Doctoral Thesis

Blowing and drifting snow in alpine terrain
A physically-based numerical model and related field measurements

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Blowing and Drifting Snow in Alpine Terrain: 
A Physically-Based Numerical Model 
and Related Field Measurements

A dissertation submitted to the
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presented by

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Abstract

Blowing and drifting snow influences human activities in various ways. For example, in mountainous regions, snow drift is a key factor in the formation of slab avalanches; the quality of avalanche danger mapping and land-use planning depends significantly on the correct assessment of snow redistribution by drift in the avalanche release zones.

Conceptually, drifting snow is a turbulent multi-phase flow which consists of a continuous and a dispersed phase. For each phase—air as the continuous phase and the snow particles as the dispersed phase—the principles of mass and linear momentum balance ought to be formulated and to be solved numerically; however, for simulations of extended areas with complex topography which have to be considered in practical applications, the computational effort becomes too large. Essentially, two transport modes contribute to snow drift: saltation and suspension. Whereas in the suspension mode the particle concentration is small and a simple mixture theory is applicable, the concentration within the so-called saltation layer is much higher and the particle–air flow interaction needs to be considered. This interaction is of importance as it leads to a self-regulation of the mass flux. For steady-state conditions, a saturated transport is established.

A physically-based numerical model of snow drift has been developed, where the involved processes are separated into two layers. One describes the transport in suspension and the wind field. The wind field is modeled by the Reynolds averaged Navier-Stokes equations, using $e-e(k-e)$ model for the turbulent closure. Suspended snow is modeled by an additional scalar equation. The second layer describes the transport due to saltation, including erosion and deposition of snow. Here, the conservation of mass and momentum is formulated for the mixture of snow and air. Parameterizing of quantities defining the saltation layer is founded on particle trajectory calculations. Both layers are mutually coupled by boundary conditions. A two-way coupling between particles and air flow is taken into account.

To validate the model, simulations were compared with wind tunnel experiments and real snow-drift events. For this purpose, the wind field and the redistribution features of the snowpack in a complex Alpine terrain have been investigated. To this end, an experimental site 2 km north from the SFISAR Institute building at Weissfluhjoch, Gaudergrat ridge (2300 m a.s.l.), was equipped with instruments to measure meteorological and snow parameters. Wind profile measurements were carried out at five points in the surrounding of the ridge with a high time resolution to determine the wind field during snow drift episodes. The areal snow redistribution was determined by taking soundings before and after drift episodes.

A comparison between simulations and wind tunnel experiments gives good agreement. The comparison with the field measurements at Gaudergrat shows that the model is suitable to predict the new-snow distribution in extended alpine areas ($\approx 6 \cdot 10^5 \text{ m}^2$).
Kurzfassung

Schneeverschüttung beeinflußt die Lebensbedingungen im Gebirgsraum auf verschiedenste Weise. So ist zum Beispiel Schneeverschüttung ein Faktor für die Entstehung von Schneebrettlawinen; die Qualität von Lawinengefahrenkarten und Raumnutzungsplänen hängt entscheidend von der korrekten Abschätzung der durch den Wind beeinflußten Schneeerteilung in der Lawinenanrisszone ab.


Ein Vergleich zwischen numerischen Simulationen und Windkanalexperimenten zeigt gute Übereinstimmung. Der Vergleich mit den Feldmessungen am Gaudergrat zeigt, daß das Modell geeignet ist die Neuschneeverteilung in ausgedehnten alpinen Gelände ($\approx 6 \cdot 10^5 \text{ m}^2$) zu berechnen.
Figure 1.1: Leeward slopes are prime areas for avalanches to start. The wind deposited snow forms compact and brittle layers; already a small additional loading can cause the triggering of a slab avalanche. (Photo E. Wengi/SLF)

1 Introduction

Blowing and drifting snow affects human activities in various ways. In alpine regions, the influence on the avalanche formation is to be mentioned in the first place. This applies to tourist avalanches as well as to catastrophic avalanches. Snow drift causes uneven snow redistribution, strongly influenced by the local topography. It tends to even out the topography by filling in hollows and by erosion of the snowpack on humps, brows or windward crest areas. Beyond crest lines often large cornices are observed which can trigger avalanches if they collapse. Wind deposited snow forms compact and brittle layers. Deposited on a weak layer, already a small loading can cause an avalanche release. Deposition of drifted snow is one of the most frequent reasons for serious and fatal accidents. Thus, knowledge about the occurrence of snow drift is important for avalanche warning. On the other hand, the quality of avalanche danger mapping and land-use planning depends significantly on the correct assessment of snow redistribution by drift in the release zones, as the run-out distance of large avalanches is strongly related to the available snow mass. Snow drift can cause an increase of the snow mass by factors of about 1.5–2.5.

Less dramatic than the influence of the avalanche activity and avalanche effects, but still of economic importance is the effect of blowing and drifting snow on every-day life. For example, reduced visibility causes airports to close, snow drift constitutes a hazard to traffic and additional road clearing is necessary. Or additional snow loads on structures have to be considered.

Also the relevance to hydrology is worth mentioning. The snow transport due to wind causes redistribution of the precipitation. Water loss due to sublimation can be considerable during snow drift. Pomeroy et al. [58] mentioned sublimation losses in the range of 40 to 74% of the annual snowfall for their study site. R.A. Schmidt [77] reported evaporation of 39% of the transport rate corresponding to an averaged sublimation rate of 1.7 mm d$^{-1}$ water equivalent (w.e.) during blizzards over a flat plane.
1.2.a: Avalanche fracture line at the summit of Albristhorn (2762 m a.s.l.). Obvious is the erosion, windward of the ridge crest, and the formation of a cornice leeward. The estimated fracture depth is about 3 m. (Photo S. Keller/SLF)

1.2.b: Damage to the forest in the valley bottom (1416 m a.s.l.) and in the reverse slope caused by the powder-snow part of the Albristhorn avalanche. (Photo S. Keller/SLF)

Figure 1.2: Example of the effect of snow drift: Avalanche action at Albristhorn (Canton of Bern) from 30 January 1995. It is probable that the fracture line area was considerably loaded by deposition of drifted snow.

In comparison, Föhn [18] gives evaporation rates from 0.15 to 0.6 mm w.e. d	extsuperscript{-1} over a snow surface without drift for the Weissfluhjoch test site (Switzerland; 2540 m a.s.l.).

Readers interested in comprehensive informations on avalanche and snow safety, and current understanding about snow and avalanches are referred to, e.g., [44].

The aim of this work was the development of a physically-based numerical model for snow transport due to wind. The mean focus was on the simulation of the snow distribution in alpine terrain during or shortly after precipitation periods with strong winds.

In the remainder of this chapter, an overview of previous work will be given. In Chapter 2, the basic physics of blowing and drifting snow will be described, and in Chapter 3 a numerical model will be presented. Chapter 4 shows the results of field measurements which where carried out in a complex alpine terrain and used for the validation of the numerical model, presented in Chapter 5.
1.1 Physical Concept

Before reviewing the work already done on blowing and drifting snow and sand, a short outline of the current understanding of the process is given. This will help to understand the different directions in recent research.

![Diagram of transport modes in snow drift](image)

Figure 1.3: Modes of transport in blowing and drifting snow: creeping, reptation, saltation and suspension. The term *drifting snow* is used to describe the near surface transport whereas the expression *blowing snow* is reserved to situations where particles rise to a height of 1.8 m and more. Typical values for the different processes, such as the heights, the volumetric concentration, the required wind speeds or the grain sizes are shown as well.

It is common to distinguish between three or four transport modes in snow drift, respectively. For the classification of the different modes, the definition given by Anderson et al. [5] will be used.

- **Surface creeping**: The motion of grains whose displacement is not directly affected by wind forces. The rearrangement is caused by particle impact. The amount of snow transported in this way contributes weakly to the transport rate, particularly, if we take into account that snow is cohesive.

- **Reptation**: To this transport mode belong grains, which are splashed up, but do not gain enough energy from the wind to rebound or eject other grains on impact.
• **Saltation and modified saltation:** Grains follow ballistic trajectories determined by the time-averaged wind profile and are not influenced by turbulence of the wind. This motion type is also called *pure saltation*. If the trajectories are somewhat modified by turbulent fluctuations of the wind, the term *modified saltation* is used. Grains in saltation are capable of rebounding and/or of splashing up other grains.

• **Suspension:** The transport mode where the grains are lifted far away from the surface by vertical gusts and are transported over long distances without contact with the soil surface, is referred to as suspension.

These different transport modes are depicted in Figure 1.3. The figure indicates the typical order of magnitude of the different concentrations, heights and grain sizes involved. The concentration strongly depends on the height above the surface. This dependency can be clearly seen in Figure 1.4, showing measurements of Dingle et al. in Antarctica presented by Mellor and Fellers [51]. Evidently, the mass concentration within the saltation layer (typical heights, $h_s \approx 10-25$ mm) is limited to an upper value of approximately $1\text{ kg m}^{-3}$.

In comparison, during calm snowfall, the volumetric concentration, $c$, is approximately $10^{-7}-10^{-6}$, corresponding to a mass concentration, $c_m$, of about $10^{-4}-10^{-3}\text{ kg m}^{-3}$. If we assume a mean fall velocity of snow grains $W_f \approx 0.7\text{ m s}^{-1}$ and a density of the new snow $\rho_{NS} = 100\text{ kg m}^{-3}$, these concentrations correspond

![Figure 1.4: Probable values of mass concentration as a function of height $z$ and wind speed $M_{10}$; plotted against $z$ on logarithmic scales, with $M_{10}$ as parameter (after Mellor and Fellers [51]).](image)
to an accumulation rate on a drift-free horizontal area of

$$\frac{dHN}{dt} = \frac{\rho_p c W_f}{\rho_{NS}} \approx 0.2-2.5 \text{ cm h}^{-1}$$

(1.1)

where the particle density, $\rho_p$, is approximately 917 kg m$^{-3}$. Typical accumulation rates are about 1--2 cm h$^{-1}$ corresponding to about 1--2 mm w.e. h$^{-1}$.

Saltation starts at wind speeds $M_{10}$ of about 5--8 m s$^{-1}$ (wind speed measured at 10 m height), depending on the properties of the snowpack. The transition from saltation to suspension occurs when the wind speeds exceeds about 7--15 m s$^{-1}$. The contribution of the different transport modes to the total mass transport depends on the strength of the turbulent flow. The following table gives a clue of the ratios.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reptation</td>
<td>5--25%</td>
</tr>
<tr>
<td>Saltation</td>
<td>50--75%</td>
</tr>
<tr>
<td>Suspension</td>
<td>3--40%</td>
</tr>
</tbody>
</table>

At low wind speeds, saltation and reptation contribute most to the mass transport. With increasing wind speeds the portion of suspension increases whereas the relative contribution of saltation and reptation decreases.

The transport due to blowing and drifting snow can be described as follows. If the wind blowing over the snow surface becomes sufficiently strong, and wind shear exceeds a certain critical value, the so-called threshold, some grains are set in motion by the wind. Some of these grains will be lifted off the surface, and can be easily accelerated by the wind. Some of them will gain enough energy to rebound and/or eject other grains on impact. At the onset, the number of grains resulting from an impact, the so called mean replacement capacity, is, on average, larger than one. This results in an exponential increase of grains in the saltation layer. As more and more grains are in saltation, the vertical wind profile is modified due to the considerable extraction of momentum from the wind by the grains in motion. Now the grains gain less energy, and less grains will rebound or be ejected on impact. The mean replacement capacity decreases until the equilibrium value of one is reached. The number of saltating grains fluctuates around a certain value, sometimes called the saturation value. The surface properties, determining the threshold, as well as dislodgements effected by collisions of grains with the surface play an important role at the initiation of saltation. Thus, if there are already particles in the air, as it is the case during snowfall, the threshold wind speed for drift is observed to be significantly reduced. Due to turbulent and gusty wind a certain number of particles go into suspension. In all cases, gravity acts as a back-driving force on the grains in saltation and suspension.
Hence, snow drift is the result of five closely/mutually linked processes:

- aerodynamic entrainment,
- grain trajectories,
- grain-snowpack impacts,
- modification of the wind-field,
- and the transport due to turbulent suspension.

1.2 Previous work

The following overview of existing work concentrates on drifting snow and sand in air. The related processes of sediment transport in water will be ignored. In the former case the density ratio between the particles and the fluid is about 1000-2000, and the process is dominated by solid-to-solid impacts, so that the dynamic system is highly inertial. Whereas in the second case the density ratio for the immersed sand in water is only 1.6, and fluid drag is of crucial importance for the particle motion.

Existing papers mostly belong to one of three categories:

- Field measurements
- Physical simulations
- Numerical simulations

Bagnold [8] laid the foundation for the current understanding of aeolian sand transport. In his wind tunnel experiments and field measurements he concentrated essentially on the creeping and saltation modes, assuming that these modes contribute most to the mass transport. Concerning the free fall velocity, he found that sand grains behave like spheres with a suitable equivalent diameter, \( f_s \cdot d_p \), where \( d_p \) is the measured grain diameter and \( f_s \) is a shape factor. For sand, \( f_s \) is in the range of 0.75–1. Bagnold stressed the importance of the particle impact for the threshold velocity and differentiated between the fluid threshold and the impact threshold. He also recognized the coupling between the intensity of the saltation and the modification of the wind. For the mass transport due to saltation, expressed in kg per m width of the lane, per s, he proposed the relation

\[
Q = k \sqrt{\frac{d_p}{d_{\text{ref}}}} \frac{\rho_a}{g} u'^{3}
\]

(1.2)

where \( u' \) is the friction velocity aloft the saltation layer. The reference particle diameter was chosen as \( d_{\text{ref}} = 250 \mu m \), and the coefficient, \( k \), ranges from 0.15 to 0.35 and even more, depending on the grading of the sand. His classical perception of the process is still valid, even if it has been modified and sharpened.
Owen [55] noted the strong influence the saltation process exerts on the near-bed wind profile. He tried to explain the self-regulation of the saltation process, using the two hypotheses:

i) *The saltation layer behaves, so far as the flow outside it is concerned, as an aerodynamic roughness whose height is proportional to the thickness of the layer;*

ii) *The concentration of particles within the saltation layer is governed by the condition that the shearing stress borne by the fluid falls, as the surface is approached, to a value just sufficient to ensure that the surface grains are in a mobile state.*

Based on published data and assuming that the saltation layer height scales as $u^2_*/(2g)$, he proposed a relation for the roughness length,

$$ z_0 = k \frac{u^2_*}{2g} \text{, where } k = 0.02 \quad (1.3) $$

For a review of current conceptual understanding of aeolian sand transport, the reader is also referred to Anderson et al. [5] or McEwan and Willets [47]. In the following, an exemplary representation of the different directions of research, emphasizing works done on snow, will be given, starting with the field measurements.

### 1.2.1 Field experiments

Most field work has been carried out on planes, e.g., in Antarctica in the case of the measurements of R. Dingle, U. Radok and W. Budd reanalyzed by Mellor and Fellers [51] (see Fig. 1.4). R.A. Schmidt made his measurements on nearly flat terrain with sparse, short grass vegetation for at least 1 km upwind [77] and on Lake Diamond (Wyoming) with an approximately 10 km flat long upwind catchment area [79]. Similar conditions were chosen by Takeuchi [88] for his measurements near the mouth of the Ishikari River in Hokkaido (Japan). The 300 m wide ice-free river provided an effective boundary across which snow transport was negligible. The main goal of these investigations was the determination of the transport rate. The investigators gave several empirical expressions for this rate, some examples are summarized in Table 1.1. In all cases, the flux was determined using drift flux traps. R.A. Schmidt also used a snow particle counter. In [77], Schmidt also presented a determination of the sublimation rate of drifting snow.\(^1\)

Common to all expressions is the nonlinear behavior, with the power ranging approximately from 2–4. But obviously, there are large differences in the drift transport for the same wind speeds, e.g., for a wind speed, $M_1 = 10 \text{ m s}^{-1}$, the transport

---

\(^1\)A corrections of his sublimation formula is given in [79]
Table 1.1: Some examples for empirical expressions for the mass flux rate $Q = \int_{0}^{\zeta} c_m(z) U_P(z) \, dz$. Subscripts denote the regarded heights, measured in m. (Similar expressions exist for the transport of sand.)

<table>
<thead>
<tr>
<th>Source</th>
<th>$Q , [\text{kg m}^{-1} , \text{s}^{-1}]$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radok [61]</td>
<td>$Q_{200} = 1 \cdot 10^{-8} (1.18 + 0.080 M_{th})$</td>
<td></td>
</tr>
<tr>
<td>R.A. Schmidt [77]</td>
<td>$Q_1 = 1.635 \cdot 10^{-4} (M_1 - M_{th})^3$ where $M_{th} = 680 , d_p^{0.25} \log(30/d_p)$ with the mean particle diameter $d_p$ at 0.01 m</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$Q_1 = k_{a2} (u_* - u_{*th}) \left( u_t^2 - u_{*th}^2 \right)$ here $u_{*th} = \kappa M_{th0.25} / \ln \left( 0.25/\zeta_0 \right)$ where $M_{th0.25} = M_0.25 - \left( \frac{d_p0.25}{100} \right)^4 - 4$; $k$ is in the range 0.45–31.37</td>
<td></td>
</tr>
<tr>
<td>Takeuchi [88]</td>
<td>$Q_2 = 2.10^{-4} M_1^{2.7}$ (old firm snow)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$Q_2 = 2.9 \cdot 10^{-6} M_1^{4.16}$ (settled dry snow)</td>
<td></td>
</tr>
<tr>
<td>Kobayashi [31]</td>
<td>$Q = 3 \cdot 10^{-6} M_1^3$ $5 &lt; M_1 &lt; 12 , \text{ms}^{-1}$</td>
<td></td>
</tr>
<tr>
<td>Nishimura et al. [54]</td>
<td>$Q = 4.2 \cdot 10^{-1} u_t^2$ wind tunnel experiments</td>
<td></td>
</tr>
<tr>
<td>Pomeroy</td>
<td>$Q_{salt} = k_{a2} u_{*th} (u_t^2 - u_{*th}^2)$ cited in [10]</td>
<td></td>
</tr>
</tbody>
</table>

rate, $Q$, range approximately from 0.03 kg m$^{-1}$ s$^{-1}$ to 0.12 kg m$^{-1}$ s$^{-1}$. Partly, these differences arise from the different snowpack properties. Takeuchi remarked that the discrepancies could also be due to different degrees of saturation. In his study, he investigated the development of the transport rate and concluded that the drift transport increases until the saturated state is achieved, which depends on the prevailing wind speed and snow conditions. He observed that the drift rate increased sharply from the beginning of the fetch up to 150–200 m, and continued to increase even at distances over 300 m until it reached a steady state (see Fig. 1.5).

Castelle [10] pointed out that due to the gusty character of the wind and the non-linear relationship between the transport rate and the wind speed, the empirical formulas based on the mean wind speed clearly underestimate the mass transport.

1.5.a: Horizontal distribution of drift-snow transport as measured with snow traps ($U_{100}$ is here the velocity at 1 m).

1.5.b: Horizontal distribution of snow depth downwind from boundary.

Figure 1.5: Development of the drift transport rate (from Takeuchi [88]).
in mountain areas. On the other hand, the drift transport will seldom, if ever reach the steady state. Among other things, Castelle made extensive measurements of the snow distribution at an alpine test site, a large pass (Col du Lac Blanc, France, 2700 m a.s.l.). To this end, he used photogrammetry, and the weather conditions were measured by an automatic station. He also made measurements of the snow concentration during drift periods. As an example, Figure 1.6 shows the measured mean saltation concentration 0.01 m above the surface.

Snow drift in a complex terrain was studied by Föhn [19], Meister [49] and R.A. Schmidt et al. [80]. They made drift flux measurements on a crest near

Figure 1.6: Mean mass concentration within saltation as function of the friction velocity for the cases with $u_{*\text{th}}$ of 0.2–0.35 m s$^{-1}$ (re-plotted from Castelle [10, Fig. 3.30]).

---

![Figure 1.6: Mean mass concentration within saltation as function of the friction velocity for the cases with $u_{*\text{th}}$ of 0.2–0.35 m s$^{-1}$ (re-plotted from Castelle [10, Fig. 3.30]).](image1.png)

Figure 1.7: Typical drift density profiles at the top of a crest. Measured drift-density profiles: left-hand side shows the effect of increased winds; right-hand side illustrates translatory shift by increased snow fall rates (from Föhn [19]).

---

![Figure 1.7: Typical drift density profiles at the top of a crest. Measured drift-density profiles: left-hand side shows the effect of increased winds; right-hand side illustrates translatory shift by increased snow fall rates (from Föhn [19]).](image2.png)
SFISAR's building at Weissfluhjoch/Davos (Switzerland, 2692 m a.s.l.). They showed that the drift density profile at the top of the crest, presented in Figure 1.7, significantly differs from profiles on a flat area, e.g., Figure 1.4.

The influence of a crest line on the snow distribution in the surrounding area was investigated by Föhn and Meister [20]. They made soundings of the snow depth on both sides of two ridges (Gaudergrat and Schwarzhorngrat) near Weissfluhjoch after drift periods. Figure 1.8 shows an example of the measurements at Gaudergrat. Föhn as well as Meister tried to quantify the wind transported snow mass, so Föhn [19] proposed the relation

$$\frac{dH_N}{dt} = k \overline{M}^3 \quad [\text{m d}^{-1}]$$

(1.4)

where \(\frac{dH_N}{dt}\) is the surplus snow accumulation rate on leeward slopes compared with drift-free horizontal areas. The coefficient \(k\) is set to \(8 \times 10^{-5} \text{s}^3 \text{d}^{-1} \text{m}^{-2}\) and \(\overline{M}\) is the mean daily wind speed. The relation is said to be valid for snow-storm conditions and for periods of 24 h. Meister [50] gave a relation for transported mass, \(\Delta m\), to leeward slopes using the expression

$$\Delta m = \left( \frac{M_2}{M_{th}} \right)^3 \left( \frac{p_r + 2 \text{ mm w.e. h}^{-1}}{200} \right) \Delta t \quad [\text{kg m}^{-2}] \text{ or } [\text{mm w.e.}]$$

(1.5)

relating the mass to the mean wind speed, a threshold wind speed (\(M_{th}\) found to be 5 m s\(^{-1}\)), the precipitation rate \(p_r\) measured in mm w.e. h\(^{-1}\) and the time period \(\Delta t\). For the investigated area, the best agreement was found for \(\Delta t = 8\ h\).

The former investigators only considered the overall effects of transport. In [31], Kobayashi studied the low-level transport of snow. He measured the drift rate and examined the particle trajectories in the saltation layer. For this, he made outdoor
1.9.a: Example of saltation at very low wind speeds during a light snow fall
($M_1 = 3.8 \text{ m s}^{-1}$)

1.9.b: Example of saltation of typical shallow drift in absence of snowfall
($M_1 = 5.0 \text{ m s}^{-1}$)

Figure 1.9: Example of the grain motion within the saltation layer. The
mean length of the saltation paths of snow particles ranges from 0.05–0.14 m
for a wind speed $M_1 = 5 \text{ m s}^{-1}$ to 0.11–0.3 m for a wind speed $M_1 = 10 \text{ m s}^{-1}$
(from Kobayashi [31]).

experiments using a photographic technique. Figure 1.9 gives an impression of the
particle motion in the saltation layer. The drift rate was determined using box-type
drift gauges or with a trench method. He emphasized the contribution of the saltat­
ing grains to the low-level drift. An example of the measured drift rates against
the wind speed at 1 m is re-plotted in Figure 1.10. In comparison to Takeuchi (see
above), he found that 90% of the saturated low-level drift rate were reached after a
distance of 30–60 m.

Figure 1.10: Snow-drift
rate as function of the
mean wind speed at 1 m,
$M_1$, measured using a
trench method; $\oplus$ with­
out snowfall; $\bullet$ with snow­
fall; particle diameters
range from 100 $\mu$m to
500 $\mu$m (re-plotted from
Kobayashi [31]).

S. Schmidt, J.D. Dent and R.A. Schmidt investigated the electrostatic force acting
on saltating snow particles, measuring the charges carried by a single grain [73, 74]
as well as the electric field above the snowpack [75, 93]. They concluded that the
trajectory of a grain could be significantly altered by this force. Similar experiments had been done by Latham and Montagne [33] to examine the contribution of electrostatic force to the formation of cornices.

### 1.2.2 Physical simulation

Physical simulations of blowing and drifting snow or sand are useful in two respects. They can help us gain a better understanding of the interaction of the different processes during drift, and they can be an aid in practical work, e.g., placement of snow fences to prevent critical snow accumulations in avalanche release zones. Particularly the second case is limited by similarity conditions which ought to be fulfilled. Some scaling requirements were examined by Naaim-Bouvet [53]. She compared the requirements given by the four investigators Kind [30], Iversen [25], Tabler [87] and Anno [6] with full scale measurements, wind tunnel simulations and outdoor modeling.

Two examples of wind tunnel experiments on the deposition pattern around snow fences are depicted in Figure 1.11. The figure shows an experiment for a solid fence without bottom gap and one for a porous fence with bottom gap.

![Figure 1.11: Examples of wind tunnel experiments on the deposition pattern around snow fences](image)

1.11.a: Profile view, showing the drift shapes for a solid fence without bottom gap (from Iversen [26]).

1.11.b: Change in the cross section of the model snow drift formed around the model snow fence as a function of time (from Anno [7]; fence with a porosity of 18.4% and a bottom gap of 1/7 of the fence height).

Figure 1.11: Examples of wind tunnel experiments on the deposition pattern around snow fences

Castelle [10] describes an example how wind tunnel experiments can be used in a practical work. He examined the effect of snow fences on a avalanche defense structure at the Couloir de Chäller (Canton of Valais), see Figure 1.12. Castelle also made wind tunnel simulations of snow drift at Col du Lac Blanc and compared
these with his field measurements mentioned above.

Another example for work directed towards practical applications is the study of snow fences in mountain terrain by Voegeli [94]. He carried out outdoor physical simulations, hoping to avoid certain scaling problems related to the properties of snow, e.g., effects of cohesion.

Most investigators used sand, glass or plastic particles for their wind tunnel experiments. Only a few times snow was used, notable by the group around Maeno [32, 37, 38, 54]. Among other things they studied the grain motion, the change of the wind velocity during drift, rebound and ejection of particles as well as the charge carried by particles. Also Kikuchi [29] worked with snow in his wind tunnel study of the aerodynamic roughness associated with drifting snow.

Interesting wind tunnel studies concerning the impact of particles were also made by Rice et al. [64, 65, 102]. They investigated the interaction of a sand bed and impacting sand grains.

Martinez [39], and Naaim and Martinez [52] carried out extensive wind tunnel experiments to determine the total transport rate. For this purpose, they used image processing technique. The images were made using a CCD camera. These measurements are supposed to serve in the development of a numerical snow drift model.

Summarizing the results of the field and wind tunnel experiments we get the following picture of snow drift:
Transport modes

The significance of the two main transport modes depends on the wind speed. The transition between both modes is continuous (modified saltation). At edges or crest lines, the probability of a grain to be suspended increases (upward moving component of the flow velocity).

Type of motion

Grains follow trajectories; similarity in the mean ratio of incident to rebound velocity, in the mean ratio of incident to ejection velocity, in the mean rebound angle and in the mean ejection angle.

