Numerical study on the influence of particle inertia in turbidity currents

Report

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Publication date: 2014

Permanent link: https://doi.org/10.3929/ethz-a-010284328

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Numerical study on the influence of particle inertia in turbidity currents

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Internal Report
Completed November 2011
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Abstract

The influence of particle inertia in turbidity currents is studied by 2D simulations with a Lagrangian-Eulerian description in the lock-exchange configuration. Besides the equations solved, also the analog Eulerian-Eulerian, the equilibrium Eulerian and the vorticity equations are presented. The typical scales of turbidity currents in laboratory experiments and in nature are illustrated by examples, together with implications for settling enhancement.

Simulations are performed for five different Stokes numbers. For the higher Stokes numbers preferential accumulation is observed at the top of the current and dilution is observed inside the vortical structures above the body of the current. Further, it is observed that less vorticity is created at the initial vertical suspension/clear-fluid interface. This leads to weaker start-up vortices, which in return results in a smaller front velocity overshoot and a higher settling rate at the beginning of the inertial phase. The start-up vortices are identified as the cause for the distinct head of gravity currents in the lock-exchange configuration in contrast to other configurations such as the continuous flow setup. Additionally, the mass-loading parameter is shown to be the governing parameter for the front acceleration.
1 Introduction

Turbidity currents are a subclass of buoyancy-driven flows occurring in lakes and oceans, where the density difference stems from small suspended particles. They are responsible for many topographical features on the seafloor through erosion and sediment deposition (Meiburg & Kneller, 2010). They are important agents in the global sediment cycle as they transport sediment from near-coastal areas to the deep sea (Meiburg & Kneller, 2010). Turbidity currents contribute to the buildup of hydrocarbon reservoirs, but also present a serious hazard to marine engineering installations (Meiburg & Kneller, 2010). They can occur when transported sediment from estuaries reaches the ground or due to sediment failures on continental slopes (Meiburg & Kneller, 2010).

Typical transport distances range from a few hundred meters or less to thousands of kilometers on the continental rise and the abyssal plain (Meiburg & Kneller, 2010). Typical heights of turbidity currents are $O\left(10^{-10} \right)$ m, and typical front velocities of order $0.1-10\text{m/s}$ (Meiburg & Kneller, 2010). The suspended particulate phase consists mainly of silt and clay. The volume fractions are estimated to be around $0.1-7\%$ (Meiburg & Kneller, 2010). The ambient and the interstitial fluid is water, whereas the differences in salinity may give rise to additional density differences. A more general overview on the subject of turbidity currents can be found in the review by Meiburg & Kneller (2010).

Field measurements of turbidity currents are afflicted with severe practical problems such as the hindered access, the infrequent and practically unpredictable occurrence and the destructive nature of turbidity currents (Meiburg & Kneller, 2010). Hence, the research community depends strongly on mathematical modeling, laboratory-scale experiments and numerical simulations. Currently the most elaborate field measurement of turbidity currents was done by Xu et al. (2004) who presented vertical velocity profiles from in-situ measurements of four turbidity current events in the Monterey canyon.

Laboratory scale experiments are often performed in the lock-exchange configuration (see figure 1), where the suspension is initially separated from the clear fluid by a gate sealing the entire cross-section of the channel. Bonnecaze et al. (1993) investigated the front velocity and the final deposit of monodisperse turbidity currents for different grain sizes and mass-loadings. Gladstone et al. (1998) extended this research to bi- and polydisperse particle distributions. Gladstone & Woods (2000) also varied the interstitial fluid and compared the experimental results to the predictions of the box model. De Rooij & Dalziel (2001) performed time- and space-resolved measurements of the deposit thickness for lock-release and continuous flow setups. They found that a distinct head is formed in the lock-exchange configuration, and that the settling rate in the head is higher than in the body and tail of the current. For continuous currents, on the other hand, the head is not as distinct and the settling occurs homogeneously over the entire current. Al-Musallami & Al-Ja’aidi (2008) measured the average velocity field in lofting turbidity currents using ultrasonic Doppler velocity profiling. Gladstone & Pritchard (2010) evaluated the deposit thickness of mono- and bidisperse, lofting turbidity currents and compared their experimental results to the predictions of the box-and-cloud-model. More experimental work can be found in the review by Kneller & Buckee (2000).

The first DNS of turbidity currents in the lock-exchange configuration was presented by Necker et al. (2002). The particulate phase is modeled in an Eulerian fashion without inertia. Blanchette et al. (2005) extended the model used by Necker et al. (2002) to account for erosion and re-suspension and presented 2D simulations with this approach for different ground inclinations. 3D simulations for different ground inclinations without resuspension were performed by Blanchette et al. (2006). Necker et al. (2005) performed DNS for the deeply submerged lock-exchange configuration.

Highly resolved LES of turbidity currents were performed by Ooi et al. (2009, 2007). These
studies used a Smagorinsky-like closure model. Henniger (2011) performed highly resolved LES simulations with the Relaxation-Term model.

The aforementioned numerical studies all used an Eulerian concentration approach to describe the particulate phase, neglecting the inertia of the particles. Weak inertial effects of the particles were investigated by Cantero et al. (2008) using the equilibrium Eulerian approach for the particulate phase.

One main motivation for including particle inertia as a model feature is particle settling enhancement by turbulence. Maxey (1987a,b); Maxey & Corrsin (1986) performed analytical work on small particles in different flow-fields where it was observed that the particles settle faster and accumulate in regions of high strain and low vorticity due to the inertia of the particles. Numerical (Bosse et al., 2006; Wang & Maxey, 1993, e.g.) and experimental (Aliseda et al., 2002; Yang & Shy, 2003) studies of small and heavy particles in homogeneous and near isotropic turbulence confirmed and quantified these findings. The effective settling velocity of the particles in the turbulence was observed to increase up to 150%, depending on the turbulence intensity, the particle volume fraction and the Stokes number. One important finding is that the settling enhancement is most distinct if the Stokes number based on the Kolmogorov scale is close to unity.

A wide range of studies is performed on preferential accumulation in the absence of gravity. Elghobashi & Truesdell (1992) presented DNS of particles with one-way coupling in decaying homogeneous isotropic turbulence. Two-way coupling was performed by Boivin et al. (1998). More recently, Scott et al. (2009) studied a wider parameter range and introduced a new measure to quantify accumulation. Shotorban & Balachandar (2009) investigated the suitability of the equilibrium Eulerian approach on this subject. More on preferential accumulation and turbulence modification due to the presence of particles can be found in the reviews by Elghobashi (1994) and Balachandar & Eaton (2010).

In a similar context, Druzhinin (1995) analytically investigated three planar, canonical flow configurations with particles, where special attention was paid to the vorticity. Meiburg et al. (2000) numerically studied the influence of the two-way coupling in particle-laden shear layers with emphasis on fluid vorticity field.

The use of Lagrangian particles introduces additional tasks compared to Eulerian methods. The fluid velocity needs to be interpolated to the particle position. The accuracy of different interpolation schemes was studied by Balachandar & Maxey (1989). In the case of two-way coupling, the force from the individual particles has to distributed to the Eulerian grid. The quality of the resulting Eulerian two-way coupling term depends on the order of the Kernel function which is used to distribute the Lagrangian source-term to the Eulerian grid (discretization error), and on the number of computational particles per fluid cell (statistical error). For more on this subject, the reader is referred to Schmidt (2006).

The current work aims at studying the influence of particle inertia in turbidity currents. To this end, 2D simulations using the Lagrangian-Eulerian approach are performed in the lock-exchange configuration for different particle Stokes numbers. The domain and the related nomenclature are shown in figure 1.

The structure of the document is as follows. In section 2, the governing equations for the Lagrangian-Eulerian approach are introduced, together with the made assumptions. Further, the analog Eulerian-Eulerian equations, the equilibrium Eulerian approach, the vorticity equations and the energy balance are presented. Typical scales of turbidity currents in laboratory experiments and in nature are briefly outlined in section 3, together with the implications on settling enhancement. The numerical approach is described in section 4, and the simulation results are presented in section 5. Section 6 contains the conclusions.

