High-concentration solar trough collectors and their application to concentrating photovoltaics

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HIGH-CONCENTRATION SOLAR TROUGH COLLECTORS AND THEIR APPLICATION TO CONCENTRATING PHOTOVOLTAICS

A thesis submitted to attain the degree of

DOCTOR OF SCIENCES of ETH ZURICH
(Dr. sc. ETH Zurich)

presented by

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2014
Abstract

Due to the dilute nature of the solar radiation reaching the surface of the earth, the greatest power output that can be generated by any solar collector is around 1 kW per m² of collecting aperture. Minimization of the cost per m² of collecting aperture is therefore considered to be a major driver in achieving a cost-competitive system. Due to their line symmetry, solar trough collectors have a low-cost construction and are highly-scalable, making them good candidates for achieving this primary goal. However, this line symmetry restricts the achievable solar concentration to at most 215×, thus limiting range of useful applications and the efficiency of downstream processes.

This work investigates ways in which this concentration limit can be broken through the introduction of a novel secondary concentrator stage, while still maintaining the benefits of having a line-symmetric primary concentrator. Thus the concept of the high-concentration solar trough collector is introduced.

The starting point for a high-concentration solar trough collector should be a low-cost, high-performance trough. For this purpose, a construction based on inflated polymer membranes mounted on a rigid concrete structure is considered. The mirror comprises a stack of \( N \) polymer membranes, the topmost membrane being metallized to form the reflector surface. When appropriately dimensioned and inflated, the top membrane assumes the shape of \( N \) tangentially connected circular arcs whose shape has been denoted the “arcspline”. This inflated construction allows for primary aperture widths of up to 10 m and trough lengths of over 200 m to be realized. Improved methods to design the arcspline profile are developed. In particular, it is shown that, as a nonimaging concentrator, the arcspline can always be designed to match the concentration of a parabola.

To break the 2D limit of concentration inherent to line-symmetric systems, the line-to-point (LTP) focus optical configuration is presented. An LTP
concentrator utilizes an array of nonimaging secondary concentrators arranged along the primary focal line, thus effectively creating a three-dimensional concentrator structure which allows the 2D limit to be considerably surpassed. By incorporating an individual tracking axis perpendicular to the primary tracking axis into each secondary concentrator, it is shown that total concentrations of up to 6000× are theoretically possible. This considerable augmentation in concentration warrants the additional complexity of the system resulting from the introduction of the secondary stage.

In addition to significantly augmenting the achievable concentration, the LTP configuration breaks up the continuous line focus of the primary into a multitude of discrete point-like foci. This is a particular advantage for the application of concentrating photovoltaics (CPV), since smaller solar cells can be used, leading to higher efficiencies and more effective cell cooling. The CPV application is therefore considered in detail, and a linear semi-dense solar cell array comprising five 1 cm² triple-junction concentrator cells is developed for use in LTP systems.

Based on the proposed inflated trough and line-to-point focus concepts, the design, modeling and experimental proof-of-concept of a 600× high-concentration photovoltaic (HCPV) collector is presented. The design utilizes an arcspline primary concentrator with an array of tracking reflective nonimaging concentrators arranged along its focal line. The secondaries are based on asymmetrically truncated $\theta_1/\theta_0$ transformers crossed with hyperbolic side-walls each with an array of five triple-junction solar cells coupled to its exit aperture. The primary achieves a concentration of $C_{g,1} = 72.4\times$ while each secondary achieves $C_{g,2} = 8.14\times$, resulting in a total concentration of $C_{g,tot} = 590\times$ for the system.

An optical-electrical model of the proposed system was developed using the Monte Carlo ray-tracing technique to simulate the transfer of radiant energy through the optical system, and an equivalent circuit model to predict the electrical performance of the array. The model predicts peak optical, array and overall system efficiencies of 62.5%, 34.9% and 21.7% respectively. The optical efficiency is limited mostly by the materials used in the initial design, while the array efficiency is limited mostly by irradiance mismatch between the
subcells of the triple-junction cell, resulting from the optical transfer function of the system. By improving the optical properties of the concentrator materials, and using cells with a spectral response tailored to the spectrum at the exit of the secondaries, system efficiencies approaching 30% are expected.

An on-sun prototype of the proposed HCPV collector was constructed in Biasca, Switzerland. The prototype comprises a 1.2 m long section of the primary trough concentrator, constructed from aluminum sheet mirror pressed on a form having the exact multi-arc shape of the arcspline concentrator. At the focus of the primary, an array of ten secondary concentrator modules was placed. The peak irradiance measured over the array was 328.0 kW/m² at a direct normal irradiance of 901.9 W/m², corresponding to a mean flux concentration ratio over the array of 364 suns: the highest solar concentration ratio ever measured on a parabolic-trough-based system. A maximum solar-to-DC efficiency of 20.2% was measured for the system.

Based on the successful on-sun prototype demonstration, a 15 kW pilot plant comprising an array of 200 secondary concentrator modules mounted at the focus of a 52 m long inflated trough collector, is currently being constructed in Biasca, Switzerland.

The methods and designs presented in this work open new avenues for solar trough collectors, reducing their cost, increasing their efficiency, and widening their range of potential applications.
Zusammenfassung

Aufgrund der geringen Strahlungsdichte der auf die Erde auftreffenden Sonnenstrahlung beträgt die maximale Leistung die von einem Solarkollektor erzielt werden kann, etwa 1 kW pro m² Primäraperatur. Die Minimierung der Kosten pro m² Primäraperatur ist demzufolge einer der wichtigsten Faktoren zur Verwirklichung eines wettbewerbsfähigen Systems. Parabolrinnen-Kollektoren sind, dank ihrer Liniensymmetrie, skalierbare und kostengünstige Systeme und somit vielversprechende Kandidaten zum Erreichen dieses primären Ziels. Diese Liniensymmetrie beschränkt jedoch die maximal erreichbare Solarkonzentration auf 215× weshalb die nützlichen Anwendungen solcher Systeme und die Effizienz der nachfolgenden Prozesse begrenzt sind.

Diese Arbeit untersucht Möglichkeiten, wie das erwähnte Konzentrationslimit durch die Einführung einer neuartigen sekundären Konzentratorstufe überschritten werden kann, während die Vorteile eines liniensymmetrischen Primärkonzentrators beibehalten werden können. Somit wird das Konzept eines hochkonzentrierenden Rinnenkollektors eingeführt.

Der Ausgangspunkt für einen hochkonzentrierenden Rinnenkollektor muss eine kosteneffiziente Hochleistungs-Rinne sein. Für diesen Zweck wird eine Konstruktion basierend auf aufgeblasenen Polymemembranen, welche auf einer starren Betonkonstruktion angebracht sind, betrachtet. Der Spiegel besteht aus einem Verbund bestehend aus $N$ Polymemembranen, wobei die oberste Membran metallisiert ist und die Spiegeloberfläche bildet. Bei entsprechender Dimensionierung und Aufblähung, übernimmt die oberste Membran die Form von $N$ tangential verbunden Kreisbögen, deren Form als „Arcspline“ bezeichnet wird. Diese aufgeblasene Konstruktion ermöglicht primäre Aperturweiten von bis zu 10 m und Rinnenlängen von über 200 m. Stark verbesserte Design-Methoden für das „Arcspline“-Profil wurden entwickelt. Insbesondere wird aufgezeigt, dass dieses als nichtabbildender Konzentrator immer so gestaltet
werden kann, dass die gleiche Konzentration wie mit einem parabolischen Konzentratork erreicht wird.


Auf Grundlage der vorgeschlagenen aufgeblasenen Rinnen-Solarkollektor- und „line-to-point“-Fokuskonzepte, werden das Design, die Modellierung und der experimentelle Nachweis von einem $600\times$ Hochkonzentrations-Photovoltaikkollektor (HCPV) vorgestellt. Das Design verwendet einen „Arcspline“-Primärkonzentrator mit einer Reihe von nichtabbildenden drehenden Sekundärkonzentratoren, die entlang der Brennlinie angeordnet sind. Jeder Sekundärkonzentrator besteht aus einem asymmetrisch abgeschnittenen $\theta_i/\theta_o$ Transformer mit hyperbolischen Seitenwänden. Am Ausgang jedes Sekundärkonzentrators ist eine Reihe von fünf Triple-Junction-Solarzellen platziert. Der Primärkonzentrator erreicht eine Konzentration von $C_{g,1} = 72.4\times$ und jeder Sekundärkonzentrator erreicht $C_{g,2} = 8.14\times$, was einer Konzentration von $C_{g,tot} = 590\times$ für das Gesamtsystem entspricht.

Ein real getester Prototyp des vorgeschlagenen HCPV-Kollektors wurde in Biasca (Schweiz) aufgebaut. Der Prototyp besteht aus einem 1.2 m langen Abschnitt des primären Rinnen-Konzentrators. Der Abschnitt ist aus Aluminiumblechspiegel aufgebaut und auf einer Struktur mit der genauen „Arcspline“-Form gepresst. Im Fokus des Primärkonzentrators wurde eine Reihe von zehn Sekundärkonzentrator-Modulen montiert. Die über der Reihe gemessene Spitzenbestrahlungsstärke war 328.0 kW/m² bei einer Direktnormalstrahlung von 901.9 W/m², was einer mittleren Strahlungsflussdichtekonzentration über der Reihe von 364 Sonnen entspricht und das höchste je gemessene Sonnenkonzentrationsverhältnis mit einem auf einem Parabolrinnenkonzentrator basierenden System darstellt. Ein maximaler „solar-to-DC“ Wirkungsgrad von 20.2% wurde für das System gemessen.

Basierend auf der erfolgreichen Demonstration wird derzeit eine 15 kW-Pilotanlage in Biasca aufgebaut. Die Pilotanlage besteht aus einer Reihe von 200 Sekundärkonzentrator-Modulen, die im Fokus eines 52 m langen aufgeblasen Rinnen-Solarkollektors montiert werden.
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# Nomenclature

## Latin characters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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</thead>
<tbody>
<tr>
<td>( a )</td>
<td>apothem</td>
<td>m</td>
</tr>
<tr>
<td>( \hat{a} )</td>
<td>unit vector parallel to primary tracking axis</td>
<td>-</td>
</tr>
<tr>
<td>( a' )</td>
<td>directional acceptance function</td>
<td>-</td>
</tr>
<tr>
<td>( \bar{a}' )</td>
<td>area-averaged directional acceptance</td>
<td>-</td>
</tr>
<tr>
<td>( a_i )</td>
<td>inlet aperture full-width, half-width in Section 3.4</td>
<td>m</td>
</tr>
<tr>
<td>( a_o )</td>
<td>outlet/exit aperture full-width, half-width in Section 3.4</td>
<td>m</td>
</tr>
<tr>
<td>( A )</td>
<td>area</td>
<td>m²</td>
</tr>
<tr>
<td>( A_{active} )</td>
<td>active area of cell/array</td>
<td>m²</td>
</tr>
<tr>
<td>( A_i )</td>
<td>inlet aperture area</td>
<td>m²</td>
</tr>
<tr>
<td>( A_o )</td>
<td>outlet/exit aperture area</td>
<td>m²</td>
</tr>
<tr>
<td>( A_{receiver} )</td>
<td>receiver area</td>
<td>m²</td>
</tr>
<tr>
<td>( \mathbf{A} )</td>
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<td>-</td>
</tr>
<tr>
<td>( AM )</td>
<td>relative optical air mass</td>
<td>-</td>
</tr>
<tr>
<td>( AOI )</td>
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<td>( AR )</td>
<td>aspect ratio</td>
<td>-</td>
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<td>( b )</td>
<td>function used in designing corrector mirrors for circular primaries</td>
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<td>-</td>
</tr>
<tr>
<td>( c )</td>
<td>leaf constant in LPC design</td>
<td>-</td>
</tr>
<tr>
<td>( c )</td>
<td>constant of integration</td>
<td>-</td>
</tr>
<tr>
<td>( c_0 )</td>
<td>speed of light in vacuum</td>
<td>m/s</td>
</tr>
<tr>
<td>( c_{ijkl} )</td>
<td>component of stiffness tensor</td>
<td>Pa</td>
</tr>
<tr>
<td>( C )</td>
<td>flux concentration ratio</td>
<td>suns</td>
</tr>
<tr>
<td>( \mathbf{C} )</td>
<td>(center-) point</td>
<td>-</td>
</tr>
<tr>
<td>( C_{jk} )</td>
<td>component of stiffness matrix</td>
<td>Pa</td>
</tr>
</tbody>
</table>

1 Throughout this thesis, bold typeface is used to denote vector quantities. As the components of a vector are scalars, they are written in normal typeface with a subscript index, e.g. the \( x \)-component of the vector \( \mathbf{A} \) is written as \( A_x \).
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<td>$C_1$</td>
<td>quantity used in optical pathlength tailoring</td>
<td>m</td>
</tr>
<tr>
<td>$C_2$</td>
<td>quantity used in optical pathlength tailoring</td>
<td>m</td>
</tr>
<tr>
<td>$C_g$</td>
<td>geometric concentration ratio</td>
<td>×</td>
</tr>
<tr>
<td>$C_{g,1}$</td>
<td>primary geometric concentration ratio</td>
<td>×</td>
</tr>
<tr>
<td>$C_{g,2}$</td>
<td>secondary geometric concentration ratio</td>
<td>×</td>
</tr>
<tr>
<td>$C_{g,tot}$</td>
<td>overall geometric concentration ratio based on exit area of secondary concentrator</td>
<td>×</td>
</tr>
<tr>
<td>$C_{g,tot,active}$</td>
<td>overall geometric concentration ratio based on active cell area</td>
<td>×</td>
</tr>
<tr>
<td>CR</td>
<td>contrast ratio</td>
<td>-</td>
</tr>
<tr>
<td>ch</td>
<td>chord length</td>
<td>m</td>
</tr>
<tr>
<td>$d$</td>
<td>distance between inlet aperture and center of curvature of curved secondary concentrator inlet</td>
<td>m</td>
</tr>
<tr>
<td>d</td>
<td>differential operator</td>
<td>-</td>
</tr>
<tr>
<td>$d_{	ext{pinhole}}$</td>
<td>pinhole diameter</td>
<td>m</td>
</tr>
<tr>
<td>$d_{	ext{sun}}$</td>
<td>distance from the center of the sun to earth’s surface</td>
<td>m</td>
</tr>
<tr>
<td>D</td>
<td>point</td>
<td>-</td>
</tr>
<tr>
<td>DNI</td>
<td>direct normal irradiance</td>
<td>W/m²</td>
</tr>
<tr>
<td>$e_1$, $e_2$, $e_3$</td>
<td>basis vectors for 3D stress state</td>
<td>-</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s modulus</td>
<td>Pa</td>
</tr>
<tr>
<td>$E$ (area-averaged)</td>
<td>irradiance</td>
<td>W/m²</td>
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<tr>
<td>$E_{	ext{earth}}$</td>
<td>yearly mean irradiance at earth’s surface</td>
<td>W/m²</td>
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<tr>
<td>$E_{	ext{av}}$</td>
<td>irradiance averaged over active area of cell</td>
<td>W/m²</td>
</tr>
<tr>
<td>$\langle E_{	ext{av}} \rangle$</td>
<td>irradiance averaged over active area of module</td>
<td>W/m²</td>
</tr>
<tr>
<td>$E_{\text{ph}}$</td>
<td>photonic irradiance</td>
<td>#/(m²·s)</td>
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<tr>
<td>$E_{\text{pre-filter}}$</td>
<td>average pre-filter irradiance</td>
<td>W/m²</td>
</tr>
<tr>
<td>EQE</td>
<td>external quantum efficiency</td>
<td>-</td>
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<tr>
<td>$f$</td>
<td>objective function</td>
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<tr>
<td>$f$ (focal length)</td>
<td>focal length</td>
<td>m</td>
</tr>
<tr>
<td>$f$ (fraction of rays within acceptance angle that are accepted by an LPC)</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$f_e$</td>
<td>effective focal length</td>
<td>-</td>
</tr>
<tr>
<td>$f_n$</td>
<td>fraction of rays suffering $n$ reflections before reaching the outlet</td>
<td>-</td>
</tr>
<tr>
<td>$f(\varphi)$</td>
<td>function used in two-mirror aplanat equations</td>
<td>m</td>
</tr>
<tr>
<td>F</td>
<td>primary focal point in x-z plane</td>
<td>-</td>
</tr>
<tr>
<td>F</td>
<td>force vector</td>
<td>N</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td></td>
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<tr>
<td>--------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>FF</td>
<td>fill factor</td>
<td></td>
</tr>
<tr>
<td>g</td>
<td>design parameter for LPCs</td>
<td></td>
</tr>
<tr>
<td>$g(\phi)$</td>
<td>function used in two-mirror aplanat equations</td>
<td></td>
</tr>
<tr>
<td>$G$</td>
<td>étendue</td>
<td></td>
</tr>
<tr>
<td>$G_{s}$</td>
<td>grayscale value of pixel</td>
<td></td>
</tr>
<tr>
<td>$h$</td>
<td>Planck constant</td>
<td></td>
</tr>
<tr>
<td>$h_F$</td>
<td>height of fingers of solar cell front grid metallization</td>
<td></td>
</tr>
<tr>
<td>$H$</td>
<td>horizontal component of tension</td>
<td></td>
</tr>
<tr>
<td>$H_{s}$</td>
<td>point</td>
<td></td>
</tr>
<tr>
<td>$i$</td>
<td>branch current</td>
<td></td>
</tr>
<tr>
<td>$I$</td>
<td>current</td>
<td></td>
</tr>
<tr>
<td>$I_0$</td>
<td>diode reverse saturation current</td>
<td></td>
</tr>
<tr>
<td>$I_{0,BD}$</td>
<td>reverse saturation current of bypass diode</td>
<td></td>
</tr>
<tr>
<td>$I_{loss}$</td>
<td>current loss due to recombination</td>
<td></td>
</tr>
<tr>
<td>$I_{ph}$</td>
<td>photogenerated current (photocurrent)</td>
<td></td>
</tr>
<tr>
<td>$I_{SC}$</td>
<td>short-circuit current</td>
<td></td>
</tr>
<tr>
<td>$j$</td>
<td>index</td>
<td></td>
</tr>
<tr>
<td>$J$</td>
<td>current density</td>
<td></td>
</tr>
<tr>
<td>$J_{01,j}$</td>
<td>reverse saturation current density of diode 1 of $j^{th}$ subcell</td>
<td></td>
</tr>
<tr>
<td>$J_{02,j}$</td>
<td>reverse saturation current density of diode 2 of $j^{th}$ subcell</td>
<td></td>
</tr>
<tr>
<td>$J_{ph}$</td>
<td>photocurrent density</td>
<td></td>
</tr>
<tr>
<td>$k$</td>
<td>index</td>
<td></td>
</tr>
<tr>
<td>$k$</td>
<td>constant in LPC design</td>
<td></td>
</tr>
<tr>
<td>$k$</td>
<td>transverse membrane stiffness</td>
<td></td>
</tr>
<tr>
<td>$k_B$</td>
<td>Boltzmann constant</td>
<td></td>
</tr>
<tr>
<td>$k_{cell}$</td>
<td>cell cost per unit area</td>
<td></td>
</tr>
<tr>
<td>$k_{conc.}$</td>
<td>concentrator cost per unit collecting aperture area</td>
<td></td>
</tr>
<tr>
<td>$K$</td>
<td>design parameter for two-mirror aplanats</td>
<td></td>
</tr>
<tr>
<td>$K_{err}$</td>
<td>reduction in acceptance efficiency due to surface error</td>
<td></td>
</tr>
<tr>
<td>$L$</td>
<td>radiance (radiation intensity)$^2$</td>
<td></td>
</tr>
<tr>
<td>$L_0$</td>
<td>unstretched membrane length</td>
<td></td>
</tr>
</tbody>
</table>

2 In this thesis, the radiometry nomenclature is used instead of the radiation heat transfer nomenclature, as it is more convenient for nonimaging optical design. The exception is for the radiant power for which the symbol $Q$ is used instead of the radiometry standard $\Phi$. 
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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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</thead>
<tbody>
<tr>
<td>$L_{\text{longbranch}}$</td>
<td>length of long branch of asymmetrically truncated $\theta_i/\theta_o$ transformer</td>
<td>m</td>
</tr>
<tr>
<td>$L_{\text{pitch}}$</td>
<td>distance between fingers of solar cell front grid</td>
<td>m</td>
</tr>
<tr>
<td>$L_{\text{shortbranch}}$</td>
<td>length of short branch of asymmetrically truncated $\theta_i/\theta_o$ transformer</td>
<td>m</td>
</tr>
<tr>
<td>$L_{\text{sun}}$</td>
<td>radiance at sun's surface</td>
<td>W/(m²·sr)</td>
</tr>
<tr>
<td>$\Delta L$</td>
<td>change in length</td>
<td>m</td>
</tr>
<tr>
<td>$m$</td>
<td>slope, $dz(x)/dx$</td>
<td>-</td>
</tr>
<tr>
<td>$m$</td>
<td>number of constant-width leaves in LPC</td>
<td>-</td>
</tr>
<tr>
<td>$M$</td>
<td>number of measurement points from SC to OC on the $I-V$ curve</td>
<td>-</td>
</tr>
<tr>
<td>$M_{\text{sun}}$</td>
<td>radiant exitance at the surface of the sun</td>
<td>W/m²</td>
</tr>
<tr>
<td>$n$</td>
<td>refractive index</td>
<td>-</td>
</tr>
<tr>
<td>$n$</td>
<td>number of sides of a polygon</td>
<td>-</td>
</tr>
<tr>
<td>$n$</td>
<td>number of constant-ratio leaves in LPC</td>
<td>-</td>
</tr>
<tr>
<td>$n^*$</td>
<td>apparent refractive index for refraction at a curved surface with linear symmetry</td>
<td>-</td>
</tr>
<tr>
<td>$n_{\text{BD}}$</td>
<td>diode ideality factor for bypass diode</td>
<td>-</td>
</tr>
<tr>
<td>$n_{\text{D}}$</td>
<td>diode ideality factor for solar cell</td>
<td>-</td>
</tr>
<tr>
<td>$n_r$</td>
<td>number of reflections</td>
<td>-</td>
</tr>
<tr>
<td>$\langle n_r \rangle$</td>
<td>average number of reflections</td>
<td>-</td>
</tr>
<tr>
<td>$\langle n_r \rangle_{\text{break-even}}$</td>
<td>average number of reflections for which enclosed and conventional collectors break even in terms of optical efficiency</td>
<td>-</td>
</tr>
<tr>
<td>$\hat{n}$</td>
<td>unit normal vector</td>
<td>-</td>
</tr>
<tr>
<td>$N$</td>
<td>number of arcs in an arcspline</td>
<td>-</td>
</tr>
<tr>
<td>$N$</td>
<td>total number of leaves in one side of a LPC</td>
<td>-</td>
</tr>
<tr>
<td>$N$</td>
<td>number of cells in mini-module</td>
<td>-</td>
</tr>
<tr>
<td>$N$</td>
<td>number measured irradiance levels</td>
<td>-</td>
</tr>
<tr>
<td>$O$</td>
<td>origin</td>
<td>-</td>
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<tr>
<td>$\text{OPL}$</td>
<td>optical pathlength</td>
<td>m</td>
</tr>
<tr>
<td>$\text{OTF}$</td>
<td>optical transfer function</td>
<td>-</td>
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<tr>
<td>$p$</td>
<td>direction-cosine in $x$-direction</td>
<td>-</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$p_{2-1}$</td>
<td>interaction pressure exerted by membrane 2 onto membrane 1</td>
<td>Pa</td>
</tr>
<tr>
<td>$p_i$</td>
<td>internal pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
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<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>$p_0$</td>
<td>external pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$\Delta p$</td>
<td>pressure difference</td>
<td>Pa</td>
</tr>
<tr>
<td>$P$</td>
<td>power</td>
<td>W</td>
</tr>
<tr>
<td>$P_{\text{m}}$</td>
<td>power at MPP (maximum power)</td>
<td>W</td>
</tr>
<tr>
<td>$\mathbf{P}$</td>
<td>point</td>
<td>-</td>
</tr>
<tr>
<td>PAR</td>
<td>peak-to-average irradiance ratio</td>
<td>-</td>
</tr>
<tr>
<td>$g$</td>
<td>direction-cosine in $y$-direction</td>
<td>-</td>
</tr>
<tr>
<td>$q$</td>
<td>elementary charge</td>
<td>C</td>
</tr>
<tr>
<td>$q_{\text{in}}$</td>
<td>input power per unit area</td>
<td>W/m²</td>
</tr>
<tr>
<td>$Q$</td>
<td>radiant power</td>
<td>W</td>
</tr>
<tr>
<td>$Q_{\text{array}}$</td>
<td>radiant power reaching the active area of the array</td>
<td>W</td>
</tr>
<tr>
<td>$Q_{\text{diss}}$</td>
<td>dissipated heat load on array</td>
<td>W</td>
</tr>
<tr>
<td>$Q_{\text{in}}$</td>
<td>input power</td>
<td>W</td>
</tr>
<tr>
<td>$Q_{\text{loss}}$</td>
<td>lost power</td>
<td>W</td>
</tr>
<tr>
<td>$Q_{\text{solar}}$</td>
<td>radiant power reaching inlet of primary</td>
<td>W</td>
</tr>
<tr>
<td>$Q_{\text{useful}}$</td>
<td>useful power</td>
<td>W</td>
</tr>
<tr>
<td>$\mathbf{Q}$</td>
<td>point</td>
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<tr>
<td>$r$</td>
<td>direction-cosine in $z$-direction</td>
<td>-</td>
</tr>
<tr>
<td>$r(\phi)$</td>
<td>parametric form of a parabola</td>
<td>-</td>
</tr>
<tr>
<td>$r_0$</td>
<td>radius of absorber of circular cross-section</td>
<td>m</td>
</tr>
<tr>
<td>$\hat{\mathbf{r}}$</td>
<td>unit direction vector of ray after being reflected by primary</td>
<td>-</td>
</tr>
<tr>
<td>$\mathbf{r}$</td>
<td>ray</td>
<td>-</td>
</tr>
<tr>
<td>$R_c$</td>
<td>radius of curvature</td>
<td>m</td>
</tr>
<tr>
<td>$R_C$</td>
<td>contact resistance at interface of front grid and solar cell</td>
<td>$\Omega \cdot \text{cm}^2$</td>
</tr>
<tr>
<td>$R_{\text{earth}}$</td>
<td>radius of the earth</td>
<td>m</td>
</tr>
<tr>
<td>$R_{\text{LCL}/j}$</td>
<td>sheet resistance of lateral conduction layer $j$</td>
<td>$\Omega$/sq</td>
</tr>
<tr>
<td>$R_s$</td>
<td>series resistance</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>$R_{\text{BD}}$</td>
<td>series resistance of bypass diode</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>$R_{\text{sh}}$</td>
<td>shunt resistance</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>$R_{\text{sun}}$</td>
<td>radius of the sun (photosphere)</td>
<td>m</td>
</tr>
<tr>
<td>$R_{\text{sup}}$</td>
<td>radius of support arc</td>
<td>m</td>
</tr>
<tr>
<td>$R_{\text{TJ}/j}$</td>
<td>resistance of tunnel junction $j$</td>
<td>$\Omega \cdot \text{cm}^2$</td>
</tr>
<tr>
<td>$\mathbf{R}$</td>
<td>rotation matrix</td>
<td>-</td>
</tr>
<tr>
<td>RH</td>
<td>relative humidity</td>
<td>%</td>
</tr>
</tbody>
</table>
\(s\)  
- polygon sidelength \(\text{m}\)

\(s\)  
- design parameter for two-mirror aplanats

\(\hat{s}\)  
- unit vector pointing to center of solar disk

\(S\)  
- stretched membrane length \(\text{m}\)

\(S_0\)  
- solar constant \(\text{W/m}^2\)

\(t\)  
- time \(\text{s}\)

\(t\)  
- LPC leaf thickness \(\text{m}\)

\(t\)  
- membrane thickness \(\text{m}\)

\(\Delta t_j\)  
- change in membrane thickness at load step \(j\) \(\text{m}\)

\(T\)  
- tension per \(\text{m}\) of trough length \(\text{N/m}\)

\(\Delta T_j\)  
- change in membrane tension at load step \(j\) \(\text{N/m}\)

\(T\)  
- temperature \(\text{°C}\)

\(T_0\)  
- standard temperature \(\text{°C}\)

\(T_{\text{sun}}\)  
- effective temperature of the sun’s photosphere \(\text{K}\)

\(T\)  
- pole of corrector mirror

\(u\)  
- focal function \(\text{m}\)

\(U\)  
- cell-to-cell uniformity

\(U\)  
- transmittance

\(v\)  
- voltage across one cell/bypass diode pair in mini-module \(\text{V}\)

\(v_{\text{F}}\)  
- forward bias voltage drop across bypass diode \(\text{V}\)

\(\hat{v}\)  
- unit direction vector of a ray

\(V\)  
- voltage \(\text{V}\)

\(V\)  
- vertical component of tension \(\text{N/m}\)

\(V_{\text{m}}\)  
- voltage at MPP \(\text{V}\)

\(V_{\text{OC}}\)  
- open-circuit voltage \(\text{V}\)

\(V_{\text{R,BD}}\)  
- reverse breakdown voltage of bypass diode \(\text{V}\)

\(V_T\)  
- thermal voltage \(\text{V}\)

\(V\)  
- vertex of primary concentrator

\(w\)  
- aperture width in symmetry (extrusion) direction \(\text{m}\)

\(w\)  
- membrane weight per \(\text{m}^2\) \(\text{N/m}^2\)

\(w_\varphi\)  
- width of edge-ray pencil from polar angle \(\varphi\) at paraxial focus \(\text{m}\)

\(w\)  
- wavefront

\(W_F\)  
- width at base of fingers of solar cell front grid metallization \(\text{m}\)

\(x, y, z\)  
- primary (collector) coordinate frame
Nomenclature

$x', y', z'$ coordinate frame for tilted focal plane

$x'', y'', z''$ coordinate frame for rotated/tracking secondary concentrator

$x_0, y_0, z_0$ coordinate frame of collector in parked position

$x_L$ $x$-coordinate at left edge of arc in arcspline

$x_p, z_p$ coordinates of first mirror in two-mirror aplanat

$x_R$ $x$-coordinate at right edge of arc in arcspline

$x_s, z_s$ coordinates of second mirror in two-mirror aplanat

$X$ general optical property

$X, Y, Z$ cardinal coordinate frame

$X$ intersection point

$\dot{z}$ slope $dz(x)/dz$ of mirror profile

$z_C$ central (parabolic) solution of ideal concave mirror problem

$\dot{z}_C$ slope of central (parabolic) solution of ideal concave mirror problem

$z_L$ left solution of ideal concave mirror problem

$z_R$ right solution of ideal concave mirror problem

$\dot{z}_L$ slope of left solution of ideal concave mirror problem

$\dot{z}_R$ slope of right solution of ideal concave mirror problem

Greek characters

$\alpha$ absorptivity

$\alpha$ axial beamspread half-angle

$\alpha$ beam collimation half-angle

$\alpha_a$ axis altitude angle

$\alpha_{crit}$ maximum axial beamspread half-angle

$\alpha_{hygro}$ coefficient of hygroscopic expansion $\text{m/(m\cdot \%RH)}$

$\alpha_s$ solar altitude angle

$\alpha_{thermal}$ coefficient of thermal expansion $\text{m/(m\cdot K)}$

$\beta$ approach angle

$\gamma$ intercept factor

$\gamma_a$ axis azimuth angle

$\gamma_s$ solar azimuth angle

$\delta$ solar declination angle

$\delta$ truncation angle
<table>
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<tr>
<th>Symbol</th>
<th>Definition</th>
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<td>$\delta_{ij}, \delta_{kl}$</td>
<td>Kronecker delta</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>misalignment angle</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>strain</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>tilt angle (obliquity) of earth’s axis</td>
</tr>
<tr>
<td>$\epsilon_{I-V}$</td>
<td>rms error between experimental and simulated $I-V$ curves</td>
</tr>
<tr>
<td>$\epsilon_{\eta}$</td>
<td>rms error between experimental and simulated efficiency</td>
</tr>
<tr>
<td>$\epsilon_{V_{oc}}$</td>
<td>rms error between experimental and simulated $V_{oc}$</td>
</tr>
<tr>
<td>$\zeta_s$</td>
<td>solar zenith angle</td>
</tr>
<tr>
<td>$\eta$</td>
<td>efficiency</td>
</tr>
<tr>
<td>$\eta_{acc}$</td>
<td>acceptance efficiency</td>
</tr>
<tr>
<td>$\eta_{acc,overall}$</td>
<td>overall acceptance efficiency</td>
</tr>
<tr>
<td>$\eta_{array}$</td>
<td>array electrical efficiency</td>
</tr>
<tr>
<td>$\eta_{opt}$</td>
<td>optical efficiency</td>
</tr>
<tr>
<td>$\eta_{opt,1}$</td>
<td>primary optical efficiency</td>
</tr>
<tr>
<td>$\eta_{opt,2}$</td>
<td>secondary optical efficiency</td>
</tr>
<tr>
<td>$\eta_{opt,2,target}$</td>
<td>secondary optical efficiency based on Lambertian target</td>
</tr>
<tr>
<td>$\eta_{opt,3}$</td>
<td>tertiary optical efficiency</td>
</tr>
<tr>
<td>$\eta_{opt,overall}$</td>
<td>overall optical efficiency</td>
</tr>
<tr>
<td>$\eta_{system}$</td>
<td>system efficiency</td>
</tr>
<tr>
<td>$\theta$</td>
<td>cone half-angle</td>
</tr>
<tr>
<td>$\theta$</td>
<td>zenith angle in spherical coordinates</td>
</tr>
<tr>
<td>$\theta_d$</td>
<td>acceptance angle within dielectric</td>
</tr>
<tr>
<td>$\theta_{err}$</td>
<td>surface (angular dispersion) error</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>acceptance half-angle</td>
</tr>
<tr>
<td>$\theta_{i,90%}$</td>
<td>acceptance angle for 90% of peak performance</td>
</tr>
<tr>
<td>$\theta_o$</td>
<td>design exit angle $\theta_i/\theta_o$ transformer</td>
</tr>
<tr>
<td>$\theta_{proj}$</td>
<td>angle of ray projected into meridian plane</td>
</tr>
<tr>
<td>$\theta_{sun}$</td>
<td>angular radius of solar disk as viewed from earth</td>
</tr>
<tr>
<td>$\theta_s$</td>
<td>skew angle for one-axis trackers</td>
</tr>
<tr>
<td>$\theta_{crit}$</td>
<td>critical skew angle</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>arc angle</td>
</tr>
<tr>
<td>$\Delta\Theta$</td>
<td>arc angle span</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>curvature</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>first Lamé constant</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>λ₀</td>
<td>vacuum wavelength</td>
</tr>
<tr>
<td>μ</td>
<td>second Lamé constant</td>
</tr>
<tr>
<td>ν</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>ζ</td>
<td>parametric angle</td>
</tr>
<tr>
<td>ρ</td>
<td>reflectivity</td>
</tr>
<tr>
<td>ρ&lt;sub&gt;coated&lt;/sub&gt;</td>
<td>reflectivity of material with weather-resistant coating</td>
</tr>
<tr>
<td>ρ&lt;sub&gt;ground&lt;/sub&gt;</td>
<td>ground reflectivity</td>
</tr>
<tr>
<td>ρ&lt;sub&gt;m&lt;/sub&gt;</td>
<td>intermediate reflectivity used in 3-point fitting</td>
</tr>
<tr>
<td>ρ&lt;sub&gt;M&lt;/sub&gt;</td>
<td>resistivity of solar cell front metallization</td>
</tr>
<tr>
<td>ρ&lt;sub&gt;uncoated&lt;/sub&gt;</td>
<td>reflectivity of material without weather-resistant coating</td>
</tr>
<tr>
<td>σ</td>
<td>secondary tracking/tilt angle</td>
</tr>
<tr>
<td>σ₁</td>
<td>Stefan-Boltzmann constant</td>
</tr>
<tr>
<td>σ₂</td>
<td>first principle stress</td>
</tr>
<tr>
<td>σ₃</td>
<td>second principle stress</td>
</tr>
<tr>
<td>σₑ</td>
<td>third principle stress</td>
</tr>
<tr>
<td>σₑₑ</td>
<td>von Mises equivalent stress</td>
</tr>
<tr>
<td>σₑₑₑ</td>
<td>mode of Rayleigh-distributed angular dispersion error</td>
</tr>
<tr>
<td>σ&lt;sub&gt;ij&lt;/sub&gt;</td>
<td>stress component</td>
</tr>
<tr>
<td>σ₀</td>
<td>applied radial stress</td>
</tr>
<tr>
<td>σ&lt;sub&gt;radial&lt;/sub&gt;</td>
<td>radial stress</td>
</tr>
<tr>
<td>σ&lt;sub&gt;tangential&lt;/sub&gt;</td>
<td>tangential stress</td>
</tr>
<tr>
<td>τ</td>
<td>primary focal plane tilt angle</td>
</tr>
<tr>
<td>τ*</td>
<td>focal plane tilt angle for maximum primary concentration</td>
</tr>
<tr>
<td>Υ</td>
<td>factor used in generalized Hooke’s law for orthotropic materials</td>
</tr>
<tr>
<td>ϕ</td>
<td>polar/circumferential angle in polar/spherical coordinates, for concentrator design this is formalized as the angle that an on-axis ray makes with the optical axis as it approaches the final focus</td>
</tr>
<tr>
<td>ϕ&lt;sub&gt;L&lt;/sub&gt;</td>
<td>polar angle of ray reflected at left end of arc in arcspline</td>
</tr>
<tr>
<td>ϕ&lt;sub&gt;R&lt;/sub&gt;</td>
<td>polar angle of ray reflected at right end of arc in arcspline</td>
</tr>
<tr>
<td>ϕ</td>
<td>latitude</td>
</tr>
<tr>
<td>Φ</td>
<td>formalized (effective) rim angle, Φ = ½(Φ₂ - Φ₁)</td>
</tr>
<tr>
<td>Φ₁</td>
<td>minimum (inner) rim angle</td>
</tr>
<tr>
<td>Φ₂</td>
<td>maximum (outer) rim angle</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\Phi_{av}$</td>
<td>mean rim angle, $\Phi_{av} = \frac{1}{2}(\Phi_2 + \Phi_1)$ °</td>
</tr>
<tr>
<td>$\Phi_{opt}$</td>
<td>optimal rim angle for maximum total concentration °</td>
</tr>
<tr>
<td>$\Delta\Phi$</td>
<td>rim span, $\Delta\Phi = \Phi_2 - \Phi_1$ °</td>
</tr>
<tr>
<td>$\chi$</td>
<td>angle of incidence used in Snell’s law °</td>
</tr>
<tr>
<td>$\psi$</td>
<td>primary tracking angle °</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>maximum slope angle of curved inlet of secondary concentrator °</td>
</tr>
<tr>
<td>$\omega$</td>
<td>hour angle °</td>
</tr>
<tr>
<td>$\omega$</td>
<td>solid angle sr</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>projected solid angle sr</td>
</tr>
</tbody>
</table>

### Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>a.u.</td>
<td>arbitrary units</td>
</tr>
<tr>
<td>ASC</td>
<td>arcspline concentrator</td>
</tr>
<tr>
<td>BD</td>
<td>bypass diode, also as subscript</td>
</tr>
<tr>
<td>BoPET</td>
<td>biaxially oriented polyethylene terephthalate</td>
</tr>
<tr>
<td>BSRN</td>
<td>Baseline Surface Radiation Network</td>
</tr>
<tr>
<td>CCD</td>
<td>charge-coupled device</td>
</tr>
<tr>
<td>CPC</td>
<td>compound parabolic concentrator</td>
</tr>
<tr>
<td>CPV</td>
<td>concentrating photovoltaics</td>
</tr>
<tr>
<td>CSP</td>
<td>concentrating solar power</td>
</tr>
<tr>
<td>CSR</td>
<td>circumsolar ratio</td>
</tr>
<tr>
<td>D</td>
<td>diode in cell equivalent circuit model</td>
</tr>
<tr>
<td>DBC</td>
<td>direct bonded copper</td>
</tr>
<tr>
<td>DC</td>
<td>direct current</td>
</tr>
<tr>
<td>DCPC</td>
<td>dielectric CPC</td>
</tr>
<tr>
<td>DOF</td>
<td>degrees of freedom</td>
</tr>
<tr>
<td>DTERC</td>
<td>dielectric tailored edge-ray concentrator</td>
</tr>
<tr>
<td>E</td>
<td>east</td>
</tr>
<tr>
<td>ETFE</td>
<td>poly(ethylene tetrafluoroethylene)</td>
</tr>
<tr>
<td>FD</td>
<td>film direction</td>
</tr>
<tr>
<td>FEP</td>
<td>fluorinated ethylene propylene</td>
</tr>
<tr>
<td>GS</td>
<td>grayscale (0 – black, 1 – white/transparent)</td>
</tr>
<tr>
<td>HCPV</td>
<td>high-concentration photovoltaics</td>
</tr>
<tr>
<td>HCPVT</td>
<td>high-concentration photovoltaic thermal</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>-------------</td>
</tr>
<tr>
<td>LBLRDB</td>
<td>Lawrence Berkeley Laboratory Reduced Data Base</td>
</tr>
<tr>
<td>LPC</td>
<td>multi-foliate light-pipe concentrator</td>
</tr>
<tr>
<td>LST</td>
<td>local standard time</td>
</tr>
<tr>
<td>LTP</td>
<td>line-to-point focus concentrator</td>
</tr>
<tr>
<td>MCRT</td>
<td>Monte-Carlo ray-tracing</td>
</tr>
<tr>
<td>MCT</td>
<td>magnesium cadmium telluride</td>
</tr>
<tr>
<td>MD</td>
<td>machine direction</td>
</tr>
<tr>
<td>MPP</td>
<td>maximum power point</td>
</tr>
<tr>
<td>N</td>
<td>north</td>
</tr>
<tr>
<td>NURBS</td>
<td>nonuniform rational basis spline</td>
</tr>
<tr>
<td>PAR</td>
<td>peak-to-average irradiance ratio</td>
</tr>
<tr>
<td>PTC</td>
<td>parabolic trough collector</td>
</tr>
<tr>
<td>PV</td>
<td>photovoltaics</td>
</tr>
<tr>
<td>PCV/PES</td>
<td>vinyl coated polyester fabric</td>
</tr>
<tr>
<td>PVD</td>
<td>physical vapor deposition</td>
</tr>
<tr>
<td>PVR</td>
<td>peak-to-valley irradiance ratio</td>
</tr>
<tr>
<td>rev.</td>
<td>revolved</td>
</tr>
<tr>
<td>rms</td>
<td>root-mean-square</td>
</tr>
<tr>
<td>ROI</td>
<td>region of interest</td>
</tr>
<tr>
<td>RX</td>
<td>refraction-reflection SMS concentrator</td>
</tr>
<tr>
<td>RXI</td>
<td>refraction-reflection-total internal reflection SMS concentrator</td>
</tr>
<tr>
<td>S</td>
<td>south</td>
</tr>
<tr>
<td>SMARTS</td>
<td>Simple Model of the Atmospheric Radiative Transfer of Sunshine</td>
</tr>
<tr>
<td>SMS</td>
<td>Simultaneous Multiple Surface (Miñana-Benítez) method</td>
</tr>
<tr>
<td>SPICE</td>
<td>Simulation Program with Integrated Circuit Emphasis</td>
</tr>
<tr>
<td>SRH</td>
<td>Shockley-Read-Hall (carrier recombination)</td>
</tr>
<tr>
<td>SS</td>
<td>summer solstice</td>
</tr>
<tr>
<td>STC</td>
<td>standard test conditions</td>
</tr>
<tr>
<td>TD</td>
<td>transverse direction</td>
</tr>
<tr>
<td>TIR</td>
<td>total internal reflection</td>
</tr>
<tr>
<td>W</td>
<td>west</td>
</tr>
<tr>
<td>WS</td>
<td>winter solstice</td>
</tr>
<tr>
<td>XR</td>
<td>reflection-refraction SMS concentrator</td>
</tr>
<tr>
<td>XX</td>
<td>reflection-reflection SMS concentrator</td>
</tr>
</tbody>
</table>
Subscripts

0  parked coordinate frame
0  nominal
1  pertaining to primary concentrator
2  pertaining to the secondary concentrator
ax  pertaining to the axial direction/plane
exp  experimental
i  inlet
j  index
k  index
n  normal incidence
o  outlet
OC  open-circuit
p  pertaining to first mirror in two-mirror aplanat
ph  photogenerated
post  post-filter
pre  pre-filter
s  pertaining to second mirror in two-mirror aplanat
SC  short-circuit
sim  simulation
ss  sunset
trans  pertaining to the transverse direction/plane
λ  spectral quantity

Superscripts

'  tilted focal plane coordinate frame
"  secondary tracking coordinate frame
+  point on focal plane tilted more than τ*
-  point on focal plane tilted less than τ*
Chapter 1

Introduction

In 2010, the global yearly energy demand was $553 \times 10^{18}$ J or 13.2 billion tonnes of oil equivalent [1]. While this seems like a staggering number, it is no match for the amount of solar energy reaching the surface of the earth in one year. Solar radiation strikes earth’s outer atmosphere at a yearly average irradiance of $S = 1367$ W/m$^2$ [2], of which an average of 54% [3] passes through the atmosphere to the earth’s surface. Taking the radius of the earth as $R_{\text{earth}} = 6.371 \times 10^6$ m, the solar energy reaching the earth’s surface every second amounts to $0.54 \pi R_{\text{earth}}^2 S = 94 \times 10^{15}$ J. Multiplying by the number of seconds per year, we find that the solar energy reaching the earth’s surface in one year is $3.0 \times 10^{24}$ J, more than 5 000 times greater than the global energy demand.

With the exception of geothermal energy resulting from the residual heat from the formation of the planet, radioactive decay, and gravitational energy, solar radiation is the origin of all energy flows on the earth. All renewable energy sources (with the exception of tidal and geothermal) are driven by solar radiation. The sun drives the wind currents and the hydrological cycle, which feed wind and hydroelectric powerplants respectively. Even fossil fuels are carriers of solar energy that has been accumulated over many years in the past. Considering the vast amount of free and clean energy available from the sun, it is difficult to envision a future where solar energy does not play a major role in the global energy mix.

Nevertheless, there exist economic and technical barriers which have favored the use of other energy sources in the past. In particular, solar energy is dilute, meaning that large land and collector areas are required to collect enough solar radiation to make a significant global impact. In 2010 solar energy accounted for only 0.8% of the world’s renewable electricity generation, and less than 0.2% of electricity generation from all sources [1]. However, an average annual growth rate of 9% for solar electricity is predicted for the next
30 years, the highest for any energy source [1]. In order to afford this growth, technologies which effectively overcome the challenges of the dilute and intermittent nature of solar energy are essential.

1.1. Motivation for solar concentration

Fundamental limitations arise due to the dilute nature of solar radiation reaching the surface of the earth. The maximum energy flux of the solar radiation reaching the surface of the earth is around 1 kW/m². As a comparison, consider the energy flux of a flow of gasoline while filling up the tank of a car. Assuming a flow rate of 30 liters/minute, a heating value of 30 MJ/liter and a 20 mm nozzle diameter, the energetic flux of the gasoline flowing into the gas tank is around $50 \times 10^6$ kW/m², more than 50 million times greater than that of incident solar radiation. Such a flux limits the efficiency at which solar energy can be converted into useful forms.

Consider a receiver of a certain area $A_{\text{receiver}}$ which converts electromagnetic radiation into some useful form. There will invariably be some energy losses associated with the conversion process. In most cases, these losses depend on the intrinsic energetic potential of the receiver, but not explicitly on the energy input to the receiver. It is this important point which ultimately leads to the motivation to concentrator solar energy. Defining the energetic efficiency as:

$$\eta = \frac{Q_{\text{useful}}}{Q_{\text{in}}} = \frac{Q_{\text{in}} - Q_{\text{loss}}}{Q_{\text{in}}} = 1 - \frac{Q_{\text{loss}}}{Q_{\text{in}}} \quad (1.1)$$

we note that the efficiency can be increased by increasing the power input to the system, while holding the losses constant.

The nature of the energetic potential which drives the losses depends on the conversion process. For thermal receivers the potential is the receiver temperature, while for photovoltaic (PV) receivers it is the voltage. As an example, consider a thermal receiver which converts solar radiation into an extractable heat flow at a temperature $T$. For simplicity, we consider only thermal radiation losses which may be expressed as:

$$Q_{\text{loss}} = A_{\text{receiver}} \varepsilon \sigma T^4 \quad (1.2)$$
The energetic efficiency is:

\[ \eta = 1 - \frac{Q_{\text{loss}}}{Q_{\text{in}}} = 1 - \frac{A_{\text{receiver}} \varepsilon \sigma T^4}{A_{\text{receiver}} q_{\text{in}}} = 1 - \frac{\varepsilon \sigma T^4}{q_{\text{in}}} \]  

(1.3)

where \( q_{\text{in}} \) is the input power per unit receiver area. It is seen that the efficiency of the converter can be increased simply by increasing \( q_{\text{in}} \) while holding the potential \( T \) constant.

Now consider a photovoltaic receiver which converts solar radiation into an extractable current flow \( I \) at a voltage \( V \). Let \( I_{\text{ph}} \) be the photogenerated current, i.e. the maximum current that can be generated by the absorbed photons which can be assumed to equal \( \alpha Q_{\text{in}} \) where \( \alpha \) is a device-specific constant. The maximum power that could ever be delivered by the photovoltaic receiver would occur if the maximum current were to flow at the bandgap voltage \( V_G = E_G/q \) where \( E_G \) is the bandgap energy and \( q \) is the elementary charge. For the purposes of this discussion, it is useful to define an efficiency which describes what fraction of this limit is reached by the photovoltaic receiver:

\[ \eta = \frac{P}{I_{\text{ph}} V_G} = \frac{P}{\alpha Q_{\text{in}} V_G} \]  

(1.4)

This is not the common definition for solar cell efficiency, and is rather more analogous to the fill-factor, but is useful in this context of revealing the effect of concentration on performance. For an ideal solar cell, the loss in current due to carrier recombination is [4]:

\[ I_{\text{loss}} \approx A_{\text{receiver}} J_0 e^{V/V_G} \]  

(1.5)

where \( I_0 \) and \( V_T \) may be treated as constants in this simplified analysis. The corresponding loss in power is:

\[ P_{\text{loss}} \approx A_{\text{receiver}} V J_0 e^{V/V_G} \]  

(1.6)

The efficiency of Eq. (1.4) then becomes:

\[ \eta = 1 - \frac{P_{\text{loss}}}{\alpha Q_{\text{in}} V_G} = 1 - \frac{V J_0 e^{V/V_G}}{\alpha q_{\text{in}} V_G} \]  

(1.7)
Analogously to the thermal receiver, the efficiency of the photovoltaic convertor can be increased by increasing $q_{in}$ while holding the potential $V$ constant.

The question of how to achieve this increase of solar power input per receiver area then arises, for which solar concentration provides the answer. A solar concentrator is an optical device having an inlet and an outlet, where, by definition, the outlet is smaller than the inlet. The concentrator collects (dilute) solar radiation at its inlet, denoted the “collecting aperture”, and transports it to its outlet, at which the receiver is placed. If the energy is transported efficiently by the concentrator, then the power reaching the outlet will be approximately equal to the power at the inlet. Since the outlet is smaller than the inlet, the power per unit area at the receiver will be augmented. From the point of view of the receiver of a given area, the concentrator serves the function of increasing the power delivered to it, thus allowing the converter to operate more efficiently vis-à-vis Eq. (1.1).

1.2. Motivation for concentrating photovoltaics

In the previous section it was shown by simple considerations of the loss characteristics of solar energy converters that the potential energy efficiency can be increased when solar radiation is delivered the receiver in a concentrated form. This implies that concentration increases the quality of solar radiation. For thermal applications, this increase in the quality of solar radiation is the prime motivator, as it allows higher temperature processes to be realized, thus increasing the potential applications for the extracted heat and the efficiency of downstream processes. For photovoltaic applications, solar concentration also has the potential to increase the efficiency of energy conversion. However, this is not the prime motivator for solar concentration in the field of photovoltaics. While the efficiency of the solar cell can in fact increase with concentration, in the end, the output of a solar cell under either concentrated or non-concentrated radiation is low-voltage current. Therefore solar concentration does not open up new applications for photovoltaic conversion the way it does for thermal conversion.
The primary motivation for pursuing concentrating photovoltaics (CPV) is economy. To illustrate this motivation, consider an admittedly oversimplified economic model of a concentrating photovoltaic collector. Let $k_{\text{cell}}$ be the cost per m$^2$ of cell area and $k_{\text{conc.}}$ be the cost of the solar concentrator per m$^2$ of collecting aperture. The power output of the system is:

$$P = \eta \cdot Q_i = \eta \cdot \text{DNI} \cdot A_i$$

(1.8)

where $\eta$ is the overall energy conversion efficiency of the system, DNI is the direct normal irradiance and $A_i$ is the collecting aperture area. The overall cost of the system is:

$$K = k_{\text{conc.}} A_i + k_{\text{cell}} A_{\text{cell}}$$

(1.9)

The geometric concentration ratio $C_g$ of the system is defined as the ratio of the collecting aperture area to the receiver (cell) area:

$$C_g \equiv A_i / A_{\text{cell}}$$

(1.10)

The total cost may then be expressed as:

$$K = A_i \left( k_{\text{conc.}} + k_{\text{cell}} / C_g \right)$$

(1.11)

Dividing by the power input to the system, the system cost per Watt of output power is obtained:

$$K/P \ [\$/W] = \frac{k_{\text{conc.}} + k_{\text{cell}} / C_g}{\eta \cdot \text{DNI}}$$

(1.12)

Some very important conclusions can be drawn from Eq. (1.12). Firstly, it is seen that, keeping all other factors equal, the overall cost is reduced as the concentration is increased.\(^1\) Secondly, the direct dependence of system cost on efficiency is revealed: an $n$-fold increase in efficiency leads to an $n$-fold decrease in system cost, all other factors equal. Thirdly, it is seen that the system cost is inversely proportional to the DNI, suggesting that a given system

---

\(^1\) In reality, an increasing in concentration will lead to an increase in $k_{\text{conc.}}$ due to increased complexity and precision of the concentrator optics.
placed in a region of high insolation is effectively cheaper than an identical system placed in a region of low insolation.

The economic motivation for CPV is based on the assumption that the cost of the entire concentrator system per m² of collecting aperture can be made significantly less expensive than the cost per m² of cell area. As the cost of conventional PV systems has drastically reduced in the recent years [5], this is in fact a difficult feat. However, it has been argued [6] that due to the fact that conventional PV systems are approaching their theoretical efficiency limits, the learning curve, which is a key contributor to the successful market penetration, will slow. In contrast, CPV system can use cells with advanced structures, e.g. multijunction solar cells, for which the efficiency limits are more than doubled. The existence of this considerable room for improvement, while considering the role of efficiency in Eq. (1.12), secures the motivation for continued development of CPV systems.

Having established the economical motivation for concentrating photovoltaics, it is worthwhile coming back to the efficiency motivations discussed in Section 1.1. The greatest competitor for CPV is conventional flat panel PV, since both essentially deliver the same end product. While it is true that the efficiency of a solar cell can be increased under concentrated radiation, for purposes of comparing CPV to conventional PV it is the system efficiency that is most relevant. There are two losses inherent to CPV which do not occur for conventional PV collectors: (1) optical losses due to the concentrator system, and (2) losses due to the fact that a concentrating system can only collect direct radiation\(^2\). The second point is common for all concentrating technologies, regardless of the type of receiver. In the best case, the solar resource for a non-concentrating collector is the global normal irradiance GNI. To afford a comparison between CPV and PV, let us take the GNI as a basis for the efficiency definition for both technologies. The efficiency of a conventional PV system may be expressed as:

\[
\eta_{\text{PV}} = \frac{P}{\text{GNI} \cdot A_i} = \eta_{\text{cell}}
\]

\(^2\) Non-concentrating collectors can accept both direct and diffuse radiation.
and the efficiency for a CPV system may be expressed as:

\[ \eta_{\text{CPV}} = \frac{P}{\text{GNI} \cdot A_i} \]

\[ \frac{P}{A_i} = \text{DNI} \cdot \eta_{\text{opt}} \cdot \eta_{\text{cell}} \] (1.14)

\[ \eta_{\text{CPV}} = \frac{\text{DNI}}{\text{GNI}} \cdot \eta_{\text{opt}} \cdot \eta_{\text{cell}} \]

The two aforementioned losses are represented by the first two terms of Eq. (1.14), which are absent in Eq. (1.13). The first is the ratio DNI/GNI which accounts for the fact that predominantly only direct radiation can be collected by a solar concentrator. This ratio generally varies between 70% and 85% (see Section 2.3.4) but can be even lower for very humid sites. Therefore CPV is comparatively better in arid regions having a low diffuse fraction of radiation. This loss is sometimes overlooked when evaluating CPV systems as the standard is to define CPV system efficiency using the DNI as a basis. The DNI basis allows CPV systems to be more readily compared among each other and thus is used for the remainder of the work. However, the aforementioned losses should be kept in mind whenever comparing CPV system efficiencies to those of conventional PV systems.

The second is the optical efficiency \( \eta_{\text{opt}} \) which accounts for energetic losses in the concentrator. The state of the art silicon flat panel systems have overall solar-to-DC efficiencies of around 20%. Taking representative values of DNI/GNI = 80% and \( \eta_{\text{opt}} = 80\% \) we find that in order for a CPV system to have the same overall efficiency as a conventional PV system the CPV cell efficiency must be at least 31%.

Fortunately, significantly higher cell efficiencies are possible under concentrated solar radiation. The limits of the conversion efficiency of a photovoltaic device can be calculated under the assumption of a detailed balance [7]. For a single bandgap \( p-n \) junction solar cell operating under non-concentrated solar radiation from a 6000 K blackbody source, the limiting efficiency is 31.0% for an optimal bandgap material [8, 9]. Under maximally concentrated solar radiation this limit becomes 40.8% [9]. Additionally, concentration makes the use of multijunction devices economically feasible,
which may be considered an indirect way in which concentration increases the efficiency limit. For a solar cell with an infinite number of junction operating at the thermodynamic limit of solar concentration, the maximum of efficiency of a solar photovoltaic converter on earth is 86.8% [9]. This is very close to the Carnot efficiency of 95% which is the maximum for any cyclic work-producing device assuming the hot reservoir is at 6000 K (the sun), and the cold reservoir is at 300 K (ambient).

In addition to offering higher efficiencies, CPV systems offer several secondary benefits over their conventional PV counterparts including reduced land-use, smaller performance degradation in high-temperature environments, higher capacity factors, and generally more stable long-term performance.

1.2.1. Concentrating photovoltaic technology
In general CPV technologies can be grouped according to their geometric concentration into low-, medium- and high-concentration designs. This classification is not concrete as it is mostly dictated by the availability of solar cells capable of efficient operation under the prescribed concentration, and the costs associated with the cells, which change as manufacturing processes improve. Table 1.1 shows the classification suggested in this work.

Low-concentration systems can be thought of as extensions of conventional PV designs, where the concentrator serves the purpose of augmenting the electrical power output of a given cell/array.

Medium-concentration systems use moderately-priced specialized concentrator cells, usually crystalline silicon, which have increased front metallization to allow efficient extraction of the higher current densities generated in the cell. These systems require at least one-axis of tracking and can be passively cooled with appropriate heat sinks.

High-concentration designs are based on two-axis trackers, or line-to-point focus systems, whose introduction is a major contribution of this work. With geometric concentrations above 400×, it becomes economically feasible to use very expensive but very efficient multijunction cells whose efficiencies have surpassed 44% under concentrated irradiation [10]. These systems can use
passive cooling systems when the cells are separated by large distances, but require active cooling if the cells are densely packed [11].

In the current situation, high-concentration photovoltaic (HCPV) systems seem to be most promising of the three classes. This is due to the fact that these systems can make use of highly-efficient multijunction cells which have the greatest potential for future efficiency and cost improvements.

Concentrating photovoltaic systems emerged in the 1970s after the first oil shock [6]. The SANDIA I and II, developed in 1976 and 1977 were the first successful designs. The systems were based on a two-axis tracking parquet of acrylic Fresnel lenses each concentrating incident sunlight by 40× onto 5 cm diameter silicon concentrator cells [6]. Stationary low-concentration CPV systems, such as the CPC-based system developed by Isofotón and Universidad Polytécnica de Madrid developed in 1980, also had their footing in the early days of CPV development. The first trough-based CPV system was the EUCLIDES system whose prototype was purported to achieve a lower cost than conventional PV at the time [6, 12]. The current trend of CPV seems to be following the “third-generation photovoltaics” strategy [8] where high-concentrations and very efficiency, yet very expensive cells are preferred. This is no doubt in large part due to the rapid development of terrestrial concentrator multijunction cells [10] and the synergies shared with space technologies. There is still considerable potential for both improving the efficiency and reducing the cost of these cells.

### Table 1.1. Suggested classification of concentrating photovoltaic collectors according to the geometric concentration ratio, $C_g$.

<table>
<thead>
<tr>
<th>Class</th>
<th>$C_g$ range</th>
<th>Cell type</th>
<th>Cooling requirements</th>
<th>Tracking requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>low-concentration</td>
<td>2 – 10×</td>
<td>conventional</td>
<td>passive</td>
<td>stationary or seasonal</td>
</tr>
<tr>
<td>medium-concentration</td>
<td>10 – 100×</td>
<td>concentrator</td>
<td>passive with heat sink</td>
<td>one-axis</td>
</tr>
<tr>
<td>high-concentration</td>
<td>&gt; 400×</td>
<td>multijunction</td>
<td>passive for single-cell or active</td>
<td>two-axis or line-to-point focus</td>
</tr>
</tbody>
</table>
1.3. Revisiting the solar trough

Having established a technical and economical motivation for solar concentration, the question of what concentrator technology can best meet these goals arises. For CPV systems, the economy of concentration arises only if the concentrator can be made significantly cheaper per m² of collecting aperture than the solar cell material. For any type of solar collector, the greatest amount of energy that can be produced per m² of is around 1 kW. For photovoltaic generation, it has been shown that costs of less than $1 per peak-Watt are required in order for the cost of energy to be comparable to conventional sources [5]. Assuming a solar-to-electricity efficiency of 10 – 30%, this implies that the cost of the entire system must be less than $100 – 300 per m² of collecting aperture. These ballpark figures put into perspective the stringent cost requirements for solar collector technology. While these figures have been established based on economic considerations for photovoltaic applications, they may be considered relevant for any type of solar energy converter, since these technologies will have to deliver an end product at a comparable price.

With these figures in mind, minimization of the cost per m² of collecting aperture may be considered a primary driver for the cost-competitiveness of any solar collector technology. For concentrating collectors, the collecting aperture is the projected area of the primary concentrating device, whether it be a mirror, lens or some other optical device. Therefore we should search for primary concentrators having an inexpensive construction.

Solar concentrators may be classified as either line- or point-focus systems. Line-focus concentrators produce a focal line along which the receiver, or secondary optical components are arranged, whereas point-focus systems concentrate the radiation to a point. While point-focus concentrators require three-dimensional structures, line-focus systems can utilize concentrators with linear-symmetry. Furthermore, line-focus concentrators generally require only one axis of tracking, whereas point-focus concentrators require two-axis tracking.

Line-focus concentrators are often referred to as 2D concentrators since their optical behavior can be mostly described in two-dimensions alone. In this work, the terms “2D” or “trough” are used to denote any concentrator having
line symmetry. The most common line-focus concentrator is the parabolic trough collector (PTC). PTCs were utilized in the first large-scale solar thermal powerplant, the Solar Electric Generating Systems (SEGS), which began commercial operation in 1984 [13]. To date, the SEGS powerplant is the largest solar concentrating facility in the world. Part of the success of the system may be attributed to the robust and economical design of trough collectors.

Because of their line symmetry, trough concentrators are considerably easier to manufacture than their 3D counterparts. Additionally, troughs are very easily scaled simply by increasing the length of the trough. It is for these two important reasons that trough concentrators show promise for achieving the primary goal of minimizing the cost of the collecting aperture. Of course, troughs are not without limitation. Due to their 2D nature, trough concentrators are fundamentally bound to a concentration limit of $C_{g,\text{max,2D}} = 215\times$, whereas 3D designs may reach up to $46\ 000\times$ in theory. Two questions arise: (1) what is the achievable concentration of real trough concentrators in relation to the theoretical limit of $215\times$, and (2) is it possible to push a trough concentrator past this limit? This thesis aims to address these two questions and to ultimately produce economically viable collectors that approach the limit of solar trough performance, and beyond.

1.4. **Thesis outline**

This thesis deals primarily with the optical design of novel trough-based concentrating solar collectors. With the CPV application in mind, emphasis is placed on designs for planar receivers. However, many of the proposed designs and analysis methods will find use for other applications, and thus the optical designs are presented in a general framework.

Chapter 2 discusses the theory and background required for the understanding of the remainder of the work. Solar radiation fundamentals and nonimaging optics are reviewed. Some new methods of optical design and analysis are detailed including the source/acceptance map matching method for sizing the acceptance angle of solar concentrators, and a new method for determining the average number of reflections of any optical device. Particular
emphasis is placed on the analysis of one-axis trackers, which form the basis of trough concentrators.

Chapter 3 is devoted to the development of high-performance, low-cost trough designs. The detailed design theory for a unique inflated trough collector whose mirrors are constructed from thin metallized polymer membranes is presented. In a first design, the membrane shape is found to be spherical, and the analytical solution of secondary mirrors capable of removing the spherical aberration of the primary are developed. In a second design, the membrane mirror construction is modified such that its shape comprises a series of tangentially connected circular arcs, called the “arcspline”. It is shown that the arcspline profile can always be designed to match the performance of a parabolic trough in the nonimaging sense. The idea of nonparabolic troughs is further developed and it is demonstrated that there are infinitely many mirror profiles that match the parabolic performance. Finally two-mirror aplanatic and tailored troughs are proposed as highly compact designs which approach the 2D limit of concentration.

Chapter 4 introduces the line-to-point (LTP) focus solar concentrator which breaks the 2D limit of concentration through the introduction novel secondary concentrators, yet maintains the benefits of having a trough-based primary. Three types of LTP concentrators, distinguished by the complexity of the secondary concentrating stage, are presented. The first type, which utilizes an array of fixed secondary concentrators arranged along the focal line of the primary, has been previously investigated, but improvements to the design procedure are detailed. The second and third types are novel designs, and incorporate a range of motion into the secondary concentrator in order to significantly augment the achievable concentration beyond the 2D limit.

Chapter 5 discusses the design and characterization of photovoltaic receiver arrays to be utilized in collectors based on the optical designs of Chapter 3 and 4. The semi-dense linear array is introduced and the effect of the irradiance distribution on the array performance is investigated both analytically and experimentally.

Chapter 6 culminates the work by presenting a practical implementation of the concentrator and receiver designs discussed in the previous chapters. The
design, modeling and on-sun experimental demonstration of a full-scale 600× inflated-trough-based HCPV collector is detailed.

Chapter 7 provides a summary of the key findings of the work, and gives an outlook for recommended future investigations on high-concentration trough collectors with particular emphasis on their application to HCPV.
Chapter 2

Theory and background

In this chapter the basic theory required for the understanding of this work is outlined. For those familiar with solar radiation, nonimaging optics, and solar concentrator design, this chapter will serve as a review.

2.1. Solar radiation

The sun is a star whose radiation emission characteristics resemble that of a blackbody at an equivalent temperature of $T_{\text{sun}} = 5777$ K. The total radiant exitance at the surface of the sun is $M_{\text{sun}} = 6.31 \times 10^7$ W/m$^2$, giving an radiance of $L_{\text{sun}} = M_{\text{sun}}/\pi = 2.0 \times 10^7$ W/(m$^2$·sr) [14]. When the radiation reaches the surface of the earth, its irradiance is diluted by a factor equal to the ratio of the square of the radius of earth’s orbit to the square of the radius of the sun, resulting in an irradiance at the outside of earth’s atmosphere of 1367 W/m$^2$ [2]. While this quantity is denoted the solar constant $S$, it is not actually constant, varying by about ±3.3% over the year due to the eccentricity of earth’s orbit.

2.1.1. Sun-earth geometry

Radiation emitted by the sun appears to originate from the sun’s photosphere, a nearly spherical surface having a radius of $R_{\text{sun}} = 6.955 \times 10^8$ m. The distance between the sun and the earth ranges throughout the year from a minimum of $d_{\text{sun, min}} = 1.471 \times 10^{11}$ m at the perihelion to $d_{\text{sun, max}} = 1.521 \times 10^{11}$ m at the aphelion, with a mean of $d_{\text{sun, mean}} = 1.496 \times 10^{11}$ m [14].

While the earth orbits around the sun and not vice-versa, it is more convenient to consider the geocentric description of the sun-earth geometry for solar collection.
Figure 2.1. Cardinal coordinate frame ($X$, $Y$, $Z$) for expressing the sun-earth geometry. The sun vector $\mathbf{s}$ is the vector pointing from an earthly observer to the center of the solar disk. It may be expressed by the solar altitude angle $\alpha_s$ (or its complement the zenith angle $\zeta_s$), and the solar azimuth angle $\gamma_s$ (zero at solar noon, positive in the afternoon).

**Cardinal coordinate frame and solar angles**

The cardinal coordinate frame is a Cartesian coordinate system ($X$, $Y$, $Z$), shown in **Figure 2.1**, established in reference to a fixed earthly observer. The conventions here are biased to observers in the northern hemisphere. In this work, the $X$-axis is oriented pointing due east, the $Y$-axis pointing due north, and the $Z$-axis pointing up. The position of the sun in the sky is quantified by the unit sun vector $\mathbf{s}$ pointing from the observer to the center of the solar disk. The sun vector is parameterized by the solar zenith angle $\zeta_s$, defined as the angle between $\mathbf{s}$ and the $Z$-axis, and the solar azimuth angle $\gamma_s$, defined as the angle between the projection of $\mathbf{s}$ into the $X-Y$ plane and the $-Y$-axis, measured positive west of south (afternoon). The complement of $\zeta_s$, the solar altitude angle $\alpha_s$, is also used. It terms of the solar angles, the sun vector is:

\[
\mathbf{s} = \begin{bmatrix}
-\sin \gamma_s \sin \zeta_s & -\cos \gamma_s \sin \zeta_s & \cos \zeta_s
\end{bmatrix} = \begin{bmatrix}
-\sin \gamma_s \cos \alpha_s & -\cos \gamma_s \cos \alpha_s & \sin \alpha_s
\end{bmatrix}
\]  

(2.1)

The solar angles $\zeta_s$ and $\alpha_s$, and $\gamma_s$, may be found from [2]:
\[ \alpha_s = \arcsin \left( \cos \phi \cos \delta \cos \omega + \sin \phi \sin \delta \right) \]  
\[ \gamma_s = \text{sgn} \omega \left| \arccos \left( \frac{\sin \alpha_s \sin \phi - \sin \delta}{\cos \alpha_s \cos \phi} \right) \right| \]

where \( \phi \) is the latitude, \( \delta \) is the solar declination angle, and \( \omega \) is the hour angle (zero at solar noon, and positive in the afternoon). An additional useful relation for the solar azimuth angle is [15]:

\[ \sin \gamma_s = \frac{\cos \delta \sin \omega}{\cos \alpha_s} \]

2.1.2. Spatial distribution

In the absence of local obstructions (clouds, shading surfaces) between the sun and a collecting aperture, the incident solar radiation can be very accurately assumed to be homogeneously distributed in space. This simplifying assumption is therefore made throughout this work.

