Transportation Networks and Economic Equilibrium

Modeling Issues and Applications

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It is so easy to do something the hard way, and so hard to do it the easy way.

Abstract

This thesis is composed of three essays on integrated transport and economic equilibrium modeling. The models are used for the analysis of different parking policies and cordon tolls in Zurich.

In the first essay we give a pedagogical introduction into the field of mixed complementarity (MC) format modeling for transport and computable equilibrium models. This format allows the researcher to use general-purpose modeling software and concentrate on the model structure without the need of formulating algorithms for solving the model. We show how to set up the Wardropian transportation equilibrium model as a link flow-based MC model, and use the Arrow-Debreu framework to implement a computable general equilibrium model in MC format. Both models are then combined into an integrated model that allows the researcher to analyze the interactions between the economic and transport systems. Examples for interactions between both systems are the effects of changes in housing prices, wages on congestion and route choices, or the effects of changes in transport infrastructure on housing prices and wages. We derive each model first theoretically and then show how to implement these in GAMS, a modeling software. This tutorial should provide researchers with the tools to build integrated models that incorporate both transportation and economic features.

The second essay describes a Wardropian traffic flow model in which heterogeneous agents interact on a road network integrated with a search model for paid parking, either at the curb-side or in a parking garage. The occupancy rate influences the probability of finding curbside or garage parking slots. The decision of the agents about where to park reflects trade-offs between time (driving, search and walking) and money (fees).

We formulate the model as a link flow-based Wardropian traffic model and the integrated parking search model as a mixed complementarity problem. This concise formulation has the advantage that there is no need for a complete enumeration of all possible paths in the network, and the model can be solved without the need to decompose the model in two different submodels using readily available non-linear solvers.

The model is used to analyze different parking policies in Zurich. We find that changing the parking fee structure will lead to high efficiency gains as the cruising time is greatly reduced. Parking fees for parking garages in Zurich are clearly too high. All policies are, however, regressive and lead to an increasing burden of households with low incomes. The model is also used to see what role is played by heterogeneity in households. The effects of parking policies to reduce congestion differ only slightly between scenarios with homogeneous and heterogeneous agents. Incorporating

household heterogeneity is, nevertheless, critical for being able to investigate the distributional effects of transportation policies.

In the third essay we present an urban computable general equilibrium model with an integrated traffic equilibrium network. Contrary to existing integrated models, both equilibria are solved simultaneously and a decomposition is not necessary. The model is formulated as a mixed complementarity model and can be efficiently solved using standard optimization software. The network problem is route flow-based. An enumeration of chosen routes is, however, necessary to take into account the income effects on the economic side of the model. However, as we only need to find the actual arcs chosen given the minimal time, this is a significantly smaller and easier to solve than the tradional complete enumeration of all paths. theFurthermore, contrary to typical transport studies, agents not only optimize their choice of transport mode but can also optimize the location where they live and work. Sorting depends, among other things, on the commuting time, wages, consumption and housing prices at the different locations. An important topic of this paper is the calibration of the model to existing transport and socioeconomic data. The model is used to show the effects of cordon toll schemes for the 12 districts of Zurich and to compare the results with a pure network model without economic sorting. It includes two different household groups and two transport modes in a network with 800 origin-destination-pairs and 136 arcs. The results show that economic sorting plays a crucial role in evaluating the effects of individual behavior at the arc and node level. It also shows that cordon toll schemes are regressive if we aggregate the households in four groups, depending on where they live and work: either both locations inside or outside the cordon, or working and living in the two different regions. If we look at the regressive effects for households disaggregated according to the districts where they live and work, there is a clear regressive tendency, but for some agents with high income the relative burden is higher than for lower income households.

Zusammenfassung

Diese Dissertation enthält drei Essays über integrierte Transport- und berechenbare Gleichgewichtsmodelle. Die Modelle werden für die Analyse von politischen Massnahmen im Bereich der Verkehrspolitik benutzt.

Im ersten Essay geben wir einen didaktischen Einstieg in den Bereich der Formulierung von Transport- und berechenbaren Gleichgewichtsmodellen im MCP-Format . MCP steht für "mixed complementarity problem" und ist eine von vielen Möglichkeiten, Transportmodelle oder berechenbare Gleichgewichtsmodelle zu formulieren. MCP-Modelle können mit Standard-Optimierungssoftware gelöst werden und erlauben somit dem Forscher, sich auf die Modellstruktur zu konzentrieren da keine Notwendigkeit besteht zur Formulierung und Kodierung von Algorithmen für die Lösung des Modells. Wir zeigen, wie sich das Transportmodell als streckenbezogenes MCP-Modell formulieren lässt. Diese Formulierung kommt, im Gegensatz zu der routenbezogene Formulierung, ohne eine Bestimmung der schnellsten Routen aus. Weiter wird gezeigt, wie sich ein berechenbares Gleichgewichtsmodell als MCP-Modell formulieren lässt. Beide Modelle werden dann zu einem integrierten Gleichgewichtsmodell zusammengelegt. Ein solches integriertes Modell erlaubt die Analyse der Wechselwirkungen zwischen dem Transportsystem und dem ökonomischen System.

Der zweite Aufsatz beschreibt ein Verkehrsmodell, in dem unterschiedlichen Agenten auf einem Strassennetz agieren. Das Modell enthält ein Suchmodell für kostenpflichtige Parkplätze. Die Parkplätze befinden sich entweder am Strassenrand oder in einer Parkgarage. Die Auslastung beeinflusst die Wahrscheinlichkeit, dass man einen Parkplatz findet. Entscheidend für die Parkplatzwahl sind die Höhe der Parkgebühren, die Fahr- und Suchzeit, sowie die Zeit, welche man zu Fuss unterwegs ist. Das Modell basiert auf dem Nutzergleichgewicht nach Wardrop und ist als streckenbezogenes MCP-Modell formuliert. Das Modell wird verwendet, um verschiedene politischen Massnahmen im Bereich der Parkplatzpolitik in Zürich zu analysieren. Die vorgeschlagenen Änderungen der Parkgebührenstruktur führt zu hohen Effizienzgewinnen und Reduktionen der Reisezeit. Es zeigt sich auch, dass die Parkgebühren für Parkhäuser in Zürich zu hoch sind. Alle Massnahmen sind regressiv und führen zu einer zunehmenden Belastung der Haushalte mit geringem Einkommen. Obwohl die Auswirkungen auf den Verkehrsstau im Modell mit heterogenen Haushalten sich nur wenig unterscheiden vom Modell mit homogenen Haushalten, ist die Heterogenität wichtig für die Untersuchung von Verteilungseffekten.

Im dritten Aufsatz präsentieren wir ein berechenbares allgemeines Gleichgewichts Modell mit einem integrierten Transportmodell für die Stadt Zürich in MCP-Format. Das Modell lässt sich wiederum

effizient mit Standard-Optimierungssoftware lösen. Im Gegensatz zu bestehenden integrierte Modellen werden beide Gleichgewichte gleichzeitig gelöst und eine Dekomposition des Modells in Submodellen ist nicht erforderlich. Das Netzwerkproblem ist zwar als streckenbezogenes und nicht als routenbezogenes Problem formuliert, setzt aber trotzdem eine Berechnung der gewählten Routen voraus damit die Einkommenseffekte von politischen Massnahmen berücksichtigt werden können. Die Routen werden nicht vor, sondern nach der Lösung des integrierten Modells mit Hilfe eines linearen Optimierungsmodell berechnet. Beachte, dass wir nur die tatsächlich benutzten Routen finden müssen, was ein signifikantes kleineres und einfacheres Problem ist als das traditionale Finden aller möglichen Routen. Mit Hilfe der gewählten Routen werden die Einkommenseffekte berechnet und das integrierte Modell gelöst. Dieses Verfahren wird wiederholt bis die Resultate konvergieren. Im Gegensatz zu typischen Transportstudien, entscheiden Agenten nicht nur über die Wahl des Transportmittels (öffentlich oder privat), sondern können auch den Ort, wo sie leben und arbeiten, wählen. Diese Wahl ist abhängig von den Fahrtzeiten, Löhnen, Konsum- und Mietzinsen an den verschiedenen Standorten. Ein wichtiges Thema dieses Essays ist die Kalibrierung des Modells zu bestehenden Transport- und sozioökonomischen Daten. Das Modell wird verwendet, um die Auswirkungen zweier Road-Pricing-Szenarien in Zürich zu untersuchen. Die Resultate werden mit einem reinen Transportmodell verglichen, in dem keine Möglichkeit besteht, um den Wohn- und Arbeitsort zu wechseln. Das Modell enthält zwei nach Einkommen differenzierten Haushaltsgruppen, ein Transportnetzwerk mit 800 Quelle-Ziel-Paaren und 136 Strecken. Die Ergebnisse zeigen, dass die Möglichkeit, Wohn- und Arbeitsort wählen zu können, eine entscheidende Rolle für die Resultate spielen. Das Road-Pricing ist auf der aggregierten Ebene regressiv. Auf der disaggregierten Ebene ist eine klare regressive Tendenz vorhanden, obwohl für manche Haushalte mit hohem Einkommen die relative Belastung höher ist als für Haushalte mit einem niedrigen Einkommen.

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This project started almost five years ago as Prof. Dr. Tom Rutherford invited me to give a series of lectures on general equilibrium modeling at the ETH. I gladly agreed, but as I lacked PhD status in front of my name, he suggested to "just do your PhD with me!" In my mind I crossed out the "just" and agreed.

I am more than grateful to him for the opportunity of undertaking my PhD and teaching at the ETH. His enthusiasm, patience, modeling skills and deep knowledge of economics has always been a guideline for my own work. He truly has, as the Zen master Suzuki called, a beginner's mind: "In the beginner's mind there are many possibilities, but in the expert's there are few" [1]. I am also deeply indebted to my co-supervisor, Prof. Dr. Kay Axhausen, who encouraged my work and my teaching, gave valuable feedbacks, and introduced me to scientists in the field of transportation economics whom I only knew from their famous names. As Prof. Rutherford left for Wisconsin, he could not remain my supervisor but instead became my co-supervisor. Prof. Dr. Sebastian Rausch from the Centre of Energy Policy and Economics (CEPE), agreed without any hesitation to be supervisor. He encouraged me, gave valuable feedbacks, showed confidence and acted also as sounding board, for which I am very grateful. Also, I would like to thank my colleagues at the CEPE. My past PhD collegues who finished before me: Dr. Jan Imhof, Dr. Florian Landis and Dr. Justin Caron. It was really fun having you around! A special thanks to Rina Fichtl for the encouraging support and the administrative help. I very much appreciated the discussions on modeling and theoretical problems which I encounterd, but also the coffee breaks with my collegue, Dr. Jan Abrell. My former collegues at Ecoplan supported my idea of undertaking a PhD and gave me the opportunity to reduce my workload when I still worked with them, for which I am grateful. My PhD reserach would not have been possible without the patience and support of my wife Susann and our daughter Michèle. Without the encouragement of Susann, I would not have dared to quit a good job and return to the university. Last, but not least, I want to thank my parents, Jan und Bep van Nieuwkoop. What would I be without them?

Renger van Nieuwkoop, September 2014, Thun.

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[1] Suzuki (1970), *Zen Mind, Beginner's Mind: Informal Talks on Zen Meditation and Practice*

To my parents Jan and Bep van Nieuwkoop

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The construction of an economic model, or of any model or theory for that matter (or the writing of a novel, a short story, or a play) consists of snatching from the enormous and complex mass of facts called reality, a few simple, easily-managed key points which, when put together in some cunning way, become for certain purposes a substitute for reality itself.

Evsey Domar
Essays in the Theory of Economic Growth, 1957

Remember that all models are wrong; the practical question is how wrong do they have to be to not be useful.

George E. P. Box
Empirical Model Building and Response Surfaces, 1987

Introduction

Any serious study on the effects of transport policy measures relies on simulations with models. These models range from small spreadsheet programs to highly complex models solved with specialized modeling software using the best algorithms available. Some are directed towards the transport aspect of the question in play. The models used differ in many respects. They are used, for example, to observe effects of a policy measure on congestion, mode choice and travel times. Other models are focused on the economic, environmental and distributional effects.

One widely used class of the models used for transport policy analysis is the transport equilibrium problem (TEP) model. In these models the archetypical problem is to find the shortest route using time or generalized time costs. The equilibrium notion in the TEP stems from Wardrop (1952), in which users of the network choose an individually-rational route on the (congested) traffic network. However, these transport equilibrium models do not take into account the effects of changes in economic variables. THis may lead to serious omission of relevant economic effects. Incorporating economic effects into transportation equilibrium models requires to integrate or extend these models into a more comprehensive economic modeling framework. For example, an increase in population in a city will not only give rise to an increase in congestion, but also to changes in housing prices. These changes will cause people to change their living and working locations.

A tool that can analyze the economic dependencies is a computable general equilibrium (CGE) model. CGE models have been used extensively over the past 30 years (for an overview, see Dixon and Jorgenson, 2013). These models can be used to analyze policies in many economic fields such as trade, environmental, energy and social policy. CGE models allow the researcher to analyze the effects of policy alternatives on industrial sector-wise and aggregated prices, outputs, GDP and many other indicators and to compare them among each other or to the economy without the policy being implemented.

Most of these models are, however, used for countries or several regions (see, for example, Fujita, Krugman, and Venables, 2001) and not at the urban level. Furthermore, existing CGE models treat transportation as a production sector (like any other sector in the model) and do not take account of the explicit transportation network with its infrastructure.

In the literature, there are only a few models that explicitly link the transportation network structure of a city to the economic system of that city. Examples are found in a series of papers by Anas (Anas and Kim, 1996; Anas and Xu, 1999; Anas, 2012; Anas, 2012). These models usually consist of two separate models being solved iteratively using different algorithms. Part of the output of

one model is used as input for the other model, and hopefully, the iteration procedure converges to an equilibrium situation in both models.

The most obvious reasons why there are so few integrated urban CGE models are twofold. First, the modeler has to find solutions for two different problems. This requires extensive knowledge in two separate fields: traffic network and CGE theory, plus the modeling. The second reason is the lack of integrated model formulations that can be solved directly by (commercial) optimization software. For this reason, researchers often rely on their own coding of the models and algorithms. This practice has several drawbacks as noted by Dirkse (1994): It might be hard for other researches to read the model and the algorithm, and changing the data, the model, or switching to a different solution method can be very time consuming. Again it requires skills in two other fields, namely operations research and programing.

Another reason might be the different ways the models are formulated. TEP models are usually formulated as a variational inequality (VI) route flow problems, usually solved with a heuristic algorithm.hese models are usually solved with a heuristic algorithm. The algorithm starts with the calculation of the shortest paths without any flows on the network. Travelers are then assigned to those shortest paths, and new travel times are calculated taking into account the congestion on the arcs. In a third step, the new information on the travel times is used to recalculate the shortest paths, and the traffic assignment is updated. The whole process is repeated until an equilibrium solution is found. The disadvantage of these approaches is the neccessity of enumarating all possible paths in the process. Over the years a variety of specialized software packages to solve transportation equilibrium models have been developed to obviate the burden of coding the algorithms. In economic modeling there is a clear tendency to use the mixed complementarity format for writing down the model. This format allows the researcher to separate the modeling from the solving with such software packages as MatLab (The MathWorks, 2012) or GAMS (GAMS Development Corporation, 2014) provide the researcher with efficient algorithms. These packages allow the modelers to concentrate on the model formulation without having to write their own algorithm. Combining these different formats in one integrated model poses a further difficulty for the researcher.

The three essays in this thesis attempt to bridge the gap between the different model formulations using the mixed complementarity format for both models and applying them to transport policy measures. They furthermore show how to write transport problems as a link- instead of a flow-based problem. This formulation allows to solve the model without the need for a complete enumerating of all pathways.

The first essay, entitled "Integrated transportation and economic models in mixed complementarity format: A tutorial" gives a gentle introduction into transport modeling and computable general equilibrium modeling using the mixed complementarity format (MCP) and prepares the ground for the two following essays. It furthermore shows how to combine the transport equilibrium model with a general equilibrium model. Each section provides small examples that can be easily implemented in (demo versions of) optimization software with the capability of solving MCPs. The GAMS code for the examples are provided in the text or in the appendices. The models can be extended in

many ways. Dynamic transport modeling, where agents can choose when to travel, more realistic delay functions, where the flows on other arcs influence the travel time, imperfect competition, and environmental externalities are only a few of the many possibilities. The tutorial is aimed at researchers who have never worked with the MCP format, and might have some basic knowledge in transportation and/or computable general equilibrium models.

The second essay, entitled "A Traffic Equilibrium Model with Searching for Paid Parking". The objective of this paper is twofold. First, it develops a concise, tractable, and easily extensible, link-based formulation of the model as a mixed complementarity problem (MCP), combining a traffic assignment model in the tradition of Wardrop (1952) and a parking search model into one single model. Drivers from different user classes simultaneously decide on the choice of route and parking allocation in a road network with multiple parking facilities. The externality on the probability of finding a parking space is explicitly part of the model. We make a distinction between on-street (curbside) and off-street parking (garage). The user classes are differentiated with respect to their origin and valuation of time.

A second objective aims at using the model for the analysis of efficiency and distributional effects of different parking fee policies. From an economic point of view, the most efficient policy to reduce the congestion and other external effects of cruising, is to impose a spatially differentiated parking fee that reflects the external costs (see for example: Arnott, de Palma, and Lindsey, 1991; Verhoef, Nijkamp, and Rietveld, 1995). A parking fee can reduce cruising and will also have a positive impact on congestion, although driving through traffic is exempted, and the parking fee does not take directly into account how the cruising driver affects congestion. We use the model for the center of Zurich, the biggest city in Switzerland, and analyze several policies in which the fees for on-street parking and/or the fees for parking garages are endogenous. These policies are compared to the existing policy and a social optimum, in which the overall time costs are minimized.

The simulations show that the parking fee structure in Zurich is highly inefficient and changing this structure could lead to high efficiency gains. It also shows that the existing street parking fees relative to the parking garage fees are too low. Implementation of these policies would reduce the congestion and generalized costs of the agents, it would reduce, with the exception of the scenario with endogenous street parking fees, the tax revenue for the city by more than 60 %. Another drawback is the regressive character of all the policies: The generalized costs of poor households will increase and those of richer households will decrease, as shown for a social optimum and expected for scenarios with non-optimal taxes by Layard (1977).

Comparing the results with a model with no differences in the valuation of time of the agents, reveals no significant difference in the overall welfare effects. Incorporating household heterogeneity is nevertheless critical for being able to invstigate the distributional effects of transportation policies.

The third essay is titled "An integrated transport network-computable general equilibrium model in mixed-complementarity format for Zurich". In this paper, we propose a formulation of an integrated model that can be easily solved by readily available solvers following the techniques developed in the first essay of this thesis. We present a mixed complementarity problem (MCP) formulation of an urban general equilibrium model that embeds a closed, spatially disaggregate economic model of

housing and labor markets along the lines of the Alonso-Muth-Mills model[4]. This economic model is embedded within a model of individually-rational route choice on the (congested) traffic network (Wardrop, 1952). We use the link-flow formulation for the transport equilibrium model, which in turn makes an enumeration of possible routes unnecessary. This is however only true if there is no road pricing. If there is road pricing, the income of the households will be reduced by their toll payments. Without knowing which route they have chosen, it is impossible to infer the toll payment hence we introduce a simple procedure to infer the toll payments. We solve iteratively the integrated model followed by a simple linear optimization problem with almost no computational costs in order to infer the arcs traveled by every household, to calculate the toll payments and adjust to the income. This is repeated until the process converges (in our simulations, this process only need three iterations).

The economic aspects of the model follow the Walrasian-Arrow-Debreu paradigm. Consumers earn by providing labor to the production sectors, and they allocate their income to housing and consumption. Taxes can be applied to both residential and employer locations, such that private decisions of households and firms produce an optimal pattern of location. One of the main assumptions of the model is that households choose residence and employment locations which arbitrate differences of various locations within the urban area. This implies that consumers trade locations, to work and live on the basis of housing prices, wages, and commuting time. The model is medium-term, and we assume therefore that only a certain part of the population can change locations for living and working. The commuting costs in the model depend on the distance between the location where the household lives and where he or she works as well as on the mode chosen. The model distinguishes between public and private transport mode. The commuting time using private transport depends on the capacity and the size of the flow on the arcs. The commuting time using public transport mode is constant, as long as the capacity is not fully used. Once the demand is higher than the capacity, waiting time occurs. Transport costs for commuting are capitalized in housing values (Glazer and Van Dender, 2002) and can lead to wage differentials, depending on the zone of employment (as estimated by Darren and Wheaton, 2001).

We also show the usefulness of the integrated approach for the analysis of two cordon toll schemes in Zurich, the biggest city of Switzerland. The discussion on road pricing alternatives started in Zurich in 2001 by the Zurich City Council, which substantiated the goal of road pricing in the following years. Based on several political and research white papers and developments at the national level, the council was of the opinion that with the help of more detailed analysis, the goal of road pricing for the City of Zurich should be pursued (Bächtiger and Ott, 2010). Recently, the discussion has increasingly centered on mobility pricing instead of road pricing.[5] With mobility pricing a charge on distance traveled is levied for financing transport infrastructure. The reduction of congestion is seen as a beneficial side effect. The model is also used to show differences between the results from the integrated model and a (simple) transport equilibrium model.

The results demonstrate that incorporating economic variables and sorting can lead to significantly

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[4] For a description of the Alonso-Muth-Mills model see, for example, Glaeser (2008, Chapter 2) or Brueckner (1987).

[5] See, for example, the article in one of the leading Swiss newspapers (Hotz, 2014).

different results which may be of high relevance for policy analysis. The results show the importance of incorporating the market for houses and labor into the model. The comparison also shows that without a sorting mechanism, the effects in a pure transport model might differ significantly, especially at the disaggregated level. The model shows especially, that sorting plays an important role for the changes in the choice of mode, wages and housing rental prices. It shows that, although the cordon toll has a negative impact on the share of private transportation by making working within the cordon less attractive, changes in wages and housing rental prices can outweigh these effects. The cordon toll leads to a reduction in overall travel time costs for both transport modes.

If one groups households with respect to living and working locations within the cordon or outside the cordon, welfare changes are on average negative for people not living and working in the cordon and households with low income suffer the most from the cordon toll. However, this is not always true at the household level. In some cases the burden for high income households can be higher in some cases than for low income households.

The economic changes for the scenario with the greater cordon toll are more pronounced, but the average utility changes do not differ much between both scenarios. Which scenario is better depends on what the goal of the cordon toll is. If a reduction of congestion in the center is the main goal, the first scenario might be more efficient. If, however, the primary goal of the cordon toll is funding for transport infrastructure (which is on the political agenda for Zurich), the greater cordon toll raises more tax revenue. Note, that in the case of the transport model, the tax revenue is estimated to be much higher than in the integrated model.

An important issue of this essay is the calibration of the model to the given data. For our simulations, the model is calibrated to the travel demand and travel times from the Cantonal Transport Model (Amt für Verkehr Volkswirtschaftsdepartement Kanton Zürich, 2011) by adjusting the parameters of the delay functions. The logit demand for the two modes is calibrated to the given modal split of the origin-destination-matrix and given price elasticities. On the economic side of the model, we show how to calibrate the model to available housing stock, wages and rents.

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Integrated transportation and computable general equilibrium models in mixed complementarity format: A tutorial

1.1   Introduction

In the next ten years most large cities around the world will face substantial changes in transport, housing infrastructure and zoning regulation as well as changes in population and employment. These changes will not only impact on the locational decisions of households and firms, but also on wages, housing prices and the environment.

These changes are usually studied with the tools of urban economics. Most of these tools are models that only look at changes for a small set of variables without taking into account the ramifications on other important variables. Often, models for solving transport equilibrium problems (TEP) are used, in which the problem is to find the shortest route using generalized time costs. The equilibrium notion in TEP goes back to Wardrop (1952), in which users of the network choose an individually-rational route on the (congested) traffic network.However, these transport equilibrium models do not take into account the effects of changes in economic variables, a more comprehensive tool is needed. For example, an increase in population in a city will not only give rise to an increase in congestion, but also to changes in housing prices. These changes will cause people to change their living and working locations.

A tool that can analyze economic dependencies is a computable general equilibrium (CGE) model. CGE models have been used extensively over the past 30 years (for an overview, see Dixon and Jorgenson, 2013). These models can be used to analyze policies in many economic fields such as trade, environmental, energy, social policy, and more. CGE models allow the researcher to analyze the effects of policy alternatives on industrial sector-wise and aggregated prices, outputs, GDP and many other indicators and to compare them among each other or compare them to the economy without the implementing the policy.

Most of these models are however used for countries or several regions (see for example, Fujita, Krugman, and Venables, 2001) and not at the urban level. Furthermore, existing CGE models treat transportation as a production sector like any other sector in the model, and do not take account of the explicit transportation network with its infrastructure.

There are only a few urban computable general equilibrium models that explicitly link the transportation network infrastructure of a city with the economic part of the model. Examples are found

in a series of papers by Anas and Kim (1996), Anas and Xu (1999), Anas (2012), and Anas (2012). These models usually consist of two separate submodels being solved iteratively using different algorithms. Part of the output of one model is used as input for the other model; an iterative procedure ensures convergence of both submodels.

The most obvious reasons why there are so few integrated urban computable general equilibrium (CGE) models are twofold. First, the modeler has to find solutions for two different problems. This requires extensive knowledge in two separate fields: traffic network and CGE theory and modeling. The second reason is the lack of integrated model formulations that can be solved directly by (commercial) optimization software. For this reason, researchers often rely on their own coding of the models and algorithms. This practice has several drawbacks as mentioned by Dirkse (1994) in that it might be hard for other researchers to read the model and the algorithm, and changing the data, the model, or switching to a different solution method can be very time consuming. Furthermore it requires skills in again two other fields, namely operations research and programming.

TEP models are usually solved with a heuristic algorithm.[1] The algorithm starts with the calculation of the shortest paths without any flows on the network. Travelers are then assigned to those shortest paths, and new travel times are calculated taking into account the congestion on the arcs. In a third step, the new information on the travel times is used to recalculate the shortest paths and the traffic assignment is updated. The whole process is repeated until an equilibrium solution is found. The disadvantage of these approaches is the necessity of enumerating all possible paths in the process. Over the years a variety of specialized software packages that solve transportation equilibrium models have been developed in order to remove the burden of coding the algorithms.[2]

TEP problems can however also be solved as an optimization, a variational inequality or a mixed complementarity problem (MCP) using non-heuristic optimization algorithms. To the best of our knowledge, Aashtiani was the first to formulate the TEP as a MCP problem (Aashtiani, 1977; Aashtiani, 1979; Aashtiani and Magnanti, 1981). Although these works were published in the same time as the papers on the variational-inequality (VI) formulation of the TEP (see for example Dafermos and Sparrow (1969) and Dafermos (1980) and for an overview Nagurney (2009)), most researchers nowadays use the VI formulation for their work.[3]

Ferris, Meeraus, and Rutherford (1999) showed that the Wardropian traffic equilibrium model can be solved efficiently as a mixed complementarity programming problem using readily available optimization software. Their paper never found its way to the transport engineers, as most transport models are still being solved using self-written algorithms or transport modeling software.[4]

---

[1] See, for example, Sheffi (1985), van Vliet (1978), LeBlanc and Mustafa (1979), LeBlanc, Morlok, and Pierskalla (1975), and Ouorou, Mahey, and Vial (2000).

[2] Correa et al. (2010) mention more than ten different commercial packages.

[3] It probably was not helpful that the term "male chauvinist pig" with the same acronym as mixed complementarity problem (MCP) became the vogue in the same time period... The VI problem was lucky to have a positive connoted and much older acronym (VIP for "very important person").

[4] The variational equalities formulation could also be solved using optimization software however many researchers still prefer own-coded algorithms.

CGE models can also be formulated and solved in different ways (see for example Ginsburgh and Keyzer, 1997). In economic modeling there is clear tendency to separate the modeling from the solving by using general purpose software packages like Mathematica (Wolfram Research, 2014), MathLab (The MathWorks, 2012), GAMS (GAMS Development Corporation, 2014b), or Gempack (Harrison et al., 2014). These packages allow the modeler to concentrate on the model formulation without having to write his or her own algorithm.[5]

This tutorial lays out the basics for implementing an integrated model that combines a transport equilibrium with a general equilibrium model using the mixed complementarity format. The tutorial is divided into five parts. We first explain the basics of the mixed complementarity framework and give some smaller examples. The second part gives an introduction to the different formulations of the transportation equilibrium problem and shows how to cast the arc-flow formulation of this problem in the MCP format. This part is followed by an introduction to general equilibrium modeling using the MCP format. It starts with partial economic equilibrium models of demand and supply, and shows how to use these building blocks to construct a general equilibrium model in MCP format. The fourth part introduces the integrated model for which an example can be found in the third essay of this thesis. The last part summarizes the main contributions of this paper.

This tutorial is meant for researchers who have little or no experience working with the MCP format, and who might have some basic knowledge in transportation and/or computable general equilibrium models. Researchers with knowledge in one of these fields can skip the respective sections. Each section provides small, stylized examples that can be easily implemented in (demo versions of) optimization software with the capability of solving MCPs. The GAMS code for the examples are provided in the text or in the appendices. For those readers unfamiliar with GAMS, the tutorial by Rosenthal provides a gentle introduction to Gams (Chapter 2 in GAMS Development Corporation, 2014a).[6]

1.2 Mixed complementarity problems

1.2.1 Introduction

A complementary problem can be described as a system of (non-)linear constraints where the system variables are linked to the constraints with complementarity conditions (Ferris and Munson, 2014). More formally, given a function $F : \mathbb{R}^n \to \mathbb{R}^n$, lower bounds $l \in \{\mathbb{R} \cup -\infty\}^n$ and upper bounds $u \in \{\mathbb{R} \cup \infty\}^n$, we try to find $x \in \mathbb{R}^n$ such that precisely one of the following holds for

---

[5] We use GAMS (GAMS Development Corporation, 2014b), which stands for General Algebraic Modeling System. It is a high-level modeling system for mathematical programming and optimization and consists of a language compiler and many integrated high-performance solvers. GAMS is tailored for complex, large scale modeling applications, and allows building large maintainable models that can be adapted quickly to new situations.

[6] For researchers using MATLAB the coding can be performed done using the equations shown for every example.

each $i \in 1, \ldots, n$:

$$F_i(x_i) = l_i \quad \text{and} \quad F_i(x_i) \geq 0, \quad \text{or}$$
$$F_i(x_i) = u_i \quad \text{and} \quad F_i(x_i) \leq 0, \quad \text{or}$$
$$l_i < x_i < u_i \quad \text{and} \quad F_i(x_i) = 0$$

This means that the variable x_i is either at one of its bounds or the linked function is equal to zero.

In the mixed complementarity problem (MCP), we not only have inequalities with complementary nonnegative variables, but also equations where the associated variables are free. The complementarity conditions can then be written as:

$$F_i(x_i, x_j) \geq 0, \quad x_i \geq 0, \quad x_i F_i(x) = 0,$$
$$F_j(x_i, x_j) = 0, \quad x_j \text{ free},$$

where we partition the set n in the sets i and j.

Often the following shorthand notation is used, where the perpendicular symbol ($\perp$) indicates the complementarity slackness between the constraint and the variable:

$$0 \geq F(x) \perp x \geq 0. \tag{1.1}$$

Complementarity models can be used for solving linear, quadratic and nonlinear programs by writing the Karush-Kuhn-Tucker optimality conditions. In the case of minimizing a function $f(x)$, where $x \in \mathbb{R}^+$, the first order condition is given by:

$$\frac{\partial f}{\partial x} \geq 0, \quad x \geq 0. \tag{1.2}$$

If x is at its lower bound, we must must have that the function is increasing in x. If we have an interior solution, the derivative must be equal to zero. Combining these two pieces of information, we get the mixed complementarity formulation:

$$\frac{\partial f}{\partial x} \leq 0, \quad x \geq 0, \quad x \frac{\partial f}{\partial x}. = 0 \tag{1.3}$$

As the complementarity problem can often be formulated using the optimality conditions of the original problem, it is easy to write down the model equations. However, there is not always an optimization problem that corresponds to the complementarity conditions. This means that a MCP formulation allows us to solve a wider class of problems.

Complementary models have been used for expressing a variety of economic equilibrium models for both markets and games, where the underlying problem cannot be written down as a single optimization problem or if no equivalent optimization problem exists, for example, due to non-integrability conditions.[7] Many examples in MCP format can be found in Ferris and Munson (2014), Rutherford (1995), and Dirkse and Ferris (1995). In this paper we will show the MCP

---

[7] See the paper on this topic by the famous economist Samuelson (1950).

format with some economic partial models, a transportation problem, and a general equilibrium model.

The development of the complementarity modeling format was motivated by theoretical and practical developments in algorithms for nonlinear complementarity problems and variational inequalities. The most recent techniques are based on ideas from interior-point algorithms for linear programming (Kojima et al., 1991). Computational evidence suggests that algorithms for solving MCPs are relatively reliable and efficient, particularly for models which are not natural optimization problems. A survey of developments in the theory and applications of these methods is provided by Harker and Pang (1990).

MCP problems are closely related to variational inequality (VI) problems.[8] Formally, a VI can be defined for $K \subseteq \mathbb{R}^n$, to find $x \in K$ such that

$$F(x^*)^T(x - x^*) \geq 0 \quad \forall x \in K \tag{1.4}$$

The MCP is a special case of the VI problem. The MCP can be represented as a VI if the feasible set is the non-negative orthant ($\mathbb{R}^{n+}$).

The proof (taken from Nagurney, 1993) is as follows: If x^* solves the VI problem, one can substitute $x = x^* + e_i$ into equation (1.4), where e_i is the n-dimensional vector with 1 at position i, and 0 elsewhere. Then it follows that $F_i(x^*) \geq 0$. Substituting $x = 2x^*$ and after that $x = 0$, one finds that $F(x^*)^T x^* = 0$, and we have the MCP formulation. Conversely, if x^* satisfies the complementarity problem, then $F(x^*)^T(x - x^*) \geq 0$ because $x \in \mathcal{R}^n_+$ and $F(x^*) \geq 0$.

Many optimization problems, especially transport problems, are cast in the variational inequality format, and we will come back to this in Chapter 3.2.

1.2.2 MCP examples

Market equilibrium in MCP format

MCP is best explained by providing some examples. The first example is from economics: We want to solve a competitive equilibrium in which one good is produced and sold on the market. The producer will want to maximize its profits. Assume that if P is the market price he will receive for selling the good at the market, and that the cost function for producing that good is quadratic. In equilibrium, the first order condition will be that the market price will be less or equal to the (marginal) cost:

$$a + bQ^S \geq P \quad \perp Q^S, \tag{1.5}$$

---

[8] An over 1400 pages long discussion of all aspects of finite-dimensional VI and MCP problems can be found in Facchinei and Pang (2003).

where Q^S is the produced quantity.[9] One would expect that the price should be higher than the costs, but this cannot be an equilibrium as more producers will enter the market and drive down the market price until the price is equal or less than cost. The quantity supplied in equilibrium is complementary to this condition: If cost exceed the market price, the producer will incur losses, and therefore decide to produce nothing ($Q^S = 0$). If the marginal cost and market price are equal, goods will be supplied to the market. We draw the inverse of the (marginal) cost function, which is the supply function.[10]

On the demand side we can derive a demand function (this will be done in section 1.4.1). For this example, we just assume that demand given by a downward sloping function of the price ($D(P) = c - dP$). When the price falls, demand increases. When the price is zero (a free good), demand is free. This is captured by the following equation:

$$Q^D \geq c - dP \quad \perp \quad Q^D. \tag{1.6}$$

This equation is complementary to the quantity demanded. The last equation necessary is the market equilibrium condition:

$$Q^S \geq Q^D \quad \perp \quad P. \tag{1.7}$$

which says that in equilibrium supply should be greater or equal to demand. The complementary variable is the market price. Now have three equations and three variables (P, Q^S, Q^D).

Depending on the specification of the supply and demand curve, we can have three possible outcomes: First, if both curves intersect in the positive quadrant (Figure 1.1-a), the market equilibrium, is given where supply equals demand. The second possible outcome is if the supply curve lies in the positive orthant everywhere above the demand curve (Figure 1.1-b). The costs of supplying a good are too high. In this case we have a corner solution, where quantity will be zero at a positive market price. The good is too expensive. The third possibility is shown in Figure 1.1-c, where the produced good is available in abundance, and the price is 0. (Figure 1.1-c)

Notice that we have a system of (linear) equations. The problem could have also been written down as an optimization problem, using the economic surplus. The code in Listing (1.1) shows the implementation of this small MCP example in GAMS.[11] The assumptions on the supply and demand functions are given in Table 1.1. This table shows also the equilibrium solutions for the

---

[9] Because of the quadratic cost function, the left hand side of the first order condition is linear.

[10] For historical reasons, economists reverse the independent and dependent axes: "Readers trained in other disciplines often wonder why economists plot demand curves with price on the vertical axis. The normal convention is to put the independent variable on the X axis and the dependent variable on the Y axis. This convention calls for price to be plotted on the horizontal axis and quantity on the vertical axis. The axis reversal — now enshrined by nearly a century of usage — arose as follows. The analysis of the competitive market that we use today stems from Leon Walras, in whose theory quantity was the dependent variable. Graphical analysis in economics, however, was popularized by Alfred Marshall, in whose theory price was the dependent variable. Economists continue to use Walras' theory and Marshall's graphical representation and thus draw the diagram with the independent and dependent variables reversed — to the everlasting confusion of readers trained in other disciplines. In virtually every other graph in economics the axes are labeled conventionally, with the dependent variable on the vertical axis." (Lipsey, 1979, p. 79)

[11] All examples can be run using a demo version of GAMS (freely downloadable from www.gams.com).

(a) Interior solution   (b) Corner solution ($Q^* = 0$)   (c) Corner solution ($P^* = 0$)
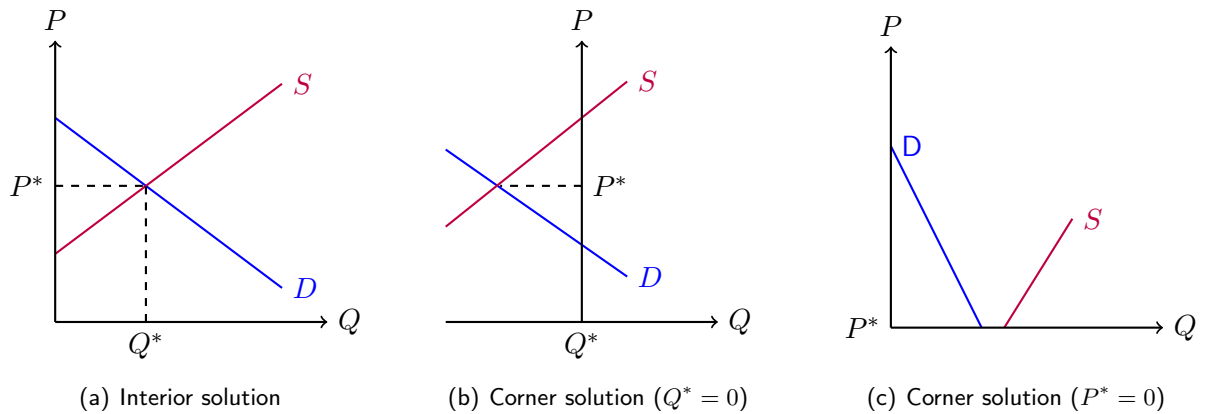
Figure 1.1 – Competitive market equilibrium

price, and the quantities supplied and demanded. Note, that in the case of a free good, we have

Demand curve	Supply curve	P^*	Q^{D*}	Q^{S*}
$P = 4.5 - 0.75Q^D$	$P = 1.5 + 0.75Q^S$	3.0	2.0	2.0
$P = 1.7 - 0.7Q^D$	$P = 4.5 + 0.8Q^S$	4.5	0.0	0.0
$P = 4.0 - 2Q^D$	$P = -4 + 1.6Q^S$	0.0	2.0	2.5

Table 1.1 – Example interior and corner solutions

Listing 1.1 – Market equilibrium with corner solutions

```
1   $title Market Equilibrium with Corners

    positive variables
        D       Equilibrium demand,
        S       Equilibrium supply,
6       P       Equilibrium price;

    equations
        mkt     Market clearance,
        demand Demand function,
11      supply Supply function;

    demand.. P =E= 4.5 - 0.75 * D;

    supply.. P =E= 1.5 + 0.75 * S;

    mkt..    S =g= D;

    model mkt_interior /supply.S, demand.D , mkt.P/;

21  *    Initialize the variables

    P.L = 1; D.L = 1; S.L = 1;

    solve mkt_interior using MCP;

    equations
        demand_cq Demand function,
        supply_cq Supply function;

31  demand_cq.. P =g= 1.7 - 0.7 * D;

    supply_cq.. P =g= 4.5 + 0.8 * S;

    model mkt_cornerQ /demand_cq.D, supply_cq.S, mkt.P/;
36  P.L = 1; D.L = 1; S.L = 1;

    solve mkt_cornerQ  using MCP;

    equations
41      demand_cp Demand function,
        supply_cp Supply function;
```

```
   demand_cp.. P =g=  4 − 2.0 * D;
46 supply_cp.. P =g= −4 + 1.6 * S;
   model mkt_cornerP /supply_cp.S, demand_cp.D, mkt.P/;
   P.L = 1; D.L = 1; S.L = 1;
   solve mkt_cornerP  using MCP;
```

The Hitchcock-Koopmans transportation problem

Another often used example, is the Hitchcock-Koopmans transportation problem from Dantzig (1963). This classical transportation problem was first presented by Hitchcock 1941 and independently, by Koopmans in 1947 (Koopmans, 1949). In this example there is supply from two plants and demand of a single commodity on three markets (see Figure 1.2-a). The unit shipment costs are given for the different routes and the economic question to be solved is, how much should be shipped between every plant and market while minimizing total shipping cost. In the figure "d" is demand and "s" is capacity (for presentation purposes San Diego is moved to the east).

a. Network with supply, demand and distances   b. Solution
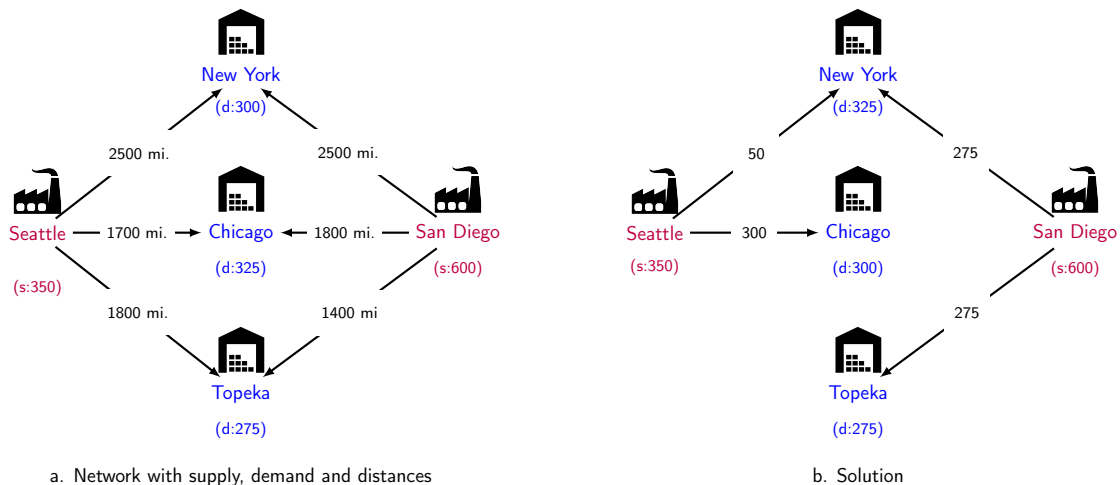
Figure 1.2 – Dantzig's transportation equilibrium problem (a) and its solution (b).

The original linear optimization problem is given by:

$$\text{Objective function:} \quad \max OBJ = \sum f d_{ij} X_{ij}$$

$$\text{Supply constraint:} \quad \sum_{j} X_{ij} \leq a_i$$

$$\text{Demand constraint:} \quad \sum_{i} X_{ij} \geq b_j,$$

where X_{ij} is the flow from plant i to market j, f is the unit transportation cost, d_{ij} is the distance between two nodes, a_i is supply and, b_j is demand. The GAMS code is shown in Listing (1.2).

Listing 1.2 – LP formulation of the transportation problem

```
  Sets
2  i canning plants / Seattle, San−Diego /
   j markets / New−York, Chicago, Topeka / ;
```

```
     Parameters
      s(i) capacity of plant i in cases
 7   / Seattle 350
      San—Diego 600 /

      d(j) demand at market j in cases
      / New—York 325
12    Chicago 300
      Topeka 275 / ;

     Table dist(i,j) distance in thousands of miles
      New—York Chicago Topeka
17   Seattle 2.5 1.7 1.8
      San—Diego 2.5 1.8 1.4;

     Scalar f freight in dollars per case per 1000 miles /0.09/ ;

22   Variables
      X(i,j) shipment quantities in cases
      OBJ total transportation costs in thousands of dollars ;

     Positive Variable X ;

     Equations
      cost define objective function
      supply(i) observe supply limit at plant i
      demand(j) satisfy demand at market j ;

     cost .. OBJ =e= sum((i,j), f * dist(i,j) * X(i,j)) ;

     supply(i) .. s(i) =G= sum(j, X(i,j)) ;

37   demand(j) .. sum(i, X(i,j)) =g= d(j) ;

     Model transport /all/ ;

     Solve transport using lp minimizing OBJ ;
```

If we write the Lagrangian

$$\min \mathcal{L}(X_{i,j}, C_i, P_j) = \sum d_{ij} f X_{ij} + W_i \left[\sum_j X_{ij} - s_i \right] + P_j \left[b_j - \sum_i X_{ij} \right] \qquad (1.8)$$

and derive the first order conditions with respect to the flows, we get:

$$d_{i,j} f + W_i - P_j \geq 0. \qquad (1.9)$$

This problem can now be interpreted as a (spatial) market equilibrium problem in which the dual multiplier W_i represents the price in supply market i, and the second dual multiplier P_j represents the price in demand market j. The first order condition can be interpreted as the zero profit condition for transportation firms that buy commodities at the plants, and ship them to the markets. As long as a transportation firm makes a positive profit, other firms will enter and drive down the profits to zero. The problem can now be formulated as a linear complementarity problem of the form:

$$W_i + f d_{ij} \geq P_j \quad \perp X_{ij} \quad \forall i, j$$

$$\sum_j X_{ij} \leq s_i \qquad \perp W_i \quad \forall i$$

$$\sum_i X_{ij} \geq b_j \qquad \perp P_j \quad \forall j$$

The implementation in GAMS is shown in Listing 1.3.

Listing 1.3 – MCP formulation of the transportation problem

```
positive variable
 P(j) Shadow price of demand at market j,
 W(i) Shadow price of supply at plant i;

equation
 zeroprofit;

zeroprofit(i,j)..
 dist(i,j)*f + W(i) − P(j) =g= 0;

model transportmcp /supply.W, demand.P, zeroprofit.X/;

solve transportmcp using mcp;
```

The solution of this problem is shown in Figure 1.2-b. We see that nothing is shipped from Seattle to Topeka, and from San Diego to Chicago, because this would cause higher overall costs. If however one would like to send one unit from Seattle to Topeka, overall costs will increase by $36, shown as the marginal in the GAMS output (see Listing 1.4). This can be seen when we calculate the costs for shipping this commodity. Shipping this unit from Seattle to Topeka will increase cost by $126 (1400 miles * $0.09). As Seattle is already working at capacity, we have to reduce supply to New York by one (minus $225). This can be counteracted by shipping one unit more from San Diego to New York, costing $162. Topeka has demand only for 275 units, hence we have to reduce the amount shipped from San Diego (minus $225). If we sum up, we get the marginal costs as reported by Gams. For one additional unit from San Diego to Chicago one will find an increase in total costs of $9.[12]

Listing 1.4 – Solution and marginals of the transportation problem

```
  LOWER LEVEL UPPER MARGINAL

Seattle .New−York . 50.0000 +INF .
Seattle .Chicago . 300.0000 +INF .
Seattle .Topeka . . +INF 0.0360
San−Diego.New−York . 275.0000 +INF .
San−Diego.Chicago . . +INF 0.0090
San−Diego.Topeka . 275.0000 +INF .
```

1.3 Transport equilibrium modeling: from VI to MCP

1.3.1 Introduction

We now turn to transport equilibrium modeling using MCP. We will first describe the network and the behavior of the road users. After this description we will discuss the different formulations of the model. Transportation equilibrium problems can be formulated either as a route-flow or as a link-flow problem. In the route-flow formulation, the TEP is formulated with respect to the routes taken from origin to destination.

The aim of a traffic equilibrium model is to find an optimal flow pattern on the transportation

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[12] An easy way to check this in GAMS is fixing the flow from Seattle to Topeka by 1 (X.FX("San-Diego)","Topeka"). GAMS will show the optimal solution with the adjusted amounts.

network, given the information on where the commuters live and work (origin-destination matrix), the network structure, the travel modes and the travel costs on the arcs of the network.

A transportation network consists of a set of nodes $\mathcal{N}$ and directed arcs $\mathcal{A}$. Agents in the network travel from their origin node to a destination node, where $\mathcal{OD}$ is the set of all OD-pairs p. Other nodes are the junctions in the network. In the following i, j and k are indices, which will be used as aliases to describe the nodes in the network. The arcs correspond to the streets in the network and are defined by a start and end node. $\mathcal{R}_w$ is the set of possible routes between the OD-pair p. Each route consists of a sequence of arcs $a \in \mathcal{A}$. We assume that routes are acyclic (no route uses a node twice). The time delay or generalized costs on the arcs are given by a cost function $\tau_a(f_a)$ of the total flow f_a the arc. The number of agents using route r between OD-pair p is given by x_{wr}.

In order to find the optimal network assignment of the agents, who are distinguished by their origin and destination, we assume that agents seek to minimize their individual travel cost when traveling to their destination. This optimality notion goes back to Wardrop's first behavioral principle (Wardrop, 1952, p. 345):[13] "*The journey times on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route.*" We will also make use of the second principle of Wardrop which states that the average journey time is a minimum. These principles, as noted by Patriksson (1994), coincide respectively with the idea of user equilibrium (UE) and social optimum (SO) coined by Dafermos and Sparrow (1969).[14] In the UE all road users minimize their travel costs independently and as a consequence all used routes between two nodes will have the same minimal travel costs. In the SO the overall travel costs are minimized and the travel costs on the routes for an OD-pair can differ. (Dafermos and Sparrow, 1969) note that the equilibrium flow in the user equilibrium is a solution of the Nash non-cooperative game among the various users of the network. Rosenthal (1973) showed this equivalence for a network model with integer flows and Devarajan (1981) extended this result to the continuous case.

In the following chapters we will use the most simple version of the transportation equilibrium problem assuming that demand is fixed and that the cost on an arc is not influenced by the traffic on other arcs (separability).

1.3.2 The route-flow formulation

We start with the route-flow formulation of the user equilibrium. This problem is given by four equation groups and a group of equilibrium conditions. For every OD pair p the conservation of

---

[13] Apart from the Wardropian user equilibrium, there are several other characterizations of a traffic network equilibrium. Marcotte and Patriksson (2007) mention equilibrated, user, normal and user optimized equilibrium. Under the assumption of separable, continuous link cost functions, these characterizations are equal to the Wardropian user equilibrium (see Marcotte and Patriksson, 2007).

[14] Patriksson also notes that the principles of Wardrop were already discussed by the famous economist A.C. Pigou (1877 - 1959) 30 years earlier.

flow requires that the demand d_p should be equal to the sum of the flows over all routes r between this OD-pair:

$$d_p = \sum_{r \in \mathcal{R}} x_{pr} \quad \forall p \in \mathcal{OD}. \tag{1.10}$$

The second group of equations defines the total flow on an arc as the sum over all OD-pairs and routes of all flows on this arc:

$$f_a = \sum_{p \in \mathcal{OD}} \sum_{r \in \mathcal{R}_p} x_{pr} \delta_{ar}, \tag{1.11}$$

where δ_{ar} is an element of the route-incidence matrix and will be equal to 1 if the arc is part of the route, and 0 otherwise. The total flow on all arcs is used in the continuous, monotonically increasing cost function associated with this arc:

$$\tau_a = \tau_a(\vec{f}) \quad \forall a \in \mathcal{A}. \tag{1.12}$$

where $\vec{f}$ is the vector of all flows on all arcs.

The total cost on a route is the sum of all costs on the arcs used:

$$C_{pr} = \sum_{a \in \mathcal{A}} \tau_a(\vec{f}) \delta_{ar}. \tag{1.13}$$

If the demand is variable and depending on the travel costs, one can replace d_p in equation (1.10) by a demand function $d_p(\vec{C}_p)$, where $\vec{C}_p$ is the vector of costs on all routes for OD-pair p.

The equilibrium conditions for the user equilibrium can be derived from the Wardropian principle: If a route is chosen, the traveling time is minimal and if the traveling time on a route is not minimal, agents do not choose this route:

$$
\begin{aligned}
C_{pr}(x_{pr}) &= C_p^{min} \text{ if } x_{pr} > 0 \\
C_{pr}(x_{pr}) &\geq C_p^{min} \text{ if } x_{pr} = 0,
\end{aligned}
\tag{1.14}
$$

or, written more compactly:

$$x_{pr} \left[C_{pr}(x_{pr}) - C_p^{min} \right] = 0 \text{ and } x_{rp} \geq 0. \tag{1.15}$$

Note that if we write the user equilibrium as a non-cooperative game with the following pay-off function:

$$C(f_a) = \sum_{a \in \mathcal{A}} \int_0^{f_a} \tau_a(s) \, \mathrm{d}s, \tag{1.16}$$

and as constraints Equations (1.10) and (1.11), the first order condition is equal to the Wardropian equilibrium conditions.

Equation (1.14) was first formulated as a variational inequality problem by (Dafermos, 1980) and can be written (in vector notation) as:

$$\vec{C}_p(\vec{x_p}^*)(\vec{x_p} - \vec{x_p}^*) \geq 0, \tag{1.17}$$

where $\vec{f}$ are non-negative flows and $\vec{d}$ are demands that fulfill the constraints (1.10) and (1.11). The vector pair $\vec{f}^*$ and $\vec{d}^*$ are a feasible solution to the UE problem. This formulation says that for any small deviation $d(\vec{x_p} = \vec{x_p} - \vec{x_p})$ from the solution, the increase in costs $(\vec{C_p}d\vec{x_p})$ must be positive (and in the flexible demand case, must exceed the marginal benefits).

The VI-formulation is mostly used for TEP models. However, there are two more possibilities of writing the model. Under the following assumptions, Beckmann, McGuire, and Winsten (1956, pp. 63-64) showed how to transform the UE with elastic demand to an optimization problem:

- If both the demand and cost functions are separable, i.e. the demand for an O-D pair only depends on the travel rate for that O-D pair, the cost on an arc only depends on the flow on that arc,

- and the demand functions are symmetric and can be inverted.

. The total costs are given by the following sum of integrals:

$$C(f_a, d_w) = \sum_{a \in \mathcal{A}} \int_0^{f_a} \tau_a(s) \, \mathrm{d}s - \sum_{p \in \mathcal{OD}} \int_0^{d_p} g_p(s) \, \mathrm{d}s, \tag{1.18}$$

where g is the inverted demand function. These costs are minimized subject to the constraints (1.10), (1.11) and the non-negativity constraints for the variables. In our case with fixed demand the second integral drops out. Writing the Lagrangian with the Lagrange multiplier λ_p associated with the constraint (1.10), and deriving the first order conditions with respect to x_{pr}, we get the Wardropian conditions of a user equilibrium for every route $r_p \in \mathcal{R}_p$:

$$x_{pr} \left[\sum_a \tau_a(f_a)\delta_{pra} - \lambda_p \right] = 0$$
$$\sum_a \tau_a(f_a)\delta_{pra} - \lambda_p \geq 0. \tag{1.19}$$

The term with the sum equals the costs of traveling along route r. If these costs are greater than λ_p, the flow will be zero, if they are equal this route will be chosen. This means that the route with the lowest costs is chosen and that the costs will be equal to λ_r. Therefore, the shadow price is equivalent to the minimal cost.

If the above assumptions do not hold, the integral of the inverse demand function will depend on the path of integration and is not uniquely determined.