Threshold wind speeds, $M_{10}$

5–8 m s\(^{-1}\) (corresponding to friction velocities, $u_*$, of approximately 0.25–0.5 m s\(^{-1}\) 7–15 m s\(^{-1}\)

Typical grain size

100–250 μm with a mean of about 200 μm  $\leq$ 200 μm

Typical heights

0.01–0.02 m  0.1–several 10 m

Typical mass concentrations

0.25–0.5 kg m\(^{-3}\)  $10^{-4}$–$10^{-2}$ kg m\(^{-3}\) depending on the height (more or less an exponential decrease). In the lowest meter over a flat plane, the decrease for a wind speed $M_{10} < 15$ m s\(^{-1}\) is $O(10^3)$. 

Typical transport rates, $Q$

0.001–0.03 kg m\(^{-1}\) s\(^{-1}\)  0.01–0.1 kg m\(^{-1}\) s\(^{-1}\)

Example for measured total transport rates, $Q$, for a wind speed $M_1 = 10$ m s\(^{-1}\)

0.03–0.12 kg m\(^{-1}\) s\(^{-1}\)

Table 1.2: Compilation of field and wind tunnel experiments
1.2.3 Numerical modeling

Like the physical simulations, the work on numerical simulation of snow and sand drift differs widely in scope and focus. On one side, specific processes were studied, e.g., by Werner and Haff [96] simulating the grain impact, but also models based on empirical rules for avalanche forecasting, like ELSA [40, 41, 82] have been developed.

Haff et al. [4, 21, 22, 96] examined the grain impact on a loose bed and the bed response using Particle Dynamic Methods (PDM). Based on the results from many such individual simulations, they derived so-called splash functions, statistical descriptions of the bed response.

Werner [95], Anderson and Haff [2, 4], McEwan and Willets [45] as well as Sørensen [83] combined such splash functions with a fluid model of the wind to saltation models. Common to most of these models is that the trajectories of the particles in saltation are explicitly calculated and their effect on the wind field is taken into account (two-way coupling). These saltation models could be classified as Lagrangian-Eulerian models. Simulations of this kind are useful for gaining a better understanding of particular issues, e.g., the entrainment mechanism, force between grains, and so forth. An example of such results is shown in Figure 1.13.

1.13.a: Simulated evolution of the saltating population, showing the number of grains leaving the bed per square meter per second due to impacts and aerodynamic entrainment, and the number of grains impacting the bed. At steady state the ejection rate equals the impact rate (on the order of $10^7$ m$^{-2}$ s$^{-1}$), and aerodynamic entrainment rate has fallen to zero. Steady state is achieved in roughly 2 seconds.

1.13.b: Simulated evolution of mass flux for each of 4 cases with $u_*$'s of 0.4-0.7 m s$^{-1}$. In all cases $d_p = 0.25$ mm, and the grain density is that of quartz, $\rho = 2650$ kg m$^{-3}$. In each case steady flux is achieved in order 1-2 s, slightly lower times for higher shear velocities.

Figure 1.13: Two examples of PDM simulation for the investigation of saltation (both from Anderson and Haff [4]).
Pomeroy’s “Prairie Blowing Snow Model (PBSM)” [57] describes snow transport in terms of the mass flux in the saltation layer and in the suspension layer:

\[ Q_{\text{salt}} = \frac{k_1 \rho_a}{u_a g} \left( u_{*a}^2 - u_{*th}^2 + w_{*n}^2 - w_{*th}^2 \right) , \]  

\[ Q_{\text{susp}} = \frac{u_a}{\kappa} \int_{z_0}^{z_1} \rho_p c(z) \ln \left( \frac{z}{z_0} \right) dz \]  

where \( u_{*a} \) denotes the stress to the non-erodible surface, the coefficient \( k_1 \approx 0.7 \text{ m s}^{-1} \) and the concentration \( c(z) = c_{ref} \left( z/z_{ref} \right)^{2\Re_s} \). Here, Pomeroy defines the Rouse number as

\[ \Re_s = \frac{\left( J_{b_2} + E_{z_j \rightarrow z_{j+1}} \right)}{\kappa u_a \rho_p c} - W_f \]  

where \( E_{z_j \rightarrow z_{j+1}} \) is the sublimation rate of drifting snow between heights \( z_j \) and \( z_{j+1} \); \( W_f \) is the absolute value of the mean terminal fall velocity of drifting snow particles; and \( J_{b_2} \) is the boundary flux. He assumes a logarithmic wind profile, feedback coupling is considered to the extent that he modifies the roughness length according to

\[ z_0 = k_2 \frac{u_a^2}{2 g} + 0.5 \text{ m} N_{st} A_{st} \]  

in which the coefficient, \( k_2 \), is set to 0.1203 (see [58]). The second term on the right hand side describes the roughness created by exposed vegetation (see Lettau [34]) where \( N_{st} \) is the number of vegetation elements per unit area and \( A_{st} \) is the exposed silhouette area of a single element. The first term on the right hand side corresponds to Owen’s proposal (see Equation (1.3)). Actually, Pomeroy’s PBSM is a non-dynamical model based on field observations and empirical assumptions.

In 1983, Decker and Brown [14, 15] presented a two-dimensional model for the atmospheric mixture of snow and air. The model is based on the principles of conservation of mass and momentum. They solved the both balance equations for constituent snow, assuming a logarithmic wind profile or a predefined airflow, and neglecting transient effects. In the case of the assumed logarithmic wind profile, the model is restricted to the case of neutral stability over a plane. The model is one-way coupled. Only the wind-added deposition is regarded, erosion of snow is ignored. A finite-difference technique was used to solve the equations for the snow phase.

In their first snow drift model, a group around Uematsu [92] used a finite-element calculation for the snow transport in saltation. They calculated the two dimensional wind field using the vorticity equation

\[ \frac{\partial \omega}{\partial t} + \left( \frac{\partial \psi}{\partial z} \frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \omega}{\partial z} \right) = \nu \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial z^2} \right) \]  

(1.10)
where
\[ -\omega = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} \]
and \( \psi \) is the stream function \( (u = \partial \psi / \partial z, v = -\partial \psi / \partial x) \) and \( \nu \) the eddy diffusivity. The drift rate due to saltation is parameterized using the empirical formula
\[ Q_{\text{salt}} = k \frac{W_f}{u_{\ast \text{th}}} u_{\ast}^2 (u_{\ast} - u_{\ast \text{th}}) \]
(1.11)
where the friction velocity was determined by
\[ u_{\ast} = \frac{u(z) \kappa}{\ln(z/z_0)} \]
(1.12)
the constant \( k \) was assumed to be equal to 2.1 and the threshold shear stress set to \( u_{\ast \text{th}} = 0.15 \text{ m s}^{-1} \). Again, this model is only one-way coupled. In a next step, Uematsu et al. [28, 93] suggested a three-dimensional model. For the transport in the saltation mode, they proposed the same ansatz as above with some modifications in [91], but now they also considered the transport in suspension. They solve the Navier-Stokes equations including an additional concentration equation for the suspension:
\[ \frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0 \]
(1.13)
\[ \frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \rho K_m \frac{\partial u_i}{\partial x_j} \right) \]
(1.14)
\[ \frac{\partial \rho c}{\partial t} + \frac{\partial \rho c u_i}{\partial x_i} = \frac{\partial}{\partial x_i} \left( K_s \frac{\partial \rho c}{\partial x_i} \right) + \frac{\partial \rho c W_f}{\partial x_i} \delta_{ij} \]
(1.15)
They used a first-order turbulent closure, with the eddy viscosity given by
\[ K_m = l^2 \sqrt{\left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2} \]
(1.16)
where the mixing length \( l = \kappa r \) and \( r \) the distance to the nearest wall. The eddy diffusivity, \( K_s \), is assumed to be equal to \( K_m \).

Liston, a co-worker of R. Brown, introduced a one-and-a-half order closure in snow drift modeling [Liston et al. [35, 36]]. He used the \( \epsilon-\epsilon \) closure (or \( k-\epsilon \) closure in the engineering literature) for his model. He considered a steady, two-dimensional flow. The model is restricted to the case where wind speeds are low enough to neglect the transport in suspension. As for the description of the saltation mode, he used the following empirical formula
\[ Q_{\text{salt}} = 0.68 \text{ m s}^{-1} \frac{W_f}{u_{\ast \text{th}}} \left( \frac{u_{\ast \text{th}}}{u_{\ast}} \right) (u_{\ast}^2 - u_{\ast \text{th}}^2) \]
(1.17)
As solving technique for the nonlinear system of equations, Uematsu et al. [28, 91, 93] (see above) as well as Liston used a finite control-volume method. Both models are still one-way coupled.

Clappier and Castelle [12] proposed a semi-empirical model for the transport within the saltation layer, where they described the horizontal mass concentration transport by

$$\frac{\partial c_m}{\partial t} + \frac{\partial c_m U_{p, salt}}{\partial x_\alpha} = \frac{\partial}{\partial x_\alpha} \left( K_s \frac{\partial c_m}{\partial x_\alpha} \right) + j_{eros}$$  \hspace{1cm} (1.18)

here \( \alpha \) take on 1 and 2, and the \( j_{eros} \) is the erosion/deposition rate set to

$$j_{eros} = \frac{c_{m, sat} - c_m}{t_{sat}}$$  \hspace{1cm} (1.19)

The saturation concentration, \( c_{m, sat} \), and the time, \( t_{sat} \), to reach saturation, and the particle velocity, \( U_{p, salt} \), within the saltation layer are empirical values which they gained from the experiments by Takeuchi [88] (see page 8) and by Pomeroy. Castelle [10] used this model to recalculate the experiments by Takeuchi [88] (see Fig. 1.5). For it, he included the equation

$$-\frac{\partial c_{m, susp}}{\partial z} = \frac{\partial}{\partial z} \left( K_s \frac{\partial c_{m, susp}}{\partial z} \right)$$  \hspace{1cm} (1.20)

for the transport due to suspension. Then, he took (1.18), (1.20) and a logarithmic-wind profile as one's starting-point for derivation of the algebraic form of the mass flux within the lowest 0.3m above the snowpack

$$Q_{0.3} = h_s c_{m, sat} (U_{p, salt} + \Xi)$$  \hspace{1cm} (1.21)

with

$$c_{m, sat} = c_{m, sat} \left( 1 - \exp \left( \frac{x}{U_{p, salt} t_{sat}} \right) \right)$$  \hspace{1cm} (1.22)

where \( x \) is the distance from the starting point and

$$\Xi = \frac{u_s}{\gamma \kappa} \left( \left( \frac{0.3 \text{ m}}{z_0} \right)^\gamma \ln \left( \frac{0.3 \text{ m}}{z_0} \right) - \ln \left( \frac{h_s}{z_0} \right) - \left( \frac{0.3 \text{ m}}{h_s} \right)^\gamma \right)$$  \hspace{1cm} (1.23)

with \( \gamma = 1 - \frac{W_r}{\kappa u_s} \). \( \Xi \) is derived using Equation (1.7). The roughness length was set to 0.001 m, and the saltation height, the particle velocity and the saturation concentration are given by

$$h_s = k_1 \frac{u_{*-1}^2}{2g}$$

$$U_{p, salt} = k_2 u_{*th}$$

$$c_{m, sat} = 2 \rho_a k_3 \left( 1 - \frac{u_{*th}^\gamma}{u_s^\gamma} \right)$$  \hspace{1cm} (1.24)
where the empirical constants were set to $k_1 = 1.6$ and $k_2 = 1.6$, and the empirical parameter $k_3$ describing the efficiency was chosen to be $k_3 = 1$ for $u_s = u_{sth}$ and $k_3 = (0.31 \text{ m s}^{-1}/u_s)$ for $u_s \geq 1.5 \text{ u}_{sth}$. Castelle and Clappier neglected the feedback coupling (one-way coupling).

For the recalculation of his wind tunnel measurements, Martinez [39] restricted himself to consider only the transport due to suspension. To this end, he added a concentration equation to the Navier-Stokes equations. For turbulence closure, he also employed the $\varepsilon-\omega$ model. He used the commercial flow solver CFX4.1 (formerly called FLOW3D; it should be distinguished from the code FLOW-3D used by Sundsbo, see below). This program is based on a finite-volume technique.

Recently, Sato et al. [71] proposed a 2-dimensional random-walk approach for modeling blowing snow. The wind field is still calculated using the Navier-Stokes equations and $\varepsilon-\omega$ closure, but for the description of the snow motion a Lagrangian approach is used. For that, a stationary wind field is assumed and the particles are tracked by

$$x_\alpha(t + \Delta t) = x_\alpha(t) + (u_\alpha + W_{\alpha}) \Delta t + u'_\alpha \Delta t$$

(1.25)

where $x_\alpha$ is the position vector of the snow particle ($\alpha = 1$ or $2$), $W_{\alpha}$ is the vector of settling velocity, $u'_\alpha$ is a random fluctuation velocity, obtained for each time step from a Markov chain simulation after:

$$u'_\alpha(t + \Delta t) = u'_\alpha(t) R(\Delta t) + \sqrt{1 - R(\Delta t)^2} \sigma_\alpha \omega$$

(1.26)

where $\omega$ is random number from the Gaussian distribution with zero mean and unit variance. The variance, $\sigma_\alpha$, of the velocity fluctuation is related to the friction velocity by $k_\alpha u_*$. $R$ is the Lagrangian autocorrelation function given by

$$R(\Delta t) = \exp(-\Delta t/t_L)$$

(1.27)

where $t_L$ is the Lagrangian time scale. The back-reaction of the particles on the flow is not considered (one-way coupling).

Sundsbo and Hansen [86] described their model as a Lagrangian-Eulerian approach to the simulation of snow drift around man-made structures. The wind field is determined by the Navier-Stokes equations and particles are tracked according to the equation of particle motion, in [86] given by

$$\frac{dU_{Pi}}{dt} = -\frac{3}{4} \frac{C_d}{dP} \frac{\rho}{\rho_p} \left[ U_{Pj} - u_j \right] \left( U_{Pi} - u_i \right) - g \delta_{i3}$$

(1.28)

To create a random particle behavior, they superimposed a kind of diffusion velocity to mean particle velocity: $U_{Pi} = U_{\text{mean}i} + U_{\text{diff}i}$. The diffusion velocity is given as:

$$U_{\text{diff}i} = \sqrt{\frac{4k}{\Delta t}} \text{erf}^{-1}(\omega) \delta_i$$

(1.29)
where $k$ is the particle diffusion coefficient, $\Delta t$ is the time step size and $\varpi \in [0, 1]$. $\delta_i$ denotes the unit vector. The erosion and deposition is controlled by the threshold shear stress, no difference is made between aerodynamic entrainment and ejection due to impacts. They emphasized the engineering aspects. To save computation time, they enhance the volume of deposited particles by order of $10^9$. There is no feedback from the snow to the air. The drift model is implemented in the transient flow code FLOW-3D.

Up to here, in all numerical models the description of the fluid is based on the Navier-Stokes equations. Masselot and Chopard [42, 43] chose a totally different approach. They propose a lattice Boltzmann model for the simulation of snow drift. The starting-point of their cellular automata approach is the assumption that the fluid as well as the snow grains can be modeled by a set of particles moving synchronously on a regular lattice. At each time step and for each lattice cell, a new local distribution of the snow and air particles is computed. This recalculation is based on rules for “particle” collisions conserving mass and momentum, and so within certain limits the Navier-Stokes equations will be fulfilled. Erosion and deposition are modeled by heuristic rules for the impact on solid boundaries. At this time, the model is still 2-dimensional. One advantage of this numerical approach is the high potential for parallelization.

To complete this overview, two empirical rule based models are worth mentioning: ELSA [40, 41, 82] belonging to CEMAGREF/Grenoble and a model by Purves et al. [60]. Both models determine the snow distribution in an area, by means of heuristic rules concerning the topography and some information on the mean wind. These models are implemented as modules in geographical information systems (GIS), such as ARC/INFO.

Table 1.3 shows a compilation of the numerical models. At present, almost all models are one or two dimensional. Models based on empirical expressions for the saltation layer, which describe the steady-state transport, are strictly speaking only valid for fetches long enough to achieve steady-state conditions. In most models, with exception of the saltation models, feedback is disregarded, thus they are only one-way coupled. This will cause an overestimation of the transport rate. As suspension can have a significant contribution, models which disregard this mode will underestimate the transport rate. Treating erosion, most models do not distinguish between aerodynamic entrainment and ejecta due to impact.
<table>
<thead>
<tr>
<th>Model</th>
<th>Suspension</th>
<th>Deposition</th>
<th>Wind Field</th>
<th>Comp 1</th>
<th>Empirical</th>
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<td>McEwan et al.</td>
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Table 1: Compilation of the existing numerical models.

The empirical formulations are typically of the form \( Q = C_0 (tL^*)^3 \), with \( tL^* \) derived from field-measurements on planes and \( C_0 \) being a constant that can be derived from laboratory experiments on horizontal planes for steady-state conditions (see Table 1.1).
Due to the limited validity of used simplifications, the presented models are not able to concern complex topographies like an alpine terrain and non-steady wind fields, to this end. Hence, to overcome some of the weak points of the previous models, in this work attention is focused on:

- fully 3-dimensional non-steady-state modeling;
- dynamical treatment of the saltation transport;
- dynamical modeling of deposition and erosion, distinguishing between aerodynamic entrainment and ejecta due to particle impacts;
- two-way coupling;
- including the suspension transport.
2 The basic physics

Blowing and drifting snow is strongly related to the atmospheric boundary layer (ABL). The erosion, the transport and the deposition of snow depend on the wind field and the turbulence within the lowest 10–100 m of the boundary layer, the so-called surface layer (SL). As already mentioned, we can identify five mutually linked processes responsible for the redistribution of snow:

- aerodynamic entrainment,
- grain trajectories,
- grain-snowpack impacts,
- modification of the wind field,
- and the transport due to turbulent suspension,

for which the boundary layer flow is the driving force. On the other hand, the drifting snow acts like an increased roughness length and serves as feedback onto the wind field.

In this section, the physical basis of the different processes will be explained. We start with the governing equations for the boundary layer which define the settings for suspension as well as for saltation. Then we will focus on the motion in the saltation layer and the modification of the wind field. Based on this physical background in Chapter 3 a numerical model for blowing and drifting snow will be presented.
Before starting, here are some conventions used throughout the text:

- As usual in geophysical approaches, a Cartesian coordinate system is used aligned such that the \((x, y, z)\) axes point (east, north, up) and \((u, v, w)\) are the velocity components in the \(x-, y-,\) and \(z-\)directions. \(\delta_1 = \hat{i}, \delta_2 = \hat{j},\) and \(\delta_3 = \hat{k}\) are the corresponding unit vectors. Exceptions from this convention will be made for description of the saltation layer. Here \(u\) is pointing in direction of the mean flow direction parallel to the surface and \(w\) perpendicular to the surface.

- Einstein summation notation will be used. Readers not familiar with this notation are referred to appendix C for a short introduction.

2.1 Boundary-layer equations

Five equations form the foundation of boundary layer meteorology: the equation of state, and the conservation equations for mass, momentum, moisture and heat. In the following, Reynolds averaging is already applied in almost all cases to get the equations for the mean variables within the turbulent flow. To simplify notation, overbars indicating mean values are dropped, only primes marking the turbulent (perturbation) parts and overbars indicating averages of products of turbulent parts are kept. For a complete derivation of these equations, the reader is referred to the literature, e.g., [56, 85].

2.1.1 Equation of state (Ideal Gas Law)

The ideal gas law adequately describes the state of gases in the boundary layer:

\[
\frac{p}{\rho_a} = \rho_a T_u + \rho_a' T_u' \tag{2.1}
\]

where \(p\) is the pressure, \(\rho_a\) is the density of moist air, \(T_u\) is the virtual absolute temperature, and \(R_a\) is the gas constant of dry air. The virtual absolute temperature \(T_u\) is usually given as

\[
T_u = T (1 + 0.61 r) \tag{2.2}
\]

where \(T\) is the absolute temperature, and \(r\) the water-vapor mixing ratio of the air parcel \([\text{kg water vapor kg}^{-1}]\). For saturated air with a saturation mixing ratio \(r_{sat}\) and a liquid/solid-water mixing ratio \(r_L\), the virtual potential temperature is

\[
T_v = T (1 + 0.61 r_{sat} - r_L) \tag{2.3}
\]

The virtual temperature \(T_v\) is defined as the temperature that dry air must be at order to have the same density as moist air at temperature \(T\). The approximation in (2.2) and (2.3) are only valid for \(r \ll 1\), which is not always true in the case of heavy rain or snow-loaded air. Hence, it is better to use the original definition

\[
T_v = T \left( \frac{r_{sat} + 0.622}{0.622} \right) \frac{1}{1 + r_{sat} + r_L} \tag{2.4}
\]
Usually, the second term on the right hand side in equation (2.1) is much smaller than the first, so
\[
\frac{p}{R_a} = \rho_a T_v
\]
(2.5)
is a reasonable approximation for the equation of state.

2.1.2 Conservation of mass (Continuity)

The continuity equation can be written as
\[
\frac{1}{\rho_a} \frac{d\rho_a}{dt} + \frac{\partial \tilde{u}_j}{\partial x_j} = 0
\]
(2.6)
where \(\tilde{()}\) mark quantities including the mean and the perturbation part \((\tilde{a} = a + a')\). Businger [9] showed that
\[
\frac{1}{\rho_a} \frac{d\rho_a}{dt} \ll \frac{\partial \tilde{u}_j}{\partial x_j}
\]
(2.7)
is valid, if the following conditions are met:

- \(M^2 \ll 1\), where Mach number \(M = U/C_s\), and \(U\) is a typical velocity scale and \(C_s\) the speed of sound;
- \(M/F \ll 1\), the Froude number, \(F = U^2/(Lg)\), and \(L\) is a length scale for the boundary layer;
- \(L f_p/C_s \ll 1\), where \(f_p\) is the frequency of any pressure wave that might occur.
- \(L/H_{atm} \ll 1\), \(H_{atm}\) is the scale height of the atmosphere \((\approx 12\text{ km})\)

Since these conditions are generally met for all turbulent motions smaller than meso \(\gamma\) scale \((L \ll 12\text{ km})\), (2.6) reduces to the incompressibility approximations:
\[
\frac{\partial \tilde{u}_j}{\partial x_j} = 0
\]
(2.8)
\[
\frac{\partial u'_j}{\partial x_j} = 0
\]
(2.9)

2.1.3 Conservation of momentum

The conservation of momentum is expressed by the following three equations
\[
\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho_a} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_i \partial x_j} + f_c \epsilon_{ijk} u_j - \delta_{ij} g - \frac{\partial u'_i u'_j}{\partial x_j}
\]
(2.10)
where \(\nu\) is the kinematic viscosity, the Coriolis parameter, \(f_c\), is defined as \(f_c = 2\omega \sin \Phi\). The angular rotation rate of the Earth, \(\omega\), is \(7.27 \times 10^{-5}\text{ rad s}^{-1}\), and for the Swiss Alps, the latitude, \(\Phi\), is approximately \(47^\circ\), and so \(f_c = 10^{-4}\text{ s}^{-1}\). \(\delta_{ij}\) and \(\epsilon_{ijk}\) are the Kronecker Delta and the Alternating Unit Tensor, respectively.
2.1.4 Conservation of moisture

The specific humidity is defined as the ratio of the mass of water per unit mass of moist air \([\text{kg}_{\text{water}}/\text{kg}_{\text{moist air}}]\). The total specific humidity, \(q_T\), can be split into vapor, \(q\), and non-vapor, \(q_L\), parts using \(q_T = q + q_L\), where the non-vapor part can exist of liquid or solid water, respectively. In the case of snow, the solid water content is given by \(c_m = \rho_p c\), where \(c_m\) is the mass concentration, \(\rho_p\) is the density of an ice grain, and \(c\) is the volumetric concentration. The conservation of moisture can now be written, assuming incompressibility, as a pair of coupled equations

\[
\frac{\partial q}{\partial t} + u_j \frac{\partial q}{\partial x_j} = \nu_q \frac{\partial^2 q}{\partial x_j \partial x_j} + \frac{S_q}{\rho_a} + \frac{E}{\rho_a} \frac{\partial u_j}{\partial x_j} \tag{2.11}
\]

\[
\frac{\partial q_L}{\partial t} + u_j \frac{\partial q_L}{\partial x_j} = \frac{S_{qL}}{\rho_a} - \frac{E}{\rho_a} \frac{\partial u_j}{\partial x_j} \tag{2.12}
\]

where \(\nu_q\) is the molecular diffusivity for water vapor in air, and \(E\) represents the mass of water vapor per unit volume per unit time being created by phase change from liquid or solid. In (2.12), the molecular diffusivity term is dropped as it is usually negligible. The terms including \(S_q\) and \(S_{qL}\) are net moisture source terms (sources/sinks) not already included in the equations, so falling of precipitation (non-advecting transport) could be included as part in \(S_{qL}\).

2.1.5 Conservation of heat

A common procedure in micro-meteorology is the use of a conservation equation for the potential temperature instead of the conservation equation for the enthalpy. The potential temperature \(\Theta\) is defined as

\[
\Theta = T \left( \frac{P_0}{p} \right)^{R_a/C_p} \tag{2.13}
\]

where \(P_0\) is a reference pressure, set to 1000 hPa (for calculations, sometimes an adapted reference pressure is used) and \(R_a/C_p \approx 0.28\). It is a measure for the sum of the potential and internal energy of an air parcel. The conservation equation for the potential temperature can be written as

\[
\frac{\partial \Theta}{\partial t} + u_j \frac{\partial \Theta}{\partial x_j} = \nu_\Theta \frac{\partial^2 \Theta}{\partial x_j \partial x_j} - \frac{1}{\rho_a C_p} \frac{\partial I_j}{\partial x_j} - \frac{L_p E}{\rho_a C_p} \frac{\partial u_j}{\partial x_j} \tag{2.14}
\]

here \(\nu_\Theta\) is the thermal diffusivity, and \(L_p\) is the latent heat associated with the phase change of \(E\). The specific heat for moist air at constant pressure, \(C_p\), is approximately related to that of dry air \(C_p = C_{pd}(1 + 0.84q)\). \(I_j\) is the vector of the net radiation.

The equations (2.5), (2.6), (2.10), (2.11), (2.12) and (2.14) form the basis for the description of the wind field and the suspension layer in the numerical model (see section 3.1).
2.2 Motion within the saltation layer

The motion within the saltation layer is in two respects crucial for the snow transport: Firstly, the snow concentration is highest within the saltation layer and so saltation itself contributes significantly to the mass transport. Secondly, saltation serves as a source for suspension.

