Societal decision-making for optimal fire safety

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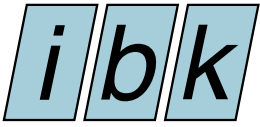
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Foreword

Fire safety measures reduce the consequences of building fires, but have their costs. The total costs of fire and fire safety must be minimized to optimize the use of limited societal resources. For the same reason, also acceptance criteria for investments into life safety must be based on efficiency considerations.

This IBK report written as doctoral thesis by Katharina Fischer focuses on the optimization of societal investments for building fire safety. It begins with the formulation of a generic decision problem consisting of the monetary optimization and a societal acceptance criterion. The acceptability of life safety decisions is evaluated using the marginal life saving costs principle. This allows becoming independent of the detailed requirements given in prescriptive codes and to benefit from the full choice of measures given by a performance-based approach. The report proceeds by deriving quantitative design objectives based on a generic risk-informed framework for optimizing fire safety. Fire safety decisions are made under uncertainty and thus require risk assessment. The report explores the use of statistical data to reduce the modelling bias. A framework for model calibration is developed, which allows combining engineering knowledge with observations from fire events.

Zurich, September 2014   Mario Fontana

Abstract

Fire safety measures save lives and reduce economic losses caused by building fires. However, these benefits come at a cost, because fire safety is not free of charge. An economic optimum is achieved when the total costs of fire and fire safety are minimized. Of course, fire safety decisions cannot be based only on economic reasoning. The safety of building occupants is an important boundary condition for monetary optimization. Societal resources for life saving measures are limited and should be invested where the largest risk reduction can be achieved. Thus, also the definition of acceptance criteria for decisions regarding investments into life safety should be based on efficiency considerations.

The focus of this thesis is on the optimization of societal investments for preventive building fire safety. The starting point is the formulation of a general decision problem consisting of two parts: monetary optimization and societal risk acceptance. The optimization may be performed either by a private decision-maker or at societal level. The acceptability of fire safety decisions with respect to life safety, on the other hand, is always evaluated from a societal point of view. Quantitative acceptance criteria can be derived based on the marginal life saving costs principle, which ensures that societal resources are directed to the most efficient risk reduction measures available.

Decisions on fire safety measures are generally made by the owner of a building. At societal level, investments into building fire safety are controlled mainly based on codes and regulations. The owner is free to optimize fire safety using his own objective function, provided that he fulfils the minimum requirements defined by the code. Traditionally, fire safety is regulated based on prescriptive rules defining in detail which measures have to be taken to reduce fire risk. In order to increase the flexibility of code-based fire safety design, a number of countries around the world have adopted performance-based codes, which specify the design objectives, but leave the concrete choice of measures to the designers. Unfortunately, the code objectives are rarely formulated in quantitative terms. In this thesis it is shown how quantitative safety goals for code-based design may be derived from a generic risk-informed framework for balancing the costs and benefits of fire safety. Following this approach, both prescriptive and performance-based fire safety codes can be based on the same principles of monetary optimization and acceptable life safety.

Fire safety decisions are decisions under uncertainty. Optimizing fire safety thus requires risk assessment for evaluating the effect of safety investments on the expected monetary and human consequences of fire. For a comparison between the uncertain benefits of fire safety measures and their costs, the risk has to be assessed in absolute terms, with as little bias as possible. The present thesis explores the use of statistical data to reduce the modelling bias resulting from assumptions and simplifications used to estimate the risk. A framework for the calibration of engineering fire risk models with data collected by, for instance, fire brigades or insurance companies is developed. The proposed approach allows a combination of engineering knowledge with observations from real fire events, making the best use of both sources of information.

The applicability of the general principles and approaches discussed in the theoretical part of the thesis to real-world decision problems is shown with the aid of three case studies. The examples chosen are from different fields of fire safety engineering: The first case study compares egress route designs with and without active fire safety in the context of prescriptive regulation. In the second case study, target reliabilities for performance-based structural fire safety are derived based on a generic optimization approach. Finally, in the last case study the economic and life-saving effects of home smoke alarms are evaluated and compared to their costs.

Zusammenfassung

Brandschutz rettet Menschenleben und reduziert wirtschaftliche Schäden aus Gebäudebränden. Diese Vorteile haben allerdings ihren Preis, denn Brandschutz ist nicht kostenlos. Im wirtschaftlichen Optimum wird die Summe aller Schäden und Kosten minimiert. Brandschutzentscheidungen können natürlich nicht ausschließlich ökonomisch begründet werden. Der Personenschutz muss stets als wichtige Rahmenbedingung für die Optimierung berücksichtigt werden. Die gesellschaftlichen Ressourcen zur Verbesserung der Personensicherheit sind begrenzt und sollten dort eingesetzt werden, wo die größte Risikoreduktion erreicht werden kann. Entscheidungen im Bereich des Personenschutzes sollten daher ebenfalls auf Basis von Effizienzbetrachtungen gefällt werden.

Der Fokus dieser Doktorarbeit liegt auf der Optimierung der gesellschaftlichen Investitionen für vorbeugende Brandschutzmaßnahmen in Gebäuden. Als Ausgangspunkt dient die Formulierung eines Entscheidungsproblems bestehend aus zwei Teilen: Wirtschaftliche Optimierung und gesellschaftliche Risikoakzeptanz. Die Optimierung kann aus Sicht eines privaten Entscheidungsträgers oder auf gesellschaftlicher Ebene durchgeführt werden. Die Akzeptierbarkeit von Entscheidungen im Bereich des Personenschutzes wird dagegen immer aus gesellschaftlicher Sicht beurteilt. Quantitative Akzeptanzkriterien können auf Basis des Grenzkostenprinzips formuliert werden. Die für den Personenschutz verfügbaren Mittel werden hierbei für die Risikoreduktionsmaßnahmen mit der höchsten Kosteneffizienz eingesetzt.

Entscheidungen über Brandschutzmaßnahmen werden normalerweise vom Gebäudeeigentümer getroffen. Auf gesellschaftlicher Ebene werden Brandschutzinvestitionen hauptsächlich durch Vorschriften und Normen bestimmt. Der Eigentümer kann frei über das Brandschutzniveau in seinem Gebäude entscheiden, solange er die Mindestanforderungen aus den Brandschutzvorschriften erfüllt. Brandschutz wird häufig mit Hilfe von präskriptiven Normen reguliert, in denen genau definiert ist, welche Maßnahmen zur Reduktion des Brandrisikos erforderlich sind. Um die Flexibilität der Vorschriften zu erhöhen, haben einige Länder leistungsorientierte Brandschutznormen eingeführt. Diese definieren die Brandschutzziele, jedoch nicht die Auswahl von Maßnahmen zur Erreichung dieser Ziele. Leider sind die Schutzziele in der Regel nur qualitativ beschrieben. In der vorliegenden Arbeit wird ein generischer risikobasierter Ansatz vorgestellt, mit dessen Hilfe quantitative Brandschutzziele festgelegt werden können. Die Prinzipien der wirtschaftlichen Optimierung und der gesellschaftlichen Akzeptanz von Entscheidungen im Bereich des Personenschutzes können so für die Festlegung sowohl von präskriptiven als auch von leistungsorientierten Brandschutzvorschriften verwendet werden.

Brandschutzentscheidungen sind Entscheidungen unter Unsicherheit. Die erwarteten wirtschaftlichen Schäden und die Personengefährdung müssen daher mit Hilfe von Methoden zur Risikobewertung abgeschätzt werden. Um den Nutzen von Brandschutzmaßnahmen mit ihren Kosten vergleichen zu können, muss das berechnete Risiko möglichst genau den erwarteten Schäden entsprechen. In der vorliegenden Doktorarbeit wird untersucht, wie der systematische Fehler aufgrund von Annahmen und Vereinfachungen in der Risikomodellierung mit

Hilfe von statistischen Daten reduziert werden kann. Hierzu wird unter anderem die Kalibrierung von Risikomodellen mit Feuerwehr- oder Versicherungsdaten untersucht. Der vorgeschlagene Ansatz ermöglicht die Kombination von Ingenieurwissen mit Beobachtungen aus realen Gebäudebränden. Beide Informationsquellen können so optimal genutzt werden, um die durch Brandschutzmaßnahmen erreichte Risikoreduktion möglichst genau abzuschätzen.

Die Anwendbarkeit der grundlegenden Prinzipien und Ansätze aus dem theoretischen Teil der Arbeit auf reale Entscheidungsprobleme wird mit Hilfe von drei Fallstudien gezeigt. Die untersuchten Fragestellungen beziehen sich auf unterschiedliche Bereiche des vorbeugenden Brandschutzes: Die erste Fallstudie vergleicht maximale Fluchtweglängen in Gebäuden mit und ohne aktive Brandschutzmaßnahmen im Rahmen von präskriptiven Vorschriften. In der zweiten Fallstudie werden optimale Versagenswahrscheinlichkeiten für leistungsorientierte Vorschriften im Bereich des baulichen Brandschutzes hergeleitet. In der letzten Fallstudie werden der Nutzen und die Effizienz von Heimrauchmeldern in Bezug auf den Personenschutz und die Reduktion der wirtschaftlichen Schäden untersucht.

Contents

Chapter 1

Introduction

1.1 Current strategies and state of the art

Societal investments into preventive fire safety help saving human lives and reducing the economic costs of fire. However, these benefits do not come free of charge. According to international estimates summarized by The Geneva Association (2012), the costs of building fire safety amounts to 2.5-7% of the overall construction costs. For Switzerland, this corresponds to 1.5-4.3 billion CHF per year or around 0.26-0.73% of the Gross Domestic Product (based on 2012 numbers provided by BFS, 2013). With direct fire losses summing up to more than 600 million CHF each year (estimate provided by BfB, 2013), one might hypothesise that the level of fire safety in Switzerland is rather high. However, comparing the safety costs with the observed losses is not meaningful at all for judging the efficiency of societal investments into preventive fire safety. Instead, the costs would have to be compared with the risk reduction achieved, which cannot be determined by direct observation.

Reducing the fatality count is often regarded to be the most important goal of societal investments into fire safety. International comparison shows that the fatality rate in Switzerland is among the lowest with around 0.3 fire deaths per 100'000 inhabitants and year (average 2007-2009, The Geneva Association, 2012). But is this low figure something to be proud of? Or does it mean we are wasting societal resources to improve life safety in fire while neglecting other areas where more lives could be saved at the same cost? Societal resources for life saving measures are limited and should be invested where the corresponding risk reduction is highest. Thus, knowing the efficiency of different life saving measures is much more important than the absolute level of safety that can be achieved.

It may be argued that the observed level of fire safety is the outcome of a political decision process and can be regarded to be optimal, or at least acceptable, from a societal point of view. Society has aimed at mitigating fire risk for hundreds of years, starting with simple, mostly organisational measures to prevent fire ignition and to facilitate first-aid fire fighting. Current fire safety regulations have been developed mainly based on experience, with a focus on lessons learnt from major fire incidences. This "design by disaster" strategy may, at least in the long run, discover problems with existing design procedures but it is certainly not an efficient approach

to risk mitigation especially in less prominent, but more frequent fire events. Moreover, there is a tendency to add new requirements without questioning the existing ones, which increases the cost of compliance (Bennetts and Thomas, 2002). In summary, the past decision process can hardly be regarded to be rational and the observed level of fire safety should be subjected to a more rigorous analysis instead of postulating optimality without proof.

Prescriptive and performance-based fire safety regulation

Until recently, prescriptive regulation has been the main approach to control fire risk at societal level. Thus, fire safety codes define which safety measures have to be adopted without specifying the top-level code objectives behind the individual requirements. The advantage of prescriptive fire safety codes is that they are easy to apply both for the designers and the authorities involved in checking code compliance. When properly designed, prescriptive regulation can be an efficient approach to control fire safety in the majority of buildings. However, it does not provide enough flexibility for the design of highly specific or new types of buildings.

While prescriptive fire safety codes are still in use in most countries, including Switzerland (VKF, 2003a), there is a world-wide movement towards performance-based design using fire safety engineering methods. Performance-based regulations specify the desired outcome of a design without prescribing how the code objectives may be achieved. It has often been argued that offering more flexibility to the designers will lead to more cost-efficient, innovative solutions while maintaining the level of safety inherent in the prescriptive codes (Hadjisophocleous et al., 1998). Yet after performance-based fire safety codes were introduced in several countries around the world, some authors have expressed their concerns that society might have lost control over the level of safety in buildings designed based on the new regulations, see e.g. Lundin (2005), Babrauskas (2000) or Brannigan (2000). A major drawback of the performance-based fire safety codes currently used in practice is that most of them do not specify the code objectives in quantitative terms (Beller et al., 2002). Therefore, prescriptive "deemed-to satisfy" design is often the only reference point available to define an acceptable level of safety. However, the performance-based design option is often used for very large and highly specific buildings, where the prescriptive solution cannot provide satisfactory results. Here a performance-based approach with clearly specified quantitative safety goals would certainly be much more efficient in finding the most appropriate solution.

Another source of variability in the level of fire safety resulting from performance-based regulation is the method used for design verification. In practice a performance-based design is often justified based on deterministic, scenario-based engineering calculations, see Hadjisophocleous et al. (1998). The term "fire hazard assessment" has been used by Bukowski (2003, 2006) to distinguish an analysis focussing on one or few scenarios from fire risk methods estimating the risk from all possible scenarios (Hall, 2003). The outcome of a fire hazard assessment depends strongly on the input variables describing the scenario. These are for example the design fire, ventilation conditions, occupant characteristics and reliability of fire protection systems (see e.g. Rein et al., 2009 or Tonegran and Ryber, 2010). Uncertainty is dealt with by choosing "worst

case" scenarios and by applying safety factors in the calculations. Such a simplified approach can be justified if the scenario and safety factors have been derived based on quantitative, risk-informed reasoning, as in the Eurocode approach to structural fire safety (see EN 1991-1-2 and Schleich et al., 2002, Hosser et al., 2008 or Weilert et al., 2008). Unfortunately this is hardly the case in other fields of fire safety engineering (Magnusson, 1997).

Uncertainty and probabilistic risk assessment

It is widely accepted that designing buildings for fire safety requires some treatment of uncertainty (see e.g. Magnusson et al., 1995, 1997 or Notarianni, 2000, 2002). Uncertainties in fire safety engineering arise both from the inherent randomness of building fire events and from our incomplete knowledge of the underlying phenomena. Probabilistic risk assessment allows to consistently account for both types of uncertainties. The overall goal of quantitative fire risk assessment is to support decisions on risk reduction measures by estimating their impact on the expected monetary and human consequences of all possible fire scenarios.

Two different sources of information may provide the basic input for the development of fire risk methods: statistical data and engineering models. Data-driven models as described by Ramachandran (1980) or Tillander (2004) can be used for simple cost-benefit studies, see e.g. Juås and Mattsson (1994), Ramachandran (1998) or Rasbash et al. (2004). However, the application of these models for engineering decision-making is strongly restricted by the information content of the available data. To provide sufficient detail for practical decision problems, statistical models thus usually have to be combined with engineering models and, where nothing else is available, with expert judgement.

Engineering methods for fire risk assessment have been described by Hasofer et al. (2007), Yung (2008) and Ramachandran and Charters (2011), among others. The most comprehensive fire risk models available today were developed for the purpose of design optimization in the context of performance-based regulation, see e.g. Beck (1991, 1997), Beck and Yung (1990, 1994) or Fraser-Mitchell (1994). The main focus is on verifying equivalence to a prescriptive, deemed-to-satisfy solution; i.e. for a relative, comparative risk assessment. The advantage of this approach is that the modelling bias resulting from assumptions and simplifications used to estimate the risk may be expected to be at least roughly the same for all regarded design options. A more scrutinized assessment is required if the goal is to estimate fire risk in absolute terms, e.g. when comparing the uncertain benefits of a fire safety measure to its (usually certain) costs. In this case, the fire risk model has to assess the expected loss of property or life with as little bias as possible.

Normative decision-making and societal risk acceptance

Probabilistic risk assessment provides an ideal framework not only for the design of individual buildings, but also to improve societal decision-making for optimal fire safety. At the highest level, this requires the formulation of general decision rules based on societal preferences regarding both monetary and non-monetary consequences of fire safety decisions. Here, the basic

principles of rational, normative decision-making (Von Neumann and Morgenstern, 1944, Pratt et al., 1965) and welfare economics (Hicks, 1939, Kaldor, 1939) provide the foundation.

Formulating decision rules for investments into life safety is certainly more difficult and controversial than monetary optimization, see e.g. Ramachandran (2002), Linnerooth (1975) or Reid (2000). From the plethora of different approaches for the definition of quantitative acceptance criteria for risk to life, the marginal life saving costs principle may be regarded to be most consistent with rational decision-making and economic theory. This principle requires the evaluation of different life saving investments by the cost of marginal risk reductions (Tengs et al., 1995). The approach recognises that societal resources for life saving activities are limited and have to be invested efficiently.

For societal decision-making, the marginal life saving costs are compared with a "Willingness To Pay" (WTP) for small increments in life safety. Focussing on small mortality changes rather than lives saved avoids the difficulty of assigning a "value of life", but nevertheless provides a clear rationale for decision-making aiming at an efficient allocation of societal resources (Schelling, 1968). A convincing approach for estimating society's WTP in practice was proposed by Nathwani et al. (1997), who introduced the Life Quality Index (LQI) to support decisions on life saving investments based on socio-economic considerations. Examples for the application of the LQI can be found, for instance, in the context of natural hazard risk management (Lentz, 2007, Nathwani et al., 2009) and structural safety (Rackwitz et al., 2005, Schubert and Faber, 2009), to mention just a few. A first application of the LQI for the evaluation of a fire safety system has been presented by Hasofer and Thomas (2008), but the approach is still largely unknown in fire safety engineering.

In practice many fire safety decisions are not made at societal level, but during the design of individual buildings. In a market economy it may be a viable approach to let the building owners decide on the level of property protection in their premises. The resulting level of safety may, however, not be optimal from a societal point of view due to positive effects of fire safety benefiting third parties, e.g. building occupants or the owners of neighbouring properties (Meeks and Brannigan, 1996). The need for societal interaction gets most obvious if it is not the property, but the life safety of others that is at stake. The building owners should thus be allowed to decide on the level of fire safety only within certain bounds imposed by society. Fire safety regulation, whether prescriptive or performance-based, should be designed to translate societal preferences for life safety, and to a certain extent also for property protection, into practical decision rules.

Code calibration and the need for quantitative safety goals

When aiming at a more rational approach to the formulation of fire safety codes, it is worthwhile to draw a comparison to code-based design of structures and structural elements. Modern structural design codes provide the end-users with a deterministic design method based on prescribed design equations and corresponding input values. Uncertainty is dealt with by applying partial safety factors to load and resistance variables, i.e. based on a semi-probabilistic approach.

To achieve consistency across a wide range of different design situations, the safety factors are calibrated to a clearly specified target level of safety, see e.g. Faber and Sørensen (2003) or Faber et al. (2003). This target reliability may be determined by calibration to current design practice, by optimization or based on a combination of both (Sørensen et al., 1994, Rackwitz, 2000, Vrouwenvelder, 2002). Thus, an optimal and acceptable level of safety is achieved using a simplified design approach and reliability-based code calibration.

Compared to the field of structural design, fire safety engineering is a young discipline. Setting up a rational framework for code-based fire safety design may still require a considerable amount of research, especially in the field of fire risk assessment. As has been noted by Babrauskas (2000) and Lundin (2008), a one-by-one application of the methods and procedures developed in the field of structural safety without considering the problems specific to fire safety engineering is certainly not appropriate. However, this involves mainly technical issues; from a philosophical point of view the approach followed in structural design is applicable in the area of fire safety, too. In fact, some work has already been done towards the development of simplified design methods with safety factors calibrated to a pre-specified target level of safety, see e.g. Thomas (1986), Schleich et al. (2002), Hosser et al. (2008), Frantzich et al. (1997) or Hasofer and Beck (2000). However, a common rationale for defining quantitative safety goals with respect to both property protection and life safety in building fire events is still missing.

1.2 Rational decision-making for optimal fire safety

A rational approach to optimal fire safety treats societal decision-making as a problem of resource allocation: Societal resources for risk reduction measures are limited and should be spent in an efficient way. Figure 1.1 illustrates the monetary consequences of fire safety decisions. The decision parameter p may represent fire safety investments or any design parameter that is positively correlated with the level of safety. The more we invest into fire safety, the lower the fire losses will be. However, the marginal benefit of additional investments decreases with growing safety level. Summing the two cost components illustrated in Figure 1.1 allows the optimization of societal investments into fire safety simply by minimizing the total cost of fire. Obviously, the same approach is valid if the optimization is performed by a private decision-maker, such as e.g. the owner of a building.

From a societal point of view, it is not acceptable to base fire safety decisions on monetary optimization alone. To prevent avoidable risk to life, the optimization has to be complemented by suitable acceptance criteria for decisions regarding investments into life safety (JCSS, 2008, Eurofer, 1993). The protection of life and limb is a societal issue, while monetary optimization may, depending on the context, be performed either at the level of society or by a private decision-maker. Societal risk acceptance thus enters the decision problem as a boundary condition defining the "acceptable region" within which monetary optimization is admissible (Figure 1.1). In consistency with the marginal life saving costs principle, also the definition of societal risk acceptance in quantitative terms should be guided by efficiency considerations to ensure that societal resources are invested in a way that maximizes the life saving benefit achieved.
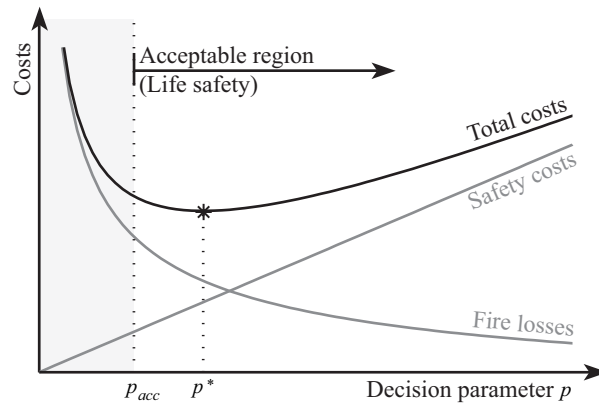
Fig. 1.1: Optimal and acceptable fire safety decisions.

Outline and scope of the thesis

The goal of the present thesis is to translate the basic principles of rational decision-making into practice. The work is structured to discuss three basic steps towards a risk-informed approach for societal decisions in preventive building fire safety:

— The definition of the general decision problem based on societal preferences

— The derivation of practical design guidance from the general decision problem

— The use of risk assessment for optimizing societal investments into fire safety

The first step involves the definition of the decision problem in terms of societal preferences for both monetary and human consequences of fire safety decisions. The formulation of the objective function for monetary optimization is relatively straight-forward, but a more detailed discussion is required for the derivation of societal acceptance criteria for decisions affecting life safety. Finally it is shown how the two parts of the decision problem can be combined. The general principles of monetary optimization and societal risk acceptance, as well as their interaction, are discussed in *Chapter 2*.

The second step is related to societal risk management. The focus is on managing fire risk based on regulations, which so far has been the main approach to societal risk control in the context of building fire safety. In *Chapter 3*, it is discussed how the principles of economic optimization and societal risk acceptance may be translated into code-based design both in the context of prescriptive and performance-based codes. A large part of this chapter is dedicated to the formulation of quantitative safety goals for performance-based design based on generic optimization approaches developed in the field of structural safety. The derivation of simplified design methods to achieve a specified target level of safety is not within the scope of this thesis.

Optimizing fire safety investments requires a comparison between the uncertain benefits of different risk reduction measures and their costs. The expected fire losses thus have to be estimated with as little bias as possible. This requires the use of methods for absolute risk assessment. Another requirement for fire risk models to support societal decision-making for

optimal fire safety is that they have to be applied at the level of non-homogeneous building portfolios. This implies that the models have to account for the effect of building-specific characteristics relevant for estimating the risk and, more importantly, the efficiency of risk reduction measures. The development of new fire risk models is beyond the scope of this thesis. Instead, the discussion in *Chapter 4* explores the use of statistical data to facilitate absolute risk assessment either by referring to simple data-based models or by calibrating engineering models to observations from real fire events. Engineering risk assessment is discussed mainly with respect to modelling fire risk at portfolio level, which is also a prerequisite for model calibration using statistical data.

The chapters related to the three steps discussed above comprise the theoretical part of the thesis. To prove the applicability of the basic concepts to real-world decision problems, they are complemented by three case studies illustrating the concepts discussed in Chapter 2 to 4.

The first case study is related to the design of egress routes (*Chapter 5*). It shows how prescriptive code design may be improved by allowing for alternative design solutions which are expected to be too costly for the average building, but may nevertheless be optimal in a specific building. The definition of quantitative safety goals for performance-based codes is illustrated in the second case study, where target reliabilities are derived for structural fire safety (*Chapter 6*). Finally, in the third case study a generic fire risk model for single family houses is calibrated to statistical data to allow for an absolute risk assessment (*Chapter 7*). The model is used to judge the efficiency of home smoke alarms for reducing monetary losses at portfolio level. In addition, also the life saving effect of home smoke alarms is estimated and used to illustrate the application of risk acceptance criteria in connection with monetary optimization on behalf of society.

The thesis closes with a short summary and conclusions in *Chapter 8*. This chapter also contains an outlook and recommendations for future research efforts.

Chapter 2

Optimal and acceptable fire safety decisions

Parts of this chapter are based on a series of papers by the author; see Kraemer et al. (2010), Fischer and Faber (2012) and Fischer et al. (2011, 2013b) (note that my family name has changed from Krämer to Fischer in 2011). In the following, these references will only be mentioned where more details can be found in one of the articles.

2.1 Societal optimization in a market economy

An important first step in risk-based decision-making is a clear definition of the decision problem. The system representation may be based on the guidelines for risk assessment in engineering provided by the Joint Committee on Structural Safety, see JCSS (2008). It includes the identification of the decision maker and his preferences, which is the focus of the present chapter.

Decisions made on a societal level may affect different groups or individuals. In the context of building fire safety, the following actors can be identified:

— Building occupants

— Building owners and tenants

— Fire insurance companies

— Fire brigades

— Fire prevention authorities

— Designers and contractors

In a free society most daily decisions are left to the individuals. In principle this should include also decisions regarding investments into fire safety: The building owners and tenants should generally be free to invest as much as they want into safety measures to protect their property and themselves from the adverse effects of fire. A basic assumption of market economy is that individual optimization in a free market leads to results that are optimal also from a societal point of view. However, there are good reasons to optimize fire safety also at societal level. Obvious examples are investments that are made using public resources, e.g. for an effective fire brigade. Yet even if the costs of fire safety are borne by individuals, the benefits are often

shared by others. As a result, the level of safety selected by the individual decision makers may be suboptimal from the point of view of society as a whole. This situation is particularly problematic if it is not only the property, but also the life of others that is at risk. The individual optimization should therefore at least be restricted to guarantee a certain minimum level of life safety that is acceptable from a societal point of view.

Another reason for optimization at societal level could be that the individual decision-makers do not always have the knowledge to correctly assess the risk of fire. For many building owners it is in fact efficient to follow a code of practice without questioning the code requirements. Therefore, the code should, at least for an typical, "average" building, be cost-efficient and acceptable in terms of life safety. Increasing the flexibility of the code can improve the resulting fire safety design for non-standard buildings. Code-making decisions should thus be made from a societal point of view, but leaving a certain share of freedom and responsibility with the individual decision-makers.

Whether fire safety codes should focus on life safety only or include also requirements to reduce the economic risk of fire also depends on how monetary losses are distributed at societal level. In industrialized countries, the risk of fire is usually traded on the insurance market. If the risk is correctly evaluated and priced, the insurance rates should depend on the safety measures implemented by the building owners. To a certain extent, this may reduce the knowledge problem mentioned above, as the building owners profit from the expertise of the insurance companies. However, due to market imperfections in the insurance sector it may be questioned whether the pricing mechanism always leads to optimal results. This is particularly true if the insurance markets are strongly regulated as e.g. in parts of Switzerland, where building fire insurance is provided by a public monopoly. In such a case, it is the responsibility of the regulator to ensure that societal resources for fire safety are invested in an optimal way.

Societal decisions on fire safety investments can be split into two main parts (Figure 1.1):

— Monetary optimization

— Societal acceptance criteria

As discussed above, monetary optimization may either be performed by the individuals (using their own decision criteria) or at a societal level, see Section 2.2. The optimization is bounded by societal acceptance criteria for risks that are difficult to measure in monetary terms. In the context of fire safety, this concerns especially decisions regarding investments into life safety. In Section 2.3, it is discussed how quantitative risk acceptance criteria may be derived from the societal preferences for life safety.

2.2 Monetary optimization of fire safety investments

2.2.1 The societal costs of fire and fire safety

From a societal point of view, optimal decisions can be found by taking into account all consequences of the decisions relevant to society as a whole. A list of all costs and benefits accruing to

the different actors mentioned in Section 2.1 can serve as a good starting point for defining these "societal" consequences of a decision. When optimizing societal investments into fire safety, this list will usually contain only cost components; the benefits of fire safety measures are in fact cost reductions.

Studies on the "total cost of fire" have been published in different countries, see e.g. Ashe et al. (2009), Goodchild et al. (2005), Hall (2011), Møller (2001), Schaenman et al. (1995) or Weiner (2001). The list of cost items presented in Table 2.1 was developed in a Swiss project aiming at the optimization of fire safety investments from a code-making point of view (Fischer et al., 2012b). The list is less comprehensive than those used by some of the authors cited above because the focus was on the evaluation of preventive measures for building fire safety. In this context it is not necessary to know the total costs of fire to society in absolute terms; instead the marginal costs resulting from a change in the code requirements must be estimated. This implies that cost components that are not affected by the code-making decisions do not have to be regarded. Which cost components are relevant for the analysis depends on the safety measures to be evaluated. In many cases it may be sufficient to compare the reduction of the monetary losses implied by a fire safety measure with its (marginal) costs.

Tab. 2.1: List of monetary cost components relevant for societal optimization of building fire safety.

Prevention costs	**Reaction costs**
- Costs of fire safety measures (installation, inspection, maintenance) - Fire brigade availability costs (facilities, equipment, training, fire water supply) - Research expenditures, fire safety education and training - Administration costs (authorities, insurance administration)	- Costs of fire brigade actions - Costs of response to false alarms **Monetary fire losses** - Direct losses to buildings, contents and surroundings; clean-up costs - Indirect (consequencial) losses incurring at societal level

For societal decision-making, all cost components in Table 2.1 have to be evaluated from the point of view of society as a whole. Probably the most simple way to achieve this is to sum up all costs and benefits accruing to individual members of society, but without double-counting. Some costs relevant to one member of society may be offset by benefits accruing to others. As an example, the indirect losses resulting from a large fire in one company (e.g. loss of profit or market share) are often considerable for the affected firm. However, at a macroeconomic level, the loss is largely offset by the profits gained by competing firms filling the gap, see Ramachandran (1998).

Treating society as the sum of all its members is only one possible way to formulate a societal objective function. Another view is that societal decisions should focus on those aspects that

are (or should be) managed at societal level. The following questions may help to decide which cost components should be taken into account:

— What resources/consequences are shared at societal level?

— Which cost components are generally managed and borne by the individuals?

— What cannot be effectively managed by market forces?

— Who has the knowledge to assess the consequences of the decision?

Deciding which cost components from Table 2.1 have to be taken into account is the first step towards the formulation of a societal objective function for monetary optimization of fire safety investments. It is clear that also non-monetary consequences like the loss of cultural heritage, damages caused to the environment or human consequences (injuries and fatalities) must be considered in societal decision-making. These intangible values are generally hard to assess in monetary terms. Nevertheless, also here a quantitative risk assessment should always be aimed at, e.g. by introducing boundary conditions that restrict monetary optimization to an acceptable region (see Section 2.3).

2.2.2   Uncertainty and risk in fire safety engineering

Decisions in the area of fire safety are decisions under uncertainty. This is caused both by the inherent randomness of building fire events and by the fact that we are not able to fully understand and model the underlying phenomena. The sources of uncertainty in fire safety engineering calculations are manifold, see e.g. the taxonomy proposed by Notarianni (2002) or the discussion in Lundin (2005). Throughout this thesis, two types of uncertainties are distinguished:

— *Aleatoric uncertainty* related to intrinsic randomness and variability

— *Epistemic uncertainty* resulting from our incomplete knowledge

The distinction between these two types of uncertainties is not always clear-cut; it depends on the focus of the analysis and on time. To give an example, the location within a building where a fire may start is associated with aleatoric uncertainty from the point of view of the designer assessing fire risk a priori. However, for the fire brigade arriving at the scene, locating the fire is clearly connected to epistemic uncertainty only. From a pragmatic point of view, it is reasonable to define aleatoric uncertainty as uncertainty that cannot be reduced by collecting additional information (Faber, 2012).

In principle, both types of uncertainties can be treated using probabilistic methods and the distinction may be irrelevant for many practical decision problems. However, the identification of epistemic uncertainties can be helpful e.g. to direct research and data collection efforts in order to improve future predictions. In addition, the modeller should be aware that epistemic uncertainties may introduce dependencies between variables commonly assumed to be uncorrelated, see e.g. Der Kiureghian and Ditlevsen (2009) or Schubert and Faber (2008, 2012). In this

case, the distinction between aleatoric and epistemic uncertainties is clearly relevant as it may influence the decisions to be made based on a probabilistic model.

In engineering decision-making it is generally accepted that risk should be defined in terms of both probability and consequences. According to JCSS (2008), risk may be defined as follows:

$$R = \mathrm{E}[Y] = \sum_{i=1}^{n} p_i \cdot y_i \tag{2.1}$$

Here, $i = 1, ..., n$ denote n different uncertain events with occurrence probabilities p_i and consequences y_i. Consequences may be defined e.g. in terms of monetary losses or human fatalities, depending on the focus of the analysis. A continuous version of Equation 2.1 is achieved by replacing the event probabilities p_i with a probability density function $f_Y(y)$ for the random consequences Y and integrating over the domain D_Y of Y:

$$R = \mathrm{E}[Y] = \int_{D_Y} y f_Y(y) \, \mathrm{d}y \tag{2.2}$$

Mathematically, Equation 2.1 and 2.2 are equivalent to estimating the expected value $\mathrm{E}[Y]$ of the uncertain consequences.

2.2.3   Risk neutral decision-making

A classical result of normative statistical decision theory is that preferences between different options can be compared based on their expected utility, see Von Neumann and Morgenstern (1944), Savage (1972) or Pratt et al. (1965). The optimal decision is the one that maximizes expected utility:

$$\mathrm{E}[u(Y)] = \sum_{i=1}^{n} p_i \cdot u(y_i) \tag{2.3}$$

The utility function $u(y)$ quantifies the preferences of the decision-maker at hand. Typical utility functions for normal goods are monotonously growing ($u'(y) > 0$), indicating that utility grows for increasing gains or, equivalently, disutility grows for increasing losses. According to the second derivative $u''(y)$, decision-makers are generally classified into three different groups according to their risk attitudes:

— *Risk averse behaviour*: Concave utility function ($u''(y) < 0$)

— *Risk neutral behaviour*: Linear utility function ($u''(y) = 0$)

— *Risk-seeking behaviour*: Convex utility function ($u''(y) > 0$)

In economic theory it is typically assumed that individuals are risk averse decision-makers. The notion of risk aversion expressed by a concave utility function is not to be confused with other concepts like aversion to events with very low probability but high consequences or "ambiguity aversion", aversion to epistemic uncertainties encountered in the quantification of probabilities and consequences during the risk assessment. These concepts are relevant in the context of descriptive decision theory, i.e. for predicting actual behaviour of irrational decision-makers (see

e.g. Kahneman and Tversky, 1979). They are misleading in a normative context where the purpose is to support rational decision-making.

In contrast to other concepts of risk aversion, the choice of a concave utility function is consistent with the axioms of decision theory. However, Maes and Faber (2007) argue that non-linear utility functions can almost always be explained by indirect or "follow-up" consequences not included in the risk assessment. For a more transparent formulation they propose to explicitly account for those indirect consequences instead of applying a concave utility function, see also the framework developed by Schubert et al. (2007). Using a linear utility function in the context of societal decision-making is also supported by Arrow and Lind (1970), who showed that in most situations governments can act as risk-neutral decision-makers. The reason is that governments have a large capacity to bear risks due to the effects of pooling (investment into a great number of diverse projects) and, more importantly, spreading to a large number of individuals (the share of the risk born by each individual being negligible). The result is valid for random outcomes that are independent of other components of national income. This can be achieved by carefully accounting for all indirect consequences that may occur at macro-economic level.

It can be concluded that it is appropriate to use a linear utility function for describing the risk attitude of a societal decision-maker. If all direct and indirect costs to society are properly accounted for, the optimal decision is found by maximizing the expected total (net) benefit. If the consequences of a decision comprise only cost components, this is equivalent to minimizing the expected value of the (total) costs. Thus, the definition of risk given in Equation 2.1 and 2.2 is clearly relevant for societal decision-making under uncertainty.

The focus so far was on the risk attitude regarding consequences that can be measured in monetary terms. Risk-neutral decision-making is, however, also relevant in the context of societal decision-making for life safety (Section 2.3). Here, risk averse behaviour would lead to an uneven distribution of life saving resources, as has been pointed out by Schubert et al. (2007): More money would be spent where accidents with many fatalities have to be expected even though the same number of persons may be at risk in several smaller accidents elsewhere. Only with a linear utility function, every fatality is valued in the same way and an efficient resource allocation is achieved.

2.2.4 Time effects in the objective function

In practice, the cost components listed in Table 2.1 often accrue at different, possibly random points in time. Thus, the objective function used for monetary optimization has to account for time effects. A simple way to achieve this is to discount all cost components back to the time of the decision, $t = 0$. Summing up all discounted costs accruing at different points in time t_i leads to the present value (PV) formulation of the total costs:

$$PV = \sum_{i=0}^{n} \frac{c_{t_i}}{(1+r)^{t_i}} \tag{2.4}$$

Here, r denotes the discount rate relevant for the reference period of one year. A continuous version of Equation 2.4 is derived as follows:

$$PV = \sum_{i=0}^{n} c_{t_i} \exp\left(-\gamma t_i\right) \tag{2.5}$$

The continuous discount rate γ in Equation 2.5 can be transformed to an equivalent annual rate $r = \exp\left(\gamma\right) - 1$. The advantage of continuous discounting is that costs accruing at any point in time are handled consistently. This is helpful especially for discounting expected fire losses, which may be evaluated as a "loss rate" by multiplying the losses conditional on a fire event with the fire occurrence rate. Replacing the discrete future costs c_{t_i} in Equation 2.5 by a continuous cost rate $c(t)$ leads to the following formulation:

$$PV = \int_0^{t_u} c(t)\exp(-\gamma t)\,\mathrm{d}t \tag{2.6}$$

Where t_u denotes the time horizon of the decision. It is of course also possible to combine Equation 2.5 and 2.6, e.g. if the total costs are composed of initial safety investments, expected fire losses and future maintenance costs accruing at discrete points in time. With a constant cost rate $c(t) = c$, Equation 2.6 simplifies to

$$PV = \int_0^{t_u} c \cdot \exp\left(-\gamma t\right)\,\mathrm{d}t = c\frac{\exp(\gamma t_u) - 1}{\gamma \exp(\gamma t_u)} \tag{2.7}$$

For $t_u \to \infty$, the factor on the right-hand side in Equation 2.7 converges to $1/\gamma$.

Finally, it should be noted that the different cost components, the time horizon t_u and the discount rate γ are generally uncertain. Decisions should thus be based on the expected present value of total costs and the following objective function has to be minimized:

$$\mathrm{E}\left[PV\right] = \mathrm{E}_{T_u,\gamma}\left[PV\left(\mathrm{E}_C\left[c(t)\right]\right)\right] \tag{2.8}$$

For simplicity it is often assumed that the discount rate γ is deterministic. Hence the first expectation in Equation 2.8 has to be performed only with respect to time effects. Uncertainty in the time horizon t_u may be accounted for in different ways, of which the most intuitive approach is possibly the one indicated in Equation 2.8. Depending on the problem at hand, it may however be helpful to extend the present value formulation to more than one renewal cycle, as will be discussed in Section 2.3.9. A renewal theoretic formulation is fully equivalent to Equation 2.8 provided that appropriate probabilistic models are used to model the uncertainty regarding time effects (see Section 3.3 and 3.4 for more detail).

2.2.5 Choosing a societal discount rate

In societal decision-making, it is generally not appropriate to simply use the rates observed on the financial market for discounting future costs and benefits. In the following, it is discussed how a societal discount rate γ_S can be deduced from the market rate γ_M.

According to the neoclassical approach by Ramsey (1928), the market discount rate γ_M under perfect market conditions can be derived as:

$$\gamma_M = \varepsilon\delta + \rho \tag{2.9}$$

The market discount rate is composed of two effects: The real economic growth rate per capita δ multiplied by the elasticity of the marginal utility of consumption ε and the pure time preference rate ρ. The first term accounts for the fact that a society's wealth increases with time leading to diminishing marginal utility for an extra amount of money or consumption. ε can be derived from the (societal) utility function and is typically set equal to one, which is the exact solution for a logarithmic utility function. In the present thesis, however, ε is chosen in consistency with the Life Quality Index (LQI) utility function introduced in Section 2.3.4. This leads to $\varepsilon = 1 - q$, with q defined as in Equation 2.12 (see Fischer et al., 2013b for a derivation). There exists no theoretical basis for quantifying the pure time preference rate ρ, which represents the psychological phenomenon that people value future utility less than what they can enjoy now, see Frederick (2006). A typical value for ρ assumed in the literature is 3%. With a real economic growth rate δ of $2 - 3\%$ and $\varepsilon = 1 - q \approx 0.8$ (see Section 2.3.7), a value of about 5% can be derived as a rough estimate for the market discount rate γ_M.

For the derivation of the societal discount rate γ_S, the needs of future generations have to be taken into account. In discounting schemes accounting for intergenerational equity, the market rate γ_M is applied only for intra-generational discounting, see e.g. Bayer and Cansier (1998), Faber and Nishijima (2004) or Rackwitz et al. (2005). For intergenerational discounting, the pure time preference rate ρ is set to zero. In general, γ_S fulfils the following inequality:

$$\varepsilon\delta \leq \gamma_S \leq \varepsilon\delta + \rho \tag{2.10}$$

With the values used above for the estimation of the market discount rate γ_M, the societal discount rate γ_S lies approximately between 2% and 5%. A value of 3% is assumed in the following.

2.3   Societal preferences for life safety

2.3.1   Investments into life safety: The ethical dilemma

Human life is of immeasurable value. This basic norm, resting on our modern society's fundamental value settings, impedes the treatment of risk to life as just another item to be considered in monetary optimization: The ethical dimension of the problem makes us hesitate to quantify loss of life in monetary terms. At the same time it is clear that a certain risk to life and limb can never be avoided. Furthermore, we have to recognize that societal resources for life saving activities are scarce and need to be invested in the most efficient risk reduction measures available. Decisions affecting risk to life have to be made in all kinds of fields, including health economics, traffic safety, natural hazard protection and engineering decision-making. A common basis for judging the acceptability of these decisions is needed to guarantee an efficient allocation

of societal resources. Several approaches have been proposed in the past to find a way out of the ethical dilemma of life-saving:

— *Vision zero*: Reduce as much as possible

— *Acceptable risk*: Absolute criteria for acceptable risk

— *ALARP*: As low as reasonable possible / practical

— *Willingness To Pay*: Pricing marginal safety changes

At a first glance, the *Vision Zero*, i.e. aiming at zero risk to life, seems to be the only risk management principle that is ethically justifiable (see Elvik, 2008 for discussion). In practice this means that any measure to improve life safety should be implemented, no matter at what cost. However, applying this strategy separately in different fields leads to an inefficient resource allocation: Budget constraints may stop investments in areas where highly efficient life saving measures are available while in other fields societal resources are "wasted" for investments with a small risk reduction benefit. Maximizing risk reduction at a societal level thus requires a more structured approach for decisions regarding life safety.

The most simple way to define quantitative criteria for societal risk acceptance is the principle of *Acceptable risk*. The idea here is that risks below a certain threshold may be accepted. The difficulty is to find a rationale for defining this threshold. One possible approach is to analyse the level of safety inherent in current codes and standards or observed in accident statistics (see e.g. Rasbash, 1985 or the Code of Practice BS PD 7974-7 issued by the British Standards Institution for observed levels of fire safety). An implicit assumption is that decisions in the past have been made in accordance with societal preferences for life safety, which introduces a circularity into the problem. Finally, focussing on the absolute level of risk without knowing the available options for risk reduction cannot support rational decision-making. A nice example discussed by Viscusi (1995) is the probability of death due to an asteroid impact, that has often been proposed as a benchmark for acceptable risk. Yet the fact that we accept this risk is not a matter of magnitude: There is simply nothing we can do about it.

The *ALARP* approach combines absolute risk acceptance criteria with a consideration of the possibilities for risk reduction. Absolute criteria for the magnitude of risk define whether a specific activity lies in the "acceptable", the "non-acceptable" or the "ALARP" region. In the latter, an activity is termed "tolerable" if all reasonable risk reduction measures available have been implemented. Without a clear rationale for judging which risk reduction measures are "reasonable", risk regulation based on the ALARP approach does, however, always require some sort of subjective judgement or negotiation, see Melchers (2001) for discussion.

In the economics literature, the *Willingness To Pay* (WTP) approach is considered to be the state of the art for evaluating life saving decisions. The idea here is to monetize minor changes in life safety (or the risk of dying, respectively), which allows to include them in economic analysis (Schelling, 1968). The approach has often been misunderstood due to the somewhat misleading wording. The "Willingness To Pay" in economic theory may simply be understood as a way to

express the trade-offs we make in daily decisions, for example when purchasing additional safety devices for a car. It refers to the marginal amount of money an individual is willing to give away for an additional unit of a desirable good, e.g. safety.

In the following, it is discussed how the ethical dilemma of life saving can be solved based on the marginal life saving costs principle. A rational acceptance criterion for decisions regarding life safety is derived based on the WTP approach. The difficulty when using this approach is how to quantify the WTP for safety. In this thesis the so-called "Life Quality" method is used. This method is introduced in Section 2.3.4 followed by a discussion of more practical issues arising in the application of the marginal life saving costs principle.

2.3.2   The marginal life saving costs principle

The marginal life saving costs principle is derived from the basic resource allocation problem already mentioned in the previous section: Societal resources for life saving measures are limited and have to be invested in an optimal way in order to maximize the benefit that can be achieved with our technical and financial constraints. Therefore, investments into risk reduction measures should be efficient not only in financial terms, but also in terms of their life saving effects.

The efficiency of life saving measures is assessed by evaluating the marginal life saving costs, i.e. the investments necessary for a small increase in life safety, or, in other words, the costs of saving an additional life. Investments into risk reduction measures generally have decreasing marginal rates of return: The risk reduction benefit achieved by an additional investment will be the lower, the higher the safety level was before the investment. Figure 2.1 shows this behaviour for two different activities, e.g. traffic safety and building fire safety. The marginal life saving costs correspond to the first derivative of the curves. Imagine a decision-maker with a fixed budget that he can spend to reduce risk to life in both activities. The optimal resource allocation strategy is to use the budget such that the marginal life saving costs for the last investment are the same in both curves. This leads to different absolute levels of risk to life, as illustrated by the dashed lines in Figure 2.1.
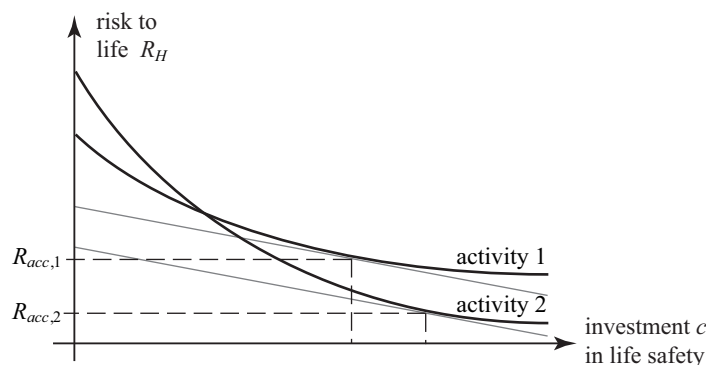
Fig. 2.1: Illustration of the marginal life saving costs principle.

A similar approach can be followed when applying the marginal life saving costs principle for the definition of risk acceptance criteria at societal level. The budget constraint as a criterion

to stop investments is then replaced by the WTP criterion: Investments into life safety should be performed (only) until the marginal life saving costs are equal to the WTP for safety. As before, this may lead to different absolute levels of "acceptable" risk to life: It is not the risk that is deemed to be acceptable, but the decisions made on investments into risk reduction.

Combining the marginal life saving costs principle with a WTP criterion allows the derivation of quantitative acceptance criteria for decisions regarding risk to life. Provided that societal resources for life saving activities are limited, the approach can also be justified from an ethical point of view: Aiming at an optimal resource allocation is in fact equivalent to saving as many lives as possible given a society's financial constraints.

Figure 2.1 illustrates the marginal life saving costs principle for continuous decision parameters, but it can also be applied to combinations of discrete risk reduction measures. The continuous risk-cost curves are then discretized into several linear sections starting with the most efficient risk reduction measures available. The acceptance threshold is achieved when the last measure added has average life saving costs higher than (and close to) the WTP. The procedure has to account for interactions between different safety measures, see Schubert (2009).

2.3.3 Quantifying the Societal Willingness To Pay

The definition of the investment threshold in Figure 2.1 has to be based on societal preferences, as expressed by the WTP or SWTP (Societal Willingness To Pay, see Section 2.3.4). The determination of preferences for intangible goods like safety is not a trivial task. Different approaches proposed in the literature can broadly be classified as belonging to one of the following two groups (see e.g. ASCC, 2008, Schubert, 2009 or Pliefke, 2010):

— Stated preference approaches

— Revealed preference approaches

In *stated preference* surveys, the WTP is determined by asking questions on people's choices in hypothetical situations. The advantage of these approaches is their flexibility: Being independent of existing markets, they may be applied for the pricing of any non-tradeable good. However, the results strongly depend on the survey design, i.e. the way the questions are asked. The *revealed preference* approaches rest on the observation of people's choices when facing trade-offs in real-world situations. The WTP for life safety can be derived e.g. by analysing compensating wage differentials, product choices or housing decisions. Revealed preference methods are generally preferable to stated preference studies. Yet also in real-world situations it may be questioned whether the observed behaviour of people reveals their real preferences or whether a bias is introduced by market imperfections (e.g. due to asymmetries or incomplete information) or irrational behaviour (e.g. due to the inability of people to judge small probabilities).

Normative decisions should be based on the real preferences of people in society. To point out the difference to an individual's actual (revealed or stated) preferences, Harsanyi (1997) introduced the term *informed preferences*, the preferences an individual would show when basing

his decisions on full information. Due to their hypothetical nature, informed preferences are not observable. They may only be approximated by the actual (preferably revealed) preferences.

In the following, the Life Quality method for describing societal preferences for life safety is introduced. This method may be seen as a special type of revealed preference approach. However, the focus is shifted from the valuation of small changes in risk to life to trade-offs between wealth and (free) time. The basic assumption is that utility is derived from *life time* rather than "life", treating time as "the ultimate source of utility" (Zeckhauser, 1973). It is not possible for us to "save" human lives in the long term; even the best life saving measure can only postpone death. Another big advantage of the Life Quality method is that people's choices regarding their labour-leisure trade-off can be expected to be much better informed than when deciding on small changes in risk to life. It may thus be assumed that the Life Quality method yields WTP estimates close to the ones that could be derived if people's true, informed preferences were known.

A detailed discussion of different approaches to derive the WTP for safety can be found e.g. in ASCC (2008) or Pliefke (2010). Each method has its advantages and disadvantages. For practical decision-making the exact WTP value is not crucial, see Lind and Nathwani (2012). More important than the method used to quantify the WTP is that decisions for life safety are based on the marginal life saving costs principle. The consistent application of this principle guarantees that societal resource allocation is optimized by promoting the most efficient risk reduction measures and avoiding the most inefficient ones.

2.3.4   The Life Quality Index and societal preferences for life safety

The Life Quality Index (LQI) first introduced by Nathwani et al. (1997) is a socio-economic indicator composed of the GDP per capita g, the life expectancy e and the fraction of total lifetime spent for work, w. In a simple form, the LQI can be written as:

$$LQI = g^q e(1 - w) \tag{2.11}$$

The exponent q defines the trade-off between wealth (as a prerequisite for consumption) and leisure time $e(1 - w)$. It can be derived based on the assumption that people optimize their leisure-work ratio, which leads to the following definition (assuming that w is in an optimal state):

$$q = \frac{1}{\beta} \frac{w}{1 - w} \tag{2.12}$$

Here, β is the output elasticity of labour in a Cobb-Douglas production function and describes the contribution of human labour to a society's economic output (see Cobb and Douglas, 1928 or any macroeconomics textbook, e.g. Mankiw, 1994).

The indicators used in the LQI derivation are carefully chosen to incorporate a variety of aspects contributing to the quality of life, see Nathwani et al. (1997, 2009) for discussion. The GDP per capita g and the life expectancy e describe not only the conditions under which people live and die, but also their opportunities to live the life they want. Both indicators are widely available and have a clear meaning in the context of evaluating life saving activities. The

fraction of life time spent for work w contains information on people's preferences by observing their trade-off between wealth and free time.

Information on the verification of the LQI with observed data from different countries can be found e.g. in Kübler and Faber (2005) and Rackwitz (2008). For a full LQI derivation the reader is referred to Nathwani et al. (1997, 2009), Pandey et al. (2006), Rackwitz (2008), Lentz (2007) or Pliefke (2010). The approach followed by the different authors is not always the same, but the results are similar. A distinct view was provided by Ditlevsen (2004) who derived the Life Quality Time Allocation Index (LQTAI), an alternative version of the LQI expressed in time units (see also Ditlevsen and Friis-Hansen, 2005). Despite some conceptual differences, the LQTAI produces very similar results when used to assess societal preferences for life safety and is therefore not discussed in this thesis. In the following, the interpretation of the LQI and its relation to the marginal life saving costs principle will be shortly discussed.

The LQI as life time utility function

In its original derivation by Nathwani et al. (1997) the LQI was regarded to be a compound social indicator containing information on the development of society, similar to the Human Development Index (HDI). Besides the possibility to compare the quality of life in different nations, the LQI may also be used to support societal decisions affecting health and life safety. This second use was further supported by an alternative LQI derivation, see Pandey et al. (2006). By embedding the LQI in the theory of welfare economics, this derivation showed parallels between the LQI and the "life-time utility" approach developed in the field of health economics, see e.g. Shepard and Zeckhauser (1984), Johannesson et al. (1997) or Johansson (2001). By defining the utility of income (or consumption, respectively) as g^q, the expected life-time utility of a person at age a can be formulated as follows:

$$LU(a) = \int_a^{a_u} g^q \cdot S(t|a) \cdot \exp[-\gamma(t-a)]\,\mathrm{d}t = g^q e_d(a) \tag{2.13}$$

Here, a_u is the last age in the life table, $S(t|a)$ is the conditional survival probability of a person that has already reached age a, γ is an appropriate discount rate and $e_d(a)$ is the discounted remaining life expectancy of a person at age a. The implicit assumption that g, q and $e_d(a)$ are constant in time is discussed in Section 2.3.6. Age-averaging gives the expected life-time utility for an average member of society:

$$\mathrm{E}_A[LU(a)] = g^q \int_0^{a_u} e_d(a) \cdot h(a)\,\mathrm{d}a = g^q \bar{e}_d = LQI \tag{2.14}$$

Where $h(a)$ is defined as the (observed) age probability density function for the society at hand. When ignoring the constant factor $(1-w)$, Equation 2.14 resembles the original LQI definition 2.11; only the life expectancy e is replaced by the age-averaged discounted remaining life expectancy $\bar{e}_d$, see Lentz (2007) for discussion. Discounting future life years has not been without critique, see Ditlevsen and Friis-Hansen (2007). In the life-time utility approach for deriving the LQI, it becomes necessary to account for the fact that future consumption is valued

less than what can be enjoyed today. This notion is clearly consistent with economic theory. However, discounting should be applied only to the remaining life years of a person at age a, which requires age-averaging when defining the LQI at societal level.

