Improving geometric calibration methods for multi-axis machining centers by examining error interdependencies effects

Author(s): Bringmann, Bernhard

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Improving Geometric Calibration Methods for Multi-Axis Machining Centers by Examining Error Interdependencies Effects

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presented by
BERNHARD BRINGMANN
M. Sc.
born December 4th 1977
citizen of Germany

accepted on the recommendation of
Prof. Dr. K. Wegener, examiner
Prof. Dr. U. Meyer, co-examiner
Dr. W. Knapp, co-examiner

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May 2007
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Abstract

For machining processes (e.g. milling, grinding, electrical discharge machining), there has been a trend for many years towards ever increasing demands for workpiece accuracies, i.e. closer form, dimensional and location tolerances.

Kinematically any machine tool for such processes can be understood as an arrangement of different linear and rotary axes that cause a relative motion between tool and workpiece. The movements of these axes have a multitude of different errors, affecting the workpiece (e.g. positioning or straightness errors, roll and tilt motion errors, squareness and parallelism errors). These axes errors cause a major part of the form, dimensional and location deviations of the workpiece to be machined.

Today’s state of the art of geometric testing and calibrating machine tools is widely based on sequential measurements of single axes errors. For machining centers the problems of such an approach are pointed out as an example in this work. Since a multitude of different measuring devices and measuring setups are necessary, many different instruments and much time are required. More important are interdependency effects between errors. Several errors may have a similar influence on a measurement result, making the identification of different errors difficult.

A method for separating different errors despite their interdependencies is presented for 3-axis machining centers. With a ball plate that can be brought into different defined locations in space a spatial grid of measuring points is created. With this measurement different errors can be distinguished easily. Design details for achieving a measurement uncertainty as small as possible are presented.

For complex kinematics, where such a clear distinction between the single errors is not possible, an approach is presented to quantify the resulting uncertainty of the identification of an error due to the influence of measurement uncertainty and other geometric and non-geometric errors. Thereby a systematic planning and improving of calibration procedures is made possible. The geometric behavior of a machine that can be expected after calibration can be predicted.
Machining centers are the examples for the methods presented here. The considerations made here can be applied easily on other types of machine tools as well as on robots. As examples for the calibration 3- and 5-axis serial kinematic machines as well as the parallel kinematic Hexaglide are used here.
Kurzfassung

Bei spanenden und erosiven Bearbeitungsverfahren (z. B. Fräsen, Schleifen, Elektroerosion) gibt es seit vielen Jahren einen Trend zu immer höheren Genauigkeitsanforderungen, das heisst zu immer engeren Form-, Mass- und Lagetoleranzen, die am Bauteil einzuhalten sind.


Der heutige Stand der Technik der geometrischen Prüfung und Kalibrierung von Werkzeugmaschinen basiert weitgehend auf einem sequentiellen Messen von einzelnen Achsabweichungen. Ausgehend hiervon werden im Rahmen dieser Arbeit beispielhaft an Bearbeitungszentren die Probleme dieser Art der Kalibrierung aufgezeigt. Durch die Vielzahl an benötigten Messmitteln und Messaufbauten ist ein hoher instrumenteller Aufwand und ein hoher Zeitbedarf für solche Messungen nötig. Wichtiger noch ist die gegenseitige Beeinflussung von Abweichungen. So können mehrere Abweichungen einen ähnlichen Einfluss auf ein Messresultat haben, was eine Identifikation der einzelnen Abweichungen erschwert.


Für komplexe Kinematiken, wo eine solche eindeutige Unterscheidung der einzelnen Abweichungen nicht möglich ist, wird eine Methode vorgestellt, die resultierende Unsicherheit

Als Beispiele für die hier präsentierten Methoden dienen Bearbeitungszentren. Die hier angestellten Überlegungen sind ohne weiteres auch auf andere Werkzeugmaschinenarten sowie auf Roboter übertragbar.

Als Beispiele für Kalibrierungen dienen 3- und 5-achsige seriellkinematische Maschinen sowie die Parallelkinematik Hexaglide.
In order to show a sufficient geometric performance, as stated for example in [1], every machining center has to be calibrated geometrically before it may come into operation. The geometric machine errors have to be identified. They may afterwards either be compensated mechanically or numerically in the machine control.

Furthermore, the geometric performance has to be rechecked periodically and after mechanical collisions. The quality of such calibrations is one of the decisive factors on the workpiece tolerances that can be achieved with a certain machine tool.

The geometric calibration of machine tools is a time consuming and demanding task. Many different geometric errors influence the geometric performance of machine tools. To check these errors, many different measuring approaches and devices do exist. Unfortunately the separation of individual errors is difficult due to interdependencies between errors.

In this work systematic improvements for geometric machine tool calibration are proposed in order to cover error interdependencies.

With a new measuring device presented here single machine errors can be separated easily despite their interdependencies at least for the three linear axes of conventional machining centers. Therefore improved calibration results can be achieved.

Wherever such a calibration is not sufficient, a new method for taking into account the effect of such interdependencies is presented. With it a systematic approach for planning and optimizing the measurement for geometric performance is introduced. The effect of the machine tool design (e.g. kinematic buildup, utilized components such as guideways)
on the overall geometric performance can be quantified. Special attention has been given to easy applicability of the new method.

In this introductory chapter an overview over the types of errors to be calibrated is given (chapter 1.1). The state of the art of all relevant realms of geometric machine tool metrology approaches is described in chapter 1.2. Deficiencies of the current state of the art are shown in chapter 1.3.

Basically the new methods presented here are applicable for different types of machine tools (e.g. turning centers, multi-tasking machine tools, electro discharge machine tools). The work has been done for machining centers. They always serve as examples in this work.

1.1 Geometric errors of machining centers

Before methods to identify machine errors can be discussed, it should be pointed out what different types of geometric machine tool errors do exist and how they affect each other. Here only motion errors are addressed. Motion errors include all errors that cause imperfect motion between tool and workpiece. Errors of the machine components - like e.g. a flatness error of the machine table - are not taken into account, because for most applications such errors are irrelevant.

The concept of describing geometric errors of machine tools is based on the view of a machine tool as a kinematic composition of different linear and rotary axes. The geometric errors of the different axes cause relative displacements between tool and workpiece. How the different errors add up depends on the arrangement of the axes.

1.1.1 Location errors

Location errors are defined as deviations of positions and orientation between two different axis motions. Examples are parallelism or squareness deviations between the movements of two linear axes or offsets of a rotary axis from its nominal position in the respective coordinate system. They are constant. That means that they can be described by just one parameter (although the deviation each parameter causes in the workspace may be position dependent).
1.1. Geometric errors of machining centers

1.1.1. Location errors of linear axes

As an example the location errors of a linear Z axis can be seen in figure 1.1. The three location errors of a linear Z axis are:

- **Z0Z**: Zero position
- **A0Z**: Squareness of Z to the Y axis of the coordinate system
- **B0Z**: Squareness of Z to the X axis of the coordinate system

![Figure 1.1: Location errors of a linear Z axis](image)

1.1.1.2 Location errors of rotary axes

As an example the location errors of a rotary C axis can be seen in figure 1.2. The 4 location errors of a rotary C axis are (according to [2]):
\( X0C \): X position of C

\( Y0C \): Y position of C

\( A0C \): Squareness of C to the Y axis of the coordinate system

\( B0C \): Squareness of C to the X axis of the coordinate system

Actuated rotary axis made for positioning have an additional location error:

\( C0C \): Zero angular position

![Figure 1.2: Location errors of a rotary C axis according to [2]](image)

1.1.2 Number of axis location errors

The total number of axis location errors equals the sum of location errors over all axes minus the number of parameters necessary to define the one or more coordinate systems of the machine tool. This is not a simplification of the model but the consequence of the fact
1.1. Geometric errors of machining centers

that the machine coordinate systems can be chosen arbitrarily. A systematic approach how many location errors do exist for a certain kinematic configuration is given in [5].

Examples on the number of location errors for different kinematics can be found in the calibration examples (chapters 3.5.7, 3.5.8).

1.1.3 Component errors

Component errors are motion errors of the components themselves. Examples are axial and radial joint motions, straightness deviations of axes or positioning deviations of actuators. Their magnitude is usually not a simple function of position. E.g. a straightness deviation of a nominally perfectly straight axis motion is a complex course over the length of the motion. Mathematically every geometric component error $CE$ can be described by a Fourier series:

$$CE(\omega) = \sum_{j=1}^{\infty} (A_j \cos(j\omega) + \phi_j)$$ (1.1)

The number of parameters necessary to describe exactly one component error is infinite. For every harmonic two parameters have to be known (amplitude and phase angle). Furthermore non-geometric factors like friction have an influence on component errors. So they are e.g. also depending on directions of motion.

Examples on the number of component errors for different kinematics can again be found in the calibration examples (chapters 3.5.7, 3.5.8).

1.1.3.1 Component errors of linear axes

Linear axes can have 6 component errors in general, one for each possible degree of freedom in space.

As an example the component errors of a linear Z axis can be seen in figure 1.3. The 6 component errors of a linear Z axis are (according to [6]):

---

Example: Consider a conventional serial kinematic machine tool with three nominally orthogonal axes. Any arbitrary point with its actuator positions can be defined to be the origin of the machine coordinate system (so three zero positions of the axes are set to zero). The mean movement of the longest axis on the machine bed is defining an axis of the coordinate system (so it has no squareness errors - two parameters are set to zero). The second coordinate axis must be square to the first and lie in a plane created by the mean movements of the first machine axis and the second one. That means that the second machine axis can only have one squareness error. Examples for defining the coordinate systems of parallel kinematic machine tool is given in [3],[4].
Chapter 1. Introduction and State of the Art

$EXZ$: Straightness of $Z$ in $X$ direction (or horizontal straightness)

$EYZ$: Straightness of $Z$ in $Y$ direction (or vertical straightness)

$EZZ$: Positioning of $Z$ (only for actuated axes)

$EAZ$: Tilt motion of $Z$ around $X$ (Pitch)

$EBZ$: Tilt motion of $Z$ around $Y$ (Yaw)

$ECZ$: Roll of $Z$

Figure 1.3: Component errors of a linear $Z$ axis according to [6]

### 1.1.3.2 Component errors of rotary axes

As well as linear axes, rotary axes can also have 6 component errors in general, one for each possible degree of freedom in space.

As an example the component errors of a rotary $C$ axis can be seen in figure 1.4. The 6 component errors of a rotary $C$ axis are (according to [2]):

$EXC$: Radial motion of $C$ in $X$ direction

$EYC$: Radial motion of $C$ in $Y$ direction

$EZC$: Axial motion of $C$

$EAC$: Tilt motion of $C$ around $X$

$EBC$: Tilt motion of $C$ around $Y$
1.1. Geometric errors of machining centers

**ECC**: Angular positioning error of C (only for actuated axes)

![Component errors of a rotary C axis according to [2]](image)

Figure 1.4: Component errors of a rotary C axis according to [2]

### 1.1.4 Interdependencies of geometric errors

With the geometric errors defined above, the geometric performance of a machine tool can be fully described. A practical problem are the interdependencies of geometric errors on each other. This is depending on the kinematic buildup of the machine tool.

As an example a 3-axes machining center is considered. The X axis carries the machine table with the workpiece. The Y axis is attached to a portal. This axis carries the Z axis with the spindle and the tool. One example for error interdependencies is the yaw movement of the linear X axis (see chapter 1.1.3.1). The effect of a tilt motion error \( ECX \) (yaw; in this example linearly increasing in magnitude with the axis position of the X axis) is shown in figure 1.5. Depending on the horizontal offset in Y direction from the axis the effective positioning error will change. Furthermore the effective squareness measured between the X and Y axis is depending on the positions of the lines measured. When e.g. the squareness error C0Y is measured, the separation of the real location error squareness from the component error yaw is difficult.
Figure 1.5: Distorted workspace due to a linear yaw error $EC_X$ of the X axis ($5 \mu m/m$ on axis movement of $500 mm$, magnification $20000x$); scale at $Y=0$

Another example is a tilt motion error $EB_X$ (pitch) of the X axis. It changes the positioning depending on the vertical offset. Here the measured squareness between the X and the Z axis will change depending on the selected lines. An example again for an error linearly depending on the X axis position is shown in figure 1.6.

A lot of similar effects like these exist. For component errors changing nonlinearly with the axis position the interdependencies become very complex. It is critical therefore to define a reference value for each geometric error on a specific measuring line. E.g. measuring lines in the middle of the workspace or on the machine table could be chosen as reference.

Good descriptions about such interdependencies for serial kinematic machining centers with three linear axes can be found in [7], [8].
1.2 State of the art in geometric machine tool calibration

Before going into details of the calibration concepts presented in this work, the state of the art in geometric calibration of machining centers has to be described as a benchmark for the new approaches.

1.2.1 Geometric measurements

Geometric measurements are today the state of the art and the most important measurements for calibrating machining centers.

Here only one machine axis is moving at a time during the measurement. Usually only one error of the machining center is measured (one degree of freedom). The machine errors are measured consecutively (e.g. first positioning of one axis, thereafter horizontal...
straightness, thereafter vertical straightness, etc.). Some geometric measurement devices allow the simultaneous measurement of two or more geometric errors when moving one machine axis (e.g. simultaneous measurement of pitch and roll with an autocollimator, see [6]).

Today doing a multitude of geometric measurements is still the most established approach for calibrating machine tools. The measuring devices are well known. Generally the measurement uncertainties are sufficiently small for the task.

Doing a sequence of measurements is of course very time consuming. Furthermore the measurement results of any geometric measurement is only valid on the measuring line used (see chapter 1.1.4). The different geometric errors do affect each other, depending on the actual measuring line. So drawing the right conclusions from a sequence of measurements and combining them in the right way is a demanding task for specialists.

A multitude of geometric measurements does exist. Well established examples of such measurements for linear axes are:

- Measurement of positioning deviations (e.g. $E_{ZZ}$, see figure 1.3) with a laser interferometer (see figure 1.7) or a comparator scale. Such measurements are described in the standards ISO 230-1 [6] and ISO 230-2 [9].

Figure 1.7: Positioning measurement with a laser interferometer according to [10]

- Measurement of straightness deviations (e.g. $EXZ$ or $EYZ$, see figure 1.3) with straightedges (see figure 1.8), taut wire (for horizontal straightnesses), local square-
ness measurements (for some machine kinematics only, see [11], [12], [13] and [14]). An overview over these methods can be found in ISO 230-1 ([6]) and [14].

- Measurement of angular deviations tilt (pitch and yaw for horizontal axes) and roll (e.g. $EAZ$, $EBZ$ and $ECZ$, see figure 1.3) with precision levels (for roll and pitch of horizontal axes), autocollimators (for pitch and yaw) and laser interferometers (for pitch and yaw; see figure 1.9, [6], [1]).

- Measurement of the location errors of linear axes (squareness and parallelism errors) with straightedge and square (see figure 1.10, [6]).

Figure 1.8: Straightness measurement $EYX$ with a straightedge according to [1]
For rotary axes, geometric tests where only one axis is moving at a time cannot be implemented for every axis error - especially for swivel axes where the axis of motion is in or below the workpiece table (e.g. the A axis in the example in chapter 3.5.7). There no measuring artefact can be aligned with the axis. Therefore geometric tests can be performed only during the machine setup. Otherwise only multi-axes tests can be used (see chapters 1.2.2 and 1.2.3).

The following geometric measurements are established for rotary axes (see also [2],[15], [16]):
1.2. State of the art in geometric machine tool calibration

- Measurement of positioning errors (e.g. $ECC$) with a laser interferometer and a rotary indexer (master rotary table) or with an optical polygon and an autocollimator (e.g. [6], [17]).

- Measurement of radial motion errors (e.g. $EXC$, $EYC$), axial motion errors (e.g. $EZC$) and tilt motions (e.g. $EAC$, $EBC$) of a rotary axis. Conventionally the measurements use a test mandrel and only one probe in X or Y direction (for $EXC$, $EYC$). The measurements are repeated with different Z offsets to see any influence of $EAC$ and $EBC$ from the difference between the first and the second set of measurements (see [6]). Another measurement is necessary for $EZC$. Today up to 5 sensors are used with two precision spheres to measure the 5 component errors simultaneously (see [2], [15]) focusing only on the axis movements, not the run-out of the test mandrel.

Most errors of rotary axes can in practice only be measured while moving synchronously with other axes. Therefore these so-called kinematic measurements are more important for rotary axes (see [16]).

1.2.2 Kinematic measurements

Kinematic measurements are defined as tests where at least two axes move simultaneously. Such tests are usually closer to the actual axis movements during machining than geometric measurements (see chapter 1.2.1). Since several axes are used at the same time, such tests are suitable as quick interim checks (e.g. for periodic testing, testing after collisions).

A lot of machine errors affect the result of such measurements. To distinguish between different possible error sources is not easy, in some cases even impossible (see e.g. [12], [8], [18], [19], [20]).

Established kinematic measurements for machining centers are:

- The circular test with a circular master and a two dimensional probe [21], [8], with a double ball bar [22], [23] (extension with rotary encoder in [24]) or with a cross-grid encoder. Two or three linear axes are moved synchronously on a nominally circular path. Geometric machine tool errors cause typical distortions of the circles measured. This way error sources can be identified. These error patterns where examined in literature, e.g. in [12], [8], [18], [23].

- The laser diagonal test for two or three linear axes (measurement of positioning on a face or body diagonal; see [13], [19], [20]).
• Measurement of a synchronous motion of two linear and a rotary axis: a probe or a reference item is moved by a rotary axis. Two linear axes follow so that nominally there is no relative motion between the probe/reference item to its counterpart. Axis deviations cause relative displacements between the probe and the reference that can be measured. Assuming that parts of the relative deviations measured originate from either one linear axis or from the rotary axis, parameters like the positions and orientation errors of the rotary axis or the squareness deviations between the linear axes can be determined.

Several methods do exist for such measurements:

– Measurement with a double ball bar between rotary and linear axes (see [25],[26],[27],[28],[6]). Normally this measurement has to be done with three setups to measure relative displacements in radial, tangential and axial direction of the rotary axis.

– Measurement with a 2D probe in tool position relative to a reference block moving with the rotary axis (see [16],[27]). To measure relative displacements in radial and tangential direction the measurement has to be done with two setups.

– More recently an approach where three probes are mounted together in one holder are measuring the relative displacements of a precision sphere in tool position simultaneously in X, Y and Z direction (“Radial test” according to [6]; see [29], [30], [31],[32]) has been developed. With this system only one measurement is necessary in order to measure radial, tangential and axial relative displacements (with respect to the movement of the radial axis).

To derive exact values for location errors (e.g. squareness errors between linear axes) or even the exact courses of component errors (e.g. of straightness errors) is a difficult task (see chapter 1.1.4). Approaches to derive such information e.g. from laser diagonal tests (see [33], [34]) have recently been restrained (see [19], [35], [20]). For circular tests it was mentioned already in [8] that some errors cannot be separated from others.

1.2.3 Model-based calibration

With a model-based calibration approach, a multitude of machine errors is supposed to be identified numerically by doing only a small number of measurement setups.

To do this, a multi axes measurement is made. A set of parameters \( \mathbf{p} \), a subset of all geometric machine tool errors \( \mathbf{E} \), is selected for evaluation. The relative displacements of
the Tool Center Point (TCP) at certain measuring poses are emulated with a kinematic simulation model of the machine tool. This is done by changing the parameters $p$ iteratively until the relative TCP displacements that these parameters would cause at the measuring poses are closest to the real relative displacements measured. This is normally done with a least squares algorithm:

$$\text{Min} \sum_{j=1}^{m} (\Delta X_{\text{measured}} - \Delta X_{\text{calculated}}(p))^2$$

(1.2)

The identified set of parameters $p$ is assumed to be the exact kinematic configuration of the machine.

This procedure has been implemented for 3-axes machining centers [36], [37] and 5-axes machining centers [38], [39], [40], [41]. The basic procedure is better known in robotics, where much effort has been taken to evaluate the mathematical condition of the numerical optimization problem (see [42], [43], [44], [45], [46]) and for parallel kinematic machines (e.g. [47], [48], [49], [50], [51], [3], [52], [53], [54]).

Model-based calibration approaches do have the advantage that a calibration takes only very little time. This would allow frequent recalibrations of machining centers and would save time for machine manufacturers during machine production and for the end user for inspection. Due to the short measuring time required, model-based calibration approaches may be an interesting option to identify the changes in the geometric configuration due to thermal drift, thus allowing very frequent re-calibrations of machine tools, resulting in a better geometric performance. Model-based calibration approaches show even the potential that the calibration can be performed automatically in the future.

Furthermore for some kinematics - like parallel kinematic machines (PKM) - conventional direct (geometric) calibration cannot be used. The relative TCP displacements may be measured on a certain measurement path. But what combination of machine tool errors causes these deviations remains unclear. So the errors cannot be compensated for. If e.g. a straightness deviation on a line in the workspace is measured, a combination of many different errors cause this deviation (e.g. strut length errors, zero position errors of the joints, straightness errors of guideways). These different geometric errors add up differently at every point of the workspace. So a direct identification of one or more geometric error is not possible.

Despite the potential of model-based calibration approaches, an important question concerning such calibration methods has not been answered conclusively so far:

Can a certain geometric performance be achieved when using a model-based calibration with a certain measurement device and approach? The geometric behavior can only be
roughly estimated with an heuristic approach based on mathematical condition issues today [44]. An uncertainty estimation for the parameters identified cannot be done so far. In order to achieve a geometric performance that is expected of a machining center (unlike e.g. robots) this is an insufficient approach. The actual realization of the machine tool to be calibrated (e.g. component errors, repeatability, thermal drift) and of the measurement (measurement uncertainty) have so far not realistically been taken into account. This makes the planning of a calibration task difficult - for example to determine what machine errors should be identified, how many measuring poses are needed or how measurement uncertainty and machine errors that are not included in the set of parameters $p$ do affect the result.