2.2.1 Aerodynamic entrainment

Exactly how saltation starts in the absence of any external mechanical disturbance is still incompletely known. There are only few studies which focus their attention on the aerodynamic entrainment, e.g., Williams et al. [103]. Anderson and Haff [4] proposed a linear relation between the number of entrained grains per unit area and unit time, $N_{ae}$, and the excess shear stress

$$N_{ae} = \zeta (\tau_a - \tau_e) \tag{2.15}$$

where $\tau_a$ is the so-called air-borne shear stress and is assumed to be equal to the shear stress of the air flow at the bed, $\tau_e$ is the critical fluid shear stress for entrainment, and $\zeta$ is an empirical constant. Again very little is known about the dependency of the critical shear stress on the snowpack properties, such as the cohesion, and on the meteorological conditions. R.A. Schmidt [76] gave an estimation on the mean value of the critical shear stress for a plane. For a slope with slope angle $\phi$, his statement might be modified considering the balance of the angular momentum due to the forces acting on a grain which will be computed for the motion around the contact point $o$ (see Fig. 2.2).

$$\tan \beta' \tan \phi = \frac{F_D}{F_G} \tag{2.16}$$

where $F_D$ is the required drag force to move the grain, $F_L$ is the lift force approximated by 0.85 $F_D$, $F_G$ is the gravitational force, and $F_C$ is the cohesion. The angle

![Figure 2.2: Sketch of the snowpack for the analysis of the critical shear stress, $\tau_c$](image-url)
of repose $\beta$ is assumed to be $33^\circ$ and $\beta'$ determines the mean drag level ($\approx 24^\circ$). If we assume that the drag force acts on the cross-section $A = \pi d_r^2/4$, we get

$$
\tau_c = \frac{\eta}{\Gamma} \left( 0.666 \left( \rho_P - \rho_a \right) g d_F (\cos \phi - \sin \phi/\tan \beta) + \frac{F_C}{A} \right) \tan \beta'
$$

(2.17)

Determining the mean critical shear stress, Schmidt includes two constants $\eta = 0.21$ representing the packing ratio and $\Gamma = 2.5$ describing the ratio of the maximum to the mean turbulent impulse. Typical values for the cohesion $F_C$ range from a low of the order $10^{-8}$ N, soon after contact between small particles at low temperatures, to $10^{-2}$ N, for large sintered particles. Hence, the critical shear stress $\tau_c$ for aerodynamic entrainment, assuming a grain size of $d_r \approx 200 \mu m$, is in the range from 0.04 to 9000 Pa. In comparison, typical values for the wind shear stress/Reynolds stress, $\tau_a$, during snow drift are in the range from 0.06 to 1 Pa, so we can see that aerodynamic entrainment is only possible within small bounds. On the other hand, the snowpack is not homogeneous, so it is reasonable to assume that bond strength of single grains have a certain distribution. The mean and the variation of the distribution might depend on snowpack properties. Thus, even for old settled snow, one can find some few grains which could be effected by the wind.

The model above gives a rough estimation of the threshold, but little is known about how exactly the snowpack properties and the meteorology influence the threshold, for example:

- **Cohesion**
  The cohesion of a single grain is given by the bond strength and the number of bonds to neighboring grains (3-dimensional, coordination number). With increasing time after deposition, both bond size and coordination number will increase depending on the meteorological conditions. Thus, e.g., Schmidt emphasized a strong temperature effect on cohesion soon after contact between ice particles, where with decreasing temperature the cohesion decreases. He cited the relation

$$
\frac{F_C}{F_C(-15^\circ)} = 8 \exp (0.141 T)
$$

(2.18)

here $T$ is in Celsius degrees. He also mentioned that initial strength of the bond is a strong function of humidity. Thus, the initial contact area is apparently smaller at low humidities, and, for a completely dry atmosphere, the strength of the bonds decreases to a negligible value below $-5^\circ C$.

- **Time scale of gusts**
  As snow grains sinter soon after contact with the snowpack and the bond strength increase with the time, the time scale of gusts will be of significance for the aerodynamical entrainment.
Grain shape
Nothing is known about the influence of the grain shape. It is reasonable to assume that stellar crystal and large surface hoar crystals can easily be broken by wind forces and the initial threshold value will be low. On the other hand, wind-drifted, fragmented crystals will form compact layers with high threshold values.

2.2.2 Grain trajectories (equations of grain motion)
After the initiation of the grain motion by the wind, some of the grains start to jump over the snowpack following ballistic trajectories. It is widely accepted that the saltation trajectories are governed by:

- **Gravitation**
  The submerged weight of a particle is given by
  \[
  F_{G_i} = \frac{4}{3} \pi \left( \frac{d_p}{2} \right)^3 (\rho_p - \rho_a) g_i
  \]
  where the gravity vector \( g_i = (0, 0, -g) \) (for reference see, e.g., Clift et al. [13, Chapter 11]).

- **Drag force**
  The force due to the viscous stress acting on a particle is expressed by the drag force
  \[
  F_{D_i} = -\frac{1}{2} \pi \left( \frac{d_p}{2} \right)^2 \rho_a C_D \left\| U_{e_j} \right\| U_{e_i}
  \]
  where the relative velocity \( U_{e_i} = (U_{p_i} - u_i) \), \( \left\| U_{e_i} \right\| \) is the vector (Euclidean) norm of the relative velocity vector, e.g., \( \left\| U_{e_i} \right\| = \sqrt{U_{e_j} U_{e_j}} \), and \( C_D(Re_p) \) is the drag coefficient (for reference see, e.g., Clift et al. [13, Chapter 11]). Several approximations exist for \( C_D \), valid for different ranges of the particle Reynolds number \( Re_p = \left\| U_{e_j} \right\| d_p/\nu \). In the following, the expression
  \[
  C_D(Re_p) = \frac{24}{Re_p} + \frac{4}{Re_p^{1/3}}
  \]
  is used, given in [67, p. 387]. This approximation is valid for \( Re_p < 400 \) which is sufficient for our purpose.
• **Lift Force**

The lift force, \( F_L \), is one of the pertinent dynamic forces acting on small spheres in a shear flow. Based on the derivation of Saffman [69, 70], Harper and Chang [23] generalized his analysis and wrote

\[
F_{Li} = 9 \pi^2 d_F^2 \rho_a \nu^{0.5} S^{0.5} L_{ij} U_{\dot{\epsilon}j}
\]

where the lift tensor is found to be

\[
L_{ij} = \begin{pmatrix}
5.01 & 0 & 3.29 \\
0 & 3.73 & 0 \\
1.82 & 0 & 1.73
\end{pmatrix} \cdot 10^{-2}
\]

The lift force is proportional to the magnitude of the velocity gradient \( S \) (in the case of a simple shear flow equal to \( \left| \frac{\partial U_\dot{\epsilon}}{\partial z} \right| \)). This lift force might have a special importance for the initiation of the particle motion, when the particle launched from the surface. Here, close to the surface, the wind shear is greatest, and hence the lift force. Saffman’s derivation is only valid for small Reynolds numbers, however \((\frac{S}{d_F^2/\nu} \ll 1, \frac{1}{U_\dot{\epsilon}U_{\dot{\epsilon}}/d_F^2/\nu} \ll 1 \) and \( \frac{\rho_a(d_F^2/\nu)}{\dot{\epsilon}} \ll 1 \).

• **Magnus force**

The Magnus force, \( F_M \), is determined by the particle spin during saltation, resulting in additional lift. White [97, 98], and White and Schulz [99] give the following expression for the acting force

\[
F_{Mi} = \pi \left( \frac{d_F}{2} \right)^3 \rho_a \epsilon_{ijk} \Omega_{\dot{\epsilon}j} U_{\dot{\epsilon}k}
\]

where \( \Omega_{\dot{\epsilon}j} \) is the relative angular velocity vector. \( F_M \) acts orthogonal to the directions of \( \Omega_{\dot{\epsilon}j} \) and \( U_{\dot{\epsilon}i} \).

The following forces are usually not included:

• **Added (virtual) mass force**

This term depends on the relative acceleration of the particle with respect to the surrounding fluid resulting in an apparent mass, in addition to the real mass of the particle. The force is proportional to the mass of the displaced fluid times the relative acceleration (for reference see, e.g., Clift et al. [13, Chapter 11]).

\[
F_{Ai} = -\frac{2}{3} \pi \left( \frac{d_F}{2} \right)^3 \rho_a \frac{d U_{\dot{\epsilon}i}}{dt}
\]
• **Pressure gradient term**

The pressure gradient term describes the force acting on a particle due to a pressure differential across the particle (pressure gradient of the ambient flow), also effective in an inviscid flow. It represents the force required to accelerate the fluid which would occupy the volume of the particle if the particle is absent. This term is given by (for reference see, e.g., Clift et al. [13, Chapter 11])

\[ F_{P_i} = \frac{4}{3} \pi \left( \frac{d_P}{2} \right)^3 \rho_a \frac{du_i}{dt} \]  

(2.26)

• **Electrostatic forces**

Up to now electrical effects had not been considered in the description of the transport of drifting sand or snow. D.S. Schmidt et al. [72-75] mentioned that electrical force can significantly influence the particle motion. The force acting on a charged particle in an electrostatic field is given by

\[ F_{E_i} = q_e E_i \]  

(2.27)

where \( q_e \) is the charge on a grain and \( E_i \) is the electric field strength.

• **Basset history term**

The influence of the history of the flow of the particle motion, as the effect of acceleration on the viscous drag and the boundary layer development, is expressed by the Basset history term (for reference see, e.g., Clift et al. [13, Chapter 11]).

\[ F_{B_i} = -6 \rho_e \left( \frac{d_P}{2} \right)^2 \sqrt{\pi} \nu \int_0^t \frac{\frac{dU_{gl}}{dt'}}{\sqrt{t-t'}} dt' \]  

(2.28)

Figure 2.3: Forces and velocities associated with a saltating particle.
Combining these forces yields the equations of the particle motion in saltation:

\[ m_p \frac{dU_i}{dt} = F_{G_i} + F_{D_i} + F_{L_i} + F_{M_i} + F_{A_i} + F_{P_i} + F_{E_i} + F_{B_i} \]  

(2.29)

To get an estimate of the relevance of the different terms, it is useful to make a scaling. For that reason all terms will be made non-dimensional by dividing or multiplying by characteristic scales, respectively. Here the following scaling is used:

\[ (U_P, W_P, U_t, u) \rightarrow (U_P^*, U W_P^*, U U_t^*, U u^*) \]

\[ t \rightarrow \frac{u}{g} t^* \]

\[ \Omega \rightarrow \Omega \Omega^* \]

\[ S \rightarrow S S^* \]

\[ q_e \rightarrow q_e q_e^* \]  

(2.30)

Dimensionless values are marked with \((\cdot)^*\) and are of order \(O(\lesssim 1)\). In the following, it is assumed that the mean flow is along the x-axis and the z-axis is perpendicular to the plane surface, so a simple shear flow is considered. After a short calculation the equations of motion, (2.29), can be rewritten as

- in x-direction

\[ \frac{dU_P^*}{dt^*} = - \frac{3}{4} C_D \frac{\rho_a}{\rho_P g d_P} U^2 |U_e|^* (U_P^* - u^*) \]  

III

\[ + 0.54 \pi \frac{\rho_a}{\rho_P} \frac{U}{d_P g} S^{0.5} (U_P^* - u^*) \]  

(5.01 (U_P^* - u^*) + 3.29 (W_P^* - w^*))  

(2.31)

\[ + \frac{3}{4} \frac{\rho_a}{\rho_P} \left( \frac{U \Omega}{g} \Omega_P^* - \frac{1}{2} \frac{U S}{g} S^* \right) W_P^* \]  

IV

\[ - \frac{1}{2} \frac{\rho_a}{\rho_P} \frac{d(U_P^* - u^*)}{dt^*} + \frac{\rho_a}{\rho_P} \frac{du^*}{dt^*} \]  

V

\[ - \frac{g}{\sqrt{\pi} d_P} \frac{\rho_a}{\rho_P} \sqrt{\frac{U}{g}} \int_0^{t^*} \frac{d(U_P^* - u^*)}{\sqrt{t^* - t^*}} dt^* \]  

VIII
in $z$-direction

$$
\frac{dW_p^*}{dt^*} = -\frac{\rho_p - \rho_a}{\rho_p} - \frac{3}{2} C_D \frac{\rho_a}{\rho_p} \frac{U^2}{g_d} \| U_e \| W_p^* \\
+ 0.54 \pi \frac{\rho_a}{\rho_p} \frac{S^{0.5} \mathbf{U} \cdot S^{0.5}}{d_p g} (1.82 (U_p^* - u^*) + 1.73 (W_p^* - w^*))
$$

$$
- \frac{3}{4} \rho_a \left( \frac{\Omega^* P}{g} - \frac{1}{2} \frac{\mathbf{S} \cdot \mathbf{S}^*}{g} (U_p^* - u^*) \right)
$$

$$
- \frac{1}{2} \frac{\rho_a}{\rho_p} \frac{dW_p^*}{dt^*} + \frac{q_e E^*}{m_p g} q_e E^*
$$

$$
- \frac{9}{\sqrt{\pi} d_p g} \frac{\rho_e}{\rho_p} \sqrt{\nu U} \int_0^{t^*} \frac{dW_p^*}{dt^*} \frac{\delta x}{\sqrt{t^* - t^*}} dt^*
$$

(2.32)

where the relation $\Omega^* = \Omega_P - \frac{1}{2} \Omega$ is used for the relative angular velocity, with the particle angular velocity, $\Omega_P$. The second term corresponds to the intrinsic angular velocity $\frac{1}{2} \epsilon_{ijk} \frac{\partial u_k}{\partial x_j}$ of a sphere in a shear flow [68]. The electric field is assumed to be pointing perpendicular from the surface. To get an impression of the order of the different terms, the following estimates for the characteristic scales are used (a compilation is given in Table 2.1):

- $\mathbf{U} = 1\text{–}5 \:\text{m}\cdot\text{s}^{-1}$: If we assume that the saltation layer height is about 0.01 m, then a value of 5 m s$^{-1}$ corresponds to a wind speed of about $M_{10} = 15 \:\text{m}\cdot\text{s}^{-1}$, assuming a logarithmic wind profile with $z_0 = 10^{-4} \:\text{m}$. This seems to be reasonable during snow drift, likely on the high end. The characteristic value for $U_e$ is likely to be lower.

- $t = \mathbf{U}/g$: This value corresponds approximately to the jump time of a rebounding particle, if we assume that the particles are launched with a speed of 0.5 $\mathbf{U}$.

- $d_p$: Measurements of drifting snow particles [59, 78] showed that typical particle diameters are 150–250 $\mu$m.

- $C_D$: Typical values for the drag coefficient range from approximately 1 to 30 ($Re_p \approx 1\text{–}50$).

- $S$: Within the viscous sublayer ($z \leq 10 \nu / u_* \approx 5 \cdot 10^{-4} \:\text{m}$), $S \approx u_*^2 / \nu \approx 15000 \:\text{s}^{-1}$. Above the viscous sublayer, $S \approx u_* / (\kappa z) \approx 100\text{–}2500 \:\text{s}^{-1}$. Again, this value might be on the high end because during snow drift the wind profile
Table 2.1: Typical values of quantities within the saltation layer for the
determination of the characteristic scales.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air density at 2,500 m a. s. l.</td>
<td>$\rho_a$ 0.98 kg m$^{-3}$</td>
</tr>
<tr>
<td>Kinematic viscosity of air ($T_{air} = 273$ K)</td>
<td>$\nu$ 1.74 $\cdot$ 10$^{-5}$ m$^2$s$^{-1}$</td>
</tr>
<tr>
<td>Density of snow-ice particles</td>
<td>$\rho_p$ 917 kg m$^{-3}$</td>
</tr>
<tr>
<td>Particle diameter</td>
<td>$d_p$ 200 $\mu$m</td>
</tr>
<tr>
<td>Drag coefficient</td>
<td>$C_D$ 10</td>
</tr>
<tr>
<td>Velocity scale</td>
<td>$U$ 3 m s$^{-1}$</td>
</tr>
<tr>
<td>Shear scale</td>
<td>$S$ 1000 s$^{-1}$</td>
</tr>
<tr>
<td>Angular velocity of the particle</td>
<td>$\Omega/(2\pi)$ 300 s$^{-1}$</td>
</tr>
<tr>
<td>Electric field strength</td>
<td>$E$ 200 kV m$^{-1}$</td>
</tr>
<tr>
<td>Charge to mass ratio</td>
<td>$q_e/m_p$ $\pm$50 $\mu$C kg$^{-1}$</td>
</tr>
</tbody>
</table>

within the saltation layer will be modified due to particle-fluid interaction and
the shear will be reduced.

- **$\Omega$:** Wind tunnel experiments done by White and Schulz [97, 99] showed that
saltating particles can have spin rates of about 100–500 s$^{-1}$ corresponding to
angular velocities of about 600–3000 s$^{-1}$.

- **$E$:** To estimate the contribution due to the electrical force the approxi­
ma­tion $E = 200$ kV m$^{-1}$ is used. This approximation corre­sponds to the expres­sion for a calculated electric field strength $E(z) = 373 z^{-0.92}$ kV m$^{-1}$ given by
D.S. Schmidt and Dent [72] and $z \approx 1$ mm. In comparison, their mea­sure­ments [73] of the field strength yield $E(z) = 2800 z^{-0.46}$ kV m$^{-1}$ (calculated
from their Figure 2). The measured particle charge-to-mass ratios, $q_e/m_p$, differ from $-208$ $\mu$C kg$^{-1}$ to 72 $\mu$C kg$^{-1}$ [72], or from $-1$ $\mu$C kg$^{-1}$ to 1 $\mu$C kg$^{-1}$
[38], exhibiting a temperature dependence, among other factors.

Based on these values, it is possible to determine the order of the terms I–VIII and
to get an estimation of their relative contribution to particle motion.

<table>
<thead>
<tr>
<th>Term</th>
<th>Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$O(\approx 1)$</td>
</tr>
<tr>
<td>II</td>
<td>$O(\lesssim 1)$</td>
</tr>
<tr>
<td>III</td>
<td>$O(\lesssim 0.7)$</td>
</tr>
<tr>
<td>IVa</td>
<td>$O(\lesssim 0.5)$</td>
</tr>
<tr>
<td>IVb</td>
<td>$O(\lesssim 0.2)$</td>
</tr>
<tr>
<td>V</td>
<td>$O(\approx 10^{-3})$</td>
</tr>
<tr>
<td>VI</td>
<td>$O(\approx 10^{-3})$</td>
</tr>
<tr>
<td>VII</td>
<td>$O(\lesssim 1)$</td>
</tr>
<tr>
<td>VIII</td>
<td>$O(\lesssim 0.7)$</td>
</tr>
</tbody>
</table>
The approximations for the terms III, IV, VII and VIII are rather conservative, so it is reasonable that their contribution to the particle motion is more likely smaller. Thus, let us neglect all terms with the exception of the terms I and II in a first approximation. These simplifications might not be justified along the whole particle trajectory, particularly close to the surface, but for our purpose the simplified equations will be sufficient\(^2\). Thus, Equations (2.31) and (2.32) reduce to a balance of inertial, drag force, and gravity.

\[
\begin{align*}
\frac{dU_p}{dt} &= -\frac{1}{2m_p} \pi \left( \frac{d_p}{2} \right)^2 \rho_a C_D \left| U_p \right| (U_p - u) \\
\frac{dW_p}{dt} &= -\frac{1}{2m_p} \pi \left( \frac{d_p}{2} \right)^2 \rho_a C_D \left| U_0 \right| W_p - g
\end{align*}
\]

These equations form a system of differential equations which can be solved for given initial conditions \((U_{p0}, W_{p0})\) and known wind field or \(u(z)\). They build the basis for the parameterizing of the saltation layer, described in Section 3.2.

### 2.2.3 Grain–snowpack impacts

Grain–snowpack impacts play an important role during aeolian sediment transport. Impacts are responsible for ejection of new particles, and a large number of particles in saltation rebound on impact. The rebounding particles lose only part of their energy. Their next jump may thus be higher, so they get even more accelerated by the faster flow. Some of these particles will be caught by a gust and carried away in suspension, others are fast enough to eject one or more particles on impact. Figure 2.4 shows a sketch of such a particle impact.

---

\(^2\)Above all, the disregarding of the electrostatic force can be critical. To this end, it is not clear if electrostatic forces really significantly influence the trajectories of saltating grains. On the other hand, it is reasonable that electrostatic force contribute to the formation of cornice [33], but this is not further considered in this study.
Several investigations concerning the particle-bed impact have been made throughout the years. For these purpose, physical simulations, e.g., [32, 64, 65], as well as numerical simulations, e.g., [4, 96], have been carried out. All these studies confirm that impacting particles behave in similar way, in that they have a certain probability to rebound, and the mean impact and launch angles as well as the ratio of impact to rebound velocity vary only in a small band. Also ejected particle behave in a certain manner. Typical values of the quantities describing the particle-bed impact will be summarized in Table 2.2.

\[ \zeta_R = \frac{U_R}{U_I} \approx 0.5 - 0.6 \]
\[ \zeta_E = \frac{U_E}{U_I} \approx 0.1 \]
\[ \alpha_I \approx 10^\circ - 20^\circ \]
\[ \alpha_R \approx 35^\circ - 45^\circ \]
\[ \alpha_E \approx 50^\circ - 70^\circ \]

Table 2.2: Typical values of the quantities describing the particle-bed impact.

The experiments also show that the probability of particles to rebound is approximately 0.6-0.9 for the investigated impact speeds. Anderson and Haff [4] gave the following expression for this probability:

\[ P_R = 0.95 \left( 1 - \exp\left( -\xi \| U_I \| \right) \right) \tag{2.35} \]

where \( \xi \) is an inverse velocity scale set to \( 2.0 \text{ m}^{-1} \text{s} \).

To estimate the number of particles ejected per impact, \( N_{EpI} \), it is reasonable to assume that this is a function of the difference of the kinetic energy of a particle before \( (E_I) \) and after \( (E_R) \) the impact, of the total kinetic energy of the ejected particles \( (N_{EpI} E_E) \), the energy dissipated \( (E_D) \) into the bed during the impact and, if we include cohesion, of the total energy of the bonds \( (N_{EpI} E_B) \) which had to be broken. Out of the energy balance, we get

\[ N_{EpI} = \frac{E_I - E_R - E_D}{E_B + E_E} \tag{2.36} \]

Experiments carried out by Dietrich [16] confirm this estimation. He found that wind-blast abrasion is mainly controlled by the effective kinetic energy of the projectiles and the bond strength of the target.
2.2.4 Modification of the wind field

In 1941, Bagnold [8] was the first who noted the feedback between particle movement and the driving wind. A common approach to characterize this feedback uses a logarithmic profile to fit the horizontal wind at any height \( z \) above the saltation layer:

\[
M(z) = \frac{u^*}{\kappa} \ln \left( \frac{z}{z_0} \right) \tag{2.37}
\]

Bagnold observed that the roughness length during sand movement differs from that over a flat sand surface. In his first hypothesis Owen (see also section 1.2) stated that the saltation layer behaves as an aerodynamic roughness to the flow outside. As already mentioned, he assumed that the saltation layer height scales as \( L_s \sim (z_0/2g) \) and so the roughness length is given by

\[
z_0 = k \frac{u^*_s}{2g} \tag{2.38}
\]

where \( k \) is a constant. Owen proposed \( k = 0.02 \), whereas e.g., Rasmussen et al. [62] reported values of 0.14 and 0.18 for field experiments. The expression above is similar to Charnock’s relationship [11] for the roughness length of a sea surface. Here \( k \) is equal to 0.013.

Up to now this relation has almost always been used to parameterize the interaction between the particles in the saltation layer (extraction of momentum) and the wind above. However, there are two weak points:

- It is unreasonable that the roughness length only depends on the saltation layer height and that there is no dependency on the number of particles within the saltation layer. In this case, ten particles jumping 5 cm high will have a greater influence on the wind speed as thousands of particles jumping 1–2 cm.

- Equation (2.37) cannot explain that the wind speed just above the top of the saltation layer (\( h_s \approx 1–2 \text{ cm} \)) decreases more rapidly than predicted—an effect which could be observed in Bagnold’s classical wind velocity plot (see Fig. 2.5).

The primary effect of the particles on the airflow is to provide a spatially distributed momentum sink, with a sink strength \( d\tau_a(z)/dz \cong -d\tau_g(z)/dz \) per unit volume of fluid. \( \tau_a(z) \) is the flux of fluid momentum across the level \( z \), and \( \tau_g(z) \) represents the net flux of horizontal grain momentum across that level. \( d\tau_g(z)/dz \) corresponds to the interaction (drag) forces on the wind, due to the horizontal acceleration the grains by the air. The total shear stress is given by

\[
\tau_{hs} = \tau_a(z) + \tau_g(z) \tag{2.39}
\]

where \( \tau_{hs} \) is the stress at the top and above the saltation layer.
Figure 2.5: Classical wind velocity data, re-plotted from Bagnold [8].

In addition, the figure shows also a fit of the data aloft the assumed saltation height after (2.40) using:

<table>
<thead>
<tr>
<th>org. Bagnold</th>
<th>fit after (2.40)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v'_w$ [m s$^{-1}$]</td>
<td>$u_*$ [m s$^{-1}$]</td>
</tr>
<tr>
<td>0.19$^+$</td>
<td>0.19</td>
</tr>
<tr>
<td>0.40</td>
<td>0.42</td>
</tr>
<tr>
<td>0.62</td>
<td>0.67</td>
</tr>
<tr>
<td>0.88</td>
<td>0.77</td>
</tr>
</tbody>
</table>

$^+$ According to Bagnold, this value corresponds to the dynamic threshold.

Raupach [63] argues that vegetation canopies offer an analogy for saltation layers in that both flows have similar momentum sink distributions. In boundary layer meteorology, it is customary to describe the effect of vegetation on the flow above with a generalized logarithmic law [27, 90, 101]

$$M(z) = \frac{u_*}{\kappa} \ln \left( \frac{z - d}{z_0} \right)$$

(2.40)

where $d$ is the displacement height. Jackson [27] gave the following physical interpretation of the length scales $z_0$ and $d$: the roughness length $z_0$ expresses the magnitude of forces which act on the surface, whereas $d$ is related to the distribution of these forces. Thom [90] gave a similar explanation. Hence, for a known distribution of the interaction (drag) forces $F_{Dw}(z)$ within the saltation layer the displacement height
can be calculated using the ansatz [81, 90]

\[
d = \left( \frac{\tau_{hs} - \tau_{sa}}{\tau_{hs}} \right) \frac{\int_{h_s}^{zs} z F_{DW}(z) \, dz}{\int_{h_s}^{zs} F_{DW}(z) \, dz}
\]

(2.41)

where \( \tau_{hs} \) and \( \tau_{sa} \) are the stresses at the top of the saltation layer and at the soil surface, respectively. The term in brackets corrects for the stress at the ground [81]. It corresponds to the ratio of the total change in horizontal momentum of all grains ejected or rebounded from a unit area in a unit time to the available momentum at the top of the saltation layer. If we assume that the particles do not influence each other, then we can write for the interaction force

\[
F_{DW}(z) = \sum_{k=0}^{N(z)} \frac{1}{z} \rho_{0} C_{D}(z) \pi \left( \frac{d_{p}}{2} \right)^{2} |U_{\varepsilon z_{k}}(z)| U_{\varepsilon z_{k}}(z)
\]

(2.42)

where \( N(z) \) is the number of particles in a height \( z \) with a horizontal relative velocity \( U_{\varepsilon z_{k}}(z) \). Here, only the acceleration of the grains due to the wind is regarded. In the case of vegetation, for the roughness length the following ansatz is often proposed

\[
z_{0} = \lambda (h - d)
\]

(2.43)

The values for \( \lambda \) range between 0.1 and 0.36 [27, 81, 90]. As \( (\tau_{hs} - \tau_{sa})/\tau_{hs} \) is not constant during unsteady saltation, it is reasonable to assume that \( z_{0} \) also depends on this ratio. Hence, here the expression

\[
z_{0} = \lambda \left( \frac{\tau_{hs} - \tau_{sa}}{\tau_{hs}} \right) (h_s - d)
\]

(2.44)

is suggested. Putting (2.41) in (2.44), we get an equation which is quadratic in \( (\tau_{hs} - \tau_{sa})/\tau_{hs} \), where \( (\tau_{hs} - \tau_{sa}) \) depends on the stage of drift. At the onset of drift, \( (\tau_{hs} - \tau_{sa}) \approx 0 \) and drifting particles cause little changes to the roughness length. With the development of drift, \( (\tau_{hs} - \tau_{sa}) \) as well as the roughness length increase. Beyond a maximum, \( z_{0} \) will decrease again. An increase of \( z_{0} \) with the transport rate was observed by Schmidt [79], whereas Wiberg and Rubin [100] also reported the decrease of \( z_{0} \) for bed load transport (near-bed sediment transport in a river). In this case, the top of saltation layer seemingly works like a solid-looking surface when viewed from the air, and tends to seal the layer aerodynamically. For dense vegetation is this an observed effect.

