Doctoral Thesis

Optimal decision making for secure and economic operation of power systems under uncertainty

Author(s):
Vrakopoulou, Maria

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OPTIMAL DECISION MAKING FOR SECURE AND ECONOMIC OPERATION OF POWER SYSTEMS UNDER UNCERTAINTY

A thesis submitted to attain the degree of

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(Dr. sc. ETH Zurich)

presented by

MARIA VRAKOPOULOU

Dipl.-Ing., Univ. of Patras

born on 25.02.1985

citizen of

GREECE

accepted on the recommendation of

Prof. Göran Andersson, examiner
Prof. Antonio Conejo, co-examiner
Prof. John Lygeros, co-examiner

2013
Extending my gratitude and thanks to the many people that have contributed one way or the other to my Ph.D life is probably the most pleasant part of this dissertation.

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Abstract

Operating power systems in a secure way constitutes a critical task for ensuring a well functioning society. However, security comes at the expense of additional investment and operational cost. The additional costs incurred to maintain a desired security level are expected to increase further due to the integration of Renewable Energy Sources (RES). This highlights the necessity to revisit certain operational concepts and construct novel design methodologies to achieve a better trade-off between a secure and an economic operation of the power network.

This dissertation concentrates on quantifying this trade-off and proposes a mechanism for optimal decision making in the presence of uncertainty. We first introduce the concept of probabilistic security, where the system and security constraints are allowed to be violated with a prespecified probability. To achieve this, we formulate an optimal power flow problem with N-1 security constraints and infeed uncertainty (e.g. wind power, loads) as a chance constrained optimization program. We then incorporate a generation-side reserve decision mechanism in the developed probabilistic security constrained optimal power flow framework. We model the deployed reserves as piecewise linear functions of the uncertainty and we optimize with respect to the coefficients of these functions and determine the required reserve capacity by the extreme values of the deployed reserves. In this way, another product of our production and reserve scheduling algorithm is the construction of a reserve strategy that can be deployed in real time operation.

Analogously to the generation-side reserves, we also provide a representation for the reserves offered by demand response. Exploiting demand-side capabilities for reserve provision allows a more economic operation of power systems since it results in lower total cost compared to the case
where only generation-side reserves are taken into account. In view of re-
ducing the total cost, the potential of certain network components other
than demand response to provide corrective control actions are also ex-
plored. Emphasis is given to HVDC links, and it is shown that, by
appropriately modeling their post-disturbance operating point, the de-
sired security level can be achieved at a lower operational cost. Finally,
we extend the framework of probabilistic security to an AC optimal
power flow set-up and exploit the controllability of the AVR system by
introducing a corrective control scheme that imposes post-disturbance
control actions for AVR set-point.

Since in all cases uncertainty is present, all proposed formulations are
formulated as chance constrained optimization programs. To solve the
resulting problems we employ recently developed algorithms based on
uncertainty sampling that offer a-priori guarantees regarding the prob-
ability of constraint satisfaction.
Zusammenfassung

Der sichere Betrieb elektrischer Energiesysteme ist eine wichtige Aufgabe für die Sicherstellung einer gut funktionierenden Gesellschaft. Allerdings erfordert Netzsicherheit zusätzliche Investitions- und Betriebskosten. Die Kosten, die zusätzlich anfallen, um ein bestimmtes Sicherheitsniveau zu gewährleisten, werden in der Zukunft wegen der zunehmenden Integration von erneuerbaren Energiequellen weiter steigen. Dies hebt die Notwendigkeit hervor, bestimmte Betriebsskonzepte neu zu bewerten und neue Planungsmethoden zu gestalten, um einen besseren Trade-Off zwischen einem sicheren und einem wirtschaftlichen Netzbetrieb zu erreichen.


Analog zu den Einspeisungsreserven liefern wir auch eine Darstel-
Zusammenfassung


Da in allen Fällen Unsicherheit vorliegt, werden alle vorgeschlagenen Optimierungsaufgaben als Optimierungen mit stochastischen Nebenbedingungen formuliert. Um diese Probleme zu lösen, verwenden wir kürzlich entwickelte Algorithmen, die auf einem Sampling der Unsicherheiten basieren. Diese Algorithmen bieten a-priori-Garantien bezüglich der Wahrscheinlichkeit, mit der die Nebenbedingungen erfüllt werden.
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List of Acronyms

**AC**  Alternating Current
**AGC**  Automatic Generation Control
**AVR**  Automatic Voltage Regulator
**CSC**  Current Source Converter
**DC**  Direct Current
**DR**  Demand Response
**FACTS**  Flexible AC Transmission Devices
**HVDC**  High-Voltage Direct Current
**IT**  Information Technology
**OPF**  Optimal Power Flow
**pSC-OPF**  Probabilistic Security-Constrained Optimal Power Flow
**PV**  Photovoltaic
**RES**  Renewable Energy Sources
**SC-OPF**  Security-Constrained Optimal Power Flow
**SCADA**  Supervisory Control And Data Acquisition
**SVC**  Static Var Compensator
**TCL**  Thermostatically Controlled Load
**TSO**  Transmission System Operator
VSC  Voltage-Source Converter
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Chapter 1

Introduction

1.1 Power systems operation

Power systems constitute a large-scale infrastructure of critical importance in the modern society. A potential disruption of power provision may lead to undesirable societal and economic consequences. Therefore, the main objective in power systems operation is to supply consumers with electric power in a reliable way. Typically, reliability refers to the ability of the system to provide the desired level of service continuously over an extended time horizon apart from only a few time instances where this service is interrupted \cite{40, 41, 9}. On the other hand, a power system is considered to be secure at a given instance of time if it is able to withstand nonanticipated disturbances. Therefore, to ensure reliability, the power system needs to be secure most of the time.

Due to the fact that power systems are time-varying and in view of achieving a secure, and hence reliable, performance, certain requirements need to be satisfied, i.e. frequency and voltage should be maintained at certain levels and equipment overloading should be avoided even at the occurrence of a disturbance.

In the sequel we provide more details about the different states of power system operation \cite{41} and discuss different control schemes that have been introduced to enhance its performance.


1.1.1 States of operation

Depending on the ability of the network to withstand disturbances, five different states of operation are defined [41]. Specifically the power system is in a normal state if all operational limits are satisfied and is operating in a secure way, i.e. no system limits are violated if a disturbance occurs. In this stage, automatic voltage and frequency control take place to keep the system within the safety margins, following generation and load fluctuations. These automatic control loops are also present in the other states of operation. In normal state, another goal is to minimize the operational costs. For this purpose, network and generation changes may take place.

If the system, operating with no limit violations, would fail to operate within its limits after a disturbance, it is considered to be in an alert state. In this state, security is at stake. To return in the normal state, preventive control actions need to be employed. Typical preventive control measures involve generation adjustments, increased system reserves, topological changes, etc.

If the system operates in the alert state and an disturbance that leads
to operational limit violations occurs, the system enters the *emergency state*. In this case *corrective* (or *emergency*) *control actions* are required to lead the system back to the normal or at least to the alert state. Typical corrective control measures involve exciter control, fast generation reduction or increase, generation tripping, HVDC modulation, system protection devices, load curtailment, etc.

If the aforementioned actions are not effective the system enters the *in extremis state* where cascading events occur and parts of the system may be disconnected. To prevent widespread blackouts, actions like load shedding and controlled islanding should be taken. Finally, in the *restorative state*, control actions are taken to reconnect the lost parts of the system and eventually operate again in a normal state. A schematic diagram summarizing the different states of operation is shown in Fig. 1.1.

### 1.1.2 Real-time operation

Interconnected power systems are divided in control areas and typically each area represents one country. The Transmission System Operator (TSO) is the responsible entity for the security of a single control area. Each area is system-wide supervised and controlled by the TSO through an IT infrastructure, referred to as Supervisory Control and Data Acquisition (SCADA) systems. The SCADA system measures data through remote devices installed throughout the grid and gathers the information at a control center through communication channels. After computer processing, it identifies the system’s operating state and decides on control commands to be sent from the center back to the system. Additionally, the system is equipped with local control devices to protect the equipment and to provide system-wide services after certain co-ordination. Voltage and frequency control and the evaluation of the security level are the main tasks so as to keep the system in the normal state.

**Voltage control:** Voltage control is mainly offered by the generating units, where by means of the Automatic Voltage Regulator (AVR) the voltage at the terminal of the units is maintained at the scheduled value. This control loop is local and the setpoint of the terminal voltage can be adjusted through the SCADA system. Other resources of voltage control include tap-changing transformers, Static Var Compensators (SVCs), synchronous condensers, etc.
**Frequency control:** Frequency is a common quantity through the system and depends on the active power balance. If a generation-load active power mismatch occurs, frequency will deviate from its nominal value. Any component that essentially influences the active power balance could be a potential resource of frequency control. Generating units are mainly used to provide frequency control since they can increase or decrease their active power production fast enough. The frequency regulation is mainly divided in two parts; the primary control and the secondary control (i.e Automatic Generation Control (AGC) or Load Frequency Control).

Primary frequency control refers to a local proportional controller that measures frequency deviations and adjusts the active power production of the corresponding generating unit. The response is in the scale of seconds and fast generating units are participating in this control action. The AGC scheme is performed through the SCADA system and involves pre-selected generating units in the control area. The goal is to bring frequency deviation back to zero and also to maintain the power flow on the tie lines that connect it with the other control areas at its scheduled value. The response is in the scale of minutes, so it is possible for slower units to participate.

The pre-disturbance production setpoint of the generators participating in frequency control should have a sufficient margin from their capacity limits that could be used once the frequency control is activated. Therefore, each generating unit could have a reserve capacity margin for primary and/or secondary frequency control.

After the activation of these automatic frequency control loops, tertiary frequency control takes place. This is a manually activated action and the main purpose is to release the deployed primary and secondary control reserves while performing an economic dispatch.

**Security assessment:** A commonly used security measure is the N-1 security criterion, where the system is supposed to be secure if it can withstand a predefined set of credible single contingencies. Typically, the examined contingencies include outages that are expected to occur with higher probability, like a single outage of a line, a transformer or a generator, and at some cases simultaneous component outages. The definition of security implies that both at the post-contingency steady state point of operation and during the transient phase, no operational limits violation should occur. Therefore, we can further distinguish
between a *static security assessment* and a *dynamic security assessment*.

In real-time operation, the TSO evaluates the security of the system through the SCADA system either continuously (practically every couple of minutes) or only when it is necessary. In practice, a static security assessment is performed by computing the post-contingency power flow for a large set of contingencies (i.e. contingency screening). The most severe contingencies are used for the dynamic security assessment which involves dynamic simulations and evaluation of the transient state trajectory. In case the system is insecure, the TSO identifies and applies preventive control actions (e.g. adjustment of the generation schedule) to restore the system to a secure operation. If a disturbance occurs before the security is restored the state of the system may enter the emergency state and additional actions should be taken as described in the previous subsection.

### 1.1.3 Day-ahead planning and economic operation

Towards a secure real-time operation, the TSO is performing various activities in advance that can be divided in three main categories: day-ahead operation planning, short-term planning, long-term planning [87]. In the long-term planning, tasks among others include the load forecast and identification of the new system conditions, investigation of system extensions and planning for control actions (preventive, corrective). The short-term planning includes approval of maintenance decisions, procurement of reserve power, etc. Security assessment is performed not only in real-time operation but also in any of the planning phases. In the latter, security is evaluated over different possible scenarios of the system conditions.

In the day-ahead operation planning, the TSO is responsible to schedule a day ahead unit commitment, a generation dispatch and, in certain control areas, also decide about the reserve procurement, while minimizing the operational costs and satisfying the security requirements. The identification of the optimal generation dispatch while satisfying the network constraints is traditionally referred to as Optimal Power Flow (OPF). The OPF problem, apart from the generation dispatch, may include additional control variables (e.g. setpoints for tap changers, phase shifters, generation terminal voltage, etc.) that could be adjusted to result in a more optimal performance in terms of cost. If static security
Chapter 1. Introduction

constraints are included in the OPF, the problem is referred to as Security Constrained Optimal Power Flow (SC-OPF). In the SC-OPF the control variables that correspond to the pre-contingency case (i.e. the case of no outage) represent preventive control actions. This implies that given the optimal values of these variables, the system after a contingency will result in a secure steady state point depending only on the existing automatic control loops. On the other hand, different control variables corresponding to each contingency represent corrective control actions. Under this concept, the system after a contingency will result in a secure steady state point of operation depending not only on the existing automatic control loops but also on the new set-points that are a result of certain corrective actions. It is of critical importance to distinguish between fast and slow corrective control actions and apply them appropriately given the time duration for which the components are allowed to be overloaded. For instance, power flow line limits could be roughly distinguished in two levels. First, the steady state limits should be satisfied for a permanent operation. These limits could be violated for several minutes as long as they satisfy the emergency limits, which if violated, the line will be tripped. Hence, for a more optimal in terms of cost but secure operation, devices with different time constants could be scheduled to offer corrective control dealing with different component limits.

Most TSOs do not incorporate security requirements into the optimization problem that concerns the day-ahead schedule, but only perform an a-posteriori security analysis. In the case that the security assessment shows that the system will be insecure, they employ a sequence of preventive control actions until a secure dispatch is identified. The post-contingency power flow analysis can incorporate also new post-contingency set-points for the devices that offer corrective control. The decision on which corrective control measures should be employed for a given outage is predefined in a long-term run. Note that satisfying the security requirements implies that sufficient reserve power is available to balance the system after a contingency. Typically, the amount of the reserves should be sufficiently high to supply the demand in the case that the largest generation unit is tripped and/or greater than or equal to a portion of the peak load.

It should be apparent from the discussion so far that the problem of optimal day-ahead planning is of great importance to ensure that the system is secure. Due to the complexity of the problem, the different un-
1.2 Motivation and problems to be addressed

The day-ahead operation of power systems involves designing a secure and economic schedule for the generating units and the reserves. However, security comes at the expense of additional investment and operational costs, thus revealing the trade-off between a secure and an economic system operation. In an ideal setup, where the underlying system is considered deterministic, there is a common consensus on how to identify the minimum cost operating point while satisfying the desired security level. However, power systems processes are inherently stochastic since they are subjected to stochastic power infeed, load uncertainty, unpredictable component outages, etc. Operating power system under uncertainty has attracted significant attention from both the power systems and operation research community. However, there is still no widely accepted approach and no unified framework to quantify the trade-off between a secure and an economic performance.

Toward this direction, decision making in the presence of uncertainty has often resorted to ad-hoc or rule based methodologies, leading either to a design that is conservative in terms of cost for the desired security level or, if uncertainty is ignored in the design phase, to a solution where security might be at stake. Such a performance might be acceptable if the level of uncertainty of the system is relative low. The uncertainty level, however, is increasing since environmental concerns and the pursuit of sustainable energy sources have resulted in policy measures toward high shares of renewable generation. Renewable Energy Sources (RES), such as wind and photovoltaic power generation, are non-dispatchable, fluctuating and uncertain. Therefore, employing suboptimal measures to account for the stochastic nature of RES may result in an undesirable economic effect. On the other hand, performing a deterministic design by ignoring the uncertainty will lead to an unacceptable reliability level. It should thus be apparent that it is necessary to develop a mechanism for optimal decision making in the presence of

derlying market mechanisms and the level of system uncertainty, different alternative implementations have been already proposed; however, obtaining a satisfactory solution is still subject of ongoing research.
uncertainty that takes into account the multi-objective nature of the problem, i.e. the trade-off between security and economic operation.

The increasing share of RES results in an increasing amount of required reserves, and hence it may have an opposite effect both from an environmental and economic point of view. The latter raises the need of cheaper and environmentally friendlier reserves providers. Demand side resources have already been used to provide certain control services but the full exploitation of their controllability has recently become an emerging research topic. Demand response and storage resources could be utilized to offer ancillary services including reserve provision. Promising technologies such as electric vehicles and thermostatically controlled loads could contribute with a large amount of reserve capacity and hence allow for the integration of high shares of RES. However, these technologies include uncertainty, mainly introduced due to human behavior and weather conditions, rendering their successful exploitation challenging.

Taking the uncertainty into account in the decision making mechanism introduces additional operational costs compared with a deterministic solution. To alleviate this, the controllability of certain network components other than the loads could also be exploited. These components could be utilized for preventive and corrective control actions. Some examples of controllable components are FACTS devices, HVDC lines, transformers etc. These components do not provide reserve capacity but their setpoint can be modulated in a post-disturbance situation, thus leading to lower operating costs.

This dissertation deals with the problem of developing a unified stochastic framework for optimal decision making, taking the uncertainty due to RES and the demand side into account, while exploiting the controllability of certain network components. Specifically, the following problems are addressed.

1. Probabilistic security: Probabilistic variants of deterministic security constrained optimal power flow problems need to be developed, providing enough flexibility to quantify the trade-off between security and economic system operation.

2. Production and generation-side reserve scheduling: Within a security constrained probabilistic framework standard day-ahead planning problems like production and reserve scheduling need to be revisited.
1.3. Contributions and organization of the thesis

3. Exploiting demand response for reserve provision: In an uncertain environment, demand-side resources should be taken into account in a decision mechanism to provide ancillary services while reducing the cost that would occur if reserves were solely purchased from the generating units.

4. Exploiting component controllability: Corrective control actions offered by certain network components could result in a more economic operation of the network, especially in cases where the level of uncertainty is increasing.

5. Development of new algorithms and tools: To address the problem of taking optimal decisions in the presence of uncertainty new algorithms for stochastic scheduling with guaranteed performance need to be developed, and the (probabilistic) properties of the obtained solutions should be re-interpreted.

1.3 Contributions and organization of the thesis

This dissertation aims at addressing the problems defined in the previous section. Specifically, we propose a framework that allows us to take optimal decisions regarding energy and reserve provision in the presence of generation and load-side uncertainty. We provide novel modeling formalisms to represent reserves and component controllability, constructing a new architecture that could be deployed in real time operation. Moreover, we base our solution methodologies on recent advances in stochastic optimization, employing algorithms with guaranteed (in a probabilistic sense) performance. Specifically, the contributions of the dissertation can be summarized as follows:

1. We introduce the concept of $N-1$ security with probabilistic guarantees and formulate the problem of probabilistic security constrained DC optimal power flow as a chance constrained optimization program. The proposed formulation provides us the means to quantify the trade-off between a secure and an economic operation.

2. A novel reserve scheduling methodology is proposed. We represent the reserves by piecewise linear decisions rules, which constitute a
strategy that can be deployed in real-time operation. The contingency dependant nature of the developed strategy provides a corrective control mechanism.

3. We propose a strategy to model uncertain reserves offered by the demand response. The proposed solution can be also used for real-time reserve deployment.

4. In view of a more economic operation we exploit component controllability to provide corrective control actions in a security constrained optimal power flow framework.

5. We develop a probabilistic security constrained AC optimal power flow framework, including a corrective control scheme modeling the post-disturbance set-point of the Automatic Voltage Regulator (AVR).

6. We propose the use of recently developed stochastic optimization techniques that are based on sampling to compute a solution to the developed chance constrained optimization programs, while offering a-priori guarantees regarding the probability of constraint satisfaction.

The remainder of the thesis is organized as follows:

1. **Chapter 2: Probabilistic security-constrained DC optimal power flow.**
   In this chapter we focus on a DC power flow set-up and revisit the problem of designing an N-1 secure day-ahead generation dispatch for power systems with RES, and in particular wind power. N-1 security is interpreted as the ability of the system to withstand single component outages. Here, we introduce the concept of probabilistic security and allow for system limits and security constraints to be violated with a low probability. The value of this probability is chosen a-priori in the design phase and provides the means to trade reliability to economic operation. The lower this value is, the higher is the probability that the obtained solution is reliable; however, this comes at the expense of additional cost.

   The problem of probabilistic security is formulated as a quadratic optimization program with chance constraints and it is solved using recently developed sampling based technique, which is based on a
1.3. Contributions and organization of the thesis

A combination of randomized and robust optimization. This technique allows us to provide a-priori guarantees regarding the probability of constraint satisfaction. The performance of the proposed methodology is illustrated on a case study involving the IEEE 30-bus network and is compared by means of Monte Carlo simulations against the solution of a deterministic variant of the problem, where the system operator determined a secure dispatch based only on the available wind power forecast.


This chapter incorporated a reserve decision mechanism in the probabilistic security-constrained optimal power flow set-up of Chapter 2. Modelling the steady-state behavior of the secondary frequency controller (i.e. Automatic Generation Control (AGC)) leads to representing the reserves by a piecewise linear function of the total generation-load mismatch. Optimizing over the coefficients of these piecewise linear rules, we also identify a reserve strategy according to which we can deploy reserves in real time operation. The contingency dependant nature of the latter offers an intuitive way to introduce corrective control actions in a decision mechanism.

The overall problem is formulated as a chance constrained, bilinear program. To alleviate the bilinearity issue we propose a heuristic algorithm and a convex reformulation, whereas to deal with the chance constraint the sampling based methodology of Chapter 2 is employed. The effectiveness of the proposed algorithms is illustrated by means of Monte Carlo simulations on the IEEE 30-bus network.

3. Chapter 4: Exploiting uncertain reserves from demand response.

In this chapter we exploit demand response for reserve provision in power systems with uncertainty. As for the generation-side reserves, we devise a strategy for the reserves offered by the demand response that could be deployed in real time operation. Optimizing over demand response reserves, even though they are uncertain, results in lower total cost compared to the case where only generation-side reserves are taken into account. The overall problem is formulated as an optimization program with chance constraints and we solve it using a variant of the sampling methodology that was employed in the previous chapters. To demonstrate the performance of our methodology we apply the developed algorithms on the IEEE 30-bus network and carry out Monte Carlo simulations.
4. **Chapter 5: Exploiting component controllability.**
   In this chapter we build on the probabilistic framework developed in Chapters 2-4 and extend it to incorporate corrective control actions offered by certain network components. Specifically, we focus on HVDC links and model their post-disturbance operating point. We show by means of a simulation based study that introducing component controllability in a security constrained optimal power flow set-up allows us to reduce the additional cost incurred due to a secure design and achieve a more economic operation. An extensive analysis is carried out and the proposed algorithmic modification is compared in terms of cost and performance against the algorithms of Chapters 2-4 on the IEEE RTS96 network, appropriately modified to include HVDC links and wind power infeed.

5. **Chapter 6: Probabilistic security-constrained AC optimal power flow.**
   This chapter extends the work on probabilistic security of Chapter 1 to an AC power flow set-up. Specifically, we employ recent results on convex AC optimal power flow relaxations and augment them to include N-1 security constraints. We also enhance the flexibility of the system by introducing a corrective control scheme that imposes post-disturbance control of the Automatic Voltage Regulator (AVR) set-point. The latter proposes a strategy for the AVR set-point and is analogous to the reserve strategy that was proposed in Chapters 3 and 4. To illustrate the performance of the proposed probabilistic security constrained AC optimal power flow we compare it against the DC formulation of Chapter 2. Using Monte Carlo simulations we show that it results in lower operational cost compared with the base case where the AVR set-points are constant.

6. **Chapter 7: Conclusion.**
   This chapter summarizes the contributions of the dissertation and provides direction for possible extensions and future research on related topics.

7. **Appendix:**
   In the appendix we provide detailed information about the sampling based optimization techniques that are employed throughout the dissertation. Different implementation alternatives are discussed and are compared in terms of the constraint violation guarantees they provide and the computational effort they require.
1.4 Publications

The work presented in this dissertation relies mainly on the following publications.

