n = 2$ ------ (green), higher azimuthal modes in hierarchical order.

the recirculation zone increases in amplitude for increasing Mach number
for swirling jets undergoing vortex breakdown ($0.45 \leq Ma \leq 0.5$) and decreases again possibly due to the development of a secondary vortical structure described above ($Ma \geq 0.6$). A third local maximum located in the inner shear-layer surrounding the breakdown bubble decreases in amplitude for increasing Mach number possibly due to the upstream movement of the recirculation zone.

At the outer side of the nozzle, the flow is governed by the axisymmetric and single-helix type instability independent of the Mach number. The region of the nozzle lip is dominated by azimuthal modes of higher wave number. This region extends in the streamwise direction along the
outer shear-layer of the swirling jet being shifted radially outwards for a
higher Mach number due to the upstream movement of the recirculation
zone. At the centreline of the swirling jet a single-helix type instability
dominates the flow. In the front part of the recirculation zone along
the inner shear-layer surrounding the vortex breakdown bubble single-
and double-helix type instabilities dominate with azimuthal modes of
higher wave number for increased Mach number \( Ma \geq 0.6 \). The nozzle
flow regime is governed by a single-helix instability together with an
axisymmetric mode overwhelming the first more and more for increasing
Mach number. For \( Ma \geq 0.7 \), a double-helix type instability is observed
additionally in the front section of the nozzle.

Figs. 6.42 and 6.43 show the maximum amplitude of the Fourier-
transformed streamwise velocity fluctuations. For \( Ma = 0.4 \), local
maxima are found at the centreline in front of the minimum centreline
streamwise velocity (see Fig. 6.35), which are globally dominant, and
in the shear-layer of the swirling jet. For the occurrence of vortex
breakdown (\( Ma = 0.45 \)), the amplitude dramatically increases in front
of the recirculation zone possibly connect to the precessing vortex core.
For \( Ma \geq 0.5 \), the recirculation zone moves upstream and the thickness
of the inner shear-layer decreases while amplitudes increase in this
region. The same is true for the outer shear-layer of the swirling jet. The
overall maximum amplitude is found at the centreline of the swirling jet
for \( Ma \leq 0.45 \) and in the inner shear-layer for higher Mach numbers.
Amplitudes decrease generally for Mach number \( Ma = 0.5 \) and \( Ma = 0.6 \)
compared to \( Ma = 0.45 \) and increase again for \( Ma \geq 0.7 \) due to the
continued decrease in the shear-layer thickness.

Strouhal numbers and associated amplitudes of the overall dominating
single-helix type instability (see Fig. 6.38) are given in Tab. 6.9 together
with the values at the nozzle lip. The Strouhal numbers of the dominant
structures are small for the entire Mach number range investigated with
an approximately ten-times larger value for \( Ma = 0.6 \). The associated am-
plitudes differ with Mach number with highest amplitude for \( Ma = 0.45 \),
see Figs. 6.42 and 6.43 for comparison. The Fourier-amplitudes at the
nozzle lip are comparably small for all cases, while the Strouhal numbers
differ depending on the thickness of the outer shear layer of the swirling
jet (Müller, 2007).
### Table 6.9: Overall dominant Strouhal number ($St = f \cdot R/w_c$) with amplitude and location and Strouhal number at nozzle lip with amplitude.

<table>
<thead>
<tr>
<th>$Ma$</th>
<th>$St_{dom}$</th>
<th>$A(St_{dom})$</th>
<th>$(r, \theta, z)(St_{dom})$</th>
<th>$St_{lip}$</th>
<th>$A(St_{lip})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>0.005</td>
<td>0.634</td>
<td>(0.248, 0, 3.064)</td>
<td>0.796</td>
<td>0.005</td>
</tr>
<tr>
<td>0.45</td>
<td>0.002</td>
<td>2.410</td>
<td>(0.037, 0, 1.690)</td>
<td>1.230</td>
<td>0.003</td>
</tr>
<tr>
<td>0.5</td>
<td>0.007</td>
<td>0.554</td>
<td>(0.999, 0, 2.294)</td>
<td>0.732</td>
<td>0.014</td>
</tr>
<tr>
<td>0.6</td>
<td>0.078</td>
<td>0.403</td>
<td>(1.101, 0, 7.734)</td>
<td>0.903</td>
<td>0.026</td>
</tr>
<tr>
<td>0.7</td>
<td>0.002</td>
<td>0.852</td>
<td>(1.050, 0, 1.304)</td>
<td>1.296</td>
<td>0.007</td>
</tr>
<tr>
<td>0.8</td>
<td>0.002</td>
<td>0.707</td>
<td>(1.050, 0, 1.250)</td>
<td>1.626</td>
<td>0.008</td>
</tr>
</tbody>
</table>
Figure 6.40: Maximum of $A_\theta(r, z)$ over all azimuthal modes $n_i$ and associated mode number (contour lines). Violet-blue contour lines indicate change from $(n = 0)$-dominance to $(n = 1)$-dominance, Green-red contour lines the change from $(n = 1)$-dominance to $(n = 2)$-dominance. Sponge layer regions are hatched in black, nozzle wall indicated in grey.
Figure 6.41: For caption see Fig. 6.40.
Figure 6.42: Maximum amplitude of the Fourier-transformed instantaneous streamwise velocity fluctuations. Regions where sponges act on the flow field are shaded in white.
6.2 Mach number effects

