Extrinsic Self Calibration of a Camera and a 3D Laser Range Finder from Natural Scenes

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Abstract—In this paper, we describe a new approach for the extrinsic calibration of a camera with a 3D laser range finder, that can be done on the fly. This approach does not require any calibration object. Only few point correspondences (at least 4) are used, which are manually selected by the user from a scene viewed by the two sensors. The proposed method relies on a novel technique to visualize the range information obtained from a 3D laser scanner. This technique converts the visually ambiguous 3D range information into a 2D map where natural features of a scene are highlighted. These features represent depth discontinuities and direction changes in the range image. We show that by enhancing the features the user can easily find the corresponding points of the camera image points. Therefore, visually identifying laser-camera correspondences becomes as easy as image pairing. Once point correspondences are given, extrinsic calibration is done using the well-known PnP algorithm followed by a non-linear refinement process. We show the performance of our approach through experimental results. In these experiments, we will use an omnidirectional camera. The implication of this method is important because it brings 3D computer vision systems out of the laboratory and into practical use.

I. INTRODUCTION

One of the basic issues of mobile robotics is the automatic mapping of the environments. Digital 3D models of the environment are needed in autonomous navigation, rescue and inspection robotics, facility management, and architecture. Autonomous mobile robots equipped with 3D laser range finders are well suited for this task. Recently, several techniques for acquiring three-dimensional data with 2D range scanners installed on a mobile robot have been developed. A popular approach is to use multiple scanners that point towards different directions [2]. An alternative is to use pan/tilt devices that sweep the range scanner in an oscillating way [3, 4]. More recently, techniques for rotating 2D range scanners have been developed [5]. However, to create realistic virtual models, visually-perceived information from the environment has to be acquired and it has to be precisely mapped onto the range information. To accomplish this task, camera and 3D laser range finder must be extrinsically calibrated, that is, the rigid transformation between the two reference systems must be estimated.

Most previous works on extrinsic laser-camera calibration concern calibration of perspective cameras to 2D laser scanners [6, 7, 8, 9]. Furthermore, some of this works uses visible lasers [6]. In contrast to previous works, in this paper, we consider the extrinsic calibration of a general camera with a three-dimensional laser range finder where the laser points are invisible to the camera. Because of the recent development of 3D laser scanners, only little work about extrinsic calibration of camera and 3D scanners exists. Furthermore, the process of external calibration is often poorly documented. This process usually requires some modification of the scene by introducing landmarks that are visible by both the camera and the laser. For instance, some approaches use high reflectance surfaces while other approaches use 3D spheres as markers. In the latter, using a fitting procedure along with prior knowledge of the radius of the sphere, the 3-D coordinates of the sphere center can be estimated quite accurately on condition that there are sufficient laser returns off its surface. The spheres can also be easily detected in an image. Then, extrinsic calibration can be done using these 3-D-2D correspondence pairs of the sphere centers. However, the spheres need to be quite big to be accurately detected by the laser. Thus, this method is not portable.

Well-documented work about extrinsic calibration of camera and 3D scanners can be found in [10] and [11]. However, in [10], the authors deal with the case of visible laser traces. Conversely, for the case of the invisible laser in [11], the authors propose a method for fast extrinsic calibration of a camera and a 3D scanner which makes use of a checkerboard calibration target, like the one commonly used for the internal calibration of a camera. Furthermore, they provide a useful laser-camera calibration toolbox for Matlab that implements the proposed calibration procedure [12]. Their method requires the user to collect a few laser-camera acquisitions where the calibration grid is shown at different positions and orientations. Then, the user is asked to manually select the region of points in the camera and laser images, which contain the grid. Finally, a plane is fitted to the selected points of each view pair and calibration is done by minimizing the difference in orientation and distance of the planes observed by the two sensors. This technique however

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needs several camera-laser acquisitions of the grid for a sufficiently accurate external calibration of the system.

The work described in this paper also focuses on the extrinsic calibration of a camera and a 3D laser range finder but the primary difference is that we do not use any calibration pattern. We use only the point correspondences that the user hand selects from a single laser-camera acquisition of a natural scene. As we use no calibration target, we name our technique self-calibration.