2.1.3. Directional distribution

When viewed from just outside earth’s atmosphere, the sun appears to be a disk of near constant brightness subtending angular radius ranging from \( \theta_{\text{sun,min}} = \arcsin \left( R_{\text{sun}}/d_{\text{sun,max}} \right) = 0.262^\circ = 4.57 \text{ mrad} \) at aphelion to \( \theta_{\text{sun,max}} = 0.271^\circ = 4.73 \text{ mrad} \) at perihelion, with a value of \( \theta_{\text{sun,mean}} = 0.266^\circ = 4.65 \text{ mrad} \) at the mean sun-earth distance. For solar concentrator design, it is common to take the mean \( \theta_{\text{sun}} = 0.266^\circ \) as a representative value (the maximum being only \( \sim 2\% \) higher).

Due to scattering by particles and molecules in the atmosphere, not all of the solar radiation at the top of the atmosphere reaches the surface of the earth directly. The portion of radiation that appears to originate from the solar disk is called the direct solar radiation. The portion that arrives at the surface of the earth after being scattered by the atmosphere is called the diffuse solar radiation. Although the distinction between direct and diffuse radiation is not absolute – there exists a transition region between the solar disk and the sky hemisphere called the circumsolar aureole – the radiance usually drops very
quickly when moving away from the solar disk, such that the two components can essentially be separated. The circumsolar aureole is usually taken as part of the direct radiation, and not the diffuse.

For concentrating applications, the directional distribution of incident solar radiation within the solar disk and circumsolar region can have consequences on the optical efficiency of a collector and on the spatial distribution of concentrated solar radiation in the focal region. The directionality is almost always assumed to be circumferentially symmetric about the sun vector, and thus can be described by the radiance \( L(\theta) \) as a function of the angle \( \theta \) between a given direction and the sun vector. When normalized by the intensity at \( \theta = 0 \), this function is called the sunshape \( L(\theta)/L(0) \).

**Extraterrestrial sunshapes**

Extraterrestrial sunshapes exclude the effect of atmospheric attenuation and scattering. The simplest is to assume that the radiance is constant over the solar disk, and zero outside the solar disk:

\[
\frac{L(\theta)}{L(0)} = \begin{cases} 
1 & \theta \leq \theta_{\text{sun}} \\
0 & \theta > \theta_{\text{sun}} 
\end{cases}
\]  

(2.5)

Slight geometric improvements to this model can be made by considering the sun to be a diffusely emitting disk or sphere, but the differences are negligible. The main source of error of the extraterrestrial models is the assumption of diffuse-opaque surface emission. In reality, the sun is gaseous and thus optical thickness changes as the line of sight moves from the center of the solar disk to the edge. This effect is called limb darkening. Common models for the limb darkened sunshape may be found in [14].

**Terrestrial sunshapes**

While extraterrestrial sunshapes are relatively constant and can be accurately measured by satellites, they do not include the important effects of atmospheric attenuation and scattering. Small-angle forward scattering in the atmosphere creates a halo around the solar disk (the circumsolar aureole), and wide-angle scattering illuminates the sky hemisphere. For very clear skies, the effect of the
atmosphere may be small, but for hazy conditions, there may be a significant amount of radiation arriving from the circumsolar aureole.

To date, the most successful characterization of terrestrial sunshapes was performed in the 1970s in the USA using a circumsolar telescope. The data has been compiled into what is known as the Lawrence Berkley Laboratory Reduced Data Base (LBLRDB) [16-17].

For modeling purposes the terrestrial sunshape model proposed by Buie, Monger and Dey [18] is the most common in current use, and is thus adopted in this work.

2.1.4. Spectral distribution

For solar absorbers relying on quantum effects, e.g. solar cells, the spectral distribution of incident solar radiation is of importance.

*Extraterrestrial solar spectrum*

When the solar emission spectrum is smoothed between the absorption lines, it is referred to as the solar continuum. A simplified spectrum can be found in [14]. The American Society for Testing and Materials (ASTM) has standardized the extraterrestrial solar spectrum, referred to as the air mass zero (AM0) spectrum, in the standard ASTM E-490 [19].

*Terrestrial solar spectra*

For terrestrial solar collection, atmospheric attenuation and scattering may have a significant impact on the solar spectrum. The terrestrial spectrum depends greatly on the pathlength that light must pass through the atmosphere and the atmospheric composition along the entire path. The pathlength is quantified by the relative optical air mass (AM), defined as the ratio of the optical pathlength at a given sun position to that when the sun is directly overhead. There exist standard terrestrial solar spectra, making standardized comparisons possible. For concentrating solar collectors, the most relevant standard is the ASTM AM1.5 direct + circumsolar spectrum included in standard ASTM G-173 [20].

For non-standard conditions, probably the most widely used model amongst the solar community is the Simple Model of the Atmospheric Transfer of Sunshine (SMARTS) developed by C. Gueymard [21]. This open-source code
allows the solar spectrum to be estimated with good accuracy under a wide range of clear-sky conditions, and also serves as the basis for the ASTM G-173 standard. The SMARTS model is used in this work wherever non-standard terrestrial solar spectra are required.

2.1.5. Polarization

In this work, it is assumed that solar radiation is unpolarized, i.e. that it has equal proportions of parallel $p$ and perpendicular $s$ polarization states. For reflective surfaces (mirrors), the effect of polarization is small. Additionally, for normally incident radiation on refractive surfaces, the reflectivity of the dielectric interface is theoretically independent of the polarization state. However, for non-normal incidence, the polarization may play an important role in the overall behavior of refractive surfaces. The majority of the systems discussed in this thesis use reflective surfaces, and therefore polarization effects are neglected.

2.2. Fundamentals of nonimaging optics

Nonimaging optics is a branch of geometrical optics that deals with the design of optical devices for the efficient transformation of the spatial and angular distributions of electromagnetic radiation. Solar concentrators serve the general function of intercepting a large area of nearly collimated solar radiation and transporting it to a small receiver area as efficiently as possible. Along with the design of illumination systems, solar concentrator design forms the backbone of the discipline of nonimaging optics.

2.2.1. Geometric and flux concentration

An important distinction is made between the geometric and flux concentration ratios. The geometric concentration ratio\footnote{For brevity, the word “ratio” is often omitted when discussing both geometric and flux concentrations.}, $C_g$, is the ratio of the area of the inlet aperture of an optical system, to that of the outlet/exit aperture:

$$C_g = A_i / A_o$$ (2.6)
Being a ratio of areas, $C_g$ is dimensionless. Throughout this work it is given the dimensionless unit $\times$ read as the letter “$x$”. The flux concentration ratio $C$ is the ratio of the (average) irradiance at the inlet aperture to that at the exit aperture:

$$C = \frac{E_o}{E_i} \quad (2.7)$$

The geometric and flux concentrations are related by:

$$C = \frac{E_o}{E_i} = \frac{O_o/A_o}{Q_i/A_i} = \frac{O_o}{Q_i} C_g = \eta_{\text{opt}} \cdot C_g \quad (2.8)$$

where $\eta_{\text{opt}}$ is the optical efficiency, i.e. the ratio of radiant power at the outlet to that at the inlet, which is rigorously defined in Section 2.5.1. While $C_g$ can be made arbitrarily large by decreasing the outlet area while keeping the collecting aperture area constant, the flux concentration is bound by physical laws. Examination of Eq. (2.8) reveals that an increase of $C_g$ above the physical limit of $C_{\text{max}}$ must be accompanied by a reduction of optical efficiency of comparable magnitude. In the limit of $\eta_{\text{opt}} = 1$, the geometric and flux concentration are equal in magnitude. We refer to this limit as “full-collection” or “full-intercept” since all of the radiant power arriving at the inlet of the optical system reaches the exit. The maximum full-collection geometric concentration $C_{g,\text{max}}$ follows the same physical laws as the maximum flux concentration $C_{\text{max}}$ and can be considered equivalent. This duality is useful since it is often more convenient derive concentration limits from geometric arguments (conservation of étendue) rather than thermodynamic ones.

### 2.2.2. Conservation of étendue

The basic radiometric quantity is the radiance $L$, defined as the radiant power per unit surface area normal to the direction of travel, per unit solid angle about that direction:

$$L = \frac{dQ}{dA_p d\omega} = \frac{dQ}{dA \cos \theta d\omega} \quad (2.9)$$

In radiation heat transfer, this identical quantity is referred to as the radiation intensity, and given the symbol $I$. The radiometry nomenclature is preferable
for nonimaging optical design and is thus adopted here. The radiance may also be considered as a spectral quantity by taking the radiance emitted in a small wavelength interval \( d\lambda \) about a certain wavelength \( \lambda \). The subtleties of nonimaging optics arise from geometric dependencies rather than wavelength dependencies, and therefore we deal here only with total quantities, i.e. quantities integrated over all wavelengths.

To determine the amount of radiant power leaving or arriving at a particular surface \( A \) from a certain cone of directions subtending a solid angle \( \omega \), Eq. (2.9) may be integrated:

\[
Q = \int_{\omega} \int_{A} \frac{dG}{L dA \cos \theta d\omega}
\]  

(2.10)

The bracketed quantity, which is purely geometrical, is the differential étendue \( dG \). The differential étendue exists of two parts: (1) a spatial component, the differential surface area \( dA \); and (2) an angular component, the differential projected solid angle \( d\Omega = \cos \theta d\omega \).

By examining the radiation exchange between two blackbodies at thermal equilibrium, it may be shown that in an ideal optical system, both the radiance \( L \) and the differential étendue \( dG \) are conserved [22]. For electromagnetic radiation propagating through a medium with refractive index \( n \neq 1 \), the differential étendue is \( n^2 dA \cos \theta d\omega \), which, along with the basic radiance \( L^* = L/n^2 \), is conserved.

In nonideal systems, the radiance and étendue are not necessarily conserved. However, due to the directionality of energetic processes dictated by the second law of thermodynamics, the radiance and étendue must follow a set of rules, even in a nonideal system. The radiance may either be conserved or decrease. The étendue may either be conserved, may increase, or may be lost. The étendue cannot be “decreased” without a loss in radiant power of comparable magnitude. The distinction between these processes may be illustrated by considering some common nonideal optical systems.

Consider an aperture stop illuminated by a cone of radiation from blackbody source of constant radiance. The aperture stop clips the beam, allowing only the central region to pass and the outer region to be blocked. The
Radiance is conserved, but the étendue of the outer region is lost by this system.

Consider a beam of light that specularly reflects off a mirror with reflectivity \( \rho < 1 \). The specular nature of this reflection conserves the geometric nature (étendue) of the beam, but the radiance is decreased due to absorption of energy at the mirror surface.

Consider a Lambertian diffuser that is illuminated by a collimated beam of radiation and diffuses it into a hemisphere. Such a system decreases the radiance and increases the étendue of the beam.

### 2.2.3. Ray-representation in direction-cosine space

A ray is a geometrical construct describing the propagation of electromagnetic radiation in the limit of geometric optics. In 3 dimensions, a ray \( \hat{\mathbf{v}} \) is a 6-dimensional quantity (not including the refractive index \( n \)) fully defined by a point \( \mathbf{Q}(x, y, z) \) and a unit direction vector \( \hat{\mathbf{v}}(p, q, r) \). Figure 2.2(a) shows two generic rays \( \mathbf{u} \) and \( \mathbf{v} \) striking different points \( \mathbf{Q} \) on an aperture (x-y plane) with different directions. In 3-space, it is sufficient to only prescribe the first 2

![Figure 2.2: Diagram showing rays in 3-space and in direction-cosine space.](image)

**Figure 2.2.** (a) In 3-space a ray is defined by a point \( \mathbf{Q}(x, y, z = 0) \) and a direction \( \hat{\mathbf{v}}(p, q, r = -[1-p^2-q^2]^{1/2}) \); (b) In \( p-q \) direction-cosine space a ray is represented solely by \( p \) and \( q \), the \( x \) and \( y \) components of its unit direction vector (direction-cosines) respectively. The rays \( \mathbf{u} \) and \( \mathbf{v} \) shown in Cartesian coordinates in (a) have the representation in \( p-q \) space in (b). Any ray having the same direction would appear identically in \( p-q \) space, regardless of the point \( \mathbf{Q} \) where it strikes aperture.
components of the unit direction vector, \( p \) and \( q \), with \( r \) following from \( p^2 + q^2 + r^2 = 1 \). Furthermore, if the position of rays is restricted to a plane aperture (or more generally any two-parameter surface), a local coordinate system can be setup with \( z = 0 \) defining the aperture plane such that only the \( x \) and \( y \) components of \( \mathbf{Q} \) are needed. An important additional simplification can often be made for nonimaging applications: it is sufficient to consider only the direction of a ray striking the aperture, and neglect the point \( \mathbf{Q} \) where it strikes the aperture. For example, an ideal nonimaging concentrator designed for an infinite source will accept all rays having certain directions \((p, q)\) regardless of the point at which they strike the inlet aperture. The concept can still be useful for nonideal concentrators, where angular behavior varies in space over the inlet aperture, if we restrict the analysis to the worst case point on the inlet aperture with the goal of designing for full-collection. It is convenient then to plot only the direction-cosines of the rays in \( p-q \) space as shown for rays \( \mathbf{u} \) and \( \mathbf{v} \) in Figure 2.2 (b). Any ray arriving at the aperture with the same direction will be represented by the same point in \( p-q \) space. All rays in direction-cosine space must be inside the unit circle.

2.2.4. Phase-space representation of étendue

At the collecting aperture \( A_i \), the differential étendue \( dG_i \), of a source \( \Sigma_i \) with differential angular extent \( d\Omega_i = dpdq \), and differential spatial extent \( dA_i = dx dy \) is [23]:

\[
dG_i = n_i^2 dx dy dp dq = n_i^2 dA_i d\Omega_i
\]

(2.11)

where \( dx dy dp dq \) has the physical interpretation as an elemental volume in \( x-y-p-q \) phase space [23]. This can be reconciled with the definition of Eq. (2.10) by performing a change of variables from \( p-q \) space to spherical coordinates. Let \( \theta \) be zenith angle, and \( \phi \) the azimuth angle measured counterclockwise from the \( p \)-axis. The coordinate transformation from \( p, q \) to \( \theta, \phi \) may be expressed as:

\[
p = \sin \theta \cos \phi
\]

\[
q = \sin \theta \sin \phi
\]

(2.12)

The differential angular extent \( dp dq \) is equal to:
\[ dpdq = \det (J) = \begin{vmatrix} \frac{\partial p}{\partial \theta} & \frac{\partial p}{\partial \phi} \\ \frac{\partial q}{\partial \theta} & \frac{\partial q}{\partial \phi} \end{vmatrix} d\theta d\phi \]

which reveals that the differential angular extent is equal to the differential projected solid angle \( d\Omega = dpdq = \cos \theta \sin \theta d\theta d\phi = \cos \theta d\omega \). Integration over any ray bundle implies that a finite spatial extent is equivalent to a finite projected solid angle. The merit of the direction-cosine representation of a ray bundle is that its angular extent, or projected solid angle, is simply the “area” that this ray bundle occupies in a \( p-q \) space plot (see for example Figure 2.10).

The étendue is analogous to the four-dimensional volume of a 4-parameter bundle of rays in \( x, y, p, q \) phase space. From Fermat’s principle, it can be shown that in any “lossless”\(^2\) optical system the differential étendue of a ray bundle is conserved [23, 24], such that the differential étendue, \( dG_o \), of the same ray bundle at the exit aperture \( \Sigma_o \) is conserved:

\[ dG_o = n_o^2 dA_o d\Omega_o = n_o^2 dxdydpdq = dG_i \]  

(2.14)

This relation is formally known as the integral invariant of Poincaré [23]. By integrating for all ray bundles at the collecting aperture and exit aperture the conservation of integral étendue follows:

\[ G_i = n_i^2 \int_{\Sigma_i} dA_i d\Omega_i = G_o = n_o^2 \int_{\Sigma_o} dA_o d\Omega_o \]  

(2.15)

where it has been assumed that the material is homogenous such that the refractive index can be brought outside of the integral.

**Spatial isotropy in phase space**

The integral of Eq. (2.15) can be written as an iterated integral with integration over angular and spatial dimensions separated:

\[ G_\Sigma = n^2 \int_{\Omega_\pi(A)} \int_{\Sigma_\pi} d\Omega dA \]  

(2.16)

\(^2\) The term “lossless” implies that all surfaces are perfectly reflective and there are no internal aperture stops in the optical system.
where the limits of the angular integral are written as $\Omega \Sigma(A)$ since, in general, the limits of the angular integral may depend on the value of the spatial variable. If the angular limits are uniform over the area, then a condition of spatial isotropy exists, and the integral can be separated:

$$G_{\Sigma, \text{spatially isotropic}} = n^2 \left( \int_{\Omega_x} d\Omega \right) \left( \int_{A_x} dA \right) = n^2 A\Omega \quad (2.17)$$

In the spatially anisotropic case, for given maximum angular and spatial ranges $\Omega$ and $A$, the integral of Eq. (2.16) must be less than the product $A\Omega$:

$$\int_{\Omega_x(A_x)} d\Omega dA \leq A\Omega \quad (2.18)$$

To quantify the inequality of Eq. (2.18), it is convenient to introduce the degree of isotropy\(^3\) $g$, here defined in a slightly different manner than previously done in [25, 26]:

\(^3\) The degree of isotropy represents a fundamental intrinsic geometric loss of the optical concentration system. It is often manifested in other forms of optical aberration; the well known coma present in any parabolic concentrator is a source of anisotropy since different spatial regions of the focal plane have different angular distributions, i.e. the hotspot receives predominantly paraxial radiation, whereas the coma region receives only rays emerging from large rim angles.
where $g$ has the range 0 (full spatial anisotropy) to 1 (spatial isotropy).

Consider a plane aperture collecting incident direct solar radiation as shown in Figure 2.3. Assuming no shading of the aperture (by clouds or other obstructions), each spatial point on the collecting aperture receives the same angular distribution of rays as in Figure 2.3 (a). Therefore the phase space is spatially isotropic, and the source étendue is $G_i = n^2 A_i \Omega_i$. This is assumed to hold for any aperture collecting direct solar radiation throughout the remainder of this work.

2.2.5. The limit of concentration

The theory of maximum concentration of solar energy has been examined from several different perspectives. The derivations follow closely the works of Luque and Miñano [25], and Brunotte et al. [27].

The analysis is restricted to systems having full-collection, meaning that the exit aperture is sized to be large enough to collect all ray bundles from the source. We wish to find the smallest area $A_o$ that is just large enough to intercept all rays at the exit aperture $\Sigma_o$. For the case of maximum concentration, étendue is conserved:

$$G_o = n_o^2 g_o \Omega_o A_o = G_i = \Omega_i A_i$$

Equation (2.20) reveals that maximum concentration is achieved when the angular extent at the exit aperture is maximized under conditions of spatial isotropy. The maximum extent of $\Omega_o$ is a circle of radius 1, yielding:

$$C_{g,\text{max}} \equiv A_i / A_o = n_o^2 g_o \Omega_o / \Omega_i$$

Equation (2.21) reveals that maximum concentration is achieved when the angular extent at the exit aperture is maximized under conditions of spatial isotropy. The maximum extent of $\Omega_o$ is a circle of radius 1, yielding:
Equation (2.22) gives a convenient form for determining the maximum permissible concentration once the projected solid angle $\Omega_i$ of the source is known. The projected solid angle $\Omega_i$ of a source is of fundamental importance to this work.

Although Eq. (2.22) is a very powerful form of the limit of concentration, it is more common to see the limit of concentration expressed for particular source geometries. For example, for a source occupying a cone of rays subtending an (acceptance) half-angle of $\theta_i$, $\Omega_i = \pi \sin^2 \theta_i$ and Eq. (2.22) takes the familiar form:

$$C_{g, \text{max}} = \frac{n_o^2}{\sin^2 \theta_i}$$

(2.23)

**Maximum concentration in 2D**

If a particular optical system possesses linear symmetry, then it is convenient to treat étendue as a two-dimensional quantity. The two-dimensional étendue is:

$$\text{d}G_{2D} = n \text{d}x \cos \theta \text{d} \theta = n \text{d}x \text{d}p$$

(2.24)

Defining a 2D concentration based on aperture widths instead of areas, we arrive at the limit of concentration in 2D:

$$C_{g,2D,\text{max}} = \frac{a_i}{a_o} = \frac{n_o \Delta p_o}{\Delta p_i} = \frac{2n_o}{\Delta p}$$

(2.25)

For a source comprising a ray fan subtending a half-angle $\theta_i$, $\Delta p_i = 2\sin \theta_i$, and we recover the familiar formula for the maximum concentration of a 2D system:

$$C_{g,2D,\text{max}} = \frac{a_i}{a_o} = \frac{n_o}{\sin \theta_i}$$

(2.26)

A note on the refractive index $n_o$

In all of the designs considered in this thesis, there exists at most one dielectric interface, always occurring at the outlet of the system (i.e. the receiver being
immersed in the dielectric). For this reason, we omit the subscript “o” on the refractive index of the material in which the exit aperture is immersed.

2.3. Concentrating solar collectors

2.3.1. Acceptance angle and the need for tracking

In Section 2.2.3, it was shown that the achievable concentration is inversely proportional to the projected solid angle of the cone of light that the concentrator is able to accept. Simply put, the higher the acceptance angle, the lower the achievable concentration. Since to an earthly observer the sun appears to move through the sky, this establishes the need of tracking for concentrating solar collectors.

Consider a stationary solar concentrator designed to collect all radiation, direct and diffuse, from the sky hemisphere. The smallest acceptance angle occurs when the concentrator is facing up, for which the required acceptance angle is \( \theta_i = 90^\circ \). From Eq. (2.23), the maximum achievable concentration is \( C_{g,\text{max}} = 1 \times \) (no concentration) for a hollow design and \( n^2 \) for an immersed one. Clearly this is not satisfactory.

Consider a stationary solar concentrator designed to collect all direct radiation throughout the year. The smallest required acceptance angle occurs when the concentrator is tilted southward from horizontal by an angle equal to the latitude [28]. The resulting projected solid angle of the effective source is [28]:

\[
\Omega_i = 2\varepsilon + \sin 2\varepsilon
\]

(2.27)

where \( \varepsilon = 23.44^\circ \) is the obliquity of earth’s axis. From Eq. (2.22), the maximum concentration for collection of direct radiation by a stationary concentrator is therefore:

\[
C_{g,\text{max}} = \frac{\pi n^2}{2\varepsilon + \sin 2\varepsilon}
\]

(2.28)

which equates to \( 2.0 \times \) for \( n = 1 \) and \( 4.5 \times \) for \( n = 1.5 \). To achieve higher concentrations, it is necessary to reduce the required acceptance angle, or more
correctly the accepted projected solid angle. One option is to collect radiation for only a portion of the day, e.g. for $2t$ hours of the equatorial day. Then the achievable concentration given by Eq. (2.28) is augmented by a factor $\csc(360° \cdot t/24 \text{ hours})$ [28]. To achieve concentrations higher than this, it is necessary to track the apparent motion of the sun through the sky.

2.3.2. Tracking schemes

By tracking the apparent motion of the sun, the projected solid angle traced by the solar disk throughout the year with respect to the aperture can be significantly reduced, allowing higher concentrations to be achieved. The extent of this depends on the tracking scheme employed. Tracking schemes may be either discrete or continuous. Discrete tracking schemes utilize concentrators with acceptance angles considerably larger than the angular size of the solar disk, such that it is sufficient to periodically adjust the alignment of the collector, e.g. hourly, daily or seasonally.

High concentrations require continuous tracking such that the acceptance angle can be made on the order of the angular size of the solar disk. Continuous tracking schemes are classified by the number of angular degrees of freedom (DOF). Two-axis trackers have two angular DOF making it possible to achieve any orientation in three-dimensional space. Two-axis trackers normally operate by making the normal of the collecting aperture parallel to the sun vector. One-axis trackers have one angular DOF and therefore the collecting aperture cannot be made to directly face the solar disk at all sun positions. Different tracking strategies are employed for one-axis trackers classified according to the orientation of the tracking axis. The most axis orientations are: horizontal N-S; horizontal E-W; and polar (axis parallel to the earth’s rotation axis). One-axis trackers form the basis of trough concentrators and are discussed in detail in Section 2.4.

2.3.3. Acceptance of diffuse radiation

In Section 2.3.1 it was shown that a concentrator designed to collect all diffuse radiation from the sky hemisphere can achieve only negligible concentration. It was therefore concluded that concentrating collectors must be designed to collect direction radiation only. However, it should be should be noted that
concentrating collectors can collect a portion of the diffuse radiation. To determine fraction of diffuse radiation collected by a concentrator, consider a simplified model of the sky hemisphere where the diffuse radiance $L_{\text{diffuse}}$ is constant in all directions. Further assume the ground reflectance to be 1, such that the ground appears to have the same radiance as the sky. Under these assumptions, the diffuse fraction of radiation collected by a concentrator is:

$$\frac{E_{\text{diffuse, collected}}}{E_{\text{diffuse}}} = \frac{\int \Sigma L_{\text{diffuse}} \cos \theta \sin \theta \, d\theta \, d\varphi}{\int_{\text{hemisphere}} L_{\text{diffuse}} \cos \theta \sin \theta \, d\theta \, d\varphi} = \frac{\Omega_i}{\pi} = \frac{n^2}{C_{g,\text{max}}} \quad (2.29)$$

from which we come to the elegant conclusion that the fraction of diffuse radiation collected by a concentrator is inversely proportional to the concentration. In reality, the collected diffuse fraction will differ slightly due to the anisotropy of the diffuse sky radiance and imperfect ground reflection. Nevertheless, Eq. (2.29) serves as a very useful rule-of-thumb and indicates that, for high-concentration collectors, the collected fraction of diffuse radiation is very small.

2.3.4. The solar resource for concentrating collectors

As was shown in Section 2.3.3, concentrating collectors cannot accept an appreciable amount of diffuse radiation. This is a fundamental limitation and must be considered when comparing them to non-concentrating collectors which can, in theory, collect radiation from the whole hemisphere. A comparison of the available solar radiation for non-concentrating and concentrating collectors having different tracking schemes was performed to elucidate this fundamental limitation. For such a comparison, it is most accurate to use ground based irradiation measurements. For this purpose the Baseline Surface Radiation Network (BSRN) database was used. Table 2.1 shows the characteristics of the 11 sites selected for the comparison. These sites were selected based on the availability of radiation data, and to establish a diverse selection of site latitudes and climates.

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4 This material has been extracted from J. Müller, “The solar resource for concentrating photovoltaic (CPV) systems as a function of tracking range and acceptance angle,” Semester Project, ETH Zurich, 2013, conducted under the direct supervision of T. Cooper.
Figure 2.5 shows the yearly available solar irradiation for the 11 BSRN sites for four different collector configurations: a stationary south-facing latitude-tilt non-concentrating collector; a N-S one-axis concentrating collector; a two-axis concentrating collector; and a two-axis non-concentrating collector. The concentrating collectors are assumed to collect only direct radiation, while the non-concentrating collectors are assumed to collect all sky and ground radiation (ground reflectivity is assumed to be 0.2) reaching their aperture. The values for E-W oriented horizontal one-axis trackers are not shown, but are generally 10% lower than the values for N-S collectors due to the higher yearly-average cosine loss.

The two-axis non-concentrating collector is indicative of the greatest possible radiation availability, and serves as a basis for comparing the other collectors. The radiation collected by an ideal two-axis non-concentrating collector is referred to as the global normal irradiation (GNI). Interestingly, except for very low irradiance sites, the two-axis concentrator collects more than the stationary non-concentrating collector. This serves as a motivation for pursuing concentrating photovoltaics. The N-S one-axis tracker collects the least for all but the two highest irradiation sites.

Table 2.1. Baseline Surface Radiation Network (BSRN) databases considered for the evaluation of the solar resource for concentrating collectors.

<table>
<thead>
<tr>
<th>Site</th>
<th>Label</th>
<th>Latitude [°]</th>
<th>Longitude [°]</th>
<th>Elevation [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alice Springs, Australia</td>
<td>ASP</td>
<td>−23.7980</td>
<td>133.8880</td>
<td>547</td>
</tr>
<tr>
<td>Bermuda</td>
<td>BER</td>
<td>32.2670</td>
<td>−64.6670</td>
<td>8</td>
</tr>
<tr>
<td>Carpentras, France</td>
<td>CAR</td>
<td>44.0830</td>
<td>5.0590</td>
<td>100</td>
</tr>
<tr>
<td>Cener, Navarra, Spain</td>
<td>CNR</td>
<td>42.8160</td>
<td>−1.6010</td>
<td>471</td>
</tr>
<tr>
<td>Southern Great Plains, USA</td>
<td>E13</td>
<td>36.6050</td>
<td>−97.4850</td>
<td>318</td>
</tr>
<tr>
<td>Izaña, Tenerife, Spain</td>
<td>IZA</td>
<td>28.3093</td>
<td>−16.4993</td>
<td>2373</td>
</tr>
<tr>
<td>Sede Boqer, Israel</td>
<td>SBO</td>
<td>30.8597</td>
<td>34.7794</td>
<td>500</td>
</tr>
<tr>
<td>Solar Village, Saudi Arabia</td>
<td>SOV</td>
<td>24.9100</td>
<td>46.4100</td>
<td>650</td>
</tr>
<tr>
<td>Payerne, Switzerland</td>
<td>PAY</td>
<td>46.8150</td>
<td>6.9440</td>
<td>491</td>
</tr>
<tr>
<td>Tamanrasset, Algeria</td>
<td>TAM</td>
<td>22.7903</td>
<td>5.5292</td>
<td>1385</td>
</tr>
<tr>
<td>Toravere, Estonia</td>
<td>TOR</td>
<td>58.2540</td>
<td>26.4620</td>
<td>70</td>
</tr>
</tbody>
</table>
Figure 2.5. Available yearly solar irradiation for different collector types: Latitude-tilt non-concentrating (flat-panel); horizontal N-S one-axis tracking concentrating; two-axis tracking concentrating; and two-axis tracking non-concentrating (flat-panel), based on 11 selected sites of the BSRN database. A ground reflectance $\rho_{\text{ground}} = 0.2$ is assumed.

Figure 2.4. Collectable fraction of the GNI for two-axis and one-axis N-S concentrating collectors for the 11 BSRN sites considered, plotted as a function of the GNI.
Figure 2.4 shows the collectable fraction of the GNI for two-axis and N-S one-axis concentrating collectors plotted as against the GNI. The plot shows a trend of increasing collectable fraction with increasing GNI. Simply put, the higher the radiation availability at a given site, the better is that site for solar energy collection in general, and the better a concentrating collector will fare against a non-concentrating collector. Concentrating solar collectors are therefore best suited for deployment in high-insolation areas.

2.4. One-axis trackers

2.4.1. Coordinate system

A one-axis tracker is conveniently oriented within the cardinal frame of Figure 2.1 by specifying the unit direction vector $\hat{a}$ pointing along its tracking axis as shown in Figure 2.6. The axis vector $\hat{a}$ may be parameterized in analogous manner to the sun vector by an axis altitude angle $\alpha_a$ and axis azimuth angle $\gamma_a$.

---

\[ \mathbf{a} = \begin{bmatrix} -\cos \alpha \sin \gamma_a, & -\cos \alpha \cos \gamma_a, & \sin \alpha_a \end{bmatrix} \]  

(2.30)

Within the cardinal coordinate frame, a new coordinate system shown in Figure 2.7 is set up, which is more conveniently oriented with respect to the tracking primary aperture. First the parked coordinate frame \((x_0, y_0, z_0)\) defining the zero position of the tracking primary aperture is established. It is oriented within the cardinal coordinate frame according to two criteria: (1) \(y_0\) is parallel to \(\mathbf{a}\); and (2) \(z_0\) points in the direction of the aperture normal \(\mathbf{n}\) when the sun is at solar noon \((\gamma_s = 0)\).

As the concentrator tracks, its aperture is rotated about the \(y_0\) axis by an angle \(\psi\). It is convenient to have a tracking coordinate frame \((x, y, z)\) which follows the rotation of the aperture. In this tracking frame, the \(z\)-axis is always parallel to the aperture normal and defines the optical axis of the primary concentrator. The principle planes of the tracking aperture, defined in the tracking \((x, y, z)\) coordinate frame, are: the aperture \(x\)-\(y\) plane; the transverse \(x\)-\(z\) plane; and the axial \(y\)-\(z\) plane. Only collectors having their tracking axis coplanar with the collecting aperture are considered in this work.
Chapter 2

2.4.2. The skew angle for one-axis trackers

Sun-tracking serves the general function of minimizing the incidence angle between the aperture normal \( \hat{n} \) and the sun vector \( \hat{s} \). For a one-axis tracker, there are insufficient degrees of freedom to obtain normal incidence for all sun positions, resulting in a time dependent nonzero incidence angle: the skew angle \( \vartheta \) defined in Figure 2.8. \( \vartheta \) is minimized when \( \hat{s} \) and \( \hat{n} \) are coplanar with the axis vector \( \hat{a} \), as in Figure 2.7.

Minimization of the incidence angle implies that the axis vector, the normal vector and the sun vector are all coplanar, yielding:

\[
\hat{s} \cdot (\hat{n} \times \hat{a}) = 0
\]

which can be solved for the normal vector. Once the normal vector is known, the skew angle may be found from:

\[
\cos \vartheta = \hat{s} \cdot \hat{n}
\]

However, with the aid of Figure 2.9 the skew angle may be more conveniently found from:

\[
\sin \vartheta = \hat{s} \cdot \hat{a}
\]
which has the advantage both of preserving the sign of the skew angle, and of being readily calculated without knowledge of the normal vector. Expanding out Eq. (2.33) in terms of the components of $\mathbf{s}$ and $\mathbf{a}$ we obtain a general formula for the skew (incidence) angle for one-axis trackers:

$$\sin \vartheta = \cos \alpha_s \cos \alpha_a \cos (\gamma - \gamma_a) + \sin \alpha_s \sin \alpha_a$$  \hspace{1cm} (2.34)

Equation (2.34) boasts a twofold improvement over the formulas for tracking aperture incidence angles commonly found in the literature [2, 15, 29]: (1) it is general for all one axis trackers provided that the tracking axis is parallel to the aperture plane; and (2) the result is expressed in terms of $\sin \vartheta$ rather than $\cos \vartheta$, such that the sign of the skew angle is preserved.

Further expanding $\alpha_s$ and $\gamma_s$ in terms of latitude $\phi$, declination angle $\delta$, and hour angle $\omega$ yields:

$$\sin \vartheta = \sin \delta \left( \sin \phi \sin \alpha_a - \cos \phi \cos \alpha_a \cos \gamma_a \right)$$
$$+ \cos \delta \left[ \cos \phi \cos \omega \sin \alpha_a + \cos \alpha_a \left( \sin \phi \cos \omega \cos \gamma_a + \sin \omega \sin \gamma_a \right) \right]$$  \hspace{1cm} (2.35)

The maximum and minimum skew angles for polar, horizontal E-W, and horizontal N-S one-axis trackers are derived in the following sections following Eq. (2.35). Derivations are performed for positive latitudes (northern hemisphere), but may appropriately adapted for the southern hemisphere. The
Table 2.2. Minimum and maximum skew angles for common one-axis trackers. The values for a horizontal N-S tracker are a function of latitude $\phi$, whereas for the polar and horizontal E-W trackers they are constant.

<table>
<thead>
<tr>
<th></th>
<th>Polar</th>
<th>Horizontal N-S</th>
<th>Horizontal E-W</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_{\min}$</td>
<td>$-23.44^\circ$</td>
<td>$-\arcsin[\sin(23.44^\circ)/\cos\phi]$</td>
<td>$-90^\circ$</td>
</tr>
<tr>
<td>$\theta_{\max}$</td>
<td>$+23.44^\circ$</td>
<td>$23.44^\circ +</td>
<td>\phi</td>
</tr>
</tbody>
</table>

results are summarized in Table 2.2. In a collector field, the maximum and minimum angles may differ slightly from those listed due to shading restrictions from adjacent concentrations.

**Polar tracker**

For a polar tracker, $\gamma_a = 0^\circ$ and $\alpha_a = -\phi$. Subbing into Eq. (2.34) yields:

$$\sin \vartheta = \cos \alpha_s \cos \gamma_s \cos \phi - \sin \alpha_s \sin \phi$$  \hspace{1cm} (2.36)

Subbing in Eq. (2.3), Eq. (2.36) simplifies to the trivial solution:

$$\vartheta = -\delta$$  \hspace{1cm} (2.37)

The minimum and maximum values are $\theta_{\min,\text{polar}} = -\varepsilon = -23.44^\circ$ and $\theta_{\max,\text{polar}} = +\varepsilon = +23.44^\circ$ occurring a solar noon on the solstices.

**Horizontal E-W tracker**

For a horizontal E-W tracker, $\gamma_a = 90^\circ$ and $\alpha_a = 0^\circ$ for which Eq. (2.34) gives:

$$\sin \vartheta = \cos \alpha_s \sin \gamma_s$$  \hspace{1cm} (2.38)

Subbing in Eq. (2.4):

$$\sin \vartheta = \cos \alpha_s \frac{\cos \delta \sin \omega}{\cos \alpha_s} = \cos \delta \sin \omega$$  \hspace{1cm} (2.39)

The sunrise and sunset hour angle is [2]:

$$\cos \omega_{ss} = -\tan \phi \tan \delta$$  \hspace{1cm} (2.40)
On the equinoxes, $\delta = 0^\circ$ and $\omega_{ss} = 90^\circ$. Plugging these values in to Eq. (2.39) we find that the minimum and maximum skew angles for an E-W tracker are $-90^\circ$ and $+90^\circ$, occurring on the equinoxes at sunrise and sunset respectively. Note that since these extrema occur at sunrise/sunset, they will not occur for collectors in a field, due to shading by neighboring troughs.

**Horizontal N-S tracker**

For a horizontal N-S tracker, $\gamma_a = 0^\circ$ and $\alpha_a = 0^\circ$ for which Eq. (2.34) gives:

$$\sin \vartheta = \cos \alpha_s \cos \gamma_s$$  

(2.41)

Substituting in Eqs. (2.2) and (2.3), and applying some trigonometric simplifications we get:

$$\sin \vartheta = \sin \phi \cos \delta \cos \omega - \cos \phi \sin \delta$$  

(2.42)

The minimum occurs at sunrise/sunset on summer solstice (or winter solstice in the southern hemisphere) where:

$$\cos \omega_{ss} = -\tan \phi \tan \delta = -\tan \phi \tan \epsilon$$  

(2.43)

yielding:

$$\vartheta_{\min} = \arcsin \left( -\sin \phi \cos \epsilon \tan \phi \tan \epsilon - \cos \phi \sin \epsilon \right)$$

$$= -\arcsin \left( \sin \epsilon \sec \phi \right)$$  

(2.44)

Since this minimum occurs at sunrise/sunset, it will not occur in practice for collectors in a field, due to shading by neighboring troughs, but nevertheless serves as an absolute possible minimum. The maximum skew angle occurs at solar noon on winter solstice (or winter solstice in the southern hemisphere), yielding:

$$\vartheta_{\max} = \arcsin \left( \sin \phi \cos \epsilon + \cos \phi \sin \epsilon \right) = |\phi| + \epsilon$$  

(2.45)

The absolute value in the final equality has been added such that the formula can be used both for the northern and southern hemispheres.
2.4.3. The effective source map for one-axis trackers

The effective source map for a one-axis tracking aperture is found by mapping out the directions of all rays from the solar disk arriving at the aperture from the minimum to the maximum skew angles. For normal incidence, the direction of all rays reaching the aperture from the edge of the solar disk may be described parametrically by:

\[
\hat{v}(\xi) = \begin{bmatrix} \sin \theta_{\text{sun}} \sin \xi & \sin \theta_{\text{sun}} \cos \xi & \cos \theta_{\text{sun}} \end{bmatrix}
\approx \begin{bmatrix} \sin \theta_{\text{sun}} \sin \xi & \sin \theta_{\text{sun}} \cos \xi & 1 \end{bmatrix}
\]

where \( \xi \) is a parameter ranging from 0 to \( 2\pi \). For nonzero skew, we rotate \( \hat{v}(\xi) \) about the \( x \)-axis by the skew angle \( \vartheta \):

\[
\hat{v}(\vartheta, \xi) = R_x(\vartheta) \hat{v}(\xi)
\]

When plotted in \( p-q \) space, the directions of rays arriving from the edge of the solar disk at a given skew angle form an ellipse with center \((0, \cos \theta_{\text{sun}} \sin \vartheta) \approx (0, \sin \vartheta)\) and with semi-diameters \( \sin \theta_{\text{sun}} \) and \( \sin \theta_{\text{sun}} \cos \vartheta \) in the \( p \) and \( q \) directions respectively, as shown in Figure 2.10 (a). By superimposing the individual maps from \( \vartheta_{\text{min}} \) to \( \vartheta_{\text{max}} \) we obtain the effective source map shown in Figure 2.10 (b).

2.4.4. Maximum concentration of a one-axis tracker

Knowing the effective source map of a one-axis tracker it is straightforward to calculated the maximum geometric concentration at full-collection. Here we make no \( a \ priori \) assumptions about the geometry of the concentrator.

Referring to Eq. (2.22), the maximum concentration of a one axis tracker can be determined once the projected solid angle \( \Omega_i \) of the effective source map
is known. This is simply the area in $p$-$q$ space of the effective source map, Figure 2.10 (b):

$$\Omega_i \approx 2 \sin \theta_{\text{sun}} \left( \sin \vartheta_{\max} - \sin \vartheta_{\min} \right)$$

(2.48)

The maximum concentration of a one-axis tracker is therefore

$$C_{g,\text{max}} = \frac{\pi n^2}{\Omega_i} \approx \frac{\pi n^2}{2 \sin \theta_{\text{sun}} \left( \sin \vartheta_{\max} - \sin \vartheta_{\min} \right)}$$

(2.49)

The skew angle range for common axis orientations was given in Table 2.2. This limit was previously derived in [27] for the specific case of a polar tracker. The smallest skew range, and highest maximum concentration, occurs for a polar tracker, or a horizontal N-S axis tracker at the equator, and evaluates to $425n^2$.

2.4.5. Maximum concentration of a one-axis tracking trough

In Section 2.4.4 the maximum concentration of a one-axis tracker was derived based on conservation of étendue, making no restrictions on the concentrator.

---

**Figure 2.10.** (a) Source map of the solar disk at a skew angle of $\vartheta$ for a one-axis tracker. $q$ points in the direction of the tracking axis; (b) the effective source map for a one-axis tracker. The effective source map is formed by superimposing the individual source maps (a) over all sun positions seen by the collector. In both schematics the angular size of the solar disk has been intentionally oversized to emphasize its shape.
geometry. Most often, one-axis tracker utilize trough (line-symmetric) concentrator geometries. This symmetry imposes a stricter limit on the achievable concentration. To derive the limit for troughs, we introduce the rigorous procedure of source/acceptance map matching, which will prove to be a useful tool for the sizing of the acceptance angle of concentrators in general.

**Hollow trough**

First consider a hollow (reflective) trough, as will be present in the vast majority of cases. Figure 2.11 shows a generic ray \( \hat{v} \) approaching the inlet of a generic trough concentrator with line symmetry along the \( y \)-axis. A 2D concentrator will accept any ray if the angle between the \( x-z \) projection of the ray and the optical axis \( z \) is within the acceptance angle \( \theta_i \) of the concentrator. This condition may be expressed mathematically as:

\[
\sin |\theta_{\text{proj}}| = \frac{p}{\left(p^2 + r^2\right)^{1/2}} \leq \sin |\theta_i|
\]  

(2.50)

Squaring both sides and applying the identity \( p^2 + q^2 + r^2 = 1 \) yields:
Plotting out Eq. (2.51) in $p$-$q$ space, we obtain the acceptance map of a 2D concentrator, which is an ellipse with semi-minor axis $\sin\theta_i$ in the $p$-direction and semi-major axis 1 in the $q$-direction (symmetry axis), as shown in Figure 2.11. For an ideal linear 2D concentrator (e.g. 2D CPC), all rays within the acceptance map are accepted by the concentrator and all rays outside are rejected. For a non-ideal 2D concentrator (e.g. parabolic trough), all rays within the acceptance map are accepted by the concentrator as long as the acceptance angle for full-collection is used for $\theta_i$.

In the source/acceptance map matching method, the acceptance angle is sized such that the acceptance map just envelopes the effective source map, resulting in the intersection “design point” shown in Figure 2.12. This point may be expressed mathematically by:

$$\frac{p^2}{\sin^2 \theta_i} + q^2 \leq 1$$  \hspace{1cm} (2.51)

Plotting out Eq. (2.51) in $p$-$q$ space, we obtain the acceptance map of a 2D concentrator, which is an ellipse with semi-minor axis $\sin\theta_i$ in the $p$-direction and semi-major axis 1 in the $q$-direction (symmetry axis), as shown in Figure 2.11. For an ideal linear 2D concentrator (e.g. 2D CPC), all rays within the acceptance map are accepted by the concentrator and all rays outside are rejected. For a non-ideal 2D concentrator (e.g. parabolic trough), all rays within the acceptance map are accepted by the concentrator as long as the acceptance angle for full-collection is used for $\theta_i$.

In the source/acceptance map matching method, the acceptance angle is sized such that the acceptance map just envelopes the effective source map, resulting in the intersection “design point” shown in Figure 2.12. This point may be expressed mathematically by:

$$\frac{\sin^2 \theta_{\text{sun}}}{\sin^2 \theta_{i,\text{max}}} + \sin^2 \left| p \right|_{\text{max}} = 1$$  \hspace{1cm} (2.52)
Rearranging, we obtain:

$$\sin \theta_{t,1} = \sin \theta_{\text{sun}} \sec |\vartheta|_{\text{max}}$$ \hspace{1cm} (2.53)

From Figure 2.12 it is evident that for a full-collection trough primary some of the available étendue is unused, as evidenced by the regions between the acceptance map and the effective source map. From this we can make the very important conclusion that, from the point of view of conservation of étendue, no one-axis tracking trough concentrator can be ideal. This fundamental limitation is due to the mismatch in shape between the source and acceptance maps.

The maximum concentration of a hollow one-axis tracking is then simply:

$$C_{g,\text{max}} = \frac{1}{\sin \theta_{t,1}} = \frac{\cos |\vartheta|_{\text{max}}}{\sin \theta_{\text{sun}}}$$ \hspace{1cm} (2.54)

It is evident that the maximum skew angle plays a predominant role in the maximum concentration that can be achieved with a one-axis tracking trough. For $|\vartheta|_{\text{max}} = 90^\circ$, the full-collection geometric concentration vanishes.

**Dielectric-filled trough**

If the trough is filled with a dielectric material (e.g. a 2D DCPC [30]), and refraction occurs only on planar surfaces, then the acceptance map is described by the ellipse:

$$\frac{p^2}{\sin^2 \theta_i} + \frac{q^2}{n^2} \leq 1$$ \hspace{1cm} (2.55)

where $\theta_i$ is the acceptance angle outside of the dielectric. Subbing in the design point [$\sin \theta_{\text{sun}}$, $\sin |\vartheta|_{\text{max}}$] we obtain:

$$\frac{\sin^2 \theta_{\text{sun}}}{\sin^2 \theta_i} + \frac{\sin^2 |\vartheta|_{\text{max}}}{n^2} = 1$$ \hspace{1cm} (2.56)

Solving for $\theta_i$, we obtain:
\[
\sin \theta_i = \frac{n \sin \theta_{\text{sun}}}{\sqrt{n^2 - \sin^2 |\vartheta|_{\text{max}}}}
\]  
(2.57)

which reduces to Eq. (2.53) for \(n = 1\). The achievable concentration is:

\[
C_{g,\text{max}} = \frac{n}{\sin \theta_i} = \frac{\sqrt{n^2 - \sin^2 |\vartheta|_{\text{max}}}}{\sin \theta_{\text{sun}}}
\]  
(2.58)

Interestingly, unlike for the case of a hollow trough, the concentration of a dielectric filled trough, does not vanish for \(|\vartheta|_{\text{max}} = 90^\circ\), but rather reaches a limiting value of \((n^2 - 1)^{1/2}/\sin \theta_{\text{sun}}\).

2.4.6. Skew dilation

The maximum concentration formula for a hollow one-axis tracking trough, Eq. (2.53), was derived by the rigorous method of source/acceptance map matching. It is also possible to derive this formula through simple geometric argument, which gives an intuitive interpretation of the \(\cos |\vartheta|_{\text{max}}\) term in Eq. (2.53). The general formula for the maximum concentration of a hollow 2D trough-like concentrator is:

\[
C_{g,\text{max,2D}} = 1/\sin \theta_i
\]  
(2.59)

where \(\theta_i\) is the acceptance half-angle of the concentrator in the plane of linear symmetry (transverse plane). At normal incidence, \(\theta_i\) may be chosen based on the angular radius of the sun \(\theta_{\text{sun}}\), which in practice is increased by some factor to account for optical and tracking errors. For one-axis trackers however, the apparent angular size of the solar disk is increased due to the presence of nonzero skew angles. This phenomenon, which we denote skew dilation, was detailed in the classic report from SERI [31], and is represented by Figure 2.13. At normal incidence, the angular radius of the sun is:

\[
\theta_{\text{sun}} = \arcsin \left( \frac{R_{\text{sun}}}{d_{\text{sun}}} \right)
\]  
(2.60)

with a value of 0.266° at the yearly mean sun-earth distance [14]. At nonzero skew, the effective angular radius of the sun in the \(x\)-\(z\) plane is found from:
By projecting the ray-trace diagram of skew rays into the x-z plane \[31\], it is found that the required acceptance angle of the primary for collecting all rays from the solar disk year-round must be:

\[
\sin \theta_{\text{eff}} = \frac{R_{\text{sun}}}{d_{\text{sun}} \cos \vartheta} = \sin \theta_{\text{sun}} \sec \vartheta
\]

By projecting the ray-trace diagram of skew rays into the x-z plane \[31\], it is found that the required acceptance angle of the primary for collecting all rays from the solar disk year-round must be:

\[
\sin \theta_i = \sin \theta_{\text{sun}} \sec |\vartheta|_{\text{max}}
\]

which is identical to Eq. (2.53) derived by the more rigorous source/acceptance matching method.

2.5. Some solar concentrator performance metrics\(^6\)

2.5.1. Optical efficiency

Along with the geometric concentration ratio, the principal performance metric for a solar concentrator is the optical efficiency, defined as the fraction of radiant power at the inlet of the concentrator that reaches the outlet:

\[
\eta_{\text{opt}} = \frac{Q_o}{Q_i}
\]

---

The optical efficiency gives the relationship between geometric and flux concentrations of the system:

\[ \eta_{\text{opt}} = \frac{Q_o}{Q_i} = \frac{E_o}{E_i} = \frac{A_o}{A_{\text{g}}} \]  

(2.64)

2.5.2. Phase-space quantities

The phase-space quantities completely describe the geometric passage through a perfectly shaped concentrator of a ray striking the inlet aperture at position \((x, y, z = 0)\) and direction \((p, q)\). The first is the directional acceptance function:

\[ a'(x, y, p, q) = \begin{cases} 1 & \text{if ray accepted} \\ 0 & \text{if ray rejected} \end{cases} \]  

(2.65)

and the second the number of reflections function:

\[ n_i(x, y, p, q) = \text{integer \# of reflections suffered on route to outlet or back to inlet} \]  

(2.66)

2.5.3. Acceptance efficiency

The area-averaged directional acceptance is:

\[ \bar{a}'(p, q) = \frac{\int_A L(x, y, p, q) a'(x, y, p, q) \, dx \, dy}{\int_A L(x, y, p, q) \, dx \, dy} \]  

(2.67)

If the inlet aperture is uniformly illuminated, then:

\[ \bar{a}'(p, q) = \frac{I(x, y, p, q) \int_A a'(x, y, p, q) \, dx \, dy}{L(x, y, p, q) \int_A \, dx \, dy} = \frac{\int_A a'(x, y, p, q) \, dx \, dy}{A_i} \]  

(2.68)

The 1D transmission-angle curves may be calculated from the area-averaged directional acceptance by changing to polar coordinates and then averaging over the circumferential angle:

\[ \cos \theta = \sqrt{1 - p^2 - q^2} \]

\[ \tan \varphi = q/p \]  

(2.69)
\[
\bar{a}'(\theta) = \frac{\int_{0}^{2\pi} a'(\theta, \phi) d\phi}{2\pi}
\]

(2.70)

In the general case, we refer to the integral quantity of the acceptance performance of a concentrator as the acceptance efficiency \(\eta_{\text{acc}}\). It may be calculated from:

\[
\eta_{\text{acc}} = \frac{\int_{A} \int_{\Omega_{i}} L(x, y, p, q) a'(x, y, p, q) dx dy dp dq}{\int_{A} \int_{\Omega_{i}} L(x, y, p, q) dx dy dp dq}
\]

\[
= \frac{\int_{A} \int_{\Omega_{i}} L(x, y, p, q) a'(x, y, p, q) dx dy dp dq}{Q_i}
\]

(2.71)

This quantity can be thought of as the optical efficiency of the system, but considering only geometric losses (shading, blocking, spillage, and ray-rejection). It is the optical efficiency that would be obtained for the case of perfect optical properties (100% transmittance, 100% reflectance, no absorption) for all surfaces.

2.5.4. Average number of reflections

From the phase-space quantities, the average number of reflections is defined as:

\[
\langle n_r \rangle = \frac{\int L(x, y, p, q) a'(x, y, p, q) n_t(x, y, p, q) dx dy dp dq}{\int L(x, y, p, q) a'(x, y, p, q) dx dy dp dq}
\]

(2.72)

Note that in this definition, rays that are rejected by the concentrator are not included since reflections suffered by these rays do not affect the optical efficiency. Evaluation of \(\langle n_r \rangle\) by means of the fundamental definition of Eq. (2.72) is possible by ray-tracing but requires a detailed accounting of which rays reach the exit aperture and which are rejected, which can be tedious. We therefore propose an alternative method based on evaluation of the slope of the curve of optical efficiency vs. reflectivity near \(\rho = 1\). Following Rabl [32], and extending to account for the fixed loss (quantified by \(\eta_{\text{acc}}\)) due to rejection:
\[ \eta_{\text{opt}}(\alpha) = \eta_{\text{acc}} \cdot (1 - \alpha \cdot \langle n_t \rangle + O(\alpha^2)) \]  

(2.73)

Taking the derivative of \( \eta_{\text{opt}} \) with respect to \( \alpha \) yields:

\[ \frac{\partial \eta_{\text{opt}}(\alpha)}{\partial \alpha} = \eta_{\text{acc}} \cdot (-\langle n_t \rangle + O(\alpha)) \]  

(2.74)

Rearranging for \( \langle n_t \rangle \):

\[ \langle n_t \rangle = -\frac{1}{\eta_{\text{acc}}} \frac{\partial \eta_{\text{opt}}(\alpha)}{\partial \alpha} + O(\alpha) \]  

(2.75)

By taking the limit as \( \alpha = 1 - \rho \to 0 \), the error term vanishes giving:

\[ \langle n_t \rangle = -\frac{1}{\eta_{\text{acc}}} \lim_{\alpha \to 0} \frac{\partial \eta_{\text{opt}}(\alpha)}{\partial \alpha} = \frac{1}{\eta_{\text{acc}}} \lim_{\rho \to 1} \frac{\partial \eta_{\text{opt}}(\rho)}{\partial \rho} = \frac{1}{\eta_{\text{acc}}} \frac{\partial \eta_{\text{opt}}(\rho)}{\partial \rho} \bigg|_{\rho = 1} \]  

(2.76)

This expression gives a simple method to evaluate the average number of reflections based solely on the optical efficiency which is easily calculated by ray-tracing. For this purpose, the optical efficiency is calculated at \( \rho_1 = 1 \) and \( \rho_2 = 1 - \Delta \rho \), where \( \Delta \rho \ll 1 \). The average number of reflections is then found from:

\[ \langle n_t \rangle \approx \left. \frac{1}{\eta_{\text{acc}}} \frac{\Delta \eta_{\text{opt}}}{\Delta \rho} \right|_{\rho = 1} = \frac{1}{\eta_{\text{acc}}} \frac{\eta_{\text{opt}}(\rho_1 = 1) - \eta_{\text{opt}}(\rho_2 = 1 - \Delta \rho)}{\Delta \rho} \]  

(2.77)

Note that when evaluating \( \eta_{\text{opt}} \) by Monte-Carlo ray-tracing, it is important that sufficient rays are used such that the random error in \( \eta_{\text{opt}} \) is significantly lower than \( O(\Delta \rho) \).

2.5.5. Effect of reflectivity on optical efficiency

The common approximation for the effect of reflectivity on optical efficiency is [32, 33]:

\[ \eta_{\text{opt}} = \eta_{\text{acc}} \cdot \langle \rho^{n_t} \rangle = \eta_{\text{acc}} \rho^{\langle n_t \rangle} \]  

(2.78)
which holds for high $\rho$ and low $\langle n_r \rangle$ [32]. Here we have included the pre-factor $\eta_{\text{acc}}$ to account for a fixed fraction of ray-rejection. The optical efficiency may alternatively be expressed exactly as an infinite power series in $\rho$ [32] where the coefficients $f_n$ are the fraction of accepted rays suffering $n_r$ reflections before reaching the exit aperture:

$$\eta_{\text{opt}}(\rho) = \eta_{\text{acc}} \cdot (f_0 + f_1\rho + f_2\rho^2 + \cdots) = \eta_{\text{acc}} \sum_{n_r=0}^{\infty} f_{n_r} \rho^{n_r} \quad (2.79)$$

If $f_n$ is sufficiently small for $n > 2$, then to a good approximation the series may be truncated to quadratic:

$$\eta_{\text{opt}}(\rho) \approx a + b\rho + c\rho^2 \quad (2.80)$$

The coefficients may be fitted either by least-squares regression or by a 3-point fitting, with points at $\rho = 0, \rho = \rho_m, \rho = 1$, for which:

$$a = f_0$$

$$b = \frac{f_0 + \rho_m^2 \cdot (\eta_{\text{acc}} - f_0) - \eta_{\text{opt}}(\rho_m)}{\rho_m \cdot (1 - \rho_m)} \quad (2.81)$$

$$c = \frac{f_0 \cdot (\rho_m - 1) - \eta_{\text{acc}} \rho_m + \eta_{\text{opt}}(\rho_m)}{\rho_m \cdot (\rho_m - 1)}$$

For the 3-point fitting, Eq. (2.80) is exact for $\rho = 0, \rho = \rho_m$ and $\rho = 1$.

2.6. Summary

In this chapter, the prerequisite background information for the understanding of this work has been presented. In particular limits of concentration were derived based on the principle of étendue conservation. One-axis trackers were discussed in detail, and the conventions adopted for the remainder of this thesis were presented. The most important metrics for the assessment of solar concentrator performance were outlined.
Chapter 3

Low-cost, high-performance solar troughs

In Chapter 1, it was proposed that minimization of the cost per m\(^2\) of collecting aperture should be a primary driver in the design of an economical solar collector. Solar troughs are line-symmetric, having curvature in only one direction, and thus can be manufactured more easily than three-dimensional designs. In this chapter, solar trough designs achieving this low-cost objective, without sacrificing optical performance are investigated.

3.1. Inflated trough collectors

One method to reduce the cost of a solar trough concentrator is to construct it from a low-cost material. Raw-material cost can be reduced by reducing the thickness of the mirror surface. If the material is made sufficiently thin, then it will act like an elastic membrane, whose shape may then be controlled by applying a pressure difference across the membrane, i.e. by inflation.

By analogy to photovoltaics, inflated designs may be considered the “thin-film” of concentrating solar collectors. Their technical advantages include superior wind resistance when the double-bubble configuration is used (Section 3.1.1), and very low mirror surface roughness leading to a high specularity (as evidenced in Appendix C). In addition to benefits appearing during the operation, membrane mirrors offer distinct benefits at the beginning- and end-of-life of the collector, since the material can be transported in rolls, greatly simplifying the logistics for installations in remote areas.

3.1.1. Basic construction of an inflated trough

The basic objective of an inflated trough is to produce a line-symmetric concave reflector surface by inflating a thin metallized membrane. Figure 3.1 shows the basic construction of an inflated trough collector. The system comprises a minimum of two membranes: a transparent topsheet and a
reflective mirror membrane, both clamped along rigid supports spanning the length of the trough, and sealed at the ends. A pressure is applied between the topsheet and mirror membranes, creating an inflated envelope.

While these two membranes are sufficient to produce a concave mirror, the underside of the mirror membrane would be exposed to the environment and thus susceptible to deformations caused by wind loads. By placing a bottom sheet underneath the mirror membrane, and applying a pressure between the bottom sheet and the mirror membrane, a second envelope is formed. With this “double-bubble” configuration, the mirror membrane is almost entirely isolated from wind loads [15], thus permitting the use of very thin mirror membrane sheets.

Because the membranes are essentially weightless, the scale of inflated trough collectors can be made significantly larger conventional troughs, which may potentially reduce the balance of system costs. An inflated trough with a collecting aperture width of greater than 9 m [34] and a trough length of greater than 200 m has been experimentally demonstrated. With these dimensions, the solar power input to a single trough collector is ~2 MW.

The inflated construction has the disadvantage of an immediate optical loss due to the fact that incident radiation must pass through the topsheet. In order to avoid a doubling of this loss, the focus should be placed inside the envelope. In this case, the inflated collector has the major advantage of creating an enclosed

Figure 3.1. Basic construction of an inflated trough.
environment in which to house the downstream optical equipment and the receiver. The topsheet transmission loss it at least partially offset by the fact that the membrane mirror metallization can be essentially uncoated, e.g. pure silver, resulting in a very high specular reflectivity that would not be possible if the mirror were not enclosed. In the case that a reflective secondary optical element is used, the topsheet loss may in some cases be entirely offset by the fact that uncoated indoor reflector materials may be used for the secondary. The optical efficiency of multi-reflection enclosed and conventional systems may be estimated from:

\[ \eta_{\text{opt, enclosed}} \approx U_{\text{topsheet}} \cdot \rho^{(n_t)}_{\text{uncoated}} \cdot \eta_{\text{acc}} \]  

\[ \eta_{\text{opt, conventional}} \approx \rho^{(n_t)}_{\text{coated}} \cdot \eta_{\text{acc}} \]  

Setting equal and solving for \( \langle n_r \rangle \) yields the average number of reflections at which the two designs break even:

\[ \langle n_{\text{break-even}} \rangle = \frac{\log U_{\text{topsheet}}}{\log \left( \rho_{\text{coated}} / \rho_{\text{uncoated}} \right)} \]  

As an example consider an inflated collector having a topsheet with a transmittance of 92% and with all mirror surfaces being uncoated silver with a reflectivity of 96%. For comparison, consider a conventional trough construction, where the mirror elements are exposed, such that an aluminum or coated silver mirror with reflectivity of 93% must be used. From Eq. (3.3), the break-even average number of reflections is 2.6 implying that a system having more reflections than this on average would benefit from an enclosed collector. The maximum expected topsheet transmittance is \( \sim 96\% \) (50 μm thick FEP) for which the break-even \( \langle n_r \rangle \) is 1.3 considering the same reflectivities as before. This implies that even a two-mirror primary (i.e. aplanat or cylinder with corrector mirror) may benefit from an enclosed construction.

In Section 3.1.3, it will be demonstrated that a mirror constructed from a single long inflated membrane results in a circular cross-section. Alone, this configuration suffers from severe spherical aberration for distant sources (the sun) as shown in Figure 3.2, unless the rim angle is very small which would
result in a very incompact design. However, as discussed in Section 3.2, with the introduction of a small corrector mirror, the spherical aberration of the system can be completely eliminated with little additional cost. Therefore it is worthwhile to investigate the design of simple one-membrane inflated troughs. Moreover, the membrane design principles presented here will be applicable to the more advanced multi-membrane inflated concentrators discussed in Section 3.3.

3.1.2. Membrane materials

Major mechanical, thermal and optical properties of the optically active membrane materials for inflated trough collectors are given in Table 3.1. Detailed optical properties are presented in Appendix C.

*Transparent topsheet*

The topsheet material must have a high solar transmittance, have a good long-term weather resistance, have a low moisture permeability, be durable, and have a low cost. Fluoropolymers satisfy the functional requirements and have the additional benefits of being very scratch resistant and chemically stable. Due to their very low coefficient of friction, fluoropolymers also exhibit self-cleaning properties, thus reducing the required frequency of cleaning the collector.
Of the commercially available fluoropolymers, fluorinated ethylene propylene (FEP) and poly(ethylene tetrafluoroethylene) (ETFE) are the most promising due to their very high transmittance in the visible and near infra-red frequencies. Due to its lower refractive index, FEP has the higher transmittance between the two, but has a lower tensile strength and is more expensive.

**Mirror membrane**

The mirror membrane material has relaxed weather-resistance requirements since it is enclosed by the envelope, but has very strict dimensional stability requirements in order that the mirror shape can be accurately obtained. Biaxially oriented polyethylene terephthalate (BoPET), commonly referred to

---

**Table 3.1.** Major mechanical, thermal and optical properties of membrane materials used in inflated trough collectors.

<table>
<thead>
<tr>
<th>Property</th>
<th>ETFE</th>
<th>FEP</th>
<th>BoPET</th>
</tr>
</thead>
<tbody>
<tr>
<td>application</td>
<td>topsheet</td>
<td>topsheet</td>
<td>mirror</td>
</tr>
<tr>
<td>thickness [μm]</td>
<td>100</td>
<td>50</td>
<td>23</td>
</tr>
<tr>
<td>density [g/cm³]</td>
<td>1.70</td>
<td>2.15</td>
<td>1.39</td>
</tr>
<tr>
<td>Young’s modulus, $E$ [MPa]</td>
<td>$830^a$</td>
<td>480</td>
<td>$4807/5003^b$</td>
</tr>
<tr>
<td>Poisson’s ratio, $ν$</td>
<td>–</td>
<td>–</td>
<td>0.38</td>
</tr>
<tr>
<td>tensile strength [MPa]</td>
<td>41</td>
<td>21</td>
<td>196/235$^b$</td>
</tr>
<tr>
<td>yield strength [MPa]</td>
<td>–</td>
<td>12</td>
<td>98$^c$</td>
</tr>
<tr>
<td>dynamic coefficient of friction</td>
<td>0.2 – 0.3</td>
<td>0.1 – 0.3$^d$</td>
<td>–</td>
</tr>
<tr>
<td>refractive index, $n$</td>
<td>1.4</td>
<td>1.341 – 1.347</td>
<td>–</td>
</tr>
<tr>
<td>normal solar transmittance, $U_{n,solar}$</td>
<td>0.92</td>
<td>0.96</td>
<td>N/A</td>
</tr>
<tr>
<td>solar specular reflectivity, $ρ_{solar}$</td>
<td>N/A</td>
<td>N/A</td>
<td>0.89/0.93$^e$</td>
</tr>
<tr>
<td>melting point [$°C$]</td>
<td>260 – 280</td>
<td>260 – 280</td>
<td>254</td>
</tr>
<tr>
<td>thermal conductivity [W/(m·K)]</td>
<td>0.24</td>
<td>0.195</td>
<td>0.155</td>
</tr>
<tr>
<td>cost [€/m²]</td>
<td>6 – 7</td>
<td>12 – 14$^f$</td>
<td>0.7</td>
</tr>
<tr>
<td>Refs.</td>
<td>[35]</td>
<td>[36]</td>
<td>[34]</td>
</tr>
</tbody>
</table>

---

$^a$ Flexural modulus

$^b$ MD/TD, where MD is the machine (rolling) direction and the TD transverse direction

$^c$ Strength at 5% elongation

$^d$ Film against steel

$^e$ aluminized/silvered

$^f$ For a thickness of 100 μm
as Mylar®, has a high yield strength, excellent creep resistance, and a cost of €0.70 per m², making it an ideal mirror membrane material. The neat membrane may be metallized by physical vapor deposition (PVD) with pure aluminum, pure silver, or alloys. Having a surface roughness of ~30 nm, BoPET results in a mirror surface with excellent specularity when PVD metallized (see Appendix C).

**Bottom sheet**

As it is not an optical component, the material requirements for the bottom sheet are far less strict and for the topsheet and mirror membrane. The most important property is durability and cost. Vinyl coated polyester (PVC/PES) is a suitable material with a track record in fabric structure roofs and buildings.

3.1.3. Static equilibrium in a long inflated membrane

In order to design inflated trough collectors, it is necessary to understand the mechanical behavior of a long inflated membrane. The derivations here are independent of the elastic properties of the membrane and result simply from the static force equilibrium in an inflated membrane that is thin enough such that it can transmit only tensile forces. The theory of elasticity is only required when choosing the initial dimensions of the membrane before it is inflated. This is discussed briefly in Section 3.1.4 and detailed in Appendix A.