The social optimum (SO) according to Wardrop is an optimization problem in which the total time on all routes is minimized and consists of equations (1.10) -(1.13), and the total cost function:

$$\min \sum_{p,r} C_{pr}. \tag{1.20}$$

We use a small example taken from Chapter 4.4 in Nagurney (1993): A small network (see Figure 1.4) with two nodes and five directed links of which three connect node $n1$ with node $n2$ and

two nodes in the opposite direction. The travel demand is given and is equal to 10 users in both directions. Table 1.2 shows the user cost functions for the links. Note that the costs not only depend on the flow on the arc itself, but also on flows on other arcs. Nagurney describes how to solve this model using either a projection mode or a relaxation mode (both algorithms are described in detail in Chapter 2 of the book). We will, as Nagurney suggested, use a quadratic programming problem for one of the subproblems.
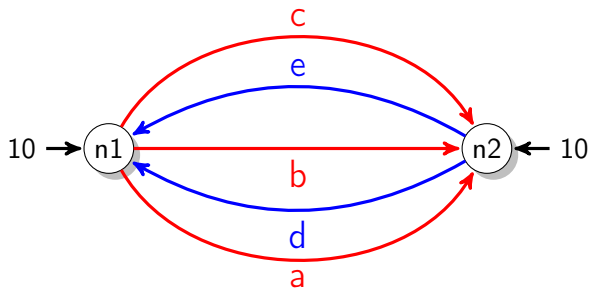
Figure 1.3 – Network example

OD	Arc	User cost function
(1,2)	a	$c_a = 5f_a + f_d + 5$
(1,2)	b	$c_b = 10f_b + 5f_d + 5$
(1,2)	c	$c_c = 10f_c + 5f_e + 110$
(2,1)	d	$c_d = 5f_d + 2f_a + 150$
(2,1)	e	$c_e = 4f_e + 3f_c + 10$

Table 1.2 – User cost functions

Figure 1.4 – The small network model from Nagurney (1993)

The projection method starts with the initialization of a feasible flow and demand pattern (F_0 and D_0), and selecting a symmetric, positive definite matrix G which satisfies certain properties.[15] In the following loop, the values for the vector h are constructed:

$$h^{k-1} = \rho C(F^{k-1}) - GF^{k-1}, \tag{1.21}$$

where k is the iteration number, C is the cost function, and F is the vector of flows. In the next step, the user equilibrium is calculated using an adjusted cost function with the form:

$$C^{k-1}(F) = GF + h^{k-1}, \tag{1.22}$$

the flow balance equation:

$$D_p = \sum_p F_p, \tag{1.23}$$

and the quadratic objective function:

$$OBJ = \sum_p 0.5 G_{p,p} F^2 + h_p F_p. \tag{1.24}$$

---

[15] In our example we use, as proposed by Nagurney, a diagonal matrix with the values of the coefficients of the user cost function for the flow on the specific arc: 5, 10, 10 ,5, and 4.

The implementation in GAMS is shown in Listing (1.5).[16]

Listing 1.5 – VI Implementation of transport model from Nagurney (1993)

```
$TITLE Model from Chapter 4.4, Nagurney 1993

sets
    n       nodes       /1,2/
    p       path        /a,b,c,d,e/,
    iter    iteration   /1*5/;

alias(p,ap);alias(n,o,oo,d,dd);

parameter nod(o,d)    OD matrix;

nod("1","2") = 10;
nod("2","1") = 10;

table c(p,*)  Coefficient user cost function
        a   b   c   d   e   const
a       5   0   0   1   0   5
b       0   10  0   5   0   5
c       0   0   10  0   5   110
d       2   0   0   5   0   150
e       0   0   3   0   4   10;

set odp(o,d,p)    Possible routes;

odp("2","1","a") = YES;
odp("2","1","b") = YES;
odp("1","2","c") = YES;
odp("1","2","d") = YES;
odp("1","2","e") = YES;

**    VI formulation
parameters
    g(p,ap)             G—Matrix from algorithm
    f(p)                Start values flow
    prevf(p)            Flow solution from previous iteration
    rho                 Multiplicative constant      /1/,
    convergence         Convergence criterium
    hnew(p)             Correction term user cost function;

*   Values for G set to diagonal elements of cost matrix
g(p,p) = c(p,p);

variable
    OBJ  Objective of quadratic programming problem;

positive variables
    FLOW(p)  Flow on arc p,
    COSTA(p) Adjusted user cost on arc p;

equations
    eq_costa(p)    Adjusted user cost function,
    bal_node(o,d)  Flow balance
    eq_quadratic   Quadratic programming objective;

eq_costa(p)..
    COSTA(p) =E= sum(ap,g(ap,p) * FLOW(ap)) + hnew(p);

eq_quadratic..
    OBJ =E= sum(p, 0.5 * g(p,p) * FLOW(p)**2 + hnew(p)*flow(p));

bal_node(o,d)$nod(o,d)..
    nod(o,d) =E= sum(p$odp(o,d,p), FLOW(p));

model nagurney_vi /eq_quadratic, bal_node, eq_costa/;

*   Inititialization
```

---

[16] Since the development of EMP (Extended Mathematical Programming; see Ferris, Dirkse, et al. (2009)), even the implementation of a VI model in GAMS is possible. EMP is not a solver but a framework for automated mathematical programming reformulations. EMP supports the modeling of Bilevel Programs, Variational Inequalities, Disjunctive Programs, Extended Nonlinear Programs and Embedded Complementarity Systems. In this case, one tells GAMS which are the equations that build the VI problem (cost function, flow definition, demand and flow balance) and solves the problem with the following code:

```
model mVI / cost, flow,demand, flowbalance/;
file myinfo / '\%emp.info\%' /;
putclose myinfo 'vi C F '; solve mVI using EMP;
```

where C is the variable cost and F the flow.

```
67  f(p)         =   0;
    f("a")       =  10;
    f("d")       =   5;
    f("e")       =   5;
    prevf(p)     = f(p);
72  hnew(p)      = rho * sum(ap, c(p,ap) * f(ap)) + c(p,"const") − sum(ap, g(ap,p) * f(
       ap));
    convergence  = 1;

    while(convergence ge 1e−6,
    *    Solve quadratic problem
77       solve nagurney_vi minimizing OBJ using nlp;
    *    Recalculate the adjustment term in user cost function
         hnew(p)          = rho * COSTA.L(p);
    *    Calculate the maximal difference between new and old flow values
         convergence = smax(p, abs(FLOW.L(p) − prevf(p)));
82       prevf(p) = FLOW.L(p)
    );
```

We now turn to the MCP formulation of the same model. The structure of this model consists of flow balance, cost function and the rationale for choosing the path with the shortest time (Equation (1.15) above)

$$C_p = C(F_p)$$

$$D_p = \sum_p F_p \perp C^{min}$$

$$C_p \geq C^{min} \perp F_p.$$

The implementation as shown in Listing (1.6), is straightforward.

Listing 1.6 – MCP Implementation of transport model from Nagurney (1993)

```
    variables
2       cost(o,d,p) User cost on path  traveling from o to d,
        minc(o,d)   Minimal cost on path traveling from o to d;

    positive variable
        flowod(o,d,p) Flow on path p  traveling from o to d;

    equation
        eq_cost(o,d,p)      User cost function,
        eq_rational(o,d,p)  Rational choosing path p,
        bal_nodeod          Node balance;

    eq_cost(o,d,p)$odp(o,d,p)..
      COST(o,d,p) =E= sum((ap,oo,dd)$nod(oo,dd), c(p,ap) * FLOWOD(oo,dd,ap)) + c(p,"
          const");

    eq_rational(o,d,p)$odp(o,d,p)..
17      COST(o,d,p) − MINC(o,d) =G= 0;

    bal_nodeod(o,d)$nod(o,d)..
        nod(o,d) =E= sum(p$odp(o,d,p), FLOWOD(o,d,p));

22  model nagurney_mcp /bal_nodeod.minc, eq_rational.flowod,  eq_cost.cost/;

    *    Fix variables that are not part of the model
    flowod.fx(o,d,p)$(not odp(o,d,p)) = 0;
    *    Drop the equations with fixed variables from the output
27  nagurney_mcp.holdfixed = 1;

    solve nagurney_mcp using mcp;
```

.

Listing (1.7) shows the solution of this problem. Only path e is used for traveling from node 1 but not from node 2. GAMS gives the marginals for the paths not used. These marginals shows the increase in time, if one would change from path e to either of paths c or d. For od-pair (1,2), if both routes are used and consistent with the Wardropian principle, the travel time is equal.

Listing 1.7 – Solution of MCP model formulation

```
1  ———— VAR cost   User cost on path  traveling from o to d

                LOWER           LEVEL           UPPER          MARGINAL

   1.2.c        −INF         160.0000          +INF              .
6  1.2.d        −INF         163.3333          +INF              .
   1.2.e        −INF          50.0000          +INF              .
   2.1.a        −INF          38.3333          +INF              .
   2.1.b        −INF          38.3333          +INF              .

11 ———— VAR minc  Minimal cost on path traveling from o to d

                LOWER           LEVEL           UPPER          MARGINAL

   1.2          −INF          50.0000          +INF              .
16 2.1          −INF          38.3333          +INF              .

   ———— VAR flowod  Flow on path p  traveling from o to d

                LOWER           LEVEL           UPPER          MARGINAL

   1.2.c          .              .             +INF          110.0000
   1.2.d          .              .             +INF          113.3333
   1.2.e          .          10.0000           +INF              .
   2.1.a          .           6.6667           +INF              .
26 2.1.b          .           3.3333           +INF              .
```

Comparing both formulations, we see that the MCP formulation is equal to the original problem formulation. There is no need for coding the algorithm, as GAMS sends the model to a solver which returns the solution to the user. Moreover, we can also see that the great drawback of the path-flow formulation is the necessary enumeration on available paths. This can be a tedious task in a network problem with many nodes. Although there are fast path-seeking algorithms, like the famous Dijkstra algorithm, [17] the researcher would have to implement these or is bound to specific software packages.

1.3.3 Arc-flow formulation of the transportation problem

The route-flow formulation is directly based on the Wardropian principles that are stated in terms of routes. Another formulation is the arc-flow formulation, in which the flows on the arcs are at the center of the problem and the equations have to be adjusted accordingly.

The total flow from node i to j is defined as the sum over agents traveling from node i to node j with any destination k (X_{ijk}):

$$F_{i,j} = \sum_k X_{i,j,k} \quad \forall (i,j) \in \mathcal{A}. \tag{1.25}$$

The flow conservation for every node j for the number of people traveling from this node to destination k is given by:

$$\sum_i X_{i,j,k} - \sum_i X_{j,i,k} = N_{j,k}, \tag{1.26}$$

where $N_{j,k}$ is the total flow of people of household type h traveling from node j to node k.

---

[17] See for example van Vliet (1978) and Magzhan and Hajar (2013) for a description and comparison of the Dijkstra, Floyd-Warshall, Bellman-Ford and Genetic Algorithms.

The second Wardropian principle requires that:

$$\tau_{ij} + T_{jk} \geq T_{ik} \quad \perp \quad X_{ijk}. \tag{1.27}$$

On the right-hand side of Equation (3.4) we have the minimal time $T_{i,k}$ for traveling from node i to k. This travel time should be less than or equal to the sum of the travel time τ on an arc starting at node i to any of the adjacent nodes j, and the minimal time T from traveling from the adjacent node j to the destination node k. The time minimization equations are associated with the non-negative variable $X_{i,j}$ as complementary variables. This variable is only positive for those adjacent arcs where the traveling time from i to k is minimal. If this is not the case, the flow on that arc will be zero.

This equilibrium condition can be derived by solving the equilibrium problem as a non-cooperative game when we replace f_a by F_a in the payoff function of the route-flow formulation (Equation (1.16), and using Equations (3.1) and (3.3)).

Note that a complete enumeration of all possible routes from node i to node k is now not necessary. The information on the fastest routes from the adjacent nodes j to the destination k is given by the corresponding minimum time equations for traveling from j to k.

The time minimization equation is explained in more detail in Figure 1.5. As an example, we are looking at the minimal time for traveling from node i to node k. The times for traveling to the adjacent nodes j_1 to j_3 are 6, 3 and 5 minutes, respectively. From these nodes, the minimal times for traveling to node k are 7, 8 and 7 minutes. This information is coming from the Wardropian equations for these nodes and is calculated simultaneously. For our traveler the fastest route is equal to 11 minutes, and the left hand side of the equation is equal to the minimal time on the right hand side. He travels from node i to node j_2 and then to k. This means that $X_{i,j_2,k} > 0$. For the two other routes the time traveling from node i over j_1 or j_3 to node k is greater than the minimal time and therefore, the flow on these arcs will be equal to zero.

The big advantage of the MCP formulation of the transport problem is that we do not have to specify in advance which arcs will be used. Ferris and Munson (2014, p. 4) state that this is the key property of a complementarity problem over a system of equations: "*If we know what arcs to send flow down, we can just solve a simple system of linear equations. However, the key to the modeling power of complementarity is that it chooses which of the inequalities satisfy as equations.*"

Figure 1.5 – The second Wardropian principle for node j

24
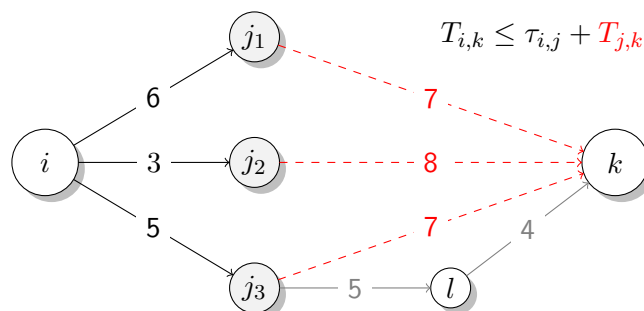
1.3.4 Example

We will show the different formulations of the TEP with the help of a simple, but interesting example: the Braess paradox (Braess, 1968) which states that by closing an arc in a network this can in some cases increase overall performance of the network. The example of the closure of 42nd Street in New York in 1990 that unexpectedly reduced instead of increased congestion in the area around this street (Kolata, 1990/12/25) is well known. This is because the user equilibrium of such a system is not necessarily optimal.

The network consists of four nodes, labeled O ("Origin"), D ("Destination"), A and B (see Figure 1.6-a). The travel costs are linear and also shown in the figure. In the original example, a highway is built to reduce the congestion on the other roads (see Figure 1.6-b). Travel time is fixed and is only 5 minutes.

(a) Network without highway

(b) Network with additional highway between A and B
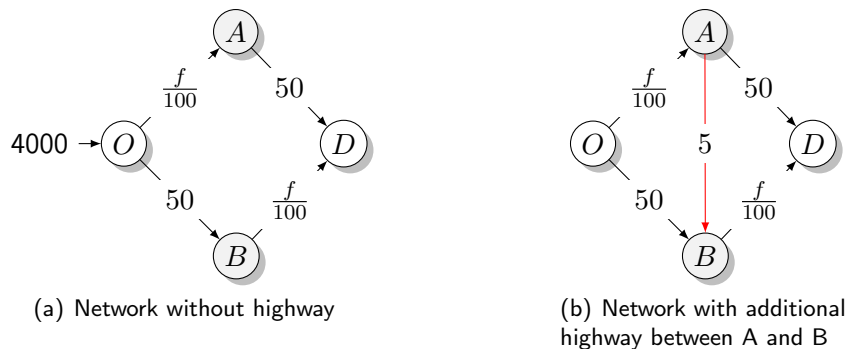
Figure 1.6 – Braess example: travel times

4000 people enter the city at node O and travel throughout that city with destination D. Without the additional highway the social optimum and the user equilibrium result in the same equilibrium (marginal and average costs are the same; 1.7-c). Half of the people choose route OAD and the other half route OBD. The travel time is 70 minutes. There is no incentive to change the route.

As soon as the highway is opened, the situation changes (Figure 1.7-b). In the user equilibrium everybody wants to use the route $OABD$. This is a Nash equilibrium as the time used to travel this route is 85 minutes. If one person would change its route to OBD, this would increase the travel time by 5 minutes. The user equilibrium without the highway is therefore better.

The social optimum is given in Figure 1.7-c. The travel times are either 50 minutes ($OABD$) or 72.5 minutes (OAD, OBD). The total travel costs are around 20 hours less than in the case without the highway.

The following GAMS code shows the implementation of the social optimum as NLP problem and the user equilibrium as MCP (the full code can be found in Appendix 1.A).

Listing 1.8 – Social optimum NLP and user equilibrium MCP formulation of Braess

```
equations
    flowbalance(i,j)  Flow balance
    rational(i,j,d)   Wardropian conditions
    cost(i,j)         Cost function
```

(a) Social Optimum and user equilibrium without highway

(b) User equilibrium with highway

(c) System Optimum with highway
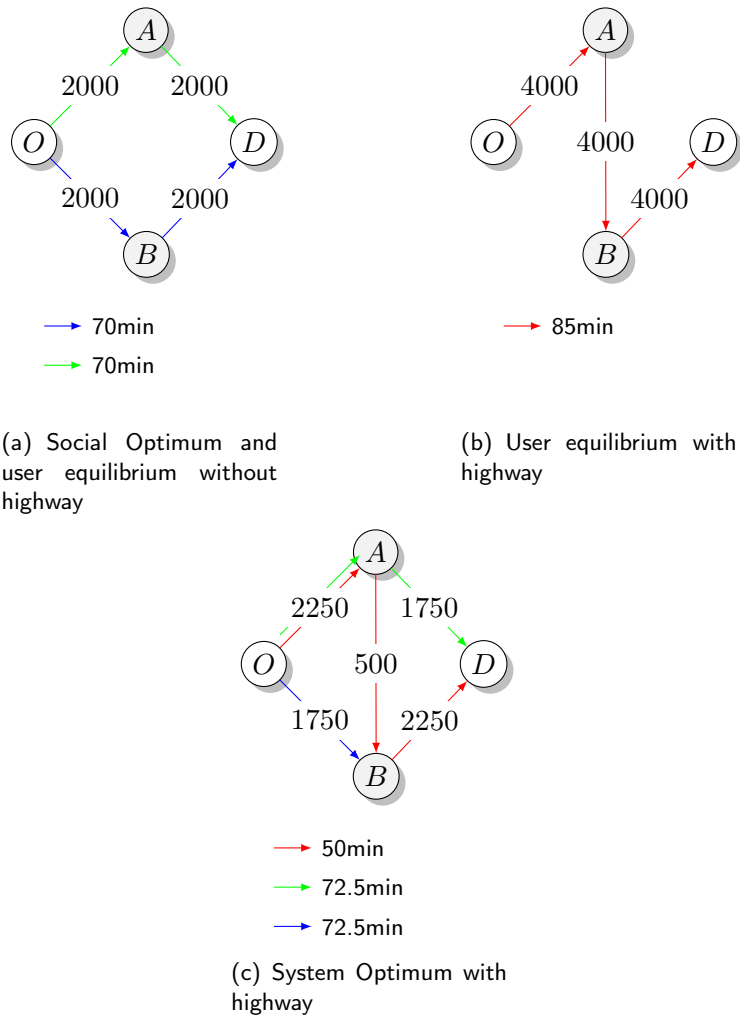
Figure 1.7 – Solutions with and without highway

```
      flow(i,j)          Flow definition
      objdefSOC          Objective function for SO;

9   set k(i)  Active destinations;

    k(i)$sum(o, od(o,i)) = YES;

    positive variables
14      X(i,j,d)    Flow on arcs
        T(i,d)      Minimal time
        C(i,j)      Cost functions
        F(i,j)      Flow
        T(i,j)      Minimal time;

    variable
        TAX(i,j)    Endogenous tax,
        OBJSOC      Objective;

24  flow(a)..
        F(a)   =E= sum(k, X(a,k));

    flowbalance(j,k)$(not sameas(j,k))..
        sum(i, X(j,i,k)) - sum(i, X(i,j,k))  =E= od(j,k);

    cost(a)..
        C(a) =E= a0(a)  + a1(a)*F(a);

    rational(i,j,k)$a(i,j)..
34      C(i,j) + T(j,k) =G= T(i,k);

    objdefSoc..
        objSoc =E= sum((a,d), C(a)*X(a,d));
```

```
39  model braess_UE /flow.F, flowbalance.T  , cost.C, rational.X/;
    model braess_SO /flow.F, flowbalance    , cost, objdefSOC/;
```

An additional example taken from Nagurney (2009) can be found in Appendix 1.B. The user equilibrium is formulated as a VI-problem, a MCP and a NLP-problem. This code contains also two additional model formulations: A mathematical program with equilibrium constraints (MPEC) that solves the optimal subsidy on the roads to reach the social optimum. The other model formulation calculates one of the many possibilities of imposing a road toll to reach the social optimum.

1.4 Computable general equilibrium models demystified

1.4.1 Introduction

In the following we will show that building a simple CGE model using the mixed complementarity format is easy. We introduce the two most important building blocks of a CGE model, the household and producer optimization problems, and use these partial models to build a CGE model. The implementation of these small models in optimization software like GAMS or MATLAB is straightforward and for every example the GAMS program listing is shown.[18]

A general equilibrium problem can be described as finding factor and commodity prices, production levels and household demand for goods such that each producer in the economy maximizes profits and each household maximizes its utility from consumption. CGE models allow economists to analyze the effects of policy changes on, for example, the welfare of households and the allocation of resources. In the case of the integrated model of the next chapter, it will allow the researcher to analyze the impact of transport policy measures on housing prices, wages and distributional effects, which are not captured by pure transport equilibrium models.

CGE models can be formulated and solved in different ways (see for example Ginsburgh and Keyzer, 1997, Chapter 3). CGE models are often written as an optimization problem. This has the drawback, that the formulation of the necessary equations can be very time consuming and error prone. Furthermore, corner solutions are not easily dealt with and often need artificial constructs to solve the model (for example, dummy variables which are maximized, or explicit Lagrange multiplicators). The mixed complementarity problem-formulation of a CGE model permits writing CGE models as a system of nonlinear equations, complementary problems or variational inequalities in a natural format. This formulation allows a broader range of CGE models to be implemented.

---

[18] Computable general equilibrium models go back to the Walrasian general equilibrium structure formalized in the 1950s by Arrow and Debreu 1954 and Debreu 1959 and several other economists. The interested reader can refer to a number of good books on microeconomics and CGE modeling. For CGE modeling the classic introduction by Shoven and Whalley (1992) and by Hosoe, Gasawa, and Hashimoto (2010) might serve as a good start. An introduction to microeconomic theory and the foundations of general equilibrium are in Varian (1993) or Nebchya (2010). An advanced textbook on microeconomics is by Mas-Colell, Whinston, and Green (1995).

The MCP model is formulated as a nonlinear system of (weak) inequalities corresponding to the three classes of equilibrium conditions associated with the Arrow-Debreu general equilibrium: The first class consists of the market clearing conditions: supply minus demand for every commodity should be non-negative. The second class are built of zero-profit-conditions: no sector earns a positive profit. The third class of conditions contains the income conditions: income should be greater than or equal to total expenditure. The fundamental unknowns of the system are three vectors consisting of non-negative prices (commodities and factors like capital and labor), production levels (production indices) and household incomes. In equilibrium each of these variables is linked to one of the inequalities mentioned above: a commodity price with a market clearing condition, an activity level with an exhaustion of the product constraint, and a consumer income variable with an income definition equation. The standard Arrow-Debreu framework can be easily extended to reflect market imperfections such as taxes, administered prices and quantities.

Households and firms

A CGE model must at least have one household that maximizes its utility given its income. Utility is a function of the consumed goods. In CGE modeling the most widely used functions are those of the class of constant elasticity of substitution (CES). These functions allow to capture how easily a household substitutes one consumption good for another if, for example, their relative prices change. CES functions allow the goods, depending on the elasticity, to range from perfect substitutes (with elasticity of substitution of ∞) to perfect complements (with elasticity of substitution of zero). The "constant" in CES refers to the fact that the substitution elasticity is equal for all bundles of consumption.

The CES utility function for a bundle of two goods is given by:

$$U(X_m, X_n) = \left(\alpha X_m^{-\rho} + (1-\alpha)X_n^{-\rho}\right)^{-\frac{1}{\rho}}, \tag{1.28}$$

where X_i is the demand for good i, $0 < \alpha < 1$, and $1 \le \rho \le \infty$. The substitution parameter ρ is linked to the substitution elasticity σ by $\sigma = 1/(1+\rho)$.

We will now use a simple example showing how to use the MCP format to solve this maximization problem. We will use the Cobb-Douglas utility function, which is a special case of the CES function with a substitution elasticity of 1. We can write the utility maximization problem as a partial equilibrium model, in which we assume that income I and commodity prices p_m and p_n are given:

$$U(X_m, X_n) = X_m^\alpha X_n^{1-\alpha}$$
$$\text{s.t. } INC \ge p_m X_m + p_n X_n.$$

where X α is expenditure share for good m.

Although this model is easily solved as an optimization problem, it will be difficult to implement it if we have more than one household. With multiple households there is more than one maximization problem and it is not easy to generate an overall function that maximizes all these single utility

functions, let alone solve it.[19] If we write the problem with multiple households as a MCP, we can solve for several households at the same time, as the MCP is a set of non-linear equations without a function to be optimized.

The MCP formulation of the utility maximization can be derived by writing the Lagrangian and using the first order conditions (FOC). If we go one step further and derive form the FOCs the demand equations, we can write the equations in the Arrow-Debreu format with market clearing conditions and the income constraint (as supply is exogenously given, there is no zero-profit condition).

The Lagrangian is formulated as follows:

$$\mathcal{L}(X_m, X_n, \lambda) = U(X_m, X_n) + \lambda[INC - P_m X_m - P_n X_n], \tag{1.29}$$

where λ is the Lagrange multiplier.

The first order conditions with respect to X_n and X_m are:

$$\alpha X_m^{\alpha-1} X_n^{1-\alpha} - \lambda P_m = 0$$
$$(1 - \alpha) X_m^{\alpha} X_n^{-\alpha} - \lambda P_n = 0$$

If we divide the first equation by the second, and solve for X_m we get:

$$X_m = \frac{1 - \alpha}{\alpha} \frac{P_n X_n}{P_m}.$$

Using the budget constraint to substitute for $P_n X_n$, and repeating the same procedure for X_n we can derive the demand equation for both goods:

$$D_m = \frac{\alpha INC}{P_m} \text{ and } D_n = \frac{(1 - \alpha) INC}{P_n}. \tag{1.30}$$

The complete model with fixed supply, income and commodity prices in MCP format consists of only four equations: the budget constraint, the two demand functions, and the market clearing for both goods, which is given by:

$$S_m = X_m \text{ and } S_n = M_n,$$

The following listing shows the GAMS code for this small model.

Listing 1.9 – Gams code for utility maximization in MCP format

```
set i /m  Consumption good 1,
      n  Consumption good 2/;

positive variables
    D(i)  Demand,
    P(i)  Prices,
    INC   Income,
    S(i)  Supply;

parameters
    Inclevel      Income          /100/
    p0(i)         Price levels    /m 1,   n 1/
```

---

[19] An equilibrium can be found by maximizing the weighted sum of the utility of all agents in a loop, where the weights are adjusted in every iteration (Negishi, 1960).

```
13      alpha(i)    Share of good m /m 0.3, n 0.7/
        supply(i)   Supply of goods /m 100, n 100/;

   equations
        mkt_x(i)    Market clearance,
18      demand(i)   Demand functions,
        budget      Budget constraint;

   mkt_x(i)..
        S(i) =G= D(i);

   demand(i)..
        D(i) =E= alpha(i) * INC / P(i);

   budget..
28      INC =G= sum(i, P(i) *D(i));

   *  Fix the variables that are given.
   INC.FX  = inclevel;
   P.FX(i) = p0(i);
33 S.FX(i) = supply(i);

   model util /mkt_x.P, demand.D, budget.INC/;

   solve util using mcp;
```

The second component of a CGE model is the production side. Producers minimize their costs given the production technology, market prices for their output, and input prices. We can cast this problem again in the MCP format of the Arrow-Debreu model. In the case of a Cobb-Douglas production function, the producer solves the following problem:

$$\min \overline{R}K + \overline{W}L$$

$$\text{s.t. } \overline{Y} = AK^\alpha L^\beta.$$

Writing down the Lagrangian for this minimization problem and deriving the first order conditions gives us:

$$\lambda_y r = \delta AK^{\delta-1}L^\beta = 0$$

$$\lambda_y w = \gamma AK^\delta L^{\gamma-1} = 0$$

.

Dividing the first by the second equation, and solving for L, we can derive the demand for labor:

$$L = \frac{r}{w}\frac{\gamma}{\delta}K.$$

Substituting this into the production function and solving for K gives us the factor demand equation (conditional on the level of output). The same procedure can be used to derive the factor demand functions of K and L:

$$K = \frac{1}{A}\left(\frac{\alpha}{\beta}\frac{W}{R}\right)^\beta Y$$

$$L = \frac{1}{A}\left(\frac{\beta}{\alpha}\frac{R}{W}\right)^\alpha Y.$$

The model in GAMS format is shown in the following listing.

Listing 1.10 – Gams code for cost minimization in MCP format

```
parameter alpha Share of capital in production /0.3/
          beta  Share of labor in production   /0.7/;

variables
```

```
      K    Capital input,
      L    Labor input,
      Y    Production level
8     W    Wage,
      R    Capital costs;

   equations
      demand_k, demand_l,zeroprofit, prodfun;

   demand_k..
      K =E= (alpha/beta * W/R)**beta * Y ;

   demand_l..
18    L =E= (beta/alpha* R/W)**alpha * Y;

   prodfun..
      Y =E= (K)**alpha * (L)**beta;

23 * Fix and initialize the variables
   Y.FX = 1;
   W.FX = 1;
   R.FX = 1;
   K.L  = 1;
28 L.L  = 1;

   model production /demand_L.L, demand_K.K, prodfun.Y/;

   solve production using mcp;
```

The step to a general equilibrium model even if is small is straightforward by combining the two partial models, changing some of the parameters to variables, and adding the market clearing conditions for the factors and commodities. In the utility maximization problem there is demand for two goods and therefore, we have to add an additional production function.

Listing 1.11 – Gams code for CGE model in MCP format

```
   set i /m  Consumption good 1,
         n  Consumption good 2/;

   parameters
      gamma(i) Share of good m    /m 0.5, n 0.5/
      kbar      Capital endowment /100/,
      lbar      Labor endowment   /100/,
8     alpha(i) Capital share in production /m 0.3, n 0.7/,
      beta(i)  Labor share in production   /m 0.7, n 0.3/;

   parameter  A(i) Cobb Douglas constant;

13 * Calibrate A, so we have a benchmark equilibrium with prices equal to 1

   A(i) = 100 / ((alpha(i) * 100)**alpha(i) * (beta(i) * 100)**beta(i));

   positive variables
18    D(i)  Demand,
      P(i)  Prices,
      Y(i)  Supply,
      K(i)  Capital input,
      L(i)  Labor input,
23    W      Wage,
      R      Capital cost;

   variable
      INC   Income;

   equations
      zeroprofit(i) Zero—profit condition,

      mkt_L          Market clearance,
33    mkt_K          Market clearance,
      mkt_x(i)       Market clearance,

      budget         Budget constraint,

38 * Definitions
      demand(i)      Demand functions,
      demand_k(i)    Input demand capital,
      demand_l(i)    Input demand labor;

43 zeroprofit(i)..
      P(i)*Y(I) =G= (W*L(i)+ R*K(i));
```

31

```
    mkt_L..
        lbar =G= sum(i, L(i));

    mkt_K..
        kbar =G= sum(i, K(i));

    mkt_x(i)..
53      Y(i) =G= D(i);

    budget..
        R * kbar + W * LBAR =G= sum(i, P(i) *D(i));

58  demand(i)..
        D(i)  =E= gamma(i) * INC / P(i);

    demand_k(i)..
        K(i) =E= (alpha(i) / beta(i) * W / R)**beta(i) * Y(i) /A(i) ;

    demand_l(i)..
        L(i) =E= (beta(i) / alpha(i)* R / W)**alpha(i) * Y(i)/A(i);
    *     Initialize with the benchmark values
68  Y.L(i) =100;
    P.L(i) = 1;
    W.L     = 1;
    R.L     = 1;
    K.L(i) = 1;
73  L.L(i) = 1;
    INC.L   = 200;
    P.L(i) = 1;
    Y.L(i) = 100;
    D.L(i) = 100;
78  K.L(i) = alpha(i) * 100;
    L.L(i) = beta(i) * 100;
    model smallcge /zeroprofit.Y, mkt_L.W, mkt_K.R, mkt_x.P,
                    budget.INC, demand.D, demand_L.L, demand_K.K/;

    solve smallcge using mcp;
```

A good example of a CGE model that has been used for the analysis of the US corporation income tax based on a paper by Harberger (1962), can be found in Shoven and Whalley (1972). The Harberger paper is probably the most used and seldom cited paper in CGE research as it introduced the "Harberger convention". As data is usually aggregated in CGE models, the price of a real good can be hard to define. Harberger, therefore, introduced the idea of taking the unit of a good as the amount that can be bought by one unit of the used currency (e.g. instead of one bottle of wine at a price of 10 dollars, we have 1/10 bottle as unit at a price of 1 dollar). We will use this convention for the calibration of the data in the third essay.

This example was also used in introductions of Shoven and Whalley (1984) and Shoven and Whalley (1992).[20] The description and the GAMS listing of the model can be found in Appendix 1.C.

1.5   Bringing it all together: The integrated model

We turn to the integration of the transport arc-flow equilibrium and the general equilibrium model discussed in the two previous sections. The model is formulated as a mixed complementarity problem of an urban general equilibrium model that embeds a closed, spatially disaggregate economic model

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[20] Shoven (1976) pointed out two errors in the original paper by Harberger that changed some of the original results significantly (see also the interesting reply in Harberger and Bruce, 1976).

of housing and labor markets along the lines of the Alonso-Muth-Mills model [21]. The economic model is embedded within a model of individually-rational route choice on the (congested) traffic network (Wardrop, 1952).

The economic aspects of the model follow the Walrasian-Arrow-Debreu paradigm. Consumers earn by providing labor to the production sectors, and they allocate their income to housing and consumption. The model includes both traffic congestion and co-locational externalities among firms. Taxes at the residential or employer locations, can be used to reach an optimal pattern of location. One of the main assumptions of the model is that households choose residence and employment locations which arbitrage differences of various locations within the urban area. Consumers trade locations to work and live on the basis of housing prices, wages, and commuting time. The commuting costs in the model depend on the distance between the location where the household lives and where he or she works as well as on the mode chosen. The commuting time on the arcs is defined by the capacity and the size of the flow on the arcs. Transport costs for commuting are capitalized in housing values (Glazer and Van Dender, 2002) and can lead to wage differentials, depending on zone of employment (as estimated by Darren and Wheaton, 2001).

1.5.1 The Transport Submodel

The transport equilibrium model has the same structure as the TEP model that was introduced in Section 1.3.2 and consists of four equation groups. The first defines the flows for each mode m on the arcs:

$$F_{m,i,j,k} = \sum_k X_{m,i,j,k} \quad \forall (i,j) \in \mathcal{A}, \tag{1.31}$$

with the associated free variable $F_{m,i,j}$ and $X_{m,i,j,k} \geq 0$ and $\mathcal{A}$ the set of all arcs.

The effect of the road capacity on travel times is specified by means of volume-delay functions. The most popular volume-delay function for private transport is the one found in Bureau of Public Roads (1964) and defines the travel time on an arc $\tau_{m,i,j}$ as:

$$\tau_{m,i,j} = \alpha_{i,j} + B^{m,i,j} \left(\frac{F_{m,i,j}}{\kappa_{m,i,j}} \right)^4 \quad \forall (i,j) \in \mathcal{A}, \tag{1.32}$$

where α is free flow time on the arc, B is congestion scale factor and κ is capacity of the arc.

Figure 1.8 shows the form of the volume-delay function.

The third group of equations defines the flow conservation at every node j for the number of people traveling from this node to destination k:

$$\sum_{i,m} X_{m,i,j,k} - \sum_{i,m} X_{m,j,i,k} = N_{j,k}, \forall (m,i,k) \text{ and } i \neq d, \quad \forall j \in \mathcal{N}, \tag{1.33}$$

---

[21] For a description of the Alonso-Muth-Mills model see, for example, Glaeser (2008, Chapter 2) or Brueckner (1987).
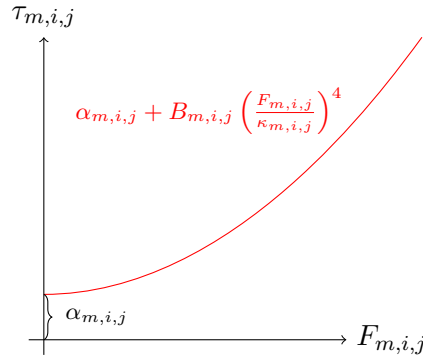
Figure 1.8 – Volume-delay function of transport modes

where $N_{j,k}$ is the total flow of people living in node $j \in \mathbb{O}$, the set of all origins, and node $k \in \mathbb{D}$, the set of all destinations. The associated free variable is the minimum time from node j to node k ($T_{m,j,k}$). The set of all nodes is $\mathcal{N}$.

The last equation group reflects the second Wardropian principle:

$$\tau_{m,i,j} + T_{m,j,k} \geq T_{m,i,k} \quad \text{and} \quad T_{m,k,k} = 0 \quad \forall (i,j) \in \mathbb{A}^m, \tag{1.34}$$

with $X_{m,i,j,k}$ as associated free variable. On the left hand side we have the minimal travel time T from node i to k. This travel time should be less than or equal to the sum of the travel time τ on an arc starting from node i to any of the adjacent nodes j, and the minimal time T from traveling from the adjacent node to the destination node k.

1.5.2 The Economic Submodel

The economic model is formulated as an Arrow-Debreu model with firms and households who maximize their profits and utility, respectively. On the production side we have at every node identical firms who use either labor and capital as inputs for producing a single output Y (see the left part of Figure 3.2). We assume a Cobb-Douglas production function with substitution elasticity equal to one.

Firms maximize their profits and the zero profit condition for production at node j can be formulated as follows:

$$\left(\frac{W_j}{\overline{W}}\right)^{1-\theta^k} \left(\frac{R_j}{\overline{R_{k,j}}}\right)^{\theta^k} \geq pc \quad \perp \quad Y_j \tag{1.35}$$

where W_j is wage for labor and R_j is rental price of capital at node j. θ_k is value share of capital. pc and Y_j are price and output level of the production sector.

Market clearing for labor demand is given by:

$$\sum_o N_{o,d} = \left(1 - \theta^k\right) \frac{pc Y_d}{W_d}, \tag{1.36}$$

Figure 1.9 – Production and utility function

(a) Production function   (b) Utility function
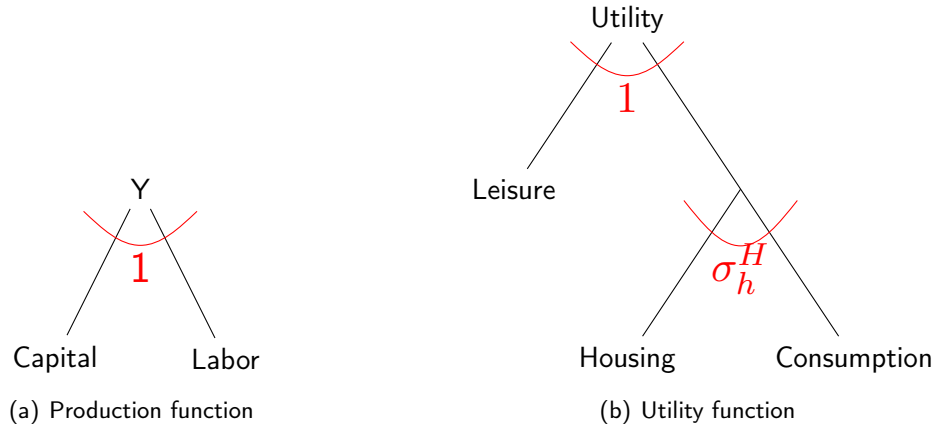
where the left-hand side of the equation represents total labor supply from all households living at nodes $o \in \mathcal{O}$ and working in $d \in \mathcal{D}$.

As the capital and housing stock are fixed, the capital stock price index can be written as follows:

$$PK = \frac{\sum_i PH_i \overline{HS}_i + \sum_j RK_j \overline{KS}_j}{\sum_i hs_i + \sum_i ks_i}, \tag{1.37}$$

where PH_j and RK_i are housing rental price and RK_i is rental price of capital services at node i. hs_i and ks_i the capital stock available at node i.

The capital market clearing condition at node j is given by the following equation:

$$\overline{KS}_j = \sum_h \theta^k \frac{pcY_j}{RK_j}, \tag{1.38}$$

where we have the capital supply on the left-hand side of the equation and the capital demand on the right-hand side of the equation. The associated free variable is RK_j.

We now turn to the household side of the economy. Households are characterized by the place they live (o), and the place they work (d). They maximize their utility level with respect to their income. The utility is given by a nested Constant-Elasticity-of-Substitution function (see Equation (3.13) and Figure 3.2).

$$U_{o,d} = \left(\theta^{ls} \sum_m LS_{m,o,d}^{\rho} + \left(1 - \theta^{ls}\right) \left[\frac{INC_{o,d}}{\overline{INCPC}_i} \right]^{\rho} \right)^{\frac{1}{\rho}} + \xi_{o,d}, \tag{1.39}$$

where ρ is the elasticity parameter of the utility function between leisure and aggregate of consumption and housing for household group. The amenity for living at node i and working at node j is given by $\xi_{o,d}$. This factor reflects the (unobserved) attributes for the living-working combinations that are not explained by the model (for example, preferences for schools or environmental amenities).

Leisure endowment is calibrated to unity when commute time equals $\overline{T}$, where T^{max} is the maximum travel time in the network:

$$LS_{m,o,d} = \frac{\overline{T}_m^{max} - T_{m,o,d}}{\overline{T}_m^{max} - \overline{T}_m}. \tag{1.40}$$

The housing market at node i clears if:

$$hs_o = \theta^H \sum_d N_{o,d} INC_{o,d} \frac{PC_o^{\sigma^H - 1}}{PH_o^{\sigma^H}}, \tag{1.41}$$

where the value share of house rent is given by θ^H.

The price for the composite of housing and consumption PCH_i is given by:

$$PCH_o = \left(\theta^H PH_o^{1-\sigma^H} + \left(1 - \theta^H\right) pc^{1-\sigma^H} \right)^{\frac{1}{1-\sigma^H}}. \tag{1.42}$$

The income of a household working in d is given by the wage income at d plus the capital income out of the capital $(\overline{K})$ and housing stock $(\overline{H})$. The fraction θ^d defines the share of local capital income.

$$INC_{o,d} = W_d + \frac{\overline{K}_d + \overline{H}_d}{n^{tot}} \left[\theta^d PK + \left(1 - \theta^d\right) \right] \tag{1.43}$$

where $INC_{o,d}$ is the associated free variable.

1.5.3   Mode Choice and Sorting

A household has to decide where to live and work as well as which transport mode it will use for commuting. These decisions are taken simultaneously as the amount of leisure in the utility function depends on the commuting time, which itself depends on the choice of where to live and work, the mode and route chosen.

We assume that a household chooses a particular combination of living and working location if the utility of this combination is at least as large as the utility from any other combination. If this utility is higher than the actual utility, the household will resort to new locations. This means that in the equilibrium the combination of living and working location (i, j) is chosen if

$$U^* \geq U_{i,j}, \tag{1.44}$$

where U is equal to the average level of utility of all households living at i and working at j:

$$U = \frac{\sum_{i,j} N_{i,j} U_{i,j}}{N} \quad \perp \quad N_{i,j}, \tag{1.45}$$

where $N_{i,j}$ is the number of people living at node i and working at node j, and $\overline{N}$ is the overall number of people. This means that if for example the attainable utility would be higher for a certain (o,d)-pair, more people will choose this node and the utility will drop until it is equal to the overall utility in the city. If $U_{i,j}^* < U^*$, $N_{i,j}^*$ will be zero as shown in Ginsburgh, Papageorgiou, and Thisse

(1985): Using the definition of U, we get $\overline{NU} = \sum_{i,j} N_{i,j} U_{i,j}$ or, in equilibrium $\sum_{i,j} N_{i,j}^* (U_{i,j}^* - U^*)$. By contradiction one can see that if $N_{i,j} < 0$, it must be that $\sum_{i,j} N_{i,j}^* (U_{i,j}^* - U^*) < 0$, which is a contradiction. Equation (3.18) is therefore complementary to $N_{i,j}$.

The mode choice is captured with a binary logit model, where the utility of traveler h using one of the two modes on arc (i, j) is given by:

$$U_{h,m,i,j}^{Transport} = \frac{T_{m,i,j} + \phi_{m,i,j}}{\lambda_{h,i,j}} + \epsilon_{h,m,i,j}, \tag{1.46}$$

where $\lambda_{h,i,j}$ is a scale parameter and, as an increase in travel time reduces utility, is less than zero. The unobserved utility $\epsilon_{h,m,i,j}$ is distributed as iid extreme value with variance $\pi^2/6$ (Gumbel). As this scale parameter goes to zero, there is infinite taste heterogeneity (the variance goes to infinity) and we have equal choice probabilities for all alternatives and therefore households choose completely randomly. If the scale parameter goes to infinity (and the variance goes to zero), the households choose deterministically the alternative with the minimal time (Ben-Akiva and Lerman, 1997). The constant $\phi_{m,i,j}$ captures the average effect on utility of the factors that are not included in our model. As only the differences in the alternatives matter, we can normalize the absolute levels of the constants by dropping the constant for one of the modes. The remaining constant now captures the average effect of all the factors not included of one mode relative to the other mode (see Train, 2009).

The share of transport is given by the following logit function:[22]

$$\theta_m = \frac{e^{\frac{T_m + \phi_m}{\lambda_{ij}}}}{e^{\frac{T_m + \phi_m}{\lambda_{ij}}} + e^{\frac{T_n}{\lambda_h}}} \tag{1.47}$$

The number of people using mode m for this location pair is now defined as:

$$N_{m,i,j} = \theta_{m,i,j} NN_{i,j}. \tag{1.48}$$

Equations (3.1) to (3.21) define the equilibrium of the integrated model. An implementation for the analysis of a cordon toll in Zurich, largest city in Switzerland, can be found in the third essay in this thesis.

1.6 Summary

This paper gave a gentle introduction to the mixed complementarity format and showed how to use it in transport and general equilibrium modeling.

In transport equilibrium modeling most models are written as a route-flow problem in VI format. This formulation has the drawback that the research either has to use software geared to transport modeling or has to implement the algorithms for finding routes and solving the VI. The MCP format

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[22] For convenience we drop the indices for start and end node.

allows to use more general modeling software like GAMS or Matlab, and there is no need for the enumeration of possible routes.

The MCP format is becoming more and more the workhorse for computable general equilibrium models as it allows to use the Arrow-Debreu formulation. This formulation has a clear structure with three groups of equations: income definitions, zero-profit and market clearing conditions. The implementation of a simple, but complete, computable equilibrium model was shown.

Using the MCP format for both type of models also allows to integrate both models to a more complete model. This integrated model can give insights in the interaction between the economic and transport system and show pitfalls if one relies either on the results of only one model.

Every model introduced in this paper was first derived theoretically and then implemented in GAMS. The transport, and economic models and the integrated models in MCP format can be easily extended in many directions to more realistic models. Dynamic transport modeling, where agents can choose when to travel, more realistic delay functions, where the flows on other arcs influence the travel time, imperfect competition, and environmental externalities are only a few of the many possibilities.

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Appendix 1.A   The Braess Paradox

Listing 1.12 – The Braess Paradox model formulations

```
1   $TITLE The Braess paradox
*   User equilibrium MCP formulation
*   SO NLP formulation

    sets
6       n       nodes /O,A,B,D/;

    alias(n,i,j,o,d);

    sets
11      arcs(i,j)  /(O.A),(O.B),(A.D),(B.D),(A.B)/;

    alias(a,arcs);

    parameter
16      a0(i,j) constant in link cost function,
        a1(i,j) multiplicator in link cost function,
        od(o,d) O—D matrix;

    parameter
21      od(i,j)          Demand from node i to node j;

    od("O","D") = 4000;

    a0("O","B") = 50;
26  a0("O","A") = 0;
    a0("A","D") = 50;
    a0("B","D") = 0;
    a0("A","B") = 5;

31  a1("O","B") = 0;
    a1("O","A") = 0.01;
    a1("A","D") = 0;
    a1("B","D") = 0.01;
    a1("A","B") = 0;


    equations
        flowbalance(i,j) Flowbalance
        rational(i,j,d)  Wardropian conditions
41      cost(i,j)        Cost function
        flow(i,j)        Flow definition
        objdefSOC        Objective function for SO;

    set k(i)  Active destinations;

    k(i)$sum(o, od(o,i)) = YES;

    positive variables
        X(i,j,d)   Flow on arcs
51      T(i,d)     Minimal time
        C(i,j)     Cost functions
        F(i,j)     Flow
        T(i,j)     Minimal time
        TAX(i,j)   Endogeneous tax;

    variable
        OBJSOC     Objective;

    flow(a)..
61      F(a)   =E= sum(k, X(a,k));

    flowbalance(j,k)$(not sameas(j,k))..
        sum(i, X(j,i,k)) — sum(i, X(i,j,k))  =E= od(j,k);

66  cost(a)..
        C(a) =E= a0(a) + a1(a)*F(a);

    rational(i,j,k)$a(i,j)..
        C(i,j) + T(j,k) + TAX(i,j) =G= T(i,k);

    objdefSoc..
        objSoc =E= sum(a, C(a)*F(a));

    model braess_UE /flow.F, flowbalance.T  , cost.C, rational.X/;

    model braess_SO /flow.F, flowbalance  , cost, objdefSOC/;

    TAX.FX(i,j) = 0;
```

```gams
   X.FX(i,j,k)$(not a(i,j)) = 0;
81 T.FX(i,i) = 0;

   braess_UE.holdfixed = 1;
   braess_SO.holdfixed = 1;

86 parameter results(*,*)  Results;

   * Solve user equilibrium without additional road by
   * setting costs for this arc very high.
   a0("A","B") = 100000;
91 solve braess_UE using mcp;

   results("UE without highway","Total time") = sum((a(i,j),k), X.L(i,j,k)* C.L(i,j))
      /60;

   solve braess_SO minimizing OBJSOC  using nlp;
96 results("SO without highway","Total time") = sum((a(i,j),k), X.L(i,j,k)* C.L(i,j))
      /60;

   * Solve the user equilibrium and social optimum with arc
   a0("A","B") = 5;

101 solve braess_UE using mcp;
   results("UE wit highway","Total time") = sum((a(i,j),k), X.L(i,j,k)* C.L(i,j))/60;

   solve braess_SO minimizing OBJSOC using nlp;
   results("SO wit highway","Total time") = sum((a(i,j),k), X.L(i,j,k)* C.L(i,j))/60;
106 display results;

   * MPEC formulation of MCP

   model mMPEC /cost.C, flow.F, flowbalance.T, rational.X, objdefsoc /;

   TAX.LO(i,j) = -INF;
   TAX.UP(i,j) = +INF;
   TAX.FX(i,j)$(not a(i,j)) = 0;
   mMPEC.holdfixed = 1;
116 solve mMPEC  using MPEC minimizing objsoc;
   $exit
   equations equalfee;
   equalfee(i,j).. TAX(i,j) =G= 0;


   model mMPECE /def_c.C, c_flow.F, c_flowbalance.T, equalfee, c_foc.X, obj_so /;

   TAX.LO(i,j) = -INF;
   TAX.UP(i,j) = +INF;
126 TAX.FX(i,j)$(not a(i,j)) = 0;
   mMPECE.holdfixed = 1;
   solve mMPECE  using MPEC minimizing objso;
```

Appendix 1.B The Braess Paradox: MCP, NLP, VI and MPEC formulations

This example is taken from Nagurney (2009).

Listing 1.13 – The Braess Paradox model formulations

```gams
  $TITLE Model in Beckman/VI/MCP format from Nagurney 2009
2 * Network economics: a variational inequality approach, p. 13.

  option limrow = 10;
  * Model with fixed demand

7 sets
     n       nodes /1,2,3,4/
  ;

  alias(n,i,j,o,d);

  sets
     arcs(i,j)  /(1.2),(1.3),(2.4),(3.4),(2.3)/;
  alias(a,arcs);

17 positive variables
```

```
              F(i,j)       Flow on arcs,
              X(o,d,i,j) Flow on arc with destination d
              C(i,j)       Link cost function,
              T(o,d,i)     Travel time to destination
22    ;
      variables
              objso        Mimimizing objective,
              objue        Mimimizing objective,
              TAX(i,j)     Congesttion tax;

      parameter
              a0(i,j) constant in link cost function,
              a1(i,j) multiplicator in link cost function,
              od(o,d) O—D matrix;

      a0("1","2") = 0;
      a0("1","3") = 50;
      a0("2","4") = 50;
      a0("3","4") = 0;
37    a0("2","3") = 10;

      a1("1","2") = 10;
      a1("1","3") = 1;
      a1("2","4") = 1;
42    a1("3","4") = 10;
      a1("2","3") = 0;

      od("1","4") = 6;


      equations
              c_flowbalance(o,d,i)      Flow balance for node n,
              c_flow(i,j)              Total flow on arc a,
              obj_so                   Objective for SO,
52            obj_ue                   Objective for UE,
              def_c(i,j)               Link cost function,
              c_demand(d)              Demand constraint,
              c_foc(o,d,i,j)
              c_focSO;

      c_flowbalance(o,d,i)$od(o,d)..
              od(i,d) — sum(j, X(o,d,i,j)) + sum(j, X(o,d,j,i)) —od(o,d)$sameas(d,i)  =E=
                  0;

      c_flow(a)..
62            F(a)  =E= sum((o,d),X(o,d,a));

      c_demand(d)..
              sum((o,i), X(o,d,i,d)) =E= sum(o,od(o,d));

67    def_c(a)..
              C(a)  =E= a0(a) + a1(a)*F(a);

      obj_ue..
              objue =E= sum(a, a0(a)*F(a) + 0.5*a1(a)*F(a)**2);

      obj_so..
              objso =E= sum(a, C(a)*F(a));

      c_foc(o,d,i,j)$(od(o,d) and a(i,j))..
77            T(o,d,j) + c(i,j) +TAX(i,j) =G= T(o,d,i);

      c_focSO(a)..
              a0(a) + 2*a1(a)*F(a) =G= 0;

82    model SO /obj_so,c_flowbalance,def_c,c_flow/;

      model UE /obj_ue,obj_so,c_flowbalance,def_c,c_flow/;

      TAX.FX(i,j) = 0;

      X.L(o,d,a)                    = 3;
      F.L(a)                        = 3;
      F.L("2","3")                  = 0;
      X.L("1","4","2","3")          = 0;
92    C.L(a)                        = a0(a) + a1(a)*F.L(a);
      X.FX(o,d,i,j)$(not od(o,d)) = 0;
      X.FX(o,d,i,j)$(not a(i,j))  = 0;
      X.FX(o,d,i,j)$(not od(o,d)) = 0;
      C.FX(i,j)$(not a(i,j))       = 0;

      OBJSO.L                       = sum(a, C.L(a)*F.L(a));

      UE.holdfixed = 1;

102   SO.holdfixed = 1;
```

```
      option nlp = pathnlp;
      solve SO minimizing OBJSO using NLP;

      X.L("1","4","1","2") = 4;
107   X.L("1","4","1","3") = 2;
      X.L("1","4","2","3") = 2;
      X.L("1","4","2","4") = 2;
      X.L("1","4","3","4") = 4;
      F.L(i,j) =  X.L("1","4",i,j);
112   C.L(a) = a0(a) + a1(a)*F.L(a);

      OBJUE.L                           = sum(a, C.L(a)*F.L(a)**2);
      OBJSO.L                           = sum(a, C.L(a)*F.L(a));

117   solve UE minimizing OBJUE using NLP;

      * VI flow-link formulation

      model mVI / def_c, c_flow,c_demand, c_flowbalance, obj_ue, obj_so/;

      mVI.holdfixed = 1;
      * define the complementary pairs for vi
      file myinfo / '%emp.info%' /;
      putclose myinfo 'vi c_flow  F ';x

      solve mVI using EMP;

      * MCP route-flow formulation of UE

132   model mMCP /def_c.C, c_flow.F, c_flowbalance.T, c_foc.X, obj_so.objso, obj_ue.
          objue/;
      T.FX(o,d,i)$(not od(o,d)) = 0;
      mMCP.holdfixed = 1;

      * MCP link-flow formulation of UE

      equations
          flowbalance(i,j) Flowbalance
          rational(i,j,d)  Wardropian conditions
          cost(i,j)        Cost function
142   flow(i,j)        Flow definition
          ;

      set k(i)  Active destinations;

147   k(i)$sum(o, od(o,i)) = YES;

      positive variables
          Xm(i,j,d)   Flow on arcs
          Tm(i,d)     Minimal time;

      flow(a)..
          F(a)  =E= sum(k, Xm(a,k));

      flowbalance(j,k)$(not sameas(j,k))..
157       sum(i, Xm(j,i,k)) - sum(i, Xm(i,j,k))  =E= od(j,k);

      cost(a)..
          C(a) =E= a0(a) + a1(a)*F(a);
162   rational(i,j,k)$a(i,j)..
          c(i,j) + Tm(j,k) =G= Tm(i,k);

      model nagurney2009 /flow.F, flowbalance.Tm  , cost.C, rational.Xm/;

167   Xm.FX(i,j,k)$(not a(i,j)) = 0;
      Tm.FX(i,i) = 0;

      nagurney2009.holdfixed = 1;
      solve nagurney2009 using mcp;


      solve mMCP using MCP;

      * MPEC formulation of MCP

      model mMPEC /def_c.C, c_flow.F, c_flowbalance.T, c_foc.X, obj_so /;

      TAX.LO(i,j) = -INF;
      TAX.UP(i,j) = +INF;
182   TAX.FX(i,j)$(not a(i,j)) = 0;
      mMPEC.holdfixed = 1;
      solve mMPEC  using MPEC minimizing objso;

      equations equalfee;
187   equalfee(i,j).. TAX(i,j) =G= 0;
```

```
        model mMPECE /def_c.C, c_flow.F, c_flowbalance.T, equalfee, c_foc.X, obj_so /;
192     TAX.LO(i,j) = −INF;
        TAX.UP(i,j) = +INF;
        TAX.FX(i,j)$(not a(i,j)) = 0;
        mMPECE.holdfixed = 1;
        solve mMPECE  using MPEC minimizing objso;
```

Appendix 1.C   The Harberger model

The following description follows the exposition in Shoven and Whalley (1984). The Harberger model contains a manufacturing and a non-manufacturing sector. Each sector employs labor and capital, which is owned by rich capital owners and poor laborers respectively.