The LQI formulation in Equation 2.14 can be interpreted as the utility function of an average member of society in an intertemporal context. Pliefke (2010) showed that it is in fact to be seen as an indirect utility function, i.e. the maximum utility level that can be achieved by an average person given the constraint defined by the Cobb-Douglas production function. In consistency with this view, Nathwani et al. (2009) introduced the term "Societal Capacity to Commit Resources", which points to the fact that our WTP is bounded by what we can afford. Referring to a society's capacity to invest into life safety is helpful especially when introducing the LQI concept to a non-technical audience, as the term "Willingness To Pay" can easily be misunderstood. Throughout this thesis I will nevertheless use the term WTP, or SWTP (as defined in the following, see Equation 2.16), which is more common in the economics literature.

Deriving the Societal Willingness To Pay for life safety

Equation 2.13 is now used to derive the WTP for marginal changes in (discounted) life expectancy. The easiest way to achieve this is to treat the LQI as a bivariate utility function, where the exponent q defines the trade-off between wealth g and longevity $\bar{e}_d$. Requiring that the total derivative dLQI is equal to zero (i.e. that the utility level remains unchanged) leads to the following WTP formulation for a person at age a:

$$WTP(a) = -\mathrm{d}g = \frac{g}{q} \cdot \frac{\mathrm{d}e_d(a)}{e_d(a)} \tag{2.15}$$

The WTP is here defined as the marginal reduction in the GDP per capita, dg, that the person accepts in exchange for a marginal increase of the discounted remaining life expectancy d$e_d(a)$. The Societal Willingness To Pay (SWTP) is now derived by applying age-averaging and multiplying with the population size n_{pop}:

$$SWTP = \mathrm{E}_A\left[WTP(a)\right] \cdot n_{pop} = \frac{g}{q}\mathrm{E}_A\left[\frac{\mathrm{d}e_d(a)}{e_d(a)}\right] \cdot n_{pop} \tag{2.16}$$

Note that Equation 2.16 refers to yearly values for the SWTP. When judging the efficiency of a life saving measure, the costs have to be annualized before comparing with the SWTP. This involves defining a time horizon of the decision and choosing an appropriate discount rate, see Section 2.3.9 for discussion.

Combining the LQI with the marginal life saving costs principle

Nathwani et al. (1997) originally introduced the *LQI net benefit criterion* which requires that the total derivative of the LQI shall be equal to or larger than zero ($dLQI \geqslant 0$). When evaluating a safety measure with marginal life saving costs dc, this leads to the following inequality:

$$\mathrm{d}c = -\mathrm{d}g \cdot n_{pop} \leq \frac{g}{q}\mathrm{E}_A\left[\frac{\mathrm{d}e_d(a)}{e_d(a)}\right] \cdot n_{pop} = SWTP \tag{2.17}$$

Equation 2.17 is a criterion for the efficiency and affordability of life saving investments. It can be used to judge the efficiency of regulatory requirements. Only efficient safety measures with marginal life saving costs dc below or equal to the $SWTP$ should be required. Investments into more expensive safety measures lead to an overspending of limited societal resources that could be invested more efficiently. The same reasoning may, however, also be applied the other way round: Disregarding safety measures that fulfil the LQI net benefit criterion implies an unnecessary loss of human lives, as efficient risk reduction measures would be available.

This argument can be used when the goal is to define quantitative acceptance criteria for decisions on investments into life safety made by individuals (e.g. the owner of a facility or structure): A societal acceptance criterion is derived by requiring all efficient safety measures as evaluated by the LQI net benefit criterion. This means that safety investments have to be increased (starting with the most efficient safety measures) until the marginal life saving costs dc are (at least) equal to the $SWTP$. A decision maker managing risk to life based on the marginal life saving costs principle thus has to fulfil the following condition for societal risk acceptance:

$$\mathrm{d}c = -\mathrm{d}g \cdot n_{pop} \geq \frac{g}{q} \, \mathrm{E}_A \left[\frac{\mathrm{d}e_d(a)}{e_d(a)} \right] \cdot n_{pop} = SWTP \tag{2.18}$$

Higher values for dc are inefficient from a life saving point of view (i.e. wasting money) and should not be required. Nevertheless there may be good reasons (possibly other than life safety) to go into the inefficient region, e.g. if monetary optimization leads to higher safety investments than the LQI criterion. The equality in Equation 2.18 defines a minimum investment threshold for the individual decision-makers to show that their decision is acceptable from a societal point of view, see Figure 2.2. The definition of acceptability is based on the requirement that all efficient life saving measures have to be performed. The "acceptable region" within which individual decision-makers may optimize based on their own preferences is equivalent to the "inefficient region" for life saving investments.

Throughout this thesis, Equation 2.18 will be referred to as *LQI acceptance criterion* or simply as *LQI criterion*. It is identical with the LQI net benefit criterion (Equation 2.17) except for the direction of the inequality sign (d$LQI \leq 0$ as opposed to $dLQI \geq 0$ for the net benefit criterion). Similar criteria have already been used by other authors, but without a rigorous derivation. Instead, it seems that the inequality sign has sometimes been turned by making formal mistakes, see e.g. Equations (21) and (43) in Rackwitz (2008).

2.3.5 Interaction between LQI criterion and optimization

In practice, societal risk acceptance typically interacts with monetary optimization. At present, there is no clear consensus in the literature on how to combine these two parts of the decision problem. Two fundamentally different approaches can be distinguished:

— The LQI criterion can be used within the framework of monetary optimization

— The LQI criterion can be used as a boundary condition for optimization

When following the first approach, the LQI criterion is used to transform the expected loss of lives or life years into monetary units, e.g. based on the $SWTP$. The monetized life loss then enters optimization as a cost (or benefit) term, see e.g. Nathwani et al. (2009). The second approach, illustrated in Figure 2.2, uses the LQI criterion as a threshold defining the acceptable region within which monetary optimization is allowed, see Rackwitz (2008) and others.
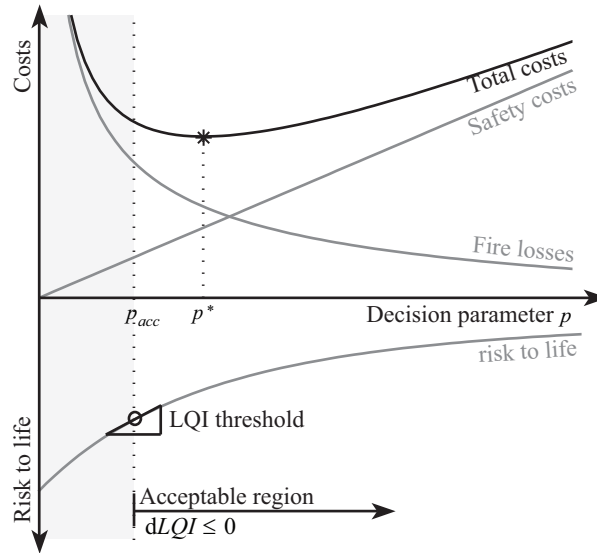
Fig. 2.2: Interaction between LQI acceptance criterion and monetary optimization.

As has been discussed in Section 2.1, the optimization may be performed either by a private or by a societal decision-maker while risk acceptance always has to be evaluated from a societal point of view. Therefore, it is reasonable to separate the two parts of the decision problem, using the LQI criterion as a societal boundary condition for monetary optimization (Ditlevsen, 2003). This approach makes sense especially if the optimization is performed by a private decision-maker. In addition to checking societal risk acceptance based on the LQI criterion, the private decision-maker may include risk to life also in the optimization problem, e.g. if a compensation has to be paid to the relatives of the victims in case of a fatality.

The situation is less clear for a societal decision-maker performing the optimization. At a first glance, in this case it seems to be more intuitive to include the LQI criterion in the optimization. The advantage of using the LQI only as a boundary condition is that the same acceptance criterion can be used for all decision-makers, private ore public. For a consistent allocation of resources in different sectors, it is thus beneficial to separate the two parts of the decision problem also for societal decision-makers. For all decisions made in society, risk acceptance is then guaranteed by imposing the LQI criterion as a boundary condition for optimization.

Independent of this conclusion, the potential loss of human lives may of course also be included in the optimization. Different approaches for monetizing human consequences in the context of optimization from a societal point of view have been reviewed and discussed in Fischer and Faber (2012). Finding a clear definition for these "societal human compensation costs" is not easy, because the notion of human compensation itself is not a meaningful concept in the

context of societal decision-making. It was concluded that the LQI should primarily be applied to define the acceptance criterion as a societal boundary condition and not to define human compensation costs for monetary optimization.

Whether the LQI acceptance criterion becomes active obviously depends on how human consequences are accounted for in the objective function. As could be shown in Fischer and Faber (2012), the optimal decision will always be acceptable according to the LQI criterion if the monetary value assigned to loss of life in the optimization is equal to or higher than the $SWTP$ derived from the LQI. A precondition for this simple conclusion is that all costs and benefits are evaluated and discounted in the same way in both parts of the decision problem, as in the case of a societal decision-maker performing the optimization.

2.3.6   Time effects in the LQI acceptance criterion

The LQI net benefit criterion and the acceptance criterion introduced in Section 2.3.4 are based on annual costs and life safety benefits. If costs and benefits accrue at different points in time they need to be discounted before comparison. Equation 2.18 is then reformulated by comparing the present values (PV) of future costs and benefits:

$$PV\left(\mathrm{d}c\right) = PV\left(-\mathrm{d}g \cdot n_{pop}\right) \geq PV\left(\frac{g}{q}\,\mathrm{E}_A\left[\frac{\mathrm{d}e_d(a)}{e_d(a)}\right] n_{pop}\right) = PV\left(SWTP\right) \qquad (2.19)$$

In real-world decision problems many of the quantities needed for the assessment of acceptability in Equation 2.19 have to be modelled as random variables. The decision should then be based on expected present values of costs and benefits. For ease of notation, the expectation operator is omitted in Equation 2.19. The choice of discount rate and time horizon to calculate the present values is discussed in Section 2.3.9.

Only changes in the LQI that result from the decision at hand are accounted for in Equation 2.19. The three indicators g, $\bar{e}_d$ and q (or w, respectively) are assumed to be constants. Obviously, this is only possible if $\mathrm{d}g(t)$ and $\mathrm{d}\bar{e}_d(t)$ are only marginal changes of the gross domestic product g and the life expectancy $\bar{e}_d$, respectively. Yet, even if the influence of the decision on those societal indicators is marginal, it is well known that both g and $\bar{e}_d$ do chgange in time due to influences external to the decision (typically both quantities are growing). Also the exponent q will be changing as people take into account the growth in wealth and longevity when optimizing their leisure-work ratio. These effects could of course be explicitly taken into account. To achieve this, functional forms for $g(t)$, $\bar{e}_d(t)$ and $q(t)$ would have to be assumed. For GDP growth, Rackwitz (2008) and Lentz (2007) adopt an exponential model. However, it is not sufficient to consider growth in GDP alone, neglecting that also the other societal indicators are time dependent. The development of e_d and q in time is much less understood than economic growth. Socioeconomic studies based on statistical data from different countries might help to establish principles how to account for future changes in the $SWTP$ but are beyond the scope of this thesis. Predicted future changes can never be much more than an extrapolation of observed historic performance and should only be relied on if the underlying phenomena are well

understood. As long as this is not the case, it is better to determine the $SWTP$ based on the most recent values for g, $\bar{e}_d$ and q without taking into account the possibility of future changes.

Besides the socioeconomic developments discussed above it has to be expected that also the best practice in risk reduction will be improving. This means that the marginal life saving costs are decreasing in the long run. In general it will be impossible to predict those changes. The assessment of conformity with societal preferences for life saving investments should thus be performed based on the best practice risk reduction measures available today.

2.3.7   Calibration of the LQI

For practical applications of the Life Quality Method, the parameters needed for the estimation of the SWTP (Equation 2.16) have to be calibrated to statistical data appropriate for the society at hand. This includes the GDP g, the exponent q and the relative life expectancy change $E_A\left[\mathrm{d}e_d(a)/e_d(a)\right]$. The LQI constants are usually estimated at national level, but it is also possible to apply the LQI at a supranational or global level (Faber and Virguez-Rodriguez, 2011). The calibration of the individual constants was subject to continuous discussions in the LQI literature, see e.g. Nathwani et al. (1997, 2009), Pandey et al. (2006), Lentz (2007), Rackwitz (2008), Ditlevsen and Friis-Hansen (2005, 2007) and Pliefke (2010). An overall consensus was never achieved. Ditlevsen and Friis-Hansen (2007) argued that the application of the LQI to derive the SWTP for safety should be regarded to be a normative approach for societal decision-making. This implies that normative choices can and have to be made in the LQI calibration, of course justified by theoretical arguments. As already mentioned in Section 2.3.2, the exact value chosen for the SWTP should not be the major concern in societal decision-making for life safety. Much more important is to base all decisions on the same, rational principle to guarantee an efficient recourse allocation at societal level. This is achieved by applying the marginal life saving costs principle to all decisions regarding investments into life safety.

The following proposal for LQI calibration is based on the author's perception and understanding of the LQI concept. The focus of this section is on the calibration of g and q. The quantification of life expectancy changes based on demographic calculations is discussed in Section 2.3.8. All parameters are summarized in Table 2.2. The values proposed have been derived based on statistical information for Switzerland provided by BFS (2013).

Tab. 2.2: LQI parameters for Switzerland (2010).

Parameter	Value	Description
g	69'887	GDP per capita $[CHF/a]$
w	0.12	Fraction of lifetime spent for work $[-]$
β	0.72	Cobb-Douglas labour elasticity $[-]$
q	0.19	LQI exponent for labour-leisure trade-off $[-]$
$\bar{e}_d$	15.0	Age-averaged discounted life expectancy $[a]$
J_Δ	13.6	Demographic constant (Equation 2.23) $[a]$
$g/q \cdot J_\Delta$	$5 Mio.$	SWTP per life saved $[CHF]$

Calibration of g

The constant g in the LQI derivation is generally related to the Gross Domestic Product (GDP). Rackwitz (2008) and Lentz (2007) proposed to define g as the "share of the GDP that is available for risk reduction", excluding regular investments necessary to maintain economic stability. The share of the GDP used for private consumption may be used as an approximation. The decision which part of the GDP can be used for investments into life safety will, however, always remain somewhat arbitrary. In theory we may choose to invest the whole GDP into risk reduction. We will of course not do this as many other important aspects of a decent life do come at a cost, too. This behaviour is described appropriately by the law of diminishing marginal utility resulting from choosing a value smaller than unity for the exponent q (see also the discussion on the calibration of w in Lentz, 2007). To the opinion of the author, it is therefore reasonable to define g as the full GDP per capita, see also Nathwani et al. (2009), Ditlevsen and Friis-Hansen (2007) or Pliefke (2010). This definition is also consistent with the derivation of the LQI based on the Cobb-Douglas production function for economic output and with the calibration of the exponent for labour-leisure trade-off discussed below.

Calibration of q

According to Equation 2.12, two parameters are needed for the calibration of q: The output elasticity of labour β and the fraction of total lifetime spent for work w. The value for β can be derived by calibrating the Cobb-Douglas production function to statistical data. A simple approximation for β is the share of the GDP that is paid for human labour (including income of self-employed workers), see e.g. Mankiw (1994) or Nathwani et al. (2009). A value for w can be derived from the product of the average number of hours worked per employed person during one year (estimated in labour market studies) and the number of people employed. The resulting number of yearly working hours in the whole economy is then divided by the population size n_{pop} and the total number of hours in one year, see also Pliefke (2010):

$$w = \frac{\text{number of employees}}{n_{pop}} \cdot \frac{\text{yearly working hours per employee}}{24 \cdot 365} \tag{2.20}$$

Rackwitz (2008) and Lentz (2007) provide a similar formula, but include a factor 9/8 to account for commuting time. However, as discussed by Pliefke (2010), it is more consistent with the theory of GDP production to take into account only paid working hours. Another issue discussed in the literature is whether w should be defined with respect to a 24h-day or a 16h-day to account for sleeping time, see e.g. Ditlevsen (2004). While it is obvious that humans have to sleep, the definition in Equation 2.20 can again be justified by the law of diminishing marginal utility, see Lentz (2007) for discussion.

Equation 2.20 cannot be applied to economies with very high unemployment rates. In these countries, w is not in an optimal state and the observed labour-leisure trade-off does not reflect the true preferences of society. An alternative way to calibrate the LQI has been presented by Kübler and Faber (2005) who derived the exponent q for different countries from the correlation

between the GDP g and life expectancy e in different years. Finally, it is always possible to simply choose a value for q, e.g. based on values derived for similar countries.

2.3.8　Assessment of life expectancy changes

The SWTP for life safety is expressed as a function of the life expectancy change implied by a safety measure, see Section 2.3.4. The remaining discounted life expectancy of a person at age a is calculated from the information on age-specific mortality $\mu(a)$ collected in life tables (see e.g. Lentz, 2007):

$$e_d(a) = \int_a^{a_u} \exp\left[-\int_a^t \mu(\tau) + \gamma\,\mathrm{d}\tau\right]\,\mathrm{d}t \tag{2.21}$$

Here, a_u is the last age in the life table and γ the interest rate used to discount future life years. The market rate γ_M (see Section 2.2.5) may be used because intra-generational discounting is relevant for Equation 2.21. Investments into safety measures imply (marginal) changes $\mathrm{d}e_d(a)$ in the age-specific discounted life expectancy. The estimation of $\mathrm{d}e_d(a)$ depends on the safety measure and can involve fairly complicated demographic calculations. However, simple approximations exist for marginal changes in mortality and some standard mortality regimes. One example is the "constant" mortality regime that is often proposed for engineering applications. In this regime it is assumed that the mortality change $\mathrm{d}\mu(a) = \Delta$ is independent of age:

$$\mathrm{d}\mu(a) = [\mu(a) + \Delta] - \mu(a) = \Delta \tag{2.22}$$

Here, the term in brackets refers to the situation after the implementation of the safety measure, while $\mu(a)$ denotes age-specific mortality before the investment. For small Δ the following approximation holds (see e.g. Rackwitz, 2006, 2008 or Lentz, 2007):

$$\mathrm{E}_A\left[\frac{\mathrm{d}e_d(a)}{e_d(a)}\right] \approx -\int_0^{a_u} h(a)\frac{\int_a^{a_u}(t-a)\exp\left[-\int_a^t \mu(\tau) + \gamma\,\mathrm{d}\tau\right]\,\mathrm{d}t}{\int_a^{a_u}\exp\left[-\int_a^t \mu(\tau) + \gamma\,\mathrm{d}\tau\right]\,\mathrm{d}t}\,\mathrm{d}a \cdot \Delta = -J_\Delta \cdot \Delta \tag{2.23}$$

Multiplying with g/q and n_{pop} gives the annual $SWTP$ for the regarded safety measure, see Equation 2.16. Mortality is defined as the expected number of deaths per inhabitant. The $SWTP$ per life saved is thus derived as $g/q \cdot J_\Delta$.

The demographic constant J_Δ is independent from the safety measure considered and can be estimated from life tables. A MATLAB code for the required calculations is provided in Appendix C. The J_Δ constant has the unit $[a]$ as mortality μ is typically defined per annum. Note that the approximation in Equation 2.23 assumes that mortality is changed by Δ and remains on the new level, see Lentz (2007) and Johannesson et al. (1997) for details. For safety measures with finite service life this implies that some kind of "renewal" assumption has to be made when comparing the marginal life saving costs with the SWTP, see Section 2.3.9.

Values for J_Δ have been estimated by different authors for a number of countries around the world, see e.g. Rackwitz (2006, 2008), Lentz (2007) and Pliefke (2010). An assumption typically made for estimating the age distribution in society $h(a)$ is that of a "stable population", i.e. assuming the birth rate and the age-specific death rates to be constant in time (Keyfitz and

Caswell, 2005). This assumption is not necessarily realistic as can be seen in Figure 2.3a where the age distribution of a stable population is compared with the observed age distribution in Switzerland. Nevertheless, the stable population assumption can serve as a good approximation if data on the real age distribution is not available. The effect of the assumption on the demographic constant J_Δ is illustrated in Figure 2.3b for different discount rates. For comparison, the graph also shows results for the age-averaged discounted remaining life expectancy $\bar{e}_d$. The calculations are based on Swiss life tables and population statistics for the year 2010 provided by BFS (2013). For a discount rate $\gamma = \gamma_M \approx 5\%$ and using the observed age distribution, the demographic constant is $J_\Delta = 13.6\,a$.
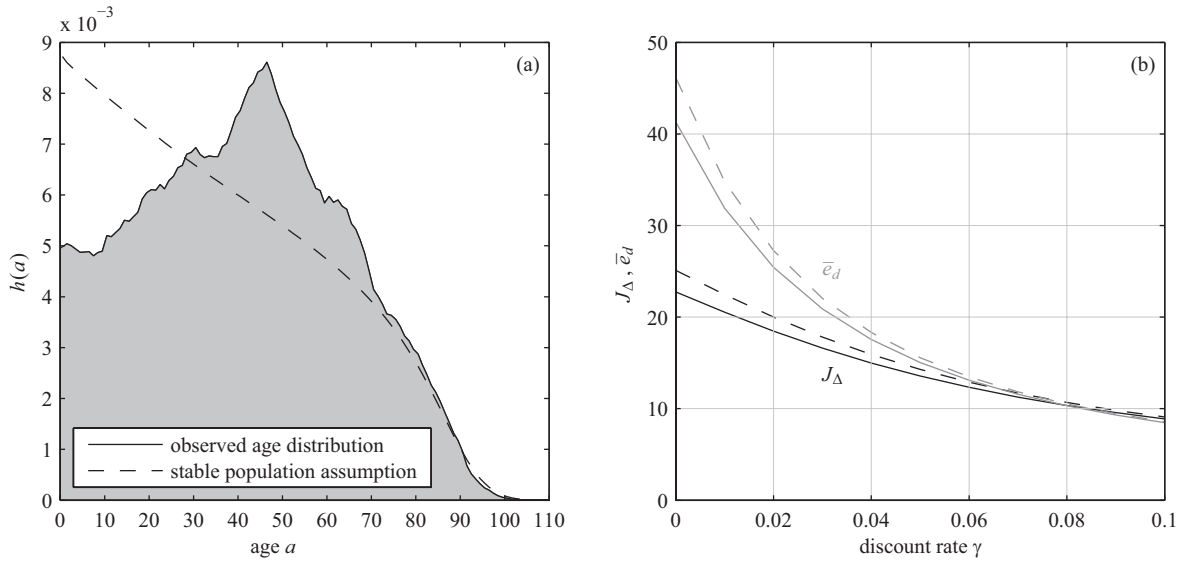
Fig. 2.3: a) Comparison of the age probability density function $h(a)$ under the stable population assumption with the observed age distribution in Switzerland. b) Effect of different assumptions for $h(a)$ on the demographic constant J_Δ and the age-averaged discounted life expectancy $\bar{e}_d$.

The constant mortality reduction regime based on an absolute mortality change Δ is often regarded to be the most "fair" and appropriate choice for the assessment of life saving effects in engineering applications, where the risk of dying can be assumed to be independent of a person's age (see e.g. Rackwitz, 2006, 2008 or Lentz, 2007). Depending on the problem at hand, it may however be justified to introduce a certain age-dependency and to estimate the demographic constant J_x for a different mortality reduction regime x. The effect of choosing a regime that assigns a higher risk to young children and especially to the elderly has been investigated by Kraemer et al. (2010). Such a regime may be appropriate for assessing the life saving effect of safety measures that prevent fire ignition, as statistics show that a person's age has an important effect on the risk of dying in case of a fire, see e.g. Barillo and Goode (1996), Hasofer and Thomas (2006) or Fischer et al. (2012b).

Accounting for age-dependency in mortality reduction leads to lower marginal life saving investments the older the beneficiaries of a safety measure are. At first glance, this seems unethical, as different population groups are treated in a different way. Yet the LQI does not

intend to set a monetary value to human life, nor to ensure that all lives saved are valuated in the same way. Instead, it quantifies the amount of money a society, in accordance with its economic situation, is willing to spend for saving a future life year of any of its members. If existing age-dependencies are taken into account consistently, different absolute levels of risk to life can result not only for different societal activities (Figure 2.1), but also for different age groups, as shown in Figure 2.4. The reason for the differentiation on the two axes is the same: The objective is to maximize the societal lifetime gain by investing the available resources into the most efficient safety measures. Efficiency is determined not only by the marginal life saving costs, but also by the lifetime gain per life saved, which depends on the age of the beneficiaries.
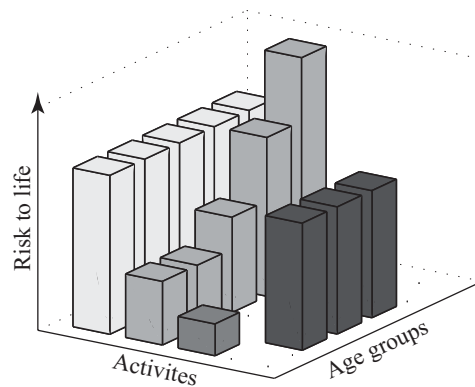
Fig. 2.4: Variation of absolute risk to life for different activities and age groups resulting from decisions that are acceptable following the LQI principle.

In practice, differentiation between age groups should be bounded to a reasonable level of detail. In general, the constant mortality reduction regime should be the first choice, and departing from it has to be justified carefully. Also, it must be emphasised that the LQI does explicitly not differentiate for how much a person is able to contribute to a society's economic output during his or her remaining life. Such a distinction would surely not be ethically justifiable. The same, average values for g and q have to be used for all members of society.

2.3.9   Assessment of marginal life saving costs

At a first glance, it seems that during the evaluation of risk reduction measures the cost quantification is trivial, at least compared to the assessment of the life saving benefit that can be achieved. Yet to facilitate a consistent application of the LQI criterion to real-world decision problems, the following issues have to be clarified:

— What kind of costs have to be taken into account?

— Which discount rate has to be used in the evaluation of future costs and benefits?

— Over which time horizon shall future consequences be accounted for?

All three questions can have a strong influence on the assessment of societal risk acceptance and will thus shortly be discussed in the following. The results are based on a clear distinction between monetary optimization performed by the individual (private or public) decision-makers and the LQI acceptance criterion in Equation 2.19, which always has to be evaluated from a societal point of view. This distinction, which was first introduced by Ditlevsen (2003), is a key concept when it comes to the quantification of marginal life saving costs and forms the basis for the following discussion. The general idea is that societal preferences for life safety should guide all decisions where human lives are at stake. Only monetary risks may be managed individually by the respective decision-makers. As has been discussed in Section 2.3.5, the conclusions derived from this distinction should remain valid also if a societal decision-maker is performing the optimization.

Definition of marginal life saving costs

As the LQI criterion is a societal risk acceptance criterion, the marginal life saving costs dc in Equation 2.19 have to be assessed from a societal point of view. In theory, they can be defined most consistently as the net marginal costs to society, i.e. $dc = dc_S - db_S$. The terms dc_S and db_S refer to the costs and benefits to society as a whole; they can in general not be deduced from the net marginal costs relevant to a private decision-maker. The correct quantification of the net marginal costs to society is not a trivial task, especially because the effect of decisions made on a project level can be regarded to be negligible from a societal point of view. The problem is further complicated by the fact that the decision-maker may be inclined to manipulate the acceptance criterion if an objective definition of costs and benefits is not available.

Yet even if we assume that the net marginal costs to society are determined correctly, the definition $dc = dc_S - db_S$ is still problematic at least for the case of a private decision-maker performing the optimization. Including the societal benefit term db_S is in fact equivalent to imposing a societal optimality criterion, where human losses are monetized based on the LQI. In a market economy, where economic optimization is generally left to the individual decision-makers, this is an unnecessary strong constraint.

It may be concluded that the definition $dc = dc_S - db_S$ is consistent with the LQI derivation, but not practical in a regulatory framework. A simple solution is to define the marginal life saving costs only in terms of the direct payments necessary to increase life safety. If we assume that all (indirect) benefits and disbenefits implied by the decision are summarized in db_S, this definition is equivalent to neglecting the benefit term in the LQI criterion, i.e. $dc = dc_S$. Cost components that are reduced by the safety investments are a (marginal) benefit of life safety and have to be neglected in the LQI criterion, too. In the fire safety problem, the marginal life saving costs should thus be derived from the prevention costs only (see Table 2.1).

Defining the marginal life saving costs at project level only in terms of the direct safety investments allows for a clear and objective definition which costs have to be taken into account in practical decision problems. Neglecting the monetary benefits of life saving investments does of course have an influence on the acceptance threshold derived from the LQI criterion.

The question is whether the actual safety level observed in practice will change considerably. To answer this question, we have to remember that the acceptance criterion serves only as a boundary condition for monetary optimization (see Section 2.3.5). The individual decision-makers may always choose to invest more into safety than required by the LQI criterion.

A basic assumption in a market economy is that individual optimization on a free market will lead to results that are optimal also from a societal point of view. It is, however, well-known that this assumption is not always true. A typical reason could be that the private decision-maker does not take into account all benefits of a safety investment if he is not the main beneficiary. The private optimum may in this case lie in a region that is not acceptable from a life safety point of view. As the decision-maker has to choose an acceptable decision he will opt for a solution that exactly fulfils the minimum requirement by the LQI criterion. This solution is generally less safe than the societal (monetary) optimum because the LQI criterion with marginal life saving costs defined as $dc = dc_S$ does not account for externalities implied by monetary benefits to society not considered in the objective function of the private decision-maker. Only the life safety externality is dealt with, which (depending on the decision problem) may lead to higher safety investments than in the situation without regulation.

Discounting marginal life saving costs and life safety benefits

Depending on the decision-maker, different discount rates have to be applied for the optimization. A private decision-maker will choose a discount rate γ_P according to his financing costs and/or time preferences. This will typically result in a higher discount rate than appropriate for societal decision-making, especially for long-term decisions (e.g. with time horizon $t_u \geq 50a$), where intergenerational equity becomes an issue, see Rackwitz et al. (2005). The private discount rate chosen by the individual decision-makers for their cost-benefit optimization is thus usually not appropriate for calculating the present values in Equation 2.19.

As discussed in Section 2.3.5, the acceptance criterion serves as a boundary condition to (monetary) optimization. A societal discount rate γ_S has to be applied because the acceptance criterion is imposed on behalf of society. For decisions with long time horizons, this discount rate should take into account the needs of future generations, following the principles of sustainable discounting (Section 2.2.5). One exception is the discount rate used to calculate the demographic constant J_x, when discounting and age-averaging is applied (Section 2.3.8). Here, future life years are discounted as a proxy for future individual consumption (being alive as a prerequisite for consumption). Discounting remaining life expectancy thus accounts for an average individual's time preferences and the market discount rate γ_M should be applied (intra-generational discounting).

Except for the calculations needed to determine the demographic constant J_x, the societal discount rate γ_S has to be used for all time-dependent quantities in the acceptance criterion, i.e. for discounting both future costs and future life saving effects. Applying different discount rates to costs and benefits leads to serious inconsistencies, see e.g. Weinstein and Stason (1977)

or Keeler and Cretin (1983). Both costs and life safety benefits contribute to the same utility function and have to be discounted at the same rate.

The LQI criterion introduced in Section 2.3.6 is thus evaluated with $\gamma = \gamma_S$ for both public and private decision-makers performing the optimization, even though the latter might have financing costs much higher than implied by the societal discount rate γ_S. Financing costs may be understood to be a transfer payment between borrowers and lenders, with high rates resulting (at least partly) from the credit shortfall risk. These effects are not relevant when regarding society as a whole. Accounting for intergenerational equity further reduces societal discount rates, as has been discussed in Section 2.2.5. The reasoning behind the application of societal discounting in the LQI acceptance criterion regardless of the private decision-maker's financing costs is the Kaldor-Hicks compensation principle in welfare economics (Kaldor, 1939, Hicks, 1939), which in this context states that welfare is increased as long as the decision is beneficial to society as a whole, no matter whether the private decision-maker is compensated for investments into the life safety of others or not.

Choosing an appropriate time horizon

When looking at an individual project, it can usually be assumed that costs and benefits of a decision will accrue only during a certain finite time frame; e.g. the service life of the safety measure or project. In such a situation it is sufficient to confine the assessment of acceptability to a finite, possibly random, time horizon T_u, which has to be chosen such that initial investments, maintenance costs and costs for the decommissioning of the safety measure as well as all future life safety benefits are taken into account. The probability distribution of the safety measure's service life T_u can have a strong influence on the acceptable level of safety. If high initial investments lead to a relatively small, but continuing future benefit, the efficiency of the safety measure will be the higher, the longer the benefits of the safety measure can be accumulated.

If the service life of the safety measure is linked to the service life of an (economic) activity or facility, the estimation of T_u will typically be influenced by aspects such as obsolescence and economic optimization rather than technical reasoning. It is thus not a trivial task to select an appropriate time horizon for the acceptance criterion and in many cases an objective estimate may not exist. When applying the marginal life saving costs principle to individual projects, this leads to difficulties from a regulatory point of view, because the individual decision-makers can easily modify the acceptance criterion based on their own interests, e.g. by assuming a short service life for the facility and then prolonging its use several times.

It can, however, be expected that a decision made in one project will be repeated in the same or a similar context also in future projects. Most societal activities do not stop at a certain point in time but continue beyond the technical or economic service lives of facilities and implemented safety measures. Technical components or whole structures are systematically renewed (including regular maintenance or repair actions). Then, the time horizon of the decision is infinite but the duration of subsequent renewal cycles will nevertheless influence the assessment of marginal life saving costs as it determines the frequency of future safety (re-)investments.

Following this line of thought, the present value of marginal life saving costs can be represented with models from the field of renewal theory, taking its basis in a seminal paper by Rosenblueth and Mendoza (1971) (see also the mathematical formulations provided in Fischer et al., 2011).

A renewal theoretic approach is especially helpful for code-making decisions affecting a whole portfolio of structures to be built in the future. In assessing the expected marginal life saving costs implied by the code, the code-maker has to account for the fact that any regulatory requirement has an impact on a non-homogeneous group of projects. The distribution of service lives of all structures addressed by the regulation (or the renewal intensity) influences the overall efficiency of investments into life safety on a population level. Yet while very low acceptable levels of safety can result when applying life saving efficiency considerations to individual structures with very short operational life times, for decisions on a population level the influence of these extreme cases is negligible.

It is of course possible to differentiate regulatory requirements by defining structural classes according to the expected service life of the structures. To give an example, we might want to address life safety in interim or existing structures with short (remaining) service lives separately from the regular design codes instead of using the same code provisions for all types of structures. When basing the assessment of risk acceptance on the LQI criterion, this leads to different absolute levels of life safety in each structural class. Nevertheless, the code-making decision is still made on the level of a (sub-)portfolio of structures, and the influence of extreme cases (individual objects with very short planned service live) on the risk acceptance criterion remains negligible also when a certain level of differentiation is introduced into the codes.

Differentiation increases the efficiency and appropriateness of code provisions, but it comes at the cost of an increased effort for designing the individual structures. For groups of similar structures or projects it is thus more efficient to optimize safety investments on a population level. Some extraordinary or new types of structures may be so specific that a portfolio of similar structures or projects does not seem to exist at a first glance. In such situations the decision may be regulated by applying the marginal life saving costs principle on the level of an individual project, which can be regarded to be an extreme level of differentiation in code provisions. The modified ALARP approach proposed by Faber and Maes (2009) (where the LQI criterion is applied only in the ALARP region) or the precautionary principle (e.g. assuming an infinite time horizon of the decision) could be applied to ensure reasonable assumptions regarding the time horizon used for the acceptance criterion. The above considerations do, however, suggest that project specific optimization should be applied only in very rare, highly specific situations. In fact, any structure or project can be regarded to belong to a portfolio of structures distributed in (future) time and space. Decisions on life saving investments for structures should therefore always be treated as decisions with respect to "generic" structures, i.e. decisions that can be reapplied for other structures under similar conditions.

Chapter 3

Managing fire risk at societal level

Parts of this chapter are taken from a paper on the derivation of target reliabilities for structural design based on the LQI acceptance criterion, see Fischer et al. (2012a).

3.1 Different approaches for managing societal fire risk

As has been discussed in Section 2.1, different actors in society are affected by decisions aiming at optimal fire safety. At the same time, some of them may also act as decision-makers. A societal decision-maker has to control a large number of individual decisions made by e.g. building owners investing into fire safety in order to manage the risk of fire at societal level. At the same time it is worthwhile to leave as much freedom of choice to the individuals as possible. Societal risk control in the area of building fire safety may be achieved using different approaches:

- — Public safety investments
- — Codes and regulation
- — Information and incentives
- — Fire insurance system

Public safety investments are the most direct approach for risk management at societal level. They are most effective where no individual decision-makers are involved, e.g. for improving the general fire water supply. Preventive building fire safety, in contrast, is generally managed by the building owners and can only be influenced indirectly by a societal decision-maker.

At societal level, building fire safety is managed mainly by codes and regulations. This implies that the owner of a building is free to optimize fire safety using his own objective function, provided that he fulfils the minimum requirements defined by the code. The level of freedom and responsibility left to the individual depends on the structure of the code and the code provisions. In practice, fire safety is often perceived to be mainly a cost factor and individual optimization is restricted to minimizing the investments necessary for code compliance.

A somewhat weaker approach than fire safety regulation is the information and education of the public, possibly combined with incentives (e.g. tax incentives). This approach is helpful especially where it is difficult to enforce regulations, e.g. when trying to prevent fire ignitions resulting from incautious behaviour of building occupants. Information and incentives may also

be used to stimulate investments into preventive fire safety if there is large uncertainty about the efficiency of the safety measures. The advantage here is that more freedom is left to the individuals than in the regulation-based approach.

Finally, also the fire insurance system can have an important effect on societal investments into fire safety. Here, the insurer plays a role similar to a societal decisions-maker by influencing the decisions of the insured building owners and tenants. All three mechanisms discussed above may be used to manage fire risk at the level of the insurance portfolio.

Whatever approach is chosen for managing fire risk in society, the decisions usually affect a whole portfolio of buildings. The efficiency of fire safety measures depends on the characteristics of the individual objects. Thus, the societal decision-maker has to deal with variability in a non-homogeneous building portfolio. A simple approach often followed in fire safety regulation is to divide the portfolio into several subgroups, e.g. according to the occupancy or construction type. Another possibility is to leave more freedom to the individual decision-makers who know more exactly the problems specific to their project. Carrying this to the extreme would mean that decisions on fire safety investments are left completely to the individual. As has been discussed in Section 2.1, individual optimization should, however, almost always be restricted by societal risk acceptance criteria, especially to ensure an acceptable level of life safety.

The focus of the following sections is on code-making decisions. In Section 3.2 it is discussed how code-based design can be optimized from a societal point of view in different regulatory regimes. Risk-informed decision-making for the design of structures is much more developed than in the area of fire safety and may be used as a starting point for the development of a rational approach to fire safety regulation. The basis for defining target reliabilities for structural design codes based on monetary optimization is reviewed in Section 3.3 and extended to the assessment of acceptable life safety based on the LQI. Finally, in Section 3.4 the framework is adapted as a basis for the definition of safety targets in the area of preventive building fire safety.

3.2   Optimizing code-based fire safety design

Fire safety regulation has a large effect on societal investments into fire safety: While there are certain options to negotiate individual code provisions during the design of a specific building, the general safety level implied by the code is usually not debatable. This means that the level of safety in a whole portfolio of buildings is defined by the code-maker. In the following, it is discussed how the fire safety codes may be optimized from a code-making point of view in two different regulatory regimes:

— *Prescriptive codes*, i.e. regulation of fire safety based on detailed specification of risk reduction measures to be implemented.

— *Performance-based codes*, i.e. codes specifying the desired outcome of a fire safety design while leaving the concrete choice of measures to the designer.

Independent of the regulatory regime, code requirements should be based on the principles of monetary optimization and acceptable life safety discussed in Chapter 2.

3.2.1 Optimizing prescriptive code requirements

Traditional fire safety codes are based on prescriptive design rules specifying e.g. the fire resistance of structural components, the number and dimensions of egress routes or the conditions under which a sprinkler system has to be installed. The code provisions were developed historically based on experience with the existing building stock and observed fire events. In order to adapt prescriptive codes to changes and innovations in the construction industry and newly emerging hazards, they are revised periodically. Code-making decisions are thus marginal decisions dealing with code changes rather than defining new code provisions from scratch.

The strength of prescriptive codes is that they are simple to use and do not require much effort and expertise by the designers and the authorities involved in checking code compliance. However, prescriptive fire safety regulation also has a number of disadvantages:

— The code objectives are not clearly specified

— The code provisions have little or no scientific foundation

— The resulting design is rarely cost-efficient

— The codes offer little flexibility to the designers

In the literature, these disadvantages are often used as an argument for the introduction of performance-based fire safety codes, see e.g Hadjisophocleous et al. (1998). At the same time it is generally recognized that also in future, most of the buildings should be designed using prescriptive codes, with performance-based design focussing on large or highly specific projects. Moreover, it can be observed in practice that the prescriptive approach is often used as a starting point for performance-based design, see e.g. Buchanan (1999) or Lundin (2005). If prescriptive codes are not completely replaced by their performance-based counterparts, it is better to treat their deficiencies rather as an agenda for improving prescriptive design than as arguments against the regulatory approach itself. In the following, it is discussed how the disadvantages listed above may be reduced without shifting to a performance-based regime.

The first step is to define and communicate the objectives of a prescriptive code. It is not sufficient to simply state the top-level goals in the introduction of the code; every code provision has to be clearly justified and linked to the code objectives. Wherever possible, this link should be established in quantitative terms. However, already a qualitative justification of the individual code provisions may be helpful as many fire safety measures fulfil several functions and it is not always obvious which one is the main intention of the code-maker. Most of this information may be provided in publicly available background documents to keep the code itself as simple as possible. Besides improving the transparency of the code-making process, knowing the objectives behind the code requirements is helpful whenever the formulation of the code is subject to discussion or not applicable to a specific project. Finally, also alternative solutions or trade-offs between different safety measures can only be justified if the basis for the prescriptive solution is known.

Justifying the code requirements is easiest if a scientific foundation for the code exists. Scientific methods can be used to establish the link between the code objectives and individual code provisions and to show that the proposed measures are cost-efficient. Based on probabilistic reasoning, it is possible to support code-making decisions also in areas where scientific knowledge is limited and the data base is poor, see e.g. Fischer et al. (2012b). Establishing a scientific foundation for every code detail may not be feasible in the near future. Combining scientific methods with the traditional, experience-based code-making process can nevertheless provide an adequate decision support for the continued development of prescriptive fire safety codes.

Improving the cost-efficiency of prescriptive fire safety design is a matter of optimizing the code requirements. It can be achieved by applying the general principles discussed in Chapter 2 to code-making decisions. The code provisions have to be optimized at portfolio level as they affect a large number of future projects. As a result, fire safety can only be optimized in average, for a whole class of buildings. This may be sufficient for many projects where the designer and/or the building owner does not expect that the cost reduction achieved by an alternative solution is large enough to justify the effort required by a performance-based design. Only for a few very specific projects, there will be an incentive for departing from the prescriptive design.

The code-maker can improve the efficiency of the code requirements by explicitly accounting for variability in the building portfolio to be regulated. This may be achieved e.g. by dividing the portfolio into different subgroups according to risk-relevant characteristics of the buildings to be designed. In principle, all variables with a large effect on the efficiency of the code provisions should have an influence on the design. It has been shown in the literature that the safety level implicit in prescriptive fire safety codes shows a large variation due to variables not considered in the code, see e.g. Lundin (2005), Fischer et al. (2012b) or De Sanctis et al. (2014). Optimizing the code provisions based on efficiency considerations will result in different absolute levels of fire safety, too. However, neglecting important risk indicators can lead to substantial variation in the cost-efficiency of a prescriptive design in different buildings.

Another option for improving the efficiency of code-based design is to leave more flexibility to the designers. This can be achieved by allowing for different design strategies in the prescriptive code. This strategy is most fruitful if the alternative solution is expected to be more costly, but safer than the standard design, as illustrated in Figure 3.1. The reason is that the costs of preventive safety measures are easy to assess and generally borne by the owner of a building (dark grey area in Figure 3.1). He may have good reasons to opt for the alternative design p^{alt}, either because of higher safety requirements than implied by the code or because the standard design turns out to be very expensive in his specific building. In this case, the building owner has all required information to make an informed choice. From a societal point of view, there is no reason to restrict individual optimization, as the fire losses, which are possibly borne by other members of society, do not increase. The same reasoning holds if the benefit of increasing fire safety is a reduction in risk to life rather than monetary losses.

Raising the level of differentiation and introducing alternative design solutions increases the complexity of the code. Writing a prescriptive code requires a trade-off between efficiency and simplicity, and thus between design costs and safety costs. Keeping the code as simple as
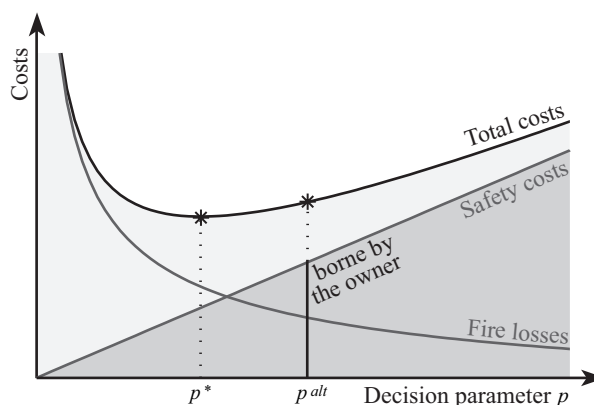
Fig. 3.1: Alternative design solution with higher safety costs.

possible reduces administrative costs during the design of a large part of the building portfolio. In practice, this may also be achieved by allowing for alternative designs not explicitly described in the prescriptive code. In many prescriptive codes, object-specific designs are regulated using so-called "equivalency clauses", see e.g. Art. 11 of the Swiss fire safety code, VKF (2003a). Whether a design is equivalent to the prescriptive solution is subject to negotiation with the authorities, which may lead to somewhat arbitrary decisions in some cases. Stating the design objectives of a prescriptive code and communicating the scientific basis of the code provisions, as discussed above, is one way to improve this process as it provides a common basis for discussions regarding code equivalency. However, the concept of code equivalency itself leads to a number of problems, as will be discussed in the following. A more structured approach is to use a performance-based code with quantitative safety goals for regulating alternative designs.

3.2.2 Quantitative safety goals for performance-based code design

To resolve the problems discussed in Section 3.2.1, a number of countries around the world have adopted performance-based fire safety codes. The new codes are based on clearly stated objectives, which may be defined very broadly, e.g. "life safety"; and functional requirements describing more specifically how to achieve the top-level objectives. Both performance objectives and functional requirements are generally stated in qualitative terms, see Beller et al. (2002). The verification of adequate performance is often made by comparing the outcome of deterministic, scenario-based engineering calculations with so-called "performance criteria" describing e.g. threshold values for ignition, flashover, occupant tenability or structural failure (Hadjisophocleous et al., 1998). Uncertainty is dealt with by referring to "worst case" scenarios and, especially where human behaviour is modelled, by applying safety factors in the calculations. However, the definition of design fire scenarios and safety factors is hardly based on quantitative risk-informed reasoning, the only exception being maybe the Eurocode approach to structural fire safety (see Section 6.1 for discussion).

An alternative approach is to verify a performance-based design using fire risk assessment. However, most fire safety codes do not specify performance criteria in terms of acceptable risk

and the deterministic performance criteria mentioned above cannot be fulfilled with certainty in a probabilistic analysis (see also Bukowski, 2006). In the absence of absolute criteria for acceptable risk, relative risk assessment is the strategy mostly followed in practical risk-based fire safety design: The fire risk resulting from an alternative design is compared to that implied by an "acceptable solution", e.g. resulting from a prescriptive design (see e.g. Lundin, 2005 or Beck, 1991). Such a comparative analysis may require less scrutiny regarding the assumptions used by the risk model, as they are the same both for assessing the alternative design and for the estimation of the level of risk deemed to be acceptable. However, the requirement of equivalency to the prescriptive code also has several pitfalls, as will be discussed in the following.

First of all, the level of safety inherent in prescriptive fire safety codes currently in practice is not optimized at all. The assumption generally made to justify equivalency clauses is that the observed fire losses and fatalities are accepted by society. However, even when taking this for granted, it may be questioned that the safety level in a building portfolio is equal to that implied by the prescriptive code due to voluntary safety investments and code enforcement issues, see Brannigan and Meeks (1995). In any case, we have to expect that prescriptive code design wastes money in some areas while not requiring sufficient investments in others. Requiring equivalency to a prescriptive code design makes sense only if the code provisions have been optimized both economically and with respect to societal investments into life safety.

Another problem is that the level of safety inherent in prescriptive code design shows large variations between different buildings. As the "prescriptive reference building" for a relative risk assessment will never be built, the designer may influence the level of fire risk deemed to be acceptable by deliberately defining the building characteristics such that the risk resulting from the prescriptive code design is high. The problem becomes even more pronounced if the building to be designed is outside the design envelope of the prescriptive code, see Lundin (2005) for discussion. The equivalency approach thus fails especially in those cases where a performance-based design is the best (or the only) solution to achieve adequate fire safety.

A final issue are "conservative" assumptions often made when assessing the risk both for the prescriptive and the alternative design. However, assumptions leading to an overestimation of fire risk are conservative only for the assessment of the alternative design. Applying the same assumptions on both sides of a relative risk assessment seems intuitively right but may lead to an unpredictable bias if their effect is different depending on the design strategy chosen.

In summary, it may be concluded that most of the benefits of performance-based regulation are offset by requiring equivalence to the prescriptive code. This can be resolved by defining quantitative risk acceptance criteria that are independent from the prescriptive code, as has been proposed by Buchanan (1994). Nevertheless, equivalence between the two types of codes can be attained by deriving both the safety goals for performance-based code design and the prescriptive code provisions from the same principles. Buildings designed using the performance-based code may be somewhat more cost-efficient and show less variation in the resulting level of safety. However, while the differences may be large for specific buildings, the overall performance at portfolio level should be roughly the same independent of the regulatory approach chosen.

Risk-informed decision-making in a performance-based code environment requires that quantitative performance criteria have to be defined in terms of risk and/or probability based on the foundations laid out in Chapter 2. This is discussed in Section 3.3 and 3.4. The optimization of societal investments into fire safety requires an absolute risk assessment with as little bias as possible, as discussed in Chapter 4. At least a bias should not be introduced intentionally by making conservative assumptions, as this will lead to a waste of societal resources for inefficient safety measures. Relative risk assessment may be used where little knowledge is available as a basis for modelling the risk, e.g. to explore trade-offs between different fire safety measures.

Also deterministic, scenario-based methods can still have a place in a risk-informed framework for performance-based fire safety. However, the scenarios and safety factors used should be defined in consistency with the quantitative performance criteria to be used in a risk-based analysis. The calibration of safety factors to target reliabilities is already state of the art in the design for structural fire safety, see Section 6.1. Similar approaches can be developed also in other areas of fire safety engineering, see e.g. Frantzich et al. (1997), Hasofer and Beck (2000) or He (2010) for the derivation of safety factors in the design of evacuation routes.

3.3 Target reliabilities for structural design codes

Before turning to the situation in fire safety design, it is discussed how to define safety targets for structural design codes based on the principles of economic optimization outlined in Chapter 2. The aim of structural design is to ensure that the probability of failure of individual components or whole structures remains below a certain threshold. Quantitative safety goals are thus conveniently defined in terms of target reliabilities β_t. Different reliability levels may be defined for different structural classes, e.g. to account for varying consequences of structural failure as in Annex B of the Eurocode, EN 1990. The Eurocode allows using these target reliabilities as quantitative safety goals for design verification using probabilistic methods. The safety factors for the standard, semiprobabilistic design are calibrated to the safety level of reliability class RC2 ($\beta_t = 4.7$ for a 1-year reference period and ultimate limit state design).

Tab. 3.1: Target reliabilities β_t for structural design and corresponding failure probabilities related to a 1-year reference period and ultimate limit states (JCSS, 2001).

Relative cost of	Consequences of failure		
safety measure	Minor	Moderate	Large
Large	$\beta_t = 3.1\ (P_f \approx 10^{-3})$	$\beta_t = 3.3\ (P_f \approx 5 \cdot 10^{-4})$	$\beta_t = 3.7\ (P_f \approx 10^{-4})$
Normal	$\beta_t = 3.7\ (P_f \approx 10^{-4})$	$\beta_t = 4.2\ (P_f \approx 10^{-5})$	$\beta_t = 4.4\ (P_f \approx 5 \cdot 10^{-6})$
Small	$\beta_t = 4.2\ (P_f \approx 10^{-5})$	$\beta_t = 4.4\ (P_f \approx 5 \cdot 10^{-6})$	$\beta_t = 4.7\ (P_f \approx 10^{-6})$

Table 3.1 shows tentative target reliabilities provided in the JCSS Probabilistic Model Code (JCSS, 2001) and corresponding failure probabilities $P_f = \Phi\left(-\beta_t\right)[1/a]$. Here, the reliability is not only differentiated according to the failure consequences, but also as a function of the relative costs of increasing safety. The JCSS target reliabilities were defined partly for consistency with

the Eurocode requirements and observed failure rates (Vrouwenvelder, 2002). Another important input was an optimization study by Rackwitz (2000), which is explained in the following.

3.3.1   Target reliabilities based on monetary optimization

A presumption of Rackwitz' study was that design codes should be optimized in terms of costs, benefits and failure consequences. His formulation of the objective function is based on a renewal theoretic approach first proposed by Rosenblueth and Mendoza (1971). In structural design optimization, renewal theory is used to evaluate the total costs implied by a series of structures to be built in the future. The renewal strategy relevant for structural design codes is systematic reconstruction (or repair) after obsolescence or failure: The need for structures continues beyond the service life of individual objects, and even if a structure is not rebuilt as an exact copy at the same place, new structures will always be built using the same or similar design codes. Therefore, the time horizon t_u of code-making decisions is generally very long and may be assumed to approach infinity ($t_u \rightarrow \infty$). The objective function is then not limited to one individual structures, but to a series of similar structures to be built one after the other.

Defining a simple objective function

The following design objective was proposed by Rackwitz (2000):

$$\max_{\mathbf{p}} \left\{ Z(\mathbf{p}) = B - C(\mathbf{p}) - A(\mathbf{p}) - D(\mathbf{p}) - U(\mathbf{p}) - M(\mathbf{p}) - I(\mathbf{p}) \right\} \tag{3.1}$$

Here, B denotes the benefit derived from the existence of the structure, $C(\mathbf{p})$ the construction costs, $A(\mathbf{p})$ the obsolescence costs, $D(\mathbf{p})$ the ultimate limit state failure costs, $U(\mathbf{p})$ the costs associated with serviceability failure, $M(\mathbf{p})$ the fatigue and other ageing failure costs, $I(\mathbf{p})$ the inspection and resulting maintenance costs and $\mathbf{p}$ a vector of decision variables to be optimized. In consistency with Section 2.2, all costs and benefits are evaluated as expected present values.

The last two cost components $M(\mathbf{p})$ and $I(\mathbf{p})$ were neglected in the optimization performed by Rackwitz and are also here not considered. Here it is furthermore assumed that the serviceability failure costs $U(\mathbf{p})$ can be neglected (see Annex B of Rackwitz, 2000 for the effect of this assumption). Rackwitz assumes the benefit B to be independent of the decision parameter $\mathbf{p}$. Maximizing $Z(\mathbf{p})$ is thus equivalent to minimizing the total costs and Equation 3.1 simplifies to:

$$\min_{\mathbf{p}} \left\{ T(\mathbf{p}) = C(\mathbf{p}) + A(\mathbf{p}) + D(\mathbf{p}) \right\} \tag{3.2}$$

The advantage of the renewal theoretic approach is that, with a few simplifying assumptions, all cost components can be estimated as a function of the construction costs $C(\mathbf{p})$. To achieve this, it is assumed that the structure is systematically rebuilt after each failure or obsolescence event. For the assessment of $A(\mathbf{p})$ and $D(\mathbf{p})$, it is assumed that obsolescence and failure are independent events and occur at random points in time with independent and identically distributed inter-arrival times for both types of renewals. For simplicity, it is furthermore assumed that the time of construction is negligibly short compared to the average lifetime of the structure (see Rackwitz, 2000 for a simple approach to account for finite construction times).