Assumptions:

- the membrane can resist only tensile loads (zero bending/compression stiffness)
- the applied pressure is uniform
- the weight of the membrane is negligible
- properties are homogenous
- no friction
- no electrostatic forces
Consider an infinitely long tubular membrane subjected to a uniform internal overpressure \( \Delta p = p_o - p_i \) as shown in Figure 3.3 (a). From simple symmetry considerations, the cross-section of the tube must be circular. If we take two cuts at point A and point B and apply, as a boundary condition, the resulting internal tension, then the resulting curve \( S \) must be a circular arc. Taking moments about \( y \) we find for any \( \Theta_A \) and \( \Theta_B \):

\[
T_A = T_B = \text{const.} = T \tag{3.4}
\]

Similarly, if the resulting segment is clamped at A and B the curve \( S \) must still remain circular and the support reactions will be \( F_A = T[\cos\Theta_A, \sin\Theta_A] \) and \( F_B = T[\sin\Theta_B, \cos\Theta_B] \). The force acting on differential element \( dS \) of the membrane at angle \( \Theta \) due to the pressure difference is:

\[
dF = \Delta p \ dS \begin{bmatrix} \sin \Theta & -\cos \Theta \end{bmatrix} = \Delta p \ R \ d\Theta \begin{bmatrix} \sin \Theta & -\cos \Theta \end{bmatrix} \tag{3.5}
\]

For any segment \( S \), a force balance in the z-direction yields:
\[ \sum F_z = 0 = T \sin \Theta_B - T \sin \Theta_A + \int_{\Theta_A}^{\Theta_B} dF_z \]
\[ = T (\sin \Theta_B - \sin \Theta_A) - \Delta p R (\sin \Theta_B - \sin \Theta_A) \]

which yields a simple expression relating the tension in the membrane to its radius and the pressure difference across it:

\[ T = \Delta p R \]

Note that due to the linear symmetry of the system, the tension \( T \) is given in units of Newtons per meter of trough length, such that Eq. (3.7) is dimensionally correct.

The effect of the boundary conditions at either end of the trough should be addressed, as the trough cannot be made infinitely long in practice. There are a number of options for how to fix the membrane at the ends of the trough. From an installation point of view, the simplest would be to clamp the membrane to a fixed circular support having an arclength equal to the unstretched membrane length. This circular support would have a slightly different radius than the design radius for the inflated membrane, and thus there would exist an end region of varying radius. Finite element simulations have predicted that this end region is small (on the order of the chord length of the trough aperture).

A preferred solution is to clamp the membrane to a fixed circular support having the exact geometry (radius and arc angle) of the inflated design. This slightly complicates installation, as it is be required to stretch the membrane onto the support. Provided the stretching is performed uniformly during installation, this would result in a boundary condition nearly identical to that of making a cut of an infinitely long trough and fixing the membrane along the cut. Therefore end effects are expected to be negligible using this end clamping method.

3.1.4. Mirror membrane design

Eq. (3.7) provided the elegant relationship between the membrane tension pressure and radius based on force equilibrium alone. In designing a membrane to achieve a prescribed radius with a prescribed pressure, it is necessary to determine the required unstretched membrane length \( L_0 \). For this purpose, the
theory of elasticity is required. Detailed derivations are provided in Appendix A, with major results summarized here.

The stretched membrane length is:

\[ S = R \cdot \Delta \Theta \] (3.8)

If the membrane is spanned between clamping points A and B, the subtended arc angle \( \Delta \Theta = \Theta_B - \Theta_A \) may be determined from:

\[ \Delta \Theta = 2 \arcsin \left( \frac{\text{ch}}{2R} \right) \] (3.9)

where \( \text{ch} \) is the chord length:

\[ \text{ch} = \| \mathbf{AB} \| = \sqrt{(B_x - A_x)^2 + (B_z - A_z)^2} \] (3.10)

**Unstretched membrane length**

Consider a stretched membrane with tension \( T \) and stretched length \( S \). If an infinitesimal reduction in membrane tension \( dT \) is applied to the membrane, then the resulting infinitesimal change in transverse membrane length \( d\Delta L \) may be found from:

\[ \frac{d\Delta L}{L} = d\varepsilon = \frac{dT}{k} \] (3.11)

where \( L \) is the instantaneous membrane length, \( k \) is the instantaneous transverse stiffness of the membrane. Under the assumption of a linearly elastic isotropic material subject to a plane strain stress state, the transverse stiffness is shown in Appendix A to be equal to:

\[ k = \frac{E}{1 - \nu^2} t \] (3.12)

where \( E \) is the Young’s modulus, \( \nu \) is Poisson’s ratio, and \( t \) is the instantaneous membrane thickness. For very small strains, Eq. (3.11) may be used to estimate the unstretched membrane length as:
\[
\frac{\Delta L}{S} = \frac{S - L_0}{S} \approx \frac{T}{k_0} \equiv \epsilon_0
\]  
(3.13)

where:

\[
k_0 \equiv \frac{E}{1-\nu^2} t_0
\]  
(3.14)

is the nominal transverse stiffness of the membrane, and \(\epsilon_0\) is the nominal strain. However, even for small strains, it is more appropriate to use the exact formula as derived in Appendix A:

\[
\frac{\Delta L_{\text{exact}}}{S} = \frac{S - L_{0,\text{exact}}}{S} = 1 - e^{-T/k_0} \equiv \epsilon_{\text{exact}}
\]  
(3.15)

which accounts for large displacements by considering the effect of the increasing length, and the reducing membrane thickness on the elastic behavior of the membrane as it is stretched. The stretched and unstretched lengths are related by:

\[
S = L_{0,\text{exact}} e^{T/k_0}
\]  
(3.16)

By comparing Eq. (3.13) by Eq. (3.15) we find that:

\[
\frac{\epsilon_0}{\epsilon_{\text{exact}}} = \frac{\epsilon_0}{1 - e^{-\epsilon_0}}
\]  
(3.17)

which is always positive for positive \(\epsilon_0\), implying that Eq. (3.13) overestimates the stretch, and thus results in too short unstretched membrane lengths.

**Strength considerations**

From Appendix A, the principal stresses in the stretched membrane are \(\sigma_1 = T/t\) in the transverse direction, \(\sigma_2 = \nu \sigma_1\) in the axial direction, and \(\sigma_3 = 0\) across the thickness. The von Mises equivalent stress is:

\[
\sigma_e = \sqrt{\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_1-\sigma_3)^2}{2}} = \sigma_1 \sqrt{1-\nu+\nu^2}
\]  
(3.18)
The von Mises stress is always less than the first principle stress, so $\sigma_1$ may be used to obtain a conservative failure assessment.

There is a minimum level of pressure that must be applied to a membrane, below which the pressure cannot be controlled to sufficient accuracy. If we take $\Delta p_{\text{min}} = 10$ Pa and a maximum radius on the order of $R_{\text{max}} = 10$ m, then the tension must be at least $T_{\text{min}} = 100$ N/m in the mirror membrane. Using a membrane thickness of $t = 23 \mu$m, the membrane stress with therefore be at least $\sigma_{1,\text{min}} = 4.3$ MPa. At a conservative membrane temperature of 50 °C, BoPET membranes have a yield strength of 69 MPa and an ultimate strength of 138 MPa, and therefore the stresses experienced in the membranes are well below failure.

While membrane failure due to static yield or rupture is not expected to be a problem, the tight shape tolerances required on the membrane imply that dimensional stability, treated in the following section, will be a major consideration in the mechanical design of the membrane mirror.

3.1.5. Dimensional stability

In Section 3.1.4 the methods to determine the unstretched membrane length were presented. To estimate the sensitivity of the concentrator performance to deviations in the membrane length, we can perform a back of the envelope calculation considering a membrane radius of $\sim 10$ m. A deviation in membrane length of 1 mm will cause a change of $\sim 0.1$ mrad in the arc angle subtended by the mirror. This implies that at the rim, the mirror normal will be offset by an angle of 0.1 mrad from its design value, and the direction of a ray reflected at the rim will be offset by an angle of 0.2 mrad from the correct direction. To judge the severity of this offset, we consider that the angular size of the solar disk is $\sim 5$ mrad. This shows that a membrane length deviation causes an error in the reflected ray direction that is $\sim 4\%$ of the angular radius of the solar disk, which on the order of an upper limit of what could be tolerated. Therefore the dimensional tolerance on the transverse membrane length should be at most 1 mm. This is in agreement with a more thorough sensitivity analysis presented in [34]. Such a tight tolerance implies that it is necessary to take into account environmental strains affecting the dimensional stability of the membrane. The
relevant environmental strains are thermal expansion, hygroscopic expansion, shrink (residual strain-relief), and creep.

**Thermal and hygroscopic expansion**

The coefficient of linear thermal expansion of BoPET is $\alpha_{\text{thermal}} = 1.7 \times 10^{-5} \text{ m/(m·K)}$. BoPET is hygroscopic and expands under the presence of increased moisture. The coefficient of linear hygroscopic expansion of BoPET is $\alpha_{\text{hygro}} = 0.6 \times 10^{-5} \text{ m/(m·%RH)}$. Thermal and hygroscopic expansion can be corrected for if the temperature and relative humidity within the collector are controlled, and those during manufacture are known. Assuming thermal and hygroscopic expansion are independent, the factory unstretched membrane length may be determined from:

$$\frac{\Delta L}{L_0} = \frac{L_{\text{factory}} - L_0}{L_0} = \epsilon_{\text{thermal}} + \epsilon_{\text{hygro}}$$  \hspace{1cm} (3.19)

$$\epsilon_{\text{thermal}} = \alpha_{\text{thermal}} \left( T_{\text{factory}} - T_{\text{collector}} \right)$$  \hspace{1cm} (3.20)

$$\epsilon_{\text{hygro}} = \alpha_{\text{hygro}} \left( \text{RH}_{\text{factory}} - \text{RH}_{\text{collector}} \right)$$  \hspace{1cm} (3.21)

$$L_{\text{factory}} = L_0 \left( 1 + \epsilon_{\text{thermal}} + \epsilon_{\text{hygro}} \right)$$  \hspace{1cm} (3.22)

**Shrinkage**

Due to the manufacturing process of BoPET, residual strain is present in the polymer chains. When heated to an elevated temperature, the strain is relieved causing the polymer chains to randomize and the material to shrink. Shrinkage is expected to be low for service temperatures less than 60 °C. Since shrinkage has the tendency to shorten the membrane length, a small amount can be tolerated considering the flexibility of membrane pressure control (Section 3.1.6). For higher service temperatures, and in the case that shrink cannot be tolerated, it is recommended to pre-shrink the material to the maximum expected service temperature before cutting the membrane to $L_{\text{factory}}$. 
Creep

Despite BoPET’s high creep-resistance, creep is expected to be the factor which will limit the service life of the mirror membrane. Creep data is limited, but it is known that creep occurs well below the yield point. For example an elongation of 0.2% after 1000 hours was measured at an applied stress of 20 MPa. The creep behavior of BoPET membranes at lower stresses will need to be assessed after more long-term field experience with inflated collectors is established. Fortunately, the effects of moderate creep can be offset by the pressure control methods discussed in Section 3.1.5.

3.1.6. Membrane pressure control

One of the unique advantages of an inflated membrane collector is that manufacturing imperfections can be at least partially corrected by controlling the inflation pressure. From Eq. (3.15):

\[
S = L_0 \exp \left[ \frac{\Delta p R \left( 1 - \nu^2 \right)}{E t_0} \right]
\]  

(3.23)

We can rearrange for the pressure difference:

\[
\Delta p = \frac{1}{R} \left[ \ln \left( R \cdot \Delta \Theta \right) - \ln L_0 \right] t_0 \frac{E}{1 - \nu^2}
\]  

(3.24)

Eq. (3.24) gives the required pressure in order to achieve the specified arc shape \((R \text{ and } \Delta \Theta)\) as a function of the membrane parameters. In theory, Eq. (3.24) can be used to correct any deviations in \(L_0, t_0, E \text{ or } \nu\) from the design values simply by changing the pressure. The most critical parameter is expected to be the unstretched membrane length \(L_0\). This value may differ from the design value due to manufacturing tolerances, due to changes in the operating conditions within the collector (temperature/humidity), and due to long-term effects such as creep. The restriction that \(\Delta p\) must be greater than 0 suggests that it is preferable to manufacture membranes that are too short than too long.

The robustness of the inflated design results from the fact that the membrane will always have a circular cross-section, as dictated by the force
equilibrium of Section 3.1.1. Regardless of the membrane parameters, increasing the pressure will have the tendency to reduce the membrane in a continuous manner. This implies that the correct radius can always be achieved by suitably choosing the pressure (provided that the unstretched membrane length is less than $R \cdot \Delta \Theta$).

3.2. Corrector mirrors for circular primaries

In Section 3.1 it was demonstrated that a single long inflated membrane yields a cylindrical trough concentrator which suffers from spherical aberration. One option to improve the geometric performance is to add a second “corrector” mirror to remove the spherical aberration of the primary. While the motivation of having a primary of circular cross-section came from the requirements of the inflated structure, these designs may also be of interest for non-inflated troughs, as the circular profile may still yield considerable manufacturing advantages over a parabolic one.

In the general sense, spherical aberration is the condition when on-axis rays striking different parts of the inlet aperture are focused to different focal points. The parabola is the only continuous curve that focuses on-axis rays to a point; all other primary mirror profiles suffer from spherical aberration. A single corrector mirror can be designed to eliminate the spherical aberration of any primary geometry. This was investigated in [38] but the resulting mirror profiles were presented in the form of a differential equation. The general recipe to design a mirror may to reflect a wavefront to a point is constant optical pathlength tailoring. This integral approach is preferred to other previous approaches, e.g. [38, 39], which are based on differential analysis where numerical [38] or analytical [39] integration is required to obtain the final mirror profiles. With the optical pathlength tailoring method the analytical parametric expressions for corrector mirror profile may be easily determined without integration.

3.2.1. Constant optical pathlength tailoring

From Fermat’s principle, optical conjugates, i.e. wavefronts of a ray bundle, are connected by rays of constant optical pathlength. This property can be used to
design reflective and refractive surfaces that transform one wavefront into another. This method is extremely useful because it gives the shape of the surface in integral form – there is no need for using differential quantities which then need to be integrated at a later stage to get the final form of the surface. Consider a ray leaving point \( P \), which is immersed in a dielectric with refractive index \( n_1 \), pointing in the direction \( \hat{v} \). The ray is to be reflected or refracted at point \( Q \) on a surface and directed to point \( F \), which is immersed in a dielectric with \( n_2 \) such that the total optical pathlength from \( P \) to \( F \) is OPL. If the OPL is taken as given, then the point \( Q \) on the reflecting/refracting surface may be found from [22]:

\[
Q = P + \frac{C_1 + \text{sgn}(n_2 - n_1) \sqrt{C_2 (n_2^2 - n_1^2) + C_1^2}}{n_1^2 - n_2^2} \]

\[
C_1 = n_1 \cdot \text{OPL} + n_2^2 \cdot (P - F) \cdot \hat{v} \\
C_2 = \text{OPL}^2 - n_2^2 \cdot (P - F) \cdot (P - F)
\]

In the case where \( n_1 = n_2 = n \) (i.e. reflection), Eq. (3.25) simplifies to:

\[
Q = P + \frac{(\text{OPL}/n)^2 - (P - F) \cdot (P - F) \cdot \hat{v}}{2(\text{OPL}/n + (P - F) \cdot \hat{v})}
\]

3.2.2. Analytical solution for the corrector mirror profile

**Figure 3.4** shows the general geometry of a corrector mirror for a circular primary. In the extreme case, the primary may span from \(-180^\circ \leq \Theta \leq 180^\circ\), but generally a smaller angular range is used.

A ray from wavefront \( w \) hits a point \( P \) on the circular primary and is reflected in direction \( \hat{v} \) towards point \( Q \) on the secondary which reflects it to the final focus \( F[0,0] \). The parametric equations of \( P \) and \( \hat{v} \) are:

\[
P(\Theta) = \begin{bmatrix} C_x + R \sin \Theta & C_z - R \cos \Theta \end{bmatrix}
\]

\[
\hat{v}(\Theta) = \begin{bmatrix} -\sin 2\Theta & \cos 2\Theta \end{bmatrix}
\]
From Eq. (3.26), constant optical pathlength tailoring gives any point Q on the corrector mirror as:

\[
Q_x (\Theta) = C_x + R \sin \Theta - b(\Theta) \sin 2\Theta
\]

\[
Q_z (\Theta) = C_z - R \cos \Theta + b(\Theta) \cos 2\Theta
\]

(3.29)

where:

\[
b(\Theta) = \frac{(\text{OPL} - R \cos \Theta)^2 - (C_x + R \sin \Theta)^2 - (C_z - R \cos \Theta)^2}{2(\text{OPL} - 2R \cos \Theta - C_x \sin 2\Theta + C_z \cos 2\Theta)}
\]

(3.30)

The polar angle of a given ray as it reaches the final focus may be found from:

\[
\phi(\Theta) = \arctan\left(-\frac{x(\Theta)}{z(\Theta)}\right)
\]

(3.31)

The effective focal length is found by extending the twice-reflected ray forwards or backwards until it intersects the initially incident on-axis ray at X. The length of the ray between F and X is the effective focal length \(f_e(\Theta)\). In terms of the rim angle:
Correctors for an on-axis circular arc

For an on-axis arc, the arc’s centerpoint will lie on the optical $z$-axis, i.e. $C_x = 0$. The choice of OPL is somewhat arbitrary, so we may replace it by a more convenient quantity. Considering the paraxial ray, the OPL from the initial wavefront $w$ to the final focus $F$ may be expressed as:

$$\text{OPL} = R + \left[ R - (C_z - T_z) \right] + T_z = 2R + 2T_z - C_z$$

(3.33)

where $T$ is the pole of the corrector mirror. Eq. (3.30) then becomes:

$$b(\Theta) = \frac{(2R + 2T_z - C_z - R \cos \Theta)^2 + 2RC_z \cos \Theta - C_z^2 - R^2}{2(2R + 2T_z - C_z - 2R \cos \Theta + C_z \cos 2\Theta)}$$

(3.34)

where the inputs are now the radius $R$ and angular span $\Theta$ of the primary, position $C_z$ of the center of the primary and the position $T_z$ of the pole of the corrector, both measured relative to the final focus $F[0,0]$. In analyzing the mirror shapes for different choices of $C_z$ and $T_z$ it is useful to organize the designs into different regimes. For this it is useful know the position where the marginal ray (ray reflected at the rim) crosses the axis. The paraxial focus $G$ the paraxial focus of the primary is:

$$G_z = C_z - \frac{1}{2} R$$

(3.35)

but due to spherical aberration, a ray at given $\Theta$ crosses the optical axis at:

$$z = C_z - \frac{1}{2} R \sec \Theta$$

(3.36)

The marginal ray crosses the optical axis at:

$$M_z = C_z - \frac{1}{2} R \sec \Theta_{\text{max}}$$

(3.37)

If we choose $C_z$ and $T_z$ such that $M_z < T_z$ then the rays cross the axis before hitting the corrector mirror. We refer to such a configuration as folded, since the $x$-component of the ray direction is reversed by the corrector. If we choose $C_z$ and $T_z$ such that $M_z > T_z$ then the rays do not cross the axis before hitting the
corrector mirror, resulting in an unfolded design. The folded and unfolded designs may also be considered pre- and post-focus correctors in the sense that they operate on the rays before and after the marginal focus $M_z$ of the primary, respectively. Additionally they may be referred to as Gregorian and Cassegrain designs respectively, due to their resemblance to the classical two-mirror parabolic-elliptic (Gregorian) and parabolic-hyperbolic (Cassegrain) systems.

Table 3.2 gives a roadmap of the type of design resulting from different parameter choices (note that $M_z$ may be used as an input parameter instead of $C_z$ with $C_z$ subsequently calculated from Eq. (3.37)). It should be noted that Table 3.2 serves as only a guide and initial assessment of suitable parameter choices. Designs with $M_z \approx T_z$ will in general be problematic.

**Table 3.2. Roadmap for corrector mirrors for on-axis circular arcs.**

<table>
<thead>
<tr>
<th>$T_z$</th>
<th>$M_z &lt; T_z$</th>
<th>$M_z &gt; T_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_z &lt; 0$</td>
<td>unphysical$^a$</td>
<td>unphysical$^a$</td>
</tr>
<tr>
<td>$T_z = 0$</td>
<td>cardioid</td>
<td>unphysical$^a$</td>
</tr>
<tr>
<td>$T_z &gt; 0$</td>
<td>folded</td>
<td>unfolded</td>
</tr>
</tbody>
</table>

$^a$ focus completely blocked by secondary

cardoid corrector ($T_z = 0$)

For $T_z = 0$, the pole of the corrector mirror is at the final focus. This configuration yields very interesting designs that were previously identified in [39, 40]. The most interesting design results when we also choose $G_z = 0$ (i.e. $C_z = \frac{1}{2}R$), such that the final focus is coincident with the paraxial focus of the arc. In this case Eq. (3.34) becomes:

$$b_{T_z=G_z=0}(\Theta) = \frac{1}{2}R$$

(3.38)

The shape of the corrector mirror is then a cardioid [39] with the form:

$$x(\Theta) = R(1 - \cos \Theta)\sin \Theta$$

$$z(\Theta) = -R(1 - \cos \Theta)\cos \Theta$$

(3.39)

and is shown in Figure 3.5. The rim angle is simply:
Low-cost, high-performance solar troughs  

\[ \phi(\Theta) = \Theta \quad (3.40) \]

and effective focal length is:

\[ f_c(\Theta) = R \quad (3.41) \]

which amazingly is a constant implying that this system is in fact aplanatic since the Abbe sine condition is fulfilled \[39\]. Unfortunately this configuration requires significant truncation making it impractical for use as a solar concentrator.

**Unfolded designs \((T_z > 0, M_z > T_z)\)**

The unfolded designs are characterized by a corrector mirror whose pole is closer to the final focus than the marginal focus. The corrector mirror must not cross the caustic of rays reflected by the primary given by which is given by:

\[ x = C_x + \frac{1}{4}(3 \cos \Theta - \cos 3\Theta) \]
\[ z = C_z - \frac{1}{4}(3 \sin \Theta - \sin 3\Theta) \quad (3.42) \]
If it does cross the caustic, the corrector mirror will have a cusp caused by the changing sign of the gradient of the OPL at the caustic. Two unfolded designs are shown in Figure 3.6.

**Folded designs** ($T_z > 0, M_z < T_z$)

Folded designs are characterized by a corrector mirror whose pole as farther from the final focus than is the marginal focus. Since the caustic terminates at the paraxial focus, caustic intersection is not usually a problem with folded designs. It is possible to have designs where the receiver is illuminated over more than a hemisphere. This would be useful for tubular or bifacial flat receivers. Three folded designs are shown in Figure 3.5, and two in Figure 3.6.

### 3.2.4. Correctors for an off-axis circular arc

The designs from Section 3.2.3 revealed that severe shading of the primary by the corrector can occur. This is especially problematic for large rim angles, which are generally required to achieve a compact design. This can be alleviated by considering a primary composed of two off-axis arcs. In this case Eq. (3.30) must be used, but for small $C_x$, Eq. (3.33) may be used as an initial estimate of the input OPL. In general the designs for an off-axis circular arc are very similar to those for an on-axis circular arc, however the shading and compactness of the concentrator as a whole can be improved.
3.2.5. Oversizing of the corrector and truncation of the primary

The corrector is designed to intercept all on-axis rays reflected by the primary and reflect them to the final focus. However, for solar concentration, the system must handle off-axis rays up to the acceptance angle $\theta_i$. Off-axis rays reflected near the rim of the primary may miss the corrector. To catch these off-axis rays, the corrector must be oversized. This can be done by specifying a larger parameter range for the corrector than the primary, $\Theta_{\text{max,corrector}} = \Theta_{\text{max}} + \Delta \Theta_{\text{oversize}}$. The exact amount of oversizing required may be determined by tracing the edge-ray striking the rim of the primary. There are two edge-rays such edge rays, one inclined at $+\theta_i$ and the other at $-\theta_i$ measured counterclockwise from the positive $z$-axis. The equation of the edge-rays after being reflected at the rim of the primary is:

$$\mathbf{r}(\pm \theta_i; t) = P(\Theta_{\text{max}}) + t\left[-\sin(2\Theta \mp \theta_i) \cos(2\Theta \mp \theta_i)\right]$$

(3.43)

The exact required amount of oversizing may then be determined by solving the system of equations:

$$\mathbf{r}(\pm \theta_i; t) = Q(\Theta_{\text{max}} + \Delta \Theta_{\text{oversize}})$$

(3.44)

for $\Delta \Theta_{\text{oversize}}$ where the $+\theta_i$ edge-ray is used for unfolded designs, and the $-\theta_i$ edge-ray is used for folded and cardioid designs.

In all cases, the corrector will shade a central portion of the primary. This portion of the primary, and its corresponding portion of the corrector, may therefore be removed by truncation. Truncation is performed by reducing the angular span of the primary $\Theta_{\text{trunc}} \leq \Theta \leq \Theta_{\text{max}}$. The truncation angle for the primary is chosen such that $P_x(\Theta_{\text{trunc}}) = Q_x(\Theta_{\text{max}} + \Delta \Theta_{\text{oversize}})$ which yields:

$$\Theta_{\text{trunc}} = \arcsin\left(\frac{Q_x(\Theta_{\text{max}} + \Delta \Theta_{\text{oversize}}) - C_x}{R}\right)$$

(3.45)

3.3. The arcspline concentrator (ASC)

In Section 3.2 secondary corrector mirrors were presented as a solution to the spherical aberration problem of the single-membrane inflated troughs presented
in Section 3.1. The analytical expression encompassing all secondary mirrors which correct the spherical aberration of the primary was presented, and the corrector mirrors which were found to come in a variety of shapes. Although several practical designs resulted, the main drawbacks of all of these designs are the increased complexity of an additional mirror surface, unavoidable shading of the primary by the corrector mirror, and that all rays must suffer an additional reflection at the corrector mirror. Since each reflection results in a loss, we strive for a single reflection solution based on inflated membranes. In this section, a multi-membrane solution which allows the elimination of spherical aberration without the use of a corrector mirror is detailed.

Eq. (3.7) provides a simple overview of the problem at hand: in order to change the radius $R$ of the inflated membrane, we must be able to change the pressure difference $\Delta p$, or the tension $T$ along the span of the membrane. To change the tension, a tangential force must be applied on the membrane. This would require pulling the membrane at some intermediate location along its span\(^1\), which would be technically challenging and difficult to control. It therefore seems that changing the pressure difference along the span of the membrane is the only feasible way to obtain a membrane of variable radius.

In previous work [34] a method to change the pressure difference, and thus the radius, along the span of a membrane was realized. The design consists of a stack of $N$ membranes clamped together at the outer end, and having separate clamping points at the inner end as shown in Figure 3.7. To facilitate control of the inner clamping point locations, an asymmetric design is utilized, whereby each membrane stack forms one half of a conventional symmetric trough collector. To obtain a symmetric collector, two identical membrane stacks, mirrored about the $z$-axis would be utilized. The topmost membrane is metallized to form a mirror surface. When this structure is appropriately inflated, the cross-section of the top membrane assumes the shape of $N$ tangentially connected circular arcs. The resulting $C^1$ continuous (continuous and continuously differentiable) curve has been nicknamed the “arcspline”. It has been shown, both theoretically and experimentally [34], that the arcspline

---

\(^1\) Alternatively a tangential force could be provided by means of friction, but this would be difficult to control and could lead to wear and electrostatic build-up.
can be designed to closely match the shape of a parabola. Although the basic concept of the arcspline is a predecessor of this work, significant improvements to the understanding of its behavior and to its design methods are reported here.

3.3.1. Basic theory of the inflated membrane stack

Suppose that we would like to approximate a continuous monotonic curve, which we call the “basis curve”, by an arcspline. If the radius of curvature $R_c$ of the basis curve is not constant, then an arcspline with more than one arc will be a better approximation of this curve.

Start with a weightless membrane, membrane 1, of unstretched length $L_{0,1}$ clamped at points $A_1$ and $N_0$ and subjected to an inflation pressure $p_1$ as in Figure 3.8 (a). The resulting membrane radius is given implicitly by Eqs. (3.16), (3.8) and (3.9). Suppose that we clamp one end of a second membrane at point $N_0$, the other end at $A_2$, and we choose its length such that it sits unstretched just underneath membrane 1 as in Figure 3.8 (b).

If we now increase $p_1$, the displacement of membrane 1 in the vicinity of $N_0$ will be restricted by the support membrane, membrane 2. If the membranes are frictionless, then only normal forces can be transferred between them. Membrane 2 will therefore appear to exert a uniform pressure, $p_{2-1}$, on the back of membrane 1 up to some point $N_1$ at which the membranes separate. As a result, membrane 1 will experience a pressure difference of $p_1$ from $A_1$ to $N_1$, and a pressure difference of $p_1 - p_{2-1}$ from $N_1$ to $N_0$. Since only normal forced
are transferred between membrane 1 and membrane 2, the tension in membrane 1 must remain constant. It therefore follow from Eq. (3.7) that at $N_1$, the radius of membrane 1 must change as shown in Figure 3.8 (c). The radius from $A_1$ to $N_1$ is given by:

$$R_1 = \frac{T_1}{p_1}$$  \hspace{1cm} (3.46)

whereas the radius from $N_1$ to $N_0$ is given by:

$$R_2 = \frac{T_1}{p_1 - p_{2-1}}$$  \hspace{1cm} (3.47)

The exchange pressure $p_{2-1}$ must be positive, and therefore comparison of Eqs. (3.46) and (3.47) reveals that $R_1 > R_2$. We have therefore achieved a change in the radius of the top membrane by introduction of an elastic support membrane 2, which effectively changes the pressure along the span on membrane 1.

The second membrane will have a radius of $R_1$ from point $N_1$ to $N_0$, and will be straight from $N_1$ to $A_2$. The magnitude of the pressure exerted by membrane 2 onto membrane 1 and vice-versa must equal:

$$p_{2-1} = \frac{T_2}{R_i}$$  \hspace{1cm} (3.48)

Knowing this, we could choose the length of membrane 2 and the position of $A_2$ such that application of pressure $p_1$ would give the desired radius $R_1$ and $R_2$ for membrane 1. However, such a design would be very susceptible to small variations in the support membrane length and the position of the clamping point $A_2$, which would be difficult to dynamically control. We strive for a system which is not overly sensitive to such manufacturing tolerances, and moreover that can be controlled to at least partially alleviate such dimensional errors.

With this in mind, consider the effect of now applying a pressure $p_2$ in between the membranes of our two membrane stack. This pressure will cause the section of membrane 2 from $A_2$ to $N_1$ to no longer be flat, but rather cause it to assume a circular shape with radius $R_2$. Additionally this pressure will cause the separation point $N_1$ to shift to the right, as shown in Figure 3.8 (c). By
introduction of this secondary pressure, the radii $R_1$ and $R_2$ and the position of the separation point $N_1$ can now be controlled to a greater degree. It was demonstrated that with a two membrane stack, the top membrane assumes the shape of two tangentially connected circular arcs. The radii and the position of the separation point $N_1$ are dictated by the applied pressures $p_1$ and $p_2$, in conjunction with the position of the clamping points $A_1$, $A_2$ and $N_0$ and the unstretched membrane lengths $L_{0,1}$ and $L_{0,2}$.

Extension of the system to three or more membranes is straightforward. For $N = 3$, the top membrane now assumes the shape of three tangentially connected circular arcs: a three-arc arcspline. There are two support membranes. The upper support membrane 2 separates from the stack at $N_2$ and is composed of three arcs. The lower support membrane 3 separates from the

---

**Figure 3.8.** Formation of a two-arc arcspline: (a) a single pressurized membrane; (b) a second membrane is placed underneath the top membrane, the two membranes separate at the separation point $N_1$; (c) the pressure is increased, the portion spanning $N_0N_1$ is supported from behind by the second membrane and thus experiences less deformation resulting in two different radii $R_1$ and $R_2$ for the top membrane; (d) a pressure $p_2$ is introduced between the two membranes allowing the position of the separation point $N_1$ to be controlled.
stack at \( N_1 \) and is composed of two arcs. In general, the formation of an \( N \)-arc arcspline requires a stack of \( N \) membranes.

3.3.2. The arcspline as a plane curve

In the previous section, it was demonstrated that the top membrane of an appropriately inflated \( N \)-membrane stack assumes the shape of \( N \) tangentially connected circular arcs, whose shape was denoted the arcspline. In this section, the arcspline is examined from a purely geometrical point of view, as a plane curve.

A single circular arc has two local DOF, its radius \( R \) and arc angle \( \Delta \Theta \). Any plane curve can be oriented by specifying three orientation DOF, e.g. the Cartesian coordinates of a point on the curve, and the slope at some point on the curve, giving a total of 5 global DOF for a plane circular arc.

An \( N \)-arcspline has \( N \) circular arcs. However, the tangency condition at the interior nodes fixes the position and slope of one arc relative to the previous, thus eliminating \( 3(N - 1) \) DOF. The arcspline as a plane curve therefore has a total of \( 2N + 3 \) DOF. These are most conveniently supplied by giving the \( N \) arc radii \( R_j \), \( N \) arc angles \( \Delta \Theta_j \), and the Cartesian coordinates and slope at one of the endpoints of the arcspline. Here we use the outer endpoint \( N_0 \) as a basis, since the rim of the concentrator is generally more critical to the optical performance of a concentrator and therefore serves as a good point to start the design.

**Arcspline geometry from known \( R_j, \Delta \Theta_j, N_0 \) and \( \Theta_0 \)**

Consider an \( N \)-arc arcspline. The center of the \( j^{th} \) arc is:

\[
C_j = \begin{bmatrix} N_{x,j-1} & N_{z,j-1} \end{bmatrix} + R_j \begin{bmatrix} -\sin \Theta_{j-1} & \cos \Theta_{j-1} \end{bmatrix}
\]  

(3.49)

The angular span of arc \( j \) is:

\[
\Theta_{j-1} - \Delta \Theta_j \leq \Theta \leq \Theta_{j-1}
\]  

(3.50)

where \( \Theta_{j-1} \) is the slope angle at node \( N_{j-1} \) (right of the arc) and \( \Theta_j \) is the slope angle at node \( N_j \) (left end of the arc). A point on arc \( j \) may be found from:
\[
P_j(\Theta) = \begin{bmatrix} C_{x,j} & C_{x,j} \end{bmatrix} + R_j \begin{bmatrix} \sin \Theta & \cos \Theta \end{bmatrix} \quad \Theta_j \leq \Theta \leq \Theta_{j-1}
\]

Eqs. (3.49) to (3.51) are then repeated for the subsequent arcs to give the full profile of the arcspline.

**Arcspline geometry from known \(N_x, R_j, m_0, N_z,0\)**

In some cases it is more convenient to describe the arcspline in terms of the nodal \(x\)-coordinates, the arc radii, and the \(z\)-coordinate and slope at \(N_0\). To determine the center point of an arc (starting from \(N_0\)):

\[
C_j = \begin{bmatrix} N_{x,j-1} & N_{z,j-1} \end{bmatrix} + \frac{R_j}{\sqrt{1 + m_{j-1}^2}} \begin{bmatrix} -m_{j-1} & 1 \end{bmatrix}
\]

(3.52)

To determine the \(z\)-coordinate of the next node:

\[
N_{z,j} = C_{z,j} - \sqrt{R_j^2 - (N_{x,j} - C_{x,j})^2}
\]

(3.53)

To determine the slope of the next node:

\[
m_j = -\frac{C_{x,j} - N_{x,j}}{C_{z,j} - N_{z,j}}
\]

(3.54)

### 3.3.3. Arcspline optical design based on curvature matching

Thus far, we have developed the basic concept of the arcspline concentrator, and have presented its basic geometrical form. The question remains as to how to best design the arcspline both geometrically and mechanically for use as a solar concentrator profile. There are two distinct steps in designing an arcspline concentrator.

The first is optical design, where the geometry of the top membrane – the arcspline – is determined in order to fulfill its function as a solar concentrator. This involves choosing the \(2N + 3\) DOF of the top mirror membrane.

The second is the mechanical design of the membrane stack such that the prescribed top mirror membrane geometry, determined from the optical design, is realized. This section deals with the optical design; the mechanical membrane design is subsequently discussed in Section 3.3.4.
There are two basic approaches for performing the optical design of an arcspline as a solar concentrator: (1) the indirect geometric approach; and (2) the direct optical approach. In the former, the arcspline curve is fitted against a reference curve with known optical performance, e.g. to closely match the shape of a reference parabola. In the latter, no \textit{a priori} assumption about the desired shape is made, and rather the arcspline is designed with regard to some optical parameter, e.g. to maximize geometric concentration, or to minimize spherical aberration. In previous work [34], a hybrid optimization approach was used.

Here we discuss a simple geometric design method based on matching of the curvature to a reference curve. It is a deterministic approach which does not require optimization. In Section 3.4, a direct optical approach, which yields a minimum-arc design of maximal geometric concentration, is presented. This direct optical approach is superior when the arcspline is to be used as a single-stage concentrator. Nevertheless, the geometric approach discussed in this section is useful because it is not restricted to the case of a single-stage concentrator. Here we will take the basis curve as a parabola, but the design method is identical for any reference curve.

The extrinsic curvature of a plane curve given explicitly as $z(x)$ is:

$$
\kappa(x) = \frac{|d^2z/dx^2|}{\left[1 + (dz/dx)^2\right]^{3/2}} \quad (3.55)
$$

For the more general case of a parametric curve given by $x(\phi), z(\phi)$:

$$
\kappa(\phi) = \frac{|dx/d\phi \cdot d^2z/d\phi^2 - dz/d\phi \cdot d^2x/d\phi^2|}{\left[ \left( \frac{dx}{d\phi} \right)^2 + \left( \frac{dz}{d\phi} \right)^2 \right]^{3/2}} \quad (3.56)
$$

For example, a parabola has the parametric form:
\[ r(\varphi) = \frac{2f}{1 + \cos \varphi} \]

\[ x(\varphi) = r(\varphi) \sin \varphi = \frac{2f \sin \varphi}{1 + \cos \varphi} = 2f \tan \left( \frac{1}{2} \varphi \right) \quad (3.57) \]

\[ z(\varphi) = r(\varphi) \cos \varphi = \frac{2f \cos \varphi}{1 + \cos \varphi} \]

The curvature of the parabola is then found from Eq. (3.56):

\[ \kappa(\varphi) = \frac{\cos^3 \left( \frac{1}{2} \varphi \right)}{2f} \quad (3.58) \]

The corresponding radius of curvature is:

\[ R_c(\varphi) = \frac{1}{\kappa(\varphi)} = \frac{2f}{\cos^3 \left( \frac{1}{2} \varphi \right)} \quad (3.59) \]

This may be equivalently expressed as a function of the \( x \)-coordinate by subbing in Eq. (3.57):

\[ R_c(x) = R_c(\varphi(x)) = \frac{f}{4} \left( 4 + \frac{x^2}{f^2} \right)^{3/2} \quad (3.60) \]

Figure 3.9 shows the radius of curvature of a parabola with \( f = 1 \) and a rim angle of \( \Phi = 90^\circ \). A circular arc has a radius of curvature simply equal to its radius \( R_c = R \), shown also in Figure 3.9. If the arc radius is chosen to match the paraxial focus of the parabola, then \( R = 2f \), as confirmed from the figure. Since it is composed of a series of \( N \) circular arcs, the radius of curvature of the arcspline is a staircase-like function with \( N \) steps, as shown in Figure 3.10.

To perform the fitting, we must choose both the step width (\( x \)-interval) and radius for each arc. We may adopt a sophisticated approach such that the absolute error, or squared error between the arc radius and the parabola, is minimized. However, such a procedure would require a numerical solution, thus defeating our goal of having a simple analytical solution for the arc intervals and radii. An alternative approach, which approximately minimizes
the difference between radius of curvature of the parabola and the arc approximation, yet has a simple analytical solution, is presented here.

Choice of $x$-intervals

The simplest choice is to choose $N$ evenly spaced $x$-intervals:

$$\Delta x = \frac{x_R - x_L}{N}$$

Figure 3.9. Radius of curvature normalized by the focal length $R_c/f$ for a primary of circular and parabolic cross-section.
Alternatively, the arc $x$-intervals may be chosen such that each interval has the same difference in parabola curvature. The total difference in curvature is divided equally in $N$ parts:

$$\Delta R_c = \frac{R_c(\phi_R) - R_c(\phi_L)}{N} = \frac{2f}{N} \left( \sec^3 \frac{1}{2} \phi_R - \sec^3 \frac{1}{2} \phi_L \right)$$ (3.62)

where $\phi_L$ and $\phi_R$ are the polar angles at the left (inner) and right (outer) ends of the parabola respectively. The radii of curvature at the nodes are then determined from:

$$R_{c,j} = R_0 - j \cdot \Delta R_c$$ (3.63)

where the zeroth node corresponds to the right node of the first interval. The corresponding $x$-coordinates of the nodes are then calculated from Eqs. (3.59) and (3.57).

**Choice of arc radii**

Once the $x$-interval of each arc is known, the radius of each arc must be chosen. Two simple options are to set the arc radius equal to the radius of curvature at the left node of each arc yielding $R_j = R_c(x_{L,j})$, or the right node of each arc $R_j = R_c(x_{R,j})$. A better approach is to use the median radius of curvature:

$$R_j = \frac{R_c(x_{L,j}) + R_c(x_{R,j})}{2}$$ (3.64)

An even better approach is to use the average radius of curvature of the parabola on the interval $x_{L,j} \ldots x_{R,j}$:

$$R_j = \frac{1}{\phi_R - \phi_L} \int_{\phi_L}^{\phi_R} R_c(\phi) \, d\phi = \frac{2f}{\phi_R - \phi_L} \left[ \ln \left( 1 + \frac{2}{\cot \frac{1}{4} \phi - 1} \right) + \tan \frac{1}{2} \phi \right]_{\phi_L}^{\phi_R}$$ (3.65)

The radius of curvature of an arcspline designed to match a $90^\circ$ rim angle parabola using each of these methods is shown in Figure 3.10.
Chapter 3

Arcsplines designed by curvature matching

With the radii and nodal x-coordinates specified, all intrinsic DOF have been defined. The three extrinsic DOF may be chosen simply by specifying that the arcspline is coincident and tangent to the parabola at and point, e.g. \( x_L \) or \( x_R \). With this condition, the arcspline geometry is completely determined by the curvature matching method. Figure 3.11 shows exemplary arcspline geometries for \( N = 4 \) with the radius of each arc determined from the left, right, median and average radius of curvature of the underlying parabola interval.

3.3.4. Membrane stack mechanical design

In Section 3.3.1, the basic physics behind the formation of the arcspline from a stack of inflated membranes was qualitatively presented. The task of determining the resulting membrane shape (radii and separation points) given the clamping points, unstretched membrane lengths, and applied pressures

Figure 3.10. A series of circular arcs can approximate the curvature of a parabola. The diagram here shows the curvature of a parabola of rim angle 90° being approximated by 4 arcs. The \( x \)-interval of the arcs is chosen such that the difference between the radius of each arc and the maximum and minimum radius of curvature on that interval is equal c.f. Eq. (3.62). The radius of each arc may be chosen as the radius at the left node or right node, or as the median or average radius between nodes, as indicated in the figure.
results in a system of nonlinear equations, which cannot be simply formulated or solved in the general case.

Fortunately, the task of designing the membrane stack, that is, determining the required pressures, tensions and unstretched lengths given the complete geometry of the arcspline is straightforward. This is because the nonlinearities of the system are purely geometrical, resulting from Eqs. (3.10) and (3.9). Therefore if the arcspline geometry is completely defined, for example using the curvature matching method of Section 3.3.3, the quantities resulting from Eqs. (3.10) and (3.9) can be thought of as given, such there is no need to include these nonlinear equations in the system, resulting in a linear system that is trivial to solve.

Consider a $N = 4$ membrane arcspline as shown schematically in Figure 3.12. All membranes in the stack are assumed to have identical properties, and all assumptions of Section 3.1.1 apply. The knowns are the $N$ radii of the top arc, the clamping points $A_1$ and $N_0$, and the nodes $N_1\ldots N_{N-1}$ (note that $N_N = A_1$). The unknowns are:

![Figure 3.11. Arcspline geometries ($N = 4$) designed by matching the parabola curvature at the left edge of each interval, the right edge of each interval, the median curvature across the interval, and the average curvature across each interval. A single arc with the curvature matched at $x = 0$ is also shown. The average curvature method is shown to produce the best overall agreement between the shape of the parabola and the arcspline.](image-url)
In this formulation the pressures, tensions, and support radii are chosen independently of the inner clamping points. Note that $p_5$ is assumed to be known. If it is unknown, the system can be solved assuming $p_5 = 0$ and increasing the resulting pressures by the given $p_5$ once its value is known. The inner clamping points are then chosen by choosing the span (arc angle) of the support membranes. By performing a force balance akin to Eq. (3.7) on each arc of the top membrane, we obtain $N$ equations, one for each arc $j$ of the top membrane:
\[ \sum_{k=1}^{j} T_k = (p_i - p_{k+1}) R_{N-k+1} \] 

(3.66)

By performing a similar force balance across each support arc \( j \) we obtain, we obtain the \( N-1 \) support arc equations:

\[ T_j = (p_j - p_{j+1}) R_{\text{sup},j} \] 

(3.67)

The basic formulation of the system involves \( 3N-1 \) unknowns:

\[ \frac{N-1}{R_{\text{sup},j}}, \frac{N}{T_j}, \frac{N}{p_j} \] 

(3.68)

in the \( 2N-1 \) equations of Eqs. (3.66) and (3.67). This gives \( N \) free design parameters. Possible design parameter choices include:

- \( p \)-method: all \( N \) pressures are prescribed
- \( T \)-method: all \( N \) tensions are prescribed
- mixed-method: setting all tensions equal and prescribing \( p_1 \)

These three methods are explored for the case of \( N = 4 \).

**p-method: all \( N \) pressures are prescribed**

Eq. (3.66) forms a linear system:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 \\
1 & 1 & 1 & 0 \\
1 & 1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4
\end{bmatrix}
= 
\begin{bmatrix}
R_4 \left(p_1 - p_2\right) \\
R_3 \left(p_1 - p_3\right) \\
R_2 \left(p_1 - p_4\right) \\
R_1 \left(p_1 - p_5\right)
\end{bmatrix}
\] 

(3.69)

which has the solution:

\[
\begin{bmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4
\end{bmatrix}
= 
\begin{bmatrix}
R_4 \left(p_1 - p_2\right) \\
p_1 \left(R_3 - R_4\right) + p_2 R_4 - p_3 R_3 \\
p_1 \left(R_2 - R_3\right) + p_3 R_3 - p_4 R_2 \\
p_1 \left(R_1 - R_2\right) + p_4 R_2 - p_5 R_1
\end{bmatrix}
\] 

(3.70)
The support membrane radii are then found from Eq. (3.67).

**T-method: all N tensions are prescribed**

Taking \( p_5 = 0 \), Eq. (3.66) forms a linear system:

\[
\begin{bmatrix}
1 & -1 & 0 & 0 & \vdots & \vdots \\
1 & 0 & -1 & 0 & \vdots & \vdots \\
1 & 0 & 0 & -1 & \vdots & \vdots \\
1 & 0 & 0 & 0 & \vdots & \vdots \\
\end{bmatrix}
\begin{bmatrix}
p_1 \\
p_2 \\
p_3 \\
p_4 \\
\end{bmatrix}
= 
\begin{bmatrix}
\frac{T_1}{R_1} \\
\frac{(T_1 + T_2)}{R_2} \\
\frac{(T_1 + T_2 + T_3)}{R_3} \\
\frac{(T_1 + T_2 + T_3 + T_4)}{R_4} \\
\end{bmatrix}
\]

(3.71)

which has the solution (now offsetting by \( p_5 \)):

\[
\begin{bmatrix}
p_1 \\
p_2 \\
p_3 \\
p_4 \\
\end{bmatrix}
= 
p_5 
+ 
\begin{bmatrix}
\frac{(T_1 + T_2 + T_3 + T_4)}{R_1} \\
\frac{(T_1 + T_2 + T_3 + T_4)}{R_1} - \frac{T_1}{R_4} \\
\frac{(T_1 + T_2 + T_3)}{R_3} - \frac{(T_1 + T_2)}{R_3} \\
\frac{(T_1 + T_2 + T_3 + T_4)}{R_4} - \frac{(T_1 + T_2 + T_3 + T_4)}{R_2} \\
\end{bmatrix}
\]

(3.72)

If all \( T \) are equal, then Eq. (3.72) has the simplified form:

\[
\begin{bmatrix}
p_1 \\
p_2 \\
p_3 \\
p_4 \\
\end{bmatrix}
= 
p_5 
+ 
T 
\begin{bmatrix}
\frac{4}{R_1} \\
\frac{4}{R_1} - \frac{1}{R_4} \\
\frac{4}{R_1} - \frac{2}{R_3} \\
\frac{4}{R_1} - \frac{3}{R_2} \\
\end{bmatrix}
\]

(3.73)

In either case, the support radii are found from Eq. (3.67).

**Mixed-method: all tensions equal and \( p_1 \) prescribed**

Eq. (3.66) simplifies to:

\[
j \cdot T = \left( p_1 - p_{j+1} \right) R_{N-j+1}
\]

(3.74)

which has the solution:

\[
T = \frac{\left( p_1 - p_2 \right) R_4}{4}
\]

(3.75)

Then from Eq. (3.66):
\[
\begin{bmatrix}
  p_2 \\
  p_3 \\
  p_4
\end{bmatrix}
= \begin{bmatrix}
  p_1 - T/R_4 \\
  p_1 - 2T/R_3 \\
  p_1 - 3T/R_2
\end{bmatrix}
\]  \quad \text{(3.76)}

The support membrane radii are then found from Eq. (3.67). Interestingly, with the constraint of all tensions being equal, the support membrane radii are fixed by the arcspline geometry and are independent of pressure and tension:

\[
\begin{bmatrix}
  R_{\text{sup,2}} \\
  R_{\text{sup,3}} \\
  R_{\text{sup,4}}
\end{bmatrix}
= \begin{bmatrix}
  \frac{R_3 R_2}{2R_4 - R_3} \\
  \frac{R_2 R_1}{3R_3 - 2R_2} \\
  \frac{R_1 R_4}{4R_2 - 3R_1}
\end{bmatrix}
\]  \quad \text{(3.77)}

\textit{Choosing the inner clamping points}

Up to this point, all tensions, pressures and support membrane radii have been determined. The arcspline mechanical design may then be completed by defining the position of the inner clamping points $A_j$. The inner clamping points may be defined by specifying the arc angle subtended by the support arcs. These may be chosen such that the inner clamping points are collinear, or preferably they may be chosen to match the unstretched lengths of the membranes. The unstretched membrane length is:

\[
S = L_{0,\text{exact}} e^{T/k_0}
\]  \quad \text{(3.78)}

For the top membrane:

\[
S_1 = \sum_{j=1}^{N} R_j \cdot \Delta \Theta_j
\]  \quad \text{(3.79)}

Combining Eqs. (3.78) and (3.79), the unstretched length of the top membrane can be determined. For the support membranes:

\[
S_j = R_{\text{sup,j}} \cdot \Delta \Theta_{\text{sup,j}} + \sum_{k=1}^{j} R_k \cdot \Delta \Theta_k
\]  \quad \text{(3.80)}
From Eqs. (3.78) and (3.80), the $\Delta \Theta_{sup,j}$ can be chosen to match the unstretched lengths of all membranes. Note that if all tensions are equal, it is sufficient to specify that the stretched lengths of all membranes are equal.

**Design for given clamping points**

In this formulation support membranes are fitted to existing inner clamping points and an arcspline. This method is useful in the case that a new arcspline design is to be installed in an existing frame. With the inner clamping points $A_j$ prescribed, the support membrane radii and arc angles are calculated. The chord length of the support membrane is:

$$\text{ch}_j = \sqrt{\left( A_{x,j} - N_{x,N-j+1} \right)^2 + \left( A_{z,j} - N_{z,N-j+1} \right)^2} \quad (3.81)$$

The radius and arc angle may be found from:

$$R_{sup,j} = \frac{\text{ch}_j \sqrt{1 + m_{N-j+1}^2}}{2 \left[ A_{z,j} - N_{z,N-j+1} - m_{N-j+1} \left( A_{x,j} - N_{x,N-j+1} \right) \right]} \quad (3.82)$$

$$\Delta \Theta_{sup,j} = 2 \arcsin \left( \frac{\text{ch}_j}{2R_{sup,j}} \right) \quad (3.83)$$

We then have $2N$ unknowns and $2N - 1$ equations from Eqs. (3.66) and (3.67), leaving 1 free design parameter, for which we set $p_1$. The solution is:

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} = \begin{bmatrix} \frac{r_1 \left( p_1 - p_5 \right) \left( R_3 - R_{sup,2} \right) \left( R_2 - R_{sup,3} \right) \left( R_1 - R_{sup,4} \right)}{\left( R_4 - R_{sup,2} \right) \left( R_3 - R_{sup,3} \right) \left( R_2 - R_{sup,4} \right)} \\ \frac{R_{sup,2} \left( p_1 - p_5 \right) \left( R_4 - R_3 \right) \left( -R_2 + R_{sup,3} \right) \left( -R_1 + R_{sup,4} \right)}{\left( R_4 - R_{sup,2} \right) \left( R_3 - R_{sup,3} \right) \left( R_4 - R_{sup,4} \right)} \\ \frac{R_{sup,3} \left( p_1 - p_5 \right) \left( R_3 - R_2 \right) \left( R_1 - R_{sup,4} \right)}{\left( R_3 - R_{sup,3} \right) \left( R_2 - R_{sup,4} \right)} \\ \frac{R_{sup,4} \left( p_1 - p_5 \right) \left( R_2 - R_1 \right)}{R_2 - R_{sup,4}} \end{bmatrix} \quad (3.84)$$
Table 3.3. Complete geometric design of the reference arcspline.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$f$ [m]</th>
<th>$F_x$ [m]</th>
<th>$F_z$ [m]</th>
<th>$\Phi_i$ [$^\circ$]</th>
<th>$\Phi_o$ [$^\circ$]</th>
<th>$\Delta\Phi$ [$^\circ$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.25</td>
<td>0.29526</td>
<td>3.23141</td>
<td>37.26</td>
<td>78.17</td>
<td>70.87</td>
<td></td>
</tr>
</tbody>
</table>

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<td>78.17</td>
<td>70.87</td>
<td></td>
</tr>
</tbody>
</table>

where the pressures may then be found from Eq. (3.72).

3.3.5. Example design

Having developed the arcspline design theory, we now give the dimensions of an example design. This example arcspline geometry was designed using the hybrid geometric/optical approach outlined in [34], prior to the development of the more advanced design procedures (see in particular Section 3.4) discussed here. Table 3.3 gives the complete specifications of the example design. The geometry is that shown in Figure 3.12.
3.3.6. Finite element analysis

The arcspline design theory has been developed under a number of previously mentioned simplifying assumptions. In order to ascertain the validity of the theory under more realistic conditions, a series of structural finite element simulations were performed. The finite element simulation was used to predict the shape of the example design of Section 3.3.5. The optical performance of the resulting shape was then ascertained by ray-tracing in the commercial

\[ p_1 - p_2 \]

\[ p_2 - p_3 \]

\[ p_3 - p_4 \]

\[ p_4 - p_5 \]

Legend:

- \( \rightarrow \) prescribed displacement
- \( \Rightarrow \) zero displacement
- \( \rightarrow \) uniform pressure

Figure 3.13. Schematic of the finite element model in ANSYS showing boundary and loads at: (a) first load step; (b) second load step. For clarity, only the boundary conditions for the top membrane are shown. The boundary conditions on the remaining membranes are identical. The inlay shows a detail of the mesh.

\[ p = 1 \text{ Pa} \]

---

2 Material from this section has been extracted from M. Bernini, “Structural analysis of inflated membrane trough concentrators,” Master Thesis, ETH Zurich, 2014, conducted under the direct supervision of T. Cooper.
LightTools code and ultimately compared to the ideal shape resulting from the theory.

**Finite element model**

Finite element (FE) modeling was performed using ANSYS 15.0. To reduce the size of the grid, a 1 m long section of the trough was modeled with symmetry boundary conditions applied at the ends. **Figure 3.13** shows a schematic of the finite element model showing the geometry, mesh, boundary conditions, and loads.

Modeling of the initial unstretched arcspline geometry proved to be a difficult task. One option would be to choose the initial configuration as the final theoretical configuration, e.g. **Figure 3.16**. However this poses two difficulties: (1) it would be necessary to assign a pre-stress to the membranes; (2) this requires meshing of multi-arc surfaces. The second difficulty was found to lead to serious mesh problems in ANSYS. The problems arise because the mesh tool treats multi-arc surfaces, even if tangent, as separate entities with a connectivity constraint at the intersection, and these connectivities are unstable for very thin shells. To obtain a robust mesh, and avoid the necessity of providing pre-stress in the initial configuration, it was decided to use a simplified initial geometry. Each membrane is assigned its correct unstretched length and an assigned a radius such that the inner clamping points are close to their real positions, and such that the membranes do not contact each other along their span. The model is then loaded in two steps.

In the first step, a small pressure (~1 Pa) is applied to each membrane, and the clamping points are moved to their correct position by means of a fixed displacement boundary condition at the inner clamping points. The model is solved and the resulting geometry then serves as the initial configuration for the second loading step. In the second step, the correct inflation pressures are applied and the model is again solved. Gravitational effects have been omitted, but could easily be introduced into the model at a later stage.
Optical model

The finite element solution geometry is imported into CAD software (SolidWorks) from which a NURBS surface through the nodes is imported into the commercial ray-tracing code LightTools. Additionally, the ideal arcspline geometry, as predicted from the theory of Section 3.3.1 is also simulated in LightTools. The collecting aperture of the arcspline is uniformly illuminated by radiation having a half-angular subtense of 0.266°. The irradiance distribution on a target centered along the focal line and tilted to bisect the inner and outer rim angles of the arcspline was used as a basis of comparison between the FE and theoretical solutions.

Results

Figure 3.15 shows a comparison of the irradiance distribution using the arcspline shape predicted by theory to that simulated by the FE model. In Figure 3.15 (a) the source comprises a set of parallel on-axis rays uniformly distributed over the inlet aperture. In Figure 3.15 (b) the sun is modeled as a distant uniform disk subtending an angular radius of 0.266°. In both cases the agreement is very good, supporting the correctness of the theory. The discrepancies in Figure 3.15 (a) result mainly from slope deviations at the nodes, which may be due to contact modeling approximations.

The finite element model should be extended to include secondary effects which may affect the arcspline shape. It is expected that electrostatic and
friction forces between membranes are the most critical effects remaining to be studied.

### 3.4. Nonparabolic troughs

The previous section introduced the arcspline trough concentrator whose shape is described by $N$ tangentially connected circular arcs. It was demonstrated that this shape can be designed to closely approximate that of a parabola. The question then arises of how well this arcspline shape, and nonparabolic shapes in general, perform as solar concentrators.

Conservation of étendue sets the well-known fundamental limit of concentration [41] for a hollow reflective geometry for an infinite source

$$C_{g,\text{ideal,2D}} = \frac{a_i}{a_o} \frac{\sin \theta_o}{\sin \theta_i}$$

where $a_i$ and $\theta_i$ are the aperture width and maximum angle of incidence (acceptance angle) at the inlet aperture, and $a_o$ and $\theta_o$ are those at the outlet.

---

aperture, at which the receiver is placed. Throughout this section, \( a \) is taken as the aperture half-width. For three-dimensional geometries, \( C_g \) is the ratio of inlet to receiver areas and has the corresponding limit \( \sin^2 \theta_o / \sin^2 \theta_i \). Although this thesis deals primarily with trough concentrators, the results of this section are of general importance for solar concentrator designs and thus the analysis is presented in a general framework. Implicit in Eq. (3.85) is the condition of full-collection: every ray within \( \pm \theta_i \) incident on the inlet aperture must reach the receiver (with an angle \( \leq \theta_o \)). Certain preimposed restrictions on the concentrator and receiver geometry will enforce stricter limitations on the concentration than the fundamental limit.

### 3.4.1. The limit for focusing concave mirrors

Figure 3.16 shows the cross-section of a focusing concave mirror whose profile is described by an arbitrary continuous function \( z(x) \), for a receiver of (a) flat or (b) circular cross-section both centered at the origin. This 2D profile may then be either extruded along the \( y \)-axis to obtain a line-symmetric (trough) concentrator, or revolved about the \( z \)-axis yielding an axisymmetric (dish) concentrator. The concentrator is to be designed to collect all radiation from an infinite source of finite half-angular subtense \( \theta_i \). The scale is set by choosing the receiver half-width \( a_o \) or radius \( r_o \).

The maximum concentration attainable from such a device was previously derived in [42] and follows directly from the geometry of Figure 3.16 once \( \theta_i \)
and Φ have been specified, since these parameters fix the location of the point \(\mathbf{P}_0(x_0, z_0)\) and thus the largest possible inlet aperture \(a_i\). For a flat receiver \(\mathbf{P}_0 = [a_o \sin(2\Phi)/\sin(2\theta_i), -a_o(\cos2\Phi + \cos2\theta_i)/\sin2\theta_i]\) which yields

\[
C_{g,\text{max,2D,concave,flat}} = \frac{a_i}{a_o} = \frac{\sin \Phi \cos \Phi}{(\sin \theta_i \cos \theta_i)} - 1
\] (3.86)

where the receiver is considered to be one-sided (only receiving radiation from \(z < 0\)) such that it partially shades the mirror. For a receiver of circular cross-section \(\mathbf{P}_0 = [r_o \sin\Phi/\sin\theta_i, -r_o \cos\Phi/\sin\theta_i]\) yielding:

\[
C_{g,\text{max,2D,concave,circular}} = \frac{a_i}{(\pi r_o)} = \frac{\sin \Phi}{(\pi \sin \theta_i)}
\] (3.87)

Here the receiver is considered to receive radiation also from the top such that shading need not be considered. Without loss of generality, we set \(a_o = r_o = 1\) throughout the remainder of this section.

Eqs. (3.86) and (3.87) are the maximum concentrations that can be achieved for a single continuous mirror contour where all rays within the acceptance angle suffer exactly one reflection on their way to the receiver. It is essential to differentiate this from edge-ray condition used to design ideal concentrators in 2D, for example the 2D CPC. In the CPC all edge-rays suffer exactly one reflection on their way to the outlet, but all other rays may suffer anywhere from zero to infinitely many reflections on their way to the outlet. This is a characteristic of nonimaging concentrators designed by the string method. For small acceptance angles, edge-ray designs may have a very large number of reflections. For example, it has been shown [32] that the average number of reflections for small acceptance-angle 2D CPCs is approximately equal to \(\frac{1}{2}\ln(1/\theta_i)\), where \(\theta_i\) must be specified in radian measure.

Inasmuch as it is not possible to design a single-stage focusing concave mirror contour that achieves a concentration higher than Eq. (3.86) or (3.87) while collecting all rays within \(\pm \theta_i\), we may consider such a device to be an ideal concave mirror in the nonimaging sense. We consequently refer to Eqs. (3.86) and (3.87) as the concave limit for flat and circular receivers respectively. It may also be appropriately referred to as the one-reflection limit. This limit, which pertain to the specific case of a focusing concave mirror, is always less than the fundamental limit of Eq. (3.85).
3.4.2. A unifying equation for all ideal concave focusing mirrors

As the limits of concentration were derived considering the rim point $P_0$ alone, the question of the mirror shape is yet unanswered. The problem is then to find a mirror profile with its rim at $P_0$ that reflects all rays within $\pm \theta_i$ to the receiver. We need only consider the portion of the mirror extending from $P_0$ to the truncation point $T[x_T, z_T]$ where the $-\theta_i$ edge-ray strikes the mirror after just passing the right side of the receiver. This gives the interval $[x_T, x_0]$ over which we must design the mirror, since the portion $-x_T < x < x_T$ is completely shaded by the receiver, and the portion $x \leq -x_T$ follows from symmetry about the $z$-axis. Note that point $T$ is not unique for a given choice of $\theta_i$ and $\Phi$, but rather depends on the specific geometry of the mirror contour. Its location, however, does not affect the geometric concentration of the mirror. For simplicity, the mirror may be designed on the interval $[0, x_0]$ and later truncated to point $T$. By the construction of Figure 3.16, there are no practical designs for which point $T$ lies on the left side of the $z$-axis, that is $x_T > 0$ always holds for an ideal concave mirror regardless of the choice of $\theta_i$ and $\Phi$.

A ray incident at an angle $\theta$ measured counterclockwise from the optical axis strikes point $x$ on the mirror and is reflected towards the receiver. It is convenient to parameterize the mirror profile by the slope angle $\Theta = \arctan(\dot{z})$, where $\dot{z} = dz(x)/dx$. The equation of the reflected ray is then:

$$\tilde{r}(\Theta, t; \theta) = P(\Theta) + \hat{v}(\Theta)t$$

(3.88)

where $P[x(\Theta), z(\Theta)]$ is the point at which the ray strikes the mirror, $\hat{v}(\Theta) = [-\sin(2\Theta - \theta), \cos(2\Theta - \theta)]$ is the unit direction vector of the reflected ray, and $t$ is a parameter representing the distance travelled by the ray.

For a flat receiver, the point where the reflected ray strikes the receiver may be expressed by $X_{\text{flat}}(u, 0)$. Solving the intersection of Eq. (3.88) with $X_{\text{flat}}$ for $u$ yields:

$$u_{\text{flat}}(\Theta; \theta) = x(\Theta) + \tan(2\Theta - \theta)z(\Theta)$$

(3.89a)

$$u_{\text{flat}}(x; \theta) = x - z \frac{\dot{z}(\dot{z} \sin \theta + 2 \cos \theta) - \sin \theta}{\dot{z}(\dot{z} \cos \theta - 2 \sin \theta) - \cos \theta}$$

(3.89b)
We denote Eq. (3.89) the focal function of the concentrator, shown in parametric form in Eq. (3.89a) and nonparametric form in Eq. (3.89b), obtained by subbing in \( \Theta = \arctan(z) \).

For the circular receiver, the focal function may be similarly derived by considering the intersection of Eq. (3.88) with a circular receiver parameterized by \( X_{\text{circ}}(u \cos \phi, u \sin \phi) \). The smallest receiver radius \( u \) required to intercept any given ray occurs when the ray is tangent to the receiver, such that \( \phi = 2\Theta - \theta \). Solving the intersection for \( u \) subject to this tangency constraint yields the focal function for the circular receiver:

\[
\begin{align*}
  u_{\text{circ}}(\Theta; \theta) &= x(\Theta) \cos(2\Theta - \theta) + z(\Theta) \sin(2\Theta - \theta) \\
  u_{\text{circ}}(x; \theta) &= \{[x - \dot{z}(xz - 2z)] \cos \theta \\
                   &- [z - \dot{z}(\dot{z} + 2x)] \sin \theta \}/(1 + \dot{z}^2)
\end{align*}
\]

The angle \( \Theta \) has been restricted to \([0, \pi]\) such that rays striking the bottom of the receiver have \( u < 0 \), thus giving the same range of \(-1 \leq u \leq 1\) as for the flat receiver.

The condition for the mirror to collect all radiation within \( \pm \theta_i \) may then be described by the following system of differential inequations:

\[
\begin{align*}
  u(x; \theta = +\theta_i) &\leq 1 \\
  u(x; \theta = -\theta_i) &\geq -1 \\
\end{align*}
\]

where \( u \) is taken from Eq. (3.89) for a flat receiver and Eq. (3.90) for a circular receiver. Any mirror profile spanning to \( P_0 \) and satisfying Eq. (3.91) is an ideal nonimaging concave mirror, since it reaches the maximum \( C_g \) set by Eq. (3.86) or (3.87) and collects all rays within \( \pm \theta_i \). The concept of the focal function and the corresponding system of differential inequations describing the condition for full-collection may be generalized to any receiver geometry.

3.4.3. Canonical solutions

Let us first tackle the case of a flat receiver and consider the geometrically intuitive solution of a parabola with its axis collinear with the optical axis. From the geometry of Figure 3.16, this parabola will have a focal length of
\[ f = \frac{1}{2} x_0 \cot(\frac{1}{2} \Phi) \] and its paraxial focus will lie at \( F[0, z_0 + x_0 \cot \Phi] \). Figure 3.17 shows the profile, slope and focal function of this parabola, as computed by Eq. (3.89). Figure 3.17 (c) verifies that Eq. (3.91) is satisfied and thus this parabola is indeed a solution, which we denote the central solution \( z_C \). It is interesting to note that when the receiver is centered at the paraxial focus, as is often done in practice for solar concentrators, the maximum concentration is \( C_g = \sin \Phi \cos(\Phi + \theta_i)/\sin \theta_i - 1 \) [43], which is always smaller than the concave limit given by Eq. (3.86). In this light, we note that this maximum concentration for the parabola is achieved when the receiver is shifted down slightly from the focal plane, as previously mentioned in [22].

This parabola is not the only curve capable of fulfilling Eq. (3.91) for a flat receiver, and it is therefore not the only ideal concave mirror. In the extreme
case we may consider a mirror profile which focuses all of the $+\theta_i$ family of edge-rays incident on the mirror to point A on the receiver. In this case, the mirror profile, denoted the right-extreme solution $z_R$, will be a parabola with focus A having its axis parallel to the $+\theta_i$ edge-rays. This solution may be found by transforming the first inequality of Eq. (3.91) into the differential equation $u(x, +\theta_i) - 1 = 0$ and solving. The left-extreme solution $z_L$ may be similarly found by solving $u(x, -\theta_i) + 1 = 0$, which results from transforming the second inequality of Eq. (3.91) into an equality. $z_L$ is a parabola with focus B and its axis parallel to the $-\theta_i$ edge-rays. The extreme solutions are shown alongside $z_C$ in Figure 3.17.

For the circular receiver, we can construct three analogous canonical solutions, shown in Figure 3.18. Like the case of the flat receiver, the central solution $z_C$ is a parabola with its axis collinear with the optical axis, this time
with a focal length of \( f = (1 + \cos \Phi)/(2\sin \theta_i) \) and its paraxial focus at the origin. Unlike the case of the flat receiver, the maximum \( C_g \) is achieved when the receiver is centered at the paraxial focus of the parabola.

As for the flat receiver, the extreme solutions may be found by solving 
\[ u(x, \pm \theta_i) \pm 1 = 0, \]
where \( u \) is now specified by Eq. (3.90). The resulting curves reflect the \( +\theta_i \) and \( -\theta_i \) edge-rays such that they are always tangent to the top and bottom of the receiver respectively. Such curves are known as macrofocal parabolas [22, 44]. The right-extreme solution \( z_R \) is a “winding” [22] macrofocal parabola with its axis parallel to the \( +\theta_i \) edge-rays. The left-extreme solution \( z_L \) is an “unwinding” macrofocal parabola with its axis parallel to the \( -\theta_i \) edge-rays.