Households maximize their utility, which has a constant elasticity of substitution (CES):

$$U_c = \left[\sum_i \alpha_{c,i}^{\frac{1}{\sigma_c}} X_{c,i}^{\frac{\sigma_c-1}{\sigma_c}} \right]^{\frac{\sigma_c}{\sigma_c-1}} \tag{1.49}$$

subject to their budget constraints:

$$P_L w_{c,L} + P_K w_{c,K} \geq \sum_i P_i X_{c,i} \tag{1.50}$$

with the following values for the parameters:

| | Endowment | | Parameters | | |
Household	K	L	α_M	α_N	σ^c
Rich	25	0	0.5	0.5	1.5
Poor	0	60	0.3	0.7	0.75

Table 1.3 – Endowments and parameters of the utility functions

Deriving the first order conditions and some tedious algebra will result in the demand functions:

$$X_{c,i} = \frac{\sigma_{c,i} I_c}{P_i^{\sigma_c} \left(\sum_i P_{c,i}^{1-\sigma_c} \right)} \tag{1.51}$$

Firms minimize their costs (or maximize their profits) subject to a Constant-Elasticity of Substitution (CES) function:

$$Q_i = \phi_i \left[\delta_i L_i^{\frac{\sigma_i-1}{\sigma_i}} + (1-\delta_i) K_i^{\frac{\sigma_i-1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i-1}} \tag{1.52}$$

Here the first order conditions can be used to derive the factor demand functions for labor and capital:

$$L_i = \frac{Q_i}{\phi_i} \left[\delta_i + (1 - \delta_i) \left(\frac{\delta_i P_K}{(1 - \delta_i) P_L} \right)^{1-\sigma_i} \right]^{\frac{\sigma_i}{1-\sigma_i}} \tag{1.53}$$

$$K_i = \frac{Q_i}{\phi_i} \left[\delta_i \left(\frac{(1 - \delta_i) P_L}{\delta_i P_K} \right)^{1-\sigma_i} + (1 - \delta_i) \right]^{\frac{\sigma_i}{1-\sigma_i}} \tag{1.54}$$

where Table 1.4 contains the values for phi_i and δ_i.

Sector	ϕ_i	δ_i	σ_i
Manufacturing	1.5	0.6	2.0
Non-Manufacturing	2.0	0.7	0.5

Table 1.4 – Parameters of the production functions

To complete the model, we only have to introduce the market clearing conditions for capital, labor and goods:

$$\overline{K} \geq \sum_i K_i \perp P_K \tag{1.55}$$

$$\overline{L} \geq \sum_i L_i \perp P_L \tag{1.56}$$

$$Q_i \geq \sum_c X_i \perp P_i \tag{1.57}$$

and, using the factor demand equations, the zero profit condition

$$P_i Q_i \geq P_K K_i + P_L L_i \perp Q_i \tag{1.58}$$

The complete model consists of Equations (1.50), (1.51), and (1.53) to (1.58) .

Listing 1.14 – The Harberger model in MCP format

```
$title Compare MPSGE and its algebraic representation
set
    i Sectors     /m   Manufacturing,
                   n   Non—manucfacturing/,
    c Households /owner, worker/,
    f Factors     /K, L/;

alias(i,j);

parameters
    sigma(i)    Substitution elasticity in production  /m 2,    n 0.5/,
    phi(i)      Constant in production                 /m 1.5, n 2/,
    delta(i)    Shares in CES production               /m 0.6, n 0.7/,
    sigmac(c)   Substitution elasticity in consumption /owner 1.5, worker 0.75/;

parameter sharetax(c)  Share of tax refund;;

sharetax("worker") = 0.6;
sharetax("owner")  = 0.4;

table alpha(c,i) Parameter of utility function
```

```
            m    n
   owner  0.5 0.5
24 worker 0.3 0.7 ;

   parameter
       wl(c)  Labor endowment
       wk(c)  Capital endowment;

   wl("worker") = 60;
   wk("owner")  = 25;

   parameter tauk(i)  Tax on capital services ;
34 tauk(i) = 0;

   positive variables
       Q(i)   Production level

39     L(i)   Labor demand
       K(i)   Capital demand

       P(i)   Consumer good price
       PK     Capital rent
44     PL     Wage

       X(c,i) Demand

       INC(c) Income
49     TAXR   Tax revenue;

   Equations
       def_Q(i)   Production function,
       def_L(i)   Labor demand definition,
54     def_K(i)   Capital demand definition,
       def_X(c,i) Consumer demand definition

       prf(i)     Zero profit conditions,

59     mkt_l      Market clearing for labor,
       mkt_k      Market clearing for capital,
       mkt_q(i)   Market clearing for goods,

       budget(c)  Income constraint,

       taxrevenue Tax revenue definition;

   def_Q(i)..
       Q(i) =E= phi(i) * (delta(i) * L(i) ** ((sigma(i) - 1)/sigma(i))
69          + (1 - delta(i)) * K(i) ** ((sigma(i) - 1)/sigma(i))) **(sigma(i)/(sigma(i
              )-1));

   def_L(i)..
       L(i)  =E= 1/phi(i) * Q(i) * (delta(i) + (1 - delta(i)) *(
           delta(i) * (1 + tauk(i)) * PK / ((1 - delta(i)) * PL))**(1-sigma(i)))**(
              sigma(i)/(1-sigma(i)));

   def_K(i)..
       K(i)  =E= 1/phi(i) * Q(i)  * ( delta(i) * ((1 - delta(i))* PL /(delta(i) *
           (1 + tauk(i)) * PK))**(1-sigma(i)) + (1-delta(i)))**(sigma(i)/(1-sigma(i))
              );

79 def_X(c,i)..
       X(c,i) =E= alpha(c,i) * INC(c) /(
           P(i)**sigmac(c)*(sum(j, alpha(c,j) * P(j)**(1-sigmac(c)))));

   prf(i)..
84     P(i) * Q(i) =G= (1 + tauk(i)) * PK * K(i) + PL * L(i) ;

   mkt_l..
       sum(c, wl(c)) =G= sum(i, L(i));

89 mkt_k..
       sum(c, wk(c)) =G= sum(i, K(i)) ;

   mkt_q(i)..
       Q(i) =G= sum(c, X(c,i));

   budget(c)..
       PL * wl(c) + PK * wk(c) + TAXR * sharetax(c) =G= sum(i, P(i) * X(c,i));

   taxrevenue..
99     TAXR =E= sum(i, tauk(i) * PK * K(i));

   model harberger /prf.Q, mkt_l.PL, mkt_K.PK, mkt_Q.P, def_L.L,def_K.K, def_X.X,
       budget.INC, taxrevenue.TAXR/;

   * Initialize
```

```
      Q.L(i)    = 30;
      L.L(i)    = 30;
      K.L(i)    = 20;
      PK.L      = 1;
109   PL.FX     = 1;
      P.L(i)    = 1;

      INC.L(C) = wl(c) + wk(c);
      X.L(c,i) = 1;

*  Set the numeraire
PL.FX = 1;
*  Solve the benchmark equilibrium
solve harberger using MCP;

*  Solve the scenario with tax on capitial use in  manufacturing
tauk("m") = 0.5;
solve harberger using MCP;
```

A traffic equilibrium model with paid-parking search

2.1   Introduction

Downtown traffic congestion is a problem in most large cities around the world. Congestion is often aggravated by scarce parking facilities, causing drivers to cruise and spend time searching for parking spaces and thereby congesting the roads even more. In his seminal monograph on parking, Shoup (2011, Chapter 11) summarizes the studies for cruising in 13 cities around the world and finds that between 8 and 74 percent of cars are cruising and that the average searching time to find a park space is in the range 3.5 to 19 minutes. Apart from congestion, parking behavior can also cause an externality on others by reducing the probability of finding a parking space and thereby increasing the time of cruising. Small and Verhoef (2007) estimate, based on several studies, that the social average costs of office parking can be as high as US$0.45/km. Notably, these costs are much higher than the social road costs of congestion (US$0.09/km). As any vehicle-based trip ends with parking and as parking causes externalities, the inclusion of parking behavior in transport models is essential for understanding the impacts of parking policies.

The objective of this essay is twofold. First it develops a concise, tractable and easily extensible, link-based formulation of the model as a mixed complementarity problem (MCP),[1] combining a traffic assignment model in the tradition of Wardrop (1952) and a parking search model into one single model.[2] Drivers from different user classes simultaneously decide on the choice of route and parking allocation in a road network with multiple parking facilities. The externality on the probability of finding a parking space is explicitly part of the model. We make a distinction between on-street (curbside) and off-street parking (garage). The user classes are differentiated with respect to their origin and valuation of time.

A second objective is using the model for the analysis of efficiency and distributional effects of different parking fee policies. From an economic point of view, the most efficient policy to reduce the congestion and other external effects of cruising is to impose a spatially differentiated parking fee that reflects the external costs (see for example Arnott, de Palma, and Lindsey, 1991; Verhoef, Nijkamp, and Rietveld, 1995). A parking fee can reduce cruising and will also have a positive impact on congestion although driving through traffic is exempted and the parking fee does not

---

[1] For an introduction to the formulation of MCP models, see the first essay of this thesis.

[2] An earlier version of the model was used in Bodenbender (2013).

take directly into account how the cruising driver affects congestion. We will use the model for the center of Zurich, the biggest city in Switzerland, and analyze several policies in which the fees for on-street parking and/or the fees for parking garages are endogenous. These policies are compared to the existing policy and a social optimum, in which the overall time costs are minimized.

Traditional approaches have concentrated on formulating traffic assignment models as path-based models with solution procedures that rely on heuristic algorithms, or the models are formulated as variational inequality (VI) problems (see for example Patriksson, 1994; Nagurney, 2009). The MCP formulation is a special case of the VI formulation. Dafermos and Sparrow (1969) were the first to formulate the assignment problem as a VI-problem and a few years later Aashtiani (1977) formulated it as a MCP. With the exception of a paper by Ferris, Meeraus, and Rutherford (1999), the MCP formulation in spite of its advantages is seldom used.[3] Disadvantages of the heuristic and VI procedures are on the one hand the necessity of finding the shortest paths and on the other hand, the necessity of either using specialized and costly transport model software or tedious coding of the algorithms used for solving the model. Integrating these models with a parking search submodel can be difficult or even impossible if specialized software has to be used, or can make the coding even more tedious. The advantages of formulating the model as a link-based MCP are manifold as the integrated model can be solved by standard modeling software like MATLAB (The MathWorks, 2012) or GAMS (GAMS Development Corporation, 2014). This allows the researcher to concentrate completely on the model formulation and the model can be easily extended and shared with other researchers. Furthermore, the link-based formulation has the advantage of elimination the necessity of finding the shortest paths.

Our simulations show that the parking fee structure in Zurich is highly inefficient and that changing this structure can lead to high efficiency gains. The simulations also show that the existing street parking fees relative to the parking garage fees are substantially too low, and the implementation of these policies would reduce the congestion and generalized costs of the agents. It would reduce the tax revenue for the city by more than 60% (with the exception of the scenario with the endogenous street parking fees). Another potential problem of the investigated policies is their regressive character: The generalized costs of fees paid and time used for driving and searching with respect to poor households increase, while richer households gain. To investigate the distributional effects, we incorporate heterogeneous households in the model by distinguishing households by their valuation of time. It must be emphasized that although the simulations rely on actual data, the model results should be interpreted with caution as agents in the model do not have the possibility to choose another transport mode or another departure time.

This essay is organized as follows: Section 2 briefly reviews the existing literature on studies undertaking with parking models. Section 3 proceeds with a short introduction to mixed complementarity problems and describes the model features. Section 4 assesses the efficiency and distributional effects of different parking policies in a model for the center of Zurich. Finally, Section 6 offers some insights into the modeling exercise as well as directions for further research.

---

[3] More on the differences in the several approaches can be found in the first essay of this thesis.

2.2   Literature overview

Existing models differ highly in the way they treat traffic and parking.[4] Few of these models consider parking and congestion in a traffic network equilibrium model setting or study the potential efficiency gains from parking policies. In the following we discuss some of the more interesting models that are relevant for the present analysis and applications.

In a series of papers, Arnott and several co-authors were among the first to study parking in network equilibrium models.[5] In the following we will discuss the main findings of these papers and some other papers. Arnott, de Palma, and Lindsey (1991) use the rush-hour congestion model developed among others by Vickrey (1969) in which identical commuters travel to work at the center of a long, narrow city with zero lateral travel costs. There is only one bottleneck with congestion, where queuing can occur. People can use on-street or off-street parking. The authors find that parking fees can be at least as efficient as an optimal time-varying road toll and that competitively set parking fees are relatively inefficient. They state further that an efficient parking fee policy may be easier to implement than an efficient tolling policy for several reasons. One of these reasons is that a road toll may be regressive. With a parking fee, most low-income workers would try to avoid paying higher fees by parking further away. The authors note that a parking fee might make parking fees more progressive than road tolls when the assumption of identical commuters would be dropped. In this paper, however, we see that changing the existing parking fee structure in Zurich is highly regressive. Contrary to their model, which treats the city as long and narrow with no lateral travel costs, and one bottleneck, we use a more realistic network, where congestion can occur on any arc.

Arnott and Rowse (1999) develop a general equilibrium model focusing on stochastic aspects of parking, especially cruising for parking. Although the authors consider this analytical model as simplistic, they find that sound analytical work on parking policies is discouragingly difficult and thereby suggest the need for numerical simulations in the context of practical parking simulation models. Using simulation-based analysis, we find that analytical results on distributional effects for simple networks do not necessarily hold for more realistic networks.

Arnott and Inci (2006) explore the properties of a steady state model of interaction between downtown free parking and traffic congestion. They find three interesting results: First, cruising for parking results is pure dead weight loss. Secondly, the parking fee should be raised to the point where cruising for parking is eliminated, but parking remains saturated. Thirdly, when the level of the parking fee is fixed, the amount of curbside parking should be increased to the point where cruising is eliminated and parking is saturated. They note however, that if all agents travel the same distance and park for the same length of time, the parking fee serves as a first-best congestion toll. If these assumptions are relaxed, decentralization of the social optimum requires both a parking fee and road pricing. For the case of Zurich described in this paper, endogenous pricing of parking

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[4] For a classification of different parking models see Young (2008).

[5] Arnott, de Palma, and Lindsey (1991), Arnott and Rowse (1999), Arnott and Inci (2006), Arnott (2006), Arnott and Rowse (2009), and Arnott and Inci (2010).

indeed produces efficiency effects that are very close to the social optimal solution.

Arnott (2006) develops an interesting model where the parking garage operator's optimal policy is treated with regard to constructing and pricing. He shows that since parking garage operators have market power, the spatial equilibrium is inefficient and parking policy can be used to reduce the distortions. Arnott additionally adds underpriced on-street parking and mass transit and shows that a triple dividend may arise if the meter price is raised. The first dividend relates to the reduction in cruising, the second dividend reduces the level of distortionary taxation and the third dividend refers to a reduction in overall congestion. In the example for Zurich we see that implementing better parking policies is likely to lead to high gains in efficiency and reductions in congestion.

Arnott and Rowse (2009) combine the models developed in Arnott and Inci (2006) and Arnott (2006) and simulate a curbside parking policy for a representative medium-sized city with identical agents. They find that garage parking fees are overpriced and increase the existing distortions of underpricing of on-street parking and inefficient spacing of garage parking. One interesting question they raise is why cities do not set parking fees for on-street parking higher as this would generate welfare gains. In our simulations we see that the actual parking fee policy is highly inefficient and that the parking garage fees are too high.

Anderson and de Palma (2004) analyze the pricing of parking economics more formally. They show that, under certain assumptions, the social optimum can be achieved if parking lots are privately owned. Their model explicitly treats the link between occupancy rate and search costs by a stochastic process. In our model the probability of finding a spot is endogenous and linked to the number of persons searching and the number of parking spots available.

Calthrop, Proost, and Delder (2000) use the Trenen model, an urban general equilibrium model with 20 alternative transport markets, and an aggregated speed-flow function without an explicit network. Parking costs are exogenous costs added to the generalized cost functions of the cars. The model is calibrated to Brussels in order to examine the efficiency gains from various parking policies with or without a cordon toll. They find that a combination of parking fees and cordon toll results in higher welfare gains than parking pricing or cordon tolling. Parking pricing alone produces higher welfare gains than cordon tolling alone.

Two other more recent models are those from Lam et al. (2006) and Balijepalli, Shepherd, and May (2008). Lam et al. (2006) developed a model in which travelers simultaneously decide on the choice of departure time, route, parking allocation (on- or off-street) and parking duration in a road network with multiple user classes and multiple parking facilities. Searching time delay is modeled using a Bureau of Public Roads function. They use a hypothetical example to show that the model generates a time-dependent network equilibrium solution as well as how the results differ from a static version of their model. Their model is formulated as a variational inequality problem which is solved by a heuristic algorithm. They stress in their conclusions that case studies on real networks are necessary.

Balijepalli, Shepherd, and May (2008) also use an integrated model with multiple user classes and different arrival and departure times. They use the model to study the choice of car parks in a

small, hypothetical network as well as in managing the demand between two car parks in Leeds, UK. Their model can be solved with any standard transport modeling software such as TRIPS, EMME/3 or SATURN. This however reduces the flexibility of the model.

The studies mentioned show important insights and ideas for modeling parking search and analyzing parking policies in network equilibrium models, but most of the studies lack more realistic examples.

2.3 Methodology

2.3.1 Mixed complementarity problems

Complementary problems can be described as systems of (non)linear constraints where the system variables are linked to the constraints with complementarity conditions (Ferris and Munson, 2014). More formally, given a function $h : \mathbb{R}^n \to \mathbb{R}^n$, lower bounds $l \in \{\mathbb{R} \cup -\infty\}^n$ and upper bounds $u \in \{\mathbb{R} \cup \infty\}^n$, we try to find $x \in \mathbb{R}^n$ such that precisely one of the following holds for each $i \in 1, \ldots, n$:

$$x_i = l_i \quad \text{and} \quad h_i(x_i) \geq 0, \quad \text{or}$$
$$x_i = u_i \quad \text{and} \quad h_i(x_i) \leq 0, \quad \text{or}$$
$$l_i < x_i < u_i \quad \text{and} \quad h_i(x_i) = 0$$

This means that the variable x_i is either at one of its bounds or the linked function is equal to zero.

In the mixed complementarity problem (MCP) we not only have inequalities with complementary nonnegative variables, but we also have equations where the associated variables are free. In this case the complementarity conditions become:

$$h_i(x, x) \geq 0, \quad x_i \geq 0, \quad x_i h_i(x) = 0,$$
$$h_j(x, y) = 0, \quad x_j \text{ free},$$

where we partition the set n in the sets i and j.

Complementarity models can be used for solving linear, quadratic and nonlinear programs by writing the Karush-Kuhn-Tucker optimality conditions. Complementarity models can also be used for expressing a variety of economic models for both markets and games, where the problems cannot be written as a single optimization problem, or if no equivalent optimization problem exists. Examples are the famous transport problem by Dantzig, the Walras equilibrium and the von-Thunen land model. A model formulation of these examples can be found in Ferris and Munson (2014) and more examples can be found in Rutherford (1995), and Dirkse and Ferris (1995). A good introduction to engineering and economic applications of complementarity problems can be found in Ferris and Pang (1997). A complementarity problem can often be formulated using the optimality conditions of the original problem. However, there is not always an optimization problem that corresponds to the complementarity conditions and this is the advantage of the MCP formulation. This means that an MCP formulation allows us to solve a wider class of problems. The development of the

complementarity modeling format was motivated by theoretical and practical developments in algorithms for nonlinear complementarity problems and variational inequalities. The most recent techniques are based on ideas from interior-point algorithms for linear programming (Kojima et al., 1991). Computational evidence suggests that algorithms for solving MCPs are relatively reliable and efficient, particularly for models which are not natural optimization problems. A survey of developments in the theory and applications of these methods is provided by Harker and Pang (1990).

There are at least two major reasons why a MCP formualtion may be superior to the variational inequality (VI) formulation. First, the MCP formulation of the (link-based) transport problem does not rely on knowing in advance which routes are used. Ferris and Munson (2014, p. 3) state that this is the key property of a complementarity problem over a system of equations: "*If we know what arcs to send flow down, we can just solve a simple system of linear equations. However, the key to the modeling power of complementarity is that it chooses which of the inequalities satisfy as equations.*" Second, a MCP model can be solved with readily available "all-purpose" software packages, like MATLAB (The MathWorks, 2012) or GAMS (GAMS Development Corporation, 2014). Using these packages allows the researcher to concentrate on the model formulation and removes the burden of writing the algorithms to solve the model. Although stand-alone transport equilibrium models can be solved by a variety of specialized software packages,[6] these packages are geared to predefined models and cannot be easily extended in other directions. In our case it is relatively straightforward to add the parking search sub-model. Another example can be found in the third essay in this thesis, where we describe how to extend the Wardropian model with a fully-fledged Alonso-Muth-Mills model.

2.3.2 Traffic equilibrium model with parking search

The starting point for the our model is the static traffic equilibrium problem in which agents are fully informed and look for the fastest route from their origin to destination (Wardrop, 1952). In order to avoid complete enumeration of all the network paths, we use the link-flow formulation of the problem. In the link-flow formulation each origin-destination flow is treated as a different commodity.

The parking model consists of a street network and multiple types of agents. The network is represented as a directed graph with nodes and arcs. The agents interact on the network by searching for parking space. Households differ with respect to their origin, destination and their valuation of time. Nodes in the network can have parking garages nearby and arcs can have curbside parking facilities with a fixed and exogenously given capacity. After parking their car agents are assumed to walk to their destinations.

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[6] Correa et al. (2010) mention the following non-exhaustive list: AIMSUN, CUBE, CONTRAM, DYNAMIT, DYNAS-MART, EMME/2, PARAMICS, TRANSCAD, TRANSIMS, TSIS-CORSIM, SATURN, VISUM-VISSIM, VISTA, and UROAD-UTPS.

The probability of finding a parking place on an arc or in a garage depends on the capacity and the number of agents looking for a parking space. Decisions about where to park reflect trade-offs between time (driving, search and walking), and money (parking fees).

In the following description, the sets describing the model are street junctions (nodes $i \in \mathcal{N}$) and agents are distinguished by their origin and destination pair ($(o,d) \in \mathcal{OD}$). The set of origins is a subset of the nodes. Nodes are referenced by either i or j. Arcs in the network correspond to distinct pairs of nodes. Arcs are alternatively referenced by a or by the start-end pair $(i,j) \in \mathcal{A}$.

The network problem has two classes of complementarity equilibrium conditions. The first class of conditions governs the conservation of flow for every node j by origin-destination pair. These conditions require that the number of all inflowing agents and agents having this node as an origin should be equal to the sum of the outflowing agents. The incoming agents are either passing through without the intention of parking in a garage or at the curbside, or they were unsuccessful in finding a curbside spot on the arc in the direction of the node. The outgoing agents either drive to the next node without searching, start searching for a curbside place on their way to the next node, or try to leave their car in a nearby parking garage. This condition can be formulated as follows, where we use the perpendicular symbol ($\perp$) to indicate the complementarity slackness between the constraint and the variable (see Figure 2.1):

$$
\underbrace{\sum_{(i,j)\in\mathcal{A}} X_{odij}}_{\text{net inflow not searching}} + \underbrace{\sum_{(i,j)\in\mathcal{A}} \pi_{ij}^{S} Y_{odij}}_{\text{inflow of unsuccessful curbside searching}} + \underbrace{s_{dj}}_{\text{starting at node}} =
$$

$$
\underbrace{\sum_{(j,i)\in\mathcal{A}} X_{odji}}_{\text{net outflow not searching}} + \underbrace{\sum_{(j,i)\in\mathcal{A}} Y_{odji}}_{\text{searching curbside on outgoing arcs}} + \underbrace{\sum_{(j,k)\in\mathcal{K}} (1-\pi_{j}^{G}) Z_{odjk}}_{\text{parking at parking garage}} \qquad (2.1)
$$

$$
\perp \quad T_{odj} \geq 0, \quad \forall j \in \mathcal{N},
$$

where X are the agents driving on the arc without searching, Y are the agents searching for a curbside place and Z are those searching for a place in the parking garage surrounding the node. Agents starting their trip at node j with destination d are given by s_{dj}. The probabilities for not finding a curbside spot or a spot in the parking garage are given by π_{ij}^{S} and π_{j}^{G}.
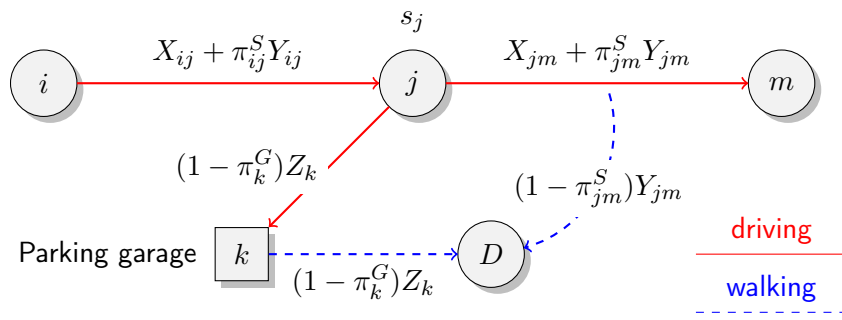
Figure 2.1 – Node balance for node j for all agents traveling from o to d (for convenience, all OD-indices have been dropped).

Note that in our model agents walk from their parking spot to their final destination. Therefore,

there are no explicit node balance equations for the destinations. The complementarity in this equation exists between the flow of agents traveling from o to d through node j and the equilibrium expected time T_{odj} for these agents to reach their destination.

The probability of not finding a curbside spot is endogenous and given by one minus the fixed total number of parking spaces at the arc $(\overline{C}_a^S)$ divided by the number of agents searching:

$$\pi_a^S = 1 - \frac{\overline{C}_a^S}{\sum_{(o,d)\in\mathcal{OD}} Y_{oda}}, \quad \forall a \in \mathcal{A}.$$

However, instead of using this definition, we use a complementarity equation for the probability of not finding a spot. This provides us with a lower and an upper bound for the probability of 0 and 1, respectively:

$$\underbrace{\pi_a^S}_{\text{probability of not finding a spot}} + \underbrace{\frac{\overline{C}_a^S}{\sum_{(o,d)\in\mathcal{OD}} Y_{oda}}}_{\text{probability of finding a spot}} \leq 1 \quad \perp \quad \pi_a^S \geq 0, \quad \forall a \in \mathcal{A}. \qquad (2.2)$$

For the probability of not finding a spot in the parking garage, we have an equivalent complementarity condition:

$$\pi_k^G + \frac{\overline{C}_k^G}{\sum_{(o,d)\in\mathcal{OD}} Z_{odk}} \leq 1 \quad \perp \quad \pi_k^G \geq 0, \quad \forall k \in \mathcal{K}, \qquad (2.3)$$

where $\mathcal{K}$ is the subset of all nodes that have a parking garage.

The second class of complementarity conditions ensures that, if there is a positive flow on an arc or if drivers decide to place their car in a parking garage, the corresponding time to reach the destination is minimized. There are three arbitrage conditions reflecting the three choices (passing through, curbside and garage parking) of the agents.

An agent with OD-pair (o,d) who is driving to an adjacent node j without looking for curbside parking, the first option, has the following arbitrage condition:

$$\tau_{ij} + T_{odj} \geq T_{odi} \quad \perp \quad X_{odij} \geq 0, \quad \forall (i,j) \in \mathcal{A}, \quad \forall (o,d) \in \mathcal{OD}, \qquad (2.4)$$

where τ_{ij} is the time for traveling from node i to j, T_j is the minimal time from node j to the final destination and T_{odi} is the minimal time from node i to the final destination. This condition is the typical arbitrage condition in the classical network problem. Note that this arbitrage condition is complementary to the number of agents deciding to drive to the next node (X_{odij}). In equilibrium this condition requires that the time for traveling to the final destination from node i using the arc (i,j) is either bigger or equal to the minimal time for traveling from node i to the destination. If the time in the case of using this arc is longer, the number of agents using this link is zero. In the case of equality, there is a positive flow on this arc.

The second option is starting to search for curbside parking on the adjacent arcs around node i. The expected time to reach the destination includes the time spent searching for a parking place, the expected time spent walking to the destination plus the parking fee expressed in time units if

a parking place is found and the expected time in continuation from this intersection should no parking place can be located given by. In the simulations we introduce a parking fee for the curbside parking places, which is denoted by PF_i^S.

The second arbitrage condition can be stated as:

$$vs\,\tau_{ij} + \left(1 - \pi_{ij}^S\right)\left(w_{ij}^S + \frac{PF_{ij}^S}{v_{od}}\right) + \pi_{ij}^S T_{odj} \geq T_{odi} \quad \perp \quad Y_{odij} \geq 0 \quad \forall (i,j) \in \mathcal{A}, \qquad (2.5)$$

where τ_{ij} denotes the travel time on the arc, vs is a multiplier for searching time in the garage and w_{ij}^S the walking time from the parking space to the destination. If the time for traveling the arc plus the time for walking to the destination; plus the parking fee in time units; plus the minimum time for traveling from the end of the arc to the destination, if the agent does not find a curbside place equals the minimum time from node the beginning of the arc to the destination; the number of agents searching on this arc is positive, otherwise this number is zero.

The third option the agents have is to park at the parking garage and pay the parking fee. The arbitrage condition for this choice is given by:

$$\frac{vs\,S_k^P + \left(1 - \pi_k^P\right)\left(w_k^G + \frac{PF_k^G}{v_h}\right)}{1 - \pi_k^P} \geq T_{odk} \quad \perp \quad Z_{odk} \geq 0 \quad \forall k \in \mathcal{K}, \quad (o,d) \in \mathcal{OD}. \qquad (2.6)$$

The parking fee PF^G is expressed in time units using the valuation of time of the specific agent, S_k^P is the searching time in the parking garage, and w_k^G is the walking time from the node to the destination. We assume that agents do not travel along the arc, but go straight from the node into a parking garage (see Figure 2.1). If the time for searching in the parking garage and the time costs of parking are greater than the minimal time for reaching the destination from this node, agents opt for not parking and the complementarity variable Z_{odk} is zero. If the total time for parking in the garage is equal to the minimal time, agents decide to do so and Z_{odk} is positive.

The search costs for a place in the parking garage, are modeled as the widely used Bureau of Public Roads (1964) function and depend on the occupancy rate:

$$S_k^P = \alpha_k^G S^P \left[1 + \left(\frac{\sum_{(o,d) \in \mathcal{OD}} Z_{odk}}{C_k^P}\right)^{\beta^G}\right] \quad \forall k \in \mathcal{K}, \qquad (2.7)$$

where S_k^P is the search time and α_k^G and β^G are constants.

The last two equations of the model define the aggregate arc flow and the travel time on the arcs. The aggregate flow $(F_{a_{ij}})$ on a given arc is equal to the number of agents, both those who are simply transiting the network from i to j (X_{odij}) and those who are driving from i to j in search of an available parking space (Y_{odij}):

$$F_{a_{ij}} = \sum_{od} \left(X_{odij} + Y_{odij}\right), \qquad (2.8)$$

and the travel time on arc a_{ij} is an increasing function of the arc flow. Here we assume once again the Bureau of Public Roads (1964) function:

$$\tau_a = \alpha_a \left(1 + \beta_a \frac{F_a^4}{C_a} \right), \tag{2.9}$$

where α_a is the uncongested travel time, β is the congestion coefficient and C is the capacity of arc a.

Equations (2.1) to (2.9) form a mixed complementarity problem consisting of nine equations and can be used to solve the user equilibrium.

2.4 Simulations

2.4.1 A numerical example

We apply the model to Zurich with a resident population of almost 400,000 people and more than 1,000,000 people live in the agglomeration. Additionally to the resident population, every day about 200,000 people come to Zurich for work, and about 100,000 people visit the city for leisure activities (Misteli, 2008). We focus on a simplified network version of the center of the city (see Figure 2.2). The network consists of a directed graph with 128 nodes, 88 two-way and 61 one-way arcs.

The key figures of the network are summarized in Table 2.1. We assume that most of the inner part of the network is a car-free zone and only accessible by public transport or by foot. The total parking capacity of the network is 4,311 of which there are 2,652 in parking garages and 1,652 on-street. We assume that over 3,000 drivers search for a parking space for a duration of two hours.

Description	Value	Description	Value
Nodes	126	Off-street parking	1441
Origins	6	On-street parking	1652
Destinations	4	Total parking capacity	3093
Arcs	149	Agents	3449
Two-way arcs	88	Area size (in hectare)	257
One-way arcs	61	Length network (in km)	21

Table 2.1 – Information on the network.

The capacity and the fees of the 12 parking garages are summarized in Table 2.2. The hourly parking fees for on- and off-street parking are based on Oswald (2012). Although there is some

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[7] The coordinate system used is based on the old Swiss reference system CH1903 introduced in 1903 (Federal Office of Topography (Swisstopo), 2008). The system comprises the definition of a reference ellipsoid (Bessel 1841) fixed in position and orientation to the old observatory in Bern (long=0m,lat=0m).
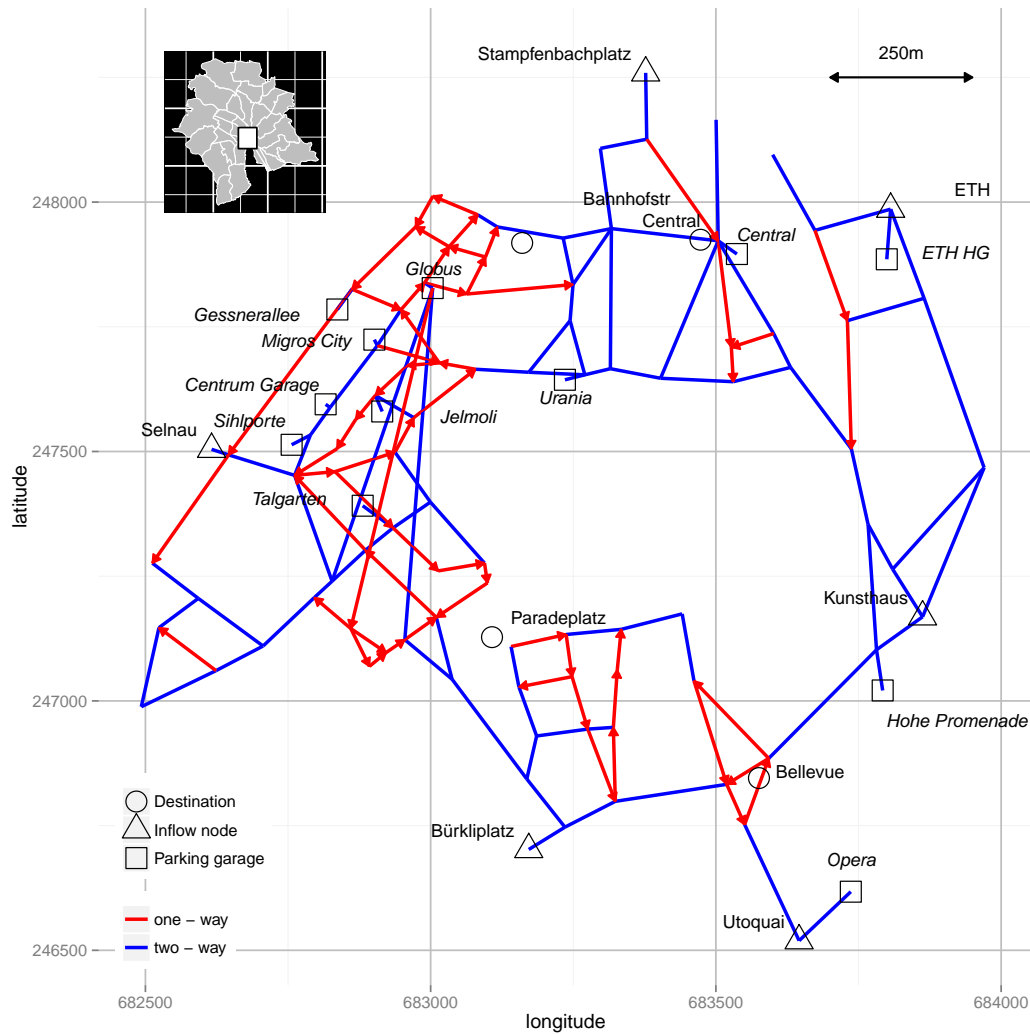
Figure 2.2 – Area of interest and the zoomed-in network with origins, destinations and arcs (Map of Zurich as inlay).[7]

differentiation in the on-street parking fee, we assume for simplicity an on-street parking fee of 2.5 Swiss Francs per hour (CHF/h) throughout the network. Note that most of the parking garages are located in the northeastern part of the network (see Figure 2.2). Only the parking garages "Hohe Promenade" and "Opera" are located in the south-eastern part of the center.

Agents enter the network at six different inflow nodes (ETH, Kunsthaus, Utoquai, Bürkliplatz, Selnau and Stampfenbachplatz) and can choose between four different destinations in the city center (Bahnhofstrasse, Central, Bellevue and Paradeplatz). The destinations and inflow nodes are also depicted in Figure 2.2. Table 2.3 shows the Origin-Destination matrix. We assume that 80% of all on- and off-street parking spots are used. The number of people entering the network is determined by this utilization rate. Note that the inflow from the southeastern nodes (Utoquai and Kunsthaus) is much lower than from the other inflow nodes.

Agents not only differ with respect to their origin, but also by their valuation of time. König, Axhausen, and Abay (2001) estimated the value of travel time savings in Switzerland for motorized

Parking Garage	Capacity	CHF/h	Parking Garage	Capacity	CHF/h
Globus	170	3.50	Migros City	56	3.00
Hohe Promenade	502	4.00	ETH HG	146	2.00
Talgarten	110	4.00	Central	49	4.00
Gessnerallee	608	4.00	Jelmoli	218	3.50
Sihlporte	40	4.50	Urania	450	3.90
Centrum Garage	11	8.00	Opera	299	4.50

Table 2.2 – Capacities and parking fees for parking garages.

	Central	Bellevue	Paradeplatz	Bahnhofstr	Total
ETH	164	164	164	164	655
Kunsthaus	86	86	86	86	345
Utoquai	69	69	69	69	276
Buerkliplatz	155	155	155	155	621
Selnau	164	164	164	164	655
Stampfenbachplatz	224	224	224	224	897
Total	862	862	862	862	3449

Table 2.3 – Origin-Destination Matrix.

and public travel by trip purpose using a stated-choice survey of around 1,200 people. This study shows, like a review of 226 studies undertaken by Abrantes and Wardman (2011), that the valuation of time is rising with income or wage. Using the estimates from König, Axhausen, and Abay (2001) and the distribution of income in Zurich from Troxler (2004) we roughly estimate the valuation of time for the different agent groups (see Table 2.4).

Inflow node	Valuation of time (CHF/min)
ETH	0.600
Kunsthaus	0.900
Utoquai	0.800
Buerkliplatz	0.700
Selnau	0.500
Stampfenbachplatz	0.400

Table 2.4 – Valuation of time by household type (origin).

2.4.2 Scenarios

For the policy analysis we consider five pricing scenarios. The first scenario is the reference scenario ("Cruising"), which reflects today's parking policy in Zurich: people pay a fixed parking fee that is always lower for on-street parking (2.5 CHF/h) than for off-street parking. The off-street parking fee ranges between 2 and 8 CHF per hour (see Table 2.2). All other scenarios are compared with this reference scenario. The other scenarios are:

- "Pricing lot" refers to the case of demand-responsive pricing for garage parking: The on-street fee is fixed to the level of the reference scenario. The demand-responsive adjustment of the fees for garage parking assures that the probability of finding a spot at the garage of choice is equal to 100%.

- "Pricing street" refers to the case of demand-responsive pricing for on-street parking: Here the fees for garage parking are fixed to the reference level and the on-street parking fees adjust to ensure a probability of finding a preferred spot is equal to 100%.

- "Pricing" refers to the case of demand-responsive pricing: The parking fees for on-street and garage parking adjust to the actual demand: The higher the demand relative to the capacity, the higher the parking fee.

- "Minimal time": In this scenario we solve the model for the minimum of the overall time costs, given by

$$
\sum_{od} v_o \left[\sum_a \tau_a (X_{oda} + Y_{oda} + (1 - \pi_{oda}) w_a^C Y_{oda}) + \sum_k (1 - \pi_k^P) w_k^P Z_{odk} \right]
$$

and set the fees to zero. This scenario is comparable to Wardrop's social optimum. This scenario allows us to evaluate the other scenarios relative to the social optimum and to assess the suitability of the pricing scenarios. The closer the overall time costs are to the value in this scenario, the better the policy is.

2.4.3   Efficiency results

We first have a look at the overall results for the times traveled and distances in all scenarios. Table 2.6 shows the times spent in passing through, searching and walking by the agents for the scenario "Cruising" in hours (first column) and the changes in these times for the other scenarios. In the cruising scenario, searching for a parking spot on the streets is with a total of 271 hours the main source of driving activity in the inner city of Zurich. The total time for searching in a parking house (161 hours) is much smaller than for searching on the street reflecting the fact, that the distance driven in a parking garage is limited. On average people need around 18 minutes to reach their destination. Of these 18 minutes, they spend around 11 minutes in their car.

	Cruising	Pricing street	Pricing lot	Pricing	Minimal time
Passing time	191.5	1.7%	7.3%	2.0%	-6.7%
Searching time - street	271.5	-92.1%	-86.1%	-94.4%	-94.1%
Searching time - garage	160.8	3.0%	-51.5%	-76.6%	-78.6%
Walking time	411.1	-2.6%	-3.9%	-6.5%	-5.7%
Total time	1034.8	-24.4%	-30.8%	-38.9%	-40.4%

Table 2.5 – Traveling times (absolute values in hours and percentages of changes).

	Cruising	Pricing street	Pricing lot	Pricing	Minimal time
Passing time	191.5	1.7%	7.3%	2.0%	-6.7%
Searching time street	271.5	-92.1%	-86.1%	-94.4%	-94.1%
Searching time garage	160.8	3.0%	-51.5%	-76.6%	-78.6%
Walking time	411.1	-2.6%	-3.9%	-6.5%	-5.7%
Total time	1034.8	-24.4%	-30.8%	-38.9%	-40.4%

Table 2.6 – Traveling times (absolute values in hours and change).

The results for the "Minimal time" scenario are the most favorable as was expected. Total time for the agents is reduced by 40.4%. The gains in time are mainly due to an almost complete reduction in searching (94.1% reduction for searching for a street parking spot and 78.6% reduction for searching in the parking garages). Walking is only reduced by 5.7%, because the majority of parking garages are within a small distance from each other and on-street parking is evenly distributed over the center of the city.

Changing the parking fee structure leads in all scenarios to a sizable adjustment of between -39% and -24% of the total time spent in the network. In the scenario with endogenous parking fees for on- and off-street parking ("Pricing"), the total reduction is close to the optimum. The actual parking policy therefore seems to be highly inefficient.

If we compare the two scenarios with only one endogenous price ("Pricing street" and "Pricing lot"), we see that setting optimal fees for the parking garages is much more efficient than endogenizing the fee for on-street parking.

	Cruising	Pricing street	Pricing lot	Pricing	Minimal time
Searching	2734.5	-90.3%	-79.3%	-92.4%	-92.0%
Passing	3134.6	12.7%	11.4%	11.4%	5.6%
Total (km)	5869.0	-35.3%	-30.9%	-37.0%	-39.9%

Table 2.7 – Vehicle kilometers (absolute and percentage change).

Figure 2.3 shows the increase or decrease in traffic flows on the arcs compared to the reference scenario. In order to distinguish the changes on bi-directional flows, the flows pointing south are depicted on the left, those pointing north are on the right. The red arcs show a decrease, the blue arcs an increase in flows. We see an overall reduction in congestion in all scenarios on almost all arcs. In almost all the scenarios the neuralgic zones in the network are in the north-eastern and north-western parts of the center. Below we will see that this is caused by a massive increase in off-street parking due to high on-street parking fees and a substitution away from on-street parking. Most of the parking garages are situated in the north-western part of the city center and therefore an increase in congestion is to be expected in this case. In the north-eastern parts people with destination "Central" try to park in the "ETH HG" parking garage, causing congestion on the arcs leading to this garage.

Table 2.7 shows the results for the number of kilometers driven and the percentage change with
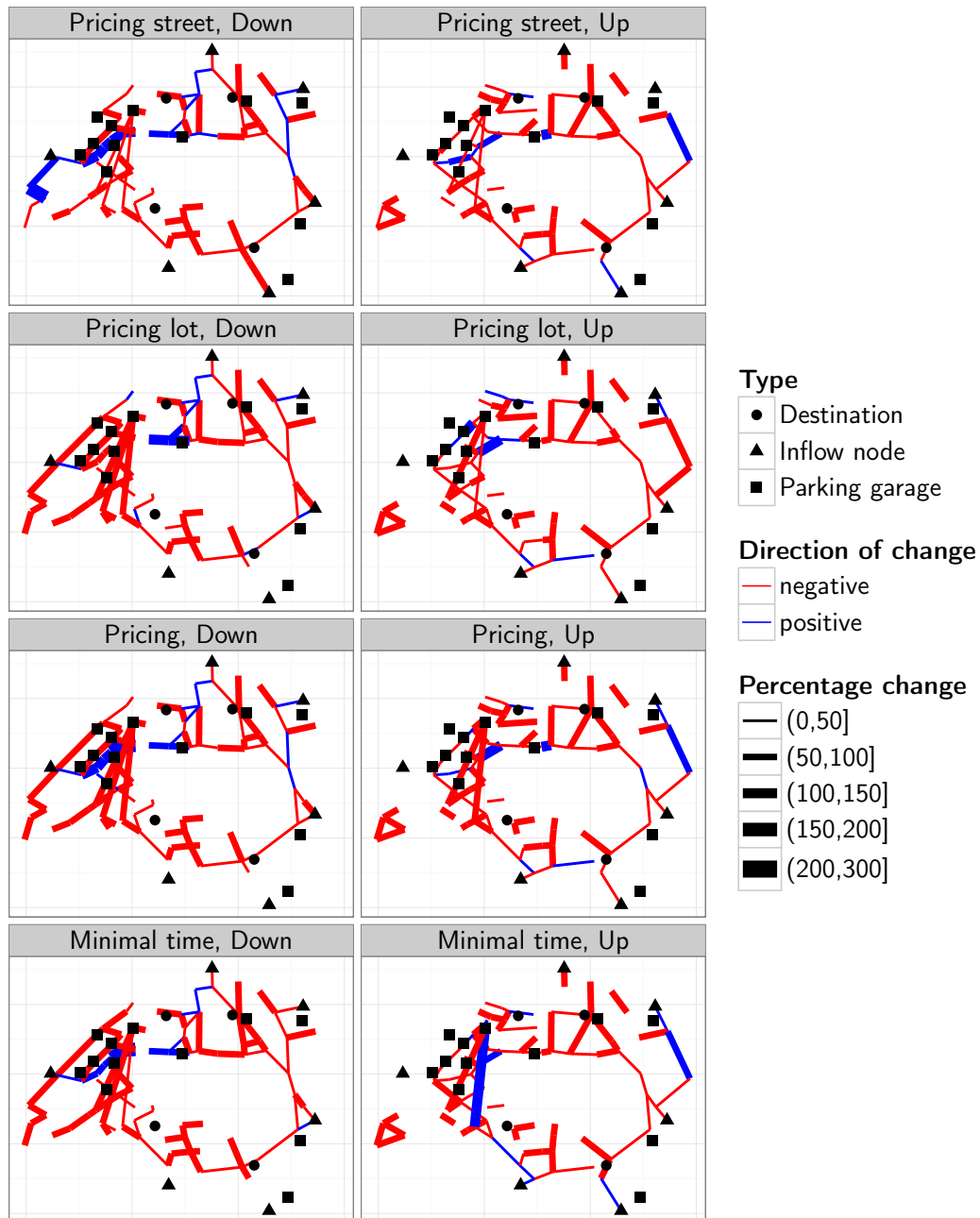
Figure 2.3 – Changes in flows on the arcs (red arcs show a decrease, blue arcs show an increase in flow).

respect to the values in scenario "Cruising". In the reference scenario agents drive almost 6000 km in the network. Of this value 47% is due to searching on the network (the distances driven while searching in the parking garages are not included). The overall reduction in kilometers in searching, driving and walking is in all scenarios more than 30%. The reduction in distances driven is caused by a reduction in searching while the passing through distances increase in all scenarios. This is because the endogenous parking fees cause a more even distribution of the agents over the network, causing longer passing through distances. Note that the reduction in kilometers is not proportional to the reduction in time. As shown in Table 2.6, the reduction in time in the scenario "Pricing

lot" is greater than in the scenario "Pricing street", but with regard to the traveled distances the results are the other way around. This means that if a city aims at reducing congestion as well as car emissions with a parking fee policy, the choice of the best scenario might not be obvious. Note however, that gasoline consumption of cars increases with income in Zurich and as shown in Section 3.4.6 the reduction in traveling in the scenario "Pricing" is higher for people with higher incomes than in the scenario "Pricing lot" and therefore the overall reduction in emissions might also be higher in this scenario.

Table 2.8 shows the generalized cost (absolute values measured in 1,000 CHF and the respective percentage changes) for all scenarios. The generalized costs are here defined as the sum of the valuated time costs and the costs of parking fees.

	Cruising	Pricing street	Pricing lot	Pricing	Minimal time
Generalized costs (1,000 CHF)	57.5	57.1	35.5	28.7	21.6
Percentage changes	0.0%	-0.8%	-38.3%	-50.0%	-62.4%

Table 2.8 – Generalized costs (total in 1,000 CHF and percentage changes).

The scenario with completely endogenous parking fees ("Pricing") fares best. The high efficiency gains in the scenario "Pricing lot" are once again expressed in the high reduction, although the differences in both scenarios with the scenario "Minimal time" are greater than the reduction of total traveling time (Table 2.6). The scenario "Pricing street" only has a slight improvement. The reductions in the time costs for driving and searching are counterbalanced by the increase in parking fees. We will come back to the generalized costs, when we discuss the difference between models with heterogeneous and homogeneous agents (next section) and the distributional effects of the scenarios (Section 3.4.6).

Tables 2.9 and 2.10 show the parking fees for off-street and on-street parking (average, minimal and maximal fee) for the scenarios with endogenous on-street parking fees. The results are similar to the findings of Arnott and Rowse (2009) that found that garage parking fees are overpriced and increase the distortions. In the scenarios with endogenous fees for parking in a garage, the fees are almost all lower than in the reference scenario. The fees for parking on-street are on average much higher in the scenario "Street pricing" and on average smaller in the scenario "Pricing".

The changes in relative prices for off- and on-street parking are reflected in the overall shares as shown in Figure 2.4. In all scenarios with the exception of scenario "Street parking" the share of garage parking increases. In the scenario "Pricing lot" this share increases by 25% points to 72.9%. The increase in the scenario "Pricing" is less, although the fees for parking in a garage are higher than in the scenario "Pricing lot". This is because agents' decisions are based on the relative and not on the absolute prices.

We have seen that the scenario with fully endogenous taxes shows the highest reduction in time, general time costs and also brings about the lowest fees for both garage and on-street parking. If however the goal of the parking policy is not only to reduce congestion, but also to collect money (for example to raise funds for infrastructure investments), this scenario is far from optimal (Table

Parkhouse	Cruising	Pricing street	Pricing lot	Pricing
Central	4.00	4.00	5.82	3.92
Globus	3.50	3.50	2.81	1.86
HohePromenade	4.00	4.00	1.32	0.44
Talgarten	4.00	4.00	1.31	0.15
Gessnerallee	4.00	4.00	0.00	0.00
Sihlporte	4.50	4.50	0.00	0.00
CentrumGarage	8.00	8.00	0.00	0.00
MigrosCity	3.00	3.00	1.81	0.84
ETHHG	2.00	2.00	2.46	1.07
Jelmoli	3.50	3.50	0.55	0.00
Urania	3.90	3.90	2.31	1.29
Opera	4.50	4.50	0.00	0.00

Table 2.9 – Hourly prices for garage parking in CHF per hour.

	Cruising	Pricing street	Pricing lot	Pricing
Mean	2.50	5.29	2.50	1.26
Minimum	2.50	2.62	2.50	0.00
Maximum	2.50	7.85	2.50	3.15

Table 2.10 – Parking fees on-street parking in CHF per hour.
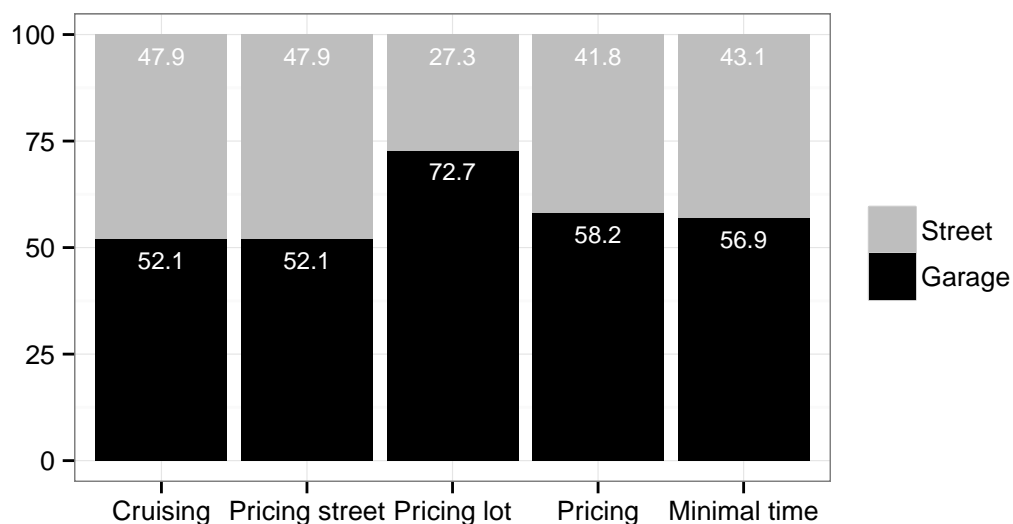
Figure 2.4 – Shares in off- and on-street parking.

2.11). With the exception of the scenario "Pricing street" the city or parking garage owners incur high income losses.

Parameter	Cruising	Pricing street	Pricing lot	Pricing	Minimal time
Parking garage	13.3	13.3	6.4	3.1	0.0
Street parking	8.3	17.7	4.7	4.4	0.0
Total	21.5	30.9	11.1	7.4	0.0

Table 2.11 – Tax revenue (in 1,000 CHF) by alternative instruments across scenarios.