The expected present value of the obsolescence costs is evaluated as:

$$
\begin{aligned}
A(\mathbf{p}) &= (C(\mathbf{p}) + A) \sum_{i=1}^{\infty} \mathrm{E}_{T_i} \left[\exp\left(-\gamma T_i\right) \right] \\
&= (C(\mathbf{p}) + A) \sum_{i=1}^{\infty} \int_0^{\infty} \exp\left(-\gamma t_i\right) f_{T_i}\left(t_i\right) \, \mathrm{d}t_i
\end{aligned}
\tag{3.3}
$$

Here, $C(\mathbf{p})$ denotes the expected (re-)construction costs, A the demolition costs and γ the discount rate relevant for the (private or societal) decision-maker performing the optimization. Equation 3.3 assumes that the structure is systematically rebuilt after obsolescence, i.e. whenever it is regarded to be inefficient from an economic point of view or not able to fulfil changing user requirements anymore. f_{T_i} is the probability distribution function for the time T_i to the ith obsolescence event. The sum in Equation 3.3 is equal to the Laplace transform of the renewal density $h^*(\gamma)$ and may be simplified as follows (see e.g. Cox, 1962):

$$
h^*(\gamma) = \sum_{i=1}^{\infty} \int_0^{\infty} \exp\left(-\gamma t_i\right) f_{T_i}\left(t_i\right) \, \mathrm{d}t_i = \sum_{i=1}^{\infty} f_{T_i}^*(\gamma) = \sum_{i=1}^{\infty} \left[f_T^*(\gamma)\right]^i = \frac{f_T^*(\gamma)}{1 - f_T^*(\gamma)}
\tag{3.4}
$$

With $f_{T_i}^*(\gamma)$ and $f_T^*(\gamma)$ denoting the Laplace transforms of the probability density functions for the time to the ith renewal T_i and the inter-renewal times T. Laplace transforms for different distribution types are given in Annex A of Rackwitz (2000). A very simple result is achieved by assuming a homogeneous Poisson process for the occurrence of obsolescence events, which leads to exponentially distributed inter-renewal times. The expected present value of the obsolescence costs is then assessed as a function of the mean obsolescence rate $\omega = 1/E[T]$:

$$
A(\mathbf{p}) = (C(\mathbf{p}) + A) \, h^*(\gamma) = (C(\mathbf{p}) + A) \frac{\omega}{\gamma}
\tag{3.5}
$$

Even though the Poisson assumption for obsolescence events is probably not very realistic, Equation 3.5 is used to study the influence of the (mean) service life on the optimization results; at least qualitatively, the behaviour will be similar for other probabilistic models. Disregarding the obsolescence costs is not acceptable, as this is equivalent to assuming an infinite service life (unless failure occurs), which is clearly not realistic. Only the demolition costs A are neglected in the following, as they are anyway assumed to be independent of $\mathbf{p}$.

A similar approach is used to estimate the expected present value of the failure costs, $D(\mathbf{p})$. For simplicity, it is assumed that failure can only occur at random disturbances (e.g. earthquakes, storms, fires) that follow a Poisson process with intensity λ. This leads to:

$$
D(\mathbf{p}) = (C(\mathbf{p}) + H) \frac{\lambda P_f(\mathbf{p})}{\gamma}
\tag{3.6}
$$

Where H denotes the costs that accrue in case of failure in addition to the costs of reconstruction and $P_f(\mathbf{p})$ is the probability of structural failure conditional on the disturbance event. Assuming that failure is caused by random disturbances is clearly reasonable for the optimization of structural fire safety, see Section 3.4. For failures due to time-variant loads, Equation 3.6 is a helpful simplification (see Rackwitz, 2000 for a description of different failure processes). For his

parameter study, Rackwitz (2000) assumed $\lambda = 1$, such that $P_f(\mathbf{p})$ refers to the yearly probability of failure or (unconditional) failure rate. For ease of notation, λ is omitted in the following as long as the focus is on the normal (non-fire) design situation.

Rackwitz estimated $P_f(\mathbf{p})$ from a simple limit state function with the only decision parameter $\mathbf{p} = p$ being defined as the central safety factor in structural design:

$$P_f(p) = P\left[p \cdot R - S < 0\right] \tag{3.7}$$

Both the resistance R and the load effect S are modelled as random variables with coefficients of variation V_R and V_S, respectively. Equation 3.7 assumes that both random variables are normalized with their expected value, such that $\mathrm{E}[R] = \mathrm{E}[S] = 1$.

The construction costs $C(p)$ are modelled as a linear function of the central safety factor p:

$$C(p) = C_0 + C_1 p \tag{3.8}$$

Where C_0 refers to the part of the construction costs that is independent of structural design. The marginal safety costs C_1 may be estimated by calibrating Equation 3.8 to the cost of a design based on the codes currently in practice. To give an example, the Eurocode target reliability $\beta_t = 4.7$ relates to a central safety factor around $p = 7$ assuming lognormal distributed loads and resistances and $V_R = V_S = 0.3$. The costs for the structure may be assumed to be proportional to the central safety factor. They typically amount to a few percent of the whole building costs. Assuming e.g. $C_1/C_0 \cdot p = 3.5\%$ leads to "relative costs of the safety measure" around $C_1/C_0 = 0.035/7 = 5 \cdot 10^{-3}$. This result is of course not exact, but it shows how at least the order of magnitude for C_1/C_0 may be determined based on a few simple assumptions.

Using Equation 3.5 to 3.8, the design objective in Equation 3.2 is reformulated as follows:

$$\min_p \left\{ T(p) = (C_0 + C_1 p) + (C_0 + C_1 p) \cdot \omega/\gamma + (C_0 + C_1 p + H) \cdot P_f(p)/\gamma \right\} \tag{3.9}$$

Parameter study

By relating all cost components to the "fixed" construction cost C_0, optimal reliabilities can be defined as a function of the relative safety costs and failure consequences. Figure 3.2 shows the optimal failure probability as a function of the key parameters H, C_1, V_R and V_S. Figure 3.2a illustrates the influence of the relative failure consequences H/C_0 on the optimal probability of failure. The dotted vertical lines depict the definition of the consequence classes in Table 3.1, which are formulated in terms of the ratio between the total failure costs $C_0 + C_1 p + H$ and the (fixed) construction costs C_0. Ratios larger than 10 (i.e. $H/C_0 > 9$) are beyond the scope of Table 3.1; for these structures, JCSS (2001) recommends a full cost-benefit study.

The effect of the relative safety costs C_1/C_0 is illustrated in Figure 3.2b for the range of H/C_0 referred to as "moderate" consequences of failure in Table 3.1. JCSS (2001) does not define the cost classes in quantitative terms. It is, however, clear that their definition should be based on the relative (marginal) safety costs C_1/C_0. Figure 3.2 furthermore shows that increasing the coefficients of variation V_R and V_S has a similar effect as increasing the cost parameter C_1. Also
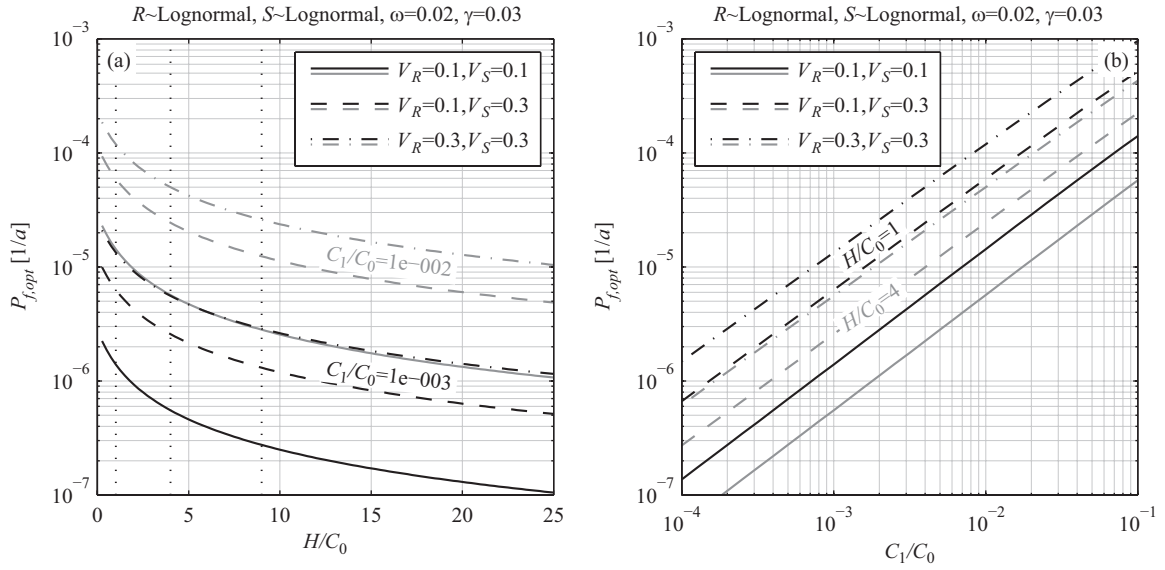
Fig. 3.2: Optimal failure probabilities $P_{f,opt}$ as a function of (a) the relative failure consequences H/C_0 and (b) the relative costs of safety measure C_1/C_0 for varying coefficients of variation V_R and V_S (R and S lognormally distributed).
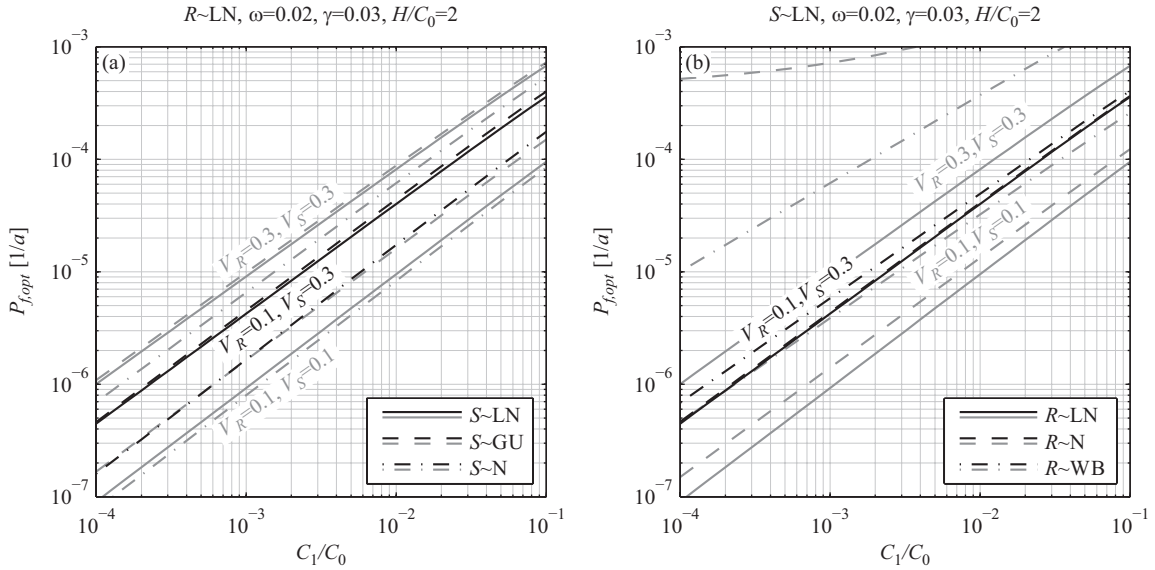
Fig. 3.3: Optimal failure probabilities $P_{f,opt}$ for different distributional assumptions (a) for the load effect S (with R lognormally distributed) and (b) for the resistance R (with S lognormally distributed).

the obsolescence rate ω and the discount rate γ may be seen as increasing the cost of structural safety, see Rackwitz (2000) and JCSS (2001).

The results in Figure 3.2 are valid for lognormal (LN) distributed resistance and load effect. The influence of distributional assumptions is investigated in Figure 3.3. Figure 3.3a shows the effect of changing the distribution of the load effect S to Gumbel max (GU) or Normal (N). In Figure 3.3b, the distribution of the resistance R is changed to Normal (N) and Weibull min (WB).

From the two graphs it can be concluded that the influence of distributional assumptions on the optimization results is relatively small for coefficients of variation $V \leq 0.3$. The distribution of the resistance R has an important effect only if the coefficient of variation is $V_R = 0.3$, which is already fairly high for a resistance model. Moreover, the lognormal distribution is in fact the model of choice for many material resistance variables, see JCSS (2001).

Discussion and open questions

The biggest advantage of the formulation proposed by Rackwitz (2000) is its simplicity, which makes it easy to understand the influence of different parameters used by the model. In spite of a number of simplifying assumptions, Rackwitz' approach is able to take into account all relevant aspects that may have an effect on the optimal failure probability at least to an extent that allows for a rough "order of magnitude" estimate to be used in code-based design.

What remains a bit unclear is the interpretation of the resulting target reliabilities in terms of component and system design. Rackwitz (2000) proposed to use his results as a substitute for direct optimization at the level of structural components while the target reliabilities given in the Probabilistic Model Code are meant to be used for the design of structural systems, see JCSS (2001). However, it only makes sense to use the same target reliabilities both for component and system design if system failure is dominated by one component or failure mode.

In principle, the optimization approach discussed above is applicable to both component and system failure. The only requirement is that the marginal safety costs C_1 and the failure consequences H are estimated for the same failure event. The cost estimation is easiest when looking at ultimate limit state failure at component level. Performing the consequence assessment at component level does, however, require taking into account structural robustness.
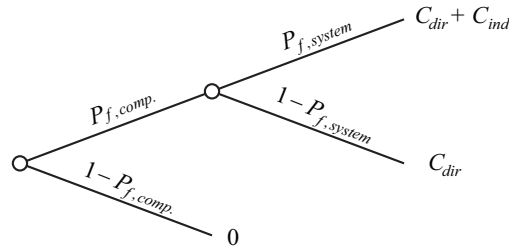
Fig. 3.4: Event tree illustrating the direct and indirect consequences of component and system failure.

Figure 3.4 shows an event tree illustrating the different types of consequences due to failure at component level ("direct consequences") and at system level ("indirect consequences"). The probability of system failure is assessed conditional on the event of component failure. Structural robustness minimizes the contribution of indirect risk to the total system risk, see Baker et al. (2008). An optimal design should take into account both direct and indirect risk, e.g. by defining the failure cost function $D(\mathbf{p})$ as follows:

$$D(\mathbf{p}) = [(1 - P_{f,system}) \, C_{dir} + P_{f,system} \, (C_{dir} + C_{ind})] \, P_{f,comp.}(\mathbf{p})/\gamma \qquad (3.10)$$

Based on this consequence assessment, target reliabilities at component level could be formulated for different types of structural systems and components according to the conditional probability of system failure. A possible simplification would be to assume that the contribution of direct risk is negligible. However, taking $P_{f,system}$ as given ignores the possibility of risk reduction by increasing structural robustness. An alternative, though also heuristic approach discussed by Baker et al. (2008) is to optimize target reliabilities for structural components based on direct risk only. In a second step, structural robustness is optimized separately by balancing the marginal costs of additional risk reduction with the marginal benefit of reduced indirect risk.

3.3.2 Target reliabilities based on the LQI acceptance criterion

Human consequences of structural failure are discussed only qualitatively during the definition of consequence classes provided in the Probabilistic Model Code (JCSS, 2001). Risk to life can be taken into account quantitatively by introducing compensation costs for human fatalities in the consequence assessment. The social acceptability of the structural design in terms of risk to human life is, however, not necessarily guaranteed when relying on the JCSS target reliabilities in Table 3.1. In the following, the acceptable region for monetary optimization is defined by deriving target reliabilities from the LQI criterion. They should be understood as minimum requirements that may be complemented by the target reliabilities provided in Table 3.1 if monetary optimization is not performed explicitly during a probabilistic design of the structure. The approach chosen for defining the LQI target reliabilities should thus be consistent with the monetary optimization studies performed by Rackwitz (2000) forming the basis for the JCSS target reliabilities in Table 3.1.

Deriving the marginal life saving costs from the objective function

For the formulation of the LQI acceptance criterion, the marginal life saving costs have to be extracted from the objective function in Equation 3.2. As discussed in Section 2.3.9, the failure costs $D(\mathbf{p})$ are not taken into account in the acceptance criterion, because reductions in this cost components can be regarded to be a monetary benefit of increasing safety. For the LQI criterion it is sufficient to quantify the marginal increase in construction costs $C(\mathbf{p})$ and obsolescence costs $A(\mathbf{p})$ and the respective change in risk to life as a function of the decision parameter $\mathbf{p} = p$. The acceptable region is then defined by the following inequality:

$$\frac{\mathrm{d}\left(C(\mathbf{p}) + A(\mathbf{p})\right)}{\mathrm{d}\mathbf{p}} \geq -\frac{\mathrm{d}N(\mathbf{p})}{\mathrm{d}\mathbf{p}} \frac{1}{\gamma_S} \cdot \frac{g}{q} J_x \tag{3.11}$$

Here, risk to life is defined as the expected number of fatalities $N(\mathbf{p})$ per year. The Societal Willingness To Pay (SWTP) to save one additional life, $g/q \cdot J_x$, is derived from the LQI. Using the same assumptions as for the monetary optimization discussed in Section 3.3.1, the LQI criterion is evaluated as follows:

$$C_1 \left(1 + \omega/\gamma_S\right) \geq -\frac{1}{\gamma_S} \cdot \frac{g}{q} J_x \cdot N_F \cdot \frac{\mathrm{d}P_f(p)}{\mathrm{d}p} \tag{3.12}$$

Where $\mathbf{p} = p$ again refers to the central safety factor (see Equation 3.7). Risk to life is now quantified in terms of the failure probability $P_f(p)$ and the expected number of fatalities given structural failure, N_F. As the LQI criterion is a boundary condition imposed by society, the societal discount rate γ_S has to be used.

Criteria similar to Equation 3.11 and 3.12 have already been introduced by Rackwitz and Streicher (2002) and Streicher and Rackwitz (2003, 2006) with some differences regarding discounting and the treatment of obsolescence costs. To combine it with the formulation developed for monetary optimization, Rackwitz and Streicher related the LQI criterion to the "relative costs of safety measure" C_1/C_0 by making assumptions on the absolute value of the fixed construction costs C_0. These assumptions make their results scale dependent. In the following, a more generic approach is developed by relating the safety costs to the $SWTP$ for the life safety benefit achieved by the investment. Rearranging Equation 3.12 leads to the following criterion:

$$\frac{\mathrm{d}P_f(p)}{\mathrm{d}p} \leq \frac{C_1\,(\gamma_S + \omega)}{g/q \cdot J_x \cdot N_F} = K_1 \tag{3.13}$$

The numerator on the right-hand side of the inequality, $C_1\,(\gamma_S + \omega)$, indicates how much the yearly safety costs rise with a unit increase in the global safety factor p (γ_S can be understood as financing cost and ω as the cost of rebuilding the structure in case of obsolescence). For the denominator the human consequences of structural failure N_F have been transformed into monetary units by multiplying with the $SWTP$ to save one additional life. LQI target reliabilities can now be derived as a function of the constant K_1.

For a specific structure, the K_1 constant may be estimated using assumptions on the relative costs of safety measure, as defined in Section 3.3.1. Table 3.2 shows values for different relative safety costs C_1/C_0 and failure consequences N_F for the example of two types of structures with different absolute values for the construction costs C_0. The K_1 values in Table 3.2 are based on a yearly obsolescence rate $\omega = 2\%$ and a societal interest rate $\gamma_S = 3\%$. The $SWTP$ per life saved, $g/q \cdot J_x$, is set to $5Mio.CHF$ (see Table 2.2).

Tab. 3.2: K_1 values for two types of Swiss structures with different construction costs C_0 as a function of the relative costs of safety measure C_1/C_0 and the human consequences of failure N_F.

(a) Office building, $C_0 = 2'000CHF/m^2$

N_F/m^2	C_1/C_0		
	0.001	**0.01**	**0.1**
0.0001	$2\cdot10^{-4}$	$2\cdot10^{-3}$	$2\cdot10^{-2}$
0.001	$2\cdot10^{-5}$	$2\cdot10^{-4}$	$2\cdot10^{-3}$
0.01	$2\cdot10^{-6}$	$2\cdot10^{-5}$	$2\cdot10^{-4}$
0.1	$2\cdot10^{-7}$	$2\cdot10^{-6}$	$2\cdot10^{-5}$

(b) Bridge, $C_0 = 10Mio.CHF$

N_F	C_1/C_0		
	0.001	**0.01**	**0.1**
0.1	10^{-3}	10^{-2}	10^{-1}
1	10^{-4}	10^{-3}	10^{-2}
10	10^{-5}	10^{-4}	10^{-3}
100	10^{-6}	10^{-5}	10^{-4}

The K_1 values contain information on the safety costs, on the failure consequences and on the SWTP for life safety. The resulting LQI target reliabilities are further influenced by the assumptions made regarding the distributions of the basic random variables in the limit state function, R and S. This is discussed in the following.

Parameter study

As illustrated in Figure 3.5, the problem of quantifying the LQI acceptance threshold $P_{f,acc}$ can be divided into three main parts: Safety costs, failure consequences and limit state function. The assumptions made during the quantification of the marginal safety costs and the human consequences in case of failure are summarized in the constant K_1. Therefore, in the following the focus is on the influence of the probabilistic models used in the limit state function (Equation 3.7) for estimating the failure probability.
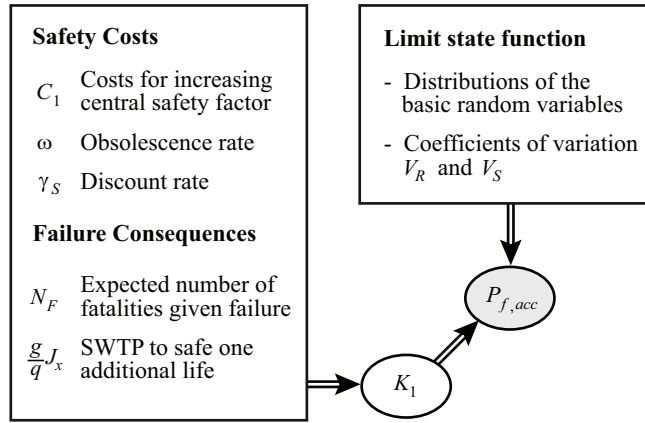
Fig. 3.5: Parameters and assumptions influencing the LQI acceptance threshold $P_{f,acc}$.

In consistency with Rackwitz (2000), it is first assumed that both the resistance R and the load effect S are lognormally distributed. Figure 3.6a shows the LQI acceptance threshold $P_{f,acc}$ as a function of the constant K_1. A straight line on a log-log scale represents a power law. The special case of a linear dependency between $P_{f,acc}$ and K_1 can be assumed only for coefficients of variation V_R and V_S smaller than 0.3. This becomes clear in Figure 3.6b where the ratio $P_{f,acc}/K_1$ is plotted as a function of K_1.

Figure 3.6 furthermore shows that the acceptable failure probability is higher for large coefficients of variation V_R and V_S. This can be explained by the fact that it is more costly to reduce the probability of failure if the variability of the basic random variables R and S is high. Therefore, large coefficients of variation have a similar effect as large safety costs C_1 (which in turn leads to a larger constant K_1). The influence of the coefficients of variation on the LQI acceptance threshold $P_{f,acc}$ was further investigated in Fischer et al. (2012a). The influence of one coefficient of variation (V_R or V_S) on the acceptance threshold is largest if the variability of the other random variable is small.

The results presented in Figure 3.6 are valid for lognormally distributed random variables R and S. Figure 3.7 shows the effect of relaxing this assumption for the load effect S (Figure 3.7a) or the resistance R (Figure 3.7b). Black curves refer to a situation with small, grey curves to large coefficients of variation. It can be seen that the influence of distributional assumptions for the load effect S is relatively small when compared to the effect of assumptions regarding the coefficients of variation or the constant K_1. The same holds for the distribution of the resistance
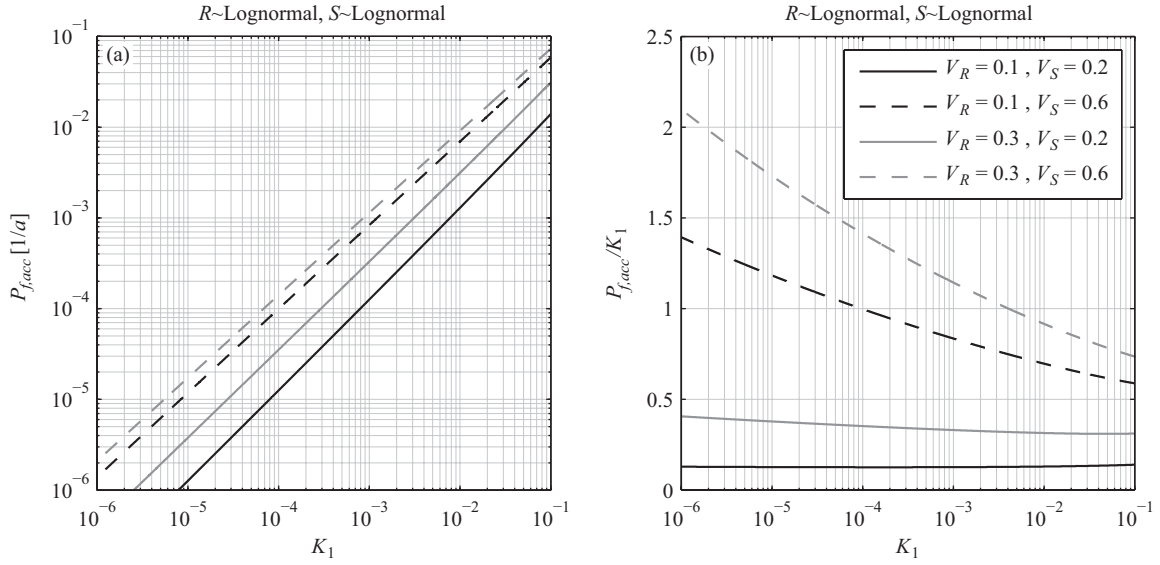
Fig. 3.6: LQI threshold $P_{f,acc}$ as a function of the constant K_1 (Equation 3.13) for different coefficients of variation of the resistance R and the load effect S (both random variables lognormally distributed).
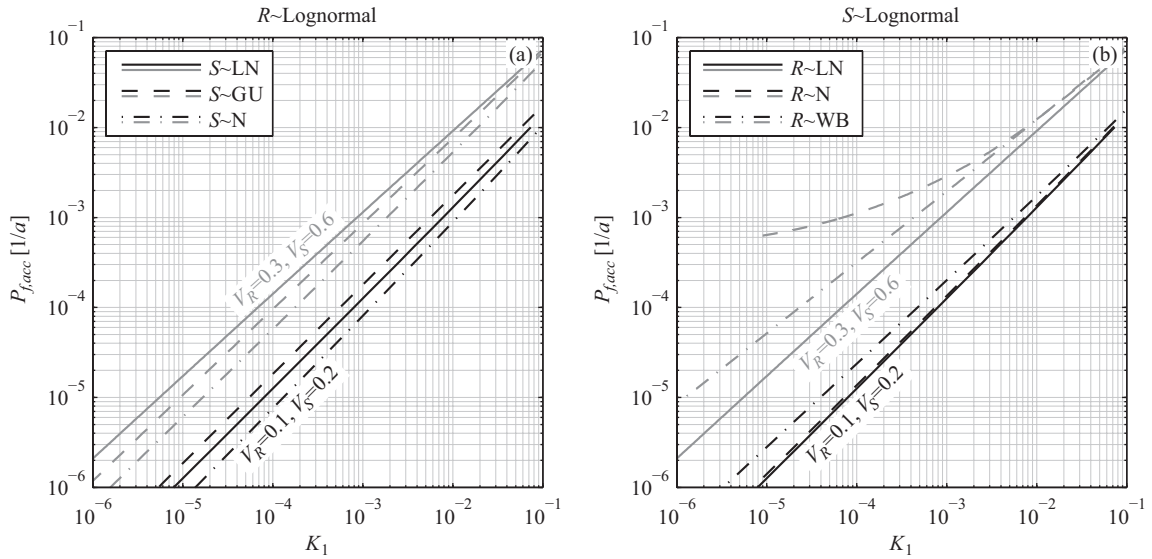
Fig. 3.7: LQI threshold $P_{f,acc}$ for different distributional assumptions (a) for the load effect S (with R lognormally distributed) and (b) for the resistance R (with S lognormally distributed).

R as long as the variability of the basic random variables is small. However, in situations with large variability, the distributional assumptions for the resistance R can have a large impact.

Despite the inevitable differences caused by assumptions regarding the limit state function in Equation 3.7, a general trend can be observed regarding the dependence of the LQI acceptance threshold $P_{f,acc}$ on the constant K_1: Increasing K_1 by an order of magnitude leads roughly to an order of magnitude increase in the failure probability deemed to be acceptable by the LQI. This observation is valid in a broad range of situations and can therefore be used to derive a simple

LQI target reliability format. Such a format is introduced in the following after discussing the interaction of the LQI criterion with monetary optimization.

Interaction with monetary optimization

As has been discussed in Section 2.3.5, the LQI criterion defines the acceptable region within which monetary optimization is admissible. A higher safety level than defined by the LQI target reliability should be aimed at if this is preferable from a monetary optimization point of view. The JCSS target reliabilities (Table 3.1) may be used as a substitute for direct optimization. In the following, a simple rule is derived for checking whether the monetary optimum is an acceptable decision according to the LQI. The derivation is valid both for private and societal decision-makers performing the optimization. It is assumed that both types of decision-makers use the same estimate for the construction costs $C(p)$; only the failure costs H and the discount rate γ may differ.

A necessary condition for the optimal design p^* is that the first derivative of the objective function must be equal to zero at p^*. Using the objective function in Equation 3.2, the following condition can be derived $(\mathrm{d}T(p^*)/\mathrm{d}p = 0)$:

$$\frac{\mathrm{d}C(p^*)}{\mathrm{d}p} \cdot (1 + \omega/\gamma) + \frac{\mathrm{d}C(p^*)}{\mathrm{d}p} \cdot P_f(p^*)/\gamma + \frac{\mathrm{d}P_f(p^*)}{\mathrm{d}p} \cdot (C(p^*) + H)/\gamma = 0 \qquad (3.14)$$

The second term can be neglected as long as $P_f(p^*) \ll \gamma$. Rearranging leads to the following equality:

$$\frac{\mathrm{d}C(p^*)}{\mathrm{d}p} = -\frac{C(p^*) + H}{\gamma + \omega} \cdot \frac{\mathrm{d}P_f(p^*)}{\mathrm{d}p} \qquad (3.15)$$

Inserting Equation 3.15 into the LQI acceptance criterion in Equation 3.11 gives the following result (note that $\mathrm{d}P_f(p)/\mathrm{d}p < 0$ for all p):

$$\frac{g/q \cdot J_x \cdot N_F}{C(p^*) + H} \leq \frac{\gamma_S + \omega}{\gamma + \omega} \qquad (3.16)$$

This criterion can be used for checking whether the monetary optimum p^* is an acceptable decision from a societal (life saving) point of view. It is valid for all decisions regarding investments that aim at decreasing the probability of an adverse event leading to both monetary and human consequences. The denominator on both sides of the inequality is evaluated from the point of view of the (private or societal) decision-maker performing the optimization while the numerator always quantifies societal preferences.

The failure costs H in Equation 3.16 may include human compensation costs $H_C N_F$, with $H_C > 0$ denoting the compensation per fatality paid by the decision-maker. The amount of human compensation costs H_C depends on a number of different factors. For a private decision-maker (e.g. the owner of the structure), the amount paid per fatality is typically defined in a court sentence or based on negotiations with the relatives of the victim. Therefore, the compensation costs for a private decision-maker can vary considerably. In addition, the range of compensations paid depends on the legal system of a country. Also in the case of a societal decision-maker performing the optimization, there is no general consensus on how to quantify

and include human losses in monetary optimization, see Fischer and Faber (2012). Therefore the ratio $H_C / (g/q \cdot J_x)$ is introduced as a variable. By rearranging and multiplying with $H_C N_F$, Equation 3.16 can be reformulated as follows:

$$\frac{N_F H_C}{C(p^*) + H} \leq \frac{\gamma_S + \omega}{\gamma + \omega} \cdot \frac{H_C}{g/q \cdot J_x} \tag{3.17}$$

This criterion is defined in terms of three ratios: The first term is the ratio between human compensation costs in case of failure, $N_F H_C$, and the total failure costs $C(p^*) + H$. This ratio is always smaller than one. The second ratio is a function of the societal discount rate γ_S and the discount rate γ that is used by the decision-maker performing the optimization. For societal decision-making both the optimization and the LQI acceptance criterion are evaluated from a societal point of view, such that $\gamma = \gamma_S$ and $(\gamma_S + \omega) / (\gamma + \omega) = 1$. For private decision-makers the ratio typically becomes smaller than one because $\gamma > \gamma_S$. Finally, the third ratio relates to how loss of life is transformed into monetary terms for the optimization (compensation costs H_C) and for the LQI acceptance criterion ($SWTP$ to safe one life $g/q \cdot J_x$).

The behaviour of Equation 3.16 and 3.17 has been further investigated in Fischer et al. (2012a). For many applications it may be sufficient to assume that $(\gamma_S + \omega) / (\gamma + \omega) \approx \gamma_S/\gamma$ (especially if the obsolescence rate ω is small) and to check only whether the discount rate ratio γ_S/γ is larger than the ratio between the $SWTP$ to prevent a loss of N_F lives and the total failure costs $C(p^*) + H$ (Equation 3.16). The higher the private discount rate γ and the lower the compensation H_C paid in case of a fatality, the more likely it is that the LQI criterion becomes active and that the decision-maker has to adopt a solution that is safer than the monetary optimum.

For a societal decision maker γ_S/γ equals one and Equation 3.16 simplifies to checking whether $g/q \cdot J_x \cdot N_F$ is larger than $C(p^*) + H$. This situation can be investigated further based on Equation 3.17. It can be shown that it is only necessary to impose the LQI criterion as a boundary condition to monetary optimization performed by a societal decision-maker if the $SWTP$ used in the acceptance criterion is larger than the human compensation costs H_C entering monetary optimization (see Fischer et al., 2012a or Fischer and Faber, 2012 for a derivation).

Deriving a simple target reliability format from the LQI

A simple format for defining minimum target reliabilities from the LQI acceptance criterion can be derived by relating the LQI threshold reliability to the constant K_1 as defined in Equation 3.13. Table 3.3 presents results for different "relative life saving costs classes".

The cost classes are defined in terms of a range for the K_1 constant (second column of Table 3.3) and are valid for medium variabilities of the total loads and resistances (i.e. $0.1 \leq V \leq 0.3$). The target probability of failure may be increased by a factor 5 for higher coefficients of variation of the basic random variables. For low variabilities, on the other hand, it should be reduced by a factor 2. The values in Table 3.3 have been derived based on lognormally distributed loads

Tab. 3.3: LQI target reliabilities related to a 1-year reference period and ultimate limit states.

Relative life saving costs	Range for K_1 constant	LQI target reliability
Large	$10^{-3} \div 10^{-2}$	$\beta_t = 3.1 \ \left(P_f \approx 10^{-3}\right)$
Normal	$10^{-4} \div 10^{-3}$	$\beta_t = 3.7 \ \left(P_f \approx 10^{-4}\right)$
Small	$10^{-5} \div 10^{-4}$	$\beta_t = 4.2 \ \left(P_f \approx 10^{-5}\right)$

and resistances. However, they can serve as a good approximation if other distribution types are used (see Figure 3.7 and discussion).

The approach followed in Table 3.3 is to define structural classes according to the relative life saving costs. Alternatively, due to the almost linear dependence between the constant K_1 and the LQI threshold for small coefficients of variation V_R and V_S, it is also possible to estimate the LQI target reliability based on the following approximate formula (again for $0.1 \leq V \leq 0.3$):

$$P_{f,acc} \approx \frac{K_1}{5} = \frac{1}{5} \cdot \frac{C_1 \left(\gamma_S + \omega\right)}{g/q \cdot J_x \cdot N_F} \tag{3.18}$$

Independent of the approach followed, the LQI target reliability format should be accompanied by a simple rule for checking whether risk to life is extraordinarily high or, at the other extreme, negligibly small. Such a rule could be formulated in terms of the expected number of fatalities N_F and should be applied especially in situations where the constant K_1 is very small or large (e.g. below 10^{-5} or above 10^{-2}). If the human consequences in case of failure are very high, the simplified format introduced in this section should not be applied. Instead, the marginal life saving costs should be estimated based on direct risk assessment.

The LQI target reliabilities can be used either as a boundary condition for direct monetary optimization or in combination with the JCSS target reliabilities. The optimal design should be aimed at if the resulting reliability is higher than the LQI threshold. Simple criteria for checking whether the optimum is also acceptable following the LQI criterion are given by Equation 3.16 and 3.17. It has to be emphasized that the cost classes in Table 3.3 are not equivalent to those defined in the JCSS table (Table 3.1): The constant K_1 is a measure relating the safety costs to the monetized human consequences in case of failure. The risk reduction costs in Table 3.1, on the other hand, are related to the total construction costs and the consequence assessment is used to choose one of the consequence classes.

Discussion and open questions

The derivation of LQI target reliabilities as a function of the "relative life saving costs constant" K_1 allows for the definition of acceptable levels of safety as a minimum requirement for structural design optimization. As the $SWTP$ to safe an additional life, $g/q \cdot J_x$, depends on the socioeconomic capacity of a society for investments into life safety, the resulting design targets may vary when applying the proposed format in different countries. This variation is, however, consistent with the principle of optimal resource allocation at national level.

The approach chosen is derived from the optimization study by Rackwitz (2000) discussed in Section 3.3.1 and shares its strengths and weaknesses. The generic formulation is applicable to a broad range of decision problems where the yearly probability of "failure" (an adverse event leading to both human and monetary consequences) can be reduced at a cost. In the context of structural safety it may be applied both for defining minimum target reliabilities for structural components or whole structures. However, the marginal safety costs C_1 and the human consequences given failure, N_F, must be estimated for the same failure event. The consequence assessment has to account for structural robustness if the proposed target reliability format is used for component design (see Section 3.3.1 for discussion).

3.4   Safety targets for fire safety design

The aim of the present section is to discuss how the approach outlined in Section 3.3 may be adapted for defining safety targets for performance-based fire safety design. To achieve this, the fire safety costs and fire consequences have to be modelled as a function of the key input variables relevant for different design situations. The development of such models for all types of safety measures is beyond the scope of this thesis. Instead, the focus is on the formulation of a general framework that has to be filled with appropriate physical models to provide results that may be used in practice. In Chapter 6 (Case study 2) the framework is applied to the definition of target reliabilities for structural fire safety. Other types of safety measures are discussed only briefly in Section 3.4.3.

The discipline of fire safety engineering is much less developed than structural design, especially with respect to risk-based reasoning. Due to the lack of appropriate models, it may not be possible to apply the principles discussed in this chapter to all fields of fire safety design, at least not in the near future. The framework introduced in the following should be understood mainly as a "roadmap" aiming at rational decision-making for optimal fire safety. However, even the application only to a few isolated problems where risk-based methods are sufficiently developed can already improve decision-making in fire safety design.

3.4.1   Fire safety targets based on monetary optimization

For the formulation of the objective function, Equation 3.1 is used as a starting point. Not all cost components are relevant for the optimization of fire safety. The safety investments may comprise construction costs $C(\mathbf{p})$, reconstruction (obsolescence) costs $A(\mathbf{p})$ and maintenance costs $I(\mathbf{p})$, depending on the safety measures regarded. The failure costs $D(\mathbf{p})$ are replaced by the expected present value of all monetary fire losses $F(\mathbf{p})$. Other cost components, e.g. fire brigade reaction costs (see Table 2.1), may be relevant for some problems of fire safety design but will be neglected in the following. The design objective is thus formulated as follows:

$$\min_{\mathbf{p}} \left\{ T(\mathbf{p}) = C(\mathbf{p}) + A(\mathbf{p}) + I(\mathbf{p}) + F(\mathbf{p}) \right\} \tag{3.19}$$

Where $\mathbf{p}$ denotes a vector of decision variables relevant for fire safety design. Like in Equation 3.1, all cost components have to be evaluated as expected present values. The service life of

many fire safety measures is shorter than that of the building they are designed for. This has to be accounted for during the quantification of the reconstruction costs $A(\mathbf{p})$, e.g. by assuming a higher renewal (obsolescence) rate. A simple optimality criterion is derived from $dT(\mathbf{p})/d\mathbf{p} = 0$:

$$\frac{\mathrm{d}\left(C(\mathbf{p}) + A(\mathbf{p}) + I(\mathbf{p})\right)}{\mathrm{d}\mathbf{p}} = -\frac{\mathrm{d}F(\mathbf{p})}{\mathrm{d}\mathbf{p}} \tag{3.20}$$

The level of safety is higher than optimal if the marginal safety costs are larger than the marginal loss reduction achieved and vice versa. Equation 3.20 is valid for continuous decision variables. Fire safety design often requires discrete decisions, e.g. whether to invest into a sprinkler system or not. In this case, the efficiency of a safety measure is judged using the following inequality:

$$\Delta(C + A + I) \leq -\Delta F \tag{3.21}$$

Where ΔX refers to the change in the cost components X implied by the decision. Combining different safety measures is of course also possible; in this case, Equation 3.19 should be used to judge which measure or combination of measures minimizes the total costs $T(\mathbf{p})$. The evaluation of costs and benefits has to account for dependencies between different measures. This may be achieved by the following iterative procedure (see also Schubert, 2009): First, all measures that fulfil Equation 3.21 are sorted according to their cost-efficiency. After selecting the most efficient measure, the remaining measures are re-evaluated conditional on the first measure being implemented. This procedure is repeated until no efficient measures are left.

The monetary fire losses $F(\mathbf{p})$ may be broadly classified into two groups (see also Chapter 7 / Case study 3): Losses resulting from small fires ("minor losses") and those resulting from "major" fire events. The term "major losses" can be defined differently depending on the context, e.g. relating to fire spread beyond the room of fire origin (as in Section 7.2) or to fully developed fires with or without flashover. The latter definition is used in the following. The mean value of the loss size probability distribution is dominated by large losses, see e.g. Fontana et al. (1999) or Fischer et al. (2012b). It is thus a reasonable simplification to neglect minor losses in the optimization. The expected present value of monetary fire losses is then estimated as follows:

$$F(\mathbf{p}) = \lambda_I(\mathbf{p})P_{F|I}(\mathbf{p})\left[\left(1 - P_{f|F}(\mathbf{p})\right)G(\mathbf{p}) + P_{f|F}(\mathbf{p})H(\mathbf{p})\right]\frac{1}{\gamma} \tag{3.22}$$

Here, $\lambda_I(\mathbf{p})$ denotes the rate of fire ignition (which may reasonably be modelled using a homogeneous Poisson process) and $P_{F|I}(\mathbf{p})$ the conditional probability of a fully developed fire given ignition. The term in square brackets refers to the expected monetary fire loss given a major fire. Two different situations are distinguished: The fire may spread without causing structural collapse and lead to an (expected) loss $G(\mathbf{p})$, which will typically be smaller than the loss that is assumed to occur in case of structural failure, $H(\mathbf{p})$. Following the same approach as in Section 3.3.1, it may be assumed that $H(\mathbf{p}) = C(\mathbf{p}) + H$, with H denoting the costs that accrue in case of failure in addition to the costs of reconstructing the building, $C(\mathbf{p})$. The probability of structural failure $P_{f|F}(\mathbf{p})$ is defined conditional on a fully developed fire. Equation 3.22 may be used to optimize fire safety using different types of risk reduction measures summarized in the vector of decision variables $\mathbf{p}$. The effect of individual measures is discussed in Section 3.4.3.

3.4.2   Fire safety targets based on the LQI acceptance criterion

For the application of the LQI acceptance criterion to the general fire safety problem, first the marginal life saving costs and benefits have to be defined in a similar way as for the monetary optimization. The monetary fire losses are reduced by increasing safety and therefore have to be neglected when assessing the marginal life saving costs, see Section 2.3.9 for discussion. Using these assumptions in Equation 3.11 leads to the following acceptance criterion:

$$\frac{\mathrm{d}\left(C(\mathbf{p}) + A(\mathbf{p}) + I(\mathbf{p})\right)}{\mathrm{d}\mathbf{p}} \geq -\frac{1}{\gamma_S} \cdot \frac{\mathrm{d}N(\mathbf{p})}{\mathrm{d}\mathbf{p}} \cdot \frac{g}{q} J_x \tag{3.23}$$

Here, $\mathbf{p}$ is a vector of decision variables having an effect on risk to life in case of fire. Equation 3.23 again refers to continuous decision variables. Turning the inequality sign leads to the following efficiency criterion for discrete fire safety measures(see Section 2.3.4 for discussion):

$$\Delta(C + A + I) \leq -\frac{1}{\gamma_S} \cdot \Delta N \cdot \frac{g}{q} J_x \tag{3.24}$$

Interactions between different safety measures have to be accounted for when evaluating combinations of measures, see Section 3.4.1 and Schubert (2009). An acceptable solution is achieved when all available safety measures that fulfil the efficiency criterion in Equation 3.24 have been implemented. It is of course also possible to go even further, i.e. to invest into safety measures judged to be inefficient from a life saving point of view. This may be reasonable e.g. if additional investments are efficient from a monetary optimization point of view.

The expected number of fatalities $N(\mathbf{p})$ per year may under certain conditions be estimated using the following simplified formula:

$$N(\mathbf{p}) = \lambda_I(\mathbf{p})N_O\left[(1 - P_E(\mathbf{p}))(1 - P_R(\mathbf{p}))\right] \tag{3.25}$$

Here, $\lambda_I(\mathbf{p})$ denotes the rate of fire ignition, N_O the number of occupants and the term in square brackets estimates the share of occupants that die in case of a fire. For a single person, the probability of dying is estimated from the probability of safe egress, $P_E(\mathbf{p})$, and the probability of being rescued by the fire brigade if safe egress is not possible, $P_R(\mathbf{p})$. The approach is also valid for $N_O > 1$ if it can be assumed that the egress and rescue probabilities are the same for each person and that the survival of different occupants can be modelled independent of each other (see e.g. Hasofer and Beck, 2000). A more complex modelling is required where this is not the case, e.g. in buildings with high occupant density (see Section 5.3.1 for discussion).

Interaction with monetary optimization

Whether the monetary optimum is also acceptable from a societal (life saving) point of view is checked by inserting the optimality criterion (Equation 3.20) into the LQI acceptance criterion in Equation 3.23:

$$-\frac{\mathrm{d}F(\mathbf{p})}{\mathrm{d}\mathbf{p}} \geq -\frac{1}{\gamma_S} \cdot \frac{\mathrm{d}N(\mathbf{p})}{\mathrm{d}\mathbf{p}} \cdot \frac{g}{q} J_x \tag{3.26}$$

The interpretation is straightforward: The LQI criterion will not become active if the marginal reduction of the monetary fire losses achieved by the safety measures is larger than the SWTP for the marginal life saving benefit.

3.4.3 Application to different types of safety measures

The acceptance and optimality criteria in Section 3.4.1 and 3.4.2 are able to account for a variety of different risk reduction measures. The effect of safety investments on the monetary and/or human risk depends on how a safety measure influences the fire process, see Figure 3.8.
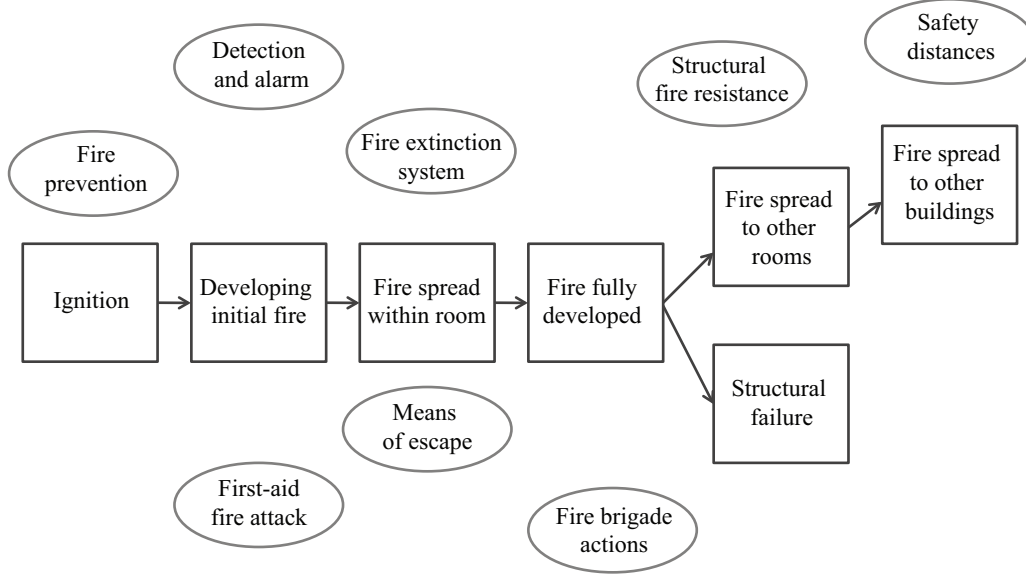
Fig. 3.8: Influence of different safety measures in the fire process.

Defining quantitative performance criteria for the whole system of building fire safety would require a consideration of all measures affecting the financial and human risk in fire events. Due to the high number of possible decision variables and numerous interactions, such an optimization would be a formidable task. However, a simplified approach can be followed by applying the framework developed in Section 3.4.1 and 3.4.2 to the definition of safety targets at a lower, more operative level. A categorization of different safety measures according to their effect on the fire process can be found in Table 3.4. In a simplified optimization approach, probabilistic performance criteria may be defined for each category of measures separately.

Measures preventing fire occurrence

An obvious way to reduce fire risk is to reduce fire occurrence rate λ_I. Many of these measures have no effect on the fire development once an ignition has occurred. Assuming that the fire risk conditional on the ignition is known, Equation 3.22 simplifies to:

$$F(\mathbf{p}) = \frac{\lambda_I(\mathbf{p})}{\gamma} R_{M|I} \tag{3.27}$$

The conditional risk $R_{M|I}$ is defined as the expected monetary consequences in case of fire and may be quantified using simplified models, e.g. on the basis of statistical data (see Section 4.2.3). A similar formula is achieved for the risk to human life by simplifying Equation 3.25 based on a rough estimate for the expected number of fatalities given fire, $R_{H|I}$:

$$N(\mathbf{p}) = \lambda_I(\mathbf{p}) R_{H|I} \tag{3.28}$$

Tab. 3.4: Overview of the effect of different types of fire safety measures on the monetary fire losses $F(\mathbf{p})$ (Equation 3.22) and the fatality risk $N(\mathbf{p})$ (Equation 3.25).

Goal of safety measure	Effect on $F(\mathbf{p})$	Examples
Prevent fire occurrence	Reduce $\lambda_I(\mathbf{p})$	RCD switches, lightning protection, consumer product safety
Early fire extinction	Reduce $P_{F\|I}(\mathbf{p})$	Fire extinguishers, sprinklers, smoke alarms, fire brigade
Limit fire spread	Reduce $G(\mathbf{p})$	Fire compartments, fire doors, shutters, safety distances
Structural fire safety	Reduce $P_{f\|F}(\mathbf{p})$	Gypsum board insulation, intumescent paint, increased cross-sections

Goal of safety measure	Effect on $N(\mathbf{p})$	Examples
Safe egress of occupants	Increase $P_E(\mathbf{p})$	Smoke alarms, egress route design, smoke management
Rescue of trapped occupants	Increase $P_R(\mathbf{p})$	Fire alarm systems, fire service access routes, structural fire safety

Measures for early fire extinction

Given that a fire occurs, it may be extinguished already in an early stage of its development. Measures for early fire extinction reduce the probability of a fully developed fire, $P_{F\|I}$. With $R_{M\|F}$ denoting the monetary fire risk in a major fire event, Equation 3.22 simplifies to:

$$F(\mathbf{p}) = \frac{\lambda_I P_{F\|I}(\mathbf{p})}{\gamma} R_{M\|F} \tag{3.29}$$

Depending on the decision problem, it may be sufficient to regard only one part of the expected losses in case of a fully developed fire, see e.g. Chapter 6 (Case Study 2) for the optimization of structural fire safety. Some measures may have an effect also in other parts of the fire process, e.g. a sprinkler that reduces the fire intensity even if it was not successful in extinguishing the fire. This can be accounted for by introducing a dependency on $\mathbf{p}$ also in other parts of Equation 3.22.

The probability of a fully developed fire is not the most relevant quantity for estimating risk to human life in case of a fire. Nevertheless, there is of course a positive effect of early fire extinction on the safety of the building occupants. This may be accounted for by modelling the influence of the safety measure on the probability of safe egress or, alternatively, by introducing a probability for a "large" fire event also in Equation 3.25. These major fires should, however, be defined in a different way pointing to the life-threatening effects of fire events.

Measures limiting fire spread

The main effect of this class of safety measures is to limit the losses in major fire events, e.g. fully developed fires in the room of fire origin. In Equation 3.22 this is summarized in the variable $G(\mathbf{p})$ describing the expected loss in a major fire event. For simplicity, it may be assumed that the probability of structural failure $P_{f|F}$ equals zero, which leads to the following formulation for the expected present value of fire losses:

$$F(\mathbf{p}) = \frac{\lambda_I P_{F|I}}{\gamma} G(\mathbf{p}) \tag{3.30}$$

Measures preventing smoke spread require a different modelling approach. Their evaluation with respect to life safety is discussed below.

Measures for structural fire safety

Investments into structural fire safety reduce the probability of structural failure $P_{f|F}$ in case of a fire event or, more precisely, given a fully developed fire. The effect on the monetary fire consequences $F(\mathbf{p})$ may be assessed using the following simplified formulation:

$$F(\mathbf{p}) = \frac{\lambda_I P_{F|I}}{\gamma} P_{f|F}(\mathbf{p}) \left(C(\mathbf{p}) + H \right) \tag{3.31}$$

Equation 3.31 assumes that the monetary loss in fires without structural failure is negligible ($G(\mathbf{p}) \to 0$). This leads to an overestimation of the effect of structural fire safety, but simplifies the analysis to a large extent. A more complex model could be developed based on Equation 3.22. The analysis in Chapter 6 (Case Study 2) is based on Equation 3.31.

The life saving effect of structural fire safety results mainly from an improved situation for the fire brigade rescue actions (see "Measures facilitating rescue of trapped occupants"). An effect on the egress probability P_E can be expected only in special cases (e.g. high-rise buildings or hospitals with very long total evacuation time).

Measures allowing for safe egress of occupants

Most life saving investments in fire safety design aim at increasing the probability of safe egress P_E for the occupants of a building. When assessing the effect of these measures on risk to life due to fire, it is often assumed that the safe egress of a person is possible if the Available Safe Egress Time (T_{ASET}) is larger than the Required Safe Egress Time (T_{RSET}):

$$P_E(\mathbf{p}) = P\left[T_{ASET}(\mathbf{p}) - T_{RSET}(\mathbf{p}) \geq 0 \right] \tag{3.32}$$

The decision variables in $\mathbf{p}$ may affect the distribution of T_{ASET} (e.g. smoke control), T_{RSET} (e.g. egress route design) ore both. It is important to note that it is not sufficient to assess the probability of any death: For decision-making in consistency with the marginal life saving costs principle, the expected number of deaths has to be estimated. In Chapter 5 (Case Study 1), the mortality risk due to fire is approximated by the following simplified equation:

$$N(\mathbf{p}) = \lambda_I N_O \left(1 - P_E(\mathbf{p}) \right) \tag{3.33}$$

Equation 3.33 is a reasonable approximation if the evacuation of individual occupants can be assumed to be independent of each other, i.e. only in buildings with low occupant densities. Otherwise P_E should be interpreted as the expected share of all occupants that will escape successfully in case of a fire event. Another assumption is that the chances of being rescued if the evacuation fails are zero, i.e. $P_R = 0$. This leads to a slight overestimation regarding the effect of the safety measures summarized in $\mathbf{p}$. A more accurate estimate is achieved using Equation 3.25.

Measures facilitating rescue of trapped occupants

Occupants that were not successful in escaping on their own ("trapped" occupants) may still survive if they can be rescued by the fire brigade. In order to estimate the effect of measures improving the chances of being rescued, Equation 3.25 is used as follows:

$$N(\mathbf{p}) = \lambda_I N_O \left[(1 - P_E) \left(1 - P_R(\mathbf{p}) \right) \right] \tag{3.34}$$

Some measures affect both evacuation and rescue, e.g. a fire alarm system that notifies the occupants and issues an alarm to the fire brigade. In this case also the escape probability $P_E(\mathbf{p})$ in Equation 3.34 has to be modelled as a function of $\mathbf{p}$ unless it may be assumed that one effect is predominant.

Chapter 4

Absolute risk assessment for societal decision support

Parts of this chapter are taken from a paper on the calibration of fire risk models with statistical data, see Fischer et al. (2013a). The data analysis section is based on Fischer et al. (2012b).