### 3.4.4. A reformulation: designing in derivative space

Figures 3.17 and 3.18 show that the extreme solutions form an envelope (in which \( z_C \) lies), hinting that we might be able to construct additional solutions by fitting curves inside this envelope. Consider an arbitrary mirror profile whose slope lies between the slopes of the extreme solutions of Figure 3.17 (b) or Figure 3.18 (b), that is:

\[
\dot{z}_R \leq \dot{z} \leq \dot{z}_L \quad \forall x \in [x_T, x_0] \tag{3.92}
\]

By integrating from \( x_0 \) to \( x \) it follows that:

\[
z_L \leq z \leq z_R \quad \forall x \in [x_T, x_0] \tag{3.93}
\]

Consider the first inequality of Eq. (3.92) for a flat receiver. The right-extreme solution is a tilted parabola whose profile \( z_R \) and slope \( \dot{z}_R \) are known analytically. Subbing an analytical expression for \( \dot{z}_R \) into the first inequality of Eq. (3.92) gives:

\[
\dot{z}_R = \frac{(x-1)\sin \theta_i + z_R \cos \theta_i + [(x-1)^2 + z_R^2]^{1/2}}{(x-1)\cos \theta_i - z_R \sin \theta_i} \leq \dot{z} \tag{3.94}
\]

Since \( z \leq z_R \) from Eq. (3.93), we can replace \( z_R \) by \( z \) in Eq. (3.94) and the inequality must still hold. After making this replacement, we can rearrange Eq. (3.94) to obtain:
which is precisely the first inequation of Eq. (3.91) for a flat receiver! Following the same procedure, it can be shown that \( \dot{z} \leq \dot{z}_L \) implies that the second inequation of Eq. (3.91) is satisfied. Therefore our imposed condition on the slope, Eq. (3.92), guarantees that Eq. (3.91) is satisfied. For a circular receiver, it can be analogously shown that all mirror profiles satisfying Eq. (3.92) are solutions to Eq. (3.91).

It is important to note that the substitution of \( z \) for \( z_R \) into Eq. (3.94) sets a stricter limit on the mirror profile. Therefore Eq. (3.92) is a sufficient but not necessary condition for the mirror to be a solution of Eq. (3.91). There will be some mirror profiles that are indeed solutions to Eq. (3.91) but fail Eq. (3.92). As an illustrative example, consider a concave mirror for a flat receiver and with \( \Phi = 60^\circ, \theta_i = 5^\circ \) whose slope is prescribed by perturbing that of the central solution by a damped sine wave:

\[
\dot{z}(x) = \dot{z}_c(x) + 0.0029 \exp[-0.75(x-x_0)]\sin[1.8\pi(x-x_0)]
\]

The profile, slope and focal function of this mirror are superimposed in Figure 3.17. It is observed that although Eq. (3.92) is not satisfied, the focal function is within \(-1\) and \(1\) such that this curve is indeed a solution to Eq. (3.91).

Nevertheless, Eq. (3.92) provides an extremely convenient statement of the problem: any mirror contour whose slope lies between that of the extreme solutions (which are known analytically) is a solution to Eq. (3.91). In the limit as \( \theta_i \to 0 \), the two extreme curves collapse into the central solution. But for finite \( \theta_i \), the fact that the extreme curves form an envelope proves that there are infinitely many ideal concave mirror geometries! We can easily construct solutions by specifying the slope of a curve such that it lies within the envelope of Figure 3.17 (b) or Figure 3.18 (b).

3.4.5. Design of ideal arcspline concentrators

Since the motivation of this work was to find shapes of easy manufacture, let us focus on a practical application to the design of an arcspline trough.
concentrator (ASC) [34]. Previous attempts to design ASCs [34] have been based on parabola approximation, requiring somewhat arbitrary fitting procedures, and yielding designs inferior to the parabola. The curvature matching method of Section 3.3.3 is deterministic and thus the problem of arbitrariness is removed, but still the resulting designs are in general inferior to the parabola. We will show that using the theory outlined here an ASC can always be designed to reach the concave limit.

Let the arcs of the arcspline be labeled from \( j = 1 \) to \( N \), with \( 1 \) being the outermost arc as shown in Figure 3.19 (a). A point on the \( j \)th arc of the arcspline may be found from:

\[
Q(\varphi) = C_j + R_j (\sin \Theta, -\cos \Theta)
\]

where \( R_j \) and \( C_j [C_{x,j}, C_{z,j}] \) are the radius and center of the \( j \)th arc respectively and \( \Theta \) serves as a parameter spanning from a maximum value at the outer endpoint \( \Theta_{out,j} \) to a minimum value at the inner endpoint \( \Theta_{in,j} \). The endpoints of each arc, referred to as the nodes, may be found from \( N_{j-1} = Q(\Theta_{out,j}) = Q(\Theta_{in,j-1}) \), e.g. \( N_0 = P_0 \). Subbing Eq. (3.97) into Eq. (3.89) yields the focal function for the \( j \)th arc:

\[
\begin{align*}
\quad u_{flat,ASC}(\Theta; \varphi) &= C_{x,j} + R_j \sin \Theta + \tan(\Theta - 2\varphi)(C_{z,j} - r_j \cos \Theta) \\
&= N_{x,j-1} + R_j (\sin \Theta - \sin \Theta_{out,j}) \\
&\quad + \tan(\Theta - 2\varphi)[N_{z,j-1} - R_j (\cos \Theta - \cos \Theta_{out,j})]
\end{align*}
\]

Since the number of arcs in an arcspline is equal to the number of polymer membranes required in the stack, technical and economic considerations mandate the use of as few arcs as possible. Based on the concept of the focal function, we develop a procedure to design an arcspline that reaches the concave limit and satisfies Eq. (3.91) with the minimum number of arcs.

We outline the design procedure by designing an exemplary ASC with \( \theta_i = 0.5^\circ, \Phi = 60^\circ \) for a flat receiver. The final result achieves the limit \( C_{g,max,2D,concave,flat} = 48.6 \times \) and is shown in Figure 3.19 (a). The arcspline is designed starting from \( x = x_0 \) and working leftwards to \( x = x_T \). The outer node of arc 1 is fixed by \( N_0 = P_0 \), and the slope angle is fixed by \( \Theta_{out,1} = \frac{1}{2}\Phi \). The
radius $R_1$ of the first arc is then chosen such that the focal function, Eq. (3.98), has a maximum of $u = 1$ for the $+\theta_i$ edge-ray, as shown in Figure 3.19 (b). With $R_1$ specified, the first arc can be fully defined by choosing $\Theta_{in,1}$. For the minimum arc solution, we would like the largest possible span of the arc without violating Eq. (3.91). This is achieved by choosing $\Theta_{in,1}$ such that $u_{flat,AS}(\Theta_{in,1}, -\theta_i) = -1$ also seen in Figure 3.19 (b). From the tangency condition of adjacent arcs it follows that $\Theta_{out,j+1} = \Theta_{in,j}$. This procedure of choosing the radius such that $\max[u_{flat,AS}(\Theta_{out,j}, +\theta_i)] = 1$ and then choosing the inner arc angle from $u_{flat,AS}(\Theta_{in,j}, -\theta_i) = -1$ continues until the arcspline crosses the $-\theta_i$ edge-ray (point $T$) at which point it is terminated. The index $j$ at this termination point gives the number of arcs, e.g. $N = 3$ for this design.

Because the resulting ASC spans $[x_T, x_0]$ and by design satisfies Eq. (3.91), it is an ideal concave nonimaging mirror. An analogous procedure for circular receivers can be derived by subbing Eq. (3.97) into Eq. (3.90) and following the
above steps. Using the above design procedure, the minimum number of arcs
required to reach the concave limit for a given $\Phi$ and $\theta_i$ can be determined.
Results for flat and circular receivers for common acceptance and rim angles
are shown in Table 3.4 and Table 3.5 respectively.

### Table 3.4. Geometric concentration (top subrow) and required number of arcs (bottom subrow) as a function of rim angle $\Phi_{\text{rim}}$ and acceptance angle $\theta_i$ for arcspline concentrators with a single horizontal flat receiver.

<table>
<thead>
<tr>
<th>$\theta_i$ [°]</th>
<th>$\Phi_{\text{rim}}$ [°]</th>
<th>0.266°</th>
<th>0.5°</th>
<th>0.75°</th>
<th>1°</th>
<th>1.5°</th>
<th>2°</th>
</tr>
</thead>
<tbody>
<tr>
<td>10°</td>
<td>35.8×</td>
<td>18.6×</td>
<td>12.1×</td>
<td>8.8×</td>
<td>5.5×</td>
<td>3.9×</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
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</tr>
<tr>
<td>15°</td>
<td>52.9×</td>
<td>27.6×</td>
<td>18.1×</td>
<td>13.3×</td>
<td>8.6×</td>
<td>6.2×</td>
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<td></td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20°</td>
<td>68.2×</td>
<td>35.8×</td>
<td>23.6×</td>
<td>17.4×</td>
<td>11.3×</td>
<td>8.2×</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
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<tr>
<td>25°</td>
<td>81.5×</td>
<td>42.9×</td>
<td>28.3×</td>
<td>21.0×</td>
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<td>2</td>
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<tr>
<td>30°</td>
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<td>48.6×</td>
<td>32.1×</td>
<td>23.8×</td>
<td>15.5×</td>
<td>11.4×</td>
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<td>3</td>
<td>2</td>
<td>2</td>
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<td>40°</td>
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Table 3.5. Geometric concentration (top subrow) and required number of arcs (bottom subrow) as a function of rim angle $\Phi_{\text{rim}}$ and acceptance angle $\theta_i$ for arcspline concentrators with a circular receiver.

<table>
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<th>$\Phi_{\text{rim}}$ $[{}^\circ]$</th>
<th>$\theta_i$ $[{}^\circ]$</th>
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<th>0.5°</th>
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3.5. Two-mirror aplanatic and SMS troughs

In the previous section, the maximum concentration that can be achieved for a single mirror concentrator for which every ray within the acceptance angle suffers exactly one reflection on its way to the receiver was shown to be given by Eq. (3.86) for a flat absorber. It is evident that under this condition, the concentration necessarily falls short of the fundamental 2D limit of $n/\sin\theta_i$ given by conservation of étendue. It has been demonstrated [45] that with two sequential optical surfaces, it is possible to transform any two wavefronts into

---

any two other wavefronts. Together with the edge-ray principle [23], this implies that two sequential optical surfaces can be designed to reach the fundamental limit. The general procedure for designing these surfaces is the simultaneous multiple surface (SMS) method [45, 46].

It has also been demonstrated that two sequential optical surfaces, designed entirely using imaging strategies, can approach the fundamental limit for small acceptance angles [47]. These imaging designs are free of the two lowest-order optical aberrations, namely spherical aberration and coma, meaning that on-axis rays are all reflected to a single focal point, and have equal magnification there. This is achieved when the conditions of: (1) constant optical pathlength; and (2) the Abbe sine condition are satisfied for all on-axis rays. The condition of being spherical aberration and coma free is known as aplanatism, and these systems are referred to as aplanats. Of particular interest are two-mirror aplanats, comprising two sequential reflective surfaces. The analytical parametric equations of the primary and secondary mirror profiles for all two-mirror aplanats was previously derived in the context of telescope optical design.

3.5.1. Parametric equations of the two-mirror aplanat

Figure 3.20 shows the geometry of a generic two-mirror aplanat. The basic shape of the aplanat is governed by two parameters: $s$, the separation between the vertex of the second mirror and the final focus; and $K$, the separation between the vertices of the first and second mirrors. The parametric equations [48] of the first mirror profile are:

$$x_p = \sin \varphi$$
$$z_p = s - \cos^2 \left( \frac{1}{2} \varphi \right) + g(\varphi) \cos^4 \left( \frac{1}{2} \varphi \right) \left[ 1 - K f(\varphi) \right] / s$$

(3.99)

The parametric equations of the second mirror profile are:

$$x_s = \frac{2s K f(\varphi) \tan \left( \frac{1}{2} \varphi \right)}{K f(\varphi) \tan^2 \left( \frac{1}{2} \varphi \right) + g(\varphi)}$$
$$z_s = -x_s \cot \varphi$$

(3.100)
The parameter \( \phi \) is the polar angle, i.e. angle between the \( z \)-axis and an on-axis ray as it approaches the focus. In general the range for \( \phi \) is [48]:

\[
\delta \leq \phi \leq \Phi
\]

upward-facing absorber

\[
180^\circ - \Phi \leq \phi \leq 180^\circ - \delta
\]

downward-facing absorber

where \( \delta \) is the truncation angle, which may be chosen such that the portion of the primary shaded by the secondary is removed, i.e. \( x_p(\delta) = x_s(\Phi) \). In practice, the secondary should be designed with a slightly larger \( \phi \) range than the primary (e.g. \( \delta - \theta_{i,1} < \phi < \Phi + \theta_{i,1} \)) in order to prevent off-axis rays from missing the secondary.

The geometric concentration of a two-mirror aplanat may be estimated by considering the width of an aberration-free image. For a focal length of \( f = 1 \) the half-width of the primary focal plane is:

\[
a_{o,1} = \sin \theta_{i,1}
\]
The half-width of the primary inlet aperture is, from Eq. (3.99):

\[ a_{i,1} = x_p(\Phi) - x_p(\delta) = \sin \Phi - \sin \delta \quad (3.105) \]

The geometric concentration of a two-mirror aplanat is therefore:

\[ C_{g,1} = \frac{a_{i,1}}{a_{o,1}} = \frac{\sin \Phi - \sin \delta}{\sin \theta_{i,1}} \quad (3.106) \]

### 3.5.2. Compact, small rim angle designs

Two-mirror aplanats designed for maximum solar concentration have been thoroughly explored in the literature [47, 48]. An additional application of the two-mirror aplanat, particularly of interest in the present work, is compact, small rim-angle designs. Whereas parabolic troughs become very incompact for small rim angles, two-mirror aplanats can essentially be made to reach the compactness limit of \( AR = 0.25 \) (see Section 4.1.1) [48] for any rim angle.

**Figure 3.21.** Two compact small rim angle two-mirror aplanats with: (a) \( \Phi = 10.5^\circ \) with \( s = 0.084, K = 0.084 \); and (b) \( s = -0.101 \) and \( K = -0.101 \). Both designs have a final focus at the vertex of the primary mirror, and approach the compactness limit of \( AR = 0.25 \). The truncation angle is chosen to eliminate blocking and shading (\( \delta = 1^\circ \) for both designs).
Figure 3.21 shows two compact two-mirror aplanats of small rim angle ($\Phi = 10.5^\circ$ for both cases). Both are designed with $s = K$ such that the final focus is at the vertex of the primary, and both essentially achieve a compactness of $AR = 0.25$. For $\Phi = 10.5^\circ$ and $\theta_{i,1} = 0.266^\circ$, the thermodynamic limit is $39.3\times$. Both designs shown in Figure 3.21 achieve 90% of this limit ($C_{g,1} = 35.5\times$) with the concentration loss being due to the shading of the primary by the secondary.

3.5.3. Aplanats as the limiting case of SMS designs\(^5\)

An analogy between the condition of aplanatism and the edge-ray condition can be made. Aplanatism implies that, to a first order approximation, all off-axis rays inclined at the same angle $\theta_i$ to the optical axis hit the same point at the receiver. As $\theta_i \to 0$ the first order approximation becomes exact, and therefore equivalent to the edge-ray condition that all rays inclined at the acceptance angle must be focused to the edges of the exit aperture. The fact that SMS optics converge to aplanatic designs as $\theta_i \to 0$ has been proven for the specific case of SMS RX [49] optics which have a refractive surface followed by a reflective one. Here we investigate this relationship for the two-mirror aplanat and SMS XX optic, which has two sequential reflective surfaces. The use of X for a reflective surface comes from the Spanish “reflexión” [22].

Figure 3.22 shows a Cassegrain-style two-mirror aplanat with $s = 0.43$, $K = 0.0385$ and $\sin\Phi = 90^\circ$, and an SMS XX concentrator designed to have the same optical pathlength. The aplanat is an imaging design and thus the geometry does not change for different acceptance angles. In contrast, the SMS design depends on the choice of $\theta_i$. For very small acceptance angles as in Figure 3.22 (a) there is negligible difference between the aplanat and the SMS XX designs. At larger acceptance angles, as in Figure 3.22 (b) the difference becomes more pronounced. For large acceptance angles ($\gtrsim 1^\circ$) it becomes necessary to oversize the secondary of the aplanat to avoid spilling many rays past the secondary concentrator.

\(^5\) Material in this section has been extracted from F. Dähler, “Application of the simultaneous multiple surface method to solar concentrator design,” Master Thesis, ETH Zurich, 2013, conducted under the direct supervision of T. Cooper.
Figure 3.23 shows the acceptance efficiency of 3D axisymmetric versions of the designs of Figure 3.22 as a function of the acceptance angle as determined by Monte Carlo ray-tracing using the VeGaS code. For $\theta_i \to 0$ the aplanat and SMS XX are identical and thus achieve the same performance. For increasing acceptance angles, the SMS XX design considerably outperforms the aplanat.

3.6. Summary

In this chapter the design of some high-performance line-focus systems, based on a low-cost inflated construction were presented. When moderate concentrations are required, a single-reflection device is satisfactory. The limits of concentration for a single-reflection concave mirror are derived and the conditions under which this mirror is ideal in the nonimaging sense are presented. It was demonstrated that the problem of finding suitable mirror geometries may be formulated as a system of differential inequalities, for which there exist infinitely many solutions, one of which is the classical parabola. The results here show that concave mirrors of nonparabolic profile can match the geometric concentration of parabolic reflectors for an infinite source of finite...
$\pm \theta_i$, thereby allowing use of more easily constructed geometries without loss of concentration. This presents new opportunities for reducing the cost of solar collectors and other nonimaging devices. As an additional benefit, the added design freedom may be used to tailor the spatial irradiance distribution on the receiver, e.g. to achieve a higher level of uniformity than possible with the parabola.

When concentrations greater than the concave limit are desired, a two-mirror primary must be used. For small acceptance angles, the two-mirror aplanats produce compact designs approaching the fundamental limit of concentration in 2D. For larger acceptance angles, the SMS XX concentrators show considerable performance improvement over the aplanatic designs.
Chapter 4

Line-to-point focus solar concentrators

Line-focus parabolic trough systems may be regarded as the workhorses of concentrating solar power (CSP). Their prevalence stems from their benefits of: (1) having simple constructions based on line-symmetric trough geometries; (2) requiring only one axis of tracking; and (3) being easily scaled to nearly any capacity by increasing the length of the trough. The principal drawback of line-focus systems is that their concentration ratio is constrained by the 2D limit of $C_{\text{max,2D}} = 1/\sin\theta_{\text{sun}} \approx 215$, whereas the corresponding limit for 3D concentrators is $C_{\text{max,3D}} = 1/\sin^2\theta_{\text{sun}} \approx 46\,000$. This limitation has implications on the temperature at which solar thermal energy can be absorbed and efficiently converted to work, which in turn affects the economics of CSP. Additionally, for concentrating photovoltaics (CPV) applications, this limit dictates the cell area required to deliver a given amount of solar electricity, thus directly influencing system cost. Many solar receiver concepts, thermodynamic cycles and processes require concentration ratios falling in-between the limits posed by purely 2D and by 3D geometries. While it is possible to obtain intermediate concentration ratios with 3D geometries by oversizing the acceptance angle, such a concentrator would still exhibit the inherent manufacturing complexities of a 3D design. It would therefore be convenient to have a solar concentrator design with a concentration limit greater than that of a purely 2D design while still maintaining some of the advantages of trough geometry. The line-to-point (LTP) focus solar concentrator achieves these objectives.

Figure 4.1 illustrates the generic goal of a LTP concentrator: the line-focus is split into a number of point-like foci spanning along its length. The main advantage of so doing is a major augmentation of the geometric concentration.

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since the overall area of the array of point-like foci is much smaller than that of a continuous line-focus of the same width. For certain applications, the fact that the continuous line-focus has been split into a number of discrete point-like foci may be an equally important advantage. For example, for concentrating photovoltaic applications, this allows a number of single cells or smaller arrays to be placed at each point focus, thus increasing the flexibility in arranging the cell interconnections.

If we wish to maintain a 2D trough as the primary concentrator, then it is clear that a line-to-point focus system will require some form of secondary concentrating stage. Depending on the desired concentration ratio of the system, the complexity of the required secondary concentrator stage may be vastly different. Regardless of the design of the secondary stage, the primary stage for the line-to-point focus systems considered here is always a one-axis tracking line-focus concentrator. As the primary concentrator represents by far the largest component of the solar collector as a whole, the major advantages of the line-focus primary are maintained by the system overall. Of course, the introduction of a secondary concentrating stage inevitably introduces some additional complexities and losses to the system. However, more exotic materials and designs can be utilized for the secondary concentrators without significantly impacting the overall cost of the solar collecting field.

**Figure 4.1.** A line-to-point (LTP) focus concentrator (b) differs from a traditional line-focus trough concentrator (a) in that the focal line is split into a number of point-like foci spanning along the length of the trough concentrator.
Figure 4.2. Three types of line-to-point (LTP) focus solar concentrators classified by the degrees of freedom of the secondary concentrator stage: (a) fixed secondary optics; (b) discrete-switching secondary optics; and (c) continuous-tracking secondary optics.
In order to achieve a higher concentration, line-to-point focus concentrators leverage the fact that, due to their 2D nature, line-focus primary concentrators focus incident solar radiation predominantly in one direction. To illustrate this fact, consider solar radiation arriving as a nearly collimated bundle of rays parallel to the optical axis of a trough concentrator, i.e. normal incidence. The collimated beam is focused by the trough in the transverse (cross-sectional) plane, but the beam remains collimated in the axial plane. Hence, there is still potential to concentrate radiation in the axial plane as it arrives at the primary focus. Unfortunately, the situation is not quite so simple since, for a one-axis tracker, solar radiation rarely arrives at normal incidence. Rather there exists a time dependent incidence angle between the incident solar beam and the optical axis of the trough in the axial plane, the so-called skew angle $\theta$, of Figure 2.8.

The path traced by solar radiation reaching the concentrator at nonzero skew angles is of particular importance in the determination of the potential maximum concentration of a line-to-point focus system. The secondary stage must be able to accept radiation from up to the rim angle in the transverse plane, and from the maximum and minimum skew angles in the axial plane. Since the degree to which radiation can be concentrated is inversely related to the acceptance angle, it is evident that the larger the range of skew angles seen by the secondary stage the smaller the concentration achievable.

Figure 4.2 shows three main classes of line-to-point focus solar concentrators, classified by the degrees of freedom of secondary concentrating stage. In order of increasing maximum concentration the classes are: (a) line-to-point concentrators with fixed secondary optics (fixed-secondary LTP); (b) those with discrete-switching secondary optics (switching-secondary LTP); and (c) those with continuous-tracking secondary optics (tracking-secondary LTP).

This chapter aims to develop the theory of the three classes of line-to-point focus concentrators in a unified way. First, considerations for primary concentrator design – relevant for all three classes – is discussed in Section 4.1. Then fixed secondary concentrates are discussed in Section 4.2. Although the basic theory is not new, it is extended from the restriction of a polar tracking primary in Brunotte [27], to any primary one-axis tracking scheme. Discrete-switching designs are presented for the first time in Section 4.3. Continuous
tracking secondary are discussed in Section 4.4. Considerable detail is given to this class of concentrators as it serves as the basis for a practical implementation presented in Chapter 6.

4.1. Considerations for primary concentrator design

Line-to-point focus concentrators are general based on one-axis tracking trough primary concentrators. In particular for tracking-secondary LTP concentrators, it is found (see Section 4.4.2) that the overall concentration is maximized for very small primary rim angles (\( \Phi \approx 10^\circ \)). This poses a challenge for the design of the primary concentrator, as small rim angle primaries are usually very incompact. This section discusses potential primary trough geometries for use in LTP collectors. Particular emphasis is placed on searching for compact, small rim angle primaries.

4.1.1. Parabolic trough

The simplest primary geometry is the parabolic trough concentrator. The profile of the parabolic trough with the origin at the focus is conveniently represented by:

\[
\begin{align*}
x &= 2f \tan \left( \frac{1}{2} \phi \right) \\
z &= -2f \cos \phi \left/ (1 + \cos \phi) \right.
\end{align*}
\]

for \(-\Phi \leq \phi \leq \Phi\). The compactness of a concentrator is quantified by its aspect ratio \( AR \). The aspect ratio is found by drawing a rectangle about the concentrator, including the focus, and then taking the ratio of the height of this rectangle to its width:

\[
AR = \frac{\Delta z}{\Delta x}
\]

where smaller values indicate a more compact design. For a parabolic trough with \( \Phi \leq 90^\circ \):

\[
AR = \frac{f}{4f \tan \left( \frac{1}{2} \Phi \right)} = \frac{1}{4} \cot \left( \frac{1}{2} \Phi \right)
\]
which has a minimum value of 0.25 for $\Phi = 90^\circ$. **Figure 4.3 (a)** shows the profiles of parabolic troughs for different rim angles. For $\Phi < 30^\circ$, the aspect ratio quickly increases yielding awkward designs, thus making the parabolic trough of limited practicality for high-concentration tracking-secondary LTP collectors.

### 4.1.2. Asymmetric parabolic trough

One possibility to improve compactness of small rim angle primaries while simultaneously eliminating shading of the primary is to consider asymmetric or off-axis designs. Brunotte et al. [27] used an asymmetric parabolic trough primary in their fixed-secondary LTP system. The asymmetric parabolic trough is governed by the same parametric equations, Eq. (4.1), as the parabolic trough, but with the polar angle spanning from $0 \leq \Phi_1 \leq \varphi \leq \Phi_2$. For the extreme case of the semi-parabolic trough, $\Phi_1 = 0$, but in practice a value on the order of $\theta_{i,1}$ may be chosen to eliminate shading losses.

In order to properly apply the concentration limits derived to asymmetric designs, it is necessary to formalize the rim angle of an asymmetric concentrator. For a symmetric trough, the rim angle is half of the angular span of the whole aperture. Following the same definition, the rim angle of an asymmetric trough is formalized as:

$$
\Phi = \frac{1}{2} \Delta \Phi = \frac{1}{2} (\max \varphi - \min \varphi) = \frac{1}{2} (\Phi_2 - \Phi_1) \\
$$

(4.4)

Note that for a symmetric trough, Eq. (4.4) also holds with $\Phi_1 = -\Phi$ and $\Phi_2 = +\Phi$. The aspect ratio of the semi-parabolic trough with $\Phi_2 \leq 90^\circ$ is:

$$
AR = \frac{f}{2f \tan \left(\frac{1}{2} \Phi_2\right)} = \frac{1}{2} \cot \Phi \\
$$

(4.5)

which is always slightly less (more compact) than a symmetric trough having the same rim angle as seen in **Figure 4.3 (b)**. A further two-fold improvement of the compactness can be easily achieved by considering a two-wing design obtained by placing two semi-parabolic troughs side-by-side, each with a focal
plane tilted toward its own wing. With this configuration, aspect ratios better than 0.5 can be achieved with rim angles smaller than 30°.

For symmetric parabolic troughs, the width of the focal image is minimized when the focal plane is parallel to the aperture plane \(x-z\). For asymmetric parabolic troughs, however, the width of the focal image is minimized when the focal plane is tilted with respect to the aperture plane \([50]\). The derivations for the achievable secondary concentration are in general performed for an un-tilted focal plane, thereby raising the question whether these limits hold when the focal plane is tilted. In Appendix D it is shown for the specific case of the tracking-secondary LTP concentrator that the same limits for secondary concentration apply also for primaries with tilted focal planes, if the tilt angle is chosen as \(\tau = \Phi_{av} = \frac{1}{2}(\Phi_1 + \Phi_2)\) and the formalized rim angle of Eq. (4.4) is used in Eq. (4.53) when calculating the secondary concentration. The same can also be said for fixed- and discrete-switching LTP concentrators. Note that this

Figure 4.3. Mirror profiles for (a) parabolic and (b) semi-parabolic primary concentrators for various rim angles \(\Phi = \frac{1}{2}(\Phi_2 - \Phi_1)\). For a given \(\Phi\), the parabolic and semi-parabolic concentrators have the same secondary concentration limits, but the semi-parabolic trough is more compact. For a two-wing design, the aspect ratios of the semi-parabolic troughs are half the values listed here.
is the tilt angle that maximizes secondary and overall concentration, but not primary concentration. The tilt angle maximizing primary concentration is slightly larger; see Appendix D and [50]. As derived in Appendix D, the primary concentration of an asymmetric parabolic trough with a focal plane tilted at \( \tau = \Phi_{av} = \frac{1}{2}(\Phi_1 + \Phi_2) \) is:

\[
C_{g,i,\text{asym. parab.}}(\tau = \Phi_{av}) = \frac{1}{2} \csc \theta_{i,1} \left[ \tan(\frac{1}{2} \Phi_2) - \tan(\frac{1}{2} \Phi_1) \right] \left( 1 + \cos \Phi_2 \right) \cos \Phi \quad (4.6)
\]

which is slightly lower than that of a symmetric parabolic trough having the same rim angle.

4.1.3. Two-mirror aplanatic trough

The asymmetric parabolic trough, especially when used in a two-wing configuration, allows for improved compactness for relatively small rim-angles. However, for rim angles near that for maximum concentration (\( \Phi \approx 10^\circ \)), these designs are still not compact enough for practical use. To improve compactness, it is necessary to have an additional degree of freedom in the primary design, which can be achieved by use of a two-mirror primary. The two-mirror aplanatic concentrators, discussed in Section 3.5, are essentially capable of reaching the compactness limit of AR = 0.25 can for any rim angle. Aplanatic troughs also have the advantage that the focus can be placed at the vertex of the first mirror of the two-mirror primary. In particular, the two small rim angle designs presented in Section 3.5.2 are of interest for LTP systems. Both designs have \( \Phi = 10.5^\circ \) (which is near the optimum of for tracking-secondary LTP concentrator), are very compact (AR \( \approx 0.25 \)), and achieve a concentration which is 90% of the thermodynamic limit of Eq. (4.10). These designs may be adapted to achieve similar aspect ratios for different rim angles by minor adjustment of the aplanat parameters \( K \) and \( s \). The obvious drawbacks of the two-mirror aplanatic primary are the additional cost due requirement of the second mirror, and the additional optical loss as each ray must suffer two reflections on its way to the focus.
4.1.4. Primary concentration

Knowing the angular size of the solar disk and the skew angle range, the primary acceptance angle required for full-collection, and subsequently the concentration limit, may be determined. As shown in Section 2.4.5, for full-collection, the acceptance angle of a one-axis tracking trough must be at least:

\[
\sin \theta_i = \sin \theta_{\text{sun}} \sec \| \theta \|_{\text{max}}
\]  

(4.7)

With \( \theta_i \) determined from Eq. (4.7), it is straightforward to determine the achievable primary concentration if some assumptions are made about the primary geometry. We consider three common primary trough concentrators: the parabolic trough, the cylindrical trough, and the ideal aplanatic trough. The motivation for considering a cylindrical trough comes from the fact that the highest overall concentrations for tracking-secondary LTP concentrators occur for very small primary rim angles, for which the spherical aberration of cylindrical troughs becomes tolerable. Linear Fresnel concentrators [51] are also well suited as primary concentrators for tracking-secondary LTP systems, but are omitted here since their analysis is more involved than the above three proposed continuous surface primaries.

When the focal plane is centered at the paraxial focus (note this is slightly lower than the convex limit when the focal plane is shifted down slightly from the paraxial focus, see Section 3.4), the geometric concentration of a parabolic trough considering full-collection is [43]:

\[
C_{g,1, \text{parab.}} = \frac{\sin \Phi \cos (\Phi + \theta_i)}{\sin \theta_i} - 1
\]

(4.8)

Analogously, the concentration of a circular trough (determined from the caustics of the edge rays) is:

\[
C_{g,1, \text{circ.}} = \frac{4 \sin \left( \frac{1}{2} \Phi \right)}{3 \sin \left( \frac{1}{2} \Phi \right) - \sin \left( \frac{5}{2} \Phi - 2 \theta_i \right)} - 1
\]

(4.9)

and that of an ideal coma-free (aplanatic) trough is:
Equation (4.10) represents an upper limit for the aplanatic trough as there will be unavoidable shading and/or blocking losses specific to the chosen design. In practice, values around 90% of those computed from Eq. (4.10) may be achieved.

4.2. Fixed secondary concentrators

The fixed-secondary solution shown in Figure 4.2 (a) was investigated in detail at the Fraunhofer ISE, Germany [27, 52], where a 300× system was developed to an on-sun prototype stage. This collector, nicknamed “BICON”, utilizes an array of fixed dielectric crossed (square-aperture) $\theta_i/\theta_o$ secondary concentrators arranged along the focal line, with silicon concentrator cell receivers coupled to the exit of each secondary. The main limitation of a fixed-secondary stage is that it sees the full range of skew angles and thus the secondary concentration is limited to $5n^2$ [27]. The total concentration of a fixed-secondary LTP concentrator is then limited to $425n^2$. These limits are for a polar tracker or, equivalently, a N-S one-axis tracker at the equator. For any other one-axis tracking configuration, the range of skew angles seen by the concentrator is significantly larger, and thus the achievable concentrations significantly lower. For example, the maximum concentration for a fixed-secondary LTP system on a N-S one-axis tracker situated at 30°-latitude is $268n^2$. Since polar trackers are limited in scale and equatorial sites suitable for solar energy collection are limited, the fixed-secondary LTP is limited in its practicality for wide-spread deployment.

In Section 2.4.5, the maximum concentration of a one-axis tracking trough was derived. It was shown that the achievable concentration is less that the limit for a one-axis tracker having no symmetry restrictions, derived in Section 2.4.4. One may envision a design where this symmetry is broken, e.g. by the introduction of a secondary concentrator with a 3D structure, such that the 2D limit no longer applies to the system as a whole. The question then arises as to what concentration is achievable by such a system. The upper limit
Line-to-point focus solar concentrators

for any one-axis tracker is given by Eq. (2.49). In [27] this was taken to be the upper limit of a line-to-point focus system with fixed secondaries. This limit would require that all concentrating stages be ideal. However, in a LTP concentrator, the primary is always taken to be a one-axis tracking trough concentrator, which was shown in Section 2.4.5 to never be able to reach ideality. The question then arises as to what concentration may be achieved by a fixed-secondary LTP collector.

4.2.1. Effective source map for the secondary stage

Consider a generic trough primary concentrator. By parameterizing the primary geometry by the polar angle that on-axis rays make with the $z$-axis at the focus, the following analysis is general for all linearly symmetric geometries, regardless of the number and shape of the mirror surfaces, provided that spherical aberration is small. The goal is to map the direction that on-axis rays, incident at a given skew angle $\vartheta$ and hitting a certain location on the trough $\phi$, make when they reach the focus.

The direction of a ray approaching the concentrator is:

$$\hat{v} = -\hat{s} = [0 \sin \vartheta - \cos \vartheta]$$

Due to the translational symmetry of the trough, the $y$-component of the surface normal is zero. From conservation of linear momentum, more specifically Noether’s theorem [53], the $y$-component of the optical momentum of any ray travelling through the optical system is conserved. The $y$-component of the ray as it reaches the focus is unchanged, and the $x$ and $z$ components are found from the polar angle $\phi$. With the aid of Figure 4.4, the direction of a ray as it approaches the focus is:

$$\hat{r} = [p \quad q \quad r] = [-\cos \vartheta \sin \phi \quad \sin \vartheta \quad \cos \vartheta \cos \phi]$$

Equation (4.12) can be verified for a parabolic trough primary with surface normal $\hat{n}$:

$$\hat{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} -\sin \frac{1}{2} \phi \\ 0 \\ \cos \frac{1}{2} \phi \end{bmatrix}$$

(4.13)
Mapping out Eq. (4.12) in $p-q$ space, we arrive at the effective source map for the secondary stage as shown in Figure 4.5.

**Tilted focal plane**

In designing secondary stages for asymmetric primaries, it becomes useful to allow the inlet plane of the secondary stage to be tilted about the focal line. A new coordinate system $x', y', z'$ is setup by rotating the $x, y, z$ coordinate system clockwise about the $y$-axis by an angle $\tau$. A clockwise coordinate system rotation is equivalent to a counterclockwise vector rotation. The direction of any ray in the primed coordinate system is then:

$$
\hat{r}' = R_y(\tau) \hat{r} = \begin{bmatrix}
\cos \tau & 0 & \sin \tau \\
0 & 1 & 0 \\
-\sin \tau & 0 & \cos \tau
\end{bmatrix} \begin{bmatrix}
-\cos \vartheta \sin \phi \\
0 \\
-\sin \vartheta \cos \phi
\end{bmatrix} (4.15)
$$

where

$$
\hat{r} = 2(\hat{s} \cdot \hat{n})\hat{n} - \hat{s}
= 2(\cos \vartheta \cos \frac{1}{2} \phi)[-\sin \frac{1}{2} \phi \quad 0 \quad \cos \frac{1}{2} \phi] - [0 \quad -\sin \vartheta \quad \cos \vartheta] (4.14)
= [-\cos \vartheta \sin \phi \quad \sin \vartheta \quad \cos \vartheta \cos \phi]
$$

Figure 4.4. The direction of a ray $\hat{r}$ reaching the primary focus is determined by the skew angle $\vartheta$ and the polar angle $\phi$. 

Figure 4.5.
Consider a symmetric parabolic trough with $\Phi = 30^\circ$ and an asymmetric parabolic trough with $\Phi_1 = 0^\circ$ and $\Phi_2 = 60^\circ$. From Eq. (4.15) it is evident that by choosing $\tau = \Phi_{av} = 30^\circ$, the directional mapping of the rays at the focal plane is identical for both the symmetric and asymmetric troughs. Therefore the secondary concentrators may be similarly designed and there is no need to distinguish between secondary concentrators for symmetric and asymmetric designs. For all subsequent designs, it is assumed that the focal plane is tilted by $\tau = \Phi_{av}$ such that the effective source map is always symmetric about the $q$-axis.

4.2.2. Maximum concentration of the secondary stage

In general the skew angle will span from $\vartheta_{\min}$ to $\vartheta_{\max}$ and the polar angle of the primary concentrator will span from $\Phi_1$ to $\Phi_2$. The concentration may be found via Eq. (2.22) if $\Omega_{i,2}$, represented by the area of the hatched region in Figure 4.5 can be calculated. The area of this region may be expressed as:

$$\Omega_{i,2} = \int_{\vartheta_{\min}}^{\vartheta_{\max}} dp dq$$  \hspace{1cm} (4.16)

To evaluate the integral parametrically, we perform a change of variables from $p-q$ to $\vartheta-\phi$:

$$dp dq = |J| d\vartheta d\phi$$  \hspace{1cm} (4.17)

where $J$ is the Jacobian. Using Eq. (4.15) for the source map, the determinant of the Jacobian evaluates to:

$$|J| = \begin{vmatrix} \partial p / \partial \vartheta & \partial p / \partial \phi \\ \partial q / \partial \vartheta & \partial q / \partial \phi \end{vmatrix} = \begin{vmatrix} \sin (\vartheta - \tau) \sin \phi & -\cos (\vartheta - \tau) \cos \phi \\ \cos \phi & 0 \end{vmatrix}$$  \hspace{1cm} (4.18)

and the integral is therefore:

$$\Omega_{i,2} = \int_{\vartheta_{\min}}^{\vartheta_{\max}} \int_{\varphi_1}^{\varphi_2} \cos^2 \vartheta \cos (\vartheta - \tau) d\vartheta d\phi$$

$$= \frac{1}{2} \left[ \sin (\varphi_2 - \tau) - \sin (\varphi_1 - \tau) \right] \left[ \vartheta_{\max} - \vartheta_{\min} + \frac{1}{2} \left( \sin 2\vartheta_{\max} - \sin 2\vartheta_{\min} \right) \right]$$  \hspace{1cm} (4.19)
Assuming that the radiation arriving at the focal plane is spatially isotropic, the maximum achievable geometric concentration at full-collection is

\[ C_{g,\text{max},2} = \frac{\pi n_o^2}{\Omega_{i,2}} = \frac{2\pi n_o^2}{\sin(\Phi_2 - \tau) - \sin(\Phi_1 - \tau)} \left( \sin(\vartheta_{\text{max}}) - \sin(\vartheta_{\text{min}}) \right) + \frac{1}{2} \left( \sin(2\vartheta_{\text{max}}) - \sin(2\vartheta_{\text{min}}) \right) \]  

(4.20)

As an example, consider a symmetric parabolic trough with rim angle \( \Phi_2 = -\Phi_1 = 45^\circ \), and a skew range of \( \vartheta_{\text{min}} = -20^\circ \), \( \vartheta_{\text{max}} = 50^\circ \). The maximum secondary concentration from Eq. (4.20) for such a primary is \( 2.2n_o^2 \).

4.2.3. Designs with only axial secondary concentration

In the previous section, we derived the concentration limit for the secondary stage of a fixed-secondary LTP concentrator. In doing so, we made no consideration as to whether it is actually possible to design a concentrator to meet this limit. There is no general prescription for designing a concentrator to accept a given distribution in \( p-q \) space [28], and therefore we must take a known concentrator geometry and try to match it to the effective source map for the secondary stage. As a starting point, we choose to consider a secondary concentrator which concentrates in the axial plane only. For example, consider an array of 2D CPCs with extrusion axes parallel to the \( x \)-axis, arranged along the focal line of the trough. The acceptance map of an ideal 2D concentrator is easily described, thus simplifying the design procedure.
According to Table 2.2, a N-S one-axis tracker has a smaller skew range than an E-W oriented axis, and thus N-S one-axis trackers are preferred for line-to-point focus systems. In terms of sun-earth geometry, a polar tracker is equivalent to a N-S tracker at the equator, and thus does not require a separate treatment. In general, the skew range for N-S one-axis trackers is asymmetric, and thus leads to an asymmetric source map for the secondary stage c.f. Figure 4.5. To overcome this asymmetry, we consider a secondary concentrator that is tilted about the x-axis by an angle $\sigma$ as shown in Figure 4.6. The acceptance map of an ideal 2D concentrator can be described parametrically by

$$p = \cos t$$

$$q = \sin \theta_i \sin t$$

$$r = -\cos \theta_i \vert \sin t \vert$$

where $p-q$ is parallel to the inlet aperture plane. Rotating about $x$ by an angle $\sigma$: 

\[ Figure 4.6. \] Coordinate system and geometry for a generic secondary concentrator used in a discrete-switching LTP concentrator.
\[ p = \cos t \]  
\[ q = q \cos \sigma - r \sin \tau = \sin \theta_i \sin t \cos \sigma + \cos \theta_i \sin t \sin \sigma \]  

To handle the absolute value, we consider two branches of the acceptance map: one spanning \(0 \leq t < 180^\circ\) and the other \(180 \leq t < 360^\circ\). For the first:

\[ q_1 = \sin \theta_i \sin \sigma \cos \tau + \cos \theta_i \sin \sigma \sin \tau = \sin \left( \theta_i + \sigma \right) \sin t \]  

For the second:

\[ q_2 = \sin \theta_i \sin t \cos \sigma - \cos \theta_i \sin t \sin \sigma = \sin \left( \theta_i - \sigma \right) \sin t \]  

We find that the acceptance map of the tilted concentrator is described by two half-ellipses one with a semi-minor axis \(\sin(\theta_i + \sigma)\), the other with a semi-minor axis of \(\sin(\theta_i - \sigma)\) in the \(q\)-direction, and both with a semi-major axis of 1 (or \(n\) for a dielectric-filled concentrator) in the \(p\)-direction.

Knowing the acceptance map of the tilted concentrator, we may size its acceptance angle to envelope the effective source map as shown in Figure 4.7. From Figure 4.6 we note that \(\theta_i + \sigma = \beta_{\text{max}}\) and \(\theta_i - \sigma = \beta_{\text{max}}\) and it therefore suffices to determine the \(\beta\) values. There are three cases that need to be considered. If the maximum and minimum skew angles are both positive then:

\[ \beta_{\text{min}} = \vartheta_{\text{min}} \]  
\[ \tan \beta_{\text{max}} = \tan \vartheta_{\text{max}} \sec|\varphi - \tau|_{\text{max}} \]  

If the maximum and minimum skew are both negative:

\[ \tan \beta_{\text{min}} = \tan \vartheta_{\text{min}} \sec|\varphi - \tau|_{\text{max}} \]  
\[ \beta_{\text{max}} = \vartheta_{\text{max}} \]  

If the maximum skew is positive and the minimum negative:

\[ \tan \beta_{\text{min}} = \tan \vartheta_{\text{min}} \sec|\varphi - \tau|_{\text{max}} \]  
\[ \tan \beta_{\text{max}} = \tan \vartheta_{\text{max}} \sec|\varphi - \tau|_{\text{max}} \]
For fixed-secondary LTP concentrators, the third case is most relevant since it occurs for all one-axis tracking primaries. This case corresponds to acceptance map shown in Figure 4.7. Nevertheless, the first and second case will prove useful for the design of discrete-switching LTP concentrators, and have thus been presented.

The maximum concentration for a concentrator with an asymmetric acceptance angle range is [54]:

\[
C_{g,2} = \frac{2n}{\sin \beta_{\text{max}} + \sin \beta_{\text{min}}} \tag{4.34}
\]

This can be reconciled with the symmetric case by considering an ideal symmetric 2D concentrator, e.g. a CPC. The CPC is tilted to bisect the angles \( \beta_{\text{min}} \) and \( \beta_{\text{max}} \).

\[
\sigma = \frac{1}{2}(\beta_{\text{max}} - \beta_{\text{min}}) \tag{4.35}
\]

The acceptance angle of this CPC is:

\[
\theta_i = \frac{1}{2}(\beta_{\text{max}} + \beta_{\text{min}}) \tag{4.36}
\]

And the concentration based on the tilted inlet aperture \( a \) is:
The tilted CPC may be extended to the inlet plane by a flat section, without changing the acceptance angle. The concentration based on $a_i$ is then:

$$C_{g,i} = \frac{a}{a_i} = \frac{1}{\sin \theta_i}$$

(4.37)

The tilted CPC may be extended to the inlet plane by a flat section, without changing the acceptance angle. The concentration based on $a_i$ is then:

$$C_{g,i} = \frac{C_{g,\perp}}{\cos \sigma} = \frac{1}{\sin \theta_i \cos \sigma}$$

$$= \frac{1}{\sin \left[ \frac{1}{2} (\beta_{\text{max}} + \beta_{\text{min}}) \right] \cos \left[ \frac{1}{2} (\beta_{\text{max}} - \beta_{\text{min}}) \right]}$$

$$= \frac{2}{\sin \beta_{\text{max}} + \sin \beta_{\text{min}}}$$

(4.38)

The total concentration depends on the type of primary chosen. Results for the best case of an aplanatic primary are shown in Figure 4.8.
4.2.4. Designs with axial and transverse concentration

If the rim angle of the primary concentrator is small, then some degree of transverse re-concentration can be incorporated into the secondary concentrator design. As the resulting concentrator must concentrate in both the axial and transverse directions, it must take on a 3D structure.

The most obvious arrangement is a \(1 \times N\) array of 3D secondary concentrators extending along the axial direction of the trough as was proposed in [27]. Since the concentrators are placed side-by-side, it is necessary that the shape of the inlet aperture can be tessellated in order to avoid large gaps between the inlet apertures of the secondary concentrator array. Secondary concentrators with square inlet apertures, e.g. the crossed CPC [27, 55, 56], are a logical choice.

The introduction of transverse concentration considerably complicates the design procedure for the secondary stage since there the acceptance map is no longer easily defined as for the case of an ideal 2D concentrator. Therefore the design procedure consists of choosing a known concentrator design and sizing it such that its acceptance map envelops the effective source map for the secondary stage.

*Square crossed CPC*

A logical candidate for the secondary concentrator design is the square crossed CPC. The acceptance maps of CPCs with polygonal inlet apertures, including those of square shape, have been investigated systematically in this work, with results detailed in Appendix B. The acceptance maps of crossed CPCs of different acceptance angles are shown in Figure B.5. It is seen that the contours of high-acceptance for CPCs with square inlets are diamond-shaped.

To illustrate the design procedure, we will pursue a design based on a polar tracking symmetric parabolic trough. Noting that the acceptance map of the crossed CPC is bilaterally symmetric, we choose the rim angle of the primary such that the effective source map is (nearly) bilaterally symmetric as well. This is done by imposing that \(p\) and \(q\) are equal for a ray at maximum skew reflected from the rim. Following Eq. (4.14) this condition can be written as
For the polar mount, $\Phi$ evaluates to 25.8°. This choice of rim angle is preferred over the choice of 23.5° used in [27]. Using the acceptance maps of Figure B.5, we design a square-crossed CPC secondary to just envelop the source map as shown in Figure 4.9. The resulting acceptance angle is $\theta_i = 42^\circ$. The total concentration of the system is:

$$C_{g,\text{tot}} = C_{g,1} C_{g,2} = \frac{n^2 \sin \Phi \cos \Phi}{\sin \theta \sin^2 \theta_i}$$

(4.40)

which evaluates to 171× for $n = 1$ and 385× for $n = 1.5$. Interestingly, this is comparable to the case of axial concentration alone shown in Figure 4.8. The reason for this is elucidated by Figure 4.9 which shows that the acceptance map of a square-crossed CPC is particularly poorly suited to the source map of the primary. This point was previously demonstrated in the work of Brunotte, et
al. [27], who instead used (dielectric-filled) $\theta_i/\theta_o$ transformers due to their more amenable acceptance map shape.

One possible alternative method to improve the matching between source and acceptance map shape would be, instead of using a $1 \times N$ array, to use an array of $M \times N$ concentrators each rotated by $45^\circ$ about the $z$-axis, such that the source and acceptance maps are better matched. A conceptual rendering of such a design is shown in Figure 4.10. There will be some losses due to spillage at the jagged edges, but these decrease with increasing $M$. Overall, an improvement of up to 1.5 in the secondary concentration is expected for such an arrangement. Other practical advantages of this arrangement include smaller concentrators, and receivers which may offer benefits for CPV applications in terms of cell efficiency and cooling.

4.3. **Discrete-switching secondary concentrators**

It is evident from Figure 4.8 that fixed designs are only practical in low latitudes ($\lesssim 30^\circ$), due to the large skew range occurring at high latitudes. The switching-secondary LTP concentrator, shown in Figure 4.2 (b), overcomes the limitation of the large skew angle range experienced by a fixed secondary stage by employing a series of non-imaging secondary stages each of which is designed to operate over a subset of the full skew angle range. In this case, the
receivers remain fixed, and the appropriate secondary stage is switched into place depending on the position of the sun. Since the total skew angle range is split between a plurality of secondary stages, the secondary concentration can significantly surpass that of fixed-secondary LTP designs. For dielectric designs, the concentrator must generally be in intimate optical contact with the absorber. Therefore the implementation of discrete switching dielectric secondaries is complex and would likely require some sort of liquid dielectric interface between optic and absorber. Nevertheless, for the purposes of generality, we do not rule out dielectric designs.

4.3.1. Skew splitting and maximum axial concentration

To simplify the designs, and allow for large primary rim angles, we consider only axial concentration. We take, as a starting point, Figure 4.7. The idea of the discrete switching concept is, rather than designing a single secondary concentrator to accept the whole effective source map, to split the effective source map into \( N \) subranges, and design a different secondary concentrator for

![Figure 4.11. Design of a 4-row hollow discrete switching secondary stage at a latitude of 45° N for a primary rim angle of \( \Phi = 45° \). The dashed line is the unit circle; shaded regions are the effective source maps of rays from the primary for each skew subrange; colored lines are the acceptance maps of each ideal 2D stage. This secondary stage achieves an axial concentration of 4.6×.](image)
each subrange. The secondary stage, shown conceptually in Figure 4.2 (b), will then consist of $N$ rows of secondary concentrators running along the length of the trough, where each concentrator in a given row is identical. Depending on the skew angle at a particular time of day, the appropriate secondary concentrator row would be switched into place by a lateral motion.

In order to share the same receiver, the concentration of each row must be equal:

$$C_{g, 2, 1} = C_{g, 2, 2} = C_{g, 2, j} = C_{g, 2, N} = C_{g, 2}$$  \hspace{1cm} (4.41)

We must therefore split up the effective source map of Figure 4.7 into $N$ regions, such that each row, when designed to match its portion of the source map, has equal concentration. This is done by assuming that each row will operate over a certain skew range $\theta_{\text{min}, j}$ to $\theta_{\text{max}, j}$, where $\theta_{\text{min}, 1} = \theta_{\text{min}}$ and $\theta_{\text{max}, N} = \theta_{\text{max}}$ such that the full skew range is accepted. Additionally, we impose that $\theta_{\text{min}, j+1} = \theta_{\text{max}, j}$ such that there are no intermediate skew angles that cannot be accepted. With these constraints, we have $N - 1$ unknown $\theta_{\text{max}, j}$s for the first $N - 1$ row, and also the unknown $C_{g, 2}$ from (4.41) leading to a total of $N$ unknowns. The concentration can for each row can be found from:

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.12.png}
\caption{Maximum achievable secondary axial concentration for a discrete-switching secondary LTP concentrator based on a N-S one-axis tracking primary with a rim angle of 45° as a function of the latitude and number of secondary rows for (a) $n = 1$, and (b) $n = 1.5$.}
\end{figure}
which provides $N$ equations, but introduces $2N$ unknown quantities $\beta_{\text{min},j}$ and $\beta_{\text{min},j}$. Referring to Eqs. (4.28) to (4.33) we can write 2 equations for each row relating $\beta_{\text{min},j}$ and $\beta_{\text{max},j}$ to $\vartheta_{\text{min},j}$ and $\vartheta_{\text{max},j}$, thus closing the system. This formulation comprises a nonlinear system of $3N$ unknowns: $\beta_{\text{min},j}$, $j = 1 \ldots N$; $\beta_{\text{max},j}$, $j = 1 \ldots N$; $\vartheta_{\text{max},j}$, $j = 1 \ldots N - 1$; and $C_{g,2}$, which can be solved numerically.

Figure 4.11 shows the splitting of the source map for an exemplary 4-row solution for a N-S one-axis tracking primary with a rim angle of $45^\circ$ situated at a latitude of $30^\circ$. This particular design achieves a secondary (axial) concentration of $4.6 \times$. Figure 4.12 shows the attainable secondary concentration for discrete-switching LTP concentrators for a $45^\circ$ rim angle primary as a function of the site latitude an number of secondary rows for both reflective ($n = 1$) and dielectric ($n = 1.5$) secondaries.
In the previous section we outlined the procedure for determining the required acceptance angle range $\beta_{\text{min},j}$ and $\beta_{\text{max},j}$ for each row of secondary concentrators in a discrete-switching LTP concentrator, and the resulting secondary axial concentration for the system. Now we consider some practical concentrator designs capable of achieving this concentration.

**Rotated 2D CPC**

The simplest design solution is a rotated CPC akin to that used in Section 4.2.3. Two additional complexities arise from the fact that the locations of the focal plane of the primary and the receiver are fixed in space: (1) the orientation of the exit of the CPC of each unit will be different with respect to the fixed receiver plane and thus an angle rotator must be used to bring the radiation to the receiver; (2) the height of the concentrator in the $z$ direction will be different for each design owing to the different acceptance angle and tilt of each

![3D representation of the secondary concentrating stage shown schematically in Figure 4.13. The scale corresponds to a receiver width of 1 a.u. in the $y$-direction and 10 a.u. in the $x$-direction. The second unit from the left is currently in range with the receiver at its exit plane. Side reflecting walls are omitted for clarity.](image)

**Figure 4.14.** 3D representation of the secondary concentrating stage shown schematically in Figure 4.13. The scale corresponds to a receiver width of 1 a.u. in the $y$-direction and 10 a.u. in the $x$-direction. The second unit from the left is currently in range with the receiver at its exit plane. Side reflecting walls are omitted for clarity.
unit and thus extension or truncation of each unit is necessary. Extension vs. truncation is a tradeoff between loss of concentration in the case of truncation, and increased wall reflections in the case of extension. Here we consider only the case where the concentrators are extended.

**Figure 4.13** shows a schematic of a design based on the solution of **Figure 4.11** using rotated 2D CPC secondaries. A 3D representation of the four rows of the discrete-switching secondary stage is shown in **Figure 4.14**.

**Asymmetric CPC**

The need for all rows to share a common receiver required that angle rotators be used to bring the radiation at the exit of each secondary to the same plane. In order to avoid the use of an angle rotator, and the additional reflection losses that it brings, it is possible to use an asymmetric 2D CPC [43]. Like those based on rotated CPCs, secondary-stages designed with asymmetric CPCs reach the concentration limits discussed in Section 4.3.2. In comparison to the rotated CPC solution, designs based on asymmetric CPCs are slightly longer, but do not require an angle rotator to bring the radiation at the exit of the secondary to
a common plane. Figure 4.15 shows a schematic of a design based on the solution of Figure 4.11 using asymmetric CPCs.

4.4. Continuous-tracking secondary concentrators

While the switching-secondary solution allows for higher concentrations and operation with horizontal axis trackers at high latitudes, practical considerations limit the number of rows of secondary concentrators to around 3–5, and therefore the concentration cannot be increased without bound. The tracking-secondary solution, shown in Figure 4.2 (c), provides the highest possible concentration from a line-to-point focus system by allowing the non-imaging secondary concentrators to individually track the skew angle of the sun on an axis perpendicular to the primary tracking axis. This rotational degree of freedom of the secondary concentrators creates a quasi-2-axis tracking system not constrained by the concentration limits for one-axis trackers. The technical implementation of individual secondary tracking axes may seem difficult, but it turns out not to be so: once a pivot point for each secondary has been established, the tracking for all secondary concentrators may be established by a single actuator. What does turn out to be a challenging task is the design of an efficient, high-concentration secondary concentrator that can track the full range of skew angles without interfering with its neighboring concentrators as they rotate in unison.

Additionally, since in most secondary concentrator designs the position of the receiver relative to the concentrator is fixed, the fact that the secondary tracks implies that the receiver must move or at least pivot. If strictly necessary, it is possible to maintain a fixed receiver by incorporating a nesting angle rotator similar to that proposed for convection-suppressing CPC receivers [43]. Such a device suffers no geometric concentration loss and no ray rejection, but does increase the complexity and reflection losses of the system. Tracking-secondary LTP systems are currently being investigated as low-cost alternatives for high-concentration photovoltaics [57, 58]. They are also of interest for increasing the operating temperature of thermal systems, in particular for trough CSP systems utilizing cavity type absorbers [59]. In general, tracking-secondary LTP collectors may be used in conjunction with any receivers that
are plate- or cavity-like and can be arranged in a discrete array along the length of the trough.

4.4.1. Maximum axial concentration of the secondary stage

Figure 4.16 shows the path traced by on-axis (i.e. originating from the center of the solar disk, parallel to $\hat{S}$) skew rays reflected at different polar angles from a trough primary. Ray $VF$ reflects off the vertex of the primary, ray $AF$ reflects off the rim, and ray $BF$ reflects at point $B$ at some intermediate polar angle $0 < \phi < \Phi$. In the transverse view, all rays are imaged to the paraxial focus $F$. In the axial view, the rays are similarly reflected to a single point $F$ as follows from Fermat’s principle. However, since they reflect off different parts of the primary, the approach angle $\beta$ between the projection of the ray in the axial plane and the optical $z$-axis is a function of $\phi$. It is seen that although the beam arrives at the primary perfectly collimated, the reflected beam has an angular spread of $2\alpha$ in the axial plane. We denote this phenomenon axial beamspread.

Now consider an array of secondary concentrators arranged along the focal line of the primary. Each concentrator is allowed to continuously rotate by an angle $\sigma$ about a rotation axis parallel to the $x$-axis, as shown in Figure 4.16. For describing the secondary geometry, a coordinate frame ($x''$, $y''$, $z''$), rotated by $\sigma$ about $x$-axis along with the secondary, is set up. Figure 4.17 shows a schematic of a generic secondary concentrator in the ($x''$, $y''$, $z''$) coordinate frame. The
secondary will require two different acceptance angles: $\theta_{i,2,ax}$ in the axial plane $y''-z''$ and $\theta_{i,2,trans}$ in the transverse plane $x''-z''$.

Referring to Figure 4.16, for full-collection the axial acceptance angle of the secondary must be at least $\theta_{i,2,ax} = \alpha + \theta_{\text{sun}}$, where $\alpha_{\text{crit}}$ is the largest axial beamspread of the beam reflected by the primary. The transverse acceptance angle of the secondary must be at least $\theta_{i,2,trans} = \Phi + \theta_{\text{sun}}$ in order to accept rays from the rim of the primary at zero skew. In general, $\theta_{i,2,ax}$ will be much smaller than $\theta_{i,2,trans}$ since $\alpha_{\text{crit}}$ is found in the subsequent analysis to be much smaller than $\Phi$. Designing a device to efficiently concentrate in two directions with largely different acceptance angles is challenging, and has no general recipe [28]. Considering that $\theta_{i,2,trans}$ is much larger than $\theta_{i,2,ax}$, the potential for concentration in the transverse direction is comparatively small. We therefore mandate that the secondary concentrate only in the axial direction, resulting in a linear 2D secondary design with $x''$ being the symmetry (extrusion) axis.

This choice of a linear 2D secondary has a threefold benefit: (1) improved manufacturability; (2) the concentrator accepts rays from any transverse angle
(\theta_{i,2,\text{trans}} = 90^\circ); and (3) the limits of concentration can be determined in a straightforward manner. The obvious downside is that we forego the possibility for secondary concentration in the transverse direction. To some extent, it may be possible to incorporate some element of re-concentration in the $x''$-$z''$ plane through the use of a crossed secondary concentrator design [43] or a freeform 3D design, and achieve concentrations higher than the limits derived here. However this is best done through optimization for a particular system, which is beyond the scope of this treatment.

In determining $\theta_{i,2}$ (as only the axial secondary acceptance angle is of interest, the ax subscript has been dropped), we strive to find the choice of secondary rotation angle $\sigma$ such that $\alpha_{\text{crit}}$ is minimized. Referring to Figure 4.16, the direction of an incident on-axis skew ray in the coordinate system of the tracking aperture is:

\[
\hat{v} = -\hat{s} = [0 \ -\sin \vartheta \ \cos \vartheta]
\]

Due to the linear symmetry along the $y$-axis, the $y$-component of the ray direction (i.e. optical momentum) is conserved. Referring to Figure 4.4, the direction of the ray as it approaches the focus is therefore:

\[
\hat{r} = [p \ q \ r] = [-\cos \vartheta \sin \phi, \ \sin \vartheta, \ \cos \vartheta \cos \phi]
\]

where $p$, $q$, $r$ are the direction-cosines of the ray. The approach angle $\beta$ is simply found from:

\[
\tan \beta = \frac{M}{N} = \tan \vartheta \sec \phi
\]

For any given skew angle, the minimum and maximum (in terms of absolute value) $\beta$ occur for rays reflected from the vertex ($\phi = 0$) and the rim ($\phi = \Phi$) respectively, yielding:

\[
\beta_{\text{min}} = \vartheta
\]

\[
\beta_{\text{max}} = \arctan (\tan \vartheta \sec \Phi)
\]
An intuitive choice for the secondary rotation angle $\sigma$ is the one causing the optical $z''$-axis to bisect $\beta_{\text{min}}$ and $\beta_{\text{max}}$:

$$\sigma = \frac{1}{2}(\beta_{\text{max}} + \beta_{\text{min}}) = \frac{1}{2} \left[ \arctan(\tan \vartheta \sec \Phi) + \vartheta \right]$$  \hspace{1cm} (4.48)

In Section 4.4.3, this choice of rotation angle is shown to be the correct choice to minimize the required acceptance angle of the concentrator. The axial beamspread is:

$$\alpha = \frac{1}{2}(\beta_{\text{max}} - \beta_{\text{min}}) = \frac{1}{2} \left[ \arctan(\tan \vartheta \sec \Phi) - \vartheta \right] = \sigma - \vartheta$$  \hspace{1cm} (4.49)

which is zero for normal incidence ($\vartheta = 0$), and nonzero for $\vartheta \neq 0$. Setting the derivative of Eq. (4.49) with respect to $\vartheta$ to 0 and solving for $\vartheta$ yields the skew angle for which axial beamspread reaches a maximum:

$$\tan \vartheta_{\text{crit}} = \left( \cos \Phi \right)^{1/2} = \cos^{1/2} \Phi$$ \hspace{1cm} (4.50)

Subbing Eq. (4.50) into Eq. (4.49), the maximum axial beamspread is:

$$\alpha_{\text{crit}} = \frac{1}{2} \left[ \arctan \left( \frac{1}{\cos^{1/2} \Phi} \right) - \arctan \left( \cos^{1/2} \Phi \right) \right]$$ \hspace{1cm} (4.51)

$$= 45^\circ - \vartheta_{\text{crit}} = 45^\circ - \arctan \left( \cos^{1/2} \Phi \right)$$

For full-collection, the acceptance angle of the secondary concentrator in the axial direction must therefore be:

$$\theta_{1,2} = \alpha_{\text{crit}} + \theta_{\text{sun}}$$  \hspace{1cm} (4.52)

where the angular size of the sun has now been added to the acceptance angle to account for the fact that the above derivations were performed assuming a perfectly collimated beam from the sun.

Figure 4.18 shows the axial beamspread as a function of the skew angle as calculated from Eq. (4.49). Interestingly, the largest axial beamspread occurs not necessarily at the maximum skew angle, but at some intermediate critical skew angle $\vartheta_{\text{crit}}$, lying along the dashed line of Figure 4.18. As the achievable
secondary concentration depends on the largest axial beamspread, the critical skew defines a design point for which the secondary acceptance angle must be sized.

As with the primary acceptance angle, the acceptance angle of the secondary may be derived by the more rigorous source/acceptance map matching method. The derivations, shown in Section 4.4.3, recover Eq. (4.51) for hollow secondaries \((n = 1)\), but produce a slightly different result for \(\sigma\) and \(\alpha\) for dielectric-filled secondaries, with \(\alpha\) being slightly smaller when derived using the source/acceptance matching method. However, since the difference is small, Eq. (4.51) is deemed suitable as a general solution for any \(n\), with the resulting acceptance angle being slightly oversized (conservative) for \(n > 1\).

If we consider that an ideal 2D non-imaging concentrator is used for the secondary, then the maximum achievable secondary concentration in the axial direction is:

\[\text{Figure 4.18.} \text{ Axial beamspread } \alpha \text{ as a function of the skew angle } \vartheta \text{ for different primary rim angles } \Phi, \text{ computed from Eq. (4.49). The curves are symmetric about the abscissa. The dashed curve shows the locus of critical skew } \vartheta_{\text{crit}} \text{ and critical beamspread } \alpha_{\text{crit}} \text{ from Eqs. (4.50) and (4.51).} \]
Interestingly, the secondary axial concentration does not depend on the latitude, as long as $\theta_{\text{crit}}$ determined from Eq. (4.50) is less than $\theta_{\text{max}}$ determined from Table 2.2.

### 4.4.2. Overall concentration

For any primary or secondary design, the overall concentration is:

$$C_{g,\text{tot}} = C_{g,1}C_{g,2,\text{ax}}$$

(4.54)

where Eqs. (4.8) to (4.10) are used for $C_{g,1}$ depending on the primary type, and Eq. (4.53) is used for $C_{g,2}$. **Figure 4.19** shows the resulting overall concentration as a function of the primary rim angle for a system based on a N-S one-axis primary at a latitude of $30^\circ$. In general, small rim angles are
preferred for maximum concentration. This is due to the strong effect that the rim angle has on the axial beamspread as evidenced by Figure 4.18. The rim angle for maximum overall concentration, $\Phi_{\text{opt}}$, may be found by setting the derivative of Eq. (4.54) with respect to $\Phi$ to 0, and solving for $\Phi$. For the ideal aplanatic primary, $\Phi_{\text{opt}} = 10.8^\circ$, independent of the latitude. For the parabolic and circular primary, $\Phi_{\text{opt}}$ depends slightly on latitude. For the parabolic primary, $\Phi_{\text{opt}} \approx 10.4^\circ$. For the circular primary, $\Phi_{\text{opt}}$ ranges from $7.9^\circ$ for $\phi = 0^\circ$ to $10.4^\circ$ at $\phi = 60^\circ$. It should be noted that for low rim angles, the secondary concentration is higher than the primary concentration. This has a consequence on the receiver shape as discussed in 4.4.4.

Figure 4.20 shows the overall geometric concentration of a tracking-secondary LTP collector of optimal primary rim angle as a function of latitude. The maximum values, all occurring at $\phi = 0^\circ$ are 4065$, 3990$, and 3352$ for an ideal aplanatic, parabolic, and cylindrical primary, respectively, all for an ideal hollow secondary concentrator. For a dielectric-filled secondary concentrator, the concentration limits are $n$ times higher than those shown in
Figure 4.19 and Figure 4.20, where \( n \) is the refractive index of the dielectric material in which the receiver is immersed.

4.4.3. Source/acceptance matching for the secondary

In the previous section the secondary acceptance angle was chosen based on the largest axial beamspread. For this, it was necessary to make an \textit{a priori} choice of the secondary tracking angle, i.e. we specified via Eq. (4.48) that the secondary concentrators track with an angle \( \sigma \) such that their aperture normal bisects the approach angle range. The secondary tracking and acceptance angles may be alternatively found by source/acceptance map matching, from which the optimal \( \sigma \) follows automatically. The results are found to be identical to those in the previous section for the case of hollow secondaries. For the case of dielectric-filled secondaries, however, the source/acceptance matching method allows a slightly higher concentration to be obtained. Therefore the limits for dielectric-filled secondaries are slightly higher than those previously presented.

The effective source map at the inlet of the secondaries is obtained by mapping out the directions of all rays reaching the secondaries over the full range of skew angles. The directions of all on-axis rays reaching secondary inlet aperture in the frame tracking secondary concentrators are:

\[
\begin{bmatrix}
p'' \\
q'' \\
r''
\end{bmatrix} = R_x(\sigma) \hat{r} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \sigma & -\sin \sigma \\
0 & \sin \sigma & \cos \sigma
\end{bmatrix}
\begin{bmatrix}
-\cos \vartheta \sin \varphi \\
\sin \vartheta \cos \sigma - \cos \vartheta \cos \varphi \sin \sigma \\
\sin \vartheta \sin \sigma + \cos \vartheta \cos \varphi \cos \sigma
\end{bmatrix} = \begin{bmatrix}
-\cos \vartheta \sin \varphi \\
\sin \vartheta \cos \sigma - \cos \vartheta \cos \varphi \sin \sigma \\
\sin \vartheta \sin \sigma + \cos \vartheta \cos \varphi \cos \sigma
\end{bmatrix}
\]

(4.55)

where \( \sigma \) is some function of \( \vartheta \), e.g. Eq. (4.48). Plotting \( p'' \) and \( q'' \) for all \( \vartheta \) and \( \varphi \) gives the effective source at the secondary inlet aperture map (if the angular size of the sun were considered, the effective source map would be slightly larger by an amount \( \sim \sin \theta_{\text{sun}} \) in the \( q'' \) direction; this is reconciled simply by increasing the resulting acceptance angle by \( \theta_{\text{sun}} \)). Figure 4.21 shows the resulting map for a primary with rim angle \( \Phi = 60^\circ \) for \( \vartheta \) ranging from \( 0^\circ \) to \( 90^\circ \), where Eq. (4.48) has been used to determine \( \sigma(\vartheta) \). The acceptance map of
an ideal 2D hollow \((n = 1)\) and dielectric-filled \((n = 1.5)\) secondary concentrator with an acceptance angle of \(\theta_i\) in the \(q''\)-direction are also shown.