2.4.4 Heterogeneous versus homogeneous agents

We now make a short digression and compare the overall results for heterogeneous agents with the results from the same model, but this time with a uniform valuation of time ('homogeneous agents'). The homogeneous valuation of time (0.6 CHF/min) is calibrated such that the total generalized costs in scenario "Cruising" are equal to the value in the model with heterogeneous agents.

If we compare the results for heterogeneous agents (Table 2.6) with the results for homogeneous agents (Table 2.12) we see that in all scenarios heterogeneity leads to a higher reduction in travel time.

	Cruising	Pricing street	Pricing lot	Pricing	Minimal time
Passing time	184.4	1.0%	13.4%	2.1%	-2.1%
Searching time street	247.9	-91.3%	-82.5%	-93.3%	-93.1%
Searching time garage	151.9	6.6%	-39.3%	-73.1%	-75.9%
Walking time	410.3	-3.4%	-4.3%	-6.5%	-5.0%
Total time	994.5	-23.0%	-25.9%	-36.7%	-37.2%

Table 2.12 – Traveling times for homogeneous agents (absolute values in hours and change).

Table 2.13 shows the absolute value and the percentage change of the generalized costs. This confirms, as shown by Glazer (1981), that the net gain (loss) is greater (smaller) if heterogeneity of agents is assumed. Note however, that the differences in time and generalized cost savings are rather small.

	Cruising	Pricing street	Pricing lot	Pricing	Minimal time
Generalized costs (1,000 CHF)	57.2	58.8	37.2	32.6	22.3
Percentage change	0.0%	2.8%	-34.9%	-42.9%	-61.0%

Table 2.13 – Generalized costs for homogeneous agents (total in 1,000 CHF and percentage changes).

2.4.5 Distributional effects

Distributional effects of parking fee policies can play an important role in the political process. The discussion on how regressive or progressive a congestion tax is critical however most of the discussion

concentrates on comparing scenarios for agents, who choose the same route and therefore travel the same distance. In reality of course agents not only choose different routes, but they also differ in where they live and we have to take into account the number of kilometers traveled.

From a political point of view, the question which is of more interest is different: How does a policy change, reduce or increase the existing burden of the agents? We see that in the analyzed scenarios, there is a clear tendency of increasing the burden for agents with a lower valuation of time. As we assume that a higher valuation of time is positively correlated to higher income, the analyzed policies have a regressive effect. Although this is known for situations where we have a social optimum (see for example Layard, 1977; Santos and Rojey, 2004)[8] it is also known that this might not be the case for scenarios with non-optimal taxes. As Layard (1977) states, that "... *this issue can only be settled by empirical work*". For all other scenarios, there is no clear relation between valuation and the reduction in time.
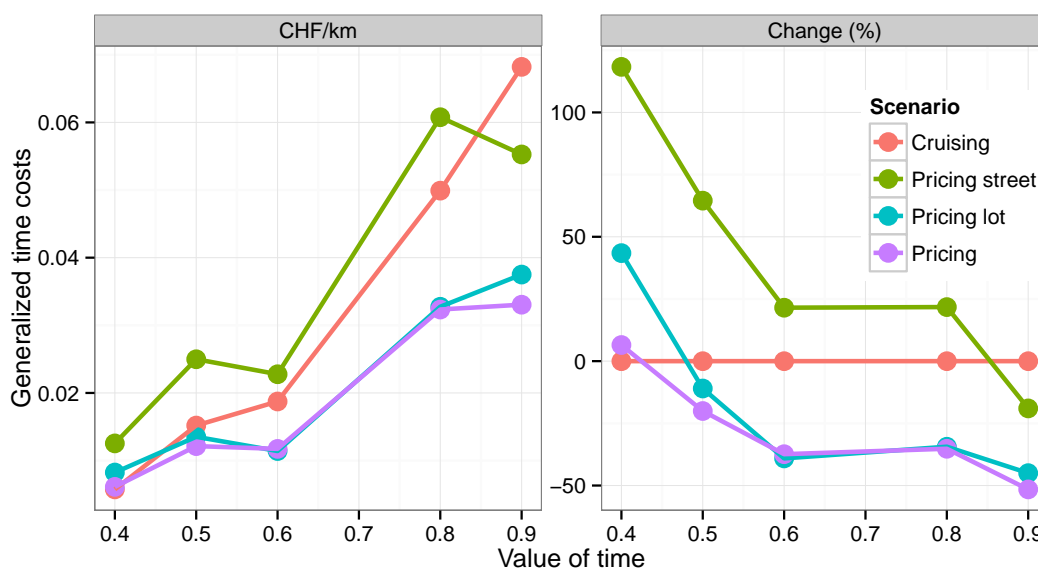
Figure 2.5 – Generalized costs per kilometer by valuation of time (absolute value on the left, percentage change on the right).

The left panel of Figure 2.5 shows the generalized costs per kilometer for different levels of time valuation. We have divided the overall costs by the number of kilometers driven to make the costs comparable. With some exceptions, the average costs in all scenarios increases with the valuation of time. As Figure 2.5 shows, there is a progressive trend. However, in some cases we see a regressive effect, especially in the bracket between a time valuation of 0.5 and 0.6 CHF/min. The right panel of Figure 2.5 shows the percentage change of the general time costs. One can see that in almost all scenarios the relative change decreases or remains almost the same as the valuation of time increases. All policies are therefore clearly regressive for the low income groups and more or less indifferent for the middle income groups with respect to the change in generalized costs per kilometer.

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[8] Although these papers investigate the distributional effects of congestion taxes, the results are also applicable for parking fees.

Origin	Valuation	Cruising	Pricing street	Pricing lot	Pricing	Minimal time
Stampfenbachplatz	0.40	20.38	16.88	14.52	13.81	11.47
Selnau	0.50	17.96	14.89	14.56	12.16	10.93
ETH	0.60	17.73	15.14	11.89	10.60	10.32
Buerkliplatz	0.70	17.02	11.14	10.50	9.20	10.79
Utoquai	0.80	16.74	10.18	11.44	9.40	11.14
Kunsthaus	0.90	15.21	6.91	8.54	6.79	8.75

Table 2.14 – Average time (in minutes).

In our numerical example demand is fixed and therefore the regressive effects in the minimal time scenario cannot be caused by refraining from traveling of the low income groups with the low valuation of time. The main driver in the pricing scenarios is the route choice. Agents with a low valuation of time use routes that take more time (see Table 2.14). In the scenario "Minimal time" there is no clear relation between the valuation of time and the average travel time.

2.5   Conclusions

In this paper, we have developed a concise and tractable model of parking search in a traffic network with heterogeneous agents who jointly decide on on-street or off-street parking and the routes they choose. The model is formulated as a mixed-complementarity problem. This format allows the researcher to concentrate completely on the model formulation as the model can be solved by readily available software.[9]

The numerical example for the inner city of Zurich with 128 junctions, 88 two-way and 61 one-way arcs shows that the model can give useful insights for the analysis of different parking policies. The model was used to analyze parking policies in which the fees for on-street parking and/or the fees for parking garages are endogenous. These policies are compared to the existing policy and a social optimum in which the overall time costs are minimized.

The simulations show that the parking fee structure in Zurich is highly inefficient and changing this structure could lead to high efficiency gains. It also shows that the existing street parking fees relative to the parking garage fees are too low. Implementation of these policies would reduce the congestion and generalized costs of the agents. It would reduce, with the exception of the scenario with endogenous street parking fees, the tax revenue for the city by more than 60%. Another drawback is the regressive character of all the policies: The generalized costs of poor households will increase and those of richer households will decrease, as shown for a social optimum and expected for scenarios with non-optimal taxes by Layard (1977).

Comparing the results with a model with no differences in the valuation of time of the agents, reveals no significant difference in the overall welfare effects. Incorporating household heterogeneity is nevertheless critical for being able to investigate the distributional effects of transportation policies.

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[9] The time used for solving the five scenarios on a business notebook computer is less than 5 minutes.

Transit traffic is not modeled. This might be a serious drawback as shown by Glazer and Niskanen (1992). In their analysis the authors show that if "...these types of drivers constitute a significant fraction of all traffic, an increase in the parking fee may have no, or even perverse, effects on congestion." However, as we analyze the policies for the center of Zurich, this is not a major drawback: Transit travelers are better off driving around the center hence there will be almost no transit traffic.

One of the main limitations of the model used in this paper is the rather restricted choice for agents: they can only choose between on-street or off-street parking and the routes they take. What is lacking is the choice between different modes (car, public transport and walking) and the possibility to either choose to refrain from a trip, choose another departure time or choose a different parking duration. Lam et al. (2006) show for a hypothetical example in a time-dependent network equilibrium model that the parking durations differ if people have more choice options, but the differences are small. One of the advantages of the MCP format is that these kinds of extensions can be easily implemented and future research in this direction is planned. Another possible direction for future research is the inclusion of different car types with respect to gasoline consumption, so that the model can be used to analyze policies with respect to emission reductions.

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Nomenclature

Acronyms

BPR   Bureau of Public Research

MCP   Mixed Complementarity Problem

TEP   Transport Equilibrium Problem

VI   Variational Inequality

Parameters

α_k^G Constant in BPR function for search time in garage

α_a Constant of BPR congestion function (uncongested travel time)

β^G Constant in BPR function for search time in garage

β_a Constant of BPR congestion function (uncongested travel time)

$\overline{C}_a^S$ Number of parking spaces at arc a

$\overline{C}_k^G$ Number of parking spaces of garage at k

s_{dj} Agents starting their trip at node j with destination d

v_{od} Valuation of time for agent (o, d)

vs Multiplier for the searching time

w_k^G Walking time from the parking garage to the destination

w_{ij}^S Walking time from the parking space to the destination

Sets

$\mathcal{A}$ Set of arcs

$\mathcal{K}$ Set of parking garages

$\mathcal{N}$ Set of nodes

$\mathcal{OD}$ Set of pairs of destination and origin

a Arc identifier

d Destination identifier

i, j, k Node identifier

k Parking garage identifier

o Origin and households identifier

Variables

π^G Probability of not finding a space in the parking garage

π^S Probability of not finding a curbside parking space

τ_{ij} Time for traveling from node i to j

PF_g^G Parking fee for off-street parking at garage k

PF_a^S Parking fee for curb-side parking on arc a

X_{odij} Throughout flow on the arc (i, j) of agents travelling from o to d

Y_{odij} Flow on the arc (i, j) of agents travelling from o to d

        searching for a curbside parking space

Z_{odj} Agents travelling from o to d parking their car in garage at node j

Appendix 2.A   Network code

Listing 2.1 – Network

```gams
$title  Network Data for Zurich

*@ Project information
* ===============================================================================
* Project: Parking
* Theme:   Network
* Contact: Renger van Nieuwkoop, renger@modelworks.ch
* ===============================================================================
* Version Control Information:
* $Id: network.gms 3049 2014—03—14 20:14:43Z renger $
* ===============================================================================

*   Set the listing directory for Gams—mode in Emacs
*   gams—lst—dir: .\listings\

$if not set resultsdir $set resultsdir results\

set     ik      All nodes and lots (numeric) /1*200/,

        i(ik)   /1*24,26*94/,

        k       Lots /
                Globus
                HohePromenade
                Talgarten
                Gessnerallee
                Sihlporte
                CentrumGarage
                MigrosCity
                ETHHG
                Central,
                Jelmoli,
                Urania,
                Opera/,

        d       Destinations /
                "Central",
                "Bellevue",
                "Paradeplatz",
                "Bahnhofstr" /,

        xy      /x,y/,

        knode(k,i)      Locations for parking lots /
                Globus.65,
                HohePromenade.29,
                Talgarten.86,
                Gessnerallee.13,
                Sihlporte.59,
                CentrumGarage.58,
                MigrosCity.60,
                ETHHG.2,
                Central.20,
                Jelmoli.55,
                Urania.71,
                Opera.4/,

        onode(i)        Locations where drivers originate /
                2
                3
                4
                6
                15
                27 /,

        h       Households (origins) /
                ETH
                Kunsthaus
                Utoquai
                Buerkliplatz
                Selnau
                Stampfenbachplatz /;

alias (i,j,ii,jj),  (garage,k);

table loc(ik,xy)

                x               y
1               683500.326      248164.909
```

75

```
           2              683806.315         247985.794
           3              683862.289         247168.58
           4              683645.858         246519.287
 83        5              683518.984         246832.739
           6              683171.948         246702.134
           7              683235.385         246746.913
           8              682892.706         247069.297
           9              682761.476         247452.179
 88       10              682948.054         247784.289
          11              683082.391         247974.599
          12              683004.028         248011.915
          13              682862.228         247825.336
          14              682645.797         247493.227
 93       15              682615.945         247504.421
          16              682832.376         247459.643
          17              682959.249         247672.342
          18              683078.659         247664.879
          19              682970.444         247567.858
 98       20              683504.058         247922.357
          21              683630.931         247668.610
          22              683315.014         247666.000
          23              683316.862         247946.836
          24              683297.462         248107.579
103      *25              683055.425         248185.178
          26              683378.757         248126.055
          27              683376.909         248259.083
          28              683766.755         247353.753
          29              683781.529         247102.474
108       30              682512.227         247276.154
          31              682706.234         247109.984
          33              682492.766         246988.274
          32              682523.796         247146.871
          34              682623.781         247060.677
113       35              682592.751         247205.483
          36              682827.199         247239.961
          37              682999.587         247398.558
          38              683530.541         247639.901
          39              683402.974         247646.796
118       40              683323.676         246798.648
          41              683037.512         247043.438
          42              683334.019         247143.423
          43              683140.945         247108.946
          44              683168.527         246843.468
123       45              683320.228         246946.901
          46              683185.766         246929.662
          47              683440.9           247174.453
          48              683461.586         247039.991
          49              683592.601         246884.842
128       50              682885.811         247302.02
          51              683009.93          247167.558
          52              683099.572         247236.513
          53              682954.766         247122.737
          54              682937.527         247498.543
133       55              682903.05          247612.319
          56              682835.818         247505.438
          57              682868.572         247564.05
          58              682823.751         247588.184
          59              682789.273         247533.02
138       60              682906.497         247712.304
          61              683015.102         247677.826
          62              682973.729         247950.199
          63              683096.124         247889.863
          64              683063.37          247815.736
143       65              682989.243         247838.147
          66              683249.549         247834.699
          67              683232.31          247927.788
          68              683116.811         247950.199
          69              683244.378         247762.296
148       70              683171.975         247658.863
          71              683270.236         247653.692
          72              683527.094         247708.856
          73              683601.221         247736.438
          74              683673.623         247943.303
153       75              683730.511         247762.296
          76              683864.974         247807.117
          77              683737.407         247505.438
          78              683599.497         248095.005
          79              683970.131         247467.513
158       80              683809.81          247264.095
          81              683237.482         247133.080
          82              683247.825         247048.610
          83              683275.407         246943.453
          84              683154.736         247027.924
163       85              683327.124         247062.401
          86              682934.079         247346.841
```

```
            87                  683013.378      247260.647
            88                  683094.400      247276.162
            89                  682859.953      247145.147
168         90                  682796.169      247207.207
            91                  682922.012      247096.879
            92                  683550.366      246752.103
            93                  683032.340      247910.550
            94                  682886.673      247183.935;

     table oloc(onode,xy)          Origin nodes
                       x              y
            2                  683806.315      247985.794
            3                  683862.289      247168.58
178         4                  683645.858      246519.287
            6                  683171.948      246702.134
            15                 682615.945      247504.421
            27                 683376.909      248259.083;

183  table kloc(k,xy)          Location of the parking lots
                       x              y
            Globus         683003.576      247827.627
            HohePromenade  683792.344      247020.947
            Talgarten      682880.876      247391.280
188         Gessnerallee   682836.421      247784.707
            Sihlporte      682756.328      247513.423
            CentrumGarage  682815.931      247594.610
            MigrosCity     682901.335      247724.231
            ETHHG          683799.525      247885.349
193         Central        683536.824      247895.860
            Jelmoli        682915.121      247580.224
            Urania         683235.342      247643.756
            Opera          683736.484      246617.497

198  table    dloc(d,xy)        Destination locations
                       x                        y
            Central        683473.034              247924.588
            Bellevue       683575.456              246845.388
            Paradeplatz    683107.551              247127.856
203         Bahnhofstr     683160.379              247918.119;

     set      ua(ik,ik)  Undirected arcs /
            1.20,  20.73, 73.21, 21.38, 72.38, 73.72, 20.72, 20.39, 39.38, 39.22,
            22.23, 23.24, 24.26, 26.27, 23.67, 67.68, 68.11, 93.11, 63.93,
208         63.68, 64.63, 65.64, 65.93, 10.65, 13.10, 11.12, 12.62, 62.13, 13.14,
            14.15, 14.30, 30.35, 35.32, 32.33, 33.34, 34.32, 34.31, 31.35, 31.90,
            90.36, 89.90, 89.8, 8.91, 36.94, 89.91, 64.66, 66.67, 66.23, 66.69,
            69.71, 69.70, 71.70, 22.71, 20.23, 26.20, 70.18, 18.61, 61.17, 17.55,
            55.19, 54.19, 55.57, 57.56, 56.9, 16.54, 54.37, 37.88, 88.52, 52.51,
213         51.50, 50.9, 50.36, 53.51, 51.41, 41.53, 86.87, 86.37, 86.50, 87.88,
            9.36, 9.14, 9.59, 59.58, 58.60, 60.10, 60.61, 61.10, 16.86, 43.81,
            81.82, 82.84, 84.43, 84.46, 46.83, 82.83, 46.44, 83.45, 45.85, 85.42,
            42.81, 40.45, 44.7, 7.40, 42.47, 47.48, 49.48, 48.5, 5.40, 7.6, 44.41,
            5.92, 49.5, 49.29, 29.3, 3.80, 80.28, 28.29, 80.79, 3.79, 28.77,
218         77.21, 79.76, 76.75, 75.77, 74.75, 74.2, 2.76, 74.78, 92.49, 19.18,
            93.62, 9.16, 91.53, 94.89, 94.53, 83.40, 92.4 /

     *        Add lots as numeric indices to node locations and the arc list:

223  *        Add lot locations to the node list:

     set      kmap(ik,k)      Pointer to nodes from lots;
     kmap(ik,k) = yes$(ik.val = card(i)+ord(k));
     option kmap:0:0:1;
228  display kmap;

     loop(kmap(ik,k), loc(ik,xy) = kloc(k,xy));
     loop(knode(k,i), loop(kmap(ik,k), ua(i,ik) = yes;));
     option ua:0:0:1;
233  display ua;

     set      oneway(i,j) /
            72.38, 73.72, 20.72, 93.11, 63.93, 63.68, 64.63, 65.64, 65.93, 10.65,
            13.10, 11.12, 12.62, 62.13, 13.14, 14.30, 34.32, 89.90, 89.8, 8.91,
238         89.91, 64.66, 26.20, 18.61, 61.17, 17.55, 54.19, 55.57, 57.56, 56.9,
            16.54, 88.52, 52.51, 51.50, 50.9, 53.51, 86.87, 87.88, 60.61, 61.10,
            16.86, 43.81, 81.82, 82.84, 82.83, 45.85, 85.42, 40.45, 49.48, 48.5,
            5.92, 49.5, 75.77, 74.75, 92.49, 19.18, 93.62, 9.16, 91.53, 94.89,
            83.40/;

     set      onewaybug(i,j);
     onewaybug(i,j)$oneway(i,j) = yes$(not (ua(i,j) or ua(j,i)));
     abort$card(onewaybug) "Unrecognized arc in oneway list.",onewaybug;

248  set      a(i,j)  Directed arcs;
     a(i,j) = yes$(ua(i,j) or ua(j,i));
```

```
      a(oneway) =  yes;
      a(i,j)$oneway(j,i) = no;
      option a:0:0:1;
253   display a;

      parameter         sref(i,j)        Benchmark speeds (km per hour),
                        dist(*,*)        Distances (km),
                        dista(*,*)       Distances arcs;

      dist(a(i,j)) = sqrt(sum(xy,sqr(loc(i,xy)-loc(j,xy))))/1000;
      dista(a) = dist(a);
      dist(i,k)     = sqrt(sum(xy,sqr(loc(i,xy)-kloc(k,xy))))/1000;
263   dist(k,i)     = dist(i,k);
      dist(i,d)     = sqrt(sum(xy,sqr(loc(i,xy)-dloc(d,xy))))/1000;
      dist(d,i)     = dist(i,d);
      dist(k,d)     = sqrt(sum(xy,sqr(dloc(d,xy)-kloc(k,xy))))/1000;
      dist(d,k)     = dist(k,d);

      *        Benchmark speeds:

      sref(a) = 15$(not oneway(a)) + 20$oneway(a);
      display sref, dist;

      parameter         iloc(i,xy)       Location of individual nodes;
      iloc(i,xy) = loc(i,xy);

      *        Need to include these lines to update the Excel file
278   *        when the network changes:

      execute_unload '%resultsdir%network.gdx',i,d,k,h,ua,a,loc,iloc,kloc,dloc,knode,
          oneway, onode, oloc,dista;

      $exit

      $onecho >gdxxrw.rsp
      par=loc         rng=nodes!a1 rdim=1 cdim=1
      par=kloc        rng=lots!a1 rdim=1 cdim=1
      par=dloc        rng=destinations!a1 rdim=1 cdim=1
288   par=iloc        rng=roadnetwork!a1 rdim=1 cdim=1
      set=oneway      rng=oneway!a1 cdim=0
      set=ua          rng=arcs!a1 cdim=0 values=nodata
      $offecho
      execute 'gdxxrw i=%resultsdir%network.gdx o=%resultsdir%RouteReport.xlsm @gdxxrw.
          rsp';

      *        Verify that it is feasible to read any node in the city
      *        from any source node:

      set    io(i)   Origin nodes (specific origins depends on calibration)
          /2,3,6,15,27/,
298       id(i)   Destination nodes;

      id(i) = yes$sum(a(j,i),1);
      display id;

303   nonnegative
      variables         Q(i,j,ii,jj)      Flow on the i-j arc destine to k
                        MF(ii,jj)         Maximum flow between ii and jj;

      variable          OBJ               Objective function;

      equations         maxflow, xflow;

      maxflow..         OBJ =e= sum((io(ii),jj), MF(ii,jj));

313   xflow(i,io(ii),id(jj))..  sum(a(i,j),Q(i,j,ii,jj)) + MF(ii,jj)$sameas(i,jj)
               =e= sum(a(j,i), Q(j,i,ii,jj)) + MF(ii,jj)$sameas(i,ii);

      Q.FX(i,ii,ii,jj) = 0;
      Q.FX(jj,i,ii,jj) = 0;
318   MF.UP(i,j) = 1;
      MF.FX(i,i) = 0;

      model feasibility /maxflow, xflow/;
      feasibility.holdfixed = yes;

      equations         timecost;

      timecost..        OBJ =e= sum((a,io(ii),id(jj)), 60*dist(a)/sref(a) * Q(a,ii,jj));

328   model primal /timecost, xflow/;

      nonnegative
      variables         T(i,j)            Minimum time from i to reach k;
```

```
333  equations        objtime, arbitrage;

     objtime..              OBJ =e= sum((i,id(j)), T(i,j));

     arbitrage(a(i,j),id(jj))..     60*dist(a)/sref(a) + T(j,jj) =g= T(i,jj);

     model shortest /objtime, arbitrage/;

     T.FX(i,i) = 0;
     shortest.holdfixed = yes;

*         Verify feasibility and then compute shortest routes and
*         times:

     solve feasibility using lp maximizing OBJ;
348  abort$(feasibility.modelstat>1) "Network is infeasible.";

     MF.FX(i,j) = MF.L(i,j);
     solve primal using lp minimizing OBJ;
     solve shortest using lp maximizing OBJ;

     set     route(*,i,j,ii,jj)      Optimal route choice;
     route("bmk",io,id,ua(i,j)) = yes$(Q.L(i,j,io,id) or Q.L(j,i,io,id));
```

Appendix 2.B Calibration code

Listing 2.2 – Calibration

```gams
$title  Calibrate to Benchmark Flow Rates

*@ Project information
* ===========================================================================
* Project: Parking
* Theme:   Calibration
* Contact: Renger van Nieuwkoop, renger@modelworks.ch
* ===========================================================================
* Version Control Information:
* $Id: calib.gms 3127 2014-04-20 22:10:29Z renger $
* ===========================================================================

*   Following line is just for Emacs as are the comments with *@, *@@ and *@@@:
*   gams-lst-dir: .\listings\

*   Set the directory for the savepoints (basedir) and the results
*   (resultsdir)
$if not set basedir $set basedir bases\
$if not set resultsdir $set resultsdir results\

*   Check if the directories exist
$if not dexist %basedir% $call mkdir %basedir%
$if not dexist %resultsdir% $call mkdir %resultsdir%
$if not dexist listings $call mkdir listings

*   Choose between heterogeneous(he) or homeogeneous (ho) housholds
$if not set hh  $set hh he

*   We use savepoints. For a clean start, set restart to "yes" and all
*   the savepoints will be deleted.
$if not set restart $set restart no
$if %restart%==yes $call 'del %basedir%calib\%hh%_mcpcalib_p.gdx'

*   Option to scale the prices and quantities
$if not set scaled  $set scaled yes

parameter
    scaling         Scaling of prices and valuation of time /1/,
    scalingN        Scaling of numbers /1/;

*@ Parameters and sets
parameter       utilization /0.8/;

$include network

set
    hnode(h,i)      Locations where drivers originate /
    ETH.2
    Kunsthaus.3
    Utoquai.4
    Buerkliplatz.6
    Selnau.15
    Stampfenbachplatz.27 /,

    ht(h,d) Households (origin and destination)
      /(ETH,Kunsthaus,Utoquai,Buerkliplatz,Selnau,Stampfenbachplatz).
      (Central,Bellevue,Paradeplatz,Bahnhofstr) /;
;
parameter
    hloc(h,xy)      Location of households,
    volume(h)       Rough cut at traffic inflow shares /
    ETH                     0.19
    Kunsthaus               0.1
    Utoquai                 0.08
    Buerkliplatz            0.18
    Selnau                  0.19
    Stampfenbachplatz       0.26/,

    gp(k)           Parking capacity lots/
    Globus          170,
    HohePromenade   502,
    Talgarten       110,
    Gessnerallee    608,
    Sihlporte       40,
    CentrumGarage   11,
    MigrosCity      56,
    ETHHG           146,
    Central         49,
    Jelmoli         218,
```

```
80       Urania          450,
         Opera           299/,

         pz(k)           Parking prices lots/
         Globus          7,
85       HohePromenade   8,
         Talgarten       8,
         Gessnerallee    8,
         Sihlporte       9,
         CentrumGarage   16,
90       MigrosCity      6,
         ETHHG           4,
         Central         8,
         Jelmoli         7,
         Urania          7.8,
95       Opera           9/,

         py(i,j)         Street parking fee (for counterfactual solution)

           sp(i,j)       Street parking on arc i—j /
100      72.38 17,    73.72 15,    20.72 44,    63.93 16,    63.68 12,
         64.63 19,    13.10 2,     62.13 14,    13.14 1,     14.30 14,
         34.32 23,    89.90 18,    89.8  15,    8.91  10,    89.91 25,
         64.66 16,    26.20 23,    54.19 6,     57.56 4,     56.9  8,
         16.54 14,    52.51 12,    51.50 24,    50.9  29,    87.88 15,
105      60.61 10,    61.10 7,     16.86 30,    43.81 22,    81.82 35,
         82.84 32,    82.83 53,    45.85 44,    85.42 53,    40.45 41,
         49.48 25,    48.5  23,    75.77 19,    19.18 6,     93.62 12,
         91.53 13,    94.89 20,    83.40 62,    1.20  2,     73.20 33,
         21.73 6,     39.20 14,    38.39 3,     39.38 4,     22.23 15,
110      23.22 15,    26.27 2,     35.32 12,    32.33 13,    33.32 13,
         90.31 4,     36.94 9,     94.36 10,    67.66 5,     23.66 10,
         66.69 15,    69.70 5,     70.69 5,     71.70 24,    22.71 10,
         18.70 3,     54.37 12,    50.36 7,     51.41 3,     41.51 3,
         41.53 8,     53.41 8,     86.37 6,     37.86 7,     86.50 3,
115      59.9  22,    58.59 18,    60.58 24,    46.83 23,    83.46 23,
         83.45 14,    45.83 15,    42.81 62,    44.41 9,     41.44 9,
         29.49 20,    49.29 3,     3.80  4,     29.28 26,    28.29 27,
         28.77 24,    77.28 25,    77.21 37,    76.75 15,    75.76 16,
         78.74 19,    94.53 12,    53.94 13/;

     abort$sum((i,j)$(not a(i,j)), sp(i,j)) "Parking spots on non—existent arcs?",sp;

     loop(hnode(h,i), hloc(h,xy) = loc(i,xy););

125  *    Set on—street rates equal to the 5 chf per two hours:
     py(a)$sp(a) = 5;

     parameter
         parkcapacity  Total number of parking spots,
130      ntot          Total number of arrivals;

     parkcapacity("street") = sum(a(i,j), sp(i,j));
     parkcapacity("lots")   = sum(k, gp(k));
     parkcapacity("total")  = parkcapacity("street") + parkcapacity("lots");
135  display parkcapacity;

     *    Total number of cars:

     ntot = parkcapacity("total")*utilization;
140  display ntot;

     parameter
         incoming(i)             Arrivals at node i
         arrivals(h,d,j)         Arrivals at node j destine to i
145      lwt(h,d,k)              Walking time from this lot (min)
         swt(h,d,i,j)            Walking time from street parking spot (min)
         streetloc(i,j,xy)       Location of street parking
         vw                      Time cost multiple for walking time /2/
         vs                      Time cost multiple for searching time /1.75/
150      v(h)                    Value of time CHF per min/
         Stampfenbachplatz       0.4
         Selnau                  0.5
         ETH                     0.6
         Buerkliplatz            0.7
155      Utoquai                 0.8
         Kunsthaus               0.9/;


     *    Stampfenbachplatz       0.5
160  *    Selnau                  1.0
     *    ETH                     1.25
     *    Buerkliplatz            1.5
     *    Utoquai                 2.0
     *    Kunsthaus               2.5/;
```

```
        incoming(i)           = ntot * sum(hnode(h,i),volume(h));
        streetloc(a(i,j),xy) = 0.5*(loc(i,xy)+loc(j,xy));

        parameter
170         vh     'Homeogeneous value of time';

        vh =  1.015 * sum(h,volume(h)* v(h))  /sum(h,volume(h));
        display vh;

175     $if %hh% == "ho" v(h) = vh;

        display incoming;

        *   Assume that we walk 5 km per hour:

        lwt(ht(h,d),k) = 60/5*dist(k,d);
        swt(ht(h,d),a) = 60/5*(sqrt(sum(xy,sqr(dloc(d,xy)−streetloc(a,xy)))))/1000;

        option lwt:1:0:1, swt:1:0:1;
185     display lwt,swt;

        *@ Model solved for minimizing total time costs (SO)

        variable
190         OBJ     Objective function;

        nonnegative variables
            F(i,j)           Aggregate flow,
            X(h,d,i,j)       Household flow,
195         Y(h,d,i,j)       Parking on the i—j arc,
            Z(h,d,k)         Parking at lot k,
            TAU(i,j)         Travel delay on the i—j arc,
            OMEGA(d)         Fraction destination to d,
            PI(i,j)          Probability of not finding a street parking place,
200         PIZ(k)           Probability of not finding a garage parking place;

        equations
            objdef           Objective function,
            flowdef          Flow definition,
205         inlot            Capacity constraint parking garages,
            onstreet             Capacity constraint on—street parking,
            balance          Node balance,
            simplex          Simplex constraint for shares of destination;

210     *   As an intial pass, determine traffic flows consistent with minimum
        *   system cost. For this we assume that people have a probability of
        *   finding a parking spot of 1 and the time for travelling an arc is
        *   equal to the minimum flow time.

        PI.FX(i,j)  = 0;
        PIZ.FX(k)   = 0;
        TAU.FX(a)   = 60*dist(a)/sref(a) + eps;

220     parameter
            tauz(k)              Time required to look for a garage parking place,
            alphaz(k);

        tauz(k) = 1.5;

        objdef..
            OBJ =e=  sum(ht(h,d), v(h) *(
                     sum(a, TAU(a)*(X(ht,a) + vs*Y(ht,a))) +
                     sum(k, tauz(k)* vs * Z(ht,k)))   +
230                  sum(a,(1−PI(a))*vw*swt(ht,a)*PY(a)/v(h)*Y(ht,a)) +
                     sum(k,(1−PIZ(k))*vw*lwt(ht,k)*PZ(k)/v(h)*Z(ht,k)));

        flowdef(a)..
            F(a) =e= sum(ht, X(ht,a) + Y(ht,a));

        onstreet(a)$sp(a)..
            sp(a) =g= sum(ht, (1−PI(a)) * Y(ht,a));

        inlot(k)..
240         gp(k) =g= sum(ht, (1−PIZ(k)) * Z(ht,k));

        balance(ht(h,d),i)..
            sum(a(i,j), X(ht,a) + Y(ht,a)) +
            sum(knode(k,i),(1−PIZ(k))*Z(ht,k)) =e=
245         sum(a(j,i), X(ht,a) + PI(a)*Y(ht,a)) + (incoming(i)*OMEGA(d))$hnode(h,i);

        simplex..
            sum(d, OMEGA(d)) =e= 1;

250     model optimal /objdef, flowdef, onstreet, inlot, balance, simplex/;
```

```
        Y.FX(ht,a)$(sp(a) =0) = 0;
        OMEGA.FX(d)            = 0.25;
        option savepoint      = 1;

        $if %scaled% == yes scaling      = 1/smax(k,pz(k));
        $if %scaled% == yes scalingN     = 1/smax(i,incoming(i));

        parameter
260         scalingR     'Factor for reverting scaling'
            scalingRN    'Factor for reverting scaling';

        scalingR     = 1/scaling;
        scalingRN    = 1/scalingN;

        pz(k)        = scaling * pz(k);
        py(a)        = scaling * py(a);
        v(h)         = scaling * v(h);
        sp(a)        = scalingN * sp(a);
270     gp(k)        = scalingN * gp(k);
        incoming(i) = scalingN * incoming(i);

        option nlp = conopt;
        $if exist "%basedir%calib\%hh%_optimal_p.gdx" execute_loadpoint "%basedir%calib\%
            hh%_optimal_p.gdx"
275     solve optimal using nlp minimizing OBJ;

        *   mv is one of the gams posix utilities. The option −f forces the move
        *   (overwrites the file, if it exists).
        execute 'mv −f optimal_p.gdx %basedir%calib\%hh%_optimal_p.gdx';

        parameter
            lotreport       Summary of lot garages,
            streetreport    Summary of street garages;

285     lotreport(k,"Z")       = sum(ht,scalingRN * Z.L(ht,k));
        lotreport(k,"gp")      = scalingRN * gp(k);
        lotreport(k,"p")       = scalingRN * pz(k);
        lotreport(k,"Marginal") = inlot.m(k);
        display lotreport;

        loop(a$sum(ht,Y.L(ht,a)),
            streetreport(a,"F")        = scalingRN * F.L(a);
            streetreport(a,"X")        = sum(ht, scalingRN * X.L(ht,a));
            streetreport(a,"Y")        = sum(ht, scalingRN * Y.L(ht,a));
295         streetreport(a,"places")   = scalingRN * sp(a);
            streetreport(a,"Marginal") = onstreet.m(a);
        );
        display "Optimal pricing allocation:",streetreport;

300     arrivals(ht(h,d),j)$hnode(h,j) = OMEGA.L(d)*incoming(j);

        *@ Solve mcp model

        nonnegative variables
305         T(h,d,j)        Minimum time to destination;

        equations
            opt_x           FOC w.r.t. cruising,
            opt_y           FOC w.r.t. on−street parking,
310         opt_z           FOC w.r.t. garage parking,
            pibalance       Zero flow balance with reference failure rate;

        opt_x(ht,a(i,j))..
            TAU(a) + T(ht,j) =g= T(ht,i);

        opt_y(ht(h,d),a(i,j))$sp(a)..
            vs*TAU(a) + (1−PI(a))*(py(a)/v(h)
            + vw*swt(ht,a)) + PI(a)*T(ht,j) =g= T(ht,i);

320     opt_z(ht(h,d),k)..
            vs*tauz(k) + (1−PIZ(k))*(vw*lwt(ht,k) + pz(k)/v(h))
            + PIZ(k)*sum(knode(k,i), T(ht,i))
            =g= sum(knode(k,i), T(ht,i));

325     pibalance(ht,j)..
            sum(a(j,i), X(ht,a) + Y(ht,a)) + sum(knode(k,j), Z(ht,k))
            =g= arrivals(ht,j) + sum(a(i,j), X(ht,a) + PI(a)*Y(ht,a)
            + sum(knode(k,j), PIZ(k)*Z(ht,k));

330     model mcpcalib /opt_x.X, opt_y.Y, opt_z.Z, flowdef.F,
                        onstreet.PI, inlot.PIZ, pibalance.T/;

        T.L(ht,j) = balance.m(ht,j);
```

83

```
335   PI.LO(a)                = 0;
      PI.UP(a)$sp(a)          = 1;
      PI.FX(i,j)$(not a(i,j)) = 0;
      PI.FX(a)$(not sp(a))    = 0;
      PIZ.LO(k)               = 0;
340   PIZ.UP(k)               = 1;
      Y.FX(ht,a)$(not sp(a))  = 0;

      mcpcalib.savepoint = 1;
      option savepoint   = 1;
345   $if exist "%basedir%calib\%hh%_mcpcalib_p.gdx" execute_loadpoint  "%basedir%calib
           \%hh%_mcpcalib_p.gdx"
      solve mcpcalib using mcp;
      execute 'mv -f mcpcalib_p.gdx  %basedir%calib\%hh%_mcpcalib_p.gdx'

      *@ Reporting of calibrated network
350   parameter
          lotreport         Summary of lot garages;

      lotreport(k,"Z")   = sum(ht,scalingRN * Z.L(ht,k));
      lotreport(k,"PIZ") = PIZ.L(k);
355   lotreport(k,"gp")  = scalingRN * gp(k);
      lotreport(k,"p")   = scalingR * pz(k);
      display lotreport;

      loop(a$sum(ht,Y.L(ht,a)),
360       streetreport(a,"F")       = scalingRN * F.L(a);
          streetreport(a,"X")       = sum(ht, scalingRN * X.L(ht,a));
          streetreport(a,"Y")       = sum(ht, scalingRN * Y.L(ht,a));
          streetreport(a,"places")  = scalingRN * sp(a);
          streetreport(a,"PI")      = PI.L(a);
365       streetreport(a,"PY")      = scalingR * py(a);
      );
      display streetreport;

      parameter
370       fref(i,j)         Reference flow,
          tauref(i,j)       Reference flow delay,
          alpha(i,j)        Open road travel delay,
          beta(i,j)         Congestion delay in reference equilibrium,
          piref(i,j)        Reference probabilities of not finding a slot (street),
375       pizref(k)         Reference probabilities of not finding a slot (garage),
          tref(h,d,j)       Reference travel times
          xref(h,d,i,j)     Reference values of X
          yref(h,d,i,j)     Reference values of Y
          zref(h,d,k)       Reference values of Z
380       vref(h)           Reference valuation of time;

      fref(a)             = sum(ht, X.L(ht,a) + Y.L(ht,a));
      tauref(a)           = TAU.L(a);
      alpha(a)            = 60 * dist(a)/50;
385   beta(a)             = tauref(a) - alpha(a);

      parameter
          echoarc  Echo print of benchmark arcs,
          echoarc2 Echo print of benchmark arcs;

      echoarc(a,"fref")        = scalingRN * fref(a);
      echoarc(a,"tauref")      = TAU.L(a);
      echoarc(a,"alpha")       = alpha(a);
      echoarc(a,"beta")        = beta(a);
395   echoarc(a,"compl")       = PI.L(a)*sum(ht,scalingRN * Y.L(ht,a));
      echoarc2(a,"Total lots")    = scalingRN * sp(a);
      echoarc2(a,"Left lots")     = scalingRN * sp(a)-(1-PI.L(a))*sum(ht,scalingRN * Y.L
          (ht,a));
      echoarc2(a,"Used lots")     = (1-PI.L(a))*sum(ht,scalingRN * Y.L(ht,a));
      echoarc2(a,"Nothing found") = sum(ht,PI.L(a)*scalingRN * Y.L(ht,a));
400   echoarc2(a,"Probability")   = PI.L(a);
      display echoarc, echoarc2;

      parameter echonode  Echo print of benchmark nodes;

405   echonode(k,"Total lots")    = scalingRN * gp(k);
      echonode(k,"Left Lots")     = scalingRN * gp(k)-(1-PIZ.L(k))*sum(ht,scalingRN * Z.
          L(ht,k));
      echonode(k,"Used lots")     = sum(ht,(1-PIZ.l(k))* scalingRN * Z.L(ht,k));
      echonode(k,"Nothing found") = sum(ht,(PIZ.l(k))* scalingRN * Z.L(ht,k));
      echonode(k,"Probability")   = 1-PIZ.L(k);
410   display echonode;

      piref(a)            = PI.L(a);
      pizref(k)           = PIZ.L(k);
      xref(ht,a)          = X.L(ht,a);
415   yref(ht,a)          = Y.L(ht,a);
      zref(ht,k)          = Z.L(ht,k);
```

```
      tref(ht,j)              = T.L(ht,j);

      display arrivals, echoarc;

      execute_unload '%resultsdir%%hh%_calib.gdx',F.L,X.L,Y.L,Z.L,TAU.L,OMEGA.L, PI.L,
          PIZ.L,echoarc, echonode,scaling,scalingN, h,ht,lwt,swt,v,vw,vs, fref,alpha,
          beta,piref,pizref,xref,yref,zref,tref,tauref,sp,gp,py=pyref,pz=pzref,arrivals,
          hloc,tauz,alphaz;

      *    Add household locations to the network geometry:

425   *execute 'gdxxrw i=%resultsdir%%hh%_calib.gdx o=%resultsdir%results.xlsm par=hloc
          rng=origins!a1 rdim=1 cdim=1';

      *execute 'gdxdiff.exe %resultsdir%%hh%_calib %resultsdir%%hh%_calibunscaled
          diffile';

      $exit
430   *@ Check why this is not used
      model empbal /objdef, flowdef, onstreet, inlot, pibalance /;

      file info /'%emp.info%'/;
      put info / 'equilibrium';
435   put      / 'min obj F X Y Z objdef flowdef inlot pibalance';
      putclose / 'vi onstreet PI';
      empbal.holdfixed = yes;

      PI.LO(a)                = 0;
440   PI.UP(a)$sp(a)          = 1;
      PI.FX(i,j)$(not a(i,j)) = 0;
      PI.FX(a)$(not sp(a))    = 0;
      PIZ.LO(k)               = 0;
      PIZ.UP(k)               = 1;
445   Y.FX(ht,a)$(not sp(a))  = 0;
      *solve empbal using emp;
```