4.1 The need for fire risk assessment in absolute terms

Risk-based decision-making for optimal building fire safety requires the use of comprehensive risk models to evaluate the effect of risk reduction measures on the monetary and human consequences of fire. The development of such models is not a simple task and requires a thorough understanding of different phenomena such as fire dynamics, structural response or human behaviour, which are dealt with by different fields of science. In addition, many of the underlying phenomena are not yet well understood even by specialists in the relevant fields and the available data is generally not sufficient to work with a pure empirical approach. Thus, the risk analyst is forced to fill the gaps with expert judgement and simplifying assumptions.

In practice, fire risk methods are mainly used for relative, comparative risk assessments (see Section 3.2.2 for discussion). In this context, fire safety engineers often rely on conservative assumptions to solve the problem of missing data and/or scientific knowledge. Yet even a model without such intentional conservatisms will always contain a certain bias resulting from the lack of knowledge discussed above. To improve the available methods towards an absolute risk assessment, the modeller should aim at reducing this bias. Especially in the context of optimization, when comparing the uncertain benefit of a safety measure to its cost, the expected loss of property or life has to be assessed with as little bias as possible.

In this chapter it is discussed how fire risk models may be improved to allow for a more accurate, unbiased risk assessment. First, the use of Swiss fire insurance data for risk assessment purposes is explored. While data-driven methods provide a fairly unbiased estimate derived from observations in real fire events, they are usually not detailed enough for decision-making and have to be complemented by engineering methods. An important goal of the present chapter is to show how both sources of information, engineering knowledge and statistical data, may

be combined by calibrating engineering risk models to data from real fire events. To facilitate calibration with data collected by e.g. fire brigades or insurance companies, a risk model has to be applicable at the level of non-homogeneous building portfolios. Also for societal decision-making, it is important to consistently deal with variability in the portfolio of objects affected by a decision. Therefore, the application of engineering models at portfolio level is briefly discussed before turning to the problem of model calibration.

4.2 Deriving simple risk estimates based on statistical data

A simple way to build fire risk models facilitating absolute risk assessment is to rely on statistical data from real fire events. In general, the usefulness of such models is strongly restricted by the information content of the data. Nevertheless, data-driven models may be fruitfully applied e.g. to provide simplified conditional risk estimates required for the definition of quantitative safety targets for performance based codes (Section 3.4.3). In addition, data-based models can be used as benchmark or sub-models in engineering risk assessment.

A classical application for data-based models is to predict future losses in an insurance portfolio. The portfolio fire loss is estimated as the sum of N_L independent fire losses C_i. Models for the loss number N_L and the loss size C_i are typically developed at portfolio level. However, as can be shown by data analysis, both the ignition frequency and the expected fire loss given ignition grow with the size of the buildings. Therefore, here the probability of fire occurrence and the financial loss are defined at the level of the individual building. The two models are then assumed to be independent conditional on the building characteristics.

4.2.1 Analysis of Swiss fire insurance data

As a first step in data-based modelling, the information content of the available data should be explored, see e.g. Fischer et al. (2012b). In 19 of the 26 cantons (states) of Switzerland, building fire insurance is obligatory and provided by public insurance monopolies. The loss data collected by the public insurers includes also small losses, as no excess (deductible) is borne by the policy holders. Three different data sources were used for the analysis:

— *VKF data*: Portfolio and loss data 2000-2007 collected by VKF and IRV, two joint organisations of the 19 public insurance companies.

— *AGV data*: Portfolio and loss data 1999-2008 collected by AGV, the public building insurance company of Aargau, one of the Swiss cantons.

— *ECA data*: Loss data 1995-2009 collected by ECA Vaud, the building insurance company of Vaud, another Swiss canton.

The first source provides the largest sample size, as the data from all 19 cantons with public building fire insurance are summarized in the VKF data. VKF also collects information on fire fatalities in all Swiss cantons. However, the VKF data contain only little information on each

building and fire event and it is not possible to link portfolio and loss data. In addition, it may be questioned whether the data from different cantons are comparable, see Fischer et al. (2012b) for discussion. Therefore, a part of the data analysis is based on the AGV data, which contain more information and allow for a link between portfolio and loss data. Finally, the ECA data are used to analyse losses to contents, as in Vaud also the insurance of contents is mandatory; The VKF and AGV data only provide information on losses to the building structure.

Fischer et al. (2012b) provide a comprehensive analysis of Swiss fire loss data; only some key results relevant for model building are recapped in the following.

Fire occurrence

Figure 4.1a shows that the fire rate grows with the volume of the buildings. The dependency is, however, not linear as the ignition frequency per m^3 decreases with the building size (Figure 4.1b). Plotting the fire rate as a function of the building's insured value leads to similar results. The fire rate furthermore depends on the building's occupancy as can be seen from the comparison between residential and agricultural buildings in Figure 4.1. It may be assumed that the presence of people is an important factor, as many fires are caused by the building occupants.
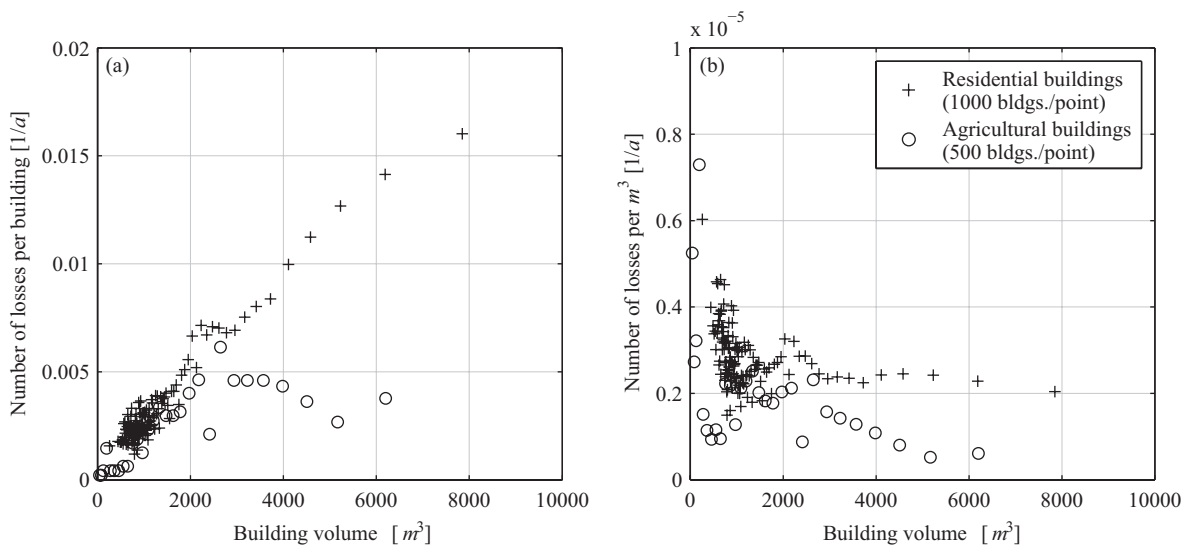
Fig. 4.1: Fire occurrence rate per building (a) and per m^3 (b) as a function of the building's volume and occupancy class (AGV data, excluding lightning damages).

Building fire loss

The empirical distribution of the building fire loss given fire occurrence is shown in Figure 4.2a for different occupancy classes. In Figure 4.2b, the fire loss C is related to the insured value V. The vertical line at $10^0 = 1$ depicts total losses resulting from fires that destroyed the building completely. The total loss probability is considerable especially for agricultural buildings.

At a first glance it seems that the loss ratio C/V is more applicable to compare the fire risk in different buildings than the absolute fire loss C. However, most fires are extinguished
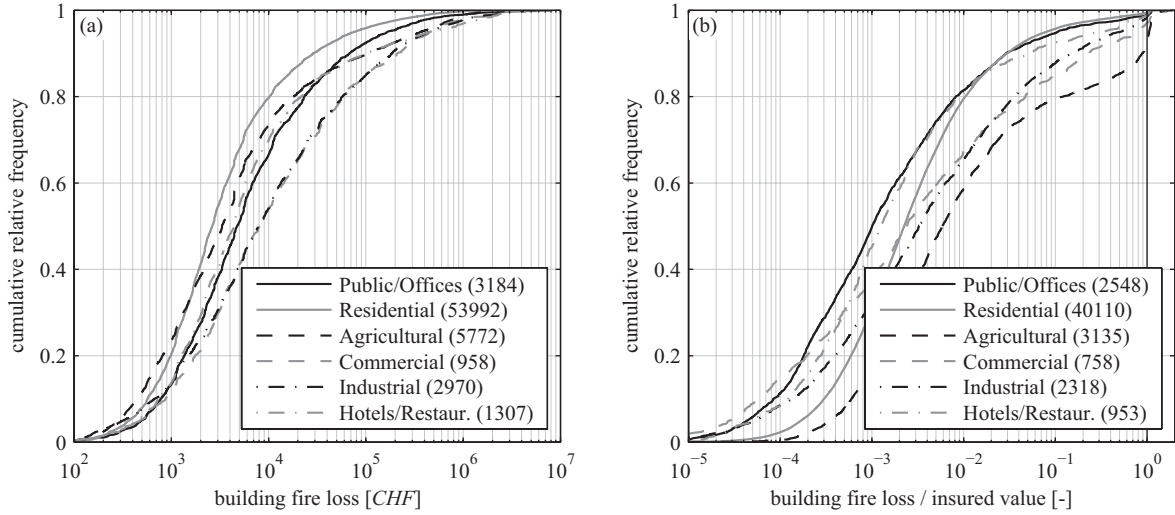
Fig. 4.2: Empirical cumulative distribution function of the building fire loss (a) and the fire loss divided by the insured value (b) in different occupancy classes (VKF data excluding lightning damages, sample size in brackets).

before leaving the room of fire origin. In these cases, the insured value has no effect on the fire loss. Only for large fires, it becomes relevant as an upper bound, which may be exceeded only by clean up costs. This is seen in Figure 4.3a: Below the diagonal, the fire loss C seems to be largely independent of the insured value V. The scatterplot for the loss ratio C/V in Figure 4.3b does, however, show a strong dependency of the insured value.
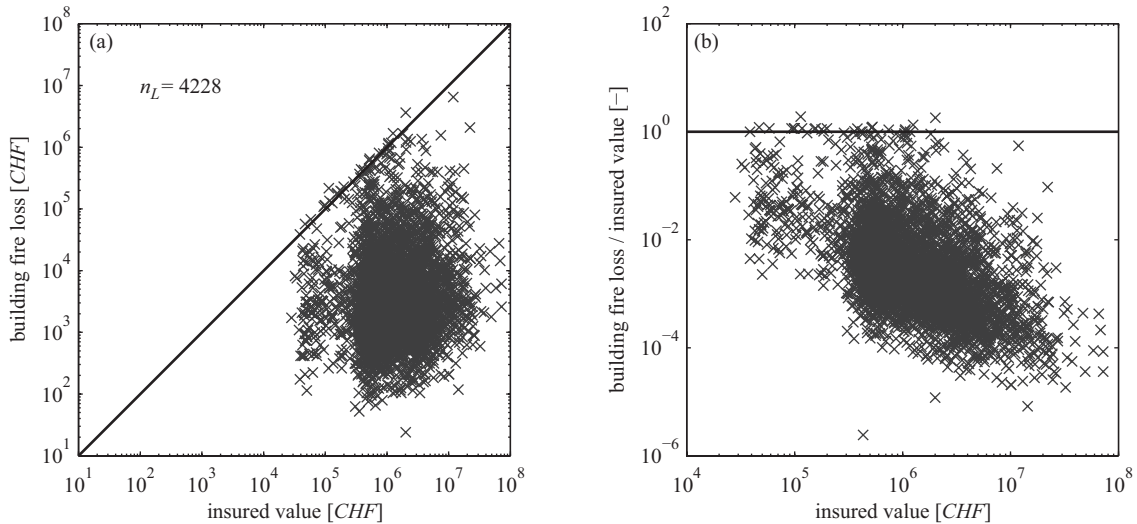
Fig. 4.3: Scatterplots for the dependency between the insured value V and the building fire loss C (a) or the loss ratio C/V (b) in residential buildings (VKF data, excluding lightning damages).

From Figure 4.3 it may be concluded that the insured value of a building has an effect on large fires only. Total losses are rare, but have a large effect on the expected loss and the sum of all fire losses in a building portfolio. This is illustrated in Figure 4.4 where it is shown what

share of the portfolio loss (on the y-axis) results from e.g. the largest 10% of the losses (on the x-axis). The corresponding loss amount is indicated in three individual points. It is seen that only a small percentage of fires (0.3%) leads to losses larger than 1 Mio. CHF. Nevertheless, almost 30% of the total loss in the building portfolio results from these fire events.
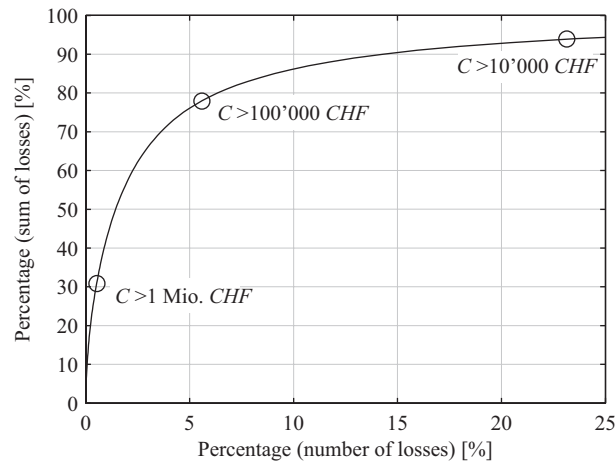
Fig. 4.4: Contribution of losses with decreasing fire loss amount C to the sum of fire losses in a portfolio of buildings (VKF data, excluding auxiliary buildings and lightning damages).

Losses to contents

So far, only losses resulting from damages at the insured buildings have been discussed. Based on a data set from the canton Vaud, it is possible to investigate the relation between building and content losses in the same fire event. The data base is described in detail in Fischer et al. (2012b); here, only the data referring to events resulting in both types of losses is analysed. In Figure 4.5, the contents loss is plotted against the building fire loss. Most data points are below the diagonal, which leads to the conclusion that the insured loss to contents is often smaller than the building fire loss. It should, however, be noted that the scatter of the points is strongly reduced visually by the logarithmic scale of the axes; in some fire events the contents loss may be much higher than the building fire loss. Figure 4.5 is based on data from all occupancy classes; the results are similar when regarding individual groups of buildings.

Fire fatalities

Finally, also the data from fire events with fatalities was analysed. A full description of the data base is found in Fischer et al. (2012b). Figure 4.6a illustrates the fatality risk for different age groups. Data without information on the age of the fire fatality were excluded from the analysis. Therefore, the results cannot be interpreted directly as probabilities. Nevertheless, the graph clearly shows the high fatality risk for the elderly, especially for persons older than 80 years. Men are more likely to die due to fire than women, which may only partly be explained by fire risks at the workplace; apparently there are behavioural aspects that influence the risk.
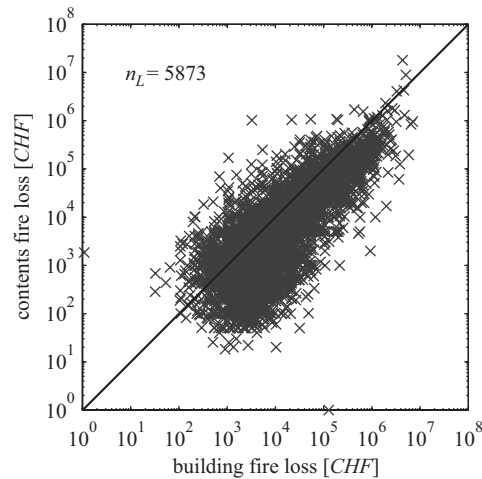
Fig. 4.5: Losses to contents and building fire loss in the same fire event (ECA data without lightning damages, all occupancies).
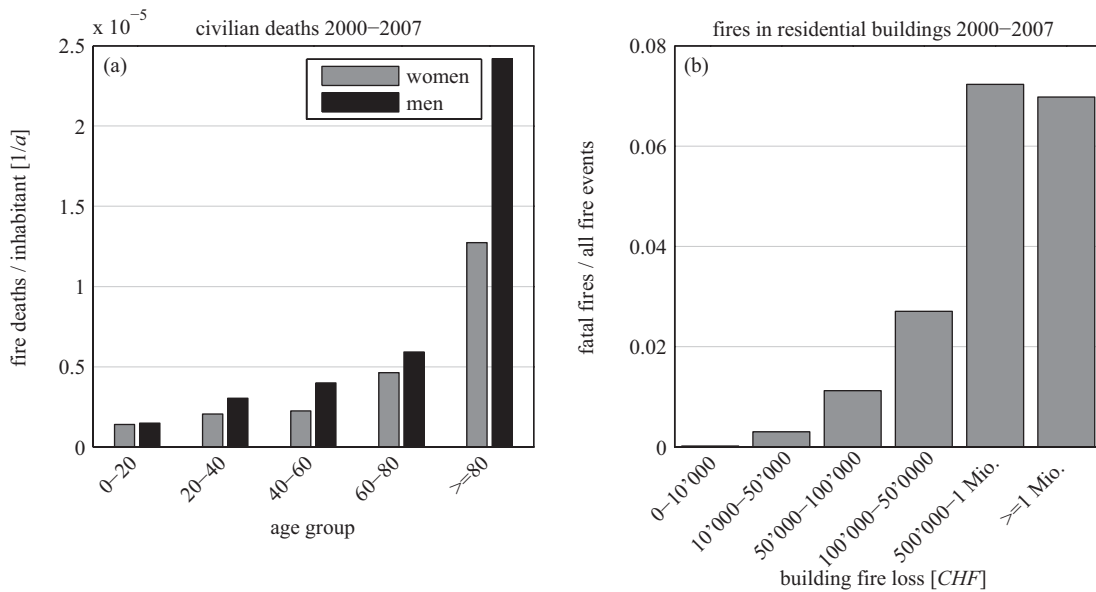
Fig. 4.6: Fatal fire events (VKF data, 8-year average): Fatality rate in different age groups (a) and (b) as a function of the fire loss in residential fires.

The effect of the fire severity on the fatality risk in residential buildings (where more than 80% of the fatalities occurred) is illustrated in Figure 4.6b. Not surprisingly, human consequences are highest if also the monetary losses in a fire event are high. It is, however, also possible to observe a fatality in a fire event without monetary losses. These fatalities have been excluded for Figure 4.6b, as it is not clear from the data base whether the building fire loss was zero or unknown. The results can thus again only serve as a rough approximation to the real probability of a fatality given a fire event.

4.2.2 Modelling fire occurrence

As has been shown in Figure 4.1, the fire occurrence rate may be modelled as a function of the size of a building. In the following this is done using the AGV data, as these allow determining the number of fires for each building individually by establishing a link between portfolio and loss data. When modelling fire occurrence based on data, it has to be clear how the event "fire" is defined. The AGV insurance data include information on all fires leading to an insured loss, i.e. a loss resulting from damages to the building structure. When modelling fire occurrence based on insurance data, it is important to choose a corresponding model for the loss given a fire event. In Section 4.2.3, this is achieved by using the same data base also for modelling the fire consequences. For comparison with international data (see e.g. Tillander, 2004), one would have to know which fires were attended by the fire brigade. Unfortunately, this information is currently not available. However, an order of magnitude for the difference between alternative data sources may be derived from Figure 4.2a showing the share of losses smaller than a certain loss threshold, e.g. 10'000 CHF.

Modelling fire occurrence as a function of the size of a building or compartment

The probability of fire occurrence is often modelled as a function of the floor area A_f of the building or compartment considered in the analysis. The basic idea is that the number of ignition sources grows with the size of a building. A functional form for this dependency can be derived based on a simple statistical approach proposed by Ramachandran (1980):

$$P_t(A_f = a_f) = \frac{n_L}{n_P} \frac{f_{A_f}^L(a_f)}{f_{A_f}^P(a_f)} \tag{4.1}$$

Here, $P_t(A_f = a_f)$ denotes the probability of fire occurrence in a time period t (e.g. 1 year) for a building with floor area $A_f = a_f$. The mean ignition frequency is estimated from the number of fires n_L (loss data) in the period t divided by the number of buildings n_P (portfolio data). In order to model the dependency of A_f, the formula furthermore contains the probability density function of the total floor area in the whole building portfolio ($f_{A_f}^P(a_f)$) and in all buildings where a fire occurred ($f_{A_f}^L(a_f)$). The two distributions $f_{A_f}^P(a_f)$ and $f_{A_f}^L(a_f)$ have different moments. To give an example, the mean floor area in the loss data is expected to be larger than the mean floor area in the portfolio data due to the increased ignition frequency in large buildings. Ramachandran (1980) suggests to use the Pareto or the Lognormal distribution for the floor area in both the portfolio and the loss data. He showed that based on the Pareto assumption, Equation 4.1 simplifies as follows:

$$P_t(A_f) = k_1 A_f^{k_2} \tag{4.2}$$

For consistency in terms of units, the floor area A_f (in $[m^2]$) has to be normalized by a reference area of $1m^2$. The constants k_1 and k_2 may be calculated from the distribution parameters of the two Pareto distributions, see Ramachandran (1980). The same power law

can also be derived when assuming two Lognormal distributions for $f_{A_f}^P(a_f)$ and $f_{A_f}^L(a_f)$. This does, however, require the additional assumption that the dispersion parameter (the standard deviation of $\ln(A_f)$) of the two distributions is identical.

When applying Ramachandran's approach to Finnish data, Tillander and Keski-Rahkonen (2002) observed that the total floor area distribution cannot be described using a Pareto or Lognormal distribution. This observation was confirmed by analysing Swiss data, see Annex A of Fischer et al. (2012b). Thus, the assumptions made by Ramachandran (1980) are not applicable. To resolve this problem, Tillander and Keski-Rahkonen (2002) proposed an expanded model, the so-called Generalized Barrois Model for the ignition probability:

$$P_t(A_f) = k_1 A_f^{k_2} + k_3 A_f^{k_4} \tag{4.3}$$

At a first glance, the model in Equation 4.3 seems superior over the one in Equation 4.2. However, we have to bear in mind that modelling the ignition frequency only based on the size of a building is already fairly simplistic. This cannot be changed by choosing a more accurate functional form for the dependency between the floor area A_f and the ignition frequency. The statistical uncertainty grows with the number of model parameters to be estimated from data. Therefore, it is proposed to use the simple model by Ramachandran (1980) (Equation 4.2) to model the ignition frequency.

The Swiss data do not contain information on the floor area. However, the basic concept in Equation 4.1 can also be applied using the volume Vol in m^3 or the insured value V in CHF as an indicator for the size of a building.

Generalized Linear Model (GLM) and Maximum Likelihood estimation

In a specific building, fire is a rare event. It is thus reasonable to model fire ignition with a (homogeneous) Poisson process. The number of fires N_L in a period t then follows a Poisson distribution with parameter λt. As discussed above, the ignition frequency λ may be modelled as a function of the size of a building. Replacing the floor area A_f in Equation 4.2 by the building's volume Vol leads to the following model (note that the volume Vol_i again has to be normalized by $1 m^3$):

$$N_{L,i} \sim \text{Poisson}\left(\lambda_i t_i\right); \quad \lambda_i = k_1 \left(Vol_i\right)^{k_2} \tag{4.4}$$

The index i refers to a specific building. With data containing information on $N_{L,i}$, t_i and Vol_i for a large number of buildings, the model parameters k_1 and k_2 can be estimated based on a Poisson regression with λ_i as a hidden variable. The form of the regression is linearised by applying the natural logarithm:

$$\ln\left(\lambda_i\right) = \ln(k_1) + k_2 \ln\left(Vol_i\right) = \alpha + \beta \ln\left(Vol_i\right) \tag{4.5}$$

With $\alpha = \ln(k_1)$ and $\beta = k_2$. Thus, Equation 4.4 represents a Generalized Linear Model (GLM) with Poisson distributed data and logarithmic link function. Using the Maximum Likelihood method for estimating the model parameters α and β allows to quantify also the statistical

uncertainty in the parameter estimation. An introduction to this method can be found e.g. in Rychlik and Rydén (2006). The application is easiest if it is possible to link portfolio and loss data in order to determine the number of fires for each building individually, see Appendix B. It is, however, easy to adapt the method for data without that link or even for data containing only information on buildings where a fire has been observed, see Rydén and Rychlik (2006).

The model parameters estimated from the AGV data are summarized in Table 4.1. Besides the Maximum Likelihood estimates α^* and β^*, Table 4.1 also contains information on the statistical uncertainty, see Appendix B. Lightning damages were excluded from the analysis as they comprise a large number of so-called "indirect" lightning strikes, damages at electrical devices resulting from lightning currents in the electricity network; the number of fire events resulting from "direct" lightning strikes is relatively small. Only occupancy classes where the data base is relatively large are shown in Table 4.1; see Fischer et al. (2012b) for more groups.

Figure 4.7 illustrates the estimation results for residential buildings. The regression model given by the Maximum Likelihood parameters α^* and β^* is represented by the solid line. A 95% confidence interval for $\lambda \mid Vol$ is found between the dashed lines, illustrating statistical uncertainty in the parameter estimates. For comparison with the data, the graph also contains the observed fire rate for buildings with different volumes. Each point has been calculated based on data for 1000 buildings. Due to the limited sample size, the points scatter around the real occurrence rate. The dotted lines show a 95% confidence interval for the observed ignition frequency in 1000 buildings under the Maximum Likelihood model assumption (i.e. using α^* and β^*). The dotted lines thus illustrate aleatoric uncertainty only, neglecting epistemic (statistical) uncertainty in the estimation of the regression parameters.
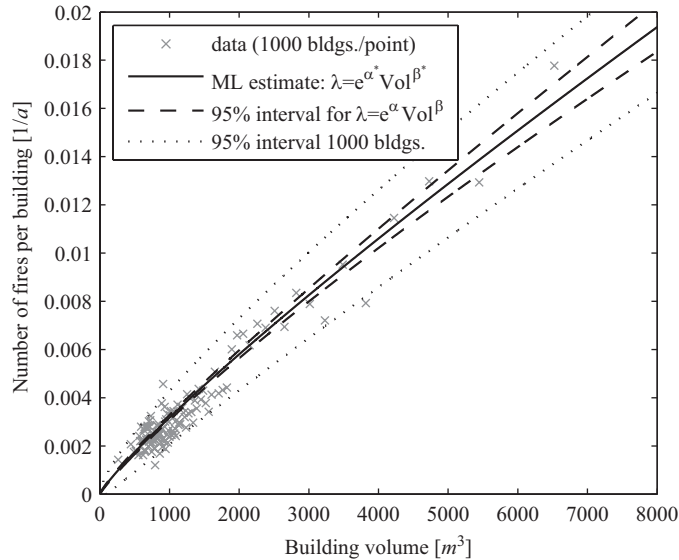
Fig. 4.7: Comparison of the Generalized Linear Model (GLM) for fire occurrence (Equation 4.4) with AGV data for residential buildings.

The fire occurrence model for different occupancies is illustrated in Figure 4.8. The curves are based on the Maximum Likelihood parameters for the GLM summarized in Table 4.1. For

residential buildings, the dependency between the building's volume and the fire rate is nearly linear (β^* close to one). This may be explained by the fact that larger residential buildings are usually apartment blocks where the number of ignition sources (and the number of building occupants) is roughly proportional to the number of housing units.
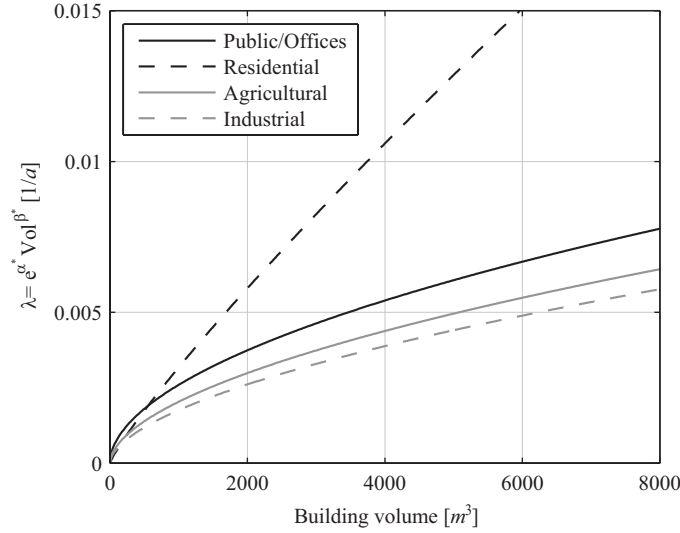
Fig. 4.8: Maximum Likelihood estimates for the fire occurrence rate λ based on AGV data as a function of the building's volume in different occupancies.

Tab. 4.1: Estimated parameters of the Generalized Linear Model (GLM, Equation 4.4) and the Generalized Linear Mixed Model (GLMM, Equation 4.6) in different occupancies based on AGV data (sample size in brackets).

	α^*	β^*	σ_ϵ^*	$\text{Var}[\alpha]$	$\text{Var}[\beta]$	$\text{Var}[\sigma_\epsilon]$	$\rho_{\alpha\beta}$	$\rho_{\alpha\sigma_\epsilon}$	$\rho_{\beta\sigma_\epsilon}$
Public/Offices ($n_P = 6'864$, $n_L = 319$)									
GLM	-9.60	0.527	-	0.1271	0.0017	-	-0.988	-	-
GLMM	-10.03	0.525	0.939	0.1565	0.0019	0.0173	-0.941	-0.325	0.025
Residential ($n_P = 125'763$, $n_L = 4'799$)									
GLM	-11.76	0.870	-	0.0136	0.0002	-	-0.992	-	-
GLMM	-12.03	0.871	0.725	0.0166	0.0003	0.0020	-0.956	-0.266	0.020
Agricultural ($n_P = 15'070$, $n_L = 405$)									
GLM	-10.02	0.554	-	0.1457	0.0024	-	-0.991	-	-
GLMM	-10.51	0.562	0.906	0.1799	0.0025	0.0241	-0.938	-0.392	0.077
Industrial ($n_P = 11'428$, $n_L = 378$)									
GLM	-10.30	0.572	-	0.0749	0.0009	-	-0.982	-	-
GLMM	-10.90	0.587	0.963	0.1132	0.0011	0.0162	-0.925	-0.481	0.156

Generalized Linear Mixed Model (GLMM) for fire occurrence

A strong assumption of the Poisson regression model (Equation 4.4) is the equality of mean and variance, $\text{Var}[N_{L,i}] = \text{E}[N_{L,i}] = \lambda_i t_i$. As the ignition frequency is modelled as a function of only one covariate, the volume Vol, one could expect that the data set should be overdispersed, i.e. $\text{Var}[N_{L,i}] > \text{E}[N_{L,i}]$. However, informal checks based on residual plots (see Gelman and Hill, 2008) or Negative Binomial models fitted to the data (see Cameron and Trivedi, 1998) indicated no or only modest overdispersion. Another possibility to account for a variance different from the mean is to use a hierarchical model allowing for inter-subject-variability. These models, also called Generalized Linear Mixed Models (GLMM), have a very consistent and intuitive uncertainty structure and are thus valuable also in a context where overdispersion is not a big issue. The structure of the model is as follows:

$$N_{L,i} \sim \text{Poisson}(\lambda_i t_i)$$
$$\ln(\lambda_i) = \alpha + \beta \ln(Vol_i) + \epsilon_i \qquad (4.6)$$
$$\epsilon_i \sim \text{Normal}(0, \sigma_\epsilon)$$

The error term ϵ_i is subject-specific while α and β are estimated for all buildings as before. The error standard deviation σ_ϵ can be seen as a third population parameter quantifying to what extent inter-subject variability that cannot be explained by variation in the volume Vol_i is found in the data. Different fire rates λ_i in two buildings with the same volume and occupancy may result e.g. from known or unknown variables not considered in the model. The model structure of the simple Poisson regression (GLM) and the "mixed" model (GLMM) is illustrated in Figure 4.9. The "box" in both models draws a line between subject-specific and population-specific variables: The model parameters α, β and σ_ϵ are estimated at portfolio level whereas the fire rate λ_i is defined at the level of the individual building.
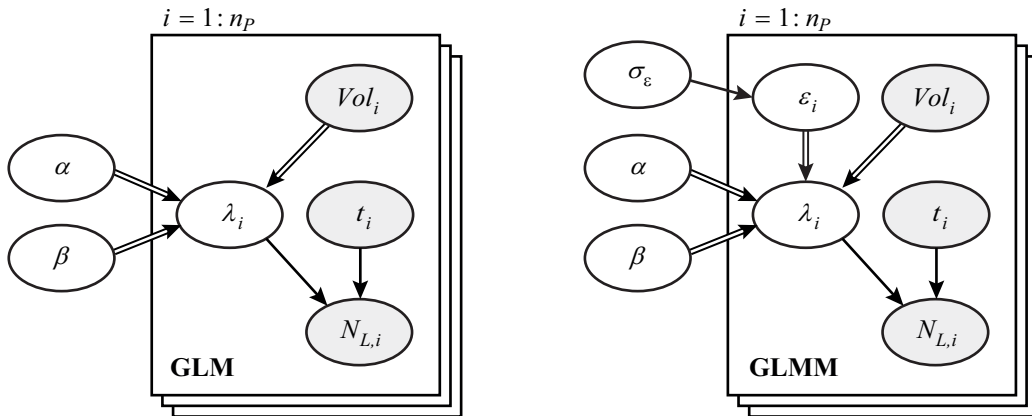
Fig. 4.9: Illustration of the Generalized Linear Model (GLM, Equation 4.4) and the Generalized Linear Mixed Model (GLMM, Equation 4.6) for fire occurrence. Double-contoured arrows denote deterministic, simple arrows probabilistic relationships. The AGV data contain evidence on the grey nodes.

Inference for the GLMM is possible with the aid of Bayesian Markov Chain Monte Carlo (MCMC) methods as implemented e.g. in the software WinBUGS (Spiegelhalter et al., 2003).

The following vague prior distributions for the parameters α, β and error standard deviation σ_ϵ have been assumed (α^* and β^* denoting the Maximum Likelihood estimates; see Appendix A for parameter definitions):

$$
\begin{aligned}
\alpha &\sim \text{Normal}(\mu = \alpha^*, 1/\sigma^2 = 10^{-6}) \\
\beta &\sim \text{Normal}(\mu = \beta^*, 1/\sigma^2 = 10^{-6}) \\
\sigma_\epsilon &\sim \text{Uniform}(a = 0, b = 10)
\end{aligned}
\tag{4.7}
$$

Convergence of the MCMC simulation has been checked informally by running multiple chains and monitoring mixing based on trace plots of sample variables. The estimation results are summarized in Table 4.1. When comparing the parameters of the GLMM and the GLM e.g. in the group of residential buildings (the occupancy class with the largest sample size), it stands out that the estimate for α^* differs between the two models. This can be explained by the nonlinear link function used in Equation 4.5 and 4.6. In the GLMM, the deterministic function for λ_i is replaced by a lognormal distribution conditional on the covariate Vol_i and the population parameters α, β and σ_ϵ. The expected value of λ_i is calculated as follows:

$$
\text{E}[\lambda_i \mid Vol_i, \alpha, \beta, \sigma_\epsilon] = \exp\left(\alpha + \beta \ln(Vol_i) + \frac{\sigma_\epsilon^2}{2}\right)
\tag{4.8}
$$

The term $\frac{\sigma_\epsilon^2}{2}$ thus adds an offset to the mean value of λ_i, which explains the difference between the two estimates for α^*. Despite the different α^* values, the expected value of the estimated ignition frequency for a given volume Vol_i is the same same in both models.

Table 4.2 shows a comparison of different information criteria providing assistance for the decision which model to use. The Akaike Information Criterion AIC (Akaike, 1974) may be used to compare the predictive ability of different models. For the GLM it is defined from the likelihood l and the number of model parameters k as $AIC = -2l(\alpha, \beta \mid \hat{\mathbf{x}}) + 2k$, lower values being favoured. In hierarchical models, neither the definition of the likelihood nor of the number of parameters is straight-forward. The AIC for the GLMM in Table 4.2 is based on the marginal likelihood (derived from integrating out ϵ_i) and the number of hyperparameters (α, β and σ_ϵ). According to Lunn et al. (2012), this "marginal likelihood" AIC is relevant when the focus of the analysis is on the hyperparameters, i.e. on predicting fire ignitions at portfolio level. If the focus is on the fire occurrence rate λ_i for individual buildings, they recommend using the Deviance Information Criterion (DIC), a generalization for hierarchical models proposed by Spiegelhalter et al. (2002). It is estimated from the likelihood of the data given the estimated λ_i and an "effective" number of parameters, see Spiegelhalter et al. (2002) or Lunn et al. (2012) for details. For non-hierarchical models and large samples, the DIC is approximately equal to the AIC, thus DIC $\approx$ AIC for the GLM. For the GLMM, the DIC is estimated by WinBUGS. It is seen from Table 4.2 that the more complex "mixed" model is better only if the goal is to predict the fire rate (and its uncertainty) at the level of the individual buildings. For most practical applications, the simple Poisson regression (GLM) should thus be sufficient.

Tab. 4.2: Akaike Information Criterion (AIC) and Deviance Information Criterion (DIC) for the Generalized Linear Model (GLM) and the Generalized Linear Mixed Model (GLMM) based on AGV data.

	GLM	GLMM	
Occupancy	AIC	AIC	DIC
Public/Offices	2′457	2′740	2′392
Residential	38′684	41′208	38′303
Agricultural	3′633	3′967	3′566
Industrial	2′960	3′313	2′865

4.2.3   Modelling the building fire loss given fire occurrence

Also the financial and human consequences given a fire event may be estimated using simple data-based models. The focus of this section is on the monetary loss related to damages at the building structure (without contents), because the data base in Switzerland is best for this fire consequence. For consistency with the ignition model, the AGV data are used also for estimating the building fire loss given fire occurrence.

The insured value as the maximum possible loss

According to Ramachandran (1980), the expected building fire loss $E[C]$ follows a power relationship with the insured value V (e.g. in CHF):

$$E[C \mid V] = k_1 V^{k_2} \tag{4.9}$$

The exponent k_1 is generally smaller than one which implies that the ratio $E[C \mid V]/V$ decreases for increasing insured value V, see Ramachandran (1998). The reason is that fires in small buildings are more likely to involve the whole building than fires in large buildings, where the fire brigade has more time to extinguish the fire before a total loss occurs. The data analysis in Section 4.2.1 suggests that below the total loss limit, the building fire loss C is largely independent of the insured value V (see Figure 4.3a). It is thus sensible to assume independence from V at least for small fire losses. Only for large fires, the insured value has an influence by defining the maximum possible loss. A simple approach for modelling the fire loss amount could thus be to fit a probability distribution to loss data from all buildings and to consider an effect of the insured value V only in the tail of the loss size distribution.

In order to derive the exact functional form of Equation 4.9, Ramachandran (1998) proposed to truncate the fire loss distribution at the insured value of a building. However, a truncated probability model does not allow for a discrete probability mass at $C = V$, as implied by the empirical cumulative distribution functions of the ratio C/V in Figure 4.2b. A more realistic model can be built based on a "censored" probability distribution function, see Figure 4.10. The dashed line indicates an uncensored distribution for the fire loss amount C, which is used for all buildings within a certain occupancy class. This distribution is curtailed at the vertical line representing the building-specific insured value V. For a truncated probability model, the

probability distribution function would have to be rescaled to ensure that the integral over the domain of C remains equal to one. This is not the case for the distribution model in Figure 4.10, where only the probability mass of the upper tail is redistributed to account for the fact that the total loss $C = V$ may only be exceeded by clean up costs. The resulting building-specific loss model is illustrated by the grey area in Figure 4.10.
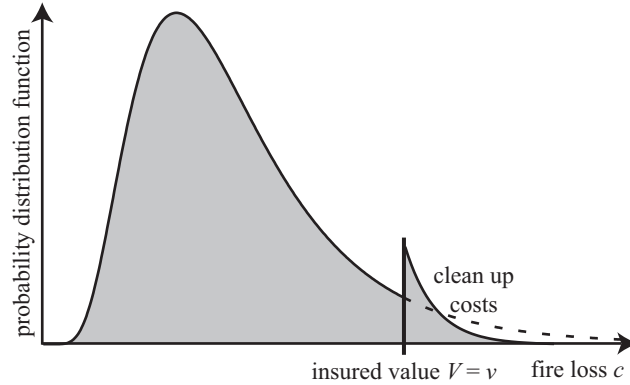
Fig. 4.10: Illustration of a "censored" probabilistic fire loss model.

Neglecting clean up costs, the probability distribution function for the financial loss given fire is formulated by allocating a discrete probability mass to the total loss $C = V$ ($f'_C(c)$ denotes the "censored" probability distribution function and $f_C(c)$ the uncensored model):

$$f'_C(c) = \begin{cases} f_C(c) & \text{for } c < v \\ 1 - F_C(c) & \text{for } c = v \\ 0 & \text{for } c > v \end{cases} \tag{4.10}$$

Choosing a probability distribution for the building fire loss

For a quantitative evaluation, a probability distribution $f_C(c)$ for the (uncensored) fire loss C has to be chosen. It makes sense to consider the natural logarithm of C, as the skewedness of the distribution disappears to some extent on a logarithmic scale. According to Ramachandran (1998), $\ln(C)$ follows a distribution of the exponential type including Exponential, Normal, Lognormal or Gamma, among others. To assess the applicability of different distributions, the Maximum Likelihood method was used to fit the models to the AGV loss data. Fire losses exceeding the insured value of the building were included as censored data points. A comparison of the log-likelihoods of different models for $\ln(C)$ is found in Table 4.3. It is seen that the Lognormal distribution is the best simple distribution model for most occupancy classes; only for industrial buildings, the likelihood of the Gamma distribution is slightly higher.

Figure 4.11a shows the complementary cumulative distribution functions of the fitted models versus the empirical distribution for residential buildings. None of the simple models fits the data well, especially in the important upper tail. A simple solution to this problem would be to fit different models to the body and the tail of the data, see e.g. Hasofer and Thomas (2002). However, when applying the "censored" distribution model (Equation 4.10) to a variety
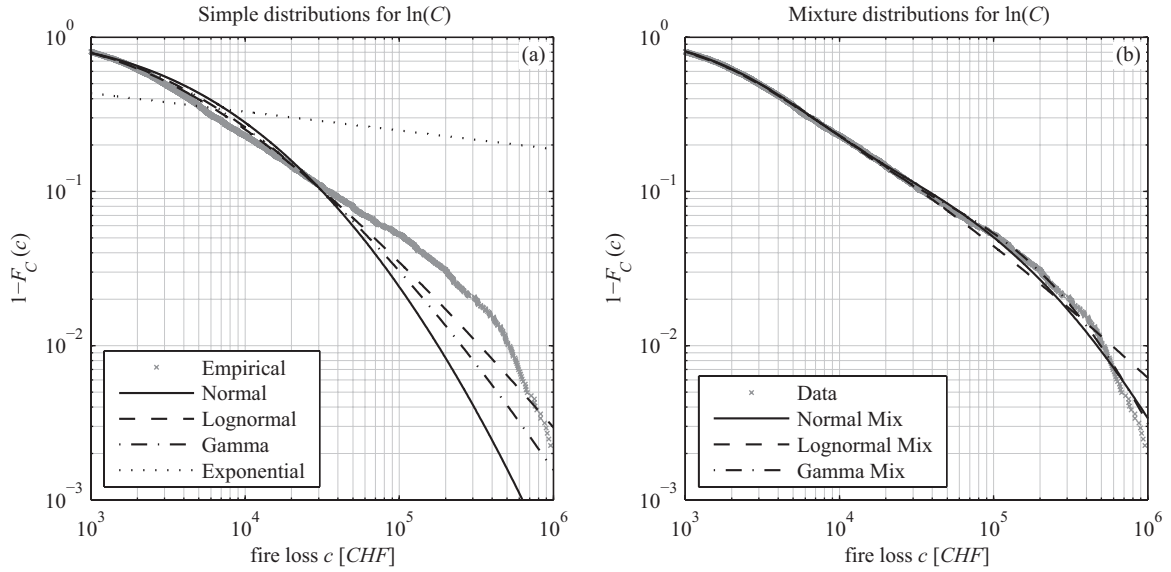
Fig. 4.11: Comparison of different (a) simple and (b) mixture distributions for the natural logarithm of the building fire loss, $\ln(C)$, with AGV data for fires in residential buildings.

of buildings with different insured value, it is convenient to approximate the whole distribution by the same model. To achieve this, a mixture of two distributions is investigated:

$$f_C(c) = p \cdot f_1(c) + (1 - p) \cdot f_2(c) \tag{4.11}$$

Figure 4.11b shows that mixtures of two distributions from the same family but with different parameters can already provide a fairly accurate estimate of the fire loss amount distribution. Therein, mixtures of two Normal, Lognormal and Gamma distributions are compared to the tail of the empirical loss distribution. The parameters of $f_1(c)$ and $f_2(c)$ as well as the mixing probability p were estimated from data using WinBUGS (Spiegelhalter et al., 2003).

The log-likelihoods of different mixtures are given in Table 4.3. For some models (e.g. all mixtures applied to the data for industrial buildings), the MCMC simulations did not converge, indicating a mixing probability close to $p = 1$; these models are not included in the table. Due to the different number of parameters, it is not possible to compare the remaining mixture models with the simple distribution models based on the likelihoods. Therefore, the Akaike Information Criterion (AIC) for the best simple distribution and the best mixture model are given in the last column of Table 4.3. It is concluded that the Normal Mixture is best for all occupancies except for the industrial buildings, where a simple Gamma distribution is sufficient to model $\ln(C)$ (also a Lognormal distribution is supported by the data, which was used in Fischer et al. (2012b) for consistency with two more occupancy classes not regarded in Table 4.3).

The parameters and distribution types of the best models for different occupancy classes are summarized in Table 4.4. The distribution parameters have been estimated from the data based on the "censored" model assumption illustrated in Figure 4.10 (neglecting clean up costs). The effect on the estimation result is small due to the small number of total losses found in the data.

Tab. 4.3: Log-Likelihoods for different simple and mixture distribution models fitted to AGV loss data and corresponding Akaike Information Criterion (AIC) for the models with the highest likelihood.

| | Log-likelihood l | | | |
	Normal	Lognormal	Gamma	AIC
Public/Offices ($n_L = 319$)				
Simple distribution for $\ln(C)$	-605.1	-586.6	-591.0	$1'177$
Mixture distribution for $\ln(C)$	-569.8	-	-569.9	$1'150$
Residential ($n_L = 4'799$)				
Simple distribution for $\ln(C)$	$-8'526.2$	$-8'296.6$	$-8'344.5$	$16'597$
Mixture distribution for $\ln(C)$	$-8'177.4$	$-8'189.9$	$-8'177.9$	$16'365$
Agricultural ($n_L = 405$)				
Simple distribution for $\ln(C)$	-819.2	-799.9	-803.5	$1'604$
Mixture distribution for $\ln(C)$	-767.3	-	-	$1'545$
Industrial ($n_L = 378$)				
Simple distribution for $\ln(C)$	-799.3	-793.9	-793.3	

Tab. 4.4: Summary of the best loss models for each occupancy class fitted to AGV loss data (distributions for the natural logarithm of the building fire loss in CHF, $\ln(C)$; see Appendix A for definitions).

| | sample size n_L | distribution type | Estimated parameters / central moments | | | | |
			p	μ_1	σ_1	μ_2	σ_2
Public/Offices	319	Normal Mix	0.607	7.64	0.97	10.25	1.70
Residential	$4'799$	Normal Mix	0.801	7.71	1.12	10.45	1.59
Agricultural	405	Normal Mix	0.801	8.16	1.43	11.87	1.16
Industrial	378	Gamma	1.0	9.59	2.10	-	-

Based on the distributions in Table 4.4, the expected loss given fire can be estimated. Figure 4.12 illustrates the dependency between the expected loss $E[C \mid V]$ and the insured value V. Black curves are derived from the "censored" model (Equation 4.10) and grey curves from the corresponding truncated distribution. The difference is large especially for industrial buildings, which may be explained by the high likelihood of large fire events in this occupancy class. In general, the effect of assuming a "censored" or a truncated probability model decreases with growing insured value V, as also the total loss probability decreases with the size of the building.

4.2.4   Limitations of data-based modelling

Data-driven models like the ones introduced in Section 4.2.2 and 4.2.3 provide a fairly unbiased estimate of the observed fire risk. However, the application of these models for engineering decision-making is restricted by the information content of the data. Information on the relevant decision variables is often missing, as in the case of the Swiss insurance data providing no
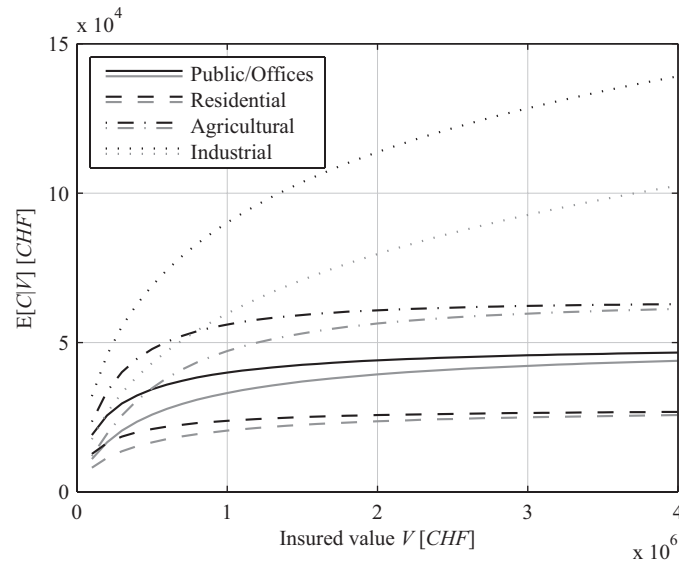
Fig. 4.12: Expected building fire loss $\mathrm{E}\left[C \mid V\right]$ given fire occurrence based on the "censored" model in Equation 4.10 (black) and the corresponding truncated distribution (grey); see Table 4.4 for distributional assumptions.

information at all on the risk reduction measures installed in the buildings. Documenting more information for each fire event would of course be possible, but at the cost of an increased data collection effort. Thus, a trade-off is required between the cost and the value of the information gathered. Yet even if data collection was completely free of charge, there would always remain questions that cannot be answered by analysing historical records from observed fire events. The introduction of new materials, construction types and safety measures are typical examples. In addition, most buildings in a data set tend to be designed based on the same or very similar fire safety concepts due to the regulatory framework of the country or region. It is thus difficult to base code-making decisions on data analysis alone.

A limited sample size can also be a problem for decision-making based on data. The sample size for "normal" fire events is generally large, but problems arise when regarding rare events like structural collapse or multiple death fires, which are often the focus of engineering decision-making. In addition, sample size problems may also arise if several variables in the data set are expected to influence the efficiency of the safety measures to be regarded. A simple way to deal with these interdependencies is to divide the data set into subgroups, which reduces the sample size available for each group. To a certain extent, this problem may be reduced by more advanced data analysis techniques, but the fact that for some combinations of variables only few observations are available cannot be changed.

Accounting for all relevant variables is required to reduce the bias of data-based models. This task is generally straight-forward if the data set provides sufficient information on the important variables. However, the risk may also be influenced by variables not in the data set. Problems arise especially if there is a data selection bias, i.e. if the collection of fire events described by the data was implicitly or explicitly chosen according to some risk-relevant characteristics.

Examples are data sets containing information only on fires leading to a loss of more than 1 Mio. CHF or fires attended by the fire brigade. Data analysis results derived from such data sets have to be interpreted with care; in general, only conditional statements are possible.

It is common to use information from several data sources together with expert judgement and engineering methods for fire risk assessment. Combining all available information is the only way to solve practical fire safety engineering problems. It can, however, introduce additional problems due to different interpretations of variables and events described by the data. In Fischer et al. (2012b) it was shown that even the comparability of the data sets compiled by public fire insurance companies in different Swiss cantons has to be questioned due to different interpretations of the most basic variables. Transferring results derived from international data is even more questionable, but often becomes necessary in practical decision-making if national data sources do not provide sufficient information.

Data analysis results are often regarded to represent "the truth". However, the above discussion shows that data-based models comprise a considerable amount of uncertainty, too. Nevertheless, it may be expected that in many circumstances the bias due to assumptions made in engineering models is larger than the bias resulting from the data analysis issues mentioned above, especially in a "young" profession like fire safety engineering, where physical models are still under development. A good solution is to combine engineering knowledge and data in order to benefit from the strengths of both approaches. In Section 4.4 it is discussed how this may be achieved by calibrating engineering models to statistical data.

4.3 Portfolio fire risk assessment based on engineering methods

Engineering approaches to fire risk assessment break down the problem of modelling fire risk into several sub-problems; e.g. ignition, fire spread, structural fire resistance, smoke spread and human behaviour. In contrast to statistical methods for fire risk assessment, engineering models are not restricted to the information contained in statistical data: The sub-models and their interactions may be modelled based on physical understanding, empirical methods, engineering judgement or a combination of these.

Several computer-based tools for engineering fire risk assessment were developed in the last decades, notably CESARE-RISK in Australia (Beck, 1991, 1997), FiRECAM in Canada (Beck and Yung, 1990, 1994) and CRISP II in the UK (Fraser-Mitchell, 1994). The programs are applicable at object level, i.e. for assessing fire risk in buildings with known characteristics (see e.g. Bénichou, 2006). The application for societal decision support requires focussing on "representative" buildings, e.g. when evaluating the effect of changing code requirements (e.g. Beck and Yung, 1994). In addition, even though the models aim at cost-effective fire safety design, currently the main application is risk comparison, e.g. to prove equivalency with prescriptive design. Using the models for absolute risk assessment, as required for optimizing fire safety at societal level, is still difficult due to the bias introduced by simplified models and conservative assumptions, see e.g. Beck and Zhao (2000). In Section 4.4 it is shown how the model bias may be reduced by calibrating engineering fire risk models to statistical data. Prior to that, it is

briefly discussed how to model fire risk in non-homogeneous building portfolios using engineering methods. The application at portfolio level is also a prerequisite for model calibration with data collected at portfolio level, e.g. by fire brigades or insurance companies.

4.3.1 Principles of generic fire risk assessment

When using a fire risk model at portfolio level, it has to be applicable to all buildings within a certain category without a user interaction required to adapt the model to the specific situation in the individual buildings. This may be achieved by developing engineering models based on the principles of generic risk assessment as described by JCSS (2008); see De Sanctis et al. (2011, 2013a) for an introduction of these principles in the context of fire risk assessment. The aim of generic fire risk modelling is to estimate the risk based on a set of indicators describing the system, e.g. the building at hand. According to JCSS (2008), a risk indicator may be understood as any measurable or observable characteristic of a system or its components containing information on the risk. In fire risk assessment, risk indicators can provide information on the characteristics of the building and/or the fire event. With data containing information on the risk indicators in a portfolio of buildings, the application of a generic risk model at portfolio level is straight-forward. Yet even where this information is not available, it can be helpful to model fire risk as a function of a set of indicators, e.g. when defining code provisions in prescriptive code design according to some risk-relevant characteristics of the buildings.
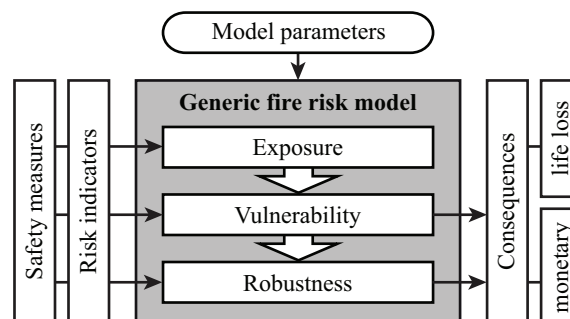
Fig. 4.13: Illustration of a generic fire risk model with system definition based on JCSS (2008).

A simple example for a generic risk model is the fire occurrence model described in Section 4.2.2, where the building's volume and occupancy class are the only two indicators used to model fire ignition. In general, a fire risk model will involve several sub-models and more than two risk indicators. The framework proposed by JCSS (2008) may be helpful to structure more complex models. This framework is illustrated in Figure 4.13, where the grey box refers to the generic risk model which estimates the distribution of the model output (the fire consequences) as a function of a set of risk indicators. Within the model, the consequences of an exposure event (e.g. fire ignition) are modelled using a hierarchical approach, with a vulnerability model estimating the direct effects of the exposure and a robustness model assessing the indirect consequences, see JCSS (2008) or De Sanctis et al. (2011) for details. Consequences may be assessed in monetary terms or e.g. in terms of loss to life and limb (number of fatalities and/or injuries).

