LOW FREQUENCY MAGNETIC METAMATERIALS AND WIRELESS CONTROL OF FORCES AND TORQUES

A dissertation submitted to

ETH Zurich

for the degree of
Doctor of Sciences

presented by

MUSTAFA BOYVAT

M.Sc. ETH Zurich, Switzerland
born on February 9, 1985
citizen of Turkey

accepted on the recommendation of

Prof. Dr. Christian Hafner, examiner
Prof. Dr. Juerg Leuthold, co-examiner
Prof. Dr. Bradley Nelson, co-examiner

2014
ACKNOWLEDGMENTS

This research was done in the ‘Institute of Electromagnetic Fields’ (IEF) at ETH Zurich, Switzerland. Here, I would like to write my thanks to the people who helped this work.

First of all, I would like to express my gratitude to my supervisor Prof. Christian Hafner for giving me the opportunity to carry out this research and supporting me through this work. He was always there when I wanted to talk or ask anything. He gave me the freedom to think of new ideas in a large frame and this was one of the most important factors which lead to the interesting results in this dissertation. I thank him for his supervision with very kind and warm attitude during these years.

I would like to thank Prof. Juerg Leuthold for the valuable time we spent after he came to ETH Zurich. I thank him for his support, for keeping his door open and for being a referee of my defense.

I am grateful to Prof. Bradley Nelson for fruitful discussions and for accepting to be an examiner in my defense.

Special thanks go to Christian Beyer for sharing his equipment and experience. I thank Dr. Selman Sakar for useful discussions we had, Claudio Maggio for mechanical designs, Aldo Rossi and Hans-Rudolf Benedickter for their help for electronic equipment and measurements. I acknowledge the Electricity Company of Zurich (EWZ) for supporting this project.

I thank all my colleagues at IEF for the nice environment they made and all my friends for the time we spent together. Finally, I would like to express my great thanks to my family for all the support they have given me in my life.

Zurich, Switzerland
2014

Mustafa Boyvat
ABSTRACT

Metamaterials are artificial materials which offer a wide range of electromagnetic properties extending out of the natural borders. They are composed of so-called meta-atoms similar to atoms in natural materials but these meta-atoms are much larger, designable, man-made structures. Metamaterials have attracted significant scientific attention with the introduction of two extreme applications, namely, the perfect lens and the invisibility cloak. Since then, a large amount of research has been performed on metamaterials, but almost all of the research has aimed at high frequencies. However, recent work on metamaterials at frequencies going down to DC proved that the very low frequency end of the electromagnetic spectrum is also highly interesting for metamaterials.

In this dissertation, it is shown that metamaterials at low frequencies can be used to manipulate magnetic fields, taking low frequency magnetic field shielding as the main application. The reason to focus on shielding is that shielding of low frequency magnetic fields is an important issue due to the related regulations and because conventional shielding techniques can be undesirably costly. This also covers the magnetic fields produced by power distribution systems which are almost everywhere.

It is numerically demonstrated that metamaterials composed of lossless meta-atoms can shield magnetic fields and it is shown that the shielding can be improved considerably by inhomogeneous metamaterials, designed by a constrained linear least squares optimization. After the numerical demonstrations, it is shown experimentally that low frequency metamaterials made of simple LC resonators formed by coils and capacitors can indeed reduce magnetic fields, but in a limited amount due to the losses in the resonators. Using resonators with higher quality factors can reduce these losses but this increases the costs, reduces the bandwidth and causes high currents and voltages. Another way to compensate these losses is to use active circuits. However, active loss compensation also has serious problems because active circuits may be bulky, expensive, and they have stability problems, need external power sources and cooling of the active components. In order to overcome these problems, an advanced purely passive meta-atom is designed based on a phase shifting concept. This new meta-atom consists of a small network of six capacitors and two rather small coils. Despite its high shielding
performance, the new meta-atom is neither bulky nor costly, does not lead to high currents and voltages. Furthermore, the bandwidth is comparable to that of LC resonators with low quality factors. The significant shielding improvement by the new metamaterial is also shown experimentally.

LC resonators sense mechanical effects due to the induced currents in their coils. Resonance enhances these currents and thereby the forces and torques acting on the resonators. The question of whether it is possible to obtain self-adaptive meta-atoms to increase the shielding performance in arbitrary external fields leads to the detailed study of these mechanical effects. Since the analysis of mechanical interactions in numerous coupled resonators is extremely complicated and demanding, simple configurations are studied to improve the understanding of these mechanical effects. Namely, the forces and torques on a single resonator are studied in detail. The direction of the forces and torques on the resonator can be changed and different stable orientations can be obtained by tuning the frequency of the external field. The analysis of systems of two and more coupled resonators is already very challenging because of the coupling between the resonators. But, such systems offer much stronger and more advanced mechanical effects. A highly attractive outcome of these mechanical effects is a ‘wireless motor’ obtained from a system of two coupled resonators. The interaction of two resonators causes one of the resonators to sense a torque that always points in the same direction. From this, continuous rotation is obtained. The basic effects of a single resonator and of a system of two resonators, including the ‘wireless motor’, are also demonstrated experimentally.

These interesting mechanical effects lead to fascinating ideas going beyond metamaterial area. The resonators are proposed for wirelessly powered actuator/agent systems consisting of independently controllable elements and even a single resonator is considered as a functional device. The strong frequency selectivity of LC resonators makes them attractive for multiple element systems with individually addressable elements, which is important in numerous areas.
ZUSAMMENFASSUNG


Resonators und die des Systems aus zwei Resonatoren, einschliesslich dem "drahtlosen Motor“, werden auch experimentell gezeigt.

# TABLE OF CONTENTS

1. **Introduction** ......................................................................................................................... 1
   
   The Outline of the Dissertation ............................................................................................... 7
   Bibliography ............................................................................................................................. 8

2. **Numerical Study of Low Frequency Magnetic Field Shielding by Metamaterials** .......................................................... 17
   
   2.1 Anisotropic Materials for Magnetic Shielding at Low Frequencies ........................................ 18
   2.2 Metamaterials for High & Low Permeability and Anisotropy .................................................. 21
   2.3 Inhomogeneous Anisotropic Metamaterial ................................................................... 26
   2.4 Practical Aspects ............................................................................................................. 29
   2.5 Comparison with Commonly Used Shielding Techniques ..................................................... 30
   2.6 Conclusion ....................................................................................................................... 31
   Bibliography ............................................................................................................................ 32

3. **Improved Meta-Atoms for Magnetic Shielding by Metamaterials** ............................................................. 35
   
   3.1 Shielding Principle of a Magnetic Meta-Atom ................................................................. 36
   3.2 Improved Meta-Atom with Active Circuits ................................................................. 40
   3.3 Improved Meta-Atom with Advanced Passive Circuitry ................................................... 41
   3.4 Experimental Demonstration of Shielding by Metamaterials ................................................. 44
   3.5 Conclusion ....................................................................................................................... 51
   Bibliography ............................................................................................................................ 53

4. **Mechanical Effects on LC Resonators** .............................................................................. 55
   
   4.1 Calculation of Forces and Torques in a Time Harmonic Field .................................................. 56
   4.2 Single Resonator ............................................................................................................. 58
   4.3 Pair of Identical Coupled Resonators ............................................................................. 62
   4.4 Different Coupled Resonators ....................................................................................... 66
   4.5 Trapping Points Selected By Frequency ..................................................................... 71
   4.6 Off-Axis Arrangement And Motor ................................................................................ 72
   4.7 Conclusion ....................................................................................................................... 77
5. Experiments in Mechanical Effects on LC Resonators........ 81

5.1 Enhanced Translational Forces on Identical LC Resonators .............................................................................. 82
5.2 Bidirectional Forces on LC Resonators.......................... 87
5.3 Rotational Effects on a Single Resonator......................... 89
5.4 Continuous Rotation by Using Coupled Resonators ........... 93
5.5 Scaling of Resonators...................................................... 95
5.6 Conclusion............................................................................. 98

6. Conclusion and Outlook..................................................... 101

6.1 Conclusion............................................................................. 101
6.2 Outlook............................................................................... 103
1. INTRODUCTION

Humans were always eager to understand nature. Electromagnetism is a result of this curiosity and stands at the core of many natural phenomena happening in nature from visual perception to rainbow and lightning. Electromagnetism has not only helped humans to understand some of the interesting phenomena in nature but also to make their technical contributions. Nowadays, countless devices and systems - light bulbs, phones and computers are only a few examples - owe their existence to electromagnetism.

In the control of electromagnetic fields, materials have a key role, because they can interact with electric and magnetic fields and they are able to manipulate these fields. The variety of electromagnetic properties of materials enriches the control over electric and magnetic fields. Electromagnetic properties of natural materials are mainly due to their different atomic and molecular structures. Since controlling atomic and molecular structures of materials is difficult, effects offered by the readily available materials are rather limited.

Metamaterials are materials which extend the range of electromagnetic properties found in natural materials and they enable engineering of these properties. Metamaterials are usually artificial materials, which are composed of artificial unit elements, which are much larger than atoms. These large unit elements, so called meta-atoms, make the electromagnetic properties designable. The electromagnetic properties depend on both meta-atoms and geometrical arrangement of meta-atoms.
1. Introduction

The property of metamaterials which first attracted scientific attention is ‘negative refractive index’, which is not found in nature. When a material has a negative permittivity and a negative permeability, its refractive index is negative and the phase velocity and the energy propagation directions of an electromagnetic wave in this medium are opposite to each other\textsuperscript{1}. Such a wave is known as a ‘backward wave’ and it has a long history going back to 1904\textsuperscript{[1–4]}. Mandel’shtam\textsuperscript{[5]}, Sivukhin\textsuperscript{[6]} and Veselago\textsuperscript{[7]} studied negative index materials. Veselago’s study was rediscovered by Smith\textsuperscript{[8]} in the end of the 20\textsuperscript{th} century\textsuperscript{[1]}. A crucial step in the negative index material subject was performed by John Pendry showing that a lens made of negative refractive index materials (such a lens had also been considered in Veselago’s paper\textsuperscript{[7]}) may not be limited by the diffraction limit\textsuperscript{[9]}.

Permittivity engineering and negative permittivity are not new phenomena. When Smith showed the composite medium with simultaneously negative permittivity and permeability in 2000\textsuperscript{[8]}, he was inspired by Rotman’s work in 1962\textsuperscript{[10]} and Pendry’s work in 1996\textsuperscript{[11]} and he used an array of wires (such an array was also studied back in 1953 by Brown to obtain a refractive index less than unity\textsuperscript{[12]}) to obtain a negative permittivity\textsuperscript{[2]}. Smith benefited from Pendry’s work in 1999 to obtain negative permeability\textsuperscript{[8,13]}. Pendry had shown that magnetic properties extending to negative permeability can be obtained by split-ring resonators\textsuperscript{[13]}, which can be found back in the early 1980s\textsuperscript{[14–16]}. The experimental verification of negative refraction at microwave frequencies by a metamaterial composed of an array of wires and an array of split-ring resonators came again by Smith in 2001\textsuperscript{[17]}.

Later, different types of unit-cells for metamaterials have been proposed, either based on the principle of mixing two lattices to control permeability and permittivity or with one lattice to control the electrical and magnetic activity together\textsuperscript{[18–29]}. In a short time, the magnetic activity obtained by metamaterials climbed up to optical frequencies, at which almost no magnetic activity is observed in ordinary materials\textsuperscript{[1,30]}. After the first demonstrations in the GHz range, metamaterials with magnetic activities were

\begin{footnotesize}
\begin{enumerate}
\item In general, dielectric permittivity \(\varepsilon\), magnetic permeability \(\mu\) and refractive index \(n\) are complex numbers with real parts \(\varepsilon'\), \(\mu'\) and \(n'\); and imaginary parts \(\varepsilon''\), \(\mu''\) and \(n''\) respectively. Imaginary parts are related to the loss in the material. In an isotropic material, \((n'<0, \varepsilon' |\mu'| + \mu' |\varepsilon'|<0)\) must be satisfied to obtain a negative real part of the refractive index. When both \(\varepsilon'\) and \(\mu'\) are negative, the inequality is always satisfied but note that it is not a necessary condition for a negative refractive index\textsuperscript{[1]}.
\end{enumerate}
\end{footnotesize}
obtained at 1 THz, 6 THz, 65 THz, 100 THz, at telecommunication wavelength 1.5 µm (200 THz) and in the visible range. Negative refractive index at optical frequencies was also demonstrated.

Another acceleration of metamaterials happened with the transformation optics paper by Pendry in 2006. The paper of Leonhardt accompanied it in the same issue of the same journal. Note that the relation between the propagation of light and the geometries of effective space-time goes back to the 1920s and the basics of transformation optics was published in the 1960s. The proposal of the invisibility cloak by Pendry excited scientists. In this interesting proposal, naturally difficult or impossible electric permittivity and magnetic permeability values and anisotropy were needed. Additional to these, engineering of these parameters was necessary and all these requirements in the invisibility cloak brought metamaterials to the stage again. Soon, the experimental studies on cloaking followed. Some other interesting ideas based on transformation optics, such as an optical black hole, a space-time cloak to hide events rather than objects and optical illusion to make an object appear as a different object, also emerged.

While new ideas on metamaterials arise and fabrication technologies develop, the sub-wavelength imaging, which played an important role at the beginning of metamaterials, has also moved on since Pendry’s paper in 2000. Encouraging experiments on Pendry lens and magnifying super-lenses were performed. A huge obstacle of metamaterials, the loss problem, was also studied and it was demonstrated that it is possible to remove the loss by active metamaterials.

Another important direction in metamaterial studies has been the tuning of metamaterials. Tunable metamaterials are important because tuning may be helpful in compensating the effects of fabrication errors and in adapting the metamaterial to different conditions. Furthermore, it can bring new functionalities, such as tunable filtering, modulation and switching. A way of tuning a metamaterial is the modification of the structural geometry of the metamaterial. Another method to tune is to change the material properties using materials such as semiconductors, liquid crystals and superconductors.

The fabrication of three dimensional metamaterials and integration of them into photonic devices are challenging. Planar and thin metamaterials form a very attractive group of metamaterials because the fabrication and integration
1. Introduction

of them is easier. The electromagnetic response of ultra-thin metamaterials or so called metasurfaces may be spatially invariant or variant. Frequency selective surfaces (FSS) known from microwaves, and considered in the photonic crystal group which is described below, are also instances of metasurfaces.

Photonic crystals form a very important group of metamaterials. Note that the term ‘photonic crystal’ is older than the term ‘metamaterial’. Nevertheless, we consider photonic crystals as metamaterials. A photonic crystal can be defined as a structure in which the electromagnetic properties change periodically in space with a spatial periodicity comparable to the wavelength of the electromagnetic wave. In photonic crystals, the geometrical arrangement is the main source of the interesting electromagnetic properties. The major difference of photonic crystals from the rest of the metamaterials is that interesting electromagnetic effects of photonic crystals are due to its crystal-like structure, which has a periodicity in the order of the wavelength. For many other metamaterials, meta-atoms and the separation distance between them are much smaller than the wavelength. This makes defining effective electromagnetic properties, namely electric permittivity and magnetic permeability, possible for them. However, the definition of an effective permittivity and permeability in photonic crystals in the regime in which the interesting electromagnetic effects are observed is ambiguous. In the scientific community, there are still discussions on whether considering photonic crystals as a subgroup of metamaterials or putting them in a separate group is more reasonable. Although the term ‘metamaterial’ was born later than the term ‘photonic crystal’, the term ‘meta’ means ‘beyond’ and is a general term. Thus, this dissertation does not exclude the photonic crystals from the metamaterial group.