This work was motivated while working in the First European Land-Robot Trial [13]. In that contest, we presented an autonomous Smart car equipped with several 3D laser range finders and cameras (both omnidirectional and perspective cameras). The goal was to produce 3D maps of the environment along with textures [1]. Especially when working in outdoor environments, doing several laser-camera acquisitions of a calibration pattern can be a laborious task. For each acquisition, the pattern has to be moved to another position and this process usually takes time. Furthermore, weather conditions (e.g. wind, fog, low visibility) can sometimes perturb or even alter the calibration settings. Hence, the calibration must be done quickly. Because of this, we developed the procedure presented in this paper. The advantages are that now we need only a single laser-camera acquisition and that the calibration input are point correspondences manually selected from the laser-camera acquisition of a natural scene. Once point correspondences are given, the extrinsic calibration problem becomes a camera pose-estimation problem which is well known in computer vision and can be solved using standard methods. The difficulty resides in visually identifying the point correspondences because range images in general lack in point features. To bypass this problem, we process the range data so that we can highlight discontinuities and orientation changes along specific directions. This processing transforms the range image into a new image that we call Bearing Angle image (BA). Using BA images, we will show that visually identifying point correspondences between the laser and the camera outputs becomes as easy as image pairing. To show the generality of the methodology, in our experiments we will use an omnidirectional camera. The BA images and the application of the method to an omnidirectional camera are the two main contributions of this paper.

This document is organized as follows. Section II describes the projection model of the camera and 3D laser scanner. Section III defines the concept of BA images and explains how to compute them. Section IV describes the calibration procedure. Finally, section V presents some calibration results.

II. LASER-CAMERA PROJECTION MODEL

A. Camera Model

In this work, we deal with central cameras either perspective or omnidirectional. Central cameras satisfy the single effective viewpoint property, that is, they have a single centre of projection (see Fig. 1). For the case of catadioptric omnidirectional cameras, such property can be achieved using hyperbolic, parabolic, or elliptical mirrors [16]. In the last years, central omnidirectional cameras using fisheye lenses have also been built [17].

We assume that the camera has already been calibrated. Some Matlab toolbox to quickly calibrate these sensors can be found in [19, 20, 21].

![Fig. 1: Pin-hole model used for perspective cameras (a). Image formation model for central catadioptric cameras (b). Unified spherical projection model for central omnidirectional cameras (c): every pixel in the sensed image measures the irradiance of the light passing through the effective viewpoint in one particular direction. The vectors are normalized to 1.

Assuming the camera is already calibrated, given a pixel point \((u,v)\) on the camera image plane, we can recover the orientation of the vector \(X\) emanating from the effective viewpoint to the corresponding 3D point \((1)\). Conversely, given a 3D point \(\lambda X\), we can reproject it onto the camera image plane \((u,v)\) \((2)\):

\[
\lambda X = \lambda [x,y,z] = F(u,v),
\]

\[
[u,v] = F^{-1}(X),
\]

where \(\lambda\) is the depth factor and \(|X|=1\). Function \(F\) depends on the camera used. Some proposed formulations for \(F\) can be found in [17, 18, 22, 23]. We assume that the origin of the camera coordinate system coincides with the single effective viewpoint. This corresponds to the optical center for perspective cameras and the internal focus of the mirror, in the catadioptric case. The x-y plane is orthogonal to the mirror axis for the catadioptric case or to the camera optical axis for the perspective case (see more in [8], [18], [23]). Without loss of generality, in this paper, we consider omnidirec-
tional cameras, but the same considerations also apply to perspective cameras. According to what we have mentioned so far, in the remainder of the document we assume that for every sensed pixel we know the orientation of the corresponding vector $X$ on the unit sphere centered in the mirror frame (Fig. 1.c)

### B. Laser Model

3D laser range finders are usually built by nodding or rotating a 2D scanner in a stepwise or continuous manner around its lateral or radial axis. Combining the rotation of the mirror inside the 2D scanner with the external rotation of the scanner itself, spherical coordinates of the measured points are obtained. However, since in reality it is impossible to adjust the two centers of rotation exactly on the same point, the measured parameters are not spherical coordinates and offset values exist. These offset values have to be estimated by calibrating the 3D sensor by considering its observation model. The approach presented in this paper for the extrinsic calibration of a 3D laser scanner with a camera is general and does not depend on the sensor model. The approach described in this paper assumes that the laser is already calibrated. In the following paragraph, the scanner model used in our experiments is explained, but a different sensor setup can also be used along with its corresponding observation model.