The definition of direct and indirect consequences depends on the focus of the analysis. In the context of fire risk assessment, indirect consequences may be defined e.g. as losses outside the room or compartment of fire origin, as losses resulting from fire spread to neighbouring buildings or as consequential losses resulting from business interruption due to a fire event.

Risk indicators may have an effect on the exposure, vulnerability or robustness of the system and relate to different model components. Risk reduction measures are introduced in a similar way, the only difference being that they are not given but represent choices available to the decision-maker. Finally, the model outcome generally depends on a set of model parameters, which in contrast to the risk indicators are not observable. Model parameters can be defined based on engineering knowledge or estimated from statistical data during model calibration.

4.3.2   Risk aggregation and portfolio dependencies

A generic risk modelling strategy at object level is an ideal starting point for estimating risks at portfolio level: If all object-specific characteristics needed to estimate the risk are captured by risk indicators, the same model can be applied to every individual object within a non-homogeneous building portfolio, see e.g. Faber et al. (2007). Summing the random model output over all objects gives an estimate for the aggregated portfolio consequences, see Figure 4.14. This can be used to support decisions made at portfolio level, e.g. regarding public fire brigade investments or the formulation of code provisions in prescriptive regulation.
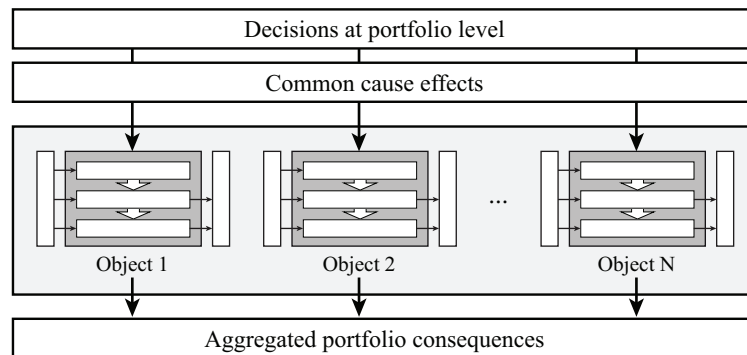
Fig. 4.14: Using generic risk models for risk aggregation and decision-making at portofolio level.

Based on the hierarchical approach illustrated in Figure 4.14 it is also possible to model portfolio dependencies resulting from so-called "common cause effects", variables or processes affecting more than one object in a portfolio. Different sources of common cause effects have been discussed by Schubert and Faber (2008, 2012). The most obvious example are common external conditions, e.g. hazard events or weather conditions. Also common economic conditions can have an effect on the risk at portfolio level, if e.g. maintenance procedures are not properly performed or resources for public fire safety are lacking. All these effects result from common aleatoric processes affecting fire risk for different objects e.g. in the same geographical region. Portfolio dependencies can, however, also arise due to common epistemic uncertainties associated with the use of the same models for the design and assessment of all objects in a portfolio.

Common cause effects do not affect the expected value of the portfolio loss. For societal (i.e. risk neutral) decision-making it is thus only relevant to consider portfolio dependencies if large portfolio losses lead to indirect consequences at societal level. Such societal "follow-up" consequences have to be expected whenever the functioning of society is questioned due to an adverse event. This is generally not the case with typical building fires even when regarding large fire events or fires affecting several adjacent buildings. Only fires in certain facilities like nuclear power or chemical plants can have a major impact on society as a whole. The same is true for critical infrastructure objects like e.g. international airports. Such cases have to be treated individually and should not be the focus of a portfolio risk analysis. Nonlinear effects due to large portfolio losses become relevant only if a large number of buildings is affected as in the example of bushfires or post-earthquake fires. Building fire losses here have to be regarded as (follow-up) consequences caused by a natural hazard.

The situation is different if a portfolio model is used in the context of building fire insurance. In this case, the variance and tail of the portfolio loss distribution are generally more relevant than its mean. Common cause effects increase the variance of the portfolio loss and the probability of observing very high losses. Accounting for portfolio dependencies is thus critical especially for estimating and minimizing the probability of ruin.

Common aleatoric effects play a major role when modelling fire risk in the context of natural hazards such as bushfires or post-earthquake fires, where the same event affects a large number of buildings. "Normal" fire ignitions can be regarded to be roughly independent provided that fire spread to neighbouring buildings is properly accounted for at object level; only seasonal events (Christmas, New Year's Eve) or weather related phenomena such as lightning strikes or heating fires in cold winters (Chandler, 1982) may introduce certain dependencies. The influence of common cause effects resulting from epistemic uncertainties is investigated below.

Portfolio dependencies resulting from common epistemic uncertainties

As mentioned above, portfolio dependencies can not only result from common external conditions (aleatoric effects), but also from common epistemic uncertainties. Two different types of epistemic uncertainties may be distinguished: Statistical uncertainty and model uncertainty. The effect of both on a portfolio risk assessment is discussed in the following, using the fire occurrence model developed in Section 4.2.2 as an example.

Statistical uncertainty arises whenever a model is calibrated to data with limited sample size and/or information content. It can be reduced by collecting more data. In a probabilistic model, statistical uncertainty can be accounted for by assigning a probability distribution to the model parameters, see e.g. Table 4.1 for the fire occurrence model. When applying the same model to all objects in a portfolio risk assessment, the "real" parameters are assumed to be unknown but identical for all objects. The influence of this "epistemic common cause effect" at portfolio level depends on the structure of the model and on the non-homogeneity of the portfolio at hand. To give an example, the GLM and the GLMM for fire occurrence are different with respect to their implied dependency structure: For both models, the (hyper-)parameters α and β (and σ_ϵ

for the GLMM) are regarded to be characteristics of the whole portfolio of buildings, but the GLMM also includes a building-specific random effect ϵ_i to account for variability not related to the building's volume and occupancy.

The application of the fire occurrence model at portfolio level with different assumptions regarding the statistical uncertainty is illustrated in Figure 4.15a. The graph shows the exceedance probability for the number of losses n_L for different portfolios of 10'000 buildings. Black curves refer to a non-homogeneous portfolio with building volumes selected randomly from the AGV portfolio data, while grey curves represent a homogeneous portfolio with $Vol = 6'121m^3$ for all buildings (the mean volume calculated from the "real" portfolio). Comparing the results for the two portfolios shows how important it is to account for the variation of building-specific characteristics (risk indicators) at object level instead of relying on models for "representative" buildings. The difference results from the non-linearity of the ignition model and can be explained by Jensen's inequality.

For each portfolio, the effect of different assumptions regarding the statistical uncertainty in the parameter estimates is investigated. Bold lines are derived from the simple Poisson regression (GLM) with Maximum Likelihood parameters α^* and β^* from Table 4.1 (neglecting statistical uncertainty). Accounting for parameter uncertainty in the GLM framework leads to the dashed curves representing the predictive distribution of the portfolio loss. The "epistemic common cause effect" discussed above introduces a certain dependency among the individual objects, but the effect on the portfolio loss distribution is small. For the non-homogeneous portfolio, these results are compared with the portfolio loss distribution derived from the "mixed" model. As expected, the GLMM with parameters α^*, β^* and σ_ϵ^* (neglecting statistical uncertainty) gives the same result as the GLM with Maximum Likelihood parameters. The dotted line is based on the GLMM with uncertain hyper-parameters. The "random effect" variability in ϵ_i is object-specific and vanishes in large portfolios. Nevertheless, the resulting exceedance probabilities are slightly higher than those estimated from the GLM. This can be explained by the fact that due to the increased complexity of the "mixed" model, the statistical uncertainty in the hyper-parameters is higher than that of the GLM parameters. Note that from the model comparison in Table 4.2 it was concluded that the simple GLM rather than the GLMM should be used for prediction at portfolio level.

Accounting for statistical uncertainty in a portfolio risk assessment is relatively straight-forward as it can usually be quantified based on standard statistical methods. However, statistical uncertainty in parameter estimates can only be quantified conditional on the model structure. Additional uncertainties derive from modelling assumptions such as the inclusion or exclusion of variables or the choice of mathematical relations and simplified method used to estimate the risk. Quantifying these uncertainties is much more difficult than in the case of statistical uncertainties. In practice, model uncertainties are therefore often estimated based on expert judgement rather than data, see JCSS (2001).

A simple way to consider model uncertainties in an engineering risk analysis is to apply multiplicative or additive random variables to intermediate model stages or to the final outcome of the model. It may be argued that, when using the same model for all objects within a portfolio,
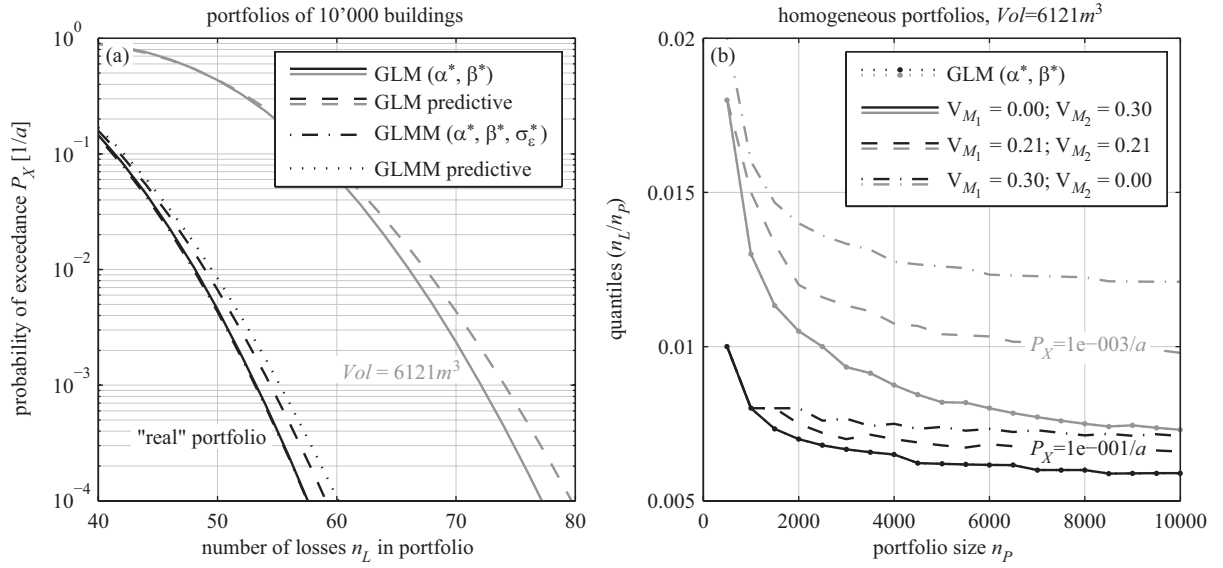
Fig. 4.15: Effect of epistemic common cause effects on the portfolio loss distribution for the example of the fire ignition model for offices/public buildings: Different assumptions regarding (a) the statistical parameter uncertainty and (b) systematic (M_1) and unsystematic (M_2) model uncertainties.

the model uncertainty variable should take the same realisation for all objects modelled in a portfolio risk analysis. Assuming that the model uncertainties for different objects are fully correlated is clearly justifiable when modelling homogeneous portfolios. However, real portfolios are rarely homogeneous. For non-homogeneous portfolios, it is less straight-forward how to model the dependency structure implied by common model uncertainties. In general, the model uncertainty may realise differently depending on the known or unknown characteristics of the individual objects; only a certain share of it must be expected to be identical for all objects in the portfolio. Thus, the model bias across objects will be dependent, but not fully correlated.

In the case of the fire occurrence model, model uncertainties may result e.g. from the choice of the regression model (i.e. the power function for the fire rate λ) or from the decision to model fire occurrence only as a function of the building's volume and occupancy class. The model bias introduced by the mathematical function used to estimate λ depends on the volume Vol; when regarding non-homogeneous building portfolios, it is thus fully correlated only for two buildings with the same size. Yet even a portfolio that is homogeneous in terms of Vol can be non-homogeneous in terms of omitted (possibly unknown) variables. To give an example, it may be assumed that fire occurrence depends to a large extent on the presence of people. The fire rate should thus be the higher, the more populated a building is. Excluding the occupant density from the fire occurrence model leads to an unknown model bias that depends on a building-specific random variable both in terms of sign and magnitude. Assuming full correlation for this type of model uncertainty thus clearly overstates the problem of common cause effects resulting from epistemic uncertainties.

To illustrate this point, the Maximum Likelihood fire occurrence model $\lambda^* = e^{\alpha^*}(Vol)^{\beta^*}$ (neglecting statistical uncertainty) is multiplied with a random variable M to account for model

uncertainty. For the sake of simplicity it is assumed that M follows a Lognormal distribution with mean value $\mathrm{E}[M] = 1$ and coefficient of variation $V_M = 0.3$. The resulting dependency structure is investigated in Figure 4.15b showing two different quantiles (with exceedance probability $10^{-1}/a$ and $10^{-3}/a$) for the number of losses n_L divided by the portfolio size n_P. The portfolio is assumed to be homogeneous in terms of the volume Vol, but not in terms of other (omitted and unknown) building-specific risk indicators. To show the effect of different assumptions regarding portfolio dependencies implied by the model uncertainty, $M = M_1 M_2$ is divided into a systematic effect M_1 (taking the same value for all buildings in the portfolio) and an unsystematic effect M_2 assumed to be independent across all objects. The individual curves in Figure 4.15b relate to different values for the coefficients of variation V_{M_1} and V_{M_2} (however, in all cases $V_M = 0.3$). It is seen that only the systematic part of the model uncertainty, M_1, has an effect on the estimation of quantiles at portfolio level: The unsystematic effect M_2 disappears already in portfolios much smaller than those investigated in Figure 4.15b. Nevertheless, the curves show a decreasing tendency due to diversification of the remaining (Poisson) uncertainty conditional on λ (without systematic epistemic uncertainty, the curves approach the mean fire occurrence rate $\lambda^* \approx 0.005$ for $n_P \to \infty$). Only the systematic model uncertainty M_1 cannot be reduced by increasing the portfolio size.

From the previous discussion it may be concluded that portfolio dependencies introduced by common epistemic uncertainties can have a considerable effect when regarding high quantiles of the aggregated loss distribution at portfolio level. Modelling common cause effects arising from statistical uncertainties in parameter estimates is relatively straight-forward; however, for the ignition model used as an example, the effect on the portfolio loss distribution is rather small (Figure 4.15a). The dependency structure resulting from model uncertainties is less clear because this type of epistemic uncertainty may depend on different building-specific risk indicators. Assuming fully correlated model uncertainties makes sense only if the portfolio is homogeneous in terms of all relevant characteristics (observed or unobserved) of the individual objects. For non-homogeneous portfolios, it is more realistic to assume that model uncertainties can be split into a systematic and an unsystematic effect, which reduces the importance of considering epistemic common cause effects in a portfolio risk assessment (Figure 4.15b). In practice, it could of course be a viable approach to assume full correlation of model uncertainties as a limiting case in order to judge whether the resulting portfolio dependencies are of any relevance for the problem at hand. Estimating what share of the model uncertainties can be regarded to be unsystematic is necessary only if a consistent modelling of common epistemic uncertainties is expected to affect the decisions to be made.

4.4   Calibration of engineering models with statistical data

As has been discussed in the previous sections, fire risk models can be based on two sources of information: Engineering models and statistical data. Data-driven models as described in Section 4.2 provide a fairly unbiased estimate of the observed fire risk. However, the application of these models for engineering decision-making is restricted by the information content of

the data available to the modeller. Engineering models, on the other hand, are based on an understanding of the physical processes leading to loss of property and life and therefore have a high potential for answering questions arising during the design of buildings for fire safety. However, these methods always include a certain bias due to assumptions made regarding the probabilistic modelling, e.g. the probability distribution functions of basic input variables and simplified methods used to model the risk.

The bias introduced by assumptions made during the modelling process can be reduced by calibrating a fire risk model to statistical data. In the following, it is discussed how this may be achieved in practice. Two main problems have to be solved when calibrating an engineering model to data collected by e.g. fire brigades or insurance companies:

— *Data collection at portfolio level*: The data is typically collected in a non-homogeneous portfolio of buildings, with a variety of factors influencing the risk.

— *Incomplete data sets*: The data does not necessarily contain all information relevant to an engineering approach for fire risk assessment.

In the following, a framework for the calibration of a generic fire risk model to statistical data is developed. The two problems mentioned above are treated separately. In Chapter 7 (Case Study 3), the framework is applied to the calibration of a generic fire risk model for single family houses to Swiss insurance data.

4.4.1   Calibration with data from non-homogeneous building portfolios

In a single building, fire is a rare event, but information on a large number of fire events in building portfolios is provided by e.g. fire insurance statistics or fire brigade reports. When using such data for model calibration, one has to bear in mind that different building characteristics may have an influence on the outcome of the fire. In the following, it is discussed how a generic fire risk model can be calibrated to statistical data on fire events collected in a non-homogeneous portfolio of buildings. The principles of generic fire risk modelling are discussed in Section 4.3.1. For the calibration procedure discussed in the following, it is sufficient to look at a generic model as a black box (see also Figure 4.13): For a given set of calibration parameters and a certain model input (e.g. a set of building-specific risk indicators), the model provides the probability distribution of the model output (e.g. the fire loss). With data containing evidence on both model input and output in a non-homogeneous portfolio of buildings, it is possible to calibrate the model on a portfolio level. In doing so, the engineering knowledge used to build the generic fire risk model is combined with information from observed fire events and the bias introduced by assumptions made during the modelling process is minimized.

Calibration based on the Maximum Likelihood method

As a starting point for formulating the calibration procedure it is assumed that there exists a generic probabilistic model assessing the distribution of the random model output $\mathbf{Y}$ conditional

on a set of risk indicators $\mathbf{X} = \mathbf{x}$ (the model input) and the calibration parameters $\boldsymbol{\Theta} = \boldsymbol{\theta}$ (bold letters denote vectors, upper case for random variables and lower case for realisations of random variables). Treating the model as a black box, it can be expressed as a conditional probability density function $f_{\mathbf{Y}|\mathbf{X},\boldsymbol{\Theta}}(\mathbf{y}|\mathbf{x},\boldsymbol{\theta})$. It is furthermore assumed that the data set used for calibration contains complete information on the model output and all risk indicators in n observed fire events. The observations are stored in a matrix $\hat{\mathbf{x}} = [\hat{\mathbf{x}}_1, ..., \hat{\mathbf{x}}_n]^\top$ for the model input and a matrix $\hat{\mathbf{y}} = [\hat{\mathbf{y}}_1, ..., \hat{\mathbf{y}}_n]^\top$ for the model output. During the calibration of the model to statistical data, the goal is to find the parameters $\boldsymbol{\theta}^*$ leading to the best representation of the observations stored in $\hat{\mathbf{y}}$ and $\hat{\mathbf{x}}$ by the model.

When applying a generic model at portfolio level, the model output for each building depends on the building-specific risk indicators (the model input) and on the calibration parameters, which are assumed to be the same for all buildings. A simple calibration procedure that is able to deal with data from non-homogeneous portfolios can be formulated based on the Maximum Likelihood method (see e.g. Rychlik and Rydén, 2006). The idea of this method is to find the parameters of a probabilistic model that maximize the "likelihood" of the observations as evaluated by the model. Statistical data on observed fire events typically contain only one observation per building. Therefore, the likelihood has to be evaluated at portfolio level. When following a generic modelling strategy, this is not problematic, because the same model can be applied to a variety of buildings or fire events; the differences between the individual observations are captured by the risk indicators $\hat{\mathbf{x}}_i$.

The likelihood L and log-likelihood l are defined as follows:

$$
\begin{aligned}
L(\boldsymbol{\theta}|\hat{\mathbf{y}},\hat{\mathbf{x}}) &= \prod_{i=1}^{n} f_{\mathbf{Y}_i|\mathbf{X}_i,\boldsymbol{\Theta}}(\hat{\mathbf{y}}_i|\hat{\mathbf{x}}_i,\boldsymbol{\theta}) \\
l(\boldsymbol{\theta}|\hat{\mathbf{y}},\hat{\mathbf{x}}) &= \sum_{i=1}^{n} \ln\left(f_{\mathbf{Y}_i|\mathbf{X}_i,\boldsymbol{\Theta}}(\hat{\mathbf{y}}_i|\hat{\mathbf{x}}_i,\boldsymbol{\theta})\right)
\end{aligned}
\tag{4.12}
$$

The Maximum Likelihood parameters $\boldsymbol{\theta}^*$ are determined by maximizing the likelihood L or, equivalently, by minimizing the negative log-likelihood $-l$:

$$
\boldsymbol{\theta}^* = \min_{\boldsymbol{\theta}}\left(-l(\boldsymbol{\theta}|\hat{\mathbf{y}},\hat{\mathbf{x}})\right)
\tag{4.13}
$$

A nice property of the Maximum Likelihood approach is that it provides not only a point estimate for the calibration parameters but also their statistical uncertainty. If the data set used for calibration is sufficiently large, it may be assumed that the uncertain parameters $\boldsymbol{\Theta}$ are asymptotically normally distributed. Their expected value is the Maximum Likelihood estimate, i.e. $\mathrm{E}[\boldsymbol{\Theta}] = \boldsymbol{\theta}^*$. The covariance matrix $\mathbf{C}_{\boldsymbol{\Theta}}$ of the parameters is determined as the inverse of the (observed) Fisher information matrix, which is defined as the negative Hessian matrix $\mathbf{H}$ of the log-likelihood function evaluated at the Maximum Likelihood estimate. For the example of

two calibration parameters, this is expressed as follows:

$$\mathbf{C_\Theta} = \begin{bmatrix} \mathrm{Var}\,[\theta_1] & \mathrm{Cov}\,[\theta_1, \theta_2] \\ \mathrm{Cov}\,[\theta_1, \theta_2] & \mathrm{Var}\,[\theta_2] \end{bmatrix}$$

$$= \{-\left.\mathbf{H}\right|_{\mathbf{\theta}=\mathbf{\theta}^*}\}^{-1} = \left\{ -\begin{bmatrix} \frac{\partial^2 l}{\partial^2 \theta_1} & \frac{\partial^2 l}{\partial \theta_1 \partial \theta_2} \\ \frac{\partial^2 l}{\partial \theta_1 \partial \theta_2} & \frac{\partial^2 l}{\partial^2 \theta_2} \end{bmatrix}_{\substack{\theta_1=\theta_1^* \\ \theta_2=\theta_2^*}} \right\}^{-1} \tag{4.14}$$

For practical applications, the Maximum Likelihood estimation can be performed using a numerical routine to solve Equation 4.13. The Hessian matrix is then typically determined as a by-product of the optimization.

4.4.2 Calibration with incomplete fire loss data

In the previous section, it was assumed that the data used for calibration contains complete information on a set of risk indicators describing the input and output of the engineering model. With real data sets, the situation is often less favourable. In the following, it is shown how a calibration can be performed with data containing only little information on the buildings and/or fire events. Also the situation where information is available only for "large" fires is shortly discussed.

Missing information on the risk indicators used by the model

Model calibration becomes very difficult or even impossible if the data contains no information at all on the specific conditions under which the observed fire losses occurred. However, in practice fire loss data typically provide some basic information on the buildings and/or fire events, although not necessarily on the risk indicators used as a model input. Such information allows at least for a rough estimation of the risk indicators needed during the calibration. Applying the calibration procedure described in Section 4.4.1 is still possible, but the uncertainty in the estimation of the risk indicators from the information contained in the data has to be quantified.

Figure 4.16 illustrates the calibration procedure for data sets with limited information content. The risk indicators used as model input are estimated from the available information using probabilistic or, if possible, deterministic assumptions. If necessary, a similar approach can be followed on the model output side.

Instead of evidence from the data, an uncertain estimate of the model input now enters the calibration. Expressed mathematically, the observed risk indicators $\hat{\mathbf{x}}_i$ in the likelihood formulation (Equation 4.12) are replaced by a random vector $\mathbf{X}_i$. The distribution of $\mathbf{X}_i$ is specified conditional on a vector $\hat{\mathbf{z}}_i$ containing the information on the building or fire event provided in the data set. The assumptions on the model input side are expressed by a conditional probability density function $f_{\mathbf{X}|\mathbf{Z}}(\mathbf{x}|\mathbf{z})$. This allows reformulating the log-likelihood based on
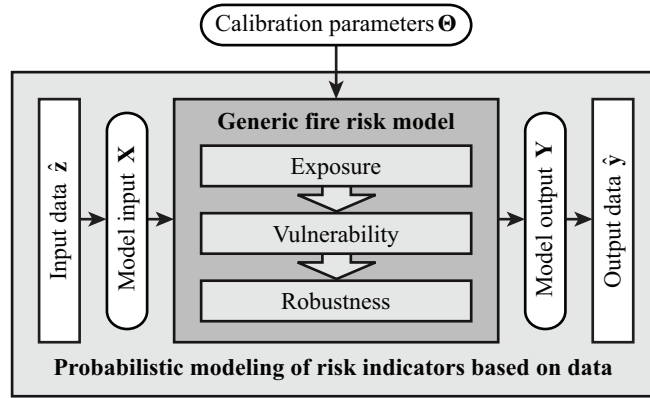
Fig. 4.16: Calibration of a generic fire risk model with data containing only limited information on the model input.

the total probability theorem:

$$l\left(\boldsymbol{\theta}\,|\hat{\mathbf{y}},\hat{\mathbf{z}}\right)=\sum_{i=1}^{n}\ln\left\{\int_{D_{\mathbf{X}}}f_{\mathbf{Y}_i|\mathbf{X}_i,\boldsymbol{\Theta}}\left(\hat{\mathbf{y}}_i\,|\mathbf{x}_i,\boldsymbol{\theta}\right)\cdot f_{\mathbf{X}_i|\mathbf{Z}_i}\left(\mathbf{x}_i\,|\hat{\mathbf{z}}_i\right)\mathrm{d}\mathbf{x}_i\right\} \tag{4.15}$$

Where $D_{\mathbf{X}}$ is the domain of $\mathbf{X}$. From a computational point of view, the likelihood formulation in Equation 4.15 is highly inconvenient: Due to the uncertainty in $\mathbf{X}$, the number of model evaluations per entry in the data set is, at least in theory, infinite. For practical applications, the probability density function $f_{\mathbf{X}|\mathbf{Z}}\left(\mathbf{x}\,|\mathbf{z}\right)$ can however be discretized and limited to a reasonable range. The level of discretization should reflect the uncertainty inherent in the distribution of the risk indicators, a rough discretization being appropriate for variables with a high degree of uncertainty. With a discrete probability mass function $p_{\mathbf{X}|\mathbf{Z}}\left(\mathbf{x}\,|\mathbf{z}\right)$ replacing $f_{\mathbf{X}|\mathbf{Z}}\left(\mathbf{x}\,|\mathbf{z}\right)$, the log-likelihood is expressed as:

$$l\left(\boldsymbol{\theta}\,|\hat{\mathbf{y}},\hat{\mathbf{z}}\right)=\sum_{i=1}^{n}\ln\left\{\sum_{j=1}^{k_i}f_{\mathbf{Y}_i|\mathbf{X}_i,\boldsymbol{\Theta}}\left(\hat{\mathbf{y}}_i\,|\mathbf{x}_{ij},\boldsymbol{\theta}\right)\cdot p_{\mathbf{X}_i|\mathbf{Z}_i}\left(\mathbf{x}_{ij}\,|\hat{\mathbf{z}}_i\right)\right\} \tag{4.16}$$

Here, k_i refers to the number of different model input combinations for the data set entry i. It depends on the number of variables in $\mathbf{X}$ and on the level of discretization used to define $p_{\mathbf{X}|\mathbf{Z}}\left(\mathbf{x}\,|\mathbf{z}\right)$.

The capabilities of the calibration approach for incomplete data sets are obviously limited by the information content of the data. Nevertheless, the calibration of a risk model can be valuable also if the data base is very poor. Even a rough calibration may help to discover inconsistencies in the engineering model, e.g. if it is not able to reproduce the observed fire and loss characteristics in different groups of buildings. Finally, the lessons learnt during the calibration of a fire risk model can help to formulate the requirements for future data collection.

Calibration with data sets limited to large fire events

The discussion above focussed on the problem of limited information content of the data used for calibration. Another problem typical for fire loss data is that information on small fire events

is missing, e.g. on those fires that are not reported to the fire brigade or insurance company. This situation can, however, easily be handled by a conditional Maximum Likelihood approach. The calibration is then still based on the likelihood formulation in Equation 4.12 or Equation 4.16, but the (unconditional) distribution of the model output, $f_{\mathbf{Y}|\mathbf{X},\mathbf{\Theta}}(\mathbf{y}|\mathbf{x},\mathbf{\theta})$, is replaced by a distribution conditional on the event determining whether the fire is included in the data base. A simple example is the case of insurance data containing only losses larger than a certain excess (deductible) y_0. The conditional distribution $f_{Y|\mathbf{X},\mathbf{\Theta},Y>y_0}(y|\mathbf{x},\mathbf{\theta},Y>y_0)$ is in this case derived from $f_{Y|\mathbf{X},\mathbf{\Theta}}(y|\mathbf{x},\mathbf{\theta})$ by truncation at y_0.

4.4.3 Benefits of the calibration approach

The calibration procedure described above provides a consistent way of combining a physical modelling approach with statistical information from real fire events. The proposed framework based on the Maximum Likelihood method is simple but nevertheless flexible enough to deal with data collection at portfolio level and incomplete data set. For societal decision-making, risk models are applied for evaluating the efficiency of fire safety measures in non-homogeneous building portfolios. Based on this goal, three different requirements can be derived that have to be fulfilled by the calibrated model:

— *Physical modelling approach*: The model assumptions and the final performance of the model have to be consistent with the physical understanding of the problem. This property of the model is important for evaluating the effect of fire safety investments, especially if no information on the risk reduction measures is contained in the data.

— *Influence of building-specific risk indicators*: The behaviour of the model for buildings with different risk indicators has to be consistent with the observations in different groups of buildings. A good fit to the data in relation to different building characteristics is also an indicator for an appropriate physical modelling.

— *Overall fit to the data*: After aggregation at portfolio level, the model has to be able to represent the observed probability distribution of the model output. This property is important for modelling the risk in absolute terms with as little bias as possible.

The Maximum Likelihood calibration of a model aims at both the overall fit to the data and an appropriate dependence on the building-specific risk indicators. Judging whether the behaviour of the model is consistent with a physical modelling philosophy remains an important task during the process of model building: Adjusting the model assumptions to achieve an optimal fit to the data while disregarding physical understanding of the fire problem is clearly not a valid approach. A good engineering model will not require much trade-off between the three requirements discussed above: The fit to the observed loss data will generally be good if the physical model is able to represent the characteristics of real fire events.

After calibration, the remaining model uncertainty depends on the sample size and the information content of the data. But even with data containing only very little information on

the problem at hand, the performance of the model can be improved with the aid of model calibration. Besides the direct bias-reducing effect, the calibration and validation of an engineering model with statistical data can also reveal inconsistencies in the model structure and foster an improved physical understanding of the problem at hand. Finally, the development and calibration of generic fire risk models helps to provide the requirements for future data collection by defining observable risk indicators that contain information on the risk. Based on this feedback loop, both the physical models and the data collection can be improved in the long run.

The calibration of engineering models is obviously limited by the data available for calibration. This holds especially for rare events like structural collapse or multiple death fires, where the data base will always remain small. But also the quality of the physical models can be a limiting factor: Calibrating a model that is not able to capture the behaviour of real fires at least qualitatively will not be successful. The strength of the calibration approach is the combination of engineering knowledge with statistical data: Observations from real fire events are most helpful in areas where the uncertainties are high and the understanding of the physical processes is poor. Engineering models, on the other hand, can be used to fill the gaps in the available data and to optimize fire safety even if the data does not contain enough information to provide the level of detail necessary for decision support.

Chapter 5

Case Study 1: Egress routes

The first case study shows how prescriptive fire safety design may be improved by adding more flexibility to the code provisions (Section 3.2.1). The calculations are based on Fischer et al. (2012b), where both the results and the underlying assumptions are discussed more in detail.

5.1   Adding flexibility to prescriptive egress route design

The design of egress routes in Swiss buildings is regulated in VKF (2003b). With respect to egress distances, the regulation prescribes a maximum distance of $35m$ to the nearest room exit ($20m$ if there is only one exit) and of $50m$ to the next staircase or exit to the outside ($35m$ if there is only one staircase). Justifying alternative egress route designs based on engineering calculations is currently not allowed by the Swiss fire safety code, see Art. 13 of VKF (2003a). Thus, the design of egress routes in Switzerland so far has been based on a very strict prescriptive approach. To increase the flexibility of the prescriptive code, alternative design solutions based on active fire safety measures were investigated in Fischer et al. (2012b). The discussion was based on example calculations for the compensation of longer egress distances with an automatic fire alarm system, which are recapped in the following.

The trade-off between egress distances and active fire protection is a nice example for an alternative design that is more expensive than the prescriptive solution (Figure 3.1): In most cases, a short egress route will be simple to achieve and therefore cheap and efficient for providing an acceptable level of life safety in building fire events. However, for some buildings the egress distances defined in VKF (2003b) can be very restrictive, e.g. for large industrial halls or existing buildings that were designed based on older regulations. In addition, the owner may consider to install a fire alarm system due to reasons other than life safety, e.g. to benefit from lower fire insurance rates. In general, active fire safety leads to a considerable reduction in monetary fire losses, though at the price of fairly high installation and maintenance costs. As has been discussed in Section 3.2.1, these costs are typically borne by the owner of the building. Therefore, from a regulatory point of view it makes sense to encourage the selection of alternative design strategies with active fire protection measures if the owner considers it to be cost-efficient for his specific project. The only prerequisite here should be equivalency in terms of life safety.

The cost-efficiency of active fire safety measures is not regarded in this case study. Instead, the focus is on assessing the level of life safety resulting from different design strategies. It is assumed that the current approach based on VKF (2003b) represents an acceptable solution for the reduction of risk to life in fire events. As a result of this assumption, the acceptability of the alternative design can be evaluated by comparison to a reference case derived from the standard, prescriptive design. Proving equivalency in terms of life safety based on a relative risk assessment is in this context a viable approach for increasing the efficiency of code-based design, e.g by allowing for a compensation of longer egress distances with active fire safety measures. Further improvements may be possible based on the marginal life saving costs principle and the LQI acceptance criterion, which requires an absolute risk assessment.

5.2   Quantifying life safety in building fire events

5.2.1   The time line approach to the safety of building occupants

The aim of egress route requirements is to allow building occupants to evacuate to a safe location before the spread of fire and smoke leads to untenable conditions. In the time line approach to the design of egress routes, occupant safety is evaluated by comparing the Available Safe Egress Time (ASET) to the Required Safe Egress Time (RSET). The evacuation of a person is considered to be successful if his or her RSET is shorter than the ASET. The time line approach is illustrated in Figure 5.1.
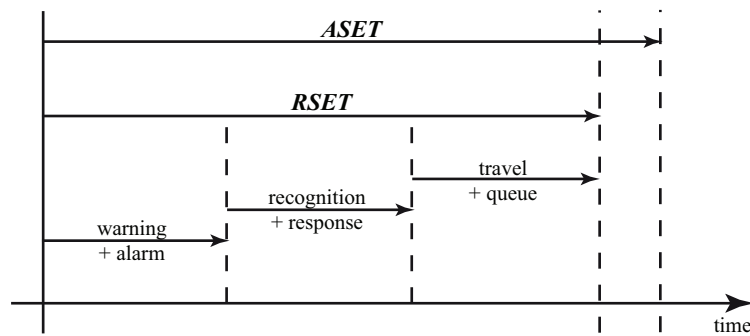
Fig. 5.1: Illustration of the time line approach to the design of egress routes.

The basic idea of the time line approach is simple and intuitive, which led to a wide acceptance in the fire safety engineering community, as reflected e.g. by the implementation in ISO/TR 16738. A major criticism is that none of the time components in Figure 5.1 is easily quantifiable, see e.g. Babrauskas et al. (2010). However, this criticism may be circumvented by modelling both the ASET and the RSET probabilistically to account for both aleatoric and epistemic uncertainties. For a single person, the probability of failed egress is evaluated as $P(T_{ASET} < T_{RSET})$. Fire safety measures may reduce this probability by affecting either the ASET or the RSET or both. Interaction between different persons may have to be accounted for when regarding groups of building occupants.

As illustrated in Figure 5.1, the *RSET* can be partitioned into three phases: Warning and alarm, recognition and response, and travel and queuing time. The definitions of the individual time components used within this case study are summarized in Table 5.1.

Tab. 5.1: Definitions for the individual time components used in the time line approach.

T_{ASET}	*Available Safe Egress Time (ASET)*: Time period between fire ignition and the onset of untenable conditions preventing the building occupants to continue their evacuation
T_{RSET}	*Required Safe Egress Time (RSET)*: Time period between fire ignition and the moment in which the building occupant arrives at a safe place within or outside the building; $T_{RSET} = T_A + T_R + T_T$
T_A	*Warning + Alarm*: Time period between fire ignition and the moment in which the building occupant gets aware of a fire alarm or another fire cue
T_R	*Recognition + Response*: Time period between the first fire cue and the moment in which the building occupant starts evacuation
T_T	*Travel + Queuing*: Time period from the evacuation start and the moment in which the building occupant arrives at a safe place within or outside the building

5.2.2 Occupant groups and scenarios regarded within this case study

When quantifying occupant safety in fire events, different scenarios and types of buildings may be distinguished e.g. in terms of the fire hazard, the occupant density, the alertness and mobility of building occupants, their familiarity with the building and the available means of egress. All these points are determined to a large extent by the occupancy class. For the present case study, the group of office buildings is selected as an example. The basic conditions are thus characterized by a medium fire hazard and by occupants that are awake and generally mobile. For simplicity, it is furthermore assumed that the occupant density is small enough to allow evacuation without queuing at the room exits, which allows representing the fatality risk with the probability of failed egress $P(T_{RSET} < T_{ASET})$, see Section 5.3.1 for discussion. The model extensions required for the assessment of life safety in rooms with high occupant density are discussed in Fischer et al. (2012b).

The following occupant groups are distinguished by their location at the time of ignition:

— Occupants in the room of fire origin

— Occupants on the same floor / in the same compartment

— Occupants in the remainder of the building

The Swiss regulations implicitly assume that the staircases remain a safe location until the whole building has been evacuated. The design of the staircases is not within the scope of this case study. The calculations are thus performed only for the first two occupant groups.

Different risk reduction measures may be used to achieve an acceptable level of safety. In this case study, the focus is on the design of egress routes in terms of the longest distance to the next room exit and to the nearest stairway or exit to the outside. It is discussed whether a longer egress route may be compensated by an automatic fire alarm system. Other active fire protection measures, notably sprinklers, are discussed only briefly in Section 5.3.4.

5.2.3   Quantitative assumptions for an evaluation with the time line approach

When using the time line approach for the evaluation of life safety in building fire events, probability distributions have to be assigned to the time periods defined in Table 5.1. The quantitative assumptions used in this case study are summarized in Table 5.2 together with the major references forming the basis for the quantification of the individual time components. In the following, the assumptions will be explained only briefly; a more detailed discussion can be found in Fischer et al. (2012b).

Simplified fire growth model

Both the ASET and the RSET depend on the fire development. The probabilistic models in Table 5.2 are based on the assumption that the heat release rate $\dot{Q}(t)$ grows quadratically with time, i.e. $\dot{Q}(t) = \alpha t^2 [kW]$. This fire model, though very popular among fire safety engineers, is a strong simplification and has been criticised as unrealistic by Babrauskas (1996). However, the advantage of the t^2 fire model is that fire growth is described by a single parameter, the fire growth parameter α. This allows modelling fire growth probabilistically by fitting a distribution for α to statistical data from real fire events, as has been done e.g. by Holborn et al. (2004). Nevertheless, the fire growth model can only serve as a rough approximation to real fire behaviour. However, for the risk comparison aimed at in the present case study the effect of this model uncertainty may be assumed to be small, as both the egress route provisions and the installation of a fire alarm system aim at reducing the RSET, not at prolonging the ASET. The absolute level of risk estimated based on the time line approach does of course strongly depend on the assumed fire growth model.

It is assumed that slowly developing fires are negligible for estimating the fatality risk in office buildings, where occupants are expected to be awake and mobile. Therefore, only fires with a fire growth parameter $\alpha > 6.6 \cdot 10^{-3} kW/s^2$ (medium, fast and ultra fast fires according to the classification used by Holborn et al. (2004)) are considered in the risk estimation.

Assumptions for quantifying the ASET

The ASET in the room of fire origin is modelled using a response surface developed by Magnusson et al. (1995) based on simulations with the two-zone model CFAST. The tenability criterion implicit in their model is defined in terms of the height of the lower, cool layer. Modelling the ASET in other rooms is more difficult and requires additional assumptions regarding e.g. open doors and the room layout. To avoid arbitrary specifications, within this case study the time

Tab. 5.2: Assumptions for a quantitative evaluation of occupant fire safety using the time line approach, based on Fischer et al. (2012b) (see Appendix A for definitions of the probability distributions and corresponding parameters).

Fire growth parameter $[kW/s^2]$				Basis
α	$\alpha \sim \text{Lognormal}(-7.1, 1.8)$			Holborn et al. (2004)
	$\alpha > 6.6 \cdot 10^{-3} kW/s^2$ (medium/fast/ultra fast)			

ASET/RSET in room of fire origin $[s]$		$\mathbf{E}\,[\boldsymbol{T}]$	$\boldsymbol{V_T}$	Basis
T_{ASET}	$T_{ASET} = 1.67\alpha^{-0.26}H_r^{0.44}A_f^{0.54}M$	237	0.20	Magnusson et al. (1995)
	H_r = room height $[m]$; $H_r = 3m$			
	A_f = floor area $[m^2]$; e.g. $A_f = 300m^2$			
	$M \sim \text{Normal}(\mu = 1.35, \sigma = 0.11)$			
T_A	$T_A \sim \text{Exponential}(\lambda = f(\alpha))$	13	1.09	Fischer et al. (2012b)
	$\lambda = 1/\mu = -\ln(0.01)\sqrt{\alpha/50kW}$			
T_R	$T_R \sim \text{Lognormal}(\lambda = 3.71, \zeta = 0.74)$	54	0.85	Proulx et al. (1996)
	assumed independent of α			
T_T	$T_T = W \cdot l$, $l = 35m$ egress distance	26	0.15	Rinne et al. (2010)
	$W \sim \text{Weibull}(a = 1.41, b = 10.14)[m/s]$			

ASET/RSET in other rooms $[s]$		$\mathbf{E}\,[\boldsymbol{T}]$	$\boldsymbol{V_T}$	Basis
T_{ASET}	$T_{ASET} = T_{ASET,room} + \Delta$	-	-	Fischer et al. (2012b)
	$\Delta = 5/10/20/30min$ "smoke spread"			
T_A	*without fire alarm*:			
	$T_A \sim \text{Lognormal}(\lambda = 5.83, \zeta = 0.22)$	348	0.22	Brennan (1997)
	with fire alarm:			
	$T_A \sim \text{Lognormal}(\lambda = f(\alpha), \zeta = 0.32)$	60	0.41	Evans and Stroup (1985)
	$\lambda = 2.51 - 0.36\ln(\alpha)$			
T_R	$T_R \sim \text{Lognormal}(\lambda = 4.20, \zeta = 0.83)$	94	1.00	Proulx et al. (1996),
	assumed independent of α			Gwynne (2007)
T_T	$T_T = W \cdot l$, $l = 50m$ egress distance	37	0.14	Rinne et al. (2010)
	$W \sim \text{Weibull}(a = 1.41, b = 10.14)[m/s]$			

to untenable conditions in other rooms is modelled by adding a constant $\Delta = 5/10/20/30min$ to the ASET in the room of fire origin. Different values for Δ may be interpreted as different smoke spread scenarios (e.g. as a function of the room arrangement or open doors) or simply as different assumptions regarding the development of critical conditions in the egress routes. It should be noted that the response surface by Magnusson et al. (1995) has been developed for large rooms ($A_f \geq 200m^2$). Scenarios with a fire starting in a small room require additional considerations, see Lundin (2005) for discussion.

Assumptions for quantifying the RSET

In Table 5.2, probability distributions for the RSET components T_A, T_R and T_T are defined. The alarm time T_A depends on the location of the building occupant relative to the fire. The distribution for the alarm time in the room of fire origin was developed based on the assumption that 99% of the occupants notice a fire at latest when the heat release rate equals $50kW$ (the order of magnitude of a waste bin fire)[1]. For occupants in other rooms, a study by Brennan (1997) describing an office fire event without internal alarm was used as the basic input. The effect of smoke alarms is assumed to be negligible in the room of fire origin, see also Section 5.2.4. Only for occupants in other rooms, a positive effect is expected. The dependency of the smoke alarm activation time on α given in Table 5.2 was developed based on a regression analysis using simulations with the computer program DETACT-t2 (Evans and Stroup, 1985), see Fischer et al. (2012b) for details.

The occupant response time T_R is assumed to be independent of the fire growth parameter α. The distributions in Table 5.2 were defined based on observed response times in evacuation drills reported by Proulx et al. (1996) and Gwynne (2007). The data may be assumed to be representative for the response of building occupants not close to the fire. Defining the response time distribution for persons in the room of fire origin required additional assumptions, see Fischer et al. (2012b) for discussion.

Finally, also the travel time T_T is modelled without considering an interaction between the building occupants and the fire event. The formulas given in Table 5.2 are based on the assumption that the queuing time at the exits is negligibly short, which is valid only for buildings with low occupant density. The walking time to the exit is calculated by multiplying the egress distance l with a random walking speed (Rinne et al., 2010). For this case study, the egress distance is used as a decision variable; the values $l = 35m$ and $l = 50m$ in Table 5.2 are examples taken from the prescriptive code, see VKF (2003b).

The overall evacuation time T_{RSET} is calculated by adding the three time components given in Table 5.2:

$$T_{RSET} = T_A + T_R + T_T \tag{5.1}$$

The first two time components T_A and T_R are much more variable and difficult to predict than the travel time T_T, as can be seen by comparing the coefficients of variation V_T given in Table 5.2. Therefore, long evacuation times are generally a result of long alarm times and/or

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[1]Equation B.1 in Fischer et al. (2012b) is wrong; the expression given in Table 5.2 is correct.

reaction times and the influence of the travel time on the tail of the distribution for T_{RSET} is small. When regarding all fire situations (including slowly developing fires), the effect of the alarm time is dominating. For $\alpha > 6.6 \cdot 10^{-3} kW/s^2$, the reaction time is the most important RSET component, see Fischer et al. (2012b) for discussion.

5.2.4 Modelling the effect of a fire alarm system on the evacuation

A fire alarm system may influence both the alarm time and the reaction time of a building occupant in a fire event. While the alarm time will generally be decreased by a fire alarm, the reaction time may be reduced or increased depending on the fire cues the person receives. Thus, the benefit of a fire alarm in terms of a reduced warning time may at least partly be offset by an increased reaction time due to difficulties with the interpretation of the alarm signal. These effects are, however, not regarded within this case study and the fire alarm is assumed to affect only the alarm time of a building occupant.

A person that is awake and located in the room of fire origin will generally discover the fire before an alarm signal is issued by an automatic fire alarm system. Therefore, it is assumed that the effect of a fire alarm on the fatality risk in the room of fire origin is negligible.

Also persons in other rooms may become aware of the fire before the fire alarm sounds. Their alarm time is defined as the minimum of the two alarm time distributions (with and without fire alarm) given in Table 5.2:

$$T_A = \min \left(T_A^{alarm}; \ T_A^{no\ alarm} \right) \tag{5.2}$$

If the fire alarm system is programmed to immediately issue an alarm signal in the whole building, this will normally be the first fire cue received by persons outside the room of fire origin. In practice, this is, however, not always the case. According to the Swiss guideline for the design of automatic fire alarm systems (VKF, 2011), the external alarm sent to the fire brigades may be delayed by at most five minutes to facilitate internal investigation. The aim of this rule is to reduce the number of fire brigade actions due to nuisance alarms. There is no requirement to directly issue an alarm to the building occupants either. In practice, the internal alarm often goes to a few responsible persons first, which may lead to a delay of several minutes for the remaining occupants. In the worst case, the building occupants get no alarm signal at all until the fire brigade arrives at the scene.

Figure 5.2 shows the effect of an internal alarm signal delayed by 0, 3, 5, and 6 minutes on the alarm time distribution of persons outside the room of fire origin. The graphs are based on simulations of the alarm time with and without fire alarm for medium, fast and ultra fast fire growth ($\alpha > 6.6 \cdot 10^{-3} kW/s^2$). Issuing an alarm to all building occupants immediately after the activation of the first smoke detector (no delay) leads to an alarm time distribution that is dominated by the fire alarm system. The same is true for a delay of three minutes. With a delay of five minutes, some building occupants may already receive other fire cues before the alarm sounds, but a positive effect of the fire alarm system can still be observed for persons who would otherwise be very late in discovering the fire. For longer delays, the effect of the fire alarm decreases further until it is restricted to the upper tail of the alarm time distribution.
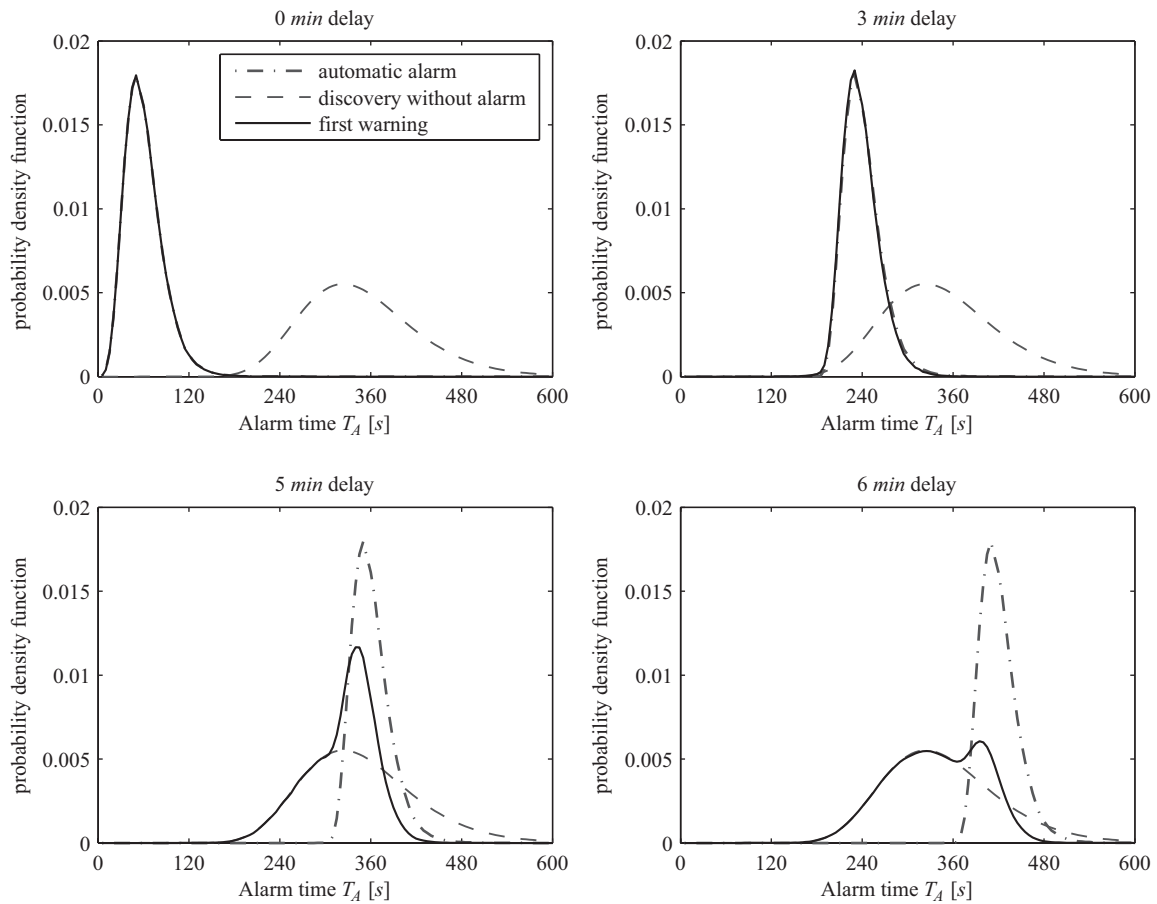
Fig. 5.2: Alarm time distribution with automatic fire alarm delayed by 0, 3, 5 or $6min$ (simulations based on the assumptions in Table 5.2 for occupants outside the room of fire origin).

5.3 Relative risk assessment results

Based on the quantitative assumptions in Table 5.2, it is possible to estimate the fatality risk for different scenarios and fire safety measures. The goal of this case study is to compare a design with fire alarm system, but longer egress routes, to the reference case without active fire protection measures and egress route design based on the current prescriptive code.

5.3.1 Representing the fatality risk with the probability of failed egress

For simplicity, in this case study the probability of failed egress $P(T_{RSET} > T_{ASET})$ is used to represent fatality risk. The failure probability is estimated for a single person, neglecting the number of occupants present during the fire event. This simplification is acceptable for rooms with small occupant densities if the goal is a relative risk assessment only: As long as it may be assumed that different persons are able to evacuate independently of each other, the expected number of fatalities given fire may be approximated by multiplying the probability of failed egress for a single person with the number of occupants (Hasofer and Beck, 2000). The number of people in the room then enters the calculation only as a constant factor and has no effect on

a relative risk assessment. The situation is different for higher occupant densities, where the dependency between the number of occupants and the fatality risk is non-linear, see Lundin (2005, 2008) and Fischer et al. (2012b) for discussion. In this case, risk has to be evaluated in terms of the expected number of fatalities.

Another simplification is that a fatality is assumed to occur if a person is not able to evacuate successfully. Strictly speaking, the time line approach for occupant safety based on a comparison of T_{RSET} and T_{ASET} evaluates only the success of self-rescue in the early phase of a fire event. The effect of fire brigade rescue actions is not considered in this case study.

5.3.2   Probability of failed egress in the room of fire origin

The effect of a fire alarm on the alarm time for occupants in the room of fire origin may be assumed to be negligible, see Section 5.2.4. Installing a fire alarm system to compensate a long distance to the nearest room exit is thus not meaningful when regarding this occupant group in isolation. Nevertheless, we may be interested in investigating the effect of a longer egress distance on the probability of failed egress in the room of fire origin.

As has been discussed in Section 5.2.3, the effect of the travel time T_T on the upper tail of the RSET distribution is small compared to that of the alarm time T_A and the reaction time T_R. Thus, the effect of a moderate increase of the egress distance l is expected to be small, at least if only persons with a very long evacuation time T_{RSET} are at risk.

The egress distance becomes more important if the ASET is short, as e.g. in small rooms, where the onset of critical conditions tends to occur earlier than in large rooms. The effect of the room area (with fixed room height $H_r = 3m$) on the probability of failed egress $P(T_{RSET} > T_{ASET})$ is illustrated in Figure 5.3: With increasing room size, the failure probability decreases and the effect of the egress distance l becomes smaller.
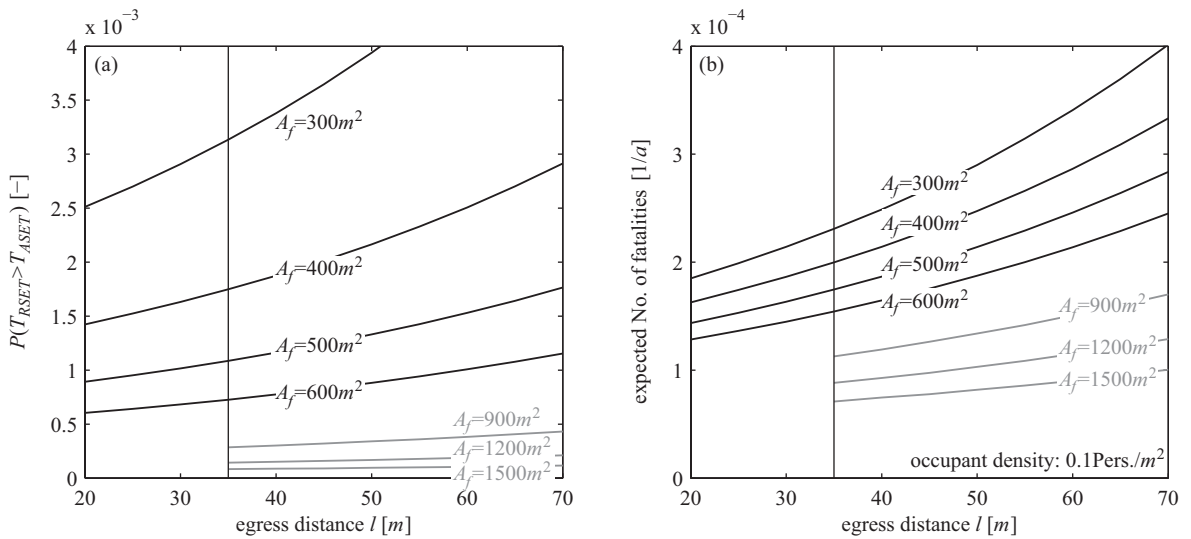
Fig. 5.3: Probability of failed egress given fire occurrence (a) and expected number of fatalities per year (b) in the room of fire origin (with $H_r = 3m$). Simulations based on the assumptions in Table 5.2.