The current work is a continuation of Häuselmann (2010). The reader is referred to this master’s thesis for the comparison of the Lagrangian-Eulerian to the Eulerian-Eulerian method for inertialess particles.
Figure 1: Sketch of the initial lock-exchange configuration in a plane channel of length $\tilde{L}_1$, width $\tilde{L}_2$ and height $\tilde{L}_3$. The shaded area illustrates the suspension reservoir (length $\tilde{L}_1$, width $\tilde{L}_2 = \tilde{L}_3$, height $\tilde{L}_3 = \tilde{L}_3$) filled with particle-laden fluid. The remaining part of the channel contains clear fluid. Gravity acts in the negative $\tilde{x}_3$ direction.
2 Equations

2.1 Governing equations

The criteria which have to be met for the following equations to be valid are that the particles are spherical, monodisperse and much smaller than the smallest flow-scales (Bosse, 2005). Further, the suspension must be dilute in order to neglect particle-particle interactions, i.e. \( \phi_{\text{max}} \lesssim 10^{-3} \) (Bosse, 2005; Elghobashi, 1994), and to neglect the volume displacement of the particles in the continuity equation (Necker, 2002). The considered particle-forces are the Stokes drag and the gravitational force. It should be noted that it was not investigated if other particle forces are relevant as well. For low particle to fluid density ratios, the added mass and the fluid acceleration term may become relevant as well, and in regions with strong vorticity (e.g. in the boundary layer or at the suspension/clear-fluid interface at the front) the lift forces may become important. For more details on particle forces, see e.g. Ferry & Balachandar (2001); Kubik (2007); Maxey & Riley (1983).

The set of equations solved in the simulations are the continuity equation for an incompressible fluid

\[
\frac{\partial u_i}{\partial x_i} = 0, \tag{1}
\]

the momentum equation for the fluid

\[
\frac{D u_i}{D t} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_m \partial x_m} - \frac{Ri}{v_s} \frac{1}{\phi_{V,0}} \sum_{j=1}^{n^r} (u_{i,j} - v_{i,j}) \cdot w_p (x - y_j), \tag{2}
\]

the momentum equation for each particle

\[
\frac{d v_{i,j}}{dt} = \frac{1}{St} (u_{i,j} - v_{i,j} - v_s \delta_{i3}), \tag{3}
\]

and the Lagrangian transport equation for each particle

\[
\frac{d y_{i,j}}{dt} = v_{i,j}. \tag{4}
\]

The function \( w_p (\xi) \) equals unity for \( |\xi| \leq r \) and zero otherwise, whereas \( r \) is the radius of the particles. Further, \( v_{i,j} \) and \( y_{i,j} \) are the velocity and the location in direction \( i \) of particle \( j \), respectively, and \( u_{i,j} \) is the fluid velocity at the particle position. \( \phi_V \) is the particle volume fraction, and \( \phi_{V,0} \) the initial particle volume fraction in the reservoir.

The governing dimensionless parameters are the Reynolds number, the Stokes number, the Richardson number, and the normalized Stokes settling velocity, i.e.

\[
Re = \frac{\bar{u}_{\text{ref}}}{\nu}, \tag{5}
\]

\[
St = \frac{2 \rho_r r^2 \bar{u}_{\text{ref}}}{9 \bar{\rho} \bar{\nu} \bar{l}}, \tag{6}
\]

\[
Ri = \frac{\bar{g} \bar{l}}{\bar{u}_{\text{ref}}^2}, \text{ and} \tag{7}
\]

\[
v_s = \frac{\bar{v}_s}{\bar{u}_{\text{ref}}}, \tag{8}
\]

respectively. The Stokes settling velocity \( \bar{v}_s \) is given by

\[
\bar{v}_s = \frac{2}{9} \frac{r^2}{\bar{\nu}} \left( \frac{\bar{\rho}_p}{\bar{\rho}} - 1 \right) \bar{g}. \tag{9}
\]
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The characteristic scales $\tilde{l}$ and $\tilde{u}_{ref}$ are taken as half the domain height

$$\tilde{l} = \frac{\tilde{H}}{2}$$

and the buoyancy velocity of the initial particle suspension in the reservoir

$$\tilde{u}_b = \sqrt{\tilde{g} \tilde{l}}$$

respectively, whereas $\tilde{g}'$ is the reduced gravity given by

$$\tilde{g}' = \phi \frac{V}{\phi V_0} \left( \tilde{\rho}_p - 1 \right) \tilde{g}.$$ 

For these specific characteristic scales, the Richardson number equals unity by definition. However, it is kept in the equations for completeness.

2.2 Derivatives of the governing equations

To interpret the results, it is illustrative to switch from a Lagrangian to an Eulerian description for the particles. This is achieved by integrating the two-way coupling term in the fluid momentum equation (2) over a small volume (e.g. the computational cell volume), dividing by that volume and by substituting the particle transport equation (4) with the Eulerian advection equation for the volume fraction of the particles. A more complete derivation can be found in e.g. Meiburg et al. (2000). By introducing the normalized volume fraction $c = \phi V / \phi V_0$, the governing equations in fully Eulerian form become

$$\frac{\partial u_i}{\partial x_i} = 0,$$

$$\frac{D u_i}{Dt} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_m \partial x_m} - \frac{R_i}{v_s} c (u_i - v_i),$$

$$\frac{D v_i}{Dt} = \frac{1}{St} (u_i - v_i - v_s \delta_{i3}),$$

and

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial x_i} (v_i \cdot c) = 0.$$ 

Unlike before, the particle velocity $v$ in the set of equations (13)–(16) is a continuous velocity field; hence the index $j$ is dropped. This requires all particles within a small volume to have the same velocity, i.e. the particle velocity field to be single-valued (Balachandar, 2009; Meiburg et al., 2000). This is the case for sufficiently low Stokes numbers. More quantitative criteria can be found in Ferry & Balachandar (2001); Meiburg et al. (2000). The total derivative in the particle momentum equation (3) becomes a substantial derivative in the Eulerian analogon (15). The subscript $v$ indicates that the material derivative is to be taken along the particle path. Hence, the advection velocity is the particle velocity $v$, i.e.

$$\frac{D_x}{D_v} (.) = \frac{\partial}{\partial t} (.) + \frac{\partial}{\partial x_m} (v_m \cdot .).$$

Notable about the particle transport equation (16) is that it does not contain a diffusion term and that the particle velocity cannot be taken out of the derivative in the advection term since the particle velocity field is not necessarily divergence-free. The diffusion term in the particle/concentration transport equation used by e.g. Henniger (2011); Necker (2002) is introduced mainly for numerical reasons. Bonometti & Balachandar (2008); Necker (2002) have found this to have a negligible influence for a Schmidt number of $Sc = 1$, given that the Reynolds
number is sufficiently high. Häuselmann (2010) has found finer structures for Sc = 10 compared to Sc = 1 for 2D simulations of lock-exchange flows with a Reynolds number of 2236. These finer structures lead to faster particle settling and to a lower front velocity.

For sufficiently small Stokes numbers, the equilibrium expansion for the particle velocity can be used to further simplify the equations (see e.g. Druzhinin, 1995; Ferry & Balachandar, 2001). For the first order approximation, the particle velocity is directly obtained from the local fluid quantities at the particle position. This expression in return can be inserted into the Stokes drag term of equation (14). Hence, the equations (13)–(16) become

$$\frac{\partial u_i}{\partial x_i} = 0,$$

$$\frac{Du_i}{Dt} = \frac{1}{1 + \Gamma} \left( - \frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_m \partial x_m} - \Gamma \cdot \frac{v_s}{St} \cdot \delta_{i3} + \Gamma \cdot v_s \cdot \frac{\partial u_i}{\partial x_j} \delta_{j3} \right),$$

$$v_i = u_i - v_s \delta_{i3} - St \cdot \left( \frac{Du_i}{Dt} - v_s \cdot \frac{\partial u_i}{\partial x_j} \delta_{j3} \right), \text{ and}$$

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial x_i} (v_i \cdot c) = 0.$$  

Balachandar (2009) suggests the first order approximation of the equilibrium expansion to be a good approximation for $St_\eta \lesssim 0.2$. The subscript $\eta$ indicates the Stokes number based on the Kolmogorov scales (see section 3). In equation (19), the mass loading parameter $\Gamma = \frac{\phi V \cdot \tilde{\rho}_p}{\tilde{\rho}}$ is introduced. This can be written in terms of the the dimensionless parameters (5)–(8) as

$$\Gamma = St \cdot Ri \cdot \frac{v_s}{c}.$$ 

The set of equations (18)–(21) are very similar to the governing equations for the non-Boussinesq lock-exchange flow with passively advected particles (see e.g. Henniger, 2011). The main difference is the last term in equation (20). This term renders the particle velocity field to be not divergence free, and hence allows for particle accumulation.