The start of photonic crystals can be accepted to be with two papers in 1987 by Yablonovitch and John, which showed the existence of a photonic band gap and the localization of light, respectively. Note that periodic dielectric structures have a longer history. However, since 1987 many research groups have worked on photonic crystals and many different types of photonic crystal structures have been fabricated. Deep holes and pillars on two dimensional lattices, slabs of holes/pillars, surface gratings, buried gratings, multilayer gratings, sphere arrays, photonic crystal fibers are examples of fabricated 2D photonic crystals whereas woodpiles, cubic lattices, oblique
holes, buried dots, stacked slabs, opals, inverse opals and spirals are examples of 3D photonic crystals\[^{104}\].

The most prominent feature of a photonic crystal is the photonic band gap, similar to the energy band gap of electrons in solid crystals\[^{104}\]. The light with a frequency falling in the photonic band gap cannot propagate in the photonic crystal. Usually the periodicity in the photonic crystal is comparable to the wavelength range of the photonic band gap and simple permittivity and permeability definition is not possible. The photonic band gap generated by photonic crystals opened the door to many interesting applications. By introducing defects in the photonic crystal, small high quality cavity resonators can be obtained and they are important for applications such as ultra-small filters, low threshold lasers, photonic chips, nonlinear optics and quantum information processing\[^{105,106}\]. It is also possible to guide light by introducing line defects in a photonic crystal. When the incident electromagnetic wave has a frequency in the photonic band gap of the photonic crystal, it can only propagate through the line defects in the photonic crystal, which enables waveguides and more complicated structures such as bends, splitters and filters\[^{104,107,108}\]. By crystal lattices with lattice constants much smaller than the wavelength, it is also possible to obtain waveguides, however these periodic structures are not classical photonic crystals but engineered structures, which can be considered as another type of metamaterial\[^{109}\].

It is not only the frequency range of the band gap which leads to fascinating functionalities. Additional to defect mode lasers operating in the photonic band gap\[^{106,110}\], band edge effects were also used to make lasers\[^{111–113}\]. Self-collimation\[^{114}\], super-prism\[^{115,116}\], negative refraction and super-lens\[^{117–119}\] are also among these fascinating effects.

While most of the metamaterial research is for high frequencies, research at very low frequencies, going down to even d.c. by using superconductors, shows that metamaterials can also find applications in the low frequency end of the electromagnetic spectrum\[^{120–126}\]. This dissertation aims to contribute to this part of the spectrum while giving also inspiration to improve the high frequency metamaterials and it focuses on shielding because of its practical importance.

Electromagnetic shielding is important for human health and for electronic devices to operate properly\[^{127}\]. It can be done by putting materials as barriers and/or to diverge the fields from the region to be shielded, or by
1. Introduction

Introducing additional sources to reduce the fields in the shielded region[127]. Most of the shielding structures use conductive materials or ferromagnetic materials. However, frequency selective surfaces and metamaterials in general can also shield electromagnetic fields but they were not studied at very low frequencies[127]. Low frequency and d.c. metamaterials in the literature are made of superconductors or include active circuits. This thesis demonstrates the shielding of low frequency magnetic fields by metamaterials made of simple inductors and capacitors, i.e. with neither superconductors nor active circuits. Due to the low frequency i.e. long wavelength, the unit elements, or so called meta-atoms, can be rather large and more advanced than their very high frequency versions. An advanced purely passive metamaterial introduced in the thesis shows a significant improvement and the method used to improve the meta-atoms can also provide inspiration for higher frequencies and for other applications than shielding.

Magnetism is one of the main mechanisms to apply forces on objects without any contacts. One knows that when there are electrical currents, there are mechanical interactions due to the magnetic forces formed. Often the mechanical interactions within meta-atoms are neglected but the LC resonators used in low frequency magnetic metamaterials also show mechanical interactions as a result of the induced magnetic forces. The analysis of numerous coupled resonators is very challenging. Simple configurations require less computational effort and form the first steps to improve the understanding of these mechanical effects. It is numerically and experimentally shown that LC resonators in a time harmonic magnetic field offer interesting mechanical effects with a strong frequency selectivity. These effects can be enhanced further and even more functionalities can be obtained by using multiple resonator systems rather than a single resonator, although the analysis gets more complicated in multiple resonator systems. The mechanical effects offered by LC resonators, even by a single one, are interesting and useful for many applications. They are attractive for applications in micro robotics and actuators[128–135], where controlling multiple of these actuators is very important[136–143]. Controlling multiple actuators independently is useful also for adaptive optics[144–146] and metamaterials[83,86,147]. Moreover, microvalves[148], microstirrers[149,150] and orientation-unlimited wireless charging of robotic endoscopic capsules[151–154] can be counted as examples of very promising applications.
1. Introduction

THE OUTLINE OF THE DISSERTATION

This dissertation is organized as follows. After the Introduction in Chapter 1, the magnetic field shielding by metamaterials is studied numerically in Chapter 2. It is demonstrated that magnetic field can be shielded successfully by homogenous anisotropic metamaterials made of ideal LC resonators as meta-atoms. It is shown that the introduction of inhomogeneity improves the shielding even further and the inhomogeneous metamaterial can be obtained in a relatively simple numerical way by a constrained linear least square optimization.

In Chapter 3, the shielding of magnetic fields at low frequencies by metamaterials is demonstrated experimentally. A method with purely passive circuits to improve the performance of the meta-atoms in the metamaterial is presented. The significant shielding improvement by the new metamaterial made of the new meta-atoms is shown experimentally. The designed passive meta-atoms are significantly advantageous over active meta-atoms because active circuits may be bulky, expensive and they may have stability problems and need external power sources.

Chapter 4 concentrates on the mechanical effects on LC resonators numerically. Both translational and rotational effects are examined. Even a single LC resonator is considered to be a functional device for motion control and configurations with multiple resonators - which are not necessarily identical - are studied for obtaining advanced mechanical systems.

Chapter 5 proves the interesting mechanical effects on LC resonators experimentally at low frequencies. Single and double resonators are studied experimentally for translational and rotational effects. A ‘wireless motor’ is demonstrated experimentally as an interesting outcome of the interactions in multiple resonator systems.

Chapter 6 concludes the dissertation with a summary and outlook.
BIBLIOGRAPHY

1. Introduction

1. Introduction

1. Introduction


1. Introduction


1. Introduction


1. Introduction


1. Introduction
2. NUMERICAL STUDY OF LOW FREQUENCY MAGNETIC FIELD SHIELDING BY METAMATERIALS

This chapter is based on the article ‘Molding the flow of magnetic field with metamaterials: magnetic field shielding’, M. Boyvat, Ch. Hafner, PIER (Progress in Electromagnetics Research), Vol. 126, pp.303-316; (2012).

The shielding of low frequency magnetic fields is of significant importance, and the most common low frequency magnetic field sources are the city power distribution systems. The stray magnetic field produced by power transformer stations in residential areas is subject to regulations and shielding is needed to satisfy the requirements[1]. In some countries, these regulations became stricter recently. For example, in Switzerland the 100 µT limit for 50Hz was reduced to 1 µT[2,3].

The city power distribution systems are not the only low frequency magnetic field sources to be shielded. The shielding of the magnetic field in wireless power transfer systems[4-6] and inductive heating systems[7,8] is also important.
In this chapter, it is numerically demonstrated that metamaterials can shield magnetic fields at low frequencies. It is shown that the natural anisotropy of metamaterials is helpful in shielding of magnetic fields and the introduction of inhomogeneity in a metamaterial allows one to improve the shielding significantly.

2.1 Anisotropic Materials for Magnetic Shielding at Low Frequencies

A way of shielding low frequency magnetic fields is to use high permeability materials. When a high permeability material is used, shielding happens by trapping field lines to a low reluctance path. In other words, the magnetic field is guided along a desired route. In principle, a low permeability material can also shield. When such a material is used, the material produces opposing magnetic field to the magnetic field entering the material.

When the field in the shielded region is considered, both methods using high and low permeability materials try to do the same thing, i.e. both try to produce opposing magnetic field in the shielded region. However, the two methods are different when the reaction of the material to the field in the material and in the close neighborhood of the material is considered. A material with high permeability attracts magnetic field and enhances the magnetic flux density $B$ in the material, whereas a material with low permeability tries to keep the magnetic flux density inside the material low.

The direction of the field lines, geometrical arrangement of the material and of the source determine which method fits the shielding problem best. If the field lines are parallel to the boundary separating the source and the shielded region, the trapping method is the suitable choice. When the field lines are perpendicular to the boundary between the shielded region and the source, the trapping method is less effective than using a low permeability material. In a realistic case, unshielded field lines are neither completely parallel nor perpendicular to the boundary separating the source and the shielded region. They can be parallel-like to the boundary in some parts whereas perpendicular-like in other parts of the space. Thus, in a realistic case, one would expect to enhance the shielding using a low permeability in the perpendicular direction.
to the boundary and a high permeability in the parallel direction to the boundary. This can be achieved by an anisotropic material.

Figure 2-1 demonstrates that shielding can be improved by introducing anisotropy. The color shows the magnitude of the magnetic flux density $B$ in logarithmic scale at $z=0$ plane, whereas $z$ axis points out of the plane. The shielding material has a rectangular box shape whose projection is seen as a black rectangle in the figure. It is seen that the maximum shielding is obtained when the material has a low permeability in the perpendicular direction to the boundary between the source and the shielded region (the relative permeability $\mu_{rx} = 0.1$) and a high permeability in the parallel directions to the boundary between the source and the region to be shielded ($\mu_{ry} = 10$ and $\mu_{rz} = 10$).

Figure 2-1c shows that even if the permeabilities in the parallel directions are equal to the vacuum permeability ($\mu_{ry} = 1$ and $\mu_{rz} = 1$) whereas $\mu_{rx} = 0.1$, the shielding is improved compared to the isotropic cases.
2. Numerical Study of Shielding by Metamaterials

Figure 2-1: Shielding with different materials. Magnitude of the magnetic flux density $B$ in logarithmic scale at $z=0$ plane. The $x$ axis is the horizontal axis in the figures and the $y$ axis is the vertical axis. The magnetic field source is a coil in the center whose projection is depicted as a black line in the figure. The material is given in a rectangular box shape with equal lengths in $y$ and $z$ directions are equal, whose projection is the black rectangle in the figure. 

a) Isotropic case with low $\mu$ ($\mu_r=0.1$), b) Isotropic case with high $\mu$ ($\mu_r=10$), c) Anisotropic case with low $\mu$ in $x$ direction ($\mu_{rx}=0.1$, $\mu_{ry}=1$ and $\mu_{rz}=1$), d) Anisotropic case with low $\mu$ in $x$ direction ($\mu_{rx}=0.1$) and high $\mu$ in $y$, $z$ directions ($\mu_{ry}=10$ and $\mu_{rz}=10$).
2. Numerical Study of Shielding by Metamaterials

2.2 Metamaterials for High & Low Permeability and Anisotropy

An approach to manufacture the above mentioned anisotropic low or high $\mu$ in practice is to use loaded conductor loops (inductors) to form metamaterials$^{[9–12]}$. Anisotropy then comes naturally due to the orientation of the loops in space. Different magnetic properties can be obtained, depending on the resonance frequency of the meta-atoms, which are modeled by RLC circuits. If the frequency of the time harmonic magnetic field is lower than the resonance frequency of the RLC circuit, the meta-atoms behave like a paramagnetic material whereas they behave like a diamagnetic material above resonance. Thus, by changing the resonance frequency, one can have different magnetic features, i.e., different effective values of the permeability. An illustration of a “naturally” anisotropic metamaterial is shown in Figure 2-2.

When the external magnetic flux density vector has a non-zero component through the loops, which build the inductances in the meta-atoms, the metamaterial reacts to the field. However, if the external field is perpendicular to the axis of the loops, i.e. $B_x$ is 0, no interaction between the external field and the metamaterial happens and no metamaterial effect is observed.

In Figure 2-2, the magnetic field can be shielded by designing the metamaterial to have a low permeability in x direction. It is also possible to add meta-atoms oriented in y and z directions to improve the shielding by having high permeability in these directions.

Calculation of the response of a metamaterial at low frequencies:

Assuming that the magnetic field caused by the source and other elements on an RLC resonator is uniform through the single turn resonator coil, i.e., the inductor $L$, the current induced in the loop is given by$^{[13]}$:

$$ I = -j\omega SB_n / \left( Z_{loop} + Z_{load} \right) , $$

(2.1)
2. Numerical Study of Shielding by Metamaterials

where $\omega$ is $2\pi$ times the frequency of the source magnetic field. $B_n$ is the axial component of magnetic flux density produced by the source and the other loops on the resonator loop. $S$ is the area of the loop. $Z_{\text{loop}} + Z_{\text{load}}$ is the total impedance of the RLC resonator.

![Figure 2-2: An illustration of a simple metamaterial that becomes “naturally” anisotropic. $B_o$ represents the magnetic flux density to be shielded. Unit elements are capacitor loaded inductors with axes in x direction, i.e., parallel to $B_o$. In this metamaterial, the relative permittivity $\mu_{rx}$ may be higher or lower than 1, depending on the selected capacitors and inductors. For obtaining strong shielding, a low $\mu_{rx}$ is desired here. Additional loaded inductors with axes in y and z directions may be created for obtaining high $\mu_{ry}$ and $\mu_{rz}$, to further improve the shielding.](image)

We can write this as:

$$I = k \cdot B_n ,$$  \hspace{1cm} (2.2)

where $k = -j\omega S/(Z_{\text{loop}} + Z_{\text{load}})$.

We can write the axial component of external magnetic field on the loop number $j$ by:
where $B_{o,n}$ is the axial component of magnetic flux density produced by the source. $x_j, y_j, z_j$ is the location of the center of the $j^{th}$ loop. $m_{ij}$ is the axial magnetic field produced by the $i^{th}$ loop with unit current on the $j^{th}$ loop. $I_i$ is the current in the $i^{th}$ loop and $N$ is the number of loops.

By (2.1), (2.2) and (2.3), we can write the current in the $j^{th}$ element by:

$$I_j = k_j \left( B_{o,n} \left( x_j, y_j, z_j \right) + \sum_{i \neq j}^N m_{ij} I_i \right),$$

After putting the terms with currents on the left and the source term on the right, we obtain the following matrix equation:

$$\begin{pmatrix} 1 & -k_1 m_{21} & -k_1 m_{31} & \cdots & -k_1 m_{N1} \\ -k_2 m_{12} & 1 & -k_2 m_{32} & \cdots & -k_2 m_{N2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_N m_{1N} & \cdots & \cdots & \cdots & 1 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{pmatrix} = \begin{pmatrix} k_1 B_{o,n} \left( x_1, y_1, z_1 \right) \\ k_2 B_{o,n} \left( x_2, y_2, z_2 \right) \\ \vdots \\ k_N B_{o,n} \left( x_N, y_N, z_N \right) \end{pmatrix}.$$  

After calculating the currents, one can calculate the field in space analytically for circular loops, assuming a quasi-static problem\[14\]. In Figure 2-3, we see the shielding performance of a homogenous array of such loops. Having a constant magnetic polarizability only in one direction for all loops, we have created a homogenous anisotropic metamaterial. The simulation was done with ideal resonators, which do not have any resistances. This means $k$ is real and unbounded. The magnetic field source is a current loop with a radius of 8 cm, the unit elements of the metamaterial are current loops with radii of 0.8 cm and the distance between elements is 7 cm. Magnetic polarizability of
loops was optimized to minimize the maximum field in the region $212 \text{ cm} > x > 132 \text{ cm}$, $150 \text{ cm} > y, z > -150 \text{ cm}$ by placing some test points in this region. The maximum magnetic flux density in this region is reduced to approximately 10% by the metamaterial. The effective relative permeability of the metamaterial according to the Clausius-Mossotti formula is found to be 0.09 \cite{13}.

