Managerial incentives for risk mitigation and the moderation of credit cycles from a macroprudential perspective

Author(s):
Aptus, Elias

Publication Date:
2014

Permanent Link:
https://doi.org/10.3929/ethz-a-010349761

Rights / License:
In Copyright - Non-Commercial Use Permitted
MANAGERIAL INCENTIVES FOR RISK MITIGATION AND
THE MODERATION OF CREDIT CYCLES FROM A
MACROPRUDENTIAL PERSPECTIVE

A thesis submitted to attain the degree of
DOCTOR OF SCIENCES of ETH ZURICH
(Dr. sc. ETH Zurich)

presented by
ELIAS APTUS
M.Sc. in Applied Mathematics, ETH Zurich

born on 02.07.1987
citizen of Germany

accepted on the recommendation of
Prof. Dr. Hans Gersbach (ETH Zurich), examiner
Prof. Dr. Antoine Bommier (ETH Zurich), co-examiner

2014
Acknowledgments

This Ph.D. thesis was written during my time as a Research Assistant at the Chair of Macroeconomics: Innovation and Policy at ETH Zurich and was supported by a grant from the Swiss National Science Foundation and by the ETH Risk Center. In the following I want to thank those people who were deeply involved in my doctorate, each of them in his or her own, unique way.

First of all, I want to thank my doctoral advisor, Professor Dr. Hans Gersbach. Teaching me scientific concepts and practical approaches, sharing his ideas and supporting me in working on my own, he truly enriched my scientific development. His wide-ranging experience and broad perspective on problems has been a valuable asset during the entire doctorate.

I also want to thank Dr. Volker Britz, who, together with Professor Hans Gersbach, co-authored the first chapter of this thesis. His technical skills as well as his ability to openly discuss any scientific issues contributed tremendously to the progress of our joint work, and also to my personal motivation to work on general ideas and nasty details. Furthermore, I want to thank my co-advisor, Professor Dr. Antoine Bommier, for his readiness to help. I am grateful to Margrit Buser for the careful proofreading of this thesis, as well as for her support in many various issues during my doctorate.

Special thanks go to Professor Dr. Hans Haller, Professor Dr. Alexey Kushnir, Professor Dr. Martin Scheffel, Dr. Maik Schneider, Dr. Oriol Tejada and all members of the ETH Risk Center for valuable comments and useful suggestions for my research, as well as for interesting discussions on broader contexts.

Besides the work on the thesis, my time as Ph.D. student has been shaped by nice atmosphere and the fun we have had with my colleagues at the chair and in the ZUE. For that I want to thank Afsoon Ebrahimi, Johannes Becker, Jürg Müller, Kamali Wickramage, Maik Schneider, Marina Iozef, Marius Lüske, Martin Scheffel, Martin Tischhauser, Max Meulemann, Noemie Hummel, Oriana Ponta, Oriol Tejada, Philippe Muller, Quirin Oberpriller, Salomon Faure, Samuel Schmassmann, Svenja Hector, Ulrich Schetter, Vitalijs Butenko, Volker Britz, Volker Hahn and Yulin Liu.

Finally, I want to thank my family for their huge and wide-ranging support during the doctorate, and Manul and my friends for the good mood.
Abstract

This thesis investigates two macroprudential tools for banking regulation and analyzes two approaches to modeling moral hazard.

After an introductory chapter that motivates the thesis, in the second chapter we examine the impact of so-called “Crisis Contracts” on bank managers’ risk-taking incentives and on the probability of banking crises. Under a Crisis Contract, managers are required to contribute a pre-specified share of their past earnings to finance public rescue funds when a crisis occurs. This can be viewed as a retroactive tax that is levied only when a crisis occurs and that leads to a form of collective liability for bank managers. We develop a game-theoretic model of a banking sector whose shareholders have limited liability, so that society at large will suffer losses if a crisis occurs. Without Crisis Contracts, the managers’ and shareholders’ interests are aligned, and managers take more than the socially optimal level of risk. We investigate how the introduction of Crisis Contracts changes the equilibrium level of risk-taking and the remuneration of bank managers. We establish conditions under which the introduction of Crisis Contracts will reduce the probability of a banking crisis and improve social welfare. We explore how Crisis Contracts and capital requirements can supplement each other and we show that the efficacy of Crisis Contracts is not undermined by attempts to hedge.

In the third chapter we analyze several extensions of the paper “ Aggregate Investment Externalities and Macroprudential Regulation” by Gersbach and Rochet (2012A). First, we show how Gersbach and Rochet’s results are affected when a macroeconomic shock hits both sectors of the economy, instead of one sector only. We provide an intuition why capital shifts between the sectors still occur in the competitive equilibrium and we analytically confirm this intuition. Furthermore, we show that the regulation of capital adjustments – which can be realized via the regulation of short-term debt – can increase social welfare. Second, we show that the results of Gersbach and Rochet with regard to the existence and uniqueness of a competitive equilibrium do not depend on the authorship of the contracts between the bankers and the investors. However, we show that the efficacy of the regulation of capital adjustments does crucially depend on the authorship of the contracts. In fact, when investors write the contracts, the regulation of capital adjustments cannot increase welfare. Third, we show that the results of Gersbach and Rochet
continue to hold qualitatively when the bankers’ moral hazard is modeled by an effort cost instead of a leisure benefit. Thus, we continue Gersbach and Rochet’s investigation of the macroeconomic consequences of the introduction of a Net Stable Funding Ratio as in Basel III.

The two mentioned approaches to modeling moral hazard are standard ones in the literature. In the fourth chapter we consider a principal-agent problem with agent-moral-hazard, in which the allocation is determined by the Generalized Nash Bargaining Solution. In this framework, we show that the two ways to model moral hazard may yield different qualitative results and are biased quantitatively into opposite directions regarding the resulting allocations. We point out the limitations of our results and illustrate how our findings may be used to build and assess models involving agent moral hazard in bargaining situations.
Zusammenfassung

Diese Dissertation untersucht zwei makroprudentielle Instrumente der Bankenregulierung und analysiert zwei Ansätze, Moral Hazard zu modellieren.


## Contents

<table>
<thead>
<tr>
<th>Contents</th>
<th>v</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of Figures</td>
<td>vi</td>
</tr>
<tr>
<td>1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>2 On the Economics of Crisis Contracts</td>
<td>7</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>7</td>
</tr>
<tr>
<td>2.2 The Model</td>
<td>11</td>
</tr>
<tr>
<td>2.2.1 A Two-period Economic Environment</td>
<td>11</td>
</tr>
<tr>
<td>2.2.2 Limited Liability and Bailout</td>
<td>12</td>
</tr>
<tr>
<td>2.2.3 The Banking Game</td>
<td>13</td>
</tr>
<tr>
<td>2.2.4 Banking Equilibrium</td>
<td>16</td>
</tr>
<tr>
<td>2.3 Second-period Subgames</td>
<td>19</td>
</tr>
<tr>
<td>2.4 Managerial Pay in the First Period</td>
<td>23</td>
</tr>
<tr>
<td>2.5 Full Risk Equilibria</td>
<td>27</td>
</tr>
<tr>
<td>2.6 Threshold Equilibria</td>
<td>29</td>
</tr>
<tr>
<td>2.7 Welfare Effects of Crisis Contracts</td>
<td>35</td>
</tr>
<tr>
<td>2.8 Capital Regulation and Crisis Contracts</td>
<td>39</td>
</tr>
<tr>
<td>2.9 Hedging</td>
<td>41</td>
</tr>
<tr>
<td>2.9.1 Risk-neutral Players</td>
<td>41</td>
</tr>
<tr>
<td>2.9.2 Differences in Risk Appetite</td>
<td>42</td>
</tr>
<tr>
<td>2.10 Ramifications and Conclusions</td>
<td>43</td>
</tr>
<tr>
<td>2.10.1 Examples</td>
<td>43</td>
</tr>
<tr>
<td>2.10.2 Extensions</td>
<td>43</td>
</tr>
<tr>
<td>2.10.3 Contract Triggers</td>
<td>45</td>
</tr>
<tr>
<td>2.10.4 Concluding Remarks</td>
<td>46</td>
</tr>
<tr>
<td>2.11 Appendix</td>
<td>47</td>
</tr>
<tr>
<td>2.12 Notation List</td>
<td>49</td>
</tr>
<tr>
<td>3 Liquidity Regulation and Aggregate Investment</td>
<td>51</td>
</tr>
<tr>
<td>3.1 Introduction</td>
<td>51</td>
</tr>
<tr>
<td>3.2 Macroshocks</td>
<td>54</td>
</tr>
<tr>
<td>3.2.1 The Model</td>
<td>55</td>
</tr>
<tr>
<td>3.2.2 Two Simple Benchmarks</td>
<td>59</td>
</tr>
<tr>
<td>3.2.3 Privately-Optimal Contracts</td>
<td>65</td>
</tr>
<tr>
<td>3.2.4 Competitive Equilibrium</td>
<td>65</td>
</tr>
<tr>
<td>3.2.5 Social Efficiency and Regulation</td>
<td>74</td>
</tr>
<tr>
<td>3.3 Contracts Written by Investors</td>
<td>85</td>
</tr>
<tr>
<td>3.3.1 Two Simple Benchmarks</td>
<td>86</td>
</tr>
</tbody>
</table>
List of Figures

3.1 Timeline of Events

4.1 Share \( b_{b,mh}^s \) of the agent, depending on the principal’s bargaining power under the *benefit approach* in the case of \( \frac{e_a}{\tau} > \frac{B}{\Delta} \).

4.2 Share \( b_{b,mh}^s \) of the agent, depending on the principal’s bargaining power under the *benefit approach* in the case of \( \frac{e_a}{\tau} < \frac{B}{\Delta} \).

4.3 Share \( b_{c,mh}^s \) of the agent, depending on the principal’s bargaining power under the *cost approach* in the case of \( \frac{e_a + C}{\tau} > \frac{C}{\Delta} \).

4.4 Share \( b_{c,mh}^s \) of the agent, depending on the principal’s bargaining power under the *cost approach* in the case of \( \frac{e_a + C}{\tau} < \frac{C}{\Delta} \).

4.5 Share of the agent under both approaches, depending on the principal’s bargaining power in the case of \( \frac{e_a}{\tau} > \frac{C}{\Delta} \).

4.6 Share of the agent under both approaches, depending on the principal’s bargaining power in the case of \( \frac{e_a + C}{\tau} < \frac{C}{\Delta} \).

4.7 Share of the agent under both approaches, depending on the principal’s bargaining power in the case of \( \frac{e_a + C}{\tau} > \frac{C}{\Delta} > \frac{e_a}{\tau} \).
1 Introduction

The financial crisis of 2008 demonstrated how much the financial sector is exposed to systemic risks (see Hellwig (2009), for instance). In general, there is more and more evidence that systemic risks affect the financial sector in several ways. Allen et al. (2010), for example, illustrate how individual banks’ incentives to diversify their risks via credit default swaps may lead to collective default and to welfare losses in the short run. Ace-moglu et al. (2013) show that at a high level of interconnectedness within the financial network, the probability of systemic failures is increased due to the risk of contagion to counterparties. They show that under such conditions, banks do not take their own impact on other banks into account, thus generating a network externality. Gersbach and Rochet (2012A) and Gersbach and Rochet (2012B) show how social welfare is lowered by aggregate investment externalities, as banks do not internalize the impact of their own credit fluctuations on the entire banking sector. Some behavioral patterns that may present serious sources of systemic risk in the financial sector are so called “peer effects” between banks, and herding, in particular (see Bonfim and Kim (2012)).

This awareness of the systemic nature of risk in the financial sector asserts the need for regulation of this sector at a systemic level. This is the core of the notion of macroprudential regulation of the financial sector. Borio (2003) provides a comprehensive description of this notion: In contrast with microprudential regulation, which aims at protecting investors and depositors by decreasing the vulnerability of individual financial institutions, macroprudential regulation aims at the reduction of losses in terms of output – such as GDP – by decreasing the vulnerability of the financial system as a whole. To reach its goals, macroprudential regulation focuses on the endogenous aspects of risk emergence within the system through a direct build-up of systemic vulnerabilities or through the interplay and amplifications of individual idiosyncratic risks at institutions at the systemic level. Borio (2011) discusses several dimensions of macroprudential regulation, and strives to strike a balance between ambitious regulatory goals and realistic expectations. In this thesis, we take a macroprudential perspective, thereby focusing on banking regulation. The Basel III regulatory framework comprises a capital regulation scheme that aims at maintaining solvency, as well as a liquidity regulation concept that aims at preventing liquidity shortages (Ingves (2014)). In this thesis, we want to consider both
these objectives from a macroprudential perspective.

We will first consider the objective of maintaining the banks’ solvency. At the systemic level, this objective includes the prevention of banking crises and the reduction of the probability of banking crises caused by solvency problems, which typically occur if the average equity ratio in the banking sector falls below a certain threshold. This prevention or reduction seems to be a sensible goal, as banking crises can lead to serious GDP losses (Laeven and Valencia (2010), for example). A large set of instruments might be useful to achieve this goal. Gersbach (2013) describes a “pecking order of buffers and insurance for banking systems” that consists of three layers. The first layer of regulation consists of equity requirements for banks. The second layer is private insurance, which means that private investors insure the banking sector against crises, either by issuing insurance contracts, or through contingent debt contracts. The third layer is public insurance, either through tradable investment contracts\(^1\), through state guarantees or through bailouts.

In Chapter 2, we will present a new regulatory tool, so-called “Crisis Contracts”, which aims at preventing banking crises. Formally, this tool appertains to private insurance against crises described in Gersbach (2013). However, the core idea of Crisis Contracts differs crucially from classical private insurance. A Crisis Contract may be viewed as a mandatory insurance between each of the bank managers on the one side, who all act as insurers, and the state on the other side, that acts as the policyholder. Note that unlike in Gersbach (2013), this insurance is mandatory for the insurer. The insured event is the occurrence of a banking crisis. If a banking crisis occurs, each manager pays the insurance-payout, which constitutes a fraction of the manager’s past earnings, to a public rescue fund. The bank managers are not financially rewarded for providing such insurance, thus the risk-premium in this insurance is zero.

For bank managers, Crisis Contracts are intended to render excessive risk-taking unattractive, an investment behavior that increases the probability of banking crises and is socially undesirable (see Hellwig (2009) and Chesney et al. (2012)). As under Crisis Contracts, the bank managers would lose a fraction of their past earnings, Crisis Contracts, unlike classical insurance, could prevent excessive risk-taking and thus prevent banking crises or reduce their probability.

Note that unlike classical insurance, which aims at a redistribution of risk in favor of both the insurer and the policyholder and in which the policyholder may be subject to moral hazard when the risk is partially endogenous, a Crisis Contract redistributes a tiny part of the risk from the policyholder to the insurer, aiming to mitigate the insurer’s

\(^1\) See Caballero (2010).
moral hazard by *setting incentives* for higher prudence in the insurer’s actions. As Crisis Contracts address an *endogenous systemic risk* of banking crises, Crisis Contracts clearly belong to the macroprudential toolbox.

We will introduce Crisis Contracts in a game-theoretic model comprising a banking sector with managers and shareholders, where the latter have limited liability and thus are in favor of excessive risk-taking. Shareholders can align the investment decisions of their managers with their own interests by offering return-dependent remuneration schemes. We will explore the interplay of bank managers and shareholders in terms of risk-taking incentives for managers and adjustments of wage offers made by the shareholders in the presence of Crisis Contracts. We will investigate the conditions under which Crisis Contracts may mitigate excessive risk-taking. Furthermore, we will inspect the relationship between Crisis Contracts and capital adequacy requirements, and analyze whether the effectiveness of Crisis Contracts might be sapped by endeavors to hedge.

Second, we will consider in this thesis the objective of banking-regulation to prevent liquidity shortage in the banking sector. For an overview of the liquidity crises that have occurred in the past, as well as of the theoretical frameworks developed both to assess and mitigate this problem, see Amihud et al. (2012). The Basel III Accord contains two liquidity requirements: the Liquidity Coverage Ratio (LCR), which regulates a bank’s liquidity level for a 30-day-horizon, and the Net Stable Funding Ratio (NSFR), which regulates a bank’s liquidity level for a time horizon of one year. We will focus on the NSFR in a way that was described by Gersbach and Rochet (2012A). They provide a framework for the analysis of the NSFR from a macroprudential perspective, in which the banking sector is subject to exogenous shocks, which, together with the bankers’ moral hazard, cause excessive credit fluctuations. Gersbach and Rochet show how under certain assumptions, the NSFR may moderate credit cycles, and thereby cause a reallocation of aggregate investment and increase social welfare. As credit cycles may cause substantial welfare losses (see, for instance, Bianchi and Mendoza (2011)), Gersbach and Rochet’s perspective may yield a macroprudential rationale for the NSFR, as well as point out some of its weaknesses. This will be the starting point of our research.

How the effects discovered by Gersbach and Rochet (2012A) vary when some of the important assumptions are varied will be addressed in Chapter 3. We will explore whether excessive credit cycles still arise and whether the NSFR remains efficient as a macroprudential tool in the following three modifications of Gersbach and Rochet’s benchmark

---

2 Note that as the bank managers’ earnings are very modest compared to the possible output losses from banking crises, the insurance-payout that goes to the public will be rather small in comparison with the public losses caused by a banking crisis and thus will be only a small side effect of Crisis Contracts.

3 Another important practical issue is the optimal calibration of the liquidity requirements to prevent liquidity shortages efficiently without excessive constraints (see Basel Committee on Banking Supervision (2013, 2014)).
model: First, we will investigate a model variant in which not the banking sector alone – as in Gersbach and Rochet (2012A) –, but the entire economy is subject to a macroeconomic shock. Second, we will explore a model variant in which investors – not bankers – write the contracts. Third, we will turn to a model variant in which the bankers’ moral hazard is not represented by a leisure benefit, but by an effort cost.

Moral hazard plays a crucial part in our model with bankers and investors presented in Chapter 3. Furthermore, one form of moral hazard plays an important role in connection with the shareholders’ limited liability in Chapter 2. According to Dembe and Boden (2000), the concept of moral hazard has been used since the nineteenth century. Kreps (1990) describes the moral hazard issue as a situation “where one party to a transaction may undertake certain actions that (a) affect the other party’s valuation of the transaction but that (b) the second party cannot monitor/enforce perfectly”. Especially, moral hazard occurs when one party has an incentive to take actions which generate costs in terms of expected output, costs that will be borne to some extent by the other party. If an employer cannot perfectly observe whether his employee exerts effort at work, for instance, the latter might be subject to moral hazard, as he could reduce his working efforts and enjoy more leisure, thereby compromising the success of the company to some extent. Yet, the employee could not be held liable for his behavior, as if the company’s success were to decrease, the employer could not observe whether this decrease is due to bad performance of the employee or to some exogenous causes.

In the literature on financial intermediation, moral hazard is extensively used to explain the functioning and failures of markets (see, for instance, the much-cited paper by Holmström and Tirole (1997)). Among different approaches to model moral hazard, there are two common ones to which we will refer as the benefit approach and the cost approach, representing two different types of incentives of the party that is subject to moral hazard: either obtaining a private benefit or avoiding a private cost. The choice between these two approaches is seldom motivated by the model builders and often, it is assumed that both approaches yield the same results.

In Chapter 4, we will explore whether these approaches to model moral hazard may yield certain differences in the results of a model when a bargaining situation between two parties is considered, in which each party has some amount of bargaining power.4 If such differences did exist, knowing the properties stemming from the choice of approach might be important both for the assessment of the results of existing models and for the design of

---

4 A party is said to have no bargaining power if in the bargaining process, it cannot obtain more resources than its own outside option would yield.
new models. We will set-up a simple two-player game between a principal and an agent who can take part in a joint production project. The project output is divided between the players according to some given distribution of bargaining power. The agent, who is supposed to exert effort on the project, is subject to moral hazard, which we model in two ways, through a leisure benefit and through an effort cost. We will explore the quantitative and qualitative differences in the allocations, depending on the modeling approach.

The thesis is structured as follows: In Chapter 2, we will address the risk of the emergence of banking crises by investigating Crisis Contracts, a potential new macroprudential regulatory tool for crisis-prevention. In Chapter 3, we will address liquidity regulation by assessing the potential of the Net Stable Funding Ratio from the Basel III Accord as a macroprudential regulatory tool to moderate credit cycles. And in Chapter 4, we will explore the properties of two approaches to modeling moral hazard in a simple two-player game.

\[^5\] In addition, note that from the perspective of behavioral economics, these two approaches might turn out to be significantly different. In contract theory, in particular, loss-aversion could generate a behavioral difference between situations with effort costs and situations with leisure benefits. For a survey of behavioral economics research on loss-aversion and on moral hazard, see Köszegi (2013).
2 On the Economics of Crisis Contracts*

2.1 Introduction

*Motivation and main insight*

In this chapter, we provide a first analysis of so-called “Crisis Contracts” as a regulatory instrument in the banking sector. Under a Crisis Contract, bank managers have to contribute a certain share of their past earnings to public rescue funds when a banking crisis occurs. Using a game-theoretic modeling approach, we will show that when they are suitably combined with capital requirements some Crisis Contracts can reduce the risk of banking crises and improve social welfare.

In the context of the 2008 financial crisis, many governments have felt compelled to provide extensive bailouts to financial institutions, implying a substantial transfer of risks from the banking sector to the government and, ultimately, the taxpayer. Moreover, there is ample evidence that excessive asset risk-taking has played a central role in this crisis (see Hellwig (2009) and Chesney et al. (2012)). This development has led both to a major debate about the regulation of risk in the financial sector and to a controversy about the remuneration of bank managers and the associated “bonus culture.” Crisis Contracts are linked to both of these issues.

The classical regulatory response to excessive risk-taking in the banking sector is to adapt capital requirements. The most recent crisis is no exception. For instance, Admati and Hellwig (2013) have proposed a drastic increase in capital requirements. In addition, there have been a number of policy proposals that would supplement capital requirements with more direct government interventions in banking activities. Some examples of such policy proposals are forced separation of retail banking from investment banking, a limit on the size of banks, or a downright ban on trading certain kinds of financial assets, such as...

---

1 Crisis Contracts were first suggested in Gersbach (2011).
2 For empirical evidence on the evolution of the compensation of bank managers before and after the crisis see Bell and Van Reenen (2014).

---

*This chapter is based on joint research with Volker Britz and Hans Gersbach and was published as CFS working paper (see Aptus et al. (2014)).*
as the prohibition of short-sales. Such direct interventions in banking activities, however, may be ineffective, as they require the regulator to identify ex ante the financial instruments to be deemed risky and hence forbidden or to be linked to high capital requirements. On the one hand, financial innovation can undermine the effectiveness of such regulation, on the other hand, an overzealous regulator might also ban the trade of assets that do play a useful role in the economy. Moreover, even if each individual bank complies with such regulations and has a sufficiently sound investment portfolio, the banking sector as a whole may still be exposed to excessive risk. This problem is discussed in Adrian and Brunnermeier (2011), who propose a modification of the common “Value-at-Risk” method that would also account for covariances.

Given the delicate nature of direct government intervention in banking activities, we propose Crisis Contracts as one alternative way to supplement capital requirements. A Crisis Contract does not intervene in the business of the bank as such, but only affects taxation of bank managers in the event of a crisis. Therefore, Crisis Contracts can also be seen as an alternative to a number of policy proposals that were made regarding the remuneration and liabilities of bank managers. We summarize some of the proposals.