Figure 6.43: For caption see Fig. 6.42.
6.2.2 Summary and conclusions

We investigated the influence of the Mach number on vortex breakdown in swirling nozzle-jet flows in the range $0.4 \leq Ma \leq 0.8$. We summarize our findings and compare our results with results found in literature.

We observe that vortex breakdown is enhanced for high subsonic Mach numbers, while below $Ma = 0.44$ (linearly interpolated) it is completely suppressed. The critical amount of swirl necessary for the occurrence of vortex breakdown increases with decreasing Mach number, a finding in contradiction to Herrada et al. (2003), Rusak & Lee (2004) and Zaitsev & Smirnov (1996) and in accordance with Melville (1996), Herrada & Shtern (2003a) and Herrada & Shtern (2003b). We want to point out that the first two references concern compressible pipe flow and therefore the comparison to our findings is of limited value. As pointed out in Melville (1996), an increased Mach number promotes vortex breakdown when the expansion rate $p_\infty/p_c = \text{const.}$ of the swirling jet is held fixed (as it is the case here). An increased jet core temperature comes along with an increasing Mach number in our investigation leading to the promotion of absolute instability (Lesshafft & Huerre, 2007) and therefore to an intensification of vortex breakdown. This temperature increase is accompanied by a decrease of the density of the swirling jet, an effect observed also by Herrada & Shtern (2003a,b). They found that the density variations induced by temperature gradients are more important for the development of the swirling jet undergoing vortex breakdown than those induced by the increase in Mach number. In addition these authors observe a promotion of vortex breakdown by a negative temperature gradient in the streamwise direction along the jet centreline. Our investigation supports the findings reported in Herrada & Shtern (2003a,b).

In the context of a swirl burner including combustion, Umeh et al. (2012) reported an increase in the streamwise velocity due to a temperature increase in the nozzle at constant inflow pressure and mean mass flow and therefore a downstream shift of the vortex breakdown region. In our case neither the inflow pressure nor the mean mass flow (due to density differences) is constant for a variable Mach number and this observation is not confirmed by the present results (albeit we observe a temperature increase in the nozzle flow when increasing the Mach number). The mean mass flux is initially substantially higher for an increasing Mach number as is the radial density gradient, while the initial expansion rate is identical for all cases. Adzlan & Gotoda (2012) investigated the vortex breakdown
behaviour of coaxial jets with higher density (lower viscosity) in the inner jet and found a promoting effect of the density difference due to a higher centrifugal force acting on the inner jet. In our investigation the density of the swirling jet is comparably smaller relative to the far-field density for higher Mach numbers. Therefore according to Adzlan & Gotoda (2012) we would expect the inverse effect, which is not observed.

Lesshafft & Huerre (2007) showed theoretically that for heated round jets an increased jet-to-ambient temperature ratio promoted absolute instability (depending on the shear-layer thickness) in agreement with Michalke (1984). Additionally, the authors showed that an increase of the Mach number prevents absolute instability. Results by Coenen & Sevilla (2012) revealed that hot jets are generically more unstable than light jets (positive radial density gradient) and that a low jet density compared to the far-field density may promote absolute instability. Lim & Redekopp (1998) observed the same trend as Lesshafft & Huerre (2007) for swirling jets and reported an additional trend towards absolute instability for increasing swirl. Additionally, Lesshafft & Huerre (2007) showed that an increase in the Reynolds number promotes absolute instability for constant jet-to-ambient temperature ratios, while an increase in the Mach number prevents absolute instability. Since it is believed that the existence of a sufficiently large pocket of absolute instability in the wake of the recirculation region is a requirement for the occurrence of vortex breakdown (Gallaire et al., 2006; Herrada & Fernandez-Feria, 2006; Liang & Maxworthy, 2005; Oberleithner et al., 2012, 2011b; Ruith et al., 2003) and for the onset of global instabilities (Lesshafft et al., 2007, 2006), especially the globally unstable single-helix type instability, we draw the following conclusion from our observations: the stabilizing effect due to an increased Mach number (as also reported in Rusak & Lee (2004)) is counter-acted by the increase in the jet-to-ambient temperature ratio. This leads to a destabilisation of the swirling jet, a promotion of absolute instability and therefore to a promotion of vortex breakdown. For an increasing Mach number, the jet-to-ambient density ratio decreases leading additionally to a destabilising effect and a promotion of absolute instability, cf. Sreenivasan et al. (1989) and Coenen & Sevilla (2012).