The 3D range sensor used in this work is a custom-built 3D scanner since such scanners are not yet frequent in the market. It is composed of a SICK LMS 200 two-dimensional laser scanner mounted on a rotating support, which is driven by a Nanotec PD4-I57 stepping motor (see Fig. 2). Therefore, the support remains fixed while tacking each 2D scan and the sensor model can be written as:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} c_i c_j - c_j s_j & s_i & c_i dx + s_i dz \\ s_j & c_j & 0 \\ -s_i c_j & s_i s_j & c_i - s_i dx + c_i dz \end{bmatrix} \begin{bmatrix} \rho_y \\ 0 \\ 0 \end{bmatrix}$$

where $\rho_y$ is the $j^{th}$ measured distance with corresponding orientation $\theta_j$ in the $i^{th}$ scan plane, which makes the angle $\phi$ with the horizontal plane (Fig. 2.b). The offset of the external rotation axis from the center of the mirror in the laser frame has components $dx$ and $dz$ (as observed in Fig. 2.b). $[x,y,z]^T$ are the coordinates of each measured point relative to the global frame (with its origin at the center of the rotation axis, the $x$-axis pointing forward and the $z$-axis toward the top). The sensor is calibrated as discussed in [26] based on a known ground truth.

### III. BEARING ANGLE IMAGES

In this subsection, we will describe how to highlight depth discontinuities and direction changes in the range image so that the user can easily find the corresponding points of the camera image points. Such features in the range image are called image details. Fig. 3.a shows the 3D scan of an office-like environment. In such an environment, we would like emphasizing key points like corners arising from the plane intersections of walls, tables, chairs, and other similar discontinuities. Fig. 3.b shows the range image while in Fig. 3.c, the result of directly applying a Sobel edge detector on
the range image can be observed. As observed, edge detection does not directly help for our task, since edges are zones of the range image where the depth between two adjacent points significantly changes. In fact, many details in the range image do not create a big jump in the measured distance. Such edges are called “roof edges” and correspond to sharp direction changes (e.g. tetrahedron shaped corners). Therefore, a measure of direction should be used to highlight all the desired details in the scene.

As representative of the surface direction, its corresponding normal vector is usually used [14, 15]. Surface normal vectors are estimated based on the neighborhood of each point. However, for our application, we avoid the use of surface normals as representatives of the direction. The reason is that we want to highlight the details of the surface along some specific directions (vertical, horizontal, and diagonal). We will show that treating each dimension separately leads to enhanced estimation of the image details.

Let the range data coming from the 3D scanner be arranged in the form of a 2D matrix where his entries are ordered according to the direction of the laser beam. This matrix will be referred to as a depth matrix. We compute the surface orientation along four separate directions of the depth matrix, namely the horizontal, vertical, and diagonal one (the latter having +45° and -45° orientation).

\[
BA_i = \arccos \left( \frac{\rho_i - \rho_{i-1} \cos \phi}{\sqrt{\rho_i^2 - \rho_{i-1}^2 - \rho_i \rho_{i-1} \cos \phi}} \right), \quad (4)
\]

where \( \rho_i \) is the \( i \)th depth value in the selected trace of the depth matrix and \( \phi \) is the corresponding angle increment (laser beam angular step in the direction of the trace). Performing this calculation for all points in the depth matrix and putting the BA values in their corresponding row and column slots will lead to an image which is referred to as a BA image.

BA images can be calculated from the depth matrix along any direction, to highlight the details of the scene in the selected direction. In our application, horizontal, vertical, and diagonal traces suffice for a successful enhancement of the details of the scene (Fig. 5). However, any other direction could be also considered depending on the application. As observed in Fig. 5, these angular measures show the geometry of the target by highlighting many details that were not distinguishable in the range image. Hence, these will be used in the next section for extracting corresponding features.