Figure 5.3a shows the probability that a person in the room of fire origin cannot evacuate safely conditional on a fire with $\alpha > 6.6 \cdot 10^{-3} kW/s^2$. For Figure 5.3b, the expected number of fatalities was approximated by multiplying this failure probability with the number of people present and the fire occurrence rate. A constant occupant density of $0.1 Pers./m^2$ was assumed, which leads to a linear relationship between the room area and the number of occupants. The fire occurrence rate was estimated as a function of the room volume as in Section 4.2.2. For simplicity, the ignition probability is not reduced to account for slowly developing fires; however, this affects only the absolute level of risk, not the comparison between rooms with different size.

5.3.3   Probability of failed egress in other rooms

For people outside the room of fire origin, a fire alarm system has a positive effect on the distribution of the alarm time T_A and thus also on the probability of failed egress in a fire event. The effect is largest if the internal alarm to all building occupants is issued immediately after the first smoke detector activation, see Section 5.2.4 for discussion. To estimate the probability of failed egress outside the room of fire origin, the time to onset of untenable conditions in the egress routes (T_{ASET}) has to be evaluated. As has been discussed in Section 5.2.3, the smoke spread to other rooms is not modelled explicitly, but using a simplified approach by adding a "smoke spread time" Δ to the ASET in the room of fire origin, with different Δ values representing different smoke spread scenarios or different assumptions for T_{ASET}.

Figure 5.4 shows the probability of failed egress with and without smoke alarm as a function of the egress distance l for a person outside the room of fire origin (with $A_f = 300m^2$ and $H_r = 3m$). Two different scenarios are distinguished for the design with fire alarm system (grey lines). The dashed line illustrates the failure probability without alarm, which becomes relevant if the fire alarm system does not work properly in case of a fire event. Assuming 100% reliability for the fire alarm system leads to the dotted line. The black line considers both scenarios weighted by the reliability of the system, assuming a conservative value of 80%; according to Ahrens (2007) and Bukowski et al. (1999), the reliability may be expected to be higher.

Figure 5.4a is based on a "smoke spread time" $\Delta = 10min$. The resulting failure probabilities $P(T_{RSET} > T_{ASET})$ are given in absolute terms. To facilitate a comparison between different assumptions for Δ, in Figure 5.4b the results are normalised by the failure probability for a reference case given by the prescriptive design ($l = 35m$, without fire alarm system). Thus, Figure 5.4b shows how the probability of failure changes if the egress distance is increased and/or a fire alarm system is installed. To give an example, for $\Delta = 5min$ the failure probability increases by 20% when changing the egress distance from $l = 35m$ to $l = 60m$. However, the installation of a fire alarm system (assuming a reliability of 80%) reduces $P(T_{RSET} > T_{ASET})$ to 30% of the failure probability in the reference case. By comparison of different assumptions for Δ, an effect already observed in Section 5.3.2 is seen: The influence of risk reduction measures aiming at a reduction of T_{RSET} is highest if the available egress time T_{ASET} is small.

The assumptions in Table 5.2 have a large effect on the absolute level of risk to life (Figure 5.4a). By relating the results to a reference case (Figure 5.4b), this problem is alleviated by
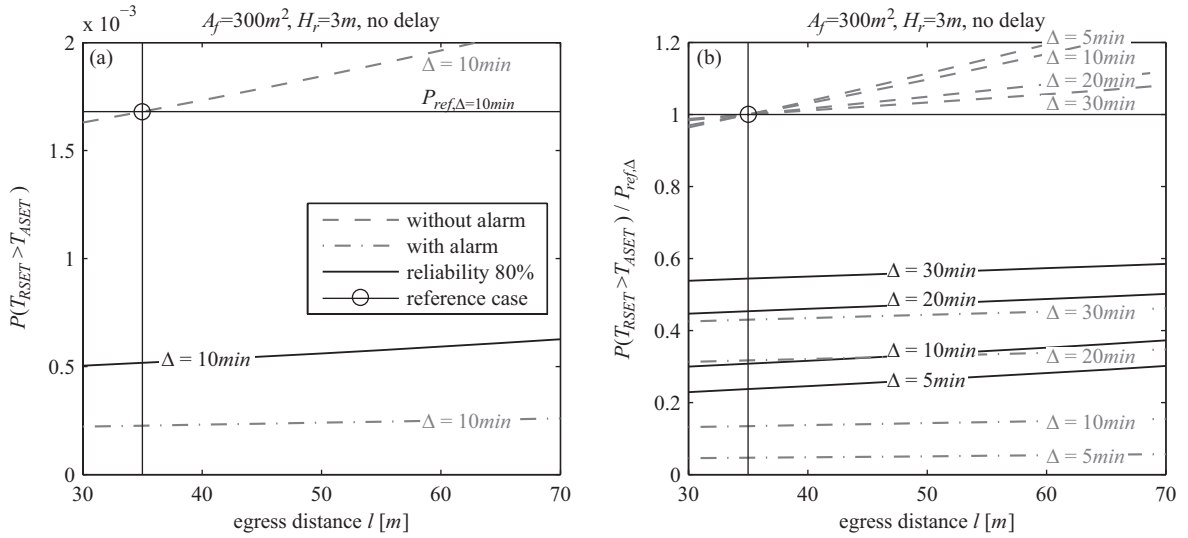
Fig. 5.4: Probability of failed egress given fire occurrence for occupants in other rooms with and without fire alarm (a) in absolute terms and (b) relative to the prescriptive reference case ($l = 35m$, without fire alarm) for different "smoke spread times" Δ. Simulations based on the assumptions in Table 5.2.
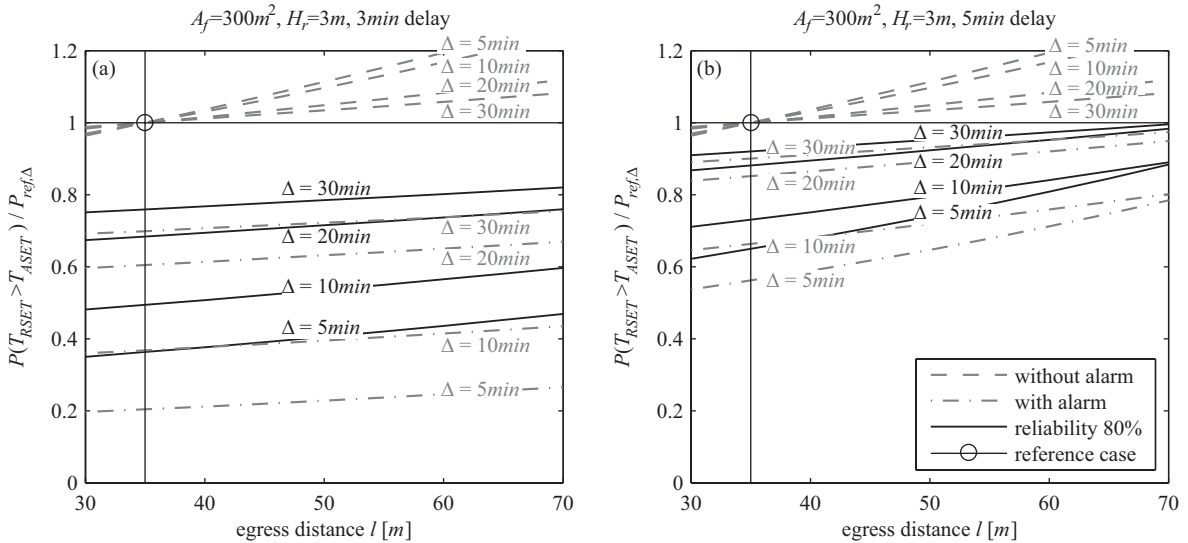
Fig. 5.5: Probability of failed egress given fire occurrence for occupants in other rooms, relative to the prescriptive reference case ($l = 35m$, without fire alarm) for different "smoke spread times" Δ and an internal alarm delay of (a) $3min$ and (b) $5min$. Simulations based on the assumptions in Table 5.2.

the fact that many assumptions, e.g. regarding the distributions of T_{ASET} or T_R, affect both the alternative solution with fire alarm system and the standard design case, see also Fischer et al. (2012b). Assumptions affecting only one of the two designs have a larger effect on the relative risk assessment. When evaluating a trade-off between egress route design and a fire alarm system, the most important assumptions are those regarding the distribution of the alarm time T_A. For example, in Figure 5.4 it was assumed that the building occupants receive a warning signal immediately after the activation of the first smoke detector. The effect of a fire

alarm delayed by several minutes is smaller, as has been shown in Section 5.2.4. Results for the probability of failed egress assuming a delay of three and five minutes are given in Figure 5.5.

In summary, it may be concluded that for occupants outside the room of fire origin the alternative design with fire alarm system is generally safer than the standard solution based on the prescriptive code: Increasing the egress route by e.g. $20m$ is generally overcompensated by the alarm time reduction achieved with the fire alarm system. Only for much longer egress routes and/or an internal alarm delay of more than three minutes, the probability of safe egress is comparable to that obtained for the reference case.

5.3.4   Discussion and implications for the prescriptive code

In the following, the relative risk assessment results presented above are discussed with respect to their implications for prescriptive code design. The quantitative results are valid for the compensation of longer egress routes with an automatic fire alarm system for the group of office buildings with low occupant density. At the end of this section, it is discussed to what extent the conclusions derived from these results may be transferred to other scenarios.

Room of fire origin: Egress distance to nearest room exit

Building occupants located in the room of fire origin, at least if awake and alert, may be expected to become aware of the fire before receiving a warning signal issued by the automatic fire alarm system. Compensating a longer within-room egress distance with an alarm system is therefore not justifiable. On the other hand, it can be shown that the effect of the egress distance on the total evacuation time T_{RSET} is relatively small. This holds particularly for persons with very long evacuation times that have the highest risk of dying in case of a fire event. Increasing the egress distance to the nearest room exit thus has an important effect only if the available egress time T_{ASET} is very short. This is to be expected mainly in small fire rooms that fill with smoke very quickly. In large rooms, e.g. with a room area of $600m^2$ or more, both the fatality risk in absolute terms and the effect of the egress distance on the risk are smaller. A similar effect is to be expected when increasing the room height. It may thus be a viable approach to differentiate the code provisions for egress route design according to the room volume (area and/or height).

Other rooms: Egress distance to nearest staircase

For occupants outside the room of fire origin, an automatic fire alarm system is clearly beneficial: The calculations show that the fatality risk in other rooms can be reduced by choosing the alternative design with fire alarm system and longer egress routes than required by the prescriptive code. The alternative solution is safer even when using a conservative estimate for the reliability of the alarm system. The positive effect of a fire alarm is, however, strongly reduced if the internal alarm to all building occupants is issued with a delay of several minutes. The alternative design is justifiable only if all persons at risk are notified of there being a fire as early as possible. At the same time, false alarms should be avoided not only for the fire brigade, but

also for the building occupants. A possible solution may be the implementation of a three-stage alarm procedure:

— Internal alarm to responsible safety personnel immediately after smoke detector activation

— Internal alarm (all occupants) raised by safety personnel or automatically after max. $3min$

— External alarm (fire brigade) raised by safety personnel or automatically after max. $5min$

Postponing the automatic alarm by several minutes allows the responsible persons to investigate the situation and to cancel possible false alarms. However, postponing the internal alarm for all building occupants by five minutes (the maximum allowed by VKF, 2011) may render it useless; the maximum delay for the internal alarm should be shorter.

Effect of scenarios and assumptions on the risk assessment results

The quantitative results presented in Section 5.3 obviously depend on the assumptions made in Section 5.2.3, see Fischer et al. (2012b) for a full discussion. The relative risk assessment may be expected to be relatively robust with respect to assumptions affecting both the reference case and the alternative design solution. Only the assumed alarm time distributions with and without fire alarm system may have a large effect on the results, as this is where the two designs differ. The qualitative statements derived from the relative risk assessment should, however, not be too sensitive to the quantitative assumptions underlying the calculations.

An important effect also on the qualitative results must be expected when regarding different scenarios. The focus of the present case study was on office buildings with low occupant density. The results are not transferable to buildings with high occupant densities (e.g. assembly halls) and to buildings where occupants may be asleep (residential buildings, hotels) or immobile (hospitals, jailhouses). For other occupancy classes, the conclusions depend on the relevant distributions for the required and available egress time (T_{RSET} and T_{ASET}), see Fischer et al. (2012b) for discussion.

Effect of other active fire safety measures

The focus of this case study was on the compensation of a longer egress distance with an automatic fire alarm system. It was assumed that the reaction time T_R is independent of the type of warning (with or without alarm) received by the building occupants. The reaction time can be decreased by installing a voice alarm system, especially in buildings that are open to the general public, where particularly long reaction times have been observed, see e.g. Gwynne (2007) or Purser and Bensilum (2001).

When evaluating the use of a sprinkler system as a compensation measure, the situation is similar as in the case of a fire alarm system: When regarding small rooms of fire origin, building occupants located in the same room derive only little benefit from a standard sprinkler system, as a considerable amount of smoke may be released before sprinkler activation (the situation is different for fast response sprinklers). For persons in other rooms, sprinklers are more beneficial:

It may be assumed that the conditions in the egress routes will not become untenable if the sprinkler is successful in extinguishing the fire. Only in case of a sprinkler failure, people outside the room of fire origin may be in danger. Figure 5.6 shows the probability of failed egress for building occupants in other rooms relative to the reference case (standard egress route design without sprinkler). A sprinkler reliability of 95% can be regarded to be a realistic estimate, see e.g. Hall (2010) or Hosser (2009), whereas a reliability of 75% is clearly conservative.
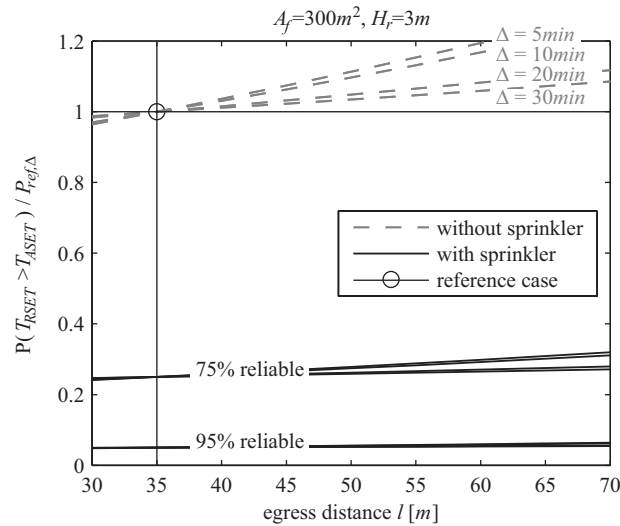
Fig. 5.6: Probability of failed egress given fire occurrence with and without sprinkler for occupants in other rooms, relative to the prescriptive reference case ($l = 35m$, without active fire safety measures). Simulations based on the assumptions in Table 5.2.

Finally, also a smoke and heat exhaust system aiming at smoke control in the room of fire origin and/or the egress routes may be considered as a compensation measure. The objective of these systems is to increase the available time T_{ASET} and to improve the conditions for the fire brigade. When focussing on self-rescue, an automatic and immediate activation of the smoke exhaust system is indispensable. Evaluating the influence of this safety measure on the fatality risk is possible based on the time line approach used in this chapter if the effect on the distribution of T_{ASET} is quantified.

Chapter 6

Case Study 2: Structural fire safety

In the second case study, the framework developed in Section 3.4 is applied to the derivation of optimal target reliabilities for a performance-based approach to structural fire safety design.

6.1 Performance-based structural fire safety

The Eurocode for structural fire safety design (EN 1991-1-2 and the relevant sections of the material-related codes) is probably the most advanced performance based fire safety code currently in practice: The safety goal is not only clear qualitatively (structures should not fail in case of fire), but also quantitatively in terms of a target reliability for a 1-year reference period. Finally, a semi-probabilistic design approach is provided based on safety factors for the fire load density that take into account several aspects relevant for structural fire safety design in a specific building, see Annex E of EN 1991-1-2. The calibration of the safety factors to the Eurocode target reliability is described in Schleich et al. (2002). A similar concept with safety factors for both the fire load density and the maximum heat release rate was developed by Hosser et al. (2008) as a basis for the German National Annex of EN 1991-1-2 (see also Weilert et al., 2008).

As discussed in Section 3.3, the Eurocode target reliabilities are roughly consistent with the principles of monetary optimization, at least when focussing on structural safety for the normal design situation. But is this safety level also appropriate in the context of structural fire safety? One obvious difference is that structural failure in fire safety design must be regarded conditional on a fire event. In the Eurocode safety concept, this is accounted for by deriving the target reliability given a fully developed fire, $P_{f,t|F}$, from the (unconditional) target failure probability for a 1-year reference period, $P_{f,t,1y}$, divided by the fire rate $\lambda_I P_{F|I}$:

$$P_{f,t|F} = \frac{P_{f,t,1y}}{\lambda_I P_{F|I}} \qquad \Rightarrow \qquad \beta_{t,F} = -\Phi^{-1}\left(\frac{P_{f,t,1y}}{\lambda_I P_{F|I}}\right) \qquad (6.1)$$

However, this is not the only difference between the fire situation and the normal design situation. An optimal fire safety design should be based on a different failure mode, different design variables, different safety costs and possibly also different failure consequences than the structural design codes. In the following, a simple optimization scheme taking into account all these aspects is developed based on the framework described in Section 3.4.

The goal of the present case study is not to develop a perfect model for structural fire safety, but to use a simple generic approach that is able to account for the most important variables with sufficient accuracy to provide at least an order of magnitude estimate for the optimal failure probability in case of fire. The approach followed is similar to the optimization studies by Holický and Schleich (2001) and Faber et al. (2004), however with some differences regarding e.g. the decision variables used in the design for structural fire safety.

6.2   Optimal and acceptable structural fire safety design

6.2.1   Defining target reliabilities in fire based on optimization

The general framework for defining a target level of structural fire safety based on monetary optimization is laid out in Section 3.4. The optimal design is found by minimizing the expected present value of the total costs $T(\mathbf{p})$, which are composed of the construction costs $C(\mathbf{p})$, the obsolescence costs $A(\mathbf{p})$, the costs for inspection and maintenance $I(\mathbf{p})$ and the monetary consequences of fire $F(\mathbf{p})$ (see Equation 3.19). The inspection and maintenance costs are neglected in the following. Using the definition for the obsolescence costs in Equation 3.5 and for the fire consequences in Equation 3.22, the design objective is formulated as:

$$\min_{\mathbf{p}} \left\{ T(\mathbf{p}) = C(\mathbf{p})\left(1 + \omega/\gamma\right) + \left(C(\mathbf{p}) + H\right)\lambda_I P_{F|I} P_{f|F}(\mathbf{p})/\gamma \right\} \tag{6.2}$$

Where $C(\mathbf{p})$ denotes the construction costs, ω the obsolescence rate, γ the discount rate, H the costs resulting from structural failure in addition to the reconstruction costs, λ_I the yearly rate of fire occurrence, $P_{F|I}$ the probability of a fully developed fire given fire ignition and $P_{f|F}(\mathbf{p})$ the conditional probability of structural failure. The vector $\mathbf{p}$ is assumed to consist of only two design variables, where p_1 relates to the structural design for the non-fire situation and p_2 to the fire safety design of the structure. In consistency with Section 3.3.1, p_1 is defined as the central safety factor for the "cold" design. The fire safety design parameter p_2 can be related e.g. to the thickness of insulating material used to protect the structure from fire. In consistency with the design procedure typically chosen in practice, it is assumed that structural safety for the normal and for the fire situation are optimized separately. For the fire safety design this means that the cross-sectional dimensions of structural members are given; In this case study this is achieved by choosing the value for p_1 that is optimal for the normal situation (i.e. following Equation 3.9). Increasing p_1 is of course also a viable strategy for improving structural safety in case of fire; the value derived from the "cold" optimization then serves as a lower bound for p_1.

Before using Equation 6.2 to derive target reliabilities for the fire situation, the functions $C(\mathbf{p})$ and $P_{f|F}(\mathbf{p})$ have to be defined. This is the focus of Section 6.3.1 and 6.3.2, where the framework is applied to structural fire safety in steel buildings.

6.2.2   Defining target reliabilities in fire based on the LQI

The optimal fire safety design according to Equation 6.2 is not necessarily acceptable from a life saving point of view. Fire safety targets that have been derived based on monetary optimization

should thus be complemented by the LQI acceptance criterion to guarantee a certain minimum level of life safety. In analogy to Section 3.3.2, the acceptance criterion is formulated as follows:

$$\frac{\mathrm{d}C(\mathbf{p})}{\mathrm{d}\mathbf{p}}\left(\gamma_S + \omega\right) \geq -\frac{g}{q}J_x \cdot \lambda_I P_{F|I} \cdot \frac{\mathrm{d}\left(P_{f|F}(\mathbf{p})N_F(\mathbf{p})\right)}{\mathrm{d}\mathbf{p}} \tag{6.3}$$

The left-hand side refers to the marginal costs of increasing structural safety in the fire situation and the right-hand side to the $SWTP$ for increased life safety. In contrast to the normal design situation (Equation 3.12), here the expected number of fatalities $N_F(\mathbf{p})$ in case of structural failure is not independent of the design variables in $\mathbf{p}$. Increasing structural fire safety does not only reduce the probability of failure $P_{f|F}$; also the time until failure occurs is increased, which improves the chances for the escape of building occupants and successful rescue actions. This can only be accounted for by allowing for time-dependent effects in the consequence modelling.

In a simplified analysis, it might of course be possible to use a rough estimate for the expected number of fatalities N_F and assume independence of $\mathbf{p}$. LQI target reliabilities could then be derived for different consequence classes allowing for different values of N_F. Nevertheless, even in such a simplified analysis the number of fatalities cannot be deduced from the number of occupants that are in the building at the time of fire ignition: Only those that are still alive but cannot escape (or be rescued) in time face the risk of dying due to structural failure.

In buildings with short evacuation routes and with occupants that are able to escape on their own, the expected number of deaths in a fire event that can be attributed to structural failure will generally be small. It may thus be assumed that in these buildings the target reliabilities derived from monetary optimization will also be acceptable according to the LQI criterion. Whether this is the case can be checked by investigating the interaction between optimality and societal risk acceptance assuming a constant value for the number of fatalities N_F.

Interaction with monetary optimization

The optimal structural design $\mathbf{p}^*$ according to Equation 6.2 is derived from $\mathrm{d}T(\mathbf{p}^*)/\mathrm{d}\mathbf{p} = 0$:

$$\frac{\mathrm{d}C(\mathbf{p}^*)}{\mathrm{d}\mathbf{p}}\left(1 + \omega/\gamma\right) + \frac{\mathrm{d}C(\mathbf{p}^*)}{\mathrm{d}\mathbf{p}}\lambda_I P_{F|I} P_{f|F}(\mathbf{p}^*)\frac{1}{\gamma} + \left(C(\mathbf{p}^*) + H\right)\lambda_I P_{F|I}\frac{\mathrm{d}P_{f|F}(\mathbf{p}^*)}{\mathrm{d}\mathbf{p}}\frac{1}{\gamma} = 0 \tag{6.4}$$

The second term may be neglected because $\lambda_I P_{F|I} P_{f|F}(\mathbf{p}^*) \ll \gamma$. Rearranging and inserting into Equation 6.3 leads to the following criterion for checking whether the optimum is acceptable from a life saving point of view:

$$-\frac{\gamma_S + \omega}{\gamma + \omega} \cdot \left(C(\mathbf{p}^*) + H\right) \cdot \frac{\mathrm{d}P_{f|F}(\mathbf{p}^*)}{\mathrm{d}\mathbf{p}} \geq -\frac{g}{q}J_x \cdot \frac{\mathrm{d}\left(P_{f|F}(\mathbf{p}^*)N_F(\mathbf{p}^*)\right)}{\mathrm{d}\mathbf{p}} \tag{6.5}$$

Assuming that N_F is independent of $\mathbf{p}^*$, as discussed above, leads to the same criterion as the one that has been derived in Section 3.3.2 for the normal design situation (Equation 3.16). If it is furthermore assumed that $(\gamma_S + \omega)/(\gamma + \omega) \approx 1$, the criterion simplifies to checking whether the monetary consequences of structural failure are larger than the human consequences N_F multiplied with the $SWTP$ for saving an additional life:

$$C(\mathbf{p}^*) + H \geq N_F \cdot \frac{g}{q}J_x \tag{6.6}$$

The expected number of fatalities that can be attributed to structural failure in case of fire, N_F, should be estimated conservatively; A more detailed analysis becomes relevant if the criterion in Equation 6.6 is not fulfilled.

6.3   Optimizing structural fire safety in steel buildings

6.3.1   A simple limit state function for structural fire safety in steel buildings

In the following, a limit state function for estimating the failure probability $P_{f|F}(\mathbf{p})$ is defined and simplified to account for only the most important input variables. A protected steel building is used as an example structure. Other building materials require different physical models but the general approach followed in this section remains valid.

In principle, the limit state function can be defined in the same way as in the normal design situation: Failure occurs if the load exceeds the resistance. The resistance is, however, reduced by the action of the fire. On the other hand, also the distribution of the load needs to be adapted as it is very unlikely that an extreme value of the mechanical load occurs at the same time as a fire event. The probability of failure in case of a fire is thus estimated as follows:

$$P_{f|F}(\mathbf{p}) = P\left[p_1 \cdot k_{\vartheta,min}\left(p_2\right) \cdot R - S_F \leq 0\right] \tag{6.7}$$

Compared to Equation 3.7, the random resistance at room temperature, R, is reduced by a factor $0 \leq k_{\vartheta,min} \leq 1$ to account for the fire action and the extreme value distributed random load S is replaced by the point-in-time distributed load S_F relevant for the fire situation.

The reduction factor k_ϑ depends not only on the fire safety design variable p_2, but also on a number of variables influencing the fire development (e.g. the fire load and the ventilation conditions), and on time. It is, however, possible to drop the dependency on time by setting $k_{\vartheta,min} = \min\{k_\vartheta(t)\}$ if it is assumed that the fire brigade has no effect on the fire development. This is only possible if the probability of a fully developed fire $P_{F|I}$ accounts for the possibility of an early fire extinction due to fire brigade actions, occupant response or a sprinkler activation. Nevertheless it should be noted that it is a simplification to neglect such time-dependent effects in a fully developed fire, see De Sanctis et al. (2014).

Natural fire model

The goal of a natural fire model is to estimate the room temperature ϑ_g during a fire as a function of time and a number of variables that govern the behaviour of real fires. For this case study, $\vartheta_g(t)$ is estimated using the Eurocode parametric temperature-time curve described in Annex A of EN 1991-1-2. The Eurocode fire curve distinguishes two phases of fire development: A heating and a cooling phase. During the heating phase, the temperature development in a room is modelled as a function of the ventilation conditions and the thermal characteristics of the enclosure. The following formula is used to estimate ϑ_g as a function of time:

$$\vartheta_g(t) = 20 + 1325\left[1 - 0.324\exp(-0.2\Gamma t) - 0.204\exp(-1.7\Gamma t) - 0.472\exp(-19\Gamma t)\right] \tag{6.8}$$

Where t is given in hours. Γ is defined by the opening factor O and a parameter b accounting for the thermal properties of the enclosure: $\Gamma = (O/b)^2/(0.04/1160)^2$ (see EN 1991-1-2 for details). For simplicity, during this case study it is assumed that $O = 0.04m^{1/2}$ and $b = 1160J/(m^2s^{1/2}K)$. This leads to $\Gamma = 1$, for which Equation 6.8 approximates a standard fire exposure.

The end of the heating phase is reached at $t_{max} = \max[(0.2 \cdot 10^{-3}q_t/O); t_{lim}]$, where q_t is the fire load density related to the total area of the enclosure, A_t. It may be estimated from the fire load density per unit floor area q_f if the ratio between A_t and the floor area A_f is known. For the present case study it is assumed that $A_t/A_f = 4$. The limiting value t_{lim} depends on the velocity of fire spread. It is assumed that $t_{lim} = 20min$, which is appropriate for a medium fire growth rate and the value typically used for the design of office buildings. The fire is ventilation controlled if $t_{max} = 0.2 \cdot 10^{-3}q_t/O < t_{lim}$ and fuel controlled if $t_{max} = t_{lim}$. In the latter case, Γ in Equation 6.8 is replaced by $\Gamma_{lim} = (0.1 \cdot 10^{-3}q_t/t_{lim}/b)^2/(0.04/1160)^2$, which leads to lower temperatures in the heating phase. Finally, the temperature development in the cooling phase is estimated based on the following expression:

$$\vartheta_g(t) = \begin{cases} \vartheta_{g,max} - 625\left(\Gamma t - x \cdot t_{max}^*\right) & \text{if } t_{max}^* \leq 0.5 \\ \vartheta_{g,max} - 250\left(\Gamma t - x \cdot t_{max}^*\right)\left(3 - t_{max}^*\right) & \text{if } 0.5 \leq t_{max}^* < 2 \\ \vartheta_{g,max} - 250\left(\Gamma t - x \cdot t_{max}^*\right) & \text{if } t_{max}^* \geq 2 \end{cases} \quad (6.9)$$

Where $\vartheta_{g,max}$ is determined from Equation 6.8 at $t = t_{max}$ and t_{max}^* is defined as $t_{max}^* = (0.2 \cdot 10^{-3}q_t/O) \cdot \Gamma$. For ventilation controlled fires x is set to one, while for fuel controlled fires $x = \Gamma \cdot t_{lim}/t_{max}^*$.

Maximum steel temperature

The Eurocode parametric fire model provides the gas temperature in the fire compartment as a function of time. In the following, the approach given in Eurocode 3 (EN 1993-1-2) for protected steel members is used to estimate the steel temperature of a component exposed to fire. The temperature increase $\Delta\vartheta_{a,t}$ of the steel member during a time interval Δt (in seconds) is determined as follows:

$$\Delta\vartheta_{a,t} = \frac{\lambda_p A_p/V}{d_p c_a \rho_a} \frac{(\vartheta_{g,t} - \vartheta_{a,t})}{(1 + \phi/3)} \Delta t - [\exp(\phi/10) - 1]\Delta\vartheta_{g,t} \quad \text{with} \quad \phi = \frac{c_p \rho_p}{c_a \rho_a} d_p A_p/V \quad (6.10)$$

Here, $A_p/V[1/m]$ denotes the profile factor of the protected steel member, $\lambda_p[W/mK]$ the thermal conductivity of the insulating material and $d_p[m]$ the thickness of the encasement. The factor ϕ refers to the amount of heat stored in the insulation, c_a and c_p to the (temperature dependent) specific heat of steel and the insulation material in $[J/kgK]$, and ρ_a and ρ_p to the unit mass of the two materials in $[kg/m^3]$. Equation 6.10 is used to determine the maximum steel temperature $\vartheta_{a,max}$ during the course of a fire event. An I-section protected by a hollow encasement exposed to fire on four sides is taken as a reference case for the calculation. The insulation consists of gypsum boards with the following properties: $\rho_p = 800kg/m^3$, $\lambda_p = 0.2W/mK$ and $c_p = 1700J/kgK$. The calculations are performed for different types of steel profiles and varying thickness of the protection material.

Reduction factor for steel resistance

The strength reduction of steel at high temperatures is accounted for by multiplying the steel resistance with a reduction factor k_ϑ, which is equal to unity for $\vartheta_a = 20°C$ and decreases to zero at $\vartheta_a = 1200°C$, see EN 1993-1-2. For this case study, the reduction factor for the yield strength of steel is used. To estimate the probability of failure based on Equation 6.7, it is sufficient to know the minimal value of k_ϑ, which is determined as $k_{\vartheta,min} = \min\{k_\vartheta(\vartheta_a(t)\} = k_\vartheta(\vartheta_{a,max})$.
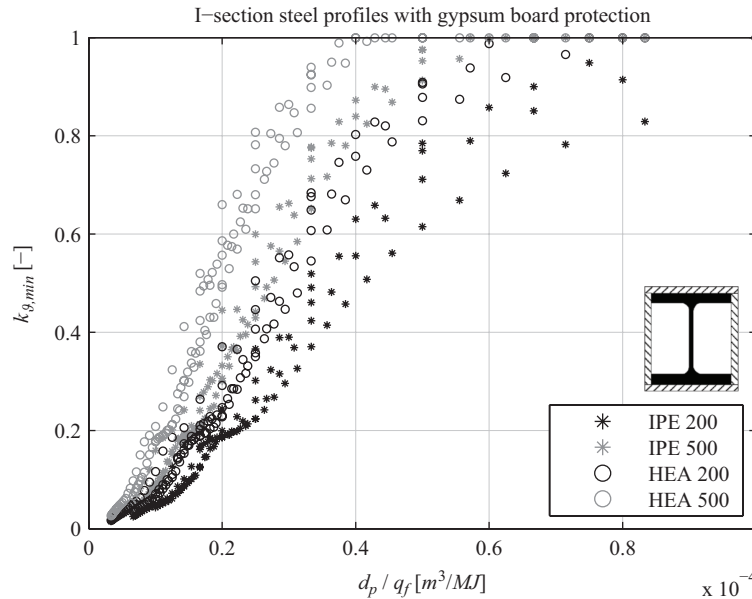
Fig. 6.1: Reduction factor for steel resistance in fire as a function of the thickness of protection d_p and the fire load density q_f.

Figure 6.1 shows $k_{\vartheta,min}$ for varying thickness of protection d_p (between 10 and $50mm$) and fire load density q_f (between 300 and $3000MJ/m^2$). The minimal reduction factor is illustrated as a function of the ratio d_p/q_f for different types of profiles. The resulting scatter plots indicate that for a given profile, $k_{\vartheta,min}$ is roughly proportional to d_p/q_f, especially when excluding high values for the reduction factor that are unlikely to lead to structural failure. This result is used for simplifying the limit state function in Equation 6.7, as will be discussed below. This implies that all assumptions made for estimating $k_{\vartheta,min}$ have to be judged only in terms of their influence on the (approximate) linearity in Figure 6.1. Without a proof it may be assumed that smaller variations in most of the variables will only affect the proportionality constant. A non-linear relationship between $k_{\vartheta,min}$ and d_p/q_f for some situations can of course not be excluded. In this case, the simplification discussed in the following may still be justified by the fact that each function can be approximated by a linear equation at least locally.

Simplifying the limit state function

In order to simplify the limit state function, a functional form must be assumed to describe the dependency between the reduction factor for the steel resistance and the ratio d_p/q_f. It is

of course not possible to achieve a reduction factor $k_\vartheta > 1$. Therefore, based on the results in Figure 6.1 it is suggested to use a bilinear model to approximate $k_{\vartheta,min}$.

$$k_{\vartheta,min} = \min\left\{k\frac{d_p}{q_f};1\right\} = \min\left\{k\frac{d_p}{\mu_{q_f}}\frac{1}{Q};1\right\} \tag{6.11}$$

Where k is a proportionality constant. For the limit state function, the random fire load density q_f is normalized by its mean μ_{q_f} to obtain the random variable Q with mean $E[Q] = 1$. Based on Equation 6.11, it would be straightforward to define the fire safety design variable p_2 as $k \cdot d_p/\mu_{q_f}$ with an appropriate restriction to ensure that $k_{\vartheta,min} \leq 1$. The definition of p_2 should, however, also account for the fact that the distribution of the mechanical load in the fire situation, S_F, is different to that of the load S relevant for the normal design situation. A simple way to achieve this is to divide by the ratio between the mean values of the two loads, $\eta_S = \mu_{S_F}/\mu_S$ (evaluated before normalization). This leads to the following definition of the fire safety design variable:

$$p_2 = k \cdot \frac{d_p}{\mu_{q_f}\eta_S} \tag{6.12}$$

And Equation 6.7 simplifies to:

$$P_{f|F,k_{\vartheta,min}\leq 1} = P\left[p_1 \cdot p_2 \cdot \frac{R}{Q} - S_F \leq 0\right] \tag{6.13}$$

Here, R, Q and S_F are normalized random variables with $E[R] = E[Q] = E[S_F] = 1$ and coefficients of variation V_R, V_Q and V_S. Equation 6.13 is valid only if $k_{\vartheta,min} = p_2\eta_S/Q \leq 1$. The bilinear model proposed in Equation 6.11 is implemented as follows:

$$P_{f|F}(\mathbf{p}) = \begin{cases} P\left[g_1 = p_1 \cdot p_2 \cdot R/Q - S_F \leq 0\right] & \text{if } p_2\eta_S/Q \leq 1 \\ P\left[g_2 = p_1 \cdot R - S_F \cdot \eta_S \leq 0\right] & \text{if } p_2\eta_S/Q > 1 \end{cases} \tag{6.14}$$

The first limit state function g_1 is taken from Equation 6.13, the second refers to the case where the fire has no effect the resistance of the steel member. The fire safety design variable p_2 does not appear in g_2 because in this situation, the reliability cannot be increased any more by further insulation. With the distribution of the random fire load Q it is possible to estimate the probability that the limit state function g_1 or g_2 is relevant in case of a fire event. The probability of structural failure is then assessed as follows:

$$\begin{aligned} P_{f|F}(\mathbf{p}) &= P\left[g_1 \leq 0\right] \cdot P\left[p_2\eta_S/Q \leq 1\right] + P\left[g_2 \leq 0\right] \cdot P\left[p_2\eta_S/Q > 1\right] \\ &\approx P\left[g_1 \leq 0\right] \cdot P\left[p_2\eta_S/Q \leq 1\right] = P\left[p_1 \cdot p_2 \cdot R/Q - S_F \leq 0\right] \cdot P\left[p_2\eta_S/Q \leq 1\right] \end{aligned} \tag{6.15}$$

The second failure mode, $g_2 \leq 0$, implies that the structure fails without any strength reduction due to the fire action ($k_{\vartheta,min} = 1$). This case is very unlikely in the fire situation and is neglected in the following (second line of Equation 6.15).

Equation 6.13 to 6.15 are not applicable if $p_2 = 0$ (no fire protection). To resolve this issue, it is assumed that $P_{f|F} = 1$ if $p_2 = 0$, i.e. the structure always fails in case of a fully developed fire. A more exact solution would require the introduction of another limit state function based on a model for unprotected steel elements exposed to fire.

For the present case study, p_2 is allowed to take on any value between zero and infinity. From Equation 6.12 it is clear that this is not realistic when regarding only one possible approach to increase structural fire safety: In practice, the thickness d_p of the gypsum boards used to protect the steel profile from the fire action cannot be much smaller than around $10mm$. Using the domain of definition $p_2 \geq 0$ implies that the gap between no fire protection at all ($p_2 = 0$) and a minimum insulation with e.g. $d_p = 10mm$ is filled by some other risk reduction measures. To give an example, small values for p_2 may be achieved by choosing a larger steel profile than required for the normal design situation, i.e. by increasing p_1. To avoid double-counting, p_1 is kept unchanged during the optimization of fire safety design and changes in p_1 are modelled implicitly by allowing for small values of p_2.

6.3.2   Defining a reference case

Based on the objective function in Equation 6.2 and the simplified limit state function introduced above, it is possible to find the optimal fire safety design parameter p_2 and the corresponding probability of structural failure in case of fire. The outcome of the optimization depends on the assumptions made regarding the random variables entering the limit state function and regarding the models to describe the fire safety costs and the consequences of structural failure. In the present section, a reference case is defined as a starting point for a parameter study, which is discussed in Section 6.3.3. The example chosen is an office building with the live load as the main mechanical load. The design is made for a protected steel structure, see Section 6.3.1.

Probabilistic models

Table 6.1 contains a summary of the probabilistic models used to describe the reference case. All random variables are normalized by their mean value and therefore can be described by their coefficient of variation and the distribution type; the mean values are included in the design variables p_1 and p_2. Only the ratio $\eta_S = \mu_{S_F}/\mu_S$ between the mean mechanical load in the fire and in the normal design situation is given in Table 6.1, as it is needed for estimating the probability of structural failure in case of fire, see Equation 6.15.

Tab. 6.1: Probabilistic models defining the reference case.

Random variable		Dist.	CoV	Mean	
R	Steel resistance	LN	$V_R = 0.1$	$\mathrm{E}[R] = 1$	
S	Mechanical load effect (1-year max)	LN	$V_S = 0.3$	$\mathrm{E}[S] = 1$	
S_F	Mechanical load effect in fire	LN	$V_{S_F} = 0.6$	$\mathrm{E}[S_F] = 1$	$\Big\}\, \eta_S = 0.5$
Q	Fire load density	LN	$V_Q = 0.3$	$\mathrm{E}[Q] = 1$	

The probabilistic models for loads and resistances are roughly based on the information provided in the JCSS Probabilistic Model Code (JCSS, 2001). The difference between the mechanical load in the normal design situation (S) and the fire situation (S_F) stems from the fact that it is very unlikely to observe a fire at the same time as the maximum mechanical load.

This can be considered by taking into account only the (point in time) sustained load for the fire situation, while for the normal design situation (1-year maximum load), it needs to be combined with the short-term intermittent load effect, see e.g. Rackwitz (1996) or Melchers (1999). The load combination approach proposed by Rackwitz (1996) yields $\mu_{S_F}/\mu_S \approx 0.7$ for η_S; the value of $\eta_S = 0.5$ in Table 6.1 is chosen for simplicity. The standard deviations of the two mechanical loads are similar which leads to $V_{S_F} = V_S/\eta_S$ (assuming $\sigma_{S_F} = \sigma_S$).

Table 6.1 assumes Lognormal distributions for all random variables, which allows the formulation of an analytic solution for Equation 6.13. Other distribution types are implemented using FORM (First Order Reliability Method) for estimating the failure probability.

Cost and consequence model

A simple approach for modelling the (marginal) costs of structural fire safety is to extend Equation 3.8 to account for both structural and fire safety design costs:

$$C(\mathbf{p}) = C_0 + C_1 p_1 + C_2 p_2 \tag{6.16}$$

Here, the marginal costs of structural safety C_1 are defined in the same way as in Section 3.3.1 while C_2 relates to the marginal costs of increasing the fire safety design parameter p_2 by one. The construction costs $C(p)$ are thus assumed to be, at least locally, a linear function of p_2.

The order of magnitude for C_2 can be estimated using a similar approach as the one described in Section 3.3.1 for estimating C_1: After calibrating the fire safety design parameter p_2 to the safety level of a design code currently in practice, the ratio C_2/C_0 is estimated based on a typical share of the total construction costs attributed to structural fire safety. To achieve this, first p_1 must be estimated from the reliability level of the code using the limit state function for the normal design situation, Equation 3.7. For the Eurocode target reliability $\beta_{t,1y} = 4.7$ (corresponding to $P_{f,t,1y} \approx 10^{-6}$ for a 1-year reference period) and the probabilistic models in Table 6.1, this leads to $p_1 \approx 4.1$.

In the Eurocode fire safety concept, the target reliability for structural failure given fire is estimated as a function of the yearly probability of a fully engulfed fire in the compartment, see Equation 6.1. Schleich et al. (2002) assume that $\lambda_I P_{F|I}$ is proportional to the compartment size. For their reference case, an office room with $A_f = 25m^2$, they get $\lambda_I P_{F|I} = 1.25 \cdot 10^{-5} [1/a]$, which leads to $P_{f,t|F} = 10^{-6}/(1.25 \cdot 10^{-5}) \approx 0.1$ and a corresponding reliability index $\beta_{t,F} \approx 1.3$. Based on this target safety level, the probabilistic models in Table 6.1 and the simplified limit state function in Equation 6.13, we get $p_1 p_2 \approx 2$ and thus $p_2 = 2/4.1 \approx 0.5$.

According to the estimates provided by The Geneva Association (2012), fire safety costs amount to around $2.5 - 5\%$ of the construction costs. These figures represent an upper limit as they include all kinds of fire safety measures, not only structural fire safety. Using 5% as an estimate for the structural fire safety costs, an upper bound for C_2/C_0 can be derived as $0.05/p_2 \approx 0.1$. Assuming $p_2 \cdot C_2/C_0 = 1\%$ yields $C_2/C_0 = 0.01/p_2 \approx 0.02$.

Table 6.2 provides a summary of all cost and consequence parameters defining the reference case for the parameter study in Section 6.3.3. All cost components are given in relation to the "fixed" construction costs C_0.

Tab. 6.2: Costs and consequence parameters defining the reference case.

Parameter	Meaning	Parameter	Meaning
$C_0 = 1$	Fixed construction costs	$H/C_0 = 2$	Failure consequences
$C_1/C_0 = 10^{-3}$	Structural safety costs	$\omega = 0.02$	Obsolescence rate
$C_2/C_0 = 10^{-2}$	Fire safety design costs	$\gamma = 0.03$	Discount rate

6.3.3 Optimization results and parameter study

In the following, the results of the design optimization are discussed together with the influence of the different parameters used in the model. The optimization is performed in two steps: First the design parameter p_1 for the normal design situation is optimized based on the approach discussed in Section 3.3.1. Given this value, the fire safety design parameter p_2 is chosen according to the design objective formulated in Equation 6.2.

As a starting point, it is assumed that in average one fully developed fire occurs each year ($\lambda_I P_{F|I} = 1/a$) and that the failure probability may be estimated using Equation 6.13 (no upper limit for $k_{\vartheta,min}$). Based on these assumptions, a rough estimate for the optimal reliability is derived on a yearly basis. Thereafter, these results are compared with a more realistic approach using different values for $\lambda_I P_{F|I}$ and Equation 6.15 for the probability of failure.

The parameters defined in Section 6.3.2 (Table 6.1 and 6.2) are used as a reference case for the parameter study, where only one or few parameters are varied at once. The obsolescence rate ω and the discount rate γ are kept constant as their influence directly becomes clear from Equation 6.2: Increasing $(\omega + \gamma)$ has the same effect as increasing the construction costs $C(\mathbf{p})$, or both C_1 and C_2.

Approximation on a yearly basis

Figure 6.2 illustrates the optimization results for the assumption $\lambda_I P_{F|I} = 1/a$ (one fully developed fire per year). In Figure 6.2a, the optimal failure probability on a yearly basis, $P_{f,t,1y}$, is plotted as a function of the marginal costs of structural fire safety C_2 for different structural safety costs C_1 and failure consequences H. The dependency on C_2 is similar to that on C_1 for the normal design situation (Figure 3.2): Increasing C_2 by an order of magnitude also leads to an approximate increase of the optimal failure probability by an order of magnitude. The effect of C_1 on the optimal fire safety design is negligible as it influences $P_{f,t,1y}$ only indirectly by the value for p_1 resulting from the "cold" optimization. The values chosen for H/C_0 correspond to the range defining the "moderate" consequences of failure in Table 3.1. Figure 6.2b shows the optimal failure probability $P_{f,t,1y}$ for different coefficients of variation V_R, V_S and V_Q. The coefficient of variation of the mechanical load in fire S_F is determined as $V_{S_F} = V_S/\eta_S$. Note that this is the only effect of η_S when estimating the probability of failure using Equation 6.13.

Some of the parameters that are varied in Figure 6.2 affect both the results for the normal design situation and for the fire safety design. Therefore, in Figure 6.3 it is explored how the optimal failure probability for the fire situation, $P_{f,t,1y}$, depends on the safety target for the
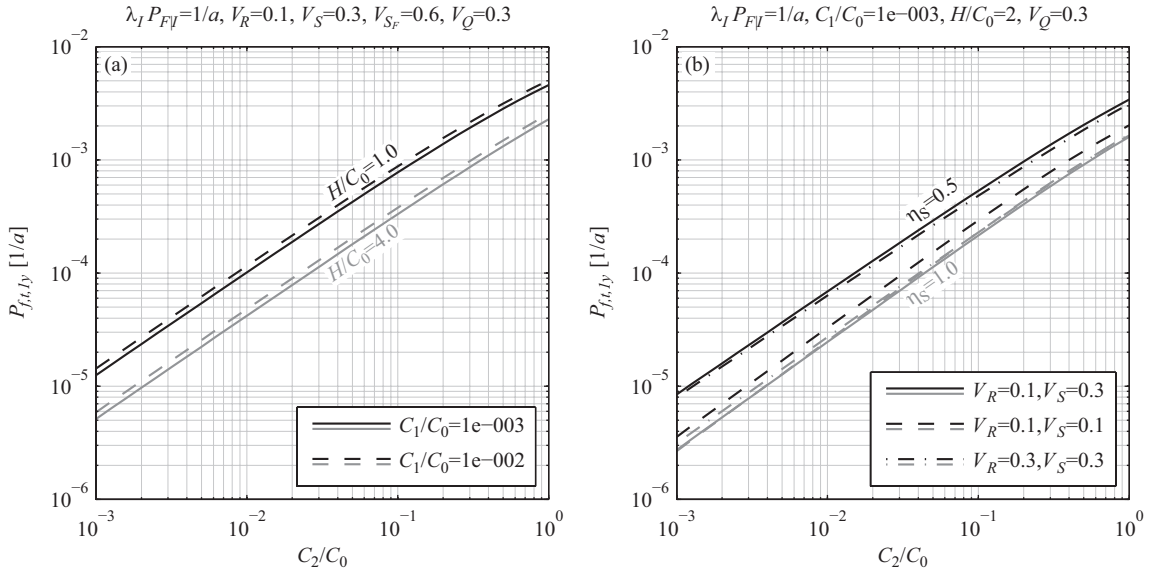
Fig. 6.2: Optimal fire safety targets on a yearly basis $P_{f,t,1y}$ (assuming $\lambda_I P_{F|I} = 1/a$) as a function of the marginal costs of structural fire safety C_2/C_0 for (a) different failure consequences H/C_0 and (b) different mechanical load ratios η_S.
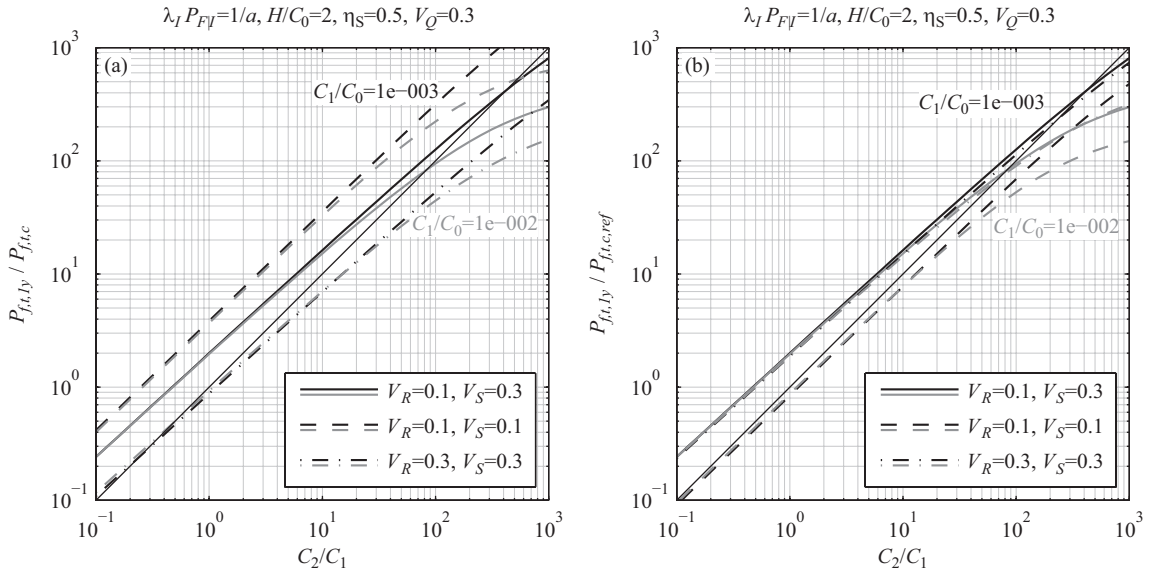
Fig. 6.3: Optimal fire safety targets on a yearly basis $P_{f,t,1y}$ (assuming $\lambda_I P_{F|I} = 1/a$) in relation to (a) the corresponding safety targets for the normal design situation, $P_{f,t,c}$, and (b) to a reference case for the cold design, $P_{f,t,c,ref}$ (based on the probabilistic models in Table 6.1).

"cold" design, $P_{f,t,c}$ (both probabilities related to a 1-year reference period). The effect of the failure consequences is identical for both situations and cancels out. Therefore, the consequence parameter is set to $H/C_0 = 2$ and kept constant in the following. Figure 6.3a shows that the ratio $P_{f,t,1y}/P_{f,t,c}$ is approximately equal to the ratio between the fire safety design costs C_2 and the costs of structural safety C_1. This rule-of-thumb can even be improved if the cold design is optimized for a reference case, see Figure 6.3b. Here, the coefficients of variation V_R and V_S (or

V_{S_F}, respectively) are varied only during the estimation of the fire safety target $P_{f,t,1y}$ while the cold design $P_{f,t,c,ref}$ is based on the reference case $V_R = 0.1$ and $V_S = 0.3$.

Qualitatively, the results shown in Figure 6.3 are intuitive: The fire safety design should be based on a lower target reliability (defined for a 1-year reference period) than appropriate for the "cold" design if it is more costly to increase structural safety in fire than in the normal design situation, and vice versa. However, the approximate equality $P_{f,t,1y}/P_{f,t,c} \approx C_2/C_1$ (or $P_{f,t,1y}/P_{f,t,c,ref} \approx C_2/C_1$) is somewhat surprising and may be specific to the modelling approach followed in this case study. A linear dependency can be expected as long as $P_{f,t,1y}$ is approximately proportional to C_2/C_0, as shown in Figure 6.2. Depending on the definition of p_2, the proportionality constant can, however, be smaller or larger than one.

Failure probability given fire

The results discussed above suggest a simple approximation for the target reliability in structural fire safety design: First, the optimal failure probability on a yearly basis is approximated as $P_{f,t,1y} \approx (C_2/C_1) \cdot P_{f,t,c,ref}$, where the target reliability for the cold design is estimated for a reference case regarding the coefficients of variation, e.g. $V_S = 0.3$ and $V_R = 0.1$. This yearly value is divided by the fire rate $\lambda_I P_{F|I}$ in order to obtain an estimate for the target failure probability given a fully developed fire, see Equation 6.1. In the following, it is investigated whether the resulting safety targets are consistent with the approach outlined in Section 6.2.1.

Figure 6.4 shows a comparison between the optimal failure probability conditional on a fully developed fire, $P_{f,t|F}$, and different approximations based on yearly safety targets. The "exact" solution for $P_{f,t|F}$ on the x-axis is derived based on the objective function in Equation 6.2 and Equation 6.15 for the failure probability (i.e. based on a bilinear model for $k_{\vartheta,min}$). The calculations were performed for different fire rates $\lambda_I P_{F|I}$. In Figure 6.4a, $P_{f,t|F}$ is compared with the approximation $P_{f,t,1y}/(\lambda_I P_{F|I})$ for different coefficients of variation V_R and V_S. The yearly value $P_{f,t,1y}$ is derived as above, i.e. assuming $\lambda_I P_{F|I} = 1/a$ and using a linear model for $k_{\vartheta,min}$ (Equation 6.13). For Figure 6.4b, the target reliability on a yearly basis, $P_{f,t,1y}$, was approximated by multiplying the optimal failure probability for the "cold" design $P_{f,t,c,ref}$ with the ratio between the marginal costs of increasing p_1 and p_2. As in Figure 6.3b, the reference values $V_S = 0.3$ and $V_R = 0.1$ were used for deriving $P_{f,t,c,ref}$. The resulting approximation is fairly good, as can be seen by comparing the curves in Figure 6.4b with the diagonal.