Publications relevant to Chapter 2:


Publications relevant to Chapter 3:


Publications relevant to Chapter 4:

Chapter 1. Introduction


Publications relevant to Chapter 5:


Publications relevant to Chapter 6:


Other publications


Chapter 2

Probabilistic security-constrained DC optimal power flow

This chapter proposes a novel framework for designing an N-1 secure day-ahead generation dispatch for power systems with a high penetration of fluctuating power sources, e.g. wind or PV power. To achieve this, we integrate the security constraints in a DC optimal power flow set-up and formulate a stochastic program with chance constraints, which encode the probability of satisfying the transmission capacity constraints of the lines. To solve the resulting problem numerically, we transform the initial problem to a tractable one by using a recently developed sampling based technique, which involves a combination of randomized and robust optimization and provides a-priori guarantees regarding the probability of constraint satisfaction. To generate wind power scenarios, a Markov chain based model is employed. To illustrate the effectiveness of the proposed algorithm, we apply it to the IEEE 30-bus network and compare it with the solution of a deterministic variant of the problem, where the operator determines a secure generation dispatch based only on the available wind power forecast. A Monte-Carlo simulation study is conducted to collect statistical results regarding the performance of our method.


2.1 Introduction

Security in power systems is an important topic of research, and it has been studied extensively over the last years. Following [86], [56], security of a power system refers to its ability to survive contingencies, while avoiding any undesirable disruption of service. To quantify security, the concept of N-1 security assessment has been developed, where the system is in a N-1 secure state, if any single component outage does not lead to cascading failures. Therefore, designing the system to be N-1 secure provides a way to prevent the network from widespread blackouts, which in most cases, at least in the final stage, develop as a result of cascading events [5].

To formulate a SC-OPF problem, the classical Optimal Power Flow (OPF) problem is extended to include additional constraints to satisfy the N-1 security criterion [65], [23]. The SC-OPF could include preventive [3] or also corrective control actions that consider set-point adjustments after a contingency [54].

Different perspectives towards the enhancement of the security of the system has been proposed. In [36], [37], the authors considered FACTS devices, and designed their setting in an optimal way considering the N-1 security criterion. In [86], a metric different from the N-1 security criterion was used, and a model predictive control scheme to achieve a high level of security against large disturbances was proposed. From a market point of view, the authors of [50] proposed a method for incorporating contingencies and stability constraints by making use of a voltage constrained optimal power flow.

The expected increase in the installed capacity of renewable energy sources, highlights the necessity of revisiting the existing N-1 security assessment method, so as to take into account their intermittent nature, which can have a major impact on the way power is distributed across the network. From an economic perspective, [11] designed a stochastic forward electricity market-clearing problem, allowing for higher wind power penetration while meeting the security requirements. The authors of [33] defined a set of security metrics to capture the different aspects of a large-scale wind integration project.

In [34] the authors attempt to quantify the value of wind at different network locations by solving a security constrained unit commitment
problem. In a similar setting, [79] formulated a stochastic unit commitment program for a combined wind-thermal system, but do not take security constraints explicitly into account in the design process. Instead, a subsequent step is performed, to measure the frequency of insecure instances. In [1], a simulation based analysis is carried out to evaluate the N-1 security of the Finish transmission system, in case of a simultaneous grid fault and a sudden decrease of the wind power infeed. Other methods dealing with the uncertainty are proposed in [27], [84], [61] where system constraints are formulated as chance constraints and solved analytically under the assumption of a Gaussian distribution of the uncertainty. However, in reality, the solution of these methods might not have the desired performance since the generation uncertainty (e.g. wind power) is not described by a Gaussian distribution [10].

In this chapter, we propose a novel framework, which provides probabilistic guarantees when designing a N-1 secure day-ahead dispatch for the generating units in systems with high amount of fluctuating power sources without making any assumption on the underlying uncertainty distribution. We integrate the security constraints, emanating from the N-1 criterion, to a DC optimal power flow program [66], and formulate a stochastic optimization problem with chance constraints, which constitute a probabilistic version of the system constraints. To transform this problem to a tractable one, we employ the recently developed sampling based technique of [44], [16], [22], that in contrast to other stochastic optimization techniques, provides a-priori guarantees regarding the probability of constraint satisfaction only based on a number of scenarios of the uncertainty. The resulting problem can be then solved easily by existing numerical tools [39]. This approach has only recently been applied to power system related problems [74, 45, 73, 75].

In the remainder of this chapter it is assumed that the fluctuating power infeed consists of wind power, and as proposed by [58], a Markov chain based model is employed to generate wind power scenarios. We consider only preventive actions for the generation dispatch. Additional controls, including changes in the topology configuration, the phase shifting transformer settings could be incorporated in the problem. To evaluate the necessity of a robust N-1 security design, we compare our method with a benchmark approach, which assumes that the network operator determines the dispatch of the generators based only on the available wind power forecast. The performance of our method is then verified via Monte Carlo simulations for different wind power realiza-
tions, using a modified version of the IEEE 30-bus network retrieved from [88].

Section 2.2 provides the problem set-up and information about how we treat uncertainty, whereas Section 2.3 states the resulting optimization program. In Section 2.4, we provide details regarding the sampling based optimization technique we employed and the Markov chain based wind power model. Section 2.6 shows the obtained simulation results to illustrate the performance of our approach. Finally, Section 2.7 summarizes our results and provides an outlook for open problems. References [74], [76] are related to this chapter.

2.2 Problem setup

For the analysis of the subsequent subsections, we consider a power network comprising of $N_G$ generating units, $N_L$ loads, $N_l$ branches, $N_b$ buses and $N_w$ wind power generators. By the term branch we refer to both lines and transformers. Moreover let $\mathcal{I}_w$ be a set that includes the bus indices where the wind power generators are connected to.

2.2.1 Power flow equations

As already mentioned, a DC power flow formulation is adopted [4], since it leads to a linear representation of the network, ensuring convexity of the developed optimization problem. It is based on the assumptions that

1. The voltage at every bus of the network remains constant at 1p.u.
2. The active power losses are neglected.
3. It is assumed that $\sin \theta_{km} \approx \theta_{km}$, where $\theta_{km}$ is the angle in radians across the branch connecting the buses $k$ and $m$.

The resulting power flow equation for every line $k \rightarrow m$ is given by

$$P_{km} = -b_{km}(\theta_k - \theta_m), \quad (2.1)$$
2.2. Problem setup

where $\theta_k, \theta_m$ are the voltage angles at the buses $k, m$ respectively, and $b_{km}$ denotes the imaginary part of the admittance of the line $k \rightarrow m$. Writing (2.1) in a matrix form, we get

$$P_f = B_f \theta,$$

where $P_f \in \mathbb{R}^{N_l}$ is a vector containing the power flows $P_{km}$ of each line, $\theta \in \mathbb{R}^{N_b}$ denotes the voltage angles at every bus, and $B_f \in \mathbb{R}^{N_l \times N_b}$.

The active power injection at a bus $k$ is given by

$$P_k = \sum_{m \in \Omega_k} P_{km},$$

where $\Omega_k = \{ m \in \{1, \ldots, N_b\} | k \rightarrow m \text{ is a line } \}$. In a more compact notation

$$P_{inj} = B_{BUS} \theta,$$

where $P_{inj} \in \mathbb{R}^{N_b}$ is the vector of the net injections $P_k$, and $B_{BUS} \in \mathbb{R}^{N_b \times N_b}$ denotes the nodal admittance matrix of the network.

In the sequel we will eliminate $\theta$ from (2.2), (2.4) so as to represent the power flows $P_f$ as a function of the power injections $P_{inj}$. Note that $B_{BUS}$ is singular, with rank $N_b - 1$, hence it is not invertible, and (2.4) can not be solved directly with respect to $\theta$. Therefore, to obtain a solution, one of the equations in (2.4) is removed, and the angle associated with this row is chosen as a reference angle and is set to zero. Without loss of generality, the last row of (2.4) is removed and $\theta_{N_b} = 0$.

Let then $\tilde{B}_{BUS} \in \mathbb{R}^{(N_b-1) \times (N_b-1)}$, $\tilde{\theta} \in \mathbb{R}^{N_b-1}$, $\tilde{P}_{inj} \in \mathbb{R}^{N_b-1}$ denote the remaining parts of $B_{BUS}$, $\theta$, and $P_{inj}$ respectively. Then,

$$\tilde{\theta} = \tilde{B}_{BUS}^{-1} \tilde{P}_{inj}.$$

Substituting (2.5) into (2.2) with $\theta = [\tilde{\theta} \quad 0]^T$, we have

$$P_f = B_f \begin{bmatrix} \tilde{B}_{BUS}^{-1} & 0 \\ 0 & 0 \end{bmatrix} \tilde{P}_{inj} = AP_{inj}.$$  

where

$$A = B_f \begin{bmatrix} (\tilde{B}_{bus})^{-1} & 0 \\ 0 & 0 \end{bmatrix}.$$  

Therefore, having represented the power flows as a function of the injections on the nodes, the optimization problem defined in the next section will only have $N_G$ decision variables.
The power injection vector $P$ can be written in a generic form as

$$P_{\text{inj}} = C_G P_{\text{gen}} + C_w P_w - C_L P_L,$$

(2.8)

where $P_{\text{Gen}} \in \mathbb{R}^{N_G}$, $P_w \in \mathbb{R}^{N_w}$, and $P_L \in \mathbb{R}^{N_L}$ denote the generating power, the wind power infeed and the load, respectively. Matrices $C_G, C_w, C_L$ are of appropriate dimension, and their element $(i,j)$ is “1” if generator $j$ (respectively wind power/load) is connected to the bus $i$, and zero otherwise.

### 2.2.2 Dynamic considerations

An outage or a deviation of the wind power from its forecasted value, which will lead to generation-load mismatch, will induce frequency deviations and activate the active power reserves of the system. To take this into account in our formulation, we consider that a new steady state is reached, as an effect of the secondary frequency control action (also known as Automatic Generation Control (AGC) or load frequency control). This is a reasonable assumption, since the optimization process is carried out in hourly steps, and hence frequency deviation settles again to zero due to secondary frequency reserve deployment. Therefore, we define $d \in \mathbb{R}^{N_G}$ to be a distribution vector, weighting the excess-deficit of power among the generating units participating in the frequency control. If a generator is not contributing to frequency control, the corresponding element in the $d$ vector is zero.

Let $P_m \in \mathbb{R}$ represent the total generation-load mismatch, which may occur due to the difference between the actual wind from its forecasted value and/or as an effect of generation-load loss. The N-1 security criterion consider any single component outage; however, a single line outage may trip a bus, and hence more than a single load, generator or wind power outage should be taken into account. In practice though, in an N-1 security set-up, one can consider a certain number of credible contingencies, where each of them could represent a set of single outages. Capturing both cases, the generation-load mismatch is determined by

$$P_m^i(P_w) = \sum_{k \in I_w/K^i} (P_{w,k} - P_{w,k}^f) - c_L^i P_L + c_G^i P_G + c_w^i P_w, \text{ for all } i \in \mathcal{I},$$

(2.9)

where $\mathcal{I}$ is a set of indices corresponding to the contingencies taken into account, whereas the “0” index corresponds to the base case of no
outage. Vectors $c^i_L$, $c^i_G$, $c^i_w$ are of appropriate dimension that based on the outage $i$ would be either zero or would include an “1” at the position that corresponds to the tripped component. In case of a wind power generator outage the amount of power produced before the outage should contribute to the generation-load mismatch, justifying the presence of the term $c^i_w P_w$ in the equation. However, the error from the forecast in this case should not be included in the mismatch. Therefore, we denote by $K^i$ a set that depends on the outage $i$ and will either be the empty set, or it will be a singleton including the index of the bus with the wind power outage. Note that the generation-load mismatch is equal to zero for the base case, i.e. $P_0^m(P^f_w) = 0$.

Given this mismatch, the new equilibrium operating point of the generators due to the secondary frequency control will be $P_G - d^i P_m^i$, where $P_G \in \mathbb{R}^{N_G}$ is the generation dispatch corresponding to the forecasted wind power $P^f_w \in \mathbb{R}^{N_w}$.

Equation (2.8), given a contingency $i \in \mathcal{I}$, can be rewritten as

$$P^{i}_{inj}(P_w) = C_G^i(P_G - d^i P_m^i(P_w)) + C_w^i P_w - C_L^i P_L,$$

(2.10)

### 2.2.3 Uncertainty Handling

There are two sources of uncertainty that are considered in the formulation of the preceding sections. The first one refers to component outages, whereas the second is the uncertain wind power production $P_w$. Therefore, we need to specify the way these uncertainties are treated, or in other words, in which sense the dispatch we will compute is robust toward component outages and wind power fluctuations. Note that no load uncertainty is considered, although the proposed framework could be extended to include such cases as well. For the component outages, a worst-case approach is adopted, that is, we enumerate all possible outages and design a generation schedule that is robust with respect to all these cases. Alternatively, a computational simpler problem may be achieved if only a few outages are selected according to some reliability index [82]. Following the same approach for the wind power uncertainty is not adequate, since the wind power takes values from a distribution with unbounded support. On the other hand, choosing some extreme values for the wind power production limits based on worst-case specifications of the wind generators may lead to very conservative conclusions or, in an optimization context, to feasibility problems. Therefore, we
Chapter 2. Probabilistic DC SC-OPF

follow a probabilistic approach and compute a generation schedule so that the limits of the generating units and the transmission capacity constraints are satisfied with a high probability. To achieve this, randomized optimization techniques are adopted [16], [44].

2.3 Optimization problem

The main objective is to design a minimum cost day-ahead dispatch, while satisfying the N-1 security constraints in a probabilistic sense. As explained earlier in this section, we take into account any contingency involving tripping of branches, loads, conventional generator units and generators with uncertain production (e.g wind plants). Note that the load profile is considered constant during each hour, and for simplicity reasons, no load uncertainty is taken into account in this section. However, this is taken into account in Sect. 4.

We consider an optimization horizon $N_t = 24$ with hourly steps, and introduce the subscript $t$ in our notation to characterize the values of the relevant quantities for a given time instance $t = 1, \ldots, N_t$. Let $c_1, c_2 \in \mathbb{R}^{N_G}$ be generation cost vectors, and by $[c_2]$ denote a diagonal matrix with vector $c_2$ on the diagonal. The resulting optimization problem is given by

$$\min_{\{P_{G,t}\}_{t=1}^{N_t}} \sum_{t=1}^{N_t} (c_1^T P_{G,t} + P_{G,t}^T [c_2] P_{G,t}),$$

subject to

**Deterministic constraints:** These constraints correspond to the case where the wind power is equal to its forecast value.

1. Power balance constraints: For all $t = 1, \ldots, N_t$

$$1_{1 \times N_h} (C_G^0 P_G + C_w^0 P_w^f - C_L^0 P_L) = 0. \quad (2.12)$$

This constraint encodes the fact that the power balance in the network should be always satisfied when $P_{w,t} = P_{w,t}^f$. In other words, the sum of all generation dispatches of the conventional units and the total wind power production, should match the total load of the system. The power balance is trivially satisfied for all other wind power instances under the assumption that sufficient
reserves are procured to allow the generation-load mismatch to be compensated by the generating units according to the distribution vectors.

2. Generation and transmission capacity constraints: For all \( t = 1, \ldots, N_t \) and all \( i \in \mathcal{I} \)

\[
-\overline{P}_f^i \leq A_i^i P_{inj}^i (P_f^w) \leq \overline{P}_f^i, \quad (2.13)
\]

\[
P_G^i \leq P_G^i - d_i^i P_m^i (P_f^w) \leq \overline{P}_G^i, \quad (2.14)
\]

where \( P_G^i, \overline{P}_G^i \in \mathbb{R}^{N_G} \) denote the minimum and maximum generating capacity of each unit and \( \overline{P}_f^i \) represents the maximum power flow on the lines. The generation-load mismatch \( P_m^i \) and the power injection \( P_{inj}^i \) is given by (2.9) and (2.10), respectively.

**Probabilistic constraints:** These constraints involve the uncertainty of the wind power forecast. For all \( t = 1, \ldots, N_t \)

\[
\mathbb{P}\left( P_w \in \mathbb{R}^{N_w} \mid -\overline{P}_l^i \leq A_i^i P_{inj}^i (P_w) \leq \overline{P}_l^i \right.
\]

\[
P_G^i \leq P_G^i - d_i^i P_m^i (P_w) \leq \overline{P}_G^i
\]

for all \( i \in \mathcal{I} \) \( \geq 1 - \varepsilon_t \), \( (2.15) \)

where the probability is taken with respect to the probability distribution of the wind power forecast uncertainty. The chance constraint (2.15) encodes the fact that, for each \( t = 1, \ldots, N_t \), the inequalities therein should be satisfied with probability at least \( 1 - \varepsilon_t \).

For each \( t = 1, \ldots, N_t \), the aforementioned problem can be written in a compact form as

\[
\min_{x_t \in \mathbb{R}^{N_x}} c_1^T x_t + x_t^T [c_2] x_t \quad (2.16)
\]

subject to

\[
F_{eq} x_t^0 + f_{eq} = 0 \quad (2.17)
\]

\[
F_d x_t + f_d \leq 0 \quad (2.18)
\]

\[
\mathbb{P}\left( \delta_t \in \mathbb{R}^{N_{\delta}} \mid F_p x_t + f_p + g(\delta_t) \leq 0 \right) \geq 1 - \varepsilon_t, \quad (2.19)
\]
Chapter 2. Probabilistic DC SC-OPF

where $x_t \in \mathbb{R}^{N_x}$ is a vector including the decision variables (in this case the generation dispatch $P_G$, i.e. $N_x = N_G$), and $\delta_t \in \mathbb{R}^{N_\delta}$ is the vector of uncertain variables (in this case $\delta = [P_w]^T \in \mathbb{R}^{N_w}$). All undefined matrices and vectors are of appropriate dimensions and are calculated based on (2.12)-(2.15). This is a stochastic program with chance constraints and a quadratic objective function. Techniques to solve this problem are described in the following section.

2.4 Dealing with the chance constraint

Consider the problem (2.16)-(2.19) for some time instance $t$. Chance constrained problems are non-convex and in general difficult to solve apart from specific cases. When the forecast error is uniform or normally distributed, analytic approaches may be employed [14]. However, recent advances in stochastic optimization provide solutions to chance-constrained problems without any requirement for the underlying distribution of the uncertainty [16]. Here we present two of these techniques and discuss their differences.

2.4.1 The “scenario approach”

The scenario approach was introduced by [16] as a methodology to solve chance constrained optimization programs. Following [16], and if the objective and the constraint functions are convex with respect to the decision variables, the chance constraint is substituted with a finite number of hard constraints corresponding to different scenarios of the uncertainty vector. It is shown that for a sufficiently high number of scenarios, one can provide a-priori guarantees that the resulting solution satisfies the chance constraint with a certain confidence. In [16], a lower bound for the number scenarios that need to be extracted is provided. Tighter bounds have been subsequently proposed by [20], [15] and [2]. Following [2], we need to extract

$$N \geq \frac{1}{\varepsilon_t} \frac{e}{e-1} \left( \ln \frac{1}{\beta_t} + N_x - 1 \right),$$

(2.20)

scenarios, where $\varepsilon_t \in (0, 1)$ (see also (2.19)) is a violation parameter determining the desired probability level, $\beta_t \in (0, 1)$ is a confidence level,
2.4. Dealing with the chance constraint
e denotes the Euler number, and \( N_x \) denotes the number of decision variables. Note that the number of the scenarios that need to be extracted increases linearly with the number of the decision variables and hence the computational cost increases as well.

The chance constraint is then reformulated with \( N \) sets of deterministic constraints each of them corresponding to a different scenario of the uncertainty. Equation (2.19) is then substituted by

\[
F_p x_t + f_p + g(\delta_t^{(s)}) \leq 0, \text{ for all } s = 1, \ldots, N.
\]

The solution of the deterministic optimization program involving only \( N \) scenarios is feasible for the chance constraint with probability at least \( 1 - \beta_t \). This reformulation relies on the assumption that both the objective and the constraint functions are convex with respect to the decision variables.

2.4.2 The “probabilistically robust design”

The probabilistically robust design is an alternative scenario based methodology to deal with the chance constraint that has been recently proposed by [44]. This method includes two steps. In the first step, the scenario approach is used to determine, with confidence at least \( 1 - \beta_t \), the minimum volume set that contains at least \( 1 - \epsilon_t \) probability mass of the uncertainty. In the second step, the robust counterpart of the initial chance constrained problem is solved, with the uncertainty enclosed in this set. In the following, we describe these steps.

**Step 1: Bounding the uncertainty**

In this step, we concentrate only on the uncertainty vector \( \delta_t \in \mathbb{R}^{N_s} \). To determine the minimum volume set that contains at least \( 1 - \epsilon_t \) probability mass of the uncertainty, a chance constrained problem is defined and is solved using the scenario approach. The decision variables are the parameters used to encode this set. We could use any set of fixed parametrization as long as its volume is a convex function of the parameter vector. It is important though to seek for a trade-off between the flexibility of the set to capture the distribution of the uncertainty and the size of the parametrization vector; the higher the number of parameters, the higher is the number of scenarios that need to be extracted (recall (2.20)) and the bigger the volume of the resulting set. A fair
compromise is to select the set to be a hyper-rectangle, parameterized by \( p_{\min}, p_{\max} \in \mathbb{R}^N \). The elements of the parameter vector represent element-wise bounds for each component of \( \delta_t \). Hence, this set can be defined by \( \Delta_t = \{ \delta_t \in \mathbb{R}^N | p_{\min} \leq \delta_t \leq p_{\max} \} \).

The chance constrained problem is then defined as

\[
\min_{p_{\min}, p_{\max}} \| p_{\max} - p_{\min} \|_1 ,
\] (2.21)

subject to

\[
P(\delta_t \in \mathbb{R}^{N_w+1} | p_{\min} \leq \delta_t \leq p_{\max}) \geq 1 - \varepsilon_t .
\] (2.22)

By minimizing the first norm of the interval lengths \( p_{\max} - p_{\min} \) which contain every uncertainty element \( \delta_t \), we minimize the volume of \( \Delta_t \). This problem is convex by construction with respect to the parametrization vectors \( p_{\min}, p_{\max} \) and we are thus able to apply the scenario approach to solve it. Denote then by \( \Delta_t^N \) the optimal solution of the scenario program that corresponds to (2.21)-(2.22), where the superscript \( N \) is included to emphasize that the resulting solution depends on \( N \) scenarios of the uncertainty vector \( \delta \). To ensure that \( \Delta_t^N \) contains at least \( 1 - \varepsilon_t \) of the probability mass of the uncertainty with confidence at least \( 1 - \beta_t \) for some \( \beta_t \in (0,1) \), it suffices to generate

\[
N \geq \frac{1}{\varepsilon_t} \frac{e}{e - 1} \left( \ln \frac{1}{\beta_t} + 2N_\delta - 1 \right) ,
\] (2.23)

scenarios of the uncertainty. Note that (2.23) is effectively invoking (2.20) with \( 2N_\delta \) in place of \( N_x \), since we have \( 2N_\delta \) decision variables in (2.21)-(2.22).