The critical skew line of the effective source map (Eq. (4.55) evaluated at \(\vartheta = \vartheta_{\text{crit}}\)) is seen to intersect both the top and bottom branch of the acceptance map for \(n = 1\), implying that this is the smallest possible secondary acceptance angle for full-collection. However, for the case of \(n = 1.5\) it can be seen that the acceptance map is slightly oversized in that the upper branch of the effective source map does not intersect the acceptance map. This implies that by slightly changing \(\sigma\) for dielectric filled secondary concentrators, the required \(\theta_i\) can be slightly reduced compared to that calculated based on Eq. (4.51). To exploit this, we determine the tracking angle as a function of skew for which the acceptance function of an ideal 2D dielectric concentrator just envelopes the effective source map with minimal acceptance angle. This procedure is valid for all \(n\), and so may be considered a rigorous solution of the required secondary acceptance angle also for hollow secondary concentrators.

From Eq. (4.55), the direction of rays arriving at the inlet of the secondary are:

\[
\begin{align*}
p'' &= -\cos \vartheta \sin \varphi \\
q'' &= \sin \vartheta \cos \sigma + \cos \vartheta \cos \varphi \sin \sigma
\end{align*}
\]  

(4.56)

where we will now solve for \(\sigma\) by source/acceptance map matching, rather than prescribing Eq. (4.48). An ideal 2D (dielectric-filled) concentrator has an elliptical acceptance map described by:

\[
\frac{p^2}{n^2} + \frac{q^2}{\sin^2 \alpha} = 1
\]  

(4.57)

It is sufficient to consider the extreme rays from the rim and from the vertex of the primary. For the vertex ray, Eq. (4.57) with \(\varphi = 0\) yields:

\[
\alpha = \sigma - \vartheta
\]  

(4.58)

and for the rim ray, Eq. (4.57) with \(\varphi = \Phi\) yields:
Eqs. (4.58) and (4.59) may then be solved to determine $\sigma$ and $\alpha$ as a function of $\vartheta$. The secondary is then designed for the critical skew giving the largest $\alpha$ (the resulting critical skew is slightly different than from Eq. (4.50) for $n \neq 1$), with the acceptance angle then taken as $\theta_{i,2} = \alpha + \theta_{\text{sun}}$. Although an analytical solution of Eqs. (4.58) and (4.59) does exits, it is too complex to be of practical use. For $n = 1$ the solution simplifies to that of Eq. (4.51) confirming that the solution presented in the previous section is correct for hollow secondaries. For dielectric-filled secondaries, the previously presented concentration limits are slightly conservative.
4.4.4. Secondary concentrator design considerations

The requirements of the tracking-secondary LTP collector pose a unique nonimaging design problem for the secondary concentrator design. In general, the secondaries must: (1) have an inlet aperture shape that can be tessellated along the focal line of the primary; (2) be compact such that individual concentrators may rotate relative to each other when placed adjacent to one another along the focal line; (3) perform well for rays outside the meridian (cross-sectional) plane of the secondary, i.e. arriving from the rim of the primary; (4) have a clear (unobstructed) inlet aperture to avoid shading losses; and (5) have a small average number of reflections to minimize absorbed power.

These criteria place restrictions on potential secondary geometry and immediately preclude certain common nonimaging designs. From criterion 1, the inlet aperture shape must essentially be rectangular. The tessellation requirement also rules out designs having virtual inlet apertures, e.g. the trumpet concentrator. From criterion 2, traditional cone type edge-ray concentrators (e.g. the CPC) cannot be used without substantial truncation. From criterion 3, designs with curved refractive surfaces pose difficulties, since such designs generally have poor performance for rays arriving from the rim. From criterion 4, use of multiple stage secondaries (e.g. SMS XX and RXI designs [23]) present difficulties due to large shading losses. Criterion 5 places limitations on reflective designs, especially compact small acceptance angle designs such as stepped flowline optics, unless total internal reflection (TIR) can be achieved.

First some general geometric considerations are presented. Secondary concentrator designs where the receiver is immersed in air, i.e. hollow designs, are discussed in Section 4.4.5. Designs where the receiver is immersed in a dielectric medium, i.e. dielectric-filled designs, are discussed in Section 4.4.6.

**Receiver shape**

The ratio of secondary to primary concentration has an important effect on the shape of the secondary concentrator. We define $a_i$ and $a_o$ as the inlet and outlet aperture widths in the direction of concentration, and $w$ the width in the
extrusion direction as depicted in Figure 4.22. For full-collection, the secondary extrusion width $w_2$ is set equal to the width of the focal image of the primary, $a_{o,1}$:

$$w_2 = a_{o,1} = a_{i,1}/C_{g,1}$$  \hspace{1cm} \text{(4.60)}$$

We are now free to choose either the inlet aperture width $a_{i,2}$ or the outlet aperture width $a_{o,2}$ of the secondary concentrator, the two being linked by the secondary concentration:

$$a_{i,2}/a_{o,2} = C_{g,2}$$  \hspace{1cm} \text{(4.61)}$$

which is fixed by the acceptance angle. In some cases, the receiver aspect ratio $w_2/a_{o,2}$ will be a driving factor in the design. It is found from:

$$\frac{w_2}{a_{o,2}} = \frac{a_{i,1}/C_{g,1}}{a_{i,2}/C_{g,2}} = \frac{a_{i,1}}{a_{i,2}} \cdot \frac{C_{g,2}}{C_{g,1}}$$  \hspace{1cm} \text{(4.62)}$$

**Figure 4.22.** Dimensioning of the primary and secondary concentrators for a tracking-secondary LTP concentrator. The secondary inlet aperture $a_{i,2}$ may be chosen freely, but generally must be much smaller than the primary inlet aperture $a_{i,1}$, such that the secondaries are at a feasible scale. With $a_{i,2}$ specified, the receiver shape is defined by Eq. (4.62). For most cases, the resulting receiver shape will be rectangular. Schematic is not to scale.
Some interesting remarks about the receiver aspect ratio can be made by inspecting Eq. (4.62). For the secondary concentrators to be of a feasible scale it is required that \( a_{i,2} \ll a_{i,1} \). Since, for most configurations, the secondary concentration will be on the order of the primary concentration (compare Eqs. (4.10) to (4.9) to Eq. (4.53)), Eq. (4.62) suggests that the resulting receiver aspect ratio will be greater than 1, implying a rectangular receiver. If it is strictly necessary to have a square receiver, then a large rim angle primary could be chosen such that \( C_{g,1} > C_{g,2} \). Alternatively an étendue squeezer or light shifter [60] could be at the exit of the secondary concentrator to reshape the concentrated beam.

**Tracking range**

The required tracking range for the secondary concentrators may be calculated from Eq. (4.48) considering the minimum and maximum skew angles in Table 2.2. Figure 4.23 shows the required secondary concentrator tracking range for full-collection as a function of latitude for the rim angle which optimizes maximum total concentration for a parabolic primary (\( \Phi = 10.5^\circ \)).
Through Eq. (4.48), the tracking range is seen to weakly increase with increasing rim angle.

*Interference envelope - plane inlet*

The secondary concentrator profile must be constructed such that there is no mechanical interference with the adjacent concentrators as they rotate in unison. The secondaries are arranged along the focal line with no gaps in between their inlet apertures. Each axis is placed at a fixed location relative to its concentrator implying that the tracking axes are separated by a distance $a_{i,2}$. By means of coordinate frame transformations it is relatively straightforward to show that to an observer sitting on one rotating concentrator, the adjacent concentrator appears to translate around the observer in a circular path with radius $a_{i,2}$ as the two concentrators rotate in unison. Since they rotate in unison, the inlet apertures of the secondaries always remain parallel to one another. It can further be shown that this relative motion is independent of the location of the tracking axis relative to each concentrator.
Figure 4.24 shows two rotating secondary concentrators placed beside each other. As the concentrators rotate, the pencil traces the path of the left edge of the inlet aperture of the concentrator on the right. In order that the concentrator on the left does not interfere with the one on the right, it must fit inside the path traced by this pencil. A similar argument may be made for rotation in the other direction. By plotting the path traced by the edges of the inlet apertures of adjacent concentrators one obtains an envelope into which the profile of each concentrator must fit to guarantee no interference during rotation. We denote this region the “interference envelope”. As seen in Figure 4.24, its shape is simply composed of two circular arcs of radius $a_{i,2}$ centered on each edge of the inlet aperture of the secondary concentrator. Interestingly, if the concentrator can track to 60° without interference, then it can also track to 90° without interference.

Interference envelope - convex inlet

In some secondary concentrator designs, e.g. the dielectric tailored edge-ray concentrator (DTERC), the concentrator features a convex inlet protruding beyond the plane inlet aperture. For such a convex inlet, more strict limits are placed on the concentrator profile. For simplicity, it is considered that the curved inlet is a circular arc subtending an arc angle $\Theta$ as shown in Figure 4.25. The shape of the interference envelope may be determined in a manner similar to the hollow case by considering the path traced by the adjacent concentrators during their relative circular orbit. The difference is that now the path traced by the whole curved inlet, rather than just the edges of the inlet aperture, must be considered. The resulting interference envelope, shown in Figure 4.25, is described by the following parametric equations:

$$
\begin{align*}
    x_1'' &= a_{i,2} \cos \xi - \frac{1}{2}a_{i,2} \\
    z_1'' &= a_{i,2} \sin \xi \\
    x_2'' &= (a_{i,2} - R) \cos \xi \\
    z_2'' &= (a_{i,2} - R) \sin \xi + d
\end{align*}
$$

$$
\begin{align*}
    0 &\leq \xi \leq \arccos \left[ \sin \left( \frac{\Theta}{2} \right) \right] \\
    \arccos \left[ \sin \left( \frac{\Theta}{2} \right) \right] &\leq \xi \leq \sigma_{\text{max}}
\end{align*}
$$

(4.63)  

(4.64)
where $R$ is the radius of the convex inlet:

$$ R = \frac{1}{2} a_{i,2} \csc \left( \frac{1}{2} \Theta \right) \quad (4.66) $$

and $d$ is the distance between the center of the circle and the inlet aperture plane:

$$ d = \frac{1}{2} a_{i,2} \cos \left( \frac{1}{2} \Theta \right) \quad (4.67) $$

Unlike for the case of the flat inlet, for $\sigma_{\text{max}} > 60^\circ$, the envelope begins to self-intersect placing stricter limits on the concentrator shape. In the limiting case of $\Theta = 180^\circ$ and $\sigma_{\text{max}} = 90^\circ$ the interference envelope is a hemisphere of radius $\frac{1}{2}a_{i,2}$. 

Figure 4.25. The interference envelope for a concentrator with a convex cylindrical inlet is constructed from three circular arcs described by Eqs. (4.63) – (4.65), and poses more strict limits on the concentrator shape than does the case of a flat inlet shown in Figure 4.24. The right pane shows the formation of the envelope for a concentrator with a curved inlet of arc angle $\Theta = 150^\circ$ for $\sigma_{\text{max}} = 60^\circ$. As with the flat inlet, the position of the tracking axis does not affect the shape of the interference envelope.
Defocus and confinement walls

If the concentrators are arranged along the focal line such that for $\sigma = 0$ their inlet apertures are coplanar with the focal plane of the primary, then for full collection, $w_2 = a_{o,1}$. As the secondary concentrators track, however, their inlet apertures will be shifted out of primary focus thus causing some of the rays from the primary to be spilled past the inlet of the secondaries. This effect may be minimized by making $a_{i,2}$ sufficiently small and placing the tracking axis at the center of the inlet aperture, but still the width $w_2$ of the secondaries would need to be oversized in order to collect all rays from the primary. A preferred solution, which does not reduce concentration or place restriction on positioning of the tracking axis or the choice of $a_{i,2}$ is to enclose the secondary concentrators between two reflective parallel confinement walls extending in the $z$-direction, or $z'$-direction in the case of a tilted focal plane (see for example Figure 4.26). These walls serve to restrict the extent of the rays in the transverse direction, thus preserving the primary concentration. Due to the linear symmetry, such walls have no effect on the beamspread from the primary, and therefore have no effect on the secondary concentrator design. This solution is adopted for all designs discussed herein.

Implications for secondary concentrator design

The requirements of a tracking secondary, in particular the interference envelopes, place very strict restrictions on potential secondary concentrator geometries. To fit inside the interference envelopes, the secondary concentrators must be very compact, where compactness is defined the overall length of the concentrator divided by the inlet aperture width, with smaller values indicating more compact devices. It is evident from Figure 4.24 that it is not possible to fit an untruncated CPC of small acceptance angle within the interference envelope. For the CPC, the restrictions can be relaxed by asymmetric truncation. While the interference envelope is more restrictive for concentrators having curved inlets (Figure 4.25), in general such concentrators can be made more compact. The shape of the interference envelope is particularly well matched to the profile of dielectric tailored edge ray concentrators (DTERCs).
4.4.5. Hollow designs

**Asymmetrically truncated 2D CPC**

One of the simplest ideal concentrators is the 2D CPC [23]. It is not possible to fit a 2D CPC of small acceptance angle within the interference envelope of Figure 4.24. However, by asymmetrically truncating the sides of a 2D CPC [61], it can be made to track the required range without interference at the expense of concentration lost due to truncation. As an example, consider the design of a secondary concentrator with acceptance angle $\theta_i = 6^\circ$ which must track from $\sigma_{\text{min}} = -30^\circ$ to $\sigma_{\text{max}} = 60^\circ$ without interference.

**Figure 4.26** shows an asymmetrically truncated CPC meeting these design requirements. The CPCs are arranged along the focal line such that for $\sigma = 0$, the right edge of each CPC is aligned with the left edge of the adjacent CPC. This arrangement leads to a small gap between the short branch and the long branch of the adjacent concentrator, but ray-tracing of the designs (Section 4.4.8) confirms that leakage through this gap is small. The rotation axis is placed at the end of the long branch. This choice ensures that: (1) the secondaries do not protrude past the inlet of the confinement walls as they rotate; and (2) the secondary inlets remain as close as possible to the primary focal plane as they rotate in order to minimize reflection losses from the confinement walls. The design procedure proceeds as follows. The truncation of the long branch is chosen first and determines the overall compactness of the design. For the example shown in **Figure 4.26**, the long branch has been truncated to half of the length of a full CPC with $\theta_i = 6^\circ$. The truncation of the short branch is then chosen such that the concentrators can track to $\sigma_{\text{max}}$ when rotated in the clockwise direction. In the anti-clockwise direction, the concentrator has a smaller tracking range due to the asymmetry. Since $|\sigma_{\text{min}}| < |\sigma_{\text{max}}|$ for all N-S trackers (see **Figure 4.23** and Table 2.2), the tracking range in the anti-clockwise direction is nevertheless sufficient to reach $\sigma_{\text{min}}$, except for extreme cases at low latitudes. The resulting geometric concentration for this design example is $C_{g,2,ax} = a/o = 7\times$ which is 25% lower than the thermodynamic maximum $1/\sin\theta_i = 9.6\times$ due to the truncation.
Figure 4.26. A secondary stage based on asymmetrically truncated CPCs. The geometric concentration of this example design is $C_{g,2,ax} = \frac{a_i}{a_o} = 7$, which is 25% lower than the thermodynamic limit $1/\sin \theta_i$ due to the truncation.

The CPCs are arranged along the focal line such that for $\sigma = 0$, the right edge of each concentrator is aligned with the left edge of the adjacent concentrator. The geometric concentration of this example is $C_{g,2,ax} = \frac{a_i}{a_o} = 7$, which is 25% lower than the thermodynamic limit $1/\sin \theta_i$ due to the truncation.
For maximum concentration for a given tracking range, the long branch should be left untruncated (at the expense of compactness). Figure 4.27 shows the achievable concentration of an asymmetrically truncated CPC as a function of the required tracking range in the clockwise direction for different acceptance angles. For small acceptance angles, the loss of concentration due to truncation can be severe. The asymmetrically truncated CPC is therefore recommended for designs having larger primary rim angles, where the secondary acceptance angle is larger.

**Other non-immersed secondary designs**

Other options for non-immersed secondary concentrators include 2D SMS RR [45] and XX [22, 23] optics. The RR optics suffer from poor performance due to the fact that the focal length of a lens changes for non-meridian rays (rays arriving from the rim of the primary). The XX optics suffer from severe

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1 Material from this section has been extracted from F. Dähler, “Application of the simultaneous multiple surface method to solar concentrator design,” Master Thesis, ETH Zurich, 2013, conducted under the direct supervision of T. Cooper.
shading losses from the second mirror (for 2D geometry). These problems may be at least partially resolved by asymmetric 2D SMS or 3D SMS [46] designs.

**Figure 4.28** shows a rendering of a secondary concentrator based on an asymmetric XX 2D SMS optic. To augment the concentration, the optic is coupled to a terminal array of dielectric \((n = 1.5)\) CPCs. This design has \(\theta_{i,2,ax} = 2.33^\circ\), can track to \(+50^\circ\), and achieves a \(C_{g,2,ax} = 42\times\) with less than 1% ray-rejection. Strictly-speaking, the design should be classified as immersed due to the introduction of the DPC tertiary optics.

The concept of using linear Fresnel lens secondaries has also been investigated. Since the focal length of a lens changes for rays out of the cross-sectional plane (non-meridian rays), a flat Fresnel lens placed at the primary focus would direct rays arriving from the rim and the vertex of the primary to...
different points. Although nonimaging Fresnel lenses can be designed with large acceptance angles out of the meridian plane, e.g. the $2^\circ/12^\circ$ lens of Leutz and Suzuki [62], such designs have relatively low optical efficiency for use as secondary concentrators, and numerical optimization is required. This solution is therefore not treated here. Another option to mitigate the poor non-meridian behavior is to curve an (imaging) 2D Fresnel lens in a circular arc and place it before the primary focus such that on-axis rays arrive normal to the curved lens. The theory derived in Section 4.4.2 applies only for secondaries placed at the primary focus and thus is not applicable for the curved Fresnel lens working before the primary focus. For this reason, the curved linear Fresnel lens solution is also not considered here.

4.4.6. Dielectric-filled (immersed) designs

While the challenges of constructing a compact, small-acceptance angle concentrator limit the potential designs for hollow secondary concentrators, some additional design possibilities are introduced for dielectric-filled concentrators. Furthermore, the concentration limits are augmented by a factor $n$ compared to those for hollow secondary designs. An additional advantage is that if rays strike the reflective surfaces shallower than the critical angle, they undergo total internal reflections (TIR) without loss. When using a dielectric-filled secondary, the receiver is immersed in the secondary concentrator medium. Therefore, such secondary concentrators are only practical for applications where the receiver can be placed in intimate optical contact with the exit of the secondary, e.g. CPV. The designs are carried out assuming a fixed refractive index $n$. Chromatic aberration resulting from dispersion is neglected.

*Multi-foliate light-pipe concentrator (LPC)*

The multi-foliate light-pipe concentrator (LPC) of Bassett and Forbes [63] is a semi-circular shaped concentrator which fits inside the interference envelope for any choice of design parameters. The LPC achieves compactness by channeling the light through slender light pipes or leaves. For sufficiently slender leaves, the designs approach ideality. Unfortunately, due to the small acceptance angles, a very large number of leaves must be used to maintain
good performance (see Appendix E). For very slender leaves, rays suffer many reflections on their way to the outlet. Fortunately for small acceptance LPCs, the angle of incidence between the rays and the walls of the leaves is well below the critical angle (for \( n \approx 1.5 \)), and thus rays undergo TIR. Since LPCs can track to any angle, they are best suited for high latitudes and large rim angles where large tracking ranges are experienced. Larger rim angles also imply larger secondary acceptance angles which are preferable for LCPs because of the smaller number of leaves required for good performance.

**Figure 4.29.** A secondary stage based on multi-foliate light-pipe concentrators (LPCs). The LPC always fits within the interference envelope and can therefore track from \( \sigma = -90^\circ \) to \(+90^\circ\) with no interference. It essentially achieves the concentration limits derived in Section 4.4.2.

**Figure 4.29** shows a secondary stage based on LPCs. The tracking axis is placed in the center of the inlet aperture and set back from the primary focal plane by a distance \( \frac{1}{2}a_{i,2}\sin\sigma_{\text{max}} \) such that the edge of the inlet aperture is coincident with the focal plane at the maximum skew angle. Since having an infinite number of leaves is impossible, the LPC must be designed for a finite number of leaves at the expense of having some rays within the acceptance angle being rejected. In Appendix E, a design procedure for choosing the required number of leaves in order to approximately meet a specified tolerable fraction of ray-rejection is presented. If it is strictly desired to have zero ray-rejection, then it may be eliminated by appropriate oversizing of the acceptance
angle. For tracking-secondary LTP collectors with LPC secondaries the limits of Figure 4.19 and Figure 4.20 can essentially be achieved.

Dielectric tailored edge-ray concentrator (DTERC)

The dielectric-filled tailored edge ray concentrator (DTERC) [64] features a curved inlet which allows it to be made substantially more compact than edge-ray designs having flat inlets, e.g. the DCPC [30], such that it can be made to fit in the interference envelope of Figure 4.25. The geometry of a secondary stage based on DTERC concentrator is shown in Figure 4.30. It is comprised of an aspheric refractive surface at the inlet and a reflective wall whose shape is determined by constant optical pathlength tailoring. The very similar, but earlier proposed, dielectric totally internally reflecting concentrator (DTIRC) [65] is not considered since it is generally less compact that the DTERC due to the constraint of having a spherical inlet.

The DTERC design procedure, outlined by Friedman and Gordon [64], is fixed by the refractive index $n$, the acceptance angle $\theta_{i,2}$, the inlet or outlet aperture $a_{i,2}$ or $a_{o,2}$, and the distance between the inlet and outlet apertures $l_{DTERC}$. For DTERCs designed with $n = 1.5$, TIR is guaranteed at all points on the side profile for $\theta_{i,2} < 9.6^\circ$. We make a slight modification to the design procedure, by specifying – instead of $l_{DTERC}$ – the slope angle at the edge of the elliptic lens $\Psi$, defined in Figure 4.31. The length may then be found from:

$$l_{DTERC} = (a_i + a_o) \tan \left( \arccos \left[ \frac{\sin (\theta_i - \chi)}{n} \right] - \Psi \right) \quad (4.68)$$

such that the Friedman and Gordon [64] procedure can be subsequently followed. The reasons for using the slope angle instead of the length as a design parameter are twofold: (1) Fresnel reflection losses generally increase with increasing incidence angle on a dielectric interface, and therefore $\Psi$ gives some indication of the relative severity of Fresnel losses between designs; and, more importantly (2) the maximum slope angle dictates the performance of the concentrator for non-meridian rays, and can be used to construct the acceptance map of the concentrator as shown in Appendix F.
Figure 4.30. A secondary stage based on dielectric tailored edge-ray concentrators (DTERCs). The design shown has an acceptance angle of $\theta_i = 5^\circ$ and can track to $\pm 50^\circ$ without interference. The maximum slope angle for this design is $\Psi = 62^\circ$ cf. Figure 4.31.
The compactness, and thus the tracking range, increases as the curvature of inlet of the DTERC, i.e. $\Psi$, is increased. Figure 4.31 shows the slope angle required in order to achieve a specified maximum tracking angle $\sigma_{\text{max}}$ for different acceptance angles is shown.

The compactness, and thus the tracking range, increases as the curvature of inlet of the DTERC, i.e. $\Psi$, is increased. Figure 4.31 shows the slope angle required in order to achieve a specified maximum tracking angle. The geometry shown in Figure 4.30 was designed for $n = 1.5$, has an acceptance angle of $\theta_i = 5^\circ$ and a maximum slope angle of $\Psi = 62^\circ$, and can track to $\sigma_{\text{max}} = 50^\circ$. For purposes of determining the interference envelope and for positioning the rotation axis, the aspherical inlet may be approximated by a circle. For the example considered in Figure 4.30 the resulting circle subtends an arc angle of $\Theta = 125^\circ$. The tracking axis is placed at the center of this circle and set back from the primary focal plane by its radius.

While the curved inlet of the DTERC creates more compact designs having the benefit of improved tracking range, it brings with it some unwanted optical characteristics. The first one is the increased Fresnel reflection due to larger incidence angles at the curved inlet. In theory this effect can be minimized through use of appropriate anti-reflective coatings. The second one, which unfortunately cannot be mitigated, is that a linear 2D concentrator with a curved refractive inlet does not have an elliptical acceptance map. Unlike the
familiar 2D CPC and DCPC which accept all rays whose projection into the $x$-$z$ plane is within the acceptance angle, the DTERC may reject non-meridian rays whose projections are within the acceptance angle. The greater the maximum slope angle $\Psi$, the more severe this effect becomes. Because the resulting acceptance map is not elliptical, Eq. (4.52) cannot be used to size the acceptance angle. Fortunately, the shape acceptance map may be calculated analytically, and is derived in Appendix F. A generalized version of the source/acceptance map matching method may then be utilized to determine the acceptance angle.

To illustrate the design procedure for a DTERC secondary stage, consider a system to be designed for $\alpha_{\text{crit}} = 0.5^\circ$ which must track to $\sigma_{\text{max}} = 44^\circ$. We estimate that required DTERC acceptance angle will be on the order of $1^\circ$. From Figure 4.31, the required slope angle may then be estimated as $\Psi \approx 55^\circ$. The acceptance maps shown in the plot do not account for the finite angular size of the sun.

**Figure 4.32.** Source/acceptance matching of a DTERC for a collector with and $\alpha_{\text{crit}} = 0.5^\circ$ and $\sigma_{\text{max}} = 43.9^\circ$. In order to track to $\sigma_{\text{max}}$, the slope angle of the inlet must be $\Psi = 55^\circ$ (cf. Figure 4.31). The curved refractive inlet causes the ideal acceptance map (dashed) to be distorted into the football-shaped map (see Appendix F). In order to envelope the source map, the acceptance angle of the DTERC must be considerably oversized to $\theta_{i,2} = 1.7^\circ + \theta_{\text{sun}}$. The acceptance maps shown in the plot do not account for the finite angular size of the sun.
map for the secondary stage, described by Eq.(4.55), to determine the required acceptance angle. The result is shown in Figure 4.32. Due to the poor matching between the shape of the effective source and acceptance maps, the acceptance angle must be considerably oversized in comparison to a concentrator having an elliptical acceptance map, also shown in Figure 4.32 for comparison. A 2D concentrator with an ideal (elliptic) acceptance map would have a concentration of $n/\sin(\alpha_{\text{crit}} + \theta_{\text{sun}}) = 112\times$, whereas the DTERC achieves $n/\sin(\theta_{i,2} + \theta_{\text{sun}}) = 44\times$. Due to the shape of their acceptance maps, DTERCs are preferred for small rim angle primaries. They perform better for lower latitudes due to the reduced maximum tracking angle allowing a less sloped elliptic lens to be used.

Other immersed secondary designs

Other immersed secondary designs include 2D SMS RX [66], XR [45], and RXI [67] optics. All of these designs suffer from poor non-meridian behavior, and the XR and RXI additionally suffer from severe intrinsic losses (shading, blocking, rays missing the primary) in 2D geometry. As with the non-immersed designs, some of these problems may be partially alleviated by considering asymmetric 2D SMS or 3D SMS designs.

Figure 4.33 shows a rendering of an asymmetric XR 2D SMS optic. This design has $\theta_{i,2,\text{ax}} = 2.33^\circ$, can track to $+50^\circ$, and achieves a $C_{g,2,\text{ax}} = 28.9\times$, which is 78% of the theoretical maximum of Eq. (4.53), with less than 1% ray-rejection.

Another option are the stepped flowline nonimaging optics [68] which may produce compact ideal designs requiring fewer surfaces and reflections than the LPC. However, since their design and analysis is considerably more involved than the LPC, stepped flowline optics are not considered here. Variable refractive index optics present additional design possibilities, but are considered to be unfeasible for solar applications.

4.4.7. Collector design and analysis

Sections 4.4.5 and 4.4.6 were devoted to developing designs capable of approaching the limits derived in Section 4.4.1 while maintaining full-collection. Here we develop some exemplary collector designs and analyze their performance by Monte Carlo ray-tracing. Since theoretical performance is
independent of system scale, all designs are constructed with a secondary outlet aperture of $a_{o,2} = 1$. For the dielectric filled concentrators, designs are carried out for a refractive index of $n = 1.5$, representative of optical glasses.

**Design 1: Asymmetric parabolic primary with asymmetrically truncated CPCs**

As a first simple design example, let us consider a line-to-point focus system based on an asymmetric parabolic trough with asymmetrically truncated CPC secondaries, designed for a N-S tracker at a latitude of 30°. Table 4.1 summarizes the major design parameters and outlines the design procedure of the system. Firstly, the skew range is calculated via Table 2.2 and $\theta_{i,1}$ determined from Eq. (4.7). To eliminate shading, the inner rim angle is chosen to be $\Phi_1 = 1°$, and to achieve reasonable compactness the outer rim angle is chosen to be $\Phi_1 = 71°$, yielding $\Phi = 35°$, thus completing the primary concentrator design. The resulting primary concentration from Eq. (4.8) is 49.1×.

Design of the secondaries begins by determining the $\sigma_{\text{min}}$ and $\sigma_{\text{max}}$ from Eq. (4.48) and $\alpha_{\text{crit}}$ from Eq. (4.51). For asymmetric CPC secondaries, the acceptance angle may be sized from Eq. (4.52), yielding $\theta_{i,2} = 3.1°$. 

![Figure 4.33. Rendering of secondary concentrator based on an asymmetric XR SMS optic. This design has $\theta_{i,2,ax} = 2.33°$, can track to +50°, and achieves a $C_g = 28.9×$ with less than 1% ray-rejection.](image-url)
Figure 4.34. Schematic of: (a) Design 1; (b) Design 2; and (c) Design 3 showing the path traced by rays incident at skew angle of $\theta = 0^\circ$ (normal incidence). Only a small section of the primary has been illuminated for clarity.
The truncation of the long branch is then selected. Here it is set at ¼ of the total length of a CPC with $\theta_i = 3.1^\circ$ yielding $L_{\text{longbranch}} = 44.4$. The short branch is then truncated to a length allowing $\sigma_{\text{max}} = 56.1^\circ$ to be reached, in this case resulting in $L_{\text{shortbranch}} = 12$. The resulting secondary concentration of the asymmetrically truncated CPC is $10.7\times$, which is 42% lower than the thermodynamic limit of $18.4\times$ due to the substantial truncation. The receiver width determines the overall scale of the secondaries, as described in Section 4.4.4. It is taken here as $w_2 = 10$. A schematic of the overall system, which achieves $C_{g,\text{tot}} = 526\times$, is given in Figure 4.34 (a).

**Design 2: Aplanatic trough primary with DTERC secondaries**

Let us explore a high concentration design for a lower latitude of $20^\circ$. With the aid of Figure 4.19, we choose a rim angle of $15^\circ$ which is near the optimal rim angle of $\sim 10^\circ$. To maintain a compact system with such a small rim angle, we choose an aplanatic trough primary, similar to that shown in Appendix D, with design parameters: $s = K = 0.115$ and $\delta = 2.2^\circ$. The primary falls short of the maximum theoretical concentration by 15% due to truncation of the first mirror in order to avoid shading by the second mirror, yielding $C_{g,1} = 34.5\times$.

For small rim angles, high concentrations, and relatively small tracking ranges, the DTERC is an appropriate choice for the secondary concentrator. The tracking range and $\alpha_{\text{crit}}$ are determined from Eqs. (4.48) and (4.51) respectively. The DTERC acceptance angle is then sized by the source/acceptance matching method, resulting in $\theta_{i,2} = 2^\circ$ and $C_{g,2} = 43\times$. Due to the high $C_{g,2}$, a receiver width of 20 is chosen to maintain a small scale for the secondary concentrator. The system, shown schematically in Figure 4.34 (b), achieves an overall concentration of $C_{g,\text{tot}} = 1482\times$. The main design parameters are summarized in Table 4.1.

**Design 3: Parabolic trough primary with multi-foliate light-pipe secondaries**

Having designed a system for a high concentration and low latitude, let us now consider a design for a higher latitude of $40^\circ$. Due to the larger tracking range required, the multi-foliate light-pipe concentrator (LPC) is well-suited for the secondary concentrator. In order to reduce the number of leaves required for the LPC, we will let the primary do more of the work, and therefore choose a larger
primary rim angle of 45°. For the primary geometry we choose a symmetric parabolic trough, which is sufficiently compact for such a large rim angle. In order to avoid shading losses, the section of the parabola spanning $-0.6 < \varphi < 0.6$ is removed, with the removed section being considered a loss in concentration and not in optical efficiency. The resulting primary concentration is $C_{g,1} = 46.5 \times$. The required secondary acceptance angle is 5.2° which we increase to 5.5° to reduce the required number of leaves and partially offset the non-ideality of having a finite number of leaves. For the LPC design, we

<table>
<thead>
<tr>
<th>Table 4.1. Design table for three exemplary LTP concentrators with tracking secondary optics.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quantity</strong></td>
</tr>
<tr>
<td>primary geometry</td>
</tr>
<tr>
<td>secondary geometry</td>
</tr>
<tr>
<td>$\phi [^\circ]$</td>
</tr>
<tr>
<td>$\varphi_{\min} [^\circ]$</td>
</tr>
<tr>
<td>$\varphi_{\max} [^\circ]$</td>
</tr>
<tr>
<td>$\theta_{i,1} [^\circ]$</td>
</tr>
<tr>
<td>$\Phi [^\circ]$</td>
</tr>
<tr>
<td>$C_{g,1,\max} [\times]$</td>
</tr>
<tr>
<td>$C_{g,1} [\times]$</td>
</tr>
<tr>
<td>$\sigma_{\min} [^\circ]$</td>
</tr>
<tr>
<td>$\sigma_{\max} [^\circ]$</td>
</tr>
<tr>
<td>$\theta_{\text{crit}} [^\circ]$</td>
</tr>
<tr>
<td>$\alpha_{\text{crit}} [^\circ]$</td>
</tr>
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<td>$\theta_{i,2} [^\circ]$</td>
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<tr>
<td>$C_{g,2,\max} [\times]$</td>
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<td>$C_{g,2} [\times]$</td>
</tr>
<tr>
<td>$C_{g,\text{tot,\max}} [\times]$</td>
</tr>
<tr>
<td>$C_{g,\text{tot}} [\times]$</td>
</tr>
</tbody>
</table>
choose $f = 0.95$ and $g = 0.1$ for the LPC, yielding a leaf constant of $c = 2.204 \times 10^{-3}$ from Eq. (E.3). This yields, from Eq. (E.9), a concentrator having $N = 2687$ leaves. The secondary concentration is $C_{g,2} = 15.6 \times$, which is slightly lower than the thermodynamic limit of $16.5 \times$ due to the slight oversizing of $\theta_{i,2}$. The receiver width is taken as 10. Additional design parameters are summarized in Table 4.1. This system, shown schematically in Figure 4.34 (c), achieves $C_{g,tot} = 770 \times$.

4.4.8. Geometric performance

The foremost figure of merit describing concentrator performance is the optical efficiency, defined by the fraction of sunlight incident on the primary collecting aperture that reaches the receiver(s). With the goal of assessing the geometric performance of the various designs developed, all optical interactions are assumed to be ideal: i.e., perfect specular reflectivity for reflective surfaces and zero Fresnel reflectivity (perfect anti-reflective coatings) on all refractive surfaces. End effects can be minimized by making the trough sufficiently long and are thus not considered. Since the only optical losses considered here are geometric losses (ray-rejection and spillage), the optical efficiency reported here is more appropriately termed the acceptance efficiency $\eta_{acc}$. In mathematical terms it may be found from:

$$\eta_{acc}(\vartheta) = \frac{Q_{receiver}}{Q_{inlet \ aperture}} \frac{E_{receiver}A_{receiver}}{E_{aperture}A_{inlet \ aperture}} = \frac{E_{receiver}}{C_g \cdot \text{DNI} \cdot \cos \vartheta} \tag{4.69}$$

where $Q$ is the radiant power, $E$ is the irradiance, $A$ is the aperture area, and DNI is the direct normal irradiance. The last equality of Eq. (4.69) provides a convenient method for determining $\eta_{acc}$ since $E_{receiver}$ may be readily determined by Monte Carlo ray-tracing. For this purpose a simulation is setup where the primary aperture is illuminated with spatially uniform radiation arriving at a skew angle of $\vartheta$ from within a cone of angle $\theta_{sun} = 0.266^\circ$ with a DNI of 1000 W/m². Simulations were performed using the in-house VeGaS code [69] with 10 million rays per simulation. A representative trace of 100 rays at $\vartheta = 0^\circ$ for the three example designs is shown in Figure 4.34. The resulting
acceptance efficiency vs. skew angle for the three designs is given in Figure 4.35.

Figure 4.35 (a) shows the acceptance efficiency of Design 1. It is seen to be nearly 100% over the design skew angle range ($-27.3^\circ \leq \vartheta \leq 53.4^\circ$), where the maximum and minimum skew angles are indicated by the dashed lines. For $\vartheta > 53.4^\circ$, there is a steep drop in $\eta_{\text{acc}}$ as the secondaries cannot track to the correct tracking angle due to interference with adjacent secondaries. For $\vartheta < -27.3^\circ$, there is a small region of high $\eta_{\text{acc}}$ until $\vartheta < -33.1^\circ$, after which there is a steep drop due to tracking interference. This is because the concentrators can actually track to $\sigma_{\text{min}} = -35.8^\circ$ ($\vartheta = -33.1^\circ$) without interference even though the minimum required tracking angle is $\sigma_{\text{min}} = -29.8^\circ$ ($\vartheta = -27.3^\circ$). There is a very slight drop in $\eta_{\text{acc}}$ at small positive skews (local minimum of $\eta_{\text{acc}} = 98\%$ @ $\vartheta < 11^\circ$) which is attributed to rays leaking through the gap between the short branch and the long branch of the adjacent concentrator.

Figure 4.35 (b) shows the acceptance efficiency of Design 2. Due to the symmetry of the secondary concentrator designs, the curve is symmetric about $\vartheta = 0^\circ$. $\eta_{\text{acc}}$ is very nearly 100% over the design range of skew angles ($-25.0^\circ \leq \vartheta \leq 43.4^\circ$). For $\vartheta > 43.4^\circ$ there is a very steep drop in $\eta_{\text{acc}}$ due to the secondary concentrators reaching their tracking limit of $\sigma_{\text{max}} = 43.9^\circ$. The drop is comparatively steeper than that of Design 1 for three reasons: (1) the acceptance angle is smaller; (2) the design is untruncated; (3) the maximum tracking angle is closer to the critical skew angle for which the axial beamspread is maximum. For $\vartheta < -25.0^\circ$, there is a region of high acceptance beyond the minimum skew angle due to the symmetry of the design.

Figure 4.35 (c) shows the acceptance efficiency of Design 3. As with that for Design 2, the curve is symmetric about $\vartheta = 0^\circ$. In comparison to the case of Design 1 and 2, the drop off in $\eta_{\text{acc}}$ for $\vartheta > \vartheta_{\text{max}} = 63.4^\circ$ is much less steep. This is due to the fact that the drop off is not caused by the tracking limit of the secondaries, as the LPC secondaries can track to any angle without interference. Rather the drop off is caused by rays missing the secondary inlet aperture due to increased skew dilation (see Section 2.4.6). This effect scales with $\cos \vartheta$ rather than the abrupt effect of secondary reaching its tracking limit.
Another predominant feature in the curve of Design 3 is a strong dip in \( \eta_{\text{acc}} \) near the critical skew of \( \vartheta = 40.1^\circ \), due to the fact that LPCs are only ideal for an infinite number of leaves. For a finite number of leaves, the LPC is designed to meet a nominal value of \( \eta_{\text{acc}} \approx f = 0.95 \). The drop in \( \eta_{\text{acc}} \) could be reduced at the expense of concentration by appropriately oversizing the secondary acceptance angle.

4.4.9. Performance considerations for real collectors

The performances reported in Section 4.4.8 considered only purely geometric losses (ray-rejection and spillage) with the goal of reporting the geometric performance of tracking-secondary LTP systems without the need of detailed considerations of specific material optical properties. The optical efficiency of
real systems will be lower than those reported in Figure 4.35 due to losses. These losses may be separated into those that are geometric in nature, i.e. they may be offset by increasing the acceptance angle, and those that are optical in nature. The geometric losses include those caused by primary mirror surface errors, tracking errors, circumsolar radiation, and chromatic aberration for refractive secondaries. These losses represent a trade-off in concentration/optical efficiency. At the discretion of the designer, they may be taken as a loss or, alternatively, the acceptance angle of the system may be appropriately oversized in order to mitigate these losses at the expense of concentration. The optical losses include: primary mirror reflectivity, secondary mirror reflectivity for hollow secondary designs, Fresnel reflection, and imperfect TIR for dielectric-filled designs. In general, these losses may not be mitigated by oversizing the acceptance angle and are rather dictated by the choice of materials.

4.5. Summary

The theory and design principles for line-to-point focus systems which break the 2D limit of their primary trough concentrators have been developed. Three types of designs were considered, classified by the degrees of freedom of the secondary concentrator. Table 4.2 shows the derived concentration limits for the three classes of LTP concentrators investigated for a system based on a N-S one-axis tracker at a latitude of 30°. Also shown is the achievable limit for a purely line-focusing system and the fundamental concentration limit for a one-axis tracker.

Of the three classes the tracking-secondary line-to-point (LTP) focus concentrator achieves the highest concentration and thus shows most promise, as the additional cost of the secondary stage is most warranted. Therefore the greatest emphasis was placed on the theory and design of these systems. A tracking-secondary LTP concentrator comprises a one-axis tracking trough primary coupled with an array of nonimaging secondary concentrators arranged at the focal plane of the primary, which are allowed to individually track on secondary tracking axes which are perpendicular to the primary axis. The concentration limits of tracking-secondary LTP concentrators have been
derived based on the condition of full collection, i.e. all rays from the solar disk are collected year-round. The achievable concentration was found to depend on the range of skew angles experienced by the concentrator, which depends on the latitude and primary tracking scheme employed. The smallest skew range is exhibited by the polar tracker, or the N-S one-axis tracker when it is required that the primary axis be horizontal. For this reason, N-S one-axis primaries are the most suitable for tracking-secondary LTP concentrators. For an idealized system based on a polar tracking primary (or equivalently a horizontal N-S tracker at the equator), the concentration limits are $C_{g,\text{max}} \approx 4000\times$ for hollow secondaries, and $C_{g,\text{max}} \approx 6000\times$ for dielectric-filled secondaries with $n = 1.5$. For a N-S one-axis tracker at a latitude of 30°, the corresponding limits are $C_{g,\text{max}} \approx 2500\times (n = 1)$ and $C_{g,\text{max}} \approx 3750\times (n = 1.5)$.

The maximum concentration is achieved for small rim angle primaries ($\Phi \approx 10^\circ$). These limits have been derived considering that both the primary and secondary are linear 2D concentrators, i.e. only concentrating in one direction. Concentrations higher than those presented here are possible if transverse re-concentration is incorporated into the secondary concentrator design (i.e. by giving it a 3D structure), especially considering that the peak overall concentrations occur for small primary rim angles where the potential for transverse re-concentration is greatest.

Table 4.2. Comparison of achievable concentration limits for different concentrator types based on a horizontal N-S one-axis tracker at a latitude of 30°. For all concentrator types, the secondary concentrator is 2D, i.e. concentrating in only one direction.

<table>
<thead>
<tr>
<th>limit/concentrator type</th>
<th>$n = 1$</th>
<th>$n = 1.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>fundamental limit for N-S one-axis tracker</td>
<td>270×</td>
<td>600×</td>
</tr>
<tr>
<td>2-stage trough</td>
<td>130×</td>
<td>190×</td>
</tr>
<tr>
<td>fixed-secondary LTP concentrator</td>
<td>170×</td>
<td>390×</td>
</tr>
<tr>
<td>discrete-switching LTP concentrator ($N = 4$)</td>
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<td>760×</td>
</tr>
<tr>
<td>continuous-tracking LTP concentrator</td>
<td>2600×</td>
<td>3900×</td>
</tr>
</tbody>
</table>
Potential primary and secondary concentrator geometries were explored. Design of the primary concentrator focused on achieving compact, small rim angle primaries. Reasonably compact small rim angle primaries were shown to be possible with asymmetric parabolic trough primaries used in a two-wing configuration. Very compact primaries of any rim angle were found to be achievable using two-mirror aplanatic primaries. Design of the secondary concentrator focused on achieving small acceptance angle primaries that are able to rotate without interference when placed next to each other. Considering potential hollow secondary designs, the asymmetrically truncated CPC was found to be the most practical solution. For dielectric-filled secondary designs, the multi-foliate light-pipe concentrator (LPC) and the dielectric tailored edge-ray concentrator (DTERC) were found to be most suitable. The LPC can approach the concentration limits above, but its practical implementation is difficult due to the large number of leaves required. The DTERC is a more practical candidate, but falls short of the concentration limits due to the non-ideal behavior of non-meridian rays refracted at its curved inlet.

Three exemplary collector designs were considered. Design 1, designed for a latitude of 30°, featured an asymmetric parabolic primary with asymmetrically truncated CPC secondaries and achieved a concentration of $C_{g} = 526\times$. Design 2, designed for a latitude of 20°, featured a two-mirror aplanatic primary with DTERC secondaries and achieved a concentration of $C_{g,\text{tot}} = 1482\times$. Design 3, designed for a latitude of 40°, featured a parabolic trough primary with LPC secondaries achieving a concentration of $C_{g,\text{tot}} = 770\times$. The geometric performance of the three exemplary designs was ascertained by Monte Carlo ray-tracing which confirmed near 100% acceptance efficiency (no ray rejection or spillage) over the full skew angle range for which each system was designed.

LTP concentrators present a compromise between (primary) mirror complexity and concentration in-between those for purely 2D and 3D geometries. They are well-suited to applications where concentration limits of purely 2D geometries are inadequate, but the levels achievable by 3D designs are not required. For such applications, tracking-secondary LTP concentrators are able to break the 2D limit while still maintaining the advantages of having a
linear trough primary, namely having a simple geometric construction and being highly scalable. They are currently being developed as low-cost alternatives for high-concentration photovoltaics [57, 58], and may find further use for boosting the concentration of any trough system where the continuous line receiver can be broken up into discrete point-like receivers.
Chapter 5

Characterization of concentrator solar cell arrays

The line-to-point focus designs presented in Chapter 4 result in a system with an array of receivers arranged along the length of the trough primary concentrator. This focal arrangement provides a unique advantage for concentrating photovoltaic applications in that it is not necessary to pack the solar cells in a dense array as would be required for line-focus concentrator or a large point-focus concentrator such as a parabolic dish. Because of the line-symmetry of the primary, and the periodicity of the secondary stage, each receiver in the array receives essentially the same input power. This is a significant advantage because it allows the receivers to be serially connected along the length of the array, allowing very high voltages to be reached.

For these reasons, we note that line-to-point focus systems are especially interesting for HCPV applications. This chapter discusses the experimental characterization and modeling of a prototype array of concentrator solar cells suitable for use in line-to-point focus systems. The analysis methods presented are quite general and therefore may find use in analyzing the performance of solar cell arrays for other concentrator types.

5.1. The semi-dense array

Depending on the ratio of secondary to primary concentration, a line-to-point focus system will produce a rectangular shaped receiver, as detailed in Figure 4.22. While this may at first seem a disadvantage since standard production terrestrial concentrator cells are square in shape, it actually turns out to be beneficial for the design of the receiver, since it allows the placement of a linear array of concentrator cells. Figure 5.1 shows a prototype of a linear array.

---

of 1 cm² GaInP/GaAs/Ge triple-junction concentrator cells (manufacturer: Azur Space GmbH, model: 3C40C) [71]. The 5-cell array is subsequently referred to as the “mini-module”.

The linear array arrangement has been denoted semi-dense array [57], and has significant advantages over 2D (i.e. square) dense arrays. In particular the cells can be arranged with their busbars, shown in Figure 5.1 (b), running to the parallel of the length of the array. This allows the cells to be placed very close together thus reducing the gap losses of the illuminated front grid regions. For the mini-module under investigation, there is a fixed inter-cell gap of 0.5 mm, yielding a ratio of active area to illumined area of 96.2% (3.8% gap loss). Additionally, circuitry and electronics can be easily positioned on either side of the linear array without optically interfering with the cells.

In this array prototype the cells are connected in series. Each cell is connected in parallel with a high-current Schottky barrier rectifier (Vishay, model SS12P2L [72]), functioning as a bypass diode. The back contacts of the cells are vacuum reflow soldered to a direct bonded copper (DBC) board into which the electrical circuit is etched. The top contacts of the cells are wire bonded to the DBC board.
5.2. Equivalent circuit modeling

Equivalent-circuit models of triple-junction cells have been reported recently in the literature with complexity ranging from simple lumped models [73], to models considering individual subcell circuits [73, 74], to complex distributed circuit models [75-78]. In the present work, a simple model similar to that of Ben Or and Appelbaum [73] is applied, which considers the three subcells as a single lumped one-diode equivalent circuit, but is nevertheless shown capable of predicting $I-V$ behavior with good accuracy.

In the lumped mode, the triple-junction cell, shown in Figure 5.1 (b), is represented by a single ideal current source $J_{ph}$ representing the photocurrent generated by the limiting subcell, in parallel with an exponential diode $D$, representing recombination, and a shunt resistance $R_{sh}$ representing current leakage [79], all connected in series with a series resistance $R_s$ representing the finite conductivity across the p-n junctions, tunnel junctions, contacts and DBC circuit. The one-diode mode of the solar cell is shown schematically in Figure 5.2 (a). Five identical cell models, each with a parallel-connected Schottky bypass diode, are connected in series to form the equivalent circuit model of the mini-module, as shown in Figure 5.2 (b). The bypass diodes are modeled as exponential diodes with characteristics taken from the manufacturer’s specifications [72], as summarized in Table 5.1.

Advantages of the lumped cell model versus the series subcell model are: (1) the number of tuning parameters is 4 compared to 10 (for both one- and two- diode forms); (2) only one short-circuit (photo-) current density is needed, compared to 3 for the subcell model; and (3) the short circuit current is by

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>reverse saturation current, $I_{0,BD}$ [A]</td>
<td>$1.6895 \times 10^{-5}$</td>
</tr>
<tr>
<td>diode ideality factor, $n_{BD}$</td>
<td>1.1637</td>
</tr>
<tr>
<td>series resistance, $R_{s,BD}$ [Ω]</td>
<td>0.0066</td>
</tr>
<tr>
<td>reverse breakdown voltage $V_{r,BD}$ [V]</td>
<td>20</td>
</tr>
</tbody>
</table>
construction equal to the photocurrent source, whereas for the series subcell model the short circuit current may exceed the limiting junction photocurrent when shunt resistances are considered. Drawbacks are: (1) reduced tunability; (2) some physical effects, including reverse bias in current limited subcells at short-circuit, are not captured by the model; and (3) the model must be re-tuned for a different incident spectrum, since subcell photocurrents are not individually assigned.

For a single-cell, the current density $J$ drawn by the external circuit, and the voltage across the cell $v$ are related by:

$$J = J_{ph} - J_0 \left( e^{q(v+JAR_s)/n_Dk_BT} - 1 \right) - \left( v + JAR_s \right) / R_{sh}$$  \hspace{1cm} (5.1)

where $J_{ph}$ is the photocurrent density generated under zero bias (short-circuit), $J_0$ is the reverse saturation current density of the diode, $n_D$ is the diode ideality

**Figure 5.2.** (a) Lumped single-diode equivalent circuit model of a solar cell; and (b) mini-module equivalent circuit model consisting of 5 series-connected cell models with parallel bypass diodes (BD).
factor (emission coefficient), $R_s$ is the series resistance, and $R_{sh}$ is the shunt resistance. When the diode represents physical recombination phenomena, the diode ideality factor is restricted to the physically relevant range of $2/3$ to 2, depending on the dominant recombination process. The theoretical values are 2, 1 and $2/3$ for Shockley-Read-Hall (SRH), radiative, and Auger recombination processes respectively [74, 79]. While in general the parameters of Eq. (5.1) may vary with cell temperature and concentration, the following simplifying assumptions are made: (1) all measurements and simulations are performed at a temperature of 25 °C such that temperature effects may be neglected; (2) the photocurrent $J_{ph}$ is linear with irradiance; and (3) recombination and resistance effects are independent of injection level such that the parameters $J_0$, $n_D$, $R_s$, and $R_{sh}$ can be considered constant over the range of concentrations considered [79]. With regard to the last assumption, the parameters are fitted across a range of concentrations ($10 − 1678 \text{ kW/m}^2$) rather than at a single concentration, such that the model achieves equally good agreement with experiment over the relevant concentration range.

The $I-V$ characteristic of the bypass diodes is described by the Shockley diode equation, including a series resistance:

$$i_{BD} = i_{0,BD} \left( e^{(qV_{f,BD}R_s)/n_D k_B T} - 1 \right)$$  \hspace{1cm} (5.2)

where the quantities are defined analogously to Eq. (5.1), but with currents, rather than current densities, being used. Together with Kirchoff’s circuit laws, Eq. (5.1) and (5.2) completely define the complete (non-linear) $I-V$ behavior of the mini-module.

### 5.3. Experimental characterization

To ascertain the performance of the proposed cell array under different irradiance levels, a thorough indoor experimental characterization of the array was performed.
5.3.1. Experimental setup

Measurements were performed using a large-area flash solar simulator (Pasan IIIa) with a modified setup to allow the irradiance to be changed continuously from 1 kW/m² to >2000 kW/m² by changing the lamp-to-cell distance [80-82]. The experimental setup is shown schematically in Figure 5.3. The mini-module was fixed to a cooling plate using a vacuum chuck to provide good thermal contact between the DBC board and the plate. Cooling water was fed continuously through the experiment to maintain the DBC board at a temperature of 25 °C ±0.1 °C. A fixture, integrated into the cooling plate was used to precisely position the filters over the mini-module. Current and voltage were measured using 4-terminal sensing via two Kelvin probes attached to the terminals of the mini-module. The cooling plate assembly was centered with the flash lamp and mounted on a moveable carriage to facilitate repeatable positioning of the test device at difference cell-to-lamp distances. A monitor cell was attached in the same plane as the test device to correct for temporal instabilities of the Xe flash lamp.

5.3.2. Experimental procedure

The setup was used to obtain the $I-V$ characteristic of the mini-module under varying irradiance levels ranging from 10 to 1678 kW/m². For each measurement, the entire $I-V$ curve of the mini-module was measured during a single 2-3 ms flash, by sweeping the voltage from short-circuit (SC) to open-circuit (OC) using a four-quadrant power supply. The irradiance was measured in reference to the known 1-sun short-circuit current assuming linear dependence of $I_{SC}$ with irradiance. Since measurements were performed on the mini-module, rather than on a bare cell, the results include the effect of contact and Ohmic resistance of the cell package, as well as the intrinsic behavior of the cell.

5.3.3. Model tuning

With $J_{ph} = I_{SC}/A_{active}$ known from the 1 kW/m² measurements, Eq. (5.1) contains 4 unknown parameters: $J_0$, $n_D$, $R_s$ and $R_{sh}$, which may be tuned vis-à-vis the experimental results. The fitting procedure requires establishment of an
objective function representative of the error between model and experiment, which is to be minimized. Several possible objective functions have been studied previously in the literature [73, 74]. Since full $I$-$V$ curves were measured, one possibility is to compute the rms relative error in current over the characteristic from $I_{SC}$ to $V_{OC}$:

$$
\epsilon_{I-V} = \sqrt{\frac{1}{NM} \sum_{N} \sum_{M} \frac{I_{\text{sim}}(V) - I_{\text{exp}}(V)}{I_{\text{exp}}(V)}}
$$

(5.3)

where the summation is performed over $M$ measured points on the $I$-$V$ curve at $N$ irradiance levels, such that the model is tuned to cover the full range of operating conditions. This objective function has the disadvantage that the relative current error becomes unbounded if points too close to OC are used. Consequently points near OC have an inflated impact on the fitting, compared to points away from OC whose current is generally of more physical

Figure 5.3. Modified large-area solar simulator for characterizing solar cells and arrays under concentrated irradiation.
importance. One solution is to weight the summand, e.g. by the power output of
the cell, such that the useful portions of the \( I-V \) curve are given more
significance in the tuning procedure.

In many cases, full \( I-V \) curves are not available, and an objective function must
be established from other available quantities. One such procedure is the
“three-point fitting” \[73\] which requires experimental fitting against short-
circuit, open-circuit, and the maximum power point. With the photocurrent set
equal to the measured short-circuit current, tuning at short-circuit is redundant,
and it is sufficient to fit against the open-circuit voltage and efficiency as in
\[74\]. For this purpose the objective function \( f \) is taken as average rms error of
the open-circuit voltage and the efficiency:

\[
\varepsilon_{V_{\text{OC}}} = \sqrt{\frac{1}{N} \sum_{N} \left( \frac{V_{\text{OC,\text{sim}}} - V_{\text{OC,exp}}}{V_{\text{OC,exp}}} \right)^2}
\]

\[
\varepsilon_{\eta} = \sqrt{\frac{1}{N} \sum_{N} \left( \frac{\eta_{\text{sim}} - \eta_{\text{exp}}}{\eta_{\text{exp}}} \right)^2}
\]

\[
f = \frac{1}{2} \left( \varepsilon_{V_{\text{OC}}} + \varepsilon_{\eta} \right)
\]

It is stressed that the error is accumulated with equal weighting over all \( N \)
measured irradiance levels such that the model well represents the full relevant
range of concentrations.

It was found that the rms error over the full \( I-V \) curve, \( \varepsilon_{I-V} \), was only slightly
lower when Eq. (5.3) was used as the objective function than when the simpler
Eq. (5.6) was used. This supports the suggestion of Ben Or and Appelbaum
[73] that three-point fitting can produce good agreement over the full \( I-V \) curve.
Furthermore, tuning with Eq. (5.6) allowed for the efficiency vs. concentration
curve to be more accurately predicted. Therefore Eq. (5.6) was used to perform
the final parameter fitting.

The parameter tuning was performed using the Nelder-Mead simplex
unconstrained non-linear optimization algorithm \[83\]. The algorithm is
implemented as the built-in function “fminsearch” in Matlab® as a general
multi-dimensional optimization solver. It requires a user-defined objective function $f$, an initial guess of the optimization variables, and convergence criteria as inputs. Details on the algorithm are available in [84]. The objective function gives the value of Eq. (5.3) for a given set of parameter estimates, and is evaluated according to the steps: (1) the first of $N$ irradiance levels is chosen; (2) the photocurrent is assigned according to $J_{ph} = J_{ph,1-sun} \langle E_{av} \rangle$; (3) using the model with the current guess of parameters, Eq. (5.1) is solved to generate the $I-V$ curve (the effect of bypass diodes may be neglected since tuning was performed against measurements at uniform irradiance); (4) $V_{OC}$ and $\eta$ are calculated from the $I-V$ curve; (5) the summands of Eq. (5.3) are calculated; and (6) steps 1 through 5 are repeated for the remaining irradiance levels, the summands of Eq. (5.3) are accumulated, and the computation of Eq. (5.3) completed.

As the method is unconstrained, bounds on the parameters or other constraints are not required; however a good initial guess of the parameters is required for convergence. The following initial guesses $J_0 = 2 \cdot 10^{-15}$ A/cm$^2$, $n_D = 3.5$, $R_s = 0.03$ Ω and $R_{sh} = 500$ Ω were used. The two convergence criteria used are the relative change (tolerance) of the parameters, and of the objective function, both of which were set to $10^{-9}$.

5.3.4. Experimental results

Measurement at an irradiance of 1-sun (1.017 kW/m$^2$ verified by a calibrated reference cell with known spectral response) revealed a short circuit current of $J_{SC,1-sun} = 14.425$ mA/cm$^2$ for the mini-module. It is expected that the 1-sun short circuit current under AM1.5d conditions would be lower, due to a lower portion of the incident power being in the spectrum of the current limiting (middle) subcell. The short circuit current extrapolated to 1 kW/m$^2$ (reported at 500 kW/m$^2$ on the manufacturers specification) is 13.22 mA, supporting the above claim. Therefore the efficiencies under AM1.5d are expected to be slightly lower than those reported here.

**Figure 5.4** shows the measured $I-V$ characteristic of the mini-module for irradiance values ranging from 10 kW/m$^2$ to 1678 kW/m$^2$. At high concentrations, an increase in the effect of series resistance $R_s$ and a possible
decrease in shunt resistance $R_{sh}$ are observed. However, the step-like behavior of the $I$-$V$ curve at 1678 kW/m$^2$ suggests that the apparent increase in $R_{sh}$ is more likely due to the increased spatial nonuniformity at high irradiances (>1000 kW/m$^2$) due to non-planar wavefronts occurring very close to the lamp. Figure 5.5 shows the resulting dependency of short-circuit current $I_{SC}$, open-circuit voltage $V_{OC}$, fill-factor $FF$ and module-efficiency $\eta$ extracted from the $I$-$V$ curves of Figure 5.4.

Figure 5.5 (a) shows a linear dependence of $I_{SC}$ with irradiance. Figure 5.5 (b) shows a logarithmic dependence of $V_{OC}$ with irradiance, with a slight downward deviation from logarithmic above 500 kW/m$^2$. The measured $V_{OC}$ agrees well with the manufacturer’s specifications (3.19 V at 500 kW/m$^2$ and 3.17 V at 1000 kW/m$^2$ for a single cell), except that a reduction of $V_{OC}$ at high irradiance was not observed. Fill-factor, shown in Figure 5.5 (c), increases with irradiance to a maximum of 88%, staying nearly constant between 25 kW/m$^2$ and 250 kW/m$^2$, and then decreasing significantly for irradiance levels above 500 kW/m$^2$, falling below 80% above 1500 kW/m$^2$. The efficiency-irradiance curve, shown in Figure 5.5 (d), peaks at 500 kW/m$^2$ with a value of 39.5% dropping quickly above 500 kW/m$^2$ due to reduced fill-factor. The efficiency is above 37% between 99 kW/m$^2$ and 1678 kW/m$^2$. The lack of
a knee at high irradiances suggests that the tunneling current limit of the tunnel junctions has not been exceeded [76].

5.3.5. Fitted equivalent circuit model

Table 5.2 shows the resulting best-fit parameters from the parameter tuning, the converged value of the objective function $f$, the error on the $V_{OC}$ and $\eta$, and the rms error over all $I-V$ curve points given by Eq. (5.3). Interestingly, the lumped cell model is capable of achieving as good fits as the more complex one- and two-diode subcell models [74] (albeit for a different cell and with no temperature dependence considered). Superimposed in Figure 5.4 are the

Figure 5.5. Measured and simulated curves of (a) $I_{SC}$; (b) $V_{OC}$; (c) FF; and (d) $\eta$ as a function of irradiance for the mini-module. Cell temperature 25 °C ±0.1 ºC.
predictions of the fitted model showing good agreement over the full $I$-$V$ characteristic and the full range of concentrations considered.

**Figure 5.5** shows the agreement between model and experiment for $I_{SC}$, $V_{OC}$, FF and $\eta$. The 1-sun measurements were omitted from the fitting since the effect of shunt resistance was found to be much less significant at 1-sun ($R_{sh}$ is estimated at ~20 k$\Omega$ at 1-sun). The fitted shunt resistance of 499.8 $\Omega$ under concentration corresponds well with the expected range for III-V triple-junction cells [85].

Attention must be given to the best-fit value of the diode ideality factor $n_D = 3.189$. This is out of the physically relevant range of 2/3 to 2 associated with recombination processes present in solar cells. The high value is attributed to the lumping of all subcells into a single model, causing the bias dependence of the dark current to be substantially reduced. If the three junctions were separately modeled, an ideality factor on the order of one-third of that calculated here may be expected. The fitted series resistance of $R_s = 0.0113$ $\Omega$ is lower than that for similar triple-junction cells [74], likely attributed to the high front metallization of the cells in question, and the vacuum reflow soldering process used to connect the cell to the DBC board.

<table>
<thead>
<tr>
<th>Table 5.2. Best-fit parameters and goodness of fit of the tuned equivalent circuit model. Tuning performed for best fit on $V_{OC}$ and $\eta$ against measurements at 25 °C for irradiance levels of 10, 22, 46, 99, 194, 496, 956 and 1678 kW/m$^2$ (equally weighted).</th>
</tr>
</thead>
<tbody>
<tr>
<td>reverse saturation current density, $J_0$ [A/cm$^2$]</td>
</tr>
<tr>
<td>diode ideality factor, $n$</td>
</tr>
<tr>
<td>shunt resistance, $R_{sh}$ [$\Omega$]</td>
</tr>
<tr>
<td>series resistance, $R_s$ [$\Omega$]</td>
</tr>
<tr>
<td>objective function value, $f$</td>
</tr>
<tr>
<td>rms error on $V_{OC}$, $\varepsilon_{V_{OC}}$</td>
</tr>
<tr>
<td>rms error on $\eta$, $\varepsilon_{\eta}$</td>
</tr>
<tr>
<td>rms error over all $I$-$V$ points, $\varepsilon_{I-V}$</td>
</tr>
</tbody>
</table>
Figure 5.6. Absolute error between the modeled and experimental $I$-$V$ characteristics of Figure 5.4 from SC to OC for uniform irradiance levels: (a) 10, 22, 46, 99 kW/m²; and (b) 194, 496, 956, 1678 kW/m².

**Goodness of fit**

Figure 5.6 shows the absolute error $|I_{\text{sim}} - I_{\text{exp}}|$ from SC to OC between the modeled and experimental $I$-$V$ curves shown in Figure 5.4 for the best-fit parameters of Table 5.2. Note that absolute error is preferred to relative error since the latter is by definition unbounded at OC. Table 5.3 summarizes the error between model and experiment for different irradiance levels under uniform irradiance. Comparing the errors across the different irradiance levels, it is seen that the agreement is good across the full range concentrations considered.

5.4. **Effect of cell-to-cell nonuniformity**

In general, the optical system of a concentrating photovoltaic collector will produce a certain distribution of irradiance $E(x,y)$ over the surface of the cell receiver. Nonuniformities in this distribution may arise from several factors including: achromatic aberrations (coma, astigmatism, etc.), asymmetry of the optical system, receiver oversizing, dichroic/chromatic effects, and the angular brightness profile of the solar disk (sunshape). In concentrating systems, improvement of the irradiance uniformity generally comes at the expense of
optical efficiency, e.g. through introduction of a homogenizing optical component. It is therefore important to have an accurate prediction of how a given irradiance distribution affects the performance of the cell receiver.

An important distinction is made between single-cell nonuniformity, which describes the spatial irradiance distribution over a single cell, and cell-to-cell nonuniformity which describes the difference in average irradiance from one cell to the next in a multi-cell module. The two are fundamentally different in the way that they affect the performance of an array of solar cells. This investigation considers only cell-to-cell nonuniformity, which may lead to severe efficiency penalties for series-connected arrays of high-efficiency cells. This is in contrast to the considerable tolerance observed for single-cell nonuniformity [86, 87], which is discussed in Section 5.5.

Cell-to-cell nonuniformity is primarily of concern for cells that are connected in series. It results from the fact that the current delivered by the array is constrained by the cell having the lowest (average) irradiance. The power loss resulting from this current-limiting behavior is usually referred to as mismatch loss. This is not to be confused with homonymous phenomenon resulting from mismatched subcell photocurrents common in lattice-matched multijunction solar cells [88]. Since current scales linearly with (cell-averaged) irradiance, mismatch loss can be significant. For cells connected in parallel, currents add across a common voltage. Since voltage scales logarithmically with irradiance, the resulting loss is negligible in comparison to serial

<table>
<thead>
<tr>
<th>$\langle E_{av} \rangle$ [kW/m$^2$]</th>
<th>10</th>
<th>22</th>
<th>46</th>
<th>99</th>
<th>194</th>
<th>496</th>
<th>956</th>
<th>1678</th>
<th>mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of I-V points, $M$</td>
<td>80</td>
<td>84</td>
<td>87</td>
<td>90</td>
<td>93</td>
<td>88</td>
<td>87</td>
<td>88</td>
<td>87</td>
</tr>
<tr>
<td>$(V_{OC,\text{sim}}-V_{OC,\text{exp}})/V_{OC,\text{exp}}$ [%]</td>
<td>0.90</td>
<td>0.65</td>
<td>0.30</td>
<td>-0.04</td>
<td>-0.20</td>
<td>-0.23</td>
<td>-0.06</td>
<td>0.37</td>
<td>0.21</td>
</tr>
<tr>
<td>$(\eta_{\text{sim}}-\eta_{\text{exp}})/\eta_{\text{exp}}$ [%]</td>
<td>-0.88</td>
<td>-0.70</td>
<td>-0.56</td>
<td>-0.37</td>
<td>-0.60</td>
<td>-0.43</td>
<td>-0.10</td>
<td>0.74</td>
<td>-0.36</td>
</tr>
<tr>
<td>$[(1/M)\Sigma((I_{\text{sim}}-I_{\text{exp}})/I_{\text{sim}})^2]^\frac{1}{2}$ [%]</td>
<td>4.82</td>
<td>6.90</td>
<td>4.90</td>
<td>0.92</td>
<td>3.93</td>
<td>5.25</td>
<td>5.35</td>
<td>3.59</td>
<td>4.46</td>
</tr>
</tbody>
</table>

*a* The point closest to $V_{OC}$ is omitted from the rms calculation since the relative error in current is unbounded at OC.
arrangements. Strictly speaking, the voltage across a parallel array is not limited to the open-circuit voltage of the cell with the lowest illumination.

Mismatch loss has been historically investigated with regard to two phenomena occurring in series-connected flat-panel arrays: (1) shading/partial shading of cells or groups of cells [89, 90]; and (2) variability of \( I-V \) behavior from one cell to another resulting from manufacturing tolerances and ageing [91-96]. These phenomena are of stochastic behavior, and their analysis has generally been approached with statistical methods. In regard to dense and semi-dense arrays for concentrating systems, the resulting behavior is similar, but the cause is due to the more deterministic effects of nonuniform focal plane irradiance distributions caused by the concentrating optics.