To complete the parameter study, Figure 6.5 shows the same comparison as in Figure 6.4b, but for different values for V_Q and η_S. Both parameters are not accounted for in the approximate solution on the y-axis. The coefficient of variation of the fire load density, V_Q does not have a strong influence on the results, as can be seen in Figure 6.5a. A larger effect is observed when varying η_S, which affects the distribution of the mechanical load in the fire situation, S_F. The effect on the mean mechanical load is accounted for in the fire safety design, i.e. when choosing p_2 (see Equation 6.12). Therefore, the main effect of η_S is that it defines the coefficient of variation of the mechanical load, $V_{S_F} = V_S/\eta_S$, which is why the target failure probability $P_{f,t|F}$ increases with growing η_S.
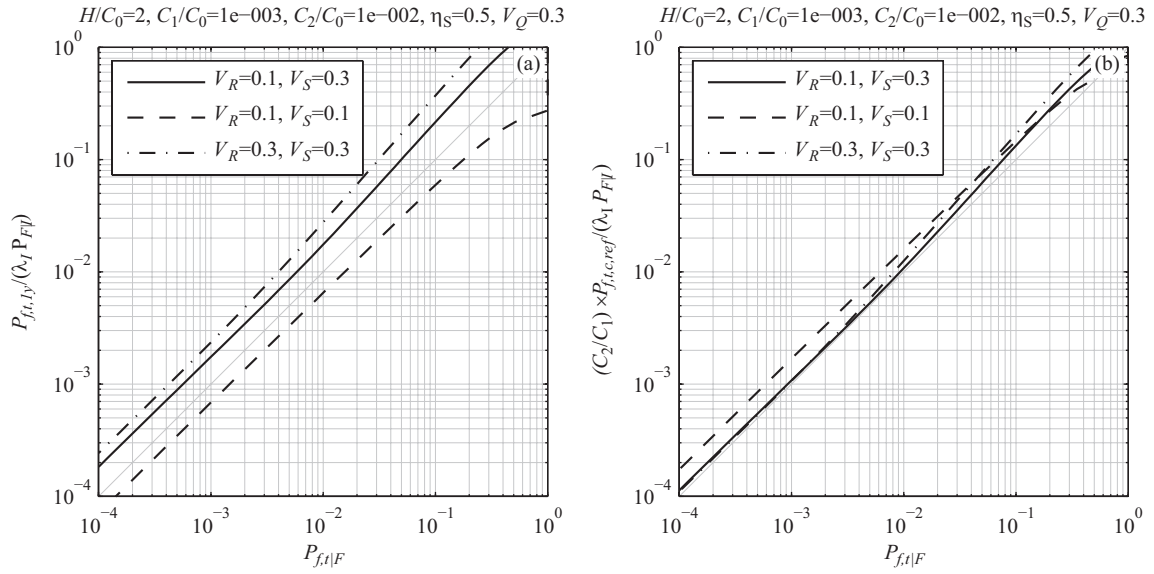
Fig. 6.4: Comparison between the optimal failure probability given a fully developed fire, $P_{f,t|F}$, and the approximation based on yearly fire safety targets derived from (a) the "hot" optimization assuming $\lambda_I P_{F|I} = 1/a$ (Figure 6.2) and (b) the approximation in Equation 6.17.
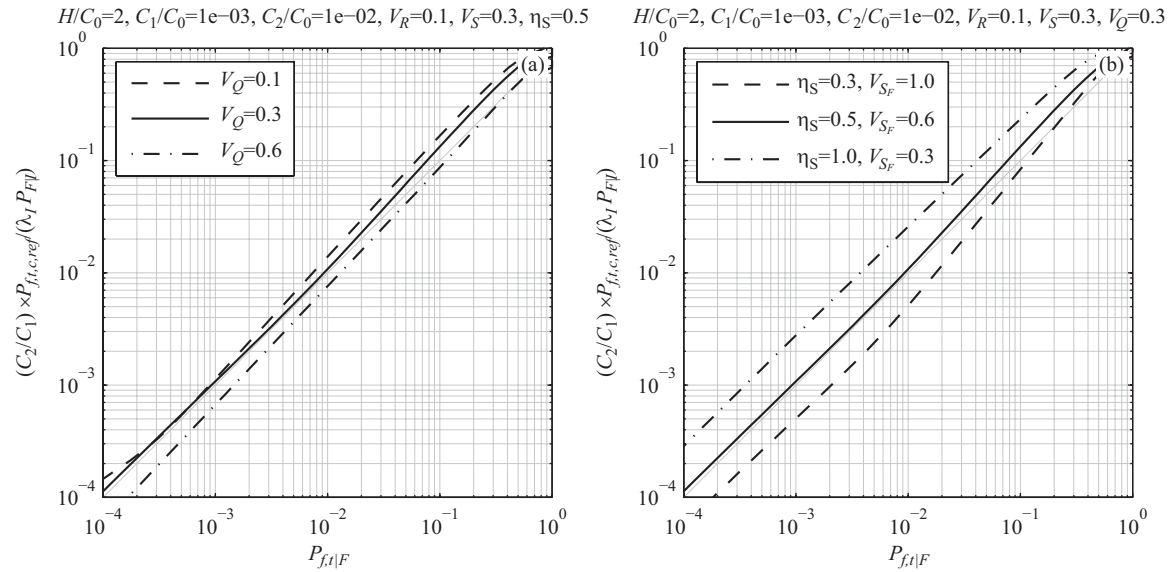
Fig. 6.5: Influence of (a) the coefficient of variation for the fire load density V_Q and (b) the mechanical load ratio η_S on the approximation in Equation 6.17.

In summary, it may be concluded that the target value for the probability of structural failure given a fully developed fire, $P_{f,t|F}$, can be approximated fairly well by scaling the optimal failure probability for the normal design situation, $P_{f,t,c,ref}[1/a]$, to account for the marginal costs of increasing structural fire safety and for the rate of fire occurrence:

$$P_{f,t|F} \approx \frac{C_2}{C_1} \frac{P_{f,t,c,ref}}{\lambda_I P_{F|I}} \tag{6.17}$$

The model parameters summarized in Table 6.1 and 6.2 are either explicitly or implicitly considered in Equation 6.17, or their effect on the results is relatively small. The only exception is the parameter η_S: Depending on the value chosen for this parameter, the approximation may over- or underestimate the "correct" value for $P_{f,t|F}$ by a factor of two to four. It should, however, be noted that the assumptions made in this case study to derive the distribution of S_F from the random variable S are fairly simplistic. In practice, it is difficult to define the relation between the two mechanical loads in a general way that is appropriate for all types of loadings. Considering that the effect of η_S is clearly below an order of magnitude for the optimal failure probability, it is proposed to neglect the influence of this parameter.

Effect of distributional assumptions

The parameter study in Figure 6.2 to 6.5 was based on the assumption that the three random variables R, Q and S (or S_F, respectively) follow a lognormal distribution. In Figure 6.6 this assumption is relaxed for the load effect S and the resistance R. As before, the optimal failure probability given a fully developed fire, $P_{f,t|F}$, is compared with the approximation based on Equation 6.17. The reference case for the normal design situation (i.e. $P_{f,t,c,ref}$) is here defined by $V_R = 0.1$, $V_S = 0.3$ and lognormal (LN) distributed random variables.
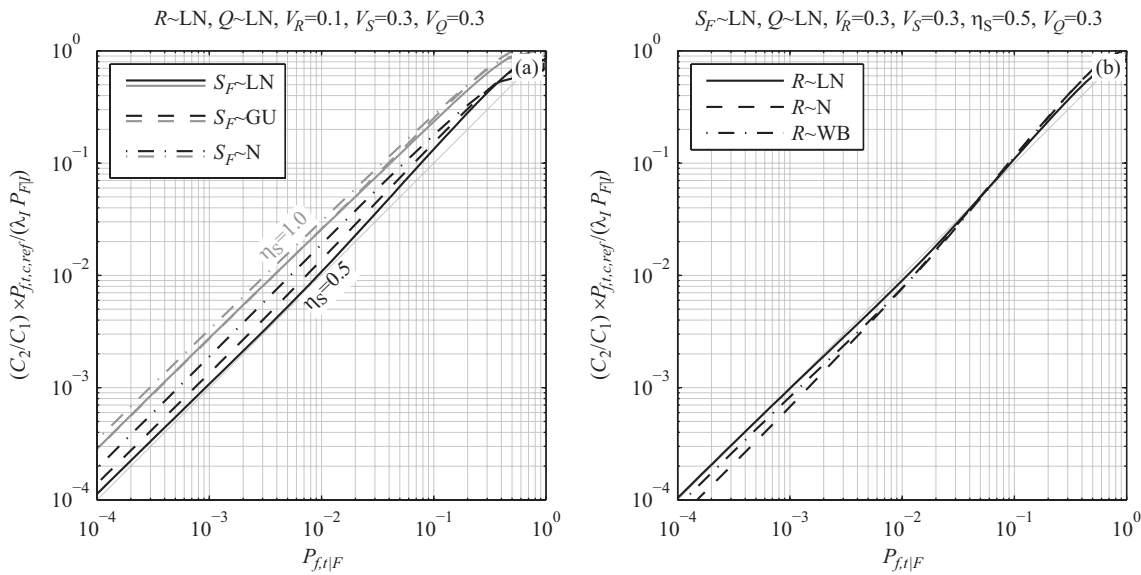
Fig. 6.6: Effect of distributional assumptions for (a) the mechanical load effect S_F (with R lognormally distributed) and (b) the resistance R (with S_F lognormally distributed) on the approximation in Equation 6.17 (Cost and consequence variables according to Table 6.2).

Figure 6.6a shows the effect of changing the distribution of the mechanical load in the fire situation, S_F, to Gumbel max (GU) or Normal (N). The difference between the distributions depends on the value chosen for η_S due to the assumption that $V_{S_F} = V_S/\eta_S$; it is, however, smaller than the effect of η_S itself. The effect of the distribution for the resistance R is investigated in Figure 6.6b, where WB refers to the Weibull min distribution. The two figures show that the distributional assumptions for the basic random variables R and S_F affect the safety targets for

structural fire safety even less than the target reliabilities for the normal design situation (see Figure 3.3 for comparison). It can be shown that this is also true for the distribution of the fire load density, e.g. when assuming a Gumbel max instead of the lognormal distribution for Q. This insensitivity to distributional assumption goes in line with the relatively small influence of the coefficients of variation V_R, V_{S_F} and V_Q observed in the parameter study above.

6.3.4 Safety targets with active fire protection

In the preceding sections, passive fire protection was assumed to be the only measure to reduce the probability of structural failure in case of fire. The framework can, however, also account for the effect of active fire protection, e.g. a sprinkler system. To achieve this, a third decision variable p_3 is introduced that takes the value $p_3 = 1$ if the building is sprinklered and $p_3 = 0$ if no sprinkler is present. Active fire protection may be assumed to reduce the probability of a fully developed fire, $P_{F|I}$, and possibly also the consequences given a large fire event. In the following, it is assumed that reducing $P_{F|I}$ is the only effect of p_3. Formally this is achieved by introducing a reduction factor χ for the probability of a fully developed fire:

$$P_{F|I}(p_3 = 1) = \chi \cdot P_{F|I}(p_3 = 0) \tag{6.18}$$

Where the factor $0 \leq \chi \leq 1$ denotes the probability that the sprinkler system fails to prevent a fully developed fire. The construction cost formula in Equation 6.16 is extended to include also the sprinkler costs:

$$C(\mathbf{p}) = C_0 + C_1 p_1 + C_2 p_2 + C_3 p_3 \tag{6.19}$$

In contrast to C_1 and C_2, which denote marginal costs, the active fire protection costs C_3 are assessed in absolute terms: C_3 is defined as the present value of all costs attributed to the sprinkler system, i.e. construction, inspection, maintenance and renewal costs.

With the assumptions in Equation 6.18 and 6.19, the cost optimization in Equation 6.2 can be performed with $p_3 = 0$ and $p_3 = 1$ to evaluate the effect of a sprinkler system on the target reliability for structural fire safety. For simplicity, the same failure model is used as before. It should, however, be noted that the limit state function in Equation 6.13 was developed for protected steel columns and is not appropriate for small values of p_2 that become relevant for the sprinkler solution. It is thus assumed that no passive fire protection is present if $p_{2,opt}$ remains below 0.1 and that this leads to $P_{f|F} = 1$, i.e. the structure always fails if a fully developed fire occurs. A more elaborate failure model based on simulations for unprotected steel columns would be necessary to replace these simplifying assumptions.

Figure 6.7a shows the total costs $T(\mathbf{p})$ according to Equation 6.2 as a function of the passive fire protection parameter p_2. The total costs with and without a sprinkler system are evaluated for two different fire rates $\lambda_I P_{F|I}(p_3 = 0)$. The sprinkler solution is more costly than the design with passive fire protection, even though it becomes more cost efficient with growing fire rates (assuming sprinkler costs $C_3/C_0 = 10^{-2}$ and reduction factor $\chi = 10^{-2}$). However, Equation 6.2 does not capture the full effect of sprinklers on the expected costs in case of a fire, as only

costs due to structural failure are accounted for. The efficiency of the sprinkler system can be evaluated more precisely based on Equation 3.22 for estimating the monetary fire losses.
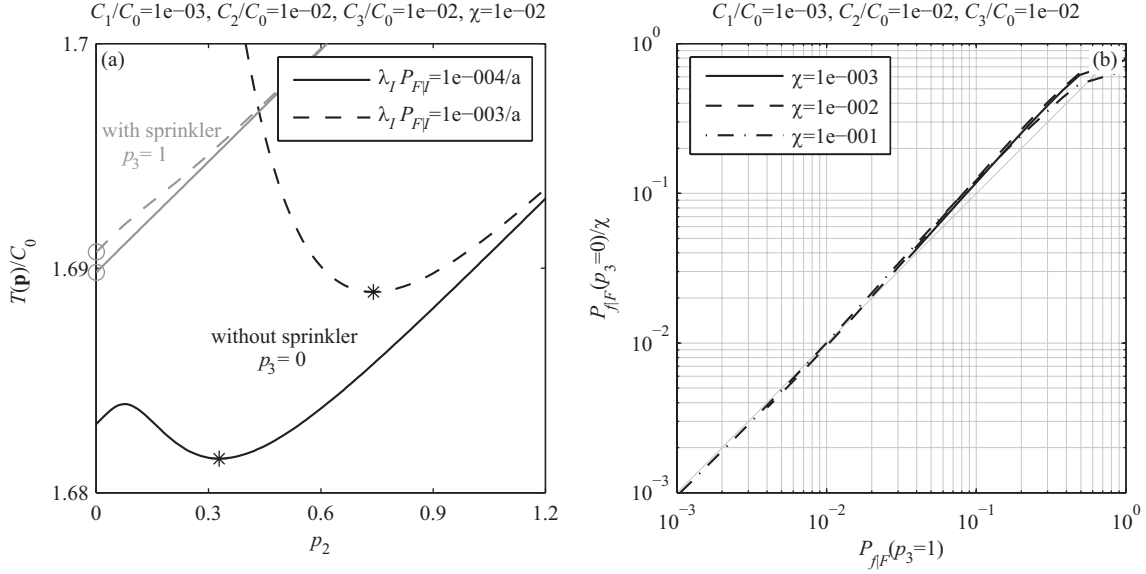
Fig. 6.7: Optimal structural fire safety with and without sprinkler: Total costs $T(\mathbf{p})$ as a function of the decision variable for structural fire safety p_2 (a) and optimal failure probability given a fully developed fire $P_{f|F}$ for varying sprinkler failure probabilities χ (b) (see Table 6.1 and 6.2 for parameter assumptions).

In the two examples shown in Figure 6.7a, $p_2 = 0$ (no passive fire protection) is the optimal design for sprinklered buildings. $p_2 > 0$ may be optimal for higher fire rates, but the sprinkler solution will always require less passive fire protection than the design without sprinkler. This difference can be accounted for by using the same annual safety targets for structural design, but a different fire rate for deriving the target reliability given fire, see Equation 6.1. The difference between the optimal structural design with and without sprinkler is then fully explained by the reduction factor for the fire rate, χ:

$$P_{f,t|F}(p_3 = 1) \approx P_{f,t|F}(p_3 = 0)/\chi \leq 1 \tag{6.20}$$

$P_{f,t|F}(p_3 = 1) = 1$ implies that no structural fire safety is required in the sprinkler solution. Figure 6.7b shows a comparison between the results of a direct optimization for $P_{f,t|F}(p_3 = 1)$ for different fire rates (on the x-axis) and the approximation according to Equation 6.20 on the y-axis. It is seen that the approximation is fairly accurate. Varying the reduction factor χ has almost no effect at all. This implies that Equation 6.20 may be used also for other types of active fire protection measures reducing the probability of a fully developed fire $P_{F|I}$ or the rate of fire ignition λ_I. Similar results are achieved when using Equation 6.17 to approximate the safety target without sprinkler, $P_{f,t|F}(p_3 = 0)$.

6.3.5   Discussion and Outlook

The results presented in this case study can be used to define target reliabilities for structural fire safety based on monetary optimization. The calculations were performed for protected steel

structures, but the approach followed is applicable also to other building materials. In the following, a simple target reliability format is proposed followed by a discussion of the most important open questions and suggestions for further research.

A simple target reliability format for structural fire safety

Target reliabilities for structural fire safety can be defined based on Equation 6.17: The annual target failure probability for the normal design situation is multiplied by the cost ratio C_2/C_1 in order to obtain the target level of safety for the fire design related to a 1-year reference period. This ensures that the fire safety targets are reduced if it is very costly to increase structural safety in case of fire compared to the normal design situation. As the marginal costs of structural safety, C_1 and especially C_2, are not quantifiable in a very intuitive way, it is probably helpful to define groups of buildings with proposed values for the ratio C_2/C_1. The consequences of structural failure are implicitly accounted for in the target reliability for the "cold" design, see e.g. the JCSS target reliabilities in Table 3.1. The influence of the coefficients of variation and distributional assumptions for the random variables summarized in Table 6.1 is small, as could be shown by the parameter study. They may, however, influence the estimate for C_2 (especially the variability of the fire load density Q).

The target reliability in the fire situation furthermore depends on the ignition frequency and the probability of a fully developed fire given ignition. This is considered by dividing the annual target probability of structural failure by the fire rate, see Equation 6.1 and 6.17. Risk reduction measures reducing the frequency of severe fire events may be accounted for in this step.

The biggest advantage of a target reliability format based on Equation 6.17 is its simplicity. It is consistent with the approach formulated by Rackwitz (2000) (see Section 3.3.1), which allows the derivation of the safety targets for the fire design from the target reliabilities for the normal design situation. As was discussed in Section 6.1, the Eurocode simply assumes the same level of safety for both design situations. This is consistent with the results presented in this case study if $C_2/C_1 = 1$. The rough estimates provided in Section 6.3.2 suggest that the ratio C_2/C_1 might in fact be larger than one, permitting a lower safety level for the fire situation. It is, however, also possible that $C_2/C_1 \approx 1$, especially when regarding other building materials than protected steelwork.

Open questions and suggestions for future research

The model used in this case study to estimate the probability of structural failure was developed based on simulations for a protected steel component in a fire event typical for office buildings. Before making generalizations, it needs to be checked whether the simple limit state function developed in Section 6.3.1 is applicable for other construction types and fire conditions, too. Here, especially the assumed linearity between the decision variable p_2, the safety margin defined by the limit state function in Equation 6.13 and the construction costs in Equation 6.16 needs to be verified. Another important simplification is that the same model was used for any level of structural fire safety, including very small values for $p_2 > 0$. This may, however, be a problem

specific to structural steelwork, where high levels of structural fire safety are typically achieved by insulation.

It is likely that different values for the ratio C_2/C_1 are appropriate when regarding different types of construction. Nevertheless, such cost differences should not be used to justify reliability differentiation between different building materials, as this would punish the construction types where structural fire safety can be achieved most efficiently. A simple solution to this problem is to first optimize the target reliability individually for all relevant types of construction. A common safety level may then be defined based on the following minimization:

$$P_{f,t} = \min_{P_{f,t}} \left\{ \sum_{i=1}^{k} w_i \left(P_{f,t,i} - P_{f,t}\right)^2 \right\} \quad \text{or} \quad \beta_t = \min_{\beta_t} \left\{ \sum_{i=1}^{k} w_i \left(\beta_{t,i} - \beta_t\right)^2 \right\} \tag{6.21}$$

Here, the counter i refers to k different construction types with respective weights w_i considering e.g. the relative shares of the building materials in the overall construction economy. The optimization approach in Equation 6.21 is similar to the one proposed by Sørensen et al. (1994) in the context of reliability-based code calibration, see also Faber and Sørensen (2003). An alternative solution would be to optimize a common target reliability β_t based on a weighted sum of the objective functions derived for different types of materials and construction.

Following Equation 6.17, the safety target for structural fire safety is derived from the target reliability for the normal design situation. Applying the results in practice requires a clear definition whether these target reliabilities relate to the design of structural components or whole structures, see Section 3.3.1 for discussion. The target reliabilities in EN 1990 (2002) are defined at the level of structural components. Taking this definition implies that the probability of a fully developed fire, $\lambda_I P_{F|I}$, is estimated for the room or fire compartment where the component is located. This definition allows to differentiate fire safety design within a building according to the probability of fire occurrence, which is consistent with the design approach currently used in practice.

The focus of this case study was on the derivation of target reliabilities for the fire situation based on monetary optimization. This may be sufficient in many buildings where it is clear that the optimal solution will also be acceptable from a life saving point of view, see Section 6.2.2 for discussion. For buildings with long total evacuation times, the approach has to be complemented by a target reliability format based on the LQI acceptance criterion. The general approach is described in Section 6.2.2. A generic model for estimating the fatality risk due to structural failure in a fire event is needed for applying the framework in practice.

Chapter 7

Case Study 3: Smoke alarms

The last case study illustrates the application of the framework developed in Section 4.4 to the calibration of a generic fire risk model with Swiss insurance data. The model is then used to estimate the effect of home smoke alarms on monetary fire losses. The efficiency of smoke alarms as a life-saving measure is evaluated based on the LQI criterion introduced in Section 2.3. The content of this chapter is based on Fischer et al. (2013a) for the calibration and Fischer et al. (2012b) for the evaluation with the LQI criterion.

7.1 Judging the efficiency of home smoke alarms

The installation of smoke alarms in residential buildings is currently not mandatory in Switzerland, but it is discussed whether the fire safety code should be changed in this regard. Smoke alarms help to detect fires at an early stage of development, providing the building occupants time to fight the fire or to evacuate and call the fire brigade. In addition, home smoke alarms are easy to install and relatively cheap compared to other fire safety measures. Therefore, it is often assumed that smoke alarms are efficient in reducing risk to life and monetary losses due to home fire events.

In the present case study, the effect of home smoke alarms on monetary losses and their life-saving effect are regarded separately. The monetary risk reduction is estimated with the aid of a generic fire risk model for single family houses developed by De Sanctis et al. (2013a). To facilitate an absolute risk assessment, the model is calibrated to Swiss insurance data using the procedure developed in Section 4.4. After calibration and validation, the model can be used to support decisions regarding investments into safety measures both at object and at portfolio level. The code-making decision regarded in this case study is a typical example for a decision made at portfolio level.

The life saving effect of home smoke alarms is estimated based on a literature review and national and international data. The efficiency of mandatory smoke alarms in residential buildings is then evaluated based on the LQI net benefit criterion (Equation 2.17). At the end of this chapter, the interaction between monetary optimization and the LQI criterion is discussed together with the implications of the results for societal decision-making.

7.2 Calibration of a generic fire risk model to insurance data

In the following, the calibration procedure described in Section 4.4 is applied to the calibration of a generic fire risk model for single family houses. The model is described in detail by De Sanctis et al. (2013a). Herein, only a short introduction to the model is provided.

7.2.1 Short introduction to the engineering risk model

In the generic risk model, each house is described by a set of building-specific risk indicators listed in Table 7.1. The table also contains the definition of some fire-specific risk indicators and the model calibration parameters. For a complete list of all indicators see De Sanctis et al. (2013a). The model estimates the probability distribution of the financial loss due to a fire for a given set of risk indicators. Loss of life and injuries are not within the scope of the model. The monetary losses refer to damages at the building structure only. Loss of contents and consequential losses are excluded for consistency with the data set used for calibration (see Section 7.2.2).

Tab. 7.1: Risk indicators and parameters of the generic risk model (upper case for random variables, lower case for realisations).

Building-specific risk indicators (Model Input)	X	x
Total building floor area $[m^2]$	A_{tot}	a_{tot}
Floor area of largest room $[m^2]$	A_{max}	a_{max}
Number of rooms $[-]$	N_R	n_R
Number of connections between rooms $[-]$	N_C	n_C
Insured value $[CHF]$	V	v
Fire-specific risk indicators		
Floor area of room of fire ignition $[m^2]$	A_0	a_0
Area of fire spread $[m^2]$	A_d	a_d
Model output	**Y**	**y**
Financial loss (building structure) $[CHF]$	C	c
Calibration parameters	$\boldsymbol{\Theta}$	$\boldsymbol{\theta}$
Distribution parameters for minor loss model	$[\Lambda, Z]^\top$	$[\lambda, \zeta]^\top$
"Fire spread coefficient"	Ψ	ψ
"Control time exponent"	K	κ

An overview on the model structure can be found in Figure 7.1. The model consists of several sub-models, each of which will be shortly described in the following.

Ignition model (Exposure)

The focus of this section is on the calibration of a model for the fire risk conditional on a fire. To assess yearly risk, the yearly rate of fire occurrence needs to be known. Both models (fire occurrence and consequences given fire) have to use the same definition of a fire event. In this
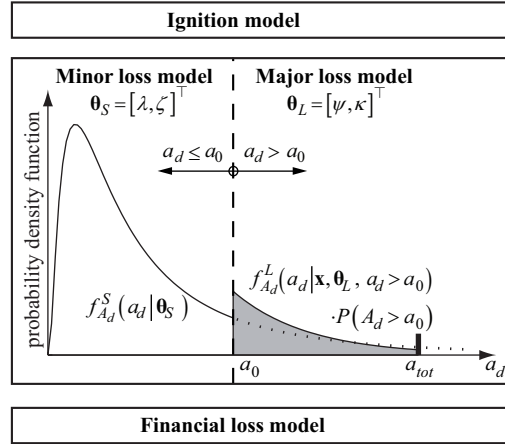
Fig. 7.1: Overview on the components of the engineering model described in De Sanctis et al. (2013a).

case study, the fire rate is determined as a function of the insured value in CHF. The parameters were fitted to Swiss insurance data for residential buildings with an insured value below 1Mio. CHF, see Fischer et al. (2012b). "Fire occurrence" implies that the fire has been reported to the insurance company. The same data is used to calibrate the loss model, see Section 7.2.2.

Minor loss model (Vulnerability)

The analysis of fire insurance statistics shows that the sum of fire losses is dominated by large losses, see e.g. Fontana et al. (1999). For the single family house model this means that small losses, e.g. those resulting from fires confined to the room of fire origin, are of minor importance for the expected loss. Therefore, these "minor losses" are modelled based on a simplified statistical approach, with engineering modelling focusing on the tail of the loss distribution, see Figure 7.1. The minor loss model $f^S_{A_d}(a_d|\boldsymbol{\theta}_S)$ uses a (shifted) log-lognormal distribution for the fire spread area A_d. The distribution is independent of the building-specific risk indicators and used only for small losses confined to the room of fire origin ($a_d \leq a_0$). The distribution parameters $\boldsymbol{\theta}_S = [\lambda, \zeta]^\top$ are estimated from the data, while the shift is fixed and introduced only to ensure positive values in the logarithm. The probability mass in the tail of the loss distribution is redistributed according to an engineering model, the "major loss model". The minor loss model here only provides the probability of fire spread beyond the room of fire origin:

$$P(A_d > a_0) = 1 - F^S_{A_d}(a_0|\boldsymbol{\theta}_S) \tag{7.1}$$

Major loss model (Robustness)

The engineering model for large losses with fire spread beyond the room of fire origin ($a_d > a_0$) is composed of a fire spread model and a fire brigade model. The calibration parameters $\boldsymbol{\theta}_L = [\psi, \kappa]^\top$ are related to these two sub-models. The fire spread model describes the development of the area damaged A_d as a function of time conditional on the risk indicators $[a_{tot}, a_{max}, n_R, n_C]^\top$ (see Table 7.1). The model is modified by the "fire spread coefficient" ψ. The influence of this calibration parameter is such that for higher ψ, the fire develops faster and vice versa.

The fire brigade model is based on a simple time-line approach: First, the starting time of fire brigade actions is defined as a random variable. Next, the average time needed to extinguish or confine the fire ("control time") is modelled as a power function of the area damaged at the starting time, which is estimated using the fire spread model. The exponent defining the shape of this control time model is the second calibration parameter κ. Finally, the area damaged A_d at the end of the fire brigade actions is again determined based on the fire spread model.

Financial loss model

Both the minor and the major loss model are defined in terms of the "fire spread area" A_d, which should be understood mainly as a proxy for the monetary fire loss, see De Sanctis et al. (2013a). For the conversion to financial losses it is assumed that the ratio between the monetary loss C and the insured value V is the same as the ratio between the area damaged A_d and the total floor area A_{tot}, i.e. $C = V \cdot A_d/A_{tot}$.

7.2.2 Description of the data set used for calibration

The data set used for calibration was extracted from the fire loss data provided by AGV, the public building insurance company in the canton of Aargau (see also Section 4.2.1). The data includes also small losses, as no excess (deductible) is borne by the policy holders. Only the building structure is insured by AGV; losses to contents and consequential losses are insured on the private market. The data provides information on all claims due to fires in single family houses (detached, semi-detached and row houses) submitted to AGV from 1999 to 2008. The resulting data set contains the following information on $n = 1996$ fire events: The building's insured value, its year of construction, its volume in m^3 and the fire loss amount.

A comparison with the risk indicators in Table 7.1 reveals that only for the insured value V and the financial loss C information is readily available. The missing building-specific risk indicators were estimated based on the evidence provided by the data. This was done partly using deterministic assumptions as in the case of the total floor area A_{tot} which was calculated from the building's volume assuming a room height of $2.7m$. For the remaining indicators N_R, N_C and A_{max}, probability distributions were derived using a two-step procedure: First the probability distribution of the number of rooms N_R was derived based on the building's volume and the year of construction. For this task, statistical information published by the Swiss Federal Statistical Office (BFS, 2013) could be used. The number of connections between rooms, N_C, and the size of the largest room relative to the total floor area, A_{max}/A_{tot} were modeled conditional on the number of rooms N_R. No statistical information could be found on these two risk indicators. In order to come to reasonable estimates, the author conducted a survey of typical single family house layouts found on the online real estate portal homegate (www.homegate.ch). These estimates can be expected to be very uncertain. An illustration of the procedure used to estimate the building-specific risk indicators is found in Figure 7.2.

The limitations of the data set used for calibration are obvious: Almost all building-specific risk indicators in Table 1 had to be estimated based on assumptions, and information on fire
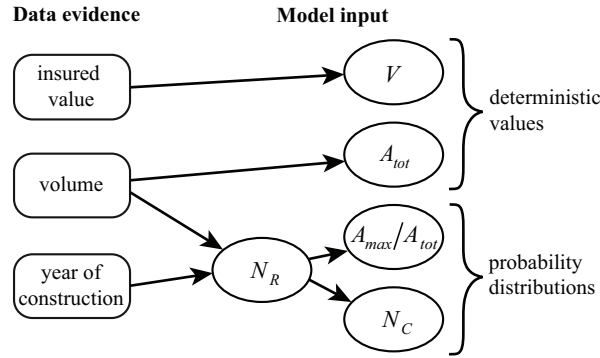
Fig. 7.2: Illustration of the assumptions made to estimate the building-specific risk indicators (model input) from the evidence provided by the data.

development, fire brigade actions and risk reduction measures is missing completely. Another factor is the small number of large losses (only 10 out of 1996 claims are larger than 500'000 CHF), especially for the calibration of model components that mainly influence the tail of the loss size probability distribution. The amount of data can be increased e.g. by extending the data set to a longer time period. Improving the quality of the data requires more effort, but could be aimed at in the future: All risk indicators listed in Table 7.1 are, at least in principle, quantifiable.

7.2.3 Application of the calibration procedure

In the following, it is shown how the single family house model was calibrated to the observed loss data. As discussed in Section 7.2.1, the model is composed of a simple minor loss model for fires confined to the room of fire origin (small losses) and a more complex engineering model for major losses. The respective calibration parameters are $\boldsymbol{\theta}_S = [\lambda, \zeta]^\top$ for the minor loss model and $\boldsymbol{\theta}_L = [\psi, \kappa]^\top$ for the major loss model. The characteristics of each building i are described by the random vector $\mathbf{X}_i = [A_{tot,i}, A_{max,i}, N_{R,i}, N_{C,i}, V_i]^\top$ (see Table 7.1 for variable definitions). As the data set contains no evidence on most of the building-specific risk indicators, the model input is generated by a discrete probability distribution $p_{\mathbf{X}|\mathbf{Z}}(\mathbf{x}|\mathbf{z})$ for the random vector $\mathbf{X}_i$ conditional on the information available in the data set. The data input is represented by a vector $\hat{\mathbf{z}}_i$ containing the insured value, the volume and the year of construction for each individual building. The model output is defined as the financial loss in case of fire, i.e. $Y_i = C_i$.

The incomplete data set requires estimating the likelihood based on Equation 4.16, which is computationally expensive. The number of model evaluations per observation could be reduced by assuming that combinations of risk indicators with $p_{\mathbf{X}_i|\mathbf{Z}_i}(\mathbf{x}_{ij}|\hat{\mathbf{z}}_i) < 10^{-6}$ are negligible. Further reductions in computation time were achieved by estimating the model parameters $\boldsymbol{\theta}_S$ of the minor loss model separately from the calibration parameters $\boldsymbol{\theta}_L$ of the major loss model: First the minor loss model is fitted to the whole data set and then the calibration of the engineering model is performed with fixed minor loss model parameters:

$$\boldsymbol{\theta}_L^* = \min_{\boldsymbol{\theta}_L} \left(-l\left(\boldsymbol{\theta}_L | \hat{\mathbf{y}}, \hat{\mathbf{z}}, \boldsymbol{\theta}_S^*\right)\right) \tag{7.2}$$

Calibration results and comparison with the data

Using the procedure described above, the calibration parameters are estimated as:

$$\boldsymbol{\theta}_S^* = [\lambda, \zeta]^\top = [-0.13, 0.91]^\top$$
$$\boldsymbol{\theta}_L^* = [\psi, \kappa]^\top = [1.73, 1.44]^\top \tag{7.3}$$

Based on the Maximum Likelihood parameters $\boldsymbol{\theta}^*$ and the building-specific information $\hat{\mathbf{z}}_i$ in the data set, the loss size probability distribution function for each building is determined:

$$f_{Y_i|\mathbf{Z}_i,\boldsymbol{\Theta}}\left(y_i|\hat{\mathbf{z}}_i, \boldsymbol{\theta}^*\right) = \sum_{j=1}^{k_i} f_{Y_i|\mathbf{X}_i,\boldsymbol{\Theta}}\left(y_i|\mathbf{x}_{ij}, \boldsymbol{\theta}^*\right) \cdot p_{\mathbf{X}_i|\mathbf{Z}_i}\left(\mathbf{x}_{ij}|\hat{\mathbf{z}}_i\right) \tag{7.4}$$

For comparison with the data, the loss size distribution is aggregated at portfolio level:

$$f_{Y,Portf}\left(y\right) = \frac{1}{n}\sum_{i=1}^{n} f_{Y_i|\mathbf{Z}_i,\boldsymbol{\Theta}}\left(y_i|\hat{\mathbf{z}}_i, \boldsymbol{\theta}^*\right) = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{k_i} f_{Y_i|\mathbf{X}_i,\boldsymbol{\Theta}}\left(y|\mathbf{x}_{ij}, \boldsymbol{\theta}^*\right) \cdot p_{\mathbf{X}_i|\mathbf{Z}_i}\left(\mathbf{x}_{ij}|\hat{\mathbf{z}}_i\right) \tag{7.5}$$

Figure 7.3 shows a comparison of the portfolio loss size distribution with the data. The cumulative distribution function $F_{Y,Portf}(y)$ is illustrated in Figure 7.3a, which allows judging the overall fit of the model in the whole range of the observed losses. Plotting the complementary cumulative probability distribution function $1 - F_{Y,Portf}(y)$ on a logarithmic scale, as in Figure 5b, puts emphasis on the important upper tail of the probability distribution. The expected value E[c] assessed with the model shows a 7% deviation from the sample mean.
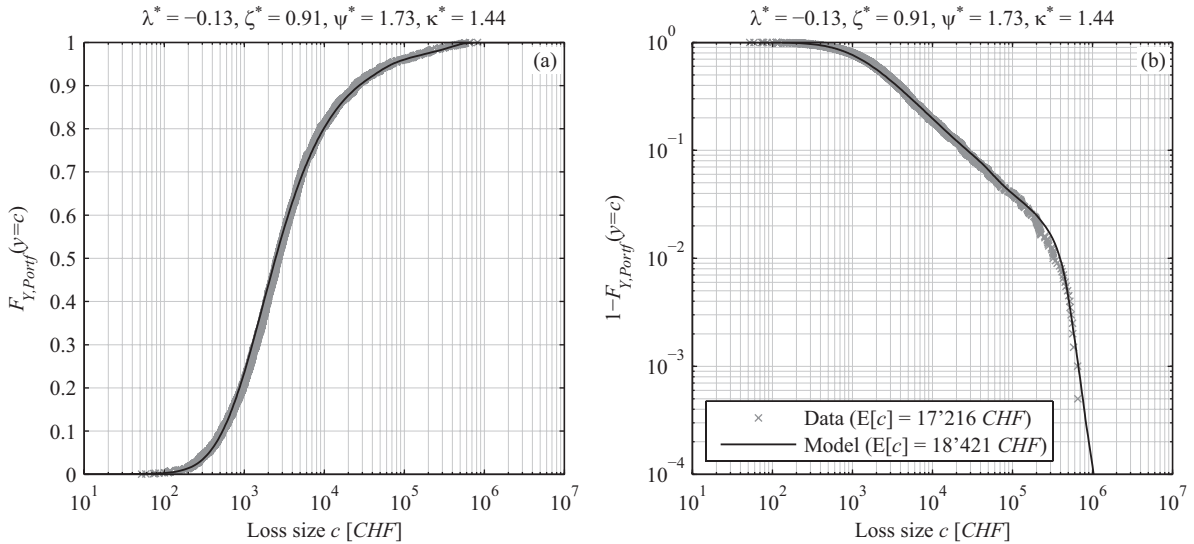
Fig. 7.3: Comparison of the calibrated model with the data set (sample size $n = 1996$ fire events): Cumulative (a) and complementary cumulative distribution function (b), evaluated at portfolio level.

A new engineering risk model cannot be expected to perfectly represent the observations in real fires right from the beginning; typically a few iterations are required to improve the model. For the single family house model, the results of the first calibration trials revealed

problems with parts of the model that could not be calibrated due to the sparse information contained in the data. An advantage of the calibration approach is that engineering knowledge can fill the gap when information is lacking in the observed data and vice versa. The choice of calibration parameters was therefore guided by the availability of quantitative engineering knowledge: Parameters with a clear physical meaning, like the time of fire spread beyond the room of fire origin, can be defined based on physical models or expert judgement. Processes that are more difficult to quantify are captured by the calibration parameters.

Effect of the calibration parameters

The effect of the two calibration parameters for the major loss model, $\boldsymbol{\theta}_L = [\psi, \kappa]^\top$, is illustrated in Figure 7.4. The lower tail of the probability distribution function remains unchanged because the parameters of the minor loss model were kept constant. The solid lines in both graphs are based on the Maximum Likelihood parameters $\boldsymbol{\theta}^*$. In Figure 7.4a only the fire spread coefficient ψ is varied. Choosing a higher value (e.g. $\psi^* + 0.5$ for the dotted line) implies increasing the velocity of the fire development in the model and therefore increases the probability of large losses, and vice versa. Setting the fire spread coefficient ψ equal to one (dashed line in Figure 7.4a) is equivalent to not calibrating the fire spread model at all. When compared to the data, this leads to an underestimation of the expected fire loss $\mathrm{E}[c]$ by almost 30%. This bias in the engineering model is strongly reduced by calibrating the model with the data at hand.
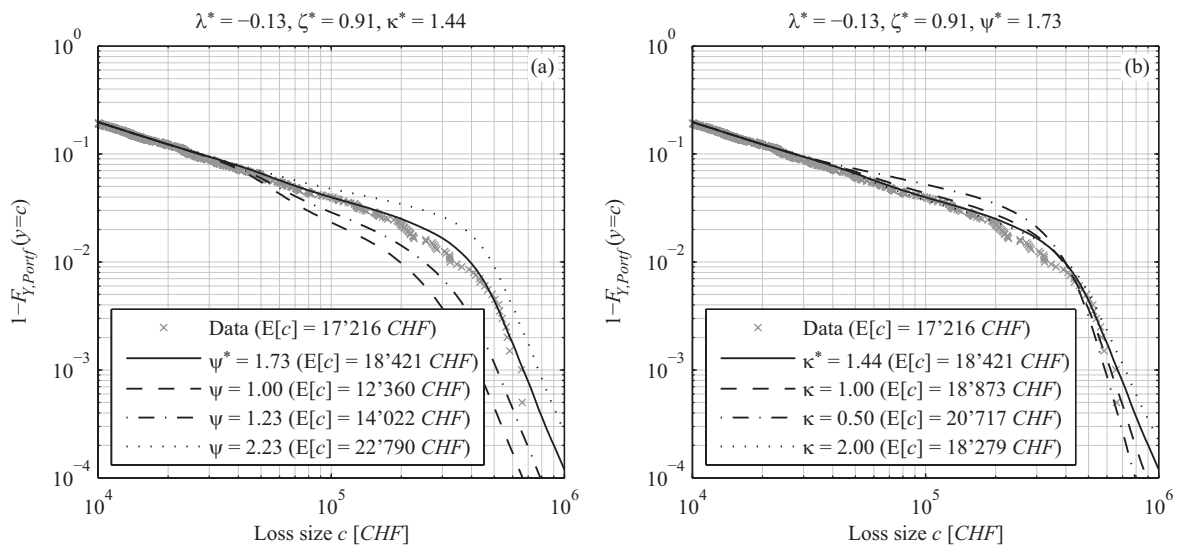
Fig. 7.4: Effect of the calibration parameters on the loss size distribution at portfolio level: Results for varying fire spread coefficient ψ (a) and fire brigade control time exponent κ (b).

Figure 7.4b shows the effect of the control time exponent κ. It is seen that this parameter has an effect mainly on the shape of the loss size probability distribution. As discussed in Section 7.2.1, the average time needed to control the fire is modelled as a function of the area damaged A_d at the starting time of fire brigade actions. The function used for this control time model is a power law with the calibration parameter κ introduced as the exponent: $\kappa = 1$ implies a

linear dependency between the fire area and the control time, $\kappa = 2$ a quadratic relationship and so on. When comparing the expected losses E[c] in Figure 7.4b, one may argue that the data can be represented better when choosing a larger value than the Maximum Likelihood estimate $\kappa^* = 1.44$. However, the aim here was not to model the "average" expected loss at portfolio level but to best represent the loss size probability distribution at object level, as a function of the building-specific risk indicators. Whether this goal was achieved is discussed in the following.

Results for different sub-portfolios

Figure 7.5 shows a comparison of the model with the observed losses for different groups of buildings. The effect of the building characteristics is illustrated by dividing the data into two equal-sized groups according to different risk indicators. For Figure 7.5a, the data set was separated according to the building's insured value V. The distribution of the loss size in both subsets is estimated using the Maximum Likelihood parameters $\boldsymbol{\theta}^*$; only the risk indicators describing the individual buildings differ. The comparison with the observed loss size distributions shows that the model is able to describe the differences between the two groups of buildings fairly well. It should be noted that the statistical uncertainty in the tail of the observed loss distribution is higher than in Figure 7.3 and Figure 7.4 because of the reduced sample size.
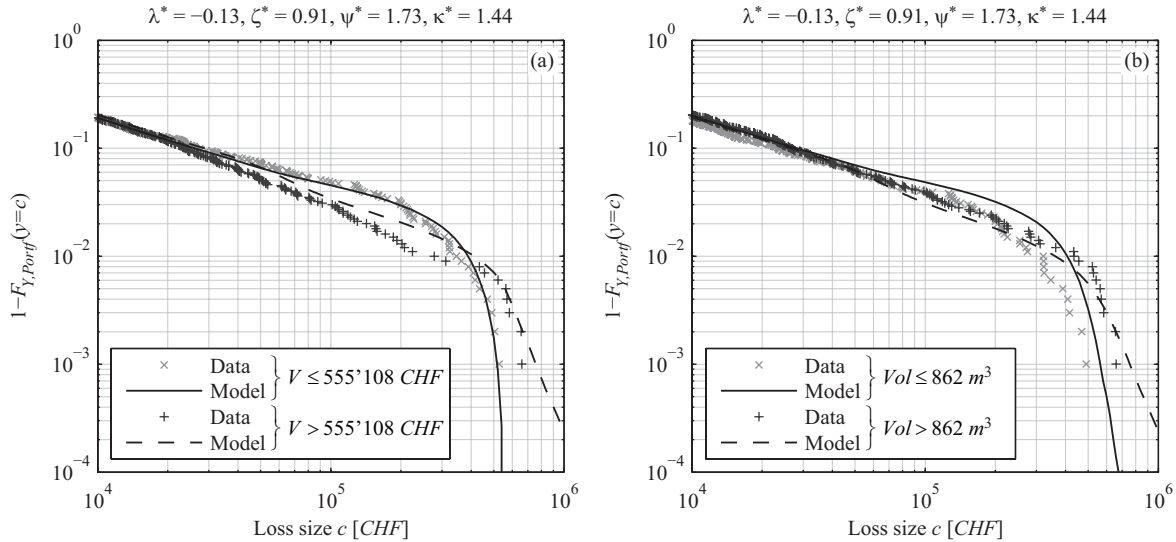
Fig. 7.5: Comparison of the model with data for buildings with (a) different insured values V and (b) different building volumes Vol.

Figure 7.5b shows a similar analysis, but the data set is now divided according to the volume of the buildings. Also here the model is at least qualitatively able to describe the different group characteristics, but not as well as in Figure 7.5a. This may be explained by the fact that the volume is not directly used as a model input. However, most of the risk indicators are directly or indirectly derived from it, see Figure 7.2. This indirect estimation of the model input introduces large uncertainties into the calibration procedure. With a data set containing evidence on all model input variables, it should be possible to produce better results.

7.2.4   Model validation

In engineering decision-making, the goal is not only to describe or explain the observations made in the past; instead, the models shall be used to model future outcomes of decisions. The aim of model validation is to judge whether it can be reasonably assumed that the model is able to estimate the fire risk in a new context after its calibration to a limited data set.

A simple approach for model validation is to calibrate the model only to a subset of the observations before applying it to the remaining data. The two subsets should be comparable in terms of the building characteristics to avoid problems introduced by the effect of different risk indicators. For Figure 7.6, 60% of the data were randomly selected as a "training set" for calibrating the model. The resulting parameter estimates are similar, but not equal to those derived from the calibration with the whole data set. The model was applied to the remaining 40% of the data; this "test set" is used to compare the prediction with observations. The same procedure was applied ten times with different random training sets; Figure 7.6 shows only two examples. The modelled loss size probability distributions of the two subsets (solid and dashed line) differ only slightly because the probability distribution of the building-specific risk indicators is similar. Therefore, the best results are achieved with random subsets where also the observed loss distributions were similar. Figure 7.6a shows an example with only small differences in the tail of the probability distributions that can easily be explained by statistical uncertainty. The worst outcome out of the ten validation trials is shown in Figure 7.6b.
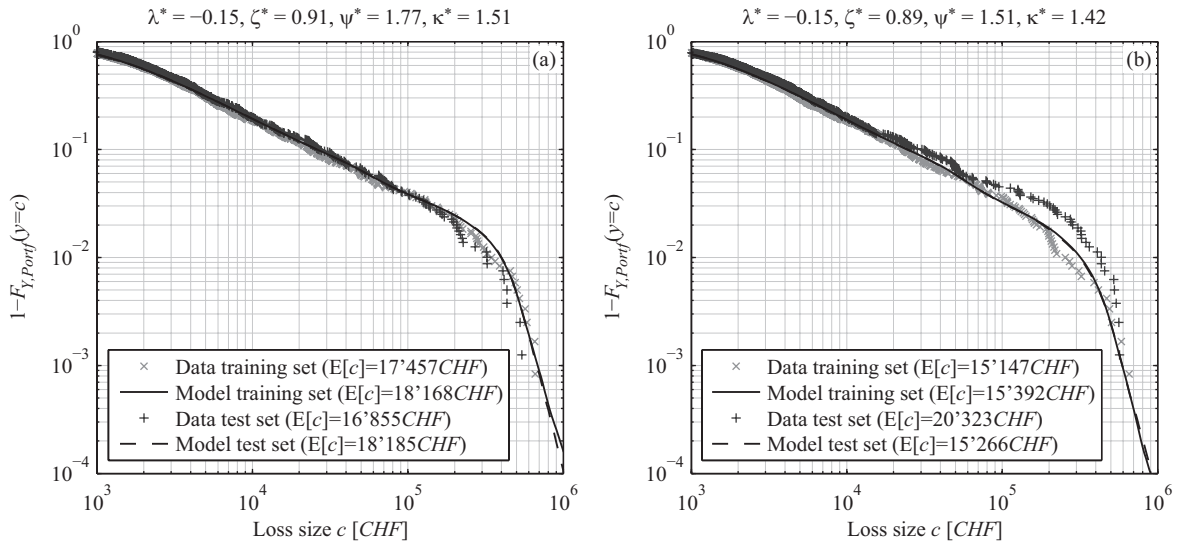
Fig. 7.6: Validation results for two different random training sets (60% of the data) and corresponding test sets (remaining 40% of the data).

No trend could be observed that might explain the differences between the model calibrated to a random training set and the observations in the corresponding test set. Instead, outcomes like the one in Figure 7.6b seem to occur completely at random. The differences can be attributed to the large variation of the loss size, of which only a small fraction is explained by the building-specific risk indicators. This leads to considerable statistical uncertainty especially

in the important upper tail of the loss size probability distribution. The uncertainty can be reduced by using the whole data set for calibration. At any rate, the differences observed in Figure 7.6b are still smaller than the bias introduced by using an engineering model without calibration, see e.g. the model with $\psi = 1$ in Figure 7.4. This shows the benefit of combining engineering knowledge with data even for relatively small data sets with limited information content.

7.3   Assessing the efficiency of home smoke alarms

In the following, the efficiency of home smoke alarms for reducing the monetary and human consequences of fire is evaluated. The generic risk model calibrated to Swiss insurance data is used to estimate the monetary risk reduction. Thereafter, the life-saving effects of home smoke alarms are estimated and evaluated with the LQI criterion. Finally, the results are discussed with respect to their implications for societal decision-making. For simplicity, the code-making decision regarded in this chapter is treated as a discrete choice: Should smoke alarms in residential buildings be mandatory or not? A more detailed analysis would require the quantification of smoke alarm effects as a function of the number of alarms installed.

7.3.1   Financial risk reduction: Monetary optimization

The effect of smoke alarms on the financial risk due to fire for an average single family house is assessed using the generic risk model calibrated to data. For simplicity it is assumed that at present none of the houses in the building portfolio is equipped with smoke alarms. For Switzerland, this assumption is not too far from reality: In Fischer et al. (2012b) it is estimated that only around 11% of Swiss households have smoke alarms installed, see also Table 7.3. Therefore, the generic fire risk model with Maximum Likelihood parameters (Equation 7.3) is used to estimate the fire risk without smoke alarms. For the risk assessment after the investment, the effect of smoke alarms must be implemented in the model. In the following, only the general idea will be discussed shortly; for details see De Sanctis et al. (2013a).

Modelling the effect of smoke alarms in the generic risk model

A smoke alarm is assumed to affect both the minor and the major loss model. In the latter, the implementation is straightforward: As smoke alarms increase the probability of early detection, their effect is modelled by adapting the distribution of the detection time in the fire brigade model, see De Sanctis et al. (2013b) for details. The effect on the minor loss model is less clear. Early detection increases the likelihood that the fire can be extinguished by the occupants or the fire brigade before spreading beyond the room of fire origin. To model this effect, the probability of a "major loss" $P(A_d > a_0)$ is multiplied by a scaling factor $0 \leq \chi \leq 1$. The minor loss model is adapted accordingly to ensure that the integral over the probability distribution function for A_d remains equal to one. An upper limit $\chi = 0.62$ was obtained by De Sanctis et al. (2013a) based on UK fire brigade statistics, which underestimate the effect of smoke alarms on small losses.

Finally, also the failure probability of the smoke alarms has to be accounted for. In De Sanctis et al. (2013a), the reliability of notification is set to 50.9% for battery-powered alarms. This estimate takes into account that the technical availability of the smoke alarm is not the only requirement for early notification in case of a fire: Even if the smoke alarm operates, there may be no person in earshot, or the building occupants may fail to respond or, in the best case, raise the alarm before the smoke alarm sounds.

Estimating the risk reduction for an average single family house

Based on the assumptions mentioned above, the effect of smoke alarms on the fire risk in the building portfolio is investigated. For simplicity, the same data is used as during the calibration, which contains information only on those buildings where a fire has been observed. Applying the model to all single family houses in the canton of Aargau would also be possible, but requires additional data processing and a high computational effort due to the large sample size.

Figure 7.7a shows the influence of smoke alarms on the loss size distribution at portfolio level for different assumptions regarding the reliability of smoke alarm notification and the scaling factor χ for the probability of fire spread beyond the room of fire origin. $\chi = 0$ is equivalent to the assumption that the "major loss probability" $P(A_d > a_0)$ is reduced to zero, while $\chi = 1$ implies that the reduction of the fire brigade notification time in the major loss model is the only positive effect of installing smoke alarms. The results in Figure 7.7a are derived from the model for the fire risk conditional on a fire. The yearly risk R_M is defined as the expected financial loss per year and can be assessed for each building by multiplying the expected value of the loss given fire with the rate of fire occurrence. Figure 7.7b shows the yearly risk reduction ΔR_M for an average building in the portfolio as a function of the scaling factor χ. The average yearly costs of $7.35 CHF$ per smoke alarm are taken from Fischer et al. (2012b), see also Table 7.3.
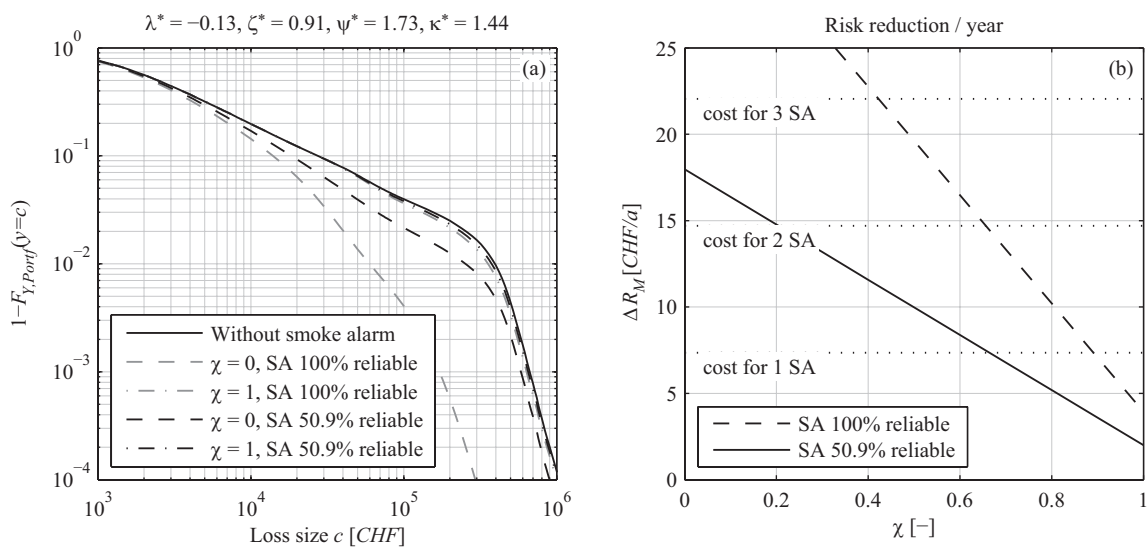
Fig. 7.7: Influence of home smoke alarms (SA) on the loss distribution at portfolio level (a) and yearly risk reduction (b) for different reduction factors χ for the major loss probability $P(A_d > a_0)$.

The risk without smoke alarms is $53CHF/year$ for an average single family house. The curve for a smoke alarm reliability of 50.9% thus corresponds to a relative risk reduction of 4-34%, or 15-34% assuming that $\chi \leq 0.62$. It is interesting to note that the result for $\chi = 1$ is very close to a relative risk reduction of 5% that was derived by Thomas (2002) based on U.S. fire brigade statistics. This value underestimates the true effect of smoke alarms, as fires extinguished by the occupants are not reported to the fire brigade. Yet even for smaller values of χ, the risk reduction is still lower than the yearly cost for three detectors, which is a reasonable level of protection for a single family house. Therefore, smoke alarms are not cost-efficient.