85

Appendix 2.C Model code

Listing 2.3 – Model

```
$title  Replicate Benchmark and Compute Some Counterfactual Simulations

*@ Project and model information
* ================================================================================
* Project: Parking paper PhD
* Theme:   Model
* Contact: Renger van Nieuwkoop, renger@modelworks.ch
* ================================================================================
* Version Control Information:
* $Id: model.gms 3155 2014-04-28 14:04:46Z renger $
* ================================================================================

* ================================================================================
*@ Parking search model description and scenarios

* Network for the inner city of Zurich with one way and twoway arcs,
* parking garages, street parking. Agents value their time
* differently. The origin-destination matrix is calibrated to a 80%
* utilization rate of all parking spaces.  There is only one mode of
* transport (cars) and people have to walk the last part of their trip.

* The model is formulated as a MCP multi-commodity network problem.

* Following counterfactuals are computed:
* - Cruising (Benchmark)
* - Endogenous pricing parking garages
* - Endogenous pricing on-street parking
* - Endogenous pricing on-street and garage
* - Minimal time cost (Social Optimum)
*
*@@ Further ideas:
* - MPEC formulation to find optimal toll and fees (tried, but unsuccesful)

* ================================================================================
*    Following line is for gams-mode in Emacs, so it knows where the listings are.
*    gams-lst-dir: .\listings\

*    In Emacs comments with *@, *@@ and *@@@ allow folding of the code in
*    sections

*    The following parameter is used in the R-Scripts

parameter
    numberscenarios 'Number of scenarios' /5/;

*    Choose between heterogeneous(he) or homeogeneous (ho) housholds
$if not set hh  $set hh he

*    We use restarts with savepoints and set the directory for the
*    savepoints (basedir) and the results (resultsdir)
$if not set basedir $set basedir bases\
$if not set resultsdir $set resultsdir results\

*    Check if the directories do exist. If not make them.
$if not dexist %basedir% $call mkdir %basedir%
$if not dexist %resultsdir% $call mkdir %resultsdir%
$if not dexist listings $call mkdir listings

*    For a clean start, set restart to "yes" and all the savepoints will be
*    deleted.
$if not set restart $set restart no
$if %restart%==yes $call 'del %basedir%%hh%_*.gdx'

*@ Sets and Parameters

*    Load all the information from the network (generated with network.gms)

sets
    i(*) 'Nodes',
    d(*) 'Destinations',
    k(*) 'Lots';

parameter
    dista(*,*)   'Distance';

$gdxin '%resultsdir%network.gdx'
$load i d k dista

alias (i,j);
```

```gams
      set
          knode(k,i)        'Locations for city lots (nodes)',
          ua(*,*)           'Undirected arcs',
84        a(i,j)            'Directed arcs';

      $loaddc knode ua a

      *   Load the calibrated model parameters (calibration done in calib.gms)
89    $gdxin '%resultsdir%%hh%_calib.gdx'

      set
          h(*)      Households (origins),
          ht(h,d) Households (origin and destination);

      $loaddc h ht

      parameter
          lwt(h,d,k)               'Walking time from this lot (min)',
99        swt(h,d,i,j)             'Walking time from street parking spot (min)',
          arrivals(h,d,j)          'Arrivals at node j destine to i',
          sp(i,j)                  'Street parking on arc i—j',
          gp(k)                    'Parking capacity lots',
          fref(i,j)                'Reference flow',
104       alpha(i,j)               'Open road travel delay',
          beta(i,j)                'Reference congestion delay',
          piref(i,j)               'Reference probabilitites of not finding a slot (
              street)',
          pizref(k)                'Reference probabilitites of not finding a slot (
              garage)',
          tref(h,d,j)              'Reference travel times',
109       xref(h,d,i,j)            'Reference values of X',
          yref(h,d,i,j)            'Reference values of Y,'
          zref(h,d,k)              'Reference values of Z,'
          pyref(i,j)               'Street parking price',
          pzref(k)                 'Lot parking price',
114       v(h)                     'Value of time ($ per min)',
          vw                       'Time cost multiple for walking time',
          vs                       'Time cost multiple for searching time',
          tauref(i,j)              'reference delay',
          tauzref(k)               'Time required to look for a garage parking place',
119       scalingRn                'Inverse of scaling of quantities used in calibrtione'
              ,
          scalingR                 'Inverse of scaling of prices used in calibrtion'
          alphaz(k)                'Coefficient for searching time garage'
      ;

124   $loaddc lwt swt fref alpha beta piref xref yref zref pizref tref sp gp
      $loaddc pyref pzref arrivals v vs vw tauref tauzref=tauz scalingR = scaling
      $loaddc scalingRn = scalingN

      alphaz(k)   = tauzref(k) /(1 + power((sum(ht,zref(ht,k))/gp(k)),4));

      *   We scaled the quantities and prices in the calibration
      *   procedure. These parameters are used to revert the values in the
      *   report writing back to the orriginal dimensions.
      scalingRn = 1/scalingRn;
134   scalingR  = 1/scalingR;

      parameter
          feps                     'Epsilon for calibration of arcs with zero flow' /1/;

139   *@ Model

      variables
          F(i,j)            'Aggregate flow',
          TAU(i,j)          'Travel delay on the i—j arc',
144       TAUZ(k)           'Travel delay at lot k';

      nonnegative variables
          T(h,d,j)          'Minimum time to destination',
          X(h,d,i,j)        'Agents driving on the i—j arc',
149       Y(h,d,i,j)        'Searching for parking on the i—j arc',
          Z(h,d,k)          'Searching for parking at lot k,'

          PI(i,j)           'Probability of not finding a parking place',
          PY(i,j)           'Street parking fee (for counterfactual solution)',

          PIZ(k)            'Probability of not finding a place in lot k',
          PZ(k)             'Lot parking fee  (for counterfactual solution)';

      equations
159       opt_x             'FOC with respect to X',
          opt_y             'FOC with respect to Y',
          opt_z             'FOC with respect to Z',
```

```
               flowdef           'Definition of flow on arc',
               taudef            'Definition of time on arc',
164            tauzdef           'Definition of search time in parking garage',
               onstreet          'Capacity constraint curb side parking',
               inlot             'Capacity constraint parking garage',
               nodebal           'Node balance';

169  opt_x(ht(h,d),a(i,j))..
               TAU(a) + T(ht,j) =g= T(ht,i);

     opt_y(ht(h,d),a(i,j))$sp(a)..
               vs*TAU(a) + (1-PI(a))*(PY(a)/v(h) + vw*swt(ht,a)) +
174            PI(a)*T(ht,j) =g= T(ht,i);

     opt_z(ht(h,d),k)..
               vs*TAUZ(k) + (1-PIZ(k))*(vw*lwt(ht,k) + PZ(k)/v(h)) +
               PIZ(k)*sum(knode(k,i), T(ht,i)) =g= sum(knode(k,i), T(ht,i));

     flowdef(a)..
               F(a) =e= sum(ht, X(ht,a) + Y(ht,a));

     taudef(a)$round(TAU.UP(a)-TAU.LO(a),4)..
184            TAU(a) =e= alpha(a) + beta(a) *
               power((F(a)+feps)/(feps+fref(a)),4);

     tauzdef(k)$round(TAUZ.UP(k)-TAUZ.LO(k),4)..
               TAUZ(k) =e= alphaz(k)* (1
189            + power((sum(ht,Z(ht,k))/gp(k)),4));

     onstreet(a)$sp(a)..
               sp(a) =G= (1-PI(a)) * sum(ht, Y(ht,a)) ;

194  inlot(k)..
               gp(k) =g= (1-PIZ(k)) * sum(ht, Z(ht,k));

     nodebal(ht,j)..
               sum(a(j,i), X(ht,a) + Y(ht,a)) + sum(knode(k,j),
199            (1-PIZ(k))*Z(ht,k)) =E= arrivals(ht,j) + sum(a(i,j),
               X(ht,a)+PI(a)*Y(ht,a)) ;

     *     Set up a first-best calculation in which we implicity price
     *     every arc in the network:

     variable
               OBJT              Objective for time,
               OBJM              Objective for money,
               OBJH              Objective for health;

     equation
               objmintime        Objective function for minimizing travel time,
               objmincost        Objective function for minimizing utilitarian cost;

214  objmintime..
               OBJT =e= sum((ht,a), TAU(a)*(X(ht,a) + vs*Y(ht,a))) +
                        sum((ht,a),(1-PI(a))*vw*swt(ht,a)*Y(ht,a)) +
                        sum((ht,k), TAUZ(k)*vs*Z(ht,k)) +
                        sum((ht,k),(1-PIZ(k))*vw*lwt(ht,k)*Z(ht,k)) ;

     objmincost..
               OBJM =e= sum(ht(h,d), v(h) *(
                        sum(a, TAU(a)*(X(ht,a) + vs*Y(ht,a))) +
                        sum(k, TAUZ(k)*vs*Z(ht,k))   +
224                     sum(a,(1-PI(a))*(vw*swt(ht,a)+ PY(a)/v(h))*Y(ht,a)) +
                        sum(k,(1-PIZ(k))*(vw*lwt(ht,k) + PZ(k)/v(h))*Z(ht,k))));

     model cruising /opt_x.X, opt_y.Y, opt_z.Z, flowdef.F, taudef.TAU,
                     tauzdef.TAUZ, onstreet.PI, inlot.PIZ, nodebal.T,
229                  objmintime.OBJT, objmincost.OBJM/;

     *@@ Model initialization

     TAUZ.L(k) = tauzref(k);
234  TAU.L(a)  = tauref(a);

     *     Before defining a scenario, all variables are unfixed(with the
     *     exception of tolls). This is done because of the risk of forgetting
     *     to unfix some of the variables after running a specific scenario.
239  $include reset.gms

     *     Initializing
     F.L(a)                     = fref(a);
     T.L(ht,j)                  = tref(ht,j);

     X.L(ht,a)                  = xref(ht,a);
     Y.L(ht,a)                  = yref(ht,a);
```

```
      Z.L(ht,k)                 = zref(ht,k);

249   PI.L(a)                   = piref(a);
      PIZ.L(k)                  = pizref(k);

      PY.FX(a)                  = pyref(a);
      PZ.FX(k)                  = pzref(k);

      OBJT.L = sum(ht(h,d),
                  sum(a, TAU.L(a)*(X.L(ht,a) + vs*Y.L(ht,a))) +
                  sum(k, TAUZ.L(k)*vs * Z.L(ht,k)) +
                  sum(k,(1-PIZ.L(k))*vw*lwt(ht,k)*Z.L(ht,k)) +
259               sum(a,(1-PI.L(a))*vw*swt(ht,a)*Y.L(ht,a)));

      OBJM.L =  sum(ht(h,d), v(h) *(
                  sum(a, TAU.L(a)*(X.L(ht,a) + vs*Y.L(ht,a))) +
                  sum(k, TAUZ.L(k)*vs*Z.L(ht,k))   +
264               sum(a,(1-PI.L(a))*(vw*swt(ht,a) + PY.L(a)/v(h))*Y.L(ht,a)) +
                  sum(k,(1-PIZ.L(k))*(vw*lwt(ht,k) + PZ.L(k)/v(h))*Z.L(ht,k))));;

      *@ Sc 1: Benchmark "Cruising"

269   *    In this scenario we have the fees fixed and the probability of
      *    finding a spot is endogenous.

      cruising.savepoint = 1;
      cruising.iterlim   = 0;

      *    This form of a display statement adds a heading in the outline mode in
      *    emacs.
      display "* Scenario 1: Cruising";

279   solve cruising using mcp;
      *    We move the savepoint file to the base directory
      execute 'mv -f cruising_p.gdx %basedir%hh%_cruising_p.gdx';

      cruising.iterlim   = 1000000;
284   option fref:1:0:1;
      Option tref:1:0:1;
      option piref:1:0:1;
      option xref:1:0:1;
      option yref:1:0:1;
289   option zref:1:0:1;
      option pzref:1:0:1;
      option tauref:1:0:1;

      abort$(cruising.objval > 1e-5) "Benchmark equilibrium does not replicate!";

      *@@ Reporting parameters

      set
          routechoice(*,*,*,*,i,i)          Route choices;

      parameter
          reportagg                 'Aggregate summary report',
          report                    'Summary report',
          lotparking                'Household parking by lot',
304       streetparking             'Streetparking',
          parkingcount              'Parking counts by destination',
          arcflow                   'Arc flows',
          flowrate                  'Arc flows by household',
          flow                      'Flow report by arc',
309       lotreport                 'Summary of lot garages',
          modelinfo                 'Info on solve statistics',
          reportagents              'Report with respect to agents'
          checks                    'Checks';

314   *    We write a report file (report.scr) that is called after each solve
      *    with batinclude. Example
      *    $batinclude %gams.scrdir%report.scr "Pricing street" "pricing_street"
      *    This call has as parameter the name of scenario (%1). An alternative
      *    way would be to do a loop over a set of scenarios. A second parameter
319   *    is the name of the model, so we can get the model statistics
      *    (e.g. modelname.status).

      $onechov        >%gams.scrdir%report.scr

324   lotreport("%1",k,"Z")         = scalingRN * sum(ht,Z.L(ht,k)+eps);
      lotreport("%1",k,"Z*(1-piz)") = scalingRN * (1-PIZ.L(k))*sum(ht,Z.L(ht,k)+eps);
      lotreport("%1",k,"gp")        = scalingRN * gp(k) +eps;
      lotreport("%1",k,"PIZ")       = PIZ.L(k) +eps;
      lotreport("%1",k,"Price")     = scalingR * PZ.L(k) +eps;

      report("%1",ht,"PZ*Z*(1-piz)") = sum(k, scalingR * PZ.L(k)*scalingRN *
          Z.L(ht,k)*(1-PIZ.L(k))) +eps;
      report("%1",ht,"PY*Y*(1-pi)")  = sum(a, scalingR * PY.L(a)*scalingRN *
```

```
                 Y.L(ht,a)*(1-PI.L(a))) +eps;

        report("%1",ht(h,d),"objT") = scalingRN *(
                    sum(a, TAU.L(a)*(X.L(ht,a) + vs*Y.L(ht,a))) +
                    sum(k, TAUZ.L(k)*vs*Z.L(ht,k)) +
                    sum(k,(1-PIZ.L(k))*vw*lwt(ht,k)*Z.L(ht,k)) +
339                 sum(a,(1-PI.L(a))*vw*swt(ht,a)*Y.L(ht,a)));

        report("%1",ht(h,d),"objm") =  v(h) *(
                    sum(a, TAU.L(a)*(X.L(ht,a) + vs*Y.L(ht,a))) +
                    sum(k, TAUZ.L(k)*vs*Z.L(ht,k))   +
344                 sum(a,(1-PI.L(a))*(vw*swt(ht,a)+ PY.L(a)/v(h))*Y.L(ht,a)) +
                    sum(k,(1-PIZ.L(k))*(vw*lwt(ht,k)+PZ.L(k)/v(h))*Z.L(ht,k)));

        report("%1",ht(h,d),"genCost") = v(h) *scalingRN * scalingR *(
            sum(k,  PZ.L(k)/v(h)* Z.L(ht,k)*(1-PIZ.L(k))) +
349         sum(a,  PY.L(a)/v(h)* Y.L(ht,a)*(1-PI.L(a))) +
            sum(a,  (X.L(ht,a) +  vs * Y.L(ht,a))* TAU.L(a)) +
            vs * sum(k,  (Z.L(ht,k)*TAUZ.L(k))) +
            vw * sum(a, (1-PI.L(a)) * Y.L(ht,a)*swt(ht,a)) +
            vw * sum(k,(1-PIZ.L(k)) *Z.L(ht,k)*lwt(ht,k)));

        report("%1",ht, "X")                          = sum(a, scalingRN * X.L(ht,a)) +eps;
        report("%1",ht, "Y")                          = sum(a, scalingRN * Y.L(ht,a)) +eps;
        report("%1",ht, "Z")                          = sum(k, scalingRN * Z.L(ht,k)) +eps;
        report("%1",ht, "F")                          = sum(a, scalingRN * X.L(ht,a)+Y.L(ht,a))
            +eps;

        report("%1",ht,"Passing time")                = sum(a, scalingRN * X.L(ht,a)*TAU.L(a))
            /60 + eps;
        report("%1",ht,"Searching time street")       = sum(a, scalingRN * Y.L(ht,a)*
            (vs*TAU.L(a)))/60 + eps;
        report("%1",ht,"Searching time garage")       = sum(k, scalingRN * Z.L(ht,k)
            *(vs*TAUZ.L(k)))/60 + eps;
        report("%1",ht,"Walking time")                =
364         (sum(a, (1-PI.L(a))*scalingRN * Y.L(ht,a)*vw * swt(ht,a)) +
            sum(k, scalingRN * (1 - PIZ.L(k))* Z.L(ht,k)*vw *lwt(ht,k)))/60+ EPS;
        report("%1",ht,"Total time")  =  report("%1",ht,"Passing time") +
            report("%1",ht,"Searching time street") + report("%1",ht,"Searching time
                garage")   +
            report("%1",ht,"Walking time");

        report("%1",ht,"Total time change")$report("Cruising",ht,"Total time") =
            (report("%1",ht,"Total time") / report("Cruising",ht,"Total time") - 1) * 100
                + EPS;

        reportagents("%1",h,"Total time") = sum(d, report("%1",h,d,"Total time"));
374     reportagents("%1",h,"Total time change")$ reportagents("Cruising",h,"Total time")
            =
            (reportagents("%1",h,"Total time") / reportagents("Cruising",h,"Total time") -
                1) * 100;

        reportagg("%1","objTa") = sum(ht,
            sum(a, scalingRN * X.L(ht,a)*TAU.L(a)) +
379         sum(a, scalingRN * Y.L(ht,a)* (vs*TAU.L(a))) +
            sum(k, scalingRN * Z.L(ht,k)*(vs*TAUZ.L(k))) +
            sum(a, scalingRN * (1-PI.L(a))* vw * Y.L(ht,a)*swt(ht,a)) +
            sum(k, scalingRN * (1 - PIZ.L(k))* vw *Z.L(ht,k)*lwt(ht,k)));
        ;

        report("%1",ht(h,d),"objT") = scalingRN *(
                    sum(a, TAU.L(a)*(X.L(ht,a) + vs*Y.L(ht,a))) +
                    sum(k, TAUZ.L(k)*vs*Z.L(ht,k)) +
                    sum(k,(1-PIZ.L(k))*vw*lwt(ht,k)*Z.L(ht,k)) +
389                 sum(a,(1-PI.L(a))*vw*swt(ht,a)*Y.L(ht,a)));

        report("%1",ht,"X*TAU")                       = sum(a, scalingRN * X.L(ht,a)*TAU.L(a))
            +eps;
        report("%1",ht,"Y*TAU")                       = sum(a, scalingRN * Y.L(ht,a)*TAU.L(a))
            +eps;
        report("%1",ht,"F*TAU")                       =
394         sum(a, scalingRN * (X.L(ht,a)+Y.L(ht,a))*TAU.L(a)) +eps+ EPS;
        report("%1",ht,"Z*TAUZ")                      = sum(k, scalingRN * Z.L(ht,k)*TAUZ.L(k))
            + EPS;
        report("%1",ht,"(1-piz)*Z*lwt")               =
            sum(k, (1-PIZ.L(k))* scalingRN * Z.L(ht,k)*lwt(ht,k))+ EPS;
        report("%1",ht,"(1-pi)*Y*swt")                =
399         sum(a, (1-PI.L(a)) * scalingRN * Y.L(ht,a)*swt(ht,a))+ EPS;

        report("%1",ht,"(1-piz)*Z")                   = sum(k, (1-PIZ.L(k))*scalingRN *Z.L(ht,k
            ))+ EPS;
        report("%1",ht,"(1-pi)*Y")                    = sum(a, (1-PI.L(a))*scalingRN *Y.L(ht,a)
            )+ EPS;
        *report("%1",ht,"(1-piz)*Z%")                 = 100 * (report("%1",ht,"(1-piz)*Z") /
            report("Cruising",ht,"(1-piz)*Z") - 1);
```

```
404  *report("%1",ht,"(1-pi)*Y%")            = 100 * (report("%1",ht,"(1-pi)*Y") /
         report("Cruising",ht,"(1-pi)Y") - 1);

     reportagg("%1","PZ*Z*(1-piz)")          = sum(ht,report("%1",ht, "PZ*Z*(1-piz)"))+
         EPS;
     reportagg("%1","PY*Y*(1-pi)")           = sum(ht,report("%1",ht, "PY*Y*(1-pi)"))+
         EPS;

409  reportagg("%1","objm")                  = sum((h,d),report("%1",h,d,"objm"))+ EPS
         ;
     reportagg("%1","objmO")                 = OBJM.L + EPS;

     reportagg("%1","objt")                  = sum((h,d),report("%1",h,d,"objT"))+ EPS
         ;
     reportagg("%1","objtO")                 = scalingRN * OBJT.L + EPS;
414  reportagg("%1","genCost (in 1'000 CHF)")          = sum((h,d),report("%1",h,
         d,"genCost"))/1000+ EPS;

     reportagg("%1","X")                     = sum(ht,report("%1",ht, "X"))+ EPS;
     reportagg("%1","Y")                     = sum(ht,report("%1",ht, "Y"))+ EPS;
     reportagg("%1","Z")                     = sum(ht,report("%1",ht, "Z"))+ EPS;
419  reportagg("%1","F")                     = sum(ht,report("%1",ht, "F"))+ EPS;

     reportagg("%1","Passing time")          = sum(ht,report("%1",ht, "Passing time"))+
         EPS;
     reportagg("%1","Searching time street")      = sum(ht,report("%1",ht,"Searching
         time street"))+ EPS;
     reportagg("%1","Searching time garage")  = sum(ht,report("%1",ht,"Searching time
         garage"))+ EPS;
424  reportagg("%1","Walking time")          = sum(ht,report("%1",ht,"Walking time"))+
         EPS;
     reportagg("%1","Total time")            = reportagg("%1","Passing time") +
       reportagg("%1","Searching time street") + reportagg("%1","Searching time garage"
         ) +
       reportagg("%1","Walking time") +  EPS;

429  reportagg("%1","Total general cost")    = sum((h,d),report("%1",h,d,"genCost"))+
         EPS;

     reportagg("%1","X*TAU")                 = sum(ht,report("%1",ht, "X*TAU"))+ EPS;
     reportagg("%1","Y*TAU")                 = sum(ht,report("%1",ht, "Y*TAU"))+ EPS;
     reportagg("%1","F*TAU")                 = sum(ht,report("%1",ht, "F*TAU"))+ EPS;
434  reportagg("%1","Z*TAUZ")                = sum(ht,report("%1",ht, "Z*TAUZ"))+ EPS;
     reportagg("%1","(1-piz)*Z*lwt")         = sum(ht,report("%1",ht, "(1-piz)*Z*lwt"))
         + EPS;
     reportagg("%1","(1-pi)*Y*swt")          = sum(ht,report("%1",ht, "(1-pi)*Y*swt"))+
         EPS;

     reportagg("%1","(1-piz)*Z")             = sum(ht,report("%1",ht, "(1-piz)*Z"))+
         EPS;
439  reportagg("%1","(1-pi)*Y")              = sum(ht,report("%1",ht, "(1-pi)*Y"))+ EPS
         ;

     lotparking("%1",ht,k,"(1-PIZ)*Z")       = (1-PIZ.L(k))*scalingRN *Z.L(ht,k)+ EPS;
     lotparking("%1",ht,k,"PIZ*Z")           = PIZ.L(k)*scalingRN *Z.L(ht,k)+ EPS;
     lotparking("%1",ht,k,"PIZ")             = PIZ.L(k)+ EPS;
444  lotparking("%1",ht,k,"PZ")              = scalingR * PZ.L(k)+ EPS;

     streetparking("%1",ht,a,"PI")$sp(a)     = PI.L(a)+ EPS;
     streetparking("%1",ht,a,"Y")$sp(a)      = scalingRN *Y.L(ht,a)+ EPS;
     streetparking("%1",ht,a,"PI*Y")$sp(a)   = PI.L(a)*scalingRN *Y.L(ht,a)+ EPS;
449  streetparking("%1",ht,a,"(1-PI)*Y")$sp(a) = (1-PI.L(a))*scalingRN *Y.L(ht,a)+ EPS;
     streetparking("%1",ht,a,"PY")$sp(a)     = scalingR *PY.L(a)+ EPS;

     flow("%1",a,"F")                        = sum(ht,scalingRN * Y.L(ht,a)+scalingRN *X.L(
         ht,a));
     flow("%1",a,"F%")$flow("cruising",a,"F") =
454      100 * (flow("%1",a,"F") / flow("cruising",a,"F") - 1);
     flow("%1",a,"F New")$((flow("Cruising",a,"F") = 0) and flow("%1",a,"F")) = 1;

     flow("%1",a, "X")             = sum(ht,scalingRN *X.L(ht,a)) + EPS;
     flow("%1",a, "Y")             = sum(ht,scalingRN *Y.L(ht,a)) + EPS;

     parkingcount("%1",i,"Z")      = sum(ht(h,d),sum(k,(1-PIZ.L(k))*scalingRN *Z.L(
         ht,k)))+ EPS;
     parkingcount("%1",i,"Y")      = sum(ht(h,d),sum(a,(1-PI.L(a))*scalingRN *Y.L(ht
         ,a)))+ EPS;

     arcflow("%1",a,"X")           = sum(ht,scalingRN *X.L(ht,a))+ EPS;
464  arcflow("%1",a,"Y")           = sum(ht,scalingRN *Y.L(ht,a))+ EPS;
     arcflow("%1",a,"X+Y")         = arcflow("%1",a,"X") + arcflow("%1",a,"Y");
     checks("%1","X-arcflow")      = sum(a, arcflow("%1",a,"X"));
     checks("%1","X-flw")          = sum(a, flow("%1",a,"X")) ;
     report("%1",ht,"Distance")    = sum(a,scalingRN * dista(a)*(X.L(ht,a)+Y.L(ht,a)
         ))+ EPS;
469  report("%1",ht,"Distance X")  = sum(a,scalingRN * dista(a)*(X.L(ht,a)))+ EPS;
```

```gams
      report("%1",ht,"Distance Y")        = sum(a,scalingRN * dista(a)*(Y.L(ht,a)))+ EPS;
      reportagg("%1","Distance")          = sum(ht,report("%1",ht,"Distance"))+ EPS;
      flowrate("%1",ht, a,"X")            = scalingRN * X.L(ht,a)+ EPS;
      flowrate("%1",ht, a,"Y")            = scalingRN * Y.L(ht,a)+ EPS;
474   flowrate("%1",ht, a,"X+Y")          = flowrate("%1",ht, a,"X") + flowrate("%1",ht, a,
          "Y") + EPS;


      routechoice("%1","X",ht,ua(i,j)) = yes$(X.L(ht,i,j)$a(i,j)>0 or X.L(ht,j,i)$a(j,i)
          >0);
479   routechoice("%1","Y",ht,ua(i,j)) = yes$(Y.L(ht,i,j)$a(i,j)>0 or Y.L(ht,j,i)$a(j,i)
          >0);
      routechoice("%1","F",ht,ua(i,j))
          = yes$(routechoice("%1","X",ht,ua) OR routechoice("%1","Y",ht,ua));

      modelinfo("%1","Time to solve")    = %2.resUsd;
484   modelinfo("%1","Status")           = %2.modelstat;
      modelinfo("%1","Equations ")       = %2.numEqu;
      modelinfo("%1","Infeasibilities ") = %2.numInfes;
      modelinfo("%1","Redefined ")       = %2.numRedef;
      modelinfo("%1","Variables")        = %2.numVar;
489   modelinfo("%1","Time in sec")      = %2.procUsed;
      $offecho

      $onechov        > %gams.scrdir%deltareport.scr
      report("%1",ht,"deltaY") =
494        report("%1",ht,"Y*TAU") — report("cruising",ht,"Y*TAU")+ EPS;
      report("%1",ht,"deltaX") =
          report("%1",ht,"X*TAU") — report("cruising",ht,"X*TAU")+ EPS;
      report("%1",ht,"deltaZ") =
          report("%1",ht,"Z*TAUZ") — report("cruising",ht,"Z*TAUZ")+ EPS;
499   report("%1",ht,"deltaF") =
          report("%1",ht,"F*TAU") — report("cruising",ht,"F*TAU")+ EPS;
      report("%1",ht,"deltaY%")$(report("cruising",ht,"Y*TAU") NE EPS)
          = 100 * (report("%1",ht,"deltaY") / report("cruising",ht,"Y*TAU"))+ EPS;
      report("%1",ht,"deltaX%")$(report("cruising",ht,"X*TAU") NE EPS)
504        = 100 * (report("%1",ht,"deltaX") / report("cruising",ht,"X*TAU"))+ EPS;
      report("%1",ht,"deltaF%")$(report("cruising",ht,"F*TAU") NE EPS)
          = 100 * (report("%1",ht,"deltaF")  / report("cruising",ht,"F*TAU"))+ EPS;

      flow("%1",ua(i,j),"deltaF") =
509        flow("%1",ua,"F") — flow("cruising",ua,"F")+ EPS;
      flow("%1",ua(i,j),"deltaX") =
          flow("%1",ua,"X") — flow("cruising",ua,"X")+ EPS;
      flow("%1",ua(i,j),"deltaY") =
          flow("%1",ua,"Y") — flow("cruising",ua,"Y")+ EPS;
514   $offecho

      $batinclude %gams.scrdir%report.scr Cruising Cruising

      *@ Sc 2: Scenario Street pricing ("Pricing street")
519   * ─────────────────────────────────────────────────────
      *    This scenario looks at the pricing of street parking, holding garage
      *    rates fixed. If the fee of street parking is endogenous, the
      *    probability of not finding a spot is equal to 0.

524   model pricing_street /opt_x.X, opt_y.Y, opt_z.Z, flowdef.F, taudef.TAU,
                        tauzdef.TAUZ, onstreet.PY, inlot.PIZ, nodebal.T,
                        objmincost.OBJM, objmintime.OBJT/;

      $include reset.gms

      PZ.FX(k)        = pzref(k);
      PIZ.UP(k)       = 1;
      PI.FX(a)        = 0;
      PY.FX(a)$sp(a) = 0;
534   PY.UP(a)$sp(a) = inf;

      $if exist '%basedir%%hh%_pricing_street_p.gdx' execute_loadpoint '%basedir%%hh%
          _pricing_street_p.gdx';
      $if %restart%==yes execute_loadpoint '%basedir%%hh%_cruising_p.gdx';

539   pricing_street.savepoint = 1;
      display "* Scenario 2: Pricing Street";
      pricing_street.holdfixed = 1;
      solve pricing_street using mcp;

544   execute 'mv —f pricing_street_p.gdx %basedir%%hh%_pricing_street_p.gdx';

      $batinclude %gams.scrdir%report.scr "Pricing street" "pricing_street"
      $batinclude %gams.scrdir%deltareport.scr "Pricing street" "pricing_street"

549   *@ Sc 3: Scenario lot parking
      * ─────────────────────────────────────────────────────
```

```
*    Another scenario looks at the pricing of lot parking, holding street
*    rates fixed and setting the probability of not finding a parking
*    garage spot to 0.

model pricing_lot /opt_x.X, opt_y.Y, opt_z.Z, flowdef.F, taudef.TAU,
                   tauzdef.TAUZ, onstreet.PI, inlot.PZ, nodebal.T,
                   objmincost.OBJM, objmintime.OBJT/;

$include reset.gms

PIZ.FX(k)       = 0;
PI.LO(a)$sp(a) = 0;
PI.UP(a)$sp(a) = 1;
PY.FX(a)        = pyref(a);

$if exist '%basedir%%hh%_pricing_lot_p.gdx' execute_loadpoint '%basedir%%hh%
    _pricing_lot_p.gdx';
$if %restart%==yes execute_loadpoint '%basedir%%hh%_cruising_p.gdx';
pricing_lot.savepoint = 1;

display "* Scenario 4: Pricing Lot";
pricing_lot.holdfixed = 1;
solve pricing_lot using mcp;

execute 'mv -f pricing_lot_p.gdx %basedir%%hh%_pricing_lot_p.gdx';

$batinclude %gams.scrdir%report.scr "Pricing lot" "pricing_lot"
$batinclude %gams.scrdir%deltareport.scr "Pricing lot" "pricing_lot"

*@ Sc 4: Scenario garage and street pricing
* ————————————————————————————————————————————————————————————
*    Another scenario looks at the pricing all parking, both on streets and
*    in lots, setting the probabilities of not finding a spot for garage
*    and street parking to 0.

model pricing /opt_x.X, opt_y.Y, opt_z.Z, flowdef.F, taudef.TAU,
               tauzdef.TAUZ, onstreet.PY, inlot.PZ, nodebal.T,
               objmincost.OBJM, objmintime.OBJT/;

$include reset.gms

PI.FX(a)        = 0;
PIZ.FX(k)       = 0;

$if exist '%basedir%%hh%_pricing_p.gdx' execute_loadpoint '%basedir%%hh%_pricing_p
    .gdx';
$if %restart%==yes execute_loadpoint '%basedir%%hh%_cruising_p.gdx';
pricing.savepoint = 1;
display "* Scenario 3: Pricing";
pricing.holdfixed = 1;

solve pricing using mcp;

execute 'mv -f pricing_p.gdx %basedir%%hh%_pricing_p.gdx';

$batinclude %gams.scrdir%report.scr Pricing Pricing
$batinclude %gams.scrdir%deltareport.scr Pricing Pricing

*@ Sc 5: Optimal policy: minimal time
* ————————————————————————————————————————————————————————————
*    Final simulations look at "optimal" policies. In this model we
*    minimize the time and the optimal fees for both garage and street
*    parking are made endogenous (probabilities exogenous).

model   firstbest /objmintime, objmincost, flowdef, taudef,
                   tauzdef, onstreet, inlot, nodebal/;

firstbest.savepoint = 1;

$include reset.gms
PI.FX(a)        = 0;
PIZ.FX(k)       = 0;
PY.FX(a)        = 0;
PZ.FX(k)        = 0;

option nlp      = conopt;

display "* Scenario 5: Minimal Time";
firstbest.holdfixed = 1;

$if exist '%basedir%%hh%_firstbest_p.gdx' execute_loadpoint '%basedir%%hh%
    _firstbest_p.gdx';
$if %restart%==yes execute_loadpoint '%basedir%%hh%_cruising_p.gdx';

solve firstbest using nlp minimizing OBJT;
```

```
634  *    Assign prices for the first—best allocation using shadow prices on the
     *    associated constraints, when solving for the time—minimizing scenario.
     *    This is somewhat ad—hoc: convert time using the weighted value of time
     *    for agents parking in these places:

639  *PY.L(a)$(onstreet.m(a) and sum(ht, Y.L(ht,a))) =
     *    onstreet.m(a) * sum(ht(h,d), v(h)*Y.L(ht,a))/sum(ht, Y.L(ht,a));
     *PZ.L(k)$(inlot.m(k) and sum(ht, Z.L(ht,k)))   = inlot.m(k) *
     *    sum(ht(h,d), v(h)*Z.L(ht,k))/sum(ht, Z.L(ht,k));

644  *    Generate reports for the minimum time solution:
     execute 'mv —f firstbest_p.gdx %basedir%%hh%_firstbest_p.gdx';

     $batinclude %gams.scrdir%report.scr "Minimal time" "firstbest"
     $batinclude %gams.scrdir%deltareport.scr "Minimal time" "firstbest"

     execute_unload '%resultsdir%%hh%_results.gdx', modelinfo, reportagg, report,
        lotreport, streetparking, lotparking, parkingcount, routechoice, arcflow, flow
        ,flowrate, numberscenarios, reportagents;
     display report;

     $onecho >gdxxrw.rsp
654  par=modelinfo           rng=flow!a2 cdim=0
     par=report              rng=report!a2 cdim=0
     par=reportagg           rng=reportagg!a2 cdim=0
     par=lotreport           rng=lotreport!a2 cdim=0
     par=streetparking       rng=streetparking!a2 cdim=0
659  par=lotparking          rng=lotparking!a2 cdim=0
     par=parkingcount        rng=parkingcount!a2 cdim=0
     par=flow                rng=flow!a2 cdim=0
     par=flowrate            rng=flowrate!a2 cdim=0
     par=modelinfo           rng=modelinfo!a2 cdim=0
664  par=reportagents        rng=reportagents!a2 cdim=0
     $offecho

     *   If the excel file with the results is open, close it

669  $call 'xlstalk.exe —S %resultsdir%%hh%_results.xlsx'
     $call 'xlstalk.exe —Q'

     execute 'gdxxrw i=%resultsdir%%hh%_results.gdx o=%resultsdir%%hh%_results.xlsx
        @gdxxrw.rsp';

674  *$call 'xlstalk.exe —S %resultsdir%RouteReport.xlsm'
     *execute 'gdxxrw i=%resultsdir%results.gdx o=%resultsdir%RouteReport.xlsm set=
        routechoice rng=Results!a1 cdim=0';

     *   Open the results file in Excel

679  *execute 'xlstalk.exe —O %resultsdir%%hh%_results.xlsx';

     *   Run the rscript that produces all the tables and graphs for the paper.
     *execute '.\R\%hh%.bat';
```

An integrated transport network-computable general equilibrium model in mixed-complementarity format for Zurich

3.1 Introduction

famous American comedian, actor and singer Groucho Marx(1890-1977) noted that "if they'd lower the taxes and get rid of the smog and clean up the traffic mess, I really believe I'd settle here until the next earthquake." Although defining traffic jams (congestion) is not straightforward, as stated in a recent study of the Organisation for Economic Cooperation and Development (2007), one thing is evident: congestion is nowadays a major problem in most big cities. Congestion not only leads to time losses and increase in vehicle operating and fuel costs, but it also has negative effects on the environment.

From an economic point of view, the best way to reduce and internalize these negative impacts is to introduce marginal cost pricing: Users should be taxed for the additional cost they impose on other users on a specific link. This link-specific tax should be equal to the difference between the marginal social cost and the average private cost. Congestion pricing has the advantage over other demand management policies, in that it sets an incentive to adjust all aspects of travel behavior. Based on the congestion tax agents may reduce the number of trips, change transport mode, time of day, route and in the long-run of where to live and work.

However, a link-differentiated toll on the entire network is often not possible. High toll collection costs or political reasons lead to suboptimal tax schemes where only certain links are being tolled. Another scheme that has been introduced in several cities around the world is the cordon toll, where a fixed toll has to be paid whenever a traveler enters a certain zone. Famous examples are the London Congestion Charge (since 2003), Stockholm cordon tax (since 2007), and Singapore's Electronic Pricing system (since 1975).[1] The cordon toll has the advantage that easily implementable technology is available. A drawback of this policy is that we do not have a first-best solution, which would require link- and distance-specific taxes as a second-best solution.

Several studies have been conducted to either analyze the optimality of cordon tolling using small examples or to study the effects of existing cordon tolling systems. Most of these studies were undertaken using transportation network models (examples are Verhoef, 2002; May et al., 2002;

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[1] An overview of existing cordon tolls and other schemes can be found in de Palma and Lindsey (2009).

Maruyama and Sumalee, 2007; Ho et al., 2005). However, as transportation decisions not only depend on traffic costs, but also on economic variables like income, housing rent prices and taxes, a more comprehensive tool is needed. Tools that can analyze all these dependencies are either spatial equilibrium or general equilibrium models. Only a few papers examine the theoretical and empirical effects of cordon pricing based on an urban spatial model. Mun, Konishi, and Yoshikawa (2005) study a non-monocentric city for which they compare the no-toll equilibrium with the first-best optimal and optimal cordon pricing. Their simulations with a fictitious urban model show that welfare gains of cordon pricing are higher for cities with an urban spatial structure that is close to monocentric, where the population density gradient is steeper, trip demand is less elastic and road capacity is larger. Surprisingly, they find that the welfare results for an optimal cordon toll scheme are close to the social optimum.

Verhoef (2005) uses a (continuous) spatial general equilibrium model with endogenous residential density and endogenous labor supply. He confirms the good performance of cordon tolling found by Mun, Konishi, and Yoshikawa (2005). This good performance even holds for different specifications of the utility function and Verhoef suspects that the monocentric city structure is responsible for driving this result. In a recent paper Brueckner (2014) uses a simple model of two congested bridges where space is discrete rather than continuous. He analytically derives the optimal cordon toll and shows that people move away from the zone immediately outside the cordon. The model presented in this essay differs with Mun, Konishi, and Yoshikawa (2005) and Verhoef (2005) in the treatment of space: in our model space is not continuous, but discrete and defined by the transport network.

Our model is more in line with the work of Anas and Hiramatsu (2013) who use RELU, a general equilibrium model calibrated for the Chicago Metropolitan Statistical Area as described in Anas and Liu (2007) to study the impact of several cordon tolling schemes.[2] The prototype-type model consists of four building, industry and labor types as well as 15 land-use zones. It has a 68-link-highway-network. Although our model differs in some important aspects (we refrain for example from a detailed housing market (no construction and demolition) the model size and structure is very similar.

A main difference is the way RELU is solved: The model is divided into two submodels (a transport model and a general equilibrium model) that are solved iteratively using a heuristic algorithm that is implemented by the researchers themselves. Our model is cast in a format that can be solved by readily available software.[3] This allows us to use the best solvers available and frees us from coding the algorithms ourselves.

In this essay, we propose a formulation of an integrated model that can be easily solved by ready available solvers following the techniques, developed in the first essay of this thesis. We present a mixed complementarity problem (MCP) formulation of an urban general equilibrium model that embeds a closed, spatially disaggregate economic model of housing and labor markets along the

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[2] Examples for other policies can be found in Anas and Kim (1996) and Anas and Xu (1999)

[3] We use GAMS (GAMS Development Corporation, 2014) with the Path solver (Ferris and Munson, 2014). GAMS has also the advantage that it adequately communicates with R (R Core Team, 2014), which was used for all figures and tables in this essay.

lines of the Alonso-Muth-Mills model [4]. This economic model is embedded within a model of individually-rational route choice on the (congested) traffic network (Wardrop, 1952). We use the link-flow formulation for the transport equilibrium model, which in makes an enumeration of possible routes unnecessary. This is however only true if there is no road pricing. If there is road pricing, the income of the households will be reduced by their toll payments. Without knowing which route they have chosen, it is impossible to infer the toll payments. We introduce a simple procedure to infer the toll payments. We solve iteratively the integrated model followed by a simple linear optimization problem with almost no time costs to infer the arcs traveled by every household, then calculate the toll payments and adjust the income. This is repeated iteratively until the process converges (in our simulations, this process only needs three iterations).

The economic aspects of the model follow the Walrasian-Arrow-Debreu paradigm. Consumers earn by providing labor to the production sectors, and they allocate their income to housing and consumption. Taxes can be applied to both residential and employer locations, such that private decisions of households and firms produce an optimal pattern of location. One of the main assumptions of the model is that households choose residence and employment locations which arbitrate differences of various locations within the urban area. This implies that consumers trade locations, to work and live on the basis of housing prices, wages and commuting time. The model is medium-term and we assume therefore that only a certain part of the population can change locations for living and working. The commuting costs in the model depend on the distance between the location where the household lives and where he or she works as well as on the mode chosen. The model distinguishes between public and private transport. The commuting time using private transport depends on the capacity and size of the flow on the arcs. The commuting time using public transport mode is constant, as long as the capacity is not fully used. Once the demand is higher than the capacity, waiting time occurs. Transport costs for commuting are capitalized in housing values (Glazer and Van Dender, 2002) and can lead to wage differentials, depending on zone of employment (as estimated by Darren and Wheaton, 2001).

We also show the usefulness of the integrated approach for the analysis of two cordon toll schemes in Zurich, the biggest city of Switzerland. The discussion on road pricing alternatives was started in Zurich in 2001 by the Zurich City Council, which substantiated the goal of road pricing in the following years. Based on several political and research white papers and developments at the national level, the council was the opinion that with the help of more detailed analysis the goal of road pricing for the City of Zurich should be pursued (Bächtiger and Ott, 2010). Recently, the discussion has centered increasingly on mobility pricing instead of road pricing.[5] With mobility pricing a charge on distance traveled is levied for financing transport infrastructure. The reduction of congestion is seen as a beneficial side effect.

An important issue of this essay is the calibration of the model to the given data. For our simulations, the model is calibrated it to the travel demand and travel times from the Cantonal Transport Model (Amt für Verkehr Volkswirtschaftsdepartement Kanton Zürich, 2011) by adjusting the pa-

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[4] For a description of the Alonso-Muth-Mills model see for example Glaeser (2008, Chapter 2) or Brueckner (1987).

[5] See for example the article in one of the leading Swiss newspapers (Hotz 2014).

rameters of the delay functions. The logit demand for the two modes is calibrated to the given modal split of the origin-destination-matrix and given price elasticities. On the economic side of the model, we show how to calibrate the model to available housing stock, wages and rents.

A final goal of this paper is to show differences between the results from the integrated model and a (simple) transport equilibrium model. The results show that incorporating economic variables and sorting can lead to significantly different results which may be of high relevance for policy analysis.

This essay is structured as follows: The first part explains the structure of the integrated model. The second part shows how the model can be calibrated to a set of data on travel times, mode shares and economic variables. In part the fourth we discuss the effects of cordon toll schemes for the city of Zurich, and finally the conclusion reviews the contributions of this paper and offers some suggestions for future research and development

3.2 Model Description

In this chapter we provide lay out of the structure of the integrated model which consists of two submodels: a spatially disaggregated economic model of housing and labor markets, and a model of individually-rational route choice on the (congested) traffic network as introduced by Wardrop (1952). Both models are embedded in a single mixed complementarity problem (MCP). We first give a short introduction into the MCP format and then discuss the two submodels. An introduction to this kind of modeling can be found in the first essay of this thesis.

3.2.1 Mixed complementarity problems

Complementary problems can be described as systems of (non)linear constraints where the system variables are linked to the constraints with complementarity conditions (Ferris and Munson, 2014). More formally, given a function $h : \mathbb{R}^n \to \mathbb{R}^n$, lower bounds $l \in \{\mathbb{R} \cup -\infty\}^n$ and upper bounds $u \in \{\mathbb{R} \cup \infty\}^n$, we try to find $x \in \mathbb{R}^n$ such that precisely one of the following holds for each $i \in 1, \ldots, n$:

$$x_i = l_i \quad \text{and} \quad h_i(x_i) \geq 0, \quad \text{or}$$

$$x_i = u_i \quad \text{and} \quad h_i(x_i) \leq 0, \quad \text{or}$$

$$l_i < x_i < u_i \quad \text{and} \quad h_i(x_i) = 0$$

This means that the variable x_i is either at one of its bounds or the linked function is equal to zero.

In the mixed complementarity problem (MCP) we not only have inequalities with complementary non-negative variables, but we also have equations where the associated variables are free and the complementarity conditions become:

$$h_i(x_i, x_j) \geq 0, \quad x_i \geq 0, \quad x_i h_i(x_i, x_j) = 0,$$

$$h_j(x_i, x_j) = 0, \quad x_j \text{ free}$$

where we partition the set n in the sets i (inequalities) and j (equalities).

Complementarity models can be used for solving linear, quadratic and nonlinear programs by writing down the Karush-Kuhn-Tucker optimality conditions. Complementary models can also be used for expressing a variety of economic models for both markets and games, where the problems cannot be written down as a single optimization problem, or no equivalent optimization problem exists. Examples are the famous transport problem by Dantzig, the Walras equilibrium and the von-Thunen land model. A model formulation of these examples can be found in Ferris and Munson (2014) and more examples can be found in Rutherford (1995) and Dirkse and Ferris (1995).

The development of the complementarity modeling format was motivated by theoretical and practical developments in algorithms for nonlinear complementarity problems and variational inequalities. The most recent techniques are based on ideas from interior-point algorithms for linear programming (Kojima et al., 1991). Computational evidence suggests that algorithms for solving MCPs are relatively reliable and efficient, particularly for models which are not natural optimization problems. A survey of developments in the theory and applications of these methods is provided by Harker and Pang (1990).

One of the main reasons why we have chosen for the MCP formulation, is that a MCP model can be solved with readily available software packages, like MathLab (The MathWorks, 2012) or GAMS (GAMS Development Corporation, 2014), that provide powerful algorithms. Using these packages allows us to concentrate on the model formulation, and takes the burden of writing the algorithms to solve the model. Although, stand-alone TEP models can be solved by a variety of specialized software packages,[6] these packages are geared to predefined models and cannot be easily extended in other directions.

3.2.2 The Transport Model

A transport problem can be defined as finding the optimal way to assign demand, usually given by an origin-destination matrix, to a network. The set of origins is denoted with $\mathcal{O}$ and the set of destinations with $\mathcal{D}$. In our model an origin is a node where a household lives, and a destination is a node where a household works. In order to reach the optimal assignment, we assume that households seek to minimize their individual travel cost when commuting to their work place. A Wardropian user equilibrium (Wardrop, 1952) is reached when no household has an incentive to choose another route.[7] We will see that in the complete model, two more notions of equilibrium are used: The first is a spatial equilibrium which is reached when no household has an incentive to change its decision where to live and where to work. The second is the economic equilibrium in which the households maximize their utility and firms maximize their profits. Overall equilibrium

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[6] See Correa et al. (2010) for a list of the most used packages.

[7] Apart from the Wardropian user equilibrium, there are several other characterizations of a traffic network equilibrium. Marcotte and Patriksson (2007) mention equilibrated, user, normal and user optimized equilibrium. Under the assumption of separable, continuous link cost functions, these characterizations are equal to the Wardropian user equilibrium (see Marcotte and Patriksson, 2007, Chapter 2.8).

in the model is reached when the conditions for all three equilibriums are met.

In the following i, j and k are indices used as aliases to describe the nodes in the network. For every transport mode m, a (directed) arc is defined by a start and an end node. The set of all arcs for a transport mode is given by $\mathcal{A}^m$.

The transport problem can be formulated as a multi-commodity flow problem as shown in Steenbrink (1974). This means that we use the link flow formulation instead of the often used route flow formulation. The advantage is that there is no need for enumerating the possible routes.

Our network consists of only four complementarity equation groups.[8] The first group of equations defines the aggregate flows of a transport mode on the arcs. The aggregate flow $F_{m,i,j}$ from node i to j is defined as the sum of all households h traveling from node i to node j with any destination k using mode m:

$$F_{m,i,j} = \sum_{h,k} X_{h,m,i,j,k} \quad \forall (i,j) \in \mathcal{A}^m, \tag{3.1}$$

with the associated free variable $F_{m,i,j}$ and $X_{h,m,i,j,k} \geq 0$. We make a distinction between private ("pr") and public ("pb") transport modes.

The second group of equations defines travel times or costs on the arcs. The travel time for private transport is a function of the traffic volume. The effect of the road capacity on travel times is specified by means of volume-delay functions. The most popular volume-delay function for private transport is the BPR function found in Bureau of Public Roads (1964) and defines the travel time on an arc $\tau_{pr,i,j}$ as:

$$\tau_{pr,i,j} = \alpha_{pr,i,j} + b_{i,j} \left(\frac{F_{pr,i,j}}{\kappa_{pr,i,j}} \right)^4 \quad \forall (i,j) \in \mathcal{A}^{pr}, \tag{3.2a}$$

where α is the free-flow time on the arc, b the congestion scale factor, and κ the reference capacity. The associated free variable is $\tau_{pr,i,j}$. The time costs increases as the arc gets more congested (see Figure 3.1a).[9] For the public transportation mode, we assume that there is a capacity limit (see figure 3.1b). The travel time is given by:

$$\tau_{pb,i,j} = \alpha_{pb,i,j} + \mu_{i,j} \quad \forall (i,j) \in \mathcal{A}^{pb}, \tag{3.2b}$$

with $\tau_{pb,i,j}$ as associated free variable. The travel time for public transport on an arc incorporates the complementary variable $\mu_{i,j}$ of the capacity constraint (see Marcotte and Patriksson, 2007, Chapter 3.2). The shadow price of the constraint can now be interpreted as the average waiting time. If the constraint is binding the average waiting time is positive, otherwise it is zero. The capacity constraint is given by:

$$\overline{F}_{pb,i,j} \geq F_{pb,i,j} \quad \perp \quad \mu_{i,j} \quad \forall (i,j) \in \mathcal{A}^{pb}. \tag{3.2c}$$

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[8] In all equations variables are written as uppercase and parameters as lower case symbols. Fixed variables are overlined.

[9] We assume separable delay-volume functions (traffic on other links influences the travel time on a specific arc). The MCP formulation allows non-separable volume-delay functions.

Figure 3.1 – Volume-delay function of transport modes
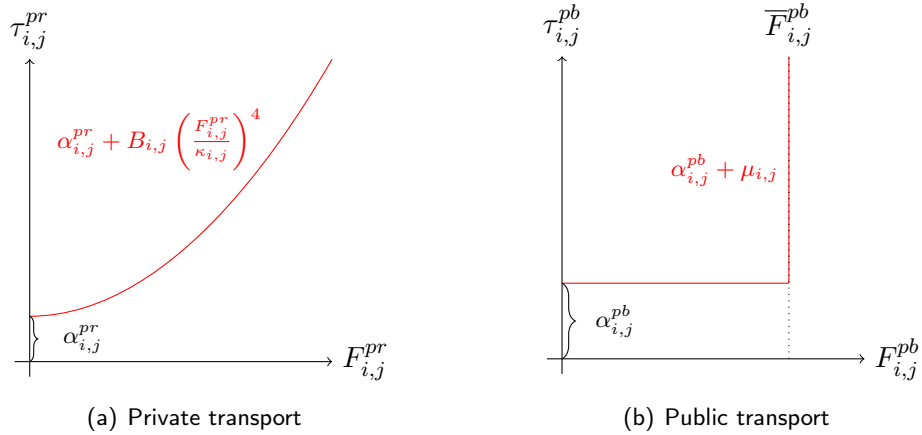
(a) Private transport

(b) Public transport

The third group of equations defines the flow conservation at every node j for the number of people traveling from this node to destination k:

$$\sum_{i,m} X_{h,m,i,j,k} - \sum_{i,m} X_{h,m,j,i,k} = N_{h,j,k}, \quad \forall(h,m,i,k) \text{ and } i \neq d, \tag{3.3}$$

where $N_{h,j,k}$ is the total flow of people of household type h starting from node j if $j \in \mathcal{O}$ and node $k \in \mathcal{D}$. The associated free variable is the minimum time from node j to node k ($T_{h,m,j,k}$).

The last equation group reflects the second Wardropian principle. In its original form it states that "*the journey times on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route*" (Wardrop, 1952, p. 345):

$$\tau_{m,i,j} + T_{h,m,j,k} \geq T_{h,m,i,k} \quad \text{and} \quad T_{h,m,k,k} = 0 \quad \forall(i,j) \in \mathcal{A}^m, \tag{3.4}$$

with $X_{h,m,i,j,k}$ as associated free variable. On the left-hand side we have the minimal travel time T for household h from node i to k. This travel time should be less than or equal to the sum of the travel time τ on an arc starting from node i to any of the adjacent nodes j, and the minimal time T is from traveling from the adjacent node to the destination node k.

The time minimization equations are associated with the non-negative variable $X_{h,m,i,j,k}$ as complementary variables. This variable is only positive for those adjacent arcs where the traveling time from i to k is minimal. If this is not the case, the flow on that arc is zero.

Note that a complete enumeration of all possible routes from node i to node k is not necessary. The information on the fastest routes from the adjacent nodes j to the destination k is given in the corresponding minimum time equations for traveling from j to k.

The big advantage of the MCP formulation of the transport problem is that we do not have to specify in advance which arcs are used. Ferris and Munson (2014, p. 3) state that this is the key property of a complementarity problem over a system of equations: "*If we know what arcs to send flow down, we can just solve a simple system of linear equations. However, the key to the modeling power of complementarity is that it chooses which of the inequalities satisfy as equations.*"

The mode choice is captured with a binary logit model, where the utility of traveler h using one of the two modes on arc (i, j) is given by:

$$U_{h,m,i,j}^{Transport} = \frac{T_{m,i,j} + \phi_{m,i,j}}{\lambda_h} + \epsilon_{h,m,i,j}, \tag{3.5}$$

where λ_h is a scale parameter and, as an increase in travel time reduces utility, is less than zero. The unobserved utility $\epsilon_{h,m,i,j}$ is distributed iid extreme value with variance $\pi^2/6$ (Gumbel). We use the normalization of the variance with a scale parameter $\lambda_{h,i,j}$ for calibrating the model to the observed elasticities. As this scale parameter goes to zero, there is infinite taste heterogeneity (the variance goes to infinity) and we have equal choice probabilities for all alternatives and therefore households choose completely randomly. If the scale parameter goes to infinity (and the variance goes to zero), the households deterministically choose the alternative with the minimal time (Ben-Akiva and Lerman, 1997). The constant $\phi_{m,i,j}$ captures the average effect on utility of the factors that are not included in our model. As only the differences in the alternatives matter, we can normalize the absolute levels of the constants by dropping the constant for one of the modes. The remaining constant now captures the average effect of all the non-included factors of one mode relative to the other mode (See Train, 2009, pp. 19-21).

The share of public transport is given by the following logit function[10]

$$\theta_{h,pu} = \frac{e^{\frac{T_{pu} + \phi_{pu}}{\lambda_{h,i,j}}}}{e^{\frac{T_{pu} + \phi_{pu}}{\lambda_{h,i,j}}} + e^{\frac{T_{pr}}{\lambda_{h,i,j}}}}, \tag{3.6}$$

and for private transport:

$$\theta_{h,pr} = \frac{e^{\frac{T_{pr}}{\lambda_{h,i,j}}}}{e^{\frac{T_{pu} + \phi_{pu}}{\lambda_{h,i,j}}} + e^{\frac{T_{pr}}{\lambda_{h,i,j}}}}. \tag{3.7}$$

3.2.3   The economic model

The economic model is formulated as an Arrow-Debreu model with firms and households who maximize their profits and utility respectively.

On the production side we have at every node identical firms who use either high- or low-skilled labor and capital as inputs for producing a single output Y_{hi} (see the left part of Figure 3.2). The index h denotes if the firm is a high- or low-skilled production sector is. We assume a Cobb-Douglas production function for which the substitution elasticity between capital and labor input is equal to one:[11]

$$Y_{s,j} = A_{h,j} K_{h,j}^{\theta_h^k} L_{h,j}^{1-\theta_h^k}, \tag{3.8}$$

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[10] For convenience we drop the indices for start and end nodes.

[11] The number of different firms at a node can be easily enlarged to reflect the actual situation in a network. The production technology is also easily replaced by another functional form and a more complex nesting structure.

where $A_{h,j}$ is the efficiency parameter, $K_{h,j}$ and $L_{h,j}$ are the capital and labor inputs. The value share of capital is denoted by θ_h^k.

Figure 3.2 – Production and utility function
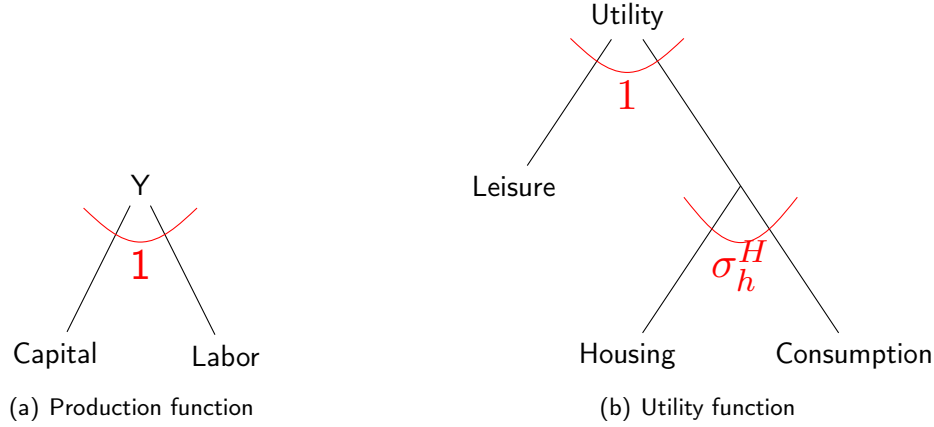
(a) Production function   (b) Utility function

Firms maximize their profits and the zero profit condition for production at node j can be formulated as follows:[12]

$$\left(\frac{W_{h,j}}{\overline{W}_h}\right)^{1-\theta_h^k} \left(\frac{R_{h,j}}{\overline{R}_{k,j}}\right)^{\theta_h^k} \geq pc_h \quad \perp \quad Y_{h,j}, \tag{3.9}$$

where $W_{h,j}$ is the wage for labor of household (skill) h; $R_{h,j}$ is the rental price of capital provided by household h at node j; θ^k is the value share of capital; pc_h and $Y_{h,j}$ are the price and the output level of the production sector.

Market clearing for labor demand is given by:

$$\sum_i \left(\overline{N}_{h,i,j} + N_{h,i,j}\right) = \left(1 - \theta_h^k\right) \frac{W_h Y_{h,j}}{W_{h,j}}, \tag{3.10}$$

where on the left-hand side we have the total labor supply from all households with skill h living at the nodes i and working in j. We assume that a part of the households are not willing to move ($\overline{N}_{h,i,j}$). For Switzerland the yearly cantonal mover rates range from 8 to 14% (ZKB-Immobilienresearch, 2011). For the USA the mover rate is around 12% (Ihrke, 2014). Statista (2014) gives estimates for the 10-year mover rate for Germany of almost 75%. As in our model not all movement can be explained, we arbitrarily set the mover rate to 40%.

As the capital and housing stock is fixed, the capital stock price index can be written as follows:

$$PK = \frac{\sum\limits_{h,i} PH_{h,i}\overline{HS}_{h,i} + \sum\limits_{h,j} RK_{h,j}\overline{KS}_{h,j}}{\sum\limits_{h,i} hs_{h,i} + \sum\limits_{h,i} ks_{h,i}}, \tag{3.11}$$

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[12] We use the calibrated share form for writing down most of the equations. This form, as developed by Rutherford (1998), allows the modeler to write down the zero-profit and market clearing conditions without the explicit need to calibrate the parameters of the underlying functional forms.

where $PH_{h,j}$ and $RK_{h,i}$ are the housing rental price according to skill segment of the housing market; $RK_{h,i}$ is the rental price of capital services at node i; $hs_{h,i}$ and $ks_{h,i}$ the capital stock available at node i.

The capital market clearing condition at node j is given by the following equation:

$$\overline{KS}_j = \sum_h \theta_h^k \frac{pc_h Y_{h,j}}{RK_{h,j}}, \tag{3.12}$$

where we have the capital supply on the left-hand side and the capital demand on the right-hand side of the equation. The associated free variable is $RK_{h,j}$.

We now turn to the household side of the economy. Households are characterized by the place where they live (i), the place where they work (j) and their skill (h). They maximize their utility level with respect to their income. The utility is given by a nested Constant-Elasticity-of-Substitution function (see equation (3.13) and Figure 3.2).

$$U_{h,i,j} = \left(\theta_h^{ls} \sum_m LS_{h,m,i,j}^{\rho_h} + \left(1 - \theta_h^{ls}\right) \left[\frac{INC_{h,i,j}}{\overline{INC}_h PC_{h,i}} \right]^{\rho_h} \right)^{\frac{1}{\rho_h}} + \xi_{h,i,j}, \tag{3.13}$$

where ρ_h is the elasticity parameter of the utility function between leisure and aggregate of consumption and housing for household group. The amenity for living at node i and working at node j is given by $\xi_{h,i,j}$. This factor reflects the unobserved attributes for the living-working combinations.

The housing market clearing at node i is given by:

$$hs_{h,i} = \theta_h^H \sum_j N_{h,i,j} INC_{h,i,j} \frac{PC_{h,i}^{\sigma_h^H - 1}}{PH_{h,i}^{\sigma_h^H}}, \tag{3.14}$$

where the value share of house rent is given by θ_h^H and σ_h^H is the substitution elasticity between housing and consumption. The price for the composite of housing and consumption $PCH_{h,i}$ is given by:

$$PCH_{h,i} = \left(\theta_h^H PH_{h,i}^{1-\sigma_h^H} + \left(1 - \theta_h^H\right) pc_h^{1-\sigma_h^H} \right)^{\frac{1}{1-\sigma_h^H}}, \tag{3.15}$$

The income of a household working in j with skill h is given by the wage income at j plus the capital income out of the capital ($\overline{K}$) and housing stock ($\overline{H}$). The fraction θ^d defines the share of local capital income.

$$INC_{h,i,j} = W_{h,j} + \frac{\overline{K}_{h,i} + \overline{H}_{h,i}}{n_h^{tot}} \left[\theta_h^d PK + \left(1 - \theta_h^d\right) \right] \tag{3.16}$$

where $INC_{h,i,j}$ is the associated free variable. Leisure supply is calibrated to unity when commute time equals $\overline{T}$:

$$LS_{h,m,i,j} = \frac{\overline{T}_{h,m}^{max} - T_{h,m,i,j}}{\overline{T}_{h,m}^{max} - \overline{T}_{h,m}} \tag{3.17}$$

3.2.4 Mode choice and sorting

A household has to decide where to live and work as well as which transport mode it uses for commuting. These decisions are taken simultaneously as the amount of leisure in the utility function depends on the commuting time, which depends on the choice of where to live and work, the mode and the route chosen. We assume that a household chooses a particular combination of living and working location if the utility of this combination is at least as large as the utility from any other combination. If this utility is higher than the actual utility, the household resorts to the new locations. That means that the combination of living and working location (i, j) is chosen if

$$U_h^* \geq U_{h,i,j}.$$

$$(3.18)$$

This equation is complementary to the number of people $(NN_{h,i,j})$ living at node i and working at node j. We further assume that the model is closed by fixing the total urban population:

$$\overline{N}_h \geq \sum_{i,j} NN_{h,i,j},$$

$$(3.19)$$

with the complementary variable U_h^*.

The share of the population for household h living at node i and working at node j choosing mode m can now be described by the following logit formulation:

$$\theta_{h,m,o,d} = \frac{e^{\frac{T_{h,m,o,d}+\phi_{h,m,o,d}}{\lambda_{h,m,o,d}}}}{\sum_{k,l} e^{\frac{T_{h,m,k,l}+\phi_{m,k,l}}{\lambda_{h,m,o,d}}}},$$

$$(3.20)$$

where the mode is given by $-\phi$ and the mean by $-\phi + \gamma\lambda$ (γ being the Euler constant) and the number of people using mode m for this location pair is:

$$N_{h,m,i,j} = \theta_{h,i,j}^m NN_{h,i,j}.$$

$$(3.21)$$

3.3 Calibration

A model is particularly useful for policy simulations if it can reproduce the observed values of the model variables. In this case, the observed variables include where people live and work, the time it takes them to travel to work, their wages and the prices for capital, housing and consumption. We solve the calibration of the model parameters in the following consecutive steps (see Figure 3.9) :

1. To get a good starting point for the calibration procedure, we solve the social optimum by minimizing total travel time. For this we use the OD-matrix with completely separated modes using equations (3.1) to (3.3) and the objective function $\sum_{h,a,k} \tau_a X_{h,a,k}$. We calibrate the BPR volume-delay functions to given travel times in a loop, where in each iteration the

model is solved and the BPR parameters are adjusted. This step is repeated until there is no significant change in the total travel time. This procedure is explained below.

2. In the next step, the user equilibrium is solved (equations (3.1) to (3.4)) using the calibrated BPR functions from the previous step, once again with completely separated modes and the calibration procedure for the BPR functions is repeated.

3. In this step, the information on the travel times on the arcs from the previous step and the shares from the OD-matrix are taken to calibrate the logit function parameters to reflect the given mode shares using equations (3.6) and (3.7).

4. The trip times and information on the exogenously given prices are then used to calibrate equations for the production and demand side of the economic model (equations (3.8) - (3.12), and (3.14) - (3.17)).

5. With the information of the previous steps, the location amenity in the utility function is calibrated.

6. Up to here, all equations are calibrated and we check if the integrated model reproduces the benchmark data.
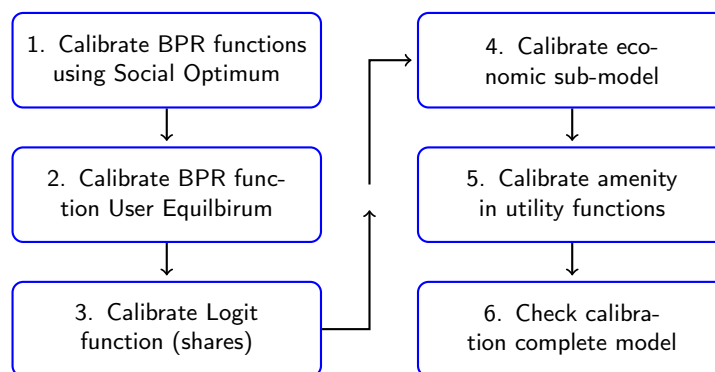
Figure 3.3 – Steps of the calibration.

We discuss the calibration step-by-step.

3.3.1 Calibration of the flow delay functions

In a first step, we calibrate the flow delay functions to the trip times we might have from a transport model. We assume that this model provides us with the following information on both public and private transport: The OD-matrix with the number of people traveling between the different nodes, the distances traveled between the OD-pairs and the travel times between the OD-pairs.

We calibrate the BPR function for both transport modes with a heuristic algorithm, where we first solve the transport model with arbitrary, but have reasonable start values for the free flow time and

the multiplicative constant in the BPR function. We then approximate these values using flows on the arcs, where we assume that we have information on a specific combination of flow and travel time. For example, we might know that if the flow on the arc is also multiplied with m_F, the flow time is multiplied by m_τ. In the next step, we run the model with these new values. This procedure is repeated until the total time on all arcs does not change further anymore significantly. The recalculation of the parameters is based on the following system of equations with the two unknowns α and B. The first equation has on the left-hand side the reported flow time value from the transport model. On the right-hand side, we have the calculated flow time based on the flow solution from the previous solution. The second equation uses the assumption on the flow time, if the flow is m_F times as big:[13]

$$\overline{\tau} = \alpha + B \left(\frac{F}{\kappa} \right)^4$$

$$m_\tau \overline{\tau} = \alpha + B \left(\frac{m_F F}{\kappa} \right)^4.$$

Solving this pair of equations results in the following expressions for the free flow parameter and the multiplicative constant:

$$B = \overline{\tau} \left(\frac{m_\tau - 1}{m_F^4 - 1} \right) \left(\frac{\kappa}{F} \right)^4$$

$$\alpha = \overline{\tau} \left(\frac{m_F^4 - m_\tau}{m_F^4 - 1} \right).$$

The algorithm is first used for calibrating the BPR function for the social optimum and then for the user equilibrium.