The secondary vortical structure in the front part of the recirculation zone increases in size and strength for an increasing Mach number leading to a larger minimum streamwise centreline velocity. Due to the growth of the secondary structure the vortex breakdown region decreases in size, being shifted upstream due to the promotion of vortex breakdown for an
increasing Mach number. For a Mach number slightly above the critical level, the recirculation zone is governed by one large vortical structure only, comparable to results reported in Sec. 6.1.2. The overall amplitudes of the azimuthal modes decrease for an increasing Mach number. While for moderate Mach numbers the highest Fourier-amplitudes are found in the wave-maker region in front of the recirculation zone, the maximum is shifted more and more into the region of the inner shear-layer of the swirling jet. A second maximum is found in the outer shear-layer between the swirling jet and the ambient fluid.

Overall, the results of our study of Mach number effects on the vortex breakdown behaviour of compressible, subsonic swirling jets are largely in agreement with results reported in literature. The modulation of the density and the temperature is the main reason for the observed changes for an increasing Mach number in the vortex breakdown behaviour of the swirling jets.
Chapter 7

Conclusions and recommendations

In this chapter we conclude our investigation and summarize the main findings of our study of compressible swirling jet flows undergoing vortex breakdown. We discuss the questions raised in Chap. 1. Recommendations are given on how to possibly continue the present investigation.

7.1 Conclusions

We investigated the vortex breakdown behaviour of swirling jets in the compressible regime by means of numerical simulations on massively parallel computing architectures. To this purpose we parallelised our numerical code PARACONCYL following a mixed parallelisation strategy consisting of the ghost-cell approach and data transposition. The numerical code shows good weak and strong scaling properties. High-order spatial as well as temporal numerical schemes were used to perform Direct Numerical and Large-Eddy Simulations (DNS/LES). A nozzle was included into the computational domain to account for more realistic inflow conditions and to investigate the flow within the nozzle and the jet flow regime at the same time. The nozzle was either in rotation with the mean flow or was kept at rest.

The effect of various combinations of inflow and outflow boundary conditions on the breakdown behaviour of a swirling jet has been studied (at $Re = 5000$, $Ma = 0.6$, $S_6 \approx 1$) for the rotating nozzle setup. While the influence of the outflow boundary conditions on the breakdown behaviour is weak, the variation of the inflow conditions leads to significant changes of the flow field of the swirling jet. The intensity of the recirculation, the size and shape of the vortex breakdown bubble as well as mode selection details differ for varied inflow boundary conditions. We decide to set all five conservative variables at the inflow and outflow of the computational domain using non-reflecting boundary conditions supplemented with sponge-layers. With these combinations of boundary conditions a minimum of artificial reflections at the domain boundaries is achieved. The choice proposed herein is in accordance with boundary conditions widely used in the computational aero-acoustics community.
A comparison of the development of the swirling jet at a swirl intensity above the critical threshold for the occurrence of vortex breakdown ($Re = 5000, Ma = 0.6, S_6 = 0.85$) for the setup with a rotating nozzle and a nozzle kept at rest reveals a significantly different instability mechanism in the nozzle flow regime. For the rotating nozzle, helical instabilities of low azimuthal wave number grow in the boundary layer at the outer side of the nozzle wall and interact with the developing conical shear layer in the breakdown region of the swirling jet. The instabilities saturate at a comparably high amplitude level. The nozzle flow stays laminar in general and turbulence is induced by upstream effects of the vortex breakdown only. The nozzle wall kept at rest leads to a transitional flow within the nozzle and helical instabilities grow within the boundary layer at the inner side of the nozzle wall. The nozzle flow regime is dominated by an instability with high azimuthal wave number. The flow state upstream of vortex breakdown is therefore of substantially different type for the two setups. Following from the differences observed upstream of the jet flow regime, the vortex breakdown configuration differs as well. Nevertheless, the mechanism leading to vortex breakdown of the swirling jet is identical for both setups.