Effect of assumptions on the results

Figure 7.7b suggests that the financial benefit of reduced fire losses does not justify a smoke alarm obligation for Swiss single family houses. It should, however, be noted that the results can be expected to underestimate the effect of smoke alarms due to two assumptions made during the analysis: Firstly, the positive effect of smoke alarms already installed in Swiss households is neglected. As already mentioned, the share of households with smoke alarm protection is fairly small in Switzerland, so the influence of this assumption should be small, too. Secondly, the model was calibrated using data on losses to the building structure only; in Fischer et al. (2012b) it is estimated that accounting for losses to contents increases the fire risk by about 50-60% in the group of residential buildings. Depending on the assumptions made regarding the scaling factor χ and the smoke alarm reliability, this may lead to a different result regarding the cost-efficiency of smoke alarms.

The results are valid for single family houses. It is not clear whether they can be transferred to multi family houses. According to Thomas (2002), the relative risk reduction is smaller in multi family houses with protected construction, but larger if the construction is not protected. The Swiss fire safety regulations require a separate fire compartment for each apartment. It may thus be assumed that Swiss single family houses are in the category "protected construction".

7.3.2   Life-saving effects: Societal risk acceptance

The efficiency of home smoke alarm as life saving measure is evaluated using the LQI criterion, see Section 2.3. Thus, the marginal life saving costs are compared with the $SWTP$ for the marginal life saving benefit achieved by requiring smoke alarm installation in all Swiss households.

Estimating the life saving effects of home smoke alarms

Not all fire fatalities can be prevented by installing smoke alarms. For some persons, e.g. those involved with fire ignition, a smoke alarm makes no difference. Others may profit from an early alarm, but the smoke alarm does not sound for various reasons. The yearly risk reduction ΔR_H (expected number of lives saved per year) resulting from the installation of smoke alarms in all Swiss households is estimated as follows:

$$\Delta R_H = R_H \cdot P_{S|A} \cdot P_A \tag{7.6}$$

Here, R_H denotes the expected number of fire fatalities per year currently observed in Swiss households, i.e. without (obligatory) smoke alarms. $P_{S|A}$ is the probability that a fatality can be prevented by early notification with an activated smoke alarm. Finally, P_A is the probability of smoke alarm activation in a fire event. The quantitative assumptions for all three variables are summarized in Table 7.2. In the following, they are discussed only briefly; see Fischer et al. (2012b) for details.

Tab. 7.2: Quantitative assumptions (Mean, Quantiles and distribution) for the variables used to estimate the life-saving effect of home smoke alarms. Probability distributions are defined in Appendix A.

Variable		Mean	$Q_{0.05}$	$Q_{0.95}$	Distribution
No. of fatalities/year without SA	R_H	24	21	27	Normal$(24, 1.8)$
Survival probability with alarm	$P_{S|A}$	0.353	0.171	0.488	Triangle$(0.08, 0.45, 0.53)$
Probability of SA activation	P_A	0.588	0.501	0.675	Normal$(0.588, 0.053)$
No. of lives saved/year	ΔR_H	5.0	2.4	7.3	Equation 7.6

The expected number of deaths without smoke alarm was derived based on the VKF fatality data for the years 2000-2007 (see Section 4.2.1). Only civil fatalities in residential buildings were considered. The Normal distribution in Table 7.2 reflects statistical uncertainty in estimating the mean from a limited data sample.

The survival probability with activated smoke alarm was derived based on several data sources. The upper and lower bound of the triangular distribution in Table 7.2 were defined based on an investigation of police reports from the canton Zürich containing a more detailed description of fire events with fatalities than the VKF data. The modus was chosen subjectively, accounting for data analysis results from Germany (Wilk, 2011) and the U.S. (Ahrens, 2009).

The probability of smoke alarm activation in a fatality fire was estimated based on U.S. data compiled by Ahrens (2009). It accounts for technical failures, low batteries and disconnected smoke alarms, but also for situations where the fire started in an room or space without detector. Therefore, P_A depends on the number of smoke alarms installed. Assuming an average number of three alarms per household (Table 7.3) is consistent with the current situation in the U.S..

Finally, the marginal life safety benefit of requiring smoke alarm protection in all Swiss households is estimated from simulations with Equation 7.6. The assumptions in Table 7.2 do not account for the fact that some households in Switzerland already possess smoke alarms. This is considered when estimating the marginal costs of mandatory smoke alarms.

Estimating the marginal costs of mandatory home smoke alarms

The marginal yearly costs to society ΔC_Y are determined by adding the installation costs in all Swiss households that are currently not protected:

$$\Delta C_Y = C_A \cdot N_A \cdot N_H \cdot (1 - P_P) \tag{7.7}$$

Here, C_A denotes the yearly costs per smoke alarm, N_A the number of alarms per household, N_H the number of households in Switzerland and P_P the share of households already protected.

Neglecting the costs of smoke alarms installed without legal requirement is consistent with the estimation of the life saving effects: Also the lives saved by smoke alarms already installed today are not considered. The assumptions made for the variables in Equation 7.7 are summarized in Table 7.3. A short discussion is provided below; see Fischer et al. (2012b) for details.

Tab. 7.3: Quantitative assumptions (Mean, Quantiles and distribution) for the variables used to estimate the marginal costs of requiring smoke alarms in all Swiss households. Probability distributions are defined in Appendix A.

Variable		Mean	$Q_{0.05}$	$Q_{0.95}$	Distribution
Costs per smoke alarm $[CHF/a]$	C_A	7.35	6.2	8.5	Normal$(7.35, 0.70)$
No. of detectors per household	N_A	3	-	-	Deterministic
No. of Swiss households [Mio.]	N_H	3.2	3.1	3.3	Uniform$(3.1, 3.3)$
Share of protected households	P_P	0.113	0.044	0.205	Triangle$(0.02, 0.7, 0.25)$
Societal costs $[Mio.CHF/a]$	ΔC_Y	62.9	51.5	74.7	Equation 7.7

The yearly costs per smoke alarm were derived based on the assumption that smoke alarms are replaced every 10 years and batteries every 1-2 years. The estimate is valid for photoelectric smoke alarms with alkaline batteries, the most common detector type on the Swiss market. The price of a detector in Switzerland lies around $20 - 40 CHF$, which is fairly high in international comparison. However, more than half of the yearly costs are battery costs. Smoke alarms with lithium batteries do not require new batteries in their lifetime of 10 years, but are more expensive than the detectors with alkaline batteries.

It is assumed that in average three smoke alarms are installed in each household. This number is probably below the recommended level of protection, but roughly consistent with survey data collected in the U.S. (Ahrens, 2009) and in Germany (Forsa, 2006).

Information on the number of households in Switzerland is provided by BFS (2013). The range of the Uniform distribution in Table 7.3 is defined based on data for the years 2000-2007 for consistency with the fatality counts used to estimate ΔR_H in Table 7.2.

There are no reliable sources for estimating the share of Swiss households already equipped with smoke alarms. The assumption in Table 7.3 is based on survey data from Germany (Forsa, 2006) and some estimates for Switzerland, see Fischer et al. (2012b) for details.

Comparing the marginal costs per life saved with the SWTP

For comparing the marginal life saving costs with the $SWTP$, the societal costs of requiring smoke alarms in all Swiss households (Table 7.3) are divided by the life saving benefit (Table 7.2). The costs per life saved are $14.3 Mio.CHF$ (mean value). From the LQI, the $SWTP$ per life saved is derived as $g/q \cdot J_x \approx 5 Mio.CHF$, see Table 2.2. Basing the decision on expected values leads to the conclusion that smoke alarms cannot be regarded to be an efficient life saving measure. This conclusion remains valid even when considering all uncertainties reflected by the probability distributions in Table 7.2 and 7.3. This is seen in Figure 7.8, where the distribution of the marginal costs per life saved is illustrated. The solid black line indicates the mean value of

the distribution, the dashed lines define a 90% confidence interval. Comparing these values with the LQI threshold (grey line) shows that it is very unlikely that the societal costs of smoke alarm installation in all Swiss households are below the $SWTP$ for the life saving benefit achieved.
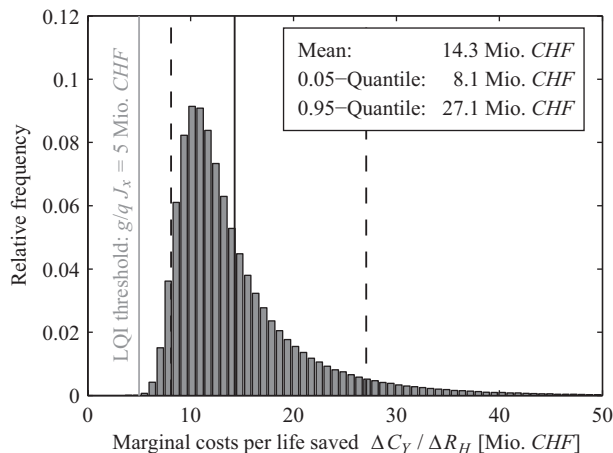
Fig. 7.8: Distribution of the yearly costs per life saved for evaluating the efficiency of home smoke alarms based on the Life Quality Index (LQI). Simulations based on Table 7.2 and 7.3.

Effect of assumptions on the results

The low efficiency of home smoke alarms as a life saving measure is somewhat surprising. It may possibly be explained by two conditions specific to Switzerland: The comparatively high prices for smoke alarms on the Swiss market and the low annual rate of only 0.3 fire deaths per 100'000 inhabitants (The Geneva Association, 2012). Of course also the assumptions made for estimating the costs and life saving benefits could affect the result.

In Equation 7.6, the probability of survival due to an early alarm, $P_{S|A}$, and the probability of smoke alarm activation P_A are the most uncertain variables. For both probabilities, the U.S. fire brigade data is the best information currently available. Yet especially for the activation probability P_A it is difficult to derive meaningful results from this database. Fire brigade data tend to underestimate the true effect of home smoke alarms, because fires extinguished by the occupants are not contained in the data set. In addition, it may be questioned whether the results derived from U.S. data are transferable to Swiss conditions. Smoke alarms of the ionization type are common in the U.S., whereas in Switzerland only optical (photoelectric) home smoke alarms are available. Photoelectric alarms respond faster to smouldering fires and are less prone to false alarms than detectors of the ionization type. In residential buildings, both properties may lead to a higher smoke alarm reliability (Berger and Kuklinski, 2001).

The cost estimation based on Equation 7.7 required less assumptions than the assessment of the life saving benefit. Only for the share of Swiss households already equipped with smoke alarms, little information is available. A more detailed modelling of smoke alarm costs and benefits as a function of the number of smoke alarms per household would of course be desirable. This is, however, difficult especially with respect to the smoke alarm activation probability P_A.

7.3.3   Implications for societal decision-making

In Section 7.3.1 and 7.3.2, the monetary loss reduction and the life saving effect of home smoke alarms were considered separately. In the following, it is shown how to combine the two parts of the decision problems for societal decision-making. Thereafter, future developments that may have an effect on the decision are briefly discussed.

Interaction between monetary optimization and societal risk acceptance

Two different approaches for combining the LQI criterion with monetary optimization were discussed in Section 7.3.1. It was concluded that, as a general rule, the LQI should be used to define a boundary condition for the optimization. As has been shown above, according to the LQI net benefit criterion home smoke alarms are not efficient as a life-saving measure. Also the monetary loss reduction does not justify the installation costs. The decision whether to invest into smoke alarms should thus be left to the individual house-owners and tenants.

One may argue that the life-saving benefit should also be considered when evaluating the monetary effects of home smoke alarms from a societal point of view. As a result, the cost-benefit ratio may look more favourable due to the combined effect of life safety improvements and monetary loss reduction. Nevertheless, it can be reasonable to leave the decision to the individuals instead of optimizing at societal level. In contrast to other fire safety measures, smoke alarms are relatively cheap and easy to install also by non-professionals. According to Hall (1985), the widespread use of smoke alarms in the U.S. was fostered mainly by manufacturer's marketing efforts, long before they became mandatory by law. On a free market, scale effects lead to price reductions if the demand for a product increases. As a result, the efficiency of smoke alarms as evaluated based on the LQI net benefit criterion may increase in the future.

Future developments that may have an effect on the decision

The analysis in Section 7.3.1 and 7.3.2 is based on assumptions reflecting the current state of knowledge. New information may change the results and improve future decisions to be made at societal level. Possibilities for improving the data base for the assessment are discussed in Fischer et al. (2012b). In addition, the efficiency of home smoke alarms may be affected by future developments e.g. in the following areas:

— Lower costs per smoke alarm   — Increased $SWTP$ for life safety

— Improved smoke alarm reliability   — Higher fire losses and fatality risk

The last item suggests that there is a potential for trade-offs between the use of home smoke alarms and other, more expensive fire safety measures. This does, however, require a more complex analysis, which is beyond the scope of this case study.

Chapter 8

Conclusion and Outlook

8.1 Summary and conclusions

The goal of the present thesis was to develop a framework for optimizing fire safety at societal level. It is argued that optimal fire safety decisions should be derived from a basic resource allocation problem: Societal resources for risk reduction measures are limited and have to be invested efficiently. This is fairly obvious when regarding only the monetary aspects of fire safety. However, also investments into life safety should be directed to those areas where the largest life-saving benefit can be derived from the available resources.

Chapter 2 lays out the general principles of societal decision-making for optimal fire safety. The basic decision problem is divided in two parts: Monetary optimization and acceptability with respect to life safety. The optimization aims at minimizing the total costs of fire and may be performed either by a private or a societal decision-maker. Acceptable fire safety decisions in terms of life saving investments always have to be defined from a societal point of view. The approach followed in this thesis takes its basis in the marginal life saving costs principle and the Life Quality Index (LQI) by Nathwani et al. (1997), which is used for deriving the Societal Willingness To Pay for life safety. Both the optimization and the acceptance criterion are based on existing work. The main contribution of this thesis lies in the formulation of guidelines for the application of the basic principles to practical fire safety decisions. An important point is the interaction between monetary optimization and societal risk acceptance, which is discussed considering that the two parts of the decision problem may be dealt with by different decision-makers.

Preventive building fire safety is managed mostly by the building owners. The choices made by these private decision-makers are, however, restricted by fire safety codes and regulations. The code provisions should thus be optimized at societal level based on the principles of monetary optimization and societal risk acceptance. Chapter 3 discusses how this may be achieved in the context of prescriptive and performance-based fire safety regulation. A large part of this chapter focuses on the formulation of a risk-informed framework for defining quantitative safety goals for code-based fire safety design. A generic approach developed by Rackwitz (2000) for the definition of optimal target reliabilities for structural design is used as a starting point.

An important contribution of this thesis is the extension of this approach to the derivation of acceptable failure probabilities from the LQI acceptance criterion. Finally, a generic framework for the definition of optimal and acceptable fire safety design targets is developed. The framework is able to account for the effect of various risk reduction measures on the monetary and human consequences of fire. The measures are categorized according to their influence on the process of fire initiation and development. It is proposed to define design targets at this operational level rather than at the highest level, related to the top-level code objectives.

The benefits of fire safety measures in terms of reduced monetary losses and fatalities prevented can be quantified using risk assessment methods. For comparison with the costs of fire safety, the risk has to be estimated in absolute terms, with as little bias as possible. Chapter 4 explores the possibilities for modelling fire risk based on Swiss insurance data. Data-based models may be assumed to provide a fairly unbiased estimate of the losses observed in real fire events. However, their application to engineering decision-making is strongly restricted by the information content of the data. For estimating the risk reduction achieved by investments into fire safety, statistical methods thus generally have to be combined with engineering knowledge. In the last part of Chapter 4 it is discussed how engineering models for fire risk assessment may be calibrated to statistical data collected by e.g. insurance companies or fire brigades. A calibration framework is formulated that is able to deal with data collection at portfolio level and incomplete data sets. To account for variability in non-homogeneous building portfolios, fire risk has to be estimated as a function of a set of building-specific risk indicators. This may be achieved by following a generic modelling strategy as described by JCSS (2008).

The application of the general principles and approaches developed in Chapter 2 to 4 to real-world decision problems is illustrated with three case studies. The first case study (Chapter 5) deals with egress route design and shows how prescriptive design can be improved by adding more flexibility to the code provisions. In the second case study (Chapter 6), target reliabilities for structural fire safety are derived based on monetary optimization. In the last case study (Chapter 7), the efficiency of home smoke alarms is evaluated as an example for the interaction between monetary optimization and societal risk acceptance. The effect of smoke alarms on monetary fire risk is assessed using a generic risk model calibrated to Swiss insurance data.

8.2   Discussion and outlook

The work presented in this thesis gives rise to new questions in several different areas. Further research is needed especially for the application of the approaches developed in Chapter 3 (societal risk management) and Chapter 4 (absolute risk assessment). This will be the main focus of the following discussion.

The framework for the definition of quantitative fire safety design targets presented in Section 3.4 is an excellent starting point for future research. In this thesis, the application of the generic formulation was restricted to a case study on the derivation of target reliabilities for structural fire safety. Defining safety goals in terms of risk or probability for other fields of fire safety design may be less straight-forward. Much work remains to be done for the development of

generic risk-based approaches e.g. for assessing the effect of safety measures limiting fire spread. Also the definition of design targets for acceptable life safety requires additional research, which may lead to refinements of the generic framework developed in this thesis. As a final step, semi-probabilistic methods should be developed and calibrated to the quantitative safety goals using the corresponding generic risk models. For consistency across different regulatory regimes, the prescriptive code provisions should be based on the same safety targets as the performance-based design option.

The development of engineering fire risk models was not within the scope of this thesis. Methods for absolute fire risk assessment are, however, key to the application of the general principles of monetary optimization and acceptable life safety to practical decision problems. Combined with the calibration procedure developed in Section 4.4, a generic modelling strategy based on observable risk indicators offers many opportunities for societal decision-making. In the last case study presented in this thesis, a generic fire risk model calibrated to Swiss insurance data was used to support the code-making decision whether smoke alarms should be mandatory in residential buildings or not. Strategic planning for fire brigade availability may be another promising area of application. In this case, a generic building fire risk model would have to be calibrated to data containing information on e.g. fire brigade arrival times or equipment. The model could then be used to support decisions e.g. on the number and location of fire stations at community level.

The three case studies presented in Chapter 5 to 7 showed the practical applicability of the research presented in the theoretical part of the thesis. The results are valid only for the specific conditions defining the case studies. To give an example, the analysis of alternative egress route designs in the first case study (Chapter 5) is restricted to office buildings with low occupant density. In addition, the risk assessment results can be expected to be fairly biased. This is acceptable for the comparative risk analysis aimed at in the case study, but more work needs to be done if an absolute risk assessment is required.

The target reliabilities for structural fire safety design derived in the second case study (Chapter 6) are valid for protected steel components in buildings with medium fire hazard, such as e.g. office buildings. The framework must be applied to other construction types and fire conditions before deriving a common target reliability format. The main focus of the case study was on monetary optimization. For occupancies where the optimal design may not be acceptable in terms of life safety, they should be complemented by minimum target reliabilities derived from the LQI acceptance criterion.

Data availability was the limiting factor during the evaluation of home smoke alarms in the third case study (Chapter 7). Another limitation is that the fire risk model used to assess the effect of smoke alarms on monetary fire losses is restricted to single family houses. It is not clear whether the results can be transferred also to multi-family houses.

Appendix A

Probability distributions and parameter definitions

Distribution type	Param.	Mean μ and Std. σ
Uniform $(a \leq x \leq b)$		
$f_X(x) = \frac{1}{b-a}$	a	$\mu_X = \frac{a+b}{2}$
$F_X(x) = \frac{x-a}{b-a}$	b	$\sigma_X = \frac{b-a}{\sqrt{12}}$
Triangle $(a \leq x < b)$		
$f_X(x) = \begin{cases} \frac{2(x-a)}{(b-a)(c-a)} & \text{for } a \leq x \leq c \\ \frac{2(b-x)}{(b-a)(b-c)} & \text{for } c < x \leq b \end{cases}$	$a \leq c$ $b \geq c$	$\mu_X = \frac{a+b+c}{3}$ $\sigma_X = \frac{\sqrt{(a-b)^2+(b-c)^2+(a-c)^2}}{6}$
$F_X(x) = \begin{cases} \frac{(x-a)^2}{(b-a)(c-a)} & \text{for } a \leq x \leq c \\ 1 - \frac{(b-x)^2}{(b-a)(b-c)} & \text{for } c < x \leq b \end{cases}$		
Normal $(-\infty < x < \infty)$		
$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$	μ	$\mu_X = \mu$
$F_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \int\limits_{-\infty}^{x} \exp\left(-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2\right) dt$	$\sigma > 0$	$\sigma_X = \sigma$
Lognormal $(0 < x < \infty)$		
$f_X(x) = \frac{1}{x\zeta\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln(x)-\lambda}{\zeta}\right)^2\right)$	λ	$\mu_X = \exp\left(\lambda + \frac{\zeta^2}{2}\right)$
$F_X(x) = \Phi\left(\frac{\ln(x)-\lambda}{\zeta}\right)$	$\zeta > 0$	$\sigma_X = \exp\left(\lambda + \frac{\zeta^2}{2}\right)\sqrt{\exp(\zeta^2) - 1}$
Gamma $(0 \leq x < \infty)$		
$f_X(x) = \frac{b^a}{\Gamma(a)} \exp(-bx)x^{a-1}$	$a > 0$	$\mu_X = \frac{a}{b}$
$F_X(x) = \frac{\Gamma(bx,a)}{\Gamma(a)}$	$b > 0$	$\sigma_X = \frac{\sqrt{a}}{b}$
$\Gamma(bx, a) = \int\limits_{0}^{bx} t^{a-1} \exp(-t)\, dt$		

Distribution type	Param.	Mean μ and Std. σ
$\Gamma(a) = \int\limits_0^\infty t^{a-1} \exp(-t)\, dt$		
Exponential $(0 \leq x < \infty)$		
$f_X(x) = \lambda \exp(-\lambda x)$	$\lambda > 0$	$\mu_X = \frac{1}{\lambda}$
$F_X(x) = 1 - \exp(-\lambda x)$		$\sigma_X = \frac{1}{\lambda}$
Gumbel max $(-\infty < x < \infty)$		
$f_X(x) = a \exp\left(-a(x-u) - \exp\left(-a(x-u)\right)\right)$	u	$\mu_X = u + \frac{0.577216}{a}$
$F_X(x) = \exp(-\exp(-a(x-u)))$	$a > 0$	$\sigma_X = \frac{\pi}{a\sqrt{6}}$
Weibull min $(0 \leq x < \infty)$		
$f_X(x) = ba^{-b}x^{b-1} \exp\left(-\left(\frac{x}{a}\right)^b\right)$	$a > 0$	$\mu_X = a\Gamma\left(1 + \frac{1}{b}\right),\ \ b \approx \left(\frac{\sigma_X}{\mu_X}\right)^{-1.09}$
$F_X(x) = 1 - \exp\left(-\left(\frac{x}{a}\right)^b\right)$	$b > 0$	$\sigma_X = a\sqrt{\Gamma\left(1 + \frac{2}{b}\right) - \Gamma^2\left(1 + \frac{1}{b}\right)}$
Poisson $(n \in \{0, 1, 2, 3, ...\}, t > 0)$		
$P(N = n) = \frac{(\lambda t)^n}{n!} \exp(\lambda t)$	$\lambda > 0$	$\mu_N = \sigma_N^2 = \lambda t$

Appendix B

Maximum Likelihood estimation for the fire ignition model

With data containing information on the number of fires $\hat{n}_i$ in a time interval $\hat{t}_i$ for a portfolio of $i = 1, \cdots, n_P$ buildings with volume $\hat{vol}_i$, the parameters for the simple ignition model without random effect (GLM, Equation 4.4 in Section 4.2.2) can be estimated based on the Maximum Likelihood method. Using the Poisson distribution, the probability of $N_i = n_i$ fires during the time interval t_i is determined as follows (for ease of notation, the symbol V is used for the building's volume instead of Vol in Equations B.1 to B.7):

$$P(N_i = n_i \mid \lambda_i, t_i) = \frac{(\lambda_i t_i)^{n_i}}{n_i!} \exp\left(-\lambda_i t_i\right); \quad \lambda_i = e^{\alpha}\left(V_i\right)^{\beta} \tag{B.1}$$

Now assume that the data is stored in a matrix $\hat{\mathbf{x}} = [\hat{\mathbf{n}}, \hat{\mathbf{t}}, \hat{\mathbf{v}}]^{\top}$. Using Equation B.1, the likelihood function $L(\boldsymbol{\lambda}(\alpha, \beta) \mid \hat{\mathbf{x}})$ is formulated as follows (see also Section 4.4.1):

$$
\begin{aligned}
L(\boldsymbol{\lambda}(\alpha, \beta)) \mid \hat{\mathbf{x}}) &= \prod_{i=1}^{n_P} \frac{(\lambda_i \hat{t}_i)^{\hat{n}_i}}{\hat{n}_i!} \exp\left(-\lambda_i \hat{t}_i\right) \\
l(\boldsymbol{\lambda}(\alpha, \beta) \mid \hat{\mathbf{x}}) &= \sum_{i=1}^{n_P} \hat{n}_i \ln\left(\lambda_i \hat{t}_i\right) - \lambda_i \hat{t}_i - \ln\left(\hat{n}_i!\right)
\end{aligned} \tag{B.2}
$$

Here, $l(\boldsymbol{\lambda}(\alpha, \beta) \mid \hat{\mathbf{x}})$ denotes the log-likelihood function. For the estimation of the parameters α and β, its first derivatives are set equal to zero. This leads to the following equation for the estimation of α:

$$
\begin{aligned}
\frac{\partial l(\boldsymbol{\lambda}(\alpha, \beta) \mid \hat{\mathbf{x}})}{\partial \alpha} &= \sum_{i=1}^{n_P} \frac{\partial \lambda_i}{\partial \alpha} \cdot \left(\frac{\hat{n}_i}{\lambda_i} - \hat{t}_i\right) = \sum_{i=1}^{n_P} \hat{n}_i - e^{\alpha} \hat{v}_i^{\beta} \hat{t}_i = 0 \\
&\Rightarrow \alpha^* = \ln\left(\sum_{i=1}^{n_P} \hat{n}_i\right) - \ln\left(\sum_{i=1}^{n_P} \hat{v}_i^{\beta^*} \hat{t}_i\right)
\end{aligned} \tag{B.3}
$$

And for estimating β:

$$\frac{\partial l(\boldsymbol{\lambda}(\alpha, \beta) \mid \hat{\mathbf{x}})}{\partial \beta} = \sum_{i=1}^{n_P} \frac{\partial \lambda_i}{\partial \beta} \cdot \left(\frac{\hat{n}_i}{\lambda_i} - \hat{t}_i\right) = \sum_{i=1}^{n_P} \ln(\hat{v}_i)\hat{n}_i - e^{\alpha}\hat{v}_i^{\beta}\ln(\hat{v}_i)\hat{t}_i = 0$$

$$\Rightarrow \frac{\sum_{i=1}^{n_P} \ln(\hat{v}_i)\hat{n}_i}{\sum_{i=1}^{n_P} \hat{n}_i} = \frac{\sum_{i=1}^{n_P} \hat{v}_i^{\beta^*}\ln(\hat{v}_i)\hat{t}_i}{\sum_{i=1}^{n_P} \hat{v}_i^{\beta^*}\hat{t}_i} \tag{B.4}$$

Equation B.4 can be solved numerically. The statistical uncertainty of the resulting estimates α^* and β^* is evaluated with the aid of the Fisher information matrix $\mathbf{I}$, which is defined as the negative Hessian matrix of the log-likelihood evaluated at the Maximum Likelihood estimate (see also Section 4.4.1):

$$\mathbf{H} = - \begin{bmatrix} \frac{\partial^2 l}{\partial^2 \alpha} & \frac{\partial^2 l}{\partial \alpha \partial \beta} \\ \frac{\partial^2 l}{\partial \alpha \partial \beta} & \frac{\partial^2 l}{\partial^2 \beta} \end{bmatrix}_{\substack{\alpha=\alpha^* \\ \beta=\beta^*}} = e^{\alpha^*} \begin{bmatrix} \sum_{i=1}^{n_P} \hat{v}_i\hat{t}_i & \sum_{i=1}^{n_P} \hat{v}_i^{\beta^*}\ln(\hat{v}_i)\hat{t}_i \\ \sum_{i=1}^{n_P} \hat{v}_i^{\beta^*}\ln(\hat{v}_i)\hat{t}_i & \sum_{i=1}^{n_P} \hat{v}_i^{\beta^*}(\ln(\hat{v}_i))^2 \hat{t}_i \end{bmatrix} \tag{B.5}$$

Approximative theory (for $n_P \to \infty$) states that α and β follow a multivariate normal distribution with mean values α^* and β^* and covariance matrix $\mathbf{C}_{\alpha,\beta} = \mathbf{H}^{-1}$. For a given volume V_i, the natural logarithm of the ignition frequency, $\ln(\lambda_i)$, is a linear function of α and β and thus also approximately normal distributed with the following parameters:

$$\mu_{\ln(\lambda_i)} = \mathrm{E}[\ln(\lambda_i) \mid V_i] = \alpha^* + \beta^* \ln(V_i)$$
$$\sigma^2_{\ln(\lambda_i)} = \mathrm{Var}[\ln(\lambda_i) \mid V_i] = \mathrm{Var}[\alpha] + (\ln(V_i))^2 \mathrm{Var}[\beta] + 2\ln(V_i)\mathrm{Cov}[\alpha, \beta] \tag{B.6}$$

It follows that $\lambda_i \mid V_i$ is lognormally distributed with parameters μ and σ. The expected value and variance of $\lambda_i \mid V_i$ are then given by the following expressions:

$$\mathrm{E}[\lambda_i \mid V_i] = \exp\left(\mu_{\ln(\lambda_i)} + \tfrac{1}{2}\sigma^2_{\ln(\lambda_i)}\right)$$
$$\mathrm{Var}[\lambda_i \mid V_i] = \left(\exp\left(\mu_{\ln(\lambda_i)} + \tfrac{1}{2}\sigma^2_{\ln(\lambda_i)}\right)\right)^2 \left(\exp\left(\sigma^2_{\ln(\lambda_i)}\right) - 1\right) \tag{B.7}$$

The Maximum Likelihood estimates for the GLM in Table 4.1 and the confidence intervals for illustrating statistical uncertainty in Figure 4.7 are based on this estimation.

Appendix C

MATLAB code for demographic calculations

The following MATLAB code was used to estimate the age-averaged discounted remaining life expectancy $\bar{e}_d$ and the demographic constant J_Δ (Equation 2.23). The life table input for Switzerland is found in Table C.1 and C.2 (Source: BFS, 2013). Some additional input is defined in the "basic input" section of the code. Under the "stable population" assumption the age probability density function $h(a)$ is estimated as follows:

$$h(a,\rho) = \frac{\exp\left[-\rho a\right] \cdot \exp\left[-\int_0^a \mu(\tau)\,d\tau\right]}{-\int_0^{a_u} \exp\left[-\rho t\right] \cdot \exp\left[-\int_0^t \mu(\tau)\,d\tau\right]\,dt} \tag{C.1}$$

The code was developed using the version R2011b (MATLAB, 2011). It is applicable for life tables with age classes $a = 0, 1, 2, 3, ...$ and requires adjustments if the mortality information is provided e.g. for 5-year cohorts.

```matlab
%%%%%%%%%%%%%%%%%%%%%% Define basic input %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%required basic input: gamma, stablepop
%only if stablepop==1: popgrowth, Psex

gamma = 0.05; %market discount rate
stablepop = 0; %use empirical age distribution

%stablepop = 1; %stable population assumption
if stablepop
    popgrowth = 0.01; % population growth rate (10 year average)
    Psex = [0.5; 0.5]; %share of two sexes in whole population
end

%%%%%%%%%%%%%%%%%%%%%% Load life table input %%%%%%%%%%%%%%%%%%%%%%%%%%%

load('LifeTables.mat');
```

```
%required life table input:
% Af, Am — Age classes a (lower bound)
% Mf, Mm — Mortality mu(a)
%only if stablepop = 0: empirical age distribution
% Hf, Hm — Age pdf h(a) related to whole pop (sum(Hf)+sum(Hm)=1)


%%%%%%%%%%%%%%%%%%%%%%% Define matrix for calculations %%%%%%%%%%%%%%%

sexes = 2; ageclasses = max(size(Af,1),size(Am,1));
Calc=zeros(ageclasses,5,sexes);

%dimensions of Calc:
% 1 — age classes
% 2 — variables (see below)
% 3 — sexes


%columns in Calc:
% 1 — age class a — lower bound
% 2 — mortality mu(a)
% 3 — age distribution h(a)
% 4 — (discounted) remaining life expectancy e_d(a)
% 5 — marginal change in e_d(a) (mortality regime Delta)

Calc(1:size(Af,1),1:2,1)=[Af,Mf]; %females: Age classes a, mortality mu(a)
Calc(1:size(Am,1),1:2,2)=[Am,Mm]; %males: Age classes a, mortality mu(a)

Au = [max(Af); max(Am)]; %Assumed maximum attainable age for both sexes

% column 3 — age distribution h(a)
if stablepop %stable population assumption
    for s = 1:sexes
        ku = find(Calc(:,1,s)==Au(s));
        for k = 1:ku
            a = Calc(k,1,s);
            Calc(k,3,s)=exp(-popgrowth*a)*exp(-sum(Calc(1:k-1,2,s)));
        end
        Calc(:,3,s)=Calc(:,3,s)./sum(Calc(:,3,s))*Psex(s);
    end
else %empirical age distribution
    Calc(1:size(Af,1),3,1)=Hf; %females
    Calc(1:size(Am,1),3,2)=Hm; %males
    if 1-sum(Hf)-sum(Hm)>1e-12
        disp('Warning: sum of age distribution h(a) not equal to unity!')
        Calc(:,3,:)=Calc(:,3,:)./(sum(Hf)+sum(Hm));
    end
end


clear Af Am Mf Mm Hf Hm %clean up
```

```matlab
%%%%%%%%%%%%%%%%%%%%%%% Define output variables %%%%%%%%%%%%%%%%%%%%%%%%

Ed = 0; %Age-averaged discounted remaining life expectancy [a]
Jx = 0; %Demographic constant (mortality regime Delta) [a]

%%%%%%%%%%%%%%%%%%%%%%% Demographic calculations %%%%%%%%%%%%%%%%%%%%%%%

%Estimate Ed and Jx
for s = 1:sexes %females/males
    ku = find(Calc(:,1,s)==Au(s));
    for k = 1:ku
        a = Calc(k,1,s);
        for i = k+1:ku+1
            if i<=ku
                t = Calc(i,1,s);
            else
                t = Calc(ku,1,s)+1;
            end
            % column 4 - discounted remaining life expectancy e_d
            Calc(k,4,s)=Calc(k,4,s)+... % (dt)
                exp(-sum(Calc(k:i-1,2,s))-gamma*(t-a)); % (dtau)
            % column 5 - marginal change in ed (mortality regime delta)
            Calc(k,5,s)=Calc(k,5,s)+... % (dt)
                (t-a)*exp(-sum(Calc(k:i-1,2,s))-gamma*(t-a)); % (dtau)
        end
    end
    %Age-averaging (whole population)
    Ed = Ed + sum(Calc(1:ku,3,s).*Calc(1:ku,4,s));
    Jx = Jx + sum(Calc(1:ku,3,s).*Calc(1:ku,5,s)./Calc(1:ku,4,s));
end

clear t a i k ku s ageclasses sexes Au %clean up

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Tab. C.1: Life table input (females) for Switzerland 2010.

Af	Mf	Hf	Af	Mf	Hf	Af	Mf	Hf
0	$4.17E-03$	$4.84E-03$	40	$8.07E-04$	$7.64E-03$	80	$4.03E-02$	$3.42E-03$
1	$3.50E-04$	$4.88E-03$	41	$8.80E-04$	$7.91E-03$	81	$4.64E-02$	$3.23E-03$
2	$2.00E-04$	$4.85E-03$	42	$9.61E-04$	$8.09E-03$	82	$5.33E-02$	$3.04E-03$
3	$1.63E-04$	$4.82E-03$	43	$1.05E-03$	$8.18E-03$	83	$6.13E-02$	$2.83E-03$
4	$1.35E-04$	$4.74E-03$	44	$1.16E-03$	$8.32E-03$	84	$7.04E-02$	$2.70E-03$
5	$1.14E-04$	$4.73E-03$	45	$1.27E-03$	$8.32E-03$	85	$8.06E-02$	$2.51E-03$
6	$9.90E-05$	$4.79E-03$	46	$1.40E-03$	$8.48E-03$	86	$9.22E-02$	$2.29E-03$
7	$8.80E-05$	$4.69E-03$	47	$1.54E-03$	$8.28E-03$	87	$1.05E-01$	$2.05E-03$
8	$8.10E-05$	$4.73E-03$	48	$1.70E-03$	$7.97E-03$	88	$1.19E-01$	$1.79E-03$
9	$7.90E-05$	$4.75E-03$	49	$1.87E-03$	$7.75E-03$	89	$1.35E-01$	$1.62E-03$
10	$8.00E-05$	$5.05E-03$	50	$2.06E-03$	$7.55E-03$	90	$1.52E-01$	$1.34E-03$
11	$8.50E-05$	$5.06E-03$	51	$2.26E-03$	$7.35E-03$	91	$1.70E-01$	$9.57E-04$
12	$9.60E-05$	$5.13E-03$	52	$2.48E-03$	$7.05E-03$	92	$1.88E-01$	$7.45E-04$
13	$1.16E-04$	$5.22E-03$	53	$2.71E-03$	$6.91E-03$	93	$2.07E-01$	$6.18E-04$
14	$1.44E-04$	$5.37E-03$	54	$2.96E-03$	$6.67E-03$	94	$2.26E-01$	$4.75E-04$
15	$1.78E-04$	$5.35E-03$	55	$3.21E-03$	$6.47E-03$	95	$2.45E-01$	$3.66E-04$
16	$2.16E-04$	$5.42E-03$	56	$3.49E-03$	$6.33E-03$	96	$2.64E-01$	$2.97E-04$
17	$2.52E-04$	$5.51E-03$	57	$3.78E-03$	$6.15E-03$	97	$2.83E-01$	$2.10E-04$
18	$2.83E-04$	$5.79E-03$	58	$4.08E-03$	$6.12E-03$	98	$3.02E-01$	$1.42E-04$
19	$3.05E-04$	$5.90E-03$	59	$4.42E-03$	$5.86E-03$	99	$3.21E-01$	$9.39E-05$
20	$3.16E-04$	$5.99E-03$	60	$4.78E-03$	$5.99E-03$	100	$3.41E-01$	$6.19E-05$
21	$3.19E-04$	$5.98E-03$	61	$5.17E-03$	$5.92E-03$	101	$3.61E-01$	$3.43E-05$
22	$3.16E-04$	$6.12E-03$	62	$5.59E-03$	$6.00E-03$	102	$3.82E-01$	$2.25E-05$
23	$3.13E-04$	$6.05E-03$	63	$6.07E-03$	$5.86E-03$	103	$4.04E-01$	$1.08E-05$
24	$3.13E-04$	$6.21E-03$	64	$6.60E-03$	$5.83E-03$	104	$4.29E-01$	$6.23E-06$
25	$3.17E-04$	$6.31E-03$	65	$7.19E-03$	$5.52E-03$	105	$4.56E-01$	$2.52E-06$
26	$3.25E-04$	$6.44E-03$	66	$7.85E-03$	$5.47E-03$	106	$4.86E-01$	$1.54E-06$
27	$3.36E-04$	$6.50E-03$	67	$8.61E-03$	$5.30E-03$	107	$5.21E-01$	$9.04E-07$
28	$3.50E-04$	$6.75E-03$	68	$9.46E-03$	$5.04E-03$	108	$5.61E-01$	$5.11E-07$
29	$3.68E-04$	$6.76E-03$	69	$1.04E-02$	$4.76E-03$	109	$6.08E-01$	$2.76E-07$
30	$3.89E-04$	$6.86E-03$	70	$1.16E-02$	$4.43E-03$	110	$1.00E+00$	$1.01E-07$
31	$4.13E-04$	$6.77E-03$	71	$1.29E-02$	$4.31E-03$			
32	$4.41E-04$	$6.69E-03$	72	$1.44E-02$	$4.16E-03$			
33	$4.72E-04$	$6.73E-03$	73	$1.61E-02$	$3.98E-03$			
34	$5.06E-04$	$6.77E-03$	74	$1.82E-02$	$4.01E-03$			
35	$5.44E-04$	$6.75E-03$	75	$2.06E-02$	$3.93E-03$			
36	$5.86E-04$	$6.92E-03$	76	$2.35E-02$	$3.81E-03$			
37	$6.33E-04$	$7.00E-03$	77	$2.68E-02$	$3.69E-03$			
38	$6.85E-04$	$7.22E-03$	78	$3.06E-02$	$3.60E-03$			
39	$7.43E-04$	$7.54E-03$	79	$3.51E-02$	$3.46E-03$			

Tab. C.2: Life table input (males) for Switzerland 2010.

Am	Mm	Hm	Am	Mm	Hm	Am	Mm	Hm
0	$5.34E-03$	$5.08E-03$	40	$1.45E-03$	$7.67E-03$	80	$6.89E-02$	$2.31E-03$
1	$4.03E-04$	$5.20E-03$	41	$1.57E-03$	$7.89E-03$	81	$7.67E-02$	$2.07E-03$
2	$3.12E-04$	$5.17E-03$	42	$1.70E-03$	$8.13E-03$	82	$8.53E-02$	$1.89E-03$
3	$2.29E-04$	$5.05E-03$	43	$1.86E-03$	$8.22E-03$	83	$9.47E-02$	$1.68E-03$
4	$1.80E-04$	$5.01E-03$	44	$2.03E-03$	$8.51E-03$	84	$1.05E-01$	$1.53E-03$
5	$1.50E-04$	$5.02E-03$	45	$2.22E-03$	$8.61E-03$	85	$1.16E-01$	$1.36E-03$
6	$1.32E-04$	$4.99E-03$	46	$2.43E-03$	$8.74E-03$	86	$1.28E-01$	$1.15E-03$
7	$1.22E-04$	$4.92E-03$	47	$2.66E-03$	$8.48E-03$	87	$1.41E-01$	$1.00E-03$
8	$1.19E-04$	$5.01E-03$	48	$2.92E-03$	$8.18E-03$	88	$1.55E-01$	$8.42E-04$
9	$1.22E-04$	$5.04E-03$	49	$3.21E-03$	$7.89E-03$	89	$1.70E-01$	$7.21E-04$
10	$1.29E-04$	$5.35E-03$	50	$3.52E-03$	$7.71E-03$	90	$1.87E-01$	$5.75E-04$
11	$1.43E-04$	$5.30E-03$	51	$3.87E-03$	$7.47E-03$	91	$2.04E-01$	$3.86E-04$
12	$1.63E-04$	$5.41E-03$	52	$4.26E-03$	$7.26E-03$	92	$2.23E-01$	$2.82E-04$
13	$1.92E-04$	$5.48E-03$	53	$4.68E-03$	$7.07E-03$	93	$2.43E-01$	$2.13E-04$
14	$2.34E-04$	$5.65E-03$	54	$5.15E-03$	$6.93E-03$	94	$2.65E-01$	$1.60E-04$
15	$2.97E-04$	$5.61E-03$	55	$5.66E-03$	$6.60E-03$	95	$2.88E-01$	$1.12E-04$
16	$3.93E-04$	$5.73E-03$	56	$6.22E-03$	$6.35E-03$	96	$3.12E-01$	$9.33E-05$
17	$5.26E-04$	$5.81E-03$	57	$6.83E-03$	$6.10E-03$	97	$3.37E-01$	$5.82E-05$
18	$6.87E-04$	$6.06E-03$	58	$7.51E-03$	$6.06E-03$	98	$3.62E-01$	$3.79E-05$
19	$8.44E-04$	$6.19E-03$	59	$8.25E-03$	$5.82E-03$	99	$3.87E-01$	$2.11E-05$
20	$9.51E-04$	$6.23E-03$	60	$9.07E-03$	$5.95E-03$	100	$4.12E-01$	$1.32E-05$
21	$9.97E-04$	$6.21E-03$	61	$9.97E-03$	$5.78E-03$	101	$4.36E-01$	$6.23E-06$
22	$9.98E-04$	$6.29E-03$	62	$1.10E-02$	$5.80E-03$	102	$4.58E-01$	$3.94E-06$
23	$9.76E-04$	$6.19E-03$	63	$1.21E-02$	$5.69E-03$	103	$4.77E-01$	$1.91E-06$
24	$9.53E-04$	$6.36E-03$	64	$1.33E-02$	$5.60E-03$	104	$4.93E-01$	$1.02E-06$
25	$9.34E-04$	$6.43E-03$	65	$1.46E-02$	$5.35E-03$	105	$5.06E-01$	$6.83E-07$
26	$9.19E-04$	$6.63E-03$	66	$1.61E-02$	$5.17E-03$	106	$5.14E-01$	$4.05E-07$
27	$9.08E-04$	$6.66E-03$	67	$1.77E-02$	$4.92E-03$	107	$5.68E-01$	$2.27E-07$
28	$9.02E-04$	$6.85E-03$	68	$1.95E-02$	$4.67E-03$	108	$1.00E+00$	$8.29E-08$
29	$9.01E-04$	$6.91E-03$	69	$2.16E-02$	$4.27E-03$			
30	$9.05E-04$	$7.01E-03$	70	$2.38E-02$	$3.84E-03$			
31	$9.17E-04$	$6.81E-03$	71	$2.64E-02$	$3.71E-03$			
32	$9.35E-04$	$6.77E-03$	72	$2.92E-02$	$3.54E-03$			
33	$9.62E-04$	$6.81E-03$	73	$3.24E-02$	$3.34E-03$			
34	$9.98E-04$	$6.74E-03$	74	$3.60E-02$	$3.25E-03$			
35	$1.05E-03$	$6.76E-03$	75	$4.01E-02$	$3.16E-03$			
36	$1.10E-03$	$6.98E-03$	76	$4.46E-02$	$3.00E-03$			
37	$1.17E-03$	$7.02E-03$	77	$4.98E-02$	$2.76E-03$			
38	$1.25E-03$	$7.30E-03$	78	$5.55E-02$	$2.65E-03$			
39	$1.34E-03$	$7.52E-03$	79	$6.18E-02$	$2.46E-03$			

Nomenclature

Abbreviations

AIC	Akaike Information Criterion (Akaike, 1974)
ALARP	As Low As Reasonable Possible / Practical
ASET	Available Safe Egress Time
CHF	Swiss Francs (8.1.2014: $1CHF \approx 0.81EUR \approx 1.1USD$)
DIC	Deviance Information Criterion (Spiegelhalter et al., 2002)
GDP	Gross Domestic Product
GLM	Generalized Linear Model
GLMM	Generalized Linear Mixed Model
LQI	Life Quality Index
MCMC	Marcov Chain Monte Carlo
MU	Monetary units
PV	Present Value (Equation 2.4)
RSET	Required Safe Egress Time
SWTP	Societal Willingness To Pay
WTP	Willingness To Pay

Notation for random variables

X, x	Random variable, realisation
$\mathbf{X}, \mathbf{x}$	Vector of random variables, realisations
$\hat{\boldsymbol{x}}$	Data set with observations of $\mathbf{X}$
$f_X(x)$	Probability density function for X
$F_X(x)$	Cumulative distribution function for X

$f_{X|Y}(x|y)$ Conditional probability density function for X given Y

$p_X(x)$ Discrete probability density (mass) function

$E[.]$ Expectation operator

$Var[.]$ Variance operator

$Cov[.]$ Covariance operator

μ_X Mean value of X

σ_X Standard deviation of X

ρ_{XY} (Linear) correlation coefficient $\rho_{XY} = Cov[X,Y]/(\sigma_X \sigma_Y)$

V_X Coefficient of variation of X, $V_X = \sigma_X/\mu_X$

Latin letters

A Age of a person $[a]$ (random variable)

A_d Area of fire spread $[m^2]$ (random variable)

A_f Floor area $[m^2]$ (random variable)

A_{max} Floor area of largest room $[m^2]$ (random variable)

A_{tot} Total building floor area $[m^2]$ (random variable)

A_0 Floor area of room of fire origin $[m^2]$ (random variable)

a Parameter of the Uniform, Triangle, Gamma, Gumbel or Weibull distribution

a_u Assumed maximum attainable age for life table calculations $[a]$

b Parameter of the Uniform, Triangle, Gamma or Weibull distribution

C Monetary costs or consequences $[MU]$ (random variable); Financial loss from damages at the building structure $[CHF]$ (random variable)

C_A Yearly costs per smoke alarm $[CHF/a]$

C_Y Yearly costs to society $[CHF/a]$

C_0 Part of the construction costs $C(\mathbf{p})$ that is independent of the design $[MU]$

C_1 Marginal costs of structural safety (normal design situation) $[MU]$

C_2 Marginal costs of structural fire safety $[MU]$

C_3 Present value of all sprinkler system costs $[MU]$

$\mathbf{C_\Theta}$ Covariance matrix of calibration parameters $\mathbf{\Theta}$

c Parameter of a Triangle distribution

d_p Thickness of the insulation material $[m]$

db_S	Marginal (monetary) benefit evaluated from a societal point of view
dc	Marginal costs / marginal life saving costs
dc_S	Marginal costs evaluated from a societal point of view
e	(Remaining) life expectancy $[a]$
e_d	Discounted remaining life expectancy $[a]$
$\bar{e}_d$	Age-averaged discounted remaining life expectancy $[a]$
g	GDP per capita $[MU/a]$
H	Expected costs of structural failure in addition to the reconstruction costs $[MU]$
H_C	Human compensation costs $[MU]$
H_r	Room height $[m]$
$\mathbf{H}$	Hessian matrix
$h(a)$	Age probability density function of the society or population at risk
J_Δ	Demographic constant $[a]$ for the constant mortality regime $x = \Delta$
J_x	Demographic constant $[a]$ for a mortality regime x
K_1	"Relative life saving costs" constant $[-]$ (Equation 3.13)
k	(Proportionality) constant
k_ϑ	Temperature-dependent reduction factor for steel resistance
$k_{\vartheta,min}$	Minimal resistance reduction factor during the course of a fire event
L	Likelihood
l	Log-likelihood; Egress distance $[m]$
M	Model uncertainty (random variable)
M_1	Model uncertainty, systematic effect
M_2	Model uncertainty, unsystemtatic effect
N_A	Number of smoke alarms per household
N_C	Number of connections between rooms (random variable)
N_F	Expected number of fatalities given structural failure
N_H	Number of households in Switzerland
N_L	Number of losses (random variable)
N_O	Number of occupants at risk
N_R	Number of rooms (random variable)
n_P	Portfolio size

n_{pop}	Population size
P_A	Probability of smoke alarm activation in a fatality fire
P_P	Share of Swiss households already protected by smoke alarms
$P_{S|A}$	Probability of survival with working smoke alarm
P_X	Exceedance probability $[1/a]$
P_f	Probability of structural failure (related to a 1-year reference period) $[1/a]$
$P_{f|F}$	Probability of structural failure conditional on a fully developed fire
$P_{f,acc}$	Acceptable failure probability $[1/a]$
$P_{f,opt}$	Optimal failure probability $[1/a]$
$P_{f,t,c}$	Target (optimal) failure probabiltiy for the normal design situation $[1/a]$
$P_{f,t,c,ref}$	Target (optimal) failure probability for the normal design situation $[1/a]$, for a reference case regarding the distributions of R and S
$P_{f,t,1y}$	Target (optimal) failure probability for a 1-year reference period $[1/a]$
$P_{f,t|F}$	Target (optimal) failure probability conditional on a fully developed fire
$P_{ref,\Delta}$	Probability of failed egress for the prescriptive design and smoke spread time Δ
p	Decision variable; Probability
p^*	Decision variable – monetary optimum
p_{acc}	Decision variable – acceptance threshold
p_1	Decision variable for structural design – central safety factor
p_2	Decision variable for structural fire safety design (Equation 6.12)
p_3	Decision variable for active fire protection ($p_3 = 1$: sprinkler; $p_3 = 0$: no sprinkler)
$\mathbf{p}$	Vector of decision variables
$A(\mathbf{p})$	Obsolescence costs $[MU]$ (expected present value)
$C(\mathbf{p})$	Construction costs $[MU]$ (expected present value)
$D(\mathbf{p})$	Structural failure costs $[MU]$ (expected present value)
$F(\mathbf{p})$	Monetary fire losses $[MU]$ (expected present value)
$G(\mathbf{p})$	Expected monetary loss given fire without structural failure $[MU]$
$I(\mathbf{p})$	Inspection and maintenance costs $[MU]$ (expected present value)
$N(\mathbf{p})$	Expected number of fatalities per year $[1/a]$
$P_E(\mathbf{p})$	Probability of safe egress of a person in a fire event
$P_{F|I}(\mathbf{p})$	Probability of a fully developed fire given ignition / fire occurence

$P_R(\mathbf{p})$	Probability that a trapped person is rescued by the fire brigade
$T(\mathbf{p})$	Total costs $[MU]$ (expected present value)
Q	Fire load density (random variable)
q	LQI exponent for labour-leisure trade-off
q_t	Fire load density related to the total area of the enclosure $[MJ/m^2]$
R	Risk; Resistance (random variable)
R_H	Human risk
R_M	Monetary risk
r	Annual discount rate
S	Mechanical load effect (random variable)
S_F	Mechanical load effect in the fire situation (random variable)
T	Time (random variable)
T_A	Alarm time $[s]$ (random variable)
T_{ASET}	Available Safe Egress Time $[s]$ (random variable)
T_R	Response time $[s]$ (random variable)
T_{RSET}	Required Safe Egress Time $[s]$ (random variable)
T_T	Travel time $[s]$ (random variable)
T_u	Time horizon of a decision (random variable)
u	Utility function; Parameter of the Gumbel distribution
V	Insured value $[CHF]$ (random variable)
Vol	Building Volume $[m^3]$ (random variable)
W	Walking speed $[m/s]$ (random variable)
w	Fraction of total lifetime spent for work
$\mathbf{X}$	Model input of a generic risk model (random vector)
Y	Consequences (random variable)
$\mathbf{Y}$	Model output of a generic risk model (random vector)
$\mathbf{Z}$	Risk indicators provided in the data set used for calibration (random vector)

Greek letters

α	Fire growth parameter in t^2 fire model $[kW/s^2]$; Parameter of fire occurrence model
β	Cobb-Douglas labour elasticity; Parameter of fire occurence model

β_t	Target reliability index related to 1-year reference period
$\beta_{t,F}$	Target reliability conditional on a fully developed fire
γ	Discount rate (continuous discounting)
γ_M	Market discount rate
γ_P	Private discount rate
γ_S	Societal discount rate
Δ	Constant additive change, finite difference; "Smoke spread time" to egress routes
δ	Real economic growth rate per capita
ε	Elasticity of the marginal utility of consumption
ϵ_i	Subject-specific error term (GLMM for fire occurrence)
ζ	Parameter of the Lognormal distribution
η_S	Mechanical load ratio $\eta_S = \mu_{S_F}/\mu_S$
$\boldsymbol{\Theta}$	Calibration parameters (random vector)
$\boldsymbol{\theta}^*$	Vector of Maximum Likelihood calibration parameters
$\boldsymbol{\theta}_L$	Vector of calibration parameters for the major loss model, $\boldsymbol{\theta}_L = [\psi, \kappa]^\top$
$\boldsymbol{\theta}_S$	Vector of calibration parameters for the minor loss model, $\boldsymbol{\theta}_S = [\lambda, \zeta]^\top$
ϑ	Temperature $[^\circ C]$
ϑ_a	Steel temperature $[^\circ C]$
ϑ_g	Gas temperature $[^\circ C]$
K	"Control time exponent" (random variable, calibration parameter)
λ	Intensity of a homogeneous Poisson process; Parameter of the Lognormal or Exponential distribution
λ_I	Rate of fire occurrence / ignition rate
μ	Mean value; Parameter of the Normal distribution
$\mu(a)$	Raw (background) mortality at age a
ρ	Pure time preference rate
σ	Standard deviation of a distribution; Parameter of the Normal distribution
σ_ϵ	Error standard deviation (GLMM for fire occurrence)
Φ	Cumulative distribution function of the Standard Normal distribution
χ	Reduction factor for the probability of a large fire event due to active fire protection (e.g. sprinkler system, smoke alarm)

Ψ "Fire spread coefficient" (random variable, calibration parameter)

ω Obsolescence rate

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