For instance, one possible policy would be to limit the fixed salary of bank managers directly or limit the extent of bonus payments or subject bonus payments to an exceptional bracket of income taxation. Some governments have tried to limit managerial pay at least in those banks rescued by a bailout. The relation between government bailouts and managerial pay in the banking sector has been studied by Hakenes and Schnabel (2010). They develop a theoretical model in which bailout guarantees by the government encourage the shareholders of banks to offer their managers very variable compensation with a large bonus for high returns on investment. This remuneration policy in its turn leads to excessive risk-taking by managers. Empirically, Hakenes and Schnabel argue in favor of a regulatory cap on bonus pay. In a similar vein, Thanassoulis (2012A) proposes a limit to bonus pay that would vary with the bank’s balance sheet. In a follow-up paper, Thanassoulis (2012B) examines the impact of such a regulation on banks’ portfolio choices and finds that the regulation would encourage asset diversification and would also create incentives to focus more on retail banking. John et al. (2000) propose a deposit insurance scheme in which the insurance premium to be paid by a bank depends on that bank’s payment practices. VanHoose (2011) gives an overview of different regulations for bankers’ pay in the United States and provides a survey of theoretical and empirical findings on the effects of such regulations.

In addition to regulations of risk management and the tax system, the judicial system can also be used to discourage excessive risk-taking. In principle, tort law could be used to hold banks, or bank managers, responsible for any damage caused by excessive
risk-taking. In reality, however, it may often be very difficult to assign individual responsibility to specific banks or managers. This is especially true in an environment where systemic risks are present at a macro level and depend among other things on the degree of interconnectedness of different banks. Moreover, tort law may only be a suitable tool if managers have taken actions explicitly or implicitly banned by the regulator. This, again, would require the regulator to discern dangerous practices ex ante. One alternative approach would leave it to the courts to determine ex post which practices can be considered “excessive,” but courts may find this task complex to the point of infeasibility.\(^3\) Armour and Gordon (2013) have recently pointed out that tort law is of limited use in internalizing social costs of banking crises.

Crisis Contracts can avoid some of the aforementioned shortcomings. In particular, a Crisis Contract holds managers in the financial sector liable collectively rather than individually. To apply a Crisis Contract, it is thus not necessary to attribute individual responsibility to a specific bank or a specific manager, so it does not involve any tort action lawsuits. Furthermore, a Crisis Contract does not require the regulator to make ex ante judgments about whether a certain investment strategy is excessively risky or not. Instead, the payments stipulated in a Crisis Contract become due if and when a crisis occurs. Hence, a Crisis Contract cannot be easily undermined by financial innovation. In addition, a Crisis Contract promotes a fairer cost sharing since managers’ previous earnings are “bailed in.” Moreover, a Crisis Contract treats all of the manager’s past earnings from the banking sector equally, rather than singling out the bonus component. Accordingly, the payments stipulated by a Crisis Contract cannot easily be circumvented by redefining a bonus as part of a fixed salary.

No Crisis Contract, however, would work well in an environment with inadequate capital requirements. Excessive risk-taking becomes extremely attractive for both shareholders and managers in such circumstances and outweighs feasible Crisis Contract disincentives. This seems to confirm the idea that in the absence of suitable capital requirements, the financial system as a whole would be highly vulnerable to small shocks that cannot be effectively dealt with by other regulatory tools (Hellwig, 2009; Gersbach, 2013). Recent conceptual contributions identifying the working and design of such requirements are Repullo (2012), Repullo and Suarez (2013), and Admati and Hellwig (2013). We show, however, that Crisis Contracts enable capital requirements to be set at lower levels than would otherwise be necessary to prevent banking crises. Thus, Crisis Contracts appear to be a useful tool for supplementing and strengthening the effects of capital requirements.

\(^3\)The authors are grateful to Roberta Romano for her helpful comments on the limitations of individual liability and tort law in restricting excessive risk-taking.
**Model and formal results**

In this chapter we introduce a stylized two-period model of a financial sector. There are a finite number of banks. Each bank is owned by a shareholder and operated by a manager. In each of the two time periods, each manager chooses between a risky and a safe investment. The shareholder cannot control the manager’s choice directly but may pay the manager depending on the return achieved on the investment. Due to limited liability, the risky investment leads to a higher expected return for the shareholder but to social losses in the event of a crisis. Such a crisis will occur if more than a critical number of managers chose the risky investment in a given period. If a crisis occurs in the second period, then the managers lose a percentage of the income they have previously earned in the banking sector. Additionally, managers have job opportunities outside the banking sector. In this model, the optimal outcome from a social welfare point of view is that a limited subcritical number of banks invest in risky high-return assets.

Our main results are the following: In the absence of Crisis Contracts, the model admits only full risk equilibria, that is, equilibria in which all managers choose the risky investment in both periods. Such an equilibrium maximizes the risk of a crisis and minimizes social welfare. The introduction of a Crisis Contract does not change shareholders’ preference for risky investments, since it directly affects the managers only. Shareholders may respond to the introduction of a Crisis Contract by offering even higher wage or bonus payments to re-align the managers’ interests with their own. It is therefore not immediately clear whether a Crisis Contract will change investment choices. In short, the question of interest is: Will an appropriately designed Crisis Contract avoid crises and improve social welfare?

We show that under certain restrictions on the model parameters, the introduction of a Crisis Contract can undo the full risk equilibria we found in the benchmark case with no Crisis Contracts. Moreover, for a suitable choice of the model parameters we show that the introduction of Crisis Contracts as tools for the regulator leads to the existence of what we will call threshold equilibria. This is a type of equilibrium in which no crisis occurs and in which social welfare is maximized. Some of the parameter restrictions necessary for the effectiveness of a Crisis Contract can be suitably interpreted as bounds on bank leverage and thus as sufficient capital requirements. We conclude that Crisis Contracts can effectively lower the level of risk-taking and enhance social welfare in an environment with sufficient capital requirements. Accordingly, Crisis Contracts may be a suitable alternative to direct government interventions in banking activities. In addition, Crisis Contracts could be perceived as enhancing fairness since in case of crises bank managers are exposed to a (limited) degree of personal liability.

We also discuss the robustness of Crisis Contracts. In particular, we show that the effi-
cacy of Crisis Contracts is not undermined by bank managers’ attempts to hedge against the crisis tax. Moreover, Crisis Contracts remain a useful regulatory tool if some shareholders are adversely affected by banking crises or if a share of smaller banks has no influence on the occurrence of banking crises.

The rest of the chapter is organized as follows: In Section 2 we give a formal description of our game-theoretic model. In Section 3 we discuss subgames played in the second period of the two-period model. We analyze the equilibrium remuneration of managers in Section 4. Then we focus on two specific kinds of equilibrium, the full risk equilibrium and the threshold equilibrium, which from a social welfare point of view turn out to be the worst and best equilibria respectively. We discuss these two kinds of equilibrium in detail in Sections 5 and 6. In Section 7 we study the effects of Crisis Contracts on risk-taking behavior and on social welfare. The relation between capital regulation and Crisis Contracts is identified in Section 8. In Section 9 we explore the possibility of hedging against Crisis Contracts. Finally, in Section 10 we discuss some ramifications and conclude.

2.2 The Model

2.2.1 A Two-period Economic Environment

We model a banking sector consisting of a finite set of identical banks \( N = \{1, \ldots, n\} \) with \( n \geq 2 \), where the members of \( N \) are sometimes indexed by \( i \) or \( j \). The economy has two periods \( t = 1, 2 \). In each period, each bank either invests in a risky asset or in a safe asset or is out of business. We capture the investment decision of bank \( i \) in period \( t \) with the indicator \( A_{it} \in \{R, S, O\} \). We denote the activity profile \((A_{11}, \ldots, A_{n1}, A_{12}, \ldots, A_{n2})\) by \( \mathcal{A} \) and write \( A_{-it} \) for the restriction of the activity profile to round \( t \) and banks \( j \in N \setminus \{i\} \). Given the activity profile \( \mathcal{A} \), we use \( n_t(\mathcal{A}) \) to stand for the number of banks choosing the risky asset in round \( t \). This variable reflects the overall level of risk-taking in the banking sector. We assume that there is a threshold \( \bar{n} \in \{1, \ldots, n - 1\} \) such that investment choices \( \mathcal{A} \) trigger a banking crisis in period \( t \) with positive probability if and only if \( n_t(\mathcal{A}) \) attains this threshold.

**Assumption 2.1**

Let the activity profile be given by \( \mathcal{A} \). If \( n_t(\mathcal{A}) \geq \bar{n} \), then a banking crisis will occur in period \( t \) with probability \( p \in (0, 1) \). If \( n_t(\mathcal{A}) < \bar{n} \), then no banking crisis will occur in period \( t \).

\(^4\) We need at least two periods to examine Crisis Contracts, as a first period is needed to generate the wage income of bank managers for future taxation when a banking crisis occurs.
The restriction $\bar{n} \leq n - 1$ implies that no single bank can prevent a crisis by going out of business or by investing in the safe asset.

Assumption 2.1 is a particular simple and stylized formalization of the idea that the risk of a banking crisis is the result of the joint behavior of banks in the system.

We use the pair of indicators $Z = (Z_1, Z_2)$, where $Z_t = 1$ if a crisis occurs at period $t = 1, 2$ and $Z_t = 0$ otherwise.

We now turn to the asset structure of the economy. The safe asset yields a sure payoff of $x_{rf}$, independently of the occurrence of a crisis. The risky asset yields a payoff of $x_g$ when there is no crisis and a payoff of $x_b$ when there is a crisis. We assume that

$$x_g > x_{rf} > 0 > x_b. \quad (2.1)$$

These inequalities clearly reflect a risk-return trade-off. The risky asset outperforms the safe asset when there is no crisis, but the safe asset performs better than the risky asset when a crisis does occur. We assume that the bank could guarantee itself a sure payoff of zero in each period by staying out of business. Therefore, the payoff $x_b < 0$ from the risky investment in the case of a crisis is suitably interpreted as an (avoidable) loss. In other words, some risk-taking in the economy (by up to $\bar{n} - 1$ banks) guarantees high returns, but major losses are possible when risk-taking is excessive. A more detailed rationale for this payoff structure can be found in Section 2.8.

### 2.2.2 Limited Liability and Bailout

We assume in this chapter that the public is to a large extent liable for the losses made by banks during a crisis. The rationale behind this assumption is that, on the one hand, a functioning banking sector is vital for the economy and thus for the public good as a whole, while on the other hand, banks are owned by shareholders whose liability is limited.\(^5\) Government-backed deposit insurance schemes and downright bailouts as in the recent financial crisis are examples of mechanisms that eventually hold the public liable for losses in the banking sector. Such explicit or implicit liability of the public creates a distortion of the risk-taking incentives. In particular, banks may have an incentive to take risks that are harmful from the social welfare point of view. In a popular phrase, banks may have the possibility to “privatize gains but socialize losses.” In order to restrict attention to those cases where such a conflict of interest does indeed arise, we assume

\(^5\) We do not consider the case of private or family-owned banks, where the owners are personally liable for losses.
henceforth that

\[ px_b + (1 - p)x_g < x_{rf} < (1 - p)x_g. \]  

(2.2)

The leftmost term is the expected payoff from the risky asset, given that a critical number of banks take risk. The rightmost term is the expected payoff from the risky asset when the possible negative realization is disregarded. In other words, the rightmost term is the expected payoff from the point of view of a shareholder with limited liability. Thus, given a sufficiently high level of risk-taking in the banking industry, the public and the shareholders have opposing interests. It is in the public’s interest to invest in the safe asset rather than the risky one, whereas the shareholders have more to gain from investing in the risky asset. In Section 2.8 we provide a simple balance sheet-based derivation of such a payoff structure.

In what follows, we model the relationship between shareholders and managers as a principal-agent problem. Shareholders do not directly control investment decisions, but can pay managers depending on the returns generated on investments.

### 2.2.3 The Banking Game

Having described the economic environment in which banks operate, we now turn to decision-making within banks. Each bank \( i = 1, \ldots, n \) is owned by a single shareholder and run on his behalf by a manager. We will refer to the shareholder and the manager of bank \( i \) as shareholder \( i \) and manager \( i \), respectively. The decision to invest in the risky or safe asset or to go out of business is the result of a strategic game (henceforth the banking game) played by the \( n \) shareholders and the \( n \) managers. More precisely, each period \( t = 1, 2 \) of the banking game proceeds as follows: First, all shareholders simultaneously offer a wage scheme to their respective manager. The wage scheme \( \omega_{it} \) offered by shareholder \( i \) to manager \( i \) in period \( t \) is a triple \( (\omega_{gt}^{it}, \omega_{rf}^{it}, \omega_{b}^{it}) \) specifying the manager’s wage conditional on asset return. The shareholder can only use the asset return to finance the manager’s wage. This budget constraint implies that the manager will earn zero if the asset return is negative. More formally, the set of possible wage schemes is \( \Omega := \{ \omega^{it} \in \mathbb{R}^3_+ | \omega^{it} \leq (x_g, x_{rf}, 0) \} \). Note that the shareholder cannot condition the wage on anything other than the realized asset return in the current period. More particularly, the remuneration of one manager cannot be conditioned on the performance of other managers.

Once the shareholders have made their wage offers, each manager \( i \) observes the wage scheme \( \omega^{it} \) but not the wage schemes \( \omega^{jt} \) offered to the other managers \( j \in N \setminus \{ i \} \). Then all managers choose simultaneously between three options: refuse the offered wage
scheme and work outside the banking sector (“opt out”), accept the wage scheme and invest in the risky asset (“take risk”), or accept the wage scheme and invest in the safe asset (“invest safely”). If manager \( i \) does not opt out, then the instantaneous utilities of shareholder \( i \) and manager \( i \) in period \( t \) are

\[
\begin{align*}
    u_{ts}^i & = \max (x_k - \omega_k, 0), \\
    u_{tm}^i & = \omega_k
\end{align*}
\]

for \( k \in \{g, rf, b\} \), respectively. If manager \( i \) does opt out, then the resulting instantaneous utilities are

\[
\begin{align*}
    u_{ts}^i & = 0 \\
    u_{tm}^i & = D > 0.
\end{align*}
\]

We interpret \( D \) as the wage that the manager could earn outside the banking industry in each period. We make the following assumptions on \( D \) in relation to the other model parameters:

\[
x_{rf} \geq D > (1 - p)x_g + px_b.
\]

These inequalities are related to the social desirability of investments by banks. The safe asset is at least as socially desirable as the manager’s outside option. However, the value of the manager’s outside option is greater than the expected payoff from the risky investment if risk-taking in the banking system is excessive. Note that the second inequality is satisfied whenever \( x_b \) is negative and sufficiently large in absolute value, that is, when the adverse consequences of a crisis are sufficiently bad.

During each period, no actions by a shareholder or a manager of one bank are observable to any other bank’s shareholder or manager. After the first period, the investment choices of the first period become publicly observable. Moreover, all shareholders and managers can observe the occurrence of a crisis. A shareholder cannot make a credible commitment in the first period to a wage scheme he will offer in the second period. Moreover, we assume that a shareholder cannot replace the manager after the first period, even if the manager rejects the shareholder’s wage offer for the second period.\(^6\)

\(^6\)This could be justified for instance by the human capital argument of Hart and Moore (1994). Once a manager is employed, the shareholder faces a loss when he replaces him, as the manager has acquired human capital to run the bank. Our current set-up with wage offers by the shareholders assigns all bargaining power to shareholders. The analysis can be performed for circumstances in which wage offers are made by managers.
We have now described how each period of the game is played. In order to complete the formal description of the incentives in this game, we now specify the intertemporal utilities of shareholders and managers. For this purpose, we will now formally introduce the **Crisis Contract**: It is a remuneration rule for bank managers which stipulates that the manager’s wage from the first period is taxed retroactively at a flat rate of \( c \in [0, 1] \) in the event of a crisis in the second period.\(^7\) Hence a Crisis Contract leads to a kind of collective liability for bank managers.

**Definition 2.1**

*Suppose that manager \( i \) has earned wage \( \omega_{i1}^k \) at \( t = 1 \), and suppose that a crisis occurs at \( t = 2 \). Then the manager will be charged a crisis tax of \( c \omega_{i1}^k \), where \( c \in [0, 1] \) and \( k \in \{g, rf, b\} \).*

Note that the Crisis Contract is only relevant if the manager has worked for the bank and obtained a strictly positive wage in the first period. A manager who takes the outside option of \( D \) in the first period will never be liable for crisis tax. We have already seen that the interests of the shareholder and the public diverge. A shareholder can use “performance pay” (i.e., condition wage payment on the return on investment) to align the manager’s interests with his own. The introduction of a Crisis Contract may allow the government to distort the alignment of shareholder and manager interests in order to better protect the interests of the public.

We assume that the manager is risk-neutral with utility being linear in income. Thus intertemporal utility of a manager is additively separable into the instantaneous utilities of the two periods and the possible crisis tax. More specifically, manager \( i \)’s intertemporal utility is given by

\[
U_{m}^i = u_{m}^{i1} + \delta u_{m}^{i2} - \delta Z_2 c u_{m}^{i3},
\]

where \( \delta \in [0, 1] \) is the discount factor. As we proceed, we will sometimes refer to \( (1 - \delta Z_2 c)u_{m}^{i1} \) as the manager’s net payoff from the first period. Note that this net payoff depends on whether or not a crisis occurs in the second period. If a crisis is expected to occur in the second period with some probability \( \tilde{p} \in [0, 1] \) and \( u_{m}^{i1} \) is manager \( i \)’s expected instantaneous payoff in the first period, then we will refer to the quantity \( (1 - \delta \tilde{p}c)u_{m}^{i1} \) as the manager \( i \)’s expected net payoff from the first period. In this context, we can also think of quantity \( (1 - \delta \tilde{p}c)u_{m}^{i1} + \delta u_{m}^{i2} \) as manager \( i \)’s expected intertemporal utility.

Intertemporal utilities of the shareholder are additively separable into instantaneous utili-

\(^7\) If the government is unsure about the feasibility of retroactive taxation, it may require the relevant share of the first period wage to be deposited in escrow or in a frozen account.
ties, and all shareholders have the same time preferences as all managers. Accordingly,

$$U^i_s = u^{1s}_i + \delta u^{2s}_i.$$  \hspace{1cm} (2.9)

Of course, the linear specification above implies that all shareholders and all managers are risk-neutral. We can think of the wage payments as a direct transfer of utility from the shareholders to the managers.

### 2.2.4 Banking Equilibrium

To complete the description of the game, we now need to specify the notion of a strategy and the solution concepts. To begin with, we focus on second-period subgames of the banking game only. The first-period history of the banking game consists of the wage schemes offered in the first period, the investment decisions taken by the managers in the first period, and the realization of $Z_1$. We use $h$ to denote such a first-period history of the banking game. The set of all first-period histories is denoted by $H \subset \Omega^n \times \{R, S, O\}^n \times \{0, 1\}$. We will only consider histories that are consistent with the rules of the game. For instance, a first-period history in which all managers have invested safely but a crisis has occurred is not consistent and hence does not belong to the set $H$.

From the information in a history $h$ one can infer the amount of crisis tax each manager $i$ would have to pay in the second period if a crisis were to happen. We refer to this amount as the looming crisis tax and treat it as a function $\tau^i(h)$ of the first-period history. We will refer to the subgame starting in the second period following first-period history $h$ as the $h$-subgame. A strategy $\sigma^h_i$ for shareholder $i$ in the $h$-subgame consists only of wage offer $\omega^i \in \Omega$. A strategy $\sigma^h_m$ for manager $i$ in the $h$-subgame is a partition of the set $\Omega$ into three subsets: the set of wage offers to which manager $i$ responds by taking risk, those to which he responds by investing safely, and those to which he responds by opting out. Suppose $\sigma^h$ is a strategy profile for the $h$-subgame. This strategy profile induces an activity profile $A^{-i2}(\sigma^h)$ indicating the second-period investment decisions of all banks other than bank $i$. Assuming that managers $j \in N \setminus \{i\}$ do indeed choose according to $A^{-i2}(\sigma^h)$, and given the wage he has realized in the first period, manager $i$ can calculate the expected amount of crisis tax he will have to pay if he takes risk, or invests safely, or opts out. If, in addition, he is given any second-period wage scheme $w \in \Omega$, manager $i$ can compute his expected payoffs from risk-taking, investing safely, and opting out. If the strategy $\sigma^h_m$ partitions the set $\Omega$ in such a way that manager $i$ chooses an action with maximal expected payoff, then we will say that manager $i$’s strategy $\sigma^h_m$ is a best response to $A^{-i2}(\sigma^h)$. Note that the optimality of $\sigma^h_m$ relates only to the activity profile, not to the
whole profile $\sigma^h$. This kind of optimal behavior is crucial for the equilibrium concept we use to solve the $h$-subgame and to which we refer as the $h$-banking equilibrium.

**Definition 2.2**

A strategy profile $\sigma^h = (\sigma^1_m, \ldots, \sigma^n_m, \sigma^1_s, \ldots, \sigma^n_s)$ in the $h$-subgame is an $h$-banking equilibrium if

1. For each manager $i \in N$, the strategy $\sigma^i_m$ is a best response to $(A^{-i2}(\sigma^h), \tau^i(h))$.

2. For each shareholder $i \in N$, the strategy $\sigma^i_s$ is a best response to $A^{-i2}(\sigma^h)$ and $\sigma^i_m$.

The next step is to introduce the strategies and the solution concept for the entire banking game. A strategy $\sigma^i_m$ for manager $i$ in the banking game consists of a strategy $\sigma^i_m$ in the $h$-subgame for every $h \in H$ and a partition of $\Omega$ into three subsets, the set of wage offers to which manager $i$ responds by risk-taking, those to which he responds by investing safely, and those to which he responds by opting out in the first period. A strategy $\sigma^i_s$ of shareholder $i$ consists of a strategy $\sigma^i_s$ in the $h$-subgame for every $h \in H$ and a wage offer $\omega^i$.1

Recall that we have assumed that at the end of the first period the shareholders and managers of bank $i$ learn about the investment choices of the remaining banks $j \in N \setminus \{i\}$ but cannot condition their behavior in the second period on the wage offers $\omega^j$ for $j \in N \setminus \{i\}$. We formalize this assumption indirectly as a restriction on the strategy space rather than directly on the information structure of the game. This is convenient, as it allows us to maintain a definition of the first-period history under which every such history is the root of a proper subgame.8 In particular, we require the strategies of the shareholders and managers to satisfy the following restriction: If two first-period histories $h', h'' \in H$ involve the same first-period investment choices by all banks and the same realization of $Z_1$, and if for bank $i$ the first-period wage payment to manager $i$ under $h'$ is equal to the first-period wage payment to manager $i$ under $h''$, then $\sigma^i_m = \sigma^i_m$ and $\sigma^i_s = \sigma^i_s$.9 A banking equilibrium is then defined as follows:

---

8 If the restriction at hand was dropped from the definition of the strategy space, then additional equilibria could arise in which the investment choice of one bank depends on earlier wage offers made in other banks. But the wage is a mere redistribution of payoffs between the shareholder and manager of a particular bank and need not in any way concern the shareholders or managers of other banks. Accordingly, such equilibria seem implausible.