**Step 2: Robust problem**

In the second step, we use the probabilistically computed set \( \Delta_t^N \) and formulate a robust problem where the uncertainty is confined in this set. The resulting optimizing problem is given by (2.16)-(2.18) and the chance constraint (2.19) is substituted by the following robust constraint

\[
F_p x_t + f_p + g(\delta_t) \leq 0 , \text{ for all } \delta_t \in \Delta_t^N .
\] (2.24)

The interpretation of (2.24) is that the constraint should be satisfied for all values of \( \delta_t \in \Delta_t^N \). Following [44], any feasible solution of this robust
problem is feasible for the chance constraint \(2.19\) with confidence at least \(1 - \beta_t\). This confidence arises from the fact that \(\Delta_t\) is chosen in a probabilistic way. To solve the resulting robust program the reader is referred to \([7]\), \([44]\).

Note that the constraints in \(2.19\) exhibit a specific structure that can be exploited to achieve a computationally simpler problem. Specifically, the uncertainty appears only in the terms \(g(\delta_t)\) that are additive in the constraint functions and no bilinearities between the uncertainty elements and the decision variables exist. Therefore, it suffices to calculate off-line the maximum values of the elements of \(g(\delta_t)\) as \(\delta_t\) varies within \(\bar{\Delta}_t^N\) and replace with it the term \(g(\delta_t)\) in the constraints. Specifically, \(g(\delta_t)\) is substituted with a vector whose elements are calculated as \(\max_s g_k(\delta_t(s))\), where \(\delta_t(s), s = \{1, \ldots, 2^{N_w}\}\), denotes the uncertainty element that correspond to the vertices of the computed set \(\bar{\Delta}_t^N\). Subscript \(k\) denotes the \(k^{th}\) element of the vector \(g(\delta_t)\). The resulting problem is of the same size as its deterministic counterpart. This is also the case even if the standard scenario approach is employed. Since in both cases (scenario approach, probabilistically robust design) the complexity of the resulting problem is the same, for the simulation study of the next section we use the probabilistically robust design since it requires generating fewer scenarios. This follows from \(2.20\), \(2.23\) since in our case study \(2N_w \leq N_x\).

### 2.5 Wind power modeling

We assume that the wind power at every time instance is the sum of the forecasted wind power, as this was disclosed to the operator at the market clearing time in a day-ahead context, and a stochastic component. To model the latter one and generate different wind error realizations, motivated by \([58]\), we use a Markov chain mechanism. We used normalized hourly measured wind power data, both forecasts and actual values, for the total wind power infeed of Germany over the period 2006-2011. Following \([58]\), we discretized the error between the forecast and the actual data to “train” the transition probability matrix, which enables us to generate various wind power error realizations.

Fig. 2.1 shows the resulting transition probability matrix, when a 41-state Markov chain is constructed. As also stated in \([58]\), the block
Figure 2.1: Top view of the transition probability matrix for the wind power error, using a 41-state Markov chain. The color coding denotes the associated transition probability; “red” corresponds to high, whereas “blue” to low probability values.

Figure 2.2: Forecast (“green”) and actual (“red”) wind power, and 10,000 wind power scenarios (“blue”) based on different error realizations, initialized with the actual wind power.

The triangular structure of the state transition matrix, suggests that the wind power error is strongly correlated in time. For a single day of the
2.6 Simulation results

In this section we evaluate the performance of our approach by applying it to a modified version of the IEEE 30-bus network [88], which includes a wind power generator connected at bus 22. A single line diagram of the system is given in Fig. 3, whereas all numerical values for the network data and the cost vectors are retrieved from [88]. It includes $N_b = 30$ buses, $N_G = 6$ generators, $N_l = 41$ lines and $N_w = 1$ wind power generator. To quantify in terms of probability the improvement afforded by the proposed method, we compare it with what we will refer to as a benchmark approach. The benchmark approach involves solving a deterministic variant of the problem defined in Section 2.3, where only the day-ahead forecast is considered. The system operator would then run a standard N-1 security routine solving (2.11)-(2.14) (i.e. without the probabilistic constraints).
Figure 2.4:  a) Four different daily load profiles. b) Forecast wind power ("green"), and the different error realizations that were used to construct the robust generation dispatch c) Probability of insecure instances, estimated by evaluating the generation dispatch, generated by solving a deterministic problem considering only the forecast, against 10,000 wind power realizations. The color coding corresponds to the different load profiles of Fig. 2.4a. d) Probability of insecure instances, estimated by evaluating the robust generation dispatch, generated by our proposed methodology, against 10,000 wind power realizations.
2.6. Simulation results

For the robust implementation of our algorithm the stochastic program (2.11)-(2.15) was solved using the scenario approach, with $\varepsilon_t = 0.1$ and $\beta_t = 10^{-4}$ for all $t = 1, \ldots, N_t$. To compute the numerical solution of the problem, both for the probabilistically robust day-ahead dispatch and the benchmark approach, the solver CPLEX [39] was used via the MATLAB interface YALMIP [43].

![Graph showing number of insecure hours per day for forecast-based and proposed robust dispatch](image.png)

Figure 2.5: Number of insecure hours per day for the forecast-based and the proposed robust dispatch, encountered by the contingency analysis using the actual wind power scenarios for 90 days.

We used the forecast of a single day from the wind power dataset, depicted (together with the 200 scenarios, generated by our Markov chain model and initialized 16 hours ahead at the market clearing time) in Fig. 2.4b. As shown in Fig. 2.4a, four different profiles, representing how the total load in the system changes over the day, are considered. For each case in Fig. 2.4a, both the benchmark forecast-based approach and our robust technique are applied. Each dispatch was then tested against 10,000 wind power realizations (not including the scenarios used for the optimization process), representing the potential “actual” wind infeed, and the number of insecure incidents was recorded. With the term insecurity we refer to the case where after the contingency analysis that was performed, one or more of the lines got overloaded (i.e. the deterministic version of constraint (2.15), for $P_{w,t}$ being the “actual” wind power, was violated), for at least one of the $|\mathcal{T}|$ outages.
Fig. 2.4c,d show the probability of insecure cases, computed as the ratio between the number of identified insecurities over the 10,000 test cases. By inspection of Fig. 2.4c,d, we can easily deduce that the forecast-based dispatch leads to significantly more line overloads compared to the proposed robust solution. It is also apparent that the magnitude of the load plays an important role on the frequency of security critical cases. In general, at the time of the day where the load is high, the lines operate closer to their limits, and hence it is more likely to end up with a constraint violation in our contingency check. Note that the load profile $L_1$ (“red”) is high over the entire day, leading to an increased number of insecure encounters. From a network perspective, the contingency analysis revealed that lines $10 \rightarrow 22$, $21 \rightarrow 22$, $22 \rightarrow 24$ are the most critical ones in terms of security. Note that bus 22 is the one where the wind power generator is connected to.

It should be also mentioned, that the robust dispatch leads to a much lower number of constraint violations compared with its theoretical guarantees, confirming that the bound on the number of necessary scenarios proposed by [16] is indeed conservative. Nevertheless, in case tighter guarantees are required, $\varepsilon_t$ could be reduced resulting in a higher value for $N$, without an unaffordable increase in the computational overhead.

To further test our algorithm, we considered the load profile $L_2$, and applied both our robust approach and the benchmark method to compute the generation dispatch for 90 different days of the 2007 data. In a post-processing phase, a contingency analysis was carried out, using the actual wind realization (extracted from the real data) for each day. The obtained results are depicted in Fig. 2.5. The “red” lines corresponds to the case where the forecast-based approach is used, and shows the total number of hours per day where the system is not N-1 secure. In contrast to our method, which achieves secure operation for all days, the forecast-based approach leads very frequently to insecure incidents. It should be noted that the cases where a low number of insecurities is recorded corresponds to situations where the forecasted wind power is very close to its actual value. Day 26 of Fig. 2.5, corresponds to the day analyzed in Fig. 2.4 for different load profiles. The average value of line overloads per day was 2% of the line capacity limits, high enough to require a preventive action by the system operator. As expected, the daily cost of the proposed solution is higher than the benchmark one, due to presence of additional constraints. However, a maximum differ-
ence of 0.6% was encountered, implying that only small adjustments were needed to design a secure day-ahead dispatch. This additional cost could be thought of as a price to pay for security in networks with high penetration of renewable energy sources (in addition to the cost of reserves).

2.7 Concluding remarks

In this chapter a new methodology for generating a probabilistically robust generation dispatch so as to ensure that the system is N-1 secure under wind uncertainty, is proposed. To obtain a solution to this problem, a chance constrained optimization program was developed, and was solved using a sampling based methodology with guaranteed performance. The efficiency of the proposed scheme was evaluated in terms of Monte Carlo simulations on the IEEE 30-bus network, and was compared against the solution corresponding to the deterministic variant of the problem.

In this work, no reserve scheduling was considered, and it was assumed that the reserves are bounded by the generating capacity of each unit. Moreover, the load was assumed to follow a fixed profile and no load uncertainty was considered. These issues are addressed in Chapters 3 and 4, respectively. It should be also noted, that increasing the wind power penetration in the network will lead to feasibility problems. Our proposed framework could be used though to provide an indication of the maximum possible wind power infeed, or as a guideline for where to place wind generation or new lines to achieve a secure system. Results toward this direction are provided in Chapter 5. The analysis of this chapter was based on a DC power flow set-up. The AC counterpart of the proposed methodology is provided in Chapter 6.
Chapter 3

Generation-side reserve scheduling

We propose a probabilistic framework to design an N-1 secure day-ahead dispatch while determining the minimum cost reserves for power systems with wind power generation. We also identify a reserve strategy according to which we can deploy the reserves in real time operation, that serves as a corrective control action. To construct a reserve decision scheme, we take into account the steady state behavior of the secondary frequency controller, and hence consider the deployed reserves to be a piecewise linear function of the total generation-load mismatch. We extend the formulation of Chapter 2 and additionally optimize over the coefficients of this function. The chance constraints encode now the probability of satisfying not only the transmission capacity constraints of the lines and the generation limits, but also the reserve capacity limits. To achieve tractability we propose a convex reformulation and a heuristic algorithm, whereas to deal with the chance constraint we use a scenario based technique. To quantify the effectiveness of the proposed methodologies and compare them in terms of cost and performance we use the IEEE 30-bus network and carry out Monte Carlo simulations.
3.1 Introduction

The expected increase in the installed wind power capacity and other fluctuating power sources as well (e.g. photovoltaic power), highlights the necessity of revisiting certain operational concepts, like N-1 security and reserve scheduling. Chapter 2 provided a framework to guarantee an optimal N-1 secure operation with the desired probability taking into account the wind power forecast uncertainty. In this framework, the power required to balance the system is compensated by each generator with a fixed percentage (i.e fixed distribution vector) and hence the reserves of each generator are then determined by the worst-case value of the power mismatch. This implies that we can calculate the required reserves that the systems operator needs to purchase via our probabilistic approach but we do not optimally distribute them to the generating units. Extending Chapter 2 to optimally allocate the reserve requirements to the generators is the objective of the current chapter.

In current electricity markets, there are different market products aiming at minimizing the generation dispatch and the reserve costs, while satisfying the network constraints. The energy market determines the optimal generation dispatch, the transmission market takes into account network constraints and determines changes in the generation dispatch so as to ensure the network security (e.g. the N-1 security criterion), and the reserve market determines the optimal allocation of a usually predetermined reserve capacity to the generating units. These markets are procured either sequentially (in the so called unbundled market systems) or in the same optimization problem (in the so called integrated market systems) [67]. Reference [81] evaluates the efficiency and the incentives of both systems. In practice, the sequential approach is the most applied one. However, from an optimization point of view, this gives a suboptimal solution of the overall problem and could even result in feasibility issues for the last problem. For instance, if the reserve schedule is first determined without taking into account the N-1 criterion and all the reserves are allocated to the cheapest generator, then there is no feasible solution for an N-1 secure energy scheduling if this generator is tripped, since no other unit could provide the reserves that are required to compensate its production. This is an extreme example and usually heuristics are used to take care of such pronounced issues, however, it highlights that the reserves may not be adequately in unbundled market systems. Hence, an integrated market mechanism enables
us to identify the optimal solution of the overall problem. Toward this direction, in this chapter we present a framework dealing with the co-optimization of energy and reserves while taking network constraints and the N-1 security criterion into account.

Toward maximizing the expected social welfare, optimization of reserve power has been addressed in [28], [12], [12], [11], in a security constrained market clearing context. Using the same framework, [55], [60] formulated a multi-stage stochastic unit commitment program, modeling the uncertain generation by means of scenarios and using reduction techniques to ensure tractability of the problem. However, these approaches do not offer a priori guarantees regarding the reliability of the resulting solution.

In this chapter, we propose a unified framework that simultaneously solves the problem of designing an N-1 secure day-ahead dispatch for the generating units, while determining the minimum cost reserves and the optimal way to deploy them. To account for the variability of wind power we follow a probabilistic methodology, providing certain guarantees regarding the satisfaction of the system constraints. We first integrate, as in Chapter 2, the security constraints emanating from the N-1 criterion to a DC optimal power flow problem and formulate a stochastic optimization problem with chance constraints. Modeling the steady state behavior of the secondary frequency controller leads to a representation of the reserves as a linear function of the total generation-load mismatch, which may be due either to the difference between the actual wind and its forecast, or to a generator/load loss. We introduce different ways to distribute reserves based on the type of mismatch, thus offering an implementation of corrective security. Therefore, the overall formulation includes both preventive and corrective control [64]. The generation dispatch and the reserve capacity determination constitute preventive control actions, whereas the case dependent strategy according to which we deploy reserves in real time operation falls in the framework of corrective control. Apart from the physical intuition, using such a strategy for the reserves has the advantage that the number of decision variables does not grow with the number of uncertainty realizations as in [55] and the resulting solution is less conservative compared with [45]. This makes our method applicable even for large scale networks. After this research was first published, similar ideas concerning the generation dispatch strategy were discussed in [8], [80].

Taking into account the security constraints, our resulting problem is
a chance constrained, bilinear program. To achieve tractability, the issues arising due to the bilinear terms and the presence of the chance constraint need to be resolved. To alleviate these difficulties we propose a heuristic algorithm and a convex reformulation, as well as two alternative techniques to deal with the chance constraint. The effectiveness of the proposed methodologies is illustrated by means of Monte Carlo simulations for a modified version of the IEEE 30-bus network [88].

In Section 3.2 we discuss how to represent reserves in an uncertain environment, whereas in Section 3.3 we formulate the security constrained reserve scheduling problem as a chance constrained optimization program. Section 3.4 provides details on how to deal with the bilinearity problem and the chance constraint and Section 3.5 illustrates the performance of the proposed approaches via a simulation study. Finally, in Section 3.6 we provide some directions for future work. References [75], [73], [46] are related to this chapter.

3.2 Problem set-up and reserve representation

As in the previous chapter, we consider a power network comprising $N_G$ generating units, $N_w$ wind power plants, $N_L$ loads, $N_l$ lines, and $N_b$ buses. Recall also that $\mathcal{I}$ is a set that includes the indices corresponding to outages of all components including also the index “0” that corresponds to the base case of no outage, and denote by $|\mathcal{I}|$ its cardinality. Moreover, denote by $\mathcal{I}_l, \mathcal{I}_L, \mathcal{I}_G$ the set of indices corresponding to branch, load and generator outages.

The problem formulation of this chapter is based on the following assumptions:

1. A standard DC power flow approximation is used.
2. Perfect load forecasts are considered.
3. Line outages do not lead to multiple generator/load failures.
4. The “on-off” status of the generating units has been fixed a-priori by solving a unit-commitment problem.
3.2. Problem set-up and reserve representation

The first assumption is standard for this type of problems. The second and third one are included to simplify the presentation of our results and could still be captured by the proposed algorithm. Removing the last assumption by incorporating the unit commitment problem would give rise to a mixed-integer problem; this can be tackled using the “probabilistically robust design” [44] that can deal with a certain class of non-convex problems (see also Section 2.4).

Reserves are needed to balance generation-load mismatches, which may occur due to a difference between the actual wind power and its forecast, or as an effect of a generator/load loss. Such mismatches between load and generation induce frequency deviations and activate the primary and secondary frequency controller (by means of Automatic Generation Control - AGC). Here we assume an ideal primary frequency control functionality compensating for any fast time scale power deviation and focus only on the steady state behavior of the AGC actions (hence on the secondary frequency control reserves). The AGC output is distributed to certain participating generators, whose setpoint is changed by a certain percentage of the active power to be compensated. In Section 2.2.2 we refer to the vector that contains the percentage weights as the distribution vector. The product of these percentage weights with the worst case imbalance results in the amount of reserves that each generating unit has to provide.

The existing set-up of the AGC loop is shown in Fig. 3.1 in the dashed box, demonstrating the role of the distribution vector. In current practice this vector results from the market that determines the secondary frequency control reserves and remains constant until the next market auction. Typically, this task is performed without taking the network constraints into account. Moreover, the distribution vector may differ between up-spinning and down-spinning reserves, but is the same for all possible outages. In this paper, in view of a corrective security control scheme, apart from distinguishing between up-spinning and down-spinning reserves, we also consider different distribution vectors depending on the outage. Optimizing then over the distribution vectors we determine an optimal reserve schedule, while taking the network security constraints into account. Our approach enables us to compute simultaneously both the minimum cost reserves per generator, and also a reserve strategy that can be deployed in real time operation. This strategy consists in using the distribution vectors, which depending on the outage and the wind power deviation dictate the amount of power
by which each generating unit should adjust its production. Therefore, the proposed methodology serves as an alternative to other methods for reserve scheduling, e.g. [55], [60], which account implicitly for real time response via their day-ahead decisions.

To encode the proposed reserve representation, we define a power correction term $R^i \in \mathbb{R}^{N_G}$ as a piecewise linear function of the total generation-load mismatch. This term is directly related to the reserves since it shows the amount of the power that each generator should adjust its production given an imbalance.

$$R^i(P_w) = d^i_{up} \max_+ (-P^i_m(P_w)) - d^i_{down} \max_+ (P^i_m(P_w)), \quad i \in \mathcal{I} \quad (3.1)$$

where $\max_+ (\cdot) = \max (\cdot, 0)$. Variable $P^i_m \in \mathbb{R}$ denotes the generation-load mismatch, which for each outage is given by

$$P^i_m(P_w) = \sum_{k \in \mathcal{I}_w} (P_{w,k} - P^f_{w,k}) - c^i_L P_L + c^i_G P_G + c^i_w P_w, \text{ for all } i \in \mathcal{I}. \quad (3.2)$$

All undefined parameters and their interpretation are discussed below (2.9) in Section 2.2.2.

Vectors $d^i_{up}, d^i_{down} \in \mathbb{R}^{N_G}$ represent the distribution vectors. The sum of their elements is equal to one and, if a generator is not contributing to the AGC, the corresponding element in the vector is zero. The indices “up” and “down” are used to distinguish between up-spinning reserves and down-spinning reserves.

If $P^i_m$ is negative, up-spinning reserves are provided and the production of the generators is increased accordingly, whereas in the opposite case the second term of (3.1) is active and hence down-spinning reserves are provided. Notice that $d^i_{up}, d^i_{down}$, may have negative elements as well. Consider for example the base case where we have no outages, the power mismatch is negative ($P^0_m < 0$) and some elements of $d^0_{up}$ are negative as well. This corresponds to a set-up where the network is congested. The interpretation of some elements of $d^0_{up}$ being negative is that the corresponding generators should provide down-spinning reserves so that congestion is relieved, while the rest of the units would provide up-spinning reserves.
3.3 Probabilistic security constrained reserve scheduling

Figure 3.1: Schematic diagram illustrating the AGC functionality required for the security constrained reserve scheduling algorithm.

3.3 Probabilistic security constrained reserve scheduling

We consider an optimization horizon $N_t = 24$ with hourly steps $^1$, and introduce the subscript $t$ to indicate the value of the quantities for a given time instance $t = 1, \ldots, T$. As in [88], we consider a quadratic form for the production cost and a linear cost for the reserves. Similarly to Section 2.3, let $c_1, c_2, c_{up}, c_{down} \in \mathbb{R}^{N_G}$ be generation and reserve cost vectors and let $[c_2]$ denote a diagonal matrix with vector $c_2$ on the diagonal.

For each step $t$ define the vector of decision variables to be $x_t = [P_{G,t}, [d^i_{up,t}, d^i_{down,t}, R^i_{up,t}, R^i_{down,t}]_{i \in \mathcal{I}}]_T \in \mathbb{R}^{N_G+4N_G(|\mathcal{I}|+1)}$, where $R^i_{up,t}, R^i_{down,t} \in \mathbb{R}^{N_G}$ denote the probabilistically worst-case up-down spinning reserves that the system operator needs to purchase for every

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$^1$Here we implicitly assume an ideal primary frequency control performance, capturing the fast time scale wind power variability. This is standard practice for day-ahead planning problems of this type [11], [55], [60].
$i \in I$. The resulting optimization problem is

$$\min_{\{x_t\}_{t=1}^{N_t}} \sum_{t=1}^{T} \left( c_1^T P_{G,t} + P_{G,t}^T [c_2] P_{G,t} \right.$$  

$$+ \sum_{i \in I} \left( c_{up}^T R_{up,t}^i + c_{down}^T R_{down,t}^i \right) \right), \quad (3.3)$$

subject to

**Deterministic constraints**

These constraints correspond to the case where the wind power is equal to its forecast. In this case, the reserves are determined by the generation-load mismatch that may occur due to an outage.

1. Forecast power balance constraints: For all $t = 1, \ldots, N_t$,

$$1^T (C_G P_{G,t} + C_w P_{w,t}^f - C_L P_{L,t}) = 0. \quad (3.4)$$

This constraint encodes the fact that the power balance in the network should always be satisfied when $P_{w,t} = P_{w,t}^f$.

2. Generation and transmission capacity constraints: For all $t = 1, \ldots, N_t$ and $i \in I$

$$-P_f^i \leq A_i^j P_{in,j,t}(P_{w,t}^f) \leq P_f^i \quad (3.5)$$

$$P_G^i \leq P_{G,t}^i + R_t^i (P_{w,t}^f) \leq \overline{P}_G^i \quad (3.6)$$

The generation capacity constraint (3.5) ensures that the scheduled generation dispatch plus the power correction term $R_t^i$ will not result in a new operating point outside the generation capacity limits.

3. Reserve capacity constraints: For all $t = 1, \ldots, N_t$ and $i \in I$

$$-R_{down,t}^i \leq R_t^i (P_{w,t}^f) \leq R_{up,t}^i, \quad (3.8)$$

$$R_{up,t}^i, R_{down,t}^i \geq 0 \quad (3.9)$$
4. Distribution vector constraints: For all \( t = 1, \ldots, N_t \) and \( i \in \mathcal{I} \)
\[
1^T d_{\text{up},t}^i = 1, 1^T d_{\text{down},t}^i = 1,
\]
(3.10)
For \( i \in \mathcal{I}_G \), the element of \( d_{\text{up},t}^i, d_{\text{down},t}^i \) corresponding to the tripped generator is equal to zero. Constraints (3.10) encode the fact that the elements of the distribution vectors should sum to one.

Probabilistic constraints
These constraints deal with the uncertainty of the wind power forecast. The reserves are characterized now by the generation-load mismatch that may occur due to the wind power forecast error and the outages. We thus have that for all \( t = 1, \ldots, N_t \),
\[
\mathbb{P}\left( P_{w,t} \in \mathbb{R} \mid -P_f^i \leq A^i P_{\text{inj},t}(P_{w,t}) \leq P_f^i \right)
\]
\[
P_G^i \leq P_{G,t}^i + R_t^i(P_{w,t}) \leq P_G^i
\]
\[-R_{\text{down},t}^i \leq R_t^i(P_{w,t}) \leq R_{\text{up},t}^i, \text{ for all } i \in \mathcal{I} \right) \geq 1 - \varepsilon_t,
\]
(3.11)
where the probability is meant with respect to the probability distribution of the wind power vector \( P_{w,t} \in \mathbb{R}^{N_w} \).