We varied the swirl intensity for the setup with a rotating nozzle and a nozzle kept at rest. For an increasing swirl intensity, the intensity of the vortex breakdown as well as the spatial extent of the recirculation region grows for both nozzle configurations. The effect of vortex breakdown on the upstream flow within the nozzle increases as well. For both setups we find a strong evidence of a bifurcation of super-critical Hopf type to a global single-helix mode, in agreement with findings reported in literature. A precessing vortex core is observed in front of the recirculation zone, which gyrates around the centreline of the swirling jet. For a higher swirl intensity, this vortical motion triggers the growth of helical instabilities—mainly the globally unstable single-helix mode—at the inner side of the conical shear-layer of the swirling jet in the region of vortex breakdown. The dominant single-helix type instability is accompanied by the double-helix type instability and the axisymmetric mode. We conclude from the observations made throughout the course of the investigation that vortex breakdown is induced by a core-mechanism.

The investigation of compressibility effects in the subsonic regime led to the following observations: compressibility slightly enhances the vortex breakdown of the swirling jet emanating from the rotating nozzle, in agreement with findings reported in literature. The suppression of vortex breakdown for a decreasing Mach number and the strong enhancement of
the break-up of the swirling jet observed for an increasing Mach number is mainly due to temperature and density modulations in the core of the swirling jet, which accompany the Mach number variation and promote the transition to a globally unstable flow. An increasing Mach number damps the growth of azimuthal instabilities leading to overall smaller mode amplitudes. The secondary vortical structure observed in the front part of the recirculation region is intensified by a Mach number increase.

7.2 Recommendations for future work

There are several possibilities to extend the present investigation. One possible way to continue the present investigation is to analyse the results for the time-averaged flow field of the DNS/LES computations by means of a local spatio-temporal linear stability analysis. The aim of such an investigation is to exactly identify the parameter set at which absolute instability sets in and to find the regions of absolute instability. By now it is not clear where the change from convective to absolute instability takes place in the compressible swirling jet flow regime. One would have to account for the non-parallelism of the underlying flow in the jet flow regime, especially for high swirl rates for which vortex breakdown takes place.

Another possible continuation of the present investigation is to linearise the numerical code utilised throughout the present investigation and to analyse the flow by means of a linear global stability analysis. Starting from a time-averaged or stationary solution of the linearised Navier-Stokes equations, one would either solve the eigenvalue problem of the global system or would apply the Dynamic Mode Decomposition (DMD) method, which is equivalent in this special case (input data set is a linear solution). This investigation would address the question whether or not the single-helix type instability dominating the flow field of the swirling jet is globally unstable and would clarify the indications found throughout the present study.

Solving the linearised Navier-Stokes equations in combination with the adjoint linear Navier-Stokes equations would open up possibilities to investigate the sensitivity and the receptivity of the underlying flow. In the context of the controllability of the swirling jet undergoing vortex breakdown it is of interest to study the receptivity of the flow to external forcing to precisely identify the regions of the flow field where the instability mechanism leading to vortex breakdown is especially sensitive.

The computational tool at hand opens up the possibility to study
aero-acoustics in the near-field of the swirling jet due to the highly accurate numerical schemes implemented. It is of interest to get insight into the contribution of vortex breakdown to the near-field noise in comparison to other noise sources such as combustion. In this context, it might be necessary to review the parallelisation strategy followed during the present investigation to improve the weak scaling properties of the code because of increased computational cost.
Appendix A

Grid convergence study

For the grid convergence study, we perform four Large–Eddy Simulations (LES) to study the effect of changing the resolution in the streamwise (G0/G1) and the radial directions (G2/G3). The boundary conditions are chosen according to the recommendations made in Sec. 4.6 for all four simulations. Details of the resolutions for the different cases are given in Tabs. A.1 and A.2. In contrast to the simulations reported in Sec. 4.4, the nozzle length for these simulations is $L = 6$. Fig. A.1 shows that changing the streamwise resolution only slightly affects the $\langle t, \theta \rangle$-averaged results for the three velocities. The effect of changing the radial resolution is higher, but still sufficiently small for our purposes (see Fig. A.2). We thus conclude that the results for the coarser grids are accurate enough for the analyses made in the present investigation.