A subset of all machine errors is supposed to be identified. All other errors do disturb the identification of the parameters $p$. They are systematic and repeatable; therefore they cannot be approximated as a white noise of uncertainty over all measuring points.

### 1.3 Deficiencies of the state of the art

All the measurements described before measure the TCP displacements in some degrees of freedom on a certain path (e.g. straight line, circle). Measuring devices and procedures with small measurement uncertainties are available (see e.g. [6]). With the information gained during one measurement, one geometric error (see chapter 1.2.1) or several geometric errors (see chapters 1.2.2, 1.2.3) are identified. Unfortunately other errors, that are normally not supposed to be identified, disturb this identification by changing the measured TCP displacements systematically. From the measurement result a falsified result is deduced.

While it is quite simple to estimate the measurement uncertainty of the device used according to well known procedures (see [55]), the uncertainties of identified geometric errors cannot be determined easily. These uncertainties in the end determine the geometric performance of a machine tool.

While the measurement uncertainty is primarily determined by the measuring principle used and by the environmental conditions during the test, the uncertainties in the identified parameters are also determined by the performance of the machine tool itself.

Without knowing how some geometric errors disturb the identification of others, it is hard to predict

- the geometric tolerances that are achievable with a certain machine tool,
1.3. Deficiencies of the state of the art

- the effect of a change in the configuration of a machine tool (e.g. new guideway systems, different kinematic setup),
- the best way to increase the geometric performance of a machine tool,
- the effect of the measurement uncertainty on a geometric error identified in comparison to the effect of the disturbances due to other geometric errors.

So the combination of the results of the single measurements for machine adjusting and compensation requires expert knowledge and a lot of experience. Still due to the interdependencies of the geometric errors, the overall uncertainty of such calibrations often remains unclear.

Here calibration approaches where the interdependencies of the geometric errors are taken into account are shown. Two basic concepts are shown:

- A concept for identifying geometric errors by performing fast spatial grid measurements. Geometric errors are measured on several lines. Thus interdependencies become apparent. The concept proposed here furthermore allows volumetric compensation of displacements in X, Y and Z direction throughout the machine tool’s three-dimensional workspace (“Space Error Compensation”, see [56]). The design issues of the measuring instrument and the overall measurement uncertainty are given special attention here.

- A concept for estimating the quality of an identification of certain geometric errors. This method is used if there is no measuring method available to resolve different error interdependencies. With this concept the effect of other geometric errors on the uncertainty of the identified geometric errors can be determined. While it cannot be avoided that error interdependencies will still have an influence on the calibration result, this way a calibration method can be planned. The result quality that can be expected is determined.

A kinematic measurement as well as a model-based calibration approach of machine tools with at least 5 degrees of freedom is shown here as examples. With a new Monte-Carlo approach the parameter uncertainties for machine errors identified with any kind of metrology can be estimated.

The potential of both methods is the improvement of the prediction of the anticipated geometric performance, the reduction of the calibration effort and the achievement of better calibration results. In both cases the aim of the calibration is to reduce parameter uncertainty, calibration time and cost when compared to the current state of the art.
The dramatic reduction of the measuring time required to geometrically calibrate a machining center has the following advantages:

- With shorter measuring times, the effect of thermal drift (e.g. due to changes in the ambient temperature, due to heat generated by moving axes) during measurement is minimized. This way it does impair the calibration result as little as possible.

- The cost of geometric calibration for both the machine tool manufacturer after assembly and for the end user for periodic inspections is reduced.

- The reduced time demands allow shorter delivery times for the machine tool manufacturer. A faster periodic inspection and recalibration increases the overall availability of the machine tool for the end user.

- Frequent recalibrations of machine tools are made possible. E.g. before the production of a part with tight tolerances, a calibration could be made. This way the effect of a change in the environmental conditions on the kinematic configuration of the machine tool could be identified and be compensated for.
Chapter 2

Error Compensation by Spatial Grid Measurements

As seen in chapter 1.3 the current state of the art in geometric machine tool calibration shows some deficiencies, mainly due to error interdependencies effects.

The geometric calibration would be much easier if a measurement could be made where the single geometric errors could be separated. With the current state of the art, this is not possible. Different measurement setups are required for most errors to be identified.

Here a new approach is presented: A measuring device that allows measurements on a spatial grid throughout the workspace. The effect of the different geometric errors may change (for component errors) on these different lines. Thus a separation of the different errors becomes possible for machine tools with a serial kinematic. Furthermore the number of measuring setups and the measuring time needed in order to calibrate a machine tool can be reduced dramatically with the new measuring device. The method described here does only work for translatory three-dimensional workspaces (no rotary degrees of freedom). For rotary axes a method as shown in chapter 3 has to be used.

The measuring device complies with the following conditions:

- The device allows to measure deviations in the three translatory degrees of freedom.
- The measuring positions are distributed regularly throughout the three-dimensional machine workspace, allowing volumetric measurement and compensation of geometric errors.
- The 3D measurement uncertainty is smaller than 5 \( \mu m \) in every spatial direction under normal shop floor conditions.
With such a device a volumetric error compensation (Space Error Compensation) is made possible. That means that the resulting Tool Center Point (TCP) deviations - caused by a combination of geometric machine tool errors, can be measured and compensated throughout the machine tool’s workspace. Instead of measuring the single geometric errors of a machine tool, the resulting TCP displacements are measured and compensated. Even if the interdependencies between the geometric errors are too complex to be resolved, still a good geometric performance can be achieved.

The laser tracker is the only device commonly used that allows the simultaneous measurement of three degrees of freedom in a three-dimensional workspace. On the downside the obtainable 3D measurement uncertainty appears to be rather large under shop floor conditions (see [57]).

The new artefact for a calibrations via precision sphere probing - presented in chapter 2.1 - allows spatial measurements with a total expanded measurement uncertainty $U(k=2)$ below 5 $\mu$m in a large three-dimensional working volume (500 x 500 x 320 mm$^3$ for the prototype). A three-dimensional grid of measuring points is created by kinematically correct relocation of a ball plate.

A suitable probing system is presented in chapter 2.2 that allows simultaneous measurements of deviations in X, Y and Z.

In chapter 2.3 calibrations done with the new measuring instrument are characterized. An overall uncertainty estimation is made.

Finally in chapter 2.4 examples where a conventional and a parallel kinematic machining center have been calibrated are shown.

## 2.1 The “3D Ball Plate” artefact

In this chapter the “3D Ball Plate” artefact is described. With it a regular spatial grid of measuring points can be created. Displacements can be measured in X, Y and Z direction at each of these points. Thus Space Error Compensation is made possible. From the TCP displacements measured the geometric errors can also be identified.

The work principle, the necessary calibration routine and design details are explained in this chapter.
2.1. The “3D Ball Plate” artefact

2.1.1 Work principle and benefits of the artefact

The “3D Ball Plate” Artefact is based on a standard 2D ball plate (see figure 2.1). A ball plate [58], [59], [60], [61] is a well-established artefact used primarily to calibrate CMMs. Usually ceramic precision spheres are attached to a steel plate in their neutral plane. After calibration the positions of each sphere center are known with a very small uncertainty. With a ball plate deviations in two or sometimes three degrees of freedom are measured in a plane.

The idea is to create a pseudo 3D artefact. The 2D ball plate should be repositioned in different known locations. With the spatial grid created this way, Space Error Compensation becomes possible. This is especially useful for machines where a calibration based on geometric measurements (see chapter 1.2.1) is difficult, like for robots with three translatory degrees of freedom and parallel kinematic machines (like tripods or triglides).

To create a suitable 3D artefact it is necessary that the exact translatory and rotatory shifts between the locations of the ball plate are known. A high repeatability is reached by using kinematic couplings, well known design elements described e.g. in [62] and [63]. Details about kinematic couplings can be found in chapter 2.1.3.2.

Depending on the type of the machine - especially the orientation of the Z axis - the ball plate should be oriented horizontally (see figure 2.2) or vertically (see figure 2.3). Here the application for vertical Z axis is examined in detail.

With a vertically oriented Z axis the ball plate should be oriented horizontally. The ball plate can rest on one kinematic coupling either directly on a base plate or on a spacer. With a set of spacers of different heights the ball plate can be vertically relocated in a defined way. The spacers are again kinematically coupled to the base plate to ensure repeatability. By using spacers of different heights the 2D grid of the ball plate is expanded into the third dimension, thus creating a 3D grid.

For machining centers with horizontally oriented Z axes the ball plate should be oriented vertically. Figure 2.3 shows a possible setup. The ball plate can be repositioned by placing it again kinematically correct into different couplings on a base plate fixed on the machine table. For this application the ball plate has to be recalibrated since the bending due to gravity, that changes the position of the spheres, is much smaller in this configuration.

With these setups a quasi- three-dimensional artefact is created. The measuring points form a regular grid with known relative position between all points (after calibration, see chapter 2.1.2). The measurements to determine the different deviations of the axes are redundant for most kinematics (e.g. positioning or squareness errors for serial kinematics can be determined on different measuring lines). With the redundant measurement
Chapter 2. Spatial Grid Measurements

In [58], [59], [60] and [61], methods using a ball plate as a pure 2D artefact in different locations are described. The ball plate is usually used oriented nominally parallel to the XY plane in two locations with different Z heights, nominally parallel to the XZ plane in two locations with different Y positions and nominally parallel to the YZ plane in two locations with different X positions. Such methods are basically sufficient to identify different geometric errors of Coordinate Measuring Machines (CMMs) assuming rigid body behavior.

This is possible for CMMs because of the possibility of having its probe oriented in basically arbitrary directions. An alignment of the probing system to the X or Y direction on machine tools would only be possible with large offsets in these directions (due to the dimensions of the machine headstock). Likewise an orientation of the ball plate parallel to three different coordinate planes is hard to imagine due to workspace restrictions of the machine.

Furthermore machines might diverge from rigid body behavior (e.g. bending due to a shift of the center of gravity). This can be checked for machining centers and CMMs with this measuring device when used as proposed here.
2.1. The “3D Ball Plate” artefact

Figure 2.2: Setup with horizontal ball plate

Figure 2.3: Setup with vertical ball plate
2.1.2 Calibration of the artefact

The calibration has to include two steps. First of all the X, Y and Z positions of the spheres on the ball plate have to be determined. This can be done according to a standard measuring procedure (chapter 2.1.2.1).

In a second step the relative location of the ball plate when it is resting on one of the spacers has to be determined with respect to its location when it is resting directly on the base plate. This procedure is described in chapter 2.1.2.2.

2.1.2.1 Calibration of the sphere positions on the ball plate

The ball plate itself can be calibrated on a Coordinate Measuring Machine (CMM). The basic procedure is outlined in the guideline from the German calibration service (DKD) [64]. The guideline recommends the ball plate to be measured at the same location in 4 positions (reversal method as proposed e.g. in [65]). After the first measurement the plate is flipped upside down for the second measurement. Thereafter the plate is rotated by 180° around the vertical Z axis for the third measurement, afterwards again flipped upside down for the last measurement. This enables to separate the X and Y ball plate coordinates from the CMM errors. For the Z coordinates (height positions) the vertical bending of the plate is identified.

Since the ball plate is not flipped over during the use in the application presented here, this standard measurement can be changed. The exact bending due to gravity does not have to be identified since the ball plate is always used in horizontal position (when used for machining centers with a vertical spindle orientation). The ball plate is still measured in 4 different poses. But instead of flipping it over it is simply turned by 90° around the vertical Z axis from measurement to measurement. If the setup for machining centers with vertical spindles should be used, it has to be calibrated separately.

With this approach the most important geometric errors of the CMM can be compensated. By measuring the ball plate in different orientations the effects of geometric errors of the CMM are affecting the measuring result in opposing directions. Thus by taking the average values of sphere positions from the 4 measurements the effects the most important geometric CMM errors (such as squareness and straightness errors) are compensated.

If for example the CMM has a squareness error of the Y axis with respect to the X axis, the positions of the ball plate spheres would be identified with respect to their actual positions as shown in figure 2.4 (red: coordinate system of the ball plate). If the measurement is repeated when the ball plate has been turned by 90°, the effect of the squareness error of
the CMM causes errors of the sphere positions in the opposite direction with respect to the coordinate system of the ball plate (see figure 2.5). Thus by taking the average value of both measurements, the effect of the squareness error is evened out. Therefore by using such reversal measurements, the measurement uncertainty of the ball plate calibration can be reduced and the measuring device (the CMM) can be checked at the same time. Details can be found in [64], [65], [66], [67].

Figure 2.4: Measurement deviations of sphere positions due to a squareness error of the CMM when the orientation of the ball plate is nominally aligned to the CMM coordinate system (solid: measurement result; transparent: real values)
2.1.2.2 Calibration of the relative locations of the ball plate on the spacers

Here the relative locations when the ball plate is resting on a spacer with respect to its location when resting directly on the base plate have to be determined. These measurements can again be done on a CMM.

The relative vectors of the 4 corner spheres of the plate when resting on a spacer (in comparison to when resting directly on the base plate) are determined. By measuring the relative position change of 4 spheres the translatory and rotary shift of the plate is measured redundantly, thus providing a crosscheck of the measurement result.

The measurement is repeated three times for each spacer. Again, the whole artefact is turned around by 90° (three times 90°) around the Z axis from measurement to measurement (base plate, spacer and ball plate). This way, the straightness and squareness errors of the Z axis of the CMM can be compensated similar to the reversal measurement of the sphere positions shown above.

From the average vectors for each corner sphere, the relative translation and rotation between the ball plate pose on one spacer with respect to the pose on the base plate can be determined with a least-squares algorithm [68].

Calibration measurements have been performed by the Swiss Federal Office of Metrology.
2.1. The “3D Ball Plate” artefact

([68]) as well as at the Institute for Machine Tools and Manufacturing of the ETH Zurich [66]). The maximum difference was 1.2\( \mu m \). The measurements of the Swiss Federal Office of Metrology and Accreditation are taken as reference due to their better environmental conditions.

2.1.3 Design details of the artefact

Although the basic build-up has been defined, some design details have to be kept in mind in order to be able to measure with a sufficiently small measurement uncertainty.

In chapter 2.1.3.1 the influence of the selected material on the measurement uncertainty will be described. In chapter 2.1.3.2 kinematic couplings for this application are focused on. Finally in chapter 2.1.3.3 the necessity for high stiffnesses of the spacers and the base plate is shown.

2.1.3.1 Material selection

When the uncertainty estimation for the artefact is considered (see chapter 2.1.4), it becomes clear that the most important contributors to uncertainty are the ones originating from imperfect environmental conditions. The thermal expansion behavior is very important.

Machining centers are usually built to have a thermal behavior close to the most important material machined on them, usually steel (linear thermal expansion coefficient \( \alpha \approx 11.6 \mu m/mK \)). This way the thermal expansion behavior of machine tool and workpiece is as similar as possible when the temperature differs from 20\( ^\circ C \) (reference temperature according to [69]).

The goal of the design of the artefact is to have a similar thermal expansion behavior like the machine tool under test. This is why the artefact itself is made out of steel (ball plate, spacers and base plate). The geometric errors due to temperature offsets from 20\( ^\circ C \) are kept to a minimum.

The steel used for ball plate and spacers is annealed to relieve internal stresses. This way the geometry of the artefact is stable.

The spheres on the ball plate and for the kinematic couplings are made of aluminum oxide, which is very resilient and corrosion insensitive.

The prisms for the kinematic coupling are made of tempered spring steel. The hardness of the material is required to avoid plastic deformations due to the high Hertzian stresses.
in the point contact zones of the kinematic couplings.

2.1.3.2 Kinematic couplings

Kinematic couplings hold a general body without symmetries in 6 points. This way the 6 degrees of freedom are constrained. For example three spheres attached to a body are in contact with two planes (a prism) each (see figure 2.6). This way the coupling is free of play, the body is not deformed. This ensures a very good repeatability, which is needed for the positioning of the ball plate on the spacers respectively on the base plate.

According to [63], kinematic couplings as used for the artefact here have been used at least since the end of the 19th century (see [70]). But until today, extensive research work is done in the field of kinematic couplings (see e.g. [71], [72], [73], [62], [74]).

First of all, for kinematic couplings a Hertzian analysis must be made to ensure that the stresses in the point contact zones do not become too high (in order to avoid plastic deformation).
For the 3D Ball Plate application, the repeatability must be as good as possible. Because of the symmetric arrangement of the spheres of the bottom side of ball plate and spacers, a "three-vee coupling" has been selected. Three spheres rest in three prisms. Each of the 6 contact points constrains one degree of freedom.

In [73] a simple estimation for the magnitude of repeatability in the coupling is given. The only relevant forces that are not repeatable for the application here are the static friction forces. Their magnitudes and directions depend on how the three spheres of the coupling slid into the three vees. The static friction force of a single Hertzian contact \( f \) is divided by the stiffness of the coupling \( k \). The friction force is depending on the coefficient of friction \( \mu \). It is assumed that the "compliance of the coupling in all directions is equal to a single Hertzian contact carrying a load \( P \) and having a relative radius \( R \) and elastic modulus \( E \)" (see [73]):

\[
\frac{f}{k} \approx \mu \left( \frac{2}{3R} \right)^{1/3} \left( \frac{P}{E} \right)^{2/3}
\]

This estimation is just for the repeatability due to the compliance of the kinematic coupling itself, not due to the compliance of the substructure. This will be addressed in chapter 2.1.3.3.

The most effective way to enhance the repeatability is to limit the coefficient of friction \( \mu \). To do this, the prisms are lubricated here.

When a coefficient of friction of \( \mu = 0.1 \) is assumed, the estimated value for the repeatability of the ball plate on the spacer is calculated to be 0.14\( \mu m \) (overall load of 500\( N \) on 6 planes, 45\( ^\circ \) inclination angle of the prism planes, mixed elastic modulus \( E = 262.5\)\( GPa \), sphere radius \( R = 12\)\( mm \)).

The relative orientations between the 6 contact planes have been arranged according to the suggestions made in [75], [63]. The vectors from one of the three spheres to the other two spheres are considered. According to these papers, the axis created by the intersection of the two planes of one vee should bisect the angle between the two vectors (vectors from the sphere center belonging to the current vee to the other two sphere centers).

2.1.3.3 Stiffness of spacer and base plate

As mentioned in chapter 2.1.3.2, the repeatability of a kinematic coupling is limited by static friction forces which elastically deform the coupling.

This static friction forces do not only deform the coupling itself, but also the spacer respectively the base plate. For a first prototype that was built (star shaped base plate and spacer element; see figure 2.7), the torsional stiffness was limited. The static friction
forces caused a moment of rotation.

The first version of the base plate had a much too low stiffness (measured standard uncertainty due to repeatability about $8\mu m$ in X and Y direction). It was replaced immediately by a base plate with much stronger elements (profiles were changed from $40 \times 12 mm^2$ to $40 \times 40 mm^2$). For the artefact with the improved base plate, a standard uncertainty ($k = 1$) due to repeatability of $1.2\mu m$ was measured with the ball plate on the spacer (see [68]). This complies very well with FEM simulations, where the static deformation due to static friction forces was estimated.

![Figure 2.7: First prototype spacer on first prototype base plate](image)

To improve the repeatability a new design has been conceived. Both base plate and spacers must show a high stiffness, especially against torsion. Therefore the elements were designed as steel rings. FEM simulations were made to estimate the static deformation due to static friction forces. For the base plate and the spacer with a small ring height, these forces cause warping of the structure (see figure 2.8). For higher spacer elements, the stiffness increases (see figure 2.9). For this reason, for the base plate and the spacer with the lowest height wall thicknesses of $40 mm$ has been selected, for the other spacers the wall thickness is only $14 mm$. The repeatability of the location of the ball plate on the spacers has been measured to be below $0.5\mu m$ (standard uncertainty). This is in accordance with FEM simulation.
2.1. The “3D Ball Plate” artefact

Figure 2.8: FEM simulation of the qualitative effect of static friction forces on a spacer with $\Delta Z = 80\, \text{mm}$ (same magnification as figure 2.9; colors indicate the radial displacements)

Figure 2.9: FEM simulation of the qualitative effect of static friction forces on a spacer with $\Delta Z = 160\, \text{mm}$ (same magnification as figure 2.8; colors indicate the radial displacements)

2.1.4 Uncertainty estimation for the artefact

For the 3D Ball Plate there are some contributors to the uncertainty of the position of the sphere centers in the artefact coordinate system. Besides the uncertainty contributors coming from the calibration of the artefact (see chapter 2.1.2) and the repeatability of the positioning of the ball plate on a spacer respectively the base plate (see chapter 2.1.3), the main contributors are dependent on the environmental conditions during calibration. Since they have a great influence on the total measurement uncertainty, the uncertainty estimation for the artefact is made for two scenarios. The first one is valid for shop floor conditions (chapter 2.1.4.1), the second one for metrology lab conditions (chapter 2.1.4.2). The values stated for the uncertainty of the artefact calibration are valid for the measurements made at the Swiss Federal Office of Metrology and Accreditation ([68]). Values for a CMM with a higher Maximum Permissible Error (according to [76]) and under a worse temperature control can be found in [66].
Together with the uncertainty of the probing the total measurement uncertainty can be determined (see chapter 2.3). The uncertainty estimation has been made according to the method described in the GUM [55].

Other uncertainty contributions are the repeatability and drift of the machine under test. Since these are not influenced by the measuring device but depend only on the machining center under test, these effects have deliberately been omitted.