By taking the curl of the momentum equation (14), the vorticity equation

$$\frac{D\omega_f}{Dt} = \left( \omega_f \cdot \nabla \right) u + \frac{1}{Re} \Delta \omega_f + \frac{Ri}{v_s} c \left( \omega_p - \omega_f \right) - \frac{Ri}{v_s} c \left( v - u \right) \times \nabla c$$

is obtained, where $\omega_f$ and $\omega_p$ are the vorticity of the fluid and the particle velocity field, respectively. The terms on the right-hand side describe the vortex stretching, the vorticity diffusion, the coupling of the particle and the fluid vorticity, and the production due to the misalignment of the slip velocity and the concentration gradient.

The transport equation for the particle vorticity is obtained by taking the curl of the particle momentum equation (15), which gives

$$\frac{D_c \omega_p}{Dt} = \left( \omega_p \cdot \nabla \right) v - \frac{1}{St} \left( \omega_p - \omega_f \right).$$
2.3 Integral quantities

The integral quantities considered here are the mass of the suspended particles $m_p$, the front position $y_f$, the front velocity $v_f$, the particle deposit $D$, and the energy budget. Here, only a short summary of the energy budget is presented for the sake of completeness. Further information can be found in Häuselmann (2010).

The energy budget for particle-driven gravity currents consists of the kinetic energy of the fluid $K_f$, the potential and kinetic energy of the particles $E_{pot}$ and $K_p$, respectively, the dissipated energy of the resolved flow-scales $E_d$, the dissipated energy of the modeled Stokes flow around the particles $E_s$ and the lost kinetic energy of the particles due to settling $Q_p$. The according relations for these quantities are

$$K_f = \frac{1}{2} \int_{\Omega} u_i u_i dV, \quad (25)$$

$$E_{pot} = Ri \cdot \frac{V_{res}}{n_{p0}} \sum_{j=1}^{n_p} y_{3,j}, \quad (26)$$

$$K_p = \frac{1}{2} Ri \cdot \frac{S_t}{v_s} \cdot \frac{V_{res}}{n_{p0}} \sum_{j=1}^{n_p} u_{i,j} v_{i,j}, \quad (27)$$

$$E_d = \int_0^t \varepsilon(\tau) d\tau, \quad \text{and} \quad (28)$$

$$E_s = \int_0^t \varepsilon_s(\tau) d\tau. \quad (29)$$

The quantities $\varepsilon$ and $\varepsilon_s$ are the dissipation rates of the resolved flow-scales and of the modeled Stokes flow, respectively. They are given by

$$\varepsilon = \int_{\Omega} \frac{2}{Re} s_{kl} s_{kl} dV = \int_{\Omega} \frac{1}{Re} u_i \cdot \frac{\partial^2 u_i}{\partial x_j \partial x_j} dV \quad \text{and} \quad (30)$$

$$\varepsilon_s = \frac{R_i}{v_s} \cdot \frac{V_{res}}{n_{p0}} \sum_{j=1}^{n_p} (u_{i,j} - v_{i,j})(u_{i,j} - v_{i,j}), \quad (31)$$

where $s_{kl}$ is the rate of strain tensor

$$s_{kl} = \frac{1}{2} \left( \frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k} \right). \quad (32)$$

Since all energy losses of the system are accessible, the total energy

$$E_{tot} = K_f + E_{pot} + K_p + E_d + E_s + Q_p \quad (33)$$

over time is constant and can be used as a measure for the quality of the simulation. To interpret the results, especially when comparing simulations with different Stokes numbers, it is advantageous to introduce the total kinetic energy in the system as well, i.e.

$$K_{tot} = K_f + K_p. \quad (34)$$
3 Typical scales of turbidity currents

3.1 Scaling with respect to settling enhancement

The main motivation for studying the influence of particle inertia in turbidity currents is the observation that turbulence can have an enhancing effect on the effective particle settling velocity. This effect is most distinct if the particle Stokes number and the dimensionless settling velocity, both based on the Kolmogorov scales, are close to unity (Aliseda et al., 2002; Bosse, 2005; Wang & Maxey, 1993; Yang & Shy, 2003).

The characteristic scales introduced in section 2.1 are characteristic for the large flow-scales, and hence the dimensionless parameters (5)–(8) are representative of these scales as well. An estimation of the particle Stokes number and the dimensionless settling velocity based on the Kolmogorov scales can be achieved by using the Kolmogorov hypotheses and an estimation of the dissipation rate based on the large scales (see e.g. Pope, 2000, chap. 6). Generally, this results in

\[ \text{St}_\eta = \text{St} \cdot \frac{\tilde{t}_{ref}}{\tilde{t}_\eta} \sim \text{St} \cdot \text{Re}^{1/2} \text{ and } \]

\[ \text{v}_{s,\eta} = \text{v}_s \cdot \frac{\tilde{u}_{ref}}{\tilde{u}_\eta} \sim \text{v}_s \cdot \text{Re}^{1/4}. \]  

(35)

(36)

The ratio of the particle diameter \( \tilde{d}_p \) to the Kolmogorov length \( \tilde{l}_\eta \) can be estimated in the same manner as

\[ \text{d}_{p,\eta} = \frac{\tilde{d}_p}{\tilde{l}_\eta} \sim \text{d}_p \cdot \text{Re}^{3/4}. \]  

(37)

This relation can be used to estimate if the particles are much smaller than the smallest flow-scales. As mentioned in section 2.1, this is a necessary requirement for the point-force approximation to be valid.

By setting \( \text{St}_\eta \) and \( \text{v}_{s,\eta} \) to unity, it follows from relations (35) and (36) that settling enhancement would be most distinct if

\[ \text{St} \sim \text{Re}^{-1/2} \text{ and } \]

\[ \text{v}_s \sim \text{Re}^{-1/4}. \]  

(38)

(39)

This implies that there is only one degree of freedom, i.e. the Stokes number and the settling velocity for which settling enhancement would be most distinct are prescribed by the specified Reynolds number. However, this conclusion must be set into perspective. Settling enhancement is also observed if \( \text{St}_\eta \) and/or \( \text{v}_{s,\eta} \) deviate from unity (see e.g. Aliseda et al., 2002; Bosse, 2005; Wang & Maxey, 1993; Yang & Shy, 2003). Further, these studies precondition (near) homogeneous and isotropic turbulence without boundaries. These two restrictions are not fulfilled in turbidity currents. Additionally, the above reasoning is based on a scaling law.

3.2 Scales in laboratory experiments

The experiments by Gladstone & Pritchard (2010) are chosen as a representative example to determine typical scales of laboratory experiments on turbidity currents. It should be noted that these experiments are on lofting turbidity currents, i.e. when the interstitial fluid is of a lower density than the ambient fluid. Further, two different grain sizes are used to investigate the influence of polydisperse particles. The necessary specifications from the experiments to calculate the characteristic scales and the dimensionless parameters (5)–(8) are summarized in table 1. It is worth pointing out that the initial particle volume fraction in the reservoir is an order of magnitude higher than the limit at which particle-particle interactions become important (Elghobashi, 1994).
Table 1: Parameters of the experiments by Gladstone & Pritchard (2010)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>channel width $\tilde{L}_2$</td>
<td>0.15 m</td>
</tr>
<tr>
<td>channel height $\tilde{L}_3$</td>
<td>0.2 m</td>
</tr>
<tr>
<td>reservoir depth $\tilde{L}_{2c}$</td>
<td>0.05 m</td>
</tr>
<tr>
<td>density of interstitial fluid $\tilde{\rho}_I$</td>
<td>914...998 kg/m³</td>
</tr>
<tr>
<td>density of ambient fluid $\tilde{\rho}_f$</td>
<td>998 kg/m³</td>
</tr>
<tr>
<td>density of the particles $\tilde{\rho}_p$</td>
<td>3217 kg/m³</td>
</tr>
<tr>
<td>kinematic viscosity $\tilde{\nu}$</td>
<td>$10^{-6}$ m²/s</td>
</tr>
<tr>
<td>particle diameter $\tilde{d}_p$ (fine)</td>
<td>$1.28 \cdot 10^{-6}$ m</td>
</tr>
<tr>
<td>particle diameter $\tilde{d}_p$ (coarse)</td>
<td>$3.65 \cdot 10^{-6}$ m</td>
</tr>
<tr>
<td>particle volume fraction $\phi_{V,0}$</td>
<td>$1.5 \ldots 5 \cdot 10^{-2}$</td>
</tr>
</tbody>
</table>

This is not an exceptional case for lock-exchange experiments. Bonnecaze et al. (1993) for instance have particle volume-fractions of up to 2% in the reservoir. Additionally it is not clear if the volume displacement should be considered in the continuity equation for these volume-fractions.