9 We note that histories $h'$ and $h''$ can only differ with respect to wage offers and realized wage payments at banks other than bank $i$ and with respect to the wage offer at bank $i$ except for the realized wage in the first period.
Definition 2.3

A strategy profile $\sigma = (\sigma^1_m, \ldots, \sigma^n_m, \sigma^1_s, \ldots, \sigma^n_s)$ is a banking equilibrium if the following holds for every $i \in \{1, \ldots, n\}$:

1. For every $h \in H$ consistent with $\sigma$, the restriction of $\sigma$ to the $h$-subgame is an $h$-banking equilibrium.

2. Given $A^{i2}(\sigma_h)$ for every $h \in H$ consistent with $\sigma$ and given $A^{-i1}(\sigma)$, the partition of $\Omega$ prescribed by $\sigma^i_m$ for the first period is optimal.

3. Given $A^{i2}(\sigma_h)$ for every $h \in H$ consistent with $\sigma$, given $A^{-i1}(\sigma)$, and $\sigma^i_m$, the wage offer prescribed by $\sigma^i_s$ for the first period is optimal.

The idea of a banking equilibrium is based on the subgame-perfect Nash equilibrium. The game-theoretic literature distinguishes between non-cooperative games with sequential and simultaneous moves. In a game with simultaneous moves, one finds Nash equilibria by fixed-point search. Each player’s strategy must be a best response to the strategies of the other players. In games with sequential moves, one finds subgame-perfect Nash equilibria by backward induction. Replacing the ultimate decision nodes by the payoffs resulting from an optimal choice at these nodes, one obtains a reduced game with a “shorter” tree. This reduction is repeated until only the initial node remains. The banking game has both simultaneous and sequential move aspects. On the one hand, the decision by the manager of a bank always follows a decision by the shareholder of the same bank. On the other hand, shareholders decide simultaneously on wage schemes, and the manager of one bank is uninformed about the moves made by the shareholders and managers of the other banks. In addition, the game is played over two periods.

The procedure for finding the banking equilibria is backward induction nested in a fixed-point argument. Initially, we fix the activities of all but one bank. Given the activities of other banks, we can find a best-response correspondence for the manager of the one remaining bank. We then proceed by backward induction to the shareholder of that bank and determine his optimal decision. In this way, we find the optimal activity of the bank under consideration. That is, we have begun with a specification of bank activities by all but one bank and obtained an optimal bank activity for the bank under consideration. If we do this for every bank one at a time, we obtain a map from a profile of bank activities to a profile of bank activities. A fixed point of that map is an $h$-banking equilibrium.

One important feature of the banking equilibrium is that it restricts the managers’ behavior at non-singleton information sets. Loosely speaking, one could describe the banking equilibrium as being “information-set perfect” rather than “subgame perfect.”

One could also think of the banking equilibrium as a Bayesian equilibrium with deterministic beliefs. We do not, however, define belief systems in this chapter, since we do not focus on the whole set of Bayesian equilibria of the banking game.
2.3 Second-period Subgames

In this section we focus on the second period and deal first with the relationship between the shareholder and the manager of each bank. Later we deal with the strategic interaction between the different banks.

Fix a first-period history $h \in H$ and a strategy profile $\sigma^h$ for the $h$-subgame. Let $\sigma^h$ be such that manager $i$ works for the bank (rather than taking the outside option) in the second period. In this case, we define manager $i$'s expected wage in the second period as

$$\mu_i^2(\sigma^h) = \begin{cases} 
\omega_i^2(\sigma^h) & \text{if } n_2(\sigma^h) \leq \bar{n} - 1 \text{ and } A_i^2(\sigma^h) = R, \\
(1 - p)\omega_i^2(\sigma^h) & \text{if } n_2(\sigma^h) \geq \bar{n} \text{ and } A_i^2(\sigma^h) = R, \\
\omega_i^2(\sigma^h) & \text{if } A_i^2(\sigma^h) = S.
\end{cases}$$

This wage is “expected” in the sense that the realization of the possible move of nature in the second period is not yet known. Observe that the expected wage has only been defined for the case where the manager works for the bank and thus forgoes his outside option. Let us now suppose that manager $i$ deviates from the strategy profile $\sigma^h$ by opting out in the second period. This has three effects on his expected payoff. First, he loses the expected wage as defined above. Second, he gains the payoff $D$. Third, if $n_2(\sigma^h) = \bar{n}$ and $A_i^2(\sigma^h) = R$, then his deviation to the outside option reduces the probability of a crisis from $p$ to zero and the expected crisis tax bill from $p\tau(h)$ to zero. Whenever the latter two effects outweigh the loss of the expected wage, the deviation to the outside option is profitable. More formally, we say that manager $i$ is pivotal under $\sigma^h$ if $n_2(\sigma^h) = \bar{n}$ and $A_i^2(\sigma^h) = R$. In words, manager $i$ is considered pivotal if his decision to take risk is responsible for attaining threshold $\bar{n}$.

For a (not necessarily pivotal) manager $i$, we define his reservation wage as

$$\kappa_i^2(\sigma^h) = \begin{cases} 
D + p\tau_i(h) & \text{if } n_2(\sigma^h) = \bar{n} \text{ and } A_i^2(\sigma^h) = R, \\
D & \text{otherwise.}
\end{cases}$$

From the construction of the reservation wage it follows that working for a bank is optimal for a manager only if the expected wage is greater than, or equal to, the reservation wage. In the sequel, our claim is that in equilibrium the expected wage is equal to the reservation wage. The intuition behind this claim is straightforward. If the expected wage were higher than the reservation wage, then it would be a profitable deviation for the shareholder to offer the manager an infinitesimally lower remuneration. This is a variant of an argument well-known in the literature on ultimatum and bargaining games, where equilibrium offers make the responding player exactly indifferent between accepting or declining the offer.
Lemma 2.1
Fix a bank $i \in \{1, \ldots, n\}$. Suppose $\sigma^h$ is a strategy profile in the $h$-subgame and $A^{i2}(\sigma^h) \neq O$. Also suppose the strategies $\sigma^h_m$ and $\sigma^h_s$ are best responses to $\sigma^h$. Then it holds that $\kappa^{i2}(\sigma^h) = \mu^{i2}(\sigma^h)$.

**Proof.** Suppose $\sigma^h$ is an $h$-banking equilibrium in the $h$-subgame and $A^{i2}(\sigma^h) \neq O$. Suppose $\mu^{i2}(\sigma^h) > \kappa^{i2}(\sigma^h)$. Let $\omega^{i2}$ be the wage offer made under $\sigma^h$. For $\varepsilon > 0$, define the triple $\hat{\omega}^{i2}$ as follows:

$$
\hat{\omega}^{i2} = \begin{cases} 
(\bar{\omega}_g^{i2} - \varepsilon, 0, 0) & \text{if } n_2(\sigma^h) \leq \bar{n} - 1 \text{ and } A^{i2}(\sigma^h) = R \\
(\bar{\omega}_g^{i2} - \frac{\varepsilon}{1-p}, 0, 0) & \text{if } n_2(\sigma^h) \geq \bar{n} \text{ and } A^{i2}(\sigma^h) = R \\
(0, \omega_t^{i2} - \varepsilon, 0) & \text{if } A^{i2}(\sigma^h) = S
\end{cases}
$$

Note that, as $A^{i2}(\sigma^h) \neq O$, for sufficiently small $\varepsilon > 0$, the triple $\hat{\omega}^{i2}$ belongs to the set $\Omega$ and is therefore a wage scheme to which the strategy $\sigma^h_m$ must assign a response from $\{R, S, O\}$. We argue that it is the same response as the one assigned to wage scheme $\bar{\omega}^{i2}$. In order to see this, first observe that if manager $i$ responds to the offer $\hat{\omega}^{i2}$ in the same way as to the offer $\bar{\omega}^{i2}$, then by the construction of $\hat{\omega}^{i2}$ he will obtain a payoff of $\mu^{i2}(\sigma^h) - \varepsilon > D$ for $\varepsilon > 0$ small enough. It is clear that manager $i$ will not respond to $\hat{\omega}^{i2}$ by opting out. Now suppose, by way of contradiction, that strategy $\sigma^h_m$ responds to $\hat{\omega}^{i2}$ by working for the bank but taking a different asset than in response to $\bar{\omega}^{i2}$. Again, by the construction of $\hat{\omega}^{i2}$ this would lead to a zero payoff for manager $i$. However, $\mu^{i2}(\sigma^h) - \varepsilon > D > 0$, so it is clear that manager $i$ will not respond to $\hat{\omega}^{i2}$ by working for the bank and choosing a different asset than in response to $\bar{\omega}^{i2}$. We have now established that strategy $\sigma^h_m$ prescribes the same response from manager $i$ to both wage schemes $\bar{\omega}^{i2}$ and $\hat{\omega}^{i2}$. To complete the proof of the lemma, observe that for shareholder $i$ it is a profitable deviation from strategy profile $\sigma^h$ to offer $\hat{\omega}^{i2}$ instead of $\bar{\omega}^{i2}$. This deviation increases shareholder $i$’s payoff by $\varepsilon > 0$.

Lemma 2.1 demonstrates how the shareholder can align the manager’s interests with his own: First, by setting the appropriate component of the wage scheme equal to zero, the shareholder can ensure that a manager who works for the bank will choose the asset preferred by the shareholder. Second, by choosing the remaining component of the wage scheme so as to equalize expected and reservation wages, the shareholder can ensure that a manager will indeed work for the bank. A manager who works for the bank always expects to receive $D$ unless he is pivotal, in which case he obtains an additional transfer payment of $pr^i(h)$ compensating him for the cost of crisis tax.

If all banks take risk under the strategy profile $\sigma^h$, then we say that $\sigma^h$ involves full risk.
If exactly \( \bar{n} - 1 \) banks take risk under strategy profile \( \sigma^h \), then we say that \( \sigma^h \) involves threshold risk.

**Lemma 2.2**

An \( h \)-banking equilibrium involves either full risk or threshold risk.

**Proof.** The proof of Lemma 2.2 rests on two contradictions. Suppose first that \( \sigma^h \) is an \( h \)-banking equilibrium but \( n_2(\sigma^h) < \bar{n} - 1 \). Then there is \( i \in N \) such that \( A^{i2}(\sigma^h) \neq R \). Clearly, manager \( i \) is not pivotal under \( \sigma^h \), so his payoff is \( D \). From the supposition that \( \sigma^h \) is an \( h \)-banking equilibrium we conclude that strategy \( \sigma^h_{im} \) prescribes taking risk as a response to offer \( (D + \varepsilon, 0, 0) \) for \( \varepsilon > 0 \) sufficiently small (and in particular \( \varepsilon < (1 - p)x_g - x_{rf} \)). If shareholder \( i \) deviates from \( \sigma^h \) by making the offer \( (D + \varepsilon, 0, 0) \), then he obtains a payoff of \( x_g - D - \varepsilon \). Under \( \sigma^h \), however, his payoff would be either \( x_{rf} - D \) or zero. Since \( x_g > x_{rf} \) and \( x_g > D \), we see that shareholder \( i \) has a profitable deviation from \( \sigma^h \). Hence we obtain a contradiction and conclude that \( n_2(\sigma^h) \geq \bar{n} - 1 \).

Now suppose secondly that \( \sigma^h \) is an \( h \)-banking equilibrium but \( n > n_2(\sigma^h) \geq \bar{n} \). Then there is \( i \in N \) such that \( A^{i2}(\sigma^h) \neq R \). Clearly, manager \( i \) is not pivotal under \( \sigma^h \), so his payoff is \( D \). From the supposition that \( \sigma^h \) is an \( h \)-banking equilibrium we conclude that strategy \( \sigma^h_{im} \) prescribes taking risk as a response to the offer \( (D + \varepsilon, 0, 0) \) for \( \varepsilon > 0 \) sufficiently small. If shareholder \( i \) deviates from \( \sigma^h \) by making the offer \( (D + \varepsilon, 0, 0) \), he obtains a payoff of \( (1 - p)x_g - D - \varepsilon \). Under \( \sigma^h \), however, his payoff would be either \( x_{rf} - D \) or zero. Since by assumption \( (1 - p)x_g > x_{rf} + \varepsilon \), shareholder \( i \) has a profitable deviation from \( \sigma^h \). Therefore we again obtain a contradiction, so \( n_2(\sigma^h) = \bar{n} - 1 \) or \( n_2(\sigma^h) = n \).

One implication of Lemma 2.2 is that no manager is pivotal in an \( h \)-banking equilibrium. But if a manager is not pivotal, his expected and reservation wage will both be equal to his outside option.

**Corollary 2.1**

In an \( h \)-banking equilibrium, the expected wage of each manager in the \( h \)-subgame is equal to \( D \).

Having excluded any other types of \( h \)-banking equilibria, we now turn to the conditions for the existence of \( h \)-banking equilibria involving full risk or threshold risk.

**Theorem 2.1**

For every \( h \)-subgame, there is an \( h \)-banking equilibrium that involves full risk.

The proof of Theorem 2.1 can be found in the Appendix.
Risk-taking by all banks in the second period can be supported by an \( h \)-banking equilibrium irrespective of the choice of the model parameters, first-period history, or a Crisis Contract. This result is driven by the fact that in the second period managers can only collectively eliminate the risk of a crisis tax. In other words, they need to coordinate on not crossing threshold \( \bar{n} \). Due to the assumption that \( \bar{n} \leq n - 1 \), no individual manager can insulate himself against the crisis tax. Indeed, once the threshold \( \bar{n} \) is crossed, each individual manager has an incentive to take risk. In the first period, where each individual manager can eliminate the risk of the crisis tax for himself by opting out, no coordination among managers is necessary.

Let us define the following threshold for the looming crisis tax:

\[
\tau^* = \frac{(1 - p)x_g - x_{rf}}{p}.
\]

It follows from our assumptions that threshold \( \tau^* \) is strictly positive.

**Lemma 2.3**

*If there is an \( h \)-banking equilibrium involving threshold risk, then \( \tau^i(h) \geq \tau^* \) for at least \( n - \bar{n} + 1 \) banks.*

**Proof.** Suppose that \( \sigma^h \) is an \( h \)-banking equilibrium and \( n_2(\sigma^h) = \bar{n} - 1 \). Then there are \( n - \bar{n} + 1 \) banks \( i \in N \) such that \( A^{i2}(\sigma^h) \neq R \). Take one such bank, say \( i \). Observe that \( i \) is not pivotal under \( \sigma^h \). Hence, from the supposition that \( \sigma^h \) is an \( h \)-banking equilibrium, we obtain that \( \mu^{i2}(\sigma^h) = D \). We argue that strategy \( \sigma^h_i \) prescribes taking risk as the response to offer \( \hat{\omega}^{i2} = \left( \frac{D + \varepsilon + p\tau^i(h)}{1 - p}, 0, 0 \right) \). To see this, note that the expected payoff to manager \( i \) from taking risk in response to \( \hat{\omega}^{i2} \) is equal to \( D + \varepsilon + p\tau^i(h) - p\tau^i(h) = D + \varepsilon \), whereas the expected payoff to manager \( i \) from investing safely or opting out in response to \( \hat{\omega}^{i2} \) is zero and \( D \), respectively. Now suppose that shareholder \( i \) deviates from \( \sigma^h \) by making offer \( \hat{\omega}^{i2} \). Then he will obtain an expected payoff of \( (1 - p)x_g - D - \varepsilon - p\tau^i(h) \), whereas his payoff under \( \sigma^h \) is \( x_{rf} - D \). We conclude that shareholder \( i \) has a profitable deviation from \( \sigma^h \) if \( (1 - p)x_g - p\tau^i(h) > x_{rf} \), or, equivalently, if \( \tau^i(h) < \tau^* \). Repeating the argument for every \( i \in N \) with \( A^{i2}(\sigma^h) \neq R \), we obtain the claim of the lemma. \( \square \)

When \( c = 0 \), then we have \( \tau^i(h) = 0 \) for all \( i \in N \) and \( h \in H \). Since \( \tau^* > 0 \), we know that an \( h \)-banking equilibrium with threshold risk does not exist in the absence of Crisis Contracts.

**Corollary 2.2**

*In the absence of Crisis Contracts (i.e., \( c = 0 \)) all \( h \)-banking equilibria in all \( h \)-subgames involve full risk.*
Lemma 2.3 states a necessary condition for an \( h \)-banking equilibrium involving threshold risk. We show next that the necessary condition is also sufficient.

**Theorem 2.2**

For every \( h \in H \), there is an \( h \)-banking equilibrium involving threshold risk if and only if \( \tau^i(h) \geq \tau^* \) for at least \( n - \bar{n} + 1 \) banks.

The proof of Theorem 2.2 can be found in the Appendix.

### 2.4 Managerial Pay in the First Period

We now turn to the first period. The expected wage of manager \( i \) in the first period can be defined analogously to the earlier definition of \( \mu^{i2} \) as follows:

\[
\mu^{i1}(\sigma) = \begin{cases} 
\omega^i_g(\sigma) & \text{if } n_1(\sigma) \leq \bar{n} - 1 \text{ and } A^{i1}(\sigma) = R, \\
(1 - p)\omega^i_g(\sigma) & \text{if } n_1(\sigma) > \bar{n} - 1 \text{ and } A^{i1}(\sigma) = R, \\
\omega^i_f(\sigma) & \text{if } A^{i1}(\sigma) = S.
\end{cases}
\]

However, the realized wage may be subject to the crisis tax in the second period. Given that some strategy profile \( \sigma \) is played throughout the banking game, we can define the ex ante probability \( \rho(\sigma) \) with which a crisis will occur in the second period. In this context, ex ante means that \( \rho(\sigma) \) has not been updated with the possible move of nature in the first period. For example, if \( \sigma \) is such that all managers invest safely in the second period, then \( \rho(\sigma) = 0 \). If, on the contrary, \( \sigma \) is such that all managers take risk in the second period, then \( \rho(\sigma) = p \). It is possible, however, for \( \rho(\sigma) \) to take other values than 0 or \( p \). For example, suppose that under strategy profile \( \sigma \) managers \( 1, \ldots, \bar{n} - 1 \) always take risk in the second period, whereas managers \( \bar{n}, \ldots, n \) take risk in the second period if and only if a crisis has occurred in the first period. Suppose further that under \( \sigma \) all managers take risk in the first period. In that case, we have \( \rho(\sigma) = p^2 \). Based on the probability \( \rho(\sigma) \) and the expected wage of manager \( i \) in the first period, we can define manager \( i \)'s expected net wage under \( \sigma \) from the first period as follows:

\[
\nu^{i1}(\sigma) = (1 - \delta c \rho(\sigma)) \mu^{i1}(\sigma).
\]

To understand this expected net wage intuitively, suppose that by the end of the first period a manager has earned a certain wage, say \( w \). Since the manager has worked for a bank in the first period, he may have to pay crisis tax in the second period. So, the realized wage of \( w \) in the first period only increases his intertemporal utility in expected terms by the amount \( (1 - \delta c \rho(\sigma)) w \).
Lemma 2.4
If \( \sigma \) is a banking equilibrium and \( A^1(\sigma) \neq O \), then \( \nu^1(\sigma) = D \).

Proof. First suppose, by way of contradiction, that \( \bar{\sigma} \) is a banking equilibrium and \( \nu^1(\bar{\sigma}) < D \). By Corollary 2.1, manager \( i \)'s expected wage at \( t = 2 \) equals \( D \). Hence, his expected intertemporal wage under \( \bar{\omega} \) is \( \nu^1(\bar{\sigma}) + \delta D < (1 + \delta)D \). However, by opting out in both periods, manager \( i \) could have obtained the intertemporal payoff of \( (1 + \delta)D \), which is a contradiction.

Second, suppose now that \( \bar{\sigma} \) is a banking equilibrium and \( \nu^1(\bar{\sigma}) > D \). Again, we show that this leads to a contradiction. Let \( \bar{\omega}^1 \) and \( \bar{\omega}^2 \) be the wage schemes offered by shareholder \( i \) associated with the supposed banking equilibrium \( \bar{\sigma} \). Moreover, consider the following alternative wage schemes:

\[
\bar{\omega}^1 = \begin{cases} 
(\bar{\omega}^1_g - \frac{2\varepsilon}{1 - \delta p(\sigma)}, 0, 0) & \text{if } n_1(\sigma) \leq \bar{n} - 1 \text{ and } A^1(\sigma) = R, \\
(\bar{\omega}^1_g - \frac{2\varepsilon}{(1-p)(1-\delta p(\sigma))}, 0, 0) & \text{if } n_1(\sigma) \geq \bar{n} \text{ and } A^1(\sigma) = R, \\
(0, \bar{\omega}^1_r + \varepsilon, 0) & \text{if } A^1(\sigma) = S,
\end{cases}
\]

for the first period, and

\[
\bar{\omega}^2 = \begin{cases} 
(\bar{\omega}^2 + \varepsilon, 0, 0) & \text{if } A^2(\bar{\sigma}) = R, \\
(0, \bar{\omega}^2_r + \varepsilon, 0) & \text{if } A^2(\bar{\sigma}) = S,
\end{cases}
\]

for the second period.

The proof strategy is to show that a unilateral deviation by shareholder \( i \) from \( \bar{\sigma} \) to the alternative wage schemes \( (\bar{\omega}^1, \bar{\omega}^2) \) is profitable. In the first step below, we demonstrate that under banking equilibrium profile \( \bar{\sigma} \), manager \( i \) will respond to both \( \bar{\omega}^1 \) and \( \bar{\omega}^2 \) with the same choice from \( \{ R, S, O \} \).

Step 1. Suppose that this is not the case. That is, suppose that strategy \( \bar{\sigma}^1_m \) assigns different responses to wage offers \( \bar{\omega}^1 \) and \( \bar{\omega}^2 \). If manager \( i \) opts out in response to \( \bar{\omega}^1 \), then his payoff from the first period is \( D \). If manager \( i \) does not opt out in response to \( \bar{\omega}^1 \), then, by construction of \( \bar{\omega}^1 \), his payoff from the first period is zero. Since \( \bar{\sigma} \) is a banking equilibrium, manager \( i \)'s expected payoff in the second period is \( D \). If he does not opt out in response to \( \bar{\omega}^1 \), his intertemporal payoff is \( \delta D \), but if he does opt out, it is \( (1 + \delta)D \). We see that manager \( i \) reacts to \( \bar{\omega}^1 \) by opting out. He obtains the intertemporal payoff of \( (1 + \delta)D \) in the banking game under \( \bar{\sigma} \). However, if manager \( i \) did respond to \( \bar{\omega}^1 \) and \( \bar{\omega}^2 \) with the same action, then, by construction of \( \bar{\omega}^1 \), his intertemporal payoff would be \( \nu^1(\bar{\sigma}) - 2\varepsilon + \delta D > (1 + \delta)D \), where the inequality follows directly from the supposition
that $\nu^{i1}(\bar{\sigma}) > D$ when $\varepsilon > 0$ is small enough. We see that manager $i$ has a profitable deviation from $\bar{\sigma}$, which yields the desired contradiction.

We conclude that if $\bar{\sigma}$ is a banking equilibrium, then manager $i$ responds by the same action to two wage offers $\hat{\omega}^{i1}$ and $\bar{\omega}^{i1}$.

**Step 2.** Now suppose that shareholder $i$ deviates from $\bar{\sigma}$ by offering $\hat{\omega}^{i1}$ in the first period and $\hat{\omega}^{i2}$ in the second period. Clearly, shareholder $i$’s payoff from the first period increases by $2\varepsilon > 0$. As we have shown, the deviation to $\hat{\omega}^{i1}$ does not have any effect on the investment choices of the banks in the first period. Hence the actions of shareholders and managers $j \in N \setminus \{i\}$ in the second period are unaffected by the deviation to $\hat{\omega}^{i1}$; consequently, banks $j \in N \setminus \{i\}$ will act according to activity profile $A^{-i2}(\bar{\sigma})$ in the second period.