How well equation (2.40) fits Bagnold’s data is shown in Figure 2.5, remembering that (2.40) is only valid outside the saltation layer. Within the saltation the wind profile depart from a logarithmic profile. Because of particles descending from faster flow levels, the velocity of the air will be enhanced. This could also be observed in Figure 2.5 and for example, it is reported by Maeno et al. in [37].
2.2.5 The transport due to turbulent suspension

The transport due to suspension as well as the precipitation can be regarded as already included in the conservation of moisture (2.12). The modification of the saltation trajectories and the transition into suspension depends on the stage of turbulence, usually characterized by the eddy length scale, \( L_e \), the eddy lifetime, \( t_e \), and the eddy velocity scale, \( U_e \). According to Faeth [17], these scales can be defined as

\[
L_e = \frac{e^{3/4} E^{3/2}}{\epsilon} \tag{2.45}
\]

\[
U_e^2 = \frac{2}{3} \epsilon \tag{2.46}
\]

\[
t_e = \sqrt{\frac{3}{2} e^{3/4} \epsilon} \tag{2.47}
\]

where \( \epsilon \) and \( \epsilon \) are the turbulent kinetic energy and the dissipation, respectively. Depending on the point of view, Lagrangian or Eulerian, \( L_e \) and \( t_e \) might differ by a factor of the order \( O(1) \). The particle motion is characterized by the three time scales

\[ t_r \] the particle relaxation time characterizing the aerodynamic response of the particle to changes of the flow, defined as

\[
t_r = \frac{4}{3} \frac{\rho_p d_p}{\rho_a \| U_e \| C_D} \tag{2.48}
\]

In the case of Stokesian drag, \( t_r \approx 0.1 \text{ s} \). Using the characteristic scales from Tab. 2.1, \( t_r \approx 0.01 \text{ s} \)

\[ t_g \] corresponding to the gravitational travel time, given by

\[
t_g = \frac{U}{g} \tag{2.49}
\]

Typical values range between 0.05–0.2 s.

\[ t_i \] is an inertial (interaction) time scale, defined as the time spent by a particle within an eddy.

These time scales can be grouped into two dimensionless numbers:

\[
\text{Stokes number} \quad \mathit{St} = \frac{t_r}{t_i} = \frac{4}{3} \frac{\rho_p}{\rho_a} \frac{d_p}{C_D U t_i} \tag{2.50}
\]

\[
\text{Froude number} \quad \mathit{Fr} = \frac{t_g}{t_i} = \frac{g t_i}{U} \tag{2.51}
\]

The ability of particles to be suspended by the fluid can now be estimated using the dimensionless form of equation (2.34)

\[
\frac{dW_p^*}{dt^*} = -\frac{1}{\mathit{St}} (W_p^* - w^*) - \frac{1}{\mathit{Fr}^2} \tag{2.52}
\]
A particle will stay in equilibrium at a given height if the dimensionless vertical fluid velocity $w^*$ equals the dimensionless fall velocity, $W_f^* = \frac{St}{Fr^2} = \frac{4}{3} \frac{\rho_p}{\rho_a} \frac{d_p g}{C_D U^2}$ \hspace{1cm} (2.53)

A particle will settle down, if $W_f^* > w^*$, else if $W_f^* < w^*$ it will be lifted by the fluid. In the case of Stokesian drag, that is $Re_P \ll 1$ and $C_D = 24/Re_P$, and assuming that $U w^*$ corresponds with the eddy velocity scale $U_e$, (2.53) yield

$$U_e = \frac{1}{18} \frac{\rho_p}{\rho_a} \frac{d_p^2 g}{\nu}$$ \hspace{1cm} (2.54)

The probability of a grain to be caught by an eddy and to be lifted, is, in a first approach, inversely proportional to the square of the diameter. For example, given a typical grain diameter of $d_p \approx 200 \mu m$, a eddy velocity of

$$U_e \gtrsim 1.2 \text{ m s}^{-1}$$ \hspace{1cm} (2.55)

is necessary to lift a grain. For non-Stokesian drag, $U_e$ will be rather smaller. If we now consider an ensemble of particles in a turbulent flow, on average a concentration gradient will established, depending on $W_f$, $U_e$, and $L_e$. Putting (2.55) into (2.46) and using the similarity relationship in the case of neutral stability $u_* \approx \sqrt{2\epsilon/\nu}$ (see [85, p. 366]), we get an estimation for the required friction velocity for beginning of particle suspension

$$u_* \gtrsim 0.6 \text{ m s}^{-1}$$ \hspace{1cm} (2.56)

which agrees quite well with observations. With the similarity for the dissipation $\epsilon \kappa z/u_*^3 = 1$, also corresponding eddy length and life time at the saltation layer height could be estimated:

$$L_e \approx 0.8 h_s$$ \hspace{1cm} (2.57)

$$t_e \approx 0.7 \text{ m}^{-1} \text{ s} h_s$$ \hspace{1cm} (2.58)
3 Numerical Model

When designing a numerical simulation scheme for blowing and drifting snow, several constraints have to be considered. For example, if we consider the saltation layer with a mass density of approximately 1 kg m\(^{-3}\) and a typical height of about 0.02 m, and if we assume a typical grain diameter of 200 \(\mu m\), we get about 5 \(\cdot\) 10\(^8\) grains per square meter. The areas of interest for snow drift simulations are at least several thousand square meters and the relevant time periods are at least several hours. On the other hand, today and in the near future workstation computing power is sufficient to follow the motion of hundreds to thousands of grains for times of several seconds real time in a particle dynamics method (PDM) simulation. Therefore, a (pseudo-) continuum formulation of the problem seems to be appropriate. The term "pseudo continuum" is used to indicate that drifting snow not really behaves like a continuum as the investigated mixture of air and discrete snow grains is diluted, and the motion of the grains occurs in band- or cloud-like structures.

Another constraint concerns the vertical resolution of transport within the saltation layer. Close to the surface (0.01-0.1 m above) the mass density is usually highest and contributes significantly to the total transport. However, if we want to resolve the saltation layer in a suitable way, the number of grid cells increases drastically and the length of the allowable time step decreases. Both constraints lead to unacceptably long computing times. To avoid this problem, a two-layer model is proposed. One layer describes the flow and the transport due to suspension. Within the second, the transport due to saltating grains is solved. The motion in the so-called modified saltation is either included in the suspension or in the saltation. Both layers are mutually coupled by boundary conditions. Figure 3.1 shows a schematic diagram of the proposed model.

Figure 3.1: Schematic diagram of the processes involved in the drift system. The symbols denote: \(M(z)\), wind speed at height \(z\); \(\tau\), shear/Reynolds stress; \(\tau_a\), airborne shear stress; \(\tau_g\), grain-borne shear stress; \(J_{ca}\), aerodynamic entrainment; \(J_{e(ej)}\), ejection rate; \(J_{de}\), deposition rate; \(J_{ct}\), turbulent entrainment; \(J_{cs}\), settling rate due to gravity; \(J_m\), horizontal mass transport rate; \(J_{pr}\), precipitation rate; \(F_{DW}\), interaction force (feedback). These notations will be used later on.
3.1 Modeling of the suspension layer

The equation of state (2.5), and the conservation equations for mass (2.6), momentum (2.10), moisture (2.11) and (2.12), and heat (2.14) constitute a system of partial differential equations for the description of the suspension layer. An examination of these equations reveals that almost all equations involve unknown correlation terms of turbulent quantities, e.g., the Reynolds stress, \(-u_i' u_j'\). Hence, the number of unknowns in the set of equations for the turbulent flow is larger than the number of equations. This is the well known closure problem in the description of turbulent flows. Here, the \(\epsilon-\epsilon\) model (also known as \(k-\epsilon\) model) is proposed to resolve this problem.

3.1.1 Turbulent closure

The \(\epsilon-\epsilon\) model forms a local one-and-half-order closure and is based on the eddy viscosity and eddy diffusivity hypotheses, which means that all turbulent fluxes are linearly related to the mean gradients (for reference see, e.g., [66, 85]). For example, the Reynolds stress can be expressed as

\[
-\overline{u_i' u_j'} = \nu_T \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \epsilon \delta_{ij} \tag{3.1}
\]

and the Reynolds flux of the moisture will be written as

\[
-\overline{u_i' q_L'} = \frac{\nu_T}{\sigma_T} \frac{\partial q_L}{\partial x_i} \tag{3.2}
\]

where \(\nu_T\) is the turbulent viscosity and \(\sigma_T\) is an empirical constant of \(\mathcal{O}(1)\). The turbulent viscosity is now parameterized as

\[
\nu_T = c_\mu \frac{\epsilon^2}{\epsilon} \tag{3.3}
\]
where the empirical constant, \( c_p \), is usually set to 0.09, and \( \epsilon \) and \( \epsilon' \) are the turbulent kinetic energy and the dissipation, respectively. Both quantities are still unknown. In the approach of the \( \epsilon-\epsilon' \) closure, balance equations for both will be formulated. Thus, we write the budget of the turbulent kinetic energy

\[
\frac{\partial \epsilon}{\partial t} + u_j \frac{\partial \epsilon}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \left( \nu + \frac{\nu_T}{\sigma_e} \right) \frac{\partial \epsilon}{\partial x_j} \right) - \frac{\rho_a}{\rho} \frac{\partial}{\partial x_j} \left( \frac{\partial \epsilon}{\partial x_j} \right) - \frac{g}{\rho_a} \frac{\partial}{\partial x_j} \frac{u_i}{\delta_{ij}} \delta_{kl} - \epsilon \tag{3.4}
\]

and the dissipation budget

\[
\frac{\partial \epsilon'}{\partial t} + u_j \frac{\partial \epsilon'}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \left( \nu + \frac{\nu_T}{\sigma_e} \right) \frac{\partial \epsilon'}{\partial x_j} \right) \\
+ c_{1\epsilon} \frac{\epsilon}{\epsilon'} \left( \frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j} - \frac{g}{\rho_a} \frac{\rho_a}{\rho} \frac{u_i}{\delta_{ij}} \delta_{kl} \right) \left( 1 + c_{3\epsilon} \mathcal{R}_f \right) - c_{2\epsilon} \epsilon' \epsilon \tag{3.5}
\]

where \( c_{1\epsilon}, c_{2\epsilon}, c_{3\epsilon}, \sigma_e \) and \( \sigma_e \) are empirical constants, e.g., given by Rodi [66] (a compilation is also given in Table B.5 in Appendix B). \( \mathcal{R}_f \) is the so-called flux Richardson number and is usually defined as minus the ratio of the buoyancy production of turbulent kinetic energy, \( \epsilon \), to the shear production.

\[
\mathcal{R}_f = \frac{g \rho_a}{\nu a_h} \frac{u_l}{u_j \frac{\partial u_i}{\partial x_j}}
\tag{3.6}
\]

The flux Richardson number is a stability parameter of the flow. For a statically stable flow \( \mathcal{R}_f > 0 \), for neutral stability \( \mathcal{R}_f = 0 \), and for unstable flow \( \mathcal{R}_f < 0 \).

### 3.1.2 Assumptions and approximations I

The equations (2.5), (2.6), (2.10), (2.11) (2.12), (2.14) as well as (3.4) and (3.5) constitute a complete set of equations for the description of the flow within the surface layer. The following assumptions and approximations will allow us to make useful simplifications in the case of snow drift simulations:

- Typical wind velocities, \( U \), during snow drift range from 5 m s\(^{-1}\) to 30 m s\(^{-1}\).
- A restricted area within the surface layer will be considered, so typical values are \( L \approx 1000 \) m for the length-scale and \( H \approx 100 \) m for the scaling height.
- In this case the Rossby number, defined as

\[
\mathcal{R}_o = \frac{U}{L \sqrt{f \epsilon}}
\tag{3.7}
\]

which describes the influence of the rotation of the Earth and only arises in a scaled version of the momentum balance (2.10) as the coefficient of the dimensionless Coriolis term \( \frac{1}{\rho_a f_c} \epsilon_{ij} u^* \), is of the order \( \mathcal{O}(> 50) \). Hence, the Coriolis force is negligible.
The Reynolds number
\[ \Re = \frac{UH}{\nu} \]
is of the order \( O(10^7) \), thus contribution due to the kinematic viscosity, \( \nu \), have little influence on the flow, except close to the surface and in the dissipation terms \( \epsilon = \nu \left( \frac{\partial u_i'}{\partial x_j} \right)^2 \).

- The case of neutral stability is considered, \( \text{Re}_f \approx 0 \). This will simplify Equation (3.5).
- No radiation effects will be considered (\( I = 0 \)).
- It will be assumed that the density change in response to the temperature and pressure change with increasing altitude is negligible within the considered volume. Thus a constant hydrostatic pressure and a constant potential temperature are assumed over the whole domain.
- Sublimation will be disregarded during snow drift, so no phase change is considered (\( E = 0 \)). In this case, we can disregard the balance equation for the vapor part of the humidity, (2.11). This approximation is not valid during clear-day drift.
- From the three points above, it follows that the conservation of heat (2.14) need not be computed for our purpose.
- The density fluctuations are caused by the fluctuations of the moisture, so the buoyancy flux \( g \frac{\varrho_0 u'_i}{\varrho_0} \approx g \varrho_L u'_i \).
- Although the snow grains differ in size within a certain range, only one kind of snowflakes and snow grains will be considered. It is assumed that all particles behave as spheres with a particle diameter of approximately 200 \( \mu m \).
- Snowflakes and snow grains in suspension travel with a velocity \( U_{Pi} = u_i - W_f \delta_{i3} \), where \( W_f \) is the absolute value of the free fall velocity of a snowflake or grain.
- The momentum transfer between the wind field and the snow grains due to particle acceleration is negligible. This is reasonable if we assume that the particles in suspension travel more or less with wind speed (see above) and that acceleration of particles which will be suspended occurs within the saltation layer.

### 3.1.3 Summary of the equations for the simulation of the wind field and the suspension layer

The assumptions and approximations given above are now incorporated into the following summary of the governing equations for the wind field as well as the suspension layer:
• Equation of state

\[ \frac{p}{R_a} = \rho_a T_v \quad \text{with} \quad T_v = T \frac{r_{sat} + 0.622}{1 + r_{sat} + r_L} ; \quad (3.9) \]

• Incompressibility approximation

\[ \frac{\partial u_i}{\partial x_j} = 0 ; \quad (3.10) \]

• Conservation of momentum

\[ \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho_a} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} - \frac{\partial u_i u'_j}{\partial x_j} - g \delta_{i3} ; \quad (3.11) \]

• Conservation of moisture (solid part)

\[ \frac{\partial q_L}{\partial t} + u_j \frac{\partial q_L}{\partial x_j} = -\frac{\partial}{\partial x_j} \left( q'_L u'_j - q_L W_j \delta_{j3} \right) ; \quad (3.12) \]

• Budget of the turbulent kinetic energy

\[ \frac{\partial e}{\partial t} + u_j \frac{\partial e}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \left( \nu + \frac{\nu_T}{\sigma_e} \right) \frac{\partial e}{\partial x_j} \right) \right) - u'_i u'_j \frac{\partial u_i}{\partial x_j} - g \frac{q'_L u'_i}{\delta_{i3}} - \epsilon ; \quad (3.13) \]

• Dissipation Budget

\[ \frac{\partial \varepsilon}{\partial t} + u_j \frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \left( \nu + \frac{\nu_T}{\sigma_e} \right) \frac{\partial \varepsilon}{\partial x_j} \right) \right) + c_1 \frac{\varepsilon}{\varepsilon} \left( u'_i u'_j \frac{\partial u_i}{\partial x_j} - g \frac{q'_L u'_i}{\delta_{i3}} \right) - c_2 \frac{\varepsilon}{\varepsilon} ; \quad (3.14) \]

where the Reynolds stress,

\[ -u'_i u'_j = \nu_T \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \epsilon \delta_{ij} , \quad (3.15) \]

the Reynolds flux of the moisture,

\[ -u'_i q'_L = \frac{\nu_T}{\sigma_T} \frac{\partial q_L}{\partial x_i} , \quad (3.16) \]

and the turbulent viscosity is given by

\[ \nu_T = c_\mu \frac{\epsilon^2}{\varepsilon} . \quad (3.17) \]
3.2 Modeling of the saltation layer

The phenomenon of blowing and drifting snow is strictly speaking a multi-phase flow that consists of a continuous and a dispersed phase. For each phase—air as the continuous and the snow grains as the dispersed phase—the principles of the mass and linear momentum balance ought to be formulated. In the present model, for the suspension layer only the mass balance is formulated for both phases; the momentum balance is given for the mixture. This is a common procedure in the description of passive tracers and in air pollution modeling, where the momentum transfer between the different phases is negligible. Within the saltation layer, the assumption of a negligible momentum transfer breaks down, as the acceleration of massive grains imposes an additional force on the wind.

In this Section, the governing model equations for the saltation will be derived, based on the finite-volume description of the mass and momentum balances. Then, the particle trajectories resulting from (2.33) and (2.34), need to be suitably parameterized so that they can cast into a continuum form.

3.2.1 Governing model equations for the saltation layer

The saltation layer is described by the mass balances and the momentum equations of the individual phases, air and snow. Using the volumetric concentration of the snow within the saltation layer, \( c = V_{\text{snow}}/V \), and the partial density of the air, \( \rho_a = (1 - c) \rho_a \), as well as the partial density of the snow phase, \( \rho_P = c \rho_P \), the conservation equations of the two phases can be written as

\[
-\rho_a \frac{\partial c}{\partial t} + \rho_a \frac{\partial (1 - c) u_i}{\partial x_i} = E \quad (3.18)
\]

\[
\rho_P \frac{\partial c}{\partial t} + \rho_P \frac{\partial c U_P}{\partial x_i} = -E \quad (3.19)
\]

It will be assumed that the intrinsic air density \( \rho_a \) and the grain density \( \rho_P \) are constant. As in the suspension, sublimation is no longer considered \( (E = 0) \). In the
conservation equation of momentum for the air an additional term, $F_{DWi}$, appears. This term also appears in the momentum equation for the particle phase with opposite sign and describes the momentum transfer between the two phases. Thus, the momentum equations are given by

$$\begin{align*}
(1 - c) \rho_a \frac{du_i}{dt} &= -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + (1 - c) \rho_a g_i - F_{DWi} \\
c \rho_p \frac{dU_{Pi}}{dt} &= \frac{\partial \sigma_{Pi}}{\partial x_j} + c \rho_p g_i + F_{DWi}
\end{align*}$$

(3.20) 

(3.21)

where $\sigma_{Pi}$ denotes the stress due to particle collisions. As body force for the particle phase, only gravitation is included (see Section 2.2.2). If we add (3.20) and (3.21), we get the momentum equation of the mixture

$$\begin{align*}
(1 - c) \rho_a \frac{du_i}{dt} + c \rho_p \frac{dU_{Pi}}{dt} &= -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + \frac{\partial \sigma_{Pi}}{\partial x_j} + (1 - c) \rho_a g_i + c \rho_p g_i \\
\end{align*}$$

(3.22)

A scaling of (3.18) and (3.19) using the scales

$$\begin{align*}
c &\rightarrow C c^* \\
x_i &\rightarrow \frac{U}{g} x_i^* \\
\left( U_{Pi}, u_i \right) &\rightarrow \left( U_{Pi}^*, U_{a}^* \right) \\
\tau_{ij} &\rightarrow \rho_a \frac{U^2}{T^*} \tau^* \\
\frac{\partial p}{\partial z} &\rightarrow \rho_a g \frac{\partial p^*}{\partial z} \\
t &\rightarrow \frac{U}{g} t^* \text{ or } t_{SL} t^*
\end{align*}$$

(3.23)

$t_{SL}$ is a characteristic time scale for wind within the surface layer. The velocity scale, $U \approx 3 \text{ m s}^{-1}$ and the concentration scale, $C \approx 10^{-3}$. The scaling shows that the mass conservation of the air reduces to the incompressibility approximation $\partial u_i^*/\partial x_i^* \approx 0$

$$\begin{align*}
C \frac{\partial c^*}{\partial t^*} + C \frac{\partial c^*}{\partial x_i^*} = (1 - C c^*) \frac{\partial u_i^*}{\partial x_i^*} = 0 \\
\mathcal{O}(10^{-3}) &+ \mathcal{O}(10^{-3}) + \mathcal{O}(1)
\end{align*}$$

(3.24)

Using the above, the mass conservation of the mixture can be approximated by

$$\begin{align*}
\frac{\partial c^*}{\partial t^*} + \frac{\partial c^* U_{Pi}^*}{\partial x_i^*} = \\
\frac{\rho_a}{\rho_p} \frac{\partial c^*}{\partial t^*} + \frac{\rho_a}{\rho_p} \frac{\partial c^* U_{Pi}^*}{\partial x_i^*} - \frac{\rho_a}{\rho_p C} \frac{\partial u_i^*}{\partial x_i^*}
\end{align*}$$

(3.25)

$\mathcal{O}(10^{-3}) \quad \mathcal{O}(10^{-3}) \quad \mathcal{O}(1) \approx 0$
Here, use is made of the ratio of the intrinsic densities, \( \rho_P / \rho_a \approx 1000 \). Thus, the mass conservation of mixture is primarily determined by the conservation of the snow mass

\[
\frac{\partial \rho_c^*}{\partial t^*} + \frac{\partial \rho_c^* U_{P_i}^*}{\partial x_i^*} = 0
\]

(3.26)

Concerning the momentum equation (3.22), the boundary layer approximations \( (\partial/\partial z) \approx (\partial/\partial x, \partial/\partial y) \) will be made. We assume a diluted mixture, so that the particle stress, \( \sigma_{Pij} \), is negligible. This is a common procedure regarding diluted mixtures. A rough estimation of the mean free path \( \ell \approx d_p / (6 \sqrt{2} \sigma) \) shows that \( \ell \) is of the range 0.02 m. In comparison, typical saltation lengths will be in the order \( \mathcal{O}(0.1-0.5 \text{ m}) \). Hence, with a concentration of \( 10^{-3} \), the assumption that it is a diluted mixture starts to be questionable. The above approach disregards the prevailing direction of particle motion, so it is reasonable to assume that the mean free path is longer. Sorensen and McEwan [84] noted that mid-air collisions can influence the feedback mechanism and hence the transport rates at high wind speeds; nevertheless, this effect will be ignored here. Hence, we write

\[
\rho_a \frac{U}{\rho_p} \left( \frac{1}{C} - c^* \right) \frac{du_i^*}{dt^*} + c^* \frac{dU_{P_i}^*}{dt^*} = -\frac{\rho_a}{\rho_p C} \frac{\partial \rho_c^*}{\partial x_i^*} \delta_{ij} + \frac{\rho_a}{\rho_p C} \frac{U^2}{\partial \tau^*_{j3}} \frac{\partial \tau^*_{j3}}{\partial x_3^*} - \frac{\rho_a}{\rho_p C} g_i^* - \frac{\rho_p - \rho_a}{\rho_p} c^* g_i^*
\]

(3.27)

where \( \Pi \) is turbulence intensity, defined as the ratio of the standard deviation of the wind speed to the mean wind speed (in our case \( \Pi \approx \sqrt{u'/v'/U^2} \)). \( \Pi \) is usually in the range of 0.1 to 0.5. \( g_i^* \) denotes the dimensionless gravity component. Since the acceleration of an air parcel is usually small in comparison to the acceleration of a particle, that is \( U / t_{SL} \ll g \), the first term on the left hand side is negligible. Further, if we assume that the mean state is in hydrostatic equilibrium \( (\partial p/\partial z = -\rho_a g) \), then the mixture momentum equation (3.27) simplifies to

\[
c^* \frac{dU_{P_i}^*}{dt^*} = \frac{\rho_a}{\rho_p C} \frac{U^2}{\partial \tau^*_{j3}} \frac{\partial \tau^*_{j3}}{\partial x_3^*} - \frac{\rho_p - \rho_a}{\rho_p} c^* g_i^*
\]

(3.28)

Thus, the reduced mixture momentum equation describes the balance of the force necessary to accelerate the saltating particles and the driving forces, represented by the Reynolds / shear stress and gravity. If we now consider the horizontal component over a plane, the second term on the right-hand side vanishes, and (3.28) reduces to a balance between the acceleration of grains and the Reynolds / shear stress. As all terms should be of the order \( \mathcal{O}(1) \), we gain an estimation of the saturation value of the mass concentration

\[
\rho_P C_{sat} \approx \rho_a \Pi^2 \approx 0.25 \text{ kg m}^{-3}
\]

(3.29)

This value might underestimate the maximum saturation, but, in comparison with measurements, it is of the correct order, e.g., see Fig. 1.4 or Fig. 1.6.
For the model description of the saltation layer, we use the finite-volume form of equation (3.19), with \( E \) set to zero

\[
\int_V \rho_P \frac{\partial c}{\partial t} \, dV + \int_A \rho_P c U_{P_i} n_i \, dA = 0
\]  

(3.30)

where \( V \) is the considered control volume, \( n_i \) is normal vector on the surface area \( dA \). Here, Gauss' Theorem was used in the form

\[
\int_A ()_1 n_i \, dA = \int_V \frac{\partial ()_j}{\partial x_j} \, dV
\]

(3.31)

In a similar way, we can derive the momentum equation for the control volume

\[
\int_V \frac{d}{dt} (c \rho_P U_{P_i}) \, dV = \int_A \tau_{ij} n_i \, dA + \int_V c \rho_P g_i \, dV
\]

(3.32)

3.2.2 Assumptions and approximations II

The next step will be to parameterize the saltation layer quantities. But before doing so, a summary of the approximations and assumptions will be given here. These assumptions were used, or will be used later on, for the derivation of governing model equations of the saltation layer.