The characteristic scales and the governing dimensionless parameters are obtained by inserting the specifications in table 1 into the equations (5)–(12), which gives

\[
\tilde{\nu} = 0.1 m, \quad (40)
\]

\[
\tilde{u}_b \approx 0.14 m/s, \quad (41)
\]

\[
Re \approx 10^4, \quad (42)
\]

\[
St \approx 10^{-5} \ldots 10^{-4}, \quad (43)
\]

\[
v_s \approx 5 \cdot 10^{-4} \ldots 5 \cdot 10^{-3}, \quad \text{and} \quad (44)
\]

\[
\Gamma_0 \approx 5 \cdot 10^{-2}. \quad (45)
\]

Noticeable are the extremely low Stokes numbers, indicating that particle inertia is fairly negligible for the large scales of the current. Estimating the Stokes numbers, the settling velocities and the dimensionless particle diameters based on the Kolmogorov scales with the relations (35)–(37), respectively, gives

\[
St_\eta \sim 10^{-3} \ldots 10^{-2}, \quad (46)
\]

\[
v_{s,\eta} \sim 5 \cdot 10^{-3} \ldots 5 \cdot 10^{-2}, \quad \text{and} \quad (47)
\]

\[
d_{p,\eta} \sim 1.28 \cdot 10^{-2} \ldots 3.65 \cdot 10^{-2}. \quad (48)
\]

The Stokes numbers based on the Kolmogorov scales are still very small. This would suggest that particle inertia is most likely negligible for laboratory-scale turbidity currents. The low particle diameter to Kolmogorov length ratios indicate that the point-force approximation would be justified. However, it should be pointed out that the above reasoning is based on a scaling law and that the calculations are only performed for one specific set of experiments. Hence the conclusions drawn must be treated with caution.

3.3 Scales in nature

As in section 3.2, the typical scales of turbidity currents are illustrated here by an example. More general information can be found in Meiburg & Kneller (2010).
Table 2: Assumed parameters for estimating the scales of a turbidity current in nature.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>front velocity $\tilde{v}_f$</td>
<td>0.6 m/s</td>
</tr>
<tr>
<td>buoyancy velocity $\tilde{u}_b$</td>
<td>$0.6 \cdot \tilde{v}_f$</td>
</tr>
<tr>
<td>reference length $\tilde{l}$</td>
<td>50 m</td>
</tr>
<tr>
<td>density ratio $\tilde{\rho}_p/\tilde{\rho}$</td>
<td>2.6</td>
</tr>
<tr>
<td>kinematic viscosity $\tilde{\nu}$</td>
<td>$10^{-6}$ m$^2$/s</td>
</tr>
<tr>
<td>gravitational acceleration $\tilde{g}$</td>
<td>9.81 m/s$^2$</td>
</tr>
<tr>
<td>particle diameter $\tilde{d}_p$</td>
<td>$10^{-5} \ldots 10^{-4}$ m</td>
</tr>
</tbody>
</table>

Xu et al. (2004) present in-situ measurements of horizontally averaged vertical velocity profiles of four gravity currents in the Monterey Canyon. Based on the plots for the first event at the mooring R3, the front velocity can be estimated as 60 cm/s, and the height of the current as 50 m. Together with estimations for the particle to fluid density ratio $\tilde{\rho}_p/\tilde{\rho}$, kinematic fluid viscosity $\tilde{\nu}$, gravitational acceleration $\tilde{g}$ and the particle diameter $\tilde{d}_p$, everything else is obtained using the definitions of the characteristic scales and the dimensionless parameters in section 2.1 and the equations based on the scaling analysis in section 3.1. Further, the buoyancy velocity is estimated as $\tilde{u}_b = 0.6 \cdot \tilde{v}_f$ based on the simulations by Necker (2002). The estimations made here are given in table 2. The assumption for the density ratio is based on quartz and water. The choice for the particle diameter range is inspired by the grading of silt by ISO 14688, which is 0.002 mm–0.063 mm (Wikipedia, 2011). Silt, together with clay, was the main component of the failed sediment which formed the gravity current off the Grand Banks of Newfoundland in 1929 (Meiburg & Kneller, 2010).

Three parameters are independent of the particle diameter, namely the Reynolds number, the initial volume fraction, and the mass-loading coefficient. With the above assumptions, these become

$$Re \approx 5 \cdot 10^7,$$  \hspace{1cm} (49)

$$\phi_{V,0} \approx 1.3 \cdot 10^{-3}, \text{ and}$$  \hspace{1cm} (50)

$$\Gamma_0 \approx 3.3 \cdot 10^{-3}. \hspace{1cm} (51)$$

For the particle diameters considered, the Stokes numbers according to equation (6) ranges from $3 \cdot 10^{-7}$ to $3 \cdot 10^{-5}$. The settling velocity according to equation (6) ranges from $10^{-4}$ to $10^{-2}$.

The Stokes numbers, settling velocities and dimensionless particle diameters, all based on the Kolmogorov scales, are plotted in figure 2. Due to the higher Reynolds number these values are larger than in laboratory-scale experiments. For the larger particles considered, the Stokes number $St_{\eta}$ and the settling velocity $v_{s,\eta}$ are both of the order of 0.1 . . . 1 for which settling enhancement may start to become relevant according to the studies by Aliseda et al. (2002); Bosse (2005); Wang & Maxey (1993); Yang & Shy (2003). However, the physical interpretation of settling enhancement being most distinct for $St_{\eta} \approx v_{s,\eta} \approx 1$ is that the particles mainly interact with the smallest turbulent scales. From figure 2, one can see that the particle diameter $d_{p,\eta}$ is of the same order, violating the assumption of the particles being much smaller than the smallest turbulent scales.

As in the two previous subsections, it should be stressed that the conclusions drawn here must be treated with caution. Additionally to the previously mentioned uncertainties in section 3, all the values in table 2 are estimates. Further, the above reasoning implies similarity for the lock-exchange flow and the turbidity current measured by Xu et al. (2004), especially by assuming...
\[ \tilde{u}_b = 0.6 \cdot v_f. \] This is not the case due many reasons, one of them being the fact that large scale turbidity currents are known to re-suspend sediment from the ground (Meiburg & Kneller, 2010), whereas e.g. Necker (2002) and Gladstone & Pritchard (2010) claim this to be unlikely due to the low Shields numbers in their cases. Further, there are most likely major differences in the geometry such as the slope, the curvature of the canyon, obstacles on the ground and so forth.
Figure 2: Estimates for the Stokes number $St_\eta$, settling velocity $v_{s,\eta}$ and dimensionless particle diameter $d_{p,\eta}$ according to the relations (35)–(37), respectively, plotted for the parameters in table 2.
4 Numerical approach

In order to discretize the two-way coupling term in equation (2), $w_p$ has to be substituted by a kernel $G(\xi)$. This is done by integrating the two-way coupling term over a computational cell and dividing by the cell volume. As $w_p$ is zero except inside the particle, the velocity difference takes the value of the velocity difference at the position of the particle since the particle is much smaller than the smallest flow scales. The integration of $w_p$ equals the particle volume. The resulting expression is amended by the desired kernel under the condition, that the kernel $G$ integrated over the domain equals unity. Hence, one obtains

$$\frac{D u_i}{Dt} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i}{\partial x_m \partial x_m} - \frac{Ri}{v_s} \frac{Vres}{V_c} \frac{1}{n_{p0}} \sum_{j=1}^{n_p} (u_i - v_{i,j}) G(x - y_j, \Delta),$$

where $V_c$ is the cell volume, $n_{p0}$ the initial number of particles in the reservoir and $\Delta$ the kernel width vector of the kernel $G$.

At each sub-time step the fluid velocity has to be interpolated to the particle position. This is done by a trilinear interpolation from the surrounding grid points in the 3D and by bilinear interpolation in the 2D case. This results in a second order interpolation scheme. This is found to be accurate in the sense that Häuselmann (2010) found a good agreement of the Lagrangian and the Eulerian particle method.

On the other hand, figure 3 shows that unphysical clustering of particles can occur in regions with high velocity gradients. The figure shows a particle scatter and a concentration contour plot of a region at the head of the current. The particles are modeled without inertia, i.e. the particle velocity equals the local fluid velocity plus the steady-state settling velocity. Since this results in a divergence free velocity field, particle accumulation should not occur. Still, particle clustering on a scale which is smaller than the grid resolution can be seen close to the interface, whereas in the bulk the particles are more homogeneously distributed. The concentration also shows larger variation close to the boundary than in the bulk. Unphysical particle clustering can hence be seen as a reduction of the effective number of particles in regions with strong velocity gradients, and thus to reduce the quality of the results in regions of interest.