Figure 2-4 shows that a single layer of meta-atoms also can shield the magnetic field. A single layer can be seen as a special case of the metamaterial in Figure 2-3, which has multiple layers. Obviously, multiple layer structures offer more parameters which can be optimized, such as the number of layers and the distance between the layers.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2-3.png}
\caption{Shielding effect of a homogenous metamaterial. The color shows the magnitude of magnetic flux density $B$ in the $z=0$ plane in logarithmic scale.}
\end{figure}
By putting the condition of homogeneity and keeping the geometric arrangement the same, we limit the best shielding performance which can be obtained. Thus, to improve the shielding more, one has to either optimize also the geometry (such as radii of resonators, shape of resonators, location and orientations of them in space) or remove the homogeneity condition. In the next section, it is shown that only removing the inhomogeneity condition from the metamaterial can already give significant shielding performances. Additional to its satisfactory shielding, its optimization, which is the optimization of numerous parameters (each of which corresponds to a different resonator), is a constrained linear least squares problem, which is relatively simple.
2. Numerical Study of Shielding by Metamaterials

2.3 Inhomogeneous Anisotropic Metamaterial

To improve shielding, we introduce inhomogeneity in addition to anisotropy. The common method of designing a practical metamaterial is first to design a continuous material, which does not have the discrete structure of a metamaterial yet. Then, benefiting from effective medium approximations and homogenization methods, this continuous material is implemented by discrete meta-atoms and possibly, some optimization/tuning steps follow. Here, instead of coming from a continuous medium approach, we directly solve for the current values in the loops and skip the design of a continuous medium for the desired application. Although designing a continuous medium at the beginning may help to have an intuition, it is an intermediate step and the final aim is to have a system showing the desired effects, which is magnetic shielding of a certain area in this particular case.

The current values in the metamaterial loops are unknowns to be solved and equating magnetic flux density at some test points, we obtain a linear matrix equation.

At all test points, the total magnetic flux density shall be minimized in order to obtain strong shielding. We may set the flux densities in the test points equal to zero and express it in a linear matrix equation form. For the $i^{th}$ test point we have

$$\sum_{j=1}^{N} \vec{B}_{ji} = \vec{B}_{oi} + \sum_{j=1}^{N} I_{j} \vec{B'}_{ji} = 0,$$

(2.6)

where $\vec{B}_{oi}$ is the source magnetic flux density vector at the $i^{th}$ test point, $\vec{B}_{ji}$ is the magnetic flux density produced by the $j^{th}$ element at the $i^{th}$ test point, $\vec{B'}_{ji}$ is the magnetic flux density produced by the $j^{th}$ element carrying unit current at the $i^{th}$ test point and $I_{j}$ is the current in the $j^{th}$ element. Writing this equation for 3 components of $\vec{B}$ for all the test points, we have the following matrix equation:
2. Numerical Study of Shielding by Metamaterials

\[
\begin{pmatrix}
B'_{11,x} & B'_{21,x} & \cdots & B'_{N1,x} \\
B'_{11,y} & B'_{21,y} & \cdots & B'_{N1,y} \\
B'_{11,z} & B'_{21,z} & \cdots & B'_{N1,z} \\
\vdots & \vdots & \ddots & \vdots \\
B'_{1M,y} & \cdots & B'_{NM,y} \\
B'_{1M,z} & \cdots & B'_{NM,z}
\end{pmatrix}
\begin{pmatrix}
I_1 \\
I_2 \\
\vdots \\
I_N
\end{pmatrix}
= 
\begin{pmatrix}
-B_{o1,x} \\
-B_{o1,y} \\
\vdots \\
-B_{oM,y}
\end{pmatrix}
+ 
\begin{pmatrix}
\text{Error}
\end{pmatrix}
, \quad (2.7)
\]

where \( N \) is the number of loops and \( M \) is the number of test points.

Note that using an overdetermined system of equations, i.e., more test points than necessary for a linear matrix equation because using more test points helps delocalizing shielding effect. In this case, the fields at the test points may not be set equal to zero anymore. Instead it is minimized in the least square sense. In addition, we put a maximum limit for the current values to prevent too high values and jumps in currents within the current loops of metamaterial. Thus, the problem turned into the following constrained linear least squares problem\(^{[15]}\):

\[
\min_{I} \| E \|_2^2 = \min_{I} \| CI - D \|_2^2 \quad \text{such that} \quad -I_{\text{max}} < I < I_{\text{max}} ,
\]

where \( I_{\text{max}} \) is the maximum limit for the current magnitude in metamaterial loops. This problem was solved in MATLAB\(^{[15]}\).

The new shielding effect can be seen in Figure 2-5. The maximum magnetic field in the shielded region is approximately 0.42\% of the maximum unshielded field in the same region. Additional to this significant shielding, the maximum current in the resonators in the inhomogeneous metamaterial is less than the maximum current in the homogenous metamaterial in Figure 2-3. Figure 2-6 shows the shielding of a single layer of meta-atoms and again inhomogeneity improves the shielding compared to the shielding by a single layer shown in Figure 2-4. Since the meta-atoms in an inhomogeneous metamaterial can have different polarizabilities, having more layers brings a significant advantage and this is clearly observable when Figure 2-4 and Figure 2-6 are compared.
2. Numerical Study of Shielding by Metamaterials

**Figure 2-5:** Shielding by an inhomogeneous metamaterial.

**Figure 2-6:** Magnetic shielding by a single inhomogeneous layer of meta-atoms.
Inhomogeneity can be obtained by tuning the resonance frequencies of elements separately for obtaining the desired current values. This can be achieved 1) by changing the capacitance values, 2) by changing the inductors, or 3) by using two inductors in series and changing the mutual inductance between them by changing the distance, as mentioned in the next section. Figure 2-7 depicts an inhomogeneous metamaterial obtained by changing capacitance values.

**Figure 2-7:** An illustration of an inhomogeneous anisotropic metamaterial built by capacitances that depend on the location. \( C_{m,n,p} \) corresponds to the capacitance in the element having indices \( m,n \) and \( p \), where indices \( m, n \) and \( p \) run in \( x, y \) and \( z \) directions respectively.

### 2.4 Practical Aspects

In theory, the magnetic flux density of an ideal LC loop has only the same phase or the opposite phase to the external magnetic flux density. However, in practice, resistance introduces reduction in current values and also shifts the phase. Thus, a perfect shielding is not possible with a non-zero resistance. The polarizabilities of the LC loops in the preceding sections are possible only with zero resistance. Since the method aims very low frequencies (for example, 50-60 Hz), compensation of the resistance is possible with some additional electronic circuits. Using a negative impedance converter, the
resistance of the coil can be compensated or the circuit can be converted to a Non-Foster circuit which has more bandwidth and a gyrator would allow one to replace the capacitor by a second inductor (which would also couple to the magnetic field)\(^{16-18}\). However, active circuits have problems such as stability, requirement of an external power source and the need for cooling in case of large currents. In the next chapter, it is shown that the undesired effect of the resistance can be also compensated in a purely passive way, which does not have the problems of active circuits.

A meta-atom unit element which has an inductor connected to a capacitor is limited by commercially available components and does not provide a fine tuning after assembling. Using more than one inductor enables tuning of inductance by mutual coupling, thus tuning of resonance frequency and current in the meta-atom. The total inductance of two inductors connected in series depends on the mutual inductance between coils as well as individual self-inductances\(^{19}\). By changing the relative positioning of two coils connected in series, the total inductance in the circuit, hence the resonance frequency can be adjusted to the desired value.

Instead of using mechanical tuning to tune resonance frequency by changing mutual inductance, the active circuits mentioned above to compensate resistance may also be designed in a way to tune the impedance/current values electronically.

### 2.5 Comparison with Commonly Used Shielding Techniques

Magnetic field shielding techniques can be categorized in two main groups: passive and active shielding\(^{1}\). In passive shielding, no external power sources are added into the shielding system and shielding is done by material properties only. There are two traditional shielding mechanisms for passive shielding. One is to trap/guide the magnetic field using materials with high relative permeability and the second mechanism is eddy current cancellation by highly conductive materials, where the source magnetic field induces some current loops in the conductive material which oppose the source magnetic field. The use of materials with very high permeability (mu-metal is a typical
example) provides better shielding performance than eddy current cancellation at low field values in general. However, permeability and shielding performance at high field strengths decreases\textsuperscript{[20,21]}. In addition to this, mu-metal comes with the disadvantage of high cost\textsuperscript{[20]}.

Active shielding is a suitable method for local shielding in general. It is based on applying some controlled currents in the system to cancel the source magnetic field\textsuperscript{[22,23]}. The metamaterial shielding is similar to passive shielding when there are only RLC resonators and no external circuit, because the shielding is done in principle by effective material properties. When some active components are added to the RLC circuits to control the resonance frequency or to provide some external power to the circuits for loss compensation, the metamaterial starts resembling an active shield. However, it is different from active shielding techniques which continuously monitor the magnetic field at some test points and provide current values in the loops with a reasonable reaction time to cancel the magnetic field\textsuperscript{[23]}. The metamaterial method presented here, requires the source magnetic field profile but not a continuous monitoring of it. If the strength of the magnetic source varies, the electronic steering of the active shielding must react and tune the currents in the loops. No such steering is required in the metamaterial approach because the meta-atoms react automatically on variations of the field strength.

2.6 Conclusion

Numerical studies for resistance-free LC resonators show that metamaterials, which are known mostly for high frequency applications, are able to shield magnetic fields at low frequencies. In shielding of low frequency magnetic fields, anisotropy brings advantages and metamaterials are naturally anisotropic. Since meta-atoms, which can be simple LC resonators, are designable, a controlled inhomogeneity also can be obtained. The introduction of inhomogeneity in the metamaterial improves the shielding capability significantly. Numerical studies have been performed for resistance-free LC resonators as meta-atoms although resistance in a normal LC resonator is unavoidable in practice. However, it is possible to form advanced meta-atoms to compensate the negative effects of resistance, thanks to the large wavelength at low frequencies, as demonstrated in the next chapter.
2. Numerical Study of Shielding by Metamaterials

BIBLIOGRAPHY

2. Numerical Study of Shielding by Metamaterials


2. Numerical Study of Shielding by Metamaterials
3. IMPROVED META-ATOMS FOR MAGNETIC SHIELDING BY METAMATERIALS

This chapter is based on the article ‘Magnetic Field Shielding by Metamaterials’, M. Boyvat, Ch. Hafner, PIER (Progress in Electromagnetics Research), Vol. 136, pp.647-664; (2013).

Magnetic field shielding at low frequencies is a problem of high importance that is known for a long time. Metamaterials, which are known from fancy applications such as the so-called perfect lens and cloaking, also offer a new way to create efficient magnetic shielding by means of anisotropic metamaterials with low permeability in one direction. This was demonstrated numerically in Chapter 2.

Similar to natural materials, which are composed of atoms and molecules, metamaterials are composed of subwavelength units, which are called 'meta-atoms'[1]. Metamaterials to shield low frequency magnetic fields can be constructed by assembling arrays of relatively simple LC circuits. LC circuits as meta-atoms provide strong effects thanks to resonance but they typically come with high loss and narrow bandwidth, being two major issues of metamaterials[2,3]. There have been attempts to overcome these problems by using active elements. When the frequency is low enough, large wavelengths
3. Improved Meta-Atoms for Magnetic Shielding

allow one to use lumped elements such as inductors and capacitors but also electronic parts within a meta-atom\cite{4-10}.

In this chapter, we first discuss the basic principles of a magnetic meta-atom and show that typical resistive losses in the coils and capacitors of the LC circuits reduce the shielding quality. Then, we consider the possibility of active electronic loss compensation and discuss the drawbacks of this concept. Finally, we propose a purely passive way that benefits from the inhomogeneity of the magnetic field to be shielded and show the shielding improvement experimentally.

### 3.1 Shielding Principle of a Magnetic Meta-Atom

When the frequency is very low, a meta-atom, which must have subwavelength size, can be manufactured rather easily because of the long wavelength. For example, it can consist of a simple LC resonator consisting of standard lumped circuit elements, i.e., a coil and a capacitor as shown in Figure 3-1, whereas its optical analogue requires advanced fabrication techniques\cite{11,12}.

For the analysis, the quasi-static approximation can be used. This simplifies the metamaterial analysis considerably. The working principle of an LC resonator as a meta-atom can be explained as follows:

The coupling to the field happens through the inductor, i.e., the coil. When there is a time varying magnetic field through a conductive loop, a current is induced on the loop. Assuming that the magnetic flux is the same for all loops of the coil, this current is given by

\[
I = -N j \omega \phi / (Z_{load} + Z_{loop}),
\]

(3.1)

where \(N\) is the number of turns in the coil, \(\omega\) is the angular frequency, \(\phi\) is the magnetic flux through the coil, caused by the source and the other meta-atoms, \(Z_{load}\) and \(Z_{loop}\) are the impedances of the load connected to the coil and the coil itself. This current also produces a magnetic field and the total magnetic field at a point in space is the superposition of the source magnetic field and the fields caused by all meta-atoms. When the meta-atom is a simple
LC resonator with some resistance, the total impedance and the current in the resonator are given by the following relations:

\[
Z_{\text{total}} = Z_{\text{load}} + Z_{\text{loop}} = j\omega L + 1/(j\omega C) + R, \quad (3.2)
\]

\[
I_{\text{res}} = \frac{-j\omega}{j\omega L + 1/(j\omega C) + R} \int B_{\text{ext}} dS. \quad (3.3)
\]

Figure 3-1: A meta-atom at low frequencies, composed of a capacitor (1 µF) and a coil (1 mH).

When the meta-atom size is small enough, \( B_{\text{ext}} \) can be assumed to be uniform in a meta-atom and the relation between the resonator current and the external magnetic field can be written as \( I_{\text{res}} = \alpha \cdot B_{\text{ext},n} \). Here, \( B_{\text{ext},n} \) is the axial component of the external magnetic flux density at the center of the coil and \( \alpha \) is \( j\omega A/(j\omega L + 1/(j\omega C) + R) \), where \( A \) is the area of the loop. When \( R \) is 0, the resonator produces in-phase magnetic field with the external magnetic field just below the resonance frequency, thus enhances the magnetic field, whereas it is in opposite phase just above the resonance frequency and reduces the magnetic field if the magnitude of the counteracting flux density is less than the double of the external magnetic flux density. When there is a non-zero resistance, one can still observe enhancement below resonance frequency and shielding above resonance frequency, but the magnetic field produced by the meta-atom can never have exactly 0 or 180 degree phase difference with respect to the incident field. The field enhancement and reduction mechanism of a meta-atom can be seen in the phasor diagram shown in Figure 3-2.
3. Improved Meta-Atoms for Magnetic Shielding

\( B_x \) is the x component of magnetic flux density, at a point in space where the x-axis is the axis through the coil. \( B_0 \) represents the external magnetic flux density, \( B_{\text{res}} \) is the field produced by the meta-atom, and \( B_{\text{tot}} \) is the total magnetic flux density. When the frequency changes, the magnitude and the phase of the current in the coil, thus the magnitude and the phase of the magnetic flux density produced by the meta-atom changes.