5.4.1. Irradiance mismatch in serial arrays

In series, voltages add through a common current. Consider a string of \( N \) cells connected in series with cell-averaged irradiance\(^2\) \( E_{av,j} \) incident on the \( j_{th} \) cell. If there are no bypass diodes present between the series-connected cells, then the current delivered by the \( N \) cell string is limited to that of the cell having the lowest \( E_{av,j} \). Letting \( \langle E_{av} \rangle = Q/A_{active} \) be the average irradiance over the module (for no gaps between cells), then for perfect cell-to-cell uniformity, \( \langle E_{av} \rangle = E_{av} \). Consider the simple demonstrative case of fill factor \( FF = 1 \). The total power delivered by the \( N \) series-connected cells under perfectly uniform irradiance is:

\[
P_{\text{uniform}} \approx I_{SC} \cdot N \cdot V_{OC} \propto E_{av} \cdot N \cdot V_{OC} = \langle E_{av} \rangle \cdot N \cdot V_{OC}
\]  

(5.7)

where \( V_{OC} \) is the open-circuit voltage of a single cell. For the same average irradiance but now nonuniformly distributed from cell-to-cell, the power is:

\[
P_{\text{nonuniform}} \approx \min(I_{SC}) \cdot N \cdot V_{OC} \propto \min(E_{av}) \cdot N \cdot V_{OC}
\]

(5.8)

The ratio of power output for the uniform and nonuniform distributions is:

---

\(^2\) In this Chapter, the cell-averaged irradiance is denoted \( E_{av} \) to emphasize that it is not a local value, whereas in the rest of the thesis, \( E \) is used for average irradiance, except where it is obvious that it is a local value. The angled brackets \( \langle \rangle \) are used throughout this Chapter to denote the irradiance averaged over the active cell areas in the array.
The cell-to-cell uniformity, \( U = \min(E_{av})/\langle E_{av} \rangle \), is, for the case of a serial array with no bypass diodes, roughly proportional to the reduction in power.

**Series-connected arrays with bypass diodes**

The behavior of series-connected cells under cell-to-cell nonuniformity is slightly different when each series connection is coupled with a parallel bypass diode. Whereas in the case of no bypass diodes, the current is always limited to that of the cell with lowest illumination, the presence of bypass diodes allows an alternate path for the excess current to flow. If the forward voltage of the bypass diode \( v_F \) is considerably lower than the open-circuit voltage of a single cell \( v_{OC} \), then the short-circuit current of the series-connected array is very nearly that of a single cell having the highest illumination. When the voltage across the terminals of the array is zero (array short-circuit), the cell with highest illumination operates in low forward bias, delivering nearly its short-circuit current. The remaining cells are weakly reverse-biased, admitting slightly more than their short-circuit current, with the excess current flowing in parallel through the bypass diodes. As the voltage across the array is increased, the cells are one-by-one required to operate in forward bias to meet the prescribed voltage. If the cells are ordered from 1 to \( N \) in terms of decreasing irradiance, i.e. cell \( j \) has the \( j^{th} \) highest irradiance, the voltage at which the \( j^{th} \) cell switches from reverse to forward bias is approximated by:

\[
V_j = (j-1)v_{OC} - (N-j+1)v_F
\]

where \( V_j \) is the voltage across the terminals of the array, \( v_{OC} \) is the open-circuit voltage\(^3\) of a single cell at a representative (e.g. module average) uniform irradiance, and \( v_F \) is the voltage drop across a single bypass diode. At this voltage, the \( j^{th} \) cell operates in short-circuit, and the total current flowing through the terminals of the array is limited to the short-circuit current of that

---

\(^3\) Lowercase \( v \) is used to denote voltage difference across a single cell or diode, uppercase \( V \) is used to denote the voltage difference across the terminals of the mini-module.
cell. As the voltage across the array is swept from short- to open-circuit, the result is the characteristic stepped $I-V$ behavior typified by irradiance mismatch in series-connected arrays.

5.4.2. The no-cells-bypassed limit

For sufficiently high cell-to-cell nonuniformity, the maximum power point (MPP) occurs at a point where one or more of the cells is operating in reverse bias. The gain in voltage from the current limiting cell is insufficient to make up the loss in current by activating this cell, resulting in a power curve that decreases as the cell is switched to forward-bias. This case is intolerable because the cells operating in reverse bias do not provide any useful power, and – in the case of bypass diodes – the current must flow through the diode thus inducing a voltage drop. This case therefore serves as a useful occurrence from which to set a lower limit to the acceptable cell-to-cell nonuniformity on serial arrays.

Consider a set of $N$ cells connected in series, where the cells are numbered in terms of decreasing cell-averaged irradiance, i.e. cell 1 having the highest, and cell $N$ having the lowest. Most convenient is to have a relation for the limit of $U = \min(E_{av})/\langle E_{av} \rangle$ for which the criterion of no cells bypassed is just met, i.e. the power from all $N$ cells is equal to that delivered by the first $N-1$ cells having highest irradiance. To obtain a simple relation we consider each cell as having ideal square $I-V$ behavior, i.e. $FF = 1$, and neglect the logarithmic dependence of $v_{OC}$ on irradiance and the voltage drop across bypass diodes in forward bias $v_F$. Here $v_{OC}$ is the open circuit voltage of a single cell. The power delivered by the first $N-1$ cells having the highest cell-averaged irradiance is:

$$P_{1:N-1} = I_{SC,N-1} (N-1)v_{OC} = E_{av,N-1} I_{SC} (1 \text{ kW/m}^2)(N-1)v_{OC} \quad (5.11)$$

The power delivered by all $N$ cells is:

$$P_{1:N} = I_{SC,N} \cdot N \cdot v_{OC} = E_{av,N-1} I_{SC} (1 \text{ kW/m}^2) \cdot N \cdot v_{OC} \quad (5.12)$$
To just meet the condition of no-cells-shaded, the power delivered by all $N$ cells, given by Eq. (5.12), is equal to that delivered by the first $N-1$ cells, given by Eq. (5.11):

\[
P_{1,N} = P_{1,N-1}
\]  \hspace{1cm} (5.13)

\[
E_{av,N}I_{SC} \text{ (1 sun)} N\nu_{OC} = E_{av,N-1}I_{SC} \left(1 \text{ kW/m}^2\right)(N-1)\nu_{OC}
\]  \hspace{1cm} (5.14)

\[
\frac{E_{av,N}}{E_{av,N-1}} = \frac{N-1}{N}
\]  \hspace{1cm} (5.15)

Eq. (5.13) provides a relation between the cell with lowest irradiance to that with the second lowest irradiance. To obtain the relation in terms of $U$, we consider the most uniform distribution $\vartheta$ which just passes the no-cells-shaded requirement. This occurs when all 1 to $N-1$ cells have the same cell-averaged irradiance equal to $E_{av,N-1}$. This yields a module average irradiance of:

\[
\left\langle E_{av} \right\rangle = \frac{1}{N} \left[(N-1)E_{av,N-1} + E_{av,N} \right]
\]  \hspace{1cm} (5.16)

Eliminating $E_{av,N-1}$ from Eqs. (5.13) and (5.16) yields:

\[
U_{\text{critical}} = \frac{E_{av,N} \left\langle E_{av} \right\rangle}{N + 1}
\]  \hspace{1cm} (5.17)

The simple expression of Eq. (5.17) we denote the no-cells-shaded limit. Under realistic $I-V$ behavior this limit is conservative due to reduced voltage from the first $N-1$ cells due to finite voltage drop across the bypassed $N^{\text{th}}$ cell. The slightly less conservative limit considering a voltage drop across the bypass diode $v_F$ is (by a similar derivation):

\[
U_{\text{critical}} = \frac{N \left(v_F/\nu_{OC} - N + 1\right)}{v_F/\nu_{OC} - N^2 + 1}
\]  \hspace{1cm} (5.18)

Eqs. (5.17) and (5.18) are useful in selecting permissible or target levels of cell-to-cell nonuniformity for series connected arrays at a very early stage in the design, and may be used to determine the feasibility of optical configurations.
5.4.3. Experimental methods

Line-to-point focus systems produce unique irradiation distributions at the exit of their secondaries. In particular the cell-to-cell nonuniformity can be quite severe if attention is not given to homogenizing the distribution. Therefore a thorough experimental investigation of the effects of irradiance mismatch on the performance of the proposed mini-module was carried out. The irradiance distributions shown here are based on an early design of the InPhoCUS HCPV collector discussed in detail in Chapter 6.

The method to determine the effect of cell-to-cell nonuniformity on the cell performance consists of three steps: (1) optical modeling to determine the spatial distribution of irradiance over the array; (2) production of filters with a transmittance profile matching the normalized irradiance distribution; and (3) indoor measurement of the $I-V$ curve at different irradiance levels (Section 5.3.2).

**Optical modeling**

The spatial irradiance distribution on the mini-module resulting from an exemplary line-to-point focus optical system (See Chapter 6) was determined by Monte Carlo ray-tracing (MCRT). The distribution was resolved on a 20 by 20 grid over each cell (500 by 500 μm grid element size), with 100 million rays, using the in-house VeGaS code [69]. Figure 5.7 (a) shows a resulting spatial irradiance distribution $E(x, y)$ over the mini-module at a DNI of 1000 W/m$^2$. Since, in the scope of this experimental investigation, only cell-to-cell nonuniformity is considered, the distribution of local irradiance is averaged to obtain the average irradiance $E_{av,j}$ over each cell, as shown in Figure 5.7 (b), where:

$$E_{av,j} = \frac{1}{A_{cell}} \int_{A_j} E(x, y) \, dA_j$$  \hspace{1cm} (5.19)
Filter production

Filters were produced with a transmittance profile, Figure 5.7 (c), matching the normalized cell-averaged irradiance distribution, shown in Figure 5.7 (b). The (constant) transmittance of the filter over the $j^{th}$ cell is calculated as:

$$\tau_j = \frac{E_{av,j}}{E_{av,max}} \frac{\tau_{max}}{\tau_{max}}$$

where $E_{av,j}/E_{av,max}$ is the average irradiance over cell $j$ normalized by the maximum cell-averaged irradiance for all cells and $\tau_{max}$ is the maximum transmittance that can be achieved with the filter (bare substrate transmittance with no filter ink). For a given pre-filter irradiance $E_{\text{pre-filter}}$, the post-filter irradiance reaching each cell is then:

$$E_{av,j} = E_{\text{pre-filter}} \tau_j$$

The module-averaged post-filter irradiance is:

---

Note that the “av” subscript is omitted from the pre-filter irradiance since it is essentially uniform.
\[ \langle E_{av} \rangle = E_{pre-filter} \frac{1}{N} \sum_{j=1}^{N} \tau_j = E_{pre-filter} \langle \tau \rangle \] (5.22)

Filters were produced using a Xerox ColorQube 8750 printer by depositing black solid wax ink on a semi-transparent polyvinyl chloride film. Filter patterns were created by a computer script which generated a PostScript file that could be directly interpreted by the printer. Ink-jet and xerographic printing techniques were also considered, but the solid wax ink was found to give the best repeatability. The solid wax filters were found to degrade above 500 kW/m², therefore the filtered measurements were limited to a pre-filter irradiance of 500 kW/m². If measurements of higher irradiance are required, xerographic printing is preferred. The most important figures of merit for the filters are: spectral neutrality; linearity; contrast; and repeatability.

The spectral transmittance of similarly produced filters was previously analyzed and showed no significant non-gray behavior in the wavelength range 300 to 1200 nm [97]. The transmittance of the filter was controlled by changing the grayscale (GS) when generating the filter postscript image. Ideally, the correlation of GS to \( \tau \) would be linear, but in practice \( \tau \) depends on the printing process employed (filter substrate, ink, dithering algorithm). To determine this dependency, the filter transmittance was calibrated by placing filters of known GS in front of the module and measuring the resulting short-circuit current. The transmittance may then be found from:

\[ \tau(GS) = \frac{I_{SC}^{\text{unfiltered}}}{I_{SC}^{\text{filtered}}} \] (5.23)

The calibration was performed for a set of filters with grayscale ranging from 0 (maximum ink density) to 1 (bare substrate) resulting in a calibration curve of transmittance as a function of grayscale \( \tau = f(GS) \), shown in Figure 5.8. The grayscale required for a given transmittance is then \( GS = f^{-1}(\tau) \).

The contrast ratio is defined as the ratio of minimum to maximum transmittance that can achieved with the filter:
max \quad \min

\begin{align}
\tau = \frac{f (\text{GS} = 1)}{f (\text{GS} = 0)}
\end{align}

and indicates the highest peak-to-valley ratio (PVR) of the irradiance distribution that can be reproduced by the filter. For a single-layer filter, the maximum and minimum transmittances were 93.1% and 14.3% resulting in a contrast ratio of 6.5. Double and triple-layer filters, created by stacking identical single-layer filters of a given grayscale, were also investigated. For the double-layer filter the maximum and minimum transmittance were 88.0% and 2.3% respectively, resulting in a contrast ratio of 38.5 which is sufficiently high to reproduce the PVR range relevant for most CPV optical systems. For triple-layer filters, the contrast ratio was above 100 allowing recreation of distributions with a PVR of two orders of magnitude.

The repeatability of the filter production method was assessed by comparing the transmittance of a group of filters of equal nominal transmittance (grayscale). A set of seven filters of GS = 0.5 were compared. The standard deviation of the measured transmittance for the samples was below 1% of the mean transmittance value of 0.62.

Figure 5.8. Measured calibration curve for filter transmittance as a function of grayscale for single-layer, double-layer and triple-layer filters. Transmittance computed from Eq. (5.23). The double and triple-layer filters consist of a stack of identical filters each with the indicated grayscale.
For the InPhoCUS collector, discussed in detail in Chapter 6, the optical configuration changes with different skew angles. The nominal operating range of the InPhoCUS collector is from $\vartheta = -20^\circ$ to $+50^\circ$. To gain insight into the expected performance of the mini-module over the full skew range, the irradiance distribution on the mini-module was simulated for 8 skew angles every 10° on the interval $-20^\circ$ to $+50^\circ$. From the 8 simulated fluxmaps, 8 filters representing the cell-averaged irradiance distributions over the mini-module for different operating points were considered. The precise transmission profile of each filter was verified post-measurement by comparing the unfiltered and filtered $I$-$V$ curves at 200 kW/m$^2$ using the method outlined in [97]. In this method, the transmittance of the filter over each cell is taken as the ratio of unfiltered short-circuit to local short-circuit taken at a voltage calculated from Eq. (5.10). The resulting characteristics of the final filters are outlined in Table 5.4.

**I-V measurement**

A set of measurements analogous to those for uniform irradiance was then performed with the apodizing filters installed in front of the mini-module at irradiance levels of 99, 194, and 496 kW/m$^2$. For the measurement of filtered modules, the average irradiance incident on the cell is attenuated by a factor $\langle \tau \rangle$. The average pre-filter irradiance for the flash is found by comparing the monitor cell response to that of the reference measurement at the same cell-to-lamp distance:

$$E_{\text{pre-filter}} = E_{\text{av,ref}} \frac{E_{\text{av,monitor}}^{\text{monitor}}}{E_{\text{av,ref}}^{\text{monitor}}}$$  \hspace{1cm} (5.25)

Since the goal is to examine the effect of cell-to-cell nonuniformity at a given array input power, the relevant metric is the post-filter module-averaged irradiance, which may be determined from Eq. (5.22). The power at maximum power point $P_m$, fill factor $FF$, and efficiency $\eta$ were calculated from:

$$P_{\text{max}} = \max \left( I(t) \cdot V(t) \right)$$  \hspace{1cm} (5.26)
Table 5.4. Transmittance characteristics of the apodizing filters used to represent cell-to-cell nonuniformity present in a line-to-point focus concentrator (see Chapter 6).

<table>
<thead>
<tr>
<th>Filter</th>
<th>$\vartheta$</th>
<th>transmittancea [%]</th>
<th>PVRb</th>
<th>PAR</th>
<th>U</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\tau_1$</td>
<td>$\tau_2$</td>
<td>$\tau_3$</td>
<td>$\tau_4$</td>
</tr>
<tr>
<td>1</td>
<td>-20°</td>
<td>93</td>
<td>92</td>
<td>91</td>
<td>90</td>
</tr>
<tr>
<td>2</td>
<td>-10°</td>
<td>93</td>
<td>91</td>
<td>89</td>
<td>88</td>
</tr>
<tr>
<td>3</td>
<td>0°</td>
<td>93</td>
<td>91</td>
<td>87</td>
<td>87</td>
</tr>
<tr>
<td>4</td>
<td>10°</td>
<td>93</td>
<td>92</td>
<td>90</td>
<td>89</td>
</tr>
<tr>
<td>5</td>
<td>20°</td>
<td>93</td>
<td>92</td>
<td>89</td>
<td>88</td>
</tr>
<tr>
<td>6</td>
<td>30°</td>
<td>93</td>
<td>90</td>
<td>88</td>
<td>86</td>
</tr>
<tr>
<td>7</td>
<td>40°</td>
<td>93</td>
<td>89</td>
<td>86</td>
<td>81</td>
</tr>
<tr>
<td>8</td>
<td>50°</td>
<td>93</td>
<td>82</td>
<td>77</td>
<td>64</td>
</tr>
</tbody>
</table>

a Estimated uncertainty in transmittance: ±1% relative.
b PAR: peak-to-valley ratio (ratio of highest to lowest irradiance); PAR: peak-to-average ratio (ratio of highest to average irradiance); U: cell-to-cell uniformity (ratio of lowest to average irradiance).

\[
FF = \frac{P_{\text{max}}}{I_{\text{sc}} \cdot V_{\text{oc}}} \tag{5.27}
\]

\[
\eta = \frac{P_{\text{max}}}{A_{\text{active}} \cdot \langle E_{\text{av}} \rangle} \tag{5.28}
\]

For the area of Eq. (5.28), only the active area ($A_{\text{active}} = 5 \times 1 \text{ cm}^2 = 5 \text{ cm}^2$) was considered such that the efficiency does not include the fixed relative gap loss of 3.8%. It must be stated that the reported efficiency values are intended for comparing the mini-module performance under conditions of uniform and nonuniform irradiance, and are not intended as a report of the performance of the cells used.
Figure 5.9. Measured and simulated $I$-$V$ curves under nonuniform cell-averaged irradiance imposed by filters for a pre-filter irradiance of 194 kW/m$^2$. Fit is similar for 99 and 496 kW/m$^2$. Dashed lines of $V_j$ indicate approximate locations of local short-circuit for the cell with the $j^{th}$ highest irradiance. Cell temperature 25 °C ±0.1 °C.
Table 5.5. Relative errors between model and experiment under different nonuniform irradiance levels. Values are the average for all 8 filters.

<table>
<thead>
<tr>
<th>$\langle E_{av,pre}\rangle$ [kW/m²]</th>
<th>99</th>
<th>194</th>
<th>496</th>
<th>mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(V_{OC,sim} - V_{OC,exp})/V_{OC,exp}$ [%]</td>
<td>0.05</td>
<td>−0.11</td>
<td>−0.11</td>
<td>−0.06</td>
</tr>
<tr>
<td>$(\eta_{sim} - \eta_{exp})/\eta_{exp}$ [%]</td>
<td>−0.77</td>
<td>−0.53</td>
<td>−0.52</td>
<td>−0.61</td>
</tr>
<tr>
<td>$(\sqrt{(1/M)\Sigma((I_{sim} - I_{exp})/I_{sim})^2})$ [%]$^a$</td>
<td>3.29</td>
<td>1.99</td>
<td>3.50</td>
<td>2.93</td>
</tr>
</tbody>
</table>

$^a$ The point closest to $V_{OC}$ is omitted from the rms calculation since the relative error in current is unbounded at OC.

5.4.4. Results

Figure 5.9 shows the measured $I$-$V$ curves with filters placed in front of the mini-module to reproduce the cell-to-cell nonuniformity at a pre-filter irradiance of 194 kW/m². Superimposed are the $I$-$V$ curves predicted by the tuned model discussed in Section 5.2 considering the nonuniform cell-averaged irradiance distributions. Both experiment and simulation show a characteristic stepped $I$-$V$ behavior typified by cell-to-cell nonuniformity in series connected arrays [98]. In Figure 5.9 (f), lines at voltage $V_{SC,2}$ to $V_{SC,5}$ are added to help explain the shape of the stepped $I$-$V$ curve, where $V_{SC,2}$, $V_{SC,3}$, $V_{SC,4}$, and $V_{SC,5}$ correspond to the voltages, calculated from Eq. (5.10) at which the cells with the second, third, fourth, and fifth highest average irradiance are in local short-circuit, respectively. Here, values of $V_{OC} = 15.4$ V and $V_F = 0.35$ V were used.

The simulated results match the stepped $I$-$V$ behavior over the full curve with good accuracy, serving as an additional experimental validation of the equivalent-circuit model. Table 5.5 shows the relative error between simulated and measured $V_{OC}$ and $\eta$, and the rms error over the $I$-$V$ curve for each filter at a pre-filter irradiance of 194 kW/m². The match is comparatively worse for Filter 8 which exhibits the largest nonuniformity. The errors are on the order of that for the unfiltered results suggesting that the model can equally well predict the $I$-$V$ curve under conditions of cell-to-cell nonuniformity. Comparing the values of Table 5.5 to those of Table 5.3 it is seen that the model is capable of describing the nonuniform curves with the same level of accuracy as the uniform curves for which the parameters were tuned.
The validated model allows for additional insight into the operation of the array under conditions of irradiance mismatch, which would be otherwise be available only by demanding experimental probing. Figure 5.10 (a) shows the voltage across each stage, i.e. cell/bypass diode pair, as a function of the voltage across the mini-module. For clarity, the stages are labeled according to the cell-averaged irradiance (1 = highest irradiance, 5 = lowest irradiance), rather than using their physical location in the mini-module.

To understand the behavior of the mini-module under conditions of irradiance mismatch, consider the case where cell 1 with highest irradiance is operating in short-circuit, i.e. \( V_1 = 0 \), and delivers a current \( I_{SC,1} \). This point may be visualized in Figure 5.10 (a) as the intersection of the \( V_1 \) curve with the dashed line at abscissa \( V = 0 \). Due to the serial arrangement of the mini-module, all of the current flowing through cell 1 must flow through each cell-bypass diode pair. Since \( E_{av,2} < E_{av,1} \), the second cell must operate in reverse bias.

Figure 5.10. Simulated (a) voltage across each cell/bypass diode pair in the series string; and (b) current through cell (thick lines) and bypass diode (thin lines) branches as a function of the voltage across the mini-module. Cells are numbered from highest (1) to lowest (5) average irradiance. Cell temperature 25 °C ±0.1 °C.

\[ V_1, V_2, V_3, V_4, V_5 \]

\[ i_1, i_2, i_3, i_4, i_5 \]

\[ i_{BD,1}, i_{BD,2}, i_{BD,3}, i_{BD,4}, i_{BD,5} \]

\[ V_F \]

---

1 It was confirmed by simulation that the physical order of the cells has no effect on the resulting \( I-V \) curve.
(\nu_2 < 0\) in order to admit the excess current, as seen by dropping a vertical line to the \(\nu_2\) curve in Figure 5.10 (b). With no bypass-diode, cell 2 would be likely forced to reverse breakdown in order to accommodate this excess current. However since the cell and bypass-diode have reverse polarity, with the second cell in reverse bias, bypass-diode 2 now operates in forward-bias and can admit a large current due to the exponential behavior of Eq. (5.2). The current is then split between the cell with \(i_2 \approx i_{SC,2}\), and the diode with \(i_{BD,2} \approx i_{SC,1} - i_{SC,2}\), as seen in Figure 5.10 (b). The voltage drop \(\nu_2\) across the cell/bypass-diode 2 is then the solution of Eq. (5.2) for \(i_{BD,2}\), which may be approximated as the forward voltage drop of the diode at a representative current, \(\nu_F \approx 0.35 \text{ V}^2\). The same behavior occurs for cells 3, 4 and 5, such that the total voltage across the mini-module is \(V \approx -4 \cdot \nu_F = -1.4 \text{ V}\). This value is confirmed from Figure 5.10 (a) by dropping a vertical line from the intersection of \(\nu_1\) and the zero of the abscissa (stage voltage) to intersect the ordinate (mini-module voltage). Therefore, to achieve short-circuit across the terminals of the mini-module, cell 1 must operate at a forward bias of \(\nu_1 \approx 1.4 \text{ V}\).

Following similar reasoning, the \(I-V\) curve of the mini-module under conditions of cell-to-cell nonuniformity may be constructed by taking the \(I-V\) curves of the individual cells under the corresponding cell-averaged irradiance \(E_{av,j}\) and shifting the voltage scale of the \(j^{th}\) cell by:

\[
\sum_{k=1}^{j-1} \nu_{OC,k} - (N - j)\nu_F
\]

The resulting set of curves will then resemble the set of \(i_j\) curves in Figure 5.10 (b). The overall \(I-V\) curve of the mini-module is then the upper bound of this set of curves, as confirmed by comparison of Figure 5.10 (b) and Figure 5.9. It is important to note that while the presence of bypass diodes increases the short-circuit current of a serial array, it does not increase current at the maximum power point (and hence efficiency) unless the MPP occurs at a voltage below \(V_{SC,N}\). For \(V_{MPP} < V_{SC,N}\), the cell with the lowest irradiance is

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\(^2\) Due to the on-off exponential diode behavior, \(\nu_F\) may be treated relatively constant for a large range of currents.
bypassed indicating that in this case the irradiance is necessarily worse than the no-cells-bypassed limit (Section 5.4.2).

**Figure 5.11** shows the cell efficiency vs. irradiance for uniform and nonuniform distributions. To allow comparison for equal radiant power input over the mini-module, the results are plotted against the (post-filter) module-averaged irradiance. The model predicts the experimental results well and shows that a significant reduction in efficiency is expected due to cell-to-cell nonuniformity, particularly for filter #7 and #8 representing the irradiance distribution at skew angles of 40° and 50°, respectively.

5.4.5. Prediction of InPhoCUS mini-module efficiency

In Chapter 6, the InPhoCUS line-to-point focus collector is presented in detail. This system is used as an example to demonstrate how the results of the preceding sections can be used to estimate the array efficiency under different operating conditions. The mini-module investigations were actually performed
in advance to the finalization on the InPhoCUS design, and thus the values reported here do not correspond exactly to the final design of Chapter 6. The methods and general trends are, however, the same. The module averaged irradiance for a given direct normal irradiance DNI, skew angle $\vartheta$, geometric concentration $C_g$, and overall optical efficiency $\eta_{opt, overall}$ is given by:

$$\langle E_{av} \rangle = \eta_{opt, overall} \cdot C_g \cdot DNI \cdot \cos \vartheta$$

(5.30)

The optical efficiency of the InPhoCUS collector is a weak function of the skew angle over the range $-20^\circ \leq \vartheta \leq 50^\circ$ and is here assumed constant at 78%. Combining Eq. (5.30) with the $\eta(\langle E_{av} \rangle)$ curves of Figure 5.11, the mini-module efficiency can be determined for any irradiance and $\vartheta$. Representative results are presented in Figure 5.12. With a geometric concentration of $\sim 600 \times$, it is clear that high DNI values are preferred, since as visible in Figure 5.11 and Figure 5.5, the mini-module efficiency peaks at around 500 kW/m$^2$. At high skew angles ($> 30^\circ$), the efficiency is degraded due primarily to irradiance

**Figure 5.12.** Prediction of InPhoCUS mini-module efficiency as a function of the skew angle $\vartheta$ and direct normal irradiance DNI, considering combined effect of optical efficiency, cosine loss, concentration, and cell-to-cell nonuniformity. Cell temperature 25 °C ±0.1 °C.
mismatch, with reduced concentration due to cosine loss causing a secondary reduction.

5.4.6. InPhoCUS mini-module redesign

The results show a considerable degradation of performance at high skew angles resulting from increased cell-to-cell nonuniformity. One possible solution is optical redesign; however, for the given configuration, improved uniformity generally comes at the expense of optical efficiency. It was therefore decided to change the mini-module design to reduce the effect of the nonuniformity. The catchall solution, mitigating all nonuniformity related effects, would be to connect cells in separate serial strings along the length of the trough, i.e. cell 1 from one mini-module would be connected in series to cell 1 of the adjacent mini-module, and so on. Due to the translational symmetry of the primary concentrator, the irradiance would be well-matched along series-connected cells in this configuration. However, this design would require complex cabling, expensive connectors at each mini-module, and many inverter inputs, and therefore was considered impractical.

A simpler solution is to connect the cells in each mini-module in parallel, and each mini-module in series along the length of the trough. It was found that for all expected operating conditions, the difference in efficiency between a parallel connected mini-module and the above catchall solution was negligible. Furthermore, with a parallel arrangement it should be possible to significantly reduce the gap loss. Superimposed in Figure 5.12 is the efficiency vs. skew angle curve for the re-designed mini-module with parallel connections, showing a clear improvement in efficiency and constant performance over the entire operating skew range.

5.5. Effect of single-cell nonuniformity

Single-cell nonuniformity leads to an increase in local current density, which in turn increases Ohmic losses due to the finite resistivity of the cell. It has been

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3 Material from this section has been extracted from I. Nageleisen, “Development of a 3D distributed equivalent circuit model for triple-junction concentrator cells,” Semester Project, ETH Zurich, 2012, conducted under the direct supervision of T. Cooper.
studied from both experimental [75, 99-102] and analytical [103-106] perspectives. Some related methods to modify the distribution of irradiance, enabling experimental characterization are presented in [107, 108]. The filtering method considered in Section 5.4.3 may be easily extended to measure single-cell nonuniformity. However, it is usually more convenient to estimate the effect of single-cell nonuniformity by simulation.

5.5.1. 3D distributed equivalent circuit model

To study the effects of the local irradiance distribution on triple-junction solar cells, a 3D distributed equivalent circuit model has been developed in the SPICE code. The model is based on that in [76] and divides the solar cell into a grid of elementary volumes each representing a small part of the solar cell. A two-region model is employed meaning that the illuminated regions of the cell are treated differently than the dark regions underneath the front metallization. The two elementary volumes are shown schematically in Figure 5.13.
5.5.2. Grid design

The grid should be made small enough such that its dimensions are not larger than the typical dimensions of the physical processes taking place in the cell. In particular, the grid on the busbars and fingers at the boundary with the illuminated regions should be smaller than the metal transfer length, and the grid should be smaller than the minority carrier diffusion length in the illuminated region [76]. This implies a grid pitch on the order of 2.5 μm in the boundary regions and 30 μm in the illuminated regions. A section of a grid based on these dimensions is shown in Figure 5.14 (a).

The majority of previous distributed equivalent circuit models have considered 1 mm² cells. For cells of this size, a maximum grid size of 160 000 would be required. The area of the cells considered here is 100 times larger (1 cm²) and thus a prohibitively large grid would be required at these small grid pitches. For example, the solution of 8 I-V curve points on a grid with 100 000 nodes took 14.5 hours on two 3 GHz cores. Furthermore, the simulation time was found to increase superquadratically with the number of nodes.

For this reason, it was deemed necessary to increase the elementary volume above the physically recommended size in order to keep simulation times manageable. The coarsened grid is shown in Figure 5.14 (b). The width of the
illuminated elementary cells has been made smaller in the vertical direction since the gradient of the irradiance in this direction is expected to be larger than that in the horizontal direction.

5.5.3. Model verification

To verify that the coarsening of the simulation grid does not significantly affect the results of the simulation, the results of the fine grid were compared to the coarse grid. For this purpose, the 1 mm² cell considered in [76] was used as a basis such that the currently developed model could also be compared to the results presented there. Figure 5.15 shows the current-voltage and power-voltage characteristics under uniform illumination at 847 kW/m². Excellent agreement between the fine mesh and the results of [76] indicate that the model is working correctly. Moreover, it is seen that the error between the fine mesh and coarse mesh is small and that the coarse mesh is conservative in estimating the maximum power point.

Figure 5.16 shows the power-voltage curve under the Gaussian nonuniform illumination pattern considered in [76]. Again the agreement for the fine mesh is very good, and the error for the coarse mesh is tolerable. Importantly, the coarse mesh is conservative in predicting the performance under nonuniform
Figure 5.16. Power-voltage characteristic for a Gaussian irradiance distribution for the fine (square markers) and coarse (triangle markers) grids, in comparison to the simulations of Steiner (line).

illumination. Based on the comparisons between the fine and coarse meshes, we conclude that the coarse mesh is capable of predicting the cell $I-V$ and $P-V$ curves with acceptable accuracy under both uniform and nonuniform illumination. The coarse mesh is therefore utilized for all subsequent simulations.

5.5.4. Model tuning and validation

The verified model was tuned against the set of experimental results for the mini-module under uniform illumination presented in Section 5.4.3. Table 5.6 shows the resulting best-fit parameters for the distributed equivalent circuit model. Figure 5.17 shows the efficiency vs. irradiance curve for the mini-module comparing the measurements, lumped equivalent circuit model predictions (Section 5.2), and the predictions of the distributed equivalent circuit model. It is evident that the 3D distributed equivalent circuit model is superior to the lumped model in its ability to reproduce the experimental results. In particular, the irradiance at which maximum efficiency occurs and then starts to decline is better matched by the distributed model.
Table 5.6. Best fit parameters of the 3D distributed equivalent circuit model used for the simulation of the 1 cm² III-V triple-junction concentrator cells. See also Figure 5.13.

<table>
<thead>
<tr>
<th>Subcell</th>
<th>param.</th>
<th>value</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>front grid</td>
<td>$\rho_M$</td>
<td>$1.8 \times 10^{-6} , \Omega , \text{cm}$</td>
<td>resistivity of metallization</td>
</tr>
<tr>
<td>front grid</td>
<td>$W_F$</td>
<td>10 $\mu$m</td>
<td>width at base of grid fingers</td>
</tr>
<tr>
<td>front grid</td>
<td>$h_F$</td>
<td>5 $\mu$m</td>
<td>height of grid fingers</td>
</tr>
<tr>
<td>front grid</td>
<td>$L_{\text{pitch}}$</td>
<td>125 $\mu$m</td>
<td>distance between grid fingers</td>
</tr>
<tr>
<td>front grid</td>
<td>$R_C$</td>
<td>$3.5 \times 10^{-5} , \Omega , \text{cm}^2$</td>
<td>contact resistance</td>
</tr>
<tr>
<td>top</td>
<td>$R_{LCL1}$</td>
<td>250 $\Omega$/sq$^a$</td>
<td>sheet resistance of lateral conduction layer 1</td>
</tr>
<tr>
<td>top</td>
<td>$J_{01,1}$</td>
<td>$5.0 \times 10^{-18} , \text{A/cm}^2$</td>
<td>radiative recombination</td>
</tr>
<tr>
<td>top</td>
<td>$J_{02,1}$</td>
<td>$1.0 \times 10^{-12} , \text{A/cm}^2$</td>
<td>SRH recombination</td>
</tr>
<tr>
<td>top</td>
<td>$R_{LCL2}$</td>
<td>500 $\Omega$/sq</td>
<td>sheet resistance of lateral conduction layer 2</td>
</tr>
<tr>
<td>tunnel junction 1</td>
<td>$R_{TJ1}$</td>
<td>$5.0 \times 10^{-4} , \Omega , \text{cm}^2$</td>
<td>resistance of tunnel junction 1</td>
</tr>
<tr>
<td>middle</td>
<td>$R_{LCL3}$</td>
<td>65 $\Omega$/sq</td>
<td>sheet resistance of lateral conduction layer 3</td>
</tr>
<tr>
<td>middle</td>
<td>$J_{01,2}$</td>
<td>$4.0 \times 10^{-20} , \text{A/cm}^2$</td>
<td>radiative recombination</td>
</tr>
<tr>
<td>middle</td>
<td>$J_{02,2}$</td>
<td>$2.0 \times 10^{-11} , \text{A/cm}^2$</td>
<td>SRH recombination</td>
</tr>
<tr>
<td>middle</td>
<td>$R_{LCL4}$</td>
<td>298 $\Omega$/sq</td>
<td>sheet resistance of lateral conduction layer 4</td>
</tr>
<tr>
<td>tunnel junction 2</td>
<td>$R_{TJ2}$</td>
<td>$5.0 \times 10^{-4} , \Omega , \text{cm}^2$</td>
<td>resistance of tunnel junction 2</td>
</tr>
<tr>
<td>bottom</td>
<td>$R_{LCL5}$</td>
<td>17 $\Omega$/sq</td>
<td>sheet resistance of lateral conduction layer 5</td>
</tr>
<tr>
<td>bottom</td>
<td>$J_{01,3}$</td>
<td>$1.0 \times 10^{-15} , \text{A/cm}^2$</td>
<td>radiative recombination</td>
</tr>
<tr>
<td>bottom</td>
<td>$J_{02,3}$</td>
<td>$1.0 \times 10^{-7} , \text{A/cm}^2$</td>
<td>SRH recombination</td>
</tr>
</tbody>
</table>

$^a$ The unit $\Omega$/sq “ohms per square” is used in specifying the resistance of two-dimensional structures such as thin films. It represents the resistance across a two-dimensional slab having a square aspect ratio.
5.5.5. Irradiance distributions

The validated distributed equivalent circuit model was used to investigate the effect of realistic irradiance distributions on the performance of the cell. For this purpose, the three nonuniform distributions, shown in Figure 5.18, were considered. These distributions were determined by Monte Carlo ray-tracing of the optical system of the InPhoCUS line-to-point focus collector described in detail in Chapter 6. The three cases considered were designed to represent the best and worst-case conditions of single-cell nonuniformity expected to occur in the InPhoCUS collector.

The distribution shown in Figure 5.18 (a) is for the middle cell of the mini-module at zero skew and assuming an angular dispersion error of $\sigma_{\text{err},2} = 2$ mrad for the secondary concentrator. The resulting distribution has the highest peak irradiance and peak-to-average irradiance ratio (PAR) occurring in the system, and is thus indicative of the worst-case performance.
The distribution in Figure 5.18 (b) is also at zero skew, but assumes a less specular secondary mirror material \((\sigma_{\text{err},2} = 6 \text{ mrad})\) leading to a more spread out irradiance distribution.

The distribution shown in Figure 5.18 (c) is for the middle cell of the mini-module at \(\theta = 50^\circ\) \((\sigma_{\text{err},2} = 2 \text{ mrad})\) and is indicative of the lowest PAR (most-uniform distribution) occurring in the normal operation range of the collector.
In addition to the simulated distributions for the secondary concentrator, circular Gaussian distributions, e.g. Figure 5.18 (d), having the same peak and average irradiance as the simulated distributions were considered. As most conventional CPV concentrating systems produce a Gaussian-like irradiance distribution on the cell, this affords a comparison between the behavior of the optical system considered and that of other CPV concentrators.

5.5.6. Results

Figure 5.19 shows the $I$-$V$ curves for the irradiance distributions considered as predicted by the distributed equivalent circuit model. Comparing the distributions for normal incidence ($E_{av} = 495 \text{ kW/m}^2$) we see that the uniform case has the best performance as expected. However, the $I$-$V$ curves for the simulated distribution, for both $\sigma_{err,2} = 2 \text{ mrad}$ and $\sigma_{err,2} = 6 \text{ mrad}$, are very close to the $I$-$V$ curve for uniform illumination. In contrast, the circular Gaussian distribution having the same PAR as the $\sigma_{err,2} = 2 \text{ mrad}$ distribution shows a considerable drop in performance. Calculating the efficiency from the maximum power point of the $I$-$V$ curve we find that the efficiency drop for the
Gaussian distribution is −6.7% relative, while the drop for the simulated distributions is a mere −0.85% for the 2 mrad distribution and −0.63% for the 6 mrad distribution. Similar trends are visible for the distributions at \( \theta = 50^\circ \) \( (E_{av} = 338 \text{ kW/m}^2) \) where only a very slight difference between the uniform and simulated nonuniform distributions can be seen.

The results indicate that the irradiance distributions occurring on linear semi-dense arrays situated at the outlet apertures of line-to-point focus systems have negligible adverse effects on cell performance. This can be seen as an unexpected benefit of the line-to-point focus systems, which enable the use of linear semi-dense arrays where the hotspots tend to be situated near the busbars of the cells.

5.6. Summary

We have introduced the semi-dense array which of particular interest for use in HCPV systems based on line-to-point focus concentrators. The array performance was measured at irradiance levels ranging from 1 to \( >1500 \text{ kW/m}^2 \) using a modified large-area flash solar simulator. The results were used to tune a lumped single diode equivalent circuit model of the triple-junction solar cell of interest.

A method based on printed apodizing filters with calibrated transmittance was developed to recreate conditions of spatial irradiance nonuniformity present in real optical systems without the need for fabrication or prototyping of the optical system. The method was used to determine the effect of mismatch losses resulting from cell-to-cell uniformity, estimated by Monte Carlo ray-trace of the optical system, on a linear array of 5 series-connected 1 cm\(^2\) triple-junction concentrator cells under a wide range of concentrations. An equivalent circuit model consisting of a single lumped single-diode model for each triple-junction cell was developed and fitted against open-circuit voltage and efficiency measured under uniform irradiance. The rms error over the full set of \( I-V \) curves from 10 kW/m\(^2\) to 1650 kW/m\(^2\) was 4.75%. The model was then validated against the experimentally measured \( I-V \) curves under nonuniform irradiance. An rms error of 3.66% was obtained over the full set of
measurements for 8 representative irradiance distributions at pre-filter irradiance levels of 99, 194, and 496 kW/m$^2$.

Both experiment and model revealed the characteristic stepped $I-V$ behavior associated with current mismatch in series-connected arrays. The validated model was then used to predict the efficiency vs. concentration curve from 10 to 2000 kW/m$^2$ for the cell-to-cell nonuniformity imposed by the representative irradiance distributions from the InPhoCUS collector (Chapter 6). The results indicate that for solar incidence angles (skew angles) above 30°, the efficiency of the InPhoCUS collector is significantly reduced due to cell-to-cell nonuniformity. Based on this conclusion, the InPhoCUS mini-module was re-designed with parallel cell interconnections to mitigate the effects of cell-to-cell nonuniformity.

To determine the effects of single-cell nonuniformity, a 3D distributed equivalent circuit model was developed. The model predicts that the single-cell nonuniformity for the representative line-to-point focus system has negligible impact on cell performance. This may be viewed as an unexpected advantage of line-to-point focus systems in comparison to conventional point-focus HCPV system which produce a central hotspot on the cell which can significantly reduce cell performance.
Chapter 6

A practical implementation: the InPhoCUS HCPV collector

This chapter presents the culmination of many of the ideas presented in the preceding chapters into the practical implementation of a high-concentration photovoltaic collector. The design of this collector has been carried out in collaboration with Airlight Energy Manufacturing SA, located in Biasca, Switzerland under the project title Inflated Photovoltaic Ultra-light mirror concentratorS (InPhoCUS).

Concentrating photovoltaic (CPV) collectors offer the advantage over conventional flat panel PV of requiring a much smaller cell area for the same output power. This is achieved by intercepting incident solar radiation with a collecting aperture, which may be a mirror, lens or other optical device, and then focusing it onto a solar cell or array receiver. However, this is only advantageous if the cost per m$^2$ of collecting aperture is significantly less than the cost per m$^2$ of solar cell. Minimization of the cost per m$^2$ of the collecting aperture may therefore be seen as a fundamental driver for the cost-competitiveness of CPV collectors.

As discussed in Section 1.2.1, CPV systems are classified according to their geometric concentration ratio, $C_g$, defined as the collecting aperture area to the active cell area. In high-concentration photovoltaic (HCPV) collectors, geometric concentrations greater than 400× are utilized [109], enabling the use of high-efficiency concentrator cells [10, 110], whose application in lower concentration systems would be prohibited due to their high cost. The efficiencies of triple-junction cells have reached 44% under concentration [10]

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1 Material in this chapter has been submitted for publication as T. Cooper, G. Ambrosetti, F. Malnati, I. Nageleisen, A. Pedretti and A. Steinfeld, “Full-scale experimental demonstration of a high-concentration photovoltaic system based on a parabolic trough with tracking secondary optics.”
with further improvements foreseen with four- and five-junction cells. The majority of optical systems for HCPV collectors utilize three-dimensional concentrators, e.g. Fresnel lenses, two-mirror systems, or parabolic dishes, mounted on two-axis trackers [109]. While these systems can easily obtain the required concentration ratios with large acceptance angles [6], their three-dimensional geometries present a barrier to the primary requirement of minimizing the cost per m² of collecting aperture. We propose an alternative approach to HCPV based on parabolic trough primary collectors (PTCs), which, due to their linear symmetry, have a lower-cost construction in comparison to three-dimensional designs. Furthermore, PTCs can be very easily scaled simply by increasing the length of the trough. These properties make parabolic troughs an attractive alternative for achieving the primary objective of aperture area cost minimization.

Parabolic-trough-based CPV collectors have been considered in the past [6], but due to the fundamental concentration limit of $1/\sin \theta_{\text{sun}} \approx 215 \times$ for a 2D trough-like design [23], these systems have generally been restricted to medium concentration applications, which cannot make effective use of high-efficiency multijunction cell technology. The highest concentration achieved on a parabolic-trough-based system was of $300 \times$, on the innovative line-to-point (LTP) focus system [27, 52] comprising an asymmetric parabolic trough with an array of 3D dielectric-filled secondary concentrators arranged along its focal line.

The LTP systems presented in Chapter 4, allow significantly higher concentration to be achieved on a trough-based system. In particular it was demonstrated that by incorporating a secondary axis of tracking into each secondary concentrators, resulting in an optical system theoretically capable of achieving geometric concentrations exceeding $6000 \times$ [111]. Of course the introduction of a tracking axis into each secondary inevitably introduces additional complexities into the system, but as the primary concentrator represents by far the largest component of the solar collector as a whole, the cost-implications of the increase secondary concentrator complexity are minor. Tracking-secondary LTP systems therefore provide new avenues for reducing the cost per m² of HCPV collectors.
This chapter presents the design, modeling and experimental demonstration of an all-reflective HCPV collector based on a line-to-point focus concentrator with tracking secondary optics. The proposed collector aims to significantly reduce the cost per m² of collecting aperture in comparison to conventional HCPV collectors through: (1) implementation of the tracking-secondary LTP concentrator arrangement; and (2) use of a low-cost inflated polymer membrane construction for the primary trough concentrator.

6.1. Design

6.1.1. System configuration

The basis of the collector is an inflated parabolic trough concentrator, shown in Figure 6.1, originally designed for thermal concentrating solar power applications [34]. The collector utilizes a cost-effective polymer membrane mirror construction based on the arcspline design discussed in Section 3.3 and in [34]. The primary concentrator is split into two wings, each having an individual focal line, as seen in Figure 6.1 (a). Each wing is constructed from a stack of four 23 μm thick polymer (BoPET) membranes whose shape is
controlled by adjusting the inflation pressures between membranes [34]. The topmost membrane is PVD aluminized to form the primary mirror surface. This inflated membrane structure forms a four-arc arcspline concentrator (ASC) whose general design characteristics were detailed in Section 3.3.

The membrane mirror is enclosed in a balloon comprising a 100 μm thick transparent self-cleaning, scratch-resistant ethylene tetrafluoroethylene (ETFE) topsheet and a vinyl coated polyester bottom sheet. An overpressure of ~200 Pa is maintained within the balloon allowing a stable convex shape to be maintained by the mirror membrane stack. The balloon creates a controlled environment in which to house the sensitive CPV equipment and isolates the membrane mirror stack from wind loads on the balloon [15].

The inflated trough is mounted on a cast concrete frame with an integrated one-axis tracking system. The concrete structure offers improved rigidity and wind-resistance in comparison to a steel frame. This construction allows primary aperture widths of 10 m and trough lengths of 200 m to be achieved. The large scale of the primary tracking wheel allows for primary tracking errors below 0.1°. The system construction offers some logistical advantages in comparison to competing PTC technologies: the concrete components can be cast on or near site, creating opportunities for local labor content; and the mirrors and balloon materials can be shipped in rolls, a significant advantage remote sites.

As mentioned, in order to augment to the concentration into the realm of high-concentration photovoltaics (HCPV), the tracking-secondary line-to-point focus optical configuration as discussed in detail in Section 4.4 and [111], has been adopted. The optical configuration is shown schematically in Figure 6.2. Radiation incident on the collector passes through the transparent balloon and is reflected by each wing of the primary concentrator to one of two primary focal lines. Along the focal line of the primary an array of nonimaging secondary concentrators, which further concentrate the sunlight to a solar cell array situated at the exit of each secondary, is placed. To achieve a high-concentration regardless of the position of the sun, the secondary concentrators are allowed to rotate about individual rotation axes, perpendicular to the N-S tracking axis of the primary concentrator.
A key factor in designing the optical system is the skew angle $\vartheta$, defined in Figure 2.8 and shown also in Figure 6.2. If the tracker is restricted to be horizontal (polar trackers are not practical for large-scale troughs), the skew angle range is minimized when the tracking axis is oriented N-S. From Table 2.2, the minimum and maximum skew angles for a horizontal N-S one-axis tracker are then given by

$$
\begin{align*}
\vartheta_{\text{min}} &= -\arcsin \left( \sin 23.44^\circ / \cos \phi \right) \\
\vartheta_{\text{max}} &= 23.44^\circ + |\phi|
\end{align*}
$$

(6.1)

The proposed collector is designed for a latitude of $30 - 35^\circ$ corresponding to North Africa or Southwestern United States. From Eq. (6.1), the skew angle range for $\phi = 35^\circ$ is $-29^\circ$ to $58^\circ$. Figure 6.3 shows the probability distribution of daylight hours (solid line) as a function of skew angle for a horizontal N-S one-axis tracking collector at $\phi = 30^\circ$, giving an indication of how many hours the collector will operate per year at a given skew angle. The curve integrates to
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4380 hours, i.e. the total number of daylight hours per year, and its shape is demarcated by the four key positions (vertical dashed lines) corresponding to at sunrise/sunset and solar noon on the summer (SS) and winter (WS) solstices. Also shown is the probability distribution of collectable irradiation $dE = DNI \cdot \cos \vartheta \cdot dt$, assuming a constant DNI year round (dashed line); and assuming Hottel’s clear-sky model for a mid-latitude climate (dotted line).

Figure 6.3. Probability distribution of daylight hours $t$ (solid line) as a function of the skew angle for a horizontal N-S one-axis tracker at a latitude of 30°. The curve integrates to 4380 hours, and its shape is demarcated by the four vertical dashed lines at sunrise/sunset and solar noon on the summer (SS) and winter (WS) solstices. Also shown is the probability distribution of collectable irradiation $dE = DNI \cdot \cos \vartheta \cdot dt$, assuming a constant DNI year round (dashed line); and assuming Hottel’s clear-sky model for a mid-latitude climate (dotted line).

A more relevant quantity is the distribution of available irradiation $dE = DNI \cdot \cos \vartheta \cdot dt$ with respect to the skew angle. The dashed curve shows the available radiation as a function of skew angle considering a constant DNI of 1000 W/m$^2$, i.e. only considering cosine loss. The dotted line shows the available radiation when the DNI is described by Hottel’s clear-sky model [112] for a mid-latitude climate. It is seen that (in the northern hemisphere) the majority of collectable solar radiation occurs for positive (southward) skew angles. A nominal operational skew angle range of $-20^\circ$ to $+50^\circ$ is chosen for
6.1.2. Primary concentrator design

The basis of a line-to-point focus system is a one-axis tracking trough primary concentrator. As shown in Section 4.4 and [111], the achievable overall geometric concentration is strongly dependent on the primary rim angle, with rim angles around $10^\circ$ yielding the maximum $C_g$. In order to achieve a small effective rim angle, which maintaining a compact design, an asymmetric trough with a two-wing configuration was utilized, as seen in Figure 6.4 (a). The detailed design of the primary concentrator geometry is shown in Figure 6.4 (a), and the main parameters are listed in Table 6.1. The inlet aperture width of one wing of the primary concentrator is $a_i,1 = 4.85$ m, and the inner and outer rim angles are $\Phi_1 = 7.1^\circ$ and $\Phi_2 = 77.9^\circ$, respectively, yielding an effective rim angle of $\Phi = \frac{1}{2}(\Phi_2 - \Phi_1) = 35.4^\circ$. In order to maximize the achievable secondary concentration (Section 4.4 and Appendix D), the primary focal plane was tilted from horizontal by an angle $\delta = \frac{1}{2}(\Phi_i + \Phi_o) = 42.5^\circ$. The shield shown in the detail of Figure 6.4 (a) is to protect the downstream electronics from spilled radiation and is not an active concentrating component.

The required primary acceptance angle to collect all radiation year-round from within the solar disk is (see Section 2.4.5):

$$\theta_{i,1,\text{full-collection}} = \arcsin\left(\frac{\sin \theta_{\text{sun}}}{\cos \delta_{\text{max}}}\right) = 0.41^\circ$$

(6.2)

From Appendix D, the maximum full-collection geometric concentration of an asymmetric parabolic trough with its focal plane tilted to bisect the inner and outer polar angles is:

$$C_{g,1,\text{full-collection}} = \frac{1}{2} \csc \theta_{i,1} \left[ \tan \frac{1}{2} \Phi_2 - \tan \frac{1}{2} \Phi_1 \right] (1 + \cos \Phi_2) \cos(\Phi) = 51.4 \times$$

(6.3)

which corresponds to a focal plane width of $a_o,1 = 9.4$ cm for the 4.85 m inlet aperture width of the given primary. In order to achieve a high overall geometric concentration, it was decided to decrease the focal plane width to 6.5 cm, resulting in a primary concentration of $C_{g,1} = 72.4 \times$, at the expense of
some spillage losses. Solving Eq. (6.3) for $\theta_{i,1}$ given $C_{g,1} = 72.4\times$ gives a nominal acceptance angle for the primary concentrator of 0.29°.

6.1.3. Secondary concentrator design

Potential secondary concentrator designs for tracking-secondary line-to-point focus concentrators are asymmetrically truncated compound parabolic concentrators (CPCs), multifoliate light-pipe concentrators (LPCs), and dielectric tailored edge-ray concentrators (DTERCs) [111]. To achieve an all-reflective optical system the asymmetrically truncated CPC was deemed most appropriate. However, in order to limit the angle of incidence (AOI) on the cells, a $\theta_i/\theta_o$ transformer [113] with $\theta_o = 70°$ was utilized in lieu of the conventional CPC, at the expense of a slightly reduced concentration. The minimally required acceptance angle of the secondary concentrator is:

$$\theta_{i,2,ax,\text{full-collection}} = 45° - \arctan(\cos^{1/2} \Phi) + \theta_{\text{sun}} = 3.2°$$  \hspace{1cm} (6.4)

corresponding to a maximum axial secondary concentration of $C_{g,2,ax,\text{max}} = 17.2\times$. Multiplying this with Eq. (6.3), gives the theoretical
maximum concentration for full-collection of \(C_{g,tot,max} = 884 \times 2\). The secondary concentrator was designed with a slightly oversized acceptance angle of \(\theta_{i,2,ax} = 4^\circ\) in order to reduce the required manufacturing tolerances. In line-to-point focus systems, the secondary outlet aperture width is a free parameter which governs the overall scale of the secondary concentrator. A secondary outlet aperture full-width of \(a_{o,2,ax} = 1\) cm was selected to facilitate the use of standard production terrestrial concentrator cells.

The tracking angle of the secondary concentrator is not identical to the skew angle but is rather chosen to bisect skew rays reflected at \(\Phi_i\) and \(\Phi_o\):

\[
\sigma = \frac{1}{2} \left[ \arctan \left( \tan \vartheta \sec \Phi \right) - \vartheta \right]
\]  

(6.5)

which yields a minimum and maximum secondary tracking angles of \(\sigma_{\min} = -22.0^\circ\) and \(\sigma_{\max} = +52.8^\circ\) for the design skew angle range. In order to permit rotation of the \(\theta_i/\theta_o\) secondary concentrators when arranged along the focal line of the primary, they must be asymmetrically truncated. The lengths of the long and short branch of the secondary dictate both the geometric concentration and achievable rotation range of the secondary concentrator.

\[\footnote{This value considers secondary axial concentration only. It may be increased by incorporating a secondary concentrator with a 3D structure having additional transverse concentration. Additionally, the axial concentration can be augmented by a factor \(n\) if a dielectric-filled secondary concentrator is used.}\]

**Table 6.1.** Main dimensions of the primary arcspline concentrator (ASC).

<table>
<thead>
<tr>
<th>parameter</th>
<th>index, (j)</th>
<th>(R_j) [m]</th>
<th>(P_{x,j}) [m]</th>
<th>(P_{z,j}) [m]</th>
</tr>
</thead>
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<td>12.183</td>
<td>5.252</td>
<td>2.124</td>
</tr>
<tr>
<td>(a_{i,1})</td>
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<td>9.522</td>
<td>4.080</td>
<td>1.282</td>
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<tr>
<td>(a_{o,1})</td>
<td>3</td>
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<td>2.920</td>
<td>0.657</td>
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<tr>
<td>(\Phi_1)</td>
<td>4</td>
<td>6.785</td>
<td>1.759</td>
<td>0.239</td>
</tr>
<tr>
<td>(\Phi_2)</td>
<td>5</td>
<td>-</td>
<td>0.402</td>
<td>0.013</td>
</tr>
</tbody>
</table>
Figure 6.5 shows a contour plot of the axial geometric concentration, $C_{g,2,ax}$, of an asymmetrically truncated $\theta_i/\theta_o$ transformer ($\theta_i = 4^\circ$, $\theta_o = 70^\circ$) for different truncation lengths ($L_{longbranch}$, $L_{shortbranch}$) and truncation lengths ($\theta_i = 4^\circ$, $\theta_o = 70^\circ$, $\alpha_{o,2} = 1$ cm). Also shown are contours of constant tracking range ($\sigma_{min} = -20^\circ$, $-30^\circ$ and $\sigma_{max} = +50^\circ$, $+60^\circ$). For a fixed overall length ($L_{longbranch}$), increasing $L_{shortbranch}$ (moving to the right) results in a less asymmetric design which increases the concentration, while decreasing $L_{shortbranch}$ (moving to the left) results in a more asymmetric design with an increased tracking range. Based on the design requirements, a design with $L_{longbranch} = 12$ cm and $L_{shortbranch} = 6.2$ cm (marked by the x in Figure 6.5), achieving $C_{g,2,ax} = 6.3\times$, $\sigma_{min} = -22.6^\circ$ and $\sigma_{max} = +51.7^\circ$, was selected. The detailed geometry of the secondary concentrator is presented in Figure 6.4 (b).

From Figure 6.5 it is evident that higher concentrations are possible; e.g. a design with $L_{longbranch} = 24$ cm and $L_{shortbranch} = 9$ cm would achieve $8\times$ and $\sigma_{min} = +50^\circ$, however it was chosen to restrict the overall length of the secondary to 12 cm to maintain a compact design.
While the theory in Section 4.4 only gives provisions for secondary concentration in the axial $y'-z'$ plane, it was found by ray-tracing that some transverse re-concentration could be incorporated into the secondary concentrator with minimal ray-rejection. The width of the inlet of the secondary concentrator in the transverse direction $a_{i,2,trans}$ is set equal to the focal plane width of the primary concentrator $a_{o,1} = 6.5$ cm. By introducing a secondary transverse concentration of $C_{g,2,trans} = 1.25\times$, the transverse exit aperture could be reduced to $a_{o,2,trans} = 5.2$ cm facilitating the placement of an array of five 1 cm$^2$ concentrator cells at the exit of the secondary. Flat, compound parabolic, and hyperbolic cross-sections were considered for the secondary transverse concentrator profile. The flat and hyperbolic profiles were found to achieve the lowest ray-rejection, but the hyperbolic “trumpet” profile gave the additional advantage of improving the irradiance distribution on the receiver, and was thus selected. The total geometric concentration of the secondary concentrator is $C_{g,2} = A_{i,2}/A_{o,2} = C_{g,2,ax} \times C_{g,2,trans} = 8.1\times$ based on its exit aperture area. The overall geometric concentration of the optical system, found by multiplying the primary and secondary geometric concentrations is $C_{g,tot} = 590\times$.

The secondary concentrator was fabricated from a 0.5 mm thick silvered aluminum sheet mirror material (Almeco V98100). Due to its crossed design, all surfaces of the secondary have 1-dimensional curvature such that they can be easily shaped by simple bending. A photograph of the fabricated secondary concentrator module is shown in Figure 6.6 (a).

6.1.4. Array design
The receiver comprises an array of five 1 cm$^2$ lattice-matched III-V triple-junction concentrator cells, and is shown in Figure 6.6 (b). It is an evolution of the mini-module presented in Chapter 5. The cells are arranged in a line with their busbars running parallel to the length of the array. This configuration has been termed the linear semi-dense array [57] and has the advantages over rectangular dense arrays of reduced gap losses between cells and increased flexibility for placing the cell interconnects and auxiliary electronics. The intercell gap is 0.05 cm such that the array fits at the exit of to be placed at its 1 cm by 5.2 cm exit aperture of the secondary concentrator. Basing $C_g$ on the
active cell area, rather than the secondary exit aperture area as in Section 6.1.3, gives \( C_{g,tot,active} = 613 \times \).

The back contacts of the cells are vacuum reflow soldered to the DBC substrate and the front contacts are achieved by wire bonding the busbar to the DBC. Each cell is placed in parallel with a Schottky bypass diode [72], and each cell/bypass diode pair is brought individually to a discrete wire connector at the back of the array allowing parallel, serial and other cell interconnections to be easily investigated a single array by swapping the connections. In a real installation, the parallel connection is preferred since it mitigates the effects of irradiance mismatch between cells in the array [57].

6.1.5. Secondary concentrator assembly

An exploded view of the secondary concentrator assembly showing its main components is presented in **Figure 6.6 (c)**. The secondary concentrator sits on a cast zinc body (also serving as a heat sink for the secondary concentrator

**Figure 6.6.** Photograph of the: (a) assembled secondary concentrator module; (b) dismantled module; (c) 5-cell array.
mirrors) and is supported on either side by two arms which provide the bearing surface for rotation of the secondary concentrator assembly. The array is fixed to the back of the heat sink and is actively cooled by seven water jets impinging directly on the back of the DBC substrate. The impinging jet cooler is fabricated from injection molded plastic and also serves as the attachment point for a linear actuator which controls the secondary tracking. Figure 6.6 (a) shows the location of the secondary tracking axis with respect to the secondary concentrator. The tracking axis is coincident with the end of the short branch of the $\theta_i/\theta_o$ transformer and the opening of the trumpet. With respect to the primary mirror, the secondary assemblies are arranged such that this tracking axis is coplanar with the primary focal plane.

6.2. Optical modeling

An accurate simulation of a solar concentrating system requires modeling of the energy transport from source to end product. In this case the source of energy is the sun, and the end product is DC electrical power (DC-to-AC inversion is not considered). For the purposes of simulating performance of the InPhoCUS collector over a wide variety of conditions, a detailed numerical model was developed. The model is split into two parts: (1) optical modeling of the concentrator optical system to determine the fraction of radiation incident on the collecting aperture that reaches the cell surface, discussed in this section; and (2) modeling of the array to determine its electrical output, discussed in Section 6.3. The combined model allows the prediction of the optical and electrical performance of the system for a given site, date and time of day.

6.2.1. Monte Carlo ray-tracing

Optical modeling of the concentrator system is performed using the Monte Carlo ray-tracing technique as implemented in the in-house VeGaS code [69]. The model traces the path of rays that are uniformly distributed in space on the collecting aperture of the system until they are finally absorbed by a component of the optical system, or lost to the surroundings. The terrestrial solar spectrum is determined by modeling the atmospheric transfer using the open-source SMARTS code [21, 115], version 2.9.5. The directional distribution of the
incident solar radiation is modeled by specifying the radiance as a function of the angular radius from the center of the solar disk, allowing realistic sunshape models [18] to be utilized. The geometry of all optical components of the system – topsheet, primary mirror, secondary concentrator, cell absorber surface – are described analytically in the model.

The ETFE topsheet is assumed to act as a thin optically-smooth single-layer window of circular cross-section. The narrow-angle spectral normal transmittance $U_{\lambda,n}$ in the range 250 – 4000 nm of a sample of the ETFE was measured using a modified setup of an in-house spectroscopic goniometry system [116]. Details of the measurement procedure and the results are presented in Appendix C. Together with a Sellmeier fit to refractive index measurements $n_{\lambda}$ [117], this allows the directional transmittance $U_{\lambda}(\theta)$ and reflectance $R_{\lambda}(\theta)$ for any non-normal ray to be determined through Snell’s, Fresnel’s and Bouguer’s laws. The solar transmittance of the ETFE at normal incidence, considering an incident solar spectrum following the ASTM AM1.5d [20] reference spectrum, was calculated to be 91.6%.

The primary concentrator is modeled according to its ideal shape of four tangentially connected cylindrical segments with a specified spectral narrow-angle reflectivity which was measured in-house on a stretched sample of aluminized BoPET. Details are provided in Appendix C. Based on the measured curve the resulting solar reflectivity was determined to be 89.4% for the ASTM AM 1.5d spectrum. Microscopic imperfections in the mirror surface are treated probabilistically by assuming that the reflected ray is deviated from the ideal specular direction by an angular dispersion error. It is assumed that the angular dispersion error is uniform in the circumferential direction, and Rayleigh-distributed in the zenith direction with mode $\sigma_{err}$ [118]. The value of $\sigma_{err}$ was measured on the stretched BoPET sample using the variable slit method [119] and was determined to be 0.25 mrad.

The geometry of the secondary concentrator is modeled according to its theoretical shape with any deviations from the ideal shape handled stochastically as for the primary mirror. The spectral specular reflectivity of the secondary concentrator mirror material was determined by scaling the manufacturer’s spectral hemispherical reflectivity curve by a constant factor.
0.95 representing the reported ratio between total specular and total hemispherical reflectance. The resulting solar reflectivity was calculated to be 89.1%. For the angular dispersion error of the secondary mirror, a conservative value of $\sigma_{\text{err}} = 2$ mrad was used. The cells are modeled individually as black absorbers representing their active area.

Due to the linear symmetry of the system, it is sufficient to simulate only a representative section of the collector. In order to properly account for the shading of a secondary concentrator by its neighbors, a minimum of three secondary concentrator assemblies must be modeled, with the middle concentrator then giving the representative performance of the secondary concentrator array on a long trough. Simulations were performed with 10 million rays unless otherwise noted.

6.2.2. Results
The optical model was used to determine the optical performance of the collector under different operating conditions. The variable having the greatest effect on the optical performance is the skew angle $\vartheta$. The incident solar spectrum has only a small effect, manifested through a small change in the total transitivity and reflectivity of the optical surfaces. For all the results presented here the incident solar radiation was assumed to have a spectral distribution of the ASTM AM 1.5d reference spectrum and a directional distribution following the Buie, Monger and Dey sunshape model [18] with a circumsolar ratio (CSR) of 5%.

Optical efficiency
The overall optical efficiency $\eta_{\text{opt,overall}}$ gives the fraction of radiant power incident on the collecting aperture of the primary concentrator $Q_{\text{solar}}$ that reaches the active area of the array $Q_{\text{array}}$. The basis for the calculation is one secondary module and therefore $Q_{\text{solar}}$ is determined based on the area of the primary collecting aperture corresponding to one secondary module, which equates to $A_{i,1} = C_{g,\text{tot,active}} \cdot A_{\text{active}}$. It is assumed that the trough can be made sufficiently long (lengths exceeding 200 m are attainable with such an inflated structure) such that end effects are negligible. The overall optical efficiency can then be determined from:
\[ \eta_{\text{opt,overall}} = \frac{Q_{\text{array}}}{Q_{\text{solar}}} = \frac{Q_{\text{array}}}{\text{DNI} \cdot \cos \vartheta \cdot A_{i,1}} \] (6.6)

To gain insight into the source of optical losses, \( \eta_{\text{opt,overall}} \) may be split into its component efficiencies:

\[ \eta_{\text{opt,overall}} = \frac{U_{\text{topsheet}} \cdot \rho_1 \cdot \gamma \cdot \eta_{\text{opt,2}} \cdot \eta_{\text{opt,3}}}{\eta_{\text{opt,1}}} \] (6.7)

where: the primary optical efficiency \( \eta_{\text{opt,1}} \) is the fraction of radiation incident on the collecting aperture that reaches the inlet of the secondary concentrator, which can be further decomposed into the product of the total transmittance of the ETFE topsheet, \( U_{\text{topsheet}} \), the total specular reflectivity of the primary mirror \( \rho_1 \), and the intercept factor \( \gamma \); the secondary optical efficiency \( \eta_{\text{opt,2}} \) is the fraction of radiation at the inlet of the secondary concentrator that reaches its exit aperture; and a tertiary optical efficiency \( \eta_{\text{opt,3}} \) has been introduced, despite the fact that there are only two optical stages, to account for gap and spillage losses encountered on the way from the secondary exit aperture to the active area of the array.

Figure 6.7 shows the overall optical efficiency and its components as defined in Eq. (6.7). Also shown is \( \eta_{\text{acc,overall}} \) which considers only geometric losses (spillage and ray-rejection), and is the limiting case of \( \eta_{\text{opt,overall}} \) occurring for perfect transmittance and reflectivity of the optical surfaces. The overall optical efficiency \( \eta_{\text{opt,overall}} \) peaks at 62.5% and remains relatively constant over the operating skew range \(-20^\circ\) to \(+50^\circ\). The curve of \( \eta_{\text{acc,overall}} \) shows a similar shape, but peaks at 88.1% indicating that transmission and reflection losses are the most significant source of losses in the system. The primary optical efficiency \( \eta_{\text{opt,1}} \) shows symmetric behavior with a peak of 78.1% at \( \vartheta = 0^\circ \), remaining relative constant to \( \pm40^\circ \) and then dropping off at larger skew angles due to both increased reflection from the topsheet and decreased intercept factor. The secondary optical efficiency \( \eta_{\text{opt,2}} \) is high over the operational skew range. The dip in \( \eta_{\text{opt,2}} \) near \( \vartheta = 0^\circ \) is due to a small amount of leakage between the secondary concentrators (see Section 4.4.8), and the increase towards larger skew angles is due to a reduction of the angular spread of the beam in the
transverse $x''$-$z''$ plane allowing the hyperbolic transverse concentrator to operate more efficiently. The tertiary optical efficiency $\eta_{opt,3}$ is low, with a peak value of 93.7%, indicating that improvements in coupling the array to the exit of the secondary concentrator may be possible.