**Step 3.** We now want to show that manager $i$ responds to wage offer $\hat{\omega}^{i2}$ by the same investment choice with which he responds to $\bar{\omega}^{i2}$. Notice that under $\bar{\sigma}$, manager $i$’s payoff from the second period is $D$. If he responds to $\hat{\omega}^{i2}$ with the same action as to $\bar{\omega}^{i2}$, then the resulting payoff will be $D + \varepsilon$. If manager $i$ opts out in response to $\hat{\omega}^{i2}$, then his expected payoff from the second round is $D$. If he works for the bank, but makes a different investment choice than under $\bar{\sigma}$, then, by construction of $\hat{\omega}^{i2}$, his payoff is zero. We see that following the unilateral deviation by shareholder $i$ to wage offers $(\hat{\omega}^{i1}, \hat{\omega}^{i2})$, activity profiles $A^{i1}(\bar{\sigma})$ and $A^{i2}(\bar{\sigma})$ remain unchanged.

**Step 4.** We have considered a unilateral deviation by shareholder $i$ from the supposed banking equilibrium $\bar{\sigma}$ to wage offers $(\hat{\omega}^{i1}, \hat{\omega}^{i2})$. We have shown that this deviation leaves activity profiles $A^{i1}(\bar{\sigma})$ and $A^{i2}(\bar{\sigma})$ unchanged in both rounds. But then, by the construction of $(\hat{\omega}^{i1}, \hat{\omega}^{i2})$, the deviation increases shareholder $i$’s net expected payoff from the first round by $2\varepsilon$ and decreases shareholder $i$’s expected payoff from the second round by $\varepsilon$. Indeed, the deviation increases shareholder $i$’s utility by the amount $2\varepsilon - \delta\varepsilon \geq \varepsilon > 0$; it is thus profitable. We have obtained the desired contradiction, and the proof of the lemma is complete. \[\square\]

The above lemma shows that in the first period as well, a shareholder will extract all surplus created by the investment. A manager receives a payment that makes him indifferent between working for the bank and opting out. If a banking equilibrium is such that a manager has to pay crisis tax with positive probability, then a shareholder will compensate the manager for the expected crisis payment by an increase in wage in the first period. The expected intertemporal payoff for the manager is always equal to the payoff from taking the outside option in both periods.
Corollary 2.3
In a banking equilibrium, each manager receives an expected intertemporal payoff equal to $(1 + \delta)D$.

Corollary 2.4
In a banking equilibrium, each manager’s instantaneous payoff in the first period is at least $D$.

Suppose that manager $i$ receives wage offer $\omega$ in the first period and accepts this offer. Observe that the manager’s realized wage can then never exceed $\max\{\omega_g, \omega_{rf}\}$. We will say that a wage scheme $\omega \in \Omega$ is insufficient if $D > \max\{\omega_g, \omega_{rf}\}$. We know from the above corollary that in a banking equilibrium no manager works under an insufficient wage scheme, as accepting an insufficient wage scheme is a strictly dominated strategy for every manager. Hence from now on we restrict managers’ behavior by the assumption that an insufficient wage scheme will not be accepted.\(^{11}\)

Assumption 2.2
A manager will not accept an insufficient wage scheme.

Suppose that strategy profile $\sigma$ is such that $A^{11}(\sigma) \neq O$. If the banking game is played according to $\sigma$ and if no crisis occurs in the first period, then this assumption guarantees that manager $i$ will realize a wage of at least $D$ in the first period. The importance of this assumption is technical. In the rest of the chapter, we will discuss entire strategy profiles that are banking equilibria. Such strategy profiles must specify actions to be taken in the second period after any first-period history. However, we have introduced the restriction that the shareholders and managers of an individual bank cannot condition their behavior on the wage offers of other banks but only on their own investment choices. Without the assumption above, one would have to specify second-period actions for players $j \neq i$ after a history in which a manager $i$ has worked for an insufficient wage, and these actions would not be allowed to differ from those taking place after a history in which the manager has received a sufficient wage.

Here we conclude our discussion of managerial pay in the banking game. In the next two sections, we will consider two specific types of banking equilibria, which from the social welfare point of view turn out to be the best and worst equilibria, respectively.

\(^{11}\)Note that whether or not a wage scheme is insufficient is defined purely on the basis of $D$, which is a primitive of the model. In particular, a wage scheme that leads to an expected wage lower than the reservation wage need not be insufficient. Conversely, an insufficient wage scheme always leads to an expected wage that is lower than the reservation wage.
2.5 Full Risk Equilibria

We refer to a banking equilibrium as a full risk equilibrium if it involves full risk in both periods on the equilibrium path of play. In particular, a full risk equilibrium involves risk-taking by all managers in the second period, irrespective of the realization of $Z_1$. If $\sigma$ is a full risk equilibrium, then we have $\rho(\sigma) = p$. In this section we give necessary and sufficient conditions for the existence of a full risk equilibrium. We will also be interested in the “uniqueness” of such equilibria.

To understand intuitively when a full risk equilibrium does or does not exist, let us consider an individual bank. If the bank is out of business in the first period, the manager’s payoff is $D$ and the shareholder’s payoff is zero. Recall that the manager’s outside payoff of $D$ will never be subject to crisis tax, as a Crisis Contract applies only to wages earned in the banking sector in the first period. Now we turn to the case where the bank invests in the risky asset in both periods. Here the expected net payoff from bank activities in the first round is equal to $(1 - p)x_g - \delta p(1 - p)c x_g$, which is shared by the manager and the shareholder of the bank under consideration. The first term is simply the expected return on the risky asset in the first period. The second term is the expected utility loss from the crisis tax. A crisis tax will only have to be paid if there is no crisis in the first period but a crisis does occur in the second period. Given that all banks take the risky investment in both periods, the probability of this event is $p(1 - p)$. If this event takes place, crisis tax $c x_g$ has to be paid. It may now seem intuitive that a full risk equilibrium exists if $(1 - p)x_g - \delta p(1 - p)c x_g \geq D$ or, equivalently, if $c$ does not exceed the threshold $c'$ defined as

\begin{equation}
    c' = \frac{(1 - p)x_g - D}{\delta p(1 - p)x_g}.
\end{equation}

A formal analysis, however, is more involved, since our model includes strategic interaction between the shareholder and the manager of each bank and crisis tax is levied on the manager’s realized wage, not on asset returns. Nevertheless, the following theorem confirms our first intuition, indeed it establishes that $c \leq c'$ is both a necessary and a sufficient condition for the existence of a full risk equilibrium.

**Theorem 2.3**

A full risk equilibrium exists if and only if $c \leq c'$.

**Proof** ($\Leftarrow$). Suppose $c \leq c'$. The proof is constructive. Consider a strategy profile $\bar{\sigma}$, under which every shareholder $i \in N$ makes wage offers
\[ \bar{\omega}^{i1} = \left( \frac{D}{(1 - p)(1 - \delta pc)}, 0, 0 \right) \] and
\[ \bar{\omega}^{i2} = \left( \frac{D}{1 - p}, 0, 0 \right). \]

The managers’ choices under \( \sigma \) are as follows:

- At \( t = 1 \), each manager \( i \in N \) opts out in response to the wage scheme \( \omega^{i1} \in \Omega \) if and only if \( D > \max \{ (1 - p)(1 - \delta pc)\omega^{i1}_y, (1 - \delta pc)\omega^{i1}_{rf} \} \). Conditional on not opting out, manager \( i \) will take risk if and only if \( (1 - p)(1 - \delta pc)\omega^{i1}_y > (1 - \delta pc)\omega^{i1}_{rf} \).

- At \( t = 2 \), each manager \( i \in N \) opts out in response to the wage scheme \( \omega^{i2} \in \Omega \) if and only if \( D > \max \{ (1 - p)\omega^{i2}_y, \omega^{i2}_{rf} \} \). Conditional on not opting out, manager \( i \) will take risk if and only if \( (1 - p)\omega^{i2}_y > \omega^{i2}_{rf} \).

Moreover, under \( \bar{\sigma} \) each manager \( i \) will choose to work at his bank in the second period if \( (1 - p) \max(\bar{\omega}^{i2}_y, \bar{\omega}^{i2}_{rf}) \geq D \). If he works at his bank in the second period, he will take risk if and only if \( \bar{\omega}^{i2}_y \geq \bar{\omega}^{i2}_{rf} \). In the first period, each manager \( i \) chooses to work at his bank if \( (1 - p)(1 - \delta pc) \max(\bar{\omega}^{i1}_y, \bar{\omega}^{i1}_{rf}) \geq D \). If he works at his bank, he will take risk if and only if \( \bar{\omega}^{i1}_y \geq \bar{\omega}^{i1}_{rf} \) and invest safely otherwise.

For any history \( h \in H \), the relevant restriction \( \bar{\sigma}^h \) of strategy profile \( \bar{\sigma} \) is an \( h \)-banking equilibrium in the \( h \)-subgame. This has been demonstrated in the proof of Theorem 2.1. Hence we need only consider unilateral deviations in the first period. It is straightforward to see that the managers’ decisions in the first period are optimal. We show that no shareholder has a profitable unilateral deviation from \( \bar{\sigma} \) in the first period. Under \( \bar{\sigma} \), shareholder \( i \)'s payoff in the first period equals \( (1 - p)x_g - \frac{D}{1 - \delta pc} \). Suppose that shareholder \( i \) deviates from \( \bar{\sigma} \) by offering some \( \tilde{\omega}^{i1} \) to which manager \( i \) responds by investing safely. Since manager \( i \) does not opt out, it must be true that \( (1 - \delta pc)\tilde{\omega}^{i1}_{rf} \geq D \). Then, however the shareholder’s payoff in the first period is bounded from above by \( x_{rf} - \frac{D}{1 - \delta pc} \). Since \( (1 - p)x_g > x_{rf} \), this deviation is not profitable. Now suppose that shareholder \( i \) deviates from \( \bar{\sigma} \) by offering some \( \tilde{\omega}^{i1} \) to which manager \( i \) responds by opting out. In that case, the payoff to shareholder \( i \) in the first period is zero. By rewriting the supposition that \( c \leq c' \), it holds that \( (1 - p)x_g - \frac{D}{1 - \delta pc} \geq 0 \), so this deviation is again not profitable. Indeed, \( \bar{\sigma} \) is a banking equilibrium.

**Proof \( (\Rightarrow) \).** Suppose that a full risk equilibrium \( \sigma \) exists. Then \( \nu^{i1}(\sigma) = D \) implies \( \mu^{i1}(\sigma) = \frac{D}{1 - \delta pc} \). Hence the payoff of shareholder \( i \) from the first period is \( (1 - p)x_g - \frac{D}{1 - \delta pc} \). Since shareholder \( i \) could guarantee a payoff of zero by proposing \( (0, 0, 0) \) to manager \( i \),
it must be true that \((1 - p)x_g - \frac{D}{1 - \delta pc} \geq 0\). Indeed, rearranging this inequality yields \(c \leq c'\).

We now turn to the question whether the full risk equilibrium is “unique” in the sense that all banking equilibria are full risk equilibria. We have shown previously that, without Crisis Contracts, the banking equilibrium unambiguously predicts full risk in the second period. This is the result of Corollary 2.2. However, if full risk will always prevail in the second period, then we can conclude that the expected wage of a manager in the first period must equal \(\frac{D}{1 - \delta pc}\) irrespective of the investment choices made in the first period. This observation yields

**Theorem 2.4**

*In the absence of Crisis Contracts, all banking equilibria are full risk equilibria.*

**Proof.** Suppose that \(c = 0\). Let \(\bar{\sigma}\) be a banking equilibrium. By Corollary 2.2 it holds that \(A^{i}(\sigma^h) = R\) for all \(h \in H\) and \(i \in N\). Consequently, a deviation from \(\bar{\sigma}\) in the first period has no effect on risk choices or wages in the second period.

Suppose, by way of contradiction, that there is \(i \in N\) so that \(A^{i}(\bar{\sigma}) \neq R\). Either \(A^{i}(\bar{\sigma}) = O\) and then the shareholder will earn zero, or \(A^{i}(\bar{\sigma}) = S\) and then the shareholder will earn \(x_{rf} - D > 0\). If \(c = 0\) and if \(\bar{\sigma}\) is a banking equilibrium but not a full risk equilibrium, then at least one shareholder will earn \(x_{rf} - D\) in the first period.

Consider wage offer \(\tilde{\omega}^{i} = (\frac{D + \varepsilon}{1 - p}, 0, 0)\). Manager \(i\) would respond to this proposal by taking risk. However, if shareholder \(i\) were to deviate from \(\bar{\sigma}\) by proposing \(\tilde{\omega}^{i}\), then the resulting expected payoff for shareholder \(i\) would be either \(x_g - \frac{D + \varepsilon}{1 - p}\) or \((1 - p)x_g - D - \varepsilon\). Clearly, the latter term is smaller than the former, so the deviation yields shareholder \(i\) a gain of at least \((1 - p)x_g - D - \varepsilon - x_{rf} + D = (1 - p)x_g - x_{rf} - \varepsilon\). This gain is positive when \(\varepsilon > 0\) is chosen sufficiently small. Shareholder \(i\) then has a profitable unilateral deviation from \(\bar{\sigma}\). This is a contradiction. \(\square\)

Theorem 2.4 establishes a benchmark for our further analysis of Crisis Contracts. We have now demonstrated that, in the absence of Crisis Contracts, the banking equilibrium unambiguously predicts full risk, so a crisis will occur with positive probability in both periods.

### 2.6 Threshold Equilibria

We define a *threshold equilibrium* as a banking equilibrium in which threshold risk is played in both periods on the equilibrium path. Note that in a threshold equilibrium the probability of a crisis in either period is equal to zero. A strategy profile which is
a threshold equilibrium may prescribe full risk in the second period after a crisis has occurred in the first period. Such first-period histories are not on the equilibrium path. Let

\[ c'' = \frac{(1 - p)x_g - x_{rf}}{pD}. \]  

(2.11)

**Theorem 2.5**

A threshold equilibrium exists only if \( c \geq c'' \).

**Proof.** Suppose that \( \bar{\sigma} \) is a threshold equilibrium. Let \( \bar{h} \) be the first-period history induced by playing according to \( \bar{\sigma} \) in the first period. Then, by the definition of a threshold equilibrium, we know that \( \bar{\sigma}^h \) is an \( h \)-banking equilibrium that involves threshold risk in the \( \bar{h} \)-subgame. By Theorem 2.2, this implies that \( \tau^i(\bar{h}) \geq \tau^* \) for at least \( n - \bar{n} + 1 \) banks. Since under \( \bar{\sigma} \) the crisis probability in either period is zero, we have \( \mu^i(\bar{\sigma}) = \nu^i(\bar{\sigma}) = D \) for all \( i \in N \), and the realized wage in the first period is \( D \) for all \( i \in N \). Hence \( \tau^i(\bar{h}) = cD \) for all \( i \in N \). It follows that \( cD \geq \tau^* \). Substituting from the definition of \( \tau^* \) and rearranging this inequality, we find that \( c \geq c'' \), as desired. \( \square \)

We have established a necessary condition for the existence of a threshold equilibrium. For the next theorem we shall now derive a set of sufficient conditions. Let us define \( \hat{n} = (\bar{n} - 1)/n \). This ratio tells us which share of the banks in our banking sector can take the risky investment without running the risk of triggering a crisis. We can interpret \( \hat{n} \) as a measure for the stability of the banking sector. We will also make use of the following condition:

\[ x_{rf} \geq x_g \left( 1 - p \left( \frac{1 + \delta p}{1 + \delta p - \delta} \right) \right). \]  

(2.12)

Note that for any choice of parameters \( x_g, x_{rf} \), and \( p \), the above inequality is satisfied when \( \delta \) is sufficiently close (or equal) to one.

Assuming that \( \hat{n} \) is sufficiently large and that inequality (2.12) holds, we now construct a threshold equilibrium of the following kind: On the equilibrium path of play, exactly \( \bar{n} - 1 \) banks take the risky asset in each period, while the remaining banks invest in the safe asset. All banks investing safely in the first period take the risky asset in the second period. (Clearly, this is only possible when \( \hat{n} \geq \frac{1}{2} \).) If some bank deviates from the equilibrium path in the first period by investing in the risky rather than the safe asset, then the game enters into a “punishment mode.” If a crisis occurs in the first period, then all banks take the risky asset in the second period. If the game is in the punishment mode, but no crisis has occurred in the first period, then the bank that has deviated from the equilibrium path
2.6 Threshold Equilibria

in the first period is among those banks taking the safe asset in the second period. Since
the punishment occurs in the second period, the shareholders and managers need to care
sufficiently about the future payoff for the punishment to be effective. This makes it
intuitively clear why inequality (2.12) is crucial for the result.

Theorem 2.6

Suppose that \( \hat{n} \geq \frac{1}{2} \), inequality (2.12) holds, and \( c \geq c'' \). Then a threshold equilibrium
exists.

Proof. Define a strategy profile \( \bar{\sigma} \) as follows: In the first period, shareholders make the
following wage offers:

\[
\bar{\omega}^{j_1} = (D, 0, 0), \quad i = 1, \ldots, \hat{n} - 1,
\]

\[
\bar{\omega}^{j_1} = (0, D, 0), \quad j = \hat{n}, \ldots, n.
\]

Furthermore, under \( \bar{\sigma} \) managers \( i = 1, \ldots, \hat{n} - 1 \) respond to the wage offer \( \omega^{j_1} \) by opting
out if and only if \( D > \max \{ \omega^{j_1}_g, \omega^{j_1}_{rf} \} \). If a manager does not opt out, he will respond by
taking risk if and only if \( \omega^{j_1}_g \geq \omega^{j_1}_{rf} \) and invest safely otherwise. Furthermore, under \( \bar{\sigma} \)
manager \( j = \hat{n}, \ldots, n \) will react to the wage offer \( \omega^{j_1} \) in the following way: He opts out
if and only if \( D > \max \{ (1-p)\omega^{j_1}_g, \omega^{j_1}_{rf} \} \). Conditional on not opting out, manager \( j \) will
take risk if and only if \( (1-p)\omega^{j_1}_g > \omega^{j_1}_{rf} \) and invest safely otherwise.

We now define the restriction \( \bar{\sigma}^h \) of \( \bar{\sigma} \) to the \( h \)-subgame for each \( h \in H \). If all share-
holders and all managers play according to \( \bar{\sigma} \) in the first period, then there will be no crisis.
That is, play according to \( \bar{\sigma} \) induces a unique history, which we will denote henceforth by
\( \bar{h} \in H \). To define \( \bar{\sigma}^h \), we distinguish three cases.

1. Suppose that \( h \) is a first-period history involving the same investment choices by
all banks as in \( \bar{h} \). By Assumption 2.2, it holds that \( \tau^k(h) \geq cD \) for all \( k \in N \). By
the supposition that \( c \geq c'' \), it follows that \( \tau^k(h) \geq \tau^* \) for all \( k \in N \). There exists
an \( h \)-banking equilibrium involving threshold risk, in which all banks \( j = \hat{n}, \ldots, n \)
take risk. Let \( \bar{\sigma}^h \) be that \( h \)-banking equilibrium.

2. Suppose that history \( h \) is such that no bank has been out of business in the first pe-
riod and \( Z_1 = 0 \). Moreover, suppose that the investment choices under \( h \) differ from
those under \( \bar{h} \) with regard to exactly one bank, say \( k' \in N \). Again, by Assumption
2.2 it holds that \( \tau^k(h) \geq cD \) for all \( k \in N \). By the supposition that \( c \geq c'' \), it
follows that \( \tau^k(h) \geq \tau^* \) for all \( k \in N \). There exists an \( h \)-banking equilibrium in-
volving threshold risk in which bank \( k' \) makes the safe investment. Let \( \bar{\sigma}^h \) be such
an \( h \)-banking equilibrium.
3. For any other histories $h \in H$, let $\bar{\sigma}^h$ be an $h$-banking equilibrium involving full risk.

Now we need to show that $\bar{\sigma}$ is in fact a banking equilibrium. By construction, it holds for all $h \in H$ that $\bar{\sigma}^h$ is an $h$-banking equilibrium of the $h$-subgame. Hence, one only has to verify that there is no profitable unilateral deviation in the first period. Moreover, it holds by construction of $\bar{\sigma}$ that a unilateral deviation by shareholder $k'$ or manager $k'$ will lead to an $h$-banking equilibrium in which no crisis tax is to be paid by manager $k'$. This implies that manager $k'$ expects a payoff of $D$ in the second period, regardless of whether bank $k'$ makes the first-period investment choice prescribed by $\bar{\sigma}$ or whether bank $k'$ is the only bank to make a different investment choice. Consequently, a deviation by a manager in the first-period can only be profitable if it increases that manager’s instantaneous payoff in the first period. It is now straightforward to see that there is no profitable unilateral deviation from $\bar{\sigma}$ in the first period for any manager.

What is left to show is that no shareholder can gain from unilateral deviation from $\bar{\sigma}$ in the first period.

First, consider a shareholder $i = 1, \ldots, \bar{n} - 1$. Clearly, wage offer $\bar{\omega}^{i1}$ is optimal among all those wage offers to which manager $i$ responds by taking risk under $\bar{\sigma}$. Suppose that shareholder $i$ deviates from $\bar{\sigma}$ by offering a wage scheme $\omega^{i1}$ to which manager $i$ responds by investing safely. Then shareholder $i$’s payoff is bounded from above by $(1 + \delta)(x_{r_f} - D)$. However, under $\bar{\sigma}$, shareholder $i$’s payoff is $x_g - D + \delta(x_{r_f} - D)$. Since $x_g > x_{r_f}$, this deviation is not profitable. Now suppose that shareholder $i$ deviates from $\bar{\sigma}$ by offering a wage scheme $\omega^{i1}$ to which manager $i$ responds by opting out. Then shareholder $i$’s payoff is $0 + \delta(x_{r_f} - D)$. Since $x_g - D > 0$, this deviation is not profitable.