The circle shown in Figure 3-2 is the trace of the resonator magnetic flux density phasor when the frequency is swept from 0 to infinity. At DC, the resonator magnetic flux density is zero and the resonator flux density phasor sits at the tip of \( B_0 \). When the frequency increases, the tip of the resonator flux density follows the red circle in clock-wise direction. At resonance, the magnitude of the current in the resonator and the resonator magnetic flux density reaches its maximum value. At this frequency, the magnetic flux density produced by the meta-atom has 90 degree phase difference with the external magnetic flux density and the superposition of these two gives more magnitude than the external flux density, which means there is a field enhancement. However, having maximum resonator current does not lead to the maximum total flux density because of the phase difference. Maximum enhancement occurs at a frequency \( f_a \) below resonance frequency, where \( B_{\text{tot}} \) reaches its maximum, which is shown by point M in Figure 3-2. Similarly, maximum reduction occurs at a frequency \( f_b \) above resonance frequency, where \( B_{\text{tot}} \) reaches its minimum, as shown by point N in Figure 3-2. When another point in space is taken, the frequencies \( f_a \) and \( f_b \) also change, because the weights of external and meta-atom flux densities change.

The maximum and minimum flux densities can be found using geometrical properties. If we call the maximum resonator magnetic flux density \( B_{x,\text{res,max}} \), which is the diameter of the circle in the phasor diagram, the maximum and minimum flux densities are given by the following relations:

The maximum x component of total magnetic flux density (see point M in Figure 3-2) is given by

\[
B_{x,\text{max}} = \sqrt{B_0^2 + B_{x,\text{res,max}}^2 / 4 + B_{x,\text{res,max}} / 2},
\]

(3.4)
and the minimum x component of the total magnetic flux density (see point N in Figure 3-2) is given by

\[ B_{x,\text{min}} = \sqrt{B_0^2 + B_{x,\text{res,max}}^2 / 4} - B_{x,\text{res,max}} / 2 \]  \hspace{1cm} (3.5)

One can see that the maximum enhancement in percent is always larger than the maximum reduction in percent. Also it is obvious that it is never possible to have zero magnetic flux density with a non-zero resistance.

**Figure 3-2:** Response of an RLC meta-atom: Phasors of the source (blue, continuous), resonator (red, dashed), and total (black, dotted) magnetic flux density. The circle is the trace of meta-atom magnetic flux density for frequencies from 0 to infinity. M and N points show the points at which the total magnetic flux density is maximum and minimum respectively.
3. Improved Meta-Atoms for Magnetic Shielding

3.2 Improved Meta-Atom with Active Circuits

For non-zero resistance $R$, a single RLC can never shield the magnetic field perfectly, i.e., make it zero near the coil. An obvious improvement is obtained when $L$ and $C$ components with very low resistive values are used. Then, the magnetic flux density circle of the resonator in the phasor diagram enlarges and improves shielding (see Figure 3-2). However, this idea is not feasible in practice because decreasing the resistance of coils requires thicker wires, i.e., this leads to heavy, large, and expensive coils. Obviously, superconducting coils would also provide high costs and additional problems. Furthermore, the reduction of $R$ also reduces the bandwidth of the RLC circuits, which makes its tuning much more difficult.

A common method to improve the meta-atoms is to use active circuits\cite{4,8–10}, for example, by using negative impedance converters. However, active circuits may have serious problems such as stability\cite{8,14} and the requirement of an external power source. Moreover, they need cooling because they need to provide large currents if the source field is not very weak. As a result, one obtains bulky and expensive meta-atoms. A realization of an active circuit to improve the meta-atom in kHz range can be seen in Figure 3-3\cite{15}.

![Image of active circuit and RLC meta-atom](image)

**Figure 3-3:** Comparison of the sizes of an active circuit to improve the meta-atom and an RLC meta-atom.
3. Improved Meta-Atoms For Magnetic Shielding

3.3 Improved Meta-Atom with Advanced Passive Circuitry

In principle, the magnetic field of the source to be shielded already provides energy that may be delivered to the meta-atom. Thus, an external power source – as mentioned in the previous section - is not really needed. One can simply use an additional coil that delivers the energy needed by the electronic circuit. In this case, the problem of huge currents in the active circuit remains.

A more promising alternative is to take advantage of a second coil in a purely passive circuit. Namely, one can use second coil connected in series and locate it closer to the source than the first one. As a consequence, more magnetic flux couples into the meta-atom, and more current is induced. To keep the resonance frequency constant, also a second capacitor is added in series. This makes it possible to enhance the response of the meta-atom whereas the change in the bandwidth may be kept small compared to the gain in the meta-atom response.

The meta-atom then resembles an ideal LC meta-atom more, but it is still not possible to design a circuit with two capacitors and two inductors in such a way that the magnetic flux has a phase exactly opposite to the magnetic flux of the source. In order to realize perfectly opposite phase, the magnetic flux density circle in the phasor diagram (Figure 3-2) needs to be rotated by -90 degrees. This rotation also eliminates the huge field enhancement due to the enhanced current in the meta-atom below resonance frequency.

To introduce a phase shift in a signal, a ‘Lattice Phase Equalizer’, composed of inductors and capacitors was used[16,17]. By combining the idea of using a lattice phase equalizer and using a second coil, the advanced passive circuit in Figure 3-4 has been obtained. As one may see, there are four capacitors and two inductors, one of which is close to the source. More inductors have been avoided because the interaction of those inductors with the magnetic field must be considered and this makes the design more complicated and difficult.
There are two assumptions to simplify the analysis of shielding by the advanced passive circuit. The first one is that the induced voltage is only in the inductor which is close to the source, thus the induced voltage in the other coil is neglected. The second assumption is that the inductor which is close to the source does not contribute to the meta-atom magnetic field in the region to be shielded because it is far away from the region to be shielded.

![Advanced passive circuit to improve a meta-atom for magnetic field shielding.](image)

**Figure 3-4:** Advanced passive circuit to improve a meta-atom for magnetic field shielding.

To obtain the necessary phase shift, the following relation needs to be satisfied at the frequency at which the phase is 90°:

\[ I_{\text{ind},1} = -jV / (kR), \quad (3.6) \]

where \( V \) is the induced voltage in the circuit, \( I_{\text{ind},1} \) is the current in the inductor which is further from the source, \( R \) is the resistance in the initial RLC circuit and \( k \) is a positive real number.

The circuit was designed to have the relation above with reasonable component values considering also the approximate series resistances of components at the frequency for which it is designed. In the end, the component values were rounded to commercially available values. \( C_1, C_2, C_a, C_b, R_1, R_2, R_a, R_b \) values in the final design are given below. \( R_1 \) is the resistance of the coil which is close to the source, \( R_2 \) is the resistance of the other coil, \( R_a \) is the equivalent series resistance of \( C_a \), \( R_b \) is the equivalent series resistance of \( C_b \). The equivalent series resistances of \( C_1 \) and \( C_2 \) are negligible because of the high quality of these capacitors.
3. Improved Meta-Atoms For Magnetic Shielding

\[ C_1 = 1 \ \mu F \ , \ C_2 = 1 \ \mu F \ , \ C_a = 100 \ \mu F \ , \ C_b = 10 \ \mu F \ , \]
\[ R_1 = 1.31 \ \Omega \ , \ R_2 = 1.31 \ \Omega \ , \ R_a = 0.2 \ \Omega \ , \ R_b = 0.37 \ \Omega \]

An RLC and the advanced passive circuit were simulated for comparison and verification. Both were fed by 1 V with 0 phase and the frequency characteristics of both circuits were obtained sweeping the frequency. The phasor representations of currents of both circuits can be seen in Figure 3-5. In the advanced passive circuit, the current through the coil further away from the source, or in other words, closer to the region to be shielded, is plotted. From the simulation, we see that the \( k \) value of the finalized design is 2.45 (See Equation (3.6)).

**Figure 3-5:** Trace of the current phasors with changing frequency for an RLC circuit and the advanced passive circuit (modified LPE). It is known that the magnetic flux density produced by the meta-atom and the current in the meta-atom are in phase and they are proportional to each other, thus this diagram is a direct indication of the meta-atom magnetic flux density. The voltage sources in both circuits have 1 V magnitude and 0 phase. The lattice phase equalizer circuit rotates the circle in the diagram corresponding to the RLC circuit with some shape change which is not important and makes opposite phase to the source magnetic flux density possible (See Figure 3-2).
3. Improved Meta-Atoms for Magnetic Shielding

3.4 Experimental Demonstration of Shielding by Metamaterials

a. Metamaterial composed of simple RLC meta-atoms

To measure the performance of simple RLC meta-atoms, an array of 16 meta-atoms is used. The array is mounted on a square lattice with a distance of 4 cm between neighbor elements. The source is a rectangular coil with the size of 27 cm by 20.1 cm. The array is placed at 14 cm from the source. An illustration of the experimental setup can be seen in Figure 3-6.

![Figure 3-6: Illustration of the experimental setup for a metamaterial built by 16 RLC meta-atoms. The large coil is used as magnetic field source and the metamaterial is a 4x4 array of coil-capacitor pairs.](image)

The frequency characteristic of the metamaterial is shown in Figure 3-7. A Narda EFA-300 Field Analyzer with a 3 cm probe was used to measure the magnetic flux density. The magnetic field probe was placed at 30.5 cm from the source and the frequency dependence of the normalized magnetic flux density was obtained. It was verified that the observed effects are not due to the change in the current through the source coil due to back-coupling.

In the field-frequency curve, we first see a field enhancement up to 73%. When the frequency increases, the metamaterial starts to shield and maximum shielding of 45% is obtained, i.e. the field is reduced to 55%. The bandwidth in which the shielding is more than 70% of its maximum is 171 Hz.
3. Improved Meta-Atoms For Magnetic Shielding

However, the frequency characteristic around the optimum shielding frequency is not symmetric and the shielding drops to 70% of its maximum within 54 Hz towards lower frequencies. The meta-atoms were LC resonators formed by coils with inductance 1 mH and capacitors with 1 µF capacitance. The resonance frequency of a single resonator is calculated to be 5034 Hz. The frequency response curve is shifted to higher frequencies due to negative mutual coupling between elements\[^{[18]}\]. To have an intuition about this shift, one can imagine two identical meta-atoms placed symmetrically so that they have the same flux through them and carry the same current.

\[
I_{\text{res,1}}(j\omega L + 1/(j\omega C) + R) + I_{\text{res,2}}j\omega M = -j\omega \phi_1, \tag{3.7}
\]

\[
I_{\text{res,2}}(j\omega L + 1/(j\omega C) + R) + I_{\text{res,1}}j\omega M = -j\omega \phi_2. \tag{3.8}
\]

Because of the assumed symmetry, \( \phi_1 = \phi_2 \) and \( I_{\text{res,1}} = I_{\text{res,2}} \). Thus, the two equations become identical and get the following form:

![Figure 3-7: Frequency response of a metamaterial built by RLC resonators. \( B_{\text{normalized}} \) is the magnetic flux density normalized to the source magnetic flux density. First, it is seen that the metamaterial shows an enhancement reaching 73%, and when the frequency increases it starts to shield. Maximum shielding is 45%, i.e., the field is reduced to 55%.

However, the frequency characteristic around the optimum shielding frequency is not symmetric and the shielding drops to 70% of its maximum within 54 Hz towards lower frequencies. The meta-atoms were LC resonators formed by coils with inductance 1 mH and capacitors with 1 µF capacitance. The resonance frequency of a single resonator is calculated to be 5034 Hz. The frequency response curve is shifted to higher frequencies due to negative mutual coupling between elements\[^{[18]}\]. To have an intuition about this shift, one can imagine two identical meta-atoms placed symmetrically so that they have the same flux through them and carry the same current.

\[
I_{\text{res,1}}(j\omega L + 1/(j\omega C) + R) + I_{\text{res,2}}j\omega M = -j\omega \phi_1, \tag{3.7}
\]

\[
I_{\text{res,2}}(j\omega L + 1/(j\omega C) + R) + I_{\text{res,1}}j\omega M = -j\omega \phi_2. \tag{3.8}
\]

Because of the assumed symmetry, \( \phi_1 = \phi_2 \) and \( I_{\text{res,1}} = I_{\text{res,2}} \). Thus, the two equations become identical and get the following form:
3. Improved Meta-Atoms for Magnetic Shielding

\[ I_{res}.(j\omega(L + M) + 1/(j\omega C + R)) = -j\omega \phi. \]  \hspace{1cm} (3.9)

Negative mutual coupling effectively decreases the inductance term in the equation, thus increases the resonance frequency.

The spatial distribution of the magnetic flux density in the horizontal plane at the level of the center of the metamaterial layer was also measured. The shielding performance of the metamaterial layer is shown in Figure 3-8.

**Figure 3-8:** Spatial distribution of magnetic flux density without and with metamaterial. The color shows the magnetic flux density. The source coil is at (x=0, y=0). a) The magnetic flux density of the source coil. b) The magnetic flux density with the metamaterial located at x = 14 cm. c) The magnetic flux density with the metamaterial located at x = 38 cm.
3. Improved Meta-Atoms For Magnetic Shielding

b. Metamaterial composed of two coil layers and phase lattice equalizer

A metamaterial was built to demonstrate the improvement by two coil layers and the phase lattice equalizer circuits experimentally. (The new meta-atom can be seen in Figure 3-9). The first coil layer was placed at 38 cm from the source coil and the secondary coil layer, which is close to the source, were placed at 14 cm from the source. The phase and magnitude behaviors were observed with two additional sensor coils, one was to measure the phase of the source magnetic flux density and the other one was to measure the phase of the magnetic flux density in the close neighborhood of the metamaterial layer.

![Figure 3-9: Improved meta-atom with two coils and the lattice phase equalizer circuit.](image)

Figure 3-10 shows the phase change of the magnetic flux density at 43.5 cm from the source with changing frequency. The vertical axis shows the normalized induced voltages in the coils used for monitoring. All reference signals ($V_{ref}$) were normalized to 2 V peak-peak and the magnetic field signals were scaled in the same ratio to be able to compare the magnitude of magnetic field signal ($V_{LPE}$) at the point monitored, additional to the phase of the signal. Thus, both magnitudes and phases of $V_{LPE}$ in Figure 3-10 are the direct measure of the real magnetic flux density signal magnitude and phase to observe the frequency response. The figures show the signal for several frequencies from 5100 Hz to 7000 Hz. It can be seen that the phase difference between the source magnetic flux density and the total magnetic flux density where the monitoring coil was placed starts at low values, passes through 180 degrees and goes towards 360 degrees with increasing frequency. Because of mutual coupling between the coils, the optimum operating frequency of the metamaterial is different from that of individual meta-atoms\[18\].
3. Improved Meta-Atoms for Magnetic Shielding

The frequency characteristic at 63 cm from the source coil can be seen in Figure 3-11. The suppressing effect of the Lattice Phase Equalizer on the below-resonance enhancement can be seen clearly. Thus, the new meta-atom shows much better shielding performance (see Figure 3-8) and it reduces the field enhancement below resonance. Theoretically, if a meta-atom without the phase lattice filter circuit shields the external magnetic field 80%, it enhances the external magnetic field 400% in the enhancement regime below resonance (see Equations (3.4) and (3.5)). If shielding becomes 90%, the enhancement becomes 900%. With the phase lattice filter circuit, this huge enhancement in

Figure 3-10: Phase of the magnetic flux density at different frequencies from 5100 Hz (a) to 7000 Hz (i). The magnetic flux density is observed in time domain by using coils and monitoring the induced voltages on them. \( V_{\text{ref}} \) is the normalized voltage on the coil placed very close to the source coil and \( V_{\text{LPE}} \) is the normalized voltage on the coil placed in the region to be shielded. One can see that the phase of the magnetic flux density is first almost in-phase with the source magnetic flux density. With increasing frequency it passes through 180°.
3. Improved Meta-Atoms For Magnetic Shielding

the vicinity of the shielding frequency band is strongly reduced. The bandwidth in which the shielding is more than 70% of its maximum is 109 Hz. It is 55 Hz on the lower frequency side and 54 Hz towards higher frequencies, which means that the shielding vs. frequency curve around the optimum shielding point is more symmetric than for the standard RLC metamaterial.