In addition to the comparisons discussed above we compare results of Direct and Large–Eddy Simulations (VX/DX) for the rotating nozzle and the nozzle kept at rest. Parameters are given in Tabs. A.1 and A.2, results are displayed in Fig. A.3 for the rotating nozzle wall and Fig. A.4 for the nozzle wall kept at rest. For the rotating nozzle setup the results nearly collapse indicating the high accuracy of the LES results. The results for the nozzle kept at rest show some deviations, which are strongest for the streamwise velocity $w$ around the swirling jet centreline in the downstream part of the nozzle. However, the results of the LES are accurate enough for our purposes here.
### Table A.1: Grid parameters for the simulations of the grid resolution study, GX, the assessment of boundary conditions and the parametric study, VX, and the basic cases, DX

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<tr>
<th>Parameters</th>
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<th>$N_r$</th>
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<td>360</td>
<td>360</td>
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<td>0.1174</td>
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### Table A.2: Grid parameters for the simulations of the grid resolution study, GX, the assessment of boundary conditions and the parametric study, VX, and the basic cases, DX

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<tr>
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<td>461</td>
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Figure A.1: Axial development of \langle t, \theta \rangle\text{-averaged streamwise, azimuthal and radial velocity (top to bottom). The grey bar indicates the position of the nozzle wall. G0 \ --- , G1 \ - - - .}
Figure A.2: Axial development of $\langle t, \theta \rangle$-averaged streamwise, azimuthal and radial velocity (top to bottom). The grey bar indicates the position of the nozzle wall. G2 ——, G3 ---.
Figure A.3: Axial development of $\langle t, \theta \rangle$-averaged streamwise, azimuthal and radial velocity (top to bottom). The grey bar indicates the position of the nozzle wall. Results for the basic flow configuration with rotating nozzle wall. VX ——, DX -- -.
Figure A.4: Axial development of $\langle t, \theta \rangle$-averaged streamwise, azimuthal and radial velocity (top to bottom). The grey bar indicates the position of the nozzle wall. Results for the basic flow configuration with the nozzle kept at rest. VX $\cdash\cdash$, DX $\cdash\cdash$.  

Grid convergence study
Previous publications (Luginsland et al.) related to the present work are listed at the end of this thesis.


Bibliography


Luginsland, T. & Kleiser, L. 2013b Mach number influence on vortex breakdown in compressible, subsonic swirling nozzle-jet flows. In Direct and Large-Eddy Simulation IX. (accepted).


OBERLEITNER, K., PASCHEREIT, C. O. & WYGNANSKI, I. 2007 Vortex breakdown in a swirl jet with axial forcing. 18ème Congrès Français de Mécanique (Grenoble 2007).


Publications

Parts of this thesis have been published as listed below.


# Curriculum vitae

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<td>10/12/2013</td>
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<tr>
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<th>Tobias Luginsland</th>
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Abstract

Vortex breakdown is a phenomenon which is of importance in a large variety of swirling flows. In environmental (tornadoes, hurricanes) as well as in internal (turbines, combustion chambers) and external (delta wing aircraft) flows of technical relevance it is of great interest to understand the underlying physical mechanism responsible for the occurrence of vortex breakdown and to know the parameter set at which vortex breakdown takes place. Despite decades of intense research no consensus has been found for the explanation of the instability mechanism and a widely accepted theoretical framework is lacking.

The present work deals with the numerical investigation of swirling jet flows as a model problem. The numerical code PARACONCYL is utilised to perform Direct- and Large-Eddy Simulations (DNS/LES) on massively parallel computing architectures. The compressible Navier-Stokes equations are solved on a cylindrical grid using high-order spatial and temporal numerical schemes. For the subgrid closure the Approximate Deconvolution Model (ADM) is applied. To account for a more realistic setup a nozzle is included in the computational domain modelled as an isothermal wall, which either rotates with the mean flow or is kept at rest. The effects of boundary conditions imposed at the inflow and outflow of the computational domain on the breakdown behaviour of the swirling jet are assessed in detail.

The setups with a rotating nozzle and a nozzle kept at rest are compared for identical initial integral swirl numbers. Main differences in the flow field results as well as in the vortex breakdown configuration are reported. The instability mechanism for the two setups is inherently different leading to significant differences in the mode and frequency selection. A single-helix type instability dominating the vortex breakdown configuration is found for both setups.

The Mach number is varied for the setup with a rotating nozzle wall to investigate its impact on the vortex breakdown behaviour of the swirling jet flow. The effect of the swirl intensity on the stability of the swirling jet is studied for both the rotating nozzle and the nozzle kept at rest.

Although the swirling jet is highly sensitive to the parameters investigated, the instability mechanism leading to vortex breakdown is remarkably robust.