Now consider a shareholder $j = \bar{n}, \ldots, n$. Clearly, wage offer $\bar{\omega}^{j1}$ is optimal among all those wage offers to which manager $j$ responds by choosing the safe investment under $\bar{\sigma}$. Suppose that shareholder $j$ deviates from $\bar{\sigma}$ by offering a wage scheme to which manager $j$ responds by opting out. The resulting payoff for shareholder $j$ is $0 + \delta(x_{r_f} - D)$. However, under $\bar{\sigma}$, shareholder $j$’s payoff would be $x_{r_f} - D + \delta(x_g - D)$. Since $x_g > D$, this deviation is not profitable. Finally, suppose that shareholder $j$ deviates from $\bar{\sigma}$ by offering a wage scheme to which manager $j$ responds by taking risk. Then shareholder $j$’s payoff is bounded from above by

$$(1 - p)x_g + \delta p(1 - p)x_g + \delta(1 - p)x_{r_f} - (1 + \delta)D.$$

To see this, recall that after a change in the investment choice of a single bank in the first period, no crisis tax will have to be paid by that bank in the second period. This explains the last term in the expression above. If bank $j$ makes the risky investment in the first
2.6 Threshold Equilibria

period, then a crisis will happen with probability \( p \). So the expected asset return in the first period is \( (1 - p)x_g \), explaining the first term in the expression above. If a crisis does occur in the first period, then we have a situation where an \( h \)-banking equilibrium with full risk is played in the second period, hence the second term. If, by contrast, no crisis occurs in the first period, then we have a situation where the \( h \)-banking equilibrium with threshold risk is played in the second period and bank \( j \) makes the safe investment, hence the third term. In order to complete the proof, we need to show that

\[
(1 - p)x_g + \delta p(1 - p)x_g + \delta(1 - p)x_{rf} - (1 + \delta)D \leq (x_{rf} - D) + \delta(x_g - D).
\]

Rearranging this inequality yields (2.12), which holds by the supposition of the theorem.

\[\square\]

Theorem 2.6 shows that if an additional restriction on \( \hat{n} \) is satisfied and if the discount factor is sufficiently high, \( c \geq c'' \) is a necessary and sufficient condition for the existence of a threshold equilibrium. We note that the threshold equilibrium constructed in the proof of Theorem 2.6 can coexist with a full risk equilibrium. In Theorem 2.7 below, we present an existence result that addresses these shortcomings, meaning that it does not require a condition on \( \hat{n} \) and cannot coexist with a full risk equilibrium. We construct a threshold equilibrium that differs from the previous one. This threshold equilibrium exists irrespective of the choice of \( \hat{n} \). It requires the inequality

\[
x_g \leq \frac{x_{rf}(1 + \delta) - D}{\delta(1 - p)} \tag{2.13}
\]

to be satisfied. Furthermore, the threshold equilibrium we are now going to construct requires that \( c > c' \), so it can only exist when there is no full risk equilibrium. The idea behind the threshold equilibrium to be constructed is that in both periods the banks \( 1, \ldots, \bar{n} - 1 \) make the risky investment, while the other banks make the safe investment. If one of the banks \( \bar{n}, \ldots, n \) deviates from the equilibrium path of play in the first period by choosing the risky investment, then all banks will invest in the risky asset in the second period as a “punishment.” Note that the threat of such a punishment is always credible since, in any second period subgame, risk-taking by all banks is consistent with the banking equilibrium described in Theorem 2.1. This type of threshold equilibrium can be constructed even in the extreme case of \( \bar{n} = 1 \). The idea behind this threshold equilibrium is as follows: The punishment mechanism ensures that full risk will be played in the second period if more than \( \bar{n} - 1 \) managers take risk in the first period. Anticipating this punishment, a manager would only be willing to take risk in the first period if the shareholder offers him a compensation for the potential crisis tax payment. However, if the crisis tax rate is sufficiently high, then the shareholder cannot afford such a
compensation.

**Theorem 2.7**

Suppose that $c > c'$, $c \geq c''$, and that inequality (2.13) holds. Then a threshold equilibrium exists.

**Proof.** The proof is constructive. Define the strategy profile $\bar{\sigma}$ as follows: In the first period, shareholders make the wage offers

$$\bar{\omega}^i_1 = (D, 0, 0), \ i = 1, \ldots, \bar{n} - 1,$$

$$\bar{\omega}^j_1 = (0, D, 0), \ j = \bar{n}, \ldots, n.$$

A manager $i = 1, \ldots, \bar{n} - 1$ will opt out in response to wage offer $\omega^i_1$ if and only if

$$D > \max\{\omega^i_g, (1 - \delta pc)\omega^i_{rf}\}.$$  

Conditional on not opting out, he will select the risky investment if and only if

$$(1 - \delta pc)\omega^i_g \geq \omega^i_{rf}.$$  

A manager $j = \bar{n}, \ldots, n$ will opt out in response to wage offer $\omega^j_1$ if and only if

$$D > \max\{(1 - p)(1 - \delta pc)\omega^j_g, \omega^j_{rf}\}.$$  

Conditional on not opting out, he will take risk if and only if

$$(1 - \delta pc)(1 - p)\omega^j_g \geq \omega^j_{rf}.$$  

Now we define $\bar{\sigma}_h$ for every $h \in H$. Note that $\bar{\sigma}$ induces a unique first-period history, say $\bar{h}$. We distinguish two cases.

1. If $h$ is a history involving the same investment choices by all banks in the first period as $\bar{h}$, then there has been no crisis. By Assumption 2.2, it holds that $\tau^h(\bar{h}) \geq cD$ for all $k \in N$. From the supposition that $c \geq c''$ it follows that $\tau^h(\bar{h}) \geq \tau^*$ for all $k \in N$. There exists an $h$-banking equilibrium involving threshold risk in which each shareholder makes the same wage offer at $t = 2$ as at $t = 1$. Let $\bar{\sigma}_h$ be that $h$-banking equilibrium.

2. If $h$ is a history that does not involve the same investment choices by all banks in the first period as $\bar{h}$, then let $\sigma^h$ be an $h$-banking equilibrium involving full risk.

Now we need to show that $\bar{\sigma}$ is a banking equilibrium. By construction, $\sigma^h$ is an $h$-banking equilibrium for every $h \in H$. We need to verify the absence of any profitable unilateral deviation in the first period. Note that by construction of $\bar{\sigma}$, a crisis tax will only have to be paid if some bank $j = \bar{n}, \ldots, n$ has made the risky investment in the first period but there has been no crisis in that period. This reveals that there is no profitable unilateral deviation from $\bar{\sigma}$ for any manager. It remains to be shown that no shareholder has a profitable deviation from $\bar{\sigma}$ in the first period.

First, consider a unilateral deviation by shareholder $i = 1, \ldots, \bar{n} - 1$ in the first period. Clearly, $\bar{\omega}^i_1$ is optimal among all those wage schemes to which manager $i$ responds by
taking risk. Suppose shareholder $i$ deviates from $\bar{\sigma}$ by offering some wage scheme $\omega^{i1}$ to which manager $i$ responds by investing safely. Shareholder $i$’s payoff from this deviation is bounded from above by $x_{rf} - \frac{D}{1-\delta pc} + \delta(x_{rf} - D)$. However, his payoff under $\bar{\sigma}$ is $x_g - D + \delta(x_g - D)$. We see that the deviation is not profitable. Now suppose that shareholder $i$ deviates from $\bar{\sigma}$ by offering some wage scheme $\omega^{i1}$ to which manager $i$ responds by opting out. The resulting payoff to shareholder $i$ is $0 + \delta(1-p)x_g - \delta D$. Again, we find that this is strictly less than the (expected) payoff $x_g - D + \delta(x_g - D)$ for shareholder $i$ under $\bar{\sigma}$. Therefore the deviation is not profitable.

Now consider a unilateral deviation by shareholder $j = \bar{n}, \ldots, n$. Clearly, $\bar{\omega}^{j1}$ is optimal among all those wage offers to which manager $j$ responds by investing safely. Suppose shareholder $j$ deviates from $\bar{\sigma}$ with a wage offer $\omega^{j1}$ to which manager $j$ responds by opting out. Then shareholder $j$’s payoff is $0 + \delta(1-p)x_g - \delta D$. But his payoff from $\bar{\sigma}$ is $(x_{rf} - D)(1+\delta)$. Rearranging inequality (2.13) yields $\delta(1-p)x_g - \delta D \leq (x_{rf} - D)(1+\delta)$. We see that the deviation is not profitable. Finally, suppose that shareholder $j$ deviates from $\bar{\sigma}$ by offering a wage scheme $\omega^{j1}$ to which manager $j$ responds by taking risk. But manager $j$ will only respond to $\omega^{j1}$ by taking risk if $\omega^{j1}_g \geq D(1-p)(1-\delta pc)$. By the shareholder’s budget constraint we have $x_g \geq \omega^{j1}_g$, so $x_g \geq \frac{D}{(1-p)(1-\delta pc)}$. Appropriately rearranging this inequality, we find $\epsilon \leq \epsilon'$ - a contradiction to the supposition of the theorem. We conclude that there is no profitable unilateral deviation from $\bar{\sigma}$ for any shareholder in the first period. Hence $\bar{\sigma}$ is a banking equilibrium.

One feature of the threshold equilibrium constructed in the proof of Theorem 2.7 is that it cannot coexist with a full risk equilibrium. By contrast, the threshold equilibrium constructed in the proof of Theorem 2.6 can coexist with a full risk equilibrium. This will be important for the analysis of welfare gains from Crisis Contracts in the next section.

### 2.7 Welfare Effects of Crisis Contracts

We now conduct a comparative statics analysis of the effects of Crisis Contracts. In particular, we will be interested in whether the introduction of Crisis Contracts enhances social welfare. Social welfare is defined as follows:

In period $t$ ($t \in \{1, 2\}$), instantaneous social welfare is given by

$$y^t = \sum_{i=1}^{n} \left\{ (u^{it}_s + u^{it}_m) + \min(x^t_k, 0) \right\},$$

(2.14)

for $k \in \{g, rf, b\}$. The instantaneous utilities $u^{it}_s$ and $u^{it}_m$ are as specified in equations (2.3), (2.5), (2.4), and (2.6). The term $\min(x^t_k, 0)$ captures the social losses that occur in case of a crisis, which are neither internalized by the shareholders nor by the managers.
It is clear from the above utility functions that $y_t$ depends solely on the activity profile at $t$. As we have seen, for given investment choices the wage payments are pure redistributions from shareholders to managers; they do not affect social welfare accounting. We also consider tax revenue as part of social welfare, so the payment of crisis tax does not affect social welfare either. The above notion of social welfare implies that the managers’ income is part of social welfare, whether or not they work in the banking sector. Hence, in the absence of any banking activity, the instantaneous social welfare level would be

$$y_0 = nD.$$  \hfill (2.15)

Under the above definition, instantaneous social welfare in period $t$ is maximized when $\bar{n} - 1$ managers take risk whereas $n - \bar{n} + 1$ managers invest safely, i.e. social welfare is maximal under threshold risk. The maximal level of instantaneous social welfare is

$$\tilde{y}_t = (\bar{n} - 1)x_g + (n - \bar{n} + 1)x_{rf}.$$  \hfill (2.16)

On the other hand, risk-taking by all managers leads to instantaneous expected social welfare given by

$$\bar{y}_t = n(1-p)x_g + npx_b.$$  \hfill (2.17)

Observe that $\tilde{y}_t > y_0 > \bar{y}_t$. This reflects the fact that our model assumptions enable the banking sector both to enhance and to harm social welfare in expected terms as compared to a situation in which no bank activity takes place.

It remains to define aggregate social welfare over the two periods, denoted by $Y$. We assume that

$$Y = y^1 + \delta y^2.$$  \hfill (2.18)

We assume the same time preference for society at large as for the shareholders and managers. However, the results in the sequel will hold for any positive social discount factor.

Under the above notion of social welfare, we can view the full risk and threshold equilibria as, respectively, the worst and best outcomes of the banking game. So far in this chapter, we have conducted a comparative statics analysis to see how the existence of these two types of equilibria depends on the model parameters, and in particular on crisis tax rate $c$. In what follows, we discuss how an appropriate choice of $c$ by the regulator can improve social welfare.

Let $\mathcal{E}(c)$ be the set of banking equilibria when the crisis tax rate is $c \in [0, 1]$. We use notations $\sigma' \succeq \sigma''$ to indicate that strategy profile $\sigma'$ leads to at least as much expected
social welfare as strategy profile $\sigma''$ and $\sigma' \succ \sigma''$ to indicate that social welfare under $\sigma'$ is strictly greater than under $\sigma''$.

**Definition 2.4**

A Crisis Contract with a tax rate of $c > 0$ is weakly beneficial if the following two conditions hold:

1. For all $(\sigma', \sigma'') \in \mathcal{E}(0) \times \mathcal{E}(c)$, it holds that $\sigma'' \succeq \sigma'$.

2. There is $\sigma' \in \mathcal{E}(c)$ such that $\sigma' \succ \sigma''$ for all $\sigma'' \in \mathcal{E}(0)$.

The first part of the definition requires that the introduction of the Crisis Contract does not lead to an equilibrium which is worse than some equilibrium without Crisis Contracts. The second part of the definition requires that the introduction of the Crisis Contract leads to some equilibrium which is strictly better than any equilibrium without Crisis Contracts.

If an increase in social welfare can be obtained for any selection from the set of banking equilibria, then the Crisis Contract is considered to be *strictly beneficial*, as formalized in the next definition.

**Definition 2.5**

A Crisis Contract with a tax rate of $c > 0$ is strictly beneficial if $\mathcal{E}(c) \neq \emptyset$ and $\sigma'' \succ \sigma'$ for all $(\sigma', \sigma'') \in \mathcal{E}(0) \times \mathcal{E}(c)$.

Of course, a Crisis Contract that is strictly beneficial is also weakly beneficial.

We have seen that the existence of full risk and threshold equilibria depends on how the tax rate $c$ relates to some threshold values $c'$ and $c''$. In our model, the regulator is free to choose any tax rate $c$ from the interval $[0, 1]$. Consequently, the power of the regulator to influence the existence of full risk and threshold equilibria hinges on whether $c'$ and $c''$ fall into this interval. Using equations (2.10) and (2.11), we can express conditions $c' < 1$ and $c'' < 1$ as restrictions on $x_g$ relative to the other model parameters. Intuitively, the regulator’s ability to improve social welfare through Crisis Contracts requires that $x_g$ be not too big relative to payoffs $x_{rf}$ and $D$ and to probability $p$. More precisely, the relevant restrictions on $x_g$ are captured in the following two inequalities:

\[ x_g < \frac{D}{(1 - \delta p)(1 - p)}, \]  
\[ x_g < \frac{pD + x_{rf}}{1 - p}. \]  

In general, neither of these inequalities implies the other.
Theorem 2.8
If inequalities (2.13), (2.19), and (2.20) are satisfied, then there exists a strictly beneficial Crisis Contract. Moreover, under this Crisis Contract the socially optimal outcome of the banking game is an equilibrium.

Proof. Note that, due to Theorem 2.4, all elements of $E(0)$ are full risk equilibria and thus induce the strictly lowest social welfare among all equilibria. On the other hand, due to Theorem 2.3 and inequality (2.19), there exists $c \in (0, 1)$ such that $E(c)$ does not contain a full risk equilibrium. Due to inequalities (2.13), (2.19), and (2.20), by Theorem 2.7 we may choose the value of $c$ such that $E(c)$ contains a threshold equilibrium, which leads to the socially optimal outcome of the banking game. Hence $E(c) \neq \emptyset$. □

Note that if $p$ is close enough to 1, there will always exist a strictly beneficial Crisis Contract.

To illustrate Theorem 2.8, we provide the following numerical example:

Example 2.1
Consider an example where the discount factor is $\delta = 1$ and the crisis probability $p = 0.5$. Let the payoffs be $D = 1$ and $x = (3, 1.4, -1.5)$.

In the benchmark scenario with no Crisis Contracts, all banking equilibria are full risk equilibria. The social welfare is predicted to be $2n(1 - 0.5)3 + (2n)(0.5)(-1.5) = 3n - 1.5n = 1.5n$. However, if there was no bank activity, the social welfare level of $2n$ could be reached. In the example at hand, the banking equilibrium unambiguously predicts a welfare loss from bank activity in the absence of Crisis Contracts.

Note that inequality (2.13) holds in this example. Now suppose we introduce a Crisis Contract into the example. We calculate $c' = \frac{2}{3} \approx 0.66$ and $c'' = 0.2$. Take a crisis tax rate of $c = 0.67$. The introduction of such a Crisis Contract will lead to the existence of a threshold equilibrium. The social welfare in this equilibrium is given by $2((\bar{n} - 1)3 + (n - \bar{n} + 1)1.4) > 2n$. Note that this holds independently of the value of $\bar{n}$. Finally note that the Crisis Contract rules out the full risk equilibria. Hence it is strictly beneficial.

The following theorem establishes conditions under which a weakly beneficial equilibrium Crisis Contract exists, even if at least one of the inequalities (2.13) and (2.19) does not hold.

Theorem 2.9
Suppose that $\hat{n} \geq \frac{1}{2}$. If inequalities (2.12) and (2.20) hold, then there is a weakly beneficial Crisis Contract. Moreover, under this Crisis Contract the socially optimal outcome
of the banking game is an equilibrium.

**Proof.** Note that, due to Theorem 2.4, all elements of $E(0)$ are full risk equilibria. Due to inequalities (2.12) and (2.20), by Theorem 2.6, we may choose the value of $c$ such that $E(c)$ contains a threshold equilibrium, which leads to the socially optimal outcome of the banking game. Hence $c$ fulfills the first condition in Definition 2.4. Since any full risk equilibrium induces the strictly lowest social welfare among all equilibria, $c$ fulfills the second condition in Definition 2.4. Hence a Crisis Contract with tax rate $c$ is weakly beneficial.

Note that if $\delta$ is sufficiently small, then in each of the above theorems at least one necessary condition does not hold. This means that Crisis Contracts are ineffective. This is intuitive, since a Crisis Contract can only have an impact if managers care enough about their second-period payoffs.

To illustrate Theorem 2.9, we provide the following numerical example:

**Example 2.2**
Consider an example where the discount factor is $\delta = 1$ and the crisis probability $p = 0.5$. Let the payoffs be $D = 1$ and $x = (3, 1.2, -1.5)$.

Note that inequality (2.13) does not hold in this example. Hence Theorem 2.8, which guarantees the existence of a strictly beneficial Crisis Contract, does not apply here. However, note that due to $\delta = 1$ inequality (2.12) holds. Assume in addition that the banking system is stable to some extent, namely such that $\bar{n} \geq 0.5$ holds. It holds that $c'' = 0.6$. Then the Crisis Contract with tax rate $c = 0.61$ is weakly beneficial.

### 2.8 Capital Regulation and Crisis Contracts

In the previous section, we derived the key welfare result of this chapter. The welfare-enhancing potential of Crisis Contracts crucially hinges on a set of three conditions on the model parameters. Each of these conditions imposes upper bounds on $x_g$, the return of the risky asset when no crisis occurs. We will now argue that a moderate value of $x_g$ is suitably interpreted as resulting from stringent capital requirements. This allows us to draw conclusions about the interaction between capital requirements and our idea of Crisis Contracts in the effective regulation of financial sector risk. To do this, we first show how the incentive structure of our model can be derived from the banks’ balance sheets.
To be more precise, consider a bank that finances its activities by deposits and by equity. Equity is given and we normalize the amount of equity to one. At the beginning of each period considered in our model, households deposit a total amount of \( d \) at the bank, and the bank promises them an interest rate of \( r > 0 \) over this period. Hence, at the end of a period, the bank owes \((1 + r)d\) to its depositors. We assume that the bank can invest its entire funds \(1 + d\) in a risk-free asset, with interest rate \( r \). Hence, the payoff that we have called \( x_{rf} \) in our model can be written as

\[
x_{rf} = (1 + d)(1 + r) - d(1 + r) = 1 + r.
\]

Alternatively, the bank may invest its entire funds in a risky asset which pays an interest rate of \( r' > r \) if no crisis occurs, but which leads to a loss of fraction \( l \ (l \in [0, 1]) \) of the bank’s total capital \(1 + d\) in case of a crisis. When no crisis occurs the bank has an amount \((1 + r')(1 + d)\) at its disposal at the end of the period, while it owes \((1 + r)d\) to the depositors. After paying out the depositors, the remaining amount is available to equity holders. Then, the payoff that we have called \( x_g \) in our model can be written as

\[
x_g = (1 + d)(1 + r') - d(1 + r) = 1 + r' + d(r' - r).
\]

The payoff that we have denoted by \( x_b \) in our model can be written as

\[
x_b = (1 - l)(1 + d) - (1 + r)d.
\]

Suppose that \( d \leq d^{\text{crit}} := \frac{1 - l}{r' + l} \) and thus \( x_b \geq 0 \). Then even in the worst case, bank equity would be sufficient to redeem all obligations towards depositors. If, however, \( d > d^{\text{crit}} \) and thus \( x_b < 0 \), a bank which takes risk cannot honor all obligations towards its depositors in a crisis. The shortfall will have to be covered by a public-bailout fund. This is the scenario we have considered in our model.

We see that \( x_g \) tends to infinity, as \( d \) grows without bound. As equity is given, capital requirements impose an upper limit on \( d \). Therefore, we can say that the inequalities which require \( x_g \) to be below certain bounds can be interpreted as requiring sufficiently strong capital requirements. As long as capital requirements are sufficiently strong in the aforementioned sense, however, Crisis Contracts can serve as a substitute to some extent. More precisely, recall that we have defined thresholds \( c' \) and \( c'' \) for the tax rate \( c \), so that a Crisis Contract has to impose at least this threshold tax rate to be effective. Observe that these thresholds are increasing in \( x_g \). If an effective Crisis Contract is in place, and one relaxes capital requirements somewhat, then the Crisis Contract can maintain its effectiveness if the associated tax rate \( c \) is increased. On the other hand, note that both
inequalities (2.12) and (2.13) — each of which is a necessary condition for the existence of one type of threshold equilibrium — are fulfilled as long as \( x_g \) is not too large. Hence, the relationship between Crisis Contracts and capital requirements can be wrapped up as follows:

There exist thresholds \( \phi, \bar{\phi} \in \mathbb{R}_+ \) such that \( \bar{\phi} > \phi \) and

1. if the banks’ debt-equity ratio is below some threshold \( \phi \), risk-taking and banking crises do not necessitate public bailouts.

2. if the banks’ debt-equity ratio is above threshold \( \bar{\phi} \), then banks exhibit socially detrimental risk-taking which cannot be discouraged by Crisis Contracts.

3. If capital requirements are such that the banks’ debt-equity ratios are within the interval \( [\phi, \bar{\phi}] \), then Crisis Contracts are welfare-enhancing. Within this interval, the regulator can achieve the welfare-enhancing effect by using different combinations of the tax rate associated with the Crisis Contract and the debt-equity ratio induced by the capital requirements. Stricter capital requirements allow a lower tax rate in the Crisis Contract, while a higher tax rate in the Crisis Contract allows more lenient capital requirements.

2.9 Hedging

2.9.1 Risk-neutral Players

We now examine whether the players could circumvent the effectiveness of Crisis Contracts by hedging themselves against the crisis tax by buying either insurance or a put option. Suppose that manager \( i \), who has worked for a bank in the first period and has earned \( \omega^i_1 > 0 \), could buy insurance \( I(Pr, R) \), which would pay him amount \( R \) in the case of crisis if he pays premium \( Pr \) in advance. It is natural to assume that the insurance contract will be signed and the insurance premium paid by the manager in the second period, after the wage offer has been made, and before the manager makes his investment decision. Payment from the insurance to the manager is conducted after \( Z_2 \) has been observed. We assume a competitive insurance market and that insurance firms offer actuarial fair insurance premiums.

Let us focus on the interesting case of a threshold equilibrium. First suppose that a fair insurance intends to enter the market and does not account for possible changes in investment decisions by managers. We argue that the insurance will suffer losses. Supposing that the threshold equilibrium is played, a fair insurance must have \( Pr = 0 \). So let us consider an insurance contract \( I_0 = I(0, \tau^i(h)) \), i.e. the insurance is free for the
manager and pays his crisis tax in the case of a crisis occurring in the second period. We claim that $I_0$ cannot be part of an equilibrium in a game in which insurance is modeled explicitly. We do not consider such a game in detail, but provide the intuition for this case.