3.3.2 Logit calibration of the transport choice

Given the travel times and the shares from the transport model, as well as elasticities which can be taken from the literature, we can calibrate the shares from the logit demand function using ϕ and λ_h as the free parameters.

The shares are given by:

$$\overline{\theta}_{pu,o,d} = \frac{e^{\frac{\overline{T}_{pu,o,d} + \phi_{h,o,d}}{\lambda_h}}}{e^{\frac{\overline{T}_{pu,o,d} + \phi_{h,o,d}}{\lambda_h}} + e^{\frac{\overline{T}_{pr,o,d}}{\lambda_h}}}, \tag{3.22}$$

and

$$\overline{\theta}_{pr,o,d} = \frac{e^{\frac{\overline{T}_{pr,o,d} + \phi_{h,o,d}}{\lambda_h}}}{e^{\frac{\overline{T}_{pu,o,d} + \phi_{h,o,d}}{\lambda_h}} + e^{\frac{\overline{T}_{pr,o,d}}{\lambda_h}}}. \tag{3.23}$$

Dividing both equations, taking the logarithm and solving for $\phi_{h,o,d}$ results in:

$$\phi_{h,o,d} = \overline{T}_{h,pu,o,d} - \overline{T}_{h,pr,o,d} + \frac{1}{\lambda_{h,i,j}} \log \frac{\overline{\theta}_{h,pu,o,d}}{\overline{\theta}_{h,pr,o,d}} \tag{3.24}$$

---

[13] For convenience, we drop the arc and mode subscripts.

The own elasticities for mode m are given by:

$$\eta_{h,m,o,d} = -\frac{1}{\lambda_{h,i,j}} \left(1 - \overline{\theta}_{h,m,o,d}\right) \overline{T}_{h,m,o,d}. \tag{3.25}$$

Using these last equations, we can solve for $\eta_{h,m,o,d}$, $\phi_{h,o,d}$ and $\lambda_{h,i,j}$ by minimizing the deviation from known elasticity values.

3.3.3 Calibration of the economic model

In the second step we calibrate the economic sub-model using the shortest commuting time calculated in the first step. We calibrate preferences to a reference point for a given leisure value share and aggregate value of expenditure. We introduce two elasticity parameters describing the elasticity of substitution between leisure and expenditure (σ_{ls}) and the elasticity of substitution between housing and consumption (σ_h).

We adopt the Harberger units (Harberger, 1962), which allows us to express quantities in efficiency units at given prices.[14] We first calibrate the production sector to the given wages for the household groups and capital service costs equal to unity.

Labor demand at node j for skilled or unskilled labor can be taken from the OD-Matrix by summing over all modes and origins:

$$\overline{LD}_{h,j} = \sum_{h,i} N_{h,m,i,j}. \tag{3.26}$$

We calibrate technologies such that the rental rate R_k is equal to 1. Information on the reference capital value shares for the several production sectors (θ_h^k) can be obtained from other studies or from the statistical office. As we assume that there are no intermediate goods in production, the value of capital is equal to the capital value share times the output value. The aggregate value of output is given by the product of employment demand and the benchmark wages $\overline{PL}_h$ divided by the labor value share in production ($1 - \theta_h^k$). Capital stock in efficiency units is now given by:

$$\overline{K}_{h,i} = \theta_h^k \left(\frac{\overline{w}_h \overline{LD}_{h,i}}{1 - \theta_h^k}\right). \tag{3.27}$$

The imputed value of value added ($\overline{KL}_{h,i}$), capital endowment ($\overline{K}_{h,i}$) and the housing stock $\overline{H}_{h,i}$ are given by:

$$\overline{KL}_{h,i} = \frac{\sum_{j,m} \overline{W}_h N_{h,m,i,j}}{1 - \theta_h^k} \tag{3.28}$$

$$\overline{K}_{h,i} = \delta_h \overline{KL}_{h,i} \tag{3.29}$$

$$\overline{H}_{h,i} = \frac{\theta_h^h \overline{KL}_{h,i}}{1 - \theta_h^h}. \tag{3.30}$$

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[14] If we for example assume a price of one dollar for labor, the actual unit of labor is the amount that earns or costs one dollar.

The efficiency parameter of the production function can now be inferred by:

$$A_{s,i} = \frac{(\overline{K}_{s,i} + \overline{W}_s \overline{LD}_{s,i})}{K_{js}^{\theta_s^k} LD_{js}^{1-\theta_s^k}} \tag{3.31}$$

The housing stock at node i equals the value of market expenditure times the housing value share δ_h. The value of market expenditure net housing equals the value of labor and capital earnings. We can infer this income from the value of the labor supply divided by the labor share of output.

As we assume that not all capital and housing are owned by the local households, the income of the households is calibrated using shares of total local capital and housing stock owned by the residents:

$$\overline{K}_{h,i}^H = \theta_{h,i}^H \sum_i \overline{K}_{h,i} \tag{3.32}$$

and

$$\overline{H}_{h,i}^H = \theta_{h,i}^H \sum_i \overline{H}_{h,i}. \tag{3.33}$$

Reference per-capita expenditure for housing plus other goods equals the value of labor income, capital income and housing rental income:

$$\overline{INC}_h = \frac{\sum_i \left(\overline{K}_{h,i}^H + \overline{L}_{h,i} + \overline{H}_{h,i}^H \right)}{\sum_{i,j} N_{h,m,i,j}}. \tag{3.34}$$

We assume that the maximum possible commute is two times the longest commute observed in the benchmark traffic flows:

$$\overline{T}_{h,m}^{max} = 2 \max_{i,j} T_{h,m,i,j} \tag{3.35}$$

The commuter-weighted average commute time can then be used to anchor the utility function:

$$\overline{T}_{h,m}^{max} = \frac{\sum_{i,j} N_{h,i,j} T_{h,i,j}}{\sum_{i,j} \overline{N}_{h,m,i,j}} N_{h,i,j} \tag{3.36}$$

As we assume that in the benchmark situation no household has an incentive to change its living and working places (a spatial equilibrium), we can calculate the amenity factor. Households only differ with respect to their commute time to work, which is reflected in the leisure of the utility function. Therefore, without the amenity, a household which has a longer commuting time has a lower utility level then a household with a shorter commuting time. As in the spatial equilibrium, every household group has the same utility level hence we can infer the differences in the amenity parameter for the different households. If we assume that the overall utility level for each income group is equal to the maximum of the calculated utility levels without the amenity for that group

the amenity factor is given by:[15]

$$\xi_{h,i,j} = U_h^{max} - \left(\theta_h^{ls} \sum_m LS_{h,m,i,j}^{\rho_h} + \left(1 - \theta_h^{ls}\right) \left[\frac{INC_{h,j}}{\overline{INC}_h PC_{h,i}} \right]^{\rho_h} \right)^{\frac{1}{\rho_h}} \qquad (3.37)$$

3.4 Simulations

3.4.1 The City of Zurich as a numerical example

We use a stylized version of the network of Zurich as an illustration for the integrated model. Over the next ten years Zurich faces substantial changes in transport, housing infrastructure and zoning regulation, as well as changes in population and employment.[16] These changes not only have their impact on the locational decisions of households and firms, but also on wages, housing prices and the environment. Moreover, these changes also have their impact on the traffic situation. In transport, Zurich has committed itself to the strategy "Urban transport 2025" (Stadt Zürich, 2014) which aims among other things, at an increase of use of public transport, stabilizing the individual motorized transport, and reducing the energy use and CO_2 emissions in transport.

Zurich has a resident population of almost 400,000 people and more than one million people live in the Zurich agglomeration. It is divided into 12 districts and 34 neighborhoods and borders the Lake of Zurich in the south (see Figure 3.4). Zurich is of great economic importance for Switzerland: It has 11% of the total jobs in Switzerland and almost 40% of the Swiss banking institutes (Statistik Stadt Zürich, 2013; Stadt Zürich, 2012). About 14% of the total area of Zurich (92 square kilometers) is traffic related. Of this traffic, 55% is work related. Tram and bus services handle almost 60% of the overall distances traveled (20.5 per cent for Switzerland as a whole). On almost 300 kilometers of public transport about 300 million passenger trips take place per year (Statistik Stadt Zürich, 2013, Chapter 11, Section 3). Zurich is the most congested city in Zurich in which traffic jams on workdays mainly form along the entry routes (Hürzeler, 2014).

We use data on public and private transport modes for the year 2007 for the time between 7 and 8 in the morning from the Traffic model for the Canton of Zurich.[17] Data is aggregated at the district level. We have added three nodes outside of the city (North, West and East) reflecting the people commuting into and out off the city. There are 136 directed arcs that build the connections between the centroids of the districts (see Figure 3.4). The length of the links correspond to distances between the original nodes of the transport model which are weighted with the number of travelers.

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[15] In a more realistic model, one would estimate the amenity level (see for example Klaiber and Phaneuf, 2010).

[16] According to Statistik Stadt Zürich (2013) the population increases with around 10% by the year 2025.

[17] For a short description (in German) of the model see Amt für Verkehr Volkswirtschaftsdepartement Kanton Zürich (2011). The OD-matrices can be found in Appendix 3.A.
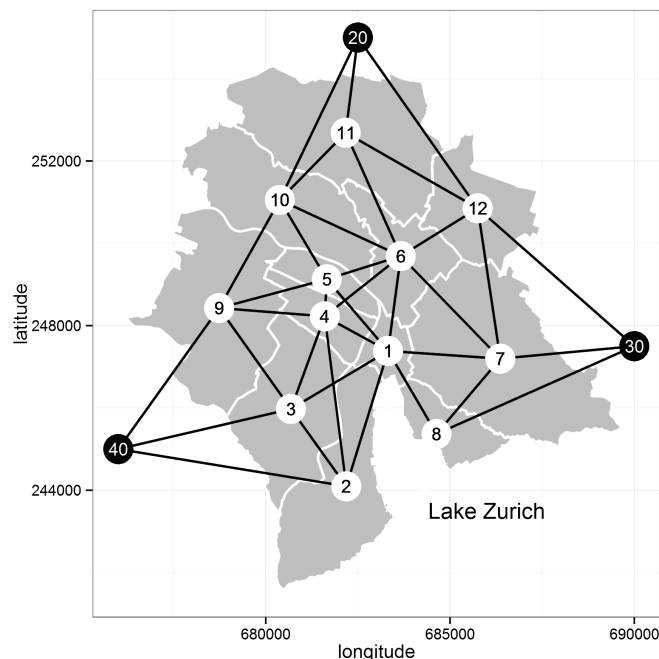
Figure 3.4 – The 12 districts and the representative nodes outside the city and the arcs between the nodes. [18]

We assume that all traffic is work-related with 70% public and 30% private transport. We make a distinction between high- and low-income groups. Matrices for both household groups can be found in Annex 3.A, Tables 3.12 and 3.15. For the two modes there is total of 816 OD-pairs.

Figure 3.5 shows the relative population density (left-hand side) and the employment density in the districts based on the OD-matrices. We can see the population density of the higher income group is higher in the south-eastern region and the density of the low-income group in the north-western part of the city. The number of people working is highest in the center of the city (where all banking institutes are), in the district to the west (machine industry) and in the northern district (education).

Table 3.1 gives an overview on the values for the most important model parameters for the household groups (elasticities, shares and wages). These values are not based on econometric estimates from the literature, but are chosen in such a way that there is a clear distinction in the characteristics of both household groups. The valuation of time for commute trips is set to half the (endogenous) wage rate (see Small, 2012, p. 16).

Table 3.2 shows some relevant economic data for the household groups based on the chosen economic parameters and the assumption that the average income of the high-skilled households is 100,000 CHF. The yearly income of the low-skilled group is 62532 CHF. The monthly housing rent (or if owned, the housing costs) for the high-skilled household is 2451 CHF and 1766 CHF

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[18] The coordinate system used is based on the old Swiss reference system CH1903 introduced in 1903 (Federal Office of Topography (Swisstopo), 2008). The system comprises the definition of a reference ellipsoid (Bessel, 1841) fixed in position and orientation to the old observatory in Bern (long=0m,lat=0m).
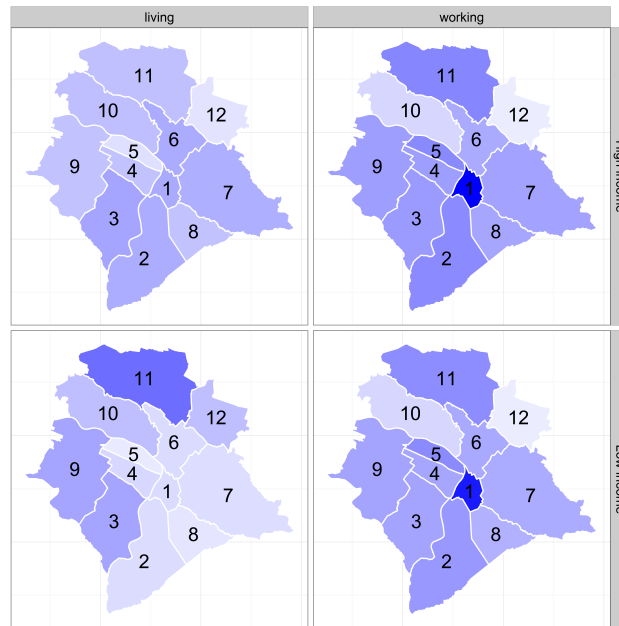
Figure 3.5 – Living (left) and working (right) density in districts of Zurich. The darker the area, the higher the density.

Parameter	Symbol	High income	Low income
Leisure value share	θ_h^{ls}	0.10	0.10
Housing value share	θ_h^h	0.29	0.34
Capital value share	θ_h^k	0.10	0.05
Wage	w_h	2.00	1.25
Elasticity of leisure	σ_h^{ls}	1.25	2.00
Elasticity of housing demand	σ_h^h	0.30	0.50

Table 3.1 – Parameter values for high and low skilled households.

for low-skilled household. This means that the income share spent on housing is 29% and 34%, respectively. Note that these figures are based on the number of people from the OD-matrix and do not count for people going to work earlier than 7 a.m. or later than 8 a.m., or people walking to their workplace. The OD-matrix only accounts for around a third of the total workforce.

Figure 3.6 shows the calibrated amenity levels for the utility function for both income groups. The longer the commute, the higher is the amenity level (the darker the shading).

The BPR-functions in the model are calibrated to the information from the Cantonal Transport Model. Only 36 of the total of 126 calibrated BPR-functions show a deviation from the given travel times. The maximum deviation for these delays is however only 0.51% of the actual travel time.[19] For the calibration of the shares and elasticities we assume for the low-income group an own-time elasticity of -1 and -1.5 for public and private transport, respectively. These values are multiplied

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[19] The travel times for the different OD-pairs can be found in Appendix 3.B.

Parameter	High income	Low income
Income	8333	5211
Housing rent	2451	1766
Consumption	5882	3445
Capital income	591	372
Wage income	7742	4839
Number of people	1	1

Table 3.2 – Economic indicators for household groups per capita and month in Swiss Francs (CHF).
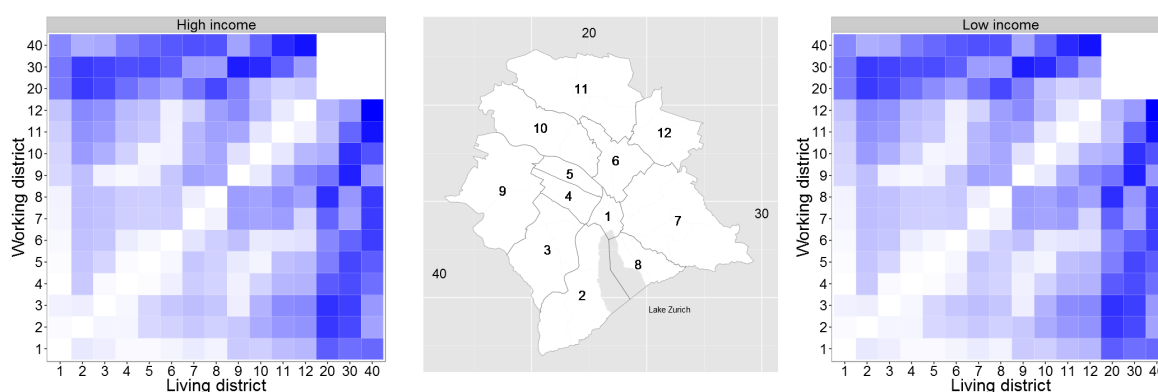
Figure 3.6 – Relative size of amenity factor (darker indicates a higher factor).

by 1.5 for the high-income households.[20]

Household	Elasticity	Mean	Median	Min	Max
high income	private	-1.33	-1.48	-1.65	-0.30
	public	-1.01	-1.04	-1.40	-0.05
	private-public	2.13	2.33	0.52	2.97
	public-private	0.62	0.65	0.03	1.11
low income	private	-1.99	-2.21	-2.48	-0.00
	public	-1.52	-1.55	-2.10	-0.00
	private-public	3.18	3.49	0.00	4.46
	public-private	0.93	0.98	0.00	1.67

Table 3.3 – Own- and cross-time elasticities.

Figure 3.7 and Table 3.3 show the distribution, mean, median, minimum and maximum of the own and cross time elasticities for the high- and low-income households.[21]

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[20] For an overview of existing estimates of transport elasticities, see Litman (2013).

[21] The own- price elasticities for all households according to their OD-combination can be found in Appendix 3.C.
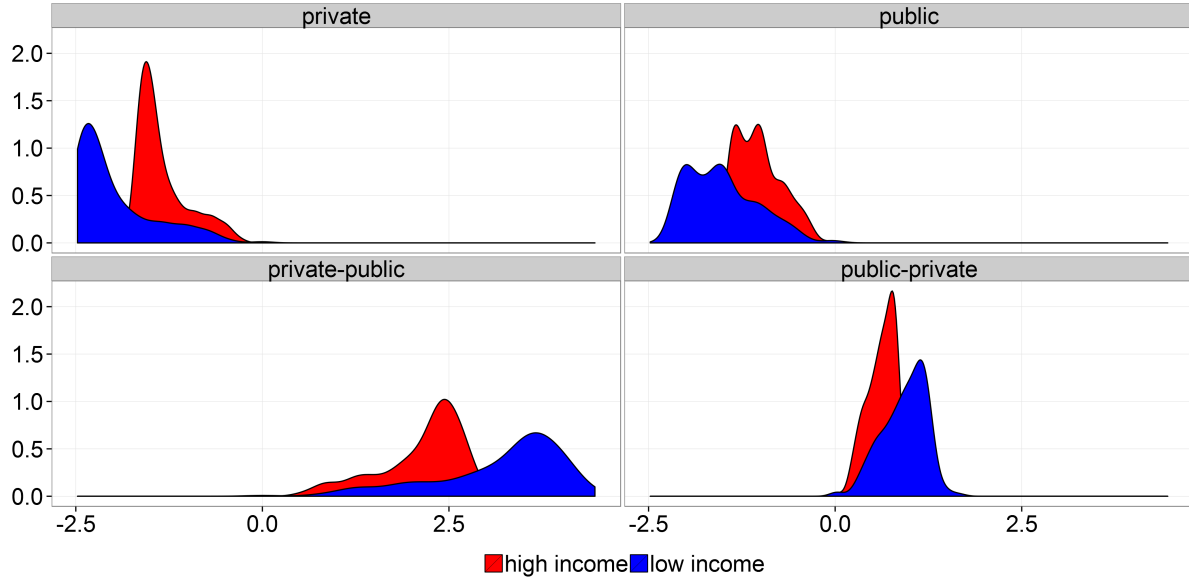
Figure 3.7 – Distribution of elasticities.

3.4.2 Scenarios

We simulate a reference scenario and two scenarios with a uniform cordon toll of 10 CHF (US $12-13) charged to vehicles assessing the cordon area between 7 a.m. and 8 a.m. The toll revenue is not redistributed or used for improvements in the transport structure. This allows us to focus on the pure effects of the toll. In the first scenario we introduce a cordon toll (CORDT) for the center of the city (districts 1, 4, 5, and 6). 25.3% percent of work-related commuting in the model takes place between the periphery and this center. The percentage of the active population living in this area is 9.3%. In the second scenario, we extend the perimeter of the cordon toll to districts 1-8 (greater cordon toll (GCORDT). The cordon region in this case covers 86% of all work-related commuting in the model. The share of the active population in this area is 42%. The perimeter of the cordon toll for both scenarios can be seen in Figure 3.8. All the results are compared with the results from the benchmark scenario (BENCHM) without a cordon toll.

The cordon toll changes the original arbitrage condition for choosing a link (Equation 3.4). The link is now only chosen if the sum of the time to travel this link, time from the end node of this link to the destination and the cost of the toll (expressed in minutes) is not greater than the fastest time from the start node of the link:

$$\tau_{i,j}^m + T_{h,j,k}^m + \frac{toll_{m,i,j}}{0.5W_{h,k}} \geq T_{h,i,k}^m \quad \text{and} \quad T_{h,k,k}^m = 0 \quad \forall (i,j) \in \mathcal{A}^m. \tag{3.38}$$

One problem with the arc-flow formulation in the integrated model is the calculation of the toll payments for each household. If a household has to pay the toll its income is reduced by this amount. However, in the arc flow formulation the route chosen by each household and therefore the toll payments are not known. We solve this problem by using the following algorithm (see Figure 3.9):
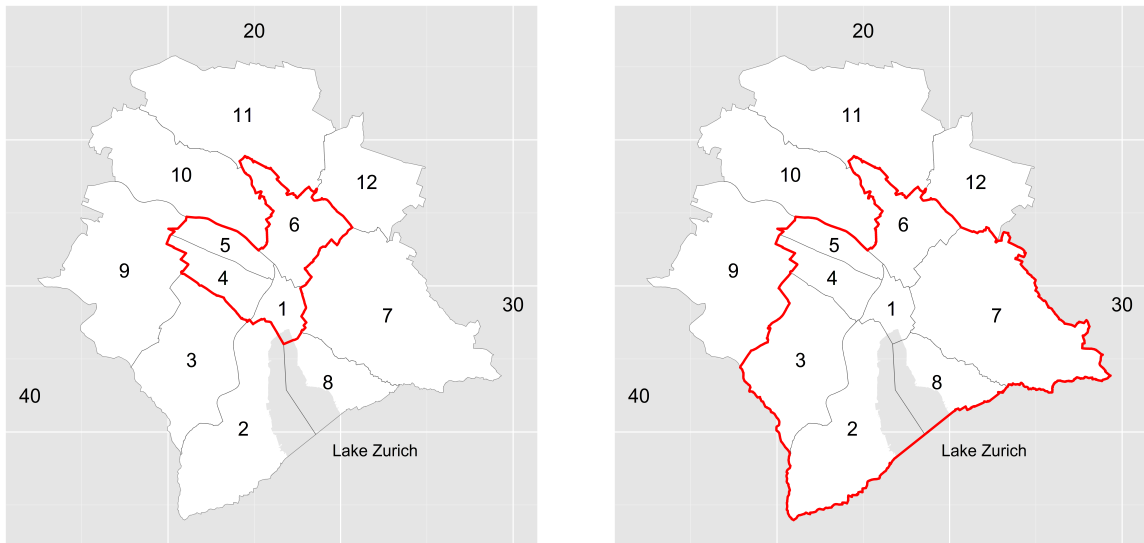
Figure 3.8 – Scenarios: Cordon toll in center (left) and greater center (right).

1. Toll payments for the households are set to zero.

2. The integrated model is solved for the scenario.

3. Using a linear programming model all chosen routes are calculated.

4. In knowing the routes the toll payments can be assigned to the households and the tax income is adjusted.

5. If the absolute value of the difference between the sum over all incomes before and after the toll assignment is above a certain threshold (1E-7) then steps 2-4 are repeated. If the difference is below the threshold then the algorithm stops.

Note that we only need to find the actual arcs chosen given the minimal time, because this is a significantly smaller problem and easier to solve than the traditional complete enumeration of all paths. The linear program for finding the arcs that belong to the routes for a household living at o and working at d has three equations. The flow balance equation is given by:

$$\sum_{a_{m,i,j}} \hat{X}_{m,i,j} = \sum_{a(m,j,i)} \hat{X}_{m,j,i} + 1_{\text{if } i=o} \quad \forall i \neq d, \tag{3.39}$$

where $\hat{X}$ is a variable that is 1 if the arc is used and 0 if not used. The (generalized) time for traveling from o to d is taken from the solution of the integrated model and should be equal to the sum of the time costs on the arcs:

$$\overline{T}_{h,m,o,d} = \sum_{(i,j) \in \mathcal{A}} \left(\frac{toll_{m,i,j}}{W_{h,d}} + \tau_{m,i,j} \right) \hat{X}_{m,i,j} \tag{3.40}$$

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The minimization or maximization objective function can be arbitrarily chosen. This procedure is used for every OD-pair and mode.[22]
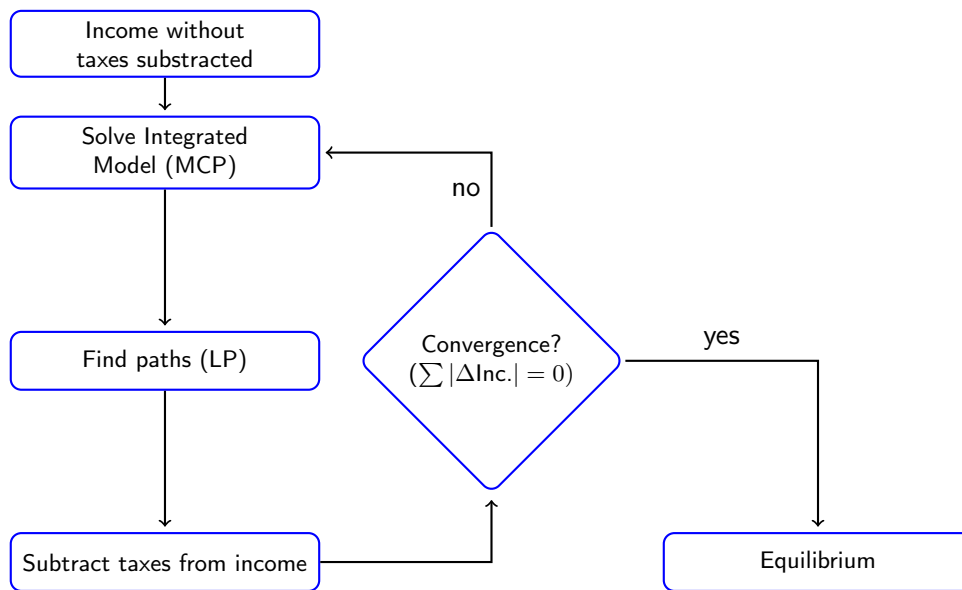
Figure 3.9 – Algorithm for including income effects.

3.4.3 Transportation and sorting effects

In this section we look at the overall results with respect to changes in total time, distances traveled and mode share. One of the advantages of the formulation of the integrated model as a MCP, is that we can easily check the model for model features that might drive the results. In order to understand the (partly counter intuitive) results, we also use a simpler version of the integrated model, where we disable the possibility of sorting and fix all economic variables. These changes reduce the integrated model to a pure traffic assignment (TAP) model (i.e., with scenarios of transport cordon toll (CORDT) and transport greater cordon toll (GCORDT)).

As will be seen, the overall results in the integrated model have little difference from the results in the transport assignment version of the model. However, there is a significant difference in the overall results with respect to how the different households react and the changes in average travel time. We will see that this difference is due to differences at the node level. Table 3.4 shows the overall results for both scenarios and model versions with heterogeneous agents.

In both versions of the model, it can be seen that total time as well as total distance traveled increase in both scenarios. Although the cordon toll reduces the total demand for private transport in all

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[22] We use the powerful CPLEX optimizers which run under GAMS. CPLEX optimizers are designed to solve large, difficult linear, quadratically constrained and mixed integer programming problems quickly and with minimal user intervention. For more than 800 combinations the algorithm takes less than a minute on a computer with an Intel(R) Core(TM) i7-3520CM CPU (2.90GHz) processor.

Parameter	TAP model		Integrated model	
	TCORDT	TGCORDT	CORDT	GCORDT
Total time	0.43%	0.39%	0.12%	0.46%
Time - public transport	2.58%	2.48%	2.48%	2.48%
Time - private transport	-5.22%	-5.07%	-6.04%	-4.82%
Average travel time	0.43%	0.39%	0.12%	0.46%
Average travel time - public	0.00%	0.41%	2.04%	1.45%
Average travel time - private	-0.71%	-1.52%	-5.34%	-3.11%
Total distance	0.20%	0.26%	0.50%	0.33%
Distance - public transport	2.00%	2.30%	2.30%	2.30%
Distance - private transport	-3.21%	-3.60%	-2.91%	-3.40%
Private transport share - high income (%-points)	-0.76%	-0.39%	0.83%	0.59%
Private transport share - low income (%-points)	-2.60%	-2.30%	-1.45%	-1.96%

Table 3.4 – Overall results for TAP and integrated model.

scenarios, this reduction is more than counterbalanced by an increase for demand in public transport (note that almost 70% of the people are using public transport). In the transport model, results with regard to reactions of the households seem at first as expected: Households with low income, which have higher cross-time elasticities, reduce their use of private transport more than the high-income households with the lower cross-time elasticities (-0.76% and -0.39% respectively). In the integrated model however the private transport share increases for the high-income households. As the low-income group switches to public transport, travel time is reduced, inducing the high-income household group to increase the demand for private transport.

Another important difference between the two models is the changes in the average travel time for both modes. In the transport version of the model, the reduction of average private travel time in both scenarios is reduced by -0.71% (TCORDT) and -1.52% (GCORDT). In the integrated model, these reductions are much higher: -5.34% (CORDT) and -3.11% (GCORDT). As public transport is near capacity, an increase in demand causes an increase in travel time everywhere.

Results for both scenarios in the transport model are almost the same. Increasing the perimeter of the cordon toll in the transport model does not have a significant impact on the overall results. In the integrated model, both scenarios differ in the reduction of the distance traveled with private transport and the average time taken with private transport (a difference of 0.49% and 1.22%, respectively).

At a more detailed level, there are more differences. Table 3.5 contains the percentage point changes in the private transport share for traffic in the cordon and the periphery as well as the private transport share for traffic between those two regions. In both models the changes are more pronounced in the scenario with the smaller cordon. In both model versions there are more possibilities to circumnavigate the cordon than in the scenario with the greater cordon. The results for the integrated model are more pronounced, because sorting plays an important role. As expected, the private transport share of people traveling into the cordon decreases in all scenarios and in both

Household	Direction	TCORDT	TGCORDT	CORDT	GCORDT
high income	Cordon -> Cordon	1.17%	0.61%	8.31%	5.18%
low income		1.74%	0.92%	7.92%	6.74%
high income	Cordon -> Outside Cordon	0.75%	0.24%	0.44%	0.24%
low income		1.12%	0.33%	0.65%	0.33%
high income	Outside Cordon -> Cordon	-2.21%	-1.30%	-1.92%	-3.70%
low income		-5.94%	-4.60%	-3.61%	-7.40%
high income	Outside Crd. -> Outside Crd.	-0.36%	-0.10%	-0.87%	-2.79%
low income		-1.13%	-0.47%	-2.69%	-3.61%

Table 3.5 – Percentage point change in share private mode.

models. The reduction in use is higher for the low-income group as for the high-income group, as the relative costs of the cordon toll are higher. Within the cordon the share of private transport increases. As less people use private transport to enter the cordon the congestion is reduced which makes the use of private transport within the cordon more attractive.

At the cordon level all changes for both households of the share in private transport have the same sign. At the district level (see Table 3.6) one can see important differences between the changes in this share for high- and low-income households. The table shows the changes in private transport shares for incoming traffic (the "i" and "o" in parenthesis after the figures indicate if the node is within or outside the cordon). At the district level the changes for the household groups can have different signs. For some of the nodes, the change in transport shares have opposite signs. In most cases the reduction in use of private transport by low income groups which reduces the congestion has a positive effect on the use of the high income groups.

Node	Household	CORDT	GCORDT	Node	Household	CORDT	GCORDT
1	High inc.	-0.24% (i)	0.00% (i)	9	High inc.	-0.99% (o)	-1.78% (o)
	Low inc.	-0.92% (i)	-0.52% (i)		Low inc.	-2.53% (o)	-4.15% (o)
2	High inc.	3.31% (o)	0.87% (i)	10	High inc.	4.50% (o)	5.44% (o)
	Low inc.	-1.00% (o)	-3.55% (i)		Low inc.	3.43% (o)	-2.72% (o)
3	High inc.	-0.11% (o)	-1.04% (i)	11	High inc.	0.47% (o)	1.03% (o)
	Low inc.	-3.09% (o)	-4.39% (i)		Low inc.	-0.43% (o)	-0.98% (o)
4	High inc.	-1.19% (i)	-0.78% (i)	12	High inc.	3.73% (o)	4.75% (o)
	Low inc.	-1.51% (i)	-1.62% (i)		Low inc.	2.87% (o)	2.42% (o)
5	High inc.	3.55% (i)	4.22% (i)	20	High inc.	2.62% (o)	-0.35% (o)
	Low inc.	1.36% (i)	1.23% (i)		Low inc.	1.33% (o)	2.96% (o)
6	High inc.	-0.92% (i)	-0.80% (i)	30	High inc.	-3.54% (o)	-0.12% (o)
	Low inc.	-1.75% (i)	-1.97% (i)		Low inc.	1.03% (o)	3.15% (o)
7	High inc.	-2.13% (o)	-1.77% (i)	40	High inc.	-0.49% (o)	-1.91% (o)
	Low inc.	-3.59% (o)	-3.24% (i)		Low inc.	-0.90% (o)	0.60% (o)
8	High inc.	-0.46% (o)	-2.06% (i)				
	Low inc.	-2.49% (o)	-2.97% (i)				

(i): inside cordon, (o): outside cordon

Table 3.6 – Percentage point changes in share for incoming private traffic.

In the integrated model, the sorting plays an important role in explaining the results. In the integrated model people can change where they work and also where they live. As the cordon toll makes the combination of living outside and working inside less attractive, in the integrated model people can decide to move to working and living places both inside the cordon, both outside the cordon or living inside and working outside the cordon. As the cordon results in a reduction of travel times within it, the center of the city becomes relatively more attractive for living and working.

Parameter	Household	CORDT	GCORDT
Working inside	High income	-13.02%	-5.56%
	Low income	-10.71%	-6.80%
Working outside	High income	7.93%	10.25%
	Low income	6.61%	12.18%
Living inside	High income	1.36%	0.63%
	Low income	2.52%	2.01%
Living outside	High income	-0.29%	-0.40%
	Low income	-0.27%	-0.61%

Table 3.7 – Percentage change in working and living location.

Table 3.7 shows the changes in where people live and work. One can see that the number of people living within the cordon increases. This is in line with the findings of Anas (2012) and Brueckner (2014). The center however becomes less attractive for working in both scenarios. These changes can be explained by the changes in the economic variables.

3.4.4 Economic effects

Sorting is not only influenced by changes in travel costs, but also by changes in wages, rents, consumption prices and housing rental prices. These changes are the mid- to long-term effects of the cordon policy and are not captured by standard transport models.

As the living population in the center increases (as shown in Table 3.7), rents go up (as shown in Figure 3.10). Housing rents go up within the cordon as more people want to live in the center which in turn drives up the rents. Outside the cordon we see the opposite effect. The changes for the low-income groups are more pronounced as the sorting in this group is higher. In the greater cordon scenario the rents change more. Although the relative changes in living locations are smaller, the absolute number of households that change is higher.

The cordon toll also impacts wages: As the number of people willing to work in the center decreases, employment gets more scarce and wages go up (see Figure 3.11). Although the reduction in sorting is higher for the high-skilled than for the low-skilled workers, the wage for the low-skilled workers increases more. This is due to higher reduction in high-skilled production (see Table 3.8), making high-skilled workers relatively less scarce.

Sorting depends on two opposing forces: The first, the cordon toll makes it less attractive to live in the periphery and work in the center. The opposing second force is the change in the economic
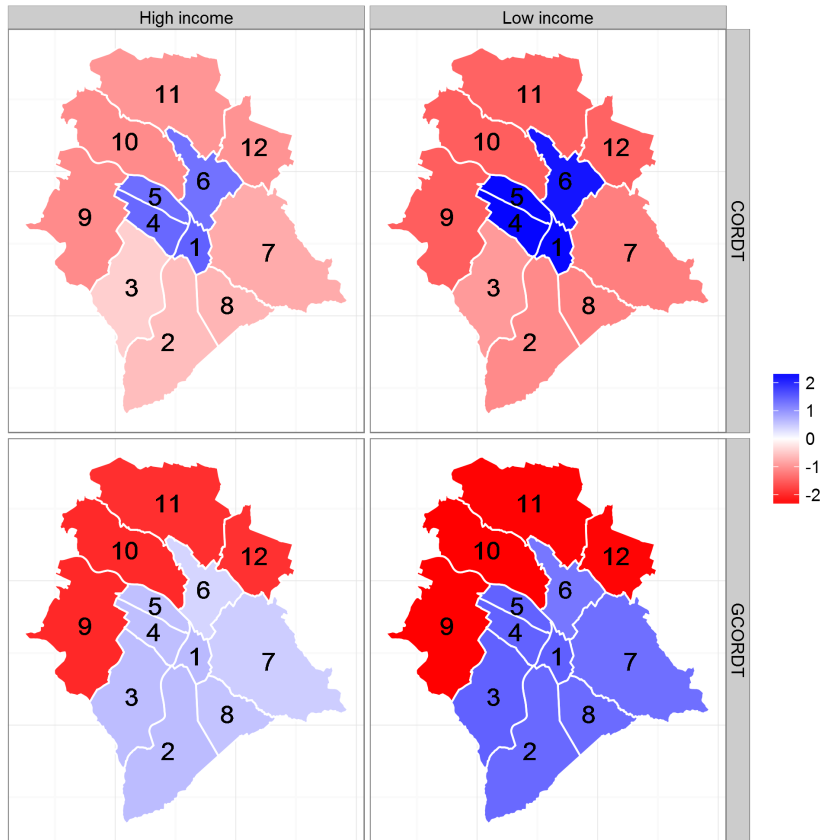
Figure 3.10 – Percentage changes in housing rents in both scenarios for high- and low-income households

variables. The effect based on changes in economic variables on the amenity of living in one district and working in another can be found by looking at the differences between the change in income (without toll costs) and cost of living (housing and consumption) for all combinations of living and working districts. Figure 3.12 shows for every combination the relative attractiveness, where blue color indicates a positive difference and red color indicates a negative difference. If this difference is positive, the amenity increases and people would tend to choose this combination above combinations with a lower difference. The decision on where to move for living and working therefore depends on the combination of cordon toll, changes in leisure time, housing rent, consumption prices and income. For example, decreasing housing rents in the periphery and increasing wages in the center are making this combination more attractive. However, households have to pay a cordon toll in this case and the net effect decides if people opt for this combination.

3.4.5 Tax revenues

Toll revenue for the greater cordon does not differ much in both models (0.82 million CHF for the transport and (0.83 million CHF for the integrated model). However, toll revenue differs
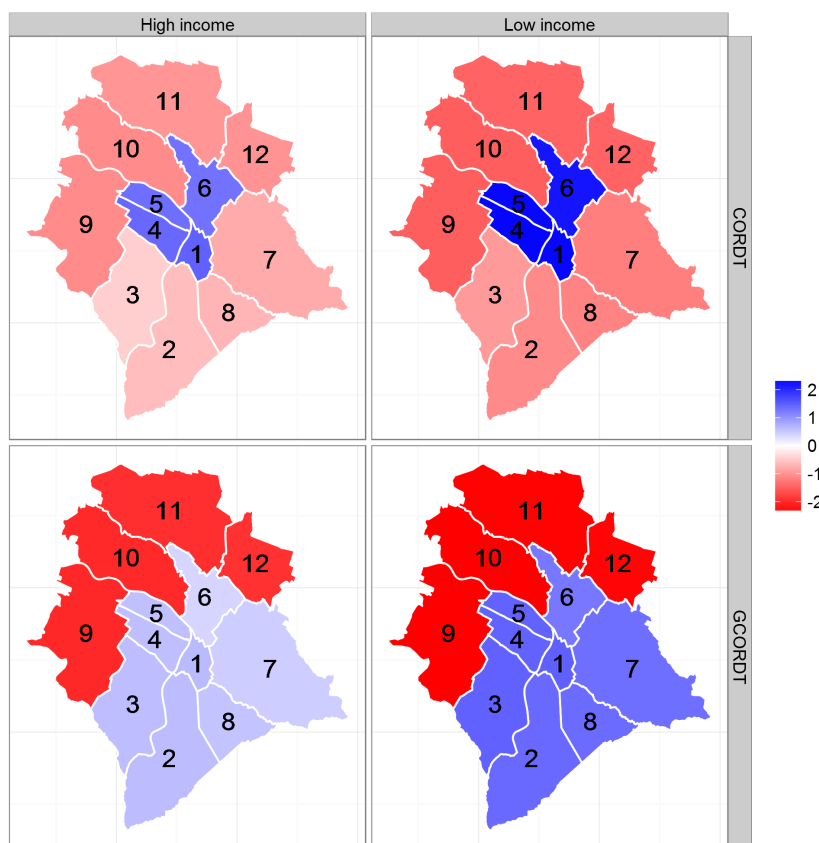
Figure 3.11 – Changes in wages (in % relative to benchmark) in both scenarios for high- and low-income households

significantly for the smaller cordon: In the integrated model the toll revenue is only 0.24 compared to 0.86 million CHF in the transport model. In the latter the difference is explained by the fact that it is easier for people to evade the toll by changing their living and working locations than it is in the second scenario.

3.4.6 Distributional effects

Distributional effects of road pricing policies can play an important role in the political process. We find that in the analyzed scenarios, there is a clear tendency of increasing the burden for agents with a lower valuation of time or income. Although this is known for situations where we have a social optimum (see for example Layard, 1977; Santos and Rojey, 2004), it is also known that this might not be the case for scenarios with non-optimal taxes. As Layard (1977) states that "... *this issue can only be settled by empirical work*".

We measure the incidence by the changes in utility. As sorting is incomplete, because of the assumption that a certain share of households does not change their living and working place, utility changes are not uniform. If everybody could sort, the spatial equilibrium with equal utility

Region	Skill	CORDT	GCORDT
Center	High skilled	-11.80%	-5.01%
	Low skilled	-8.66%	-5.48%
Outside Center	High skilled	7.11%	9.12%
	Low skilled	5.25%	9.63%

Table 3.8 – Changes in production.
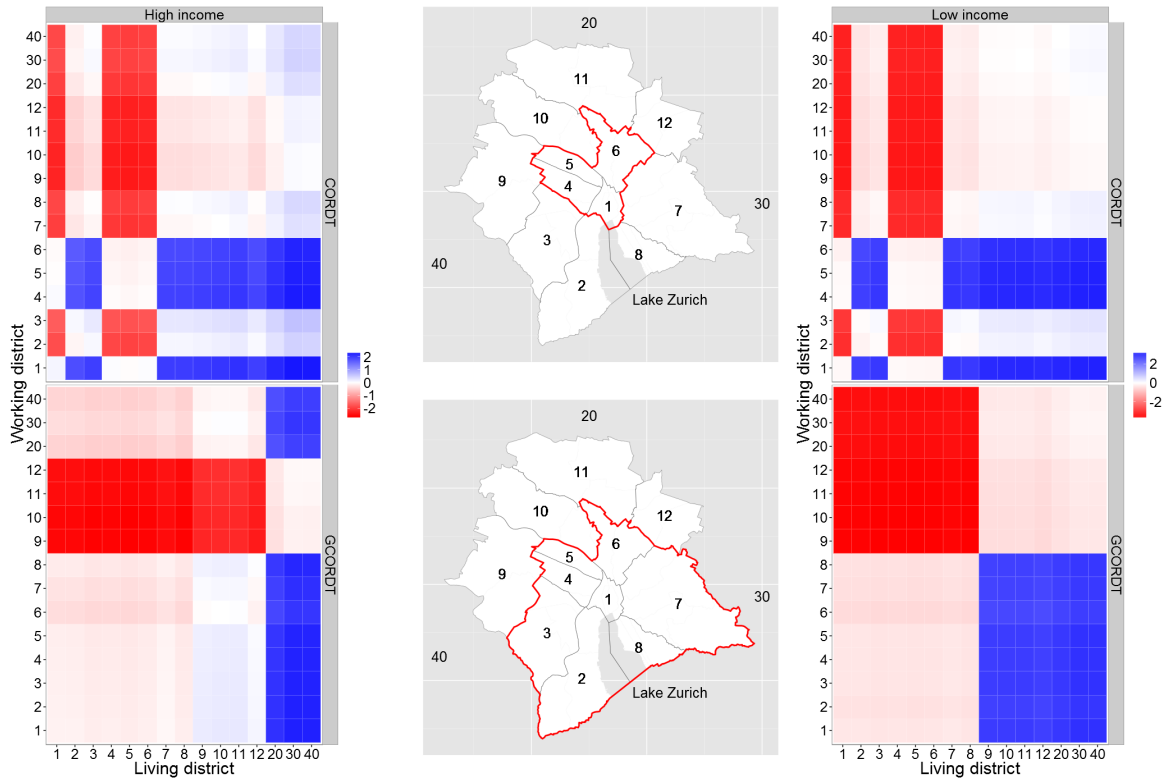
Figure 3.12 – Economic sorting: changes in attractiveness based on the difference between factor income and expenditure indices in both scenarios (left for high-income, right for low-income households).

for everybody would be the solution and the overall increase (decrease) in welfare would be the change in this uniform utility level.

Table 3.9 shows the changes aggregated at the cordon level. The policy is clearly regressive as the utility changes are more negative for the lower income group. There is no significant difference between the two household groups of people living in the periphery and working in the center. The highest losses are incurred for the households who live within the cordon and work outside of the cordon. Although these households do not have to pay the cordon toll, the increase in housing rents where they live and the decrease in wages where they work cause a reduction in utility that is higher than the combination of toll, higher wages and lower rents for people living outside and working in the center.

Does this mean that low-income groups will always fare worse? On average this is certainly the case

Direction	Household	CORDT	GCORDT
Cordon -> Cordon	High income	0.07%	-0.03%
	Low income	0.01%	-0.21%
Cordon -> Outside Cordon	High income	-1.02%	-0.54%
	Low income	-2.28%	-2.54%
Outside Cordon -> Cordon	High income	-0.02%	-0.22%
	Low income	-0.04%	-0.25%
Outside Cordon -> Outside Cordon	High income	-0.21%	-0.22%
	Low income	-0.55%	-0.52%

Table 3.9 – Changes in utility.

if we compare the weighted mean for the four groups arranged according to their living and working places with respect to the cordon. However, if we look at the distribution of the utility changes of all combinations of living and working places (see Figure 3.13), we see that the toll is only regressive on average. Some households with a low income fare better than households with a high income. Regarding the statistics on the distributions in many cases there is an overlap (see Tables 3.10 and
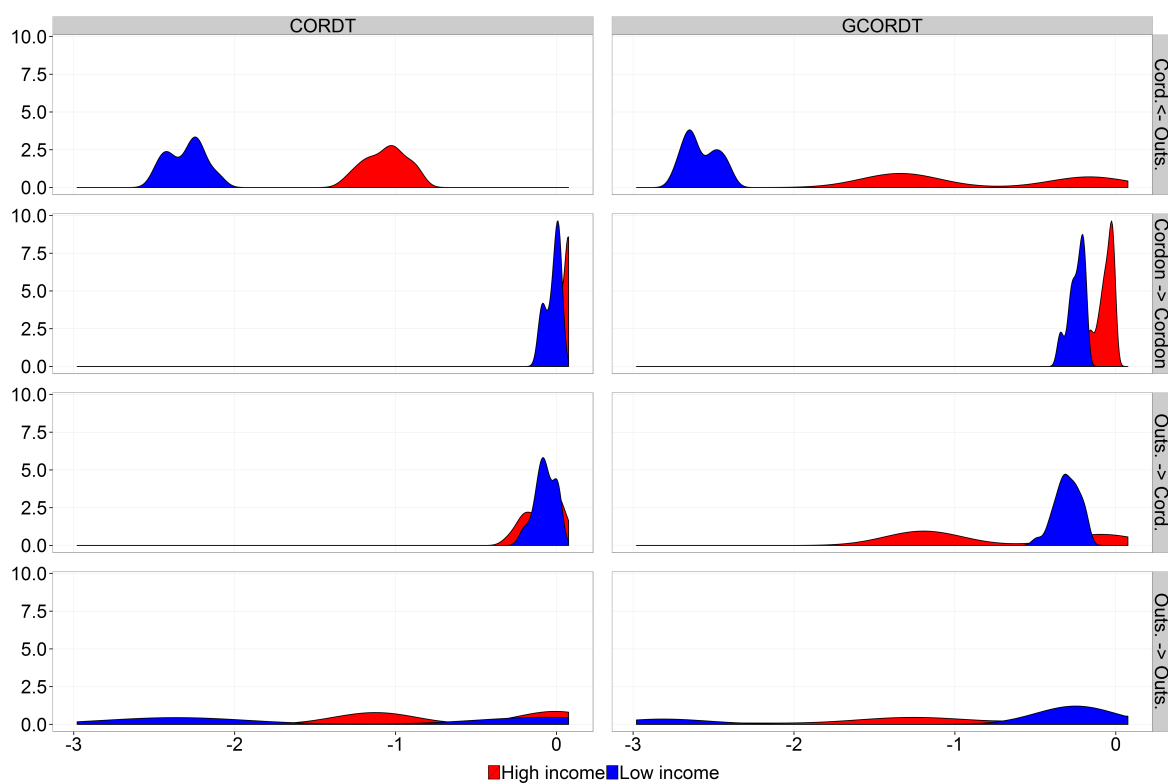
Figure 3.13 – Distributions of utility for both income groups and both scenarios.

3.11). These overlaps occur because the effects on leisure time, wages and housing rents are node specific. Figure 3.14 shows the changes in utility for the different living-working pairs and shows again that, although there is clear regressive tendency, this is not true for all households. Finally, it should be emphasized that distributional impacts could be altered if part of the tax revenue is recycled to the households.

Direction	Household	Min	Max	Mean	Median	sd
Cordon -> Cordon	High income	-0.03	0.08	0.04	0.05	0.04
	Low income	-0.10	0.02	-0.02	0.00	0.04
Cordon -> Outside Cordon	High income	-1.31	-0.86	-1.06	-1.05	0.13
	Low income	-2.50	-2.09	-2.30	-2.28	0.11
Outside Cordon -> Cordon	High income	-0.30	0.08	-0.08	-0.05	0.10
	Low income	-0.23	0.02	-0.07	-0.07	0.06
Outside Cordon -> Outside Cordon	High income	-1.50	0.08	-0.58	-0.56	0.57
	Low income	-2.77	0.02	-1.22	-1.22	1.16

Table 3.10 – Distribution information of utility changes for the scenario Center Cordon.

Direction	Hhn	Min	Max	Mean	Median	sd
Cordon -> Cordon	High income	-0.17	-0.02	-0.06	-0.05	0.05
	Low income	-0.35	-0.19	-0.24	-0.23	0.05
Cordon -> Outside Cordon	High income	-1.46	-0.08	-0.84	-1.27	0.59
	Low income	-2.74	-2.39	-2.58	-2.61	0.10
Outside Cordon -> Cordon	High income	-1.35	-0.02	-0.72	-1.07	0.55
	Low income	-0.49	-0.19	-0.30	-0.31	0.08
Outside Cordon -> Outside Cordon	High income	-2.55	-0.02	-0.75	-0.63	0.79
	Low income	-2.98	-0.19	-0.89	-0.30	1.12

Table 3.11 – Distribution information of utility changes for the scenario Greater Center.
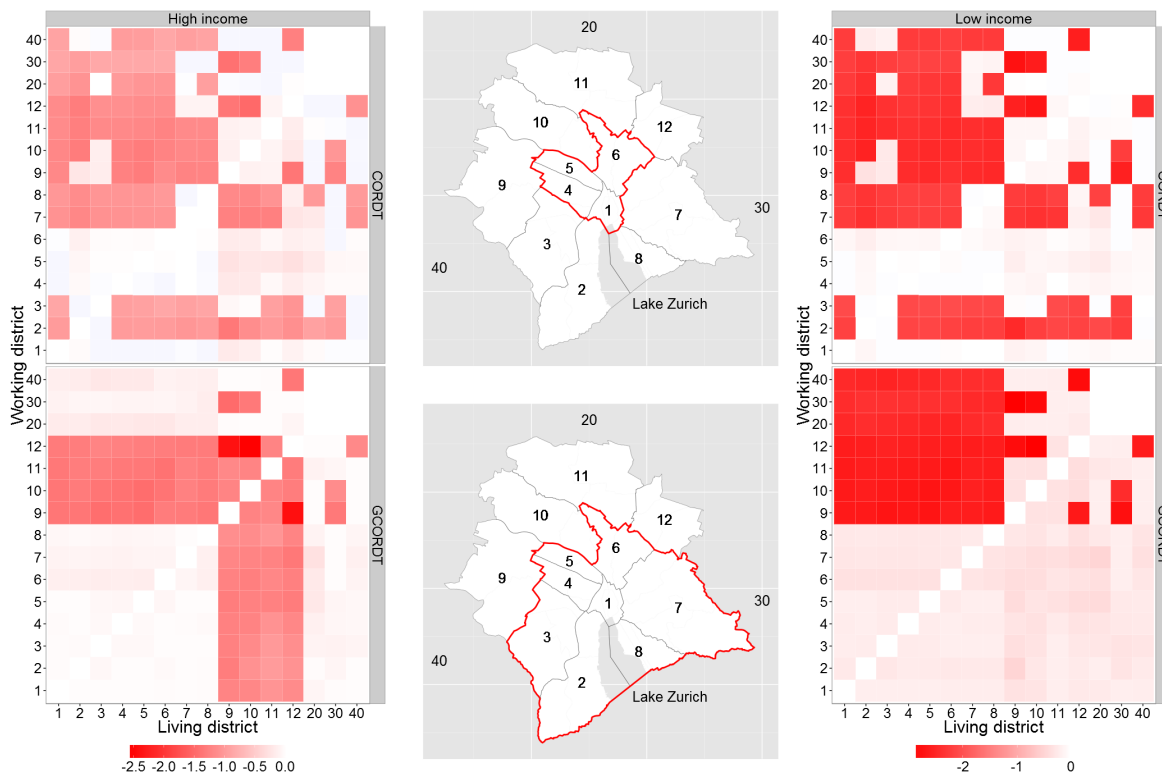
Figure 3.14 – Percentage changes in utility for both income groups.

3.5 Conclusions and further research

In this essay we developed an integrated model consisting of a transport assignment and a general equilibrium model. The model has a medium-term perspective by modeling equilibrium sorting of households on the road network. Households not only decide which route they take, but also have the possibility to change where they live and work, taking into account the prices for housing, their wage and commuting time. The model is cast in a complementarity framework that can be solved efficiently with commercial solvers. The complementarity format has as an additional advantage that the model can easily be extended in several directions which are listed below. We have also shown how to calibrate the model using several techniques to closely reproduce observed economic as well as transport data. We used stylized data for Zurich, the biggest city of Switzerland in order to run two cordon toll simulations and compared the results with those derived from a pure transport assignment model. The model has 136 directed arcs and more than 800 different households and solves four simulations in less than fives minutes on a business notebook computer.

The results show the importance of incorporating the market for housing and labor into the model. The comparison also shows that without a sorting mechanism, the effects in a pure transport model might differ significantly, especially at the disaggregated level. The model shows that sorting plays an important role for the changes in the choice of mode, wages and housing rental prices. It shows that, although the cordon toll has a negative impact on the share of private transportation by making working within the cordon less attractive, changes in wages and housing rental prices can outweigh these effects. The cordon toll leads to a reduction in overall travel time costs for both transport modes.

If the households are grouped with respect to living and working locations within or outside the cordon, welfare changes are on average negative for people not living and working in the cordon and households with low income suffer the most from the cordon toll. However, this is not always true at the household level. In some cases the burden for high-income households can be higher in some cases than for low-income households.