The last constraint in (3.11) is included to determine the reserves \( R_{\text{up},t}^i, R_{\text{down},t}^i \) as the worst case, in a probabilistic sense, value of the power correction term \( R_t^i \). The reserves that the system operator will need to purchase are then determined as
\[
R_{\text{up},t} = \max_{i \in \mathcal{I}} R_{\text{up},t}^i,
\]
(3.12)
\[
R_{\text{down},t} = \max_{i \in \mathcal{I}} R_{\text{down},t}^i,
\]
(3.13)
which denote the worst case values, given all the outages, of \( R_{\text{up},t}^i \) and \( R_{\text{down},t}^i \), respectively. Note that in (3.11) we considered the same probability level \( \varepsilon \) for each time-step \( t = 1, \ldots, N_t \); however, different probability levels per stage or a joint chance constraint for all stages could be captured by the proposed framework as well.

Following this formulation we propose an additional AGC functionality. The operator of the system needs to monitor both the production of
the tripped plant and the deviation of the wind power from its forecast, and using (3.1) as a look-up table, select the appropriate distribution vector, among those computed in the optimization problem (see Fig. 3.1).

The resulting problem (3.3)-(3.11) is a chance constrained bilinear program whose stages are only coupled due to the temporal correlation of the wind power. We could have a further coupling among the stages if a unit commitment problem was included or if ramping constraints of the generating units and minimum up and down times were modeled. The reader is referred to [46] for a set-up where all of these constraints are included. There are two main challenges when attempting to solve problem (3.3)-(3.11). The first arises from the presence of bilinear terms due to the products of $d_{i,up,t}^k, d_{i,down,t}^k$ and $P_{G,t}^i$ for $i \in I_G$, whereas the second is due to the presence of the chance constraint. These issues are addressed in the next section.

3.4 Tractable problem reformulations

3.4.1 Method 1: Heuristic algorithm

We propose here a method based on an iterative algorithm (Algorithm 1) to deal with the bilinear terms. The algorithm consists of two parts. In the first part, we attempt to identify a feasible solution to the problem starting from an arbitrarily chosen power schedule $P_{G,t}^0$. At iteration $k$ of the algorithm, we fix $P_{G,t}^{k,i}$ only in (3.1) to the value obtained in the previous iteration. Therefore, $R^i$ is still a function of the distribution vectors and the production, but this time the value of the power production term is fixed to $P_{G,t}^0$ to avoid the presence of bilinear terms. Solving then (3.3)-(3.11) a new solution $x_t^k$ is computed and $P_{G,t}^k$ is updated accordingly. If the algorithm converges, its fixed point $x_t^{k^*}$ will be a feasible solution.

At a second step, we use an iterative scheme to refine the resulting feasible solution in terms of cost. At iteration $k$ we first fix $d_{i,up,t}^{k,i}, d_{i,down,t}^{k,i}$ to the values obtained at the previous step of the algorithm only.

\footnote{For the simulation study of Section 3.5 we used $P_{G,t}^0 = 0$; we have tested other initial values as well and in all cases the algorithm converged to the same solution.}
3.4. Tractable problem reformulations

Figure 3.2: Illustration of Algorithm 1 for one hour of the simulated data, initialized with $P_{G,i,t}^0 = 0$. For the first part, the power dispatch of each unit and the obtained objective value converge after 3 iterations, whereas for the second only 1 iteration is needed.

for $i \in \mathcal{I}_G$ and obtain $[P_{G,t}^k, [d_{up,t}^{k,i}, d_{down,t}^{k,i}]_{i \in \mathcal{I} \setminus \mathcal{I}_G}, [R_{up,t}^{k,i}, R_{down,t}^{k,i}]_{i \in \mathcal{I}}]^T$ by solving (3.3)-(3.11). We then fix $P_{G,t}^k$ to the computed value in all equations it appears and solve for the decision vector $[d_{up,t}^{k,i}, d_{down,t}^{k,i}, R_{up,t}^{k,i}, R_{down,t}^{k,i}]_{i \in \mathcal{I}}]^T$. The entire process is then repeated until convergence. Note that the first part of Algorithm 1 is a heuristic scheme applied to identify a feasible solution and no convergence guarantees can be provided. The second part of the algorithm converges monotonically (this is not necessarily the case for the first part), since it is a bilinear descent iteration; however, the limit point is not guaranteed to be the global optimum of the original bilinear problem.

Fig. 3.2 shows how the power dispatch of each unit and the obtained objective value change per iteration for the benchmark problem introduced in the next section. After 3 iterations the first part converges, whereas for the second only one iteration is needed. As expected, the cost decreases monotonically in the second part.
Algorithm 1

1: Initialization – Part 1.
2: Set $P_{G,t}^0$ (e.g. $P_{G,t}^0 = 0$),
3: $k = 1$.
4: Repeat until convergence
5: Set $P_{G,t}^{k,i} = P_{G,t}^{k-1,i}$, $\forall i \in \mathcal{I}_G$, only in (3.1),
6: Compute $x_i^k$ solving (3.3)-(3.11),
7: Update $P_{G,t}^k$,
8: $k = k + 1$.
9: end
10: Return converged solution $x_i^{k^*}$
11: Initialization – Part 2.
12: Set $d_{up,t}^{0,i} = d_{up,t}^{k^*,i}$, $d_{down,t}^{0,i} = d_{down,t}^{k^*,i}$, $\forall i \in \mathcal{I}_G$,
13: $k = 1$.
14: Repeat until convergence
15: Set $d_{up,t}^{k,i} = d_{up,t}^{k-1,i}$, $d_{down,t}^{k,i} = d_{down,t}^{k-1,i}$, $\forall i \in \mathcal{I}_G$, in (3.1),
16: Compute $[P_{G,t}^k, [d_{up,t}^{k,i}, d_{down,t}^{k,i}]_{i \in \mathcal{I} \setminus \mathcal{I}_G}, [R_{up,t}^{k,i}, R_{down,t}^{k,i}]_{i \in \mathcal{I}}]^T$ solving (3.3)-(3.11),
17: Fix $P_{G,t}^k$ in (3.11),
18: Compute $[d_{up,t}^{k,i}, d_{down,t}^{k,i}, R_{up,t}^{k,i}, R_{down,t}^{k,i}]_{i \in \mathcal{I}}^T$ solving (3.3)-(3.11),
19: $k = k + 1$.
20: end

3.4.2 Method 2: Convex reformulation

Assume that in the case where $i \in \mathcal{I}_G$ we can distinguish between the mismatch that corresponds to wind deviation and the one that occurs due to a generator outage by introducing different distribution vectors. For $i \in \mathcal{I}$, the power correction term would now be

$$ R_i^t = d_{up,t}^{1,i} \max(P_{w,t}^f - P_{w,t}) - d_{down,t}^{1,i} \max(P_{w,t} - P_{w,t}^f) + d_{up,t}^{2,i} P_{G,t}^i. $$

(3.14)

By considering the optimization problem that corresponds to (3.3)-(3.11) if the additional distribution vectors are introduced, $d_{up,t}^{2,i} P_{G,t}^i$ becomes the only bilinear term, which appears both
in the constraints and the objective function. Setting \( z^i_t = d_{up,t}^2 P^i_{G,t} \in \mathbb{R}^{N_G} \) and defining the new decision vector \( \tilde{x}_t = [P_{G,t}, [d_{up,t}^i, d_{down,t}^i], i \in \mathcal{I}_G; [d_{up,t}^i, d_{down,t}^i, z^i_t], i \in \mathcal{I}_G; [R^i_{up,t}, R^i_{down,t}], i \in \mathcal{I}_T] \in \mathbb{R}^{N_G+N_G^2+4N_G(|\mathcal{I}|+1)} \), the resulting problem is linear in \( z^i_t \) and hence convex. It is of the same structure as (3.3)-(3.11) with the additional constraint

\[ 1^T z^i_t = P^i_{G,t}, \text{ for all } i \in \mathcal{I}_G. \] (3.15)

Once the solution to this problem is computed, for all \( i \in \mathcal{I}_G \), \( d_{up,t}^2 \), is calculated as \( d_{up,t}^2 = z^i_t / P^i_{G,t} \) if \( P^i_{G,t} \) is not equal to zero and is set to zero otherwise. Note that the sum of the elements of \( d_{up,t}^2 \) is constrained to be one, since \( z^i_t, i \in \mathcal{I}_G \) satisfies (3.15). For real time operation, the look-up table interpretation (discussed in Section 3.4) may be used. Then, given a mismatch \( P_{m,t}^i = (P_{w,t} - P_{w,t}^f) - P^i_{G,t} \), the participation of each unit in compensating \( P_{m,t}^i \) can be computed as \( R^i_t / 1^T R^i_t \). Note that following this procedure we convexify the bilinear terms inside the probability. The overall problem is still non-convex, however, due to the presence of the chance constraint. In Section 3.5.4 we elaborate on how to deal with this issue.

### 3.4.3 Probabilistic guarantees

The initial chance constrained problem (3.3)-(3.11) includes non-convex constraints due to the presence of bilinear terms. Concerning method 1, however, at every iteration the problem that needs to be solved is convex. Hence, we can employ any of the sampling based techniques of Section 2.4 at every iteration of the heuristic algorithm. The resulting solution (if convergence at the first part of Algorithm 1 occurs) will be feasible by construction for the bilinear problem. Therefore, for each \( t = 1, \ldots, N_t \), we can maintain the desired probabilistic guarantees that the resulting optimal solution is feasible, with certain confidence (denoted by \( \beta_t \)), for the initial chance constrained problem (3.4)-(3.11).

On the other hand, using Method 2 results in a problem where the constraints inside the chance constraints are convex by construction. Therefore, after applying Method 2, for each \( t = 1, \ldots, N_t \), the chance constrained problem would be of the form of (2.17)-(2.19), with the difference that the elements of \( F_p \) in (2.19) would depend now on the
uncertainty $\delta_t$. To solve this problem, we can follow either of the sampling based techniques described in Section 2.4. However, this enables us to claim that, with confidence at least $1 - \beta_t$, the obtained solution will be feasible for the problem where the constraints inside the probability have been convexified by applying Method 2, and not necessarily for the bilinear chance constrained problem (3.3)-(3.11). In the next section we show that under certain conditions, using method 2 in conjunction with the “probabilistically robust design”, we can obtain probabilistic guarantees regarding the satisfaction of the constraints of (3.3)-(3.11) as well.

3.4.4 Discussion on method 2

In this section we focus on a specific set-up where the the uncertainty at every stage of the optimization problem is scalar. We will show how method 2 can be used in conjunction with the “probabilistically robust design” of Section 2.4 to provide feasibility guarantees for problem (3.3)-(3.11). Consider a specific time instance $t$. Since the “probabilistically robust design” does not require convexity of the underlying problem with respect to the decision variables, with confidence of at least $1 - \beta_t$, the optimal solution of the robust counterpart of (3.3)-(3.11) is feasible for the initial chance constrained problem. Note that for each optimization stage the robust version of (3.3)-(3.11) has interval bounded uncertainty and bilinear constraints and denote by $J^*$ the optimal cost corresponding to this problem. We will show how to construct the optimal solution of this problem so as to obtain the desired probabilistic guarantees.

Using method 2, the robust counterpart of (3.3)-(3.11) can be transformed to a robust problem with linear constraints. Denote $\tilde{J}^*$ as the optimal cost of the resulting robust problem. Proposition 1 below shows that for this particular set-up the costs $J^*$ and $\tilde{J}^*$ are equal and an optimal solution that corresponds to $\tilde{J}^*$ can be used to construct an optimal solution corresponding to $J^*$. This implies that it suffices to compute a solution that leads to $\tilde{J}^*$ (this is a robust convex problem and can be solved using the approach of [7]) and use it to determine an optimal solution of the robust version of (3.3)-(3.11), thus inheriting the probabilistic guarantees of the scenario approach.
Proposition 1. If the uncertainty at every stage of (3.3)-(3.11) is scalar, then \( J^* = \tilde{J}^* \). Moreover, a solution that corresponds to \( \tilde{J}^* \) can be used to construct a solution that corresponds to \( J^* \).

Proof. Part 1: \( J^* \geq \tilde{J}^* \). For every outage \( i \in \mathcal{I}_G \), method 2 introduces different distribution vectors \( d_{up,t}^{1,i}, d_{up,t}^{2,i} \) to distinguish between the mismatch which occurs due to wind deviation and the one due to a generator outage. Therefore, the set-up of problem (3.3)-(3.11) is a special case of method 2, corresponding to the situation where for \( i \in \mathcal{I}_G \), \( d_{up,t}^{1,i} = d_{up,t}^{2,i} \). The latter implies that by construction the cost of the solution obtained by method 2 is never higher compared to the one of (3.3)-(3.11) since we have more degrees of freedom.

Part 2: \( J^* \leq \tilde{J}^* \). Since the stages are decoupled we can focus on specific time instance \( t \). Consider the terms \( P_{G,t}, [d_{up,t}^i, d_{down,t}^i]_{i \in \mathcal{I} \setminus \mathcal{I}_G}, [R_{up,t}^i]_{i \in \mathcal{I}}, [R_{down,t}^i]_{i \in \mathcal{I}} \) of the optimal solution of method 2. It suffices to show that for all \( i \in \mathcal{I}_G \) there exist vectors \( d_{up}^i, d_{down}^i \in \mathbb{R}^{NG} \), which together with these terms, constitute a feasible solution of the robust counterpart of (3.3)-(3.11). This solution would satisfy all the constraints in (3.11) for all values of the uncertainty inside the interval (scalar uncertainty was assumed) where it is confined. Due to linearity of the constraints with respect to the uncertainty, it suffices to check feasibility only for the extreme values of this interval, which is denoted here by \([\underline{P}^w,t, \overline{P}^w,t]\). We show this here only for the case where \( \underline{P}^w,t \leq P_{w,t}^f \) that results in determining \([d_{up}^i]_{i \in \mathcal{I}_G}\), whereas the proof for the other case is similar. Moreover, the cost of the constructed solution will be equal to \( \tilde{J}^* \) since the cost depends only on \( P_{G,t} \) and \([R_{up,t}^i, R_{down,t}^i]_{i \in \mathcal{I}}\). The claim follows then directly, since we will have identified a feasible but not necessarily optimal solution for (3.3)-(3.11).

To construct such a feasible solution notice that in all constraints the distribution vectors appear through the term \( R_i^t \). Therefore, consider \([d_{up}^i]_{i \in \mathcal{I}_G}\) such that the power correction term of the bilinear problem (see (3.1) for \( \underline{P}_{w,t} \leq P_{w,t}^f \)) is equal to the one obtained by method 2, i.e. for all \( i \in \mathcal{I}_G \)

\[
d_{up}^i (P_{w,t}^f - \underline{P}_{w,t} + P_{G,t}^i) = d_{up,t}^{1,i} (\underline{P}_{w,t} - \underline{P}_{w,t}) + d_{up,t}^{2,i} P_{G,t}^i.
\] (3.16)
Chapter 3. Generation-side reserve scheduling

If in addition $1^T d_{up}^i = 1$, then all other constraints of (3.3)-(3.11) will be trivially satisfied, since they are the same with those of method 2. By multiplying both sides of (3.16) with $1^T$, we get that $1^T d_{up}^i (P_{w,t}^f - P_{w,t}^k + P_{G,t}^i) = 1^T d_{up}^{1,i}(P_{w,t}^f - P_{w,t}^k) + 1^T d_{up}^{2,i} P_{G,t}^i$. Since $z_t^i = d_{up}^{2,i} P_{G,t}^i$, the last statement is equivalent to $1^T d_{up}^i (P_{w,t}^f - P_{w,t}^k + P_{G,t}^i) = 1^T d_{up}^{1,i}(P_{w,t}^f - P_{w,t}^k) + 1^T z_t^i$. We have, however, that $1^T d_{up}^{1,i} = 1$ and $1^T z_t^i = P_{G,t}^i$. Hence, $1^T d_{up}^i = 1$ concluding the proof.

Note that the assumption of scalar uncertainty was used at the second part of the proof, since by checking only the two extreme values of the interval bounded uncertainty we were able to have a unique map from $d_{up}^{1,i}, d_{up}^{2,i}$ to $d_{up}^i, d_{down}^i, i \in I_G$. In case the uncertainty is of higher dimension this proof is not always valid, unless we introduce additional distribution vectors for every vertex of the uncertainty set.

3.5 Case study

3.5.1 Simulation set-up

To generate scenarios for the wind power error, we employed the Markov chain based model of Section 2.5. To evaluate the performance of our approach we applied it to the IEEE 30-bus network [88], which includes $N_b = 30$ buses, $N_G = 6$ generators, $N_l = 41$ lines, and is modified to include a wind power generator connected to bus 22. All numerical values for the network data are retrieved from [88].

For all simulations we used $\epsilon_t = 10^{-1}$ and $\beta_t = 10^{-4}$ for all $t = 1, \ldots, N_t$. Note that based on used the sampling based technique of Section 2.4 we will choose, the number of scenarios will depend either on the number of decision variables or the number of uncertainty variables. However, according to the method used to deal with the bilinearity issue, the resulting problem will have a different number of decision variables, whereas the number of uncertainty variables remains the same. In order to perform a fair comparison between the two reformulations we selected for this case study the probabilistically robust design of Section 2.4.2. To generate scenarios we used the Markov chain based model of Section 2.5.
3.5. Case study

To collect statistical results regarding the performance of our algorithm, we carried out a Monte Carlo study, evaluating the solution of (3.3)-(3.11) (reformulated based on the proposed alternatives, i.e. method 1 and 2) against 10,000 wind power realizations, not included in the optimization process. Using the obtained reserve strategy (i.e. the power correction term (3.1) with the distribution vectors fixed according to the outcome of the optimization problem and the wind power equal to the evaluation scenario), for each of these realizations we examined whether the problem constraints are satisfied. Note that since we examine the feasibility of all constraints, all possible outages are taken into account in the evaluation phase.

Since we perform a probabilistic design, applying our reserve strategy still allows for constraint violation but with a certain probability. By constraint violation we mean the case where the wind power realization used to evaluate our solution leads to a power mismatch for which at least one of the constraints is violated. Such a violation does not necessarily correspond to the base case but to some $i \in I$. In case we violate the constraints and end up with an excess of power, we refer to the maximal such amount as power surplus, which corresponds to a potential wind power curtailment action. In the opposite case we use the term power deficit to characterize the amount of power that may not be covered by the scheduled up-spinning reserves.

In the realistic set-up of an interconnected system a fraction of this amount would be provided by the primary frequency reserves of neighboring areas (assuming the primary reserves of our area are also at saturation). If these primary reserves are not sufficient to cover the power deficit load shedding will occur. Following its definition, if no power deficit is encountered the system will always be N-1 secure.

Note that for the simulation study of the next section we differentiate among the distribution vectors based on the sign of the wind power error and the possible generator outage, thus having the same vector for all line and load outages. Our choice is motivated by a desire to minimize the number of decision variables (and hence the computational cost) in the optimization problem.

All optimization problems were solved using the solver CPLEX [39] via the MATLAB interface YALMIP [43].
3.5.2 Simulation results

We first investigate the performance of methods 1 and 2 when applying the “probabilistically robust design” for one day of our data. Fig. 3.3(a) shows the forecast (“blue”) and the actual (“red”) wind power, the wind power scenarios (“green”) that were used for the scenario based technique (generated according to (2.23) with $N_\delta = 1$ since we have one wind power generator). Fig. 3.4 provides statistical information regarding the performance of our methods using 10,000 wind power realizations, that were used for evaluation purposes and their span corresponds to the shaded region of Fig. 3.3(a).

Fig. 3.3(b) shows the scheduled cost (production + reserves) for the convex reformulation (“dashed” line) and the heuristic algorithm (“solid” line). Both methods lead to similar cost. For comparison purposes we solved the nonlinear problem (2.23)-(3.11) directly using the nonlinear solver IPOPT. In all cases the resulting solution led in slightly lower cost values compared to method 1, with a maximum difference of 1%. Therefore, method 1 provides a reliable alternative to more direct schemes based on nonlinear solvers, since it leads to a similar cost while involving the solution of a sequence of convex problems. Method 2 leads to slightly lower cost compared to the nonlinear solver (maximum difference of at most 1%). This is due to the fact that our set-up satisfies the requirements of Section 3.5.4, hence Method 2 provides an exact convex reformulation of the bilinear problem. This implies that these problems have the same optimal objective values, however, we do not have guarantees that the bilinear one (solved using the nonlinear solver) can be solved up to optimality. In the general case, where the requirements of Proposition 1 are not satisfied, Method 2 will not necessarily outperform the solution of the nonlinear solver in terms of cost since it will only be a convex relaxation of the bilinear problem. However, since Method 2 leads to a convex problem the computational cost will be much lower.

Method 1 and 2 lead to different distribution of the reserves among the various generators and to different total amount of reserves in general. Therefore, the amount of power deficit or surplus differs according to whether method 1 or 2 is employed. Figs. 3.4(a),(c) show the power surplus and power deficit when method 2 is used. The results for method 1 are similar and are omitted in the interest of space. However, in the boxplots, the “red” line corresponds to the median value, the edges of the
3.5. Case study

Figure 3.3: (a) Wind power for one day of the simulated data. Forecast (“blue”), actual (“red”), scenarios used for methods 1 and 2 (“green”), and the span of the 10,000 wind power realizations used for evaluation purposes (shaded region). (b) Total scheduled cost (production + reserves) for method 1 (“solid” line) and method 2 (“dashed” line). (c) Power deficit for method 1 (“solid” line) and method 2 (“dashed” line).

the probability of constraint violation (power deficit or surplus) depends solely on the wind power used for evaluation purposes. To see this, notice that the left or right hand-side inequalities of the last constraint inside (3.11) will always hold with equality for at least one of its rows.

box correspond to the 25th and 75th percentiles, whereas the whiskers extend to a 99% coverage. The “red” marks denote the data outliers, which lie outside the 99% confidence region.
Figure 3.4: Power deficit and surplus using the scenario approach, for one day of the simulated data, evaluated with 10,000 wind power realizations ($\varepsilon_t = 10^{-1}$ and $\beta_t = 10^{-4}$). (a) Power surplus for method 2. (b) Probability of power surplus. (c) Power deficit for method 2. (d) Probability of power deficit.
3.5. Case study

Figure 3.5: Distribution of the percentage of cost improvement, using the scenario approach, and applying method 1 against method 2 for 30 days of hourly measured data.

(possibly different rows for methods 1 and 2). From the definition of the power correction term, this implies that any wind power realization outside the span of the scenarios used for the scenario based technique will result in violating these specific constraints. Therefore, the distributions shown in Figs. 3.4(b),(d) are exactly the same both for method 1 and 2. The probabilities of these figures are calculated as the fraction of the 10,000 evaluation scenarios that resulted in power surplus and deficit, respectively. The empirical probability of constraint violation is determined by the sum of the individual probabilities of power surplus and deficit.