Meyer & Jenny (2004) investigated a similar phenomenon and found that particle accumulation can be avoided if the interpolation scheme is conservative. For a bilinear interpolation scheme on a collocated grid, the divergence can be shown to be

$$\text{div}(u) = (1 - \bar{x}_2^j) \cdot (u_1^{i+1,j} - u_1^{i,j}) + (1 - \bar{x}_1^i) \cdot (u_2^{i,j+1} - u_2^{i,j}) + \bar{x}_2^j \cdot (u_1^{i+1,j+1} - u_1^{i,j+1}) + \bar{x}_1^i \cdot (u_2^{i+1,j+1} - u_2^{i+1,j}),$$

Here, $i$ and $j$ are the indices of the grid nodes in directions 1 and 2, respectively, $u$ is the velocity whereas the subscript indicates the direction and the superscript the position. $\bar{x}_1^i$ is zero at index $i$ and unity at index $i + 1$, and $\bar{x}_2^j$ is zero at index $j$ and unity at index $j + 1$. The influence of a staggered grid and other interpolation schemes were not investigated.

The kernel $G$ in equation (52) is implemented as a linear extrapolation, which for the 1D case in the $i$-th direction is defined as

$$G_i(\xi_i, \Delta_i) = \begin{cases} \frac{\Delta_i - |\xi_i|}{\Delta_i}, & \text{if } |\xi_i| \leq \Delta_i, \\ 0, & \text{otherwise} \end{cases} \quad (54)$$
Figure 3: Detailed view of the suspension/clear-fluid interface at the front at $t = 4$ for simulation S0 (consider section 5 for details). Top: Scatter plot of the particles. Bottom: Contour plot of the concentration $c$ with the pressure grid.
with the kernel width $\Delta_i$ in the $i$-th spatial direction. For higher dimensions, the kernel is obtained by multiplying the 1D kernels of the according directions. For example, the 3D kernel can be written as $G(\xi, \Delta) = G_1 \cdot G_2 \cdot G_3$. The filter widths are chosen to be the local grid spacing in the accordant directions. This kernel is second order (Schmidt, 2006).

The governing equations are solved using IMPACT (see Henniger (2011) for details). The time-integration is performed with an explicit third-order Runge-Kutta scheme for the fluid and the particle equations. One important difference to Häuselmann (2010) is the way the dissipation rate $\varepsilon$ is obtained. In Häuselmann (2010), the expression with the rate of strain tensor $s_{kl}$ in equation (30) is used, whereas here the expression with the second derivative is employed. This fixes the error in the total energy which is observed in Häuselmann (2010). The reason for this is most likely the better numerical behavior of the second derivative if high spatial frequencies are present. High frequencies can stem from either the Lagrangian transport equation used not having a diffusion term, and/or from the statistical error due to the finite number of particles.

Considering parallelization, the Lagrangian particles are distributed among the processors depending on their current location in the domain because the local fluid velocity at the particle position is needed to integrate equation (2) for each particle. This approach guarantees that no additional communication of the fluid velocity field is necessary and thus that the scaling behavior of the solver is not influenced by the Lagrangian particles. On the other hand, load balancing issues occur when a large number of particles are not evenly distributed among the processors.
5 Simulations

5.1 Preliminaries

The boundary conditions for the fluid are chosen to be the same as used by Necker (2002). The boundaries in the $x_1$-$x_3$- and in the $x_2$-$x_3$-plane are described by symmetry boundary conditions. The top and bottom of the domain are treated as no-slip boundaries for the fluid. Particles which cross a boundary are mirrored back into the domain, except at the bottom where they fall out of the domain to model the settling. Resuspension of particles is not taken into account, as it is unlikely to happen for the Reynolds numbers considered. This can be shown using the classical Shields criterion (Necker, 2002).

The initial condition for the fluid is $u_i = 0$. The particles are initially positioned as illustrated in figure 4. The amplitude of the superimposed noise is chosen as the grid spacing of the structured auxiliary particle grid, i.e. such that the probability of a particle being positioned is the same at every point in the bulk of the reservoir. This is necessary to avoid the formation of “particle bands” as illustrated in figure 5. These particle bands can cause numerical issues, and even the simulation to crash, since many particles can come to lay on a single grid-point. The initial condition for the particle velocity is the terminal velocity, i.e. $v_{i,0} = -v_s \cdot \delta_i$. This is the same initial condition as the implied one when particle inertia is neglected and hence is chosen for the sake of direct comparison.

In addition to the time-step criteria for a fluid simulation without particles (see Henniger, 2011), further time step restrictions originate from the Lagrangian particles. The most restrictive property is found to be due to the particle momentum equation (3), which results in $\Delta t \lesssim 0.2 \cdot St$. This criterion stems from Sommerfeld et al. (2008), where also further criteria are listed. It should be mentioned that the reference is for particle-tracking in RANS-simulations; no such reference was found for DNS.

As mentioned in section 4, the parallelization-approach for the particles can lead to load-balancing issues when a large number of particles is not evenly distributed among the processors. This is especially the case in the initial phase of a lock-exchange flow. The problem can be diminished by starting with a shorter geometry and successively prolonging the domain as the front of the turbidity current moves downstream. 2D simulations with IMPACT indicate that the accuracy of the solution is not affected as long as the front does not get too close to the back boundary. A clearance of three units is found to suffice.
Figure 5: Particle scatter-plots in the braid of a Kelvin-Helmholtz vortex for two different initial conditions for the particles. Top: No noise superimposed (shaded case in figure 4). Bottom: Noise superimposed (black case in figure 4).
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Table 3: Parameters for all simulations.

<table>
<thead>
<tr>
<th>physical</th>
<th>numerical</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1$</td>
<td>15</td>
</tr>
<tr>
<td>$L_2$</td>
<td>1</td>
</tr>
<tr>
<td>$L_3$</td>
<td>2</td>
</tr>
<tr>
<td>$L_s^1$</td>
<td>1</td>
</tr>
<tr>
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<td>1</td>
</tr>
<tr>
<td>$L_s^3$</td>
<td>2</td>
</tr>
<tr>
<td>$Re$</td>
<td>2236</td>
</tr>
</tbody>
</table>

Table 4: Defined parameters ($St$ and $v_s$) and implied parameters ($\Gamma_0$, $O(St_\eta)$ and $O(v_s,\eta)$) according to relations (22), (35) and (36), respectively) for all presented simulations.

<table>
<thead>
<tr>
<th>Name</th>
<th>$St$</th>
<th>$v_s$</th>
<th>$\Gamma_0$</th>
<th>$O(St_\eta)$</th>
<th>$O(v_s,\eta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S0</td>
<td>0</td>
<td>0.02</td>
<td>0</td>
<td>0</td>
<td>0.14</td>
</tr>
<tr>
<td>S1</td>
<td>0.001</td>
<td>0.02</td>
<td>0.05</td>
<td>0.047</td>
<td>0.14</td>
</tr>
<tr>
<td>S2</td>
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<td>0.02</td>
<td>0.5</td>
<td>0.47</td>
<td>0.14</td>
</tr>
<tr>
<td>S3</td>
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<td>0.02</td>
<td>1.0</td>
<td>0.95</td>
<td>0.14</td>
</tr>
<tr>
<td>S4</td>
<td>0.03</td>
<td>0.02</td>
<td>1.5</td>
<td>1.42</td>
<td>0.14</td>
</tr>
<tr>
<td>M2</td>
<td>0.02</td>
<td>0.04</td>
<td>0.5</td>
<td>0.95</td>
<td>0.28</td>
</tr>
</tbody>
</table>

5.2 Results

The physical and numerical parameters listed in table 3 are used for all simulations. Besides the simulations where the equations (1)–(4) are solved, also one simulation with neglected particle inertia is presented. In this case, the particle velocity is directly obtained by superposing the settling velocity onto the local fluid velocity instead of solving equation (3). One property of this approach is that the resulting particle velocity field is divergence free, and hence particle accumulation can only occur due to the finite number of particles (statistical error) or numerical inaccuracy. For more details on this approach and for validation against the Eulerian method used in Henniger (2011); Necker (2002), consider Häuselmann (2010). Here, for sake of notation, this approach is referred to as having $St = 0$.

A complete overview of the differing parameters for the presented simulations is given in table 4. The simulation S0 is the reference with $St = 0$. The simulation S1 is exemplary for very weak inertial effects, and the simulations S2–S4 have Stokes numbers chosen such that $O(St_\eta)$ is around unity. For $St_\eta$ much larger than unity, consider the simulation with $St = 0.1$ in Häuselmann et al. (2011). The simulation M2 has the same implied mass loading parameter $\Gamma_0$ as S2 and the same Stokes number as S3. This is achieved by adjusting the settling velocity accordingly.