After finding the frequency (5550 Hz) at which the phase difference is 180 degree, the spatial shielding behavior of the new metamaterial is observed experimentally. The improvement in the shielding can be seen clearly in Figure 3-12 compared to the standard RLC metamaterial layer (Figure 3-8).

![Figure 3-11: Frequency response of a metamaterial built by meta-atoms with two coils and lattice phase equalizer. $B_{\text{normalized}}$ is the magnetic flux density normalized to the magnetic flux density. The suppression effect of the lattice phase equalizer on the enhancement below resonance is seen clearly.](image)

In the region to be shielded, there is a point of minimum field, at which the metamaterial field cancels the source field. Although zero field is expected at this point, only 87% shielding was measured due to alignment errors, component tolerances, ambient noise and relatively large probe size (3 cm diameter). The distance of the point of minimum field from the source depends on the magnitude of the current through the meta-atoms. Increasing the current
3. Improved Meta-Atoms for Magnetic Shielding

through the meta-atoms, the minimum field point can be shifted to larger distances, whereas decreasing the current would shift the point towards the source. The current through the meta-atoms can be controlled by the position of the secondary coils which are responsible for the induced voltages on the meta-atoms. When these secondary coils are further away from the source coil, meta-atoms have less voltage and current, and this carries the minimum point towards the source. The spatial distribution of magnetic flux density was

Figure 3-12: Spatial distribution of magnetic flux density without and with metamaterial. The color shows the magnetic flux density. The source coil is at (x=0, y=0). a) The magnetic flux density of the source coil. b) The magnetic flux density with the first metamaterial layer at x=38 cm and the layer of secondary coils at x=14 cm. c) The magnetic flux density with the first metamaterial layer at x=38 cm and the layer of secondary coils at x=16.5 cm. d) The magnetic flux density with the first metamaterial layer at x=38 cm and the layer of secondary coils at x=19 cm. One can see that the new meta-atoms improve shielding considerably (compare with Figure 3-8). One can also observe that the location of the dip point can be controlled by the position of the secondary coils.
3. Improved Meta-Atoms For Magnetic Shielding

measured for three different secondary coil positions. Figure 3-12 shows that the minimum point can be shifted towards the source by moving the secondary coils away from the source.

In Figure 3-13, the fields for different cases are plotted along the line y=0. The improvement in the shielding and the shift of the point of minimum field can be seen more clearly.

![Figure 3-13](image)

**Figure 3-13:** Magnetic flux density along the line y=0. $B_{\text{source}}$: Magnetic flux density (B) of the source, $B_{RLC,1}$: B with metamaterial made of RLCs located at x=14 cm, $B_{RLC,2}$: B with metamaterial made of RLCs located at x=38 cm, $B_{LPE,1}$: B with metamaterial made of coil layers at x=14 cm and x=38 cm, and lattice phase equalizer circuits, $B_{LPE,2}$: B with metamaterial made of coil layers at x=16.5 cm and x=38 cm, and lattice phase equalizer circuits, $B_{LPE,3}$: B with metamaterial made of coil layers at x=19 cm and x=38 cm, and lattice phase equalizer circuits.

3.5 Conclusion

Shielding of magnetic fields at very low frequencies by means of metamaterials was studied theoretically and experimentally. A promising method to improve the shielding based on passive LC circuits with a lattice
3. Improved Meta-Atoms for Magnetic Shielding

phase equalizer has been introduced, analyzed, and measured. This design does not have the drawbacks of active circuits, i.e., the resulting metamaterial is relatively cheap and has low weight and low cost. The new meta-atom shows considerably improved shielding, nice symmetry properties with respect to the maximum shielding frequency and much less undesired field enhancement below the maximum shielding frequency.
3. Improved Meta-Atoms For Magnetic Shielding

BIBLIOGRAPHY

3. Improved Meta-Atoms for Magnetic Shielding
4. MECHANICAL EFFECTS ON LC RESONATORS

This chapter is based on the article ‘Wireless control and selection of forces and torques - towards wireless engines’, M. Boyvat, Ch. Hafner, J. Leuthold, Scientific Reports 4, 5681; (2014).

The metamaterials used for the low frequency magnetic shielding application studied in the previous chapters are basically arrays of LC resonators loops consisting of current loops and capacitors. When a current loop is placed in a magnetic field, it senses forces and torques due to ‘Lorentz Force’. The strong frequency selective characteristic of these LC resonators makes them significantly valuable and attractive for multiple motion and actuation control. Powering and controlling multiple objects independently is very crucial in many fields from robotics and medicine to optics and fluid dynamics. We propose LC resonators as wirelessly powered objects/actuators which are addressable by the frequency of the external field and study the mechanical forces and torques.

Metamaterials are composed of many meta-atoms, which carry induced currents. When there are many current loops in a magnetic field, a loop in this field is not affected only by the forces and torques exerted by the source field but also the forces and torques due to the other loops, because each of them is also a magnetic field source. These interactions of meta-atoms have attracted scientific attention and some numerical and experimental studies have already
been done. However, those studies have focused on the interactions as a desired or undesired effect modifying the properties of metamaterials, which are usually composed of homogenous meta-atoms. In this chapter, we take LC resonators out of metamaterials and consider even a single LC resonator as a functional device, unlike metamaterials, which are composed of numerous LC resonators to become metamaterial. We examine mechanical responses of LC resonators to the external field and look into the interactions of them with each other from a more general perspective. We do not restrict the investigation to identical resonators, which is the usual case in metamaterials and we propose using different resonators for multiple independent control, thanks to the strong frequency selectivity of LC resonators.

The mechanical effects offered by LC resonators can find application in micro robotics and actuators\(^1\)\(^–\)\(^8\). Controlling multiple of them is very striking\(^9\)\(^–\)\(^16\). Operating multiple actuators independently is interesting also for adaptive optics\(^17\)\(^–\)\(^19\) and tunable metamaterials\(^20\)\(^–\)\(^22\). Moreover, some rotational outcomes can be used in microvalves\(^23\), microstirrers\(^24\)\(^,\)\(^25\) and orientation-free wireless charging of robotic endoscopic capsules\(^26\)\(^–\)\(^29\).

The chapter starts with the calculation of forces and torques on an induced current in a time harmonic field. After that, as the basic element, a single resonator is studied. Then, the mechanical outcomes of coupled pairs of resonators are shown for two identical and two different resonators. Trapping point activation and selection by the frequency of the external field is shown as an example of multiple resonator usage. This is followed by the study of a configuration with two identical resonators for obtaining a wirelessly powered motor. The configuration of the ‘wireless motor’ is studied also for the case with different resonators.

### 4.1 Calculation of Forces and Torques in a Time Harmonic Field

The force on a current path due to an external magnetic field is calculated by the Lorentz force equation:

\[
\vec{F} = \int d\vec{F} = \int I \, dl \times \vec{B}
\]

(4.1)
where $\overline{F}$ is the force, $I$ is the current, $\overline{dl}$ is the differential vector along the path on which the current flows, whose magnitude is the differential length. Its direction is the direction of the current. $\overline{B}$ is the external magnetic flux density vector.

The resulting torque about a point in space is calculated similarly:

$$
\overline{\tau} = \int_i d\overline{\tau} = \int_i \overline{r} \times \overline{dF} = \int_i \overline{r} \times (I \overline{dl} \times \overline{B}),
$$

(4.2)

where $\overline{\tau}$ is the torque and $\overline{r}$ is the position vector with respect to the point for which the torque is calculated.

When the fields and currents are not static but time harmonic, the forces and torques are found as follows:

$$
\overline{B} = B_0 \cdot \cos(\omega t + \varphi_1),
$$

(4.3)

$$
I = I_0 \cdot \cos(\omega t + \varphi_2),
$$

(4.4)

$$
\overline{F} = \int_i \overline{dF} = \int_i I \overline{dl} \times \overline{B} = \int_i I_0 \cdot \cos(\omega t + \varphi_2) \cdot \overline{dl} \times \overline{B_0} \cdot \cos(\omega t + \varphi_1)
$$

$$
= \int_i I_0 \cdot \overline{dl} \times \overline{B_0} \cdot \frac{1}{2} [\cos(2\omega t) + \cos(\varphi_2 - \varphi_1)].
$$

(4.5)

$$
\overline{\tau} = \int_i \overline{r} \times (I_0 \cdot \overline{dl} \times \overline{B_0}) \cdot \frac{1}{2} [\cos(2\omega t) + \cos(\varphi_2 - \varphi_1)].
$$

(4.6)

We see that force and torque have a harmonic and a static component. We assume that the frequency of the harmonic components is high enough to be neglected in mechanical considerations. Thus, the forces and torques leading to mechanical effects are the average or static forces and torques:

$$
\overline{F}_{avg} = \frac{1}{2} \int_i I_0 \cdot \overline{dl} \times \overline{B_0} \cdot \cos(\varphi_2 - \varphi_1)
$$
4. Mechanical Effects on LC Resonators

\[ = \int I_{RMS} \cdot d\ell \times \vec{B}_{RMS} \cdot \cos(\varphi_2 - \varphi_1), \tag{4.7} \]

\[ \tau = \frac{1}{2} \int r \times (I_0 \cdot d\ell \times \vec{B}_0) \cdot \cos(\varphi_2 - \varphi_1) = \int r \times (I_{RMS} \cdot d\ell \times \vec{B}_{RMS}) \cdot \cos(\varphi_2 - \varphi_1). \tag{4.8} \]

These expressions can be written also in terms of the corresponding phasors:

\[ \bar{F}_{avg} = \text{Re} \left( \frac{1}{2} \cdot \int \vec{\mathcal{I}} \cdot d\ell \times \vec{\mathcal{B}} \right), \tag{4.9} \]

\[ \bar{\tau}_{avg} = \text{Re} \left( \frac{1}{2} \cdot \int r \times (\vec{\mathcal{I}} \cdot d\ell \times \vec{\mathcal{B}}) \right). \tag{4.10} \]

4.2 Single Resonator

A single LC resonator placed in an external magnetic field is the most elementary configuration. It is assumed that a capacitor loads a circular conductive loop, which is nothing else than an LC resonator within an external magnetic field produced by a larger current loop as shown in Figure 4-1.

The self-inductance of a circular loop is given by

\[ L = r \cdot \mu_0 \cdot \left[ \ln(8r/a) - 2 \right], \]

where \( L \) is the self-inductance, \( r \) is the radius of the loop, \( a \) is the radius of the wire and \( \mu_0 \) is the vacuum permeability\(^{[30]}\). In order to have a similar radius and a similar ratio of self-inductance and mutual inductance as in the experiments in Chapter 5, we set \( r = 2.5 \) cm and \( L = 0.15 \) \( \mu \)H by selecting \( a \approx 0.23 \) mm.
The resonance frequency $f_0$ of an LC resonator is equal to $\frac{1}{2\pi\sqrt{LC}}$, where $L$ is the inductance, $C$ is the capacitance. The resonance frequency of the resonator with $L=0.15$ µH and $C=400$ pF is then 20.55 MHz. The resonator has also an inevitable resistance, which causes losses. The quality factor of a series RLC circuit (resistance, inductance and capacitance) is given by $\frac{2}{f_0L R}$, where $R$ is the resistance in the resonator. The quality factor of the resonator is set to 30, in order to have similar values as in the experiments given in the next chapter. The corresponding resistance is $R=0.646$ Ω. The magnetic field source used in the calculations is a circular loop with a 30 cm radius. It carries a time harmonic current $I_s$. The magnetic field source loop is located at $(x=0, y=0, z=0)$ and its axis is the x-axis as shown in Figure 4-1. All the calculations in this chapter are done for the same current of the magnetic field source and the units in the results showing the forces and torques are consistent throughout the whole chapter, thus they can be compared with each other directly.
4. Mechanical Effects on LC Resonators

The forces acting on the resonator can cause both translational and rotational movements. The translational movements are based on the net force on the resonator whereas the torque produces rotational effects. The first step to calculate the forces and torques acting on the resonator is to find the current through the resonator. This is done by calculating the impedance of the resonator and computing the external magnetic flux through the resonator due to the source coil by the analytical formula for the magnetic field of a circular loop, as in the second chapter\textsuperscript{[31,32]}:

$$\mathcal{Z} = \frac{-N \cdot j \cdot \omega \cdot \iint_{A} \vec{B} \cdot d\vec{A}}{j \omega L + 1/(j \omega C) + R} \cdot (4.11)$$

Here, $\mathcal{Z}$ is the current phasor, $\vec{B}$ is the external magnetic flux density phasor, $\omega = 2\pi f$, where $f$ is the frequency of the applied external magnetic field, $L$ is the inductance of the resonator, $C$ is the capacitance and $R$ is the resistance. $N$ is the number of turns of the coil used in the resonator. Throughout this chapter, it is assumed that the coil is a single conductive loop, i.e., $N=1$. The surface integration is performed over the area of the loop.

To see the translational influences on the resonator, the net force acting on the resonator is calculated by Equation (4.9). The net force on the resonator depending on the axial position and the frequency of the applied field can be seen in Figure 4-2a whereas the resonator is kept parallel to the source coil, i.e., $\theta = 0$ (see Figure 4-1). The color shows the force in arbitrary units. Below the resonance frequency, the source applies a positive force for negative x positions and a negative force for positive x positions of the resonator. Thus, the source coil attracts the resonator below the resonance frequency. When the frequency is above the resonance frequency of the resonator, the force changes the sign, i.e., the source repulses the resonator.

The force on a current loop is related to the spatial variation of the applied external field\textsuperscript{[33]}:

$$\vec{F} = \nabla (\vec{\mu} \cdot \vec{B}) \cdot (4.12)$$

Here, $\vec{\mu}$ is the magnetic dipole moment of the current loop and the magnitude of $\vec{\mu}$ is given by the multiplication of the current $I$ and the area $A$. 
of the loop. The magnetic dipole moment \( \mathbf{\mu} \) is along the axis of the loop and its direction is found by the right hand rule. This relation says that the spatial change of the field is necessary to have a net force. The net force vanishes in a uniform magnetic field. Although the larger spatial variation of the field can increase the net forces, high field gradients are not desirable\(^{12}\). Using coupled resonators overcomes this problem as discussed in the following section.