The values are valid for the artefact design as described in chapter 2.1.3 for the new ring type design. The uncertainties stated are position uncertainties valid for a reference length of 500$\text{mm}$ in X and Y direction (maximum distance of sphere centers in X and Y directions) respectively for a reference length of 320$\text{mm}$ in Z direction (nominal height difference when the highest spacer is used).

### 2.1.4.1 Shop floor conditions

The following contributions to uncertainty are relevant when the artefact is used under shop floor conditions (see table 2.1). The calibration of the artefact has always to be done under metrology laboratory conditions.

- The contribution originating from imperfect environmental conditions have been determined according to [77]. These contributions are the most important ones under shop floor conditions. There is a contribution coming from a difference in temperature between the machine and the artefact. It is assumed that this temperature difference lies within a rectangular distribution with a range of 0.5$\degree\text{C}$. The second distribution comes from a difference in the thermal expansion coefficient $\alpha$ between machine and artefact when measuring with a temperature that differs from 20$\degree\text{C}$ (here $\pm2\degree\text{C}$). Although the artefact is designed to show a thermal expansion behavior similar to a typical machine tool (see chapter 2.1.3.1), this difference in $\alpha$ is still assumed to lie rectangularly distributed within a range of 4$\mu\text{m}/\text{mK}$.

- The residual CMM standard uncertainty of the sphere center coordinates as stated in [68] consists of the repeatability of the CMM, the residual errors of the CMM after the reversal measurement and after compensation measurements for positioning errors (see chapter 2.1.2). The value stated in table 2.1 is a specification given by the Swiss Federal Office of Metrology. An extensive study about residual errors after a reversal measurement can be found in [66].

- The standard uncertainty due to the repeatability of the ball plate on a spacer respectively on the base plate. This is mainly due to the design of the artefact (see
chapter 2.1.3).

With the assumptions noted above, the estimated combined standard uncertainty for the artefact as stated in table 2.1 is $1.5\mu m$ (with independent contributors, calculated according to [55]). Take note that the contributors originating from imperfect environmental conditions are dominant, although the artefact was designed so that the influence of these contributors is as small as possible (see chapter 2.1.3.1).

Table 2.1: Artefact contributors to measurement uncertainty under shop floor conditions (rounded).

<table>
<thead>
<tr>
<th>Contributor</th>
<th>Standard uncertainty in X, Y</th>
<th>Standard uncertainty in Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainty of temperature equivalence between machine and artefact: $\Delta T = \pm 0.25^\circ C$, $\alpha = 12\mu m/m^\circ C$</td>
<td>$0.9\mu m$</td>
<td>$0.6\mu m$</td>
</tr>
<tr>
<td>Difference of $\alpha$ between artefact and machine, $\Delta \alpha \leq \pm 2\mu m/m^\circ C$ at a temperature deviation of $T - 20^\circ C &lt; 2^\circ C$</td>
<td>$1.2\mu m$</td>
<td>$0.7\mu m$</td>
</tr>
<tr>
<td>Total calibration uncertainty on sphere center coordinates</td>
<td>$0.2\mu m$</td>
<td>$0.2\mu m$</td>
</tr>
<tr>
<td>Repositioning of the ball plate on the base</td>
<td>$0.1\mu m$</td>
<td>$0.1\mu m$</td>
</tr>
<tr>
<td>Repositioning of the ball plate on the spacer</td>
<td>$0.5\mu m$</td>
<td>$0.5\mu m$</td>
</tr>
<tr>
<td>Combined standard uncertainty</td>
<td>$1.5\mu m$</td>
<td>$1.1\mu m$</td>
</tr>
</tbody>
</table>

2.1.4.2 Metrology laboratory conditions

Under metrology laboratory conditions, the contributions originating from imperfect environmental conditions decrease. The other contributors like calibration uncertainty and repeatability stay unchanged (see table 2.2).

The contribution coming from a difference in temperature between the machine and the artefact is now assumed to lie rectangularly distributed within a range of $0.2^\circ C$. The uncertainty range of the difference in the thermal expansion coefficient $\alpha$ stays the same. But the deviation from $20^\circ C$ is assumed to be only $\pm 1^\circ C$. 
Table 2.2: Artefact contributors to measurement uncertainty under metrology laboratory conditions (rounded).

<table>
<thead>
<tr>
<th>Contributor</th>
<th>Standard uncertainty in X, Y</th>
<th>Standard uncertainty in Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainty of temperature equivalence between machine and artefact: $\Delta T = \pm 0.1^\circ C$, $\alpha = 12 \mu m/m^\circ C$</td>
<td>0.3$\mu m$</td>
<td>0.2$\mu m$</td>
</tr>
<tr>
<td>Difference of $\alpha$ between artefact and machine, $\Delta \alpha \leq \pm 2 \mu m/m^\circ C$ at a temperature deviation of $T - 20^\circ C &lt; 1^\circ C$</td>
<td>0.6$\mu m$</td>
<td>0.4$\mu m$</td>
</tr>
<tr>
<td>Total calibration uncertainty on sphere center coordinates</td>
<td>0.2$\mu m$</td>
<td>0.2$\mu m$</td>
</tr>
<tr>
<td>Repositioning of the ball plate on the base</td>
<td>0.1$\mu m$</td>
<td>0.1$\mu m$</td>
</tr>
<tr>
<td>Repositioning of the ball plate on the spacer</td>
<td>0.5$\mu m$</td>
<td>0.5$\mu m$</td>
</tr>
<tr>
<td><strong>Combined standard uncertainty</strong></td>
<td>0.9$\mu m$</td>
<td>0.7$\mu m$</td>
</tr>
</tbody>
</table>

With a proper improvement of the environmental conditions, it is possible to calibrate even CMMs with a 3D Ball Plate artefact.

### 2.2 The 3D probing system for machine tools

A concept similar to the R- Test described in [29] has been chosen to detect the sphere center positions. Other probing concepts like described in [32] would in principle be also useable. A 3D touch trigger probe could also be used on many machines. With such a probe at least three positions were necessary to detect the center of one sphere. This would cause increased measuring times. Furthermore the backlash of the machine tool would have another influence on the measured position of the sphere. The probing system described in the following chapters overcomes this disadvantages.

If the 3D Ball Plate is used on CMMs, the CMM probe should be used instead of the probing system presented here.
2.2.1 Work principle of the probing system

The probing system presented here consists of 4 regularly arranged one-dimensional analogous probes with a measuring range of 12\text{mm}. They are tilted nominally 45° to the machine’s Z axis as shown in figure 2.10. The probe tips are planes. The exact orientation of the different probing planes in the coordinate system of the machine tested is calibrated before each measurement on the machine (see chapter 2.2.2). Three probes would be sufficient to find the X, Y and Z position of a sphere center (with the sphere radius known from calibration). A fourth probe is used for the following reasons:

- The geometrical configuration of the probe is matching the symmetrical mounting geometry of the spheres. They are held at 4 points in the neutral plane of the ball plate. Therefore there are 4 pockets which give room for the probes.

- Measurement deviations due to local errors like form deviations of the spheres or the planes are partially compensated, so the overall measurement uncertainty should be slightly better than with the minimal configuration with three probes. This effect has not been taken into account for the uncertainty estimation in chapter 2.2.3.

- Due to the redundancy a cross check is provided during the measurement. If a contamination on a sphere or a probing plane does occur, it can be detected. The sphere center coordinates with respect to the probing system can be calculated with a least-squares algorithm. The task is to find the center position of the sphere whose surface is closest to 4 (probing) planes. Ideally, these distances between sphere and planes would be zero. If contamination does occur, the residual distances will increase (over a certain threshold). This allows a distinction between the machine’s TCP displacements that are supposed to be measured and contamination during the probing.

2.2.2 Probe calibration

A problem of probes as described in [29],[32] are the unknown orientations of planes of the single probes in the machine tool coordinate system. This results in a rather high measurement uncertainty of the probe. The unknown orientations are due to

- a difficult alignment of the probing system to the machine coordinate system (especially around the spindle axes) of the machining center under test and

- manufacturing and assembly tolerances of the probing system.
The latter effect can be calibrated by external methods as described in [32].

Here a new approach is presented to identify the orientation of all 4 probes with respect to the machine coordinate system. This results in a much lower measurement uncertainty of the probing system. From a practical point of view there is also the advantage that the probing system can be oriented arbitrarily. That means the probing system does not need to be oriented manually with respect to the machine coordinate system. For example the probing system can be turned to avoid collisions.

To perform the probe calibration, the probes are in contact with one sphere (e.g. at one corner of the ball plate). A reference position is set here. Thereafter the machine is moved to different measuring points (here ±2\,mm) from the reference position in X, Y and Z, resulting in 6 measuring points. The displacement of the probes from the reference to the measuring positions is measured. With a least squares algorithm the orientation of the probing planes is determined. Each probe plane orientation is evaluated independently. This way the orientation of the whole probing system in the machine coordinate system as well as the actual orientations between the probing planes are found. Deviations from the nominal configuration due to the probe mounting or due to manufacturing deviations

![Figure 2.10: Probing system with 4 linear probes](image)
are measured.

Of course there are deviations from ideal probe travel during the calibration cycle due to machine tool errors (e.g. positioning, squareness, straightness errors), due to measurement errors of probes, due to form deviations of probing planes (tips of the probes) and of the sphere and due to squareness deviation between probing planes and the linear encoder of the probes (cosine influence). These errors will disturb the identification of the single probe planes’ orientation, because the motion between reference and measuring points is not perfect.

This causes systematic measurement errors, depending linearly on the travel of the probes. Since the probe travel during calibration is much bigger than during measurement (relative TCP displacements of $\pm 2\, \text{mm}$ during calibration compared to typically $< \pm 0.1\, \text{mm}$ during measurement; see chapter 2.3), the effect of the disturbance is small. If an error in the calibration reference motion of e.g. $10\, \mu\text{m}$ is assumed (due to the errors stated above), it would cause an error during measurement of only $0.5\, \mu\text{m}$. This corresponds to the ratio of the probe travel required during calibration ($\pm 2\, \text{mm}$) and measurement ($< \pm 0.1\, \text{mm}$).

If the probe travel during the measurement is bigger, e.g. for the first calibration of a parallel kinematic machine (PKM), the measurement uncertainty would increase accordingly. In this case a repetition of the measurement after the compensation is recommended. The required probe travel of the second measurement should be much smaller, thus the measurement uncertainty should decrease as described above.

### 2.2.3 Uncertainty estimation for the probing system

The dominant uncertainty contributors for the probes are (compare to table 2.3)

- deviations of the probes themselves. This uncertainty is stated by the manufacturer of the probes.

- the local form deviations of the sphere in contact and of the probing planes. It is assumed that each of these contributors lies within a uniformly distributed range of $1\, \mu\text{m}$. Monte Carlo simulations and computations according to [78] showed that a random form deviation causes an error in the computed X and Y position of the sphere center of the same size, for the Z position the error of the sphere center would be about 70% of the form deviations. This effect of reduction has been neglected in the uncertainty estimation.

- the residual error coming from the calibration (see chapter 2.2.2).
• the uncertainty of the sphere diameter. This uncertainty has a direct effect on the uncertainty in Z-direction. The probing system cannot distinguish if the probe is e.g. \(1\mu m / \sin 45^\circ\) closer to a sphere in Z-direction, or if the sphere has a diameter that is \(1\mu m\) bigger than it is supposed to be. The effective probe travel is exactly the same for the nominal configuration of the probing system. It is assumed that the Z uncertainty due to a diameter deviation of a sphere with respect to the diameter of a reference sphere (where the probe calibration has been performed) lies within a uniformly distributed uncertainty interval of \(1\mu m\).

Table 2.3: Probing contributors to uncertainty (rounded).

<table>
<thead>
<tr>
<th>Contributor</th>
<th>Standard uncertainty in X, Y</th>
<th>Standard uncertainty in Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deviations of the probes (manufacturer specification)</td>
<td>0.1\mu m</td>
<td>0.1\mu m</td>
</tr>
<tr>
<td>Local form deviation of probing planes, range 1\mu m</td>
<td>0.3\mu m</td>
<td>0.3\mu m</td>
</tr>
<tr>
<td>Local form deviation of spheres, range 1\mu m</td>
<td>0.3\mu m</td>
<td>0.3\mu m</td>
</tr>
<tr>
<td>Deviations due to errors in probing plane calibration (caused by machine tool error and form deviations of planes, spheres during calibration), total error range of 10\mu m on 2mm travel, assumed probing range during measurement 100\mu m)</td>
<td>0.1\mu m</td>
<td>0.1\mu m</td>
</tr>
<tr>
<td>Uncertainty due to sphere diameter difference between sphere at reference position and at probing position, range 1\mu m</td>
<td>-</td>
<td>0.3\mu m</td>
</tr>
<tr>
<td>Probing standard uncertainty</td>
<td>0.4\mu m</td>
<td>0.5\mu m</td>
</tr>
</tbody>
</table>

2.3 Characterization of “3D Ball Plate” calibrations

A measurement with the “3D Ball Plate” consists of the following steps:

• First the base plate is fixed on the machine table. The ball plate is put directly on it. It is roughly aligned to the machine coordinate axes (e.g. parallel within 1mm on an axis motion of 500mm).
2.3. Characterization of “3D Ball Plate” calibrations

- The orientations of the 4 probing planes in respect to the coordinate system of the machine under test have to be determined on the machine tool (see chapter 2.2.2).

- To minimize the necessary measuring range of the probes during measurement the location of the ball plate in the machine coordinate system has to be determined. The position of the 4 spheres at the corners of the ball plate can be measured in the machine coordinate frame. With a least square algorithm the orientation of the plate on the machine can be found. With this information the target positions of all spheres in every configuration of the ball plate are calculated.

- Thereafter the measurement can start with the ball plate in its first location. The machine is moved to the calculated position of each sphere. With the probing system the deviations are measured. Since position and diameter of the spheres are known, the measured deviations are caused by machine errors. After the position of each sphere has been checked the ball plate is put on the next spacer. This is repeated until all ball plate poses were checked.

- After the measurement has been finished the pose of the artefact in the machine coordinate system is changed mathematically with a least square algorithm (virtually shifted by a rigid body motion), so that the square sum of the machine deviations is minimal. These deviations can be used directly for Space Error Compensation.

The total measurement uncertainty is the combination of the uncertainty contributors of the artefact (see tables 2.1 and 2.2) and of the probing system (table 2.3). The total measurement uncertainty is stated depending on the environmental conditions. Under shop floor conditions it can be seen in table 2.4. The measurement uncertainty under metrology laboratory conditions is shown in table 2.5.

To obtain the complete test uncertainty for a certain measurement with the 3D Ball Plate, the thermal drift of the machine under test has to be added. Since this is depending on the machine, this contribution has been omitted here.
Table 2.4: Combined measurement uncertainty of the device under shop floor conditions (rounded).

<table>
<thead>
<tr>
<th>Contributor</th>
<th>Standard uncertainty in X, Y</th>
<th>Standard uncertainty in Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Artefact contributors including spacer (shop floor conditions)</td>
<td>1.5μm</td>
<td>1.1μm</td>
</tr>
<tr>
<td>Probing standard uncertainty</td>
<td>0.4μm</td>
<td>0.5μm</td>
</tr>
<tr>
<td>Total standard uncertainty</td>
<td>1.6μm</td>
<td>1.2μm</td>
</tr>
<tr>
<td><strong>Total expanded uncertainty U(k = 2)</strong></td>
<td>3.2μm</td>
<td>2.4μm</td>
</tr>
</tbody>
</table>

Table 2.5: Combined measurement uncertainty of the device under metrology laboratory conditions (rounded).

<table>
<thead>
<tr>
<th>Contributor</th>
<th>Standard uncertainty in X, Y</th>
<th>Standard uncertainty in Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Artefact contributors including spacer (metrology laboratory conditions)</td>
<td>0.9μm</td>
<td>0.7μm</td>
</tr>
<tr>
<td>Probing standard uncertainty</td>
<td>0.4μm</td>
<td>0.5μm</td>
</tr>
<tr>
<td>Total standard uncertainty</td>
<td>1.0μm</td>
<td>0.9μm</td>
</tr>
<tr>
<td><strong>Total expanded uncertainty U(k = 2)</strong></td>
<td>2.0μm</td>
<td>1.8μm</td>
</tr>
</tbody>
</table>

2.4 Measurement examples

The use of the ball plate artefact is shown for a conventional machining center (see chapter 2.4.1) as well as for a parallel kinematic machining center (see chapter 2.4.2).
2.4. Measurement examples

2.4.1 Conventional 3-axes machining center

Here the use of ball plate measurements for conventional serial kinematic machining centers is shown. First the results of a Space Error Compensation are presented. The effect of error interdependencies is made apparent in the results.

The measurement is made according to the description made in chapter 2.3. The ball plate is placed either directly on the base plate or on one of 4 spacers (nominal height difference $\Delta Z = 80\, \text{mm}$). The ball plate on the spacer with a nominal height difference of $80\, \text{mm}$ can be seen in figure 2.11, on the spacer with a nominal height difference of $320\, \text{mm}$ can be seen in figure 2.12.

After the measurement, the “distorted” workspace of the machine tool can be shown (see figure 2.13). The data contained in such figures could be used directly for Space Error Compensation. To identify single axis error motions, plots with selected information are more suitable. In figure 2.14 the change of the squareness error between Y and X axis ($C0Y$) depending on the measuring line selected is shown. This is due to a yaw error of the X axis (see chapter 1.1.4, [18], [66]). If only single geometric measurements are made (see chapter 1.2.1), there the squareness measured on one line for X and Y axis will be taken as a compensation value for the whole workspace of the machine, possibly making the overall geometric performance worse. For a calibration with conventional methods, the squareness would have to be tested on several lines (which can be difficult with a conventional setup as described in chapter 1.2.1), or the yaw error would have to be tested additionally with a completely different setup.

Another example is shown in figure 2.15. The positioning error on different lines in X direction is shown. The measuring lines are at $Y = 300\, \text{mm}$. The Z heights change (ball plate rests on different spacers). As can be seen in figure 2.15, the position error changes significantly depending on the measuring height. This indicates that the X axis has a pitch error (see chapter 1.1.4, [18]). At one Y and Z position, e.g. in the center of the workspace, the local positioning error on this line has to be defined as the positioning error of the axis. To obtain the positioning error of a single axis, normally the positioning has to be measured on up to three lines (with vertical or horizontal offset, depending on the kinematic build-up), or the pitch and the yaw movement have to be measured additionally to one positioning measurement.

With the 3D Ball Plate system used here, all geometric errors of the three linear axes of a conventional machining center can be identified. The only exception are the two tilt motions $EAZ$ and $EBZ$ of the vertical Z axis (see chapter 1.1.3) and the roll motion $ECZ$ of the Z axis (in general not relevant on 3-axes machining centers). $EAZ$ and $EBZ$
cannot be separated from the two straightness errors $EXZ$ and $EYZ$. If the distinction is relevant, they can either be separated by a different measurement or by doing a second 3D Ball Plate measurement with a different tool length.
Figure 2.12: Ball plate on spacer ($\Delta Z = 320\text{mm}$)
Figure 2.13: Exemplary measuring results (magnification 3000x)
2.4. Measurement examples

Figure 2.14: Exemplary measuring results. Change of local squareness between Y and X axis (C0Y) at different measuring lines (Z height 0\text{mm}; magnification 12000x)

Figure 2.15: Exemplary measuring results. Change of positioning error of the X axis (EXX) at Y = 300\text{mm} and different Z heights between 0\text{mm} and 320\text{mm}
2.4.2 Space Error Compensation for the PKM Hexaglide

Here the Space Error Compensation of a parallel kinematic machine tool with 6 degrees of freedom - the Hexaglide - is shown. Here 6 linear motors drive 6 slides on three nominally parallel guideways, nominally parallel to X. The actuators move 6 fixed-length struts which are connected to a platform carrying the spindle. Between platform and each strut there is a joint with three axes of rotation arranged sequentially. Between each strut and each slide there is a joint with two axes of rotation arranged sequentially. For details on the machine, see [79], [4]. The schematic kinematic build-up of the Hexaglide is shown in figure 2.16. The exact kinematic transformation of the Hexaglide is given in chapter 3.5.8.

![Figure 2.16: Schematic build-up of the Hexaglide](image)

One of the problems remaining with parallel kinematic machine tools is the uncertainty of a geometric calibration - especially if the geometric performance achieved by conventional machine tools should be reached. In contrast to serial kinematic machine tools, single errors of the machine axes usually cannot be measured separately and compensated directly due to complex error interdependencies.

For direct Space Error Compensation the 3D Ball Plate is used (see also [80]). The distorted workspace can be measured and the TCP displacements can be compensated. While with the Space Error Compensation it is sufficient for eventually receiving a good geometric performance for a purely three-dimensional use of the Hexaglide (with the platform in the same orientation as during calibration), the geometric performance when all 6 axes are used will still be insufficient. To improve that a model-based calibration is needed (see chapter 1.2.3). The example for the Hexaglide will be discussed in chapter 3.5.8.

Another problem of parallel kinematic machine tools is the shape of the workspace of the machine (see [81]). Every possible measuring point, that means every point of the spatial
grid of the 3D Ball Plate, has to be examined if it is within the workspace. The orientation of the ball plate with respect to the base coordinate system of the Hexaglide is measured before the start of the actual measurement. The spheres with the minimal and maximal feasible X respectively Y position are probed. With the displacements measured at those 4 spheres, the actual orientation of the artefact with respect to the base coordinate system is identified using a least-squares algorithm. After that, a NC program is created to probe all sphere positions of the spatial grid that lie within the workspace (with the spindle platform not rotated). The measurement of the orientation before the actual displacement measurement allows smaller required probe travels. This way the measurement uncertainty is kept minimal (compare to chapter 2.1.4). Figures 2.17 and 2.18 show the 3D Ball Plate on the machine. In figure 2.17 the ball plate is resting directly on the base plate. In figure 2.18 it rests on a spacer with a nominal Z Offset of 160 mm.