Figures 6 and 7 show contour plots of the normalized particle concentration $c$ for the simulations S0–S4, i.e. for different Stokes numbers but otherwise identical parameters. Noticeable is that no particle accumulation is observed for the simulations S0 and S1. For increasing particle
inertia, particle accumulation can be observed in the shear layer at the top of the current, especially in the front region where Kelvin-Helmholtz (KH) vortices form. Further, less particles are found inside the vortical structures in contrast to simulations S0 and S1. This agrees well with the observation that particles accumulate in regions of high strain-rate and low vorticity (e.g. Wang & Maxey, 1993).

Figure 8 shows the vorticity $\omega_f$ for the simulations S0–S4 at $t = 0.2$, 2 and 4. Generally, a vorticity sheet forms at the vertical suspension/clear-fluid interface. The endings of the vorticity sheet roll up and form the so called start-up vortices. These vortices are studied in detail by Härtel et al. (1999) for density currents (i.e. without particles). Two findings of this paper are summarized in the following list:

- In contrast to KH-vortices, the start-up vortices are independent of the Reynolds number and are not the result of an unstable shear flow. They are the rolled up endings of the vertical vorticity-sheet generated by the baroclinic production term of the vorticity equation.

- During the acceleration phase, the start-up vortices store circulation which later accelerates the front. The acceleration leads to the characteristic overshoot of the front velocity.

In the case of particle-driven gravity currents, the baroclinic production term can be split up into vorticity coupling and production due to misalignment of the slip velocity and the density gradient, as it is done in equation (23). These two terms must be responsible for the production of the vorticity sheet, and hence for the differences of this sheet for the different Stokes numbers as outlined below.

At $t = 0.2$ and 2, it can be seen that for increasing Stokes numbers less vorticity is produced and, as a consequence, that the start-up vortices are weaker. Hence, it is expected that the induced velocity field is weaker as well. This can be verified by the smaller dents at the endings of the suspension/clear-fluid interface and of the vorticity-sheet at $t = 2$ in the figures 6 and 8, respectively. These dents show the influence of the induced velocity. At the lower end of the interface, the suspension is pushed forward in the $x_1$-direction and a "suspension-bulge" forms. At the upper end, the suspension is pushed downward in the negative $x_3$-direction.

At a later stage, e.g. at $t = 4$ in figure 8, the suspension interface is more tilted and the start-up vortices are both located above the bulk of the current, together with possible KH vortices. At $t = 6$, the formerly lower start-up vortex is located around $x_1 \approx 3$, and the formerly upper one around $x_1 \approx 1$. The height of the suspension downstream of each start-up vortex is higher than upstream, as can be seen in figure 6. Upstream of the start-up vortex at $x_1 \approx 1$, a part of the bottom boundary is not even covered by the suspension anymore. This can be understood as a consequence of the induced velocity of the start-up vortices. Both vortices accelerate the suspension below, which is verified by the downstream velocity having a local maximum below each vortex. Further, the induced velocity points upwards/downwards downstream/upstream of the vortices, respectively.

The entire chain of thought from the produced vorticity sheet to the current being higher/lower downstream/upstream of the start-up vortices is supported by the tendencies for increasing Stokes numbers. As mentioned above, less vorticity is initially produced for higher Stokes numbers. From the contour plots for $t = 6$, it can be seen that the height-differences over the start-up vortices are less distinct for higher Stokes numbers. Further, the amount of bottom boundary upstream of the start-up vortex at $x_1 \approx 1$ not covered by the suspension decreases with increasing Stokes number.

Further observations can be made from the contour plots for the concentration and the vorticity. In the initial phase, when the vorticity sheet is tilting, KH vortices form between the start-up vortices for the simulations S0–S2 up till $t = 4$, but not for the simulations S3 and S4.

\footnote{The according figures are omitted in this report.}
At a later stage, as the current has developed, it can not be confirmed that more KH vortices are shed at the head of the current for lower Stokes numbers. However, the vortical structures at the top of the current are generally weaker for higher Stokes numbers, as can also be seen in figure 9. Also noteworthy is that the simulations S0 and S1 give qualitatively very similar results for the initial phase, as can be seen e.g. from figures 6 and 8. But in the figures 7 and 9, i.e. at \( t = 10 \), they differ significantly. A possible explanation could be that the formerly upper start-up vortex is dragged along with the current in the simulation S1, most likely through interaction with other vortices. In all other simulations this start-up vortex stays behind the current. At \( t = 10 \), it can be located around \( 1 \lesssim x_1 \lesssim 2 \) for all simulations except for S1. It is not clear if this is a characteristic feature of the given set of parameters or “coincidence”. Häuselmann (2010) observed significant differences in the final deposit for simulations with identical parameters, but different initial conditions for the individual particles within the reservoir. Although this does not confirm either option, it stresses that the latter can not be ruled out.

Based on the front velocity, lock-exchange gravity currents generally have three phases (Huppert & Simpson, 1980): the **acceleration phase**, the **inertial** phase with a nearly constant front velocity, and the **viscous phase** where the front velocity abruptly decreases. Figure 10 shows the front velocities for the simulations S0–S4. The simulations S0 and S1 have practically identical front velocities during the acceleration phase. For increasing Stokes numbers, the current accelerates less fast. Further, the characteristic velocity overshoot decreases in magnitude and at some point splits up into two peaks.

These differences in the magnitude and the shape of the velocity overshoot could be due to the differences in the magnitude and dynamics of the start-up vortices. As mentioned before, the start-up vortices are weaker for larger Stokes numbers. Given that the velocity overshoot is due to the start-up vortices accelerating the current as suggested by Härte et al. (1999), a decrease in the magnitude for higher Stokes numbers would be the expected consequence. The splitting up into two peaks for higher Stokes numbers could be because it takes the upper start-up vortex longer to reach the bottom boundary. In an irrotational flow, the influence of a wall on a vortex can be modeled by introducing a second vortex of equal strength, but opposite sign on the opposite side of the wall (method of images). The induced velocity field between the vortex and the wall is of largest magnitude if the vortex is close to the wall.

Figure 12 shows the front velocities for the simulations S2, S3, and M2. The simulation M2 has the same initial mass-loading parameter \( \Gamma_0 \) as S2 and the same Stokes number as S3. The front velocities of S2 and M2 are identical for the shown period of time. This indicates that not the Stokes number \( St \), but the mass-loading parameter \( \Gamma \) is the governing parameter for the front acceleration. The front velocities for the simulations S2 and M2 start to differ as soon as the faster particle settling due to the higher settling velocity \( v_s \) in simulation M2 is not negligible anymore\(^1\).

During the writing of this report the simulation M2 raised questions whether the above reasoning, that the created vorticity governs the acceleration, is correct. It appears that a little less vorticity is produced in simulation M2\(^2\) compared to simulation S2, whereas it would be expected that the same amount is produced since the front velocities match. But since the differences are rather small, another simulation was performed with the same mass-loading parameter as S2 and the same Stokes number as S4. Again, there is a good match for the front velocities. Further, it becomes more clear that the vorticity differs very little among the simulations with identical mass-loading parameter compared to the simulations with different Stokes numbers and identical settling velocities, i.e. varying mass-loading parameter.

After the acceleration phase, the front velocity is the same for all Stokes numbers up till \( t \approx 10 \). Then the front velocities for the simulations S0–S2 decrease rapidly, whereas for the simulations S3 and S4 this phenomena is much less distinct. It appears that at some point increasing the Stokes number either prolongs the inertial phase or induces a smoother transition from the inertial
Figure 6: Normalized particle concentrations $c$ at two different points in time (left column: $t = 2$, right column: $t = 6$) for five different Stokes numbers (simulations S0–S4).
Figure 7: Normalized particle concentrations \( c \) at \( t = 10 \) for five different Stokes numbers (simulations S0–S4).
Figure 8: Vorticity $\omega_f$ at three different points in time (left column: $t = 0.2$, center column: $t = 2$, right column: $t = 4$) for five different Stokes numbers (simulations S0–S4). (Positive vorticity points into the image plane and the range of the legend differs from figure 9.)
Figure 9: Vorticity $\omega_f$ at $t = 10$ for five different Stokes numbers (simulations S0–S4). (Positive vorticity points into the image plane and the range of the legend differs from figure 9.)
to the viscous phase. The consequences of the differing front velocities are visible in the plots for the front position in figure 11.