The reason is that if the insurance $I_0$ is available, every shareholder will offer the second-period wage scheme $\omega = \left( \frac{D}{1-p}, 0, 0 \right)$. Then each manager will choose risky investment and a crisis will occur with probability $p$ in the second period. Hence the insurance expects losses. Accordingly, $I_0$ is not a fair insurance and cannot be part of an equilibrium in such a game.

Second, suppose that a fair insurance takes into account possible changes of investment decisions by managers.\footnote{The same argumentation also holds in the case of a put option rather than an insurance.} Then, we argue, the threshold equilibrium cannot be destroyed by the insurance. Let us consider an insurance contract of the form $I(Pr, \tau^i(h))$. We use $\rho^{ih}(\sigma)$ to denote the probability with which a crisis will occur in the second period under strategy profile $\sigma^{-i}$, given first-period history $h$ and supposing that manager $i$ decides to take risk in the second period. Since $R = \tau^i(h)$, the expected claim payment to manager $i$ is $\rho^{ih}(\sigma)\tau^i(h)$. Suppose that a bank deviates from the threshold equilibrium by choosing the risky investment instead of the safe investment. Then $\rho^{ih}(\sigma) = p$. Hence, since the insurance is assumed to be fair, manager $i$ has to pay an insurance premium of $Pr = p\tau^i(h)$. However, to prevent manager $i$ from opting out shareholder $i$ has to increase the expected wage payment by $p\tau^i(h)$. This is equivalent to increasing the wage payment by $\tau^i(h)$ and eliminating the insurance contract. But in a threshold equilibrium there is no profitable deviation for the shareholder that consists in increasing the manager’s wage $\omega_g$ and thereby making the manager take risk. Hence, if a threshold equilibrium exists, it is not possible to annihilate it by insurance.

### 2.9.2 Differences in Risk Appetite

Even if managers were risk-averse, they would not negotiate an insurance contract that makes them worse off in terms of expected payoff, since they can always choose to opt out and obtain the riskless payoff. In fact, in equilibria with $n_t \geq \bar{n}$, shareholders would have to promise managers higher wages to retain them. We observe that manager risk neutrality is a conservative assumption in the sense that with risk-averse managers Crisis Contracts would be even more effective for establishing a threshold equilibrium.
2.10 Ramifications and Conclusions

In this section we provide a detailed discussion of further extensions, the role of contract triggers, and other aspects of Crisis Contracts. We start with some numerical examples.

2.10.1 Examples

Here we provide some additional numerical examples to illustrate the working of the model.

Example 2.3
Consider an example where the discount factor is $\delta = 1$ and crisis probability $p = 0.4$. Let the payoffs be $D = 1$ and $x = (3, 1.7, -2.1)$.

Note that $c' > 1$, so there exists no strictly beneficial Crisis Contract, since full risk equilibrium can not be ruled out by any Crisis Contract. However, $c'' = 0.25$, and due to $\delta = 1$ inequality (2.12) holds. If it holds that $\hat{n} \geq 0.5$, then a Crisis Contract with tax rate $c = 0.26$ is weakly beneficial.

Example 2.4
Consider an example where the discount factor is $\delta = 0.5$ and crisis probability $p = 0.5$. Let the payoffs be $D = 1$ and $x = (2.5, 1.2, -1.5)$.

Then $c' = 0.8$, $c'' = 0.1$, and inequality (2.13) holds. Hence a Crisis Contract with tax rate $c = 0.81$ is strictly beneficial. Note that inequality (2.12) holds in this example.

Example 2.5
Consider an example where the discount factor is $\delta = 1$ and low crisis probability of $p = 0.2$. Let the payoffs be $D = 1$ and $x = (1.5, 1.15, -1.1)$.

Then $c' = 0.83$, $c'' = 0.25$, and inequality (2.13) holds. Hence a Crisis Contract with tax rate $c = 0.84$ is strictly beneficial.

This example shows that if the difference between $x_g$ and $x_{rf}$ (and hence the potential gains from investment in the risky asset in comparison to investment in the safe asset) is not too big Crisis Contracts may be beneficial even when crisis probability is low.

2.10.2 Extensions

In this section we briefly discuss some potential extensions of the model. In reality, vulnerabilities in the financial sector seem to build up over time. In our model, one could assume that risk-taking in the first period will not trigger a crisis immediately but rather increase the risk of a crisis in the second period. This could be expressed by making
the threshold $\bar{n}$ in the second period dependent on the number of banks that took risk in the first period. We conjecture that, in such a setup, Crisis Contracts can prevent both the build-up of vulnerabilities in the first period and a crisis in the second period. The qualitative gist of the results would probably carry over to such a model, which would incorporate the following features:

- Choosing risky investments by banks in the first period does not lead immediately to a banking crisis, but increases the probability of a future banking crisis.

- The probability of a banking crisis $p$ if at least $\bar{n}$ banks take risks in the second period is higher, the more vulnerable the banking system is. Formally, $p$ is a monotonically increasing function of $n_1(A)$.

As long as the parameters fulfill the relevant conditions in both periods the qualitative results generalize to this set-up. In particular, appropriately designed Crisis Contracts can not only prevent a banking crisis in the second period but also preclude the build-up of vulnerabilities in the first period.

One more potential extension is to introduce some heterogeneity among the banks. We have assumed that all banks are of the same size and have the same impact on the triggering of a crisis. However, we could easily incorporate into our model a situation where some banks are small and have no, or only a very small, impact on crisis probability and hence neglect it when making decisions. Indeed, suppose in addition to $n$ large banks there is a continuum of small banks. First note that if we assume that they have no impact on the crisis probability, then our results obviously continue to hold. Second, assume that each small bank has an infinitesimal impact on crisis probability and that the aggregate impact of all small banks would be equal to the impact of $m$ large banks if they all chose the risky investment. Obviously, all small banks will chose the risky investment, since each of them neglects its impact on the crisis probability. Then if $m < \bar{n}$, the results on the impact of Crisis Contracts on crisis probability and on risk-taking incentives of big banks in this set-up are the same as in the model without small banks and with a smaller threshold value $\bar{n}_{\text{new}} = \bar{n} - m$.

We have assumed that all shareholders benefit from excessive risk-taking by managers. Suppose that some shareholders are harmed by a banking crisis. For instance, they may have invested in other firms and hence suffer from a banking crisis. Suppose that $\hat{n}$ shareholders (with $\hat{n} < n - \bar{n}$) are crisis-sensitive and internalize the losses $x_b$ from such

---

13 In addition or alternatively, threshold $\bar{n}$ in the second period could be a decreasing function of the number of banks that took risk in the first period.

14 Some shareholders or bank managers may also incur non-pecuniary disutilities if their investment behavior triggers a banking crisis. This would also induce crisis sensitivity.
crisis. Then we do not expect changes to the equilibrium predictions. However, the banking industry does not choose full risk, as some shareholders select the safe investment.

If the abilities of bank managers in producing different returns to shareholders differ, Crisis Contracts might help to foster the socially efficient recruitment of bank managers.

So far, we have argued that Crisis Contracts are beneficial in that they can prevent crises that would lead to social losses. To assess social benefits and costs, we have considered a very simple additive social welfare function where payoffs to citizens, shareholders, and managers are treated equally. If one views the regulator as serving the interests of the public to a greater extent than those of shareholders or managers, Crisis Contracts are even more useful. In recent public discussion, it has often been felt to be unfair that the welfare of ordinary citizens has apparently not had enough weight in the regulator’s considerations. Crisis Contracts do not suffer from this problem and may therefore be seen as a “fair” regulatory tool.

2.10.3 Contract Triggers

To implement a Crisis Contract, it is necessary to define some verifiable criteria for the occurrence of a banking crisis. There are several possibilities for defining such triggers of Crisis Contracts.

One possible trigger is bailing out a bank or banks e.g. by providing fresh equity or by guaranteeing the liabilities of banks. Using government bailout as a trigger for the execution of Crisis Contracts would probably have further effects. For instance, troubled banks may be more willing to opt for the bail-in of private debtors to avoid execution of the crisis tax. In turn, the threat to bail out may help the regulator to induce better bank equity capitalization in the banking system. Moreover, when only one or a few banks are troubled and may need to be rescued by the government, other banks may be more willing to play an active role in rescue activities.

An alternative trigger is the index of stock prices in the banking industry. Crisis Contracts are executed if the index falls below a certain threshold. A third possibility for defining a trigger is the (weighted) average of the debt-equity ratios in the banking industry. If this average exceeds a threshold, crisis taxes are due.

Each of these three possibilities defining triggers of Crisis Contracts has to be assessed in detail for the pros and cons and the further effects they may involve. The third trigger in particular is conceptually appealing, as it is rooted in the average debt-equity ratio in the banking industry. It relies on accounting information that regulators collect anyway and may be least susceptible to manipulation.
2.10.4 Concluding Remarks

We have presented an initial analysis of Crisis Contracts and have gauged their potential and limitations. This first pass of the analysis suggests that Crisis Contracts could be a useful tool in the design of a financial architecture that is significantly more resilient than in the past.

Numerous issues deserve scrutiny. While we have used a stylized model to study the functions of Crisis Contracts, in practice they have to be based on assessments of the extent of risk-taking and the likelihood of crisis in the banking industry in a calibrated model (see e.g. Chesney et al. (2012)). Erring on the conservative side will not undermine the efficacy of Crisis Contracts, but being too optimistic about the stability of the banking system will. Moreover, Crisis Contracts will likely have further effects.

Crisis Contracts may help to break peer effects when it is common in the banking system to motivate managers with high bonuses to take risks that collectively exceed socially desirable levels.\footnote{It is well-known that peer effects play a considerable role in banking. For instance, herding with regard to risk-taking is significant among the largest banks, see Bonfim and Kim (2012).} Crisis Contracts may also induce banks to become more prudent regarding their counterparties in the interbank market, which may promote stability.
2.11 Appendix

Proof of Theorem 2.1

The proof is constructive. Let $\bar{\sigma}^h$ be the strategy profile for the $h$-subgame, where all shareholders $i \in N$ make offer $\bar{\omega}^i = \left( \frac{D}{1-p}, 0, 0 \right)$ to their managers. All managers will take risk in response to any wage offer $\omega \in \Omega$ that satisfies $(1-p)\omega_g \geq D$ and $(1-p)\omega_g \geq \omega_{rf}$. If the wage offer $\omega \in \Omega$ fails to satisfy one or both of these inequalities, then the manager will invest safely if and only if $\omega_{rf} > D$. Otherwise he will opts out. We show that $\bar{\sigma}^h$ is an $h$-banking equilibrium.

Fix a bank $i \in N$. Given $A^{-i2}(\bar{\sigma}^h)$, a crisis will occur in the second period with probability $p$. For every $\omega \in \Omega$, strategy $\bar{\sigma}_m^i$ chooses an option from $\{ R, S, O \}$ that leads to a weakly higher payoff for manager $i$ than any other choice. It is straightforward to see that manager $i$ does not have any profitable deviation from $\bar{\sigma}^h$.

If manager $i$ opts out, then shareholder $i$ will obtain a payoff of zero. By construction of $\bar{\sigma}_m^i$, manager $i$ only works for the bank if his expected wage is at least $D$. Accordingly, in the case where manager $i$ invests safely, the expected payoff to shareholder $i$ is bounded from above by $x_{rf} - D$. And in the case where manager $i$ takes risk, the expected payoff to shareholder $i$ is bounded from above by $(1-p)x_g - D$. Since $(1-p)x_g > x_{rf} > 0$, it follows that given $A^{-i2}(\bar{\sigma}^h)$ the expected payoff to shareholder $i$ is bounded from above by $(1-p)x_g$. But this is the expected payoff of shareholder $i$ under $\bar{\sigma}^h$. We see that shareholder $i$ has no profitable deviation from $\bar{\sigma}^h$. □
Proof of Theorem 2.2

The “only if” part has been proved in Lemma 2.3. We now prove that an $h$-banking equilibrium involving threshold risk exists if $\tau^i(h) \geq \tau^*$ for at least $n - \bar{n} + 1$ banks.

The proof is constructive. Suppose that it holds $\tau^i(h) \geq \tau^*$ for the $n - \bar{n} + 1$ banks $j = \bar{n}, \ldots, n$. Let $\bar{\sigma}^h$ be the strategy profile for the $h$-subgame where shareholders $i = 1, \ldots, \bar{n} - 1$ all make wage offer $\bar{\omega}^{j2} = (D, 0, 0)$ and shareholders $j = \bar{n}, \ldots, n$ make wage offer $\bar{\omega}^{j2} = (0, D, 0)$. Managers $i = 1, \ldots, \bar{n} - 1$ will take risk in response to wage offer $\omega \in \Omega$ if and only if $\omega_g \geq \omega_{r,f}$ and $\omega_g \geq D$. Otherwise those managers will invest safely if $\omega_m \geq D$ and opt out if $\omega_{r,f} < D$. Managers $j = \bar{n}, \ldots, n$ will respond to wage offer $\omega \in \Omega$ by taking risk if and only if $(1 - p)\omega_g - p\tau^j(h) > \omega_{r,f}$ and $(1 - p)\omega_g - p\tau^j(h) > D$. Otherwise those managers will invest safely if $\omega_{r,f} \geq D$ and opt out if $\omega_{r,f} < D$.

Consider a manager $i = 1, \ldots, \bar{n} - 1$. Given $A^{-i2}(\bar{\sigma}^h)$, no crisis occurs. So manager $i$ chooses from the payoffs $D$ (opting out), $\bar{\omega}_i^{ih} = 0$ (investing safely), and $\bar{\omega}_g^{ih} = D$ (investing in the risky asset). Clearly, taking risk is optimal.

Consider a manager $j = \bar{n}, \ldots, n$. Given $A^{-j2}(\bar{\sigma}^h)$, there will be a crisis with probability $p$ if $j$ takes risk and with probability zero otherwise. Manager $j$ chooses from the payoffs $D$ (opting out), $\bar{\omega}_{r,f}^j = D$ (investing safely), and $(1 - p)\bar{\omega}_g^j - p\tau^j(h) \leq 0$ (taking risk). Investing safely is optimal.

Take a shareholder $i = 1, \ldots, \bar{n} - 1$. Under $\bar{\sigma}^h$, his payoff is $x_g - D > 0$. Shareholder $i$ will only receive a positive payoff if manager $i$ works. However, no manager works in the second period for an expected wage of less than $D$. Since $x_g$ is the highest possible asset return, $x_g - D$ is an upper bound on the payoff for any shareholder in any $h$-subgame. In particular, shareholder $i$ has no profitable deviation from $\bar{\sigma}^h$.

Finally, consider shareholder $j = \bar{n}, \ldots, n$. His payoff under $\bar{\sigma}^h$ is $x_{r,f} - D \geq 0$. If shareholder $j$ has a profitable deviation from $\bar{\sigma}^h$, it must involve manager $j$ working for the bank. However, manager $j$ will only invest safely for an expected wage of at least $D$ and will only take risk for an expected wage of at least $(1 - p)\omega_g^j - p\tau^j(h)$. So the payoff for shareholder $j$ from offering a wage scheme to which manager $j$ responds by investing safely is bounded from above by $x_{r,f} - D$. No such deviation can be profitable. The payoff for shareholder $j$ from offering a wage scheme to which manager $j$ responds by taking risk is bounded from above by $(1 - p)x_g - D - p\tau^j(h)$. It follows that no profitable deviation from $\bar{\sigma}^h$ is possible for shareholder $j$ if $(1 - p)x_g - p\tau^j(h) \leq x_{r,f}$. □
### 2.12 Notation List

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Set of banks</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of banks, $n \geq 2$</td>
</tr>
<tr>
<td>$i$</td>
<td>Index of a bank</td>
</tr>
<tr>
<td>$j$</td>
<td>Index of a bank</td>
</tr>
<tr>
<td>$t$</td>
<td>Index of a period, $t \in {1, 2}$</td>
</tr>
<tr>
<td>$A^i$</td>
<td>Investment decision of bank $i$ in period $t$, $A^i \in {R, S, O} = {Risk, Safe, Outside}$</td>
</tr>
<tr>
<td>$A$</td>
<td>Activity profile, $A = (A^{i1}, ..., A^{n1}, A^{i2}, ..., A^{n2})$</td>
</tr>
<tr>
<td>$A^{-i}$</td>
<td>Restriction of the activity profile to round $t$ and banks $j \in N \setminus {i}$</td>
</tr>
<tr>
<td>$n_t(A)$</td>
<td>Number of banks choosing the risky asset in period $t$ under the activity profile $A$</td>
</tr>
<tr>
<td>$\bar{n}$</td>
<td>Threshold for triggering a positive probability of a banking crisis, $\bar{n} \in {1, ..., n-1}$</td>
</tr>
<tr>
<td>$p$</td>
<td>Probability of a crisis in period $t$ under the activity profile $A$, if $n_t(A) \geq \bar{n}$</td>
</tr>
<tr>
<td>$Z_t$</td>
<td>Indicator with $Z_t = 1$ if a crisis occurs in period $t$ and $Z_t = 0$ otherwise</td>
</tr>
<tr>
<td>$Z$</td>
<td>Pair of indicators $(Z_1, Z_2)$</td>
</tr>
<tr>
<td>$x_{rf}$</td>
<td>Payoff from the safe investment in one period</td>
</tr>
<tr>
<td>$x_g$</td>
<td>Payoff from the risky investment in one period, if no crisis occurs in this period</td>
</tr>
<tr>
<td>$x_b$</td>
<td>Payoff from the risky investment in one period, if a crisis does occur in this period</td>
</tr>
<tr>
<td>$k$</td>
<td>Index of a payoff from an investment, $k \in {g, rf, b}$</td>
</tr>
<tr>
<td>$\omega^i$</td>
<td>Wage scheme offered by shareholder $i$ to manager $i$ in period $t$</td>
</tr>
<tr>
<td>$\omega_g^i$</td>
<td>Manager $i$’s wage in period $t$, if the payoff from the investment is $x_g$. The index $^i$ is omitted when redundant</td>
</tr>
<tr>
<td>$\omega_{rf}^i$</td>
<td>Manager $i$’s wage in period $t$, if the payoff from the investment is $x_{rf}$. The index $^i$ is omitted when redundant</td>
</tr>
<tr>
<td>$\omega_b^i$</td>
<td>Manager $i$’s wage in period $t$, if the payoff from the investment is $x_b$. $\omega_b^i = 0$. The index $^i$ is omitted when redundant</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Set of possible wage schemes, $\Omega = {\omega^i \in \mathbb{R}^3</td>
</tr>
<tr>
<td>$D$</td>
<td>Outside wage of a manager in one period, $D &gt; 0$</td>
</tr>
<tr>
<td>$u_{s}^i$</td>
<td>Instantaneous utility of shareholder $i$ in period $t$. If manager $i$ accepts the offer in period $t$, $u_{s}^i = x_k - \omega_k$ where $k \in {g, rf, b}$. Else $u_{s}^i = 0$</td>
</tr>
<tr>
<td>$u_{m}^i$</td>
<td>Instantaneous utility of manager $i$ in period $t$. If manager $i$ accepts the offer in period $t$, $u_{m}^i = \omega_k$ where $k \in {g, rf, b}$. Else $u_{m}^i = D$</td>
</tr>
<tr>
<td>$c$</td>
<td>Crisis Contract tax rate, $c \in [0, 1]$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Discount factor, $\delta \in [0, 1]$</td>
</tr>
<tr>
<td>$U_m^i$</td>
<td>Manager $i$’s intertemporal utility, $U_m^i = u_{m}^{i1} + \delta u_{m}^{i2} - \delta Z_2 c u_{m}^{i1}$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Meaning</td>
</tr>
<tr>
<td>--------</td>
<td>---------</td>
</tr>
<tr>
<td>$h, h', h''$</td>
<td>Variables to denote first-period histories</td>
</tr>
<tr>
<td>$H$</td>
<td>Set of all first-period histories, $H \subset \Omega^n \times {R, S, O}^n \times {0, 1}$</td>
</tr>
<tr>
<td>$\tau^i(h)$</td>
<td>Looming crisis tax given first-period history $h$</td>
</tr>
<tr>
<td>$\sigma^{ih}$</td>
<td>Strategy of shareholder $i$ in the $h$–subgame</td>
</tr>
<tr>
<td>$\sigma^{ih}$</td>
<td>Strategy of manager $i$ in the $h$–subgame</td>
</tr>
<tr>
<td>$\sigma^h$</td>
<td>Strategy profile for the $h$–subgame, $\sigma^h = (\sigma^{1h}_m, ..., \sigma^{nh}_m, \sigma^{1h}_s, ..., \sigma^{nh}_s)$</td>
</tr>
<tr>
<td>$\sigma^1_m$</td>
<td>Strategy of manager $i$ in the banking game</td>
</tr>
<tr>
<td>$\sigma^i_s$</td>
<td>Strategy of shareholder $i$ in the banking game</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Strategy profile of the banking game, $\sigma = (\sigma^1_m, ..., \sigma^n_m, \sigma^1_s, ..., \sigma^n_s)$</td>
</tr>
<tr>
<td>$\mu^{i2}(\sigma^h)$</td>
<td>Manager $i$’s expected wage in the second period, given strategy profile $\sigma^h$, under which manager $i$ works for the bank in the second period</td>
</tr>
<tr>
<td>$\kappa^{i2}(\sigma^h)$</td>
<td>Manager $i$’s reservation wage</td>
</tr>
<tr>
<td>$\tau^*$</td>
<td>Threshold for the looming tax for existence of an $h$–banking equilibrium with threshold risk</td>
</tr>
<tr>
<td>$\mu^{i1}(\sigma)$</td>
<td>Manager $i$’s expected wage in the first period under strategy profile $\sigma$</td>
</tr>
<tr>
<td>$\rho(\sigma)$</td>
<td>Ex ante probability with which a crisis will occur in the second period under the strategy profile $\sigma$(not updated with the possible move of nature in the first period)</td>
</tr>
<tr>
<td>$\nu^{i1}(\sigma)$</td>
<td>Manager $i$’s expected net wage from the first period under strategy profile $\sigma$</td>
</tr>
<tr>
<td>$c'$</td>
<td>Lower bound for $c$ excluding the existence of a full risk equilibrium</td>
</tr>
<tr>
<td>$c''$</td>
<td>Lower bound for $c$ not excluding the existence of a threshold equilibrium</td>
</tr>
<tr>
<td>$\hat{n}$</td>
<td>Measure for the stability of the banking system, $\hat{n} = (\bar{n} - 1)/n$</td>
</tr>
<tr>
<td>$y^t$</td>
<td>Instantaneous social welfare in period $t$</td>
</tr>
<tr>
<td>$Y$</td>
<td>Aggregate social welfare</td>
</tr>
<tr>
<td>$\mathcal{E}(c)$</td>
<td>Set of banking equilibria when the crisis tax rate is $c$</td>
</tr>
<tr>
<td>$\rho^{ih}(\sigma)$</td>
<td>Probability with which a crisis will occur in the second period under strategy profile $\sigma^{-i}$, given first-period history $h$ and supposing that manager $i$ decides to take risk in the second period</td>
</tr>
<tr>
<td>$e$</td>
<td>Equity of a shareholder</td>
</tr>
<tr>
<td>$d$</td>
<td>Amount of deposits of a bank</td>
</tr>
<tr>
<td>$r'$</td>
<td>Interest rate on risky asset if no crisis occurs</td>
</tr>
<tr>
<td>$r$</td>
<td>Interest rate on deposits and on risk-free asset</td>
</tr>
<tr>
<td>$l$</td>
<td>Loss of the bank’s total capital in the case of crisis</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Lower threshold on the bank’s debt-equity ratio</td>
</tr>
<tr>
<td>$\overline{\phi}$</td>
<td>Upper threshold on the bank’s debt-equity ratio</td>
</tr>
<tr>
<td>$I(Pr, R)$</td>
<td>Insurance with premium $Pr$ and payoff $R$ in the case of crisis</td>
</tr>
</tbody>
</table>
3 Liquidity Regulation and Aggregate Investment

3.1 Introduction

Motivation and main insights

Liquidity regulation of banks usually aims at the prevention or mitigation of liquidity crises. However, another relevant issue for the assessment and design of liquidity regulation is its impact on the real economy. This complementary view on liquidity regulation has been proposed by Gersbach and Rochet (2012A), who have shown that excessive investment fluctuations by the banking sector may be moderated by a limitation of short-term debt. In our paper we continue the analysis of Gersbach and Rochet (2012A) by considering three extensions of their model.