Spatial irradiance distribution

Figure 6.8 shows the irradiance distribution at the focal plane of the arcspline primary for $\vartheta = 0^\circ$ and $45^\circ$. The irradiance has been normalized by the DNI to give the flux concentration $C$ in suns, i.e. multiplying $C$ by the DNI gives the local irradiance in W/m². Also shown is the distribution for a perfect parabolic profile having the same focal length and inner and outer rim angles as the ASC. It is seen that the ASC reasonably well approximates the behavior but with a lower peak concentration and a broader base. The ASC also shows a slight asymmetric behavior indicating that the performance of the system could be improved by slightly shifting the secondary stage from the theoretical focus. The intercept factor can be calculated from the concentration curve from:
\[ \gamma \equiv \frac{Q_{\text{intercept}}}{Q_{\text{incident}}} = \int_{-\frac{a}{2}}^{\frac{a}{2}} C(x') \, dx' \bigg/ \int_{-\infty}^{\infty} C(x') \, dx' \]  

which has the visual interpretation in Figure 6.8 as the area under the curve between the vertical dashed lines divided by area under the whole curve. The intercept factors were found to be 96% and 98% for the ASC and parabola respectively at $\vartheta = 0^\circ$, and 93% and 97% respectively at $\vartheta = 45^\circ$. It should be noted that this arcspline was designed prior to the development of the nonparabolic design theory of Section 3.4. When redesigned using this theory, the performance of the arcspline would essentially be exactly the same as the parabola.

Also of interest is the irradiance distribution at the exit of the secondary concentrator, shown in Figure 6.9 for $\vartheta = 0^\circ$ and $45^\circ$. The approximate locations of the active areas of the five cells in the array are outlined. At normal incidence, the distribution is relatively symmetric about the horizontal centerline and shows two hotspot lines running parallel to the length of the
array, which are characteristic of the nonimaging design used for the secondary axial concentrator. At $\vartheta = 45^\circ$, the distribution shows a more asymmetric behavior with one of the hotspot lines being more pronounced. It can be seen that at large skew angles, the average irradiance reaching the outer cells is significantly less than the inner cells. In order to avoid losses due to the mismatch of average irradiance between the cells, a parallel cell interconnection is preferred, as was shown analytically and experimentally in Section 5.4. The absence of a pronounced central hotspot on each cell, which is prevalent in other HCPV concentrator systems [120], has a beneficial outcome on the performance of the solar cell as discussed in Section 6.3.4.

Acceptance angle

There exist two prevailing methods for determining the acceptance angle of a concentrator system. The first illuminates the collecting aperture with perfectly collimated parallel rays inclined at a certain angle $\Delta$ with respect to the optical axis [56] (the vector ideally pointing to the center of the solar disk). The second method illuminates the collecting aperture with rays having an angular distribution representative of the sunshape, and intentionally applies a degree of tracking misalignment $\Delta$ to the system [121]. In both cases the optical efficiency of the system is then plotted for different values of $\Delta$, resulting in what is commonly referred to as a transmission-angle curve [23], and the acceptance angle is taken as the angle for which the optical efficiency drops to 90% of the value for $\Delta = 0^\circ$ [121]. The first method gives a more geometrically correct interpretation of the acceptance angle, but cannot be experimentally measured due to the inability of having a source of perfectly collimated radiation. The second method is not a true acceptance angle in the sense that rays arrive at angles greater than the prescribed $\Delta$ due to the finite angular size of the specified sunshape. It may be more accurately referred to as a misalignment or tracking tolerance than an acceptance angle. Nevertheless, the second method has prevailed since it can be easily measured on-sun [121], and is therefore used here to determine the acceptance angle.
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The optical behavior of the proposed system is orthotropic and therefore the acceptance angle differs in the axial and transverse planes. The transverse and axial behaviors can be attributed mainly to the primary and secondary concentrators, respectively. Since the optical configuration of the system changes with skew angle, the acceptance angle is also a function of $\vartheta$.

![Figure 6.9](image)

Figure 6.9. Irradiance distribution at the exit of the secondary concentrator for (a) $\vartheta = 0^\circ$ and (b) $\vartheta = 45^\circ$. The approximate locations of the cell active areas are outlined. Ray-tracing conducted with 100 million rays.

The optical behavior of the proposed system is orthotropic and therefore the acceptance angle differs in the axial and transverse planes. The transverse and axial behaviors can be attributed mainly to the primary and secondary concentrators, respectively. Since the optical configuration of the system changes with skew angle, the acceptance angle is also a function of $\vartheta$.

Figure 6.10 shows the transmission-angle curves for angular misalignments of the primary and secondary in the transverse and axial planes respectively. $\Delta_{\text{trans}}$ is a weak function of $\vartheta$ and is shown for normal incidence. The acceptance angle $\theta_{i,1.90\%}$ is found to be $+0.19^\circ/-0.24^\circ$. While this is a small value, it should be noted that due to the scale of the primary tracking system, the tracking error is expected to be below $0.1^\circ$. Due to the asymmetric design of the secondary, $\Delta_{\text{ax}}$ depends on the skew angle and is shown for $\vartheta = -15^\circ$, $0^\circ$ and $45^\circ$. The values for negative misalignments are in the range of $-1.6^\circ$ to $-2.4^\circ$, and the values for positive misalignments are in the range $+1.8^\circ$ to $+3.2^\circ$. These large values indicate the liberal secondary tracking tolerances afforded by the proposed line-to-point focus system.

**Average number of reflections**

Since every reflection implies a loss of power, it is important to limit the number of reflections that incident radiation suffers before reaching the cell surface. For the primary concentrator, all rays suffer exactly one reflection. However for the secondary concentrator, rays may suffer anywhere from zero to infinitely many reflections before reaching its outlet, which is a characteristic
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It is common to quantify reflection losses by using the average number of reflections $\langle n_r \rangle$ from which the total reflection losses as a fraction of the radiant power at the inlet can be estimated as $1 - \rho^{(n_r)}$ (see Section 2.5.4). The average number of reflections for the secondary concentrator at $\theta = 0^\circ$, calculated using the optical efficiency derivative method outlined in Section 2.5.4 using 100 million rays and $\Delta \rho = 0.01$, was determined to be $\langle n_r \rangle_2 = 1.27$.

**Directional distribution**

As mentioned in Section 6.1.3, the secondary concentrator was designed to limit the AOI on the array. **Figure 6.11** shows the distribution of circumferentially averaged radiance as a function of the AOI for $\theta = 0^\circ$ and $\theta = 45^\circ$ obtained from:

$$L_{\text{circ. av}}(\theta) = \frac{1}{2\pi} \int_0^{2\pi} L(\theta, \varphi) \, d\varphi$$  \hspace{1cm} (6.9)
where $\theta$ is the zenith angle (equivalent to the AOI), $\phi$ is the circumferential angle, and $L(\theta, \phi)$ is the angular radiance distribution at the exit of the secondary obtained from the Monte Carlo ray-tracing simulation. Also shown is the cumulative power reaching the array up to a certain AOI, found by integrating the radiance distribution from 0 to a certain AOI:

$$\int_0^{\text{AOI}} \int_0^{2\pi} L(\theta, \phi) \cos \theta \sin \theta d\phi d\theta$$

From Figure 6.11, it is seen that the vast majority of the radiant power reaching the cell arrives at angles of incidence less than 40°.

6.2.3. Design improvements

There is significant potential to improve the optical efficiency of the system by using higher-performance materials for the optically active surfaces. By changing the topsheet material to a 50 μm thick fluorinated ethylene propylene (FEP) film, an increase in the normal topsheet transmittance from 91.6% to 96% is expected. Changing the PVD coating on the BoPET primary mirror from aluminum to silver would offer an improvement in the solar reflectivity from 89.4% to 95%. Additionally by switching to a secondary reflector material with a higher infrared reflectivity an improvement from 89.0% to 94% is expected. A significant loss, ranging from 6% to 12%, was found to result from poor coupling between the exit of the secondary concentrator and the array. By reducing the intercell gap (minimum of 0.2 mm) and improving the positioning of the array at the exit of the secondary, it is expected that this loss can be reduced to 2%. Combining these improvements would lead to a 25% percent relative improvement in the optical efficiency, with values approaching 80% at normal incidence.
6.3. Electrical modeling

From the optical model, the irradiance on each cell of the array can be predicted, which then serves as the input to the electrical model of the array. The array is a nonlinear circuit composed of five parallel- or serial-connected solar cells each coupled with a parallel bypass diode. The solar cell is modeled using a lumped one-diode equivalent circuit model discussed in detail in Section 5.2 and [57].

Using the one-diode lumped equivalent circuit mode, the current-voltage relationship of a solar cell, accounting for parasitic series $R_s$ and shunt resistances $R_{sh}$, is:

$$
I = I_{ph} - I_0 \left\{ \exp \left[ q \left( V + I R_s \right) / (n_D k_B T) \right] - 1 \right\} - \left( V + I R_s \right) / R_{sh} \quad (6.11)
$$

where $I_{ph}$ is the photogenerated current, $I_0$ is the reverse saturation current of the diode representing carrier recombination, and $n_D$ is its ideality factor. The
best-fit parameters to a series of indoor measurements at STC and for irradiance levels ranging from 10 kW/m$^2$ to 1678 kW/m$^2$ were determined in Section 5.3.3 to be $I_0 = 1.1745 \times 10^{-16}$ A; $n_D = 3.189$; $R_{sh} = 499.8 \, \Omega$; and $R_s = 0.0113 \, \Omega$.

Under real conditions, the solar cell performance may deviate significantly from STC. For this reason, it is necessary to modify the parameters tuned for best-fit at STC. In general, $I_{ph}$ depends greatly on spectrum and weakly on temperature. The reverse saturation current $I_0$ and the ideality factor $n_D$ depend predominantly on temperature, but will also depend on spectrum if a lumped model is used for a triple-junction solar cell, due to the effect of current-mismatch between subcells [110]. The series resistance $R_s$ is a function of temperature due to changes in resistivity of the cell and connection materials with temperature, but this effect is neglected here. The change in $R_{sh}$ away from STC is also neglected since this parameter anyway has only a small effect on the resulting $I-V$ characteristic.

### 6.3.1. Effect of spectrum

Monolithic triple-junction solar cells essentially consist of three different-bandgap subcells connected in series by tunnel junctions. When illuminated by different spectra, each subcell will have a different $I-V$ characteristic, driven by a change in its photogenerated current. If the incident spectrum and the spectral response of the cell are known, $I_{ph}$ for each subcell may be determined. The spectral response is conveniently given in terms of the external quantum efficiency EQE, which gives the probability of a photon of a given wavelength $\lambda_0$ to create an extractable electron-hole pair. The photocurrent density for the $j^{th}$ subcell is then:

$$J_{ph,j} = q \int_0^\infty \text{EQE}_j (\lambda_0) E_{ph,\lambda} (\lambda_0) \, d\lambda_0$$

(6.12)

where $E_{ph}$ is the spectral photonic irradiance:

$$E_{ph,\lambda} (\lambda_0) = E_\lambda (\lambda_0) \frac{\lambda_0}{hc_0}$$

(6.13)
If it is assumed that the EQE is independent of the carrier injection (irradiance), then the photocurrent scales linearly with total irradiance:

$$J_{ph,j} = \frac{E}{1 \text{ kW/m}^2} J_{ph,j,1 \text{ kW/m}^2}$$ \hspace{0.5cm} (6.14)

where:

$$J_{ph,j,1 \text{ kW/m}^2} = J_{ph,j} \frac{1 \text{ kW/m}^2}{\int_0^\infty E_{\lambda} (\lambda_0) d\lambda_0}$$ \hspace{0.5cm} (6.15)

With the EQE known from the manufacturer’s datasheet, Eqs. (6.12) to (6.15) allow the subcell photocurrents to be determined for any incident spectrum and irradiance level. In general the subcells will be current-mismatched, i.e. they will have unequal photocurrents. In this case, the photocurrent of the whole solar cell as a whole may then be taken as the that of the limiting subcell having that having the lowest $J_{ph,j}$:

$$J_{ph} \approx \min \left( J_{ph,j} \right)$$ \hspace{0.5cm} (6.16)

Equation (6.16) suggests that only the photocurrent of the limiting subcell has an effect on the performance of the triple-junction cell. However, the excess photocurrent generated in the other subcells has the effect of boosting the fill factor [110]. For a lumped-diode model of a triple-junction cell, increased subcell current mismatch would have the effect of reducing the $I_0$ and increasing the $n_D$. These effects are neglected in this model. A more accurate handling of spectral effects would require extending the lumped-diode model to a 3-subcell model [74].

To predict the spectrum incident on the array, it is necessary to know both the incident terrestrial solar spectrum, and the optical transfer function OTF, which represents the overall spectral transmission of the optical system. To a good approximation the OTF of the InPhoCUS system can be determined from:

$$\text{OTF}_\lambda (\lambda_0, \theta) \approx U_{\lambda, \text{balloon}} (\theta) \cdot \rho_{\lambda,1} \cdot \rho_{\lambda,2}^{(n_2)}$$ \hspace{0.5cm} (6.17)
Note that for the proposed system, the OTF depends on $\vartheta$ due to the angular dependence of the balloon transmittance, and to a lesser degree the dependence of $\langle n_2 \rangle$ on $\vartheta$. The OTF for normal incidence is shown in Figure 6.12. Table 6.2 shows the subcell photocurrents calculated from Eq. (6.12) using different incident spectra. It is seen that under real operating conditions, the cell is more current mismatched than in the indoor tests of Chapter 5 used to tune the model, meaning that the equivalent circuit model is expected to be conservative in predicting the efficiency of the array.

6.3.2. Effect of temperature

Elevated temperatures have the general effect of reducing the bandgap, leading to an increase in $I_{ph}$ and a decrease of $V_{OC}$ [79]. As described in Section 6.3.1, $I_{ph}$ is computed from the spectral irradiance and the EQE. Since temperature dependent EQE curves were not available, adjustment of $I_{ph}$ for different temperatures was done based on the experimentally derived short-circuit current temperature coefficient, applied to the current limiting subcell:
The decrease in $V_{oc}$ with temperature is due predominantly to an increase of the reverse saturation current $I_0$. The temperature dependence of the saturation current was determined based on a set of indoor measurements at 523 kW/m$^2$ with cell temperatures ranging from 25 – 75 °C. The $I_0$ was then chosen to match measured and modeled open-circuit voltages, assuming a constant $n_D$, according to:

$$I_0(T) = I_{ph}(T_0)\left(1 + \frac{dI_{ph}}{I_{ph}(T_0)} \frac{dT}{dT_0}\Delta T\right)$$

(6.18)

The resulting temperature dependence of $I_0$ is shown in Figure 6.13.

As described in Section 6.1.5, the array is actively cooled by water jets impinging directly on the back of the DBC substrate. The heat dissipation load of the array is:

$$Q_{diss} = (1-\eta_{array})Q_{array} = (1-\eta_{array})\eta_{optical,overall} \cdot C_{g,overall} \cdot A_{array} \cdot DNI \quad (6.20)$$

A nominal value of 120 W results from taking $\eta_{opt,overall} = 70\%$, $\eta_{array} = 34\%$ and a DNI of 1000 W/m$^2$. It has been demonstrated on an experimental test setup of the cooler [122] that by varying the cooling water flow up to 27.5 ml/s, the $\Delta T$ of the array over ambient can be kept below 15 °C for heat loads up to 150 W.
It is therefore expected that under most operating conditions in the real collector it will be possible to maintain the array temperature at 40 °C.

**6.3.3. Effect of angle of incidence**

The angle of incidence (AOI) of the radiation impinging on the cell changes the behavior of the antireflective coating, the interaction of the radiation with the front metallization grid, and the optical pathlength in the cell. These effects ultimately create an AOI dependence for the EQE [123]. The effect of AOI for lattice-matched triple-junction solar cells has been studied in [124], revealing only small differences for AOI < 60°. Figure 6.11 revealed that vast majority of energy reaches the cell at AOI less than 40° and thus AOI effects on the solar cell performance are neglected in the model.

**6.3.4. Effect of spatial nonuniformity**

If the spatial distribution of irradiance reaching the cell is nonuniform, then for a given average irradiance $E$, there will exist regions of the cell exposed to much higher local irradiance levels. This can be quantified by the peak-to-
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average irradiance ratio \( \text{PAR} = \max[E(x,y)]/E \), where \( \max[E(x,y)] \) is the peak local irradiance hotspot on the cell. The hotspot regions will in turn experience an elevated local current density leading to increased Joule heating and other losses. This effect has been studied experimentally [75, 99-102], and methods to predict the effect of irradiance nonuniformity based on 3D distributed equivalent circuit models have been presented [76-78]. The concentrating systems of many CPV optical systems produce a near axisymmetric irradiance distribution on the cell surface with a pronounced central hotspot trailing off in a Gaussian-like manner [120]. In contrast, the irradiance distribution of the line-to-point focus system shown in Figure 6.9 is vastly different raising the question of how this distribution affects the cell performance.

For this purpose the 3D distributed equivalent circuit model presented in Section 5.5 has been used to estimate the effect of single-cell nonuniformity on the cell performance.

Figure 6.14 shows the power-voltage characteristic predicted by the distributed equivalent circuit model for different irradiance distributions all having an average irradiance of 495 kW/m². The uniform distribution has the highest \( P_m \) and serves as a reference against which to compare the nonuniform distributions. The inlay of the figure shows the worst case distribution (\( \text{PAR} = 3.97 \)) for the proposed system as obtained from the Monte Carlo ray-tracing simulations, which was previously shown in Figure 5.18 (a). In order to finely resolve the irradiance distribution, a grid element width of 10 μm was used, for which 500 billion rays were used to reduce the statistical noise associated with such a small element size.

Also shown is the \( P-V \) curve for a circular Gaussian distribution centered on the solar cell and having the same PAR as the simulated distribution. While the Gaussian distribution leads to a relative reduction of 6.7% in \( P_m \), the simulated distribution, having the same PAR, yields a reduction of less than 1%. This is likely due to the fact that the regions of highest irradiance are located parallel and in close proximity to the cell busbars (running along the top and bottom of the cell as oriented in the inlay of Figure 6.14). Since this distribution represents the worst case PAR for any cell and any operating condition, it is expected that under real conditions, the effect of the nonuniform irradiance
distribution will be even less significant. This insensitivity to the single-cell irradiance distribution is a significant advantage of the proposed line-to-point focus system in comparison to competing CPV concentrator systems. Considering the computational expense of solving the 3D distributed equivalent circuit model in comparison to a simple lumped model, the effect of spatial nonuniformity is therefore neglected in the subsequent electrical simulations.

6.3.5. Results

Using the combined optical-electrical model, the $I-V$ curves of the array can be predicted. From these, the overall system efficiency, defined as the energy conversion efficiency from solar radiant power to DC electrical power, can be determined from:

$$\eta_{\text{system}} = \frac{P_m}{Q_{\text{solar}}} = \frac{P_m}{(\text{DNI} \cdot \cos \vartheta \cdot A_{i,1})} = \eta_{\text{opt,overall}} \cdot \eta_{\text{array}}$$  \hspace{1cm} (6.21)
Because the results depend on many inputs, a set of reference conditions for the simulations were established. The incident solar spectrum was assumed to follow the ASTM AM1.5d reference spectrum \([20]\), the DNI was set at 1000 W/m\(^2\), and the cell temperature was set at 25, 40 and 70 °C. Figure 6.15 shows the resulting array and system efficiencies as a function of \(\vartheta\) for the different reference conditions. The peak array and system efficiencies were found to be 34.9% and 21.7% respectively. The array efficiency is nearly constant over the operational skew range, while the system efficiency begins to decrease for \(\vartheta > 25^\circ\) due to the decrease in optical efficiency c.f. Figure 6.7.

6.3.6. Design improvements

As shown in Table 6.2, the subcell current mismatch is worse under the spectrum of the InPhoCUS collector than under the ASTM AM1.5d spectrum for which the cell was optimized. By tailoring the spectral response of the cell [110] it may be possible to reduce the current mismatch thus increasing the short-circuit current and efficiency of the solar cell. Together with the optical
efficiency improvements, solar-to-DC efficiencies of an optimized system are expected to approach 30% at normal incidence.

6.4. Experimental

6.4.1. Experimental setup

To provide an experimental proof-of-concept of the proposed trough-based HCPV collector, a full-scale on-sun prototype was constructed in Biasca, Switzerland (latitude 46.35°N, longitude 8.97°E, elevation 301 m). A photograph of the experimental setup during one of the on-sun tests is given in Figure 6.16. The prototype comprises a 1.2 m long section of the primary trough with an array of 10 secondary concentrator modules arranged along its focal line. Because the inflated trough can only be realized with large trough lengths, the inflated primary was mimicked by a PVD coated aluminized sheet mirror (Almeco WR293) pressed on a steel frame having the shape of the inflated arcspline concentrator. Every effort has been made to minimize the differences between the behavior of the prototype primary mirror and the real inflated construction foreseen for a real installation. Performance differences between this prototype primary mirror and the inflated membrane mirror are discussed in more detail in Section 6.5. In order to facilitate demonstration of the system small skew angles, which represent the most important part of the operating range c.f. Figure 6.3, the entire system was mounted on a two-axis tracker, allowing the skew angle to be controlled regardless of the actual position of the sun.

Of the 10 secondary modules, 9 were equipped with the semi-dense arrays described in Section 6.1.4: 4 connected in parallel (modules 3, 4, 7 and 10), 2 connected in series (modules 6 and 9), and 3 left with no intercell connections (modules 2, 5 and 8) to allow measurement of the $I-V$ curves of individual cells in the array. In module 1, the cell array was replaced by a Lambertian reflector to facilitate the measurement of the irradiance distribution at the exit of the secondary concentrator using a Basler scA640-70gm CCD camera (659×494 resolution, 70 fps).

An additional Lambertian reflector was placed at the primary focal plane, adjacent to the last secondary module, to facilitate imaging of the primary
concentrator irradiance distribution. $I-V$ curves were acquired using an MPPT-3000 multifunction testing device for photovoltaic modules (accuracy $\pm 0.2\%$) with maximum current range modified by the manufacturer to 35 A. In order to facilitate the measurement of different arrays and cells within the unconnected arrays, a multiplexing relay bank was constructed capable of switching of which cell/array was connected to the test device during the experiment. When not connected to the $I-V$ measurement device, the arrays were coupled to fixed dissipation resistors to reduce the heat-load on the cells. To verify the current measurement by the internal shunt of the $I-V$ measurement device, the parallel array of module 3 was connected to an external 50 mΩ shunt resistor. The DNI was recorded at a 1-second frequency using a Kipp & Zonen CHP-1 pyrheliometer (slope half-angle $\alpha_1 = 1^\circ$, aperture half-angle $\alpha = 2.5^\circ$, limit half-angle $\alpha_2 = 4^\circ$).
Figure 6.17. Irradiance distribution across the focal plane of the primary concentrator of the test rig: markers are experimental values measured by Gardon flux gages; solid line is measurements by CCD imaging of Lambertian target; and dashed line is simulation fitted for minimum rms error compared to the CCD curve ($\rho = 87.3\%$, $\sigma_{err} = 3.35$ mrad, rms error 2.3 suns across secondary inlet aperture width and 1.2 suns across total distribution).

6.4.2. Primary mirror characterization

The irradiance distribution at the focal plane of the prototype mirror was characterized in a separate set of initial tests conducted before the secondary concentrators were installed on the test rig. Irradiance was measured using a previously developed irradiance measurement system [127] which combines an array of Gardon flux gages with a Lambertian reflector. The measurement procedure has been previously described in [128]. Figure 6.17 shows the measured irradiance distribution across the focal plane of the primary concentrator as measured by CCD imaging of the Lambertian reflector (solid line) and the flux gages (markers). The total reflectivity and the angular dispersion error used in the optical simulation were tuned to minimize the sum of squares error between the measured and modeled distribution in the range $-3.25\,\text{mm} < x' < 3.25\,\text{mm}$, corresponding to the inlet aperture width of the
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The resulting best-fit reflectivity and dispersion error were found to be $\rho = 87.3\%$ and $\sigma_{\text{err}} = 3.35$ mrad, with an rms error of 2.3 suns across secondary inlet aperture width and 1.2 suns across total distribution.

An important observation from the primary mirror measurements was the nonuniformity of the irradiance distribution along the axial direction of the trough as seen in the fluxmap of Figure 6.18. This has implications on the performance of adjacent secondary concentrator modules will be discussed in Section 6.4.4. This asymmetry is attributed to macroscopic deviations in the mirror shape axial direction. This issue is not expected to be present in the inflated primary trough due to the linear symmetry of its supports, and the inability of a thin membrane to support bending moments.

6.4.3. Experimental procedure

The main test day was June 12, 2013. $I$-$V$ curves from each array in the prototype receiver were acquired (a minimum of 3 $I$-$V$ sweeps per array or cell) for the 9 active arrays in a clear-sky time period spanning 15:00 to 16:10 local standard time. Throughout the experiment, the cells were cooled with a flow rate of $\sim 1$ liter/min. The short-circuit current of module 3 was monitored continuously throughout the experiment to verify the current readings of the $I$-$V$ measurement device. A total of 105 CCD images of the two Lambertian targets

![Fluxmap at the focal plane of the primary concentrator of the test rig.](image)

Figure 6.18. Fluxmap at the focal plane of the primary concentrator of the test rig.
were acquired during the experiment. The peak DNI during the measurement period was 908.1 W/m².

6.4.4. Results

Figures 6.19 and 6.20 show the $I$-$V$ and $P$-$V$ curves respectively for the three modules with parallel cell interconnects. The markers show the amalgamation of the individual $I$-$V$ sweeps taken during the experiment where the currents and powers been normalized to a DNI of 1 kW/m² by scaling the current by $(1 \text{ kW/m}²)/\text{DNI}$. The dashed lines are least-squares fits of the form $I = A - B[\exp(V/C) - 1]$ and are shown to guide the eye; they are not a rigorous model.

Figure 6.19. Experimentally measured current-voltage characteristics of: (a) module 4; (b) module 7; and (c) module 10. The $I$-$V$ curve for each module is the amalgamation of 3 measurements all normalized to a DNI of 1 kW/m² by scaling the current by $(1 \text{ kW/m}²)/\text{DNI}$. The dashed lines are least-squares fits of the form $I = A - B[\exp(V/C) - 1]$ and are shown to guide the eye; they are not a rigorous model.
an elevated cell temperature. The high fill factor is attributed to the large current mismatch between subcells due to the OTF of the system (see Section 6.3.1). A significant difference in performance is seen when comparing the $I-V$ and $P-V$ curves of the different modules. This difference is attributed to the variation of the irradiance distribution of the primary concentrator in the axial direction.

The solar-to-DC conversion efficiency of the system $\eta_{\text{system}}$ can be directly determined via Eq. (6.21) from the measured DNI and with $P_m$ determined from the power curves of Figure 6.20. In order to break down $\eta_{\text{system}}$ into its components, it is necessary to know the radiant power or reaching the array, or equivalently the average irradiance over the array $E_{\text{array}}$. Since a solar cell at short-circuit is an effective photodiode, $E_{\text{array}}$ may be estimated from the sum of the short-circuit currents of the single cells in an array or directly from the short-circuit current of a parallel-connected array, thus providing a self-referenced determination of the radiant power input to the array. For serial arrays, the methods outlined in [57] may be used to determine the irradiance.

In the self-referenced method, linearity of $I_{\text{sc}}$ with $E$ is assumed [129]. The average irradiance on the cell/array is then determined from

$$E = I_{\text{sc}} / I_{\text{sc,1 kW/m}^2}$$

(6.22)

In general $I_{\text{sc,1 kW/m}^2}$ depends on the spectrum and cell, and therefore the correct value for the given spectrum and temperature must be used. The spectrum hitting the cells was estimated by determining the terrestrial spectrum during the experiment using SMARTS and then multiplying by the OTF of the prototype setup, $\text{OTF}_{\lambda} = \rho_{\lambda,1} \cdot \rho_{\lambda,2}^{(n)}$ using the spectral reflectivity of the prototype primary mirror. For the SMARTS simulation, the mid-latitude reference atmosphere [21], Shettle and Fenn rural aerosol model, ASTM E490 extraterrestrial reference spectrum [19], and a CO$_2$ concentration of 400 ppm were used. The aerosol optical depth at $\lambda_0 = 500$ nm was tuned to a value of $\tau_{a,500nm} = 0.08$ such that the predicted DNI matched the measured peak DNI of 908.5 W/m$^2$ at 14:12 LST.
Temperature effects were accounted for following the methods outlined in [130] which is commonly referred to as the $V_{OC}$-$I_{SC}$ method [131]. The detailed procedure is outlined below.

1. Estimate $E$ from the measured $I_{SC}$ and $I_{SC,1\text{ kW/m}^2}$ at STC from manufacturer’s datasheet.

2. Determine $V_{OC}(T_0)$ at this estimate of $E$. For this purpose, data at standard conditions may be used as the effect of changes in the incident spectrum on $V_{OC}$ is small [132].

3. Determine $T = T_0 + \Delta T$ using the open-circuit voltage temperature coefficient $dV_{OC}/dT$. As with the $V_{OC}$, the dependence of the coefficient on spectrum is assumed to be negligible [132].

4. Determine $I_{SC,1\text{ kW/m}^2}(T_0, E_\lambda)$ using the solar spectrum incident on the cells during the test and the EQE curve. For this purpose the incident solar spectrum has been simulated using SMARTS and multiplying by the OTF of the system.

**Figure 6.20.** Experimental power-voltage characteristics of: (a) module 7; (b) module 8; and (c) module 9 derived from the $I-V$ curves of Figure 6.19. Powers have all been normalized to a DNI of 1 kW/m$^2$. The dashed lines are shown to guide the eye and are not a rigorous model.
5. Determine \( I_{SC,1 \text{ kW/m}^2}(T) \) from Eq. (6.18).

6. Determine \( E \) from \( E = I_{SC}/I_{SC,1 \text{ kW/m}^2}(T) \).

7. If the difference between \( E \) calculated in step 6 is significantly different than in step 1, repeat steps 2 to 6 until \( E \) has converged.

Table 6.3 shows the resulting system efficiencies for the three parallel-connected modules, the two series-connected modules and for a single-cell module, calculated by summing the maximum powers from the individual cells. Also shown are the array temperatures, average irradiances on the array, array efficiencies and optical efficiencies. The maximum system efficiency measured during the tests was 20.2\% for parallel-connected module 10. The serial modules 6 and 9 showed inferior performance due to the irradiance mismatch [57], with the reduction in performance expected to be even greater at non-normal incidence. The cell temperature ranged between 52 and 66 °C. Temperatures in real operation will be lower due to the reduction of the heat load when the arrays are MPP tracked.

The peak irradiance measured over the array was 328.0 kW/m\(^2\) for module 10, at a DNI of 901.9 W/m\(^2\), corresponding to a mean solar concentration ratio over the array of 364 suns. This is the highest ever measured solar concentration on a parabolic-trough-based system, surpassing the previous record of 231 suns (184 kW/m\(^2\) @ DNI = 797.4 W/m\(^2\)) on the BICON line-to-

<table>
<thead>
<tr>
<th>module #</th>
<th>connection type</th>
<th>LST</th>
<th>DNI [W/m(^2)]</th>
<th>( Q_{solar} ) [W]</th>
<th>( T_{array} ) [°C]</th>
<th>( P_m ) [W]</th>
<th>( E_{array} ) [kW/m(^2)]</th>
<th>( C ) [suns]</th>
<th>( \eta_{array} ) [%]</th>
<th>( \eta_{opt,overall} ) [%]</th>
<th>( \eta_{system} ) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>parallel</td>
<td>14:54:07</td>
<td>864.2</td>
<td>265.1</td>
<td>51.7</td>
<td>47.3</td>
<td>277.0</td>
<td>321</td>
<td>34.2</td>
<td>52.2</td>
<td>17.9</td>
</tr>
<tr>
<td>7</td>
<td>parallel</td>
<td>14:56:11</td>
<td>870.2</td>
<td>267.0</td>
<td>55.7</td>
<td>49.7</td>
<td>292.5</td>
<td>335</td>
<td>34.1</td>
<td>54.6</td>
<td>18.6</td>
</tr>
<tr>
<td>10</td>
<td>parallel</td>
<td>14:29:19</td>
<td>901.9</td>
<td>276.7</td>
<td>61.9</td>
<td>55.9</td>
<td>328.0</td>
<td>364</td>
<td>34.1</td>
<td>59.3</td>
<td>20.2</td>
</tr>
<tr>
<td>6</td>
<td>series</td>
<td>14:31:32</td>
<td>900.2</td>
<td>276.2</td>
<td>59.1</td>
<td>50.0</td>
<td>337.2</td>
<td>375</td>
<td>29.7</td>
<td>61.0</td>
<td>18.1</td>
</tr>
<tr>
<td>9</td>
<td>series</td>
<td>14:34:21</td>
<td>892.4</td>
<td>273.8</td>
<td>65.9</td>
<td>41.5</td>
<td>285.5</td>
<td>319</td>
<td>29.2</td>
<td>52.0</td>
<td>15.1</td>
</tr>
<tr>
<td>2</td>
<td>single-cell(^b)</td>
<td>14:38:25-14:50:36</td>
<td>850.6</td>
<td>261.0</td>
<td>56.7</td>
<td>45.6</td>
<td>277.3</td>
<td>326</td>
<td>33.0</td>
<td>53.1</td>
<td>17.5</td>
</tr>
</tbody>
</table>

\(^a\) This is the maximum experimentally measured value. The actual \( P_m \) is expected to be higher due to the exact MPP being missed by the discrete sampling of the \( I-V \) characteristic.

\(^b\) Results for DNI, \( Q_{solar} \), \( T_{array} \), \( E_{array} \) and \( C \) are the average for the \( I-V \) sweeps for all cells, and \( P_m \) is calculated as the sum of the \( P_m \) for individual cells.
point focus system with fixed dielectric-filled secondaries [52]. An even higher solar concentration ratio of 375 suns was determined for module 6, but the assumptions required to extract the cell short-circuit currents from serial modules lead to some uncertainty in this value.

**Optical methods**

In addition to the parameters determined from the $I-V$ measurements, additional insight into the system performance can be obtained from the optical measurements made by CCD imaging of the Lambertian reflectors. The CCD chip is essentially a spatially resolved photon counter where the grayscale value of each pixel is linearly proportional to the number of photons reaching that pixel, which is in turn proportional to the radiance distribution of the scene mapped by the camera lens. For a Lambertian reflector, the radiance viewed in any direction is directly proportional to the incident irradiance at that point. Therefore the CCD imaging technique allows the irradiance distribution on a Lambertian reflector, or between two different Lambertian reflectors having the same optical properties, to be spatially resolved.

**Figure 6.21** shows a CCD image acquired during the on-sun tests, showing the location of the four regions-of-interest (ROIs) used for the calculation of the intercept factor and the secondary optical efficiency. A set of 156 images were acquired during the on-sun tests, and were post-processed to check for saturation within ROIs.

The intercept factor may be determined from:

$$\gamma \equiv \frac{Q_{\text{intercepted}}}{Q_{\text{incident}}} = \frac{\sum_{\text{ROI}_1} \text{GS}}{\sum_{\text{ROI}_2} \text{GS}}$$  \hspace{1cm} (6.23)

where GS is the grayscale value of each pixel within the respective ROI. The average, minimum and maximum values of $\gamma$ from 69 images acquired during the experiment were 83.1%, 79.8%, and 84.2% respectively. The low measured value of the intercept factor is attributed to micro- and macroscopic imperfections of the aluminum sheet primary mirror used on the prototype, and is a major contributor to the low optical efficiencies reported in **Table 6.3**.

The secondary optical efficiency may be expressed as:
The ratio \( C_2 = E_{o,2}/E_{i,2} \) is the flux concentration of the secondary concentrator, i.e. ratio of the average irradiance at the exit of the secondary to that at the inlet. It may be determined from the CCD image from:

\[
\eta_{\text{opt,2,target}} = \frac{Q_{o,2}}{Q_{i,2}} = \frac{E_{o,2}A_{o,2}}{E_{i,2}A_{i,2}} = \frac{C_2}{C_{g,2}} \tag{6.24}
\]

The ratio \( C_2 = E_{o,2}/E_{i,2} \) is the flux concentration of the secondary concentrator, i.e. ratio of the average irradiance at the exit of the secondary to that at the inlet. It may be determined from the CCD image from:

\[
C_2 \equiv \frac{\sum_{\text{ROI}_1} \text{GS}/A_{\text{ROI}_1}}{\sum_{\text{ROI}_4} \text{GS}/A_{\text{ROI}_4}} \tag{6.25}
\]

where the numerator and denominator represent the average grayscale value over the respective ROI. It is important to note that the Lambertian target was placed a distance of 1 mm from the exit of the secondary concentrator. This implies that the average on the irradiance on the target will be lower than that at the exit of the secondary concentrator due to the divergence of the radiation exiting the secondary. Therefore, we denote the efficiency calculated from Eqs. (6.24) and (6.25) as \( \eta_{\text{opt,2,target}} \) to differentiate it from the true secondary optical efficiency, which is expected to be approximately 6% higher (see Section 6.5.1 for further discussion). From 38 images acquired during the
experiment the average, minimum and maximum values of $\eta_{\text{opt},2,\text{target}}$ were 82.8%, 79.8%, and 84.5% respectively, confirming the correct operation of the secondary concentrators.

6.5. Model validation

6.5.1. Model adaptations

In addition to serving as an experimental proof-of-concept of the proposed HCPV collector, the experimental results serve as a basis against which to validate the optical and electrical models. To facilitate a fair comparison, it is necessary to incorporate the differences between the real collector configuration and the experimental setup into the model. The main differences are in the primary concentrator which was constructed from an aluminum sheet mirror rather than the inflated construction described in Section 6.1.2. The performance of the prototype primary mirror was experimentally measured as discussed in Section 6.4.2. The primary mirror was found to be well described by a solar reflectivity of 87.3%, with a spectrum following the manufacturer’s specification of a similar mirror material WR193, an angular dispersion error of $\sigma_{\text{err}} = 3.35$ mrad. These are different than the values expected for the inflated construction discussed in Section 6.1.2. The main implication of this is a different primary optical efficiency between the prototype construction and inflated design. However, the secondary concentrator performance, whose characterization is the main goal of this experimental investigation, is not expected to be significantly affected by the differences in the primary mirror.

6.5.2. Efficiency comparison

Figure 6.22 shows a parity plot comparing the efficiencies measured during the on-sun tests to those predicted by the adapted model. The experimental results for $\eta_{\text{system}}$, $\eta_{\text{array}}$ and $\eta_{\text{opt,overall}}$ are based on the measurements for module 10, while $\gamma$ and $\eta_{\text{opt,2,target}}$ are based on the CCD measurements. The error bars show the minimum and maximum values calculated based on all $I-V$ sweeps/fluxmaps acquired during the experiment.

The Lambertian target used to measured $\eta_{\text{opt,2,target}}$ was offset 1 mm from the exit of the secondary concentrator. Therefore an additional surface representing
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the correct position of the target was included in the ray-tracing model to allow comparison. The parity plot shows good overall agreement between the experiment and model with relative errors 

\[ \varepsilon_x = \frac{\bar{x}_{\text{experimental}} - \bar{x}_{\text{simulated}}}{\bar{x}_{\text{experimental}}} \]

\[ \varepsilon_\eta,\text{system} = +1.3\% \]

\[ \varepsilon_\eta,\text{array} = +4.8\% \]

\[ \varepsilon_\eta,\text{opt,overall} = -3.7\% \]

\[ \varepsilon_\eta,\text{opt,2,target} = -0.7\% \]

\[ \varepsilon_\gamma = -5.6\% \]

6.5.3. I-V curves

The parity plot revealed that the array performance is slightly underestimated by the model. To understand the underestimation of \( \eta_{\text{array}} \), a comparison of the shape of the modeled and experimental I-V curves for module 10 is given in Figure 6.23. The results are similar for the remaining parallel modules. In order to allow comparison at the same irradiance, the simulated curves have been scaled to match the \( I_{sc} \) of the experimental curves. Figure 6.23 reveals that the fill factor is underpredicted by the model. This is due to subcell current

Figure 6.22. Parity plot showing agreement between simulated and experimentally derived efficiencies for module 9. The experimental values for \( \eta_{\text{system}}, \eta_{\text{array}}, \) and \( \eta_{\text{opt,overall}} \) are the avg., min. and max. values for the 4 I-V sweeps, and the experimental values for \( \eta_{2,\text{Lambertian}} \) and \( \gamma \) are the avg., min. and max. values for the set of CCD images. The error bars are indicative of the experimental scatter, and not the experimental error.
mismatch, and the inability of a lumped equivalent circuit model to capture the accompanying fill factor support effect, as discussed in Section 6.3.1. The model is therefore expected to be conservative for conditions of increased subcell mismatch, which prevail due to the OTF of the optical system (see Section 6.3.1).

6.5.4. Irradiance distribution

The irradiance distribution at the exit of the secondary concentrator measured by CCD imaging also serves as a tool to validate the optical model. Figure 6.24 shows the fluxmap as predicted by the optical model in comparison to an experimental fluxmap acquired at 14:38:54 LST. Since the CCD imaging technique yields only relative irradiance values, the experimental fluxmap has been scaled to have the same average irradiance $E = 360 \text{ kW/m}^2$ as the simulated fluxmap. The simulated and measured fluxmaps show good qualitative agreement, but with the experimental distribution showing more

![Figure 6.24](image-url)
pronounced areas of low and high irradiance. This difference is attributed to imperfections in the secondary concentrator shape, and misalignments in the positioning of the secondary concentrator array in the experimental setup.

6.5.5. Cell-averaged irradiance

The cell-averaged irradiance distribution can be determined by integrating the fluxmap over the ROIs corresponding to the active area of each cell, from the $I_{sc}$ of each cell in the single-cell modules, and from the $I-V$ curves of the serial modules, following the procedure in [57]. Figure 6.25 shows the resulting average irradiance on each cell of the 5-cell array from these three experimental methods compared to that predicted by the optical simulation. The differences between model and experiment are significant and are most likely attributed to small misalignments of the receiver module, or to non-random manufacturing errors in the secondary concentrator geometry. The difficulty in predicting accurately the cell-averaged irradiance distribution is further motivation for utilizing a parallel connected module, where the effect of irradiance mismatch between the cells is mitigated [57].
6.5.6. Validation

The optical efficiency during the prototype tests was slightly overpredicted (3.7%) by model, which is attributed to a lower than expected intercept factor for the prototype primary mirror. It is expected that the inflated primary concentrator will have a more predictable behavior than the aluminum sheet mirror due to its inflated nature and the ability to actively control its shape [34]. Correcting for the differences in the intercept factor, the agreement of the overall optical efficiency is within 2%. Most importantly, $\eta_{\text{opt,2,target}}$, which is indicative of the secondary concentrator performance, is in good agreement (<1% relative error), suggesting that the optical model is capable of predicting the performance of the line-to-point focus concentrating system with good accuracy. The array efficiency is underpredicted by the model due to the inability of the lumped diode equivalent circuit model to predict the fill factor support effect for conditions of increased subcell current-mismatch. Based on

![Figure 6.25. Average irradiance reaching each cell in the 5-cell array determined from the $I$-$V$ curves of module 2 (single-cell connections), module 4 (serial connections), CCD imaging of the Lambertian target at the exit of module 1, and as predicted by the optical simulation.](image)
the results of the model validation, it is expected that the system efficiency predictions reported in Section 6.3 are slightly conservative, with underpredictions on the order of 0.5% absolute.

6.6. Summary

We have presented the design, modeling and experimental demonstration of a high-concentration photovoltaic collector based on a parabolic trough primary concentrator with tracking secondary optics. The collector uses a line-to-point focus optical configuration to allow the 2D limit inherent to all trough geometries to be considerably surpassed enabling a total geometric concentration of over 600× to be achieved. At the same time, the advantages of a trough-primary, namely simple construction and scalability are maintained.

The primary concentrator is constructed from a stack of polymer membranes, whose precise shape is made to closely approximate the shape of a parabola by controlling the inflation pressure between membranes. The secondary concentrator is constructed from silvered aluminum sheet mirror, which, due to the one-dimensional curvature of all of its surfaces, can be easily constructed by simple bending. A semi-dense array of five 1 cm² lattice-matched III-V triple-junction concentrator cells is placed at the exit of the concentrator. A parallel connection allows the effects of irradiance mismatch between the 5 cells in the array to be mitigated.

A combined optical/electrical model of the collector has been developed with the ultimate goal of predicting the solar-to-DC energy conversion efficiency of the system. The optical model was realized using the Monte Carlo ray-tracing technique, with full spectral dependence implemented for all surfaces. The spectral distribution of incident solar radiation was estimated using SMARTS, and the directional distribution is described using realistic sunshape models. The model predicts a peak overall optical efficiency of 62.5% limited mostly by the optical properties of the materials. An improvement of the optical efficiency to values approaching 80% is foreseeable through use of improved mirror and balloon materials, and by improving the optical coupling between the exit of the secondary concentrator and the array. With the initial design specifications and materials, the peak solar-to-DC conversion efficiency
of the system as predicted by the model was 21.7%. Efficiencies are expected to approach 30% with the accompanied improvements in optical efficiency and by using solar cells with a spectral response tailored to the optical transfer function of the collector.

A full-scale prototype of the system has been constructed and tested on-sun in Biasca, Switzerland. The prototype comprises a 1.2 m long section of primary trough mirror constructed from an aluminum sheet mirror with a shape replicating that of the inflated arcspline concentrator foreseen for a real installation. An array of 10 secondary concentrator modules were arranged at the focal plane of the primary, and the entire system was mounted on a two-axis tracker allowing the skew angle of the system to be controlled regardless of the actual position of the sun. The peak measured solar-to-DC efficiency was 20.2%. The mean solar concentration ratio across the array peaked at 364 suns,
the highest value ever measured for a parabolic-trough-based system. The results of the prototype measurements serve as an experimental proof-of-concept of the proposed line-to-point focus HCPV collector, and were further used to validate the optical and electrical model of the system.

Based on the successful on-sun demonstration of the prototype, a 15 kW demonstration plant, comprising an array of 200 secondary concentrator modules mounted at the focal plane of a 52 m long inflated parabolic trough primary concentrator, is currently being constructed in Biasca, Switzerland. A photograph of the current state of the collector is shown in Figure 6.26.
Chapter 7

Summary and outlook

This thesis has realized the design, analysis and experimental demonstration of the world’s highest concentration trough-based collector. Due to the dilute nature of solar radiation reaching the earth’s surface, minimization of the cost per m² of collecting aperture was identified as a key requirement for the economical conversion of solar energy into useful forms. With this as a driver, the solar trough was identified as a promising candidate due to its simple design in comparison to more complex three-dimensional geometries. However, due to their two-dimensional nature, solar trough concentrators are limited to a maximum concentration of \(~215\times\). The efficiency of downstream conversion processes generally increase with increasing solar concentration. Additionally, and especially relevant for the field of concentrating photovoltaics, higher concentrations improve the economics of the collector by reducing the required receiver size to generate a given output power. To permit use of high-efficiency triple-junction concentrator cells, concentrations above \(400\times\) are generally required. Therefore, methods to augment the concentration of solar trough concentrators into the realm of HCPV were pursued. It has been demonstrated that by using a line-to-point focus optical configuration, comprising and array of secondary concentrators arranged at the focal line of the trough primary concentrator, the concentration can be augmented well beyond the 2D limit. At the same time, the benefits of having a line-focus primary concentrator, namely simple construction and scalability are maintained. Of course the introduction of a secondary stage increases the complexity of the system, but as the primary concentrator represents the largest share of the system cost, these line-to-point systems show promise for reducing the cost of the collector as a whole.
7.1. Summary

7.1.1. Low-cost, high-performance solar troughs

A low-cost, high-performance primary is considered as a necessary prerequisite for achieving a successful high-concentration trough-based system. In this light, a novel inflated trough construction based on metallized thin polymer membrane mirrors mounted on a cast concrete frame with integrated tracking was considered. Although the concept itself is based on previous work [34, 38], major improvements in the design methods have been developed here.

When a long single membrane is inflated, it assumes the shape of a circular arc. In order to use this as a primary concentrator, it is necessary to correct the spherical aberration with a sequential corrector mirror. In this work, an analytical solution for the profile of all such corrector mirrors was developed based on optical pathlength tailoring.

Since the second reflection from a two-mirror system implies an additional optical loss, methods to control the curvature of the inflated membrane following the work of [34] were explored. By suitably inflating a stack of $N$ polymer membranes, the topmost membrane can be made to assume the shape of $N$ tangentially connected circular arcs, named the “arcspline”. By adjusting the initial membrane lengths and inflation pressures, it is possible to carefully control the shape of the topmost mirror membrane. A general method to design the shape of the arcspline to match the profile of a reference curve was developed based on the concept of radius of curvature matching.

While the curvature matching approach is useful in its generality, it produces designs that are inferior to a parabolic profile. This raised two questions: (1) what is the maximum concentration that can be achieved by a mirror profile composed of a finite number of circular arcs; and (2) how do concentrators of nonparabolic profiles compare in general to their parabolic counterparts? To answer these two questions, a new theory of solar concentrator design was developed. This theory pertains to the design of all mirror profiles where every ray within the acceptance angle undergoes exactly one reflection on its way to the focus. This is in contrast to the principles used to design ideal edge-ray concentrators, such as the 2D CPC, for which only the
edge-rays are prescribed to have exactly one reflection, and rays arriving at other angles may undergo any number of reflections from zero to infinity. The one-reflection mirrors considered here usually take on a concave shape and have thusly been called focusing concave mirrors. For planar absorbers, concave focusing mirrors where shown to have a concentration limit of:

$$C_{g,\text{max},2D} = \frac{\sin \Phi \cos \Phi}{\sin \theta_i \cos \theta_i} - 1$$

(6.26)

which is lower than the fundamental limit of $1/\sin \theta_i$ resulting from conservation of 2D étendue. Nevertheless, Eq. (6.26) represents the highest concentration that can be achieved by a single continuous mirror contour when it is imposed that every ray within the acceptance angle must undergo exactly one reflection. We denoted Eq. (6.26) the concave limit, and referred to any concave focusing mirror achieving this limit as an ideal concave mirror. The solution for the geometry of all ideal concave mirrors was posed in the form of a system of differential inequations. Though no general solution exists, it was proven that there are infinitely many mirror profiles which satisfy this system, one of which is the parabola. A convenient method to obtain solutions based on designing directly for the slope of the mirror was proposed. The nonparabolic design theory was shown to be of considerable merit for the design of arcspline concentrators. It was shown that for a finite acceptance angle, the arcspline can always be designed to match the performance of a parabola in the nonimaging sense using a finite number of arcs.

Finally, double-tailored two-mirror troughs were discussed. When designed from an imaging perspective, the resulting designs are the so-called two-mirror aplanats, whose analytical formulation has been previously derived [39]. Being aplanatic, these two-mirror systems are capable of approaching the fundamental limit of concentration for small acceptance angles. It was shown that two-mirror aplanats are also of interest when a small primary rim angle is required, since they can be made very compact for virtually any rim angle. For large acceptance angles, the performance of two-mirror aplanats was found to decrease due to higher order aberrations. To overcome this limitation, the simultaneous multiple surface method was successfully applied to tailor the
mirror shapes and maintain good performance for any acceptance angle. It was further demonstrated that the two-mirror aplanats and the SMS designs converge in the limit of $\theta_i \rightarrow 0$.

7.1.2. Line-to-point focus solar concentrators

Having developed the theory for designing high-performance solar troughs, prospects for augmenting the concentration into the high-concentration (>400×) were investigated. As the desired concentrations are in excess of the 2D limit, it is necessary to incorporate some three-dimensional structure into the concentrator to reach the desired concentration levels. It was shown this can be done by creating a line-to-point (LTP) focus configuration comprising a one-axis tracking trough primary concentrator with an array of nonimaging secondary concentrators arranged along its focal line. Three types of LTP concentrators were investigated, classified by the degrees of freedom of the secondary concentrating stage.

First, designs with fixed secondary concentrators were considered. Fixed secondary LTP systems have been considered in the past [27, 52], but were restricted to polar tracking primaries, which are less attractive than horizontal one-axis trackers for large-scale deployment. Methods for designing fixed-secondary LTP concentrators were developed, but it was found that due to the difficulty in matching a secondary concentrator to the directional distribution of rays reaching the primary focus, these designs are limited to low-latitude sites where the skew angle range is small. For a N-S one-axis tracker at a latitude of $30^\circ$, the practically achievable concentration for the fixed-secondary LTP configuration was found to be $170\times$ for hollows secondaries, and $390\times$ for secondaries filled with a dielectric of refractive index $n = 1.5$.

To overcome the limitations of fixed-secondary LTP concentrators, especially at latitudes away from the equator, the discrete-switching LTP concentrator was introduced. The discrete-switching designs utilize a set of interchangeable secondary concentrator rows that move in and out of place over a fixed receiver. Each row is designed to operate over subset of the full skew angle range, thus splitting up the required acceptance angle of the secondaries. Due to the ability to split up the skew angle range amongst a multitude of
secondary concentrator, discrete-switching LTP concentrators are capable operating effectively even at high-latitude sites.

While the discrete-switching LTP concentrator was shown to overcome the limitations of the fixed-secondary designs, the number of secondary concentrator rows that can be practically and economically incorporated into the collector is limited, and thus the concentration cannot be increased without bound. A third type of line-to-point concentrator using secondary concentrators that have a continuous range of motion was developed. These so-called tracking-secondary LTP collectors utilize an array of secondary concentrators, each having an individual tracking axis perpendicular to the tracking axis of the primary. This creates a pseudo-two-axis tracking system capable of achieving concentrations well in excess of the 2D limit. A tracking-secondary LTP concentrator based on a N-S one-axis tracking trough at a latitude of 30° can reach concentrations of up to $2600 \times$ for $n = 1$, and $3900 \times$ for $n = 1.5$. Tracking-secondary systems were deemed to be the most promising of the three considered LTP classes, since the additional cost of the secondary stage can be warranted by the significantly higher concentrations afforded. Therefore, particular emphasis was placed on developing practical secondary concentrators for these systems.

7.1.3. Characterization of concentrator solar cell arrays

The development of line-to-point focus solar concentrators led to designs having or requiring unique receiver geometries. In particular, the line-focus of the primary is split up into a multitude of point-like foci. These individual foci generally have a rectangular shape, whose aspect ratio depends on the ratio of primary to secondary concentration.

These point-like receiver geometries were found to be particularly well-suited to the CPV application. By having a multitude of point-like foci instead of a focal line, it is possible to use individual arrays coupled to each focus rather than a continuous dense array running the length of the trough. This enables the use of smaller solar cells, which in general benefit from higher efficiencies. The rectangular shape of each focus permits the use of a linear $1 \times N$ array of solar cells which has been denoted the semi-dense array. The cells
in a semi-dense array can be arranged with their busbars running parallel to the length of the array thus reducing gap losses in comparison to a \( M \times N \) dense array configuration.

A prototype 1×5 semi-dense array of 1 cm\(^2\) III-V triple-junction concentrator cells, called the “mini-module”, was developed. The performance of the mini-module under concentrated radiation was ascertained by a set of indoor experiments using a large-area module flash solar simulator. By adjusting the distance between the array and the lamp, irradiance levels ranging from 1 kW/m\(^2\) to >1500 kW/m\(^2\) were investigated.

A lumped single-diode equivalent circuit model of the triple-junction solar cell was developed and tuned vis-à-vis the experimental measurements. The lumped equivalent circuit model permitted the performance of the solar cell, and the array, to be easily determined for a wide range of concentrations.

Line-to-point focus concentrators produce a unique irradiance distribution at the exit of each secondary concentrator. The effects of this irradiance distribution on the performance of the mini-module were investigated both theoretically and experimentally. A distinction was made between cell-to-cell nonuniformity and single-cell nonuniformity in the irradiance distribution. An experimental method, based on the production of solid-wax inkjet-printed neutral density filters, was developed to measure the effect of cell-to-cell nonuniformity on the array. The experimental results were found to be in excellent agreement with the results predicted by the tuned lumped equivalent circuit model. Cell-to-cell nonuniformity was found to be of particular significance for series-connected arrays, where relative efficiency reductions of up to 25% were predicted for the considered irradiance distributions. For this reason, a parallel connection, which almost completely mitigates the effect of cell-to-cell nonuniformity, was adopted for the mini-module.

Single-cell nonuniformity resulting from the local irradiance distribution on a single cell was analyzed using a 3D distributed equivalent circuit model. It was demonstrated that, in the absence of a pronounced central peak in the irradiance distribution on the cell, the effects of single-cell nonuniformity are negligible. This was an unexpected benefit of the line-to-point focus configuration, in comparison to conventional HCPV concentrators which
produce a Gaussian-like distribution on the cell which leads to a significant reduction of cell efficiency.

7.1.4. The InPhoCUS HCPV collector

A 600× HCPV collector incorporating the designs proposed throughout the thesis was developed. The system, which has been named the InPhoCUS (Inflated Photovoltaic Ultra-light mirror concentratorS) collector, comprises an inflated arcspline primary concentrator coupled to an array of tracking nonimaging secondary concentrators, each coupled to an array of five triple-junction concentrator cells at its exit. The collector is arranged in a two-wing configuration mounted on a rigid concrete tracking integrated frame. The inflated primary allows very large primary aperture widths to be realized: 4.85 m for each wing of the concentrator giving a total aperture width of over 9 m for a single collector. The system was designed for a skew angle range of −20° to 50°, roughly corresponding to that experienced at a site latitude of 30°.

An optical model of the system was developed based on the Monte Carlo ray-tracing technique. Using the model the optical efficiency of the system was predicted over the operational skew angle range. A peak optical efficiency of 62.5% occurring at normal incidence was predicted for the system, limited mostly by the optical properties of the materials used in the initial design.

An electrical model based on that of Chapter 5, but extended to account for changes in spectrum and cell temperature, was developed. The electrical model takes as an input the irradiance distribution on the array as predicted by the Monte Carlo optical simulation, and gives as an output the I-V curve of the array, from which the maximum power and efficiency can be calculated. A peak array efficiency of 34.9%, leading to a peak overall system efficiency of 21.7% was predicted. The array efficiency is limited mostly by the subcell spectral mismatch caused by the optical transfer function of the optical system.

An on-sun prototype of the system was constructed in Biasca, Switzerland. The prototype comprised a 1.2 m long section of the primary trough concentrator, constructed from aluminum sheet mirror pressed on a form having the exact multi-arc shape of the arcspline concentrator. An array of ten secondary concentrator modules were arranged at the focus of the primary. The
entire prototype was mounted on a two-axis tracker, allowing the skew angle to be controlled regardless of the position of the sun in the sky. The peak irradiance measured over the array was 328.0 kW/m² at a DNI of 901.9 W/m², corresponding to a mean solar concentration ratio over the array of 364 suns: the highest solar flux concentration ever measured on a parabolic-trough-based system. A maximum solar-to-DC efficiency of 20.2% was measured for the system.

7.2. Outlook

7.2.1. Low-cost, high-performance solar troughs

Cylindrical primaries with sequential corrector mirrors should be reconsidered as potential designs for low-cost solar troughs. Because a single mirror naturally takes a cylindrical shape when inflated, this is expected to be a very robust design. Especially when designed for an off-axis circular arc, the corrector mirror can be made very small, and the system very compact, such that high-performance reflector materials could be used for the corrector with little impact on the overall system cost. Furthermore, two-mirror configuration makes it possible to locate the final focus closer to the pole of the primary. The off-axis performance of cylindrical primaries with sequential corrector mirrors should be investigated in detail. The existence of an aplanatic system (cardioid corrector) based on a primary with circular cross-section indicates that there may be other, more practical designs with performance surpassing the parabola.

Additional characterization of the optical properties of materials used in inflated trough collectors should be performed. The mechanical slit experiments (Appendix C) used to ascertain the angular dispersion error for stretched membrane mirrors indicate that these materials are vastly superior to aluminum sheet mirrors, however the accuracy of these measurements for low dispersion is poor due to misalignment effects. Improved accuracy may be obtained by removing the mechanical slit and using a spatially resolved detector, such as a CCD camera, to map the irradiance distribution of the reflected beam.

For the arcspline concentrator, additional efforts to experimentally characterize the mirror shape should be made. The agreement between the
theory and the 3D finite element analysis suggest that the arcspline shape can be realized. However, tight tolerances on the mirror membrane dimensions are required. In theory, the manufacturing errors can be offset by adjusting the inflation pressures from their design values. Therefore an effective control scheme for the inflation pressures is expected to be vital for the successful long-term operation of an inflated multi-membrane concentrator. Secondary effects including gravity, electrostatic, and friction forces should be incorporated into the finite element model of the arcspline concentrator. Whenever designing a new arcspline concentrator, the nonparabolic design theory should be used to yield the minimum-arc solution.

7.2.2. Line-to-point focus solar concentrators

The design theory developed for line-to-point focus solar concentrators can currently produce secondary concentrator designs which concentrate in the axial direction alone. Transverse concentration can currently be incorporated using ad hoc optimization approaches for a particular design. Ways of improving the design theory to allow for the direct design of secondary concentrators with axial and transverse concentration should be investigated. To this end, the 3D simultaneous surface method, or some extended version of it, seems to be the most promising candidate.

All of the line-to-point focus systems presented in this work have used one-axis tracking trough primary concentrators as their basis. The line-to-point focus configuration may also be of interest when the entire collector is mounted on a two-axis tracker. This would allow high concentrations to be achieved using fixed secondary concentrators. Such a design would ultimately be limited by the 3D limit of concentration $n^2/\sin^2\theta_{\text{sun}}$, yet would still maintain the benefits of having a primary concentrator with one-dimensional curvature. Two-axis tracking LTP collectors would be limited in scale since the practically achievable trough length would be shorter than for the horizontal one-axis tracking arrangement. Nevertheless, the benefits of having many multiple foci (the use of smaller solar cells/arrays leads to improved efficiency and more effective cooling) would be maintained by such a system, yet the system would require a single primary concentrator. It is recommended that two-axis tracking
line-to-point focus systems be investigated as a competitor to conventional HCPV systems based on parquets of single-cell concentrators. The design theory detailed in Chapter 4 will serve as a starting point for the development of two-axis LTP design theory.

In this work emphasis was placed on the CPV application for line-to-point focus concentrators. Consideration should be given to other applications where the unique properties of a line-to-point focus system may be beneficial.

7.2.3. Characterization of concentrator solar cell arrays

The main limitation of the equivalent circuit model developed in this work is its inability to accurately predict solar cell performance under changing spectrum. Efforts should therefore be made to extend the proposed lumped equivalent circuit model into a subcell mode which considers each subcell as a separate entity connected in series to form the multijunction cell. The difficulty in extending the model comes not from its increased mathematical complexity which can be easily handled, but rather from the lack of subcell experimental data that would be required to tune the model. Ideally \( I-V \) curves for the subcells at different concentrations and temperatures would be available. If separate component subcells representing those in the triple junction cell could be obtained from the manufacturer, then it would be possible to obtain these data using the experimental methods outlined in Chapter 5. If component subcells cannot be obtained, then it may be possible to infer subcell behavior from mathematical models, and verify the model against \( I-V \) measurements under different spectra, e.g. by filtering the lamp of the flash solar simulator.

The 3D distributed equivalent circuit model could potentially be made faster by developing a dedicated code rather than solving the system in SPICE. To experimentally validate the model, the filtering method used for cell-to-cell irradiance could be extended to study the effect of single-cell nonuniformity by printing filters with finely patterned distributions. For the validation, it is recommended to use highly nonuniform distributions that significantly affect the \( I-V \) curve, such that the nonuniformity effects are clearly discernable.
7.2.4. The InPhoCUS HCPV collector

*Experimental*

The next steps for the InPhoCUS collector are the commissioning and experimentation of the 15 kW pilot plant comprising 200 secondary concentrator assemblies. In particular the pilot plant will allow to experimental demonstration of the tracking-secondary line-to-point focus system under non-normal incidence. Additionally, the accuracy of the assumption of the symmetry of the line-focus of the primary, which is of paramount importance for the system performance when the arrays are connected in series along the length of the trough, will be revealed by the pilot plant tests.

In the prototype tests, the radiant input power to the array was measured using the self-referencing method, i.e. using the cell itself as a photodetector. While this provides a very convenient method for simultaneously measuring the input and output power of the array, it is desirable to have an independent measurement of the input radiation. For this purpose a water-cooled calorimeter to be placed at the exit of one of the secondary concentrators is currently being designed and fabricated. The calorimeter will be installed in the pilot plant to enable a measurement of the optical efficiency of the system, independent of the cell measurements.

*Modeling*

Modeling efforts should be focused on improving the accuracy of the electrical model in predicting cell performance under non-standard spectra. This will require extending the lumped equivalent circuit model with a model in which the three subcells of the triple-junction cell are separately resolved.

In addition to improvements to the electrical model, the optical model should be adapted to allow for non-random deviations in the concentrator (especially the arcspline primary) shape to be considered. This would require modeling of the primary geometry as a freeform, e.g. NURBS, surface, which is currently not possible in the VeGaS code. It is therefore recommended that the code be extended to allow the modeling of freeform surfaces, which would also prove useful for the model of other general optical systems.
Design

The results of Chapter 6 showed that the system performance is limited mostly by the materials used in the initial design. It is recommended that the materials be changed to improve the overall optical efficiency of the system. In particular, the topsheet material could be changed to FEP. However, the long-term stability of this material would need to be verified. Additionally, due to the reduced tensile strength of FEP in comparison to ETFE, the topsheet radius would need to be reduced to reduce the stress in the topsheet without increasing its thickness. This could be achieved by shaping the topsheet into multiple arcs. The possibility of using silver instead of aluminum as a primary mirror coating on the BoPET membrane should be investigated. Additionally, the secondary reflector material should be changed for one that has an improved solar reflectivity. With these material changes, it is expected that the optical efficiency of the system could be increased to around 80%.

The possibility of using multijunction cells with their spectrum tailored to match the optical transfer function of the system could lead to significant array efficiency improvements. Results of the prototype measurements indicated the presence of significant spectral mismatch between the subcells. By tailoring the spectral response of the cell, it is expected that significant increases in the array efficiency could be realized.

In addition to these minor design changes, it is recommended to consider the use of a dielectric-filled secondary concentrator, such as a DTERC. This would lead to a higher achievable overall concentration, reduced reflection losses in the secondary concentrator, and potentially improved optical coupling between the exit of the secondary and the cell array. Of course, the cost of a dielectric-filled design in comparison to a reflective one with simple construction must be considered.

Finally, the prospect of using the waste heat of the impinging jet cooling system, resulting in a hybrid high-concentration photovoltaic thermal (HCPVT) system. Provided that there is a need for low-temperature (~70 °C) heat, this extension of the system would significantly improve the overall system efficiency.
Appendix A

Stretched membrane design derivations

A.1 Elongation of an infinitely long membrane in plane strain

Assumptions (in addition to those of Section 3.1.1):

- the membrane material is linearly elastic
- the membrane is infinitely long in the axial $y$-direction resulting in a 2D stress-state of plane strain in the membrane cross-section

The transverse stiffness can be estimated using the 3D form of Hooke’s law combined with assumptions about the material isotropy and stress state. The most general form of Hooke’s law relates the stress $\sigma$ and strain $\epsilon$ tensors of an anisotropic material [133]:

$$\sigma_{ij} = \sum_{k=1}^{3} \sum_{l=1}^{3} c_{ijkl} \epsilon_{ij}$$

(A.1)

where $c$ is the $3\times3\times3\times3$ stiffness tensor, which contains 21 independent stiffness coefficients. A coordinate system is setup with $e_1$, $e_2$, $e_3$ being the basis vectors, where $e_3$ is in the axial direction ($y$-axis of the collector) and $e_1$ and $e_2$ follow the local curvature of the membrane with $e_1$ tangent to the membrane (transverse direction) and $e_2$ in the direction of the thickness (parallel to the local normal). For plane strain $\sigma_{13} = \sigma_{23} = \sigma_{31} = \sigma_{32} = \epsilon_{13} = \epsilon_{23} = \epsilon_{31} = \epsilon_{32} = \epsilon_{33} = 0$ [133, 134].