Figure 13 shows the suspended particle mass \( m_p \) and the particle settling rate \( -\dot{m}_p \) for the simulations S0–S4. During the acceleration phase, the curves are very similar for all simulations. For \( t \gtrsim 5 \), the suspended mass \( m_p \) is generally lower for increasing Stokes numbers, whereas the simulations S0 and S1 hardly show any difference in behavior. The settling rates \( -\dot{m}_p \) differ mostly during the time period \( 5 \lesssim t \lesssim 13 \), being larger for higher Stokes numbers. The rates for the simulations S0 and S1 and for the simulations S3 and S4 are qualitatively identical for the time-period \( 5 \lesssim t \lesssim 10 \), respectively.

It is illustrative to estimate the settling rate as \( -\dot{m}_p \approx \frac{L_{covered} \cdot v_s}{V_{res}} \). Here, \( L_{covered} \) is the length of the domain which is covered by the suspension, and it is implicitly assumed that the depth of the domain and the concentration \( c \) at the boundary are unity, and the vertical velocity of the particles is \( v_s \). The division by the reservoir volume is necessary since the suspended particle mass \( m_p \) is normalized to be unity at \( t = 0 \). (Similar estimations for the settling rate can be found in the literature, e.g. De Rooij & Dalziel, 2001; Necker, 2002). It follows that the derivative of the settling rate is \( -\dot{m}_p \approx \frac{L_{covered} \cdot v_s}{V_{res}} \). From the contour plots for the simulations S3 and S4 in figures 6 and 7, it becomes clear that the back of the current moves rather slowly compared to the front. If \( L_{covered} \approx v_f = 0.58 \) is assumed (\( v_f (5 \lesssim t \lesssim 10) \approx 0.58 \)), the derivative of the estimated settling rate becomes \( -\dot{m}_p \approx 5.8 \cdot 10^{-3} \). The slope of the dashed line in figure 13 is \( 5.35 \cdot 10^{-3} \). This is less than 10% error and hence a fair approximation. This indicates that the differences in the settling rates for \( 5 \lesssim t \lesssim 10 \) are due to the different amounts of bottom boundary covered by the suspension in the different simulations. The sudden drop of the settling rates for the simulations S2-S4 around \( t \approx 10 \) could be due to the long “tail” of the current having settled/diluted, which can be verified from the contour plots at \( t = 10 \) in figure 7. Above, it was argued that the amount of bottom boundary covered is interlinked with the strength of the start-up vortices. This agrees well with the reasoning and the results here.

For \( t \gtrsim 13 \) it is hard to see a general tendency; the differences are rather small and seem to be of a random nature. Noteworthy is that the slope of the settling rate for the simulation S1 decreases rapidly around \( t \approx 16 \), whereas for all other simulations this rapid decrease happens later around \( t \approx 19 \). It is tempting to link this to the back start-up vortex being dragged along in simulation S1 as pointed out above, but one can not be certain.

Figure 14 shows the deposit densities \( D \) for the simulations S0–S4 at \( t = 25 \). Necker (2002) mentions that a characteristic feature of the deposit profile is the peak around \( x_1 \approx 5 \). It should be noted that the location is based on observations in 3D simulations and experiments (e.g. Bonnecaze et al., 1993; De Rooij & Dalziel, 1998). Necker (2002) presents a deposit profile from a 2D simulation where the location of the peak is at \( x_1 \approx 4 \). For the simulation S0 this peak is at \( x_1 \approx 4 \) which is consistent with the simulation by Necker (2002). For increasing Stokes numbers, the location of the peak moves upstream. For the simulations S3 and S4, the peak is located even before the initial suspension/clear-fluid interface at \( x_1 = 1 \). This again agrees well with the amount of the bottom boundary not covered by the suspension upstream of the back start-up vortex, which is interpreted as a consequence of the variation in strength of the start-up vortices.

Figure 15 shows the different energy forms introduced in section 2.3 for the simulations S0–S4, whereas \( Q_p \) is omitted since it is nearly zero for all simulations. Häuselmann (2010) observed relatively large errors for the total energy. These are found to be due to the way the dissipated energy is calculated, as outlined in section 4. The error of the total energy for the simulations presented here is less than 0.5%.

After the acceleration phase, the potential energy \( E_{pot} \) is consistently lower for increasing Stokes numbers. This is consistent with the observation, that the current is higher at the front for lower Stokes numbers. Above, this is argued to be linked to the strength of the start-up vortices. Since the total kinetic energy \( K_{tot} \) after the acceleration phase is larger for increasing Stokes numbers...
Figure 10: Front velocities $v_f$ for the simulations S0–S4.
Figure 11: Front positions $y_f$ for the simulations $S0$–$S4$.

Figure 12: Front velocities $v_f$ for the simulations $S2$, $S3$, and $M2$. 
Figure 13: Suspended particle mass $m_p$ (unfiltered data) and settling rates $-\dot{m}_p$ (filtered data) for the simulations S0–S4. The dashed line is an approximation for the settling rates of the simulations S3 and S4 for $5 \lesssim t \lesssim 10$. 
Figure 14: Deposit density $D$ for the simulations S0–S4 at $t = 25$ (filtered data).
numbers, it can be concluded that the excess potential energy is mainly converted into kinetic energy.

The dissipated energy by the resolved flow-scales $E_d$ is generally lower for increasing Stokes numbers. For the dissipated energy by Stokes-flow around the particles $E_s$, this is also the case for $t \lesssim 3$, but then the tendency inverts and more energy is dissipated by the Stokes-flow for higher Stokes numbers.

The energy plots for the simulations S0 and S1 are nearly identical in the acceleration phase. However, they differ significantly afterwards, whereas the simulation S1 behaves differently from all other simulations in many ways. Around $t \approx 10$, the potential energy of the simulation S1 decreases and the total kinetic energy increases faster than for all other simulations. On the other hand, the kinetic energy and the dissipated energy by the particles are nearly identical for the simulations S0 and S1. This indicates a very similar behavior of the particles in simulation S0 and S1, and that the differences mentioned before are most likely because the formerly upper start-up vortex is dragged along with the current in simulation S1.
Figure 15: The potential energy $E_{pot}$, the total kinetic energy $K_{tot}$, the kinetic energy of the fluid $K_f$ and of the particles $K_p$, and the energy dissipated by the resolved flow-scales $E_d$ and by the modeled Stokes-flow around the particles $E_s$ for the simulations S0–S4.
6 Conclusions

The governing equations for the Eulerian-Lagrangian description of turbidity currents, together with the necessary requirements for validity, are presented. The equations are supplemented by the analogous Eulerian-Eulerian equations, the vorticity equations and the equilibrium Eulerian ansatz, together with the additionally made assumptions. These equations are mainly intended to aid with the interpretation of the results. The equilibrium Eulerian equations (18)–(21) show that the main difference to the non-Boussinesq equations is that the advection velocity for the particulate phase is no longer divergence free, which allows for particle accumulation. This stresses the importance of the mass-loading parameter $\Gamma$, which further indicates that some observations made for the higher Stokes numbers may also be captured with the non-Boussinesq approach. This is illustrated in section 5.2 where it is shown in figure 12 that the mass-loading parameter is the governing parameter for the initial front acceleration. This means that not the inertia of the single particles, but the inertia of the entire suspension governs the initial acceleration of the current.

One important motivation for including particle inertia in the modeling of the dispersed phase is settling enhancement by turbulence. It is pointed out in section 3.1 that the fundamental studies on settling enhancement (e.g. Aliseda et al., 2002; Bosse, 2005; Wang & Maxey, 1993; Yang & Shy, 2003) use the Kolmogorov scales as characteristic scales, whereas the equations in section 2.1 are rendered dimensionless with half the domain height and the buoyancy velocity, i.e. with the large scales. The mentioned studies all find that settling enhancement is most distinct if the particle Stokes number and settling velocity, both based on the Kolmogorov scales, are close to unity. Hence, in section 3.1 an attempt is made to estimate these parameters based on the Kolmogorov hypotheses and an estimation of the dissipation rate based on the large scales. It is found that the order of magnitude of the $St_\eta$ and $v_{s,\eta}$ are obtained by multiplying $St$ and $v_s$ with $Re^{1/2}$ and $Re^{1/4}$, respectively. Considering that settling enhancement is most distinct for $St_\eta \approx v_{s,\eta} \approx 1$, it follows that there is only one degree of freedom, i.e. that the Stokes number and the settling velocity for which settling enhancement would be most distinct are prescribed by the specified Reynolds number.