Fig. 3.5 depicts the distribution of the percentage of cost improvement, using method 1 against method 2 for 30 days of hourly measured data. Method 2 systematically leads to lower cost compared with method 1 since, as discussed above, it falls in the framework of Appendix A. However, we can not generalize this pattern if the set-up of Appendix A is not satisfied. The total amount of power deficit (surplus) for the 30 days, evaluated with the actual wind power realizations, was 37 (3) MW for method 1 and 43 (3) MW for method 2.
3.6 Concluding remarks

In this chapter a new methodology for solving security constrained reserve scheduling problems for systems with fluctuating generation is presented. Moreover, a corrective security control scheme consisting of a reserve strategy that could be applied in real time operation is introduced, and different alternatives to deal with the resulting chance constrained bilinear problem are proposed.

Despite the fact that we focus on secondary frequency control reserves, the proposed methodology could also be applied to determine the primary and tertiary reserves. Such an implementation is currently under investigation. Moreover, we focus on decentralizing the developed algorithm and on exploiting the work of [42], [83], by including a convex AC optimal power flow relaxation. Another issue is to investigate the validity of our approach for alternative market structures [68] and compare it with other benchmark methods.
Chapter 4

Exploiting uncertain reserves from demand response

Demand response (DR) can provide reserves in power systems but a fundamental challenge is that the amount of capacity available from DR is time-varying and uncertain. We propose a stochastic optimal power flow (OPF) formulation that handles uncertain energy from wind power and uncertain reserves provided by DR. To handle the uncertainty, we formulate a chance constrained optimization program and use a sampling based technique to solve it. This technique allows us to provide a-priori guarantees regarding the probability of constraint satisfaction. Additionally, we devise a strategy for the reserves, provided either by the generators or the loads, that could be deployed in real time operation. To evaluate the effectiveness of our methodology, we carry out a simulation-based analysis on the IEEE 30-bus network. Our case studies show that optimizing over the reserves provided by DR results in lower total cost compared to the case where only generation side reserves are taken into account. We also carry out a Monte Carlo analysis to empirically estimate the probability of constraint satisfaction and demonstrate that it is within the theoretical limits.
Chapter 4. Uncertain Reserves from Demand Response

4.1 Introduction

Reserve capacity is procured in electricity markets to ensure balance between supply and demand at each instance in time, given uncertain consumption, production, and events such as component outages. To schedule energy and reserves using optimal power flow (OPF) formulations, we generally assume that scheduled reserves are perfectly certain, i.e. that they will be available in real time if we need them. Some formulations take into account cases in which generators may be unable to provide scheduled reserves, for example, due to contingencies [11], [75]. However, new reserve resources, such as energy storage and demand response (DR), introduce different types of uncertainty than those considered in previous studies.

Research suggests that storage and DR could provide reserves in power systems [19, 35, 31]; however, the amount of reserves available may be time-varying and uncertain. This is especially true for DR in which both the load and the flexible portion of it may be a function of human behavior, ambient conditions such as weather, and past DR actions [47]. Therefore, for many types of loads, reserve capacity is difficult to estimate in real-time, and even harder to forecast because its based on other forecasted quantities. Moreover, it is often necessary to aggregate thousands of loads together to provide system-level reserves [18, 49] and at these scales it may be impossible to keep track of the time-varying capabilities and constraints of each load. Instead, we can use aggregated system models to approximate reserve capacity [49]; however, the mismatch between these models and the real system is another cause of uncertainty [47].

To mitigate reserve uncertainty, a DR aggregator could be conservative in offering reserves to power systems, making the reserves “practically certain”. A better option may be to explicitly take reserve uncertainty into account in our planning algorithms, which should allow us to leverage more of the available resource. In this chapter, we propose a stochastic OPF approach that allows us to consider uncertain reserves from DR. We model aggregations of DR resources as time-varying virtual energy storage units, and therefore must consider intertemporal energy constraints in the optimization problem. We assume DR could be used for both day-ahead hourly power scheduling and reserve scheduling. We formulate the problem as a probabilistic DC OPF with chance constraints and use a sampling based methodology [16], [44] to solve it. This ap-
4.1. Introduction

This work builds on earlier research results that provide methods to handle storage and uncertainty in OPF formulations. The authors in [29] formulated and proposed solution strategies for an OPF with distributed storage. The storage power and energy capacities are modeled as time-invariant, while in our paper we allow them to be time-varying. Several researchers have considered the problem of uncertain energy, for example, from wind power plants [60, 74, 75]. References [57, 30, 59] use DR as reserves in a stochastic OPF context with uncertain wind energy; however, the DR reserves are assumed certain. In [78] a formulation that allows uncertain aggregations of electric water heaters to provide a fixed amount of reserves is proposed. However, our formulation determines the optimal amount of uncertain reserves within an OPF.

The main objective of this chapter is to formulate a multi-stage day-ahead probabilistic DC OPF that optimizes for uncertain reserves from DR and certain reserves from conventional generating units. More specifically, our formulation

1. results in the optimal reserve capacity offered by generators and controllable loads,

2. offers a strategy for reserve deployment in real time operation,

3. optimizes over tertiary reserve power that will relieve secondary frequency control reserves and bring the energy state of the controllable loads back to the scheduled value, and

4. provides a-priori guarantees that the proposed solution will be reliable with a certain confidence, without requiring knowledge of the underlying distribution of the uncertainty.

To evaluate the effectiveness of our methodology, we carry out a simulation-based analysis on the IEEE 30-bus network [88]. This allows us to assess the costs associated with three OPF formulations in which we assume i) deterministic loads, ii) uncertain loads, and iii) uncertain but controllable loads that may be used for hourly scheduling and reserves. We also carry out a Monte Carlo analysis to empirically estimate the probability of constraint satisfaction.
The rest of the chapter is organized as follows. In the next section, we
detail the modeling of DR resources as uncertain reserves and describe
the power flow assumptions. In Section 4.4, we describe the stochastic
optimization problem and discuss the scenario-based methodology we
use to solve the resulting OPF. Section 4.5 provides case studies, and in
Section 4.6 we provide concluding remarks and discuss future research.
Reference [77] are related to this chapter.

4.2 Demand response resource modeling

We assume that loads can shift their consumption in time but that
the total amount of energy delivered to the load over a longer period
of time is fixed. Therefore, we model aggregations of loads as virtual
energy storage units [48]. Actions which decrease consumption relative
to the baseline consumption (i.e. the consumption that would have
occurred without scheduling) empty the storage unit and actions with
increase consumption relative to the baseline consumption charge the
storage unit. Therefore, the energy state $S$ of the aggregation evolves
as

\[ S_{t+1} = S_t + (P_{C,t} - P_{T,t})\Delta \tau, \]

where $P_{C,t}$ is the mean power consumption of the controllable portion
of the load at time step $t$ and also the optimization variable, $P_{T,t}$ is the
baseline consumption, and $\Delta \tau$ is the length of the time step.

Because the amount of controllable load within the system varies as
a function of time, the size of the virtual energy storage unit is time-
varying. Specifically, a unit’s power and energy capacity are a function
of a variety of time-dependent quantities such as ambient conditions
and human behavior. Therefore, both $P$ and $S$ are constrained by
time-varying quantities

\[ P_{C,t} \leq P_{C,t} \leq \overline{P}_{C,t}, \]
\[ 0 \leq S_t \leq \overline{S}_t, \]

where $P_{C,t}$, $\overline{P}_{C,t}$ is the aggregate upper and lower power capacity limit
and $\overline{S}_t$ is the aggregate energy capacity limit. Reference [48] describes
a method of computing these capacities and $P_T$ for an aggregation of
residential electric space heaters or air conditioners as a function of
4.2. Demand response resource modeling

outdoor air temperature \( T_t \). Here, we use this method to compute \( P_C(T_t), S(T_t) \) and \( P_T(T_t) \) for an aggregation of 1,000 heterogenous electric space heaters, as shown in Fig. 4.1. We assume \( P_C(T_t) = 0 \) for all \( T_t \).

There are many reasons why \( P_C, S, \) and \( P_T \) may be uncertain, including model error and forecasting error \([47]\). Here, we do not consider all sources of uncertainty but instead focus on just one cause: temperature forecasting error. Specifically, we assume that the values in Fig. 4.1 are accurate for a given outdoor air temperature but that our forecasts of outdoor air temperature are uncertain. Given a specific forecast of outdoor air temperature, we can use Fig. 4.1 as a look-up table to determine the expected power and energy capacity of a virtual storage unit for planning; however, the actual outdoor air temperature will dictate the actual capacities available in real-time. Note that even though we only consider this source of uncertainty in this work, our OPF formulation is also applicable to cases where we consider any and all sources of DR uncertainty.

Figure 4.1: The power and energy capacity of an aggregation of electric heaters modeled as a virtual storage unit.
4.3 Problem set-up

In our framework, we use a DC OPF for day-ahead scheduling of hourly generator power set points $P_G$, controllable load set points $P_C$, secondary frequency control capacities from both generators $R_{GS}$ and controllable loads $R_{LS}$, and intra-hour re-dispatch capacities (e.g., tertiary control in Europe or intra-hour markets in the U.S.) from generators $R_{GD}$. Under the DC power flow assumption, the power flows across the lines are given by $P_l = A P_{inj}$, where $P_{inj} \in \mathbb{R}^{N_b}$ is the net power injection at the buses and $A$ is a constant matrix that depends on the network admittances (see also Section 2.2.1 for a detailed derivation). We assume that both wind power production $P_w$ and the controllable portion of the load (both its baseline and power/energy capacity, as defined in the last subsection) are uncertain. We always explicitly handle wind stochasticity within our probabilistic OPF formulation but we compare cases in which we do and do not handle load uncertainty and controllability. Specifically, we investigate three cases: i) deterministic loads, ii) uncertain loads, and iii) uncertain but controllable loads that can be used for both hourly scheduling and reserve provision.

We assume that for each hour the intra-hour re-dispatch is activated four times, i.e. every fifteen minutes. Each 15-minute interval, the re-dispatch provides the amount of energy that would be required to return the controllable loads to the scheduled energy state (determined by the power set-point) if the secondary frequency control signal were zero over that interval, which is similar to the method proposed by the California Independent System Operator [17]. Additionally, in each 15-minute interval, the re-dispatch covers the power mismatch between the expected generation and the actual generation. Fig. 4.2 shows the action of the secondary frequency controllers and the re-dispatch for a given wind forecast error.

We do not consider security constraints in this chapter so as to demonstrate in a simpler way the new concept of the load-side reserve scheduling. However, following the methodology of Chapters 2 and 3, this framework can be extended to capture security constraints as well. Therefore, here, secondary frequency control and re-dispatch are needed only to manage wind forecast errors. We assume reserve capacities are constant over one hour, and we size the secondary frequency control reserve capacity to cover a maximum wind power deviation over a period of 15 minutes. We size the intra-hour re-dispatch capacity to cover both
4.3. Problem set-up

Figure 4.2: An example wind forecast error (top plot) and the corresponding actions of the secondary frequency controllers (second and third plot, with the fourth plot showing the energy state evolution of the loads used for secondary control) and re-dispatch (last plot), within one hour. The blue lines show the maximum wind forecast error and responses, which we plan for within the OPF. The red lines show a realistic wind forecast error and the responses. The secondary frequency controllers balance high frequency deviations while the re-dispatch is only able to produce constant outputs in each interval.
the energy required by the loads and the wind power deviations.

### 4.4 Problem formulation

#### 4.4.1 Optimization problem

In this section, we formulate an OPF which considers uncertain but controllable loads, i.e. case iii) listed in the introduction. Note that the other cases we use for comparison in the case study, i.e. i) deterministic and uncontrollable loads and ii) uncertain and uncontrollable loads, are special cases of this formulation. The objective of the optimization problem is to determine the minimum cost generation dispatch, controllable load schedules, and reserve capacities provided by both generators and reserves.

We consider a power network of $N_G$ generating units, $N_w$ wind power plants, $N_L$ loads, $N_l$ lines, and $N_b$ buses. Each load is comprised of an uncontrollable portion $P_L$, which is assumed known, and a controllable portion $P_C$, which is uncertain as described in Section 4.2. We consider an optimization horizon $N_t = 24$ with hourly steps (i.e. $\Delta \tau = 1$) and introduce the subscript $t$ in our notation to characterize the value of the corresponding quantities for a given time instance $t = 1, \ldots, N_t$. For each step $t$ we define the vector of decision variables to be

$$x_t = [P_{G,t}, P_{C,t}, R_{GS,t}^{up}, R_{GS,t}^{dn}, R_{LS,t}^{up}, R_{LS,t}^{dn}, R_{GD,t}^{up}, R_{GD,t}^{dn}, R_{GD,t0}^{up}, R_{GD,t0}^{dn}, R_{GD,t1}^{up}, R_{GD,t1}^{dn}, R_{GD,t2}^{up}, R_{GD,t2}^{dn}, d_{GD,t}^{up}, d_{GD,t}^{dn}]^T \in \mathbb{R}^{15 N_G + 5 N_L}.$$

The vectors $d$ are distribution vectors that distribute the generation-load mismatch to the resources that offer reserve capacity. The superscripts “up/dn” denote the increase/decrease of the produced or the consumed power of the generators or the loads, respectively. Hence, the up-regulating secondary reserves are characterized by the distribution vectors $d_{GS,t}^{up}$ and $d_{LS,t}^{dn}$ since the generators have to increase power while the loads have to decrease power. The total reserve capacity for up-regulating secondary reserves, for a given time step $t$, is given by $R_{GS,t}^{up} + R_{LS,t}^{dn}$. For the re-dispatch we use three sets of distribution...
4.4. Problem formulation

vectors denoted by the superscripts “0,1,2”, which are associated with energy mismatches from the previous hour, intra-hour wind power mismatch and intra-hour energy mismatches, respectively. More details on these decision variables will be given in the following paragraphs.

Let $c_1$, $c_2$ be cost vectors as defined in Section 2.3, whereas $c_{GS,up}$, $c_{GS,dn}$, $c_{LS,up}$, $c_{LS,dn}$, $c_{GD,up}$, $c_{GD,dn}$ are reserve cost vectors. The optimization problem is given by

$$\min_{\{x_t\}_{t=1}^{N_t}} \sum_{t=1}^{N_t} \left( c_1^T P_{G,t} + P_{G,t}^T [c_2] P_{G,t} ight. \
+ c_{GS,up}^T R_{GS,t}^{up} + c_{GS,dn}^T R_{GS,t}^{dn} \
+ c_{LS,up}^T R_{LS,t}^{up} + c_{LS,dn}^T R_{LS,t}^{dn} \
+ c_{GD,up}^T R_{GD,t}^{up} + c_{GD,dn}^T R_{GD,t}^{dn} \left. \right)$$

subject to

**Deterministic constraints:** All constraints presented below correspond to the forecast values, denoted with the superscript $f$, of the wind power and temperature.

1. Power constraints: For all $t = 1, \ldots, N_t$,

   $$\mathbf{1}_{1 \times N_t} P_{inj,t} = 0,$$  
   $$-P_l \leq A P_{inj,t} \leq P_l,$$  
   $$P_G \leq P_{G,t} \leq P_C(T_t^f),$$

where

$$P_{inj,t} = C_G P_{G,t} + C_w P_{w,t}^f - C_L(P_L,t + P_C,t)$$

and the $C$ matrices map the generator, wind, and load power vectors to the vector of bus injections. Constraint (4.5) guarantees power balance in the network, whereas (4.6), (4.7) encode the line and generation capacity limits, respectively. Constraint (4.8) imposes limits on the dispatch of the controllable portion of the load.
2. Energy Constraints: For all $t = 1, \ldots, N_t$,

$$0 \leq S_t \leq \mathcal{S}(T^f_t).$$

(4.10)

For all $t = 1, \ldots, N_t - 1$,

$$0 \leq S_{t+1} \leq \mathcal{S}(T^f_t),$$

(4.11)

$$S_{t+1} = S_t + (P_{C,t} - P_T(T^f_t)) \Delta \tau.$$  \hspace{1cm} (4.12)

Equation (4.12) shows the evolution of the energy state. Constraints (4.10), (4.11) are energy state capacity limits. They restrict the energy content at the beginning and at the end of hour $t$ to lie within the energy state capacity limits of the specific hour. Due to the linearity of (4.12), requiring $S_{\tau}$ to satisfy the energy state limits for $\tau = t, t + 1$ ensures that $S_{\tau}$ satisfies the energy limits for all $\tau \in [t, t + 1]$.

**Probabilistic constraints:** For every $t = 1, \ldots, N_t$, the following constraints depend on the uncertainties, i.e. the wind power $P_{w,t}$ and the outdoor temperature $T_t$, and should be satisfied with probability at least $1 - \varepsilon_t$, where $\varepsilon_r \in (0, 1)$. We split the constraints into two parts. In part a), the wind power forecast error is compensated by the secondary frequency control offered both by the generators and DR for a period of at most 15 minutes, since re-dispatch occurs every 15 minutes. In part b), the constraints impose limitations on the operating point after a re-dispatch action is performed.

a) Secondary frequency control constraints:

1. Power constraints: For all $t = 1, \ldots, N_t$,

$$-P_t \leq A P_{inj,t} \leq P_t,$$  \hspace{1cm} (4.13)

$$P_G \leq P_{G,t} + R_{GS,t} \leq \overline{P}_G,$$  \hspace{1cm} (4.14)

$$P_{C}(T_t) \leq P_{C,t} + R_{LS,t} \leq \overline{P}_{C}(T_t),$$  \hspace{1cm} (4.15)

$$-R_{dn GS,t} \leq R_{GS,t} \leq R_{up GS,t},$$  \hspace{1cm} (4.16)

$$-R_{dn LS,t} \leq R_{LS,t} \leq R_{up LS,t},$$  \hspace{1cm} (4.17)

$$R_{dn GS,t}, R_{up GS,t}, R_{dn LS,t}, R_{up LS,t} \geq 0.$$  \hspace{1cm} (4.18)
4.4. Problem formulation

\begin{align*}
1_{1 \times N_G} d_{GS,t}^{up} + 1_{1 \times N_L} d_{LS,t}^{dn} &= 1, \quad (4.19) \\
1_{1 \times N_G} d_{GS,t}^{dn} + 1_{1 \times N_L} d_{LS,t}^{up} &= 1, \quad (4.20)
\end{align*}

where

\begin{align*}
P_{inj,t} &= C_G(P_{G,t} + R_{GS,t}) + C_w P_{w,t} \\
&\quad - C_L(P_{L,t} + P_{C,t} + R_{LS,t}), \quad (4.21)
\end{align*}

\begin{align*}
R_{GS,t} &= d_{GS,t}^{up} \max(-P_{m,t}, 0) \\
&\quad - d_{GS,t}^{dn} \max(P_{m,t}, 0), \quad (4.22)
\end{align*}

\begin{align*}
R_{LS,t} &= d_{LS,t}^{up} \max(P_{m,t}, 0) \\
&\quad - d_{LS,t}^{dn} \max(-P_{m,t}, 0), \quad (4.23)
\end{align*}

\begin{align*}
P_{m,t} &= 1_{1 \times N_w} (P_{w,t} - P_{w,t}^f) \\
&\quad - 1_{1 \times N_L} (P_T(T_t) - P_T(T_t^f)). \quad (4.24)
\end{align*}

Constraints (4.13)-(4.15) are similar to the deterministic constraints (4.6)-(4.8), with the difference being that due to the uncertainty, the generation and load schedules are adjusted by the power correction terms $R_{GS,t}$ and $R_{LS,t}$, respectively. With constraints (4.16)-(4.17), we determine the probabilistically worst case values of the power correction terms (given in (4.22)-(4.23)), which represent the reserves that are penalized in the objective function.

Following Section 3.1, the power correction terms $R_{GS,t}, R_{LS,t}$ in (4.22), (4.23) are defined as piece-wise linear functions of the total mismatch between the generation and load. In our case, the mismatch is defined as the total wind power forecast error plus the total load forecast error (4.24). For example, for a negative wind power error only one of the terms in (4.22), (4.23) will be active, providing up-regulating reserves. The distribution vectors that are incorporated in this part of the formulation are $d_{GS,t}^{up}, d_{GS,t}^{dn}, d_{LS,t}^{up}$ and $d_{LS,t}^{dn}$. These vectors denote the fraction of the mismatch by which the generating units and the loads...
Figure 4.3: Evolution of the energy state of the loads. The solid lines correspond to the energy state trajectory for the case where the wind power is equal to its forecast (no reserves are needed in this case). Note that the energy state is not constant because the controllable load is dispatched above/below its baseline. The dashed lines show the evolution of the energy state for 15 minutes given that the maximum possible reserve capacity is deployed. The red lines show the capacity limits of each hour. In this paper, we assume that the lower energy bound is zero, but in principle that does not always have to be the case.

should adjust their production and consumption. Since the total mismatch is distributed both to the generating units and the loads, the sum of the elements of the corresponding up-regulating (respectively down-regulating) distribution vectors should be one. This is captured by (4.19), (4.20).

2. Energy Constraints: For all $t = 1, \ldots, N_t$,

\begin{align*}
0 \leq S_t + (P_{C,t} + R_{LS,t} - P_T(T_t)) \frac{\Delta \tau}{4} & \leq \bar{S}(T_t), & (4.25) \\
0 \leq S_t + (P_{C,t} - P_T(T_t)) \frac{3\Delta \tau}{4} \\
+ (P_{C,t} + R_{LS,t} - P_T(T_t)) \frac{\Delta \tau}{4} & \leq \bar{S}(T_t). & (4.26)
\end{align*}
For all $t = 1, \ldots, N_t - 1$,

$$0 \leq S_t + (P_{C,t} - P_T(T_t)) \frac{3\Delta\tau}{4} + (P_{C,t} + R_{LS,t} - P_T(T_t)) \frac{\Delta\tau}{4} \leq S(T_{t+1}).$$  \hfill (4.27)

Constraints (4.25)-(4.27) are sufficient conditions that ensure that the energy state remains within the desired limits regardless of the time instance within $[t, t + 1]$ when reserves are called upon. Due to the linear structure of the energy state dynamics, it suffices to satisfy the energy constraints for the first and last quarter of the hour. This can be also observed by inspection of Fig. 4.3. The solid lines correspond to the energy state trajectory for the case where the wind power is equal to its forecast (no reserves are needed in this case). The dashed lines show the evolution of the energy state for 15 minutes given that the maximum possible reserve capacity is deployed. The red lines denote the capacity limits of each hour.

b) Re-dispatch constraints:

1. Power constraints: For all $t = 1, \ldots, N_t$,

$$-P_l \leq AP_{\text{inj},t} \leq P_l,$$  \hfill (4.28)

$$P_G \leq P_{G,t} + R_{GD,t} \leq \overline{P}_G,$$  \hfill (4.29)

$$P_{C}(T_t) \leq P_{C,t} - R_{LS,t} \leq \overline{P}_C(T_t),$$  \hfill (4.30)

$$-R_{dn_{GD,t}} \leq R_{GD,t} \leq R_{up_{GD,t}},$$  \hfill (4.31)

$$R_{dn_{GD,t}}, R_{up_{GD,t}} \geq 0,$$  \hfill (4.32)

$$1 \times N_G d_{uG_{GD,t}}^{up,1} = 1,$$  \hfill (4.33)

$$1 \times N_G d_{uG_{GD,t}}^{dn,1} = 1,$$  \hfill (4.34)

$$1 \times N_G d_{uG_{GD,t}}^{up,2} = 1 \times N_L d_{LS,t}^{dn},$$  \hfill (4.35)

$$1 \times N_G d_{uG_{GD,t}}^{dn,2} = 1 \times N_L d_{LS,t}^{up},$$  \hfill (4.36)

where

$$P_{\text{inj}} = C_G(P_{G,t} + R_{GD,t})$$

$$-C_L(P_{L,t} + P_{C,t} - R_{LS,t}) + C_w P_{w,t},$$  \hfill (4.37)
\[ R_{GD,t} = d_{GD,t}^{up,1} \max(-P_{m,t}, 0) - d_{GD,t}^{dn,1} \max(P_{m,t}, 0) \]

\[ + d_{GD,t}^{up,2} \max(-P_{m,t}, 0) - d_{GD,t}^{dn,2} \max(P_{m,t}, 0). \quad (4.38) \]

The constraints above ensure that the intra-hour re-dispatch satisfies both the network constraints and the load power limits. Note that the load set point \( P_{C,t} - R_{LS,t} \) has an opposite term for the reserves compared to (4.15), which implies that the energy state returns to its scheduled trajectory (solid line in Fig. 4.3), thus satisfying the energy limits as well. Recall that the intra-hour re-dispatch capacity should cover both the energy required by the loads and the wind power deviations; this is captured in (4.33)-(4.36), (4.38). Specifically, the terms with the superscript “1” compensate the wind power error, whereas the terms with the superscript “2” compensate the energy required by the loads for providing secondary frequency control.