Gersbach and Rochet (2012A) consider an economy that consists of two sectors, the banking sector and the traditional sector. The banking sector is hit by a random shock, which can be productiveness-increasing or -decreasing. The traditional sector is not hit by the shock. This generates the impulse to reallocate capital between the sectors, depending on the size of the shock. This impulse is additionally reinforced by the presence of moral hazard in the banking sector. Gersbach and Rochet (2012A) show that under such conditions, social welfare can be increased by decreasing the absolute values of capital reallocations, values that thus turn out to be excessive. This can be done via the regulation of short-term debt. We consider an extension of the model of Gersbach and Rochet (2012A) in which the same shock hits both sectors. The question is: Will there still be excessive capital reallocations, and would regulation of short-term debt still be welfare-increasing? We answer these questions by analyzing the mentioned extension. We find that due to moral hazard, capital reallocations still would occur. Furthermore, we show that the regulation of short-term debt may increase social welfare.

In the second extension, we will vary the assumption of Gersbach and Rochet (2012A) that the contracts between bankers and investors are written by the bankers, and thus are take-it-or-leave-it offers to the investors. We consider the opposite case, in which

---

1 See Eubanks (2010), for example.
**investors** write the contracts. This extension is motivated by the observation that model results may vary significantly depending on which players write the contracts. Indeed, our findings will confirm this observation. On the one hand, the results of Gersbach and Rochet (2012A) on the conditions for the existence of different kinds of equilibria also hold in this extension. On the other hand, however, unlike in Gersbach and Rochet (2012A), regulation of short-term debt cannot increase social welfare in this variant. We will explain why this qualitative difference arises.

In the third extension, we will consider a change in Gersbach and Rochet’s (2012A) approach to modeling moral hazard. In Chapter 4 of this thesis we will show that a change in the modeling approach might significantly change model results. However, we will show that in the particular case of our extension this change does not affect the results of Gersbach and Rochet (2012A) qualitatively.

In general, the mentioned extensions yield two kinds of insights. First, they provide a robustness analysis of the results of Gersbach and Rochet (2012A) which allows to assess the range of conditions under which these results hold and which illustrates which of the assumptions in Gersbach and Rochet (2012A) are crucial for their results. These insights can be used on the one hand to assess some of Gersbach and Rochet’s possible implications on whether and how regulation of short-term debt – and especially the Net Stable Funding Ratio – should be implemented. On the other hand, being able to separate crucial from non-crucial assumptions may help future researchers to chose those hypotheses that are relevant for empirical tests. Second, analyzing the extensions yields a deeper understanding of the basic model. In particular, the extension with shocks that hit both sectors allows to better understand the nature of the capital reallocations in Gersbach and Rochet (2012A).

**Relation to the Literature**

We study the interaction between the real economy and financial markets. In particular, we explore a particular mechanism of the amplification of macroeconomic shocks by financial markets. An explanation of this amplification that complements ours is the idea of the so-called financial accelerator. This term has first been introduced by Bernanke et al. (1996):

Its basic idea is a vicious cycle: Asset prices of small firms decrease, due to a negative macroeconomic shock. Thus, investment in these firms decreases, as, due to moral hazard at firm level, investment in a firm is limited by a certain multiple of the firm’s value. Then, due to low investments, economic activity decreases in these firms. However, this again leads to a decrease of asset prices. Thus, the macroeconomic shock is amplified by the

---

2 See, for example, Balkenborg (2001) who discusses two groups of papers that yield opposite results despite the fact that their models are very similar while the main difference between them is the choice of the contract authors.
financial accelerator mechanism.

The underlying debt-deflation cycle had been described by Fischer (1933). Bernanke and Gertler (1989) identified the mechanism of the financial accelerator and Kyiotaki and Moore (1997) have further developed it by emphasizing the role of the firms’ assets and liabilities. More recent papers based on the financial accelerator principle are Lorenzoni (2008), Jeanne and Korinek (2010), Bianchi (2011) and Korinek (2011). Bargigli et al. (2014) explore the financial accelerator mechanism as an externality in the credit market modeled as an endogenously-formed network.

Although in Gersbach and Rochet (2012A), the macroeconomic shock is also amplified by the financial sector, the authors do not examine the financial accelerator, but identify another mechanism: Namely due to moral hazard at firm level, there is an oversensitive reaction to a positive or negative macroeconomic shock, which leads to excessive capital reallocations depending on the shock. This leads to excessive price fluctuations, which result in an inferior capital allocation in equilibrium. Note that Gersbach and Rochet (2012A) do not examine a decrease of economic activity at firms relative to the investment size and do not analyze a spiral as in the financial accelerator research.

A general equilibrium approach is used by Gertler and Kiyotaki (2010) to explore how crises in the real economy can be induced by disruptions in financial markets. Gertler and Kiyotaki (2010) also show how the regulation of financial intermediaries may weaken crises.

Empirical evidence for the amplification of macroeconomic shocks through credit crunches can be found in Bernanke and Lown (1991) and Peek and Rosengren (1995). Meh and Moran (2010) document that GDP growth is approximately four times less volatile than bank-lending growth in the United States. Adrian and Shin (2010) document that leverage of financial intermediaries is heavily procyclical. Jimenez et al. (2012) confirm the existence of credit cycles empirically. Égert and Sutherland (2012A) provide strong evidence for the pro-cyclicality of the banking sector in their extensive empirical study of real-business cycles and their financial counterparts in OECD countries. Together, these findings point to the existence of excessive procyclical fluctuations in bank-lending.

In the current paper, as in Gersbach and Rochet (2012A), the impact of financial intermediaries on the real economy is described as a pecuniary externality. In general, a pecuniary externality occurs when, in the presence of credit frictions, some agents trade in a market without internalizing their collective impact on prices. In their quantitative model, Bianchi and Mendoza (2011) emphasize the impact of asset fire-sales on leverage and on the occurrence of financial crises. Bianchi and Mendoza (2011) and Jeanne and Korinek (2010) provide a welfare-theoretic rationale for macroprudential regulation, and consider corresponding approaches via taxation. Following Gersbach and Rochet
(2012A), we consider a complementary approach to macroprudential regulation via the regulation of short-term debt. Caballero and Krishnamurthy (2003) explain inefficient capital allocations in investments in foreign countries by overborrowing and underinsurance without internalizing their effects on exchange rates. Subrahmanyam and Titman (2013) investigate in the context of a market entrance problem how pecuniary externalities through shocks to cash-flows affect stock prices, which, in turn, affect the aggregate economic output. Titman (2013) studies two kinds of interactions between financial market disruptions and aggregate investment. He argues that shocks to debt-markets generate externalities that differ from the externalities stemming from shocks to equity markets. Moreover, he discusses these findings in the contexts of recent boom-and-bust cycles.

Gersbach and Rochet (2012B) consider a model that incorporates a dynamical structure closely related to ours. They show that complete contingent financial markets cannot prevent excessive credit-fluctuations and argue that counter-cyclical capital ratios could prevent this inefficiency. They explore effects related to those explored in Gersbach and Rochet (2012A). However, Gersbach and Rochet (2012B) show how these effects may arise in complete contingent markets and how they might be addressed via capital regulation, an approach that is complementary to the regulation of short-term debt considered in Gersbach and Rochet (2012A) and in our paper.

This chapter is organized as follows. In the next section, we consider a modification of the basic model in which both sectors of the economy are hit by shocks. In Section 3.3, we explore the modification of the basic model in which investors – instead of bankers – write the contracts. Section 3.4 contains a robustness analysis of the results of Gersbach and Rochet (2012A), subject to a change of the approach to model moral hazard. Section 3.5 concludes.

### 3.2 Macroshocks

Gersbach and Rochet (2012A) consider a model of a two-sectoral economy in which one of the sectors is hit by an economic shock. We study a modification of this model in which the shock is inter-sectoral, which means that it hits both sectors of the economy. We will show that the main results in Gersbach and Rochet (2012A) are robust with respect to this modification. This is an important result, as the opposite result would be a serious drawback of the model of Gersbach and Rochet (2012A), since correlations between shocks in different industries have an important impact on the overall risk in an economic system.
3.2 Macroshocks

3.2.1 The Model

We use the model of Gersbach and Rochet (2012A) as a basis, and use their notation. There are three time periods labeled $t = 0, 1, 2$.

Agents

There are three types of agents: bankers, investors and entrepreneurs.

There is a continuum of measure one of bankers. At $t = 0$, each banker $i$ has some initial endowment $e_i$ of a good. When it is not needed, we will omit the index and write $e$ for $e_i$. The aggregate bankers’ endowment is $E$ with $0 < E < 1$.

There is also a continuum of measure one of investors. The aggregate investors’ endowment is $1 - E$.

The continuum of entrepreneurs also has measure one. Entrepreneurs do not have an initial endowment and do not act as strategic players in our model.

Hence, overall endowment is 1.

All agents are assumed to be risk-neutral. We assume that the discounting rate between the time-periods is 1.

Consumption is possible at $t = 0$ and $t = 2$. Aggregate consumption at $t$ is denoted by $C_t$. Social welfare is measured by the expected aggregate consumption $U = C_0 + E[C_2]$.

Technologies

There is a banking sector (BS) and a traditional sector (TS). They produce goods which can be consumed in $t = 2$. At $t = 1$, a shock occurs that impacts both sectors simultaneously. This feature of our model is the only difference to the model in Gersbach and Rochet (2012A), where only the BS is subject to a shock.

The BS has constant returns to scale and is affected by a shock $\eta$. An amount $k$, which a bank $i$ invests at $t = 0$, results in the output $\eta \tilde{R}_i k$ at $t = 2$. In this formula, $\eta$ is given by

$$
\eta = \begin{cases} 
  h & \text{with probability } q \\
  l & \text{with probability } 1 - q,
\end{cases}
$$

with $q \in [0, 1]$ and $0 < l < 1 < h$. $\tilde{R}_i$ is an idiosyncratic return modeled as a random variable, whose distribution depends on whether bank $i$ choses to exert effort at the beginning of period 2. Conditional on equal effort choice, the distribution of $\tilde{R}_i$ is identical and independent among all banks $i$. Whenever the index is not important, we write $\tilde{R} = \tilde{R}_i$. Let $\bar{R}$ denote the expectation of $\tilde{R}$ in the case that bank $i$ exerts effort. Therefore, if one unit
is invested in the BS, the expected output at \( t = 2 \) is equal to \( m\bar{R} \), with \( m = qh + (1-q)l \). Without loss of generality, we normalize \( m \) to 1.

In our model, the TS is affected by the same shock \( \eta \) as the BS. The marginal returns from investment in the TS decrease at the aggregate level. Namely, investing the amount \( X \) at \( t = 0 \) yields the output \( \eta F(x) \) at \( t = 2 \). The function \( F(X) \) is such that \( F(0) = 0 \), \( F'(X) > 0 \) and \( F''(X) < 0 \). We assume that \( F(X) \) fulfills the Inada conditions, i.e. we assume that \( \lim_{X \to 0} F'(X) = \infty \) and \( \lim_{X \to 1} F'(X) = 0 \). We assume a perfectly competitive market in the TS.

Since the shock \( \eta \) impacts the TS as well as the BS, it is an intersectoral shock. We refer to it as a Macroshock.\(^3\)

**Capital Allocation**

At \( t = 0 \), the agents who have a positive endowment decide how much of it they will consume in the current period. The aggregate amount of consumption at \( t = 0 \) is called \( C_0 \). The agents transform the rest of their endowment – cost-free – into capital, and invest it either in the BS or in the TS. We call \( K \) the aggregate amount of capital that is invested in the BS, and \( X \) the aggregate amount of capital invested in the TS. The aggregate consumption constraint \( C_0 + K + X = 1 \) has to hold.

At \( t = 1 \), after the observation of \( \eta \), it is possible for each bank to adjust its capital – i.e. the capital invested in the BS in this particular bank –, by selling or buying capital from entrepreneurs that operate in the TS at the price \( p_\eta \) per unit of capital in state \( \eta \). Capital thereby is traded against claims on the consumption good at \( t = 2 \). We assume the absence of any contractual frictions and we also assume the absence of defaults. Period 1 is the only period in which this kind of trade takes place. As a numeraire, we chose the consumption good at \( t = 0 \). We assume completeness and the absence of any frictions in financial markets.

In our model, the interest rate \( r \geq 0 \) is defined through \( \mathbb{E}[p_\eta] = 1 + r \), where the left side is obviously the ex ante rate of return on capital at \( t = 0 \). Risk-neutrality and absence of discounting imply that either \( r = 0 \) or \( C_0 = 0 \). The contingent price, which is paid at \( t = 0 \) for the promise of a delivery of one unit of the good at \( t = 2 \) in state \( l \) (or state \( h \), respectively), and nothing otherwise is equal to \( \frac{1-q}{1+r} \) (respectively \( \frac{q}{1+r} \)).
3.2 Macroshocks

Figure 3.1: Timeline of Events

- $t = 0$: Banker and investor agree on a contract $C(k, \alpha_\eta, \eta^0)$.
- $t = 1$: Banker invests joint investment $k$ in the BS. Macroshock $\eta$ hits both sectors and is observed by all players. Bank capital is adjusted by a fraction $\alpha_\eta$.
- $t = 2$: Banker exerts effort or shirks. Capital is traded at price $p_\eta$. Project outcome (success or failure) is observed. Banker and investor are paid according to the contract.

Banks

Figure 3.1 illustrates the sequence of events that each banker encounters. The variable $k$ denotes the capital invested in the BS by a typical bank before observing a shock at $t = 1$ and adjusting its investments accordingly. $k_\eta = (1 + \alpha_\eta)k$ denotes the capital invested in the BS by a typical bank after the adjustment. $\alpha_\eta$ is the rate of credit growth in the BS. It holds that $\alpha_\eta \geq -1$ for $\eta = l, h$. As stated above, capital is traded at $t = 1$ at the contingent price $p_\eta$. In addition, we assume that some adjustment costs of the amount $c\alpha^2_\eta k$ with $c > 0$ are withdrawn at $t = 2$ from gross returns on investment in the BS. These costs do not depend on the sign of the adjustment, but only on its absolute value. An example for such costs are transaction costs.

We will consider a typical banker and then translate our insights to the aggregate level. This translation is correct since the banking technology has constant returns to scale. At the beginning of period 2, the banker faces a situation with moral hazard as described in Holmström and Tirole (1997). The outcome $o$ of the banker’s project at $t = 2$ can either be success ($o = S$) or failure ($o = F$). This is reflected in two possible values which $\tilde{R}$ can take: $R^S$ and $R^F$, with $R^S > R^F$. If the banker exerts effort, the probability of $o = S$ is $\tau$ and he obtains no private benefit. If the banker exerts no effort, the probability of $o = S$ is $\tau - \Delta$, with $0 < \Delta < \tau$, and the banker obtains a private benefit $Bk_\eta > 0$.

\footnote{Note that in Gersbach and Rochet (2012A), the shock that only hits the BS is also defined as a macroeconomic shock. However, in Gersbach and Rochet (2012A), this means that this shock is due to some macroeconomic fluctuations. In our paper we say that a shock is a macroeconomic shock or a Macroshock, if and only if it impacts both sectors.}
Under a contract \( C(k, \alpha_\eta, b_\eta) \) if the shock \( \eta = \eta' \) and the outcome \( o = o' \) occur, the banker receives a payment \( k\eta' b_\eta' \) and the investor obtains \( k\eta' (1 + \alpha_\eta) R^{o'} - \frac{c}{2} \alpha^2_\eta - b_\eta' \) at \( t = 2 \). In this section we will adopt the assumption of Gersbach and Rochet (2012A) that bankers write the contracts.\(^4\) The properties of a contract that ensure that the banker exerts effort will be described in Section 3.2.2.

**Social Welfare**

We will consider the case in which banks will be paid in such a way that they will exert effort. Then, social welfare is given by

\[
U = C_0 + \mathbb{E}[C_2] = 1 - K - X + K\mathbb{E}[(1 + \alpha_\eta) \eta R - \frac{c}{2} \alpha^2_\eta] + \mathbb{E}[(X - \alpha_\eta K)]. \tag{3.1}
\]

This aggregate consumption is divided among agents in the following way:

- Investors obtain \( C_0 + (1 + r)(1 - E - C_0) \).
- Bankers obtain \( K\mathbb{E}[b_\eta(1 + \alpha_\eta)] \).
- Entrepreneurs obtain \( \mathbb{E}[\eta F(X - \alpha_\eta K) - p_\eta(X - \alpha_\eta K)] \).

Therein \( p_\eta = \eta F'(X - \alpha_\eta K) \) is the marginal productivity of capital in the TS.

Those three terms sum up to \( U \) in equilibrium. The reason for this is that in equilibrium, the expected rate of return on investment in both sectors has to be equal \( 1 + r \). In particular, since \( K - E \) is the amount invested by investors in the BS,\(^5\) this condition implies the following equilibrium condition:

\[
(1 + r)(K - E) = K\mathbb{E}[(1 + \alpha_\eta) \eta R - b_\eta'] - \frac{c}{2} \alpha^2_\eta - p_\eta \alpha_\eta].
\]

Using the equilibrium condition \( rC_0 = 0 \), we obtain

\[
U = C_0 + \mathbb{E}[C_2] = (1 + r)(1 - K - X) \quad \text{(this is } C_0) \nonumber\]
\[
+ (1 + r)(1 - E - C_0) \quad \text{(this returns to investors from TS and BS)} \nonumber\]
\[
+ K\mathbb{E}[b_\eta(1 + \alpha_\eta)] \quad \text{(this are the payoffs to bankers)} \nonumber\]
\[
+ \mathbb{E}[\eta F(X - \alpha_\eta K) - p_\eta(X - \alpha_\eta K)] \quad \text{(this are payoffs to entrepreneurs)} \nonumber\]
\[
= (1 + r)(1 - E) + K\mathbb{E}[b_\eta(1 + \alpha_\eta)] + \mathbb{E}[\eta F(X - \alpha_\eta K) - p_\eta(X - \alpha_\eta K)]. \tag{3.2}
\]

\(^4\) In Section 3.3 we will vary this assumption.

\(^5\) As Gersbach and Rochet (2012A), we assume for the rest of the paper that bankers invest all their endowment in the BS whenever there are no strictly better investment possibilities for them.
Finally, using the equilibrium conditions $\mathbb{E}[p_\eta] = 1 + r$ and $K\mathbb{E}[(1 + \alpha_\eta)(\eta\bar{R} - b_\eta^p) - \frac{c}{2}\alpha_\eta^2 - p_\eta\alpha_\eta] = (1 + r)(K - E)$, we find that (3.2) is equivalent to (3.1).

3.2.2 Two Simple Benchmarks

In analogy to Gersbach and Rochet (2012A), we now explore the following two benchmark cases: an economy without a moral hazard friction (i.e. $B = 0$), called the frictionless economy, and an economy, in which no adjustment of capital at $t = 1$ is possible, called the rigid economy. Examining those two cases yields the economic intuition necessary for the understanding of a general model that includes both moral hazard and adjustment of capital.

The Frictionless Economy

We explore the situation without moral hazard, i.e. we set $B = 0$. Of course this means that the social optimum is achieved in equilibrium.

In equilibrium, given state $\eta$, the expected marginal returns on investment at $t = 1$ have to be equal in both sectors, and, of course, be equal to the price of one unit of capital. Hence,

$$p_\eta = \eta F'(X - K\alpha_\eta) = \eta\bar{R} - c\alpha_\eta.$$  \hspace{1cm} (3.3)

Thereby, the expected marginal return on investment in the BS is equal to $\eta\bar{R} - c\alpha_\eta$ due to the following reasoning: At $t = 1$, the amount of capital invested in the BS is equal to $k(1 + \alpha_\eta) =: a$. The expected return on investment is $k(1 + \alpha_\eta)\eta\bar{R} - \frac{c}{2}\alpha_\eta^2k =: f(a)$. Thus, to find the marginal expected return on investment in the BS, we need to differentiate $f(a)$ with respect to $a$. Note that $\alpha_\eta = \frac{\alpha}{k} - 1$. Hence,

$$f(a) = a\eta\bar{R} - \frac{c}{2}(\frac{a}{k} - 1)^2k$$

$$\Rightarrow \frac{df(a)}{da} = \eta\bar{R} - \frac{c}{2}2(\frac{a}{k} - 1)\frac{1}{k}k = \eta\bar{R} - c\alpha_\eta.$$  \hspace{1cm}

On the other side, the investment in the BS at $t = 0$ should have the same expected marginal returns on capital as the investment in the TS. Thus,

$$\mathbb{E}[p_\eta] = \mathbb{E}[\eta\bar{R}(1 + \alpha_\eta) - \frac{c}{2}\alpha_\eta^2 - \alpha_\eta p_\eta].$$  \hspace{1cm} (3.4)

From equalities (3.3) and (3.4) and using the fact that $c > 0$, after simplifications, we

\[^6\] If $c = 0$, we do not obtain equation (3.4). Moreover, if $c = 0$, we do not obtain a unique equilibrium.
obtain the same result as Gersbach and Rochet (2012A):

\[ \mathbb{E}[\alpha^2 + 2\alpha \eta] = 0. \]  

(3.5)

We note that equation (3.5) is equivalent to

\[ \sigma^2 + \bar{\alpha}^2 + 2\bar{\alpha} = 0, \]

where \( \bar{\alpha} := \mathbb{E}[\alpha] = q\alpha_h + (1 - q)\alpha_l \) and \( \sigma^2 = q(1 - q)(\alpha_h - \alpha_l)^2 \) is the variance of \( \alpha \).

Using the boundary conditions \( \alpha_l, \alpha_h \geq -1 \), we conclude that

\[ \bar{\alpha} = \sqrt{1 - \sigma^2} - 1 \]

and

\[ \alpha_h = \sqrt{1 - \sigma^2} - 1 + \sigma \sqrt{\frac{1 - q}{q}} \]

(3.6)

\[ \alpha_l = \sqrt{1 - \sigma^2} - 1 - \sigma \sqrt{\frac{q}{1 - q}}. \]

(3.7)

Note that (3.7) implies that \( \alpha_l < 0 \).