A single resonator in a magnetic field also shows rotational effects, which result from the torque acting on the resonator. The torque is calculated

\[ a) \]

\[ b) \]

Figure 4-2: Force and torque acting on a single resonator. \( a) \) Axial force \( F_x \) acting on a single resonator depending on the \( x \) position and the frequency of the applied magnetic field. The color show the force in arbitrary units. It is seen that the force changes direction at the resonance frequency. The source attracts the resonator when the frequency is below the resonance frequency of the resonator and repulses it when the frequency is above the resonance frequency. \( b) \) The torque around \( z \) axis, \( \tau_z \) acting on a single resonator depending on the angle \( \theta \) (see Figure 4-1) and the frequency of the applied field. The color shows the torque in arbitrary units. The torque changes direction below and above the resonance frequency similar to the axial force acting on it. The torque vanishes at \( 0^\circ, 90^\circ, 180^\circ, 270^\circ \), which are the equilibrium angles. \( 0^\circ \) and \( 180^\circ \) are stable equilibrium angles below the resonance frequency because any deviation from these stable angles causes a torque rotating the resonator back to these angles. If the frequency is shifted above resonance, then the stable angles change to \( 90^\circ \) and \( 270^\circ \), and \( 0^\circ \) and \( 180^\circ \) become unstable.
by Equation (4.10). Assume that the resonator is placed at \((x=0,y=0,z=0)\) and that the resonator is free to rotate around the \(z\) axis. Thus, only the \(z\) component of the torque is relevant. Figure 4-2b shows the variation of \(\tau_z\) with the frequency of the applied magnetic field, \(f\) and the angle \(\theta\) of the axis of the resonator loop with respect to the \(x\)-axis (see Figure 4-1). The torque is strong near the resonance frequency and it has different directions below and above the resonance, whereas it vanishes at resonance. Two different color scales are used for positive and negative torque values to be able to see the equilibrium directions clearly. Positive torques are shown in red scale whereas negative torques are shown in blue scale.

Equilibrium angles, where the torque is zero, are 0 and 180 degrees (the resonator loop is parallel to the source loop) and 90, 270 degrees (the resonator loop is perpendicular to the source loop). At 0 and 180 degrees, the resonator loop is symmetric with respect to \(x\) axis and it does not have any preferred rotational direction. At 90 and 270 degrees, additional to the symmetry, the resonator loop is perpendicular to the source loop and the magnetic flux through the loop is zero, which means that there is no current on the resonator. Although these are equilibrium angles, they are not always stable. A stable equilibrium angle requires that the torque is negative when the angle increases and positive when the angle decreases, so that after any disturbances, the resonator loop comes back to the equilibrium angle. Thus, if the slope of the torque-angle curve is negative at a zero crossing, that angle is a stable equilibrium angle. It is seen that the stable equilibrium angles are 0 and 180 degrees below the resonance frequency and 90 and 270 degrees above the resonance frequency. Therefore, the resonator loop tries to be parallel to the source loop (in other words, it tries to keep its axis parallel to the magnetic flux density vector) below the resonance frequency and perpendicular to the source loop (in other words, it tries to keep its axis perpendicular to the magnetic flux density vector) above the resonance frequency.

### 4.3 Pair of Identical Coupled Resonators

Using multiple resonators complicates the analysis but brings also new opportunities. This section examines the effect of placing two identical resonators close to each other.
When there are multiple coupled resonators, ‘resonance splitting’ happens. In the case of two resonators, there are two resonance frequencies. While the resonance frequency of an isolated LC resonator is given by \( f_0 = \frac{1}{(2\pi \sqrt{LC})} \), the resonance frequencies of a system of two identical coupled LC resonators are found by solving an eigenvalue problem. Eigenvectors are nonzero solutions of an unexcited system:

\[
\mathcal{I}_1 \left[ j\omega L + 1 / (j\omega C) \right] + \mathcal{I}_2 \cdot j\omega M = 0 ,
\]

(4.13)

\[
\mathcal{I}_2 \left[ j\omega L + 1 / (j\omega C) \right] + \mathcal{I}_1 \cdot j\omega M = 0 ,
\]

(4.14)

where \( \mathcal{I}_1 \), \( \mathcal{I}_2 \) are the phasor forms of the currents on the resonators and \( M \) is the mutual inductance.

By multiplying both equations by \( j\omega \) and writing then in matrix form, one obtains the following eigenvalue problem:

\[
\begin{bmatrix}
L & M \\
M & L
\end{bmatrix}^{-1}
\begin{bmatrix}
1 / C & 0 \\
0 & 1 / C
\end{bmatrix}
\begin{bmatrix}
\mathcal{I}_1 \\
\mathcal{I}_2
\end{bmatrix}
= \omega^2
\begin{bmatrix}
\mathcal{I}_1 \\
\mathcal{I}_2
\end{bmatrix},
\]

(4.15)

where \( \omega^2 \) is the eigenvalue.

By solving the eigenvalue problems, two resonance frequencies of the system are found to be:

\[
f_1 = \frac{1}{2\pi \sqrt{(L+M)C}},
\]

(4.16)

\[
f_2 = \frac{1}{2\pi \sqrt{(L-M)C}},
\]

(4.17)

and the corresponding eigenfunctions are: \( \mathcal{I}_1 = \mathcal{I}_2 \) and \( \mathcal{I}_1 = -\mathcal{I}_2 \).

For the calculations, two identical resonators (they have the same parameters as in Section 4.2) are placed nearby the magnetic field source shown in Figure 4-1. One of the resonators is positioned at the center of the source coil, \((x=0, y=0, z=0)\) and parallel to the source coil. The second resonator is
4. Mechanical Effects on LC Resonators

placed at \((x=d, y=0, z=0)\), again parallel to the source coil as seen in Figure 4-3. The second resonator causes a net force on the first resonator (see Figure 4-2). The dependence of the axial force \(F_x\) on the frequency and on the distance between the resonators is shown in Figure 4-3. It is seen that the second resonator attracts the first resonator. With decreasing separation distance \(d\), the strength of the force increases and also the frequency of the excited resonance shifts to lower frequencies. The upper resonance of the system is not excited and we do not see any repulsive forces.

A coupled resonator system has advantages over a single resonator. The forces obtained in a coupled resonator system are way larger than the forces on a single resonator as seen from the results shown in Figure 4-2 and Figure 4-3. Also, the forces between multiple resonators are existent even in a uniform magnetic field. This eliminates problems due to high field gradients\(^{[12]}\).

![Figure 4-3: Attractive force for a pair of identical resonators. a) The schematic of the pair of resonators. \(d\) is the separation distance between the resonators and \(B\) is produced by a larger current loop as shown in Figure 4-1. b) The axial force acting on the resonator 1 depending on the frequency of the applied field and the separation distance \(d\). It is seen that the strength of the force decreases with increasing \(d\). The frequency of the excited resonance shifts to lower frequencies due to mutual coupling between the resonators. The shift is stronger when \(d\) is smaller because of the increased mutual coupling.](image-url)
Figure 4-4 shows the lateral forces for a pair of identical resonators depending on the lateral displacement of the first resonator and the frequency of the applied external field. Both resonators are kept parallel to the source coil, while the first resonator is free to move in y direction and the second resonator is fixed at x=4 cm, y=0, z=0. When the first resonator is disturbed in y direction and not able to move in x direction, it tries to come back to the y=0 position because the forces are in the opposite direction of the displacement. Although Figure 4-4 shows the result for y direction, the conclusion is valid for any lateral direction, i.e., any directions on the yz plane, because of the symmetry of the configuration.

![Figure 4-4](image.png)

**Figure 4-4:** Lateral forces in a pair of identical resonators. a) The schematic of the setup. b) $F_y$ depending on the frequency, $f$ of the applied field and the displacement of the first resonator in y direction, $\Delta y$. It is seen that the first resonator senses a restoring force in any disturbance in y direction, which means $y=0$ is a trapping point.

In Figure 4-5, we see how the torque $\tau_z$ on the first resonator at (0,0,0) changes with the frequency and the angular position of the first resonator. The torque $\tau_z$ is not so different from the single resonator case, except the increase in the torque. Similar to the single resonator, when the rotation angle of the first resonator is 0° (or 180°, which is identical to the configuration of 0°) or 90°, the first resonator is in equilibrium. Below the frequency of the excited
resonance, the stable equilibrium angles are 0° and 180°, whereas 90° becomes stable above the frequency of the excited resonance.

**Figure 4-5:** Torque for a pair of identical resonators. a) The schematic of the setup. b) Torque acting on the first resonator depending on the frequency $f$ of the applied field and the rotation angle $\theta$. The equilibrium directions are 0°, 90° and 180°, similar to the case with a single resonator which is shown in Figure 4-1 and Figure 4-2.

### 4.4 Different Coupled Resonators

As shown in the previous section, a pair of identical resonators offers more and stronger mechanical effects than a single resonator. However, the configuration with identical resonators does not exhibit the full capability of a pair of resonators, because the upper resonance is not excited. If the resonators in Figure 4-3 are not identical but have different resonance frequencies, which can be realized simply by connecting different capacitors, the upper resonance is also excited. Then one can obtain not only attractive forces but also repulsive forces. The resonance frequencies of the system can be found by solving an eigenvalue problem as in the previous section. For different capacitors, the solution is more complicated than identical capacitors.
The first resonance frequency is found to be

\[ f_1 = \frac{1}{2\pi} \left[ \frac{\sqrt{L^2(C_1 - C_2)^2 + 4C_1C_2M^2}}{2C_1C_2(L^2 - M^2)} \right]^{1/2}, \]  

(4.18)

and the corresponding eigen solution is:

\[ I_1 = I_2 \cdot \frac{L(C_1 - C_2) + \sqrt{L^2(C_1 - C_2)^2 + 4C_1C_2M^2}}{2MC_2}. \]  

(4.19)

The second resonance frequency is

\[ f_2 = \frac{1}{2\pi} \left[ \frac{\sqrt{L^2(C_1 - C_2)^2 + 4C_1C_2M^2}}{2C_1C_2(L^2 - M^2)} \right]^{1/2}, \]  

(4.20)

and the corresponding eigen solution is:

\[ I_1 = I_2 \cdot \frac{L(C_1 - C_2) - \sqrt{L^2(C_1 - C_2)^2 + 4C_1C_2M^2}}{2MC_2}. \]  

(4.21)

Unlike the system of two identical coupled resonators, the eigen solutions are neither symmetric (\( I_1 = I_2 \)) nor antisymmetric (\( I_1 = -I_2 \)) but a combination of symmetric and antisymmetric currents. This makes the excitation of both resonances easier.

For the following calculations, the capacitor in the first resonator is changed from 400 pF to 600 pF. As seen in Figure 4-6, having different resonators enables also repulsive forces. But, this comes at the cost of the strength of attractive forces, which is now weaker than the attractive forces for the pair of identical resonators shown in Figure 4-3. Figure 4-6 shows the variation of attractive and repulsive forces with the distance of two resonators for different quality factors. It is seen that the forces in the configuration Figure 4-6 are stronger when the distance is smaller. Also, the resonance frequencies deviate more from the individual resonance frequencies when the distance is smaller because of stronger mutual coupling. Using high quality resonators increases the force strengths but reduces the bandwidth.
4. Mechanical Effects on LC Resonators

Lateral forces are also enriched by the introduction of different resonance frequencies. Figure 4-7 shows the lateral forces for a pair of different resonators. Similar to the configuration in Figure 4-4, the first resonator is able to move in y direction and the second resonator is fixed. Below the lower resonance frequency and above the upper resonance frequency, the system offers a stable equilibrium point at y=0, to which the first resonator is attracted, similar to the pair of identical resonators shown in Figure 4-4. Between two resonance frequencies, the stable equilibrium point becomes an unstable equilibrium point, at which the first resonator moves away in any disturbance in y direction. Because of the symmetry of the configuration, the results are valid for any lateral direction, i.e., any direction in the yz plane.

The rotational effects on the first resonator in presence of the second one is shown in Figure 4-8. Both the external field and the second resonator contribute to the torque acting on the first resonator. It is seen that the upper resonance is also excited and there are stable and unstable equilibrium angles similar to the case of a single resonator.
Figure 4-6: Bidirectional axial forces in the case with two different resonators. 

a) The schematic of the two resonators. 
b) The axial force acting on the first resonator depending on the frequency $f$ of the applied field and the separation distance $d$ for $Q=30$. It is seen that both attractive (red color scale) and repulsive (blue color scale) forces are obtained by using different resonators. 
c) $Q=60$, d) $Q=90$. 

4. Mechanical Effects on LC Resonators
4. Mechanical Effects on LC Resonators

Figure 4-7: The lateral forces in the case with two different resonators. a) The schematic of the resonators. b) $F_y$ depending on the frequency of the applied field and the displacement in $y$ direction $\Delta y$. It is seen that $y=0$ is a trapping point below the lowest resonance and the above the highest resonance (shown by the green circles). Between these two resonance frequencies, this point is converted to an unstable point (shown by the white dotted circle), where the first resonator would move away in any $\Delta y$.

Figure 4-8: Torque in the case with two different resonators. a) The schematic of the resonators. b) Torque acting on the first resonator depending on the frequency $f$ of the applied field and the rotation angle $\theta$. Similar to the axial forces shown in Figure 4-6, the higher resonance is also excited. The effects of both the external magnetic field and the second resonator are significant in the resulting torque.
4. Mechanical Effects on LC Resonators

4.5 Trapping Points Selected by Frequency

Due to the strong frequency selectivity of LC resonators, LC resonators or systems of LC resonators are independently accessible. By using different frequency bands, one can address and control every LC resonator or system of LC resonators, such as a pair of resonators, separately. However, increasing the number of LC resonators is not only useful for increasing the number of controlled actuators/agents. It can also increase the functionality of a system of resonators, by benefiting from the interactions of the resonators in the system rather than focusing on the independence of them. As seen in Section 4.3 and 4.4, using a pair of resonators, which is the simplest system of multiple resonators, brings new functionalities. Here, a three resonator system is presented for a frequency controlled trapping system to exemplify that more advanced operations can be performed by increasing the number of resonators.

The trapping configuration shown in Figure 4-9 is an enriched version of the configuration shown in Figure 4-7. The first resonator is again free to move in y direction and the other two resonators are fixed at x=4 cm, y=-3 cm,
z=0 cm and at x=4 cm, y=3 cm, z=0 cm. The capacitances used in the resonators are $C_1 = 480$ pF, $C_2 = 320$ pF and $C_3 = 800$ pF. For the first resonator, trapping points are at x=0 cm, y=-3 cm, z=0 cm and x=0 cm, cm, y=3 cm, z=0 cm positions. These points can be changed from stable to unstable points by changing the frequency of the applied field.

Although, the example system here includes three resonators, the number of fixed resonators can be increased.

4.6 Off-Axis Arrangement and Motor

As it is shown in the previous sections, using a pair of resonators rather than a single resonator is advantageous. The advantages on translational forces are clearly seen. However, in terms of rotational motions, the pair configuration shown in Section 4.3 and 4.4 does not show a significant novelty compared with a single resonator. Changing the configurations in Section 4.3 and 4.4, one can get more advanced rotational effects. When we put the second resonator in a way that the symmetry with respect to the x axis is broken (see Figure 4-10), it is possible to obtain a ‘wireless motor’.

In the configuration with broken symmetry, the second resonator is placed off-axis at a distance $d$ by an angle $\beta$ and tilted by an angle $\alpha$. The first resonator is free to rotate around the z-axis whereas the second resonator is fixed. The resonators are identical and have the same parameters as in Section 4.2. It is seen that there is a frequency band for which the torque around the z-axis is strong and always positive.

Figure 4-11a shows that the average torque on the resonator is positive for all frequencies and there exists a frequency maximizing the average torque. Figure 4-11b shows the torque acting on the first resonator depending on the angular position $\theta$ at 205 MHz, which is a frequency in the ‘always-positive torque band’. It is seen that the torque is not uniform but positive for all angular positions. Having ‘always-positive torque’ is beneficial because having opposite torques may cause problems such as vibrational movements.