In section 3.2, the experiments by Gladstone & Pritchard (2010) are chosen as a representative example to determine typical scales of laboratory experiments on turbidity currents. Since even the Kolmogorov scale based Stokes number and settling velocities are approximately two orders of magnitude smaller than unity, it is questionable if settling enhancement is of importance in laboratory scale experiments or if particle inertia has any relevant influence at all. Besides the low Stokes number, also the mass loading parameter $\Gamma_0$ in the initial suspension reservoir is rather small. In fact, it is the same mass-loading parameter as in simulation S1 in this report, for which the front acceleration is practically identical to the simulation with inertialess particles. Since the front acceleration is found to be governed by the mass-loading parameter, it is probable that the mass-loading is negligible in the experiments by Gladstone & Pritchard (2010). Also noteworthy is that the initial particle volume fraction of the suspension is an order of magnitude higher than the limit $10^{-3}$ up to which e.g. Elghobashi (1994) suggests that particle-particle interactions are negligible. Further, it is not clear if the volume displacement of the particles should be considered in the continuity equation as well for the volume fractions in laboratory-scale experiments.

The scales of turbidity currents in nature are illustrated in section 3.3 by the example of the turbidity currents presented by Xu et al. (2004). Due to the much higher Reynolds number, the Stokes number and the settling velocity based on the Kolmogorov scales can be estimated to be of order unity for the larger particles considered. On the other hand, the particles then exceed the Kolmogorov length scale, rendering interaction with these scales rather improbable. However, it must be stressed that a lot of estimations and assumptions are necessary to arrive at these results, and hence any conclusion drawn is subject to doubt.
Considering the numerics, the good agreement of the Lagrangian and the Eulerian particle treatment for inertialess particles found by Häuselmann (2010) indicate a sufficiently accurate solution of the governing equations. However, unphysical particle accumulation can be observed in regions with high velocity gradients. This is most probably due to the interpolation scheme. As mentioned in section 4, Meyer & Jenny (2004) found the conservative property of an interpolation scheme to govern unphysical particle clustering. Balachandar & Maxey (1989) evaluated the performance of different interpolation schemes to gather Lagrangian turbulence statistics. One conclusion is that even for one particle statistics (e.g. turbulent diffusion) the linear interpolation scheme may be insufficient. Alternative interpolation schemes evaluated by Balachandar & Maxey (1989) are Lagrangian interpolation and the shape function method. No detailed study to evaluate either method is performed in the present work. But it is worth pointing out that the costs of the different interpolation schemes determined by Balachandar & Maxey (1989) are not directly applicable, since e.g. additionally necessary ghost-cell updates for the Lagrangian scheme or possibly already available velocity derivatives for the shape function method are not taken into account.

It may seem a little contradictory that the spatial discretization is fourth to sixth order with the motivation to only “just resolve” the Kolmogorov scales in DNS, whereas only second order linear interpolation is used to interpolate the fluid velocity to the particle position. Linear interpolation is chosen mainly because it is the computationally least expensive method and requires the least communication. Further, Bosse (2005) and Kubik (2007) both compared linear interpolation to spectral summation and found the results not to be fundamentally altered. In these studies however, higher Stokes numbers were investigated for which physical particle accumulation may have prevailed over the numerical error; but this is speculation.

Five simulations (S0–S4) are presented which differ in the Stokes number, but have otherwise identical parameters. For increasing Stokes numbers, more distinct particle accumulation is observed at the top of the suspension current, especially close to the front. Further, less particles are found inside the vortical structures above the current. This is found to agree well with the observation that particles accumulate in regions of high strain-rate and low vorticity (e.g. Wang & Maxey, 1993).

A further observation is that for increasing particle inertia less vorticity is produced in the vorticity sheet between the suspension and the clear fluid, which in return leads to weaker start-up vortices. With this observation, an attempt is made to explain several tendencies for increasing Stokes numbers:

- The velocity overshoot decays in magnitude and eventually splits up into two peaks. Härtel et al. (1999) found the velocity overshoot to be due to the stored circulation in the start-up vortices.

- Downstream of the start-up vortices the current is generally higher than upstream. The height differences over the start-up vortices decrease with increasing Stokes numbers.

- A consequence of the height difference decrease over the start-up vortices is that a larger part of the bottom boundary is covered by the suspension for a certain period of time after the acceleration phase. This may also be described as the current having a longer tail. This leads to a higher particle settling rate until the entire tail has settled.

- The longer tail decreases the width of the dip in the deposit profile close to the origin.

- The generally longer and flatter current contains less potential energy. A result of this is that more potential energy is transformed to kinetic energy.

The simulation M2 has the same initial mass-loading parameter as S2, but the same Stokes number as S3. This is achieved by adjusting the settling velocity accordingly. The front acceler-
ation of S2 and M2 are identical, indicating that the mass-loading parameter governs the front acceleration, and not the particle Stokes number.

The simulation S0 with the inertialess particles and the simulation S1 with \( St = 0.001 \) behave practically identical during the acceleration phase, and particle accumulation occurs in neither simulation. The simulation S1 however is special in the sense that it is the only simulation where the upper start-up vortex is dragged along with the current, most likely due to vortex interaction, whereas at some point it remains behind in all other simulations. It is not clear if this is a characteristic feature of the prescribed set of parameters or “coincidence”.

An important open task for future work is most definitely a detailed profiling of the particle routines in IMPACT, especially the interpolation routine \( \text{interpol coeffs} \). In the current implementation, each particle is searched for in each direction of the Eulerian grid by successively marching through the physical coordinates of the grid-points until the first one which exceeds the particle’s is found. One possibility to circumvent this global search is to store the Eulerian indices for every particle, and the next time the routine is called to only search around these indices.

As outlined above, it is hard to say if a more accurate interpolation is necessary. With a more accurate interpolation scheme, fewer particles may be necessary given that the unphysical accumulation in regions of high velocity gradients is reduced/prevented. In a related context, the question of the necessary number of particles is not satisfactorily answered. Häuselmann (2010) used the error of the total energy in the system as a criteria. However, the source of the error is found to be due to the way the dissipation \( \varepsilon \) was determined, as outlined in section 4. Further, Bosse (2005) and Häuselmann (2010) both used the two criteria introduced by Elghobashi (2000). The more restrictive criterion mainly says that the number of particles per cell in a DNS should be larger than order unity; any statistical error due to the finite number of particles is hence not taken into account. Another reference which gives a clear criterion was not found. However, Schmidt (2006) is found to be a nicely written paper which is surely a good starting point to the subject.

The presented simulations are mainly a parameter study for the Stokes number, except for simulation M2 which is not discussed in detail. For a more complete picture, the settling velocity and the Reynolds number should be varied as well whereas the implied mass-loading parameter should be kept in mind. From the scale analysis it becomes clear that to investigate particle settling enhancement higher Reynolds numbers should be considered in order to avoid influences of the inertia of the entire suspension, i.e. the mass-loading parameter. To this end, the Lagrangian particle approach may not be the most suitable approach since it is more costly than Eulerian methods. The equilibrium Eulerian approach most likely permits simulations with higher Reynolds numbers. The ability of this approach to capture particle accumulation was shown by Shotorban & Balachandar (2009). However, it must be kept in mind that the equilibrium Eulerian approach is only suitable for Kolmogorov based Stokes numbers \( St_\eta \lesssim 0.2 \) in DNS. For LES, Balachandar (2009) further states that this boundary is then for the Stokes number based on the smallest resolved scale.

The large influence of the start-up vortices stresses the need to understand the vorticity creation at the vertical suspension interface. To this end, it may be helpful to plot the different terms on the right hand side of equation (23). Further, a more quantitative measure/illustration for the amount of vorticity produced would be wishful. Simulations with free-slip boundary conditions, as performed by e.g. Härtel et al. (1999), would allow the circulation around the entire domain to be used for this purpose since the vorticity produced at the no-slip walls would not interfere.

The large influence of the start-up vortices throughout the duration of the current also raises the question if the lock-exchange configuration is suitable to study settling enhancement, or turbidity currents in general. De Rooij & Dalziel (2001) performed time- and space-resolved measurements on lock-exchange and continuous flow turbidity currents. They found that the lock-exchange configuration leads to a distinct current head from which most of the sediment is
deposited. In continuous flow turbidity currents on the other hand, no distinct head is found and the deposition occurs throughout the body of the current. Peakall et al. (2001) concluded from this, that “the use of lock-exchange experiments appears to have led to an overemphasis on the importance of the head in turbidity currents and a corresponding underemphasis of the body and tail.” They further suggest using continuous flow currents since “both the collapse of a static mass of fully suspended sediment and the relative magnitude of the countercurrent produced in most lock-box experiments are highly unrealistic of gravity current initiation in most natural situations.”
References