Figure 2.17: Ball plate resting directly on the base plate
Figure 2.18: Ball plate resting on a spacer ($\Delta Z = 160\,mm$)

The results shown in figures 2.19 and 2.20 are measured relative TCP displacements when the machine control uses the nominal kinematic transformation. Since the machine errors like e.g. strut lengths or joint zero positions are assumed to have an order of magnitude of millimeters, it is not surprising that the measured range of TCP displacement is in the same order of magnitude. For this first measurement the ranges of measured TCP
displacements are 1.98 mm in X direction, 1.45 mm in Y direction and 2.76 mm in Z direction. These displacements have to be compensated. The measurement has to be repeated once more in order to reduce measurement uncertainties (see chapter 2.2.2).

With this calibration method, the test uncertainty at the measuring points after the second calibration does only consist of the measurement uncertainty, the repeatability and the thermal drift of the Hexaglide during the measurement. The repeatability was tested to be in a range of about 5 μm (for X, Y, Z each with a nominally untilted platform). The thermal drift in a time equal to the measuring time of about 25 minutes was measured to be below 3 μm (for X, Y, Z each).

The compensation values between the measuring points can be interpolated. Short wave cyclic errors of the Hexaglide impair the quality of such an interpolation. But continuous measurements (e.g. circular tests) show that geometric TCP displacements caused by such errors are well below 10 μm.

Other measurements with the 3D Ball Plate on the Hexaglide for a model-based calibration according to chapter 1.2.3 are described in chapter 3.5.8. There, measurement results after calibration are shown.

![Figure 2.19: Measured relative TCP displacements, black: nominal grid, red: measured grid, magnification 30x, XY view; coordinate system built with least squares method](image-url)
Figure 2.20: Measured relative TCP displacements, black: nominal grid, red: measured grid, magnification 30x, YZ view; coordinate system built with least squares method
Chapter 3

Identification of Error
Interdependencies Effects

As described in chapter 1.2.1, with conventional geometric, kinematic and model-based calibration approaches certain geometric machine errors are supposed to be identified. As seen in chapter 1.1.4, such identifications are disturbed by other geometric errors. In chapter 2 a new method for calibrating translatory three-dimensional workspaces was introduced. By measuring on a spatial grid, most geometric errors can be separated for a simple standard configuration. By using a Space Error Compensation approach, the resulting TCP displacements could be identified and compensated instead of the single geometric errors.

Unfortunately for all kinematics that have more than three degrees of freedom, especially for kinematics with rotary axes, such methods do not exist. Still the geometric errors have to be identified.

Since it is known that the identification of geometric errors is disturbed by other errors, the task here is to at least quantify the effect of the disturbances. This way the uncertainty of identified geometric errors is determined. With this information a calibration can be planned and the deficiencies of the state of the art can be overcome (see chapter 1.3).

In the example here a kinematic measurement as well as a complex model-based calibration approach for machining centers with 5 or more axes is shown. Certain geometric errors are identified numerically by using only a small number of measurement setups. Measuring methods where the TCP displacements are measured with precision spheres as a reference are exemplarily used to calibrate machining centers. The estimation of the parameter uncertainty is emphasized (as described in [39]).

In chapter 3.1 a suitable probing system for machining centers with rotary axes is intro-
duced. In chapter 3.2 the geometric modeling that is the basis of quantifying error inter-
dependencies is described. In chapter 3.3 a new method for estimating the uncertainty of
the parameters identified is introduced using a Monte-Carlo approach. In chapter 3.4
kinematic measurements and in chapter 3.5 model-based calibrations are addressed.

3.1 The 3D probing system for machine tools with rotary axes

Here an exemplary probing system for calibrating geometric errors of rotary axes is shown.
It represents the current state of the art in measuring devices for machine tools with rotary
axes (see chapter 1.2.2).

The probing system is basically introduced in [29]. It is the same system as described in
chapter 2.2. The 4 linear probes mounted together in one holder are now fixed on the table
of a machining center. They are used to probe a ceramic precision sphere that represents the tool center point TCP. Different measuring poses are reached
by simultaneously moving all machine tool axes (e.g. three translatory axes X, Y and Z
and two rotary axes A and C for a conventional 5-axes machining center). The nominal
relative position between sphere and probes stays the same. Geometric machine errors
cause displacements of the sphere relative to the measuring device. These deviations can
be measured simultaneously in X, Y and Z direction. With 4 probes the measurement
is redundant – thus allowing a crosscheck for wear and dirt on the sphere or the probing
planes.

A picture of the probing system on the machine table of a conventional 5-axes machin-
ing center is shown in figure 3.1. Pictures of different measuring poses can be found in
calibration examples in chapters 3.5.7 and 3.5.8.

3.1.1 Calibration and uncertainty estimation

The probing system is calibrated in the same manner as for its use with the ball plate
artefact (see chapter 2.2.2). The orientation of the 4 single linear probes with respect to
the machine tool coordinate system is determined by measuring the probe displacements
when the linear axes are moved to measuring positions at \( \pm 2mm \) with respect to the
reference position. The uncertainty estimation is the same as stated in table 2.3. Only the
value for the uncertainty in Z direction is now the same as for the X and Y direction since
only one sphere is used now (no uncertainty contribution due to uncertainties in diameter
3.1. The 3D probing system for machine tools with rotary axes

Figure 3.1: Probing system on the table of a 5-axis machine tool of different spheres as mentioned in chapter 2.2.3). Thus the remaining measurement uncertainty is $0.8 \, \mu m$ (coverage factor $k = 2$) in all three spatial directions.

3.1.2 Sphere center offset compensation

The sphere that represents the TCP has an offset to the spindle axis due to manufacturing and assembly tolerances. This offset can be measured with the probing system by turning the whole spindle by at least $360^\circ$ while continuously taking readings. With a mechanical fixture the offset can be reduced to some micrometers. The remaining offset can be measured after the measurement cycle. With this measurement result, either the measured relative TCP displacements can be corrected (with the offset values being transformed into the coordinate system of the probing system at each measuring pose), or identified parameters can be corrected by the measured offset in X and Y direction.
3.2 General kinematic modeling of a machining center

The modeling approach presented here is necessary to quantify the effects of error interdependencies on different measurement results. Furthermore for every model-based calibration a general kinematic model is needed that allows a coordinate transformation from the machine tool axes positions into the workpiece coordinate system of the measuring device and vice versa. This means that the transformation is not only carried out for the nominal kinematic configuration of the machine tool, but also for the differing kinematic configurations that have been changed due to geometric machine errors.

3.2.1 Kinematic transformation

For serial kinematic machining centers such models have been created for a long time (see e.g. [7], [8], [16]). Also for parallel kinematic machine tools such general kinematic models exist. They have been created for a lot of different mechanisms (e.g. [82], [83], [51], [81]). Examples of kinematic modeling are shown in the calibration examples in chapters 3.5.7 and 3.5.8. Generally it must be possible to calculate the current Tool Center Point pose $X$ in all 6 degrees of freedom based on the configuration - that means on the geometric errors included in $E$ - and on the current positions $Q$ of the $n$ axes drives.

$$X = f(Q, E) \quad (3.1)$$

with

$$X = \begin{pmatrix} X & Y & Z & A & B & C \end{pmatrix}^T \quad (3.2)$$

($X \ldots C$ are the cartesian position and orientation of the tool in the workpiece coordinate system)

and

$$Q = \begin{pmatrix} Q_1 & Q_2 & \ldots & Q_n \end{pmatrix}^T \quad (3.3)$$

($Q_1 \ldots Q_n$ are the positions of the axes drives)

This correlation is generally nonlinear.
3.2.2 Error modeling

It should be possible to simulate the effects of all geometric errors $E$ of the machine tool point-wise with the generalized kinematic model. The errors change orientation vectors, axes positions and offsets. Location errors like squareness errors cause constant changes over all poses (fixed values). Component errors like straightness errors cause changes that are depending on the actual axes locations of the current pose. An orientation error of one axis changes the orientation vector of this axis as well as the orientation of every axis supported by this axis. The influence of an error is depending on the kinematic setup of the machine. Examples can be found in the appendix.

As described in chapter 1.1.3, component errors can only be approximated. The actual error value at one pose is calculated as a superposition of the first Fourier-Series harmonics (according to equation 1.1). If applicable, errors depending directly on the axis design are included. E.g. for linear axes with only indirect measuring systems, cyclic positioning errors with a wavelength corresponding to the pitch of the ball screws are included. With the same model a change of the absolute TCP pose due to changed kinematic configuration (changed assumed geometric errors $E_{\text{new}}$) can be calculated as shown before.

$$\mathbf{X}_{\text{new}} = f(\mathbf{Q}, E_{\text{new}}) \quad (3.4)$$

It is always possible to find an influence of the incremental change of just one error $E_i$ on the absolute TCP Pose. Details on this modeling can be found in the examples in chapters 3.5.7 and 3.5.8.

When a measurement is made, certain parameters $p$ are supposed to be identified. $p$ is a subset of all geometric errors $E$. The equation 3.4 changes accordingly to

$$\mathbf{X} = f(\mathbf{Q}, p) \quad (3.5)$$

From this equation the parameters in $p$ are supposed to be identified. All errors of $E$ not included in $p$ are neglected. For example if a simple squareness measurement is made with a straightedge and a square (see chapter 1.2.1), the only error included in $p$ would be the one squareness error. For the identification all other errors would be neglected. So the measured angle between the two lines used would be assumed to be the global value of the squareness error between the axes.

3.3 Parameter uncertainty estimation

The error identification of kinematic measurements or model-based calibrations ignores the effect of other errors that are present. Such errors do disturb the identification. In
this chapter a method for quantifying these disturbances is presented.

To get information about the uncertainty of the parameters identified, Monte Carlo simulations are used. Such simulations are an instrument used to assess uncertainties ([84], [85], [86]).

Here, a “real” machine tool is simulated. All geometric machine errors (see chapter 1.1) as well as the measurement uncertainty of the probing system are taken into account. If necessary, hysteresis and thermal drift can be added. Such effects change the current positions of the axes drives $Q$. The repeatability can for example be assumed to be a random value taken from within a given range at every measuring pose. The backlash effect can be a constant value for each axis, its sign depending on the direction in which a certain measuring pose was reached. The thermal drift can be assumed to be the linearly or nonlinearly changing value for each axis, depending on the time a certain measuring pose is reached. This way, the data obtained during a real measurement on the machine tool can be approximated. Component errors are simulated as a superposition of several Fourier harmonics (see equation 1.1) plus possibly cyclic deviations (depending on axis design).

For the “real” machine tool simulation the magnitude of each error is chosen arbitrarily for each run from a uniformly distributed range (specific for each error). For the harmonics of component errors, the phase angles are random. The error ranges are taken from comparative measurements, manufacturer specifications or from values stated in standards. Note that the values are depending on the machine tool used. With this model, the measurement and the identification must be simulated sufficiently often (e.g. for 1000 error combinations). The difference between the “real” parameters chosen arbitrarily and the parameter values obtained with the identification formula of the kinematic measurement (see chapter 3.4.1) can be observed. With enough runs the distribution of the difference between “real” and identified parameters becomes smooth. From this distribution the standard uncertainty of each parameter can be calculated.

The procedure can be seen in figure 3.2. Examples and results of this procedure are shown in chapters 3.5.7 and 3.5.8. Details on the modeling can be found in the appendix.
3.3. Parameter uncertainty estimation

Prerequisite:
A complete list of all geometric axis errors of the machine tool under test is available.

Error Range selection:
For each error $E_i$ (and each error harmonic) a specific error range and distribution (distribution $P_i$) is chosen (based on comparative measurements, manufacturer specifications or standards; e.g. rectangular distribution with a range of 10 $\mu m$).

Random Error Choice:
Each error $E_i$ is chosen randomly from within its specified range (including the parameters $p$; see also appendix)

$$E_{i\sim} = P_i(v_{\text{random}})$$

Measurement Simulation:
With the current error values, a measurement is simulated. The relative TCP displacements are evaluated.

$$\Delta X_{\text{sim.meas.}} = f(Q, E_{\sim})$$

Parameter Identification:
With the simulated measurement values, the selected parameters $p$ are identified (same identification formulae resp. optimization routine as used for a real measurement), e.g. for a least squares fits

$$Min \sum_{j=1}^{m} (\Delta X_{\text{sim.meas.}} - \Delta X_{\text{calculated}}(p))^2$$

Parameter Comparison:
The identified parameters $p$ are compared to the “real” parameters $p_{\sim}$ (subset of $E_{\sim}$) that have been selected randomly.

Parameter Uncertainty Evaluation:
The deviations between the “real” and the identified parameters $\Delta p = p - p_{\sim}$ are evaluated statistically. Thus a standard uncertainty of each parameter can be computed.

Figure 3.2: Procedure of Parameter Uncertainty Estimation
3.4 Error interdependencies effects on kinematic measurements

Kinematic measurements are used for identifying geometric errors using explicitly or implicitly a very simple error model of the machine tool under test (see chapter 1.2.2). In chapter 3.4.1 this error identification will be described. In chapter 3.4.2 an example for a kinematic calibration will be given. Special attention is paid to the uncertainties of the parameters identified.

3.4.1 Error identification

The error identification with kinematic measurements is based on a simple geometric error model of the machine tool. Usually the machine tool is assumed to have its nominal geometric configuration except for the one or more errors to be identified. Therefore simple formulas can be applied for identifying the errors. Good examples for this can be found in the analysis of circular tests (see e.g. [8]). If e.g. a circle in the XY-plane of a machine is measured, the linear positioning error $E_{XX}$ of the X axis (see chapter 1.1.3.1) is derived directly from the measured diameter of the circle parallel to the X direction in comparison to the nominal circle diameter. The squareness of e.g. the Y axis with respect to the X axis ($C_{0Y}$, see chapter 1.1.1.1) can be determined by the change in diameter of the circle at $+45^\circ$ and at $-45^\circ$ with respect to the X axis.

3.4.2 Calibration example

The example selected here is the kinematic measurement of all 4 location errors of a rotary C axis. These 4 errors make up the selected parameters $p$. The two translatory offset errors $X_{0C}$ and $Y_{0C}$ of the axis as well as orientation errors (squareness of C to the Y axis of the coordinate system, $A_{0C}$, and squareness of C to the X axis of the coordinate system, $B_{0C}$) are to be identified with one setup (compare to chapter 1.1.1.2).

To do this the machine table is positioned horizontally ($A = 0^\circ$). The probing system introduced in chapter 3.1 is put on the machine table with an offset from the C axis of nominally 130 mm (nominal diameter $D_{nom} = 260$ mm) at $C = 0^\circ$. 4 measuring poses are used. The relative TCP displacements are measured between the precision sphere in the spindle and the probing system. The C axis is rotated to $C = 90^\circ$, $C = 180^\circ$, $C = 270^\circ$ respectively. The linear X and Y axes are positioned to the nominal position of the probing system on the table. Two of these 4 measuring poses can be seen in figure 3.3. With the
3.4. Error interdependencies effects on kinematic measurements

![Image of measuring poses at C = 0° and C = 180°](image)

Figure 3.3: Measuring poses at $C = 0^\circ$ and $C = 180^\circ$ (poses at $C = 90^\circ$ and $C = 270^\circ$ not depicted)

In the probing system, the relative TCP displacements can be measured in the $X_{PS}$, $Y_{PS}$ and $Z_{PS}$ direction of the measuring device (see figure 3.4). These directions correspond to a radial, tangential respectively axial measuring direction relative to the motion of the C axis. For $C = 0^\circ$ these directions coincide with the machine coordinate axes.

It is very easy to deduce the parameters in $p$ from the measured TCP displacements. The

![Diagram showing change of probing system orientation with C axis rotation](diagram)

Figure 3.4: Change of probing system orientation with C axis rotation
translatory offset errors can be calculated from the measured positions:

\[
X_{0C} = \frac{X_{ps}^{0\circ} - X_{ps}^{180\circ}}{2} \quad (3.6)
\]
\[
Y_{0C} = \frac{X_{ps}^{270\circ} - X_{ps}^{90\circ}}{2} \quad (3.7)
\]

The orientation errors can be calculated using the measurement information in \( Z_{PS} \) direction:

\[
A_{0C} = \frac{Z_{ps}^{90\circ} - Z_{ps}^{270\circ}}{D_{nom}} \quad (3.8)
\]
\[
B_{0C} = \frac{Z_{ps}^{0\circ} - Z_{ps}^{180\circ}}{D_{nom}} \quad (3.9)
\]

It is clear that other geometric errors (e.g. straightness errors of the linear axes or radial or axial error motions of rotary axes) are falsifying the result obtained. The effect of such errors can be modeled. This is shown in the appendix. With the uncertainty estimation according to chapter 3.3, a standard uncertainty for the 4 parameters can be determined (see table 3.1). The ranges of the single errors have been determined according to the tolerances stated in the acceptance protocol of the machining centers and according to appropriate standards. Examples are ISO 10791-2:2001(E) “Test conditions for machining centres - Part 2: Geometric tests for machines with vertical spindle or universal heads with vertical primary rotary axis (vertical Z-axis)” ([1]) or ISO 3408-3:1992 (E) “Ball screws - Part 3: Acceptance conditions and acceptance tests” ([87]). In this example the high uncertainties are due to the very simple measurement, but also due to the limited machine performance (no direct measuring systems, cyclic deviations, etc.).

Table 3.1: Parameter uncertainties (\( k=1 \)) of a kinematic measurement from Monte Carlo Simulations (1000 runs).

<table>
<thead>
<tr>
<th>Par.</th>
<th>Unit</th>
<th>( u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_{0C} )</td>
<td>( \mu m )</td>
<td>9</td>
</tr>
<tr>
<td>( Y_{0C} )</td>
<td>( \mu m )</td>
<td>11</td>
</tr>
<tr>
<td>( A_{0C} )</td>
<td>arcsec</td>
<td>9</td>
</tr>
<tr>
<td>( B_{0C} )</td>
<td>arcsec</td>
<td>7</td>
</tr>
</tbody>
</table>
3.5 Error interdependencies effects on model-based calibrations

In this chapter, model-based calibration methods are examined. Chapter 3.5.1 addresses the different ways how geometric errors could be accounted for. Chapter 3.5.2 deals with the open questions that have to be answered to find a suitable calibration method. In chapter 3.5.3 the identification of errors is addressed. The mathematical condition of a model-based calibration is addressed in chapter 3.5.4. In chapter 3.5.5 the estimation of the uncertainty of the parameters identified is discussed using a Monte-Carlo approach. Chapter 3.5.6 addresses the issue how the geometric performance of a machine tool after model-based calibration can be checked experimentally. Finally in chapters 3.5.7 and 3.5.8 two examples for model-based calibrations via precision sphere probing are shown. The first example is a calibration of a conventional 5-axes machining center, the second one the calibration of the 6-axes Parallel Kinematic Machine (PKM) “Hexaglide”.

3.5.1 Possibilities of dealing with geometric machine errors

As first step in planning a model-based calibration, all possible geometric errors of the kinematic under test should be listed (see chapter 1.1, examples in chapters 3.5.7.2, 3.5.8.2). For a model-based calibration there are generally different ways on how to deal with the different errors:

- Errors can become machine parameters to be identified. These errors than make up the model parameter vector $p$ (see chapter 1.2.3). Usually it is a subset of the location errors. E.g. for Hexapod kinematics often the constant strut length offsets and the positions of the base and platform joints in the respective coordinate systems are used.

- Errors can be calibrated directly and compensated. With different measuring equipment a certain error can be measured, corrected or compensated. For example the positioning error of a linear axis could be measured with a laser interferometer on a specific measuring line. Besides the high effort for such calibrations, the remaining error disturbs the identification of the parameters $p$.

- Errors can simply be ignored during calibration. For example all component errors are usually ignored during calibration of parallel kinematic machines. Errors are usually ignored if their expected impact on the TCP is relatively small (due to small magnitudes of the errors or a low error propagation), if it is difficult to measure these
errors with a small measurement uncertainty or if it is difficult to model these errors (see chapter 1.1.3). Of course such errors disturb the identification of the parameters $p$.

### 3.5.2 Planning model-based calibration methods

As already mentioned in chapter 1.2.3, machine tool calibration methods have to be planned thoroughly to achieve satisfactory results of the identified geometric errors. The resulting geometric performance is depending on the measurement uncertainty, on the disturbances of other machine errors and – for complex measurements like model-based calibration approaches – on the mathematical condition of the identification of the geometric errors (How well can the parameters $p$ be separated?). As addressed in the introduction, model-based calibration methods have the potential to greatly reduce the time required for a geometrical calibration.

Yet the most important question concerning the calibration method cannot be answered conclusively so far:

Is a measurement device and method suitable to obtain a certain geometric machine performance? Heuristic or statistical approaches are used to answer the question if a calibration with a certain measuring device is good enough to find a parameter combination that leads to a sufficiently good machine behavior (e.g. [44], [45]). But the effects of several factors influencing a calibration are not taken into account sufficiently, so that some questions concerning the planning of model-based calibration methods remain open:

- How many measuring poses should be used in order to find a good compromise between calibration result and time consumption? The only obvious hint on how big this number should be is the mathematical condition of the identification problem (see chapter 3.5.4). The number of relative displacements measured has to be sufficiently bigger than the number of parameters (see chapter 3.5.3). In order to improve the calibration result, often a lot of measuring poses are used (e.g. [36]). The effect on the calibration quality of more measuring points is unclear. Often the assumption is made that only the errors that are included in the selected parameter are systematic and that all other errors and the measurement uncertainty have the effect of “white noise”. This means that they are supposed to be completely non-systematic. With this assumption an increase in the number of measuring poses would lead to a better identification of the parameters due to statistical equalization effects. But in reality this is not the case. Almost all of the TCP displacements that can be measured are systematic. The geometric errors (see chapter 1.1) are repeatable, other
errors, like e.g. thermal drift, are systematic and not random. So the assumption of an ever improving identification quality with more measuring poses must be false (see chapter 3.5.5).