To understand ‘the motor’ more clearly, the system is analyzed also without the resonance shift due to mutual coupling. It is assumed that the
mutual coupling between the resonators is 0. Figure 4-12 shows that the ‘always-positive torque band’ vanishes when the mutual coupling is neglected.

**Figure 4-10:** Off-axis arrangement with two identical resonators resulting in a motor. a) The schematic of the resonators. The first resonator is free to rotate around the z axis. The second resonators is placed at an off-axis position with a distance d and angle β and it is tilted by an angle α. b) The torque around the z axis, $\tau_z$ depending on the frequency of the applied field and the rotation angle $\theta$ for $d=4$ cm, $\beta=\alpha=45^\circ$. It is seen that there is a frequency band in which the torque $\tau_z$ is positive for all rotation angles $\theta$. Therefore, one can obtain a ‘motor’ in this frequency band.
4. Mechanical Effects on LC Resonators

Figure 4-11: a) Torque averaged over rotation angles $\theta$ (see Figure 4-10) depending on the frequency of the applied field. b) Torque vs. rotation angle $\theta$ at 205 MHz, which is a frequency at which the torque is always positive.

Figure 4-12: The torque in the motor shown in Figure 4-10, when neglecting the resonance shift due to the mutual coupling between the resonators. It is seen that the frequency band in which the torque is always positive (see Figure 4-10) disappears if the resonance shift due to the mutual coupling between the resonators is neglected.
Figure 4-13 shows the torque for different distances of the second resonator. It is seen that the ‘always-positive torque band’ shrinks and the torque strength decreases when the distance increases. Figure 4-14 shows the sensitivity of the motor arrangement to the parameters $\alpha$ and $\beta$ shown in Figure 4-10. The configuration does not offer a controllability by frequency on the rotation direction. To change the direction of the torque, one has to move the second resonator to the opposite side of the x-axis.

Figure 4-15 shows what happens when we have different resonators in the ‘motor configuration’ in the previous section. For this calculation, the capacitor in the first resonator is modified from 400 pF to 600 pF. One can see that having identical resonators is crucial in the motor system presented in the previous section, because the frequency band of always positive torque vanishes.

**Figure 4-13:** The effect of the distance $d$ on the motor (see Figure 4-10). The color shows the torque about the z axis, $\tau_z$ depending on the frequency of the applied field and the rotation angle $\theta$, for different $d$'s. $\beta=\alpha=45^\circ$. When the distance increases, the maximum torque decreases and the frequency band in which the torque is always positive shrinks.
4. Mechanical Effects on LC Resonators

Figure 4-14: The effect of $\beta$ and $\alpha$ on the motor. The color shows the torque about the z axis, $\tau_z$, depending on the frequency of the applied field and the rotation angle $\theta$, for different $\beta$’s and $\alpha$’s for $d=6$ cm.

Figure 4-15: Off-axis arrangement similar to the motor configuration shown in Figure 4-10 but with two different resonators. a) The schematic of the resonators. b) The torque around the z axis, $\tau_z$, acting on the first resonator, depending on the frequency of the applied field and the rotation angle $\theta$. $d=4$ cm, $\beta=\alpha=45^\circ$. It is seen that the result is significantly different from the result of the configuration with identical resonators shown in Figure 4-10.
4.7 Conclusion

LC resonators are not only vital elements as meta-atoms for metamaterials but they can also be tools for wireless mechanical control and they can take roles in moving agents. The major gain in using LC resonators as mechanical devices lies in the frequency selectivity of them and wireless powering and control. Frequency selectivity enables independent multiple object control, which allows for collaborating operations of many objects. Multiple resonator usage brings improved functionalities such as enlarged attractive and repulsive forces for a pair of resonators, controlled trapping in a multiple resonator system and continuous rotation which means a ‘motor’. All of these effects make LC resonators interesting for numerous applications, such as micro-robotics, adaptive optics, metamaterials, microvalves and microstirrers.
4. Mechanical Effects on LC Resonators

**BIBLIOGRAPHY**


4. Mechanical Effects on LC Resonators


4. Mechanical Effects on LC Resonators
5. EXPERIMENTS IN MECHANICAL EFFECTS ON LC RESONATORS

This chapter is based on the article ‘Wireless control and selection of forces and torques - towards wireless engines’, M. Boyvat, Ch. Hafner, J. Leuthold, Scientific Reports 4, 5681; (2014).

As discussed in Chapter 4, LC resonators coupled to external fields are not only the constituent element of metamaterials but also frequency selective mechanical devices. In this chapter, the mechanical effects on LC resonators are demonstrated experimentally at large scales as a proof of concept. It is shown that axial forces acting on resonators in a time-harmonic field can be enhanced significantly by using multiple resonators and bi-directionality can be obtained by using different resonators. Moreover, it is demonstrated that there is a torque acting on a single LC resonator in an external field and its direction can be selected by the frequency of the applied field. The rotational experiments also verify that a single resonator is stabilized at different angles depending on the frequency. The ‘motor’ obtained in a system of coupled LC resonators which was already discussed in Chapter 4 is shown experimentally.
5. Experiments in Mechanical Effects

5.1 Enhanced Translational Forces on Identical LC Resonators

When a resonating current loop is placed in a uniform magnetic field, there is no net force on the resonator. For the loop to feel a net force, the magnetic field has to be non-uniform, i.e., it has to vary in space. Putting another resonator in the vicinity of the resonator disturbs this uniformity around the resonator and also increases the field magnitudes in the neighborhood. This produces enhanced forces.

The experimental setup to show the enhancement of forces can be seen in Figure 5-1. The magnetic field is produced by a larger coil (approximately 27 cm x 20 cm) carrying a current $I$. The coils used are air core coils with approximately 0.5 Ohms DC resistance, 1 mH inductance and 4.8 cm diameter. A 1 µF capacitor is connected to each coil. These capacitors have negligible series resistance at the operation frequencies. The calculated resonance frequency of resonators with 1 mH inductance and 1 µF capacitance is 5033 Hz. The distance between the resonators is $d$. First, the force on a single

![Figure 5-1: The schematic of the experimental setup for enhanced translational forces in a pair of identical resonators with a capacitance C. The resonators are placed in a magnetic field with a separation distance d and they sense a force, F. The external magnetic field is produced by a larger source coil, carrying a current I.](image-url)
A resonator placed close to the center of the source coil is measured as a function of frequency. Then, the other resonator is placed in the vicinity of the first resonator and the force on the first resonator is measured again for different separation distances (for $d=0.75$ cm, 1.75 cm and 2.75 cm). It was verified that the field-sensor interaction does not falsify the experiment results. The current flowing through the source changes with the frequency during measurements. To compensate this, measured forces are normalized considering the fact that the force depends on the magnetic field strength (and therefore on the source current) quadratically, which will also be shown experimentally.

The measurements in Figure 5-2 show that the attractive force on the resonator coupled with another identical resonator at 0.75 cm distance at resonance is way stronger than the force on a single resonator at resonance. The peak of the force-frequency curve shifts towards lower frequencies. This is due to the mutual coupling, which is discussed in Chapter 4. When the distance between coupled resonators is increased, the force decreases and the shift of the peak becomes smaller. The peak of the force between the two coupled resonators with $d=0.75$ cm is in the order of a hundred mN for an approximately $B_{\text{rms}}=1$ mT excitation.

![Figure 5-2: Enhanced translational forces in a pair of identical resonators. It is seen that the forces in coupled resonators are way stronger than the force on a single resonator. When the distance d is larger, the forces decrease. Dashed lines are measurement and solid lines are calculations.](image)

83
5. Experiments in Mechanical Effects

The axial force between two current loops is also obtained numerically, assuming a uniform external field. Thus, there is no net force on a single resonator. For this calculation, the variation of the mutual inductance with respect to the position is used\textsuperscript{[1]}, as given in the following equation:

\[ F_x \propto I_1 I_2 \frac{\partial M}{\partial x}. \quad (5.1) \]

This relation holds not only for DC currents but also for harmonic currents, only with the addition of a phase difference term \( \cos(\varphi_1 - \varphi_2) \), as in Equation (4.7).

As seen in Equation (5.1), the axial forces are proportional to the derivative of the mutual inductance with respect to the distance between the resonators. The derivative term in the formula is calculated from experimental mutual inductance data.

The currents are found by the following equations\textsuperscript{\cite{2,3}}:

\[ \mathcal{I}_1 \left[ j \omega L_1 + 1/(j \omega C_1) + R_1 \right] + \mathcal{I}_2, j \omega M = -j \omega N_1 \Phi_1, \quad (5.2) \]

\[ \mathcal{I}_1, j \omega M + \mathcal{I}_2 \left[ j \omega L_2 + 1/(j \omega C_2) + R_2 \right] = -j \omega N_2 \Phi_2. \quad (5.3) \]

Here, \( \mathcal{I}_1, \mathcal{I}_2 \) are current phasors, \( \omega = 2\pi f \), where \( f \) is the frequency of the applied external magnetic field, \( L_1, L_2 \) are the inductances of the resonators, \( C_1, C_2 \) are the capacitances and \( R_1, R_2 \) are the resistances, \( \Phi_1, \Phi_2 \) are the external magnetic fluxes through a loop of the coils. \( N_1, N_2 \) are the number of turns of the coils used in the resonators and it is assumed that the magnetic flux is the same for all loops of a coil.

The resonators are identical and the inductance, capacitance and resistance values used in the calculations are 1 mH, 1 \( \mu \)F and 1 Ohm (approximate resistance of the coil at the operation frequencies), respectively. \( \Phi_1 = \Phi_2 \), because it is assumed that the external field is uniform. The calculated forces are forces with respect to the strength of the excitation, i.e., the right hand side of Equations (5.2) and (5.3), which is appropriate for the frequency and distance dependence of the forces. For an easy comparison of the measured and calculated force characteristics, the magnitudes of the calculated forces are normalized so that the calculated forces have the same
strengths as the measured forces. This normalization is simply done by equating the peak of the calculated force curve for $d=0.75$ cm to the peak of the experimental force curve for $d=0.75$ cm. The obtained normalization factor is then also used for the other calculated curves (curves for $d=1.75$ cm and $d=2.75$ cm). It is seen that the frequency characteristics and the effect of the distance observed in the experiments are in good agreement with the calculations, besides a slight frequency shift. This shift is within the deviation range caused by the tolerances of the inductance and capacitance values used in the experiments.

The induced forces are expected to depend on the external magnetic field strength quadratically. A force on a resonator caused by the other resonator is proportional to the current on the resonator itself and the field produced by the other resonator. Thus, it is expected that the force depends on the square of the external field strength. This can be seen also in Equation (5.1), where both $I_1$ and $I_2$ depend on the excitation field linearly. To verify this experimentally, the force is measured at a fixed frequency and position while the magnitude of the external magnetic flux density changes. In Figure 5-3, the force and the magnetic flux density is plotted in logarithmic scale. $\tilde{F}$ and $\tilde{B}$ are unitless normalized values of the force and the magnetic flux density. If the force is related to the magnetic flux density as $F = K \cdot B^n$, where $K$ is a constant, then the slope of the linear fit in the log-log plot gives $n$. The $n$ extracted from the linear fit in log-log plot is 1.98, which is approximately equal to the theoretical value 2. Figure 5-4 shows how well a quadratic representation fits the measured data in a linear scale.
5. Experiments in Mechanical Effects

Figure 5-3: The change of the force on a resonator with the magnitude of the external magnetic field in logarithmic scale and a linear fit. $\tilde{F}$ and $\tilde{B}$ are unitless normalized values of the force and the magnetic flux density. The slope of the linear fit is 1.98, which verifies the quadratic dependence of the force on the magnetic flux density.

Figure 5-4: The measured dependence of the force on the magnetic flux density and a quadratic fit.
5. Experiments in Mechanical Effects

5.2 Bidirectional Forces on LC Resonators

The enhanced forces between two resonators shown in the previous section are always attractive. Because of the symmetry, the upper resonance of the system is not excited and no repulsive forces can be observed with identical resonators. To have bidirectional forces, two different resonators can be used.

The experimental setup is similar to the setup for identical resonators as shown in Figure 5-5. One of the resonators is the same as in the previous section, but the other one has a 1.5 µF capacitor instead of 1 µF. Figure 5-6 shows the comparison of the forces for a single resonator, for a pair of identical resonators with a separation distance $d=0.75$ cm and for a pair of different resonators again with the same separation distance $d=0.75$ cm. One can see that the repulsive forces (negative forces) between the resonators show up only for the case of different resonators. Furthermore, the attractive forces between the two coupled resonators are weaker when the resonators are not identical.

![Figure 5-5: Schematic of the experimental setup for bi-directional forces. Capacitances in the resonators are different unlike in Figure 5-1. The separation distance $d$ is 0.75 cm.](image)

The repulsive forces shown in Figure 5-6 are not only weaker than the attractive forces between identical resonators but also weaker than the attractive forces in the case of different resonators. However, it is possible to increase repulsive forces while reducing attractive forces by changing the...
5. Experiments in Mechanical Effects

capacitors. By proper capacitance selection, one can design a system of resonators offering repulsive and attractive forces with equal strengths. But, by this modification, while repulsive forces increase, the attractive forces decrease even more.

![Figure 5-6: Bi-directional forces in pairs of resonators. Using different resonators enables repulsive forces (negative). The separation distance $d$ is 0.75 cm. For comparison, forces on a single resonator and on a pair of identical resonators with $d=0.75$ cm are plotted again. When using different resonators, maximum force strength is reduced. The solid lines are calculations and the dashed lines are measurements.](image)

Figure 5-6 shows that the calculations and the measured frequency characteristics are in good agreement except a slight shift in frequency. Again, the shift in the frequency can be attributed mainly to the tolerances of the inductance and capacitance values used. For the calculations, the same procedure as in the previous section is followed. The only difference is the change of the capacitance of one of the resonators to 1.5 $\mu$F.
5.3 Rotational Effects on a Single Resonator

A single resonator in a uniform magnetic field does not sense any translational forces, but there is a torque acting on it if it is neither parallel nor perpendicular to the external field. The torque on a current loop in a uniform magnetic field is given by the following formula:

\[ \vec{\tau} = \vec{\mu} \times \vec{B}, \]  

(5.4)

where \( \vec{\mu} \) is the magnetic dipole moment and \( \vec{B} \) is the magnetic flux density vector. The resonator is free to rotate around the z-axis as shown in Figure 5-7. Thus the z-component of the torque \( \tau_z \) is relevant for the rotation. From Equation (5.4) and Equation (4.8), \( \tau_z \) in a time-harmonic field can be written as:

\[ \tau_z = N \cdot A \cdot I_{\text{RMS}} \cdot B_{\text{RMS}} \cos(\Delta \phi) \sin(\theta). \]  

(5.5)

Here, \( N \) is the number of turns of the coil, \( A \) is the effective loop area, \( I_{\text{RMS}} \) is the RMS value of the current induced in the resonator, \( \Delta \phi \) is the phase difference between the magnetic flux density and the current induced in the resonator, and \( \theta \) is the angle between the magnetic flux density and the magnetic dipole moment as shown in Figure 5-7a. The calculated frequency characteristic of the torque on a resonator in a uniform magnetic field for \( \theta \neq 0^\circ \) and \( \theta \neq 90^\circ \) for 1 mH inductance, 1 µF capacitance and 1 Ohm resistance is shown in Figure 5-7b. Due to the sudden phase change of the induced current \( I_r \) around the resonance frequency, the torque changes direction. Equation (5.5) shows that the torque is zero when \( \theta=0^\circ \). The torque vanishes also at \( \theta=90^\circ \), because there is no magnetic flux through the coil and the induced current is zero, i.e., \( I_{\text{RMS}}=0 \). These are two equilibrium angles of the resonator. Note that they are not always stable. When the frequency of the applied field is below the resonance frequency, the stable angle is 0°, whereas it is 90° when the frequency is above the resonance frequency, as discussed in Section 4.2 and depicted in Figure 5-8.
5. Experiments in Mechanical Effects

Figure 5-7: a) Schematic of the experimental setup for rotational effects on a resonator. The resonator is free to rotate around the z-axis. $\vec{\mu}$ is the magnetic dipole moment and $\theta$ is the angle between the dipole moment $\vec{\mu}$ and the magnetic flux density $\vec{B}$. b) The calculated torque acting on the resonator. Due to the sudden phase change of the current $I_r$ of the resonator around the resonance frequency, the torque changes direction. Since the rotation axis is the z-axis, the z component of the torque is relevant and plotted.
5. Experiments in Mechanical Effects

Figure 5-9 shows the experiment on rotational effects when the frequency is below resonance frequency. The resonator is free to rotate and the frequency of the applied field is 4900 Hz. The details on the resonator and the source coil can be found in Section 5.1. One can see that the field tries to align the resonator parallel to the source coil applying a torque on it. The resonator senses this torque until it reaches the stable equilibrium angle and then the resonator oscillates about this stable angle for some time depending on mechanical damping. In the end, it is stabilized parallel to the source coil (see the frame for $t=3.9$ s).