2. Coupling constraints: If at the end of an hour the energy state of the load has not returned to its scheduled value (determined by the power setpoint), the re-dispatch action of the following hour should cover this remaining energy in the first quarter. To capture this, for \( t = 2, \ldots, N_t \) we impose constraints (4.29)-(4.32), (4.37) with \( R_{GD,t}^0 + R_{GS,t} \) in place of \( R_{GD,t} \), and require

\[ -R_{LS,t}^{dn} \leq R_{LS,t} - R_{LS,t-1} \leq R_{LS,t}^{up}. \]

Moreover, we require the following power and energy constraints

\[ 0 \leq P_{C,t} - R_{LS,t-1} \leq \bar{P}_C(T_t), \quad (4.39) \]

\[ 0 \leq S_t + (P_{C,t} - P_T(T_t)) \frac{\Delta \tau}{4} \leq \bar{S}(T_t), \quad (4.40) \]

where

\[ R_{GD,t}^0 = d_{GD,t}^{up,0} \max(-P_{m,t}, 0) \]

\[ - d_{GD,t}^{dn,0} \max(P_{m,t}, 0), \quad (4.41) \]

\[ 1_{1 \times N_G} d_{GD,t}^{up,0} = 1_{1 \times N_L} d_{LS,t-1}^{dn}, \quad (4.42) \]

\[ 1_{1 \times N_G} d_{GD,t}^{dn,0} = 1_{1 \times N_L} d_{LS,t-1}^{up}. \quad (4.43) \]
To facilitate the analysis of the next section, define \( x = \{ x_t \}_{t=1}^{N_t} \) to be a ‘stacked’ version of \( \{ x_t \}_{t=1}^{N_t} \) including all the decision variables, and let \( \delta_t \in \mathbb{R}^{N_w+1} \) denote the uncertainty in step \( t \), which here is the wind power \( P_{w,t} \in \mathbb{R}^{N_w} \) and the temperature \( T_t \). For every \( t = 2, \ldots, N_t \), we require the constraints that are affected either by \( \delta_t \) or by both \( \delta_t \) and \( \delta_{t-1} \) (for example, (4.39)) to be satisfied with probability at least \( 1 - \varepsilon_t \), where \( \varepsilon_t \) is a given violation level. Under this requirement, the aforementioned optimization problem can be formulated as a quadratic program with multiple chance constraints. Therefore, for every \( t = 2, \ldots, N_t \), the probabilistic constraints can be written in compact notation:

\[
P \left( (\delta_t, \delta_{t-1}) \in \mathbb{R}^{N_w+1} \times \mathbb{R}^{N_w+1} \mid F_p(\delta_t, \delta_{t-1}) x + f_p + g(\delta_t) \leq 0 \right) \geq 1 - \varepsilon_t, \tag{4.44}
\]

where all matrices and vectors are of appropriate dimension. Notice the similarities with (2.19); the basic difference however is that, due to the coupling constraints, the elements of \( F_p \) depend now on \( \delta_t, \delta_{t-1} \). For \( t = 1 \), the chance constraint is similar to (4.44), with the difference that \( F_p \) depends only on \( \delta_1 \) and the probability is with respect to the distribution of \( \delta_1 \in \mathbb{R}^{N_w+1} \). In the next section, we show how to solve this problem without introducing assumptions on the probability distribution of the uncertainty and while providing guarantees regarding the probability of constraint satisfaction.

### 4.4.2 Solution approach

The problem defined in the previous section is an optimization program with multiple chance constraints. In contrast to the previous chapters, due to the presence of coupling constraints we cannot decompose the problem in a family of \( N_t \) separate problems. To deal with the issue of multiple chance constraints we follow the “probabilistically robust design” [44] (see Section 2.4.2) and an extension of it as proposed in [85].

This method includes two steps as already described in 2.4. In the first step, for each \( t = 1, \ldots, N_t \), the scenario approach is used to determine, with a confidence at least \( 1 - \beta_t \), the minimum volume set that contains at least \( 1 - \varepsilon_t \) probability mass of the uncertainty. As in Chapter 2, we
denote this set by $\bar{\Delta}_t^N$. In the second step, we use the probabilistically computed set $\bar{\Delta}_t^N$ and formulate a robust problem where the uncertainty is confined in this set. Following the extension of this approach proposed in [85], for each $t = 2, \ldots, N$, the chance constraint (4.44) is substituted by the following robust constraint

$$F_p(\delta_t, \delta_{t-1})x + f_p + g(\delta_t) \leq 0, \text{ for all } (\delta_t, \delta_{t-1}) \in \bar{\Delta}_t^N. \quad (4.45)$$

The interpretation of (4.45) is that the constraint should be satisfied for all values of $(\delta_t, \delta_{t-1}) \in \bar{\Delta}_t^N$. For $t = 1$, the resulting constraint is similar with the difference that $F_p$ depends only on $\delta_1$ and we require the constraint to be satisfied for all $\delta_1 \in \bar{\Delta}_1^N$. Following [44], [85], any feasible solution satisfying the robust constraints (4.45) will be feasible for each chance constraint (4.44) with a probability of at least $1 - \beta_t$. To solve the resulting program standard techniques for robust optimization can be employed [7, 44].

4.5 Case Studies

4.5.1 Data and error scenario generation

Error scenarios are generated under the assumption that the wind power in-feed and the temperature are independent, which allows us to use different models for each uncertainty source.

Wind data

We use normalized forecasted and actual hourly wind power data for Germany over the period 2006-2011. To generate the appropriate number of wind power scenarios, we use the Markov chain mechanism described in [58] as further discussed in Section 2.5. In our formulation, we assume that the wind error can occur at any point during an hour, and persist for the rest of the hour, as shown in Fig. 4.2.

Temperature data

We generate temperature error scenarios using one year of forecasted and actual mean hourly temperature data from one weather station.
in Switzerland. Specifically, we generate 365 24-hour temperature error vectors and add these vectors to actual 24-hour temperature realizations to generate 24-hour temperature forecasts. This approach allows us to consider autocorrelation in the temperature errors over the course of the day; however, it does not allow us to take into account that the amount of error may be a function of the magnitude of the temperature.

We validate our approach using similar data from ten other weather stations in Switzerland. In Fig. 4.4, we plot histograms of the temperature errors in $\overline{P}_C$, $\overline{S}$, and $P_T$, generated with the data from all eleven weather stations. Note that the errors associated with $\overline{P}_C$ and $\overline{S}$ are highly non-Gaussian because of the shape of the curves in Fig. 4.1.
Table 4.1: Cost parameters.

<table>
<thead>
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<th>generator</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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</thead>
<tbody>
<tr>
<td>$c_{GS,up}$ (MU/MW)</td>
<td>6.00</td>
<td>6.75</td>
<td>7.00</td>
<td>5.25</td>
<td>5.00</td>
<td>5.00</td>
</tr>
<tr>
<td>$c_{GD,up}$ (MU/MW)</td>
<td>2.40</td>
<td>2.10</td>
<td>1.20</td>
<td>3.90</td>
<td>3.60</td>
<td>3.60</td>
</tr>
</tbody>
</table>

4.5.2 Simulation results

The methodology developed in the previous sections is applied to the IEEE 30-bus network [88], which is modified to include one wind power generator (i.e. $N_w = 1$) with capacity 35MW connected to bus 22. All loads were assumed partially controllable and therefore capable of being scheduled and providing reserves. Specifically, we assumed that over the course of a day 10% of the load, on average, is controllable and so we scaled the load of each hour accordingly. Values for the generation cost vectors can be found in [88]. Table 4.1 provides the values for the up-regulating related cost vectors. The cost for the reserves provided by the loads is equal to 1.1 for all loads. The cost vectors for the down-regulating reserves are the same as the corresponding up-regulating ones. For our simulations, we used $\varepsilon_t = 10\%$ and $\beta_t = 10^{-3}$ for all $t = 1, \ldots, N_t$. All optimization problems were solved using the solver CPLEX [39] via the MATLAB interface YALMIP [43].

Fig. 4.5 (upper plot) shows for each hour $t = 1, \ldots, N_t$ the maximum positive and negative wind power error computed from the scenarios, where the number of scenarios was determined based on Section 4.4.2. To compensate a negative wind power error, up-regulating reserves are required, whereas for a positive wind power error down-regulating reserves need to be purchased. As shown in the middle plot of Fig. 4.5, these reserves are provided either by the secondary reserves from generating units, secondary reserves from DR, or by generator re-dispatch. The total reserve cost for each case, calculated as the sum of the corresponding up and down-regulating reserves, is shown in the lower plot of Fig. 4.5. As shown, the loads, despite their uncertainty, are chosen preferentially over the generators to provide secondary frequency control because of our cost assumptions. However, the generators need to provide a substantial amount of re-dispatch capacity. This division of services makes sense as generators are better suited to slower changes in their set points while load aggregations are better suited to fast, zero-mean deviations.
Figure 4.5: Upper plot: Maximum positive (green) and negative (blue) wind power error. Middle plot: Amount of reserve power provided by the secondary reserves of the generating units, the secondary reserves of DR, and by the re-dispatch. Lower plot: Total reserve cost for each case, calculated as the sum of the corresponding up and down-regulating reserves.
Chapter 4. Uncertain Reserves from Demand Response

Fig. 4.6 (upper plot) shows the forecasted outdoor temperature as well as the temperature scenarios used in the optimization process. In the middle plot of Fig. 4.6, we show the optimal hourly schedule of the controllable portion of the load as computed by our algorithm, along with the forecasted consumption computed from the forecasted temperatures. Since the load’s power capacity changes as a function of outdoor temperature, we show values corresponding to the case when the temperature is equal to its forecast (solid red line) and the worse case error scenario (dashed red line). Note that when the temperature is low, the power capacity is high and nearly certain. The lower plot of Fig. 4.6 shows the evolution of the energy state of the load, and the interpretation of the lines is similar to that of the middle plot. As shown, the hourly schedule does not significantly deviate from the forecast; however, the additional power/energy capacity available can be used for secondary frequency control.

Fig. 4.7 shows the total cost generated by our approach, calculated as the sum of the production and reserve costs. For comparison purposes, we show the cost that would occur if the load was equal to its forecast value (deterministic) and no load controllability was taken into account. Additionally, we show the case where the load is not controllable, but is assumed to be uncertain (varying according to the scenarios of the upper plot of Fig. 4.6). This solution is higher cost than the deterministic case because we consider the load uncertainty within the OPF to ensure that the solution is robust. Note that the cost of the solution generated by our approach is always lower than the other values, highlighting the improvement resulting from incorporating load controllability in the reserve scheduling process.

To validate the guarantees offered by our approach regarding the probability of constraint satisfaction, we carried out a Monte Carlo analysis. Specifically, we fixed $x$ to the optimal solution of our optimization problem and for each $t = 1, \ldots, N_t$ we computed the empirical probability of constraint violation. The latter was calculated as the fraction of 4,000 evaluation scenarios (different from those used in the optimization process) where at least one of the constraints inside the probability in (4.44) was violated. As shown in Fig. 4.8, this empirical estimate is below the theoretical $\varepsilon_t$ guarantees for all $t = 1, \ldots, N_t$. 
Figure 4.6: Upper plot: Scenarios and forecasted values of the outdoor temperature. Middle plot: Power of the controllable part of the load (solid blue), its forecast value (dashed black), and upper limits of the load power for the case where the temperature is equal to its forecast (solid red) and for the worst case (dashed red), respectively. Lower plot: Evolution of the load energy state. The interpretation of the individual lines is the same with those of the middle plot.
Figure 4.7: Total cost, calculated as the sum of the production and reserve costs, for three cases: 1) no load uncertainty (i.e. deterministic load), no control; 2) uncertain load, no control; and 3) uncertain but controllable load.

Figure 4.8: Empirical probability of constraint violation and the theoretical limit.
4.6 Concluding remarks

We have demonstrated that uncertain reserves from DR could be considered in a day-ahead stochastic OPF to reduce the cost of dispatch. Considering uncertainty helps us to be less conservative in committing DR resources to reserve markets. Therefore, our approach allows us to better utilize the available DR resource, while still guaranteeing predefined levels of robustness. Importantly, the proposed scenario based methodology retains the structure of a deterministic problem (i.e. it remains a quadratic program) and hence is computationally tractable, and also amenable to distributed and decomposition based techniques.

Our results have important implications for the design of reserve markets. Specifically, we find that it is essential to have a mechanism to manage the energy state of load aggregations, which is in line with recent system operator proposals, for example [17]. We also find that it may be less important for a resource to be perfectly certain than to know its uncertainty so that that information can be incorporated within an OPF. Usually resources must demonstrate that they can accurately follow control signals to be able to participate in reserve markets; however, we instead suggest that the system operator should measure a range of a DR resource’s abilities in order to understand both its expected response and error distribution.

Future work concentrates on integrating N-1 security constraints into the developed framework. Moreover, distributed algorithms should be employed to ensure scalability of our approach to networks of realistic size. To conduct a more realistic analysis, we aim to extend our models for generating uncertainty realizations to capture other sources of uncertainty (e.g., human behavior, model mismatch) and also the spatial correlation of the forecast error. Another direction is to investigate the trade-off between reserve uncertainty and cost.
Chapter 5

Exploiting component controllability

Formulating a deterministic security constrained OPF results in higher operational costs compared with a solution based on a standard OPF formulation. If uncertainty is also present, performing a probabilistic security constrained OPF analysis as in Chapters 2-4, provides robustness guarantees but at the expense of additional cost. To alleviate this issue and achieve a more economic performance, we exploit the controllability of certain network components to provide preventive and corrective control actions. Concentrating on HVDC lines, we model these actions as piecewise linear policies of the uncertainty and incorporate them in a security constrained decision mechanism. We quantify the potential of our modeling approach to reduce the cost by means of various case studies using the IEEE three area RTS96 network. Our results show that by exploiting the controllability of HVDC lines we can achieve a significant reduction in the additional cost we incur when a probabilistically robust security constrained OPF design is performed. Moreover, it allows us to increase the share of RES that could be potentially integrated in the network, while maintaining the desired level of security.
5.1 Introduction

Throughout the dissertation we have considered a security-constrained optimal power flow (SC-OPF) problem in which the measure of security is based on the N-1 security criterion. As described in Chapter 1, the SC-OPF could include preventive [3] and corrective control actions [54], [64], [24]. The solution of the problem, considering preventive control actions, is in general more expensive compared to the standard optimal power flow (OPF) problem since more constraints are taken into account. However, by considering post-contingency corrective control actions, this additional cost can be reduced since more degrees of freedom are introduced. Moreover, the penetration of renewable energy sources (RES) is expected to increase significantly the next years. Due to their fluctuating nature, the level of uncertainty in the power network increases as well. Trying to obtain a robust (in a probabilistic sense) solution with respect to the uncertainty will also result in an increase of the operational cost.

To alleviate this issue, we introduce a policy-based preventive and corrective control offered by certain components in the decision making process. Typically, preventive and corrective security refers to actions that determine component setpoints for the base case (no outage) and the post-contingency cases, respectively. Our control actions though, apart from the outages, depend also on the uncertainty. In other words, the policy-based control depends on the post-disturbance operating point.

The advancements in communication systems allow the design of such control actions for various network components with response of different time scale. Some examples of controllable components are FACTS devices, HVDC lines, transformers, generator setpoints, flexible loads, etc. Note that some components, like generators and flexible loads, can be used both for reserve provision (see Chapters 3 and 4) and for preventive and corrective control. Moreover, apart from the benefits on the operating costs, incorporating such corrective control actions in a stochastic framework allows us to increase the share of RES that could be potentially integrated in the network, while maintaining the desired level of security.

The remainder of the chapter is organized as follows: Section 5.2 concentrates on HVDC lines and provides a mathematical description of the models used in our OPF framework and the type of policies we considered. Section 5.3 illustrates different aspects of our methodology by
5.2 Policies for preventive and corrective control

5.2.1 HVDC lines

High-Voltage Direct Current (HVDC) lines based on the Voltage-Source Converter (VSC) technology have the ability to control independently their active and reactive power flow with a very fast response. For that reason VSC-HVDC lines can be utilized not only for preventive but also corrective and policy-based control actions. Alternatively, Current Source Converter (CSC) HVDC can also be used, but then only active power can be controlled. Reactive power can only be controlled within certain limits, but not with the same flexibility as for VSC-HVDC lines. The formulation discussed below involves only active power control, therefore, it can be applied to both VSC and CSC-HVDC technologies.

In [25] the HVDC lines were integrated in a security-constrained AC-OPF context and provided corrective control. The formulation coupled the HVDC variables such as the DC voltage, the AC converter voltage, the HVDC line losses based on the line resistance, and the modulation factor with the AC power flow equations. Additionally, a HVDC capability curve in the form of an MVA circle was incorporated. In this section, we follow a DC-OPF formulation, and hence reactive power and DC line losses are neglected, while the voltage in the AC and DC side is always assumed equal to 1 p.u. Moreover, we do not consider a P-Q capability curve for the converters, but, similar to our assumptions for the AC lines, we assume that the maximum power transfer of the DC line is equal to the DC line active power limit.

In this work, each HVDC line is approximated by two virtual voltage sources located at the two nodes where the HVDC line is connected. The two virtual generators are constrained to have a power injection to the nodes of the same magnitude and opposite sign. The magnitude of the power injection represents the power flow on the HVDC line. Hence,
for each HVDC line, one additional variable is introduced representing the HVDC power flow.

As in the previous chapters, we consider a power network comprising of $N_G$ generating units, $N_L$ loads, $N_b$ branches, $N_w$ wind power generators, and $N_{DC}$ HVDC lines. For the N-1 security analysis we take into account any single outage involving the tripping of a line, load, conventional generator and HVDC line. Let $\mathcal{I}$ be a set that includes the indices corresponding to outages of all components including the HVDC lines and the index “0” that corresponds to the base case of no outage, and denote by $|\mathcal{I}|$ its cardinality. Moreover, let $P_{DC} \in \mathbb{R}^{N_{DC}}$ represent the vector with the power flows on the HVDC lines.

In this chapter, we augment the probabilistic SC-OPF of Chapter 2 to include HVDC policy-based preventive and corrective control actions. We assume that for any wind power forecast error and any outage the power flow setpoint is adjusted accordingly. This adjustment is represented by a set of functions $\alpha^i(\cdot) : \mathbb{R}^{N_w} \rightarrow \mathbb{R}^{N_{DC}}$ for all $i \in \mathcal{I}$, which map the wind power forecast error to the power flow setpoint adjustment. Specifically, the post-disturbance setpoint is given by $P_{DC} + \alpha^i(P_w)$.

The probabilistic SC-OPF including the corrective control action of the HVDC links is given by (2.11)-(2.15) with the additional constraint:

$$-\bar{P}_{DC} \leq P_{DC} + \alpha^i(P_w) \leq \bar{P}_{DC}, \text{ for all } i \in \mathcal{I} \hspace{1cm} (5.1)$$

where the injection vector $P_{inj}$ and generation load mismatch $P_m$ are given by

$$P_{inj}^i(P_w) = C^i_G(P_G - d^iP_m^i(P_w)) \hspace{1cm} + C^i_wP_w - C^i_LP_L + C^i_{DC}(P_{DC} + \alpha^i(P_w)),$$

$$P_m^i(P_w) = \sum_{k \in \mathcal{I}_w/K^i} (P_{w,k} - P_{w,k}^f) - c^i_LP_L + c^i_GP_G + c^i_wP_w, \text{ for all } i \in \mathcal{I}, \hspace{1cm} (5.2)$$

All undefined parameters and their interpretation are discussed below (2.9) in Section 2.2.2
5.2. Policies for preventive and corrective control

5.2.2 Piecewise-affine policies

The aforementioned problem involves optimizing over functions (i.e. \( \alpha^i(P_w) \)) which is not tractable in general. Therefore, at the expense of a suboptimal solution we restrict our attention to the specific class of piecewise affine functions.

Specifically, we consider the uncertainty space \( \mathbb{R}^{N_w} \) to be divided in \( 2^{N_w} \) regions that cover the entire space and do not overlap. These regions are chosen to be the quadrants in the \( N_w \)-dimensional space and are denoted by \( S_j, j = 1, \ldots, 2^{N_w} \). For each \( i \in \mathcal{I} \) we consider \( \alpha^i(P_w) \) to be a linear function of the uncertainty in each region \( S_j \) (it will be piecewise linear over the entire uncertainty space). We can encode this fact by representing the function \( \alpha^i(P_w) \) as

\[
\alpha^i(P_w) = K^i + \sum_{j=1}^{2^{N_w}} \Lambda^i_j \xi_{S_j}(P_w), \tag{5.4}
\]

where \( \xi_{S_j}(\cdot) \) is defined as \( \xi_{S_j}(P_w) = P_w \) if \( P_w \in S_j \) and zero otherwise. Note that for each \( P_w \in \mathbb{R}^{N_w} \) there will always be one term in the summation in (5.4) that is active.

These policies can be integrated in the optimization problem defined in the previous section. The overall problem is a chance constrained optimization program and can be addressed using the sampling based methods of Chapter 2. Specifically, when we use the probabilistically robust design of Section 2.4.2, each \( S_j \) would be a hyper-rectangular set and a robust problem is formulated. By the definition of the functions \( \xi_{S_j}(\cdot) \), the resulting robust optimization program can be transformed to a problem with multiple robust constraints, each of them concerning the region \( S_j \). These constraints can be reformulated to linear constraints using the approach of [7] or, in problems of small scale, we could enforce the constraints only for the uncertainty values that correspond to the vertices of the sets \( S_j, j = 1, \ldots, 2^{N_w} \).

In the case study presented in the next section we show that HVDC corrective control actions represented by piecewise control policies of the form of (5.4) lead to a reduction in the operational costs and can defer investments in additional AC lines when higher shares of wind power need to be integrated.
Table 5.1: Wind generation.

<table>
<thead>
<tr>
<th>Bus #</th>
<th>108</th>
<th>115</th>
<th>213</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forecast Wind Power [MW]</td>
<td>425</td>
<td>1270</td>
<td>700</td>
</tr>
</tbody>
</table>

5.3 Case studies

5.3.1 Test system

In this case study, we used the IEEE Three Area RTS-96 system, as described in [32], with some modifications. We added aggregations of wind power in-feed to three different buses (i.e $N_w = 3$), listed in Table 5.1. We also made two modifications to the topology. The HVDC link of the original system was replaced by two HVDC links between buses 109-115 and 216-316, each with a capacity of 300 MW. We consider a peak load situation, where the total system load equals 8550 MW whose 10% and the total dispatchable generation capacity (not including wind energy) is 10215 MW.