- We consider a dilute mixture. Typical volumetric concentrations, \( C_i \), within the saltation layer are at most of the order \( 10^{-3} \).
- Sublimation will not be considered \((E = 0)\).
- The acceleration of the flow within the surface layer is small in comparison to the acceleration of a particle \((U/t_{SL} \ll g)\).
- The grains do not influence each other, that is, no particle–particle impacts are considered \((\sigma_{Pij} \approx 0)\), nor will the trajectory of a single grain be disturbed due to other grains.
- Two grain classes will be distinguished: rebounding or ejected grains. The respective trajectories of the two are identical.
- Rebounding particles impact with a velocity \( U_{P_i} = \varsigma_m u_{hs} \). Here \( \varsigma_m = 0.7 \) is chosen. This approximation compares reasonably with the results of simulations by Anderson and Hallet [3], and with wind-tunnel measurements, e.g., by Maeno et al. [38].
- Within the saltation layer, the following similarity assumptions will be made. The air velocity, \( U_i \), and the concentration, \( c \), are assumed to maintain approximately similar profiles in the \( z \)-direction (perpendicular to the surface) as they change in time or in the \( x \) and \( y \) directions, that is, it is assumed that
\[ u_i(x, y, z, t) = \xi_u \left( \frac{z}{h_s} \right) u_{hs}(x, y, t) \]  (3.33)

\[ c(x, y, z, t) = \xi_c \left( \frac{z}{h_s} \right) c_{hs}(x, y, t) \]  (3.34)

where \( u_{hs}, c_{hs} \) are the wind velocity and concentration in the height of saltation layer, \( h_s \). \( \xi_u \) and \( \xi_c \) are the profile function depending on the ratio \( z/h_s \) alone. Little is known about the velocity and the concentration profiles within the saltation layer. The few measured velocity profiles show a significant departure from a logarithmic profile close to the surface, e.g., see Fig. 2.5. Also the exact concentration profile is unknown. Here, let us use the approximations

\[ \xi_u = \left( 1 + \zeta \left( 1 - \frac{z}{h_s} \right) \right)^{-2} \]  (3.35)

\[ \xi_c = 1 \]  (3.36)

\( \xi_u \) corresponds to the velocity profiles which Thom [90] proposed for the velocity profile within vegetation. \( \zeta \approx 0.35 \) is chosen to fulfill \( \zeta_m = 0.7 \) (see above). In the absence of better knowledge about the concentration profile within the saltation layer, a uniform shape is assumed. For the case of identical trajectories, Anderson and Hallet [3] proposed a similar profile with the exception that they found a sharp peak at the top of the layer. The sharp peak at the top of their concentration profile is due to the vertical concentration gradient that reflects the amount of time a particle spends in any height increment, \( dz \). This time is approximately \( dz/W_{Pt}(z) + dz/W_{Pl}(z) \), and as \( W_{Pl}(z) \rightarrow 0 \), the likelihood of finding a particle at the top of its trajectory is the highest. In the case of identical trajectories, this causes a sharp concentration peak at the top. Thus, the assumption of a uniform concentration profile relaxes to some extent the assumption of identical trajectories (see above), which is more likely.

### 3.2.3 Parameterization of saltation layer quantities

Based on the assumptions above and on the equations (2.33) and (2.34) as well as on values given in Table 2.2, it is possible to calculate the trajectory of a single grain. Fig. 3.4 shows an example of such a calculation, where the initial values are set at:

\[ U_{PR} = \xi_R \xi_m u_{hs} \cos(\alpha_R) \]  (3.37)

\[ W_{PR} = \xi_R \xi_m u_{hs} \sin(\alpha_R) \]  (3.38)

with \( \xi_R = 0.5, \xi_m = 0.7 \) and \( \alpha_R = 40^\circ \). The saltation layer height, \( h_s \), was determined by iteration \( (h_{s0} = W_{PR}^2/(2g)) \). For comparison see Fig. 1.9, showing photos of drifting snow.
Figure 3.4: Calculation of a particle trajectory; rebounding grain, $\alpha_R = 40^\circ$, and $u_{h_s} = 4 \text{ m s}^{-1}$.

These calculations of particle motion will be used to gain the necessary parameters for the continuum description of the saltation, such as the saltation layer height, $h_s$ (equal to the height of the representative trajectory), or the mean horizontal particle velocity, $\overline{U_p}$.

Figure 3.5 presents the normalized horizontal velocity of a rebounding particle as function of the normalized height for the case of ascending as well as for the case of descending. Also shown is the mean horizontal velocity, $\overline{U_p} = (U_P \uparrow + U_P \downarrow)/2$ and the assumed wind profile within the saltation layer. Obviously, the mean particle velocity $\overline{U_p} \approx 0.5 u_{h_s}$. In Figure 3.6 all of the other characterizing parameters of the particle trajectories are presented.

Figure 3.5: Normalized horizontal particle velocities as function of the normalized height for a rebounding grain, $\alpha_R = 40^\circ$, and $u_{h_s} = 4 \text{ m s}^{-1}$; ascending $U_P \uparrow$, descending $U_P \downarrow$, mean $\overline{U_p}$ and the wind speed, $u$. 
The horizontal acceleration of a particle is important for the modeling of the saltation layer, as this corresponds to the stress exerted by the grains on the surface on average.

\[ \tau_g = \frac{m_p \, dU_p}{A_s \, dt} \]  

(3.39)

where \( \tau_g \) is the so-called grain-borne shear stress, \( m_p \) is the combined mass of particles above the surface area, \( A_s \). In the model the discretized version of (3.39) is used, where we distinguish between rebounding particles or ejected ones.

\[ \tau_{gR/E} = \frac{\rho_p \, c \, V \, (U_{PI} - U_{PR/E}) \, t_{R/E}}{t_{R/E}} \]  

(3.40)

This discretized form relates the stress, \( \tau_g \), to the slope-parallel increment in linear momentum of the particles during the airborne time \( t_{R/E} \).

In addition to the parameters describing the saltation layer, we can obtain the parameters determining the modification of the wind field from the grain motion, as these are the saltation layer height and the displacement height (see 2.2.4). Knowing \( N(z) \) or \( c(z) \), respectively, we obtain from (2.42) and (2.41) a relation for \( d \left( \frac{\tau_h - \tau_a}{\tau_h} \right)^{-1} \). Here, only the transfer of horizontal momentum from the wind to the particles will be considered. Thus, (2.42) is used in the form

\[ F_{DW}(z) = \sum_{k=0}^{N(z)} \frac{1}{2} \rho_a \, C_D(z) \pi \left( \frac{dF}{2} \right)^2 \left| U_{el_k}(z) \right| \max \left( 0, -U_{el_k}(z) \right) \]  

(3.41)

The reverse transfer of momentum due to descending particles causes in the modification of the wind profile, which is assumed to be known (see (3.33)). As already mentioned, for our purpose, let us assume a homogeneous concentration profile as an approximation (see (3.34)).

The particle acceleration \( (U_{PI} - U_{PR/E}) / t_{R/E} \) as well as \( d \left( \frac{\tau_h - \tau_a}{\tau_h} \right)^{-1} \) are plotted in Figure 3.7 as functions of the wind speed at the top of the saltation layer.

In the numerical model, polynomial fits of \( h_s \), \( d \left( \frac{\tau_h - \tau_a}{\tau_h} \right)^{-1} \), the ratios \( U_{PI} / U_{PR/E} \), and the accelerations will be used.
3.6.a: Hop/saltation layer height, \( h_s \), of a rebounding grain as function of \( u_h \).

3.6.b: Hop length, \( l_s \), of a rebounding grain as function of \( u_h \).

3.6.c: Travel time, \( t_R \), of a rebounding grain as function of \( u_h \).

3.6.d: Ratio of the landing to the launch speed, \( U_{P_j}/U_{PR} \), as function of \( u_h \), for a rebounding grain.

3.6.e: Travel time, \( t_E \), of an ejected grain as function of \( u_h \).

3.6.f: Ratio of the landing to the launch speed, \( U_{Pj}/U_{PE} \), as function of \( u_h \), for an ejected grain (the sharp bends are numerical artifacts depending on the time resolution).

Figure 3.6: Trajectory parameters as function of the wind speed at the top of the trajectory, \( u_h \), for rebounding grains (\( \|U_{PR}\| = 0.35 u_h, \alpha_R = 40^\circ \)) and for ejected grains (\( \|U_{PE}\| = 0.07 u_h, \alpha_E = 65^\circ \)); Part I.
3.7.a: Horizontal acceleration, $(U_{PF} - U_{PR})/t_R$, as function of $u_h$, for a rebounding grain.

3.7.b: Horizontal acceleration, $(U_{PE} - U_{PE})/t_E$, as function of $u_h$, for an ejected grain (the sharp bends are numerical artifacts depending on the time resolution).

3.7.c: $d (\tau_h - \tau_o)/\tau_h$ as function of $u_h$, for a rebounding grain.

Figure 3.7: Trajectory parameters as function of the wind speed at the top of the saltation layer, $u_{h_s}$, for rebounding grains ($\| U_{PR} \| = 0.35 u_{h_s}$, $\alpha_R = 40^\circ$) and for ejected grains ($\| U_{PE} \| = 0.07 u_{h_s}$, $\alpha_E = 65^\circ$); Part II.
As mentioned in Section 2.2.3, it is reasonable to assume that the average number of ejected particles, $N_{Epl}$, is determined by the energy balance

$$N_{Epl} = \frac{E_I - E_R - E_D}{E_B + E_E} \quad (3.42)$$

but the current understanding of this process is poor—especially in the case of snow, where cohesion has to be included. Some experiments have been carried out in wind tunnels for sand with different sizes, e.g., [64], or for ice spheres with a diameter of 2.8 mm [32]. Here, a parameterization based on the numerical simulations of Anderson and Haff [4] is proposed. They found that the mean number of ejects increases approximately linearly with the impact speed, where the ejection rate $N_{Epl} \approx 1.5 \text{ m}^{-1} \text{s} \parallel U_{pl} \parallel$. Based on this result and the values given in Table 2.2, it is possible to approximate the dissipated energy for the sand bed ($E_B = 0$), plotted in Fig. 3.8.a. In the absence of better knowledge, let us assume that the energy dissipated during grain impact on the snowpack is similar to that on a sand bed. In this case the number of ejects for a snowpack is determined by (3.42), if $E_B$ is known. Figure 3.8.b shows a plot with $E_B$ as parameter. The bond fracture energy of a snow particle, $E_B$, depends on the snowpack properties and on the weather conditions. Typical values might range from approximately $10^{-10}$ J to $10^{-8}$ J.

3.8.a: Approximation of the energy dissipated during particle ejection in the case of sand, based on the numerical simulation carried out by Anderson and Haff [4].

3.8.b: Approximation of the ejection rate for snow particles due to particle impact as function of the impact speed; for different values of the bonding energy, $E_B$.

Figure 3.8: Parameterization of the ejection rate for snow particles due to particle impact.
3.3 Implementation

The model is implemented in the commercial flow solver CFX4.1 [1] from AEA Technology, England (formerly called FLOW3D). This program is based on the finite-volume technique and uses a body-fitted grid, which can be modified during the calculation. Thus the modification of the topography due to deposition and erosion of snow can be modeled. So-called FORTRAN USER ROUTINES, provide the programming interface for problem-specific modifications of the code and proved flexible enough to accommodate all the extra computations for the simulation of blowing and drifting snow. Hence, the program serves as a framework, providing the solver for the wind field and for the suspension mode, whereas the calculations of the saltation mode were embedded in USER ROUTINES. Although the wind field, suspension and saltation form a mutually coupled system, here a certain decoupling was aspired. Hence, the wind field and the suspension mode were solved together and provided the boundary condition for the saltation-layer calculation, which was carried out between two time steps. On the other hand, we obtain out of this calculation the boundary conditions for the next time step. In order to evaluate the snowpack, the grid is adjusted to the new snow depth at regular time intervals.

![Diagram](image)

Figure 3.9: Sketch of numerical implementation of the two layer drift model. The depicted quantities are used in the description of the respective layer. Dashed boxes mark the (boundary) interfaces between the layers.
3.3.1 Implementation of the suspension layer

The core of the suspension-layer modeling is constituted by the equations summarized in Section 3.1.3, where the equation of state is used in the form

\[ \rho_a = \frac{P_0}{\bar{R}_a} \left( 1 + \frac{T_f}{T_0} \right) \]

neglecting the change of the density due to the vapor part, \( r \). By comparison, \( r_{sat}(T = 273 \text{ K}) \approx 5 \cdot 10^{-3} \text{ kg water vapor} \text{ kg}^{-1} \text{ atm}^{-1} \) corresponding to a decrease of the density by about 0.3%.

At the boundary between the suspension and saltation layer (at the height \( h_s \)), a slip condition for the velocity is enforced, where the velocity parallel to the boundary

\[ u_{h_s \parallel i} = u_{h_s} - (u_{h_s \parallel j} n_j) n_i \]

is determined from the velocity of the previous time step. Thus we get

\[ u_{h_s \parallel i}(t) = u_{h_s \parallel i}^{(t-1)} - (u_{h_s \parallel j}^{(t-1)} n_j) n_i \]

where \( n_i \) is the normal vector to the boundary area. The velocity at \( h_s \) is calculated, assuming a logarithmic wind profile

\[ u_{h_s} = \frac{\ln \left( \frac{h_s - d}{z_0} \right)}{\ln \left( \frac{z_{ref} - d}{z_0} \right)} \]

where the reference values correspond to the values of the nearest grid cell within the suspension layer. \( h_s \) and \( d \) are obtained from the particle motion (see Section 3.2.3) on the condition that \( \tau_{h_s} \) and \( \tau_s \) are known (see Section 3.3.2). According to (2.44), the roughness length is set to

\[ z_0 = \max \left( \lambda, \left( \frac{\tau_h - \tau_s}{\tau_h} \right) (h_s - d), z_{0s} \right) \]

where \( z_{0s} \) is the aerodynamic roughness length for a snow cover, typically in the range of \( 10^{-4} \text{ m} \) to \( 10^{-3} \text{ m} \) [85, p. 380], here set to \( 3 \cdot 10^{-4} \text{ m} \). This procedure can also include the case of no drift. During no drift, \( u_{h_s \parallel i} = 0 \), corresponding to a no-slip condition.

The boundary conditions for the moisture, \( q_L \), will be described in the following Section 3.3.2.
3.3.2 Implementation of the saltation layer

The equations (3.30) and (3.32) serve as starting point for the implementation of the saltation layer. We use the following discretization for the conservation of the mixture mass

\[
\frac{(\rho P c V)^{(t)}}{\Delta t} - \frac{(\rho P c V)^{(t-1)}}{\Delta t} = - \sum_k \rho P c C_k + J_{cs} - J_{ct} + J_{ca} - J_{cd}
\]

(3.48)

where the first term on the right hand side describes the advection through the side areas \(A^k\) (for the notation see Fig. 3.10). The advection coefficients, \(C_k\), will be defined later on. The control volume might be approximated by

\[
V = h_s \parallel A_i^d \parallel
\]

(3.49)

Figure 3.10: The notation for a control volume: \(A_i^1 = A_i^d\); \(A_i^2 = A_i^N\); \(A_i^3 = A_i^U = A_i^T\); \(A_i^4 = -A_i^W\); \(A_i^5 = -A_i^S\); \(A_i^6 = -A_i^P = -A_i^F\).

The settling due to gravity, \(J_{cs}\), and the turbulent entrainment, \(J_{ct}\), determine the boundary conditions of the moisture between the saltation layer and the suspension layer. At the surface of the snowpack, the boundary conditions are determined by the deposition rate, \(J_{cd}\), and entrainment rate, \(J_{ce}\), where we distinguish between the aerodynamic entrainment, \(J_{cae}\), and the entrainment due to ejects, \(J_{cej}\). These different rates can be described as follows:

- Corresponding to (2.15), \(J_{cae}\) can be written as

\[
J_{cae} = \zeta \max(\tau_a - \tau_c, 0) \parallel A_i^1 \parallel \rho P V_p
\]

(3.50)

where \(V_p\) is the volume of a single grain and, in absence of a better knowledge, the coefficient, \(\zeta\) is set to \(10^5\) grains \(N^{-1} s^{-1}\), according to Anderson and Haff [4].
• The ejection rate is determined by the number of particle impacts per second

\[ N_I = \frac{cV}{V_F t_j} \]  

(3.51)

with \( t_j \) the duration of a particle jump, and the number of ejects per impact, \( N_{EpI} \). As rebounding and ejected grains differ in the landing speeds as well as in the travel time, a distinction is made between these two classes. Hence, we get for the ejection rate

\[ J_{eej} = f_{NpI} (\| U_{PIR} \|) \frac{\rho p cR V}{t_R} + f_{NpI} (\| U_{PIE} \|) \frac{\rho p cE V}{t_E} \]  

(3.52)

where the function \( f_{NpI} \) describes the portion of ejecta as function of the respective impact speed and is given by

\[ f_{NpI}(\| U_{PIR/I} \|) = f_e(\| U_{PIR/I} \|) \left[ N_{EpI}(\| U_{PIR/I} \|) \right] \]  

(3.53)

The number of ejecta per impact, \( N_{EpI} \), is defined by (2.36) and shown in Fig. 3.8.b as a function of the impact speed. The factor \( f_e \) is introduced to take into account that the impact speed is distributed. It is defined as the probability that \( E_I \) exceeds \( E_R \). Let us assume that the impact energies of both particle classes have a normal distribution and that the variance is \( E_I^2/2 \) \((= m_p (\| U_{PI} \|)^2/4)\). Then \( f_e \) is given by

\[ f_e = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{E_R} e^{-\frac{(E_I-E_R)^2}{E_I^2}} dE_I \]  

(3.54)

As the number of ejecta is discrete, the fit to Fig. 3.8.b is written in the form \([N_{EpI}(\| U_{PI} \|)]\).

• The deposition of a snow particle depends on its probability to rebound, \( P_R \), as \((1 - P_R)\) is its probability to be deposited. Thus, we can write for the deposition rate

\[ J_{ed} = (1 - P_R(\| U_{PIR} \|)) \frac{\rho p cR V}{t_R} + (1 - P_R(\| U_{PIE} \|)) \frac{\rho p cE V}{t_E} \]  

(3.55)

Again, we distinguish between rebounding and ejected grains. In absence of better knowledge, the relation (2.35) is used as the rebound probability. Assuming that the rebound probability for a snow grain on the snowpack is smaller than for a sand particle on the sand bed, the coefficient, \( \xi \) (in (2.35)) is set to 1.5 m\(^{-1}\)s.
• The settling rate describes the advection of snow out of the suspension layer, hence we write

\[ J_{cs} = \rho_p c_{ref} A_3 \tag{3.56} \]

where \( A_3 \) is a off diagonal matrix coefficient, defined below, and \( c_{ref} \) is the volume concentration of snow in the first cell of the suspension layer above the (boundary) interface.

• In a similar way, the turbulent entrainment into the suspension layer can be regarded as the sum of an advection and a turbulent dispersion part. Hence, let us write

\[ J_{ct} = \rho_p c \left( A_3 + C_3 \right) \tag{3.57} \]

Leap ahead of the numerical solution, the off-diagonal matrix coefficient, \( A_3 \), is set to

\[ A_3 = \max(0.5 \left| C_3 \right|, D_3) - 0.5 C_3 \tag{3.58} \]

where the advection coefficient, \( C_3 \), is given by

\[ C_3 = (u_{h,i} n_i) n_j A_j^T - W_f A_j^T \delta_j \tag{3.59} \]

and the diffusion coefficient describing the turbulent dispersion is set to

\[ D_3 = \sqrt{\frac{5 e}{11}} \| A^T \| \tag{3.60} \]

The value \( \sqrt{5 e/11} \) is obtained by similarity theory for the neutral boundary layer and corresponds to the vertical fluctuation velocity (e.g., see Stull [85]). The ansatz in Equations (3.56) and (3.57) is adapted from a hybrid differencing scheme in which central differencing is used if the mesh Peclet number \( (C/D) \) is less than 2, and upwind differencing, but ignoring diffusion, is used if the mesh Peclet number is greater than 2.

The momentum equation (3.32) is used in the following form to determine the airborne shear stress, \( \tau_a \), which causes the aerodynamic entrainment of grains (3.50). If we consider the slope-parallel part of the momentum equation, we get

\[ c_R V \frac{U_{PIR} - U_{PR}}{t_R} + c_E V \frac{U_{PIE} - U_{PE}}{t_E} = \frac{\tau_{h,i} A^T}{\rho_p} - \frac{\tau_a A^s}{\rho_p} - c V g \sin \phi \tag{3.61} \]

The left-hand side corresponds to the grain-borne shear stress and can be regarded as known (see Section 3.2.3). The first term on the right-hand side is Reynolds shear stress at the height \( h_s \), which can be expressed as

\[ \tau_{h,i} = \frac{\kappa c_R^{1/4} \rho_e e^{1/2}}{\ln \left( \frac{h_s}{\delta_0} \right)} \left| u_{ref,i} - u_{b,i} \right| \tag{3.62} \]
and the third term corresponds to the acting gravity force. Looking in the direction of the flow, the slope angle $\phi > 0$ for a rising slope and $\phi < 0$ for a falling slope. As $\tau_a$ is the only unknown, (3.61) can be solved for the air-borne shear stress.

In (3.52) and (3.61), we distinguish between $c_R$ and $c_E$, which are still undefined. To determine both quantities, we assume that the ratio of rebounding and ejected grains in a control volume will not be changed due to advection during a time step. In this case, we can write

$$c_E^{(t)} = \frac{c_{E_{calc}}^{(t-1)}}{c_{R_{calc}}^{(t-1)} + c_{E_{calc}}^{(t-1)}} c^{(t)}$$ (3.63)

$$c_R^{(t)} = \left(1 - \frac{c_{E_{calc}}^{(t-1)}}{c_{R_{calc}}^{(t-1)} + c_{E_{calc}}^{(t-1)}}\right) c^{(t)}$$ (3.64)

with the calculated concentrations $c_{E_{calc}}$ and $c_{R_{calc}}$ from previous time step.

**Numerical Solution** As time-stepping procedure a backward difference scheme is used, that is, the Equation (3.48) is written in the form

$$\frac{(\rho \rho_e V)^{(t)} - (\rho \rho_e V)^{(t-1)}}{\Delta t} = F(c^{(t)})$$ (3.65)

The terms on the left-hand side can be absorbed into a source or sink term, respectively, and the resulting equation looks like a discretized steady-state equation. This equation can then be solved by a matrix solution technique having the form

$$A_{xp} \rho \rho_e c_{xp} - \sum_{nb} A_{np} \rho \rho_e c_{np} = SU$$ (3.66)

Here $(.)_{nb}$ denotes the neighboring nodes. The matrix coefficients on the diagonal of the matrix can be written as

$$A_{xp} = \sum_{nb} A_{np} - SP + C_E - C_W + C_N - C_S + \frac{V}{\Delta t}$$ (3.67)

$SU$ and $SP$ are source terms. Equation (3.66) represents a linear equation system, which could be solved, e.g., using a SOR-method. As diffusion is negligible in the saltation mode, we can use an upwind differencing scheme for the advection through the side areas, that is, the off-diagonal matrix coefficients are

$$A_k = \begin{cases} \max (-C_k, 0) & \text{with } k = (1, 2) \\ \max (C_k, 0) & \text{with } k = (4, 5) \end{cases}$$ (3.68)

The SOR-method (successive overrelaxation) [24, pp. 471] based on iterative solution of the equation $a_{ij} b_j = a_i$, using the scheme

$$b_i^{(k+1)} = b_i^{(k)} - \frac{\omega}{a_{ii}} \left( \sum_{j=1}^{i-1} a_{ij} b_j^{(k+1)} + \sum_{j=i+1}^{n} a_{ij} b_j^{(k)} - a_i \right)$$

where $a_{ij}$ are the matrix coefficients, $\omega$ is the relaxation parameter ($0 < \omega < 2$, depending on the matrix), and $(k)$ is the present iteration number.
and advection coefficients are given by

\[ C_k = \overline{U_{p_i}} A_i^k, \quad k = (1, 2, 4, 5) \] 

(3.69)

Here, \( \overline{U_{p_i}} \) is the mean particle velocity vector at the face between the control volume surrounding the grid nodes \( x_p \) and \( n_b \). As we work on a non-uniform grid, let us use a weighted linear interpolation for the velocity

\[ \overline{U_{p_i}} = (1 - W) \overline{U_{p_i}}_{x_p} + W \overline{U_{p_i}}_{n_b}, \] 

(3.70)

where \( W \) is the interpolation 'weight, based on the relative distances \( x_p f \) and \( n_b f \)

\[ W = \frac{x_p f}{x_p f + n_b f} \] 

(3.71)

According to Fig. 3.5, the mean particle velocity is

\[ \overline{U_{p_i}} = \varsigma_q u_{h_{x_i}} \] 

(3.72)

here \( \varsigma_q = 0.58 \) is chosen.

The source term \( SU \) is made up of the fluxes \( J_{ca} \), \( J_{ce} \), \( J_{cd} \) and \( J_{cs} \) as well as

\( (\rho P V (t-1)/\Delta t) \), thus we get

\[ SU = \left( \frac{\rho P V (t-1)}{\Delta t} + J_{cs} + J_{ca} + J_{ce} - J_{cd} \right) \] 

(3.73)

The turbulent entrainment, \( J_{ed} \), (see equation (3.57)) is absorbed in the source term \( SP \). Hence the coefficient \( SP \) is set to

\[ SP = -(A_3 + C_3) \] 

(3.74)

### 3.3.3 Grid adaptation to the new snow depths

It remains the update of the bottom topography of the grid depending on the calculated new snow depths. During every time step \( \Delta HS \) is calculated for every cell of the saltation layer, according to

\[ \Delta HS = \max \left( \frac{J_{cd} - J_{ce} - J_{ca}}{\rho P c \lambda A_i^k} \Delta t, -\Delta HS_c \right) \] 

(3.75)

where \( \Delta HS_c \) is the remaining erodible snow depth and \( c \) is the snow concentration within snowpack. After a certain number of time steps, \( n \), the grid is adjusted to the new snow depth

\[ HS = HS_{old} + \sum_{i=1}^{n} \Delta HS_i \] 

(3.76)
One aim of this work was the validation of the numerical model with the help of field data. To this end, two points are of significance:

- the meteorological input, particularly the wind field as the driving force for blowing and drifting snow;
- and quantities describing the drift, e.g., flux measurements during drift or measurements of the areal snow redistribution pattern for a drift episode.

Simultaneous measurements of both, the wind field and the snow distribution, are rare, and almost all investigations concern simple topographies. Therefore, some additional measurements were carried out during the Winter 1996/97 in complex alpine terrain.

### 4.1 The Gaudergrat Experimental site

For this purpose, Gaudergrat ridge (GG), 2 km north of Weissfluhjoch (WJ), was equipped with instruments to measure meteorological and snow parameters. Figure 4.2 shows survey of the surroundings of the experimental site.