Fig. 5. BA images for a real scan (top left: vertical, top right: horizontal, and bottom: two diagonal directions). Observe that, in BA images, the scene details are very highlighted (e.g. even the corners of the picture hanged on the left wall are now well distinguishable, in the range image 3.b, they were not). In this pictures, jet colormap has been used. The color shade (from blue to red) is proportional to the BA value.

IV. EXTRINSIC LASER-CAMERA CALIBRATION

A. Data Collection

Our calibration technique needs a single acquisition of both the laser scanner and the omnidirectional camera. The acquisition target can be any natural scene with a sufficient number of distinguishable key points, i.e. roof edges or depth discontinuities. Our calibration procedure consists of three stages: firstly, we compute the BA images of the acquired range image. Secondly, we manually select several point correspondences (at least four) between the BA image and intensity image. Finally, extrinsic calibration is done using a camera pose estimation algorithm followed by a non-linear refinement process.

Observe that usually not all four BA images are needed.
Depending on the scene and the orientation of the laser scanner to the scene, only the horizontal BA image could suffice. However, the remaining BA images can be used anyway, to check if there are further details that would be worth exploiting.

At the end of the visual selection, we have \( n \) laser points \((p_1, p_2, ..., p_n) \in \mathbb{R}^3\) in the laser frame and their correspondent points on the camera image plane \((m_1, m_2, ..., m_n) \in \mathbb{R}^3\). We rewrite these points in the following way:

\[
\begin{align*}
\mathbf{g}_c &= [g_{c,1}, g_{c,2}, ..., g_{c,n}], \\
\mathbf{g}_l &= [g_{l,1}, g_{l,2}, ..., g_{l,n}], \\
d_w &= [d_{w,1}, d_{w,2}, ..., d_{w,n}]
\end{align*}
\]  

(5)

where \( \theta_c \) and \( \theta_l \) are the unit norm orientation vectors of the camera and laser points in their respective reference frames, \( d_w \) are the point distances in the laser frame.

B. Extrinsic Calibration

Extrinsic calibration of a camera and a 3D laser range finder consists in finding the rotation \( R \) and translation \( T \) between the laser frame and the camera frame that minimizes a certain error function. In photogrammetry, the function to minimize is usually the reprojection error:

\[
\min_{R,T} \frac{1}{2} \sum_{i=1}^{n} \| m_i - \hat{m}(R,T,p_i) \|^2
\]  

(6)

where \( \hat{m}(R,T,p_i) \) is the reprojection onto the image plane of the laser point \( p_i \) according to equation (2). However, the reprojection error is not theoretically optimal in our application because the resolution of the camera is not uniform. A better error function uses the Riemann metric associated to a sphere as it takes into account the spatial distribution [8, 11]. This metric minimizes the difference of the bearing angles of the camera points and the bearing angles of the laser points after reprojection onto the image, that is:

\[
\min_{R,T} \frac{1}{2} \sum_{i=1}^{n} \| \arccos( \mathbf{g}_{c,i}^T \cdot \mathbf{g}_{l,i} ) \|^2
\]  

(7)

where \( \mathbf{g}_{l,i} \) is the unit norm orientation vector of \( \hat{m}(R,T,p_i) \).

According to equation (2), each correspondence pair contributes two equations. In total, there are \( 2n \) equations in 6 unknowns. Hence, at least 3 point associations are needed to solve for \( R \) and \( T \). However, 3 point associations yield up to four solutions and thus a fourth correspondence is needed to remove the ambiguity. This problem has already been theoretically investigated for a long time and is well known in the computer vision community as PnP problem (Perspective from n Points). Some solutions to this problem can be found in [24] and [25]. We implemented the PnP algorithm described in [24] to solve the calibration problem. The output of the PnP algorithm are the depth factors of the camera points in the camera reference frame, that is \( d_c = [d_{c,1}, d_{c,2}, ..., d_{c,n}] \). Then, to recover the rigid transformation between the two point sets, namely \( R \) and \( T \), we used the motion estimation algorithm proposed by Zhang [27].

C. Non-Linear Optimization

The drawback of using the PnP algorithm is that the solution is quite sensitive to the position of the input points, which were manually selected, and also to noisy range information. Furthermore, we have to take into account that the two sensors can have different resolution and that the rigid transformation is recovered by linear least-square estimation. Thus, the solution given in section \( B \) is suboptimal.