The formulated chance constrained problem is solved using the probabilistically robust design of Section 2.4. There are $N_w + 1$ uncertain variables, corresponding to the number of wind power in-feeds and the temperature. We considered a violation level $\varepsilon = 0.1$ and a confidence level $\beta = 10^{-4}$ and generated scenarios according to 2.23. The scenarios are generated under the assumption that the wind power in-feed and the temperature are independent, which allows us to use different models for each source of uncertainty. For the wind power scenarios, we use the Markov chain mechanism described in Section 2.5. For the temperature scenarios, we use one year of hourly forecast and actual values from data of 11 sites in Switzerland to represent possible temperature forecast errors. In our single period SC-OPF, we assume the temperature is 13°C and then add the vector of forecast error to represent the temperature scenarios. To compute the numerical solutions to the problems, we use the solver CPLEX [39] via the MATLAB interface TOMLAB.

5.3.2 Operating Costs

In this case study, we investigate the cost of generation dispatch for different SC-OPF formulations incorporating different levels of forecast
5.3. Case studies

Figure 5.1: Dispatch cost with different SCOPF formulations, as a percentage of cost associated with a standard OPF. SCOPF: standard SC-OPF; +wind: pSC-OPF with wind uncertainty; +load: pSC-OPF with both wind and load uncertainty; +HVDC-corr.: pSC-OPF with wind and load uncertainty, and corrective control actions with HVDC links; +load-corr.: pSC-OPF with wind and load uncertainty, and corrective control actions with HVDC links and controllable loads.

uncertainty and controllability. We have considered wind power forecast uncertainty and a certain portion of load consisting of TCLs whose consumption is uncertain since it depends on the outdoor temperature (see Chapter 4). We exploit the controllability of generating power, HVDC lines and of TCLs for preventive and corrective control.

Fig. 5.1 shows the cost of generation dispatch for different SC-OPF formulations. We compare each cost with the one resulting from a normal OPF, where no contingency-related constraints are included. As we move towards the right, every point represents a further extension of the algorithm. Besides the standard OPF and SCOPF formulations, we include two levels of uncertainty (leading to cost increase) and two levels of controllability (leading to cost reduction).

More specifically, after executing an OPF, where no security criteria or forecast uncertainties are considered, a SC-OPF is employed, where the
N-1 security criterion is included in the constraints and hence leading to higher costs. Then, a probabilistic SC-OPF (pSC-OPF) is run using the scenario approach of 2.4, taking into account both security constraints and wind forecast uncertainties. The pSC-OPF results in higher operating costs than SCOPF or OPF, as it tries to identify a robust solution against wind forecast errors while keeping the system N-1 secure. On the other hand, controllable line flows and flexible demand help counteract this increase in the operating costs. Exploiting the controllability of HVDC lines and loads, leads to a reduction in the operational costs since we allow them react in case of contingency by offering corrective control actions.

The cost increase induced by the probabilistic constraints is due to our requirement for a robustly (in a probabilistic sense) secure performance. For an evaluation study we used 10,000 different scenarios for the wind energy in-feed to determine if there are any constraint violations. Without considering uncertainty in the SC-OPF formulation, the pre-contingency dispatch results in violations in 100% of the different wind in-feed scenarios. With a probabilistic SC-OPF formulation, the solution results in violations of less than 0.2% of the scenarios.

It should be noted that for this specific example the contribution of the second term in (5.4) is minor compared with the first one (i.e. $K^i$). Therefore, in view of a computationally simpler problem with fewer decision variables one could just differentiate among the outages and use the uncertainty independent term $\alpha^i(P_w) = K^i, i \in \mathcal{I}$. However, this might not be the case in other systems.

Concluding this case study, we observe that ensuring the security of the system under uncertainty in generation and/or demand results in higher operating costs. At the same time, corrective control capabilities offered through HVDC lines or flexible demand, have the potential to manage the additional cost due to uncertainty in RES in-feed and demand. We observe that they can limit the increase in operating costs to about 50% of the original value.

### 5.3.3 Wind power penetration

If the system operates at its limits, one way to increase the wind power penetration is to reinforce the grid with new transmission lines. However, exploiting the controllability of certain components seems to be
a promising mean to defer investments in new AC lines. In this case study, we compute the maximum wind in-feed that can be integrated into the system, without violating any security constraints for the cases with and without corrective control actions of the HVDC lines.

To calculate the maximum wind penetration in the system, we developed a variant of the probabilistic SC-OPF problem presented in Chapter 2. Specifically, we augment the decision variables to include both the generation and the wind power forecast. In this problem, we no longer minimize the generation costs but, instead, we maximize the sum of the wind power forecasts. In this case, the constraint functions include products of the decision variables and the uncertainty. Since the uncertainty is no longer additive, the problem is computationally more complex. To solve the problem, we use again the probabilistically robust design of Section 2.4, which requires in this case to take into account considerably less scenarios compared with the standard scenario approach.

Using the same test system and considering only wind power uncertainty, the probabilistic SC-OPF results in a maximum wind share of 28% of the total load. This result is also shown Fig. 5.2. By allowing the existing HVDC lines to offer corrective control actions, the maximum wind in-feed rises to a share of about 30%. This result highlights that taking advantage of the controllability offered by existing infrastructure
can have an effect equivalent to transmission expansion measures. Advantages here are that no additional investment costs would be incurred and the long licensing procedures for the building of new lines would be avoided.

5.4 Conclusion

In this chapter we proposed a probabilistic security constrained OPF formulation that takes advantage of the HVDC line controllability for post-disturbance control actions. We showed that by exploiting the controllability of HVDC lines we can achieve a significant reduction in the additional cost we incur when a probabilistically robust security constrained OPF design is performed. Moreover, we are allowed to increase the share of RES that could be potentially integrated in the network, while maintaining the desired level of security.

Current research concentrates on further investigating the performance of the policy-based preventive and corrective control and on the optimal placement of the HVDC lines towards maximizing the wind power penetration, while minimizing the operational costs.
We propose a probabilistic framework for designing an N-1 secure dispatch for AC power systems with fluctuating energy sources. This could be used in various applications, however, in this work we demonstrate our approach for a day-ahead planning problem. We extend our work on probabilistic N-1 security that was presented in Chapter 2, to incorporate recent results on convex AC optimal power flow relaxations. The problem is formulated as a chance constrained convex program; to deal with the chance constraint we follow a sampling based technique. We also enhance the flexibility of the system by introducing a corrective scheme that imposes post-disturbance control of the Automatic Voltage Regulation (AVR) set-point. This scheme allows us to inherit a-priori probabilistic guarantees regarding the satisfaction of the system constraints, unlike the base case where the AVR set-points are constant. To illustrate the performance of the proposed security-constrained AC optimal power flow we compare it against a DC power flow based formulation using Monte Carlo simulations, and also show that it results in lower operational cost compared to the case where the AVR set-points are constant.


6.1 Introduction

In Chapter 2 a probabilistic SC-OPF formulation based on the DC power flow equations was proposed. Even though using a linearized version of the network is common practice for scheduling problems [55], [60], [75], it introduces an approximation error that may lead to not sufficiently accurate results. Moreover, the solution may not meet the desired reliability levels; for example, the SC-OPF based on a DC power flow formulation may fail to meet security requirements if an AC contingency analysis is conducted.

To allow for more accurate modelling, the underlying DC power flow set-up needs to be substituted with an AC power flow formulation. Adopting an AC based SC-OPF allows to incorporate constraints regarding voltage and reactive power and introduce more control variables (i.e. tap changers, FACTS, etc.) to further reduce the cost of a secure operation. The additional controllability could be used both on a preventive and corrective control scheme [54], [64].

The AC optimal power flow problem has been studied extensively in the literature [51], [38], [52], [53]. One of the main problem is how to deal with the non-convexity issues, which arise mainly due to the nonlinear active and reactive power flow equations. This is challenging, from an optimization point of view, since there are no guarantees that the global optimum could be reached. In recent work, [6] provides a convex AC optimal power flow relaxation whose performance was tested in various IEEE benchmark networks, whereas in [42] the authors use duality concepts and provide necessary and sufficient conditions to guarantee zero duality gap between their formulation and the initial optimal power flow problem.

In this chapter we build on the work included in Chapter 2 and address the problem of computing a probabilistically N-1 secure day-ahead dispatch, while using a convex AC optimal power flow relaxation. To achieve this, we adopt the semi-definite formulation of the optimal power flow [6], [42], we incorporate the N-1 security constraints and formulate the stochastic variant of the resulting problem. This gives rise to a chance constraint semi-definite program. To deal with the chance constraint we then follow a scenario based technique, i.e. the “probabilistically robust design” [44] that was introduced in Chapter 2. We also enhance the flexibility of the system by introducing a corrective scheme that imposes post-contingency control of the Automatic
6.2 Problem formulation

Generation Control (AVR) set-point. This scheme allows us to inherit a-priori probabilistic guarantees regarding the satisfaction of the system constraints, unlike the base case where the AVR set-points are constant. To illustrate the performance of the proposed security-constrained AC optimal power flow we compare it against the work Chapter 2 using Monte Carlo simulations, and show that it results in lower operational cost compared to the case where the AVR set-points are constant.

The structure of this chapter is as follows. Section 6.2 introduces the formulation of the security-constrained optimal power flow problem as a chance constrained program, whereas Section 6.3 provides a tractable reformulation of the problem and outlines a procedure to deal with the chance constraint. Using a Monte Carlo analysis, Section 6.4 provides some preliminary results that illustrate the performance of the proposed approach against its DC power flow variant. Finally, Section 6.5 provides some concluding remarks and directions for future work. References [72], [63] are related to this chapter.

6.2 Problem formulation

6.2.1 Definitions and preliminaries

We consider a power network comprising $N_G$ conventional generating units, $N_L$ loads, $N_l$ lines, $N_b$ buses and $N_w$ wind power generators. Denote by $I_b, I_l \subseteq I_b \times I_b$, the sets of indices corresponding to the buses and the lines, respectively. Moreover, let $I_G, I_L, I_w$ be the sets that include the bus indices where the conventional units, the loads and the wind power generators are connected to. For the N-1 security analysis we take into account any single outage involving the tripping of a line, load, conventional generator and wind power generator. As in Chapter 2, denote by $I$ the set of indices corresponding to the contingencies taken into account, whereas the “0” index corresponds to the base case of no outage. The total number of outages would then be $|I| = N_G + N_L + N_l + N_w$, where $|I|$ denotes the cardinality of the set $I$. To simplify notation, we assume that at most one generator, one load and one wind power generator are connected to each bus. However, an extension to multiple units could be easily captured by the proposed framework.

Let now $X = [\Re \{V\}^T \Im \{V\}]^T \in \mathbb{R}^{2N_b}$ denote a vector that consists of the real and imaginary part of the bus voltages. In the sequel we provide
expressions for the variables that typically appear in the optimal power flow formulation, i.e. the apparent power flow $S_{lm}$, the active power flow $P_{lm}$, the active and reactive power injections $P_{k,\text{inj}}, Q_{k,\text{inj}}$, the voltage magnitude $|V_k|$ and the magnitude of the difference between the voltages of the buses connected by a line $|V_l - V_m|$. Following the formulation of [42], for all $k \in \mathcal{I}_b$ and all $(l,m) \in \mathcal{I}_l$, the aforementioned quantities can be represented by

\begin{align}
P_{k,\text{inj}} &= \text{Tr}\left\{Y_k XX^T\right\}, \\
Q_{k,\text{inj}} &= \text{Tr}\left\{\overline{Y}_k XX^T\right\}, \\
P_{lm} &= \text{Tr}\left\{Y_{lm} XX^T\right\}, \\
|S_{lm}|^2 &= \text{Tr}\left\{Y_{lm} XX^T\right\}^2 + \text{Tr}\left\{\overline{Y}_{lm} XX^T\right\}^2, \\
|V_k|^2 &= \text{Tr}\left\{M_k XX^T\right\}, \\
|V_l - V_m|^2 &= \text{Tr}\left\{M_{lm} XX^T\right\},
\end{align}

where $Y_k, \overline{Y}_k, Y_{lm}, \overline{Y}_{lm}, M_k, M_{lm}$ are constant matrices with real entries.

Then, based on the active power balance equation, the active power production of the generating unit connected at bus $k$ is given by

\begin{equation}
P_{G,k} = \text{Tr}\left\{Y_k XX^T\right\} + P_{L,k} - P_{w,k},
\end{equation}

where $P_{L,k}, P_{w,k}$ represent the load and the wind power infeed connected to bus $k$, and are respectively zero if $k \notin \mathcal{I}_L$ or $k \notin \mathcal{I}_w$. Let $P_w \in \mathbb{R}^{N_w}$ be a vector consisting of the wind power infeeds $\{P_{w,k}\}_{k \in \mathcal{I}_w}$.

In case of an outage or a deviation of the wind power from the forecasted value, the voltage profile of the system will change, and hence a different $X$ is obtained. For the rest of the paper, we introduce the superscript “$i$” in our notation to highlight the dependency of the variables on the corresponding outage. The dependency on the wind power $P_w$ will be explicit, e.g. we will use $X^i(P_w)$.

In case of active power imbalance, the frequency will deviate from its nominal value and a new steady state will be reached as an effect of the automatic secondary frequency control action. We take into account this control action and formulate the security constraints based on the
6.2. Problem formulation

resulting operating point of the system. Similarly to Section 2.2.2, we define \( d^i \in \mathbb{R}^{N_G} \) to be a distribution vector, weighting the excess-deficit of power among the generating units participating in the frequency control. If a generator is not contributing to frequency control, the corresponding element in vector \( d^i \) is zero. Typically, the distribution vector is the same for all the outages except the outages of the generators. In this case the distribution vectors are recalculated since the imbalance should be compensated only by the remaining ones.

Let \( P_m^i \in \mathbb{R} \) represent the total generation-load mismatch, which may occur due to the difference between the actual wind \( P_w \) from its forecast value \( P_{w}^f \), or/and as an effect of an outage \( i \). For every outage \( i \in I \) the total generation-load mismatch is determined by (2.9), but we repeat it here for the ease of presentation.

\[
P_m^i(P_w) = \sum_{k \in \mathcal{I}_w/K_i} (P_{w,k} - P_{w,k}^f) - c^i_L P_L + c^i_G P_G + c^i_w P_w. \tag{6.8}
\]

Vectors \( c^i_L, c^i_G, c^i_w \) are of appropriate dimension that based the outage \( i \) would be either zero or would include an “1” at the position that corresponds to the tripped component. In case of a wind power generator outage the amount of power produced before the outage should contribute to the generation-load mismatch, justifying the presence of the term \( c^i_w P_w \) in the equation. However, the error from the forecast in this case should not be included in the mismatch. Therefore, we denote by \( K^i \) a set that depends on the outage \( i \) and will either be the empty set, or it will be a singleton including the index of the bus with the wind power outage.

According now to the preceding discussion, for every \( k \in \mathcal{I}_G \) and every outage \( i \), the new operating point of the active power production at bus \( k \) is \( P_{G,k}^i = P_{G,k}^0(P_w^f) - d_k^i P_m^i \), where \( d_k^i \) is the element \( k \) of vector \( d^i \), and \( P_{G,k}^0(P_w^f) \) corresponds to the nominal case where \( P_w = P_{w}^f \) and no outage occurs. For all \( i \in \mathcal{I}, k \in \mathcal{I}_G \) and for a given wind power production \( P_w \), this fact can be encoded by imposing

\[
\text{Tr}\{Y_k^i X^i(P_w)(X^i(P_w))^T\} + P_{D,k}^i - P_{w,k}^i
= \text{Tr}\{Y_k^0 X^0(P_w^f)(X^0(P_w^f))^T\} + P_{D,k}^0 - P_{w}^f - d_k^i P_m^i. \tag{6.9}
\]
6.2.2 Probabilistic security constrained optimal power flow

We are now in a position to state the probabilistic variant of the security constrained optimal power flow problem. For all $k \in I_G$ let $c_{j,k}, j = 0, 1, 2$ be non-negative cost coefficients. We seek to minimize the total production cost

$$\min \left\{ W_i(P_w) : i \in I \right\}$$

subject to

$$\mathbb{P}(P_w \in \mathbb{R}^{N_w} | \text{equation (6.9)}, \text{constraints (6.11)} - (6.20) \text{ for } P_w = P_{w^f})$$

where $\epsilon \in (0, 1)$ is a prespecified violation level. Vector $e$ is a unit vector with its last element being “1”. In view of equation (6.9), for every outage $i \in I$ and for all $k \in I_b$ constraints (6.12), (6.13) require
6.3. Tractable reformulation

In the previous section we formulated the security-constrained AC optimal power flow problem as a chance constrained program. To ensure that the resulting problem is tractable, there are a few issues that need to be resolved. The first one refers to the structure of the objective function and the constraints which are non-convex with respect to the decision variables, whereas the second issue us due to the fact that (6.10)-(6.21) is an infinite-dimensional optimization problem since it involves optimizing over functions of the uncertainty. The last issue refers to the need of providing an efficient way to deal with the chance constraint while maintaining certain probabilistic guarantees.

6.3.1 Semi-definite constraints

Following the procedure proposed in [42], it can be shown that by using Schur’s complement formula [14], problem (6.10)-(6.20) is equivalent to

\[
\min_{\{W^i(\cdot)\}_{i \in \mathcal{I}}, \{a_k\}_{k \in \mathcal{I}_G}} \sum_{k \in \mathcal{I}_G} a_k,
\]

subject to

\[
P \left( P_w \in \mathbb{R}^{N_w} \mid \right.
\]

constraints (6.9), (6.12), (6.13), (6.14), (6.16), (6.17), (6.19),

\[
\begin{bmatrix}
(S_{lm}^{\text{max}})^2 & \text{Tr} \left\{ Y_{lm} W^i(P_w) \right\} & \text{Tr} \left\{ Y_{lm} W^i(P_w) \right\}

\text{Tr} \left\{ Y_{lm} W^i(P_w) \right\} & -1 & 0

\text{Tr} \left\{ Y_{lm} W^i(P_w) \right\} & 0 & -1
\end{bmatrix} \succeq 0,
\]

(6.24)
Chapter 6. Probabilistic AC SC-OPF

\[ W^i(P_w) \succeq 0, \quad (6.25) \]

\[ \text{rank}(W^i(P_w)) = 1, \quad (6.26) \]

for all \( k \in \mathcal{I}_b, (l, m) \in \mathcal{I}_i, i \in \mathcal{I} \) \geq 1 - \varepsilon, \quad (6.27)

constraints (6.23) – (6.26) for \( P_w = P^f_w \),

\[
\begin{bmatrix}
    c_{1,k} \text{Tr}\{Y^0_k W^0(P^f_w)\} - a_{k} + b_{1,k} \sqrt{c_{2,k}} \text{Tr}\{Y^0_k W^0(P^f_w)\} + b_{2,k} \\
    \sqrt{c_{2,k}} \text{Tr}\{Y^0_k W^0(P^f_w)\} + b_{2,k} - 1
\end{bmatrix} \succeq 0,
\]

(6.29)

where \( b_{1,k} = c_{0,k} + c_{1,k} P^D_k \) and \( b_{2,k} = \sqrt{c_{2,k}} P^D_k \). Notice that using the epigraphic formulation [14], (6.10) is substituted by (6.22) with the additional constraint (6.29), whereas (6.15) is replaced by (6.24). Constraint (6.18) is equivalent with (6.25) and (6.26).

The resulting problem is a chance constrained semi-definite program with a rank constraint due to (6.26), which raises convexity issues. In [42] the authors remove constraints (6.19), (6.26), and use duality techniques to provide necessary and sufficient conditions under which the duality gap between the optimal objective value of their formulation and the optimal value obtained by (6.22)-(6.29) is zero. In [6] the authors remove only (6.26) and by means of simulations show that for various IEEE benchmark systems the presence of (6.19) ensures that the resulting solution satisfies (6.26). In this paper we follow the second approach even though it does not provide rigorous optimality guarantees. Our choice is justified by the fact that we are mainly interested in illustrating how security constraints can be incorporated in a probabilistic AC set-up, and in comparing our results with those obtained by a DC power flow formulation.

6.3.2 Optimization over policies

Problem (6.22)-(6.29) is an infinite-dimensional optimization problem since it requires optimizing over \( \{W^i(\cdot)\}_{i \in \mathcal{I}} \) which are functions of the uncertainty vector \( P_w \). This renders the problem intractable from a numerical computation point of view. To ensure tractability and enable for numerical computations, apart from \( W^0(\cdot) \), we restrict \( W^i(\cdot), i \in \mathcal{I} \) to the class of affine policies, i.e. functions that are affine with respect
6.3. Tractable reformulation

Therefore, for all \( i \in \mathcal{I} \) we define

\[
W^i(P_w) = A^i + \sum_{k \in \mathcal{I}_w} B^i_{k} P_w, \tag{6.30}
\]

where \( A^i \in \mathbb{R}^{2N_b \times 2N_b} \) and \( B^i_{k} \in \mathbb{R}^{2N_b \times 2N_b} \) for all \( k \in \mathcal{I}_w \). As a special case, if \( P_w \) is a scalar (single wind power in-feed), then \( W^i(P_w) = A^i + B^i P_w \). By defining this class of policies we can now optimize (6.22) over the matrices \( A^i, B^i_{k}, k \in \mathcal{I}_w, i \in \mathcal{I} \), which constitute the coefficients of these affine functions. Note that by restricting our attention to a specific class of functions we come up with a suboptimal solution, albeit computable thus enabling the application of our methods to non-trivial case studies.

The choice of policies of the form of (6.30) is not arbitrary. It falls in the framework of corrective control actions from a security point of view, and emulates the way the set-points of the Automatic Voltage Regulator (AVR) vary as a function of the wind power \( P_w \) for each generating unit and each contingency \( i \).

6.3.3 Chance constraint

To deal with the chance constraint, while avoiding arbitrary assumptions on \( \mathbb{P} \) and its moments, we follow the sampling based methodologies of Section 2.4. Based on the scenario approach the chance constraint is substituted with a finite number of hard constraints, each of them corresponding to a different uncertainty realization. The number of scenarios, that one needs to generate to achieve probabilistic performance guarantees, grows linearly with respect to the number of decision variables, thus leading to computationally expensive problems for systems of high dimension as in our case. We choose then to employ the probabilistically robust design that is based on a combination of randomized and robust optimization. Note that using the policies of Section 6.3.2 the constraint functions remain convex with respect to the uncertainty \( P_w \). Therefore, to solve the resulting robust problem it suffices to enforce the constraints only at the values that correspond to the vertices of the hyper-rectangular uncertainty set. We then have guarantees that the obtained solution satisfies the chance constraints with probability at least \( 1 - \beta \), where \( \beta \in (0, 1) \) is a prespecified confidence level. For the cases where the uncertainty is of low dimension this leads to a manageable problem, however, in general the number of constraints grows
Figure 6.1: We compare in terms of cost three different alternatives; the first one is a standard optimal power flow (OPF) formulation, the second includes the N-1 security constraints (SC-OPF), whereas the third one corresponds to the proposed probabilistically robust SC-OPF approach.

exponentially with the dimension of the uncertainty vector $P_w$. Alternatively, instead of using this vertex enumeration scheme, the algorithms of [7] which provide a tractable reformulation of uncertain semi-definite programs may be adopted.