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4The experimental site was equipped in a cooperation with the project “Snowpack Modeling”.
Figure 4.2: Surrounding of the experimental site Gaudergrat (GG). The SLF building at Weissfluhjoch (WJ), the Weissfluhjoch test site (VF) and the location of the snow depth sensor at Kreuzweg (KW) are marked. (Map A. Stoffel/SLF)
Figure 4.3: Slope angle map of the Gaudergrat experimental site showing locations of the masts $M_{xx}$ (the mast numbering corresponds to the station ID) and the hut that housed the transmitting equipment; also shown the location of the snow depth sensor KW. (Map A. Stoffel/SLF)

The ridge has a rather sharp crest—the slope angles range from 28 to 38°—and might be regarded as prototypical of Alpine topography. The prevailing wind direction during strong precipitation periods is more or less north-west and thus perpendicular to the crest-line. In order to determine the wind field around the crest, the site was instrumented with five masts in the surrounding area for wind profile measurements. A sixth mast was equipped with additional sensors for measurements of meteorological and snowpack parameters. Figure 4.3 shows the locations of the installed masts and the slope angles at the ridge. A compilation of the instrumentation is given in Table 4.1.
<table>
<thead>
<tr>
<th>Station</th>
<th>ID</th>
<th>Coordinates</th>
<th>Instrumentation</th>
<th>Data capture</th>
</tr>
</thead>
<tbody>
<tr>
<td>WINDEIN</td>
<td>78</td>
<td>779529/192660/2101</td>
<td>3 x Wind Monitor (Young Model 05103-5) Ultra sonic snow depth sensor (type Brase, since winter 1997/98) (see Fig. 4.5.a)</td>
<td>Data logger (Campbell 21X) &amp; CSM1 CARD Storage Module with 2MB SRAM-CARD</td>
</tr>
<tr>
<td>SIMULUV</td>
<td>77</td>
<td>779797/192203/2269</td>
<td>Silicium Pyranometer up (Campbell Model SP1110) Silicium Pyranometer down (Campbell Model SP1110) Ultra sonic snow depth sensor (Campbell Model SR50) Relative Humidity / Temperature Probe (Rotronic Model MP-340-001C53) Infrared Thermometer (Alpug)</td>
<td>Data logger (Campbell CR10X) &amp; MD9 Multidrop Interface</td>
</tr>
<tr>
<td>WINDLUV</td>
<td>76</td>
<td>779811/19222/2262</td>
<td>3 x UVW Anemometer (Gill Model 27005RS)</td>
<td>Data logger (Campbell CR10X2M) &amp; MD9 Multidrop Interface</td>
</tr>
<tr>
<td>WINDTOP</td>
<td>75</td>
<td>779824/192200/2282</td>
<td>3 x UVW Anemometer (Gill Model 27005RS) 1 x Wind Monitor (Young Model 05103-5) Glass Pyranometer up-down (Swissteco Model SW-2) (see Fig. 4.5-b)</td>
<td>Data logger (Campbell CR10X2M) &amp; MD9 Multidrop Interface</td>
</tr>
<tr>
<td>WINDEE</td>
<td>74</td>
<td>779836/192190/2270</td>
<td>3 x UVW Anemometer (Model G17205RS) Silicium Pyranometer up (Campbell Model SP1110) Silicium Pyranometer down (Campbell Model SP1110) Ultra sonic snow depth sensor (Campbell Model SR50) Relative Humidity / Temperature Probe (Rotronic Model MP-103A) Infrared thermometer (made by SLF)</td>
<td>Data logger (Campbell CR10X2M) &amp; MD9 Multidrop Interface</td>
</tr>
<tr>
<td>SIMULEE</td>
<td>73</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WINDAUS</td>
<td>72</td>
<td>779995/192075/2192</td>
<td>3 x Wind Monitor (Young Model 05103-5)</td>
<td>Data logger (Campbell 21X) &amp; CSM1 CARD Storage Module with 2MB SRAM-CARD</td>
</tr>
<tr>
<td>Hut</td>
<td>51</td>
<td>779887/192187/2282</td>
<td>2 solar panels as power supply</td>
<td>MD9 Multidrop Interface &amp; PS512-M Modem Power Supply &amp; Radio</td>
</tr>
</tbody>
</table>
Figure 4.4: View of the Gaudergrat experimental site from the south (point 2305). One sees the hut and some masts. The erosion areas windward of the crest line (left hand side) and the formation of cornices on the leeward side are easily noticed.

4.5.a: M78, instrumented with 3 Wind Monitors (Young Model 05103-5) and ultrasonic depth sensor (type Brusa; since winter 1997/1998).

4.5.b: M75, instrumented with 3 UVW anemometers (Gill Model 27005RS), 1 Wind Monitor (Young Model 05103-5) and Glass Pyranometer up-down (Swissteco Model SW-2).

Figure 4.5: Example of the instrumentation: Masts M78 and M75.
4.2 Method and Measurements

In the winter 1996/97, six snow drift episodes could be investigated. These events took place between the mid February and the beginning of April and can be grouped into two greater periods. The first period can be identified between 14–28 February, and the second between 19 March and 7 April.

4.2.1 Meteorology

Figure 4.6 shows the meteorological conditions during the investigated periods. In Table 4.2, a compilation of new-snow sums as well as the mean new-snow densities is given for the six drift episodes. In addition, the normalized new-snow depth is given, where the new-snow depth is normalized with the average new-snow density of the six episodes. These values are related to the observation at the test site VF and will be taken as reference later on. The station VF was used as reference, because of some technical problems with the station KW which is near the experimental site. However, as can be seen in Fig. 4.6.d, both stations correlate quite well during the regarded periods, so that the choice of VF as reference is reasonable.

<table>
<thead>
<tr>
<th>Episode</th>
<th>Date</th>
<th>(\sum HN [\text{m}])</th>
<th>(\bar{\rho}_{NS} [\text{kg m}^{-3}])</th>
<th>(\sum HN_n [\text{m}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>14.02 - 16.02</td>
<td>0.35</td>
<td>91</td>
<td>0.29</td>
</tr>
<tr>
<td>E2</td>
<td>19.02 - 21.02</td>
<td>0.23</td>
<td>126</td>
<td>0.26</td>
</tr>
<tr>
<td>E3</td>
<td>27.02 - 28.02</td>
<td>0.32</td>
<td>121</td>
<td>0.35</td>
</tr>
<tr>
<td>E4</td>
<td>19.03 - 26.03</td>
<td>0.68</td>
<td>107</td>
<td>0.67</td>
</tr>
<tr>
<td>E5</td>
<td>29.03 - 31.03</td>
<td>0.57</td>
<td>96</td>
<td>0.49</td>
</tr>
<tr>
<td>E6</td>
<td>04.04 - 07.04</td>
<td>0.38</td>
<td>136</td>
<td>0.47</td>
</tr>
<tr>
<td>(\sum)</td>
<td>14.02 - 07.04</td>
<td>2.53</td>
<td>110</td>
<td>2.53</td>
</tr>
</tbody>
</table>

Table 4.2: Compilation of the new-snow depth and density for the drift episodes related to the observation at VF. \((\bar{\rho}_{NS, \text{norm}} = 110 \text{ kg m}^{-3}\) is the averaged new-snow density for all six episodes)
4.6.a: Mean wind speed $M_6$, the peak speed $VBM_6$, and wind direction $dd$ at the point M75

4.6.b: Relative humidity, $\phi_v$, at the points M77 and M73/74

4.6.c: Precipitation, RS (10 min sum, or estimated snow concentration in the air, $C_{air}$), at the point VF

4.6.d: Snow depth, $HS$, at the points VF, KW, M77 and M73/74

4.6.e: Air temperature, $T_s$, and surface temperature, $TS$, at the point M77

4.6.f: Radiation, $I$, at the point M75

4.6.g: Air temperature, $T_s$, and surface temperature, $TS$, at the point M73/74

Figure 4.6: Summary of the meteorological data from Gandergrat (2280 m a.s.l) during the period from 10.02–13.04.97.
4.2.2 Snow depth soundings

To determine the areal snow redistribution for a drift episode, soundings of the snow depth were made before and after an episode, that is, the snow depth was measured along equidistant lines across the crest, roughly 8.5 m apart and over 200 m long. The soundings were taken at 4 m intervals along the lines; in the neighborhood of the crest this distance was reduced. Thus, every field campaign resulted in around 350 data points. All measurement points were marked by thin bamboo stakes during the first campaign (see Fig. 4.7). Thus, later measurements could be taken at the same points (with an estimated horizontal deviation of less than 0.1 m). In this manner uncertainties in the depth measurements due to the small-scale topography could be minimized. For example, Figure 4.8.a and Figure 4.8.b show the mapped snow distribution before and after the episode E4, respectively.

![Figure 4.7: Profile line in the leeward (south-east) slope. The bamboo stakes mark the points of sounding. Waves and dunes imply winds coming from north-east (left) more or less parallel to the slope.](image)

To convert the depths to new-snow mass, density measurements were also made at several points. Through these measurements the recently eroded and deposited snow mass could be evaluated and mapped, see Figure 4.8.c. The figure shows a map of the recalculated new-snow distribution for the episode E4. Here, the new-snow depth is normalized with a reference depth, which corresponds to the observations at the test site VF, recalculated for the averaged snow density (see Table 4.2). A negative ratio means erosion of the old snowpack.
4.8.a: Before episode E4 (18.03.97)

4.8.b: After episode E4 (25.03.97)

4.8.c: Recalculated new-snow depth for the episode E4. The new-snow depth is normalized with the reference depth $HN_{ref} = 0.67$ m.

Figure 4.8: Map of the snow distribution around the Gaudergrat ridge before and after drift episode E4. (contour interval 10 m)
Due to increased avalanche danger after a drift episode, the soundings had to be done with a certain time lag (1–2 days). During this time the snowpack settles. The phenomenon could be observed in the snow depth plots in Fig. 4.6.d: an exponential decrease of the snow depth follows each snow fall. To take settling into account small snow pits were made at several points within the slopes to determine stratification and the density. A sketch of such a snow pit is shown in Fig. 4.9. Then, the measured snow depths were corrected according to the following procedure.

- It is assumed that the old snowpack is settled at the beginning of the first episode of each period.

- In this case, the corrected new-snow depth for the first episode can be calculated as

\[
HN_{corr_{ij}} = \frac{\rho_{SL/N}}{\bar{\rho}_N} \left( HS_{mea_{ij}} - HS_{ref_{ij}} \right)
\]

where \( \rho_{SL} \) is the density of the uppermost layer, \( \bar{\rho}_N \) is the mean new-snow density of the episode (see Tab. 4.2), and for the reference \( HS_{ref_{ij}} \), we take the sounding before the episode. The subscripts \( ()_{ij} \) mark the point of the measurement. If \( (HS_{mea_{ij}} - HS_{ref_{ij}}) < 0 \), \( \rho_{SL} \) corresponds to the density of the old-snow, else it corresponds to the measured new-snow density. In the density measurements, we distinguish between the north-west (N) facing and the south-east (S) facing slopes.

- The settling during later episodes is determined by the settling of the new-snow and by settling of snow from previous episodes. In this case, the corrected new-snow depth is calculated by

\[
HN_{corr_{ij}} = \frac{\rho_{SL/N}}{\bar{\rho}_N} \left( HS_{mea_{ij}} - HS_{refm_{ij}} \right)
\]

where the modified reference depth, \( HS_{refm_{ij}} \), was set to

\[
HS_{refm_{ij}} = HS_{ref_{ij}}
\]

if \( HN_{corr_{ij}} \) of the last episode was less than zero

\[
HS_{refm_{ij}} = HS_{ref_{ij}} - \left( 1 - \frac{\rho_{OS_{ib}}}{\rho_{OS_{ia}}} \right) HN_{corr_{ij}}
\]

where \( \rho_{OS_{ib}} \) or \( \rho_{OS_{ia}} \) are the densities of the uppermost old-snow layer before and after the episode, respectively. \( HN_{corr_{ij}} \) is the corrected new-snow depth of the last episode.
After that, all new-snow depths were related to \( \bar{\rho}_{NS,\text{norm}} \) for comparison.

\[
HN_{\text{norm},ij} = \frac{\bar{\rho}_{NS}}{\bar{\rho}_{NS,\text{norm}}} HN_{ij}
\]  

The example of the recalculated new-snow distribution has already been shown in Figure 4.8.c. Another example is given in Figure 4.10, where all six drift episodes are put together. The new-snow distribution demonstrates considerable local variability. This could particularly be seen in Figure 4.8.c, whereas in Figure 4.10 the global trend could be observed in a better way. The variability is usually determined by micro-scale effects, such as the local topography, short term changes in the snowpack properties, and the influence of the turbulent air flow. In contrast, the trend in Fig. 4.10 reflects the influence of the macro-scale effects, particularly the extensive topography. Thus, the erosion or depletion zone, about 50 m north-west of the crest, is typical for windward crest areas (see also Fig. 4.4). The variability along the crest line is related to the formation and migration of a cornice. In the leeward area, zones of deposition alternate with depletion zones. Some of these depletion zones

Figure 4.10: Map of the snow distribution around the Gaudergrat ridge. All six drift episodes are combined. The snow depth is normalized with the reference depth \( HN_{\text{ref}} = 2.53 \) m. (contour interval 10 m)
can be related to small humps or terraces interacting with the local wind field, which shows in some respects a special feature (see below). It should also be mentioned that a small slab avalanche at the beginning of the winter caused same disturbance of the measurements in the leeward area, as this area became uncovered for some time, and the deposition effected the small-scale topography.

4.2.3 Wind field measurements

The wind field is one of the great unknowns in the prediction of the snow distribution and re-distribution in a complex alpine terrain. Up-to now, almost all investigations of snow drift in mountainous regions related the drift to wind speed measurements at one point, mostly at the top of a hill or crest \[19, 50, 80\]. This is sufficient if one looks for a relation between the wind speed and the drift rate at that point. But it is not sufficient for the prediction of the snow distribution in an extended area, as this is strongly affected by the local wind field. Hence, for the validation of a numerical model, it is essential to gain an estimate of the wind field in the test area. In addition, the wind field simulation requires prescription of boundary conditions. Thus, the aim of the wind measurements during the winter 1996/97 at the Gauder-grat experimental site was the determination of reasonable boundary conditions for the numerical simulations and verification of the simulated wind field with the aid of additional point measurements.

During the drift episodes wind profile measurements were carried out at five points. These five points are aligned more or less north-west to south-east and hence perpendicular to the crest line (see Figure 4.3 and Table 4.1). At two points (M78 and M72) the horizontal wind speed and the directions were measured at the three heights (\(\approx 1\) m, \(\approx 2\) m) above the snowpack, depending on the new-snow depth, and \(5.9\) m above ground level, corresponding to a height of about \(3.8-4.7\) m above the snowpack). At the points M76, M75 and M74, the three components \(u\), \(v\) and \(w\) were measured. Here, the heights above ground were kept constant during the whole period as the changes in the snow depth were small at these points.

<table>
<thead>
<tr>
<th>Mast</th>
<th>(h_u) [m]</th>
<th>(h_m) [m]</th>
<th>(h_o) [m]</th>
<th>(HS) [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>M78</td>
<td>(\approx 1) a.s.</td>
<td>(\approx 2) a.s.</td>
<td>(5.9) a.g.</td>
<td>0.8–1.9</td>
</tr>
<tr>
<td>M76</td>
<td>2.5</td>
<td>3.5</td>
<td>6.2</td>
<td>(\approx 0.9)</td>
</tr>
<tr>
<td>M75</td>
<td>1.5</td>
<td>2.5</td>
<td>6.2</td>
<td>(\approx 0.2)</td>
</tr>
<tr>
<td>M74</td>
<td>2.5</td>
<td>3.5</td>
<td>7.2</td>
<td>(\approx 0.4)</td>
</tr>
<tr>
<td>M72</td>
<td>(\approx 1) a.s.</td>
<td>(\approx 2) a.s.</td>
<td>(6.0) a.g.</td>
<td>1.1–1.9</td>
</tr>
</tbody>
</table>

Table 4.3: Summary of the mounting heights of the anemometers during the winter 1996/97 (a.s., above the snowpack; a.g., above the ground-level; mounting height of the Gill anemometer corresponds to a middle plane of the three components).
4.11.a: Flow of a neutral boundary layer ($F = \infty$) over an isolated hill (after Stull [85]). $L_{h\text{hill}}$ is about the half-width of the barrier.

4.11.b: Sketch of wind speed-up over a gentle hill (after Stull [85]).

Figure 4.11: Sketches of an idealized flow over an isolated hill

It is known that, for strong winds and neutral stability, the air flow surmounts an isolated hill. The streamlines within the so-called region of influence are disturbed upwind and above the hill, while beyond, the flow does not "feel" the presence of the hill. Near the top of the hill the streamlines are packed more closely together, causing a speed-up of the wind. In the lee, a turbulent wake can be found. These effects are sketched in Figure 4.11. As the flow tends to surmount the barrier in circular paths, the length scale, $L_{h\text{hill}}$, of the influence zone is about half the width of the barrier. The fractional speed-up ratio, $\Delta S_D (\Delta S_{h\text{hill}})$, at the height, $h$, is defined as:

$$\Delta S_D(h) = \frac{M_D(h) - M_A(h)}{M_A(h)}$$

(4.6)

where $M_A$ is the wind-speed at point A upwind of the hill where the flow is undisturbed. Taylor and Lee [89] propose following 'rules of thumb' for the maximum speed-up values: $\Delta S_{\text{max}}(h) = 2 z_{h\text{hill}}/W_{1/2}$ for gentle 2-D ridges, $\Delta S_{\text{max}}(h) = 1.6 z_{h\text{ill}}/W_{1/2}$ for isolated 3-D hills, and $\Delta S_{\text{max}}(h) = 0.8 z_{h\text{ill}}/W_{1/2}$ for 2-D scraps, where $W_{1/2}$ is the horizontal distance from the summit to the point at which the height is half of its maximum (for notation see also Fig. 4.11.b). These estimates apply for wind speeds $\geq 6 \text{ m s}^{-1}$, horizontal scales $\leq 1 \text{ km}$, $z_{h\text{ill}}/W_{1/2} \leq 0.5$ and near neutral stability.

An example of the speed-up at the Gaudergrat site is presented in Figure 4.12. The average speed-up is about 0.6. It should be noted that, strictly speaking, none of the restrictions above holds for our situation with the exception of neutral stability.
Figure 4.13 shows an example of the wind conditions at the five anemometer stations for the episode E4. The outer mast (M78) shows the expected north-westerly wind direction. In the windward slope (M76), a small deflection towards north is visible during this episode. This indicates there is a flow component parallel to the slope. The mean wind direction at the top of the crest (M75) is again north-west. At both leeside masts M74/73 and M72, a significant change of the direction towards north-east or south-west is obvious. Also a distinctive increase of the wind speed could be observed at M72. This example clearly demonstrates the difficulty of wind field simulations over complex terrain. Instead of a recirculation zone leeward of the crest, which could be expected in the case of a long homogeneous crest line (or 2-dimensional case), we observe a fluctuating cross flow behind the crest. The reproduction of this phenomenon is a challenge for the selection of flow region to be modeled and the setting of the boundary conditions. In some cases it will be necessary to select a much greater horizontal area than is of interest. The presented example is typical of all six observed drift periods.

An example of the spectral energy density and of the cumulative frequency of the turbulent energy is given in Fig. 4.14. The calculations are performed on the basis of 30 min averages. Fig. 4.14.a shows a peak at about 10 min followed by slope $\alpha f^{-5/3}$ which can be identified as the inertial subrange. From Fig. 4.14.b it can be seen that about 75% of the turbulent kinetic energy is distributed to eddies of the duration of 1-10 min. It is reasonable to assume that these eddies significantly contributes to the mass flux. It would be of interest to do instantaneous measurements of the mass flux to resolve the relevant scales.
Figure 4.13: Example of the wind field around the experimental site Gaudergrat during the episode E4 (18.03.97 18:00 - 22.03.97 06:00). The points represent 10-second averages of the wind speed. At masts M78 and M72 only the horizontal components are available.
4.14.a: Spectral energy density, $S_e$

4.14.b: Normalized cumulative frequency of the turbulent kinetic energy; $e_0 = 0.7 \, m^2 \, s^{-2}$

Figure 4.14: Example of the spectral energy density and the cumulative frequency of the turbulent kinetic energy at mast M76 for the period from 19.03.98–21.03.98.

4.2.4 Estimation of the threshold shear stress

No special investigation for the determination of the critical shear stress, $\tau_c$, was carried out. However, some observations allow us to get a rough idea of this quantity. As can be observed in Figure 4.6.d, the change of snow depth in the north-west slope (M77) is small during the considered period. This holds also for the episodes with snow fall. But, there are three short episodes where a small increase of the snow depth and then a rapid decrease are perceptible during snowfall. This behavior suggests a change in the interaction between the snow pack and the air flow. With the aid of these episodes, an estimation of the threshold friction velocity might be possible.

Two examples are given in Figure 4.15 and Figure 4.16. The figures show a plot of the wind speed at M76 and the calculated Reynolds stress, $\tau$, where the similarity relation (see, e.g., Stull [85])

$$ u_t^2 = \frac{\overline{w'^2} + \overline{v'^2} + \overline{u'^2}}{11} \tag{4.7} $$

is used to calculate $\tau = \rho_a \, u_t^2$, based on 10 min averages. Above those plots, the measured snow depth, $HS$, at M77 and for comparison a calculated snow depth $HS_{calc}$ is given. This snow depth is recalculated using the 10 min sum of the precipitation, $RS$, measured at VF, which is also shown. In addition the measured air temperature, $T_{M77}$, the surface temperature $TS_{M77}$ as well as the relative humidity $\phi_{M77}$, are given.

Figure 4.15 shows a 30 hour interval of episode E3. At the beginning of the interval the increase of the snow depth correlates quite well with the recalculated snow
depth. Then, fluctuations appear in the measurements, pointing to migrating dunes. During that time, the Reynolds stress remains around 0.2 Pa and the surface temperature falls below $-10^\circ$C. Between midnight and six o'clock a.m., a slow decrease in snow depth is noticeable. This decrease speeds up as $\tau$ rises above approximately 0.4 Pa and slows down as $\tau$ decreases again. During the snowfall after nine o'clock a.m., small increase in snow depth is visible. At this time, $\tau$ has fallen below 0.2 Pa. Hence, it is reasonable to assume a critical shear stress in the range 0.2–0.4 Pa for this episode.

The second example is the episode E6. An interval from this episode is shown in Figure 4.16. Again, the increase in the measured snow depth at the beginning correlates quite well with the calculated one. Then, around six o'clock a.m., the Reynolds stress rises above 0.5 Pa and $HS$ starts to decrease, despite the strong precipitation observed at the same time. The erosion of the recently deposited snow stops as the stress falls below approximately 0.3 Pa. But still no new-snow is deposited until the stress drops below 0.2 Pa. During this entire, the surface temperature was $-7^\circ$C, then it started to drop, after which, no more new-snow was deposited during the following snowfalls. In this case we might distinguish between different critical values: First a threshold value for eroding the recently deposited snowpack of
Figure 4.16: Estimation of $\tau_e$ for the episode E6. Plot of the wind speed, $M_2$ (10 s averages, pointed); the Reynolds stress, $\tau$ (full line); the measured and calculated snow depth, $HS$ (30 min averages) and $HS_{\text{cal}}$ (for a better comparison an offset $\Delta = 0.55$ m is added to the leeward snow depth); the precipitation (10 min sum); and the air and surface temperature, $T_M$ and $T_{SM}$; as well as the relative humidity $\phi_{SM}$ (the last three are 30 min averages).

approximately 0.3–0.5 Pa, then the value necessary to prevent deposition of about 0.2 Pa and a value below 0.2 Pa as the surface temperature decreases.

For a comparison, Fig. 4.16 shows also the evaluation of the snow depth at the leeward mast M73. Here, the increase of the snow depth correlates quite well with calculated increase during the mean precipitation period, whereas during the later precipitation periods no deposition can be observed. The decreasing of the snow depth at the end can be related to the settling of the snowpack.

The values for the threshold shear stress are rough estimates for selected episodes only, but they are in agreement with those predicted by R.A. Schmidt [76].

The observed erosion rates are also worth mentioning. In the episode E2 the erosion rate ranges from 0.005 m h$^{-1}$ to 0.02 m h$^{-1}$, and in the episode E6 a rate of about 0.04 m h$^{-1}$ could be observed.
5 Numerical Simulations

For the purpose of validation of the numerical model presented in Chapter 3, two methods were pursued. First, the model was compared with results described in the literature. Here, the most usable results deal with wind tunnel studies or numerical simulations of saltation alone. In the next step the model was used to simulate the situation at the experimental site Gaudergrat for comparison with field measurements (see Chapter 4).

5.1 Wind tunnel simulation

The wind tunnel simulations were carried out for a wind tunnel with a test section length of 4.5 m, a width of 1.0 m and a height of 0.5 m. These dimensions correspond for example to the working area of the wind tunnel at CEMAGREF, Grenoble [39, 52, 53]. The wind tunnel was resolved by 18 000 grid cells, with a footprint of 0.05 x 0.05 m²; the cell height varied from approximately 0.04 m to 0.07 m. At the bottom, a 0.03 m deep erodible layer of cohesionless particles, with a particle diameter of 150 μm was assumed: $E_B = 0$, $\tau_c = 0.025$ Pa. At the inflow, the first 0.20 m were kept particle-free. The density of the particle was in most cases set at 917 kg m⁻³. At the inflow, the following boundary conditions were set:

Boundary conditions (upwind)

- $u(z) = u_{0,25} \frac{\ln \left( \frac{z}{z_{0s}} \right)}{\ln \left( \frac{0.25 \text{m}}{z_{0s}} \right)}$ \hspace{1cm} (5.1)
- $e(z) = \frac{u_z^2}{\sqrt{C_p}}$ \hspace{1cm} (5.2)
- $e(z) = \frac{u_z^2}{\kappa z}$ \hspace{1cm} (5.3)

where the friction velocity is

$$u_s = \frac{\kappa u_{0,25}}{\ln \left( \frac{0.25 \text{m}}{z_{0s}} \right)} \quad \text{and} \quad z_{0s} = 0.0003 \text{ m}.$$  

For the wind tunnel simulation, time steps between 0.005 s and 0.01 s were chosen.

Simulations Figure 5.1 shows an example of the spatial evolution of the mass flux,$$
Q = \sum_k \rho_p c(k) \Delta h(k) u(k) \hspace{1cm} (5.4)
$$

where $\Delta h(k)$ is the height of the $k$th grid layer, and $c(k)$ and $u(k)$ are the concentration and velocity within. In Fig. 5.1.a, the total mass flux is plotted for four cases
with $u_{0.25}$ of 3.5–7.5 m s$^{-1}$ at the inflow, whereas in Fig. 5.1.b the mass flux within the saltation layer is shown. We observe a steep increase of the mass flux, $Q_{\text{sol}}$, reaching a "steady level" after a short distance. This agrees with the prediction of Anderson and Haff [4] that the flux within the saltation layer requires at most 1–2 s or, depending on the wind speed, a distance of approximately 0.25–1.5 m to achieve a steady flux (for a comparison with the temporal evolution of the saltation see Fig. 1.13). The slower increase at lower reference velocities is due to the lower mean impact velocities of the particles and hence the lower ejection rate. On the other hand, the saltation time increases with higher wind speeds and the particles will impact after a longer distance. This leads to a shift in the curves towards greater distances with increasing reference speeds. It may be noted that, in the case of $u_{0.25} = 3.5 \text{ m s}^{-1}$, the whole mass transport occurs more or less due to saltation.

![Figure 5.1: Simulated spatial evolution of the mass flux for four case, with $u_{0.25}$ ranging from 3.5 to 7.5 m s$^{-1}$.](image)

5.1.a: Total mass flux

5.1.b: Mass flux within the saltation layer
In Fig. 5.1.a and Fig. 5.1.b, a kind of oscillation can be observed. The oscillation increase with increasing reference speed. The reason for this behavior can be found in the self-regulation. The adaption of the wind field lag in the response of saltation the concentration and vice versa, resulting in a fluctuation in the mass flux around an equilibrium value. This phenomenon could also be observed in PDM simulations, e.g., by McEwan and Willetts [46].