A.1.1 Isotropic material

For a linearly elastic isotropic material, the stiffness tensor is completely described by two moduli. In terms of the Lamé constants $\lambda$ and $\mu$, the stiffness coefficients may be found from [133]:

$$c_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu \left( \delta_{ij} \delta_{jk} + \delta_{ik} \delta_{lj} \right)$$

(A.2)
where \( \lambda \) and \( \mu \) may be found from the more common Young’s modulus \( E \) and Poisson’s ratio \( \nu \) from [133]:

\[
\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} \quad (A.3)
\]

\[
\mu = \frac{E}{2(1+\nu)} \quad (A.4)
\]

Eq. (A.1) then simplifies to [134]:

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{23} \\
\sigma_{31} \\
\sigma_{12}
\end{bmatrix}
= \begin{bmatrix}
2\mu + \lambda & \lambda & \lambda & 0 & 0 & 0 \\
\lambda & 2\mu + \lambda & \lambda & 0 & 0 & 0 \\
\lambda & \lambda & 2\mu + \lambda & 0 & 0 & 0 \\
0 & 0 & 0 & \mu & 0 & 0 \\
0 & 0 & 0 & 0 & \mu & 0 \\
0 & 0 & 0 & 0 & 0 & \mu
\end{bmatrix}
\begin{bmatrix}
\epsilon_{11} \\
\epsilon_{22} \\
\epsilon_{33} \\
2\epsilon_{23} \\
2\epsilon_{31} \\
2\epsilon_{12}
\end{bmatrix} \quad (A.5)
\]

Applying the plane strain assumption yields:

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{23} \\
\sigma_{31} \\
\sigma_{12}
\end{bmatrix}
= \begin{bmatrix}
2\mu + \lambda & \lambda & \lambda & 0 & 0 & 0 \\
\lambda & 2\mu + \lambda & \lambda & 0 & 0 & 0 \\
\lambda & \lambda & 2\mu + \lambda & 0 & 0 & 0 \\
0 & 0 & 0 & \mu & 0 & 0 \\
0 & 0 & 0 & 0 & \mu & 0 \\
0 & 0 & 0 & 0 & 0 & \mu
\end{bmatrix}
\begin{bmatrix}
\epsilon_{11} \\
\epsilon_{22} \\
0 \\
\epsilon_{23} \\
2\epsilon_{31} \\
2\epsilon_{12}
\end{bmatrix} \quad (A.6)
\]

Which can be represented as a 2D problem:

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{12}
\end{bmatrix}
= \frac{E}{(1+\nu)(1-2\nu)}
\begin{bmatrix}
1-\nu & \nu & 0 \\
\nu & 1-\nu & 0 \\
0 & 0 & \frac{1}{2}(1-2\nu)
\end{bmatrix}
\begin{bmatrix}
\epsilon_{11} \\
\epsilon_{22} \\
2\epsilon_{12}
\end{bmatrix} \quad (A.7)
\]

with the out-of-plane stress given by:

\[
\sigma_{33} = \lambda(\epsilon_{11} + \epsilon_{22}) = \frac{E\nu}{(1+\nu)(1-2\nu)}(\epsilon_{11} + \epsilon_{22}) \quad (A.8)
\]
If the membrane is very thin, it can only transmit normal stresses, and therefore the shear stress and strain are both zero. Furthermore, membrane strains across the thickness are unrestricted implying that \( \sigma_{22} = 0 \). Applying these simplifications yield the system:

\[
\begin{bmatrix}
\sigma_{11} \\
0
\end{bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix}
1-\nu & \nu \\
\nu & 1-\nu
\end{bmatrix} \begin{bmatrix}
\epsilon_{11} \\
\epsilon_{22}
\end{bmatrix}
\]  

(A.9)

which implies:

\[
0 = \nu \epsilon_{11} + (1-\nu) \epsilon_{22}
\]

(A.10)

Subbing into Eq. (A.9) and simplifying yields:

\[
\sigma_{11} = \frac{E}{(1+\nu)(1-2\nu)} \left( (1-\nu) \epsilon_{11} + \nu \epsilon_{22} \right)
\]

\[
= \frac{E}{(1+\nu)(1-2\nu)} \left( (1-\nu) \epsilon_{11} - \frac{\nu^2}{1-\nu} \epsilon_{11} \right)
\]

\[
= \frac{E}{1-\nu^2} \epsilon_{11}
\]

(A.11)

Using Eqs. (A.10) and (A.11), the axial stress can be expressed as:

\[
\sigma_{33} = \frac{E \nu}{(1+\nu)(1-2\nu)} \epsilon_{11} \left( 1 - \frac{\nu}{1-\nu} \right)
\]

\[
= \frac{E \nu}{(1+\nu)(1-2\nu)} \left( 1 - \frac{\nu}{1-\nu} \right) \sigma_{11}
\]

\[
= \nu \sigma_{11}
\]

(A.12)

Therefore the axial stress \( \sigma_{33} \) is always smaller than the transverse stress \( \sigma_{11} \), implying that \( \sigma_{11} \) is the 1st principle stress. Eq. (A.11) gives the stress caused by an infinitesimal strain of \( \epsilon_{11} \). The total change in length of the membrane can be determined by summing up the infinitesimal changes in length as the membrane is incrementally stressed to its final tension.
Assume that the final tension $T$ is applied in small incremental load-steps of $\Delta T$. As the equations of elasticity are conservative, the path of applying $T$ does not change the final solution. It is therefore appropriate to choose uniform $\Delta T$.

The incremental change of the membrane length resulting from $\Delta T$ at load-step $j$ is:

$$
\Delta L_j = \varepsilon_{11,j} L_j = \frac{1-v^2}{E} \sigma_{11,j} L_j = \frac{1-v^2}{E} \frac{\Delta T}{t_j} L_j
$$

(A.13)

and the incremental change of thickness is:

$$
\Delta t_j = \varepsilon_{22,j} t_j = -\frac{v}{1-v} \varepsilon_{11,j} t_j = -\frac{v}{1-v} \frac{1-v^2}{E} \sigma_{11,j} t_j = -\frac{v}{E} \frac{1+v}{\Delta T} \Delta T
$$

(A.14)

which is constant. Therefore the thickness at load-step $j$ may be found from:

$$
t_j = t_0 + \sum_{i=1}^{j} \Delta t_i = t_0 - j \frac{v(1+v)}{E} \Delta T
$$

(A.15)

Using this, the change in length may be expressed as:

$$
\Delta L_j = \frac{1-v^2}{E} \frac{\Delta T}{t_0 - j \frac{v(1+v)}{E} \Delta T} L_j = \frac{(1-v^2) \Delta T}{Et_0 - j \Delta T v(1+v)} L_j
$$

(A.16)

We can rearrange to obtain:

$$
\frac{\Delta L_j}{\Delta T} = \frac{(1-v^2)}{Et_0 - j \Delta T v(1+v)} L_j
$$

(A.17)

Letting $\Delta T \to 0$, we obtain:

$$
\lim_{\Delta T \to 0} \frac{\Delta L}{\Delta T} = \frac{dL}{dT} = \frac{(1-v^2)}{Et_0} L
$$

(A.18)

Rearranging and integrating:
\[
T = \frac{Et_0}{(1 - \nu^2)} \ln \frac{L}{L_0} \tag{A.19}
\]

Defining \(\sigma_0 = T/t_0\) as the nominal stress, \(k = E/(1 - \nu^2)\) as the nominal transverse stiffness and \(\epsilon_0 = \sigma_0/k\) as the nominal strain, Eq. (A.19) can be conveniently written as:

\[
\epsilon_0 = \frac{1 - \nu^2}{E} \frac{T}{t_0} = \frac{\sigma_0}{k} = \ln \frac{L}{L_0} \tag{A.20}
\]

From which we obtain the final result:

\[
L_0 = S \cdot e^{-\epsilon_0} \tag{A.21}
\]

where we have replaced the arbitrary \(L\) with the final stretched length \(S\).

A.1.2 Orthotropic material

Due to the processing of BoPET, its mechanical properties differ in the machine direction (MD), transverse direction (TD) and film direction (FD). Eq. (A.1) for an orthotropic material may be written as [134]:

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{23} \\
\sigma_{31} \\
\sigma_{12}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\begin{bmatrix}
\epsilon_{11} \\
\epsilon_{22} \\
\epsilon_{33} \\
2\epsilon_{23} \\
2\epsilon_{31} \\
2\epsilon_{12}
\end{bmatrix} \tag{A.22}
\]

where [134] \(C_{11} = E_1(1 - \nu_{23}\nu_{32})Y, \ C_{22} = E_2(1 - \nu_{13}\nu_{31})Y, \ C_{33} = E_3(1 - \nu_{12}\nu_{21})Y, \ C_{13} = E_1(\nu_{21} + \nu_{31}\nu_{23})Y, \ C_{13} = E_1(\nu_{31} + \nu_{21}\nu_{32})Y, \ C_{13} = E_2(\nu_{32} + \nu_{12}\nu_{31})Y, \ C_{44} = \mu_{23}, \ C_{55} = \mu_{13}, \ C_{66} = \mu_{12}, \) and [134]:

\[
Y = \frac{1}{1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{21}\nu_{13} - 2\nu_{21}\nu_{32}\nu_{13}} \tag{A.23}
\]

and where the Poisson’s ratios follow the reciprocity relation [134]:
\[ \nu_{ij} / E_i = \nu_{ji} / E_j \] (A.24)

Applying the simplifications of plane stress, zero shear stress, and zero normal stress in the film direction:

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33}
\end{bmatrix} \quad (A.25)
\]

This implies:

\[
0 = C_{12} \varepsilon_{11} + C_{22} \varepsilon_{22} \]

\[
\varepsilon_{22} = -\frac{C_{12}}{C_{22}} \varepsilon_{11} \quad (A.27)
\]

Therefore:

\[
\sigma_{11} = C_{11} \varepsilon_{11} + C_{12} \varepsilon_{22} \]

\[
\sigma_{11} = \left( C_{11} - \frac{C_{12}^2}{C_{22}} \right) \varepsilon_{11} \quad (A.28)
\]

### A.2  Membrane shape under combined pressure and self-weight

The exact shape of an inflated membrane spanned from a point \( P_0(x_0, z_0) \) to a point \( P_1(x_1, z_1) \) under combined isotropic pressure and weight is derived. The shape is given by a second order non-linear ODE. The assumptions of Appendix A.1 apply. A force balance in \( x \) yields:

---

1 The material in this section has been prepared in collaboration with P. Good and documented in an unpublished report entitled “Derivation of 2D membrane shape under combined pressure and weight loading”.
\[
\sum F_x = 0 = p \, ds \sin \Theta - H + H + dH \tag{A.29}
\]

where \(H\) is the horizontal component of the tension. A force balance in \(z\) yields:

\[
\sum F_z = 0 = -p \, ds \cos \Theta - w \, ds - V + V + dV \tag{A.30}
\]

where \(V\) is the vertical component of the tension. Some useful identities are:

\[
ds = \sqrt{dx^2 + dz^2} = dx \sqrt{1 + \left(\frac{dz}{dx}\right)^2} \tag{A.31}
\]

\[
\cos \Theta = \frac{dz}{ds} \tag{A.32}
\]

\[
\sin \Theta = \frac{dz}{ds} \tag{A.33}
\]

Substituting Eq. (A.33) in Eq. (A.29):

\[
\sum F_x = 0 = p \, dy + dH \tag{A.34}
\]

\[
\frac{dH}{dz} = -p \tag{A.35}
\]

\[
H = -dz + c \tag{A.36}
\]

To determine the constant we specify the boundary condition at the left endpoint \((z = 0)\), where we assume the horizontal tension \(H_0\) is known. This gives:

\[
H = -pz + H_0 \tag{A.37}
\]

Substituting Eq. (A.32) in Eq. (A.30):

\[
\sum F_z = 0 = -p \, dx - w \, dx \sqrt{1 + \left(\frac{dz}{dx}\right)^2} + dV \tag{A.38}
\]

\[
dV = dx \left( p + w \sqrt{1 + \left(\frac{dz}{dx}\right)^2} \right) \tag{A.39}
\]

From a force triangle:

\[
\tan \Theta = \frac{dz}{dx} = \frac{V}{H} \tag{A.40}
\]
\[ V = H \frac{dz}{dx} \]  
(A.41)

\[ \frac{dV}{dx} = H \frac{d^2 z}{dx^2} + \frac{dz}{dx} \frac{dH}{dx} = (H_0 - pz) \frac{d^2 z}{dx^2} - p \left( \frac{dz}{dx} \right)^2 \]  
(A.42)

Subbing Eq. (A.42) into Eq. (A.39):

\[ p + w\sqrt{1 + \left( \frac{dz}{dx} \right)^2} = (H_0 - pz) \frac{d^2 z}{dx^2} - p \left( \frac{dz}{dx} \right)^2 \]  
(A.43)

\[ \frac{d^2 z}{dx^2} = \frac{p \left[ 1 + \left( \frac{dz}{dx} \right)^2 \right] + w\sqrt{1 + \left( \frac{dz}{dx} \right)^2}}{H_0 - pz} \]  
(A.44)

This is a second-order non-linear ODE having no analytical solution. To solve numerically, we can transform into a system of first-order non-linear ODEs:

\[ \frac{dz}{dx} = m \]  
(A.45)

\[ \frac{dm}{dx} = \frac{p \left( 1 + m^2 \right) + w\sqrt{1 + m^2}}{H_0 - pz} \]  
(A.46)

Eqs. (A.45) and (A.46) constitute a boundary value problem with boundary values \( P_0 \) and \( P_1 \).

A.2.1 Membrane weight

The mass of a 10 cm \( \times \) 10 cm sample of aluminized 23 \( \mu \)m Toray BoPET was measured on a precision balance (Mettler-Toledo). Characteristics are summarized below:

- Mass, \( m = 326.5 \) mg
- Mass per unit area, \( m/A = 3.265 \) mg/cm\(^2\) = 32.65 g/m\(^2\)
- Weight per unit area (weight pressure), \( w = 0.320 \) N/m\(^2\) = 320 mPa
- Thickness, \( t = 23 \) \( \mu \)m
- Density, \( \rho = 1.4196 \) g/cm\(^3\) (assuming \( t = 23 \) \( \mu \)m)
Appendix B

Polyagonal compound parabolic concentrators

Of the known three-dimensional nonimaging concentrator designs, the first and probably most well-known is the revolved compound parabolic concentrator (CPC) [23]. The revolved CPC is nearly ideal, attaining the maximum full-collection geometric concentration of \( C_{g,\text{max}} = 1/\sin^2 \theta_i \) allowed by the conservation of étendue, while rejecting only a small fraction of radiation within the acceptance angle. However, the revolved CPC is inherently difficult to manufacture due to its 2-dimensional curvature. One common route to improved manufacturability is to consider facetted designs which are either heuristic approximations to the CPC, or whose facet arrangement is numerically optimized to improve performance.

The most complete analysis of facetted CPCs is that of Timinger et al. [135] who considered both approximations to the CPC and optimized designs, with discretization of the curvature in both the circumferential and axial directions. It was generally shown that for a reasonable number of facets, axial discretization leads to a greater penalty in optical efficiency than does circumferential. In this study, we therefore focus on designs having circumferential discretization only, with the axial profiles being continuous compound parabolic curves. These designs, which we denote polygonal CPCs, are constructed by the intersection of multiple extruded 2D CPC profiles, with the inlet and exit apertures correspondingly forming regular polygons circumscribed about the circular apertures of the underlying revolved CPC. We forego optimization of the side profile since this optimization leads to at most a 5% improvement in optical efficiency [135], and moreover, the optimal

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solutions are dependent on the wall reflectivity which may not be known *a priori* and which may degrade as the concentrator ages.

For a given acceptance half-angle, the polygonal CPC has the same geometric concentration as its axisymmetric counterpart, and the revolved CPC may be considered a limiting case of the polygonal CPC as \( n \to \infty \). Since the side walls of polygonal CPCs exhibit one-dimensional curvature, they can be constructed by the bending of mirror sheets which are commercially available with high-reflectivity aluminum and silver coatings. In addition to improved manufacture, polygonal CPCs benefit from a higher ratio of collecting aperture area to height (compactness), and the possibility of tessellation of the Euclidian plane [136, 137] or of curved planes if the inlets are slightly truncated. Of particular interest for planar tilings is the simple square or “crossed” CPC formed by the intersection of two orthogonal extruded CPCs. Specific studies of the crossed CPC were previously reported [27, 43].

This appendix presents a unified analysis of polygonal CPCs considering the effect of the number of sides, acceptance angle, reflectivity and mirror specularity on the optical performance. The analysis is performed only for hollow reflective designs, however the majority of the results may be applied also for dielectric filled designs simply by designing the CPC for an acceptance angle (within the dielectric) of \( \theta_d = \arcsin(\sin\theta/n_d) \), where \( n_d \) is the refractive index of the dielectric.

### B.1 Theory and methods

#### B.1.1 Geometric definition

The polygonal CPC is formed by first considering a 3D revolved CPC as shown in Figure B.1. The polygonal inlet and outlet apertures are then formed by circumscribing\(^2\) an \( n \)-gon about the circular inlet an outlet of the revolved CPC, such that the apothem \( a \) of the \( n \)-gon is equal to the radius \( r \) of the corresponding circular aperture of the revolved CPC. The concentration of the \( n \)-gon CPC is identical to its revolved counterpart:

\[ C_g = \frac{\theta}{\sin\theta} \]

\(^2\) A concentrator may also be formed by inscribing \( n \)-gons within the circular apertures, resulting in a concentrator with the same \( C_g \). Considering polygons with even \( n \), inscribed designs are not edge-ray designs in 2D, and such not considered here.
where $\theta_i$ is the nominal half-acceptance angle of the concentrator. The sidelength of the polygonal is:

$$s = 2R \tan\left(\pi/n\right) \tag{B.2}$$

where $s = s_i$, $R = R_i$ for the inlet and $s = s_o$, $R = R_o$ for the outlet. A polygonal CPC is more compact than its revolved counterpart since the area of a circumscribed polygonal aperture is larger than the corresponding circular aperture of the revolved CPC having the same length.
A coordinate system is set up with the concentrator inlet forming the \(x-y\) plane, and the \(z\)-axis forming the cyclic symmetry/optical axis of the concentrator as shown in Figure B.1. Where orientation is important, the polygonal apertures are oriented such that one side is parallel to the \(x\)-axis. The direction of a ray striking the inlet aperture is:

\[
\hat{v} = \frac{Q-P}{\|Q-P\|} = [p \quad q \quad r] \quad (B.3)
\]

where \(p\), \(q\), and \(r\) are the direction-cosines in the \(x\), \(y\), and \(z\) directions respectively. Note that only \(p\) and \(q\) are required to define the ray direction since the magnitude of \(r\) follows from \(p^2+q^2+r^2 = 1\) and its sign is taken as negative for rays approaching the inlet.

### B.1.2 Performance

The geometrical performance in the limit of geometric optics may be described by a series of phase-space quantities (see Section 2.5.2) describing the passage of a ray of light striking the inlet aperture at a certain position \((x, y)\) and direction \((p, q)\). However, since these quantities are four-dimensional, they are not easily communicated. Performance is more readily interpreted by integral quantities obtained by integrating the phase-space quantities over one or more dimensions. In general the integrations may be carried out using Monte Carlo ray-tracing, directly yielding the integral quantity of interest.

### B.1.3 Ray-tracing simulation

Monte Carlo ray-tracing was performed using the in-house VeGaS code [69]. Unless otherwise stated, all simulations were performed using a diffuse (equi-cosine) source of cone-angle equal to the design acceptance half-angle \(\theta_i\). A spatially uniform irradiance distribution over the inlet aperture was assumed throughout. Additionally, the following assumptions apply: the shape is ideal (except in Section B.2.6); the reflectivity is uniform, independent of incident and outgoing directions, and perfectly specular (except in Section B.2.6); and geometric optics applies. Unless otherwise stated, simulations were performed with \(10^7\) rays, for which the Monte Carlo error is estimated to be below 0.01%.
To estimate the Monte Carlo error due to the finite number of rays, a set of 20 simulations each with a different random seed were performed for \( N = 10^3, 10^5, 10^6, \) and \( 10^7 \) rays. Figure B.2 shows the scatter in the \( \eta_{\text{acc}} \) for the 20 samples (markers) for a square CPC with \( \theta_i = 30^\circ \). The result for a single simulation performed with \( 10^8 \) rays is also shown for comparison. The relative population standard deviation computed as:

\[
\frac{\sigma}{\langle \eta_{\text{acc}} \rangle} = \sqrt{\frac{1}{20} \sum_{j=1}^{20} \eta_{\text{acc},j} - \langle \eta_{\text{acc}} \rangle}
\]  

(B.4)

is a good estimator of the relative Monte Carlo error and is shown by the solid line (secondary axis). The slope in log-log scale is seen to be 1/2 indicating that a factor of \( x \) improvement in accuracy requires an increase of a factor of \( x^2 \) in the number of rays.

Figure B.2. Monte Carlo error in acceptance efficiency for a square CPC with \( \theta_i = 30^\circ \) for different numbers of rays. The markers show the scatter in the Monte Carlo results of 20 runs for each \( N \) performed with different random seeds. The line shows the relative standard deviation of the runs and is a good estimator of the accuracy of the Monte Carlo result.

B.1.4 Monte Carlo error estimation

To estimate the Monte Carlo error due to the finite number of rays, a set of 20 simulations each with a different random seed were performed for \( N = 10^3, 10^5, 10^6, \) and \( 10^7 \) rays. Figure B.2 shows the scatter in the \( \eta_{\text{acc}} \) for the 20 samples (markers) for a square CPC with \( \theta_i = 30^\circ \). The result for a single simulation performed with \( 10^8 \) rays is also shown for comparison. The relative population standard deviation computed as:

\[
\frac{\sigma}{\langle \eta_{\text{acc}} \rangle} = \sqrt{\frac{1}{20} \sum_{j=1}^{20} \eta_{\text{acc},j} - \langle \eta_{\text{acc}} \rangle}
\]  

(B.4)

is a good estimator of the relative Monte Carlo error and is shown by the solid line (secondary axis). The slope in log-log scale is seen to be 1/2 indicating that a factor of \( x \) improvement in accuracy requires an increase of a factor of \( x^2 \) in the number of rays.
From Figure B.2 it is seen that the Monte Carlo error in $\eta_{\text{acc}}$ (for the square CPC with $\theta_i = 30^\circ$) is less than $10^{-4}$ (0.01%). It may be inferred that the errors for other integral quantities (acceptance/optical efficiencies for other number of sides and acceptance angles) are in the same range.

B.2 Results

B.2.1 Acceptance efficiency

The acceptance efficiency $\eta_{\text{acc}}$ is a purely geometric quantity giving the fraction of diffuse radiation within the acceptance angle which reaches the exit aperture after any number of reflections. Figure B.3 shows the acceptance efficiency of polygonal CPCs with different numbers of sides as a function of the acceptance angle. Also shown is the reference case of a revolved CPC.

A general trend of increasing acceptance efficiency with increased acceptance angle and number of sides is observed. However, anomalous behavior is present for the square CPC ($n = 4$), with performance surpassing the pentagonal CPC for nearly all acceptance angles and the hexagonal CPC for

---

3 $\eta_{\text{acc}}$ is sometimes referred to as transmission $T$, e.g. in [23].
Polygonal compound parabolic concentrators

For $\theta_i < 17^\circ$. For $\theta_i > 30^\circ$ polygonal CPCs with $n \geq 4$ obtain acceptance efficiencies of at least 95% of the corresponding revolved design. At smaller acceptance angles, polygonal CPCs perform comparatively worse. The triangular CPC performs poorly over the full range of acceptance angles.

### B.2.2 Transmission-angle curves

The transmission-angle curves give the fraction of rays incident at an angle $\theta$ with respect to the optical ($z$) axis, from all circumferential directions, which reach the exit aperture. While traditionally referred to as transmission $T(\theta)$ [23], this fraction is referred to here as the area-averaged directional acceptance $\bar{a}'(\theta)$ to stress that it considers whether the ray is accepted (and not what fraction of energy is transmitted), and that it applies for the specific case of a uniformly illuminated inlet aperture. **Figure B.4** gives the transmission-angle curves plotted against the normalized cone angle for various polygonal and the revolved CPC for $\theta_i = 5^\circ$ and $30^\circ$. The curves for the higher acceptance angle show a more ideal step-like behavior, with a smaller transition region. A general trend of increasing ideality with increasing $n$ is observed, but the anomalous behavior of the square CPC is again visible, especially for $\theta_i = 5^\circ$. For $\theta_i = 5^\circ$, all polygonal CPCs with $n \leq 8$ fail to achieve an acceptance of 1.

![Figure B.4. Transmission-angle curves for revolved and polygonal CPCs for (a) $\theta_i = 5^\circ$; and (b) $30^\circ$.](image_url)
even for $\theta = 0^\circ$, meaning that some on-axis rays are rejected by these concentrators. This is also true for the triangular CPC with $\theta_i = 30^\circ$. The curves for polygonal CPCs show some interesting irregularities not visible in the smooth curve of the revolved CPC due to their lack of axisymmetry (see also Section B.2.3).

### B.2.3 Area-averaged directional acceptance

For axisymmetric concentrators, the directional acceptance is independent of the circumferential angle, and thus the transmission angle curves $\bar{a}(\theta)$ fully describe the directional acceptance for a uniformly illuminated inlet aperture. Polygonal CPCs are not axisymmetric and the directional performance therefore depends not only on the zenith angle $\theta$, but on the circumferential sense of the ray. The concept of the area-averaged directional acceptance may then be extended to directional components, with $\bar{a}(p, q)$ giving the fraction of uniform rays from an incident direction $(p, q)$ which are accepted by the concentrator, where $p$ and $q$ are the direction-cosines defined in Figure B.1. When plotted in $p$-$q$ space, we obtain the 2D analogue of the 1D transmission-angle curves, which we denote the (directional) acceptance map of the concentrator. The acceptance maps of some crossed CPC designs were previously analyzed in Brunotte, et al. [27] and Molledo and Luque [55]\textsuperscript{4}. These maps are especially useful for sizing the acceptance angle of a concentrator to a source of known shape in direction-cosine space, as in [27].

The area-averaged directional acceptance was calculated by tracing $10^5$ rays uniformly distributed over the inlet aperture for each ray-direction in $p, q$ space, sampled on a $101 \times 101$ grid. Contour plots of the acceptance map for various polygonal CPCs with $\theta_i = 5^\circ$, $30^\circ$ and $45^\circ$ are presented in Figure B.5. The dashed circles show the acceptance map of an ideal concentrator for an isotropic radiation source comprising a circle of radius $\sin \theta_i$ with all rays inside the circle being accepted, and all rays outside being rejected. All acceptance maps show cyclic symmetry of order $n$, with generally improved performance for increasing $n$.

\textsuperscript{4} In [27] and [55] this quantity was referred to as the directional intercept factor.
Figure B.5. Area-averaged directional acceptance for revolved and polygonal CPCs with half-acceptance angle $\theta_i = 5^\circ$ (top row); $30^\circ$ (middle row); and $45^\circ$ (bottom row). The maxima occur at $p = q = 0$ and their corresponding values are shown in the center of each map. The dashed line is a circle of radius $\sin \theta_i$ indicating the acceptance function of an ideal concentrator. The dotted lines shown for $n = 4$ and $n = 6$ define the superficial acceptance map.
Considering the orientation of the inlet apertures (analogous to Figure B.10),
the maps for the triangle, square and hexagon CPC reveal that rejection is lower
for rays aimed perpendicular to the side walls. This is due to a lower likelihood
of these rays suffering wall-to-wall reflections which contribute to ray-
rejection. The low order cyclic symmetry is the cause of the irregular bends of
the transmission angle curves (Figure B.4) prevalent for concentrators with
\( n < 8 \).

The previously discussed anomalous behavior of the square CPC is
elucidated by comparing the map square CPC to those of the other designs. For
concentrators with even \( n \), we may construct a superficial acceptance map by
intersecting the acceptance maps of the underlying 2D CPCs comprising the
concentrator walls. The 2D CPC is an ideal concentrator and therefore has an
acceptance map in the shape of an ellipse with semi-minor diameter \( \sin \theta_i \) in the
cross-sectional direction and 1 in the extrusion direction. All whose direction
lies within the ellipse are accepted and all rays outside are rejected. Since
(even) \( n \)-gon CPCs are formed by the intersection of \( n/2 \) 2D CPCs, we may
intersect the acceptance maps of the underlying 2D CPCs to form a central
bound region which we denote the superficial acceptance map. We say
superficial because although this region follows intuitively by construction, we
may easily show that some of the rays with the barrel region formed by the
intersection this region must be rejected by comparing the étendue enveloped
by this region to the maximum étendue at the exit aperture. The maximum
allowable étendue at the exit aperture is that enveloped by a circle the unit
circle in \( p-q \) space:

\[
G_{o,max} = A_o \Omega_{o,max} = \pi A_o = \pi \frac{1}{C_g} A_i = \pi \sin^2 \theta_i A_i \tag{E.5}
\]

The étendue at the inlet aperture is necessarily larger than that enveloped by a
circle inscribed within the superficial acceptance region:

\[
G_i > \pi \sin^2 \theta_i A_i \tag{E.6}
\]

Comparing Eqs. (E.5) and (E.6) it is clear that the étendue enveloped by the
superficial acceptance region is larger than the maximum allowable étendue at
the exit aperture, and thus some rays within this region must be rejected. For the square CPC, the superficial acceptance map is the barrel shaped region enveloped by the dotted line in Figure B.5. Interestingly, the square CPC alone does not accept any rays outside of its superficial acceptance region.

In Figure B.5, the resulting regions are shown for the square and hexagonal CPCs by the dotted line. It is evident that the acceptance map of the square CPC is confined to this region: any ray which would be rejected by the underlying 2D CPCs is also rejected by the square CPC. In contrast, the acceptance maps of the other polygonal CPCs are not confined in this way. This confinement leads to a sharper cutoff of the acceptance map and may be viewed as a geometric explanation of the anomalous behavior of increased acceptance of the square CPC. Interestingly, not even the revolved CPC shares this property, since its superficial acceptance region is a circle of radius $\sin \theta_i$, and as seen in its transmission-angle curve (Figure B.4) and its area-averaged directional acceptance map (Figure B.5), some rays outside of this region are accepted.

B.2.4 Average number of reflections

Referring to Section 2.5.4 the average number of reflections can be determined from:

$$\langle n_r \rangle \approx \frac{1}{\eta_{acc}} \frac{\eta_{opt} (1-\Delta \rho)}{\Delta \rho}$$ \hspace{1cm} (B.7)

For this purpose an additional simulation at $\rho = 0.99$ ($\Delta \rho = 0.01$) was used, for which the calculation is correct to $\sim 1\%$ (order $\Delta \rho$).

Figure B.6 shows $\langle n_r \rangle$ for the revolved and various polygonal CPCs as a function of the acceptance angle. The curves for $n > 5$ lie between the curves of $n = 5$ and $n = \infty$ (revolved), and are omitted for clarity. The average number of reflections is seen to increase strongly with decreasing acceptance angle, due to increased slenderness. $\langle n_r \rangle$ is generally weakly increasing with decreasing $n$, with the triangle CPC showing notably higher $\langle n_r \rangle$ for small acceptance angles. The trend of $\langle n_r \rangle$ decreasing with $n$ is somewhat counter-intuitive since slenderness increases with increasing $n$, but may be explained by an increased
occurrence of multiple reflections in the sharper corners. For $\theta_i > 30^\circ$ all designs with $n \geq 4$ have $\langle n_r \rangle$ within 5% of that of the revolved CPC.

B.2.5 Effect of reflectivity on optical efficiency

In general, the optical efficiency is a coupled function of both the geometry and the mirror reflectivity. A common approximation for the effect of reflectivity on optical efficiency follows from the pioneering work of Rabl [32, 33]:

$$\eta_{opt} = \eta_{acc} \cdot \langle \rho^{n_r} \rangle \approx \eta_{acc} \rho^{\langle n_r \rangle}$$  \hspace{1cm} (B.8)

This simple one-parameter approximation holds with good accuracy for high values of $\rho$. It is clear, however, that this approximation fails near $\rho = 0$ since $\rho^{\langle n_r \rangle}$ approaches zero but $\eta_{opt}$ actually approaches the fraction $f_0$ of rays reaching the outlet aperture directly without reflection. An alternative approach is to model the dependence of optical efficiency on reflectivity using a power series in $n_r$ (see Section 2.5.5). Here we truncate to quadratic order:

$$\eta_{opt}(\rho) = a + b\rho + c\rho^2$$  \hspace{1cm} (B.9)
where the coefficients are given by the Eq. (2.81), using \( \rho_m = 0.9 \).

**Figure B.7** shows the optical efficiency vs. reflectivity for a square CPC of different acceptance angles. The markers are the results obtained directly by ray-tracing, the dashed lines are calculated from the average number of reflections approximation using the results from **Figure B.3** and **Figure B.6**, and the solid lines are calculated using the quadratic approximation. For the square CPC, the average number of reflections approximation, Eq. (B.8), was found to be conservative for all \( \rho \), accurate to within 1\% for \( \rho \geq 0.9 \), within 3\% for \( \rho \geq 0.8 \) and generally inapplicable for \( \rho < 0.5 \). The applicability of this formula increases with decreasing acceptance angle due to a corresponding increase of \( \langle n_r \rangle \) and decrease of \( f_0 \). The quadratic approximation, Eq. (B.9), was found to have a much wider range of applicability with error below 1\% over the full range of \( 0 \leq \rho \leq 1 \) for \( \theta_i > 30^\circ \). For smaller acceptance angles, the quadratic approximation of Eq. (B.9) breaks down due to a greater fraction of radiation suffering more than 2 reflections. As a rule of thumb, for designs having \( \langle n_r \rangle > 1.5 \), extension of Eq. (B.9) to cubic is recommended if performance at low reflectivity is of interest.
Due to microscopic surface roughness and geometric imperfections, real reflectors exhibit deviations from both the true shape and from perfect specular reflection. To quantify the combined effect of all components of the surface error, we assume that the local surface normal may randomly deviate from the true direction by some angular error, which in turn causes the ray to deviate from the true specular direction. The error is assumed to be described by two orthogonal vector components each following a zero-mean normal distribution with equal rms values (standard deviation) $\sigma_{\text{err}}$ [118, 138]. This gives a one-parameter estimate for quantifying the effect of mirror surface errors, with $\sigma_{\text{err}}$ which quantify the severity of the surface error. Typical solar concentrator mirror materials exhibit an rms error in the range of $\sigma_{\text{err}} = 0.5$ to 5 mrad [119, 139].

Figure B.8 (a) shows the reduction in acceptance efficiency, $K_{\text{err}} = \eta_{\text{opt}}(\sigma_{\text{err}})/\eta_{\text{opt}}(\sigma_{\text{err}} = 0)$, of revolved and polygonal CPCs as a function of the rms error for $\theta_i = 30^\circ$. Designs having the highest acceptance efficiency (see

\[\text{Figure B.8. (a) Efficiency reduction vs. rms surface error for revolved and polygonal CPCs with } \theta_i = 30^\circ; \text{ (b) effect of acceptance angle on efficiency reduction for a square CPC.} \]

1 Note that this is equivalent to the magnitude of the vector sum of the two error components being Rayleigh distributed.
Figure B.9. Comparison of optical efficiency approximated by Eq. (B.10) (lines) and calculated directly by ray-tracing (markers) for square and hexagonal CPCs for $\rho = 0.9$ and $\sigma_{\text{err}} = 2$ mrad. The relative error is below 1% for both curves over the full range of acceptance angles considered.

Figure B.3) show the highest sensitivity to the surface error. Figure B.8 (b) shows the effect of acceptance angle for a square CPC. For smaller acceptance angles the effect of the error is considerably more severe due to the fact that the surface error represents a larger fraction of the acceptance angle. For $\theta_i > 30^\circ$, large surface errors can be accommodated without any significant reduction in acceptance efficiency.

B.2.7 Estimating the optical efficiency

Using the results of Sections B.2.1 to B.2.6, the optical efficiency, including effects of geometry, finite reflectivity, and surface errors may be estimated by a simple approximate formula which decouples the effect of each:

$$
\eta_{\text{opt}}(n, \theta_i, \rho, \sigma_{\text{err}}) \approx K_{\text{err}} \rho^{(n)} \eta_{\text{acc}}
$$

(B.10)

where $\eta_{\text{acc}}$ is taken from Figure B.3, $\langle n_i \rangle$ from Figure B.6, and $K_{\text{err}}$ from Figure B.8.

Figure B.9 shows the optical efficiency vs. acceptance angle for square and hexagonal CPCs as simulated by ray-tracing (markers) and as approximated by
Eq. (B.10) (lines). The relative error $|\varepsilon| = |\eta_{\text{opt,ray-trace}} - \eta_{\text{opt,Eq.(B.10)}}|/\eta_{\text{opt,ray-trace}}$ is below 1% over the full curve for both cases.

B.2.8 Irradiance distribution at the outlet

The normalized irradiance distributions at the outlet aperture of polygonal CPCs of different numbers of sides for an acceptance angle of 5°, 30° and 45° and are shown in Figure B.10 (a), (b) and (c) respectively. As a reference, the irradiance distribution for a revolved CPC is also shown. Note that to maximize contrast, the normalization was performed separately for each concentrator, i.e. the peak irradiance for one distribution cannot be compared directly to other distributions. To reduce noise in the distribution these simulations were performed with $10^9$ rays.

Two trends regarding the spatial uniformity of the irradiance distribution are visible from Figure B.10. First, there is a general trend of increasing irradiance uniformity with increasing $n$. For low $n$, areas of low irradiance appear near the corners of the exit aperture. The square CPC shows anomalous behavior, with less pronounced dark spots in the corners. Second, there is a trend of increasing uniformity with increasing $\theta_i$. The fine distributional features visible for $\theta_i = 5°$ are not present in the distributions for 30° and 45°. The triangle CPC for $\theta_i = 5°$ is anomalous to both of these trends, but the optical efficiency of this design is too low to be of practical use in most applications.

B.3 Summary

The optical properties of CPCs with polygonal apertures having $n = 3, 4, 5, 6, 8$ and 12 sides were determined by Monte-Carlo ray-tracing and compared to those of the revolved CPC. For $\theta_i > 30°$, the acceptance efficiency of polygonal CPCs with $n \geq 4$ were found to be within 5% of that of the revolved CPC. While performance generally increased with increasing $n$, the square CPC showed some notable anomalous behavior with $\eta_{\text{acc}}$ greater than or equal to that of the pentagonal CPC over the full range of acceptance angles considered, and greater than the hexagonal CPC for $\theta_i < 17°$, making it a particularly interesting design. The favorable anomalous behavior of the square CPC is explained by
Figure B.10. Irradiance distribution at the outlet of circular and various polygonal CPCs with half-acceptance angle (a) $\theta_i = 5^\circ$; (b) $\theta_i = 30^\circ$; and (c) $\theta_i = 45^\circ$. Calculated by Monte Carlo ray-tracing with $10^9$ rays.
the bounding of its acceptance map (Figure B.5) to the intersection of the acceptance ellipses of the underlying crossed 2D CPCs.

In terms of the average number of reflections, no considerable differences between polygonal and the revolved CPC were found, except for the triangular CPC which showed notably higher $\langle n_r \rangle$ for small acceptance angles.

The effect of normally distributed surface errors (Figure B.8) was found to be more severe for designs having high acceptance efficiencies, and the effect was found to strongly depend on the acceptance angle. For $\theta_1 \geq 30^\circ$, rms surface errors as high as 10 mrad can be accommodated by all designs with negligible decrease in acceptance efficiency. The optical efficiency, including the acceptance efficiency, the effect of surface errors, and the effect of finite reflectivity was found to be well-approximated by determining each effect individually and subsequently combining them through Eq. (B.10).

The irradiance distributions at the exit aperture of polygonal CPCs were shown to be less uniform than that of the revolved CPC, with dark spots forming near the corners, suggesting that revolved CPCs are preferred in applications where irradiance uniformity is of paramount importance.

Polygonal CPCs are of particular interest for applications where the concentrators must be uniformly tiled over a plane or sphere, since the revolved CPC would result in significant gap-losses in such an application. Especially for large acceptance angles, their performance approaches that of the revolved CPC, making polygonal CPCs good candidates for lower-cost secondary concentrators coupled to large rim-angle dish, tower, or trough primaries.
Appendix C

Optical property measurements

C.1 Materials

Four materials, one transparent and three reflective, were investigated. Their basic characteristics are listed below.

C.1.1 ETFE membrane
- Composition: poly(ethylene tetrafluoroethylene)
- Thickness: 100 μm
- Manufacturer: Toray Advanced Film Co.
- Trade name: Toyoflon
- Relevant properties: narrow-angle transmittance

C.1.2 Aluminized BoPET membrane
- Composition: PVD aluminum layer on biaxially-oriented poly(ethylene terephthalate) substrate
- Substrate thickness: 23 μm
- Manufacturer: Toray
- Usage: topsheet for inflated collectors
- Relevant properties: narrow-angle reflectivity

C.1.3 Silvered aluminum sheet
- Composition: PVD silver layer on aluminum sheet substrate
- Substrate thickness: 0.5 mm
- Manufacturer: Almeco
- Trade name: Vega V98100
- Usage: secondary concentrator/deflector
- Relevant properties: narrow-angle reflectivity

---

1 Material in this Appendix has been extracted from G. Happle, “Measurement of spectral reflectance and transmittance properties for an inflated trough-based HCPV collector,” Semester Project, ETH Zurich, 2012, conducted under the direct supervision of T. Cooper.
C.1.4 Silvered aluminum film
- Composition: PVD silver layer on copper film substrate
- Manufacturer: 3M Company
- Usage: Adhesive mirror to be pressed on preformed shape
- Relevant properties: narrow-angle reflectivity

C.2 Experimental setup and procedure
C.2.1 Membrane tensioning fixture

In order to measure the optical properties of the membrane materials, it is necessary to stretch the sample. The stretching must be applied in a uniform way to prevent wrinkling in the membrane. For this purpose, a membrane tensioning fixture was designed and fabricated. With regard to wrinkling, there are three possible stress regimes classified according to the principal stresses $\sigma_{11}$ and $\sigma_{33}$ in the plane of the membrane [140]:

1. $\sigma_{11} = \sigma_{33} = 0$: the membrane is unloaded and wrinkle-free, however the configuration is unstable.
2. $\sigma_{11} > \sigma_{33} > 0$: the membrane is stable and wrinkle-free.
3. $\sigma_{11} > 0$, $\sigma_{33} = 0$: the membrane is prone to wrinkling.

Clearly the tensioning fixture must strive to obtain the second stress regime. Figure C.1 shows a schematic of the tensioning fixture concept. The membrane sample is placed on the frame and held in place by the retention ring. The set screws are then tightened which causes the compression ring to push against the membrane and compress the O-ring, thus uniformly stretching the membrane in the radial direction. This induces a state of uniform stress in the membrane. Let $\sigma_o$ be the radial stress at the boundary induced by the stretch. In the ensuing stress state, the radial and tangential stresses are uniform and equal to the boundary stress $\sigma_{\text{radial}} = \sigma_{\text{tangential}} = \sigma_o$. By adjusting the set screws, the stress in the membrane can be carefully controlled. Figure C.2 shows an exploded view of the final membrane fixture design.
C.2.2 The RADLAB

Measurements were performed in the ETH RADLAB [116], a spectroscopic goniometry laboratory setup originally designed to measure the spectral radiative properties of participating media, namely the extinction coefficient and scattering phase function. The setup needed to be modified to allow measurement of narrow-angle optical properties. The modified setups, described in Sections C.2.3 to C.2.5 allow the precise determination of the optical properties of specular materials used in solar concentrators in the wavelength range 280 to 4000 nm.

C.2.3 Narrow-angle reflectivity

Figure C.3 shows the experimental setup for measuring narrow-angle reflectivity. Monochromatic light exits the monochromator and passes through an exit slit. The diverging beam is collimated by an $f = 50$ mm lens and then condensed by an $f = 75$ mm lens onto a 1 mm pinhole, which performs the function of reducing the étendue of the beam. The reduced beam reaches a
The collimated beam hits the sample which can be angled with respect to the beam direction. The beam is reflected to and is then reflected to an $f = 50$ mm lens which focuses the reflected beam onto the detector. In order to maintain a good system focus at all wavelengths, this lens and the $f = 250$ mm lens are mounted on translation stages to account for the change in refractive index with wavelength. The detector has a sandwiched Si/MCT construction with the Si detector suitable for wavelengths up to 1000 nm and the MCT detector suitable for wavelengths from 1000 nm to 4000 nm. The acceptance angle of the detector is defined by the detector diameter in conjunction with the focal length of the last lens by:

$$\alpha = \frac{d_{\text{pinhole}}}{2f} = \frac{1 \text{ mm}}{2 \times 250 \text{ mm}} = 2 \text{ mrad} \quad (C.1)$$

Figure C.2 Exploded view of the final membrane fixture design: (1) rotary stage; (2) retention plate; (3) frame; (4) o-ring; (5) compression ring (optional); (6) support ring; (7) set screws; (8) retention screw; (9) mount; (10) membrane sample; (11) screw; (12) screw; (13) base.
Optical property measurements

Note that the MCT detector is square. The acceptance angle in Eq. (C.2) is based on the side length. The acceptance angle based on the diagonal is $14.4 \text{ mrad}$. The acceptance angle of the detector defines the angular extent of the reflectivity being measured by the system. When perfectly aligned, the system therefore measures the narrow-angle reflectivity within an angle of $25 \text{ mrad}$ for the Si detector, and $10 \text{ mrad}$ for the MCT detector. The reported reflectivities for measurements with the Si detector are therefore expected to be slightly higher than for the MCT detector.

An absolute measurement procedure is adopted. First a reference measurement is made with no sample in place. Then the sample is placed in the setup, and the sample measurement is made. To reduce error induced by drift of the source power output, a second reference measurement was taken after the sample measurement. The ratio of the detector signals for the sample and the

\[
\alpha_{\text{Si}} \approx \frac{d_{\text{Si detector}}}{2f} = \frac{2.5 \text{ mm}}{2 \times 50 \text{ mm}} = 25 \text{ mrad}
\]

\[
\alpha_{\text{MCT}} \approx \frac{d_{\text{Si detector}}}{2f} = \frac{1 \text{ mm}}{2 \times 50 \text{ mm}} = 10 \text{ mrad}
\]

Figure C.3 RADLAB setup for measuring narrow-angle reflectivity. The setup for transmittance is similar, except that the sample measurement has the same orientation as the reference measurement.
average of the two reference measurements gives the absolute reflectivity of the sample. Each measurement consists of the average of 20 detector signal readings to reduce noise.

C.2.4 Narrow-angle transmittance

The setup for narrow-angle transmittance is nearly identical to that for reflectivity, except that the detector arm has the same position for the reference and sample measurements.

C.2.5 Optical dispersion

In addition to transmittance and reflectivity, optical dispersion was measured using the setup shown in Figure C.4. The pinhole was removed and the beam collimation was instead controlled by adjusting the monochromator slit width. This provides a highly collimated beam in the plane of the paper of Figure C.4, yet the beam remains largely uncollimated in the out-of-paper direction. This allows for measurement of the optical dispersion in one direction alone, which is useful for orthotropic materials as subsequently discussed. For all optical dispersion measurements, the monochromator slit width was set to 150 μm, which, in conjunction with the $f=150$ mm collimation lens, leads to a

![Figure C.4. RADLAB setup for measuring optical dispersion (surface scattering).](image)
collimation half-angle of 0.5 mrad in the plane of the paper. By adjusting the opening width of the mechanical slit, the collimation of the reflected beam can be profiled. The profile can then be compared against a model to extract the optical dispersion parameters of the surface. For this purpose, a Monte Carlo ray-tracing model of the setup of Figure C.4 was developed using the VeGaS code [69]. In the model, the surface is assumed to have an angular dispersion error whose magnitude follows a Rayleigh distribution with mode $\sigma_{\text{err}}$ [118]. By matching the measured and simulated beam profiles at the slit, $\sigma_{\text{err}}$ can be determined.

Due to their manufacturing process, certain materials exhibit anisotropic optical behavior. Commonly a mirror material may exhibit ridges running parallel to the length of the processing direction. This anisotropic roughness causes the optical dispersion to be dependent on the direction of the incident beam with respect to the processing direction.

Consider a nearly collimated arriving at a certain AOI with respect to the bulk surface normal. We define the plane of incidence as the plane containing the direction vector of the incident beam and the bulk surface normal. The optical dispersion will differ depending on whether the processing direction is parallel (para.) to the plane of incidence or perpendicular (perp.) to it. In general, the optical dispersion will be more severe for the perp. case. Of the considered materials, both the silvered film and sheet show visibly noticeable ridges in the processing direction. Therefore these materials were investigated in both the para. and perp. orientations. An AOI of $45^\circ$ and a wavelength of 500 nm were used for all optical dispersion measurements.

C.3 Results

Figure C.5 shows the spectral narrow-angle transmittance of the ETFE at normal incidence. Non-normal measurements were also performed, but were found to be inaccurate due to the polarization of the beam from the monochromator. To perform accurate non-normal measurements for semi-transparent materials, it would be necessary to install a linear polarizer before the sample and make measurements under both parallel and perpendicular linear polarizations.
Figure C.5. Narrow-angle spectral transmittance of ETFE at normal incidence.

Figure C.6. Narrow-angle spectral reflectivity of aluminized BoPET.
Figure C.7. Narrow-angle spectral reflectivity of silvered sheet.

Figure C.8. Narrow-angle spectral reflectivity of silvered film.
The oscillations visible at wavelengths above 1000 nm, and increasing in amplitude with increasing wavelength are due to high-order thin-film interference in the sample.

Figures C.6, C.7 and C.8 show the narrow angle reflectivity for the aluminized BoPET, silvered sheet and silvered film mirrors respectively. The curves are generally consistent with the reflectivity curves for the pure metals (aluminum and silver). However the silvered sheet, Figure C.7, showed a peculiar dip in the reflectivity around 800 nm which suggests that the investigated material in fact has an aluminum coating or an alloy. It is recommended that the measurements be repeated with a new sample.

C.3.1 Solar averaged properties

As a quick estimate of the performance of the material, it is useful to know the average value of the property over the relevant range of solar wavelengths. The so-called solar averaged properties are determined from

\[ X_{\text{solar}} = \frac{\int_0^\infty X_{\lambda}(\lambda)E_{\lambda}(\lambda)d\lambda}{\int_0^\infty E_{\lambda}(\lambda)d\lambda} \quad (C.3) \]

where \( X \) is the property of interest, e.g. the transmittance or reflectivity, and \( E_{\lambda} \) is the relevant solar spectrum. For materials used in concentrating collectors, the ASTM AM 1.5 direct + circumsolar spectrum the commonly used standard. Table C.1 gives the calculated solar weighted properties of the four materials investigated.
Figure C.5 shows the measured beam profiles in terms of the normalized detector signal as a function of the slit opening for a wavelength of 500 nm. Superimposed on the plot are the curves as predicted by the ray-tracing model for different angular dispersion errors, \( \sigma_{\text{err}} \). This allows the determination of the representative \( \sigma_{\text{err}} \) for each surface by comparing the measured and modeled curves. The discrepancy between the “no error” simulation and the reference measurements indicate the presence of some experimental effects that are not adequately captured by the model. A significant effort has been placed into tuning the model to best represent the true experimental setup. Nevertheless, some discrepancy has remained and it is therefore recommended that the reading at slit widths less than 100 \( \mu \text{m} \) be used with caution.

Additional wavelengths in the range 400 – 1100 nm were also investigated, but no significant spectral dependence in the resulting \( \sigma_{\text{err}} \) values was found, except for ETFE and BoPET which showed a reduction in the dispersion at longer wavelengths.
Table C.2. Representative values of $\sigma_{err}$ [mrad] for a Rayleigh distributed angular dispersion error. Extracted from Figure C.5.

<table>
<thead>
<tr>
<th></th>
<th>ETFE</th>
<th>alum. (para.)</th>
<th>BoPET (para.)</th>
<th>silvered foil (para.)</th>
<th>silvered foil (perp.)</th>
<th>silv. sheet (para.)</th>
<th>silv. sheet (perp.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;0.25</td>
<td>0.25 – 0.5</td>
<td>&lt;0.75</td>
<td>&lt;1.25</td>
<td>&lt;1.25</td>
<td>~5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table C.2 gives a summary of representative angular dispersion errors for the analyzed materials, inferred from Figure C.5. The dispersion error values are relatively insensitive to the wavelength.
Appendix D

Relations for asymmetric parabolic troughs

D.1 Primary concentration

Consider a parabolic trough having focal length $f$ and a polar angle spanning $\Phi_1 \leq \varphi \leq \Phi_2$ as shown in Figure D.1. The analysis presented here is only valid for reasonably small $\Phi_1$. The edge-ray pencils reflected from $\Phi_1$ and $\Phi_2$ will intersect near the paraxial focus $F$. For sufficiently small $\theta$, the intersection of these two pencils forms a parallelogram [50] $ABCD$ with centroid at the $F$. The half-width of each pencil in the vicinity of $F$ is:

$$w_\varphi = r(\varphi) \sin \theta_1 = \frac{2f \sin \theta_1}{1 + \cos \varphi} = f \sin \theta_1 \sec^2 \left(\frac{1}{2} \varphi\right) \tag{D.1}$$

From this we can calculate the half side lengths of the parallelogram:

$$\|HD\| = \frac{w_{\varphi_2}}{\sin (\Delta \varphi)} = f \sin \theta_1 \sec^2 \left(\frac{1}{2} \Phi_2\right) \csc (\Delta \Phi) \tag{D.2}$$

For full collection, both pencils must intersect the edges of the focal plane. The maximum concentration is found when the focal plane width is minimized, occurring when it is tilted by an angle $\tau^*$ and coincides with the shorter diagonal of $ABCD$. Applying the law of cosines to triangle $HFD$:

$$\|FD\|^2 = \|HD\|^2 + \|HF\|^2 - 2 \|HD\| \|HF\| \cos (\Delta \Phi) \tag{D.3}$$

$$\|FD\| = f \sin \theta_1 \csc (\Delta \Phi) \left[\sec^4 \left(\frac{1}{2} \Phi_1\right) + \sec^4 \left(\frac{1}{2} \Phi_2\right) - \frac{2 \sec^2 \left(\frac{1}{2} \Phi_1\right) \sec^2 \left(\frac{1}{2} \Phi_2\right) \cos (\Delta \Phi)}{2}\right]^{1/2} \tag{D.4}$$

---

The tilt angle may then be found from the law of sines:

\[
\sin(\angle HFD) = \frac{\|HD\| \sin(\Delta \Phi)}{\|FD\|}
\]  \hspace{1cm} (D.5)

Angle \(\angle HFD\) is equal to \(\tau^* - \Phi_2 + 90^\circ\), therefore \(\tau^*\) may be found from:

\[
\cos(\Phi_2 - \tau^*) = \frac{\sin(\Delta \Phi)}{\cos^2(\frac{1}{2} \Phi_2)} \left[ \frac{\sec^4(\frac{1}{2} \Phi_1) + \sec^4(\frac{1}{2} \Phi_2) - 2 \sec^2(\frac{1}{2} \Phi_1) \sec^2(\frac{1}{2} \Phi_2) \cos(\Delta \Phi)}{\sec^2(\frac{1}{2} \Phi_1) \sec^2(\frac{1}{2} \Phi_2) \cos(\Delta \Phi)} \right]^{-1/2}
\]  \hspace{1cm} (D.6)

An equivalent expression for \(\tau^*\) was previously derived by Collares-Pereira et al. [50]. An exact expression for the maximum concentration can be found by intersecting the actual reflected edge rays (without the small angle approximation) from \(\Phi_1\) and \(\Phi_2\).

We now consider the case when the focal plane tilt is not equal to \(\tau^*\). For \(\tau < \tau^*\), the full intercept condition is limited by the \(\Phi_2\) beam:

\[
\|FD\| = \|FB\| = \csc(90^\circ - \Phi_2 + \tau) w_{\Phi_2} = \frac{2f \sin \theta_i}{(1 + \cos \Phi_2) \cos(\tau - \Phi_2)}
\]  \hspace{1cm} (D.7)
For $\tau > \tau^*$, the full intercept condition is limited by the $\Phi_1$ beam:

\[
\|\text{FD}^*\| = \|\text{FB}^*\| = \frac{2f \sin \theta}{(1 + \cos \Phi_1) \cos (\tau - \Phi_1)} \tag{D.8}
\]

To calculate the concentration, we need the collecting aperture width:

\[
a_{i,1} = 2f \left( \tan \left( \frac{1}{2} \Phi_2 \right) - \tan \left( \frac{1}{2} \Phi_1 \right) \right) \tag{D.9}
\]

The geometric concentration for a given focal plane tilt angle $\tau$ is then:

\[
C_{g,\text{asym. parab.}} = \left( \frac{a_{i,1}}{2\|\text{FD}\|} \right) = \frac{1}{2} \csc \theta \left[ \tan \left( \frac{1}{2} \Phi_2 \right) - \tan \left( \frac{1}{2} \Phi_1 \right) \right] (1 + \cos \varphi) \cos (\tau - \varphi)
\]

where: \( \varphi = \begin{cases} 
\Phi_2 & \text{for } \tau < \tau^* \\
\Phi_1 & \text{for } \tau > \tau^* \\
\Phi_2 \text{ or } \Phi_1 & \text{for } \tau = \tau^*
\end{cases} \)

For the particular case of $\tau = \frac{1}{2}\Phi_2$ this simplifies to:

\[
C_{g,\text{APT}} (\tau = \Phi_{av}) = \frac{1}{2} \csc \theta \left[ \tan \left( \frac{1}{2} \Phi_2 \right) - \tan \left( \frac{1}{2} \Phi_1 \right) \right] (1 + \cos \Phi_2) \cos \Phi \tag{D.11}
\]

As evident from Figure D.1, $\tau^*$ is not the bisector $\Phi_{av} = \frac{1}{2}(\Phi_1 + \Phi_2)$ of the polar angles. Interestingly the decrease in concentration is less severe for $\tau < \tau^*$ as seen for the example in Figure D.2. In the next section, it will be shown that for use in tracking-secondary line-to-point concentrators, secondary concentration favors a focal plane tilt of $\tau = \Phi_{av}$. This asymmetric behavior is therefore convenient since $\Phi_{av}$ is in general less than $\tau^*$.

### D.2 Secondary concentration for tracking-secondary LTP collectors

A tilted coordinate system $x', y', z'$ is setup by rotating the $(x, y, z)$ coordinate frame about the $y$ axis by the focal plane tilt angle $\tau$. The analogous equation to Eq. (4.44) for a tilted focal plane is:
**Figure D.2.** Primary, secondary and overall concentration for a tracking-secondary LTP concentrator based on a semi-parabolic trough primary ($\Phi_1 = 0^\circ$, $\Phi_2 = 60^\circ$) as a function of the focal plane tilt angle. The system is designed for full collection at a latitude of $\phi = 30^\circ$. The secondary concentration is maximal for $\tau = \frac{1}{2}(\Phi_2 + \Phi_1) = \Phi_{av}$. The primary concentration is maximal for $\tau = \tau^* = 43.9^\circ$ as calculated from Eq. (D.6). Because the maximum secondary concentration is much more sensitive to the tilt angle, the total concentration is always maximal for the tilt angle $\tau = \Phi_{av}$ which maximizes secondary concentration.

\[
\begin{bmatrix}
\cos \tau & 0 & \sin \tau \\
0 & 1 & 0 \\
-\sin \tau & 0 & \cos \tau
\end{bmatrix}
\begin{bmatrix}
-\cos \theta \sin \varphi \\
\sin \vartheta \\
\cos \theta \cos (\varphi - \tau)
\end{bmatrix} =
\begin{bmatrix}
-\cos \theta \sin (\varphi - \tau) \\
\sin \vartheta \\
\cos \theta \cos (\varphi - \tau)
\end{bmatrix}
\] (D.12)

where $\Phi_1 \leq \varphi \leq \Phi_2$. We make use of the mean polar angle:

\[
\Phi_{av} = \frac{1}{2}(\Phi_1 + \Phi_2)
\] (D.13)

The approach angle in the tilted axial plane may be defined analogously to that for the untilted focal plane (Section 4.4.1):

\[
\tan \beta' = \frac{M'}{N'} = \tan \vartheta \sec (\varphi - \tau)
\] (D.14)
For any given skew angle, the beamspread will be minimized (in absolute value) when the focal plane is tilted by an angle \( \tau = \Phi_{av} \). The minimum and maximum approach angles for this tilt angle then:

\[
\beta'_{\text{min}} = \arctan \left[ \tan \vartheta \sec \left( \Phi_{av} - \Phi_{av} \right) \right] = \vartheta
\]  \hspace{1cm} (D.15)

\[
\beta'_{\text{max}} = \arctan \left[ \tan \vartheta \sec \left( \Phi_{av} - \Phi_{1} \right) \right] = \arctan \left( \tan \vartheta \sec \Phi \right)
\]  \hspace{1cm} (D.16)

Comparing Eqs. (D.15) and (D.16) to Eqs. (4.46) and (4.47) for the derivation of the symmetric trough, it is seen that the expressions are identical when considering the formalization of the rim angle \( \Phi = \frac{1}{2}(\Phi_2 - \Phi_1) \). Since the limits of secondary concentration may be derived from the maximum and minimum approach angles, it is concluded that the secondary axial concentration for an asymmetric trough when the focal plane is tilted by \( \tau = \Phi_{av} \) is the same as for the symmetric trough with the same (formalized) rim angle.

### D.3 Overall concentration for tracking-secondary LTP collectors

The tilt angle for maximum primary concentration \((\tau = \tau^*)\) is different than that for maximum secondary concentration \((\tau = \Phi_{av})\), thereby raising the question of which tilt angle maximizes the overall concentration. Figure D.2 shows the primary, secondary and total concentration for a semi-parabolic dish with \( \Phi_2 = 60^\circ \). It is seen that maximum is much steeper for the secondary concentration than for the primary. Therefore the optimum total concentration favors maximizing secondary concentration at the cost of primary concentration. Similar behavior occurs for any choice of \( \Phi < 90^\circ \) such that the maximum total concentration is always achieved for a tilt angle of \( \tau = \Phi_{av} \).
Appendix E

Relations for multifoliate light-pipe concentrators

Figure E.1 shows a schematic of a generic multi-foliate light-pipe concentrator. Basset and Forbes [63] derived a first order approximation for the fraction of rays within the acceptance angle for a single curved light pipe of a given thickness $t$ and inner radius $r$:

$$ f \approx 1 - \frac{t}{6r} \cot^2 \theta_d $$

(E.1)

Equation (E.1) can be used to design the leaves for a given accepted fraction $f = \text{const.}$ from each leaf:

$$ t_j = 6 \tan^2 \theta_d (1 - f) r_j = c r_j $$

(E.2)

where $\theta_d = \arcsin[\sin(\theta_i)/n]$ is the acceptance angle within the dielectric. For a constant accepted fraction $f$, this implies that the leaf thickness should be proportional to the inner radius of the leaf. The constant of proportionality $c$ is here denoted the leaf constant:

$$ c = 6 \tan^2 \theta_d (1 - f) $$

(E.3)

From Eq. (E.2), a power law for the radius may be developed. Letting $j = 1$ be the outermost leaf:

---

A power law for $t$ may be similarly developed:

$$
\begin{align*}
  r_{j+1} &= r_j - t_{j+1} = r_j - c\; r_{j+1} \\
  r_{j+1} &= \frac{1}{1+c} r_j = k\; r_j \\
  r_j &= r_0 \; k^j = \frac{1}{2} a_i \; k^j
\end{align*}
$$

(A.4)

Equation (E.4) suggests that to fulfill \( t/r = \text{const.} \), an infinite number of leaves are required. Accordingly, this condition must be relaxed towards the center of the concentrator. We consider that an inner fraction \(~g\) of the inlet aperture consists of \( m \) leaves of a fixed minimum width \( t_{\text{min}} \), with the remaining fraction \(~(1 - g)\) of the inlet aperture fulfilling Eq. (E.2). The number of leaves in the constant-ratio region is found from:

$$
\begin{align*}
  r_{j+1} &= r_j - t_{j+1} = \frac{1}{c} t_j - t_{j+1} = \frac{1}{c} t_{j+1} \\
  t_{j+1} &= \frac{1}{1+c} t_j = k\; t_j \\
  t_j &= t_0 \; k^j
\end{align*}
$$

(E.5)
\[ \frac{1}{2} a \left(1 - g\right) = \frac{1}{2} a - \frac{1}{2} a_k k^n \]

\[ n = \frac{\log g}{\log k} \]  \hspace{1cm} (E.6)

The remaining portion of the inlet aperture \(\frac{1}{2}a_i k\), is then filled up with \(m\) constant-width leaves with width equal to that of the \(n^{th}\) constant-ratio leaf:

\[ t_{\text{min}} = t_0 k^n = \frac{1}{2} a_i c k^n \]  \hspace{1cm} (E.7)

The number of constant-width leaves is:

\[ m = \frac{a_i k^n}{t_{\text{min}}} = \text{floor}\left(\frac{1}{c}\right) \]  \hspace{1cm} (E.8)

with the minimum-width then modified to \(t_{\text{min}} = \frac{1}{2} a_i k^n/m\). The total number of leaves (for one half of the concentrator) is:

\[ N = m + n = \text{floor}\left(\frac{1}{c}\right) + \text{ceiling}\left(\frac{\log g}{\log\left[1/(1+c)\right]}\right) \]  \hspace{1cm} (E.9)

**Figure E.2** shows the total number of leaves for one half of the concentrator \(N\) as a function of the leaf constant \(c\). It is seen that a large number of leaves are required for small \(c\), which corresponds to small acceptance angle, high collection concentrators. For example, a design with \(\theta_d = 5^\circ\), \(f = 0.95\) and \(g = 0.05\), gives a leaf constant of \(c = 3.66 \times 10^{-5}\) which corresponds to a total number of leaves of \(N = 1742\).
Figure E.2. Total number of leaves required for a LPC as a function of the leaf constant $c = 6\tan^2 \theta_d (1 - f)$ where $f$ is the fraction of rays within the acceptance angle which are accepted. The parameter $g$ is the fraction of the inlet aperture for which the leaves do not meet the specified $f$. 
Appendix F

Acceptance map for curved dielectric inlets\(^1\)

Throughout this work we have made convenient use of the elliptical acceptance map of an ideal 2D concentrator to aid in sizing its acceptance angle. However for concentrators with curved refractive surfaces, the acceptance map is no longer elliptical; that is, not all rays whose projection into the meridian plane is within the acceptance angle are accepted, even if the concentrator is ideal in 2D. The phenomenon causing this is the same that leads to the familiar foreshortening of the focal length of a linear lens for non-meridian rays. For the case of an ideal 2D concentrator having a single (curved) refractive surface at its inlet (e.g. DTERC, DTIRC, RX SMS optics) the acceptance map may be determined analytically, and depends only on the acceptance angle and the maximum slope angle of the curved inlet.

F.1 Projected ray paths of edge-rays for curved refractive surfaces

Consider a concentrator with linear symmetry in the \(x\)-axis. Figure F.1 shows the path of a ray \(\hat{\mathbf{v}} (p, q)\) striking the curved inlet of a dielectric filled concentrator. The ray strikes the curved inlet such that its projection into the cross-sectional \((p-q)\) plane of the concentrator has a fixed angle \(\theta_{\text{proj}}\) with respect to the optical axis, where:

\[
\sin \theta_{\text{proj}} = \frac{q}{\left(1 - p^2\right)^{1/2}} \tag{F.1}
\]

If \(\hat{\mathbf{v}}\) is a meridian ray \((p = 0)\), it refracts according to Snell’s law with \(n_1\) and \(n_2\) resulting in the ray \(\hat{\mathbf{f}}_{\text{meridian}}\) having an angle \(\chi_2\) with respect to the surface.

\(^1\) Material in this appendix has been published in T. Cooper, G. Ambrosetti, A. Pedretti and A. Steinfeld, “Theory and design of line-to-point focus solar concentrators with tracking secondary optics,” Applied Optics, 52, 2013, pp. 8586-8616.
normal. If \( \hat{\mathbf{v}} \) is a non-meridian ray (\( p \neq 0 \)), as shown by Chaves [22], its projected path may be analyzed in an equivalent 2D system with apparent refractive indices:

\[
n^* = \left( n^2 - p^2 \right)^{1/2}
\]  

(F.2)

The non-meridian ray therefore refracts according to Snell’s law with the apparent refractive indices \( n_1^* \) and \( n_2^* \) resulting in the more strongly refracted ray \( \hat{\mathbf{r}}_{\text{non-meridian}} \) having an angle \( \chi_2^* \) to the normal:

**Figure F.1.** A ray \( \hat{\mathbf{v}} \) is incident on the curved inlet of a linear concentrator. The ray has a fixed angle \( \theta_{\text{proj}} \) with respect to the optical (z) axis in the cross-sectional (y-z) plane. If the ray is a meridian ray (\( p = 0 \)), it refracts according to Snell’s law with \( n_1 \) and \( n_2 \) resulting in ray \( \hat{\mathbf{r}}_{\text{meridian}} \) having an angle \( \chi_2 \) to the normal. If the ray is a non-meridian ray (\( p \neq 0 \)), it refracts according to Snell’s law with the apparent refractive indices \( n_1^* \) and \( n_2^* \), resulting in the more strongly refracted ray \( \hat{\mathbf{r}}_{\text{non-meridian}} \) having an angle \( \chi_2^* \) to the normal. If the remainder of the optical interactions suffered by \( \hat{\mathbf{r}}_{\text{non-meridian}} \) are reflections, then this ray will have the same projected path as an equivalent meridian ray \( \hat{\mathbf{v}}_{\text{app}} \) having angle \( \theta_{\text{app}} \) with respect to the optical axis.
We consider that the concentrator has a single refractive surface (the curved inlet), with all subsequent optical interactions being reflections. By the equivalence of the 2D ray-trace of projected rays undergoing reflection [31], the subsequent projected path of a ray having the direction of the $\hat{r}_{\text{non-meridian}}$ is independent of the $p$-value of the ray. Therefore $\hat{r}_{\text{non-meridian}}$ appears to have originated from an equivalent meridian ray $\hat{v}_{\text{app}}$ having an angle $\chi_1^*$ to the normal. The incident angle of the equivalent meridian ray with respect to the optical axis is found by backwards refracting $\hat{r}_{\text{non-meridian}}$ across the inlet pretending that $p = 0$.

\[
\sin \chi_2^* = \frac{(n_1^2 - p^2)^{1/2}}{(n_2^2 - p^2)^{1/2}} \sin \chi_1 = \sin \left( \Psi - \theta_{\text{proj}} \right) \left( \frac{1 - p^2}{n_2^2 - p^2} \right)^{1/2} \tag{F.3}
\]

\[
\sin \chi_1^* = \sin \left( \Psi - \theta_{\text{app}} \right) = n \sin \chi_2^* \tag{F.4}
\]

\[
\sin \left( \theta_{\text{app}} - \Psi \right) = n \sin \left( \theta_{\text{proj}} - \Psi \right) \left( \frac{1 - p^2}{n_2^2 - p^2} \right)^{1/2} \tag{F.5}
\]
### F.2 Construction of the acceptance map

We now have an equivalent meridian ray $\hat{v}_{\text{app}}$ that has the exact same $y$-$z$ projected ray-trace through the concentrator as the non-meridian ray $\hat{v}$. Since nonimaging concentrators are ideal for meridian rays, we can use the equivalent meridian ray to determine if the non-meridian ray is accepted. Substituting Eq. (F.1) into Eq. (F.5) and considering that the ray will be accepted if $\theta_{\text{app}} \leq \theta$, we may construct the acceptance map of an ideal linear 2D concentrator with a curved refractive inlet. After some rearranging, the acceptance map in $p$-$q$ space is:

$$q = (1 - p^2)^{1/2} \sin \left( \arcsin \left( \frac{\sin (\theta - \Psi)}{\sqrt{n^2 (1 - p^2)/\left(n^2 - p^2\right)}} \right) \right) + \Psi$$

(F.6)

where the range of $p$ is chosen as:

$$|p| \leq \min \left\{ \frac{\sin \theta \sin (2\Psi - \theta)}{\sin^2 \Psi - \sin^2 (\Psi - \theta)/n^2} \right\}^{1/2}$$

(F.7)

**Figure F.2** shows the resulting acceptance map for dielectric-filled concentrators with curves inlets as given by Eq. (F.6) for different maximum slope angles $\Psi$. For $\Psi = 0$ (plane inlet) the acceptance map reduces to the familiar elliptical acceptance map. For rays in the meridian plane ($p = 0$) the concentrator behaves as ideal by construction. However, for large out-of-plane deviations ($|p| \to 1$) the non-ideality of the acceptance map in comparison to the plane inlet case can be significant. The result of this non-ideality is that the acceptance angle must be significantly oversized to meet to envelope the effective source map at the inlet of the secondaries.
References


List of publications

Journal articles


Conference proceedings


Conference presentations (oral)


**Conference presentations (poster)**


**Patents**