Figure 5-10 shows the experiment on rotational effects when the frequency is above resonance frequency. The resonator is free to rotate and the frequency of the applied field is 5200 Hz. The position of the source coil is the same as in Figure 5-9. The frequency of the applied field is 5200 Hz.
5. Experiments in Mechanical Effects

One can see that, the resonator rotates in a direction opposite to the rotation direction in Figure 5-9. Again after some oscillatory rotations, the resonator is stabilized but this time, perpendicular to the source coil.

**Figure 5-9:** The experiment demonstrating the stable parallel alignment of the resonator below the resonance frequency. The first picture shows the experimental setup. The resonator is free to rotate and excited by the magnetic field of the source coil. When the frequency of the applied field is near the resonance frequency but below it, the resonator rotates from its initial direction to the stable direction, shown in the last frame. It oscillates for a short time before getting stabilized, as one can see from frames at different times.
5.4 Continuous Rotation by Using Coupled Resonators

As explained in Section 4.6, it is possible to apply a torque in the same direction for all rotational angles by using two resonators. By this, one can obtain a continuous rotation, which leads to a ‘wireless motor’. Figure 5-12 shows the experiment verifying the ‘wireless motor’. When a second identical resonator brought close to the rotating resonator in a way to break the symmetry of the rotation directions, it starts to rotate in the direction shown by the yellow arrow in the first picture of Figure 5-12. The frames are from the starting period of the rotation and the speed of the motor increases rapidly. Placing the second resonator on the other side of the resonator as shown in Figure 5-11, the resonator senses an opposite torque.
5. Experiments in Mechanical Effects

Figure 5-12: Experiment demonstrating the ‘wireless motor’. A second (identical) resonator is brought close to the rotating resonator so that the symmetry for the rotational directions is broken, as shown in the first picture. The direction of the rotation is indicated by the yellow arrow.

Figure 5-11: The rotation in the opposite direction. When the second resonator is positioned as shown, the rotating resonator senses a torque trying to rotate the resonator in the direction shown by the yellow arrow, which is opposite to the rotation direction in Figure 5-12.
5.5 Scaling of Resonators

LC resonators offer very interesting mechanical properties. They do not need any contact or external power sources and strong frequency dependence enables controlling of several of them more or less independently, as long as their resonance frequencies are sufficiently different. These advantages become more emphasized when small scale applications such as micro-robotics are considered, where making contacts is not desired. For miniaturizing, scaling is important. In the following, we perform a rough analysis of scaling.

Let us assume that resonators are built by a single turn loop and a capacitor is connected to this loop. We also assume that the loops are much smaller than the wavelength so that we are in the quasi static regime and that the loss is only the resistive loss in the resonator. We can approximate inductance, capacitance, resistance and the resonance frequency of the resonator as follows:

\[ L = \mu_0 r [\ln(8r/a) - 2] \]  \({}^4\),  \hspace{1cm} (5.6)

\[ C = \varepsilon A_{cap} / d_{cap} \] \hspace{1cm} (5.7)

\[ R = \rho \cdot 2\pi r / A_{wire} \] \hspace{1cm} (5.8)

\[ f_0 = (1/\sqrt{LC}) / (2\pi) \] \hspace{1cm} (5.9)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L)</td>
<td>inductance of the loop</td>
</tr>
<tr>
<td>(\mu_0)</td>
<td>vacuum permeability</td>
</tr>
<tr>
<td>(r)</td>
<td>radius of the loop</td>
</tr>
<tr>
<td>(a)</td>
<td>radius of the wire</td>
</tr>
<tr>
<td>(C)</td>
<td>capacitance</td>
</tr>
<tr>
<td>(\varepsilon)</td>
<td>permittivity of the medium between the capacitor plates</td>
</tr>
<tr>
<td>(A_{cap})</td>
<td>area of the capacitor plates</td>
</tr>
</tbody>
</table>
Note that skin effect is ignored in (5.6) and (5.8). If we scale the geometry by \( k \), i.e., \( r \propto k \), \( a \propto k \), the distance between resonators \( d \propto k \), \( A_{cap} \propto k^2 \), \( d_{cap} \propto k \), \( A_{wire} \propto k^2 \), we obtain the following relations:

\[
C \propto k, \quad L \propto k, \quad R \propto 1/k, \quad f_0 \propto 1/k , \quad (5.10)
\]

\[
Q = \omega_0 L / R \rightarrow Q \propto k . \quad (5.11)
\]

where \( Q \) is the quality factor of the resonator and \( \omega_0 = 2\pi f_0 \).

When the forces are calculated for the scaled geometry assuming that currents are equal, it is seen that the forces stay the same. This means that, in the scaled version, the only factor changing the force is the current on the resonators. The magnitude of the current on the resonators is proportional to the frequency of the exciting field, to the magnetic flux through the resonator and to the inverse of the resistance:

\[
I_{res} \propto f_0 \Phi / R , \quad (5.12)
\]

where \( I_{res} \) is the current at resonance, \( \Phi \) is the magnetic flux through the resonator. \( \Phi \) is proportional to the area of the loop, thus it is proportional to the square of the scaling factor: \( \Phi \propto k^2 \). Therefore, \( I_{res} \propto k^2 \). Finally, we obtain the relation between the force between two resonators and the scaling factor:

\[
F_{max} \propto I_{res}^2 \rightarrow F_{max} \propto k^4 , \quad (5.13)
\]
for the excitation with the same magnetic flux density. Furthermore, the relation between the maximum acceleration in presence of only magnetic forces and the scaling factor is found to be:

\[ m \propto k^3, \quad a_{\text{max}} = F_{\text{max}} / m \rightarrow a_{\text{max}} \propto k. \]

(5.14)

If we assume that the wire thickness is more than the skin depth and include the skin effect for the resistance calculations, the results change:

\[ R \propto 1 / \sqrt{k} \rightarrow I_{\text{max}} \propto k^{3/2} \rightarrow F_{\text{max}} \propto k^3, \]  

(5.15)

\[ Q \propto \sqrt{k}, \]  

(5.16)

and the maximum acceleration \( a_{\text{max}} \) does not change with \( k \).

If we assume that the resonators are excited by a larger loop than the loop of the resonator and also scale the magnetic field source, then excitation field also changes. Then, we get \( F_{\text{max}} \propto k^2 \) without the skin effect, and \( F_{\text{max}} \propto k \) with the skin effect.

When the magnetic excitation strength is assumed to be unlimited (which is not realistic due to the regulations for magnetic fields and physical limitations), the only factor limiting the forces is the heating in the resonators. The small scale resonators have a clear advantage over large scale resonators in terms of heating issues. The allowed current densities in small coils are much higher than in large coils since the resistive losses heating up the conductor are proportional to the volume whereas the heat flow cooling it down is proportional to the surface\[5\].

These rough calculations show how scaling would affect the forces and movements for a single loop. However, to draw concrete conclusions for practical applications, one needs to consider the specific application, the available components and fabrication possibilities in that size and necessary forces for that application.
5. Experiments in Mechanical Effects

5.6 Conclusion

Experiments prove that LC resonators in a magnetic field show interesting mechanical effects with frequency selectivity. Translational forces on LC resonators can be enhanced and bi-directionality can be obtained in a two resonator system. It was verified that there is a torque acting on an LC resonator and its direction can be selected by the frequency of the applied field. Moreover, there are stable directions of an LC resonator relative to the direction of the applied magnetic field and these stable directions can be selected by the frequency of the applied field. Although the experiments were performed with air core coils to guarantee that observations are the effects of LC resonators, it is certainly possible to use magnetic materials in the system to further enhance the effects.

The externally selected, powered and controlled translational forces can be used in micro-robotics and actuators\cite{6-21}. The stabilization of the resonator according to the magnetic flux density direction may be used for self-adaptive metamaterials, in which the meta-atoms adapt their orientations according to the direction of the external field. The rotational stabilization according to the direction of the excitation field can also be useful for wireless power transfer where the orientation of the device is not fixed, such as wireless robotic endoscopic capsules\cite{22-25}. Moreover, the rotational features may find applications in microvalves\cite{26} and microstirrers\cite{27,28}. 

98
BIBLIOGRAPHY

5. Experiments in Mechanical Effects


6. CONCLUSION AND OUTLOOK

6.1 Conclusion

In this dissertation, the shielding of low frequency magnetic fields by metamaterials made of LC resonators was demonstrated numerically and experimentally. Moreover, methods to improve the performance were introduced. Following these, the mechanical effects on LC resonators were examined numerically and experimentally. LC resonators were proposed as addressable actuators/agents with advanced functionalities making them significantly attractive for many applications because of the possibility to simultaneously use many of them independently.

It was shown that metamaterials, which are studied mostly for high frequencies and attracted enormous scientific attention by the invisibility cloak and the perfect lens, are attractive also at very low frequencies. It was demonstrated that they can be used for shielding of low frequency magnetic fields due to the natural anisotropy and designable characteristic of metamaterials. The quasi-static approximation made the calculations easier. After the demonstration of magnetic field shielding by homogenous metamaterials made of ideal resonators without resistance, inhomogeneous metamaterials were proposed. The optimization of the inhomogeneous metamaterials was done in a constrained linear least square sense. It was demonstrated that inhomogeneous metamaterials can clearly outperform homogenous ones when properly optimized.
6. Conclusion and Outlook

Next, the shielding by metamaterials was studied experimentally. The experiments done by using simple LC resonators showed that magnetic shielding at low frequencies is indeed possible. However, the resistance of the resonators leading to a low quality factor (approximately 30), caused a rather limited shielding performance. A way of improving the shielding was the compensation of the effects of the resistance by active circuits. Nevertheless, active circuits are bulky and expensive, they may have stability and heating problems and need external power sources. Another way was to use resonators with very high quality factors. However, such resonators would be also expensive and/or bulky. They would have to be capable of carrying very high currents and voltages and would have a very narrow bandwidth due to the high quality factors. Moreover, in the enhancement frequency band in the vicinity of the shielding band, the field would go extremely high. As a very promising solution to improve the shielding, a new passive and advanced meta-atom was introduced, benefiting from the long wavelength and the magnetic field profile of the source. The new meta-atom was able to behave almost like a resonator without resistance and this brought the calculations done with resistance-free resonators closer to practice. The experiments showed that the shielding performance of the new metamaterial made of advanced meta-atoms was way better than that of the metamaterial with simple LC resonators. The new meta-atom was purely passive, which means no external power source requirement, and did not need more expensive components than those used for the simple resonators used in the experiments. Despite such a high shielding performance, it did not need to have high currents and voltages thanks to the introduction of phase shift. Operating with low current and voltages are important because it reduces heating problems, thus problems and cost of cooling. Furthermore, the shielding bandwidth of the new metamaterial was comparable to that of the metamaterial made of simple LC resonators, and the new metamaterial did not have a strong enhancement in the vicinity of the shielding frequency band.

Following the shielding, the dissertation focused on the mechanical effects on LC resonators. It was shown that there are translational forces on LC resonators excited by an external time harmonic field and these forces increase around the resonance frequencies of the resonator. Moreover, the direction of these forces changes below and above the resonance frequency. It was shown that the forces can be further enhanced by using more than one resonator. A frequency controlled trapping positioning system was given as an example of a multiple resonator system. The rotational effects also were examined. It was shown that the resonators sense torques which change direction below and
above the resonance frequency. The resonators have stable orientations with respect to the external magnetic field and these stable orientations can be controlled by the frequency of the external field. Furthermore, by using a pair of resonators, a continuously rotating system was obtained.

Mechanical effects acting on LC resonators were also demonstrated experimentally. The enhancement and direction control of the forces in a pair of resonators were shown in the experiments. It was also shown that the resonator in an external time harmonic magnetic field senses torques which change direction below and above the resonance frequency. Moreover, the frequency controlled stable orientations and the continuously rotating system or a ‘wireless motor’ were also demonstrated.

6.2 Outlook

The calculation methods shown in Chapter 2 and the principle of designing advanced metamaterials in Chapter 3 can be studied for low frequency applications of metamaterials other than shielding, e.g. the enhancement of fields. Moreover, the idea to improve metamaterials in Chapter 3 can be adapted to higher frequencies. Making the metamaterial shield self-adaptive by the mechanical effects on LC resonators also would be an interesting extension.

As shown in Chapter 4 numerically and in Chapter 5 experimentally, LC resonators offer interesting mechanical effects with a strong frequency selectivity. They are powered and controlled by the external magnetic field and this enables using LC resonators as wirelessly powered and controlled actuators/agents. The importance of such wireless agents/actuators is highly large in small scales in which wires are not desired. The strong frequency selectivity makes simultaneous and independent control of multiple wireless actuators/agents possible. Combining several resonators, more functionalities, such as continuous rotation and selective trapping, can be also obtained. These wirelessly powered multiple actuator/agent systems can find applications in many areas such as adaptive optics, tunable metamaterials, micro-valves, micro-stirrers. By placing several actuators, the shape of optical components such as mirrors can be made tunable, which can find application areas even in macro-scale, such as astronomy. By putting multiple actuators in a
6. Conclusion and Outlook

metamaterial, metamaterials can be tuned locally, which opens the way to controlled inhomogeneity. An immediate example of such a system can be a metamaterial placed on an elastic material. By changing the distance between meta-atoms, the electromagnetic properties of the metamaterial can be controlled locally. The distances can be changed by applying forces on this elastic material at different points. To apply these forces, several actuators based on LC resonators can be placed on the material and these actuators can be operated in different bands from the band of the metamaterial itself. The inhomogeneity is an important feature which is required in many transformation optics and metamaterial applications such as the invisibility cloak. Another immediately following work can be the optimization of the obtained ‘wireless motor’ for higher efficiencies.

Micro-robotics can also benefit from the concept considerably. A multiple micro robot system controlled by a global signal without any need for position knowledge to address individual robots can be an attractive application in this area.