6.4 Case study

To illustrate the performance of the proposed methodology for the security-constrained optimal power flow problem we compare it against an alternative implementation based on a DC power flow formulation (see Chapter 2) and a day-ahead planning horizon with hourly steps. The optimization problem of Section 6.3 is solved for every hour since the stages are decoupled. For our simulation study we employed the IEEE 14-bus network, which is retrieved from [88] and includes $N_b = 14$ buses, $N_G = 6$ generators, $N_l = 20$ lines. The system is modified to include one wind plant connected to bus 12. As described in the previous section, to deal with the chance constraint we used the probabilistically robust design of Section 2.4.2 with violation level $\varepsilon = 0.1$ and confidence $\beta = 10^{-4}$. To generate scenarios we used the Markov chain based model of Section 2.5.

Fig. 6.1 highlights the additional cost we incur if we impose additional
6.4. Case study

Figure 6.2: (a) Hourly load profile, (b) Wind power forecast, wind power scenarios used for the optimization process and span of 10,000 wind power scenarios used for evaluation purposes, (c) Probability of insecure instances for the SC and the DC set-up.

security requirements for one hour of the simulated data (hour 14 of Fig. 6.2). We compare three different alternatives; the first one is a standard optimal power flow (OPF) formulation, the second includes the N-1 security constraints (SC-OPF), whereas the third one corresponds to the proposed probabilistically robust SC-OPF approach. The color
coding distinguishes the cases where an AC or a DC optimal power flow set-up is adopted. As expected, both for the AC and the DC based set-up the cost increases as we constrain the problem. Specifically, the SC-OPF includes security constraints that are not present in the OPF approach, whereas the probabilistically robust SC-OPF is more constrained compared with the SC-OPF since we include additional constraints to achieve robustness against wind power uncertainty. The seemingly lower cost of the DC based approaches is at the expense of a less secure performance as it will be shown in the sequel. It should be mentioned, however, that even though the proposed approach enhances the security of the system, it leads to a formulation that is computationally more expensive compared with the DC power flow based set-up.

To evaluate the solutions generated by the AC and DC power flow based optimization programs we carry out a Monte Carlo study. Fig. 6.2(a)
6.4. Case study

Figure 6.4: Generation cost comparison between the proposed corrective scheme (policy based AVR set-point) and the base case (constant AVR set-point) for the hours that the base case optimization problem was feasible. The proposed approach was feasible for all the hours.

shows the hourly load profile that was used. For the sake of simplicity we considered a deterministic profile, while load uncertainty can be captured by the proposed framework as well. Fig. 6.2(b) depicts the wind power forecast and the wind power scenarios that were extracted and were used in the optimization process. The shaded region corresponds to the span of the 10,000 scenarios (different from those used in the optimization) that were used for evaluation purposes. For the Monte Carlo analysis we first solved the optimization problem of Section 6.3 and then imported the solution in the simulation environment of MATPOWER [88]. At a next step we performed an AC contingency analysis for the 10,000 evaluation scenarios. The fraction of these scenarios where an insecure instance is encountered serves as an empirical estimate of the corresponding probability.

For the sake of comparison the solution of the DC based approach is also imported in MATPOWER and the same procedure with AC case is followed. Since the power losses are not modeled in the DC set-up, solving an AC optimal power flow in MATPOWER would result in a different generation set-point for the unit that corresponds to the slack bus. To better interface the DC solution in MATPOWER we first calculate the losses as the power compensated by the slack bus, and then adjust the generation set-point by distributing the among the generating units according to the vector $d^0$. Such a procedure mimics the network response if the DC solution was imposed to the generating
Fig. 6.2(c) shows for every hour the probability of insecure instances for the AC and the DC based approach calculated as described above. As expected, the solution of the AC optimal power flow respects the violation level $\varepsilon$, whereas the solution of the DC based approach fails to satisfy this requirement for hours 10-18 (more loaded hours). For these hours the probability of insecure instances is almost one, which reveals the significant effect that the model mismatch between a DC and an AC design may have.

Concentrating now on the maximum line loading of each contingency analysis (corresponding to 10,000 wind power realizations), Fig. 6.3 shows for hour 14 of Fig. 6.2 how the normalized maximum loading is distributed for each line\(^1\). The line loading limits are indicated by the “dashed” lines. By inspection of Fig. 6.3(a) and Fig. 6.3(b), it can be observed that the proposed methodology leads to no line over-loadings as opposed to the approach where a DC power flow set-up is adopted.

Fig. 6.4 compares the performance in terms of cost of the SC-OPF when the policy based scheme is employed with the base case where the AVR set-points are constant. As expected, our approach leads to better cost for all time instances that the optimization problem corresponding to the base case was feasible. It should be noticed that the additional controllability that the post-contingency control offers increases the feasibility region and in this example ensured feasibility for all hours.

\section*{6.5 Concluding remarks}

In this chapter we develop a probabilistic framework to deal with the problem of N-1 security, while using a convex relaxation for the AC power flow equations. We outline a methodology which leads to a tractable problem formulation, and compare it against an alternative implementation based on a DC power flow set-up. Moreover, we enhance the flexibility of the system by introducing a corrective scheme that imposes post-disturbance control of the Automatic Voltage Regulation (AVR) set-point.

\footnote{For the boxplots, the “red” line corresponds to the median value, the edges of the box correspond to the 25th and 75th percentiles, whereas the whiskers extend to a 99% coverage. The “red” marks denote the data outliers.}
Future work concentrates toward incorporating the results of Chapter 3 on reserve scheduling in the proposed framework and exploiting policies of different complexity for the AVR set-point. Another issue is to investigate the potential of inheriting the zero duality gap guarantees of [42] for our security-constrained formulation.
Chapter 7

Conclusion

7.1 Summary

Operating power systems in a secure way constitutes a critical task for the well functioning of the society. However, security comes at the expense of additional investment and operational cost. The costs needed to maintain a desired security level are expected to increase further due to the integration of RES. This highlights the necessity to revisit certain operational concepts so as to achieve a better trade-off between a secure and an economic operation of the power network.

This dissertation concentrated on quantifying this trade-off and proposed a mechanism for optimal decision making in the presence of uncertainty. Building on a DC power flow set-up, Chapter 2 introduced the concept of probabilistic security, where the system and security constraints are allowed to be violated with a prespecified probability. We formulated an optimal power flow problem with N-1 security constraints and wind power uncertainty as a chance constrained optimization program. To obtain a solution to this problem sampling based techniques with guaranteed performance were employed. In Chapter 3, we incorporated a generation-side reserve decision mechanism in the security constrained optimal power flow framework of Chapter 2. The reserves were represented as piecewise linear functions of the uncertainty and were optimized with respect to the coefficients of these functions. Therefore, a by-product of our production and reserve scheduling algorithm was
also the construction of a reserve strategy that could be deployed in real time operation.

Analogously to the generation-side reserves, Chapter 4 introduces a representation for the reserves offered by the demand response. Exploiting demand-side capabilities for reserve provision allows for a more economic operation of power systems since it results in lower total cost compared to the case where only generation-side reserves are taken into account. In view of reducing the total cost, Chapter 5 exploited the potential of certain network components other than demand response to provide corrective control actions. Emphasis was given on HVDC links, and it was shown that, by appropriately modelling their post-disturbance operating point, the desired security level can be achieved at a lower operational cost. Chapter 6 extended the framework of probabilistic security that was developed in Chapter 2 to an AC optimal power flow set-up. In a sense analogously to Chapter 3, we enhance the flexibility of the system by introducing a corrective control scheme that imposes post-disturbance control of the AVR set-point.

In all cases we formulated the underlying optimization problems as chance constrained optimization programs. To solve the resulting problems we used recently developed algorithms based on uncertainty sampling that offer a-priori guarantees regarding the probability of constraint satisfaction.

7.2 Outlook

This dissertation proposed a unified framework for optimal decision making in power system planning problems under uncertainty. In particular, emphasis was given on studying the trade-off between a secure and an economic operation of power systems, and on exploiting corrective control actions offered by various system components. Building on the proposed framework research can be pursued in the following directions:

1. Unit commitment constraints were not included in the proposed scheduling formulations and the on-off status of the generating units was assumed to be fixed as a result of a separate optimization program. Co-optimizing with respect to binary decisions encoding the
status of the generating units would give rise to mixed-integer problems. Our framework can be directly extended to capture such cases. Results toward this direction have been documented in [46].

2. In the probabilistic security constrained optimal power flow formulation all possible single component outages are taken into account. The probability of occurrence of each outage is not taken into account. Incorporating the latter in the proposed framework would give rise to a risk-based security constrained optimal power flow formulation. Preliminary results toward this direction can be found in [62].

3. Corrective control actions have been modeled by decision rules of relatively simple complexity. More involved representations can be further investigated and their physical interpretation should be analyzed. Moreover, the effect of the particular choice in terms of cost and computational effort should be quantified.

4. Appropriate market structures providing incentives to consumers to participate in a demand response mechanism need to be developed, whereas the pricing implications of the obtained solutions require additional investigation. Moreover, game theoretic tools that capture the interaction between the production and the demand in such an environment should be incorporated in the framework of Chapter 4. Such a direction is followed in [69] but without taking network and security constraints into account.

5. Reserve decision mechanisms should be incorporated in the security constrained AC optimal power flow set-up of Chapter 6. Moreover, the scalability of the proposed convex relaxation that are based on semi-definite programming should be further investigated.

6. To allow for numerical computations in power networks of realistic size, decomposition techniques (e.g. [26]) and tools from distributed optimization need to be employed. For this purpose, recently developed algorithms based on the Alternating Direction Method of Multipliers (ADMM) [13] could be employed. However, this task is challenging due to the stochastic and non-convex nature of the underlying problems, which requires revisiting the convergence and optimality properties of the ADMM based techniques.
Appendix A

Discussion on sampling based optimization techniques

In this appendix we provide more details regarding the scenario approach and the probabilistically robust design of Section 2.4, and discuss alternative implementations that may lead to a better solution in terms of cost.

Here, we consider the following chance constrained optimization program

$$\min_{x \in \mathbb{R}^{N_x}} J(x)$$ \hspace{1cm} (A.1)

subject to

$$\mathbb{P}\left( \delta \in \mathbb{R}^{N_\delta} | f(x, \delta) \leq 0 \right) \geq 1 - \varepsilon,$$ \hspace{1cm} (A.2)

where $J(\cdot) : \mathbb{R}^{N_x} \to \mathbb{R}$ and $f(\cdot, \cdot) : \mathbb{R}^{N_x} \times \mathbb{R}^{N_\delta} \to \mathbb{R}$. Notice that all chance constrained optimization programs defined throughout the thesis correspond to special cases of this generic formulation.

If the objective function $J(x)$ is convex, and for each $\delta \in \mathbb{R}^{N_\delta}$ the constraint function $f(x, \delta)$ is also convex with respect to $x$, then the scenario approach of Section 2.4.1 could be applied to transform the chance constraint (A.2) to a finite number of hard constraints each of them corresponding to a different realization of $\delta$. This leads to the following program:

$$\min_{x \in \mathbb{R}^{N_x}} J(x)$$ \hspace{1cm} (A.3)
subject to
\[ f(x, \delta(s)) \leq 0, \text{ for all } s = 1, \ldots, N, \tag{A.4} \]

where \( N \) is chose according to (2.20) (with \( \varepsilon, \beta \) in place of \( \varepsilon_t, \beta_t \), respectively) and is proportional to the decision variables \( N_x \). Then, the optimal solution of (A.3)-(A.4) is feasible for (A.2) with probability at least \( 1 - \beta \).

On the other hand, the probabilistically robust design of Section 2.4.2 does not require convexity of the objective and constraint functions with respect to the decision variables, but requires solving the following robust program:

\[
\min_{x \in \mathbb{R}^{N_x}} J(x) \tag{A.5}
\]

subject to
\[ f(x, \delta) \leq 0, \text{ for all } \delta \in \tilde{\Delta}^N, \tag{A.6} \]

where \( \tilde{\Delta}^N \) is the minimum volume hyper-rectangle that, with probability at least \( 1 - \beta \), contains at least \( 1 - \epsilon \) of the probability mass of the uncertainty. The set \( \tilde{\Delta}^N \) is computed as described in Step 1 of Section 2.4.2, where \( N \) is chosen according to (2.23) and is proportional to the dimension of the uncertainty vector \( N_\delta \).

Note that the probabilistically robust design does not require convexity of the underlying problem with respect to the decision variables. However, it requires solving a robust program with the uncertainty being bounded in a hyper-rectangular set. To ensure that the resulting problem is solved in a tractable way, convexity and homogeneity of the constraint functions with respect to the uncertainty is required [7]. Note that even if the constraint functions do not satisfy these requirements, they may be convex and homogeneous with respect to some functions of \( \delta \) instead of \( \delta \). The probabilistically robust design would be still applicable, if the set \( \tilde{\Delta}^N \) is constructed based on the probabilistically worst-case values of the functions. The number of scenarios \( N \) that should be extracted in this case would be proportional to the number of uncertainty functions instead of \( N_\delta \).

For the problems formulated throughout the thesis, both approaches are applicable. Therefore, to choose among them involves a trade-off between the number of samples we have to generate, the computational
cost of the resulting problem and the conservatism of the resulting solution. Typically, $N_\delta \ll N_x$, therefore, the probabilistically robust design requires generating fewer samples compared to the standard scenario approach. Moreover, adopting the approach of [7] to solve the resulting robust problem results in fewer constraints compared to the scenario approach (see also [44] for a thorough analysis). However, the solution of (A.5)-(A.6) might be more conservative in terms of cost compared to the scenario approach, since it involves solving a robust program. In the following section we present alternative implementations that enable us to trade feasibility to optimality.

In the case, however, where a unit-commitment problem is appended to our formulations (see also discussion in Chapter 3), the standard scenario approach is no longer applicable. The reason is that the unit-commitment problem involves binary decisions that would give rise to a mixed-integer formulation; the latter is amenable, however, to the probabilistically robust design which does not require convexity with respect to the decision variables.

### A.1 Trading feasibility to optimality

#### A.1.1 Sampling and discarding

The scenario approach as well as the probabilistically robust design provide feasibility type probabilistic guarantees but not optimality ones. To trade feasibility to optimality, we use a subsequent development of the scenario approach, the so-called sampling and discarding methodology [21], [15]. Specifically, for the standard scenario approach, given $N$ samples of the uncertainty, $r$ of them are eliminated according to some rule and (A.3)-(A.4) is formulated with the remaining $N - r$ samples. Following [15], if $N$ is chosen according to

$$N \geq \frac{2}{\varepsilon} \left( \ln \frac{1}{\beta} + 2(N_x + r - 1) \right),$$

(A.7)

and under the assumption that the solution of the resulting problem violates the removed constraints with probability one (so that the solution is improved), the solution of the problem with the constraints corresponding to the remaining $N - r$ samples is feasible for (A.2) with probability at least $1 - \beta$. 
For the probabilistically robust design the same procedure can be employed. Given $N$ samples of the uncertainty, $r$ of them (possibly corresponding to outliers) according to some rule and the set $\bar{\Delta}^{N-r}$ is determined based only on the remaining samples $N - r$ samples. That way, the constructed set $\bar{\Delta}^{N-r}$ will have a smaller volume, thus reducing the conservatism of the solution that would have been obtained if no sample were discarded. The number of scenarios that we need to generate in this case, so that we are able to eliminate $r$ of them, is given by

$$N \geq \frac{2}{\varepsilon} \left( \ln \frac{1}{\beta} + 2(2N_\delta + r - 1) \right).$$  \hspace{1cm} (A.8)$$

Note that (A.8) is of the form of (A.7) with $2N_\delta$ in place of $N_\times$; this is due to the fact that the problem at Step 1 in Section 2.4.2 has $2N_\delta$ decision variables. Less conservative, but implicit bounds for the sample size (i.e. not solved explicitly with respect to $N$) are provided in [15], [21] and given $\varepsilon$, $\beta$ and $r$, numerical inversion is required to compute $N$.

There are multiple ways to select which $r$ samples to discard [15]; however, based on our simulation study the largest improvement in the cost is achieved when a greedy approach is adopted. We first solve the problem with $N$ constraints and identify the ones that are active for the problem where we want to determine the minimum volume hyper-rectangle, or in other words the samples that lie on its facets. We then remove the one which results in the hyper-rectangle that leads to the highest reduction in the objective value $J(x)$. In case multiple samples lie on the same facet, we remove all of them at the same time.

To illustrate the potential improvement afforded by using the sampling and discarding methodology in conjunction with the probabilistically robust design, we applied it to a variant of the problem of Chapter 2 [70]. Specifically, we considered the IEEE 30-bus network [88], modified to include two wind power generators (i.e. $N_\delta = 2$) at buses 7 and 19, and two VSC-HVDC lines connecting nodes 8 with 12, and 19 with 22. To model the HVDC lines and their controllability actions we followed the approach of Chapter 6.

Fig. A.1 shows the samples generated for the wind power of each generator. The pronounced triangular structure is due to fact that correlation was taken into account in the wind power model. When using the probabilistically robust design of Section 2.4.2 we generate samples
A.1. Trading feasibility to optimality

Figure A.1: Samples and rectangular sets generated according to the sampling and discarding methodology. The “black” and “green” color correspond to the scenario approach and the sampling and discarding approach, respectively. The outer rectangle (solid “green” line) corresponds to the case where we generate samples based on (A.8) but do not discard any of them, whereas the inner rectangle (dashed “green” line) corresponds to the situation where \( r \) out of \( N \) samples are discarded.

According to (2.23) and construct the set \( \bar{\Delta}^N \). Since we have two wind power generators, \( \bar{\Delta}^N \) is a rectangle as shown in Fig. A.1 with “black”. When employing the sampling and discarding procedure outlined above, we generate samples according to (A.8). Since now we are allowed to discard \( r \) samples, the resulting rectangle (inner “green” rectangle) has smaller volume compared to what we would have if no samples were discarded (outer “green” rectangle). Since the volume of the inner rectangle is smaller than the “black” one, we expect the solution of the robust problem to be less conservative, leading to a lower cost.

To demonstrate the efficiency of our approach we carried out a Monte Carlo analysis. The solution of the optimization problem was evaluated against 10,000 wind power realizations different from those used in the optimization process. We then computed the empirical probability of constraint violation, calculated as the fraction out of the 10,000 scenarios where at least one of the system constraints is violated. Note that the theoretical guarantees are given by \( \varepsilon \), and in this case we set \( \varepsilon = 10\% \). Fig. A.2 shows for the cases \( r = 10, 20, 40 \) how the em-
Figure A.2: Empirical probability of constraint violation calculated as the fraction out of 10,000 evaluation scenarios where the solution obtained by the optimization program violates at least one of the constraints. Three different cases are simulated $r = 10,20,40$, and for each case the samples are discarded sequentially.

The empirical probability of constraint violation changes as we progressively discard more samples. Using a sampling and discarding procedure, as the number of samples $r$ we discard increases, the empirical probability of constraint violation tends to the theoretical $\varepsilon$-type guarantees.

This is also illustrated by means of Fig. A.3. The upper panel of Fig. A.3 shows the additional cost (as determined by the optimization problems) incurred due to a probabilistically robust design, relative to the cost due to a deterministic SC-OPF (for comparison its cost is set to zero). The second case corresponds to the situation where we adopt the probabilistically robust design without performing the sampling and discarding procedure, whereas the last three correspond to problem instances where $r = 10,20,40$ samples are discarded, respectively. Notice that allowing for constraints to be discarded, a significant reduction in the cost is achieved. Similarly, the lower panel of Fig. A.3 shows the empirical probability of constraint violation. The more samples $r$ we discard the closer this value gets to the theoretical value of $\varepsilon$ ("dashed" line). Note that for the deterministic SC-OPF this value is significantly above $\varepsilon$, since no uncertainty is taken into account in the design phase.
A.1. Trading feasibility to optimality

Figure A.3: Upper panel: Additional cost incurred due to a probabilistically robust design, relative to the cost due to a deterministic SC-OPF (for comparison its cost is set to zero). The second case corresponds to the situation where we adopt the probabilistically robust design without performing the sampling and discarding procedure, whereas the last three correspond to problem instances where $r = 10, 20, 40$ samples are discarded, respectively. Lower panel: Empirical probability of constraint violation for the same problem instances with the upper panel.

A.1.2 Heuristic approach

In this section, motivated by Algorithm 1 of Section 3.4.1, we provide a heuristic procedure to improve the solution of the standard scenario approach and the probabilistically robust design in terms of cost. Consider first the standard scenario approach of (A.3)-(A.4), and let $x^N \in \mathbb{R}^{N_x}$ be its optimal solution. The solution $x^N$ would then be feasible for (A.2) with probability at least $1 - \beta$. Fix now $r$ elements of $x$ to be equal to the corresponding elements of $x^N$ and repeat the entire process considering the underlying optimization with the remaining $N_x - r$
decision variables. Therefore, we formulate (A.3)-(A.4) using only

\[
N_r \geq \frac{1}{\varepsilon_t} \frac{e}{e-1} \left( \ln \frac{1}{\beta_t} + (N_x - r) - 1 \right)
\]  

(A.9)
of the \( N \) samples that were generated according to (2.20). The constraints in (A.4) would then be only a subset of the constraints of the initial problem since \( N_r < N \) and we did not perform any resampling. Note that at every iteration of this approach the resulting solution can not degrade in terms of cost (i.e. its can be no worse that \( x^N \)), since we always use a portion of the initially generated \( N \) samples. Therefore, the resulting solution is by construction feasible, with probability at least \( 1 - \beta \), for (A.2), conditioning on the fact that \( r \) elements of \( x \) were fixed according to \( x^N \) (recall that \( x^N \) is a random a variable since it depends on \( N \) randomly extracted samples of the uncertainty). We can then repeat this process by fixing any other subset with \( r \) elements of the optimal solution of the resulting problem (alternatively one can fix fewer or more elements). Note that resampling at every iteration of the proposed heuristic is not preferable, since the solution may then degrade in terms of cost at some iteration of the algorithm.

The aforementioned procedure can be also applied to the probabilistically robust design. The difference is that we now apply our heuristic scheme at Step 1 of the approach of Section 2.4.2. Consider the problem (2.21)-(2.22) and let \( p^{\text{min}, N}, p^{\text{max}, N} \in \mathbb{R}^{N\delta} \) be its optimal solution when it is solved using the scenario approach with \( N \) chosen according to (2.23). Applying our procedure, we fix \( r \) elements of \( p^{\text{min}}, p^{\text{max}} \) to the corresponding values of \( p^{\text{min}, N}, p^{\text{max}, N} \). The resulting problem would have \( 2(N\delta - r) \) decision variables and could be solved using the scenario approach by invoking (2.23) with \( N\delta - r \) in place of \( N\delta \). The interpretation of our heuristic approach is that at every iteration we can construct a hyper-rectangular set, possibly with smaller volume compared with the initial one, thus obtaining a solution with lower cost when solving the robust problem in (A.5)-(A.6).
Bibliography


Curriculum Vitae

Feb 2009 - Dec 2013  ETH Zurich, Switzerland.
PhD studies at the Power Systems Laboratory,
Department of Information Technology and Electrical Engineering.

Oct 2002 - Feb 2008  University of Patras, Greece.
Diploma in Electrical and Computer Engineering.
Graduation with honors.

Sep 1999 - Jun 2002  High School studies, Athens, Greece.

Feb 25, 1985  born in Athens, Greece.