The case of $u_{0.25} = 7.5 \text{ m s}^{-1}$ is to some extent comparable with a simulation by Anderson and Haff [4] for the case of $u_* = 0.4 \text{ m s}^{-1}$. They found a value of about $0.007 \text{ kg m}^{-1} \text{s}^{-1}$ for the steady saltation mass flux (see Fig. 1.13.b). McEwan and Willetts [46] reported similar results for their saltation model, as they got a value of about $0.012 \text{ kg m}^{-1} \text{s}^{-1}$ for a shear stress of $u_* = 0.4 \text{ m s}^{-1}$ corresponding to a wind speed $u_{0.25} \approx 7.5 \text{ m s}^{-1}$. For the mass flux of snow in suspension, Castelle [10] measured under similar conditions values in the range of $0.009 \text{ kg m}^{-1} \text{s}^{-1}$ to $0.150 \text{ kg m}^{-1} \text{s}^{-1}$. For a wind speed $M_f = 7.77 \text{ m s}^{-1}$, Takeuchi [88] found values of $Q_{0.3} = 0.0558 \text{ kg m}^{-1} \text{s}^{-1}$ and $Q_2 = 0.058 \text{ kg m}^{-1} \text{s}^{-1}$ for the mass flux up-to $0.3 \text{ m}$ and up-to $2 \text{ m}$ above the surface, respectively (see also Fig. 1.5.a). Thus, the simulation is in good agreement with the PDM simulations and with the measurements.

As described in Section 3.2.1, the saturation mass concentration and hence the flux within the saltation layer is mainly determined by the available stress. Also the total mass flux achieves a certain equilibrium state after some a distance. As $Q$ is given by

$$Q = Q_{\text{salt}} + Q_{\text{susp}}$$

(5.5)

this state is determined by two contributions. First by the equilibrium state of the saltation layer (see above) and by the equilibrium between the turbulent diffusion and gravitational settling. How the second part can influence the total flux rate is shown in Figure 5.2. Here, the total mass flux and the saltation flux are plotted for

![Figure 5.2: Simulated spatial evolution of the mass flux for $W_f = 0.19 \text{ m s}^{-1}$ ($\rho_p = 591 \text{ kg m}^{-3}$) and $W_f = 0.35 \text{ m s}^{-1}$ ($\rho_p = 917 \text{ kg m}^{-3}$). Shown are the flux due to saltation, $Q_{\text{salt}}$, as well as the total flux.](image)
two particle densities. The saltation flux is more or less equal in both cases, but the suspension flux differs significantly. This difference depends on the different free fall velocities of the particles causing different probability to be suspended.

This dependency can be evaluated in the case of a horizontally homogeneous, steady state. Then, equation (3.12) can be resolved for the mass concentration, and with the assumption of constant $W_f$ and replacing $\nu_f/\sigma_T$ by $\kappa c_{\mu}^{1/4} e^{1/2} z$, we gain the approximation

$$c_m(z) = c_{m, ref} \left( \frac{z}{z_{ref}} \right)^{2/3}$$

(5.6)

5.3.a: Mass concentration profile for each case with $W_f$'s of 0.19 m s$^{-1}$ and 0.35 m s$^{-1}$; $u_{0, 25} = 5.0$ m s$^{-1}$.

5.3.b: Mass concentration profile for each case with $u_{0, 25}$'s of 4.0–7.5 m s$^{-1}$; $\rho_p = 917.0$ kg m$^{-3}$.

Figure 5.3: Example of the concentration profile for the cases of different densities or different inflow velocities ($x = 1$ m).
where the Rouse number $R_s = -W_f / \left( \kappa c_n^{1/4} \ell^{1/2} \right)$. Hence, with decreasing fall velocity the mass concentration increases at any fixed height. On the other hand, the mass concentration will increase with increasing turbulent kinetic energy. Both cases are depict in Figure 5.3, assuming that $c \propto u_{0.25}^3$.

Figure 5.4 shows a comparison of drift simulations with wind tunnel experiments done by McKenna Neuman and Maljaars [48]. Plotted are the mass flux against the velocity; in the simulation $u_{0.25}$ is chosen as a reference whereas McKenna Neuman and Maljaars use the free stream velocity $u_\infty$ as a reference. The difference is negligible, however. McKenna Neuman and Maljaars observe a difference in the onset of transport depending on the difference in particle diameter and hence of the mass of the particles. However, they remark also that beyond a velocity of about $8.5 \text{ m s}^{-1}$, the effect of the transport stage ($\tau / \tau_c$, where $\tau_c$ represents the threshold) has little influence so that flux curves are essentially identical for both diameters. In comparison to these experiments, the simulation gives quite reasonable results. For a comparison with real snow drift data see Figure 1.10. In that case, the reference speed was measured at 1 m and particle diameters ranged from 100 $\mu$m to 500 $\mu$m. If we take these discrepancies into account, the simulation and the measured data are in good agreement.

A logarithmic plot of the steady mass flux versus $u_{0.25}$ is given in Figure 5.5. At low reference velocities, a rapid increase in the mass flux can be observed, then the rate of increase diminishes with increasing $u_{0.25}$, but is still nonlinear. The power

$F(\mu \text{ Pa})$
is approximately 2–4. This agrees quite well with the empirical formulations as in Table 1.1. The fit in Figure 5.5 corresponds to a power of 3.1.

### 5.2 Wind-tunnel with fence

The deposition pattern around obstacles such as buildings or the effect of a snow fence are examples of problems of immediate practical interest. Thus several investigations study fences at full scale or in the wind tunnel, e.g., the experiments by Iversen [26] or Anno [7] (see Fig. 1.11). Figure 5.6 and Figure 5.7 show two examples of simulated deposition patterns around a snow fence. Here, a solid fence was assumed with a height $H = 0.05 \, \text{m}$ and a width $W = 0.5 \, \text{m}$. The first simulation was carried out for a fence without bottom gap, the second for a fence with a bottom gap of approximately $0.1 \, H$. At the inflow the reference velocity was set to $u_{0.25} = 4.5 \, \text{m} \, \text{s}^{-1}$ and a precipitation rate of about $2 \, \text{mm} \, \text{h}^{-1}$ w.e. was used. No special effort was made to obtain a better resolution close to the fence. Nevertheless, the deposition patterns are quite reasonable.
5.6.a: Cut of horizontal / vertical instantaneous wind field. The horizontal cut corresponds to the wind speed at the top of the saltation layer and the vertical cut corresponds to the middle plane.

5.6.b: 3D deposition feature.

5.6.c: Profile of the snow depth. The profile line corresponds to the middle plane.

Figure 5.6: Example of the simulated deposition features around a snow fence without bottom gap and a height of $H = 0.05\,\text{m}$. The simulated time corresponds approximately to $0.5\,\text{h}$ and $H S_{ref} = 0.35\,H$. 
5.7.a: Cut of horizontal / vertical instantaneous wind field. The horizontal cut corresponds to the wind speed at the top of the saltation layer and the vertical cut corresponds to the middle plane.

5.7.b: 3D deposition pattern.

5.7.c: Profile of the snow depth. The profile line corresponds to the middle plane.

Figure 5.7: Example of the simulated deposition features around a snow fence with bottom gap (\( \approx 0.1H \)) and a height of \( H = 0.05 \) m. The simulated time corresponds approximately to 0.5 h and \( HS_{ref} = 0.35H \).
5.3 Full-scale simulation of the Gaudergrat site

In the following, an example will be presented of a full-scale simulation of the Gaudergrat area. Therefore, a model episode was chosen which might be regarded as typical for the drift episodes during the winter 1996/97 (see Chapter 4).

The Grid  For the full-scale simulation of the Gaudergrat site, a horizontal area of $1000 \times 600 \text{ m}^2$ was chosen around the crest line. The height of the domain varies between approximately 250 m and 400 m and was chosen to resolve the region of influence, beyond which the flow does not “feel” the presence of a hill (see Fig. 4.11.a). The grid was aligned in the north-west direction, the prevailing wind direction during precipitation and storm periods. The topography was taken from the 25 m digital terrain model (DHM25) of the Swiss Federal Office of Topography. In the surrounding of the hut (see Fig. 4.3), data gained by photogrammetry were added to these DHM25 data. Through this procedure a resolution of $5 \times 5 \text{ m}^2$ could be reached in the area of interest. This procedure was undertaken as first trials had shown that the crest line was poorly resolved by the DHM25 alone. Figure 5.8 shows the simulated area and indicates the used grid. The grid consists of $120 \times 37 \times 25$ cells for the flow domain and $120 \times 37$ cells each for the saltation layer and for the snowpack. The grid spacing $\Delta x$ ranges from 15 m at the inflow and the outflow to approximately 2.5 m close to the crest line, and $\Delta y$ ranges from 20 m at the sides to approximately 8 m in the middle. The spacing in the $z$-direction varies between approximately 2 m at the bottom to 35 m at the top.

![Figure 5.8: Sketch of the grid used for the full scale simulation of the Gaudergrat area. The points mark the positions of the masts and of the hut.](image-url)
**Boundary conditions** The following boundary conditions were used for the simulation of the Gaudergrat area. The sides (the north-east and the south-west surface) were chosen as smooth walls. At the inflow (upwind), the wind speed, wind direction, the turbulent kinetic energy, the dissipation and the volumetric concentration were set as follows:

- \( u_i(z, t) = u_{i, i}(t) \frac{\ln \left( \frac{z}{z_0} \right)}{\ln \left( \frac{z_{ref}}{z_{0s}} \right)} \)  
  \( (5.7) \)

- \( dd(t) = 315^\circ \)  
  \( (5.8) \)

- \( e(z) = \frac{u_*^2}{\sqrt{c_p}} \)  
  \( (5.9) \)

- \( e(z) = \frac{u_*^2}{\kappa z} \)  
  \( (5.10) \)

and

- \( c(t) = \frac{p_r(t)}{W_f} \)  
  \( (5.11) \)

where the friction velocity is

\[ u_* = \frac{\kappa M_4(t)}{\ln \left( \frac{z_{ref}}{z_{0s}} \right)} \quad \text{with} \quad z_{0s} = 0.003 \, \text{m} \]

Here, the reference wind speed \( M_4(t) \) and the precipitation rate \( p_r(t) \) were allowed to vary with the time. The precipitation rate was determined by

\[ p_r = 1.8 \cdot 10^{-6} \, \text{min}^{-1} \, \text{kg}^{-1} \, \text{m}^3 \, RS(t) \]  
\( (5.12) \)

where \( RS(t) \) is the measured 10 min precipitation sum at VF.

For the presented model period, the chosen \( M_4(t) \) and \( p_r(t) \) are shown in Figure 5.9. Also the time evolution of the reference new-snow depth, \( H_{N_{ref}} \), is given.

At the top, wall boundary conditions were chosen, but the wind speed was set to

\[ u_{i, i} = u_i - (u_j n_j) n_i \]  
\( (5.13) \)

with \( n_i \) the exterior normal vector of the top surface and

\[ u_i = u_{i, i} \frac{\ln \left( \frac{H}{z_0} \right)}{\ln \left( \frac{z_{ref}}{z_{0s}} \right)} \]  
\( (5.14) \)
Figure 5.9: Time evolution of the boundary conditions at the inflow for the model drift period. In addition, the evolution of $HN_{\text{ref}}$ is also shown (dashed line in upper plot).

where $H$ is the maximum height of the inflow. The snowfall rate was given by (5.12).

The boundary conditions at the bottom of the flow domain are described in Sections 3.3.1 and 3.3.2.

The initial snow depth was set to 2 m for the whole area. The density was assumed to be $\rho_s = 110$ kg m$^{-3}$, and the critical shear stress was set to $\tau_c = 0.5$ Pa and $E_B = 10^{-8}$ J. For deposited new snow also a density of $\rho_{NS} = 110$ kg m$^{-3}$ was used, whereas $\tau_c = 0.05$ Pa and $E_B = 10^{-9}$ J were chosen.

For the simulations, the time step was set to 0.25 s.

The simulations prove to be very time-consuming. To reduce the required computing time, the following procedure was used: The time of the simulated model period was scaled down by a factor of 10. Whenever the grid was adjusted after a time step, change in snow depth due to deposition or erosion was scaled up by the same factor.
The simulated wind field Figure 5.10 depicts a horizontal and a vertical slice of the simulated wind field. The horizontal cut corresponds to the wind speed at approximately 2 m above the surface whereas the vertical cut is chosen to pass through the experimental area where the soundings (see Chapter 4) were made.

In the horizontal cut (Fig. 5.10.a), the expected increase of the wind speed towards the crest line could be observed. The highest wind speeds appear in the little saddle of the south is obvious. Leeward of the crest, a deflection of the wind vectors towards the south is obvious. This behavior is in agreement with the field measurement, e.g., see Fig. 4.13. In the vertical cut (Fig. 5.10.b), the highest wind speeds are found at the crest in the layers close to the surface. With increasing height the wind speeds decreases again. This speed-up is a known effect at the crest of hills (see Section 4.2.b). In the lee of the crest, a developing recirculation zone could be observed. The recirculation is superimposed on the cross-flow which is obvious in Fig. 5.10.a.

The question is now, How does the simulated wind field agree with the measurements? To investigate this, the simulated wind speeds and directions at the locations of the sensor masts were compared with the measurements of the corresponding episode. Figure 5.11 shows polar plots for the measured wind speeds with thus of the simulation. Generally, the simulations agree quite well with the measurements. While the simulated wind speeds are too high at the crest they tend to be too low at the outflow of the domain (M72). Furthermore, the cross flow in the lee is under-predicted (M74 and M72). One reason might be the rough resolution of the grid in this area. A second comparison is presented in Figure 5.12, where the wind speed and the wind direction are plotted against time for each of the five masts. This plot shows that the departure of the wind speed at the top (M75) is largest when the high inflow velocity at M78 is high. Most likely, these discrepancies are due to an underestimation of the zone of influence and thus of the necessary height of the computational domain.

Nevertheless, the simulated wind field gives reasonable results, particularly if we keep in mind that boundary conditions for this complex terrain are poorly known and so their straightforward settings.
5.10.a: Horizontal cut; velocity vectors approximately 2 m above the snowpack.

5.10.b: Vertical cut

Figure 5.10: Sketch of the simulated wind field around Gaudergrat area during the model period. Notice the change in the scaling. \( t \approx 31.5 \text{ h}; \) contour interval 20 m)
Figure 5.11: Comparison of the wind field simulation (+; 25 min averages) with the corresponding measurements (-; 5 min averages). Polar plot of the wind speed and direction at the five sensor masts.
Figure 5.12: see next page
Figure 5.12: Comparison of the wind field simulation around the Gaudergrat site during the model period and the corresponding wind measurements. Shown are the time plots for the wind speed, $M$, and wind direction, $\theta$, at the locations of the masts, approximately 4 m above the surface. 25 min averages are shown for the simulation (+) whereas 5 min averages are given for the measurements (·).
Simulation of the redistribution patterns of the new-snow layer In the following, some examples of redistribution patterns for the new-snow layer will be presented. Also a comparison between soundings (see Section 4.2.2) taken at the experimental site and the simulation will be shown.

The Figures 5.14.a and 5.14.b demonstrate the effect of additional snow drift during snowfall in comparison with a snowfall without snow drift. Figure 5.14.a shows a simulation where only the deposition of the new snow is regarded. The depth agrees more or less with the expected reference depth \(HN/\text{HN}_{\text{ref}} \approx 1\). \(\text{HN}_{\text{ref}}\) corresponds to the new-snow depth on a drift free horizontal area. The variations in the new-snow depth distribution are small and are only influenced by the flow field and dispersion effects. By the way, small scale variations could be observed, improving the scaling, but here are not considered further. In contrast, Figure 5.14.b depicts a simulation including blowing and drifting snow. Here, a significant redistribution of the new-snow occurs. The new snow is picked up in acceleration regions such as the area close to the crest line, at small humps and brows. At the crest line, erosion of the old snow pack is observed as well. Deposition occurs in the deceleration regions leeward of the saddle, in small gullies and in hollows. In this simulation deposits of twice the reference height and erosion of about 0.05 m of the old snowpack at the crest line are found.

Figure 5.15 shows the situation after approximately 35 h. In comparison to Figure 5.14.b the deposition patterns have changed with the new-snow distribution in the leeward slope evened out. The maximum deposit is still about twice the reference depth. The erosion of the old snowpack slowed down at the crest line. At the end, the maximum erosion depth is about 0.15 m corresponding to an averaged erosion rate of about 0.005 m h\(^{-1}\). During the period between 21 h–28 h, averaged erosion rates of about 0.0075 m h\(^{-1}\) could be observed.

An example of the temporal variation of the erosion rate during this period is plotted in Figure 5.13 for the point M75. The maximum erosion rate is about 0.027 m h\(^{-1}\). This value is to be compared with the observed erosion in the windward slope of 0.08 m and 0.2 m, corresponding to erosion rates from 0.005 m h\(^{-1}\) from 0.04 m h\(^{-1}\) (see Fig. 4.15 and Fig. 4.16). Thus the simulated erosion rates yields the correct order-of-magnitude.

Figure 5.13: Simulated erosion rate at point M75 during the time period between 22 h–28 h.
Figure 5.14: Comparison of the simulated snow redistribution with and without transport due to snow drift during a precipitation period. The snow depth is normalized by the reference new-snow depth; negative values mean erosion of the old snow pack. In addition a sketch of the instantaneous wind field is shown. ($HN_{ref} \approx 0.09$ m; after $t \approx 7$ h)
Figure 5.15: Simulated snow distribution at the end of the model episode. The snow depth is normalized by the reference new-snow depth; negative values mean erosion of the old snow pack. In addition a sketch of the instantaneous wind field is shown. ($HN_{ref} \approx 0.29$ m; after $t \approx 35$ h)

As it was not possible to make a one-to-one comparison of an extended area during the winter 1996/97, a comparison for a small area (about $200 \times 40$ m$^2$) will be shown in Figure 5.16 instead$^5$. To reduce the uncertainties due to the local variability of the snowpack properties during the single drift episodes for which soundings were made, all episodes were put together for the comparison. This procedure appears justified because we regarded the model episode prototypical for the drift episodes of the winter 1996/97. The measurements show great variability, but also some trends can be discerned: one can see the erosion area in the windward slope near the crest line, the formation of a cornice at the ridge, and an area just leeward of the crest where the snow depth is less than the reference depth. Down slope areas where $HN$ is twice $HN_{ref}$ alternate with depletion areas such as the small terrace on the right. It should be pointed out that the small-scale terrain features cause large differences in the erosion and deposition patterns. The simulation does not

$^5$So far, there is no suitable method available for the measurement of the snow depth of an extended area with a reasonable effort and the required accuracy. A comparison between photogrammetry and soundings showed that the accuracy of photogrammetry is in the range of $\pm 0.1$ m over an area of approximately $8000$ m$^2$. This is in the range as the new-snow depth during a typical drift period in winter 1996/97. Photogrammetry was done by Julien Vallet, Photogrammétrie, Ecole technique fédérale de Lausanne.
show such great variability, but nevertheless reproduces some characteristic features like the windward erosion area close to crest, alternation between depleted areas and areas of enhanced deposition. Apart from the uncertainties due to the poorly known snowpack properties, e.g., the critical shear stress, another reason for the discrepancy between the measurements and the simulation can be found in the history of the snowpack evolution. Thus, snow drift tends to fill in hollows and to even out the topography in the course of a winter until an equilibrium level of the snow surface is reached where erosion balances deposition. This phenomenon could explain the discrepancy in the depression throughout the windward slope, if the initial (measured) snowpack at this location was close to its equilibrium level. Nevertheless, the results of the simulation give reasonable images.

Possibly the assumed boundary conditions for the wind speed and the precipitation are another reason for the deviation between the measured and simulated distribution of the new snow in Fig. 5.16. How strongly these boundary conditions influence the snow redistribution is illustrated in Figure 5.17 which shows the new-snow depth for two time periods. Fig. 5.17.a depicts the evolution for the period 21 h–28 h and Fig. 5.17.b for the period 28 h–35 h. $H_N_{ref}$ is comparable in both cases, but in the first case, the mean wind speed at point M77 is nearly constant at $5 \text{ m s}^{-1}$ during the strongest precipitation. In the second case, the wind speed decreases during the precipitation and the minimum wind speed of about $3 \text{ m s}^{-1}$ is reached as the precipitation reaches its maximum (see Fig. 5.9). In this case, only a small band close to the crest line can be observed where $H_N$ strongly differs from $H_N_{ref}$. In the first case, the deviations are conspicuous. The erosion area starts down slope of the windward slope, in the leeward slope areas with twice $H_N_{ref}$ alternate with areas where only a fraction of $H_N_{ref}$ is deposited. This example illustrates again the importance of the snow drift and hence of the wind for the snow distribution, especially in alpine terrain.
5.16.a: Measured new-snow depths; soundings of six episodes were put together.

5.16.b: Simulation ($HN_{ref} \approx 0.29$ m; after $t \approx 35$ h)

Figure 5.16: Comparison between soundings and simulated redistribution pattern for a small area around Gaudergrat ridge. Shown are the normalized new-snow depths; negative values mean erosion of the old snowpack. (Contour interval is 10 m)
5.17.a: Simulation for the time period between 21 h and 28 h ($HN_{ref} \approx 0.1\text{ m}$).

5.17.b: Simulation for the time period between 28 h and 35 h ($HN_{ref} \approx 0.08\text{ m}$).

Figure 5.17: Comparison between two simulated time periods with different boundary conditions for the wind speed and the precipitation (see Fig. 5.9; contour interval 10 m).
6 Concluding Remarks

Figure 6.1: Slab avalanche at Gaudergrat. The photo was taken on 21 February 1997 after a snow drift period. It shows the saddle and the gully visible in Fig. 5.10, north-east from the experimental site.

Snow drift is a key factor in the formation of avalanches as well as in the effectiveness of avalanches. Hence, great interest exist in the prediction of the influence of snow drift on avalanche danger. Here, one can distinguish between the relevance for avalanche forecasting and land-use planning. In the former case, the primary interest focuses on the occurrence of snow drift and on the estimated enhancement of (new-)snow depths on a meso-scale (several 10 km$^2$) to improve avalanche warning on a regional scale. In the second case, a consequence of the intensification in land-use is the increased importance of land-use planning. In alpine regions, avalanche danger is of major significance for the restriction of land-use, and great efforts are made to develop numerical avalanche models to determine endangered zones. These models require input data on the snow mass distribution in avalanche release zones (micro-scale; several 100 m$^2$) which strongly depends on snow drift. To this end, no suitable tool is currently available to assess, analyze or forecast the effect of blowing and drifting snow in alpine topography.

The presented model is a step towards the numerical simulation of blowing and drifting snow in alpine terrain. The primary emphasis was on a physically-based description of snow drift suitable for complex alpine terrain. The model includes

- a fully 3-dimensional non-steady-state modeling;
- the modeling of the two main transport modes:
  - saltation (founded on a height-averaged dynamical description);
  - suspension;
- the dynamical modeling of deposition and erosion, distinguishing between aerodynamic entrainment and ejects due to particle impacts;
- the back-reaction of the particles in saltation on the flow (two-way coupling);
A comparison between simulations and wind tunnel experiments gives good agreement. The comparison between snow depth soundings and wind field measurements in a complex alpine terrain (Gaudergrat experimental site) and numerical simulations shows that the model is suitable to predict the new-snow distribution in extended alpine areas ($\approx 6 \cdot 10^5$ m$^2$) with a resolution of about 5 m.

However, the simulations also exhibit the major restrictions and/or problems involved in snow drift modeling:

- The computational effort increases nonlinearly with increase of the area of interest and/or spatial resolution of the area.

- The maximum possible computational time step size decreases with increasing spatial resolution.

$$\Delta t < \min \left( \frac{\min(\Delta x_i)}{\max(u_i, U_{P_i})}, \frac{\min(\Delta x_i^2)}{2 \nu_t} \right) \quad \text{(no summation)} \quad (6.1)$$

(For the presented simulations, $\Delta t$ was in the range of 0.1–0.25 s.) This numerical restriction is of particular importance for the simulation of snow drift as typical snow drift events last several hours.

- The 25 m resolution of the available digital terrain model of the Gaudergrat experimental site proved to be insufficient to resolve key topographic features, like the sharp crest line, in a reasonable way. Special effort had to be undertaken to improve the terrain model in the region of interest.

- One problem in the simulation of snow drift in a complex terrain are the often poorly known boundary conditions (e.g., the meteorological input or the snow pack properties). For these it was necessary to make rough estimations.

At this stage, the model can be used to calculate the snow distribution in avalanche release zones for typical weather situations as simulations of the Gaudergrat area demonstrate. These kind of simulations can improve the assessment of the potential fracture volume and the distribution of fracture areas, main parameters in avalanche hazard mapping. To overcome the problem with the poorly known boundary conditions, one might consider the adaptation of the output of a meso-scale weather forecast model.

First results of simulations indicate that the model is also suitable for simulating the deposition patterns around snow fences, obstacles and buildings, or the simulation of additional snow loads on buildings. These are also questions of practical importance. In this context, the major problem will be the grid handling and the resolution around the obstacle.

The computational effort limits the suitability of the presented snow drift model for operational avalanche warning. However, it can be used as a tool for the development of parameterizations for meso-scale forecasting models. Conversely, such a
A forecasting model can be used to provide the boundary conditions for simulations in hazard mapping (see above). In connection with avalanche warning, snowpack modeling should also be mentioned. Snowpack modeling depends on the correctness of the input data. One input parameter is the new-snow depth, which can significantly be influenced by snow drift, especially in slopes. Including the effect of redistribution by the wind will improve snowpack modeling for slopes. Here, the required spatial resolutions are in the micro-scale range, similar to the requirements for hazard-mapping.

For the further development of snow drift modeling (and understanding) the following topics are of interest:

- A great unknown is the influence of the snowpack on snow drift. The snowpack properties determine parameters like the erodibility, the rebound probability of particles etc. Little is known about the coupling between wind and natural snow. The parameterizing for erosion and deposition used in the presented model is mainly based on wind tunnel studies and numerical simulations for aeolian sand transport. Some research will be necessary to achieve better adaptation to the properties of snow.

- In the present model the parameterization of the saltation layer is based on experiments and calculations of particle trajectories over flat planes. Little is known about the influence of slope angles on particle trajectories. Particle Dynamic Methods simulations might be used to develop parameterization including slope effects.

- Including a sophisticated profile function for the mass concentration into the parameterizing of the saltation layer. This requires improved knowledge of the distribution of mass concentration within the saltation layer. The same applies to the wind profile. To this end, a combination of physical and numerical simulations seem to be appropriated.

- As the ability of grains to stay in suspension strongly depends on the grain size, introducing different size classes can improve the suspension modeling. However, this will significantly increase the computational effort.