- What is the exact advantage of the simultaneous measurement of several degrees of freedom? The advantage of measuring a higher number of degrees of freedom is not quantified so far. Again, only the change in the mathematical condition can be quantified. The improvement in the result is not quantified so far.

- What is the optimal number of parameters to be identified? A real machine has an almost arbitrarily high number of errors: Location errors, but also component errors like axial and radial joint movements, straightness deviations of guideways. Due to measurement uncertainties and the measuring method not all of those errors can be identified. A certain number of errors have to be chosen as a parameter set. The identification of these parameters should lead to the best geometrical performance in the workspace of a machine. More parameters may lead to smaller residual displacements at the measuring poses used during calibration, but at other poses of the workspace the behavior could worsen. Today strut lengths and joint positions are usually used as parameters to be identified for parallel kinematic machine tools. If the number of the parameters should be extended or reduced - or if other parameters should be used - can only be guessed.

- Is a measuring device/ method more suitable than another one? A comparison of measuring devices with different workspaces, different measurement uncertainties and a different number of degrees of freedom measured is hard. Usually it can not be quantified which device is better suited for a certain task.

To answer these questions the eligibility of a calibration method for a certain task has to be quantified. The Monte Carlo approach discussed in chapter 3.3 is used in the following chapters. With it a means of answering the questions above is given.

### 3.5.3 Error identification

For a given measuring device, only a subset or combination of the 6 degrees of freedom of the TCP can be measured. For example with the probing system presented in chapter 2.2, only translatory deviations can be measured (only TCP displacements in direction of $X^{\text{trans}}$) with

$$X^{\text{trans}} = \begin{bmatrix} X & Y & Z \end{bmatrix}^T$$

(3.10)
Furthermore with the given measuring device only changes of the absolute TCP displacements between the different measuring poses selected can be acquired (relative deviations between the measuring poses $\Delta X^{\text{trans}}$) since the exact location of the probing system on the machine table is unknown.

By changing the machine errors selected as parameters $p$ (see chapter 3.5.1), the measured TCP displacements can be emulated. $p$ is a subset of all geometric errors $E$.

The optimization task can be modeled as

$$\text{Min} \sum_{j=1}^{m} (\Delta X^{\text{trans}}_{\text{measured}} - \Delta X^{\text{trans}}_{\text{calculated}}(p))^2$$

In this case, this is solved with a Levenberg-Marquardt algorithm (nonlinear optimization, see [88], [89]). Due to a limited mathematical condition (mathematical separability of the single parameters from each other) and depending on the data gathered during the measuring, not all machine errors can be identified. The problematic part here is to determine which machine errors should be included into $p$. This is discussed in the following chapters.

### 3.5.4 Evaluating the mathematical condition of a model-based calibration

To evaluate the mathematical condition of an identification problem, a number of indexes using singular value decomposition or the QR decomposition of a matrix describing the linearized system can be used (e.g. [90], [44], [91], [83]). Due to the usually only mild nonlinearity of the optimization problem, the linearized correlation between a parameter of $p$ at a measuring pose is sufficient in order to evaluate the mathematical condition.

#### 3.5.4.1 Linearization of the optimization problem

If for example again the three translatory degrees of freedom are measured (see equation 3.11), the correlation between a parameter $p_j$ and the measurable TCP displacements at measuring pose $i$, $\Delta X^{\text{trans}}_{ij}$ can be calculated.

$$\Delta X^{\text{trans}}_{ij} = \begin{bmatrix} \Delta X_{ij} & \Delta Y_{ij} & \Delta Z_{ij} \end{bmatrix}^T$$

$$\approx \frac{\delta X^{\text{trans}}_{ij}}{\delta p_j} p_j = \begin{pmatrix} c_{ij}^X \\ c_{ij}^Y \\ c_{ij}^Z \end{pmatrix} p_j = c_{ij} p_j$$
3.5. Error interdependencies effects on model-based calibrations

Each $c_{ij}$ can be computed numerically for a typical value of $p_j$. The relative TCP deviations at all $m$ measuring poses due to all $n$ parameters in $p$ can be represented in one vector:

\[
\begin{pmatrix}
\Delta X_{11} + \Delta X_{12} + \cdots + \Delta X_{1n} \\
\Delta Y_{11} + \Delta Y_{12} + \cdots + \Delta Y_{1n} \\
\Delta Z_{11} + \Delta Z_{12} + \cdots + \Delta Z_{1n} \\
\Delta X_{21} + \Delta X_{22} + \cdots + \Delta X_{2n} \\
\vdots \\
\Delta Z_{m1} + \Delta Z_{m2} + \cdots + \Delta Z_{mn}
\end{pmatrix} = 
\begin{pmatrix}
\Delta X_{\text{trans}} \\
\Delta Y_{\text{trans}} \\
\Delta Z_{\text{trans}}
\end{pmatrix}
\]

(3.14)

The matrix on the right hand side, now called Influence Matrix $\mathbf{IM}$, is the Jacobian matrix which linearly relates the parameters in $p$ with the relative TCP deviations $\Delta X_{\text{trans}}$.

\[\Delta X_{\text{trans}} = \mathbf{IM} \cdot p\]  

(3.15)

With $\mathbf{IM}$ a matrix is given with elements depending on a lot of parameters of the calibration method:

- The column length of $\mathbf{IM}$ is the product of the number of measuring poses $m$ and the number of degrees of freedom measured per pose $d_m$ (in the example presented here $d_m = 3$).
- The row length of $\mathbf{IM}$ is equal to the number of parameters $n$ in $p$ to be identified.
- The value of each element of $\mathbf{IM}$ is depending on both the respective measuring pose and the reference pose chosen.

What is not taken into account are all the effects coming from the machine tool and the measuring system. These are the effects of the measurement uncertainty and of the geometric machine errors that are not supposed to be identified (not part of $p$). There effects can be examined with the method described in chapter 3.5.5.

Because of these restrictions, the matrix $\mathbf{IM}$ can be used as a relative comparison between different measuring approaches. With little effort the effect e.g. of more measuring poses or differently distributed measuring poses can be examined qualitatively. The overall uncertainty of the parameters identified cannot be evaluated.
3.5.4.2 Scaling of the parameters

If a parameter $p_j$ in $p$ would cause only very small relative TCP displacements in comparison to the other parameters, the problem of identifying $p$ would be ill conditioned. However, as long as the parameter $p_j$ always does have a small effect on the TCP, it is not important that it has to be identified with an equally small uncertainty as another parameter which causes big TCP displacements.

To avoid this apparently ill condition, the columns of $IM$ are divided by the norms of the column vectors:

$$n_j = \text{norm} \left( \begin{array}{c}
c_{1j}^T \\
\vdots \\
c_{mj}^T \\
\end{array} \right)$$

This corresponds to a scaling of the magnitudes of each parameter $p_j$. Each scaled parameter causes the same magnitude of TCP deviations over all measuring poses. So a parameter becomes as important as the TCP deviations it causes.

Details on parameter scaling can be found in [44] and [45].

3.5.4.3 Condition number as stability measure

As described before, the mathematical condition of an identification problem, a singular value decomposition (SVD) (see e.g. [44], [45]) or a QR decomposition (see e.g. [92]) of a matrix describing the linearized system can be used.

The condition number (or relative condition number) $\kappa$ is the most common stability index. Furthermore in a comparison made in [93] the use of $\kappa$ yielded good results when compared to other stability indexes.

According to an explanation in [94] here (see equation 3.15) the relative condition number $\kappa$ of perturbations of $\Delta X^{\text{trans}}$ is defined to be the maximum value of the ratio “relative change in $p$” to “relative change in $\Delta X^{\text{trans}}$”.

$$\kappa = \sup_{\delta X^{\text{calculated}}} \left( \frac{\|\delta p\|}{\|p\|} \cdot \frac{\|\delta \Delta X^{\text{trans}}\|}{\|\Delta X^{\text{trans}}\|} \right)$$

(3.18)

If $\kappa$ is high the problem is ill-conditioned. Small perturbations of the left hand side of equation 3.15 can lead to large changes of the parameters $p$. Since the measured relative
TCP displacements $\Delta X_{\text{measured}}^{\text{trans}}$ can be imagined to be a perturbed ideal $\Delta X_{\text{trans}}^{\text{trans}}$ due to machine errors not represented in the model and due to measurement uncertainties - a measure is obtained how much such effects can affect the calibration. Incorrectly identified parameters lead to an overall worse geometric performance of the machine tool.

In [94] it is shown that $\kappa$ can be calculated as the ratio of the biggest to the smallest singular value of $\text{IM}_{\text{norm}}$. \[ \kappa = \frac{\sigma_1}{\sigma_p} \] (3.19)

The singular values are the square roots of the eigenvalues $\lambda_i$ of the matrix $\text{IM}_{\text{norm}}^T \text{IM}_{\text{norm}}$. With the condition number it can be judged how a change in the measured data due to uncertainties will change the parameters to be identified.

It is an indicator of the relative performance of calibration methods which can be calculated with a relatively simple kinematic model.

A drawback is the worst-case character of the condition number. Normally perturbations of a real system have a much smaller influence on the parameters $p$ than indicated by the definition of $\kappa$ in equation 3.18.

### 3.5.5 Parameter uncertainty estimation of model-based calibration approaches with Monte-Carlo simulations

Although it is usually relatively easy to determine the measurement uncertainty of a probing system, it is hard to predict the uncertainty of the parameters identified by model-based calibration as well as the machine performance with the new parameter set.

Simulations assuming a uniformly distributed noise at each measuring pose are insufficient. With this, the number of poses would only have to be high enough so that the parameters would eventually be identified with an uncertainty smaller than the measurement uncertainty, due to statistical equalization.

A real machine tool has errors that are unknown, but they are mostly systematic and repeatable (e.g. higher order component errors), having a worse effect on the parameter uncertainty than purely random errors. To get information about the parameter uncertainty and the “Volumetric Accuracy” [6] to be expected, Monte Carlo simulations of the measurement with a “real” machine tool as described in chapter 3.3 are used.

Instead of using a simple identification formulas as for kinematic measurements with the simulated measurement, an identification routine as described in chapter 3.5.3 is used.
There the parameters $p$ can be identified. When the “real” configuration is assumed to be the actual machine performance and the identified parameters $p$ are assumed to be the model used in the machine control, the machine performance after calibration can be simulated. The procedure stays the same as shown in figure 3.2.

### 3.5.6 Experimental testing of the calibration quality

The optimization task of a model-based calibration approach is to find a set of parameters that will reproduce best the TCP displacements (see equation 1.2).

Therefore, with a mathematically correct optimization algorithm, the residue at the measuring poses will be small. If the calibration measurement is repeated with the identified parameter set, the only remaining TCP displacements will originate only from the residue plus some small contributions due to repeatability and drift of the machine tool and due to measurement uncertainty.

Therefore the repetition of the calibration measurement is not suitable for checking the calibration quality. If e.g. the parameter identification is very ill-conditioned (see chapter 3.5.4), the single parameters cannot be separated. Nevertheless at the measuring poses a parameter combination will be found that will reproduce very well the measured TCP displacements. However, the deviations of the identified parameters from their real values will be very high. When other measuring cycles are used, or when the machine tool is actually used for machining workpieces, the combination of different identified parameters (which are deviating greatly from their real values) will cause big relative TCP displacements. Thus the geometric performance of the machine tool will be unsatisfactory.

To experimentally test the calibration quality the testing measurement has to be very different from the measurement made for calibration. If e.g. for the calibration a cycle with a synchronous motion of all axes has been made, for testing the calibration quality measurements should be used where only a subset of axes are moved. Additionally e.g. another set of degrees of freedom could be measured.

Examples for such measurements can be found in chapters 3.5.7 and 3.5.8.

### 3.5.7 Calibration example: conventional 5-axes machining center

In this chapter the model-based calibration of a 5-axes machine tool is discussed. The measuring device used has been presented in chapter 3.1. The kinematic machine model
and the error modeling are shown in chapters 3.5.7.1 and 3.5.7.2. The measurement procedure is addressed in chapter 3.5.7.3. Chapter 3.5.7.4 covers the determination of the mathematical condition as described in chapter 3.5.4 as well as the uncertainty of the model-based calibration as described in chapter 3.5.5. In chapter 3.5.7.5 the results of an exemplary calibration of a 5-axes machine tool are shown. The quality of the calibration is tested with a 3D circular test. Finally in chapter 3.5.7.6 the estimated improved calibration results for a higher machine tool performance are shown.

### 3.5.7.1 Kinematic modeling

A general kinematic model is needed that allows a coordinate transformation from the machine tool coordinate system into the work piece coordinate system of the measuring device and vice versa. In this example here the machining center has a rotary C axis (carrying the mounting table for the workpiece) that rests on a swivel axis A. The following parameters have to be known (see figure 3.5):

- The kinematic build-up of the machine tool. That means the sequence of axes from the machine bed to the tool respectively to the workpiece (e.g. workpiece on the rotary table C, rotary table C on swivel axis A, swivel axis A on linear axis Z, linear axis Z on the machine bed and tool in spindle, spindle on linear axis Y, linear axis Y on linear axis X, linear axis X on the machine bed).

- The normalized direction vectors of the machine axes $\mathbf{v}_X$, $\mathbf{v}_Y$, $\mathbf{v}_Z$, $\mathbf{v}_A$ and $\mathbf{v}_C$ (nominally parallel to the respective coordinate directions). A rotation by the angle $A$ around the vector $\mathbf{v}_A$ represents a rotation from the machine tool coordinate system into the A axis coordinate system. A rotation by the angle $C$ around $\mathbf{v}_C$ represents a rotation from the A axis coordinate system into the C axis coordinate system.

- The vector $\mathbf{o}_{MT}^A$ with the Y and Z position of the A axis in the machine tool coordinate system at $X = 0$ (from the origin of the machine tool coordinate system to the A axis at $X = 0$; see figure 3.5).

- The vector $\mathbf{o}_A^C$ with X and Y offset of the C axis with respect to the A axis in the A axis coordinate system.

- The origin of the workpiece coordinate system in the coordinate system of the C axis $\mathbf{o}_{WP}^C$. 
Figure 3.5: Transformation parameters of the 5-Axes Machining Center

The axes positions $\mathbf{q}_{\text{trans}}$ / $\mathbf{q}_{\text{rot}}$ (linear axes positions $Q_X$, $Q_Y$ and $Q_Z$ / of rotary axes $Q_A$ and $Q_C$; e.g. position of the X axis $Q_X = 100mm$) must be transformed into workpiece coordinates $\mathbf{X}$ and vice versa.

$$
\mathbf{q}_{\text{trans}} = \begin{bmatrix} Q_X & Q_Y & Q_Z \end{bmatrix}^T \quad (3.20)
$$

$$
\mathbf{q}_{\text{rot}} = \begin{bmatrix} Q_A & Q_C \end{bmatrix}^T \quad (3.21)
$$

The axes themselves can have a practically arbitrary orientation to each other. They do not have to be square or parallel. If the axis positions $\mathbf{q}_{\text{trans}}$ are e.g. $[100 \ 100 \ 150 \text{ mm}]$, the TCP position in the machine coordinate system $\mathbf{q}_{\text{trans}}^{\text{MT}}$ can differ from $\mathbf{q}_{\text{trans}}$. With the normalized direction vectors the position of the TCP in the machine coordinate system $\mathbf{q}_{\text{trans}}$ can be calculated:

$$
\mathbf{q}_{\text{trans}}^{\text{MT}} = \begin{bmatrix} v_X & v_Y & v_Z \end{bmatrix} \mathbf{q}_{\text{trans}} - \mathbf{t}_{\text{MT}} \quad (3.22)
$$

$\mathbf{t}_{\text{MT}}$ represents the tool length vector in the machine tool coordinate frame.
The vector between the A axis reference point and the TCP can be calculated in the machine tool coordinate system:

\[ \mathbf{o}_{MT}^{A-TCP} = \mathbf{q}_{MT}^{trans} - \mathbf{o}_{MT}^{A} \]  

(3.23)

To transform \( \mathbf{o}_{MT}^{A-TCP} \) into the coordinate system of the A axis, it has to be rotated by the angle A around the vector \( \mathbf{v}_A \) (\( \mathbf{R}_A \) corresponds to a rotation by the angle A around \( \mathbf{v}_A \)):

\[ \mathbf{o}_{A}^{A-TCP} = \mathbf{R}_A \mathbf{o}_{MT}^{A-TCP} \]  

(3.24)

The offset between the C axis and the TCP can then be calculated in the A axis coordinate system

\[ \mathbf{o}_{A}^{C-TCP} = \mathbf{o}_{A}^{A-TCP} - \mathbf{o}_{A}^{C} \]  

(3.25)

In another transformation this offset is rotated around the vector \( \mathbf{v}_C \) to transform into the coordinate system of the C axis (\( \mathbf{R}_C \) corresponds to a rotation by the angle C around \( \mathbf{v}_C \)).

\[ \mathbf{o}_{C}^{C-TCP} = \mathbf{R}_C \mathbf{o}_{A}^{C-TCP} \]  

(3.26)

To obtain the workpiece coordinates \( \mathbf{X}^{trans} \), the translational displacement \( \mathbf{o}_{C}^{WP} \) between work piece coordinate system and C axis coordinate system has to be regarded:

\[ \mathbf{X}^{trans} = \mathbf{o}_{C}^{C-TCP} - \mathbf{o}_{C}^{WP} = [X \ Y \ Z]^T \]  

(3.27)

To calculate the axis positions from a set of workpiece coordinates, the steps just have to be followed in reverse order.

### 3.5.7.2 Geometric errors

All geometric errors of the machine tool can be simulated pointwise with this kinematic model. The errors change orientation vectors, axis positions and offsets. Location errors like squareness errors cause constant changes over all poses (fixed values). Component errors like straightness errors cause changes that are depending on the actual axis locations of the current pose. An orientation error of one axis changes the orientation vector of this axis as well as the orientation of every axes supported by this axis. The influence of an error is depending on the kinematic build-up of the machine.

The location errors of a 5-axes machining center (according to chapter 1.1.1) can be seen in table 3.2, the component errors (see chapter 1.1.3) can be seen in table 3.3.

For the model-based calibration the errors of the spindle are not identified. The spindle is not turned during measurement. They have to be identified with another measurement. The errors of the linear and rotary axes are modeled (compare to chapter 3.5.7.1):
Chapter 3. Identification of Error Interdependencies Effects

Table 3.2: Location errors of a 5-axes machining center.

<table>
<thead>
<tr>
<th>Location errors</th>
<th>Number of components</th>
<th>Errors per component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orientation errors of the motion of the linear axes (nominally parallel to X, Y or Z)</td>
<td>3 axes</td>
<td>2</td>
</tr>
<tr>
<td>Zero-position of the linear axes (zero point on each axis)</td>
<td>3 axes</td>
<td>1</td>
</tr>
<tr>
<td>Orientation errors of the motion of the rotary axes</td>
<td>2 axes</td>
<td>2</td>
</tr>
<tr>
<td>Translatory position errors of the rotary axes perpendicular to the axis direction</td>
<td>2 axes</td>
<td>2</td>
</tr>
<tr>
<td>Angular zero position of the rotary axes</td>
<td>2 axes</td>
<td>1</td>
</tr>
<tr>
<td>Position offset in X, Y of the spindle</td>
<td>1 spindle</td>
<td>2</td>
</tr>
<tr>
<td>Orientation errors (angles around X and Y) of the spindle</td>
<td>1 spindle</td>
<td>2</td>
</tr>
<tr>
<td>Definition of the machine coordinate system</td>
<td>1 coord. system</td>
<td>-3 pos.</td>
</tr>
<tr>
<td>definition</td>
<td></td>
<td>-3 orient.</td>
</tr>
<tr>
<td>Redundancies between angular zero position of first rotary axis and orientation error of the second rotary axis (if the second rotary axis rests directly on the first rotary axis)</td>
<td>1</td>
<td>-1</td>
</tr>
</tbody>
</table>

Total Number of location errors 16

- The orientation errors of the linear axes (squareness errors) change the axes vectors \( v_X, v_Y \) and \( v_Z \) in relation to each other.
- The translatory position errors of the rotary axes (offset errors) change \( o_{MT}^A / o_{C}^C \).
- The orientation errors of the rotary axes change the directions of \( v_A \) and \( v_C \).
- The positioning and the straightness errors of the linear axes change \( q^{\text{trans}} \).
- The angular errors of the linear axes (roll and tilt errors) cause angular deviations from \( v_X, v_Y \) and \( v_Z \); angular errors do also change \( q^{\text{trans}} \) depending on the actual offset between TCP and a reference line representing the guideways of the axes.
- The axial and radial movements of the rotary axes (axial and radial error motion) change \( o_{MT}^A / o_{C}^C \).
Table 3.3: Component errors of a 5-axes machining center.