The economic changes for the scenario with the greater cordon toll are more pronounced, but the average utility changes do not differ much between both scenarios. Which scenario is better depends on what the goal of the cordon toll is. If a reduction of congestion in the center is the main goal, the first scenario might be more efficient. If, however, the primary goal of the cordon toll is to accrue funding for transport infrastructure (which is on the political agenda for Zurich), the greater cordon toll raises more tax revenue. Note that in the case of the transport model, the tax revenue is estimated to be much higher than in the integrated model.

Future research might explore several extensions of the model. One limitation of the current model is the exogenously-set level of housing. As Glaeser (2008) states, the three types of actors that are needed in any comprehensive urban model are employers, builders and ordinary people choosing between different locations. Therefore, we intend to add construction firms into future research and thus make the housing stock endogenous.

Another direction of research is the dynamics of the model. The current model is static, which means that the commuters do not have the possibility of choosing the time of departure. Adding multiple daily time periods or even endogenous departures would make the model more realistic.

We assumed that demand and cost functions are separable. We explicitly assume that the demand function for an OD-pair only depends on the travel rate of that OD-pair and the costs on an arc only depend on the flows on that arc. This assumption can be easily relaxed in the MCP formulation for further research.

The data we used are stylized. For example, we have a simple structure for the production in the city. Adding more sectors and more realistic economic and transport data is another direction of future research.

We have not tested this formulation with cities having more nodes and modes hence further research will also concentrate on checking if the model solves efficiently for greater networks with more nodes and modes.

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Nomenclature

Acronyms

BPR   Bureau of Public Research

MCP   Mixed Complementarity Problem

TEP   Transport Equilibrium Problem

Parameters

$\alpha_{m,i,j}$ Free flow time in volume-delay function on arc (i,j) for mode m

$\epsilon_{h,i,j}$ Idiosyncratic taste constant in utlity function of

        household h living at i and working at j

$\kappa_{m,i,j}$ Capacity of arc (i,j) of mode m

λ Scale parameter in utility function

$\overline{F}_{pb,i,j}$ Capacity constraint on arc (i,j) of public transport)

$\overline{H}_{h,i}$ Reference housing supply at node i for household group h

$\overline{INC}_h$ Reference income level for household group h

$\overline{KL}_{h,i}$ Reference value added at node i

$\overline{K}_{h,i}$ Capital of household h living at i

$\overline{K}_{h,i}$ Reference capital services supply at node i

$\overline{LD}$ Reference labor demand

$\overline{N}_{h,j,k}$ Number of agents of group h living at j and working at k

$\overline{pc}_h$ Price of consumption good produced with skill h

$\overline{T}_{h,m}^{max}$ Maximum of commuting time for household h using mode m

$\overline{U}_{h,i,j}$ Reference utility level of household with skill

        level h living at node i and working at node j

$\overline{w}_h$ Reference wage of labor with skill level h

$\phi_{h,i,j}$ Location parameter in utlity function of household h living at i and working at j

$\phi_{m,i,j}$ Average effect on utility of the factors that are not included

ρ_h Elasticity parameter utility function between leisure and aggregate

        of consumption and housing for household group h

σ_h^H Substitution elasticity between housing and consumption

θ^d Share of local capital income

θ_h^{ls} Leisure value share in utility function for household group h

θ_h^k Value share of capital in production with skill level k

θ_h^H Housing rent value share in utility

$\theta_{h,m,o,d}$ share of the population for household h living at node i

        and working at node j choosing mode m

$\xi_{h,i,j}$ Amenity factor for living at i and working at j

$A_{h,i}$ Efficiency parameter in production at node i

B	Multiplicative constant in volume-delay function
l	Lower bound
$toll_{m,i,j}$	Road toll for mode m on arc (i,j)
u	Upper bound
v_h	Valuation of time for agent with skill h

Sets

$\mathcal{A}^m$	Set of arcs for mode m
$\mathcal{D}$	Set of destinations
$\mathcal{N}$	Set of nodes
$\mathcal{OD}$	Set of pairs of destination and origin
$\mathcal{O}$	Set of origins
$\mathcal{R}^n$	n-dimensional set of real numbers
a	Arc identifier
d	Destination identifier
h	Household group or skill
i,j,k	Node identifier
m	Transport mode identifier
o	Origin and households identifier
pb, pr	Elements of mode set (public and private transport, respectively)

Variables

$NN_{h,m,i,j}$	Number of agents of group h living at i and working at j using mode m
$\mu_{i,j}$	Shadow price of capacity constraint on arc (i,j) of public transport
$\tau_{m,i,j}$	Delay on arc (i,j) using mode m
$F_{m,i,j}$	Flow on arc (i,j) of mode m
$INC_{h,i,j}$	Income level for household group h living at i and working at j
$K_{h,i}$	Capital input at node i in production
$L_{h,i}$	Labor input at node i in production
$LS_{h,i,j}$	Leisure demand of household h living at node i and working at node j
$N_{h,i,j}$	Number of agents of group h living at i and working at j
$PH_{h,i}$	Housing rental price at node i for household group h
PK	Capital stock price index
$R_{h,i}$	Rental price at node i in production h
$T_{h,m,j,k}$	Minimum time from node j to node k for household h using mode m
U_h^*	Reference utility for sorting
$U_{h,i,j}$	Utility level of household with skill level h living at node i and working at node j
$w_{h,i}$	Wage at node i of labor with skill level h
x	Unkonwn variable in description of MCP
$X_{h,m,i,j,k}$	Throughout flow on the arc (i,j) of agents h using mode m with destination k
$Y_{h,i}$	Production level at node i of skill level h
$PCH_{h,i}$	Price for the composite of housing and consumption for household group h living at i

Appendix 3.A   OD-Matrix

Public	1	2	3	4	5	6	7	8	9	10	11	12	20	30	40
1	0	745	455	362	205	199	398	474	268	90	241	19	280	423	763
2	1101	0	349	402	294	231	302	280	186	66	151	12	154	510	735
3	1301	373	0	689	688	393	322	363	583	185	343	28	323	928	815
4	752	259	367	0	348	209	194	256	318	115	188	16	183	419	450
5	359	149	195	169	0	99	82	91	177	87	123	11	126	262	256
6	886	368	278	269	276	0	340	250	214	172	409	37	352	443	587
7	1111	359	239	225	129	341	0	435	143	86	217	32	252	273	776
8	760	280	209	212	89	167	332	0	156	52	152	22	193	272	600
9	750	231	679	601	776	235	183	218	0	263	301	27	305	1077	470
10	644	232	308	248	565	392	203	141	448	0	392	36	366	761	462
11	979	389	437	369	506	746	410	310	510	378	0	174	1193	942	1167
12	305	132	92	89	105	368	182	98	108	67	459	0	519	295	879
20	1353	504	527	486	689	698	489	432	662	418	2511	340	0	0	0
30	3413	1419	1788	1664	3041	1269	856	769	2727	809	1514	150	0	0	0
40	6034	3418	1837	1806	1754	1634	2890	2664	1145	522	2581	617	0	0	0

Table 3.12 – OD-Matrix public transport.

Private	1	2	3	4	5	6	7	8	9	10	11	12	20	30	40
1	0	745	455	362	205	199	398	474	268	90	241	19	280	423	763
2	1101	0	349	402	294	231	302	280	186	66	151	12	154	510	735
3	1301	373	0	689	688	393	322	363	583	185	343	28	323	928	815
4	752	259	367	0	348	209	194	256	318	115	188	16	183	419	450
5	359	149	195	169	0	99	82	91	177	87	123	11	126	262	256
6	886	368	278	269	276	0	340	250	214	172	409	37	352	443	587
7	1111	359	239	225	129	341	0	435	143	86	217	32	252	273	776
8	760	280	209	212	89	167	332	0	156	52	152	22	193	272	600
9	750	231	679	601	776	235	183	218	0	263	301	27	305	1077	470
10	644	232	308	248	565	392	203	141	448	0	392	36	366	761	462
11	979	389	437	369	506	746	410	310	510	378	0	174	1193	942	1167
12	305	132	92	89	105	368	182	98	108	67	459	0	519	295	879
20	1353	504	527	486	689	698	489	432	662	418	2511	340	0	0	0
30	3413	1419	1788	1664	3041	1269	856	769	2727	809	1514	150	0	0	0
40	6034	3418	1837	1806	1754	1634	2890	2664	1145	522	2581	617	0	0	0

Table 3.13 – OD-Matrix private transport.

Appendix 3.B   Travel times for OD-pairs

Start	1	2	3	4	5	6	7	8	9	10	11	12	20	30	40
1	6.71	9.00	10.77	7.44	10.95	10.28	10.46	11.51	13.50	15.46	15.94	15.81	24.89	25.70	26.42
2	13.59	11.76	12.45	13.23	16.67	18.00	17.59	16.68	17.55	20.19	23.41	26.97	30.66	25.61	27.13
3	11.68	11.25	8.13	9.68	12.70	15.12	16.71	16.26	11.88	15.34	18.70	19.78	28.01	25.15	26.71
4	9.21	9.92	9.55	6.80	9.52	12.35	14.61	14.78	10.53	12.84	15.95	17.16	25.55	24.78	25.00
5	10.73	12.67	12.89	9.11	7.68	11.01	14.55	16.64	11.58	11.53	14.29	15.48	23.29	25.25	24.68
6	11.75	14.71	15.81	12.48	11.78	9.90	13.07	16.02	16.22	11.94	12.19	13.73	22.79	28.02	24.29
7	14.17	16.10	18.90	16.48	18.48	16.82	13.92	13.89	22.31	19.57	21.16	20.68	29.03	31.94	24.84
8	11.81	13.14	16.11	14.21	17.79	15.72	12.70	9.76	20.03	20.24	21.85	22.78	29.86	30.03	22.47
9	15.98	16.33	12.64	12.52	12.74	18.08	21.53	21.61	9.75	15.56	19.99	21.93	29.31	25.53	29.11
10	14.79	16.41	16.28	13.31	11.38	12.81	16.61	20.31	13.25	10.76	14.58	16.55	23.78	24.95	24.51
11	18.89	21.18	20.99	18.00	16.02	16.21	20.55	24.26	19.01	14.75	11.27	14.47	21.02	29.36	24.71
12	18.69	21.25	22.12	18.66	17.02	17.46	20.35	23.87	20.90	17.21	12.68	9.97	22.78	34.66	21.45
20	35.61	36.55	36.47	34.33	32.93	32.30	35.99	38.72	33.69	29.12	27.36	27.64			
30	32.48	28.61	29.07	29.11	28.38	30.49	33.50	33.69	27.34	26.21	31.79	33.79			
40	32.45	32.88	34.16	32.23	32.54	30.13	29.42	29.89	33.75	29.95	29.28	24.42			

Table 3.14 – Travel times private mode for OD pairs according to Cantonal Transport Model (in minutes).

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	1	2	3	4	5	6	7	8	9	10	11	12	20	30	40
1	6.71	9.00	10.77	7.44	10.95	10.28	10.46	11.51	13.50	15.46	15.94	15.81	24.89	25.70	26.42
2	13.59	11.76	12.45	13.23	16.67	18.00	17.59	16.68	17.55	20.19	23.41	26.97	30.66	25.61	27.13
3	11.68	11.25	8.13	9.68	12.70	15.12	16.71	16.26	11.88	15.34	18.70	19.78	28.01	25.15	26.71
4	9.21	9.92	9.55	6.80	9.52	12.35	14.61	14.78	10.53	12.84	15.95	17.16	25.55	24.78	25.00
5	10.73	12.67	12.89	9.11	7.68	11.01	14.55	16.64	11.58	11.53	14.29	15.48	23.29	25.25	24.68
6	11.75	14.71	15.81	12.48	11.78	9.90	13.07	16.02	16.22	11.94	12.19	13.73	22.79	28.02	24.29
7	14.17	16.10	18.90	16.48	18.48	16.82	13.92	13.89	22.31	19.57	21.16	20.68	29.03	31.94	24.84
8	11.81	13.14	16.11	14.21	17.79	15.72	12.70	9.76	20.03	20.24	21.85	22.78	29.86	30.03	22.47
9	15.98	16.33	12.64	12.52	12.74	18.08	21.53	21.61	9.75	15.56	19.99	21.93	29.31	25.53	29.11
10	14.79	16.41	16.28	13.31	11.38	12.81	16.61	20.31	13.25	10.76	14.58	16.55	23.78	24.95	24.51
11	18.89	21.18	20.99	18.00	16.02	16.21	20.55	24.26	19.01	14.75	11.27	14.47	21.02	29.36	24.71
12	18.69	21.25	22.12	18.66	17.02	17.46	20.35	23.87	20.90	17.21	12.68	9.97	22.78	34.66	21.45
20	35.61	36.55	36.47	34.33	32.93	32.30	35.99	38.72	33.69	29.12	27.36	27.64			
30	32.48	28.61	29.07	29.11	28.38	30.49	33.50	33.69	27.34	26.21	31.79	33.79			
40	32.45	32.88	34.16	32.23	32.54	30.13	29.42	29.89	33.75	29.95	29.28	24.42			

Table 3.15 – Travel times public mode for OD pairs according to Cantonal Transport Model (in minutes).

Appendix 3.C   Elasticities

	1	2	3	4	5	6	7	8	9	10	11	12	20	30	40
1	-1.50	-1.64	-1.47	-1.52	-0.66	-1.65	-1.65	-1.64	-1.50	-1.65	-1.51	-1.63	-0.55	-0.82	-0.89
2	-1.45	-1.50	-1.32	-1.38	-1.37	-1.64	-1.63	-1.53	-1.27	-1.65	-1.65	-1.53	-1.60	-0.93	-0.69
3	-1.32	-0.68	-1.50	-1.53	-1.55	-1.63	-1.64	-1.63	-1.24	-1.64	-1.59	-1.43	-1.29	-0.72	-1.14
4	-1.39	-1.25	-1.51	-1.50	-1.35	-1.65	-1.64	-1.65	-1.37	-1.64	-1.48	-1.22	-1.11	-0.87	-1.37
5	-1.35	-1.47	-1.61	-1.49	-1.50	-1.65	-1.46	-1.63	-1.16	-1.65	-1.54	-1.52	-1.13	-1.06	-1.45
6	-1.62	-1.65	-1.57	-1.58	-1.50	-1.50	-1.55	-1.63	-1.33	-1.63	-1.52	-0.46	-0.90	-1.18	-0.92
7	-1.58	-1.63	-1.42	-1.61	-1.07	-1.57	-1.50	-1.20	-1.63	-1.57	-1.20	-0.73	-0.55	-1.48	-0.69
8	-1.61	-1.61	-1.44	-1.65	-1.27	-1.63	-1.29	-1.50	-1.63	-1.63	-1.64	-1.63	-1.37	-1.65	-1.43
9	-1.43	-1.02	-1.33	-1.53	-1.29	-1.60	-1.63	-1.65	-1.50	-1.47	-1.21	-1.14	-0.50	-0.30	-0.95
10	-1.39	-1.64	-1.55	-1.36	-1.50	-1.64	-1.45	-1.59	-0.69	-1.50	-1.05	-0.74	-0.46	-0.47	-0.99
11	-1.61	-1.64	-1.65	-1.53	-1.35	-1.65	-1.57	-1.65	-1.06	-1.47	-1.50	-1.32	-0.45	-0.61	-0.67
12	-1.61	-1.64	-1.61	-1.45	-1.10	-1.64	-1.45	-1.57	-1.41	-1.33	-1.19	-1.50	-0.33	-0.49	-0.61
20	-1.38	-1.65	-1.49	-1.26	-0.88	-1.41	-0.84	-1.60	-0.64	-0.78	-0.88	-0.50	-1.50	-1.50	-1.50
30	-1.48	-1.22	-1.00	-1.33	-1.21	-1.65	-1.60	-1.64	-0.74	-0.88	-0.89	-0.89	-1.50	-1.50	-1.50
40	-1.45	-1.44	-1.14	-1.38	-1.47	-1.60	-1.17	-1.24	-1.48	-1.55	-1.02	-1.03	-1.50	-1.50	-1.50

Table 3.16 – Own time elasticities logit function private transport of high income agents.

	1	2	3	4	5	6	7	8	9	10	11	12	20	30	40
1	-2.25	-2.47	-2.21	-2.29	-1.00	-0.00	-2.47	-2.46	-2.25	-2.48	-2.27	-2.45	-0.83	-1.23	-1.33
2	-2.18	-2.25	-1.98	-2.07	-2.06	-2.46	-2.45	-2.30	-1.91	-2.48	-2.47	-2.30	-2.41	-1.39	-1.03
3	-1.99	-1.03	-2.25	-2.30	-2.33	-2.44	-2.47	-2.44	-1.86	-2.46	-2.39	-2.15	-1.94	-1.08	-1.71
4	-2.08	-1.88	-2.26	-2.25	-2.02	-2.47	-2.46	-2.47	-2.05	-2.45	-2.23	-1.83	-1.67	-1.30	-2.06
5	-2.02	-2.21	-2.41	-2.24	-2.25	-2.48	-2.19	-2.45	-1.75	-2.48	-2.31	-2.28	-1.70	-1.59	-2.18
6	-2.43	-2.47	-2.36	-2.37	-2.24	-2.25	-2.33	-2.44	-1.99	-2.44	-2.29	-0.69	-1.35	-1.76	-1.38
7	-2.37	-2.45	-2.12	-2.41	-1.61	-2.35	-2.25	-1.80	-2.44	-2.36	-1.80	-1.09	-0.82	-2.22	-1.04
8	-2.41	-2.42	-2.16	-2.48	-1.90	-2.44	-1.93	-2.25	-2.44	-2.44	-2.46	-2.44	-2.05	-2.48	-2.15
9	-2.15	-1.53	-1.99	-2.29	-1.93	-2.41	-2.45	-2.47	-2.25	-2.20	-1.81	-1.70	-0.74	-0.46	-1.42
10	-2.09	-2.47	-2.32	-2.04	-2.25	-2.46	-2.17	-2.39	-1.04	-2.25	-1.57	-1.10	-0.69	-0.70	-1.48
11	-2.42	-2.46	-2.47	-2.30	-2.03	-2.47	-2.36	-2.47	-1.59	-2.21	-2.25	-1.98	-0.68	-0.91	-1.00
12	-2.41	-2.46	-2.42	-2.17	-1.65	-2.46	-2.18	-2.35	-2.11	-2.00	-1.78	-2.25	-0.50	-0.74	-0.92
20	-2.07	-2.48	-2.24	-1.88	-1.32	-2.11	-1.26	-2.39	-0.96	-1.17	-1.31	-0.75	-2.25	-2.25	-2.25
30	-2.21	-1.84	-1.50	-1.99	-1.82	-2.48	-2.41	-2.46	-1.11	-1.33	-1.33	-1.34	-2.25	-2.25	-2.25
40	-2.18	-2.16	-1.71	-2.07	-2.20	-2.40	-1.76	-1.86	-2.22	-2.33	-1.52	-1.54	-2.25	-2.25	-2.25

Table 3.17 – Own time elasticities logit function private transport of low income agents.

	1	2	3	4	5	6	7	8	9	10	11	12	20	30	40
1	-1.00	-0.62	-1.04	-0.96	-1.40	-0.46	-0.59	-0.63	-1.00	-0.46	-0.98	-0.33	-1.38	-1.40	-1.39
2	-1.06	-1.00	-1.20	-1.15	-1.15	-0.35	-0.69	-0.94	-1.23	-0.52	-0.57	-0.05	-0.79	-1.38	-1.40
3	-1.19	-1.40	-1.00	-0.95	-0.91	-0.29	-0.61	-0.71	-1.26	-0.65	-0.82	-1.09	-1.22	-1.40	-1.31
4	-1.14	-1.25	-0.99	-1.00	-1.18	-0.44	-0.62	-0.42	-1.16	-0.67	-1.02	-1.27	-1.32	-1.39	-1.15
5	-1.18	-1.04	-0.78	-1.01	-1.00	-0.54	-1.05	-0.68	-1.30	-0.48	-0.93	-0.97	-1.32	-1.35	-1.07
6	-0.73	-0.42	-0.87	-0.85	-1.01	-1.00	-0.91	-0.70	-1.19	-0.70	-0.96	-1.35	-1.39	-1.29	-1.38
7	-0.85	-0.68	-1.11	-0.78	-1.34	-0.88	-1.00	-1.28	-0.71	-0.87	-1.28	-1.40	-1.38	-1.03	-1.40
8	-0.78	-0.76	-1.08	-0.52	-1.24	-0.70	-1.22	-1.00	-0.30	-0.70	-0.66	-0.71	-1.15	-0.49	-1.09
9	-1.09	-1.36	-1.19	-0.96	-1.22	-0.79	-0.68	-0.44	-1.00	-1.05	-1.28	-1.31	-1.36	-1.28	-1.38
10	-1.13	-0.62	-0.92	-1.16	-1.00	-0.65	-1.07	-0.82	-1.40	-1.00	-1.35	-1.40	-1.35	-1.36	-1.37
11	-0.76	-0.37	-0.59	-0.95	-1.17	-0.44	-0.87	-0.55	-1.35	-1.04	-1.00	-1.20	-1.35	-1.39	-1.40
12	-0.78	-0.38	-0.76	-1.07	-1.33	-0.37	-1.06	-0.88	-1.12	-1.19	-1.29	-1.00	-1.30	-1.36	-1.39
20	-1.14	-0.51	-1.01	-1.25	-1.39	-1.12	-1.40	-0.81	-1.39	-1.40	-1.39	-1.37	-1.00	-1.00	-1.00
30	-1.04	-1.27	-1.37	-1.19	-1.27	-0.51	-0.79	-0.66	-1.40	-1.39	-1.39	-1.39	-1.00	-1.00	-1.00
40	-1.07	-1.08	-1.31	-1.15	-1.04	-0.79	-1.30	-1.25	-1.03	-0.91	-1.36	-1.36	-1.00	-1.00	-1.00

Table 3.18 – Own time elasticities logit function public transport of high income agents.

	1	2	3	4	5	6	7	8	9	10	11	12	20	30	40
1	-1.50	-0.93	-1.56	-1.44	-2.10	-0.00	-0.89	-0.94	-1.50	-0.68	-1.47	-0.49	-2.07	-2.10	-2.09
2	-1.59	-1.50	-1.79	-1.72	-1.73	-0.53	-1.04	-1.42	-1.85	-0.78	-0.85	-0.07	-1.18	-2.08	-2.10
3	-1.79	-2.10	-1.50	-1.42	-1.36	-0.44	-0.91	-1.06	-1.89	-0.98	-1.23	-1.63	-1.83	-2.10	-1.97
4	-1.71	-1.87	-1.49	-1.50	-1.76	-0.67	-0.94	-0.63	-1.74	-1.00	-1.53	-1.90	-1.99	-2.09	-1.73
5	-1.76	-1.56	-1.16	-1.51	-1.50	-0.81	-1.58	-1.01	-1.95	-0.72	-1.40	-1.46	-1.98	-2.02	-1.60
6	-1.09	-0.63	-1.30	-1.27	-1.51	-1.50	-1.36	-1.05	-1.79	-1.04	-1.44	-2.03	-2.08	-1.94	-2.08
7	-1.28	-1.02	-1.66	-1.17	-2.01	-1.32	-1.50	-1.92	-1.06	-1.30	-1.92	-2.10	-2.07	-1.54	-2.10
8	-1.17	-1.14	-1.62	-0.78	-1.86	-1.06	-1.84	-1.50	-0.45	-1.05	-0.99	-1.07	-1.73	-0.74	-1.63
9	-1.63	-2.04	-1.79	-1.43	-1.83	-1.18	-1.02	-0.67	-1.50	-1.57	-1.92	-1.97	-2.05	-1.93	-2.07
10	-1.70	-0.93	-1.39	-1.74	-1.50	-0.98	-1.60	-1.24	-2.10	-1.50	-2.03	-2.10	-2.03	-2.03	-2.05
11	-1.14	-0.56	-0.88	-1.42	-1.76	-0.66	-1.30	-0.83	-2.02	-1.56	-1.50	-1.80	-2.03	-2.09	-2.10
12	-1.17	-0.57	-1.13	-1.61	-2.00	-0.56	-1.59	-1.33	-1.67	-1.78	-1.93	-1.50	-1.95	-2.05	-2.09
20	-1.71	-0.77	-1.52	-1.87	-2.09	-1.68	-2.10	-1.22	-2.09	-2.10	-2.09	-2.05	-1.50	-1.50	-1.50
30	-1.55	-1.90	-2.05	-1.79	-1.91	-0.77	-1.18	-0.98	-2.10	-2.09	-2.09	-2.09	-1.50	-1.50	-1.50
40	-1.60	-1.62	-1.97	-1.72	-1.57	-1.19	-1.94	-1.88	-1.55	-1.37	-2.04	-2.04	-1.50	-1.50	-1.50

Table 3.19 – Own time elasticities logit function public transport of low income agents.

Appendix 3.D   Model code

3.D.1   Model code

Listing 3.1 – Network

```
$title  A Calibrated Urban Equilibrium Model with Multiple Households and Modes

*@ Project and model information
* =====================================================================================
* Project: Integrated model paper PhD
* Theme:   Model
* Contact: Renger van Nieuwkoop, renger@modelworks.ch
* =====================================================================================
* Version Control Information:
* $Id: modelR.gms 3414 2014—08—17 16:58:45Z renger $
* =====================================================================================

*   set the order of the nodes using an "overall" set.
set uelset /1*40/;
$eolcom  #

*@ Description of the model characteristics

*   Set heterogeneous (he) or homogeneous (ho)
$setglobal hhtype ho

*   Choose model structure
$setglobal city Smallville
*
*$setglobal city ZurichD

*   Options for deleting all restart files, sensitivity analysis
$setglobal restart yes
$setglobal sensitivity no
$setglobal calibB yes
$setglobal logit yes

*   We use restarts with savepoints and set the directory for the
```

```
   *    savepoints (basedir) and the results (resultsdir)

     $if not set basedir $set basedir .\bases\
37   $if not set resultsdir $set resultsdir .\results\
     option Savepoint = 1;

   *    If we do sensitivity analysis, don't produce huge listings.

42   $if %restart%==yes $call 'del %basedir%%hhtype%*.gdx'
     $if %sensitivity%==yes option solprint = silent;

   *    Show the scenario, loop number or other information in the DOS window

47   $SET CONSOLE CON

     FILE dosbox /TITLEMAKER.CMD/
     FILE screen /'%console%'/;

52   * =====================================================================================
     *   Following line is just for Emacs as are the comments with *@, *@@ and *@@@:
     *   gams-lst-dir: .\lst\

     set
57       h               'Households'
         i               'All Nodes'
         notcity(i)      'Nodes outside the city',
         city(i)         'City nodes',
         north(i)        'Northern quartiers (for counterfactual)',
62       center(i)       'Center of the city (for counterfactual)',
         centerg(i)      'Greater center of the city (for counterfactual)',
         noncenterg(i)   'Not in greater center of the city (for counterfactual)',
         m               'Transport modes',
         econ            'Economic parameters';

   *    We make a notational difference between nodes as origin (o),
   *    destination (d), other nodes (i,j)

72   alias(i,j,ii,jj,k,o,d,ao,ad),  (m,mm),(h,hh,ah);

     set
         a(m,i,j)   'Arcs';

77   alias(a,aa);

   *@ Get the data for the model

   *    The traffic and network is taken from the Cantonal Transport Model
82   *    (CTM). If there is no data available, we make some assumptions.

     parameter
         n0(h,m,i,j)       'Origin-Destination matrix',
         nnx(h,i,j)        'Origin-Destination matrix fixed',
87       nn0(h,i,j)        'Origin-Destination matrix free'
         n0pr(i,j)         'OD-matrix private transport total'
         n0pu(i,j)         'OD-matrix public transport total'
         arc_cost(m,i,j,*) 'Arc cost data',
         inc0high          'Benchmark actual income for high skilled household in CHF'
92       econpar(*,h)      'Parameters for economic model'
         data(i,h,*)       'Information for every node on households',
         nntot             'Total number of people',
         nntot0(h)         'Total number according to skill'
         powerm(m)         'Value of congestion function'
97       distancePUB(i,j)  'Distance between destinations public transport meters (CTM)
                          '
         distancePRI(i,j)  'Distance between destinations private transport in meters (
                          CTM)'
         distance(m,i,j)   'Distance between destinations in meters (CTM)'
         travtimePUB(i,j)  'Travelling time between destinations public transport in
                          minutes (CTM)'
         travtimePRI(i,j)  'Travelling time between destinations private transport in
                          minutes (CTM)'
102      travtime(m,i,j)   'Travelling time between destinations in minutes (CTM)'
         flagsc            'Flag for scenarios (for reporting)'
         coordinates;

   *    Extract data from xlsx-spreadheet
107  $call 'gdxxrw .\data\%city%\DataTransport%city%.xlsx sq = N  o=%basedir%%city%hh.
         gdx index=twohh         trace=3 CheckDate'
     $call 'gdxxrw .\data\%city%\DataTransport%city%.xlsx sq = N  o=%basedir%%city%n0.
         gdx index=twohhtwomode trace=3 CheckDate'
     $call 'gdxxrw .\data\%city%\DataTransport%city%.xlsx sq = N  o=%basedir%%city%.gdx
           index=twomode      trace=3 CheckDate'
     $call 'gdxxrw .\data\%city%\cantonalTM%city%.xlsx    sq = N  o=%basedir%%city%TD.
         gdx index=TD           trace=3 CheckDate'
```

```
      $gdxin  %basedir%%city%hh.gdx
112   $load h econpar
      $gdxin  %basedir%%city%.gdx
      $load i m notcity city a north center centerg econ arc_cost coordinates

      inc0high = 100000;

      *   If we use homogenous agents, we take the average from the economic parameters

      $ifi "%hhtype%" == "ho" econpar(econ,h) = 0.5 * (econpar(econ,"lo") + econpar(econ
          ,"hi"));

122   parameter sharehh(i,*)   Share of rich households, sharehh_m(i,h);
      $gdxin  %basedir%%city%n0.gdx
      $load n0pr n0pu sharehh
      display n0pr,n0pu, sharehh;
      sharehh_m(i,"hi") = sharehh(i,"share");
127   sharehh_m(i,"lo") = 1— sharehh(i,"share");

      *   We assume the following values for the overall shares of income type
      *   (low, high) and share of transport. We could have used income data
      *   according to district or precinct, but that would make the model

      *   results even more intractable. This could be done in a more realistic
      *   setting of the model.

      *   We further assume, that rich households prefer private transport more
137   *   than public transport.

      parameter
          share(m)    'Shares of households for every mode';

142   share("private") = 0.3;
      share("public")  = 0.7;
      *share("private") = 0.6;
      *share("public")  = 0.3;

147   $if %hhtype%==ho share("private") = 0.30;
      $if %hhtype%==ho share("public")  = 0.70;

      *sharehh_m(i,h) = 0.5;

152   n0(h,"private",i,j) = sharehh_m(i,h) * n0pr(i,j);
      n0(h,"public",i,j)  = sharehh_m(i,h) * n0pu(i,j);

      display n0;

157   $gdxin  %basedir%%city%TD.gdx
      $load distancePUB, distancePRI, travtimePUB, travtimePRI

      distance("public",i,j)  = distancePUB(i,j);
      distance("private",i,j) = distancePRI(i,j);
162   travtime("public",i,j)  = travtimePUB(i,j);
      travtime("private",i,j) = travtimePRI(i,j);

      powerm(m) = 4;
      $ifi "%city%" == "Smallville"  powerm(m) = 1;

      display powerm;

      option travtime:2:2:1
      display distance, travtime, a;

      *   We assume that on the public arcs the free flow parameter alpha will
      *   be equal to the travel times from the Cantonal Transport Model on
      *   these arcs. For the private transport arcs, we assume that the maximal
      *   speed on the arcs in the city is 30km/h. On the arcs outside the city
177   *   40 km/h. The free flow time is therefore the distance divided by this
      *   speed. We will calibrate the BPR function of the private transport by
      *   adjusting the multiplicative constant of the power term.

      parameter
182       freeflow(m,i,j)  'Free flow time on arcs';

      freeflow("public",i,j)$a("public",i,j)    = 0.95 * travtime("public",i,j);
      freeflow("private",i,j)$a("private",i,j)  = 0.4 * travtime("private",i,j);

187   display freeflow;


      *   We define later on an upper limit for the public mode, based on the
      *   solution of the Nash equilibrium and resolve the model using those
192   *   upper limits. We assume that in the benchmark there are no capacity
      *   limits in public transport.
```

```
      parameter
          flim0(m,i,j)    'Benchmark flow limit for public transport in Benchmark',
197       flim(m,i,j)     'Flow limit for public transport'
      ;
      flim0(m,i,j) = 0;
      flim(m,i,j)  = flim0(m,i,j) ;

202   *@ Define the transport model parameters

      *   Each arc is characterized by the uncongested travel time (alpha), the
      *   congestion scale factor (beta) and the reference capacity (kappa).

207   *   We begin with a origin—destination matrix describing where people
      *   live and work (assuming, in this first model, that all
      *   traffic flows are commuters).

      *   In the counterfactual we will introduce a road toll (cordon toll).

      parameter
          alpha(m,i,j)        'Uncongested travel time for every mode',
          kappa(m,i,j)        'Capacity coefficient',
          bigB(m,i,j)         'Traditional BPR multiplicator of power term',

          roadtoll0(m,i,j)  'Road toll in benchmark',
          roadtoll(m,i,j)   'Road toll in benchmark',

          v(h)                'Valuation of time',
222       arcsavail           'Available arcs indicator';

      alpha(a)              = freeflow(a);
      arcsavail(a)          = 1;

227   roadtoll0(m,i,j)      = 0;
      roadtoll(m,i,j)       = roadtoll0(m,i,j);

      *   Public transport has constant costs. We assume a capacity of 1 and
      *   calibrate bigB accordingly using data from the cantonal transport
232   *   model.

      bigB(a)               = 0.15;
      *bigB("public",i,j)    = 0;
      kappa(m,i,j)          = 1;

      *   No capacity limits for all modes in the reference case.
      flim(m,i,j)           = 0;

      *   Set the valuation of time equal to the wage
242   v(h)                  = 0.5 * econpar("wage",h);

      parameter scalingOD  /1000/;

      *   Scale the OD—Matrix
247   n0(h,m,i,j)           = n0(h,m,i,j) / scalingOD;

      *   We calibrate in a first step the B—coefficient (bigB) of the BPR
      *   function using the information on the freeflow time and an assumption
      *   on the flow on the arcs (the capacity is set arbitrarily to 1).

      parameter
          f0  Flow for calibrating bigB to realistic values;

      f0 = sum((h,m,i,j)$(ord(j) = 1), n0(h,m,i,j)) / 3;
257   bigB(m,i,j)$(arcsavail(m,i,j) and bigB(m,i,j))=
          (travtime(m,i,j) — alpha(m,i,j)) / (f0 ** 4);

      alias(city,acity);

262   *   Assume that people working and living in the same node have no
      *   incentive for changing their locations and traveltime within the node
      *   is zero, so we can drop them from the model.
      n0(h,m,i,i)           = 0;

267   *   Get the number of OD—pairs (just for the paper)
      parameter
          numberOD          'Number of OD—pairs',
          numberArcs        'Number of arcs',
          numberPrecincts   'Number of precincts';

      numberOD              = sum((h,m,i,j)$n0(h,m,i,j), 1);
      numberPrecincts       = card(i);
      numberArcs            = sum((m,i,j), arcsavail(m,i,j));

277   display numberOD, numberPrecincts, numberArcs;

      data(i,h,"work")      = sum((j,m), n0(h,m,i,j));
      data(i,h,"live")      = sum((j,m), n0(h,m,j,i));
```

```
282  *      Unload network information, so it can be used for plotting in R

     execute_unload "%resultsdir%%city%\%hhtype%_network.gdx", a=arcs, coordinates,
         arc_cost,n0, data, econpar, center, centerg, numberOD, numberPrecincts,
         numberArcs;

     nonnegative variables
287      X(h,m,i,j,k)              'Multicommodity flow',
         MU(m,i,j)                'Capacity—induced delay',
         T(h,m,i,k)               'Minimum time to destination',
         PL(h,k)                  'Wage rate at node k for household h';

292  variable
         TAU(m,i,j)               'Travel cost',
         N(h,m,i,k)               'Travel demand'
         NN(h,o,d)                'Number of households living at o and working at d',
         OBJ                      'Social optimum',
297      F(m,i,j)                 'Aggregate flow';

     equations
         fdef(m,i,j)              'Aggregate flow definition',
         flimit(m,i,j)            'Upper bound on arc flow',
302      taudef(m,i,j)            'Definition of delay on the arc',
         flowbal(h,m,i,k)         'Flow balance',
         rational(h,m,i,j,k)      'Cost minimization'
         objdef                   'Objective function for SO'
     ;

     fdef(a).. F(a) =E= sum((h,k), X(h,a,k));

     taudef(a)..
         TAU(a) =E= (MU(a))$flim(a) + (alpha(a) + bigB(a) * power(F(a) / kappa(a),4)
             $kappa(a));

     rational(h,a(m,i,j),k)..
             TAU(a) + T(h,m,j,k) + 60 * roadtoll(a) / (0.5 * PL(h,k)) =G=  T(h,m,i,k);

     flimit(a(m,i,j))$flim(a)..         flim(a) =G= F(a);

     flowbal(h,m,i,k)$(not sameas(i,k))..
         sum(a(m,i,j), X(h,m,i,j,k))=E=      N(h,m,i,k) + sum(a(m,j,i), X(h,m,j,i,k)) ;

     objdef..    OBJ =e= 1e—5 * sum((h,a(m,i,j),k), TAU(a) * X(h,a,k));

     MU.LO(a)                              = 0;
     MU.UP(a)                              = +inf;
     MU.FX(a)$(flim(a)                     = 0) = 0;     # No capacity constriants

327  PL.FX(h,k)                           = econpar("Wage",h);

     TAU.LO(a)                             = —INF;
     TAU.UP(a)                             = +INF;
     TAU.L(a)                              = alpha(a) ;
332  X.LO(h,m,i,j,k)                       = 0;
     X.UP(h,m,i,j,k)                       = +INF;
     X.L(h,m,i,j,k)                        = uniform(0.1,1);

     X.FX(h,m,i,j,k)$(not arcsavail(m,i,j)) = 0;     # no arc

     parameter fixshare(h) 'Fixed share of population';
     * 0.6
     fixshare("hi") = 0;
     fixshare("lo") = 0;
342  nnx(h,i,j)                            = sum(m, fixshare(h) * n0(h,m,i,j));
     nn0(h,i,j)                            = sum(m, (1—fixshare(h)) * n0(h,m,i,j));
     N.FX(h,m,o,d)$(not n0(h,m,o,d))       = 0;              # No OD—pair
     N.FX(h,m,o,d)                         = n0(h,m,o,d);
     N.FX(h,m,i,i)                         = 0;              # set inner node flows to
         zero

     NN.FX(h,i,j)                          = sum(m, (1 — fixshare(h)) * n0(h,m,i,j));
     NN.FX(h,i,i)                          = 0;

     nntot0(h)                             = sum((m,i,j), N.L(h,m,i,j));
352  nntot(h)                              = sum((m,i,j), n0(h,m,i,j));

     T.LO(h,m,i,j)                         = —INF;
     T.UP(h,m,i,j)                         = +INF;
     T.FX(h,m,i,i)                         = 0;

     *    As an initial pass solve for the least—cost flow this is equivalent to
     *    a perfectly defined set of traffic tolls. At the same time we
     *    calibrate the arc flow times to the values of the CTM. This is done by
     *    recalculating the bigB coefficient using the values of the flows and
```

```
362  *    the given delay times. The while loop stops as soon as the objective
     *    is not improving any further. We will repeat this process for the UE
     *    equilibrium.

     *@@@ Model definition "optflow"

     model optflow /fdef, flowbal, taudef, objdef /;

     F.L(a)   = 0.1;

372  optflow.holdfixed = 1;
     option nlp = conopt;

     *@ Reporting parameters

377  parameter
         solution_h(*,h,*)          'Solution for household variables',
         solution_he(*,h,*)         'Solution for economic household variables',
         solution_hu(*,h,*)         'Solution for utility household variables',
         solution_i(*,i,*)          'Solution for nodes'
382      solution_hmij              'Solution for household variables, arc and mode',
         solution_mij               'Solution for arcs',
         solution_m                 'Solution for modes',
         solution_mach              'Solution for macro indicators'
         solution_hm                'Solution for households and modes',
387      solution_hij               'Solution for households',
         solution_hi(h,i,*,*)       'Solution for nodes, households and modes',
         solution_hmi(*,m,i,j,h,*)  'Solution for nodes, households and modes',
         solution_overall(*,*)      'Overall results'
         solution_solver            'Information on convergence'
392      solve_statistics(*,*)      'Solver and model statistics',
         solsens_h(*,h,*)           'Solution for household variables',
         solsens_i(*,i,*)           'Solution for nodes'
         solsens_hmij               'Solution for household variables, arc and mode',
         solsens_hi(h,i,*,*)        'Solution for nodes, households and modes',
397      solsens_hmi(*,m,i,j,h,*)   'Solution for nodes, households and modes',
         solsens_overall(*,*)       'Overall results'
         solve_statistics(*,*)      'Solver and model statistics';

     set counterset /c1*c200/;
402  parameter convergence;

     parameter
         criterium  'Criterium for while loop',
         lastobjval 'Objective value of previous loop',
407      counter    'Loop counter';

     parameter
         checktau(m,i,j)   'Difference between CTM and actual value for delay';

412  $if %calibB%==no  $goto nocalibbigB

     display "Calibration active";

     *  If there are many loops, the listing gets rather big, so put the
417  *  solution printing off.
     *optflow.iterlim = 1;

     display bigB;

422  solve optflow using nlp minimizing obj;

     option X:2:3:2
     display X.L;

427  option solprint = silent;

     *   Initialize

     parameter multtau(m)
432          multF(m);

     multtau("private") = 3;
     multF("private")   = 2;
     multtau("public")  = 1.5;
437  multF("public")    = 2;

     criterium  = 100;
     lastobjval = 0;
     counter    = 1;
442  display kappa, F.L;

         kappa(m,i,j) = 3 * F.L(m,i,j);
         bigB(m,i,j)$(arcsavail(m,i,j) and bigB(m,i,j))=
         travtime(m,i,j) * (kappa(m,i,j) / F.L(m,i,j))**powerm(m) * (multtau(m) − 1) /(
             multF(m)**powerm(m)−1);
```

```
447         alpha(m,i,j)  = travtime(m,i,j) * (multF(m)**(powerm(m))-multtau(m))/(multF(m)
                **powerm(m)-1);

    display kappa, bigB, alpha;

    $if exist '%basedir%%hhtype%_optflow_p.gdx' Execute_loadpoint '%basedir%%hhtype%
        _optflow_p.gdx';
452 optflow.solvelink = 2;
    while(criterium gt 1e-6 AND counter LE 30,
        PUTCLOSE dosbox  "@title ## Calibrating delay times with SO  ##  Loop: ",
            counter:6:0,", Diff: " criterium,"   ##"  ;
        EXECUTE "titlemaker.cmd";
        kappa(m,i,j)$arcsavail(m,i,j) = 3 * F.L(m,i,j);
457     bigB(m,i,j)$(arcsavail(m,i,j) and bigB(m,i,j)) =
            travtime(m,i,j) * (kappa(m,i,j) / F.L(m,i,j))**powerm(m)* (multtau(m) - 1)
                /(multF(m)**powerm(m)-1);
        alpha(m,i,j)$arcsavail(m,i,j)  = travtime(m,i,j) * (multF(m)**4-multtau(m))/(
            multF(m)**powerm(m)-1)  ;
        solve optflow using nlp minimizing obj;
        criterium = abs(OBJ.L - lastobjval + optflow.numInfes + optflow.numNOpt);
462     lastobjval = OBJ.L;
        counter = counter + 1;
        display criterium, counter;
        loop(counterset$(ord(counterset) = counter),convergence(counterset) =
            criterium);
    );
467 solution_solver("Convergence",counterset,"SOE") = convergence(counterset) + EPS;
    display bigB, alpha, solution_solver;

    *   Save the bigB values in a gdx file, so we can import them back, if we
    *   skip the calibration part.

    execute_unload "%resultsdir%%city%\%hhtype%_networkbigBSO.gdx", bigB=bigBcalib,
        alpha=alphacalib, kappa=kappacalib;

    solve optflow using nlp minimizing obj;
    checktau(a) = travtime(a) - TAU.L(a);
477 execute 'mv -f optflow_p.gdx %basedir%%hhtype%_optflow_p.gdx';

    display bigB, checktau;

    $label nocalibbigB

    parameter
        bigBcalib(m,i,j)       'Calibrated value for bigB',
        alphacalib(m,i,j)      'Adjusted alpha values',
        kappacalib(m,i,j)      'Adjusted kappa';

    execute_load "%resultsdir%%city%\%hhtype%_networkbigBSO.gdx", bigBcalib,
        alphacalib, kappacalib;


    *   Use the calibrated values for bigB

    bigB(a)     = bigBcalib(a);
    alpha(a)    = alphacalib(a);
    kappa(a)    = kappacalib(a);

497 $if exist '%basedir%%hhtype%_optflow_p.gdx' Execute_loadpoint '%basedir%%hhtype%
        _optflow_p.gdx';

    solve optflow using nlp minimizing obj;

    execute 'mv -f optflow_p.gdx %basedir%%hhtype%_optflow_p.gdx';

    checktau(a) = travtime(a) - TAU.L(a);
    display "Check Tau SO",checktau;

    *   Nash equilibrium with same congestion function for public and
507 *   private transport:

    TAU.LO(a)                       = -INF;
    TAU.UP(a)                       = +INF;
    TAU.FX(m,i,j)$(not arcsavail(m,i,j)) = 0;       # no arc

    MU.LO(a)                        = 0;
    MU.UP(a)                        = +INF;
    MU.FX(a)$(flim0(a) = 0)         = 0;            # No upper bound

517 *@ Nash equilibrium decomposition procedure

    *   If the user equilibrium can't be solved due to numerical problems or
    *   its size, we use a decomposition algorithm, where we start with just
    *   one agent and solve the model. In every next loop we add an agent and
522 *   solve the model until the complete model is solved.'  Solve the model
```

```
      *   with equilibrium traffic flows (Nash equilibrium):

      *@@@ Model definition "traffic decomposition"
      model trafficdec 'Decomposed traffic model'
527         /fdef.F, taudef.TAU, rational.X, flowbal.T, objdef.OBJ/;
      alias(o,ao, d, ad, aao,aad), (a,aa), (i,ai,aj);

      *@@@ Model definition "traffic"

532   *   Nash (User) equilibrium formulation

      model traffic  'Traffic model with mode choice'
            /fdef.F, flimit.MU,taudef.TAU, rational.X, flowbal.T/;

537   traffic.holdfixed = 1;
      $if exist  '%basedir%%hhtype%_traffic_p.gdx' Execute_loadpoint '%basedir%%hhtype%
          _traffic_p.gdx';
      solve traffic using mcp;

      execute 'mv -f traffic_p.gdx %basedir%%hhtype%_traffic_p.gdx';
542   display "yeah";
      checktau(a) = travtime(a) - TAU.L(a);
      display checktau;

      traffic.holdfixed = 1;
547   $if exist  '%basedir%%hhtype%_traffic_p.gdx' Execute_loadpoint '%basedir%%hhtype%
          _traffic_p.gdx';
      solve traffic using mcp;

      execute 'mv -f traffic_p.gdx %basedir%%hhtype%_traffic_p.gdx';

552   *   Use the same procedure from above, to calibrate the delay times to the
      *   values of the CTM.

      *   Initialize

557   criterium  = 100;
      lastobjval = 0;
      counter    = 1;
      option solprint = silent;

562   $if %calibB%==no  $goto nocalibbigB2
      trafficdec.solvelink = 2;
      while(criterium gt 1e-6,
          PUTCLOSE dosbox  '@title ## Calibrating delay times with UE  ##  Loop: ',
              counter:6:0,', Diff: ' criterium:6:6,'   ##'  ;
          EXECUTE 'titlemaker.cmd';
567       kappa(m,i,j)$arcsavail(m,i,j) = 3 * F.L(m,i,j);
          bigB(m,i,j)$arcsavail(m,i,j) =
              travtime(m,i,j) * (kappa(m,i,j) / F.L(m,i,j))**powerm(m)* (multtau(m) - 1)
                  /(multF(m)**powerm(m)-1);
          alpha(m,i,j)$arcsavail(m,i,j)  = travtime(m,i,j) * (multF(m)**powerm(m)-
              multtau(m))/(multF(m)**powerm(m)-1) ;
          solve trafficdec using mcp;
572       criterium = abs( OBJ.L - lastobjval + optflow.numInfes + optflow.numNOpt);
          lastobjval = OBJ.L;
          counter = counter + 1;
          display criterium, counter;
          loop(counterset$(ord(counterset) = counter),convergence(counterset) =
              criterium);

      );
      solution_solver("Convergence",counterset,"UE") = convergence(counterset) + EPS;
      display solution_solver;
      display F.L, bigB, alpha, tau.l, travtime, kappa;

      option solprint = on;
      checktau(a) = (TAU.L(a) - travtime(a)) / travtime(a) * 100  ;
      display checktau;
      execute 'mv -f trafficdec_p.gdx %basedir%%hhtype%_traffic_p.gdx';
587   execute_unload "%resultsdir%%city%\%hhtype%_networkbigB.gdx", bigB=bigBcalib,
          alphacalib, checktau, travtime;
      display checktau, bigB;

      $label nocalibbigB2

592   parameter bigBcalib(m,i,j), alphacalib(m,i,j);

      $if %calibB%==no execute_load "%resultsdir%%city%\%hhtype%_networkbigB.gdx",
          bigBcalib, alphacalib;
      display bigBcalib, alpha;
      bigB(a) = bigBcalib(a);
597   alpha(a) = alphacalib(a);
      checktau(a) = travtime(a) - TAU.L(a) ;
      display checktau;
```

```
602   *     Install the upper bounds for public transport (120% of actual flow)
      flim0("public",i,j)    = 1.2 * F.L("public",i,j);
      flim("public",i,j)     = flim0("public",i,j);

      traffic.holdfixed      = 1;
      MU.LO(a)               = 0;
607   MU.UP(a)               = +INF;
      MU.FX(a)$(flim0(a)=0)  = 0;    # Fix if no upper bound
      $if exist '%basedir%%hhtype%_trafficUL_p.gdx' Execute_loadpoint '%basedir%%hhtype%
          _traffic_p.gdx';
      solve traffic using mcp;
      execute 'mv -f traffic_p.gdx %basedir%%hhtype%_traffic_pUL.gdx';

      variables
          MSHARE(h,m,i,j)        'Transport share of people living at i and working at j'
          THETAIJ(h,i,j)         'Share of people living and working at i and j'
      ;
617   parameters
          mshareref(h,m,j,i)     'Benchmark transport share of people living at i and
              working at j'
          thetaijref(h,i,j)      'Benchmark share of living and working'
          lambdajim(h,j,i)       'Scale parameter transport choice'
          lambdajim0(h,j,i)      'Scale parameter transport choice'
622       lambda(h)              'Scale parameter transport choice'
          lambda0(h)             'Scale parameter transport choice'
      ;

      lambdajim("hi",j,i)    = 32;
627   lambdajim("lo",j,i)    = 8;
      lambdajim0(h,j,i)      = lambdajim(h,j,i);

      *     Reference shares
      mshareref(h,m,i,j)$sum((m.local), N.L(h,m,i,j)) =
632       N.L(h,m,i,j) / sum((m.local), N.L(h,m,i,j));
      thetaijref(h,i,j)$(sum(m, N.L(h,m,i,j))) =
          sum(m,N.L(h,m,i,j)) / sum((i.local,j.local, m.local), N.L(h,m,i,j));

      MSHARE.L(h,m,i,j)                                 = mshareref(h,m,i,j);
637   THETAIJ.L(h,i,j)                                  = thetaijref(h,i,j);
      MSHARE.FX(h,m,i,j)$(not mshareref(h,m,i,j))       = 0;
      MSHARE.FX(h,m,i,i)                                = 0; # start and end the same
      THETAIJ.FX(h,i,j)$(not thetaijref(h,i,j))         = 0;
      THETAIJ.FX(h,i,i)                                 = 0; # start and end the same


      *@ Start of the economic model

      **@ Calibration of the economic model

      *     Define some value shares and elasticities for calibrating the
      *     associated economic equilibrium.  We calibrate preferences to a
      *     reference point in which the leisure value share is 10% of
      *     aggregate value of expenditure, housing is 25% of market expenditures.

      *     We calibrate technologies such that the reference capital value share
      *     is 50% of the value of output.

      *     Harberger units are adopted, assuming that the wage rate is unity and
657   *     both capital and housing are measured in efficiency units with rental
      *     rates equal to unity.

      *     We introduce two elasticity parameters, sigmals describing the
      *     elasticity of substitution between morning leisure and expenditure and
662   *     sigmah describing the elasticity of substitution between housing and
      *     consumption.

      *     Economic model parameters

667   parameter
          py(*)           'Output prices',
          thetals(h)      'Leisure value share',
          hvs(h)          'Housing value share',
          kvs(h)          'Capital value share' ,
672       wage(h)         'Wage rate',
          sigma(h)        'Top Elasticity utility function between LS and C/H',
          sigmah(h)       'Elasticity of housing demand'
          sigma0(h)       'Elasticity of leisure',
          sigmah0(h)      'Elasticity of housing demand'
677   ;

      py(h)       = 1;
      thetals(h) = econpar("thetals",h);
      hvs(h)      = econpar("hvs",h);
682   kvs(h)      = econpar("kvs",h);
      wage(h)     = econpar("Wage",h);
```

```
      sigma0(h)   = econpar("sigmals",h);
      sigmah0(h)  = econpar("sigmah",h);
      sigma(h)    = sigma0(h);
687   sigmah(h)   = sigmah0(h);


      display thetals, hvs;

      parameter
          rho(h)      Primal elasticity exponent for sigma ;

      sigma(h)$(sigma(h) = 1) = 1.25;
697   rho(h) = 1 - 1 / sigma(h);

      parameter
          ks(h,i)      Capital stock used in production at node i,
          h_ks(h,i)    Household own capital stock,
702       h_ke(h,i)    Capital endowment
          kl(h,i)      Imputed value of value add
          h_kl(h,i)    Imputed value of labor plus capital income
          ld(h,i)      Labor demand,
          hs(h,i)      Household stock an node i,
707       h_hs(h,i)    Household stock owned by household h living at i,
          hs0(h,i)     Benchmark Household stock
          h_hs0(h,i)   Benchmark Household stock;

      *    Calibrating the reference wage rate to unity, the value of
712   *    labor demand at node i equals the number of workers:

      ld(h,i) = sum((m,j), N.L(h,m,j,i));

      *    Calibrate the rental rate of capital to unity as well.
717   *    The aggregate value of output is based on employment, and
      *    the capital value share then defines capital supply in
      *    efficiency units:

      parameter kvsh(h);

      kvsh("hi") = 0.3;
      kvsh("lo") = 0.4;
      $ifi "%hhtype%" == "ho" kvsh(h) = 0.5 * (kvsh("hi") + kvsh("lo"));


      ks(h,i) = kvs(h) * wage(h) * ld(h,i) / (1 - kvs(h));
      h_ks(h,i) = 0.1 * kvsh(h) * wage(h) * sum((m,j),N.L(h,m,i,j)) / (1 - kvs(h));

      display ks, h_ks;

      *    The housing stock at node i equals the value
      *    of market expenditure times the housing value share.

      *    The value of market expenditure net housing equals the
737   *    value of labor and capital earnings.  We can infer this
      *    income based on labor supply divided by the labor share
      *    of output.

      parameter keo(h)    'capital outside'
742   ;
      hvs("hi") = 0.4;
      hvs("lo") = 0.5;


747   $ifi "%hhtype%" == "ho" hvs(h)  = 0.5 * (hvs("hi") + hvs("lo"));
      parameter hvsoh(h);
      parameter hvsi(h);
      hvsoh("hi") = 0.1;
      hvsoh("lo") = 0.05;
752   $ifi "%hhtype%" == "ho" hvsoh(h)  = 0.5 * (hvsoh("hi") + hvsoh("lo"));

      hvsi(h) = hvs(h);
      hvs(h)  = hvsi(h) / (1 + (1 - hvsoh(h))* hvsi(h));
      display hvsi,hvs;
757   kl(h,i)   = sum((m,j), wage(h) * N.L(h,m,i,j)) / (1 - kvs(h));
      h_kl(h,i) = sum((m,j), wage(h) * N.L(h,m,i,j))  + h_ks(h,i);
      h_ke(h,i) = h_ks(h,i);

      hs0(h,i)   = hvs(h) * h_kl(h,i) / (1 - hvs(h));

      display hvsi,hvs;
      *hvsi(h) = hvs(h);
      parameter check_h(h,i);

767   check_h(h,i) = hs0(h,i) - hvsi(h)* (h_kl(h,i) + hvsoh(h) * hs0(h,i));

      display check_h;
```

```
772  *h_hs0(h,i) = hvs(h) * h_kl(h,i) / (1 - hvs(h));
     hs(h,i)   = hs0(h,i);
     h_hs(h,i) =  hvsoh(h) * hs0(h,i);

     parameter
777      totals(h,*) 'Totals for economic variables',
         yref(h,i)   'Benchmark production level',
         yA(h,i)     'Efficiency parameter production';;

     yref(h,i) = ks(h,i) ** kvs(h) * ld(h,i) ** (1 - kvs(h));
782  yA(h,i) = (ks(h,i) + wage(h) * ld(h,i)) / yref(h,i);
     totals(h,"out con") = sum(i, yA(h,i) * yref(h,i));

     parameter
         mref            'Reference expenditure'
787      tref            'Reference commute time',
         tmax            'Maximum commute'
         ntot            'Total population'
         other(h)        'Other income (per capita)'
         thetad(h)       'Fraction of capital income which is local'
792      sharehs(h,hh)   'Share of owned housing';

     sharehs(h,hh)   = 0;
     sharehs("hi","hi") = 1;
     sharehs("lo","lo") = 1;
797  ntot(h,m) = sum((i,j), N.L(h,m,i,j));
     n0(h,m,i,j) = N.L(h,m,i,j);

     *   Reference per-capita expenditure for housing plus other goods
     *   equals the value of labor income, capital income and housing
802  *   rental income:

     mref(h)      = sum(i, h_kl(h,i) + h_hs(h,i)) / sum(m, ntot(h,m));

807  other(h)  = sum(i, h_ke(h,i) + sum(hh, sharehs(hh,h) * h_hs(hh,i))) / sum(m, ntot(
         h,m));

     thetad(h)    = 0.2;

     *   Assume that the maximum possible commute is twice the longest
812  *   commute observed in the benchmark traffic flows:

     tmax(h,m) = 2 * smax((i,j), T.L(h,m,i,j));

     *   Use the commuter-weighted average commute time to
817  *   anchor the utility function:

     tref(h,m) = sum((i,j), N.L(h,m,i,j) * T.L(h,m,i,j)) / sum((i,j), N.L(h,m,i,j));

     parameter
822      ph0(h,i)            'Benchmark price for housing'
         y0(h,i)             'Benchmark activity level'
         rk0(h,i)            'Benchmark capital price'
         pch0(h,i)           'Benchmark consumer price index'
         pl0(h,i)            'Benchmark wage'
827      inc0(h,i,j)         'Benchmark income'
         u0(h,i,j)           'Benchmark utility level',
         taxespaid(h,i,j)    'Taxes paid',
         incref(h,o,d)       'Reference income';

832  ph0(h,i)  = 1;
     y0(h,i)   = 1;
     rk0(h,i)   = 1;
     pch0(h,i) = 1;
     pl0(h,i)  = wage(h);
837  taxespaid(h,o,d) = 0;

     *@@ The economic model

     nonnegative variables
842      Y(h,i)       'Production at i',
         RK(h,i)      'Capital price',
         PH(h,i)      'Housing rental rate',
         CR(h,i)      'Housing capacity rent';

847  variables
         PK              'Capital income price index',
         PCH(h,i)        'Consumption/housing price index'
         INC(h,i,j)      'Aggregate income',
         LS(h,m,i,j)     'Leisure';

     equations
         mkt_pl          'Market clearing labor',
```

```
        mkt_rk          'Market clearing capital services',
        mkt_ph          'Market clearing housing',
857     mkt_py          'Market for consumptiongood'

        prf_y           'Zero profit condition production',
        pkdef           'Definition capital price index',
        pchdef          'Definition price index',

        incdef          'Income constraint',
        lsdef           'Labour supply',
        taxp            'Tax payments'
        taxr            'Tax refund',

        hs_limit        'Limit on number of people living at a certain node';

    *   Zero profit for production at node j:

872 prf_y(h,j)$ks(h,j)..
        (PL(h,j)/pl0(h,j)) ** (1 — kvs(h)) * (RK(h,j)/rk0(h,j) ) ** kvs(h) =G= py(h);

    *   Labor market clearing:

877 mkt_pl(h,j)$ld(h,j)..  sum(i, nnx(h,i,j) + NN(h,i,j)) * PL(h,j) =G= (1 — kvs(h)) *
        py(h) * Y(h,j) ;

    *   Capital market clearing

882 mkt_rk(h,j)$ks(h,j)..
        ks(h,j) * RK(h,j) =G= kvs(h) * py(h) * Y(h,j);

    *   Housing market clearing at node i:

887 hs_limit(h,i)$sum((m,j), n0(h,m,i,j))..
        1.05* sum((m,j), n0(h,m,i,j))  =G= sum((m,j), N(h,m,i,j));

    mkt_ph(h,i)$hs(h,i)..  hs(h,i) * ((PH(h,i) + CR(h,i)) ** sigmah(h)) =G=
        sum(j, (NN(h,i,j) + nnx(h,i,j)) * INC(h,i,j)) * hvsi(h) * PCH(h,i) ** (sigmah
        (h) — 1) ;

    *   Market for consumption good

    mkt_py(h,i)..  yA(h,i) * Y(h,i) * (py(h) ** sigmah(h)) =G=
        sum(j, (NN(h,i,j) + nnx(h,i,j)) * INC(h,i,j)) * (1 — hvsi(h)) * PCH(h,i) ** (
        sigmah(h) — 1) ;

    *   Capital price index:

    pkdef..  PK * (sum((h,i), hs(h,i)) + sum((h,j), ks(h,j))) =e= sum((h,i),
            (PH(h,i) + CR(h,i)) * hs(h,i)) + sum((h,j), RK(h,j)* ks(h,j));

    *   Income: Per — capta rental plus wage income for a new worker at j: If
    *   the numeraire is not equal to 1 the income of other from abroad should
    *   be multiplied with the numeraire!

907 incdef(h,o,d)$sum(m, n0(h,m,o,d))..
        INC(h,o,d) =e=  PL(h,d) + other(h) * (thetad(h) * PK
        + py(h) * (1—thetad(h))
        )  — taxespaid(h,o,d);

912 *   Cost index for market goods (housing and goods):

    pchdef(h,i)$hs(h,i)..  PCH(h,i) =E= (
        ((PH(h,i) + CR(h,i)) / ph0(h,i)) ** hvsi(h)* py(h) ** (1—hvsi(h)))$(sigmah(h)
            =1) +
        ((hvsi(h) * (PH(h,i) + CR(h,i)) ** (1 — sigmah(h)) +
917     (1 — hvsi(h)) * py(h)**(1 — sigmah(h))) ** (1 / (1 — sigmah(h))))$(sigmah(h)
            <>1);

    equation
        tcalcdef(h,m,o,d)    'Definition of traveltime';

922  variable
        TCALC(h,m,o,d)        'Traveltime on route o—d';

    lsdef(h,m,o,d)$n0(h,m,o,d)..  LS(h,m,o,d) =E= MSHARE(h,m,o,d) *
        (tmax(h,m) — T(h,m,o,d)) /
927     (tmax(h,m) — tref(h,m));

    *tcalcdef(h,m,o,d)$n0(h,m,o,d)..
    *    TCALC(h,m,o,d) * N(h,m,o,d) =E= sum((i,j), TAU(m,i,j) * X(h,m,i,j,o,d)) ;

932 *@@@ Model definition "econcalib"

    model   econcalib  'Calibration of economic part'
        /mkt_pl.PL, mkt_rk.RK, mkt_ph.PH, prf_Y.Y,  hs_limit.CR,
         pkdef.PK, lsdef.LS, pchdef.PCH, incdef.INC/;
```

```
     *   Check hs_limit

     PCH.L(h,i)          = 1;
942  PK.L                = 1;
     PL.LO(h,i)          = 0;
     PL.UP(h,i)          = +INF;
     PL.L(h,i)           = wage(h);
     RK.L(h,i)           = 1;
947  PH.L(h,i)           = 1;
     Y.L(h,j)            = sum((m,i), N.L(h,m,i,j)) * PL.l(h,j)/ (1 — kvs(h)) * py(h);
     PCH.L(h,i)          = 1;
     N.L(h,m,i,j)        = n0(h,m,i,j);
     PK.LO               = —INF;
952  INC.LO(h,i,j)       = —INF;
     INC.L(h,i,j)        = PL.L(h,j) + other(h) * (thetad(h)*PK.L
     + py(h) * (1 — thetad(h))
          );
     CR.FX(h,i)          = 0;
957  PCH.LO(h,i)         = —INF;
     LS.LO(h,m,i,j)      = —INF;
     LS.UP(h,m,i,j)      = INF;
     INC.FX(h,i,i)       = 0;
     INC.FX(h,i,j)$(not sum(m, n0(h,m,i,j))) = 0; # No OD—pair
962  incref(h,i,j)       = INC.L(h,i,j);

     totals(h,"dem con") = sum(i, ((1 — hvsi(h)) * sum(j,
         (NN.L(h,i,j) + nnx(h,i,j)) * INC.L(h,i,j)) * PCH.L(h,i) ** (sigmah(h) — 1) ));

967  totals(h,"dem housing") = sum(i, ((hvsi(h)) * sum(j,
         (NN.L(h,i,j) + nnx(h,i,j)) * INC.L(h,i,j)) * PCH.L(h,i) ** (sigmah(h) — 1) ));
     display yref, totals, hs;

     *    Fix locations of workers  / commuters in the benchmark equilibrium and
972  *    their travel times:

     N.FX(h,m,i,j)    = N.L(h,m,i,j);
     NN.FX(h,i,j)     = sum(m, (1 — fixshare(h)) * N.L(h,m,i,j));
     X.FX(h,m,i,j,d)  = X.L(h,m,i,j,d);
977  TAU.FX(a) = TAU.L(a);
     display tmax, T.L;

     LS.L(h,m,i,j)    = MSHARE.L(h,m,i,j)*(tmax(h,m) — T.L(h,m,i,j)) / (tmax(h,m) — tref
         (h,m));

982  *@@ The sorting mechanism

     equations
         udef(h,i,j)            'Defines U'
         thetaijdef(h,i,j)      'Defintion of share of people working and living at j
             and i'
987      msharedef(h,m,i,j)     'Definition of Share of transport mode on arc i j'
         usort(h,i,j)           'Sorting equation'
         population(h)          'Total population matches'
     ;
     variables
992      U(h,i,j)               'Utility for new workers living at i and working at j'
     ;
     positive variable
         UH(h)                  'Reference utility level for sorting;'
     ;
997  parameter
         uref(h,i,j)            'Reference utility level',
         t0ref(h,m,i,j)         'Benchmark value of commuting time',
         phijim(h,m,i,j)        'Scaling parameter in logit demand for transport',
         check(h,i,j,*)         'Results from calibration',
1002     deviation(h,m,i,j)     'Deviation of calibrated shares from reference shares',
         maxdev                 'Maximal deviation from share calibration',
         fuzz(h,i,j)            'Fuzz factor for scaling utility';

     t0ref(h,m,o,d)  = T.L(h,m,o,d);

     uref(h,i,j)$(sum(m, n0(h,m,i,j))) =    (sum(m, LS.L(h,m,i,j)) ** thetals(h) * (
         pch0(h,i) * INC.L(h,i,j)
         / (mref(h) * PCH.L(h,i))) **  (1 — thetals(h)))$(sigma(h)=1) +
         (((thetals(h) * sum(m, LS.L(h,m,i,j))) ** rho(h) +
         (1 — thetals(h)) * (INC.L(h,i,j) *pch0(h,i) / (mref(h) * PCH.L(h,i))) ** rho(h
             )) ** (1 / rho(h)))$(sigma(h) <> 1);

     fuzz(h,i,j)$(sum(m, n0(h,m,i,j)))  = 1;
     *uref(h,i,j)  = 1;
     parameter umax(h);

1017 umax(h) = smax((i,j), uref(h,i,j));
```

```
        display umax;

     *   Calibrate the shares of transport demand by choosing the
1022 *   location parameters accordingly. The location parameter for private
     *   transport is set to zero (only differences matter).

     parameter ownelasjim(h,m,i,j);

1027 parameter lambdajimh(h,m,i,d);

     parameter lambdajih(h,i,j);

     parameter t0m(m), sharem, ownelas, lmbd0;

     ownelasjim(h,m,o,d)       = -1;
     ownelasjim("lo",m,o,d)  = -1.5;

     $ifi "%hhtype%" == "ho" ownelasjim(h,m,o,d)  = 0.5 * (ownelasjim(h,"public",o,d)
         + ownelasjim(h,"private",o,d));
1037 ownelasjim(h,"private",o,d)    = 1.5 * ownelasjim(h,"public",o,d);

     $if %logit%==no  $goto noLogit

     variables dumelas, lmbm, phim, elasm(m);

     equations  eq_elas,  eq_phi, eq_dumelas;

     eq_elas(m)..
          lmbm * elasm(m) =E=  -t0m(m) * (1 - sharem(m));

     eq_phi..
          phim =E= lmbm*(-log(sharem("public") / sharem("private")))  + t0m("private") -
              t0m("public");

     eq_dumelas..
1052     dumelas =E= sum(m, (elasm(m) - ownelas(m))* (elasm(m) - ownelas(m)));

         model elast /eq_elas,  eq_phi,  eq_dumelas/;

     option nlp = pathnlp;
1057 option Savepoint = 0;
     option solprint = silent;
     elast.solvelink = 2;
     counter = 1;
     loop((h,o,d)$(sum(m, n0(h,m,o,d))),
1062     PUTCLOSE dosbox   '@title ## Calibrating logit function  ##  Loop: ', counter
             :6:0,', Hh: ' h.tl, " O: ", o.tl, " D: ", d.tl ,'  ##'  ;
         EXECUTE 'titlemaker.cmd';

         t0m(m) = T0ref(h,m,o,d);
         sharem(m) = mshareref(h,m,o,d);
1067     ownelas(m) =ownelasjim(h,m,o,d);

         phim.l = 20;
         lmbm.l = 12;
         lmbm.lo = 0;
1072 *    elasm.up(m) = 0.5 * ownelasjim(h,m,o,d);
         elasm.l(m) = ownelasjim(h,m,o,d);
         solve elast minimizing dumelas using nlp;
         phijim(h,m,o,d)     = phim.l;
         lambdajimh(h,m,o,d) = lmbm.l;
1077     counter = counter + 1;
     );
     option Savepoint = 1;
     execute_unload "%resultsdir%%city%\%hhtype%_Logit.gdx", phijim, lambdajim,
         lambdajimh;

1082 $label noLogit

     $if %logit%==no  execute_load "%resultsdir%%city%\%hhtype%_Logit.gdx",  phijim,
         lambdajim, lambdajimh;

     ownelasjim(h,m,o,d)$n0(h,m,o,d)        = -T0ref(h,m,o,d) / lambdajimh(h,m,o,d)*(1 -
         mshareref(h,m,o,d));

     phijim(h,"private",i,j)$mshareref(h,"private",i,j) = 0;

     parameter checkshares(h,m,o,d);
     checkshares(h,m,o,d)$(not sameas(o,d) and n0(h,m,o,d)) =      MSHARE.L(h,m,o,d) -
1092     EXP(-1 / lambdajimh(h,m,o,d) * (T.L(h,m,o,d)+ phijim(h,m,o,d))) / sum(mm, EXP
             (-1 / lambdajimh(h,m,o,d) * (T.L(h,mm,o,d) +
             phijim(h,mm,o,d))));
     display checkshares, ownelasjim;
```

```
     check(h,i,j,"Inc")      = INC.L(h,i,j);
1097 check(h,i,j,"housing")$(not sameas(i,j))
                             = hvsi(h) *  INC.L(h,i,j) * PCH.L(h,i) ** (sigmah(h) − 1) /
                                (PH.L(h,i) ** sigmah(h)) ;
     check(h,i,j,"C")        = (1 − hvsi(h)) * INC.L(h,i,j) * PCH.L(h,i) ** (sigmah(h) −
        1) /
        (py(h) ** sigmah(h));
1102 check(h,i,j,"other")$(not sameas(i,j))  = other(h) * (thetad(h)*PK.L + (1 −
        thetad(h)));
     check(h,i,j,"Exp")      = check(h,i,j,"C") + check(h,i,j,"housing");

     check(h,i,j,"Diff Inc−Exp")$INC.L(h,i,j) = check(h,i,j,"Inc") − check(h,i,j,"Exp")
        + EPS;
     check(h,i,j,"mref")     = mref(h);
1107 display nn.l, n0, check;

     parameter checknode(*,h,i)  'Values for nodes';
     mshare.fx(h,m,i,j) = mshareref(h,m,i,j);

1112 display phijim;

     positive variable

         um(h,i,j)     Utility level with amenities
1117     uoverall(h)   Utility level all agents;
     ;
     variable
         dummy
         mlive(h,i,j) Amenity of living at i
1122     mwork(h,j)   Amenity of working at j
     ;
     equations
         umdef(h,i,j)         Utility definition with amenities
         u_constraint(h,i,j)  Constraint,
1127     u_dummy;

     n0(h,m,i,i) = 0;

     umdef(h,i,j)$(sum(m, n0(h,m,i,j)))..
1132     um(h,i,j) =E=    (sum(m, LS.L(h,m,i,j)) ** thetals(h) * (pch0(h,i) * INC.L(h,i
           ,j)
         / (mref(h) * PCH.L(h,i))) **  (1 − thetals(h)))$(sigma(h)=1) +
         (((thetals(h) * sum(m, LS.L(h,m,i,j))) ** rho(h) +
         (1 − thetals(h)) * (INC.L(h,i,j) *pch0(h,i) / (mref(h) * PCH.L(h,i))) ** rho(h
           )) ** (1 / rho(h)))$(sigma(h) <> 1) +
         mlive(h,i,j);

     u_constraint(h,i,j)$sum(m, n0(h,m,i,j))..
         um(h,i,j) =E= umax(h);

     u_dummy..
1142     dummy =E= 1;

     option solprint = on;

     model ucalib /umdef, u_constraint, u_dummy/;

     solve ucalib minimizing dummy using nlp;

     parameter amen(h,i,j)  amenity of living at i and working at j;

1152 amen(h,i,j) = MLIVE.L(h,i,j);

     display amen;

     uref(h,i,j) = UM.L(h,i,j);

     udef(h,i,j)$(sum(m, n0(h,m,i,j)) and uref(h,i,j))..
         U(h,i,j) =E=
*      Cobb−Douglas formulation
     fuzz(h,i,j) * (sum(m, LS(h,m,i,j)) ** thetals(h) * (INC(h,i,j) / (mref(h) * PCH
        (h,i))) ** (1 − thetals(h)))$(sigma(h) = 1) +
1162 *  CES formulation
         fuzz(h,i,j) * (((thetals(h) * sum(m, LS(h,m,i,j))) ** rho(h) +
         (1 − thetals(h)) * (INC(h,i,j) * pch0(h,i) / (mref(h) * PCH(h,i))) ** rho(h)
           ) **
         (1 / rho(h)))$(sigma(h) <> 1) + amen(h,i,j);

1167 msharedef(h,m,o,d)$mshareref(h,m,o,d)..
         MSHARE(h,m,o,d) * sum(mm, EXP(−1 / lambdajimh(h,mm,o,d) * (T(h,mm,o,d) +
         phijim(h,mm,o,d))))
         =E= EXP(−1 / lambdajimh(h,m,o,d) * (T(h,m,o,d)+ phijim(h,m,o,d))) ;

1172 *  Sorting equations
```

```
        usort(h,i,j)$uref(h,i,j)..  UH(h) =G= U(h,i,j)/ uref(h,i,j);

        parameter mus /100/;
        parameter amult /1/;
        *usort(h,i,j)$uref(h,i,j)..  NN(h,i,j) =G= nn(h,i,j) * exp(mus*(U(h,i,j) - amult *
            amen(h,i,j))/(uref(h,i,j)- amult  * amen(h,i,j)))
        *                              / sum((ii,jj), nn(h,ii,jj)* exp(mus*(U(h,i,j) -
            amult * amen(h,i,j))/(uref(h,ii,jj)- amult * amen(h,i,j)))) * sum((ii,jj), nn0
            (h,ii,jj));



        population(h)..   nntot(h) =G= sum((i,k), NN(h,i,k) + nnx(h,i,k));


    *    Calibration of economic part of the model

        econcalib.holdfixed = 1;
        econcalib.iterlim  = 0;
        N.FX(h,m,i,j)         = N.L(h,m,i,j);
        NN.FX(h,i,j)          = sum(m,(1 - fixshare(h)) * N.L(h,m,i,j));
        T.FX(h,m,i,j)         = T.L(h,m,i,j);
        LS.L(h,m,i,j)         = MSHARE.L(h,m,i,j)*(tmax(h,m) - T.L(h,m,i,j)) / (tmax(h,m) -
            tref(h,m));

        display INC.L, mref;

        display "* Calibration of economic model";
        option solprint = on;
        solve econcalib using mcp;
        display mref, inc.l;

        execute 'mv -f econcalib_p.gdx %basedir%%hhtype%_econcalib_p.gdx';
        *abort$(abs(econcalib.objval) gt 1E-5) "**** Model econcalibdoes not calibrate";
        display kvs;

    *$include homogen.gms
        solve econcalib using mcp;

        *py(h) = 2;
        *econcalib.iterlim   = 10000;
        *solve econcalib using mcp;

        parameter
            checktotals      'Check the totals of the households';

        checknode("Production",h,i) = Y.L(h,i);
        checknode("Prod calc.",h,i) = ks(h,i) ** kvs(h) * sum(j, NN.L(h,j,i) + nnx(h,j,i))
            ** (1-kvs(h));

        checktotals("VA")          = sum((h,i), kl(h,i));
        checktotals("Houses")      = sum((h,i), hs(h,i));

        checktotals("VA + Houses")  = checktotals("VA") + checktotals("Houses");
        checktotals("Income")      = sum((h,i,j), inc.l(h,i,j));
        checktotals("LS")          = sum((h,i,j), check(h,i,j,"LS"));
        checktotals("C")           = sum((h,i,j), check(h,i,j,"C"));

        display checktotals, inc.l;

        nntot0(h)       = sum((m,i,j), N.L(h,m,i,j));
        nntot(h)        = sum((m,i,j), n.l(h,m,i,j));

        N.LO(h,m,i,k)                          = 0;
        N.UP(h,m,i,k)                          = +INF;
        N.FX(h,m,i,j)$(not n0(h,m,i,j))        = 0;
        NN.LO(h,i,j)                           = 0;
        NN.UP(h,i,j)                           = +INF;
        NN.FX(h,i,j)$(not sum(m, n0(h,m,i,j)))= 0;    # no original flows on arcs

    *    Calculate the time and utility elasticities as well as the quasi time
         elasticities.
    *    xxx check this (old code of other formulation)

        parameter
            ownelasjim(h,m,i,j)         'Own price elasticity with respect to travel time'
            crosselasjim(h,m,mm,i,j)   'Cross price elasticity with respect to travel time
                of the other mode'
            qownelasjim(h,m,i,j)        'Quasi own price elasticity with respect to travel
                time'
            qcrosselasjim(h,m,mm,i,j) 'Quasi cross price elasticity with respect to travel
                time of the other mode'
            ownelasij(h,i,j)            'Own price elasticity with respect to utility'
            crosselasij(h,i,j,ii,jj)   'Cross price elasticity with respect to utility of
                other work-live pairs';
```