To refine the solution, we minimized (7) as a non-linear optimization problem by using the Levenberg-Marquardt method [28]. This requires an initial guess of \( R \) and \( T \), which is obtained using the method described in section \( B \).

V. RESULTS

The proposed method has been tested on the custom-built rotating scanner described in section II.B and on an omnidirectional camera consisting of a KADAN 360° One VR hyperbolic mirror and a SONY XCD-SX910-CR digital camera. The camera resolution was set as 640x480 pixels.

The rotating scanner provided 360° field of view range measurements with a vertical angular resolution of 1 degree and a horizontal resolution of 0.5 degree. The calibration of the omnidirectional camera was done using a toolbox available on the Internet [20]. The calibration of the rotating scanner was done using a known ground truth as explained in [26].

We evaluated the robustness of the proposed approach with respect to the number of manually selected points. We varied the number of laser-camera correspondences from 4 to 10 and for each combination we did ten calibration trials using different input points. The results shown in Fig. 6 and 7 are the average. Observe that after selecting more than 5 points, the values of the estimated \( R \) and \( T \) become rather stable. This stability occurs when points are chosen uniformly from the entire scene viewed by the sensors. Conversely, when points are selected within local regions of the scene, the estimated extrinsic parameters are biased by the position of this region. We also tried to use more than 10 points but the estimated parameters did not deviate from the average values that had been already estimated.

In our experiments, we did not know the ground truth but
the estimated $R$ and $T$ were in agreement with the hand-measured values. Furthermore, the estimated parameters were stable against the position of the input points when these points were picked uniformly from all around the scene. In Table I, the mean and the standard deviation of $R$ (roll, pitch, and yaw angles) and $T$ ($T_x$, $T_y$, $T_z$) are shown for the case of ten correspondence pairs. The results were averaged among ten different calibration trials. In Table I, we also show the reprojection error (in pixel). For data fusion, this error is the most important. This error measures the distance between the laser points reprojected onto the image using the estimated $R$ and $T$, and the image points. In our experiments, the average reprojection error was 1.6 pixels and the standard deviation was 1.2 pixels.

The reprojection of the laser points onto the image also offers an indirect way to evaluate the quality of the calibration. To do this, we chose not to reproject all the laser points onto the image. Rather, we wanted to reproject only those laser points that represent discontinuities in the range image. To select only depth discontinuities automatically, we applied an edge detector to the BA image. The edge points, as representative of depth discontinuities, were then reprojected onto the image. The reprojection results are shown in Fig. 8. As observed, the laser edge points well reprojected on the edges of the intensity image.

In the end, using the estimated $R$ and $T$, we colored an entire 3D scan by reprojecting the scan onto the corresponding image. The results of this color mapping are shown in Fig. 9.

**VI. CONCLUSION**

In this paper, we presented a new extrinsic calibration approach to estimate the orientation and position of a central camera with respect to a 3D laser range finder, that can be done on the fly. The method uses only a few correspondent points that are manually selected by the user from a single laser-camera acquisition of a natural scene. No calibration patterns are required, nor is more than one single laser-camera acquisition necessary. The proposed method relies on a novel technique to visualize the range information obtained from a 3D laser scanner. This technique converts the visually ambiguous 3D range information into a 2D map (called BA image) where natural features of a scene are highlighted. In this way, finding laser-camera correspondences is facilitated. Once correspondence pairs have been given, calibration is done using the PnP algorithm followed by a non-linear refinement process.

Real experiments have been conducted using a central catadioptric camera and a rotating SICK scanner, but the same approach can be also applied to any other type of camera (either perspective or omnidirectional) and laser range finder. The results showed that selecting the input points uniformly from the whole scene, robust calibration can be done by using only from eight to ten correspondence pairs.

The BA images and the application of the method to an omnidirectional camera were the two main contributions of this paper. The implication of the proposed calibration approach is important because it brings 3D computer vision
systems out of the laboratory and into practical use. In fact, the proposed approach requires no special equipment and allows the user to calibrate quickly the system in those cases where special settings are difficult to be arranged.

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