<table>
<thead>
<tr>
<th>Component errors</th>
<th>Number of components</th>
<th>Errors per component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positioning errors of the linear axes</td>
<td>3 axes</td>
<td>1</td>
</tr>
<tr>
<td>Straightness errors of the linear axes</td>
<td>3 axes</td>
<td>2</td>
</tr>
<tr>
<td>Roll error of the linear axes</td>
<td>3 axes</td>
<td>1</td>
</tr>
<tr>
<td>Tilt errors of the linear axes</td>
<td>3 axes</td>
<td>2</td>
</tr>
<tr>
<td>Positioning errors of the rotary axes</td>
<td>2 axes</td>
<td>1</td>
</tr>
<tr>
<td>Radial motions of the rotary axes and the spindle</td>
<td>3 axes</td>
<td>2</td>
</tr>
<tr>
<td>Axial motion of the rotary axes and the spindle</td>
<td>3 axes</td>
<td>1</td>
</tr>
<tr>
<td>Tilt errors of the rotary axes and the spindle</td>
<td>3 axes</td>
<td>2</td>
</tr>
<tr>
<td>Total Number of component errors</td>
<td>35</td>
<td></td>
</tr>
</tbody>
</table>

- The tilt error motion cause angular deviations from \( \mathbf{v}_A \) and \( \mathbf{v}_C \).

The error modeling in detail is discussed in the appendix (see chapter A.1).

For component errors, the actual error value at one \( \mathbf{q}^{\text{trans}} / \mathbf{q}^{\text{rot}} \) is calculated as a superposition of the first Fourier-Series harmonics, plus in some cases the value of a sine function (depending directly on the axis design, like e.g. cyclic positioning error, wavelength corresponds to pitch of the ball screw). For the positioning errors of the linear axes, also a linear term is included (see also chapter A.1).

With the given measuring device (see chapter 2.2) only changes of the absolute TCP deviations between the different measuring poses can be acquired (relative displacements between the measuring poses \( \Delta \mathbf{X}^{\text{trans}} \)) since the exact location of the probing system on the machine table is unknown. So the optimization task is (in accordance with equation 3.11)

\[
\text{Min} \sum_{j=1}^{m} (\Delta \mathbf{X}^{\text{trans}}_{\text{measured}} - \Delta \mathbf{X}^{\text{trans}}_{\text{calculated}}(\mathbf{p}))^2 \quad (3.28)
\]

As described in chapter 3.5.3, the parameters in \( \mathbf{p} \), a subset of all machine errors \( \mathbf{E} \), are identified with a Levenberg-Marquart algorithm. For the example here the optimization algorithm is very stable, that means it does always find the same minimum. This has been tested by randomly changing the starting values of the optimization within sensible boundaries.
3.5.7.3 Measurement procedure

The most dominant machine errors should be identified. An acceptable machine tool performance should be achieved after one measurement. The whole calibration of the machine tool then takes only a couple of minutes, thus making frequent recalibrations possible.

The most dominant errors of a 5-axes machining center are usually the location errors (see table 3.2), errors between the relative locations of the 5-axis motion (compare to chapter 1.1.1).

Here they are

- the squareness error of the Y axis with respect to the X axis (C0Y; C orientation of Y);
- the squareness errors of the Z axis with respect to the X and Y axis (B0Z, A0Z; B/A orientation of Z);
- the translatory position errors of the A axis in Y and Z direction from its nominal position (Y0A, Z0A; Y/Z position of A);
- the orientation errors of the A axis around the Y and Z axis (B0A, C0A; B/C orientation of A);
- the translatory position errors of the C axis in X and Y direction from its nominal position with respect to the A axis coordinate frame (X0C, Y0C; X/Y position of C);
- the orientation errors of the C axis around the X and Y axis (A0C, B0C; A/B orientation of C).

To be able to include the dominant effects of thermal expansion, the positioning errors of X, Y, Z and A axis are included (linearly depending on the position; EXX, EYY, EZZ as e.g. 10μm/m and EAA as e.g. 10arcsec/rad). EXX and EYY are set to have the same magnitude. An independent identification of both parameters is not possible with this setup (IM_{norm} in equation 3.17 becomes singular if EXX, EYY and EZZ are all included into p).

In this example there are 14 parameters to be identified. Depending on the problem, other parameters can also be included. With methods described in chapter 3.3, the quality of the parameters identified can be evaluated. Other component errors can be included according to the methods described in [36].
Before the measurement, the tool length (here the distance from the spindle nose to the center of the ceramic sphere) has to be determined. This tool length has to be used as a reference length. For accurate machining results, the length difference between each tool and the sphere holder has to be determined.

The origin of the workpiece coordinate system with the probing system is at 130mm in positive X direction from the nominal center of the C axis at $C = 0^\circ$. By synchronously moving all 5 axes, 79 measuring positions are reached. The number and position of poses has been determined by considering the mathematical conditioning of the optimization problem according to chapter 3.5.4. A further increase in the number of measuring poses does not lead to a significantly improved condition, but would of course lengthen the measuring time (from less than 10 minutes for 79 poses). The relevant parts of all linear axes are used. The measuring poses are distributed over 360° of the C axis. For the A axis angles between $-95^\circ$ and the maximum of $+25^\circ$ are used. The measurements at each pose are performed automatically while the machine is stopped. To avoid backlash influences, all axes move in positive direction for the last 0.3mm/0.1° to reach each pose.

According to chapter 3.1.2, an offset compensation of the center of the sphere with respect to the spindle axis has to be made. After the optimization, the identified parameters $Y_{0A}$ (Y position of A) and $X_{0C}$ (X position of C) can be corrected by the measured offset in X and Y direction.

### 3.5.7.4 Mathematical condition evaluation and parameter uncertainty estimation

To evaluate the mathematical condition of the optimization, the method described in chapter 3.5.4 is used. For this setup and procedure, the condition number $\kappa$ is 44. According to heuristics in [44], this is a sufficiently good condition. Numerical experiments show that an equal distribution of the measuring points throughout the workspace is far more important than the actual number of points. The condition number is depending significantly on the number of degrees of freedom measured simultaneously for measurements with one setup. If e.g. twice as many measuring poses would be used, the condition number would decrease by less than 5 % If only two instead of three degrees of freedom were measured, the problem would become singular. That means the condition number would become infinite and the single parameters could not be separated at all.

According to the method described in chapter 3.5.5 the expected uncertainty of the parameters to be identified is checked.

Measurement and identification are simulated for 1000 error combinations. Parameter
standard uncertainties can be seen in table 3.4. The mean spatial TCP deviation $^1$ over 100 random [X,Y,Z,A,C]- poses with the identified parameters are calculated to check the performance of the parameter set. This is done for each of the 1000 runs. The average over all mean spatial TCP deviations is $20\mu m$, for 95% it is below $32\mu m$.

The high values for the standard uncertainties of $EXX$, $EYY$ and $EZZ$ are mainly due to the fact that they cannot be identified independently. Symmetric positioning errors (e.g. every linear axis has a linear positioning error of $10\mu m/100mm$) cannot be detected. To achieve better results, a calibration of at least one of the errors $EXX$, $EYY$, $EZZ$ with a conventional geometric measurement method (see chapter 1.2.1) would be suitable.

Table 3.4: Parameter uncertainties of a model-based calibration from Monte Carlo simulations (1000 runs).

<table>
<thead>
<tr>
<th>Par.</th>
<th>Unit</th>
<th>$u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C0Y$</td>
<td>arcsec</td>
<td>3</td>
</tr>
<tr>
<td>$A0Z$</td>
<td>arcsec</td>
<td>11</td>
</tr>
<tr>
<td>$B0Z$</td>
<td>arcsec</td>
<td>6</td>
</tr>
<tr>
<td>$Y0A$</td>
<td>$\mu m$</td>
<td>6</td>
</tr>
<tr>
<td>$Z0A$</td>
<td>$\mu m$</td>
<td>13</td>
</tr>
<tr>
<td>$X0C$</td>
<td>$\mu m$</td>
<td>3</td>
</tr>
<tr>
<td>$Y0C$</td>
<td>$\mu m$</td>
<td>3</td>
</tr>
<tr>
<td>$B0A$</td>
<td>arcsec</td>
<td>3</td>
</tr>
<tr>
<td>$C0A$</td>
<td>arcsec</td>
<td>3</td>
</tr>
<tr>
<td>$A0C$</td>
<td>arcsec</td>
<td>3</td>
</tr>
<tr>
<td>$B0C$</td>
<td>arcsec</td>
<td>2</td>
</tr>
<tr>
<td>$EXX$</td>
<td>$\mu m/260mm$</td>
<td>26</td>
</tr>
<tr>
<td>$EYY$</td>
<td>$\mu m/260mm$</td>
<td>22</td>
</tr>
<tr>
<td>$EZZ$</td>
<td>$\mu m/260mm$</td>
<td>27</td>
</tr>
<tr>
<td>$EAA$</td>
<td>arcsec/rad</td>
<td>13</td>
</tr>
</tbody>
</table>

$^1$A spatial TCP deviation is defined here to be the square sum of the deviations in X, Y and Z direction between tool and workpiece at one pose.
3.5.7.5 Experimentally testing the calibration quality

With the Monte Carlo simulation, it can be said that in most cases the calibration yields satisfactory results. With certain error combinations, the spatial TCP deviations become too big. The Monte Carlo simulations also show that the quality of the calibration cannot be tested with the same measuring device used for calibration, since parameter errors of the linear axes and the rotary axes could compensate each other (see chapter 3.5.6). To avoid this effect, the calibration quality has been tested by a purely translatory movement with a 3D circular test made with a Double Ball Bar (circle nominally lies in the [111] plane; nominal diameter 300 mm). The measurement setup can be seen in figure 3.6. Here the deviations of the identified parameters from their real values and other errors of the machining center not included into the model cause relative TCP displacements. For circular tests see chapter 1.2.2.

Before the calibration the measured unidirectional mean diameter deviation was $+106\mu m$/$+100\mu m$ (counter-clockwise/ clockwise), the circular form deviation was $39\mu m$/$45\mu m$ (all values according to [12]). After the calibration the unidirectional mean diameter deviation was $-14\mu m$/$-19\mu m$, the circular form deviation was $25\mu m$/$33\mu m$, caused largely by backlash and cyclic axes errors not calibrated.

Figure 3.6: Measurement setup for 3D circular test with a Double Ball Bar
The results of the circular test after calibration can be seen in figure 3.7.

Figure 3.7: Results of 3D circular test after calibration (magnification 1000x), blue: counter-clockwise, red: clockwise

3.5.7.6 Parameter estimation for an assumed improved machine performance

The uncertainty of the parameters to be identified has been determined in the same manner as in chapter 3.3. The same measurement cycle and the same measurement uncertainty are assumed. The kinematic build-up of the machining center stays the same. The difference in this simulation is the higher machine performance assumed, mainly much smaller positioning errors of the 5 axes (it is assumed that now the axes are equipped with state-of-the-art direct measuring systems, i.e. linear scales for the linear axes). The other difference is a better assumed repeatability and a slightly better straightness of the linear movement due to a higher preload of the bearings.
3.5. Error interdependencies effects on model-based calibrations

With the assumptions stated above, it can be seen from table 3.5 that the uncertainties of the single parameters would be much smaller. The effect of the error interdependencies is diminished. Here the benefit of improved machine equipment can be seen directly. The mean spatial TCP deviation (see chapter 3.3) has been calculated. The average value over all 1000 runs is 4μm, for 95% it is below 14μm.

Table 3.5: Parameter uncertainties with improved machine performance of a model-based calibration from Monte Carlo simulations (1000 runs). Compare to table 3.4.

<table>
<thead>
<tr>
<th>Par.</th>
<th>Unit</th>
<th>u</th>
</tr>
</thead>
<tbody>
<tr>
<td>C0Y</td>
<td>arcsec</td>
<td>3</td>
</tr>
<tr>
<td>A0Z</td>
<td>arcsec</td>
<td>4</td>
</tr>
<tr>
<td>B0Z</td>
<td>arcsec</td>
<td>4</td>
</tr>
<tr>
<td>Y0A</td>
<td>μm</td>
<td>2</td>
</tr>
<tr>
<td>Z0A</td>
<td>μm</td>
<td>4</td>
</tr>
<tr>
<td>X0C</td>
<td>μm</td>
<td>2</td>
</tr>
<tr>
<td>Y0C</td>
<td>μm</td>
<td>2</td>
</tr>
<tr>
<td>B0A</td>
<td>arcsec</td>
<td>2</td>
</tr>
<tr>
<td>C0A</td>
<td>arcsec</td>
<td>2</td>
</tr>
<tr>
<td>A0C</td>
<td>arcsec</td>
<td>1</td>
</tr>
<tr>
<td>B0C</td>
<td>arcsec</td>
<td>2</td>
</tr>
<tr>
<td>EXX</td>
<td>μm/260mm</td>
<td>8</td>
</tr>
<tr>
<td>EYY</td>
<td>μm/260mm</td>
<td>8</td>
</tr>
<tr>
<td>EZZ</td>
<td>μm/260mm</td>
<td>7</td>
</tr>
<tr>
<td>EAA</td>
<td>arcsec/rad</td>
<td>4</td>
</tr>
</tbody>
</table>

3.5.8 Calibration example: parallel kinematic Hexaglide

In this chapter a model-based calibration of the Hexaglide as an example for parallel kinematic machine tools is examined. The kinematic structure and the work principle of the Hexaglide have been explained in chapter 2.4.2. The kinematic machine model and the error modeling are shown in chapters 3.5.8.1 and 3.5.8.2. The measurement procedure is addressed in chapter 3.5.8.3. Chapter 3.5.8.4 covers the determination of the mathematical condition as described in chapter 3.5.4 as well as the uncertainty of the model-based calibration as described in chapter 3.5.5. In chapter 3.5.8.5 the results of
a calibration of the Hexaglide are shown. The quality of the calibration is tested with conical circular tests.

3.5.8.1 Kinematic modeling

The kinematic modeling of the nominal Hexaglide kinematic has been described in [95]. For arbitrary orientations of the guideways, it has been extended in [81]. Such a general kinematic model is needed. With it the corresponding actuator positions can be calculated from the work piece coordinates for a given configuration (and vice versa). The following parameters have to be known (compare to figure 3.8):

- The normalized direction vectors of the 6 nominally parallel guideways $v^{X}_{1BS}$ to $v^{X}_{6BS}$ in the base coordinate system (the left and right side of a guideway are considered to be independent to allow a more realistic modeling).

- The X, Y and Z positions of the base joints (origins of the base joint coordinate systems) in the base coordinate system $o^{B1}_{BS}$ to $o^{B6}_{BS}$ when the slides are in their reference positions.

- The X, Y and Z positions of the platform joints (origins of the platform joint coordinate systems) in the platform coordinate system $o^{P1}_{PF}$ to $o^{P6}_{PF}$.

- The lengths of the 6 struts $L1$ to $L6$ (spatial distance between e.g. $o^{B1}_{BS}$ and $o^{P1}_{PF}$ in the same coordinate system).
Figure 3.8: Kinematic of the Hexaglide, as an example $\mathbf{o}_{BS}^B$ and $\mathbf{o}_{PF}^P$ are shown in their respective coordinate systems.
For simplicity the workpiece coordinate system is assumed here to coincide with the base coordinate system. If this is not the case, an additional transformation is needed.

A set of coordinates for one point is given (position and orientation of the Tool Center Point). These coordinates determine the zero position and the orientation of the platform coordinate system with respect to the base coordinate system:

\[
X^{\text{trans}} = \begin{bmatrix} X & Y & Z \end{bmatrix}^T \\
X^{\text{rot}} = \begin{bmatrix} A & B & C \end{bmatrix}^T
\] (3.29) (3.30)

Note that a number of different definitions do exist how the three angles of \( X^{\text{rot}} \) can be defined. The two main possibilities are:

- Three sequential rotations around the axes of a fixed coordinate system (in a given order, e.g. first rotation around the X axis, second around the Y axis, third around the Z axis).

- Euler angles: Three sequential rotations about the moving axes of rotation (the current axis of rotation has been created by the rotation before) in a given order; e.g. first rotation around the X axis of the given initial coordinate system, thereafter rotation around the Y axis of the rotated coordinate system, thereafter rotation around the Z axis of the coordinate system rotated twice.

A good description of these different conventions can be found in [96]. More explanation and yet another angle definition can be found in [97]. In the example given here, Euler angles in the sequence ZYX (the initial coordinate system is the workpiece coordinate system) are used.

With this information and the workpiece coordinates the position of the 6 slides can be calculated for a given geometric configuration.

First of all the rotational transformation matrix \( R \) between the orientation of the platform coordinate system and the base coordinate system must be calculated from the given \( X^{\text{rot}} \). With the selected Euler angles \( R \) is defined as

\[
R = R_X \times R_Y \times R_Z
\] (3.31)
3.5. Error interdependencies effects on model-based calibrations

The X, Y and Z positions in the vectors of the platform joints \( o_{PF}^{P1} \) to \( o_{PF}^{P6} \) must be transformed from the platform coordinate system into the machine coordinate system. For the first joint (as an example) the equation is

\[
o_{BS}^{P1} = R \cdot o_{PF}^{P1} + X^{trans}
\]  

(3.35)

For all other joints the transformation is the same. The current position of a base joint can be calculated as an intersection of a line (motion of the joint on the guideway) with a sphere (center: position of the platform joint, radius: strutlength, see [81]). The equation for the current position of base joint 1 (\( p_{BS}^{B1} \)) on the straight line is

\[
p_{BS}^{B1} = o_{BS}^{B1} + Q_1 \cdot v_{BS}^{X1}
\]  

(3.36)

\( Q_1 \) to \( Q_6 \) are the actuator positions at the current pose that must be identified.

The equation of the current position of base joint 1 \( p_{BS}^{B1} \) on the sphere around the platform joint with the strutlength \( L1 \) as radius is

\[
\| p_{BS}^{B1} - o_{BS}^{P1} \| = L1^2
\]  

(3.37)

Eqns. 3.36 and 3.37 can be solved for the current actuator position \( Q_1 \) (see [81]):

\[
Q_{1,2} = -\frac{\|v_{BS}^{X1}(o_{BS}^{B1} - o_{BS}^{P1})\|}{\|v_{BS}^{X1}\|^2} \pm \sqrt{\left(\frac{\|v_{BS}^{X1}(o_{BS}^{B1} - o_{BS}^{P1})\|}{\|v_{BS}^{X1}\|^2}\right)^2 - \frac{\|v_{BS}^{X1}(o_{BS}^{B1} - o_{BS}^{P1})\|^2 - L1^2}{\|v_{BS}^{X1}\|^2}}
\]  

(3.38)

As expected, there are two mathematical solutions (two intersection points between a straight line and a sphere). Depending on the current actuator one solution has to be
selected. For base joints $B_1$, $B_3$ and $B_5$ the bigger one of the two solutions has to be selected, for base joints $B_2$, $B_4$, and $B_6$ the smaller solution has to be selected (to ensure that the struts are oriented “outwardly”).

When this procedure is repeated for all base joints, the actuator positions of the base joints can be calculated.

$$q = [Q_1 \ Q_2 \ Q_3 \ Q_4 \ Q_5 \ Q_6]^T$$

(3.39)

As described in [95], the forward kinematic transformation between the actuator positions $q$ and the workpiece coordinates $X^{trans}$ and $X^{rot}$ cannot be solved analytically. Starting from an estimated starting value, the workpiece coordinates are calculated numerically using an optimization method. For the Hexaglide this procedure is described in [95], [79]. For other parallel kinematic machines this is described e.g. in [98].

For a general modeling of the geometric errors of the rotary axes of the joints, the joint angles for the current pose must be known. In [81] the calculation of the joint angles is described for the joints of the Hexaglide kinematic. The following parameters have to be defined:

- The first main direction of the joint coordinate system (defined by the orientation of the first rotary axis of the joint) in the respective main coordinate system (base coordinate system or platform coordinate system). For the base joints these are $m_{BS}^{B_1}$ to $m_{BS}^{B_6}$ and for the platform joints $m_{PF}^{P_1}$ to $m_{PF}^{P_6}$. For the base joints of the Hexaglide $m_{BS}^{B_1}$ to $m_{BS}^{B_6}$ are for example defined to be $[0 \ 0 \ -1]^T$ in the nominal configuration.

- The second main direction of the joint coordinate system (coinciding with the projection of the second rotary axis of the joint on a plane perpendicular to the first main direction at the arbitrarily selected rotary angle $0^\circ$). For the base joints these are $n_{BS}^{B_1}$ to $n_{BS}^{B_6}$ and for the platform joints $n_{PF}^{P_1}$ to $n_{PF}^{P_6}$.

In the calculations below, the joints of strut 1 will always be taken as an example. For all other joints the calculation is exactly the same. So with the direction of strut 1 defined to be

$$d_{BS} = p_{BS}^{B_1} - o_{BS}^{P_1}$$

(3.40)

($o_{BS}^{P_1}$ known from equation 3.35), the first joint angle $\vartheta^{B_1}$ of the first base joint (rotation around the first axis) can be calculated as

$$\vartheta^{B_1} = \arccos \frac{n_{BS}^{B_1} \cdot (m_{BS}^{B_1} \times (d_{BS} \times m_{BS}^{B_1})))}{\|n_{BS}^{B_1}\| \cdot \|m_{BS}^{B_1} \times (d_{BS} \times m_{BS}^{B_1})\|}$$

(3.41)
For the platform joints, all relevant vectors must be known in the same coordinate system. Therefore the main direction of the platform joint coordinate systems are transformed into the base coordinate system (with $R$ taken from equation 3.31):

\[ m_{P1}^{BS} = R m_{P1}^{PF} \] \hfill (3.42)

\[ n_{P1}^{BS} = R n_{P1}^{PF} \] \hfill (3.43)

Then the calculation is similar to equation 3.41.

\[ \vartheta^{P1} = \arccos \frac{n_{P1}^{BS} \cdot (m_{P1}^{BS} \times (d1_{BS} \times m_{P1}^{BS}))}{\|n_{P1}^{BS}\| \cdot \|m_{P1}^{BS}\| \cdot \|d1_{BS}\|} \] \hfill (3.44)

The second joint angle $\varphi^{B1}$ of the first base joint (rotation around the second axis) can be calculated as

\[ \varphi^{B1} = \arccos \frac{m_{B1}^{BS} \cdot d1_{BS}}{\|m_{B1}^{BS}\| \cdot \|d1_{BS}\|} \] \hfill (3.45)

For the platform joint the solution is again similar:

\[ \varphi^{P1} = \arccos \frac{m_{P1}^{BS} \cdot d1_{BS}}{\|m_{P1}^{BS}\| \cdot \|d1_{BS}\|} \] \hfill (3.46)

The base joints do have a third axis of rotation. It is trivial to determine these angles $\psi_{B1}$ to $\psi_{B6}$. With a certain angle of the first axis of a platform joint, e.g. $\vartheta^{P1}$, the corresponding strut is turned around its axis by the same angle. So the angles of the third axis of rotation is defined to be

\[ \psi^{B1} = -\vartheta^{P1} \] \hfill (3.47)

The angle calculations are only valid for non-singular positions (see [81]).