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```
      ownelasjim(h,m,o,d)$(not sameas(o,d) and n0(h,m,o,d))      = -T0ref(h,m,o,d) /
          lambdajimh(h,m,o,d)*(1 - mshareref(h,m,o,d));
1252  crosselasjim(h,m,mm,o,d)$(not sameas(m,mm) and not sameas(o,d) and n0(h,m,o,d))  =
          T0ref(h,mm,o,d) / lambdajimh(h,m,o,d)
          *(mshareref(h,mm,o,d));
      qownelasjim(h,m,o,d)$(not sameas(o,d) and n0(h,m,o,d))      = -T0ref(h,m,o,d) /
          lambdajimh(h,m,o,d)
          *(1 - mshareref(h,m,o,d)) * mshareref(h,m,o,d);
      qcrosselasjim(h,m,mm,o,d)$(not sameas(m,mm) and not sameas(o,d) and n0(h,m,o,d)) =
          T0ref(h,mm,o,d) / lambdajimh(h,m,o,d)
1257      *(mshareref(h,m,o,d)) * mshareref(h,m,o,d);

      *ownelasij(h,o,d)            = uref(h,o,d) / lambda(h)*(1 - thetaijref(h,o,d));
      *crosselasij(h,i,j,ii,jj)$(not sameas(i,ii) and not sameas(j,jj))  = -uref(h,i,j)
          / lambda(h)
      *    *(1 - thetaijref(h,ii,jj));

      display ownelasjim, crosselasjim, qownelasjim, qcrosselasjim;

      execute_unload "%resultsdir%%city%\%hhtype%_elas_%city%", ownelasjim, crosselasjim
          , qownelasjim, qcrosselasjim;

1267  U.L(h,i,j)     = uref(h,i,j);
      U.FX(h,i,j)$(sum(m, not n0(h,m,i,j))) = 0;
      T.LO(h,m,i,d) = 0;
      T.UP(h,m,i,j) = +INF;
      T.FX(h,m,i,i) = 0;       # Start and end the same
1272  NN.FX(h,i,k)$(sum(m, N.L(h,m,i,k)) = 0) = 0;

      display uref;

      *@@ Replicate the calibrated model

      *    Read off the reference utility levels and replicate this in a model
      *    with equilibrium sorting:

      PCH.L(h,i)     = 1;
1282  PCH.LO(h,i)    = -INF;
      UH.LO(h)       = 0;
      UH.L(h)        = 1;

      CR.LO(h,i)     = 0;
1287  CR.UP(h,i)     = +INF;
      CR.L(h,i)      = 0;
      CR.FX(h,i)$(not sum((m,j), n0(h,m,i,j))) = 0;
      T.FX(h,m,i,d) = T.L(h,m,i,d);
      CR.FX(h,i)     = 0;
1292  nntot0(h)      = sum((m,i,j), N.L(h,m,i,j));
      nntot(h)       = sum((m,i,j), N.L(h,m,i,j));

      NN.LO(h,i,j)   = 0;
      NN.L(h,i,j)    = NN.L(h,i,j);


      N.FX(h,m,o,d)  = n0(h,m,o,d);
      NN.FX(h,i,k)$(sum(m, N0(h,m,i,k)) = 0) = 0;
      N.FX(h,m,i,k)$(N.L(h,m,i,k) = 0)      = 0;

      display nntot, nntot0,nN.L,N.L;

      *@@@ Model definition "econequil"

1307  model econequil   'Economic equilibrium'
          /mkt_pl.PL, mkt_rk.RK, mkt_ph.PH, prf_Y.Y, hs_limit.CR,
          pkdef.PK, lsdef.LS, pchdef.PCH, incdef.INC,
          udef.U,   msharedef.MSHARE,
          usort.NN
1312      , population.UH
          /;

      econequil.iterlim                    = 0;
      econequil.holdfixed                  = 1;

      *    Calibration of economic equilibrium

      display "* Calibration of economic equilibrium";
      solve econequil using mcp;
1322  execute 'mv -f econequil_p.gdx %basedir%%hhtype%_econequil_p.gdx';

      abort$(abs(econequil.objval) gt 1E-5) "**** Model econequil does not calibrate";

      solve econequil using mcp;

      *$include homogen.gms
```

```
      *py(h) = 2;

      econequil.iterlim                        = 1000;
1332  econequil.holdfixed                      = 1;

      solve econequil using mcp;

      *@@ The integrated model

      equation ndef(h,m,i,j)    'Constraint of population';

      ndef(h,m,i,j)..  N(h,m,i,j) =E= (NN(h,i,j) + nnx(h,i,j)) * MSHARE(h,m,i,j);

1342  *    Declare the integrated model which includes economic and traffic
      *    flow equilibrium conditions:

      *@@@ Model definition "urbanequil"

1347  model urbanequil  'Integrated model'
          /mkt_pl.PL, mkt_rk.RK, mkt_ph.PH, prf_Y.Y, hs_limit.CR,
          pkdef.PK, lsdef.LS, pchdef.PCH, incdef.INC,
          udef.U, msharedef.MSHARE,
          usort.NN,
1352      population.UH,
          ndef.N,
          fdef.F, flimit.MU, taudef.TAU,  flowbal.T,
          rational.X
          /;


      model urbanequilSO  'Integrated model'
          /mkt_pl, mkt_rk, mkt_ph, prf_Y, hs_limit.CR,
          pkdef, lsdef, pchdef, incdef,
1362      udef.U, msharedef,
          usort, population, ndef,
          fdef, flimit, taudef,  flowbal,
          objdef
          /;

      T.LO(h,m,i,d)                            = 0;
      T.UP(h,m,i,k)                            = INF;
      T.FX(h,m,i,i)                            = 0;

1372  U.FX(h,i,i)                              = 0;  # Origin and destination end the same

      X.LO(h,m,i,j,d)                          = 0;
      X.UP(h,m,i,j,d)                          = +INF;
      X.FX(h,m,i,j,d)$(not arcsavail(m,i,j)) = 0;  # no arc
1377  X.FX(h,m,i,i,d)                          = 0;  # start and end point the same

      N.LO(h,m,o,d)                            = 0;
      N.UP(h,m,o,d)                            = +INF;
      N.L(h,m,o,d)                             = n0(h,m,o,d);
1382  N.FX(h,m,o,d)$(not n0(h,m,o,d))          = 0;
      F.L(a)                                   = sum((h,d), X.L(h,a,d));
      F.FX(m,i,j)$(not arcsavail(m,i,j))       = 0;      # no arc
      TAU.LO(a)                                = -INF;
      TAU.UP(a)                                = +INF;
1387  TAU.FX(m,i,j)$(not arcsavail(m,i,j))     = 0;      # no arc

      CR.LO(h,i)               = 0;
      CR.UP(h,i)               = INF;
      CR.L(h,i)                = 0;
1392  CR.FX(h,i)$(not sum((m,j), n0(h,m,i,j))) = 0;
      *TCALC.LO(h,m,o,d)                        = -INF;
      *TCALC.UP(h,m,o,d)                        = +INF;
      *TCALC.FX(h,m,o,d)$(not n0(h,m,o,d))      = 0;

1397  urbanequil.limrow                        = 100;
      urbanequil.limcol                        = 100;
      urbanequil.iterlim                       = 0;
      urbanequil.holdfixed                     = 1;

1402  *    Check the calibrated integrated model

      parameter inccheck(h,o,d);

      u0(h,i,j)      = U.L(h,i,j);
1407  ph0(h,i)       = PH.L(h,i);
      y0(h,i)        = Y.L(h,i);
      rk0(h,i)       = RK.L(h,i);
      pch0(h,i)      = PCH.L(h,i);
      pl0(h,i)       = PL.L(h,i);
1412  inc0(h,i,j)    = INC.L(h,i,j);
      N0(h,m,i,j)    = N.L(h,m,i,j);
```

```
     MSHARE.LO(h,m,i,j)                              = -INF;
     MSHARE.UP(h,m,i,j)                              =  INF;
1417 MSHARE.FX(h,m,i,j)$(not mshareref(h,m,i,j))    = 0;
     MSHARE.FX(h,m,i,i)                              = 0;

     *    We infer the arcs used by household h traveling from o to d by using
     *    T(h,m,o,d) and tau(m,i,j) from the solution. We introduce a variable
1422 *    XR(m,i,j), which will indicate if this household took arc (i,j) and
     *    then loop over all households.

     set
         origin(i)
1427     destination(i),
         mc(m);


     nonnegative variable
1432     XR(m,i,j)        'Indicator if an arc is taken';

     variable
         TFX(m)          'Calculated time';

1437 parameter
         solroutechoice  'Solutions for route choice',
         routechoice     'Route choice',
         tfxref          'Reference value taken from benchmark solution',
         checkchoice     'Check for fractions',
1442     vc              'valuation of time';

     variable
         OPTTIME         'Minimization';

1447 equation
         objdeftime      'Objective'
         flowbalchoice   'Flow balance'
         calctime        'Calculated time';

1452 flowbalchoice(mc(m),i)$(not destination(i))..
         sum(a(m,i,j), XR(m,i,j)) =E= sum(a(m,j,i), XR(m,j,i))  + 1$origin(i);

     calctime(mc(m))..
         TFX(m) =E= sum((i,j)$arcsavail(m,i,j), roadtoll(m,i,j) / vc + TAU.L(m,i,j) *
             XR(m,i,j));

     objdeftime..
         OPTTIME =E= sum(mc(m),TFX(m));

     model route /calctime,  flowbalchoice, objdeftime/;

     *@ Simulations

     *    Define the cordons (center and greater center)
     set
1467     cordon(m,i,j)     Arcs for cordon policy in the center,
         cordong(m,i,j)    Arcs for cordon policy in the greater center,
         notcenterg(i)     Nodes not in the greater center
         notcenter(i)      Nodes not in the center;

1472 *    Define the arcs that are part of the cordon (where a toll is levied,
     *    when one enters the center)

     notcenterg(i) = not centerg(i);
     notcenter(i)  = not center(i);
1477 cordon("private",notcenter,center)$arcsavail("private",notcenter,center)       =
         YES;
     cordong("private",notcenterg,centerg)$arcsavail("private",notcenterg,centerg) =
         YES;
     alias(centerg, acenterg), (center,acenter),(cordon, acordon);
     alias(notcenterg,anotcenterg),(notcenter,anotcenter);

1482 display center, centerg, cordon, cordong;
     cordong("private",acenterg,centerg)$arcsavail("private",acenterg,centerg) = NO;

     parameter
         levelroadtoll        Overall level of road toll
1487     scroadtoll(*,m,i,j)  Scenario level of road tolls;

     *  We make the assumption that average income is 100'000 for the high income group
        .
     *  We take a road toll of 10 CHF times 200 working days. Therefore:


     parameter maxinc;
     parameter workingdays  /2000/;
     maxinc = smax((o,d), incref("hi",o,d));
```

```
1497  levelroadtoll = workingdays / inc0high * maxinc;

      scroadtoll("CordonG",m,i,j) = 0;
      scroadtoll("CordonG","private",i,j)$cordong("private",i,j) = levelroadtoll;
      scroadtoll("Cordon",m,i,j) = 0;
1502  scroadtoll("Cordon","private",i,j)$cordon("private",i,j) = levelroadtoll;

      display scroadtoll;
      PUTCLOSE dosbox  '@title ## Check Benchmark ##'
      EXECUTE 'titlemaker.cmd';

      option Savepoint = 1;
      display "* Benchmark check urban equilibrium";
      $batinclude solveproc BENCHM
      abort$(abs(urbanequil.objval) gt 1E-5) "**** Model urbanequil does not calibrate";
1512  execute 'mv -f urbanequil_p.gdx %basedir%%hhtype%_urbanequil_p.gdx';

      *py(h) = 2;
      *$include homogen.gms
      *urbanequil.iterlim = 1000000;
1517  *solve urbanequil using mcp;

      urbanequil.iterlim                             = 100000;

      urbanequil.reslim = 3600;  # Set time limit to 1h

      *  Set the modelname, so we can use it in the reporting
      $setglobal modelname urbanequil

      display "* Benchmark";
1527  *@@ Sc 1: Benchmark

      PUTCLOSE dosbox  '@title ## Benchmark ##'
      EXECUTE 'titlemaker.cmd';

1532  $if exist '%basedir%%hhtype%_urbanequil_benchm.gdx' execute_loadpoint '%basedir%%
          hhtype%_urbanequil_benchm.gdx';
      $if not exist '%basedir%%hhtype%_urbanequil_benchm.gdx' execute_loadpoint '%
          basedir%%hhtype%_urbanequil_p.gdx';

      option taxespaid:8;
      taxespaid(h,o,d) = 0;

      solve urbanequil using mcp;
      abort$(abs(urbanequil.objval) gt 1E-5) "**** Model urbanequil does not calibrate";
      execute 'mv -f urbanequil_p.gdx %basedir%%hhtype%_urbanequil_benchm.gdx';

1542  $batinclude solutionreport BENCHM econ

      urbanequil.iterlim  = 10000000;

      *@ Scenarios without economic sorting

      NN.FX(h,i,j)  = NN.L(h,i,j);
      NN.FX(h,i,j)$(not NN.L(h,i,j)) =0;
      UH.FX(h)      = 1;
      PL.FX(h,i)    = pl0(h,i);
1552  RK.FX(h,i)    = rk0(h,i);
      PH.FX(h,i)    = ph0(h,i);
      PK.FX         = 1;
      PCH.FX(h,i)   = pch0(h,i);
      Y.FX(h,i)     = y0(h,i);
1557  INC.FX(h,i,j) = inc0(h,i,j);
      INC.FX(h,i,i) = 0;
      U.FX(h,i,j)    = U.L(h,i,j);
      LS.FX(h,m,i,j)= LS.L(h,m,i,j);
      CR.FX(h,i)    = 0;

      urbanequil.holdfixed = 1;

      $ontext

1567  display "* Scenario with Transport model: Social Optimum";
      *@@ Social optimum
      PUTCLOSE dosbox  '@title ## Social optimum (no sorting) ##'
      EXECUTE 'titlemaker.cmd';
      solve urbanequilSO minimizing OBJ using NLP;
1572  $if exist '%basedir%%hhtype%_urbanequil_tcordon.gdx' Execute_loadpoint '%basedir%%
          hhtype%_urbanequil_tcordon.gdx';
      execute 'mv -f urbanequil_p.gdx %basedir%%hhtype%_urbanequil_tso.gdx';

      $batinclude solutionreport TSO econ

1577  $offtext
```

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```
       display "* Scenario with Transport model: Cordon toll";
       *@@ Cordon tolling in the center of the city (no sorting)
       PUTCLOSE dosbox  '@title ## Cordon tolling in the center (no sorting) ##'
1582   EXECUTE 'titlemaker.cmd';
       roadtoll("private",i,j) = scroadtoll("Cordon","private",i,j);
       $if exist '%basedir%%hhtype%_urbanequil_tcordon.gdx' Execute_loadpoint '%basedir%%
          hhtype%_urbanequil_tcordon.gdx';
       $if not exist '%basedir%%hhtype%_urbanequil_tcordon.gdx' Execute_loadpoint '%
          basedir%%hhtype%_urbanequil_benchm.gdx';
       solve urbanequil using mcp;
1587   execute 'mv -f urbanequil_p.gdx %basedir%%hhtype%_urbanequil_tcordon.gdx';
       convergence(counterset) = 0;
       $batinclude solutionreport TCORDT econ

       parameter checkbenchm(h,i,j);
1592   checkbenchm(h,i,j) = abs(nnx(h,i,j) + NN.L(h,i,j) - solution_hij(h,i,j,"N","BENCHM
          "));

       abort$(smax((h,i,j),checkbenchm(h,i,j) GT 1E-5)) "TCORDT: N.L(h,i,j) not constant"
          , checkbenchm;

       display "* Scenario wiht Transport model: Greater Cordon toll";
1597   *@@ Sc Cordon tolling in the greater center of the city (no sorting)
       PUTCLOSE dosbox  '@title ## Cordon tolling in the greater center (no sorting) ##'
       EXECUTE 'titlemaker.cmd';

       roadtoll("private",i,j) = scroadtoll("CordonG","private",i,j);
1602   $if exist '%basedir%%hhtype%_urbanequil_tcordong.gdx' Execute_loadpoint '%basedir
          %%hhtype%_urbanequil_tcordong.gdx';
       $if not exist '%basedir%%hhtype%_urbanequil_tcordong.gdx' Execute_loadpoint '%
          basedir%%hhtype%_urbanequil_benchm.gdx';
       solve urbanequil using mcp;
       convergence(counterset) = 0;
       execute 'mv -f urbanequil_p.gdx %basedir%%hhtype%_urbanequil_tcordong.gdx';
1607   $batinclude solutionreport TGCORDT econ


       checkbenchm(h,i,j) = abs(nnx(h,i,j) + NN.L(h,i,j) - solution_hij(h,i,j,"N","BENCHM
          "));

1612   abort$(smax((h,i,j),checkbenchm(h,i,j) GT 1E-5)) "TGCORDT: N.L(h,i,j) not constant
          ", checkbenchm;

       * solve urbanequil using mcp;

       roadtoll("private",i,j) = 0;

       *@ Scenarios with economic sorting

       UH.LO(h)      = -INF;
       UH.UP(h)      = +INF;
1622   UH.L(h)       = 1;

       U.LO(h,i,j) = -INF;
       U.UP(h,i,j) = +INF;
       U.FX(h,i,j)$(not sum(m, n0(h,m,i,j))) = 0;

       PL.LO(h,i)      = 0;
       PL.UP(h,i)      = +INF;
       PL.L(h,i)       = pl0(h,i);

1632   RK.LO(h,i)       = 0;
       RK.UP(h,i)       = +INF;
       RK.L(h,i)        = rk0(h,i);

       PH.LO(h,i)      = 0;
1637   PH.UP(h,i)      = INF;
       PH.L(h,i)       = ph0(h,i);

       PK.LO           = -INF;
       PK.UP           = INF;
1642   PK.L            = 1;

       PCH.LO(h,i)     = -INF;
       PCH.UP(h,i)     = INF;
       PCH.L(h,i)      = pch0(h,i);

       Y.LO(h,i)       = 0;
       Y.UP(h,i)       = INF;
       Y.L(h,i)        = y0(h,i);

1652   N.LO(h,m,i,j) = 0;
       N.UP(h,m,i,j) = INF;
       N.FX(h,m,i,j)$(not n0(h,m,i,j)) = 0;
```

```
      NN.LO(h,i,j)    = 0;
1657  NN.UP(h,i,j)    = +INF;
      NN.L(h,i,j)     = NN.L(h,i,j);
      NN.FX(h,i,j)$(not  NN.L(h,i,j)) = 0;

      INC.LO(h,i,j) = -INF;
1662  INC.UP(h,i,j) = +INF;
      INC.FX(h,i,j)$(not NN.L(h,i,j)) = 0;

      LS.LO(h,m,i,j) = -INF;
      LS.UP(h,m,i,j) = INF;
1667  LS.FX(h,m,i,j)$(not N.L(h,m,i,j)) = 0;

      CR.LO(h,i)      = 0;
      CR.UP(h,i)      = INF;
      CR.L(h,i)       = 0;
1672  CR.FX(h,i)$(not sum((m,j), n0(h,m,i,j))) = 0;
      T.LO(h,m,i,d) = 0;
      T.UP(h,m,i,k) = INF;
      T.FX(h,m,i,i) = 0;
      T.FX(h,m,i,j) = T.L(h,m,i,j);


*     display "* Scenario with sorting: Social Optimum";
*     *@@ Social optimum with sorting
*     PUTCLOSE dosbox  '@title ## Social optimum (with sorting) ##'
1682  *     EXECUTE 'titlemaker.cmd';
*     solve urbanequilSO minimizing OBJ using NLP;
*     $if exist '%basedir%%hhtype%_urbanequil_tcordon.gdx' Execute_loadpoint '%basedir
           %%hhtype%_urbanequil_tcordon.gdx';
*     execute 'mv -f urbanequil_p.gdx %basedir%%hhtype%_urbanequil_tso.gdx';
*     $batinclude solutionreport SO econ
1687  * T.LO(h,m,i,d)                            = 0;
*     * T.UP(h,m,i,k)                           = INF;
*     * T.FX(h,m,i,i)                           = 0;

*@@ Cordon tolling in the center of the city
1692  display "* Scenario Cordon toll";

      roadtoll("private",i,j) = scroadtoll("Cordon","private",i,j);

      $if exist '%basedir%%hhtype%_urbanequil_tcordon.gdx' Execute_loadpoint '%basedir%%
           hhtype%_urbanequil_tcordon.gdx';
1697  $if exist '%basedir%%hhtype%_urbanequil_cordon.gdx' Execute_loadpoint '%basedir%%
           hhtype%_urbanequil_cordon.gdx';
      $if not exist '%basedir%%hhtype%_urbanequil_cordon.gdx' Execute_loadpoint '%
           basedir%%hhtype%_urbanequil_benchm.gdx';

      PUTCLOSE dosbox  '@title ## Cordon tolling in the center (sorting) ##'
      EXECUTE 'titlemaker.cmd';

      $batinclude solveproc CORDT
      option solroutechoice:0:5:1;
      display solroutechoice;

1707  execute 'mv -f urbanequil_p.gdx %basedir%%hhtype%_urbanequil_cordon.gdx';

      $batinclude solutionreport CORDT econ

      display "* Scenario Greater Cordon toll";
1712  *@@ Cordon tolling in the greater center of the city
      PUTCLOSE dosbox  '@title ## Cordon tolling in the greater center (sorting) ##'
      EXECUTE 'titlemaker.cmd';

      roadtoll("private",i,j) = scroadtoll("CordonG","private",i,j);

      $if exist '%basedir%%hhtype%_urbanequil_tcordong.gdx' Execute_loadpoint '%basedir
           %%hhtype%_urbanequil_tcordong.gdx';
      $if exist '%basedir%%hhtype%_urbanequil_cordong.gdx' Execute_loadpoint '%basedir%%
           hhtype%_urbanequil_cordong.gdx';
      $if not exist '%basedir%%hhtype%_urbanequil_cordong.gdx' Execute_loadpoint '%
           basedir%%hhtype%_urbanequil_benchm.gdx';

1722  $batinclude solveproc GCORDT
      execute 'mv -f urbanequil_p.gdx %basedir%%hhtype%_urbanequil_cordong.gdx';
      $batinclude solutionreport GCORDT econ

      roadtoll("private",i,j) = 0;

      NN.LO(h,i,j) = 0;
      NN.UP(h,i,j) = +INF;
      NN.FX(h,i,j)$(not sum(m, n0(h,m,i,j))) = 0;

1732  N.LO(h,m,i,j) = 0;
      N.UP(h,m,i,j) = +INF;
```

```
       N.FX(h,m,i,j)$(not n0(h,m,i,j)) = 0;

       UH.LO(h)    = -INF;
1737   UH.UP(h)    = +INF;
       UH.L(h)     = 1;

       PL.LO(h,i) = 0;
       PL.UP(h,i) = +INF;
1742   PL.L(h,i)   = pl0(h,i);

       RK.LO(h,i) = 0;
       RK.UP(h,i) = +INF;
       RK.L(h,i)   = rk0(h,i);

       PH.LO(h,i) = 0;
       PH.UP(h,i) = INF;
       PH.L(h,i)   = ph0(h,i);

1752   PK.LO       = -INF;
       PK.UP       = INF;
       PK.L        = 1;

       PCH.LO(h,i) = -INF;
1757   PCH.UP(h,i) = INF;
       PCH.L(h,i)  = pch0(h,i);

       Y.LO(h,i)   = 0;
       Y.UP(h,i)   = INF;
1762   Y.L(h,i)    = y0(h,i);

*@ Sensitivity analysis

       option Savepoint=2;

* Show the scenario, loop number or other information in the DOS window
       Set
           sens  'Sensitivity scenarios' /sens1*sens100/;

1772   parameter
           sensitivity(*,sens)  Results of sensitivity analysis;

*   Initialize some of the values, so in case of no SA, so gdxxrw
*   doesn't raise an error if no SA is carried out.

       sensitivity(m,sens)  = EPS;

       $if %sensitivity%==no  $goto nosensitivity

1782   display "* Sensitivity analysis";
       option Savepoint=0;
       parameter
           sigma_sens(h,sens)          'SA Elasticity values for leisure',
           sigmah_sens(h,sens)          'SA Elasticity values for CES'
1787   lambda_sens(h,sens)             'SA Logit elasticity',
           lambdajim_sens(h,j,i,sens)  'SA Logit elasticity';

       loop(sens,
           sigma_sens(h,sens)        = uniform(0.5,1.5) * sigma0(h);
1792   sigmah_sens(h,sens)         = uniform(0.5,1.5) * sigmah0(h);
           lambda_sens(h,sens)       = uniform(0.5,1.5) * lambda0(h);
           lambdajim_sens(h,i,j,sens) = uniform(0.5,1.5) * lambdajim0(h,i,j);
       );

1797   parameter
           counter     'Counter for loop',
           totalloops 'Total loop number';

       totalloops = card(sens);

       counter = 0;
       loop(sens,
               sigma(h)          = sigma_sens(h,sens);
               sigmah(h)         = sigmah_sens(h,sens);
1807           lambda(h)         = lambda_sens(h,sens);
               lambdajim(h,i,j) = lambdajim_sens(h,i,j,sens);

       $include calibsens

1812           counter = counter + 1;
               PUT screen;
               PUT /;
               PUT / ' ######  Loop: ', counter:6:0, '  ######' / ;
               PUT /;
1817           PUTCLOSE;
               PUTCLOSE dosbox '@title Loop number: ', counter:6:0, ' of ',totalloops
                  :6:0;
```

```
            EXECUTE 'titlemaker.cmd';

      roadtoll("private",i,j)$cordon("private",i,j) = 4;
1822      solve urbanequil using mcp;

            sensitivity(m,sens)    = sum((h,i,j), N.L(h,m,i,j));
            sensitivity("Change in private transport",sens)  = 100 * (sensitivity("
                private",sens)
            / solution_overall("public","Benchm") − 1);
1827      sensitivity("Change in public transport",sens)   = 100 * (sensitivity("
                public",sens)
            / solution_overall("public","Benchm") − 1);

            sensitivity("Total Time",sens)              = sum(a, TAU.L(a) * sum((h,k), X
                .L(h,a,k))) / 60;
            sensitivity("Change in total delay",sens) = 100*(sensitivity("Total Time",
                sens) /
1832      solution_overall("Total Time (h)","Benchm") − 1);
      sensitivity("Sigma LS high income",sens)  = sigma("hi");
      sensitivity("Sigma LS low income",sens)   = sigma("lo");
      sensitivity("Sigma H high income",sens)   = sigmah("hi");
      sensitivity("Sigma H low income",sens)    = sigmah("lo");
1837 );


   execute_unload "resultsdir%%city%\%hhtype%_results_%city%_sens", sensitivity;

1842 $label nosensitivity

   *@ Export results to pivot tables
   execute_unload "%resultsdir%%city%\%hhtype%_results_%city%", solution_h,
      solution_he, solution_hu,  solution_hij, solution_i, solution_mij, solution_hi
      , solution_hmij,  solution_overall, solution_hm,solve_statistics,
      solution_solver;
   * solution_m,

   $onecho > taskout.txt
   par=solution_overall   rng=overall!a2     rdim=2
   par=solve_statistics   rng=solvestat!a2   rdim=2
   par=solution_i         rng=i!a2           rdim=3
1852 par=solution_h         rng=h!a2           rdim=3
   par=solution_he        rng=he!a2          rdim=3
   par=solution_hu        rng=hu!a2          rdim=3
   *par=solution_m         rng=m!a2           rdim=3
   par=solution_hm        rng=hm!a2          rdim=4
1857 par=solution_solver    rng=conv!a2        rdim=3
   par=solution_hij       rng=hij!a2         rdim=5
   par=solution_hi        rng=hi!a2          rdim=4
   par=solution_mij       rng=mij!a2         rdim=5
   par=solution_hmij      rng=hmij!a2        rdim=6
1862 $offecho

   $onecho > taskout2.txt
   par=ownelasjim         rng=eT!a2          rdim=4
   par=crosselasjim       rng=ecrossT!a2     rdim=5
1867 par=ownelasij          rng=eU!a2          rdim=3
   par=crosselasij        rng=ecrossU!a2     rdim=5
   par=qownelasjim        rng=qeT!a2         rdim=4
   par=qcrosselasjim      rng=qecrossT!a2    rdim=5
   par=phiji m            rng=phiT!a2        rdim=4
1872 var=phi_ij             rng=phiU!a2        rdim=3
   $offecho

   execute 'xlstalk −S %resultsdir%%city%\%hhtype%_results_%city%.xlsx'
   execute 'gdxxrw.exe o=%resultsdir%%city%\%hhtype%_results_%city%.xlsx i=.\results
      \%city%\%hhtype%_results_%city%.gdx EpsOut=0 @taskout.txt trace=3'
1877 *execute 'gdxxrw.exe o=%resultsdir%%city%\%hhtype%_results_%city%.xlsx i=.\results
      \%city%\%hhtype%_elas_%city%.gdx EpsOut=0 @taskout2.txt trace=3'
   execute 'xlstalk −O %resultsdir%%city%\%hhtype%_results_%city%.xlsx'
```

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3.D.2 Algorithm for finding paths

Listing 3.2 – Calibration

```
1   inccheck(h,o,d) = incref(h,o,d);
    criterium       = 1;
    counter         = 1;

    while(criterium ge 0.01 AND counter LE 30,
6       display counter;
        solve urbanequil using mcp;
        route.solvelink = 2;
        option solprint = silent;
        loop((h,o,d)$(sum(m, N.L(h,m,o,d))),
11          vc  = v(h);
            origin(i) = YES$sameas(i,o);
            destination(i) = YES$sameas(i,d);
            mc(m) = YES$N.L(h,m,o,d);
            PUTCLOSE dosbox  '@title ##  %1: Paths ##  Counter: ', counter, ', Origin:
                ', O.TL,', Destination: ' D.TL,'  ##'  ;
16          EXECUTE 'titlemaker.cmd';
            tfxref(m) = T.L(h,m,o,d);
            TFX.L(m)  = T.L(h,m,o,d);
            solve route using LP minimizing OPTTIME;
            routechoice(h,m,o,d,i,j) = XR.L(m,i,j);
21          checkchoice(h,m,o,d,i,j)$XR.L(m,i,j)= XR.L(m,i,j) − 1;
        );
        criterium = sum((h,o,d),abs(INC.L(h,o,d) − inccheck(h,o,d)));
        inccheck(h,o,d) = INC.L(h,o,d);
        counter = counter+ 1;
26      option solprint = on;
        display criterium;
        taxespaid(h,o,d)$INC.L(h,o,d) = sum((m,i,j), routechoice(h,m,o,d,i,j) *
            roadtoll(m,i,j));
        option routechoice:1:4:2;
        display checkchoice, routechoice;
31      option solprint = on;
        display taxespaid, routechoice;
        loop(counterset$(ord(counterset) = counter),convergence(counterset) =
            criterium);
    );

36  display roadtoll;
    solve urbanequil using mcp;
    solroutechoice("%1",h,m,o,d,i,j) = routechoice(h,m,o,d,i,j);
```

!

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Curriculum Vitae

Personal Information

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Education

- since November, 2009: PhD student at the Center for Energy Policy and Economics (Federal Technical University Zurich), Professor T. F. Rutherford and Professor Sebastian Rausch.

- 2006 - 2009: Advanced Studies in Applied Statistics, Swiss Technical University Zurich (Switzerland), Diploma

- 1985 - 1990: Economics and Mathematics,University Berne (Switzerland), lic. rer. pol

- 1980 - 1984: Physiotherapy, Stichting Utrechtse Paramedische Akademie, Utrecht (Netherlands), Diploma

- 1977 - 1979: Physics and Mathematics, University Utrecht (Netherlands)

- 1971 - 1977: Reformatorisch College Blaise Pascal, Zaandam (Netherlands), Diploma

Positions

- since 2008: Owner and Director Modelworks, Consulting and Coaching in Modeling and Statistics, Thun, Switzerland.

- 1992-2012: Senior Consultant Ecoplan, Specialist for modeling and statistics, IT-responsible, Berne, Switzerland.

- 1988 - 1992: Assistant the Institute for Applied Microeconomics, Prof. G. Stephan, Berne, Switzerland.

- 1987 - 1988: Physiotherapist (30 - 50%), Hospital Permance, Bern

- 1986 - 1987: Physiotherapist (25 - 50%), Praxis Peter Kunz, Wabern

- 1984 - 1985: Physiotherapist, Hospital Sonnenhof, Bern

Teaching and Coaching

- since 2012: Lecturer ETHZ (Urban and Spatial Economics)

- since 2003: Ecomod, Workshops applied computational equilibrium modeling, Brussels, Washington, Bangkok

- 2011: Urban Economics at the Technical University Zurich (assisting Prof Rutherford)

- 2008 - 2009: Workshops CGE Modeling Landcare Institute New Zealand and coaching

- 2008: Coaching for the FAO (Food and Agriculture Organization, Rome)

- 2008: Workshops on CGE Modeling for the National Bank of Poland

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