- A still poorly known process is the mutually influence between particle transport and turbulence structure of the air flow within the boundary layer (surface layer), e.g., the modification of saltation, transition to suspension, or damping of the turbulence. To this end, it would be of interest to do instantaneous profile measurements of the mass flux and the wind speed with high temporal resolutions. In this context, the questions about turbulence modeling should be mentioned on the numerical side.
References


Appendix

A Symbols

Notation

A

\[ a \] any vector
\[ a_{ij} \] any matrix coefficient
\[ A \] area \[ m^2 \]
\[ A_k^i \] area vector of cells pointing in the direction \( k \) \[ m^2 \]
\[ A^s \] surface area \[ m^2 \]
\[ A_T \] top area of the saltation layer \[ m^2 \]
\[ A_{st} \] silhouette area \[ m^2 \]
\[ A_{pD} \] off-diagonal matrix coefficients \[ m^3 s^{-1} \]

B

\[ b \] any vector

C

\[ c \] volumetric concentration \[ l \]
\[ c_m \] mass concentration \[ kg \cdot m^{-3} \]
\[ c_{sat} \] saturation concentration \[ l \]
\[ c_\lambda \] volumetric concentration of snow in the snowpack
\[ c_{\mu} \] empirical constant of \( e-\epsilon \) model
\[ c_{1\epsilon} \] empirical constant of \( e-\epsilon \) model
\[ c_{2\epsilon} \] empirical constant of \( e-\epsilon \) model
\[ c_{3\epsilon} \] empirical constant of \( e-\epsilon \) model
\[ C \] convection coefficient \[ m^3 s^{-1} \]
\[ C \] concentration scale \[ l \]
\[ C_D \] drag coefficient
\[ C_p \] specific heat at constant pressure for moist air \[ J \cdot kg^{-1} \cdot K^{-1} \]
\[ C_{pd} \] specific heat at constant pressure for dry air \[ J \cdot kg^{-1} \cdot K^{-1} \]
\[ C_s \] speed of sound \[ m \cdot s^{-1} \]

D

\[ d \] displacement height \[ m \]
\[ dd \] wind direction \[ ^\circ \]
\[ dp \] grain diameter \[ m \]
### Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
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<td>$D$</td>
<td>diffusion coefficient</td>
<td>$[\text{m}^3\text{s}^{-1}]$</td>
</tr>
<tr>
<td>$E$</td>
<td>instantaneous turbulence kinetic energy</td>
<td>$[\text{m}^2\text{s}^{-2}]$</td>
</tr>
<tr>
<td>$e$</td>
<td>sublimation rate</td>
<td>$[\text{kg} \text{m}^{-3}\text{s}^{-1}]$</td>
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<tr>
<td>$E_{z_{j}</td>
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</tbody>
</table><p>ightarrow z_{j+1}}$ | sublimation rate by Pomeroy | $[\text{kg} \text{m}^{-2}\text{s}^{-1}]$ |
| $E$    | electrical field scale                           | $[\text{V} \text{m}^{-1}]$ |
| $E_i$  | electrical field vector                          | $[\text{V} \text{m}^{-1}]$ |
| $E_B$  | bonding energy                                   | $[\text{J}]$ |
| $E_D$  | dissipated energy                                | $[\text{J}]$ |
| $E_E$  | kinetic energy of an ejected grain               | $[\text{J}]$ |
| $E_I$  | kinetic energy of an impacting grain             | $[\text{J}]$ |
| $E_R$  | kinetic energy of a rebounding grain             | $[\text{J}]$ |
| $F$    | frequency                                        | $[\text{s}^{-1}]$ |
| $f_c$  | Coriolis parameter                               | $[\text{s}^{-1}]$ |
| $f_p$  | pressure wave frequency                          | $[\text{s}^{-1}]$ |
| $f_s$  | particle shape factor                            |                     |
| $F$    | Froude number                                    | $[1]$ |
| $F_A$  | any force                                        | $[\text{N}]$ |
| $F_B$  | Basset force                                     | $[\text{N}]$ |
| $F_C$  | cohesion                                         | $[\text{N}]$ |
| $F_D$  | drag force                                       | $[\text{N}]$ |
| $F_{DW}$ | force of the particle–air interaction            | $[\text{N}]$ |
| $F_E$  | electrostatic force                              | $[\text{N}]$ |
| $F_G$  | gravitation                                      | $[\text{N}]$ |
| $F_L$  | lift force                                       | $[\text{N}]$ |
| $F_M$  | Magnus force                                     | $[\text{N}]$ |
| $F_P$  | pressure force                                   | $[\text{N}]$ |
| $G$    | acceleration due to gravity                      | $[\text{m} \text{s}^{-2}]$ |
| $g$    | vector of the acceleration due to gravity        | $[\text{m} \text{s}^{-2}]$ |
| $h$    | height                                           | $[\text{m}]$ |
| $h_s$  | saltation layer height                           | $[\text{m}]$ |</p>
## Notation

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<tr>
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<td>new snow depth (since the last observation)</td>
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<td>depth of the nth old snow layer</td>
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<tr>
<td>$HS$</td>
<td>snow depth</td>
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<td>$HS_{ne}$</td>
<td>non-erodible snow depth</td>
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</tr>
<tr>
<td>$I$</td>
<td>net radiation</td>
<td>[W/m$^2$]</td>
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<td>$i$</td>
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<td>particle deposition rate</td>
<td>[kg m$^{-1}$]</td>
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<td>precipitation rate</td>
<td>[kg m$^{-1}$]</td>
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<td>$J_{m}$</td>
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<td>erosion/deposition (source) term for the concentration</td>
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<td>wind speed at the height $z$ in meter [m s$^{-1}$]</td>
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<tr>
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<td>$N_{ac}$</td>
<td>number of aerodynamically entrained grains [m$^{-2}$ s$^{-1}$]</td>
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<td>charge scale [C]</td>
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<td>$Q$</td>
<td>mass flux rate [kg m$^{-1}$ s$^{-1}$]</td>
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### Notation

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<tr>
<td>tₚ</td>
<td>eddy life time</td>
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</tr>
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<td>tₑ</td>
<td>travel time of an ejected grain</td>
<td>[s]</td>
</tr>
<tr>
<td>tᵣ</td>
<td>gravitational travel time scale</td>
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<td>tᵣ</td>
<td>inertial (interaction) time scale</td>
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<td>tᵣ</td>
<td>travel time of grain in saltation</td>
<td>[s]</td>
</tr>
<tr>
<td>tᵣ</td>
<td>particle relaxation time</td>
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<td>tᵣ</td>
<td>travel time of a rebounding grain</td>
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<td>time scale for the surface layer</td>
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<tr>
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<td>absolute temperature</td>
<td>[K]</td>
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<td>Tᵥ</td>
<td>virtual absolute temperature</td>
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</tr>
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<td>u</td>
<td>eastward moving Cartesian wind component</td>
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<td>u₄ₑ</td>
<td>wind velocity at the saltation layer height</td>
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<tr>
<td>uᵢ</td>
<td>vector of the wind velocity (u, v, w)</td>
<td>[m s⁻¹]</td>
</tr>
<tr>
<td>uᵢ' uᵢ'</td>
<td>kinematic flux on uᵢ momentum flux in the direction i</td>
<td>[m² s⁻²]</td>
</tr>
<tr>
<td>uₑ</td>
<td>free stream velocity within a wind tunnel</td>
<td>[m s⁻¹]</td>
</tr>
<tr>
<td>uₑ</td>
<td>friction velocity</td>
<td>[m s⁻¹]</td>
</tr>
<tr>
<td>uₑₑ</td>
<td>non-erodible friction velocity</td>
<td>[m s⁻¹]</td>
</tr>
<tr>
<td>uₑₑ</td>
<td>threshold friction velocity</td>
<td>[m s⁻¹]</td>
</tr>
<tr>
<td>U</td>
<td>velocity scale</td>
<td>[m s⁻¹]</td>
</tr>
<tr>
<td>Uₑ</td>
<td>eddy velocity scale</td>
<td>[m s⁻¹]</td>
</tr>
<tr>
<td>Uₑ</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uₑ</td>
<td>particle velocity component in the main flow</td>
<td>[m s⁻¹]</td>
</tr>
<tr>
<td>Uₑ</td>
<td>direction</td>
<td></td>
</tr>
<tr>
<td>Uₑ</td>
<td>vector of the particle velocity</td>
<td>[m s⁻¹]</td>
</tr>
<tr>
<td>Uₑ</td>
<td>vector of the relative velocity</td>
<td>[m s⁻¹]</td>
</tr>
</tbody>
</table>
Notation

\[ \mathbf{V} \]

- \( v \)  
  - northward moving Cartesian wind component \([\text{m s}^{-1}]\)
- \( V \)  
  - (control) volume \([\text{m}^3]\)
- \( V_P \)  
  - volume of a single grain \([\text{m}^3]\)

\[ \mathbf{W} \]

- \( w \)  
  - upward moving Cartesian wind component \([\text{m s}^{-1}]\)
- \( W \)  
  - fence width \([\text{m}]\)
- \( W \)  
  - linear interpolation weight
- \( W_{f} \)  
  - particle free fall velocity (absolute value) \([\text{m s}^{-1}]\)
- \( W_{f_i} \)  
  - vector of the particle free fall velocity \([\text{m s}^{-1}]\)
  
\( (0,0,-W_f) \)
- \( W_P \)  
  - upward moving particle velocity component \([\text{m s}^{-1}]\)

\[ \mathbf{X} \]

- \( x \)  
  - Cartesian coordinate towards east (sometimes used in a coordinate system aligned with the mean wind direction)
- \( x_i \)  
  - represents \((x, y, z)\) for \( i = (1,2,3) \)

\[ \mathbf{Y} \]

- \( y \)  
  - Cartesian coordinate towards west

\[ \mathbf{Z} \]

- \( z \)  
  - Cartesian coordinate up
- \( z_i \)  
  - height of the boundary layer \([\text{m}]\)
- \( z_0 \)  
  - aerodynamic roughness length \([\text{m}]\)
- \( z_{0s} \)  
  - aerodynamic roughness length of a snow cover \([\text{m}]\)

\[ \text{Greek} \]

- \( \alpha \)  
  - angle \([\text{°}]\)
- \( \beta \)  
  - angle of repose \([\text{°}]\)
- \( \beta' \)  
  - angle determining the mean drag level \([\text{°}]\)
- \( \gamma \)  
  - exponent by Castelle
- \( \Gamma \)  
  - a parameter
- \( \delta_i \)  
  - unit vector
- \( \delta_{ij} \)  
  - Kronecker Delta.
- \( \epsilon_{ijk} \)  
  - Alternating Unit Tensor
- \( \epsilon \)  
  - turbulence kinetic energy dissipation rate \([\text{m}^2\text{s}^{-3}]\)
- \( \zeta \)  
  - constant in the aerodynamic entrainment rate \([\text{grains N}^{-1}\text{s}^{-1}]\)
- \( \eta \)  
  - a parameter
- \( \Theta \)  
  - potential temperature \([\text{K}]\)
# Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>von K'arm'an constant</td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>parameter</td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td>kinematic viscosity</td>
<td>$[\text{m}^2\text{s}^{-1}]$</td>
</tr>
<tr>
<td>$\nu_T$</td>
<td>turbulent kinematic viscosity</td>
<td>$[\text{m}^2\text{s}^{-1}]$</td>
</tr>
<tr>
<td>$\xi$</td>
<td>parameter in the rebound probability expression</td>
<td>$[\text{m}^{-1}\text{s}]$</td>
</tr>
<tr>
<td>$\xi_c$</td>
<td>profile function of the concentration within the saltation layer</td>
<td></td>
</tr>
<tr>
<td>$\xi_u$</td>
<td>profile function of the velocity within the saltation layer</td>
<td></td>
</tr>
<tr>
<td>$\Xi$</td>
<td>profile function for the mass flux within suspension by Castelle</td>
<td></td>
</tr>
<tr>
<td>$\pi$</td>
<td>3.14159...</td>
<td></td>
</tr>
<tr>
<td>$\varpi$</td>
<td>random number</td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>density of the mixture air and snow</td>
<td>$[\text{kg}\text{m}^{-3}]$</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>density air</td>
<td>$[\text{kg}\text{m}^{-3}]$</td>
</tr>
<tr>
<td>$\rho_{NS}^{\text{top}}$</td>
<td>density of uppermost snow layer (north-west or south-east facing slope)</td>
<td>$[\text{kg}\text{m}^{-3}]$</td>
</tr>
<tr>
<td>$\rho_{NS}$</td>
<td>density of new snow layer</td>
<td>$[\text{kg}\text{m}^{-3}]$</td>
</tr>
<tr>
<td>$\rho_{NS_{\text{norm}}}$</td>
<td>averaged new snow density for the six drift periods</td>
<td>$[\text{kg}\text{m}^{-3}]$</td>
</tr>
<tr>
<td>$\rho_{OS}$</td>
<td>density of old snow layer</td>
<td>$[\text{kg}\text{m}^{-3}]$</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>grain density</td>
<td>$[\text{kg}\text{m}^{-3}]$</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>density of the snowpack</td>
<td>$[\text{kg}\text{m}^{-3}]$</td>
</tr>
<tr>
<td>$\sigma_{p,i}$</td>
<td>particle stress tensor</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>empirical constant of e–e model</td>
<td></td>
</tr>
<tr>
<td>$\sigma_T$</td>
<td>empirical constant of e–e model</td>
<td></td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>empirical constant of e–e model</td>
<td></td>
</tr>
<tr>
<td>$\zeta$</td>
<td>parameter of the velocity profile within the saltation layer</td>
<td></td>
</tr>
<tr>
<td>$\zeta_E$</td>
<td>ratio of $</td>
<td>U_{PE}</td>
</tr>
<tr>
<td>$\zeta_m$</td>
<td>ratio of $</td>
<td>U_{PT}</td>
</tr>
<tr>
<td>$\zeta_i$</td>
<td>ratio of $\overline{U_{PT}}$ to $u_{si}$</td>
<td></td>
</tr>
<tr>
<td>$\zeta_R$</td>
<td>ratio of $|U_{PR}|$ to $|U_{PT}|$</td>
<td></td>
</tr>
<tr>
<td>$\tau$</td>
<td>shear stress / Reynolds stress</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\tau_a$</td>
<td>air borne shear stress (shear stress at the surface)</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\tau_c$</td>
<td>critical shear stress</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\tau_g$</td>
<td>grain borne shear stress</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\psi$</td>
<td>stream function</td>
<td>$[\text{s}^{-1}]$</td>
</tr>
</tbody>
</table>
### Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>slope angle</td>
<td>$[^\circ]$</td>
</tr>
<tr>
<td>$\phi_h$</td>
<td>relative humidity</td>
<td>[%]</td>
</tr>
<tr>
<td>$\Phi$</td>
<td>latitude</td>
<td>[°]</td>
</tr>
<tr>
<td>$\omega$</td>
<td>angular rotation rate of the earth</td>
<td>[rad s$^{-1}$]</td>
</tr>
<tr>
<td>$\omega_r$</td>
<td>relaxation parameter</td>
<td></td>
</tr>
<tr>
<td>$\omega_v$</td>
<td>vorticity</td>
<td>[s$^{-1}$ m$^{-2}$]</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>angular velocity</td>
<td>[s$^{-1}$]</td>
</tr>
<tr>
<td>$\Omega_v$</td>
<td>angular velocity scale</td>
<td>[s$^{-1}$]</td>
</tr>
<tr>
<td>$\Omega_p$</td>
<td>particle angular velocity</td>
<td>[s$^{-1}$]</td>
</tr>
<tr>
<td>$\Omega_R$</td>
<td>relative angular velocity</td>
<td>[s$^{-1}$]</td>
</tr>
</tbody>
</table>

### Special Symbols and Operators

- $\hat{()}$ partial quantity
- $\tilde{()}$ turbulent quantity
- $(())'$ deviation from the mean
- $\bar{()}$ average operator
- $\mathcal{O}(())$ order
- $\nabla(())$ gradient
- $(()) : (())$ scalar product
- $(()) \nabla(())$ divergence
- $(()) \times (())$ vector product
- $\frac{\partial (())}{\partial (())}$ partial derivative
- $\frac{d(())}{d(())}$ total derivative
- $\Delta(())$ difference
- $|()|$ absolute value
- $[()]$ rounded value
- $\|()\|$ Euclidean norm

### Subscripts

- $i, j, k, \ldots$ indices for summation notation (each index can take on the values 1, 2, and 3)
- $l, m, n, \ldots$ take on the values 1 and 2
- $q, r, s, t$ take on the values 1 and 2
- $\alpha, \beta$ indices for summation notation (each index can take on the values 1 and 2)
- $(())_{\text{corr}}$ corrected quantity
- $(())_E$ ejected particle
- $(())_{Ts}$ quantity at the top of the saltation layer
- $(())_I$ impacting particle
### Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>impacting ejected particle</td>
</tr>
<tr>
<td>( O_{ij} )</td>
<td>position ( i )th row, ( j )th measuring point</td>
</tr>
<tr>
<td>( O_{IR} )</td>
<td>impacting rebounded particle</td>
</tr>
<tr>
<td>( O_{\text{mea}} )</td>
<td>measured quantity</td>
</tr>
<tr>
<td>( O_{nb} )</td>
<td>neighboring nodes</td>
</tr>
<tr>
<td>( O_{NS} )</td>
<td>new snow</td>
</tr>
<tr>
<td>( O_{OS} )</td>
<td>old snow</td>
</tr>
<tr>
<td>( O_{P} )</td>
<td>particle quantity</td>
</tr>
<tr>
<td>( O_{\text{ref}} )</td>
<td>reference quantity</td>
</tr>
<tr>
<td>( O_{\text{refm}} )</td>
<td>modified reference quantity</td>
</tr>
<tr>
<td>( O_{R} )</td>
<td>rebounding particle</td>
</tr>
<tr>
<td>( O_{s} )</td>
<td>(near the) surface / snowpack</td>
</tr>
<tr>
<td>( O_{\text{sal}} )</td>
<td>saltation</td>
</tr>
<tr>
<td>( O_{\text{sus}} )</td>
<td>suspension</td>
</tr>
<tr>
<td>( O_{th} )</td>
<td>threshold value</td>
</tr>
<tr>
<td>( O_{\text{cp}} )</td>
<td>cell center</td>
</tr>
<tr>
<td>( O_{\text{r}} )</td>
<td>a relative quantity</td>
</tr>
<tr>
<td>( O_{0} )</td>
<td>initial or reference quantity</td>
</tr>
</tbody>
</table>

### Superscripts

<table>
<thead>
<tr>
<th>Superscript</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^*)</td>
<td>scaling variable</td>
</tr>
<tr>
<td>((E,N,U,\ldots))</td>
<td>representing the directions east, north, up,</td>
</tr>
<tr>
<td>((W,S,D,\ldots))</td>
<td>west, south, down</td>
</tr>
<tr>
<td>(^s)</td>
<td>superscript of the surface area</td>
</tr>
<tr>
<td>((t))</td>
<td>quantity at the time step ( t )</td>
</tr>
<tr>
<td>((t-1))</td>
<td>quantity at the time step ( t - 1 )</td>
</tr>
</tbody>
</table>
B  Physical Constants, Parameters and Conversion Factors

Table B.1: Similarity “constants”

\[ \kappa = 0.4 \quad \text{von Kármán Constant} \]

Table B.2: Geophysical parameters

\[ g = 9.8 \text{ m s}^{-2} \quad \text{acceleration due to gravity} \]
\[ \omega = 7.27 \cdot 10^{-5} \text{ rad s}^{-1} \quad \text{angular rotation rate of earth} \]
\[ f_c = 1.46 \cdot 10^{-4} \sin(\phi) \text{ s}^{-1} \quad \text{Coriolis parameter as a function of latitude} \]

Table B.3: Parameters of air

\[ \rho_a = 0.98 \text{ kg m}^{-3} \quad \text{density of (dry) air at 2500 m a.s.l (US standard atmosphere)} \]
\[ \nu = 1.74 \cdot 10^{-5} \text{ m}^2 \text{ s}^{-1} \quad \text{kinematic molecular viscosity} \quad (T = 273.15 \text{ K}) \]

Table B.4: Parameters of Snow/Ice Particle

\[ \rho_p = 917.0 \text{ kg m}^{-3} \quad \text{density of an ice particle} \]
\[ \rho_p - \rho_a = 916.0 \quad \text{relative density difference} \]
\[ d_P = 2 \cdot 10^{-4} \text{ m} \quad \text{typical diameter of a drifting particle} \]
\[ f_s = 0.75-1.0 \quad \text{shape factor} \]

Table B.5: \( e-\epsilon \) model parameter (after Rodi [66])

| Standard \( e-\epsilon \) model |
|-------------------------|-----------------|-----------------|-----------------|-----------------|
| \( c_p \) | \( c_{1e} \) | \( c_{2e} \) | \( c_{3e} \) | \( \sigma_k \) | \( \sigma_\epsilon \) |
| 0.09 | 1.44 | 1.92 | 0.8 | 1.0 | 1.3 |
### Table B.6: Compilation of the used model parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>used in</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_I = 40^\circ$</td>
<td>typical rebound angle</td>
<td>(3.37) and (3.38)</td>
</tr>
<tr>
<td>$\alpha_E = 65^\circ$</td>
<td>typical ejection angle</td>
<td>(3.37) and (3.38)</td>
</tr>
<tr>
<td>$\zeta = 10^5$ grains N$^{-1}$ s$^{-1}$</td>
<td>entrainment constant</td>
<td>(3.50)</td>
</tr>
<tr>
<td>$\xi = 1.5$ m$^{-1}$ s$^{-1}$</td>
<td>parameter in the rebound probability expression</td>
<td>(3.53)</td>
</tr>
<tr>
<td>$\xi_c$ defined in (3.36)</td>
<td>concentration profile function</td>
<td></td>
</tr>
<tr>
<td>$\xi_u$ defined in (3.35)</td>
<td>velocity profile function</td>
<td></td>
</tr>
<tr>
<td>$\varsigma = 0.35$</td>
<td>velocity profile parameter</td>
<td>(3.35)</td>
</tr>
<tr>
<td>$\varsigma_E = 0.1$</td>
<td>typical ratio of $|U_{PE}|$ to $|U_{PT}|$</td>
<td>(3.37) and (3.38)</td>
</tr>
<tr>
<td>$\varsigma_m = 0.7$</td>
<td>ratio of $|U_{PT}|$ to $|u_{hs}|$</td>
<td>(3.37) and (3.38)</td>
</tr>
<tr>
<td>$\varsigma_q = 0.58$</td>
<td>ratio of $\bar{U}<em>{P,i}$ to $u</em>{h,i}$</td>
<td>(3.72)</td>
</tr>
<tr>
<td>$\varsigma_R = 0.5$</td>
<td>typical ratio of $|U_{PR}|$ to $|U_{PT}|$</td>
<td>(3.37) and (3.38)</td>
</tr>
<tr>
<td>$z_{0s} = 3 \cdot 10^{-4}$ m</td>
<td>aerodynamic roughness length for snow cover</td>
<td>(3.47)</td>
</tr>
<tr>
<td>$\lambda = 0.1$</td>
<td>parameter</td>
<td>(3.47)</td>
</tr>
</tbody>
</table>
C  Summation Notation

Two fundamental rules apply within summation notation:

- Whenever two identical indices appear in the same one term, it is implied that there
  is a sum of that term over each value (an index can take on the value 1 and 2 in the
  case of 2-D, or 1, 2 and 3 in the case of 3-D) of the repeated index.

- Whenever one index appears unsummed (free) in a term, then that same index must
  appear unsummed in all terms in that equation. Hence, that equation effectively
  represents 2 or 3 equations, respectively, one for each value of the unsummed index.

Herein, the summation notation is used in the following way:

Vector
\[ \vec{a} = a_i = (a_1, a_2, a_3) \]  \hspace{1cm} (C.1)

Second-order tensor
\[ \vec{a} = a_{ij} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \]  \hspace{1cm} (C.2)

Coordinates unit vector \( \delta_i \) represents a unit vector in one of the three Cartesian
directions.
\[ \delta_1 = \hat{i} \quad \delta_2 = \hat{j} \quad \delta_3 = \hat{k} \]  \hspace{1cm} (C.3)

Components of the coordinates
\[ \vec{x} = x_i = (x_1, x_2, x_3) = (x, y, z) \]  \hspace{1cm} (C.4)

Kronecker Delta
\[ \delta_{ij} = \begin{cases} +1 & \text{for } i = j \\ 0 & \text{for } i \neq j \end{cases} \]  \hspace{1cm} (C.5)

Alternating Unit Tensor
\[ \epsilon_{ijk} = \begin{cases} +1 & \text{for } ijk = 123, 231, \text{or} 312 \\ -1 & \text{for } ijk = 321, 213, \text{or} 132 \\ 0 & \text{for any two or more indices alike} \end{cases} \]  \hspace{1cm} (C.6)

Scalar product
\[ \vec{a} \cdot \vec{b} = a_i b_i = a_1 b_1 + a_2 b_2 + a_3 b_3 \]  \hspace{1cm} (C.7)

Vector product
\[ \vec{a} \times \vec{b} = \epsilon_{ijk} a_j b_k = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix} \]  \hspace{1cm} (C.8)
Euclidean norm

\[ \| \vec{a} \| = \| a_i \| = \sqrt{a_i a_i} = \sqrt{a_1^2 + a_2^2 + a_3^2} \]  \hspace{1cm} (C.9)

At some places the relaxed form \( \| a \| \) instead of \( \| a_i \| \) is used always thinking \( a \) is a vector.

Gradient of a scalar (vector quantity)

\[ \nabla a = \frac{\partial a}{\partial x_i} = \left( \frac{\partial a}{\partial x}, \frac{\partial a}{\partial y}, \frac{\partial a}{\partial z} \right) \]  \hspace{1cm} (C.10)

Gradient of a vector (second-order tensor quantity)

\[ \nabla \vec{a} = \frac{\partial a_i}{\partial x_j} = \left( \begin{array}{ccc} \frac{\partial a_1}{\partial x_1} & \frac{\partial a_1}{\partial x_2} & \frac{\partial a_1}{\partial x_3} \\ \frac{\partial a_2}{\partial x_1} & \frac{\partial a_2}{\partial x_2} & \frac{\partial a_2}{\partial x_3} \\ \frac{\partial a_3}{\partial x_1} & \frac{\partial a_3}{\partial x_2} & \frac{\partial a_3}{\partial x_3} \end{array} \right) \]  \hspace{1cm} (C.11)

Divergence of a vector (scalar quantity)

\[ \nabla \cdot \vec{a} = \frac{\partial a_i}{\partial x_i} = \frac{\partial a_1}{\partial x} + \frac{\partial a_2}{\partial y} + \frac{\partial a_3}{\partial z} \]  \hspace{1cm} (C.12)

Divergence of a second-order tensor (vector quantity)

\[ \nabla \cdot \vec{a} = \frac{\partial a_{ij}}{\partial x_j} = \left( \begin{array}{ccc} \frac{\partial a_{11}}{\partial x_1} + \frac{\partial a_{12}}{\partial y} + \frac{\partial a_{13}}{\partial z} \\ \frac{\partial a_{21}}{\partial x_1} + \frac{\partial a_{22}}{\partial y} + \frac{\partial a_{23}}{\partial z} \\ \frac{\partial a_{31}}{\partial x_1} + \frac{\partial a_{32}}{\partial y} + \frac{\partial a_{33}}{\partial z} \end{array} \right) \]  \hspace{1cm} (C.13)
ACKNOWLEDGEMENTS

Figure C.1: This work was only made possible with the help of many colleagues, who leave their tracks throughout it. Many thanks to all of them.

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born August 19th, 1963
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Education and Professional Training

1994-1999  Scientific collaborator at the Swiss Federal Institute for Snow and Avalanche Research, Davos, and Ph.D. student at the Institute for Fluid Dynamic, ETH Zurich

1994  Diploma in Mechanics, TH Darmstadt, Germany

1992-1993  Teaching assistant and research assistant at the Institute for Meteorology, at the Zentrum für Interdisziplinäre Technikforschung and at the Department of Mechanics

1991-1994  Studies in Mechanics, TH Darmstadt, Germany

1991  Diploma in Electronics, TH Darmstadt, Germany

1989-1991  Teaching assistant at the Solid-State Electronics Laboratory and at the Department of Mathematics

1984-1991  Studies in Electronics, TH Darmstadt, Germany

Publications and International Conferences