### 3.5.8.2 Geometric errors

The error modeling is similar to the modeling of geometric errors for a conventional 5-axes machine (see chapter 3.5.7.2). Again all geometric errors can be simulated pointwise.

In accordance with the description of location errors in chapter 1.1.1 and component errors in 1.1.3, the geometric errors of the Hexaglide kinematic can be seen in tables 3.6 and 3.7 (see also [80]).

Besides the base coordinate system and the platform coordinate system each joint of the Hexaglide has its own joint coordinate system (12 joint coordinate systems in
The joint coordinate systems are needed to model the component errors of the rotary axes of the joints.

Geometric errors that have no influence on the TCP pose are omitted (e.g. position and orientation errors of the joint axes with respect to their joint coordinate system do only lead to small changes in the joint angles; these second order effects leave the TCP pose practically unaffected).

Table 3.6: Location errors of the Hexaglide kinematic.

<table>
<thead>
<tr>
<th>Location errors</th>
<th>Number of components</th>
<th>Errors per component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error in the fixed strut length</td>
<td>6 struts</td>
<td>1</td>
</tr>
<tr>
<td>Orientation deviations (angles around Y and Z) of the linear motion of the slides (nominally parallel to X) in the base coordinate system</td>
<td>6 slides</td>
<td>2</td>
</tr>
<tr>
<td>Zero position of the joint coordinate systems of the base joints in X,Y,Z in the base coordinate system</td>
<td>6 joints</td>
<td>3</td>
</tr>
<tr>
<td>Zero position of the joint coordinate systems of the platform joints in X,Y,Z in the base coordinate system</td>
<td>6 joints</td>
<td>3</td>
</tr>
<tr>
<td>Translatory position offset in X,Y of the spindle in the platform coordinate system</td>
<td>1 spindle</td>
<td>2</td>
</tr>
<tr>
<td>Orientation deviation (angles around X and Y) of the spindle in the platform coordinate system</td>
<td>1 spindle</td>
<td>2</td>
</tr>
<tr>
<td>Definition of the base coordinate system</td>
<td>1 coordinate system</td>
<td>-3 pos. -3 orient.</td>
</tr>
<tr>
<td>Definition of the platform coordinate system</td>
<td>1 coordinate system</td>
<td>-3 pos. -3 orient.</td>
</tr>
<tr>
<td><strong>Total Number of location errors</strong></td>
<td></td>
<td><strong>46</strong></td>
</tr>
</tbody>
</table>
3.5. Error interdependencies effects on model-based calibrations

Some errors can be defined to be zero in order to define the base coordinate system of the PKM (for example the average line of motion of one slide on its linear guideway can be determined to define the X direction of the base coordinate system, so this motion cannot have any location error; see chapter 1.1.2). Again for the model-based calibration the errors of the spindle are not identified. The spindle is not turned during measurement. The spindle errors have to be identified with another measurement.

Table 3.7: Component errors of the Hexaglide kinematic.

<table>
<thead>
<tr>
<th>Component errors</th>
<th>Number of components</th>
<th>Errors per component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positioning deviations of the slides in the base coordinate system</td>
<td>6 slides</td>
<td>1</td>
</tr>
<tr>
<td>Straightness deviations (horizontal and vertical) of the linear motion of the</td>
<td>6 slides</td>
<td>2</td>
</tr>
<tr>
<td>slides in the base coordinate system</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial error motions of each base joint axis in its respective joint coordinate</td>
<td>6 joints × 3 axes</td>
<td>1</td>
</tr>
<tr>
<td>system</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radial error motions of each base joint axis in its respective joint coordinate</td>
<td>6 joints × 3 axes</td>
<td>2</td>
</tr>
<tr>
<td>system (in two directions perpendicular to the axis orientation vector)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial error motions of each platform joint axis in its respective joint coordinate</td>
<td>6 joints × 2 axes</td>
<td>1</td>
</tr>
<tr>
<td>system</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Radial error motions of each platform joint axis in its respective joint</td>
<td>6 joints × 2 axes</td>
<td>2</td>
</tr>
<tr>
<td>coordinate system (in two directions perpendicular to the axis orientation vector)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial error motion of the spindle in the platform coordinate system</td>
<td>1 spindle</td>
<td>1</td>
</tr>
<tr>
<td>Radial error motions of the spindle in the platform coordinate system</td>
<td>1 spindle</td>
<td>2</td>
</tr>
<tr>
<td>(in two directions perpendicular to the spindle orientation vector)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tilt error motions of the spindle in the platform coordinate system</td>
<td>1 spindle</td>
<td>2</td>
</tr>
<tr>
<td>(around X and Y axis)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total Number of component errors</strong></td>
<td></td>
<td><strong>113</strong></td>
</tr>
</tbody>
</table>
The modeling of the geometric errors of the Hexaglide is shown here (compare to chapter 3.5.8.1). Note that the different component errors (here straightness deviations and axial and radial error motions) cannot be described by simple functions. The courses of such errors can be arbitrarily complex. A realistic course of the errors can only be approximated. Details can be found in the appendix (see chapter A.2):

- The orientation deviations of the linear motions of the slides change the direction vectors of the 6 nominally parallel guideways $\mathbf{v}_{BS}^{X1}$ to $\mathbf{v}_{BS}^{X6}$.
- The translatory position errors of the base joints change $\mathbf{o}_{BS}^{B1}$ to $\mathbf{o}_{BS}^{B6}$.
- The translatory position errors of the platform joints change $\mathbf{o}_{PF}^{P1}$ to $\mathbf{o}_{PF}^{P6}$.
- The positioning deviations of the slides in the base coordinate system change the values of $Q1$ to $Q6$.
- The straightness deviations (horizontal and vertical) of the linear motion of the slides in the base coordinate system change the momentary values of $\mathbf{o}_{BS}^{B1}$ to $\mathbf{o}_{BS}^{B6}$.
- The axial and radial error motions of the first base joint axis $\mathbf{r}_{B1}^{B1}$ to $\mathbf{r}_{B1}^{B6}$ change the current value of $\mathbf{o}_{BS}^{B1}$ to $\mathbf{o}_{BS}^{B6}$ directly.

\[ \Delta \mathbf{o}_{BS}^{B1} = \mathbf{r}_{B1}^{B1}(\vartheta^{B1}) \] (3.48)

- The axial and radial error motions of the second base joint axis $\mathbf{r}_{B1}^{B2}$ to $\mathbf{r}_{B1}^{B6}$ must be transformed into the base coordinate system (rotation around the negative angle of the first base joint).

\[ \Delta \mathbf{o}_{BS}^{B1} = \begin{pmatrix} \cos(-\vartheta^{B1}) & -\sin(-\vartheta^{B1}) & 0 \\ \sin(-\vartheta^{B1}) & \cos(-\vartheta^{B1}) & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathbf{r}_{B1}^{B1}(\varphi^{B1}) \] (3.49)

The first joint axis is defined to be the Z axis of the coordinate frame here (arbitrary selection).

- The axial and radial error motions of the third base joint axis $\mathbf{r}_{B1}^{B3}$ to $\mathbf{r}_{B1}^{B6}$ must be transformed into the base coordinate system (first rotation around the negative angle of the second base joint, thereafter rotation around the negative angle of the
first base joint). 

\[
\Delta o_{BS}^{B1} = \begin{pmatrix}
\cos(-\vartheta_{B1}) - \sin(-\vartheta_{B1}) & 0 \\
\sin(-\vartheta_{B1}) \cos(-\vartheta_{B1}) & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & \cos(-\varphi_{B1}) - \sin(-\varphi_{B1}) \\
0 & \sin(-\varphi_{B1}) \cos(-\varphi_{B1}) & 0
\end{pmatrix} \psi_{B1}^{B1} \tag{3.50}
\]

The first joint axis is defined to be the X axis of the coordinate frame here.

- The axial and radial error motions of the first platform joint axis \( r1_{P1} \) to \( r1_{P6} \) can be calculated similarly to equation 3.51.

\[
\Delta o_{PF}^{P1} = r1_{P1}^{P1}(\vartheta_{P1}) \tag{3.51}
\]

- The axial and radial error motions of the second platform joint axis \( r2_{P1} \) to \( r2_{P6} \) must be transformed into the platform coordinate system (rotation around the negative angle of the first platform joint).

\[
\Delta o_{PF}^{P1} = \begin{pmatrix}
\cos(-\vartheta_{P1}) - \sin(-\vartheta_{P1}) & 0 \\
\sin(-\vartheta_{P1}) \cos(-\vartheta_{P1}) & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & \cos(-\varphi_{P1}) - \sin(-\varphi_{P1}) \\
0 & \sin(-\varphi_{P1}) \cos(-\varphi_{P1}) & 0
\end{pmatrix} \varphi_{P1}^{P1} \tag{3.52}
\]

The first joint axis is defined to be the Z axis of the coordinate frame here (arbitrary selection).

As for the serial kinematic machine tool examined in chapter 3.5.7 for component errors, the actual error value at one \( X^{\text{trans}}/X^{\text{rot}} \) is calculated as a superposition of the first Fourier-Series harmonics, plus in some cases the value of a sine function depending directly on the axis design (see appendix).

With the given measuring device (see chapter 2.2) only changes of the absolute TCP deviations between the different measuring poses can be acquired (relative deviations between the measuring poses \( \Delta X^{\text{trans}} \)) since the exact location of the probing system on the machine table is unknown. So the optimization task is (in accordance with equation 3.11)

\[
\text{Min} \sum_{j=1}^{m} \left( \Delta X_{\text{measured}}^{\text{trans}} - \Delta X_{\text{calculated}}^{\text{trans}}(p) \right)^2 \tag{3.53}
\]

As described in chapter 3.5.3, this is solved with a Levenberg-Marquart algorithm.
3.5.8.3 Measurement procedure

When the number of geometric errors of the Hexaglide (see tables 3.6 and 3.7) is compared to the number of geometric errors of a conventional 5-axes machining center (see tables 3.2 and 3.3), it becomes obvious that a parallel kinematic machine such as the Hexaglide has in comparison much more possible errors. This causes a worse mathematical condition of the identification (see chapter 3.5.8.4) and more disturbances due to errors that have not been considered in the set of parameters \( p \) to be identified (see equation 3.15).

In order to find a suitable compromise, two different measurements were decided to be combined to achieve a better mathematical condition of the optimization problem. The first part of the measurement information comes from a 3D Ball Plate measurement (see chapter 2.1). The measurement is basically the same as presented in chapter 2.4.2. Now the measurement information is not used for a direct Space Error Compensation, but for a model-based calibration. Again, only changes in the absolute TCP displacements between the different measuring poses (relative displacements \( \Delta X_{BP}^{\text{trans}} \)) can be used (see chapter 3.5.3).

In a second step the probing system described in chapter 3.1 is used. It is fixed on the machine table. The Hexaglide is moved into different measuring poses, all with the same nominal translatory position \( X^{\text{trans}} \), but with different rotary angles \( X^{\text{rot}} \). Geometric errors of the Hexaglide cause relative translatory displacements between the measuring poses \( \Delta X_{RA}^{\text{trans}} \) that can be measured.

With the two measurements, the optimization task can be defined as (in accordance with equation 3.11)

\[
\text{Min} \sum_{j=1}^{m} (\Delta X_{\text{measured}}^{\text{trans}} - \Delta X_{\text{calculated}}^{\text{trans}}(p))^{2}
\]  

(3.54)

with

\[
\Delta X_{\text{measured}}^{\text{trans}} = \begin{bmatrix} \Delta X_{BP\text{measured}}^{\text{trans}} & \Delta X_{RA\text{measured}}^{\text{trans}} \end{bmatrix}^{T} 
\]  

(3.55)

\[
\Delta X_{\text{calculated}}^{\text{trans}} = \begin{bmatrix} \Delta X_{BP\text{calculated}}^{\text{trans}} & \Delta X_{RA\text{calculated}}^{\text{trans}} \end{bmatrix}^{T} 
\]  

(3.56)

Due to the high number of geometric errors, only the most important location errors are included into the set of parameters to be identified. Here they are

- the length of the struts \( L1 \) to \( L6 \);

- the \( X, Y \) and \( Z \) positions of the base joints (origins of the base joint coordinate systems; see chapter 3.5.8.2) in the base coordinate system \( o_{BS1}^{B} \) to \( o_{BS6}^{B} \).
• the X, Y and Z positions of the platform joints (origins of the platform joint coordinate systems; see chapter 3.5.8.2) in the platform coordinate system \( \mathbf{o}_{P1} \) to \( \mathbf{o}_{P6} \).

In this example there are in total 42 parameters to be identified. Only 30 are independent. 6 parameters can be used to define the base coordinate system (see chapter 1.1.2). 6 parameters can be used to define the the platform coordinate system. Nevertheless all 42 parameters are identified. The identified solution is therefore not bijective. The condition number of the identification is infinite (see chapter 3.5.4.3).

For the example here this is not important. Although an infinite number of global minima (all with the same value) does exist, the one that is identified is just as good as any other. In fact, the different minima correspond to a different definition of the base and platform coordinate systems’ origins and axes directions. The relative vectors between the different joints stay the same for all minima. They are just defined in different coordinate systems.

To be able to evaluate the relevant mathematical condition, the influence matrix \( \mathbf{IM}_{\text{norm}} \) is determined according to equation 3.17 (see chapter 3.5.4). Instead of taking the largest and the smallest singular value of \( \mathbf{IM}_{\text{norm}} \) to determine the condition number \( \kappa \), the largest and the smallest singular value corresponding to a linearly independent eigenvector of \( \mathbf{IM}_{\text{norm}}^T \mathbf{IM}_{\text{norm}} \) is taken. So with 30 independent parameters, equation 3.19 changes to

\[
\kappa = \frac{\sigma_1}{\sigma_{30}} \tag{3.57}
\]

When the ball plate is used 48 measuring poses are reached. To avoid backlash influences, all axes move in positive direction for the last 0.3\,mm to reach each pose. With the probing system ("R-Test") at one position, another 41 measuring poses are reached by tilting the platform at the same nominal position. Two exemplary measuring poses are shown in figures 3.9 (overview) and 3.10 (close up). The measurement is similar to the measurement described in chapter 3.5.7.3. With this procedure, the maximum translatory working volume with a horizontal platform is used during the 3D Ball Plate measurement. During the measurement with the tilting platform, the maximum rotary working volume at one point is used.

With a PKM it is not easy to avoid the influence of backlash during the measurement with a tilting platform. Depending on the current pose e.g. a movement of 0.3\,mm of all slides in positive direction to reach a pose (analogous to the measurement with the ball plate or with the conventional 5-axes milling machine) will cause some rotary axes of the joints to move in positive direction in some poses, while in negative in others.

To minimize the effect of backlash, each of the 41 poses is reached with the slides moving in positive direction as well as with the slides moving in negative direction to reach the
Figure 3.9: Measurement of translatory TCP deviations with a tilted platform (exemplary measuring pose)

Figure 3.10: Measurement of translatory TCP deviations with a tilted platform (close up of an exemplary measuring pose)

pose. The two sets of measured values are averaged. The averaged sets are the input for the identification according to equation 3.54.

To correct the sphere center offset (see chapter 3.1.2), the positions of the 6 platform joints in the vectors $\mathbf{o}_{PP1}^{P1}$ to $\mathbf{o}_{PP6}^{P6}$ can be corrected by the same value in X respectively Y direction.
3.5. Error interdependencies effects on model-based calibrations

3.5.8.4 Mathematical condition evaluation and parameter uncertainty estimation

The 41 measuring poses with a tilted platform (with the “R-Test”) are selected to complement the measuring information from the 3D Ball Plate ideally, so that the condition number is optimal. To do this a Simplex optimization routine is used [99]. The resulting condition number $\kappa$ according to equation 3.57 is about 200 for the optimization with the measurement of relative TCP displacements in three degrees of freedom at 48 plus 41 poses. In comparison to the model-based calibration of a conventional 5-axes machining center as described in chapter 3.5.7, it becomes obvious that the mathematical condition is much worse although the measurement procedure is much more complex (use of two instead of one measuring device). A further increase in the number of measuring poses would not lead to a significant improvement of the condition number. If only 41 measuring poses with the tilted platform were used without the ball plate, the condition number for identifying the parameters would be about 1070.

When performing a parameter uncertainty estimation according to chapter 3.3, the influences of the bad mathematical condition due to the many parameters and of the many neglected errors becomes obvious. As shown in tables 3.8 to 3.10 the uncertainties of the parameters become very high. Due to the redundancies in the identification (only 30 of the 42 parameters are independent), a fitting of the identified positions of the base and platform joints has been made to best fit the zero positions of the joints in the respective Monte Carlo generated machine kinematic (compare to chapter 3.3).

The asymmetries in the identified uncertainties of struts 1 and 2, 3 and 4 or 5 and 6 have two reasons. First of all the selected measuring points, especially for the measurement with the tilting platform, are not symmetric. Second, for the Monte Carlo simulation, the identified configuration has been selected, where the identified parameters deviate several millimeters from their nominal values.
Table 3.8: Strut length uncertainties of a model-based calibration from Monte Carlo simulations (1000 runs)

<table>
<thead>
<tr>
<th>Par.</th>
<th>Unit</th>
<th>u</th>
</tr>
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<tr>
<td>$L_1$</td>
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</tr>
<tr>
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<td>$\mu m$</td>
<td>155</td>
</tr>
<tr>
<td>$L_3$</td>
<td>$\mu m$</td>
<td>166</td>
</tr>
<tr>
<td>$L_4$</td>
<td>$\mu m$</td>
<td>181</td>
</tr>
<tr>
<td>$L_5$</td>
<td>$\mu m$</td>
<td>188</td>
</tr>
<tr>
<td>$L_6$</td>
<td>$\mu m$</td>
<td>184</td>
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</table>
3.5. Error interdependencies effects on model-based calibrations

Table 3.9: Base joint position uncertainties of a model-based calibration from Monte Carlo simulations (1000 runs), comparison of least-squares fitted parameters

<table>
<thead>
<tr>
<th>Par.</th>
<th>Unit</th>
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</thead>
<tbody>
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<td>$\mu m$</td>
<td>114</td>
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<td>$o_{B4}^{BS}$ (X dir.)</td>
<td>$\mu m$</td>
<td>71</td>
</tr>
<tr>
<td>$o_{B5}^{BS}$ (X dir.)</td>
<td>$\mu m$</td>
<td>53</td>
</tr>
<tr>
<td>$o_{B6}^{BS}$ (X dir.)</td>
<td>$\mu m$</td>
<td>79</td>
</tr>
<tr>
<td>$o_{B1}^{BS}$ (Y dir.)</td>
<td>$\mu m$</td>
<td>101</td>
</tr>
<tr>
<td>$o_{B2}^{BS}$ (Y dir.)</td>
<td>$\mu m$</td>
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</tr>
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<td>$o_{B3}^{BS}$ (Y dir.)</td>
<td>$\mu m$</td>
<td>82</td>
</tr>
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<td>$o_{B4}^{BS}$ (Y dir.)</td>
<td>$\mu m$</td>
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<td>69</td>
</tr>
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<td>$o_{B6}^{BS}$ (Y dir.)</td>
<td>$\mu m$</td>
<td>79</td>
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Table 3.10: Platform joint position uncertainties of a model-based calibration from Monte Carlo simulations (1000 runs), comparison of least-squares fitted parameters

<table>
<thead>
<tr>
<th>Par.</th>
<th>Unit</th>
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</tr>
</thead>
<tbody>
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<td>45</td>
</tr>
<tr>
<td>( o_{PF}^{P2} ) (X dir.)</td>
<td>( \mu m )</td>
<td>47</td>
</tr>
<tr>
<td>( o_{PF}^{P3} ) (X dir.)</td>
<td>( \mu m )</td>
<td>45</td>
</tr>
<tr>
<td>( o_{PF}^{P4} ) (X dir.)</td>
<td>( \mu m )</td>
<td>48</td>
</tr>
<tr>
<td>( o_{PF}^{P5} ) (X dir.)</td>
<td>( \mu m )</td>
<td>52</td>
</tr>
<tr>
<td>( o_{PF}^{P6} ) (X dir.)</td>
<td>( \mu m )</td>
<td>51</td>
</tr>
<tr>
<td>( o_{PF}^{P1} ) (Y dir.)</td>
<td>( \mu m )</td>
<td>36</td>
</tr>
<tr>
<td>( o_{PF}^{P2} ) (Y dir.)</td>
<td>( \mu m )</td>
<td>48</td>
</tr>
<tr>
<td>( o_{PF}^{P3} ) (Y dir.)</td>
<td>( \mu m )</td>
<td>19</td>
</tr>
<tr>
<td>( o_{PF}^{P4} ) (Y dir.)</td>
<td>( \mu m )</td>
<td>30</td>
</tr>
<tr>
<td>( o_{PF}^{P5} ) (Y dir.)</td>
<td>( \mu m )</td>
<td>28</td>
</tr>
<tr>
<td>( o_{PF}^{P6} ) (Y dir.)</td>
<td>( \mu m )</td>
<td>30</td>
</tr>
<tr>
<td>( o_{PF}^{P1} ) (Z dir.)</td>
<td>( \mu m )</td>
<td>25</td>
</tr>
<tr>
<td>( o_{PF}^{P2} ) (Z dir.)</td>
<td>( \mu m )</td>
<td>26</td>
</tr>
<tr>
<td>( o_{PF}^{P3} ) (Z dir.)</td>
<td>( \mu m )</td>
<td>22</td>
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<td>( \mu m )</td>
<td>20</td>
</tr>
<tr>
<td>( o_{PF}^{P5} ) (Z dir.)</td>
<td>( \mu m )</td>
<td>52</td>
</tr>
<tr>
<td>( o_{PF}^{P6} ) (Z dir.)</td>
<td>( \mu m )</td>
<td>40</td>
</tr>
</tbody>
</table>
3.5.8.5 Experimentally checking the calibration quality

For the 41 swiveling poses the TCP displacements after the calibration are within a range of $75 \, \mu m$ in X, $77 \, \mu m$ in Y and $65 \, \mu m$ in Z direction. The measured ranges of backlash at these positions were $15 \, \mu m$ in X, $20 \, \mu m$ in Y and $11 \, \mu m$ in Z direction. For the ball plate measurements the displacements are within a range of $32 \, \mu m$ in X and Y and $31 \, \mu m$ in Z direction (see figures 3.11 and 3.12). These measured displacements are mainly due to the optimization residue (the part of the measured information during the calibration measurement that could not be allocated to the parameters $p$.) Compared to the displacements measured with the nominal configuration the major part of the displacements could be accounted for (for the 3D ball plate 98.4 % in X, 97.8 % in Y and 98.8 % in Z direction, compare to figures 2.19 and 2.20).

The quality of the calibration cannot be determined by the residual error at the measuring poses used for calibration as shown above. But here a small residue at the poses of the 3D Ball Plate corresponds to a good geometric performance of the Hexaglide when used only with a untilted platform. This is different to other measuring methods usually used for model-based calibrations, where a small residue at the measured points is of hardly any practical value.

Of course with the 3D ball plate, the residual errors at its measuring poses could be compensated (Space Error Compensation). To test the quality of the calibration, an independent measurement is needed (see chapter 3.5.6).
Figure 3.11: Measured relative TCP displacements after calibration, black: nominal grid, red: measured grid, magnification 1500x, XY view; coordinate system built with least squares method.
3.5. Error interdependencies effects on model-based calibrations

Figure 3.12: Measured relative TCP displacements after calibration, black: nominal grid, red: measured grid, magnification 1500x, YZ view; coordinate system built with least squares method.
Here a continuous circular test (nominal diameter 300mm) is made where the spindle axis is describing a conical shape (the spindle axis is tilted radially by nominally 15° versus the coordinate Z-direction). Two positions on the measuring path are shown in figure 3.13. By having a constantly changing orientation of the spindle axis, the circular test is different from the measurements used for the calibration. Such a conical circular test was proposed for serial kinematic 5-axes machine tools in [26])

Figure 3.13: Two points on the measuring path of a conical circular test

The result of the circular test is shown in figure 3.14. For the counterclockwise circle the measured circular form deviation (according to [12]) was 92 μm. For the clockwise circle it was 85 μm. To achieve such a movement on a conventional 5-axes machining center, all 5 axes would have to be moved.

It is apparent that the result is worse than the residual errors at the measuring poses (for swiveling poses and 3D Ball Plate measurements). This was to be expected. Some of the effects of the errors that are not part of the 30 independent parameters in $p$ affect the optimization routine, thus falsifying the identified combination. This happens easily especially with a limited mathematical condition as with this problem here. From the parameter uncertainty estimation shown above an average circular form deviation of 93 μm was expected due to geometric effects.
Figure 3.14: Results of conical circular test after calibration (magnification 1000x), red: counterclockwise, blue: clockwise
Chapter 4

Conclusion and Outlook

4.1 Conclusion

In this work methods for improving the geometric calibration of machining centers are presented. With the 3D Ball Plate method described here, error interdependencies become apparent. Due to the spatial measurement throughout the workspace of the machine under test, a Space Error Compensation of the TCP displacement resulting from a combination of different errors becomes possible.

Wherever such a compensation is not sufficient, e.g. for 5-axes machining centers or even for parallel kinematics machine tools, a method for examining complex kinematic or model-based calibration methods is introduced, paying special attention to the resulting performance that can be expected.

For both methods measuring devices are shown that have - compared to other methods - very small measurement uncertainties, partly due to the new identification method of the probe orientations presented here. Both devices allow much faster calibrations than common today.

With the 3D Ball Plate for example three squareness measurements, three positioning measurements, 6 straightness measurements and the roll, pitch and yaw measurements for X and Y axis can be made with one setup for a machining center with a vertical spindle. All this information is gained with one measurement setup. The time required for a measurement with the 3D Ball Plate is reduced for a 3-axes machining center as shown in this work to approximately 45 minutes, while measurements to gain the same information with conventional setups are taking about two work days. Due to the arrangement of the measuring points as a regular spatial grid, the measurement interpretation is much easier and clearer when compared to a conventional approach where different measuring devices
and different evaluation methods are used.

A model-based calibration as shown for example for a 5-axes machining center takes less than 10 minutes measuring time and replaces single measurements for 14 different geometric errors, thus the number of required measurement setups is greatly reduced. The short calibration time allows frequent re-calibrations of machining centers. E.g. before the machining of parts with critical tolerances, the machine tool could be re-calibrated. Model-based calibrations are especially suitable for this task because of the extremely short measuring times and because they are suitable for automated measuring process. With such measurement methods, a changed state of the machine tool due to thermal drift could be identified before machining.

To be applicable in practice, the resulting performance of such complex measurements must be estimated. Here an uncertainty estimation method is presented that considers both the mathematical condition of the result analysis as well as the physical behavior of the machine tool. With it the effect of the geometric errors on a calibration result becomes obvious. It is the base for systematic planning and optimizing of measurements. As can be seen in examples presented in this work, the interdependencies of the errors often deteriorate the resulting performance much more than the measurement uncertainty of the measuring device, whose effect on the calibration result is often even practically negligible.

This is an advantage compared to the current state of the art where such effects are neglected completely. Thus the uncertainties of identified values of machine errors remained unknown, no systematic planning of a suitable measurement approach could be done, no direct relation between the machine build-up and quality on the calibration result was established.

With such uncertainty estimation method, it becomes apparent that the achievable calibration quality is depending primarily on the kinematic structure of the machine. The number and magnitude of the geometric errors are decisive as well as the transmission factor of geometric errors in relative TCP displacements.

The other main factor on the achievable calibration quality is determined by the machine performance. Errors that are difficult to be compensated, such as cyclic deviations with relatively short wavelength are deteriorating the quality of calibrations. Besides these effects such errors have also of course a direct influence on machined parts (“You have to pay for bad axes twice!”). The effect of improvements either of the machine tool (e.g. linear scales instead of resolvers) or of the measurement procedure (e.g. measurement in more degrees of freedom) can be quantified.
4.2 Outlook

In future work the uncertainty estimation method presented here could be used for an automated search of an optimal calibration method. Another application would be the identification of bottlenecks of machine tools that prevent better achievable geometric performances. The ideal structure of machine tools for good geometric behavior could be identified.

The 3D Ball Plate method could be expanded for machining centers with horizontal spindles. From the redundant measurements not only the geometric errors but also the elastic deviations could be identified.

This work was also published as “Fortschritt-Berichte VDI, Reihe 2 Nr. 664”, see [100].
Appendix: Modeling of geometric errors

In this chapter the modeling of geometric errors for a 5-axes machining center (see chapter A.1) as well as for the parallel kinematic Hexaglide are shown (see chapter A.2).

A.1 Modeling of geometric errors for 5-axes machining centers

In this chapter the geometric error modeling for serial kinematic 5-axes machining centers is described.

The nominal configuration of the machine has to be known (see chapter 3.5.7.1 and figure 3.5). The axis positions $q^{\text{trans}} / q^{\text{rot}}$ and the configuration are changed by geometric errors from their nominal values. A description of the errors can be found in chapter 1.1.

The axis positions and the configuration are changed in the following way:

- The axis positions of the linear axes $q^{\text{trans}}$ are changed directly by the zero positions (e.g. $X0X$) of the axis and by the positioning error (e.g. $EXX$ depending on $Q_X$).
- The straightness errors of the two other linear axes have the same effect (e.g. the straightness of the Z motion in X direction, depending on $Q_Z$ and the straightness of the Y motion in X direction, depending on $Q_Y$).

The actual value of $Q_X$, depending on these errors and the nominal value $Q_{X\text{nom}}$ can for example be computed as

$$Q_X = Q_{X\text{nom}} + X0X + EXX(Q_{X\text{nom}}) + EXY(Q_{Y\text{nom}}) + EXZ(Q_{Z\text{nom}}) \quad (A.1)$$

For the Y and Z direction the computation is done accordingly. The component errors can be modeled as the superposition of different Fourier harmonics (see equation 1.1).
In the example shown here usually the first three harmonics are used. E.g. for the straightness of Y in X direction the superposition is computed to be

\[ E_{XY}(Q_{Ynom}) = \sum_{j=1}^{3} E_{XYj} \cos(j \frac{Q_{Ynom}}{Q_{Ymax} - Q_{Ymin}} \pi + \varphi_{E_{XYj}}) \]  \hspace{1cm} (A.2)

For linear axes the reference length of the Fourier harmonics is chosen to be twice as long as the axis length (e.g. \( Q_{Ymax} - Q_{Ymin} \)). With this assumption, the first harmonic a e.g. a straightness error corresponds to half a full wave. This is suitable for modeling a simple bending or sagging of a guideway.

For the positioning errors like \( E_{XX} \) also a linear contribution \( E_{XX_{lin}} \) and a cyclic positioning error \( E_{XX_{cyc}} \), corresponding to the pitch of the axis’ ball screw \( p_{ballscrew} \) are added:

\[ E_{XX}(Q_{Xnom}) = E_{XX_{lin}} Q_{Xnom} + E_{XX_{cyc}} E_{XXpitch} \cos \left( \frac{Q_{Xnom}}{p_{ballscrew}} 2\pi + \varphi_{E_{XXpitch}} \right) + \sum_{j=1}^{3} E_{XXj} \cos \left( j \frac{Q_{Xnom}}{Q_{Xmax} - Q_{Xmin}} \pi + \varphi_{E_{XXj}} \right) \]  \hspace{1cm} (A.3)

- The axes positions of the rotary axes \( \mathbf{q}^{rot} \) are also changed directly by the zero angular positions (e.g. \( C0C \)) of the axes and by positioning errors (e.g. \( ECC \) depending on \( Q_C \)).

The actual value of \( Q_C \), depending on these errors and the nominal value \( Q_{Cnom} \) can for example be computed as

\[ Q_C = Q_{Cnom} + C0C + ECC(Q_{Cnom}) \]  \hspace{1cm} (A.4)

The positioning error can be modeled as the superposition of different Fourier harmonics (see equation 1.1), analogously to equation A.3.

- The zero position of the A axis in the machine tool coordinate system \( \mathbf{o}^A_{MT} \) is changed from its nominal value \( \mathbf{o}^A_{MTnom} \) by the translatory offsets in Y and Z position \( Y0A \) and \( Z0A \), as well as by the current axial and radial error motions \( EXA, EYA \) and \( EZA \) (depending on \( Q_A \)).

\[ \mathbf{o}^A_{MT} = \mathbf{o}^A_{MTnom} + [ 0 \ Y0A \ Z0A ]^T + [ EXA(Q_A) \ EYA(Q_A) \ EZA(Q_A) ]^T \]  \hspace{1cm} (A.5)
The error motions can again be modeled as a superposition of different Fourier harmonics, starting from the second harmonic (corresponding to a elliptic radial error motion). Here \( EYA \) is for example modeled as

\[
EYA(Q_A) = \sum_{j=2}^{4} EYA_j \cos(j \frac{Q_{Anom}}{Q_{Amax} - Q_{Amin}} 2\pi + \varphi_{EYA_j})
\]  

(A.6)

For the C axis the modeling is the same. \( Q_{Cmax} - Q_{Cmin} \) are set to be 360 ° here.

- The zero position of the C axis in the coordinate system of the A axis \( o_{C}^{A} \) is changed from its nominal value \( o_{C}^{Anom} \) by the translatory offsets in X and Y position \( X0C \) and \( Y0C \), as well as by the current axial and radial error motions \( EXC, EYC \) and \( EZC \) (depending on \( Q_{C} \)).

\[
o_{C}^{A} = o_{C}^{Anom} + [\begin{array}{ccc} X0C & Y0C & 0 \end{array}]^T + [\begin{array}{ccc} EXC(Q_{C}) & EYC(Q_{C}) & EZC(Q_{A}) \end{array}]^T
\]  

(A.7)

The component errors can be modeled in the same way as shown in equation A.6.

- The orientation of the linear axes (given in \( \mathbf{v}_X, \mathbf{v}_Y, \mathbf{v}_Z \)) is changed by rotations of different angles around the X, Y and Z axes. These angles are given by the roll respectively tilt motion of the axis and the squareness errors of the current axis to the machine tool coordinate system. The three angles for the X axis are for example

\[
\mathbf{w}^X = [\begin{array}{ccc} w_X^X & w_Y^X & w_Z^X \end{array}]^T
\]  

(A.8)

with

\[
\begin{align*}
 w_X^X &= EAX(Q_X) \\
 w_Y^X &= B0X + EBX(Q_X) \\
 w_Z^X &= C0X + ECX(Q_X)
\end{align*}
\]  

(A.9-11)

With these three angles a matrix of rotation \( \mathbf{R}_X \) can be determined. The angles can for example be defined to be Euler angles as shown in equation 3.31. Due to the very small magnitudes of the angles (in the range of 10 arcsec), the sequence of the rotation can be chosen arbitrarily. It has no relevant impact on the result.

Such a matrix of rotation is determined for every axis. The orientation vectors, e.g. \( \mathbf{v}_X \) are changed by their own matrix of rotation as well as by the matrices of rotation of every axis on which the current axis rests. If for example the X axis is resting on the Y axis which is resting on the machine bed, the orientation vector of the X axis is calculated from its nominal value \( \mathbf{v}_{Xnom} \) as

\[
\mathbf{v}_X = \mathbf{R}_Y \mathbf{R}_X \mathbf{v}_{Xnom}
\]  

(A.12)
Appendix: Modeling of geometric errors

while for the Y axis

\[ \mathbf{v}_Y = R_Y \mathbf{v}_{Y,nom} \]  \hspace{1cm} (A.13)

The current change in the orientation also changes the translatory position of the TCP. This effect is depending on the offset from a reference line (e.g. the position of the guideway). It can be calculated as

\[ \Delta q^{trans} = \omega^X \times \mathbf{o}^{MT}_{TCP} \]  \hspace{1cm} (A.14)

If the current axis is on the workpiece side, the relevant offsets are from a reference line (e.g. the position of the guideway) to the TCP, e.g. \( \mathbf{o}^M_{TCP} \) for the X axis.

If the current axis is on the tool side, the relevant offset includes only the offset from the reference line to the TCP at the machine in zero position and the axis positions of axes that are carried by the current axis.

The result from equation A.14 has to be added to the result of equation A.1 for the final value of \( Q_X \). For the other linear axes, the computation is analogous.

- The orientation of the rotary axes (given in \( \mathbf{v}_A, \mathbf{v}_C \)) is changed by rotations of different angles around the X, Y and Z axes of the respective coordinate system. These angles are given by the angular positioning errors respectively tilt motion of the axis and the squareness errors of the current axis to the respective coordinate system. The three angles for the A axis are for example

\[ \mathbf{w}^A = [ w^A_X \ w^A_Y \ w^A_Z ]^T \]  \hspace{1cm} (A.15)

with

\[ w^A_X = EAA(Q_A) \]  \hspace{1cm} (A.16)
\[ w^A_Y = B0A + EBA(Q_A) \]  \hspace{1cm} (A.17)
\[ w^A_Z = C0A + ECA(Q_A) \]  \hspace{1cm} (A.18)

With these three angles a matrix of rotation \( \mathcal{R}_A \) can be determined as shown for the linear axes. The angles can for example be defined to be Euler angles as shown in equation 3.31. Due to the very small magnitudes of the angles (in the range of 10 arcsec), the sequence of the rotation can again be chosen arbitrarily. It has no relevant impact on the result.

Such an matrix of rotation is determined for every axis. The orientation vectors, e.g. \( \mathbf{v}_A \) are changed by their own matrix of rotation as well as by the matrices of rotation of every axis on which the current axis rests (linear and rotary axes). If for example the A axis is resting on the X axis which is resting on the Y axis on the machine
The orientation vector of the A axis is calculated from its nominal value $\mathbf{v}_{Anom}$ as

$$\mathbf{v}_A = R_Y R_X R_X \mathbf{v}_{Anom} \quad (A.19)$$

### A.2 Modeling of geometric errors for the parallel kinematic Hexaglide

In this chapter the geometric error modeling for parallel kinematic Hexaglide is described. The nominal configuration of the machine has to be known (see chapter 3.5.8.1 and figure 3.8). The axis positions $\mathbf{q}$ and the configuration are changed by geometric errors from their nominal values. A description of the errors can be found in chapter 1.1.

The axis positions and the configuration are changed in the following way:

- The lengths of the struts are changed directly from the nominal values, e.g. for $L1$:

$$L1 = L1_{nom} + L10 L1 \quad (A.20)$$

- The orientation vectors of the linear motion of the slides $\mathbf{v}_{BS}^{X_1}$ to $\mathbf{v}_{BS}^{X_6}$ are changed by rotations of different angles around the Y and Z axes. These angles are given by orientation errors of the slide motions with respect to the base coordinate system (e.g. $B0Q1, C0Q1$). Since the reference line is going through the origin of the joint coordinate systems, there is no influence of the roll and tilt error motions (see chapter 3.5.8.2). The angles for the Q1 axis are for example

$$\mathbf{w}_{Q1} = [0 \quad B0Q1 \quad C0Q1]^T \quad (A.21)$$

With the angles e.g. in $\mathbf{w}_{Q1}$, a matrix of rotation, here $R_{Q1}$ is determined. This can be done analogously to the procedure shown in equation 3.31. Again the angles in e.g. $\mathbf{w}_{Q1}$ can for example be defined as Euler angles.

- The positions of the base joints $\mathbf{o}_{BS}^{B1}$ to $\mathbf{o}_{BS}^{B6}$ are changed by the zero position errors (at the chosen reference position) of the Hexaglide (e.g. $\Delta o_{BSzero}^{B1}$) and the straightness errors of the slides in the current position (e.g. $EYQ1(Q1)$).

$$\mathbf{o}_{BS}^{B1} = \mathbf{o}_{BS}^{B1_{nom}} + \Delta o_{BSzero}^{B1} + [0 \quad EYQ1(Q1) \quad EZQ1(Q1)]^T \quad (A.22)$$

The straightness errors are modeled in the same manner as shown in equation A.2. Furthermore the values of $\mathbf{o}_{BS}^{B1}$ to $\mathbf{o}_{BS}^{B6}$ are changed by the axial and radial error
motions of the three joint axes (e.g. $\mathbf{r}^{B_1}(\dot{\vartheta}^{B_1})$, see equations 3.48 to 3.50). These error motions can be modeled again as a superposition of different Fourier harmonics as shown before.

- The positions of the platform joints $\mathbf{o}^{P_1}_{PF}$ to $\mathbf{o}^{P_6}_{PF}$ are changed by the zero position error of the Hexaglide (e.g. $\Delta \mathbf{o}^{P_1}_{PFzero}$). Furthermore the values are changed by the axial and radial error motions of the two joint axes (e.g. $\mathbf{r}^{P_1}(\dot{\vartheta}^{P_1})$, see equations 3.51 and 3.52). These error motions can be modeled again as a superposition of different Fourier harmonics as shown before (see equation A.6).

- The axis positions of the linear axes $\mathbf{Q}$ are changed directly by positioning errors (e.g. $EXQ_1$ depending on $Q_1$). The actual value of e.g. $Q_1$, depending on these errors and the nominal value $Q_{1\text{nom}}$, can for example be computed as

$$Q_1 = Q_{1\text{nom}} + EXQ_1(Q_{1\text{nom}}) \quad (A.23)$$

The positioning error can be modeled similar as shown in equation A.3.


Curriculum Vitae

Bernhard Ulrich Bringmann, born December 4th, 1977 in Schrobenhausen, Germany.

1984 - 1988  Grundschule Aichach, Germany
1988 - 1997  Deutschherrengymnasium Aichach, Germany
1997 - 1998  Military service (function: motor and tank mechanic), Murnau, Germany
1998 - 2001  Undergraduate at Faculty of Mechanical Engineering at Technische Universität München, Germany (Graduation with degree Bachelor of Science in Mechanical Engineering)
2001 - 2002  Graduate student at Department of Mechanical, Aerospace and Nuclear Engineering at Rensselaer Polytechnic Institute in Troy, NY, USA (Graduation with degree of Master of Science in Mechanical Engineering)
since 2003  Ph. D. student and research associate at the Institute of Machine Tools and Manufacturing (IWF), ETH Zurich
summer 2006  Exchange student at Shinno Group, Tokyo Institute of Technology, Japan