We will assume for the rest of the paper that if the banker exerts effort, the banking technology is profitable even when capital adjustment is not possible. Since returns on investment in the BS are given by \( m\bar{R} \), and since we have normalized \( m \) to 1, we can state this assumption as follows:

**Assumption 3.1**

\( \bar{R} > 1. \)

Assumption 3.1 implies that in the first-best allocation, it holds that \( C_0 = 0 \) and \( r > 0 \). Equation (3.3), together with the condition \( 1 + r = \mathbb{E}[p_{\eta}] \), allow to determine the value of \( r \):

\[ 1 + r = \mathbb{E}[\eta F'(X - K\alpha_{\eta})] = \mathbb{E}[\eta \bar{R} - c\alpha_{\eta}]. \]

\( C_0 = 0 \) implies that \( X + K = 1 \). Thus, \( X = 1 - K \). Hence, we obtain the condition on \( r \):

\[ 1 + r = \mathbb{E}[\eta F'((1 - K(1 + \alpha_{\eta})))] = \mathbb{E}[\eta \bar{R} - c\alpha_{\eta}]. \]

(3.8)

So far, we have only examined conditions which have to hold on the expected returns. In equilibrium, however, the supply and demand for capital also have to match. We focus now on the case where \( C_0 = 0 \). For a given price \( p \), we define the supply of capital \( S_{\eta}(p) \) for the BS if the shock \( \eta \) occurs through

\[ \eta F'(1 - S_{\eta}(p)) = p, \]

(3.9)
where \(1 - S_\eta(p)\) is the amount of capital, that is invested in the TS. \(S_\eta(\cdot)\) is an increasing function with \(S_\eta(0) = 0\) and \(\lim_{p \to \infty} S_\eta(p) = 1\). This is economically reasonable, as the following explanation shows: A very high value of \(p\) – the price for capital at which entrepreneurs in the TS are willing to buy a unit of capital – can only occur if a very small share of capital is invested in the TS, and thus if the marginal productivity in the TS is very high. Hence, it must hold that a very big share of capital is invested in the BS. The higher the price \(p\), the closer this share is to 1. The monotonic increase of \(S_\eta(\cdot)\) means that the higher the price of capital at which banks are ready to sell capital to entrepreneurs in the TS, the less capital is bought by entrepreneurs at this price, and, consequently the more capital is invested in the BS. Finally, \(S_\eta(0) = 0\) means that if bankers are ready to give capital to entrepreneurs in the TS for free, the entrepreneurs will take the entire capital, and there will be no capital left to invest in the BS. Note, however, that in this paper, as in Gersbach and Rochet (2012A), only those situations are examined in which \(p \geq 1\). If \(p < 1\), investors would consume all their endowment at \(t = 0\). Also note that in the case \(C_0 = 0\), the function values \(S_\eta(p)\) do not depend on any other equilibrium values than \(p\).

In Gersbach and Rochet (2012A), it holds that \(S(p) = 1 - (F')^{-1}(\cdot)\). In our set-up, it holds that \(S_\eta(p) = 1 - (F')^{-1}(\eta)\). Since \(F'(\cdot)\) is a strictly decreasing function, \((F')^{-1}(\cdot)\) is strictly decreasing. Therefore, \(1 - (F')^{-1}(\eta) < 1 - (F')^{-1}(p) < 1 - (F')^{-1}(\xi)\) and

\[
S_h(p) < S(p) < S_l(p). 
\]

The relation \(S_h(p) < S_l(p)\) may seem somewhat unintuitive, but in fact, it is not. In fact, this relation only means that if the capital price is kept fixed exogenously, then there is more investment in the TS in the state \(\eta = h\) than in the state \(\eta = l\). The functions \(S_\eta(\cdot)\) describe the supply of capital for the BS, which is only determined by the demand for capital in the TS. Note that (3.10) does not imply that in equilibrium if the state \(\eta = h\) occurs, less capital is invested in the BS than if the state \(\eta = l\) occurs. Indeed, in equilibrium, the price \(p\) is an endogenous variable, and not fixed exogenously. Put differently, (3.10) does not imply that \(S_h(p_h) < S_l(p_l)\).

Equation (3.3) means that

\[
K(1 + \alpha_\eta) = S_\eta(\eta \bar{R} - c\alpha_\eta).
\]

Hence,

\[
K = \frac{S_h(h \bar{R} - c\alpha_h)}{1 + \alpha_h} = \frac{S_l(l \bar{R} - c\alpha_l)}{1 + \alpha_l}.
\]

This equation determines \(K\) and inserting the expressions (3.6) and (3.7) determines a
unique solution for $\sigma$, which consequently determines unique values for $K, \alpha_h, \alpha_l, r, p_h, p_l$. This allocation is Pareto optimal, as it maximizes the aggregate social welfare given by (3.1). We obtain the following proposition:

**Proposition 3.1**

*In the economy without frictions, there is a unique competitive equilibrium which is also Pareto optimal. The equilibrium is determined by the equations (3.5), (3.8), (3.9) and (3.11).*

Moreover, the following proposition holds:

**Proposition 3.2**

*In the economy without frictions, the unique competitive equilibrium is given by

\[
\begin{align*}
\alpha_\eta &= 0 \text{ for } \eta = h, l, \quad (3.12) \\
K &= 1 - (F')^{-1}(\bar{R}), \quad (3.13) \\
p_\eta &= \eta \bar{R} \text{ for } \eta = h, l \text{ and } \quad (3.14) \\
1 + r &= \bar{R}. \quad (3.15)
\end{align*}
\]

**Proof.** Inserting the values $\alpha_\eta = 0$ into equation (3.11), (3.9) and (3.8) yields the values for $K, p_\eta$ and $r$. Clearly, thereby, equation (3.5) holds. Thus, the obtained values comprise an equilibrium. By Proposition 3.1, this equilibrium is unique. \(\square\)

Proposition 3.2 yields a deeper understanding of the nature of the capital reallocations in the basic model in Gersbach and Rochet (2012A). In this model, capital reallocations occur due to the interaction of two effects: On the one hand, as the BS is hit by a shock while the TS is not, it is natural that there should be some capital reallocations between the sectors after the observance of the shock. In the absence of moral hazard, in particular, capital reallocations also occur. Of course, the reason for the excessive reallocations is the presence of moral hazard. However, as no analytical solutions are considered, it is hard to identify those very components of the reallocations that are due to moral hazard. In our extension comprising Macroshocks, the fact that moral hazard generates excessive reallocations is illustrated in a simpler way, as by Proposition 3.2, there are no reallocations in the absence of moral hazard. Thus, all the reallocations in the equilibrium that we will find in Section 3.2.4 and which cause the inefficiency of this equilibrium are due to moral hazard.
The Rigid Economy

We now consider the same model with moral hazard, as in Subsection 3.2.1, but remove the possibility to reallocate capital at \( t = 1 \). So, at this period, the shock \( \eta \) is observed, and nothing else happens. The following analysis of this set-up is very similar to the corresponding analysis in Gersbach and Rochet (2012A). As we will see, the reason for this similarity is the property that \( \mathbb{E}[\eta] = 1 \).

So assume that it is exogenously-given that \( \alpha_\eta = 0 \) for \( \eta = h, l \). As in Gersbach and Rochet (2012A), we make the following assumption for the rest of the paper:

**Assumption 3.2**

(i) \( \bar{R} + B - \Delta(R^S - R^F) < 1 \).

(ii) \( l\Delta(R^S - R^F) - B \geq 0 \).

The first part of Assumption 3.2 states that if a banker does not exert effort, investment in banking activity is not profitable. The second part of this assumption states that shirking is not socially desirable.

The same results as in Gersbach and Rochet (2012A), (p. 84-85) hold:

- “The optimal contract between a banker and investors specifies a bonus payment \( \frac{B}{\Delta}k \) when \( R^S \) is realized and nothing when \( R^F \) occurs. This causes the banker to exert effort and his expected payment per unit of assets is \( b := \frac{\tau B}{\Delta} \).

- The optimal size of the bank is the maximal value of \( k \) that satisfies the participation constraint of investors \( (k - e)(1 + r) \leq k(\bar{R} - b) \).

- Investors in the market impose the capital ratio \( \frac{e}{k} \geq \rho_0(r) := 1 - \frac{\bar{R} - b}{1 + r} \).

- Regulatory intervention regarding the size of banks is socially undesirable. Indeed, the only thing a regulator could do is reduce \( k \) (recall that transfers are ruled out) with respect to the level imposed by investors. Since \( \bar{R} > 1 + r \), this would decrease social welfare.”

Furthermore, as in Gersbach and Rochet (2012A), we also make the following assumption for the rest of the paper about the severity of moral hazard:

**Assumption 3.3**

\( \bar{R} < 1 + b \).
A proposition analogous to Proposition 2 in Gersbach and Rochet (2012A) also holds in our set-up. Namely

**Proposition 3.3**

The rigid economy has a unique equilibrium, characterized as follows:

- When \( E < E^* = S(1)\rho_0(0) \): \( K = \frac{E}{\rho_0(0)} \), \( X = 1 - S(1) - C_0, C_0 > 0, r = 0 \),
- when \( E^* \leq E \leq E^{**} = \frac{b}{R} S(\bar{R}) \): \( K = \frac{E}{\rho_0(r)} = S(1 + r), C_0 = 0, r > 0 \),
- when \( E > E^{**} \): \( K = S(\bar{R}) = 1 - X, C_0 = 0 \).

The main part of the proof is given in Gersbach and Rochet (2012A). In our set-up, however, one detail has to be clarified. At the beginning of the proof of Proposition 2 in Gersbach and Rochet (2012A) (p. 97), the condition

\[
\frac{E}{\rho_0(r)} = K = S(1 + r)
\]

is stated. The first identity follows exactly as in Gersbach and Rochet (2012A). The second identity requires some clarification. Note that although in the rigid economy, there is no trade and hence no capital price at \( t = 1 \), the supply of capital is still equal to \( S(1 + r) \), as if \( 1 + r \) were the capital price. This can be explained as follows: In the framework of Gersbach and Rochet (2012A) it has to hold in equilibrium that

\[
F'(X) = 1 + r.
\]

But since \( X = 1 - C_0 - K \), this implies

\[
F'(1 - C_0 - K) = 1 + r.
\]

In the rigid case, in it has to hold in equilibrium in the presence of Macroshocks that

\[
E[\eta F'(X)] = 1 + r
\]

\[
\Rightarrow F'(X) = 1 + r.
\]

Thus, we also obtain

\[
K = S(1 + r).
\]

Having this clarification in mind, the proof of the proposition is the same as in Gersbach and Rochet (2012A).
Finally, note that \( \mathbb{E}[\eta] = 1 \) is the reason why the corresponding section in Gersbach and Rochet (2012A) implies the same allocation as in our set-up. Thus, even if we change the productivity of the TS by factor \( \eta \) (we have \( \eta F'(X) \) instead of \( F'(X) \)), in the rigid economy – where only the expected value \( \eta \) determines the actions, as when \( \eta \) is realized at \( t = 1 \), no actions can be undertaken –, we obtain the same results as Gersbach and Rochet (2012A). Put differently, \( \mathbb{E}[\eta] = 1 \) implies that our set-up and the set-up in Gersbach and Rochet (2012A) are equivalent in terms of decision-making at \( t = 0 \).

### 3.2.3 Privately-Optimal Contracts

Let us now leave the benchmark cases from the previous section and return to our general model described in Section 3.2.1. This section corresponds entirely to Section 4 in Gersbach and Rochet (2012A), so we just provide their result, which we are going to use:

**Proposition 3.4**

"Given the vector \( p = (p_h, p_l) \) of capital prices and the banker’s initial wealth \( e \), the optimal banking contract is characterized by

\[
k(p, e) = \frac{e}{\rho(p)}, \quad \text{where}
\]

\[
\rho(p) = 1 - \mathbb{E}[(1 + \alpha_h(p))(\eta \bar{R} - b) - \frac{c}{2}\alpha^2_h(p) - p\eta\alpha_h(p)]
\]

and \( \alpha_h(p), \alpha_l(p) \) maximize

\[
\frac{b\mathbb{E}[1 + \alpha_h(p)]e}{1 - \mathbb{E}[(1 + \alpha_h(p))(\eta \bar{R} - b) - \frac{c}{2}\alpha^2_h(p) - p\eta\alpha_h(p)]}.
\]

Note that the optimal contract is implementable in our set-up in exactly the same way as in Gersbach and Rochet (2012A) (Section 4.3), where, in particular, short-term debt is used to force a banker to reduce the size of his bank by the amount \( k\alpha_l \) if state \( l \) occurs.

### 3.2.4 Competitive Equilibrium

**Properties**

In the following, the variables \( \alpha_h(p), \alpha_l(p), k, \rho(p) \) correspond to the values from Proposition 3.4.
Properties of Competitive Equilibria
As in Gersbach and Rochet (2012A), a competitive equilibrium in the economy with capital reallocation and moral hazard is an array \( \Sigma = \{ C_0, C_{2h}, C_{2l}, K, X, \alpha_h, \alpha_l, p_h, p_l \} \) such that

(i) a banker endowed with \( e \) receives \( k - e \) from investors, where \( k = \frac{e}{\rho(p)} \), at \( t = 0 \), the total capital invested in the BS amounts to \( K = \frac{E}{\rho(p)} \);

(ii) after state \( h \) (respectively state \( l \)) has been observed at \( t = 1 \), the fraction \( \alpha_h(p) \) (respectively \( \alpha_l(p) \)) of capital in the BS is reallocated to the TS;

(iii) \( p_{\eta} = \eta F'(X - \alpha_{\eta}(p)K), \eta \in \{h, l\} \);

(iv) \( E[p_{\eta}] = q p_h + (1 - q) p_l = 1 + r \);

(v) \( K + X + C_0 = 1 \);

(vi) \( C_{2\eta} = K ((1 + \alpha_{\eta})\eta \bar{R} - \frac{\xi}{2} \alpha_{\eta}^2) + \eta F(X - \alpha_{\eta}K) \);

(vii) \( C_0 \geq 0, K \geq 0, X \geq 0 \);

(viii) \( r C_0 = 0 \).

Existence and Uniqueness
In the socially-optimal case, as we have seen in Section 3.2.2, it holds that \( r = R - 1 > 0 \). In the next proposition, we characterize a competitive equilibrium, in which \( r = 0 \) and \( C_0 > 0 \) holds:

Proposition 3.5
The set of interior competitive equilibria which incorporate \( r = 0 \) is a singleton. The variance of credit growth \( \sigma^2 = q(1 - q)(\alpha_h - \alpha_l)^2 \) may be used to parametrize it. The only such competitive equilibrium is characterized by

- \( \sigma^E \) is the unique solution of \( G(\sigma) = E \),
- \( \alpha^E = \sqrt{1 - (\sigma^E)^2} - 1 \),
- \( p^E_h = (h - 1)\bar{R} + 1 - c\sigma^E \sqrt{\frac{q}{1 - q}}, p^E_l = (l - 1)\bar{R} + 1 - c\sigma^E \sqrt{\frac{q}{1 - q}} \), and
- \( \frac{E}{K^E} = \sqrt{1 - (\sigma^E)^2(1 - \bar{R} + b + c\sqrt{1 - (\sigma^E)^2} - c)} \).
3.2 Macroshocks

Thereby $G(\sigma)$ is a function given by

$$G(\sigma) := \sqrt{q(1-q)(\frac{1}{\sigma^2} - 1)(1 - \bar{R} + b + c\sqrt{1 - \sigma^2} - c)} \cdot \left\{ S_h \left((h - 1)\bar{R} + 1 - c\sigma\sqrt{\frac{1-q}{q}}\right) - S_l \left((l - 1)\bar{R} + 1 - c\sigma\sqrt{\frac{q}{1-q}}\right) \right\}.$$

**Proof.** We follow the logic of the corresponding proof in Gersbach and Rochet (2012A). The following optimization problem is solved by the banks in equilibrium:

$$\max_{K,\alpha_h,\alpha_l} KbE[1 + \alpha_h]$$
subject to

$$K E[(1 + \alpha_h)(\eta\bar{R} - b) - \frac{c}{2}\alpha_h^2 - p_\eta\alpha_h] = K - E.$$

Let $L$ be the corresponding Lagrangian, in which $\nu$ denotes the multiplier. In the solution of the optimization problem, it has to hold that

$$\frac{\partial L}{\partial K} = bE[1 + \alpha_h] + \nu E[(1 + \alpha_h)(\eta\bar{R} - b) - \frac{c}{2}\alpha_h^2 - p_\eta\alpha_h] - \nu = 0,$$

$$\frac{\partial L}{\partial \alpha_h} = K Pr(\eta)[b + \nu(\eta\bar{R} - b - c\alpha_h - p_\eta)] = 0,$$

where $Pr(h) = q$ and $Pr(l) = 1 - q$.

It is straightforward that in equilibrium it has to hold $K > 0$. Thus, the second condition implies $\nu \neq 0$. Furthermore, the second condition implies

$$p_\eta = \eta\bar{R} - b - c\alpha_h + \frac{b}{\nu}. \quad (3.19)$$

From this equation and the first condition, we obtain

$$\frac{b}{\nu} = 1 - \bar{R} + b - \frac{c}{2}E[\alpha_h^2]. \quad (3.20)$$

With Assumption 3.1, this implies that $\nu > 1$ or $\nu < 0$. Furthermore, with Assumption 3.3, this implies – in analogy to Gersbach and Rochet (2012A) – that $\nu > 1$,\(^7\) which will be used in the proofs of forthcoming propositions.

\(^7\)For this implication to hold, we need to assume that $c$ is sufficiently small. In Gersbach and Rochet (2012A) the assumption that $c$ is sufficiently small compared to the returns on investment in the BS is stated on page 85 (footnote 12) and used on page 99 after equation (B6) to derive $\nu > 1$. For the rest of the paper we also assume that $c > 0$ is sufficiently small. This allows to compare our results with those of Gersbach and Rochet (2012A). Note that $c > 0$ is a natural and important restriction that avoids arbitrary shifting of capital in period 1. This restriction is necessary to obtain a unique equilibrium.
All following steps in the proof are identical to the ones of Gersbach and Rochet (2012A), namely all steps from equation (B7) to equation (B14) in Gersbach and Rochet (2012A), so we omit them and just state the resulting parametrization of the prices and the capital adjustments. We obtain

\[\begin{align*}
\alpha^E_h &= \sqrt{1 - \sigma^2} - 1 + \sigma \sqrt{\frac{1-q}{1-q}} \\
\alpha^E_l &= \sqrt{1 - \sigma^2} - 1 - \sigma \sqrt{\frac{q}{1-q}}
\end{align*}\] (3.21)

and

\[\begin{align*}
p^E_h &= (h - 1)\bar{R} + 1 - c\sigma \sqrt{\frac{1-q}{q}} \\
p^E_l &= (l - 1)\bar{R} + 1 + c\sigma \sqrt{\frac{q}{1-q}}.
\end{align*}\] (3.22)

Hence, \(E[\alpha^E_\eta] = \sqrt{1 - \sigma^2} - 1\).

We continue at equation (B14) in Gersbach and Rochet (2012A). This equation is the equilibrium condition that states that the price of capital in state \(\eta\) has to be equal to the returns from investment in the TS, given that \(X - \alpha_\eta K\) is invested in the TS. We start from this step with the proof of the proposition. In our case, this equilibrium condition reads

\[p_\eta = \eta F'(X - \alpha_\eta K), \eta = h, l,
\]

where \(K\) satisfies

\[\frac{E}{K} = \sqrt{1 - \sigma^2}(1 - \bar{R} + b + c\sqrt{1 - \sigma^2} - c).\] (3.23)

Therefore, we obtain two conditions on the capital supply:

\[\begin{align*}
1 - S_h(p_h) - C_0 &= X - \alpha_h K, \\
1 - S_l(p_l) - C_0 &= X - \alpha_l K.
\end{align*}\]

Eliminating \(X + C_0\) yields

\[(\alpha_h - \alpha_l)K = S_h(p_h) - S_l(p_l).\] (3.24)

Inserting the expressions for \(\alpha_h, \alpha_l, K, p_h, p_l\) in dependence of \(\sigma\) we obtain

\[\begin{align*}
\frac{1}{\sqrt{q(1-q)}} \frac{E\sigma}{\sqrt{1 - \sigma^2(1 - \bar{R} + b + c\sqrt{1 - \sigma^2} - c)}} &= S_h \left( (h - 1)\bar{R} + 1 - c\sigma \sqrt{\frac{1-q}{q}} \right) - S_l \left( (l - 1)\bar{R} + 1 + c\sigma \sqrt{\frac{q}{1-q}} \right).\end{align*}\] (3.25)
Note that this equation implies that $\alpha_l = \alpha_h$ cannot hold in equilibrium, as we will show now. Suppose $\alpha_l = \alpha_h$. Then, it holds $\sigma = 0$. Then, equation (3.25) implies

$$0 = S_h((h - 1)\bar{R} + 1) - S_l((l - 1)\bar{R} + 1).$$

This, together with the definition of $S_\eta(\cdot)$, yields

$$1 - (F')^{-1}\left(\frac{(h - 1)\bar{R} + 1}{h}\right) = 1 - (F')^{-1}\left(\frac{(l - 1)\bar{R} + 1}{l}\right)$$

$$\Leftrightarrow \frac{(h - 1)\bar{R} + 1}{h} = \frac{(l - 1)\bar{R} + 1}{l}$$

$$\Leftrightarrow \bar{R}(h - l) = h - l$$

$$\Leftrightarrow \bar{R} = 1,$$

which is a contradiction.

Equation (3.25) is equivalent to

$$E = \sqrt{q(1 - q)}\frac{1}{\sigma^2} - 1)(1 - \bar{R} + b + c\sqrt{1 - \sigma^2} - c)\cdot$$

$$\left\{S_h\left( (h - 1)\bar{R} + 1 - c\sigma\sqrt{\frac{1 - q}{q}} \right) - S_l\left( (l - 1)\bar{R} + 1 + c\sigma\sqrt{\frac{q}{1 - q}} \right) \right\}.$$

(3.26)

We consider the right-hand side of this equation as a function of $\sigma$ and denote it by $G(\sigma)$. This function differs from the corresponding function $H(\sigma)$ in Gersbach and Rochet (2012A) only in the capital supply functions $S_\eta$. Note that the function $G(\sigma)$ is strictly decreasing on $(0, 1)$ and continuous on $(0, 1)$ with $\lim_{\sigma \to 0} G(\sigma) = \infty$ and $\lim_{\sigma \to 1} G(\sigma) = 0$. As these are the only properties of $H(\sigma)$ that are used in the last part of the proof in Gersbach and Rochet (2012A), the rest of the proof in our set-up is completely analogous to the one in Gersbach and Rochet (2012A), replacing the function $H(\sigma)$ by $G(\sigma)$. □

**Corollary 3.1**

*It holds that $\alpha^E_l < 0$, $\alpha^E_h > 0$ and $E[\alpha^E_\eta] < 0$.***

**Proof.** Since $\sigma \neq 0$, as shown in the proof of Proposition 3.5, equation (3.21) implies that $\alpha^E_l < 0$. On the other hand, $\sigma \neq 0$ implies $E[\alpha^E_\eta] = \sqrt{1 - \sigma^2} - 1 < 0$. Since $\alpha^E_l < 0$ and $\alpha^E_h < 0$ cannot occur simultaneously in an equilibrium, it holds $\alpha^E_h > 0$. □

Finally, note that corollaries analogous to Corollaries 1, 2 and 3 in Gersbach and Rochet (2012A) also hold in our set-up.
Explanation of the Occurrence of Non-zero Capital Adjustments under Macroshocks

When the shock only hits the BS as in Gersbach and Rochet (2012A), the occurrence of capital adjustments between the sectors at \( t = 1 \) is intuitive. Indeed, since the TS is not hit by a shock, its performance improves compared to the BS in the case of a negative shock, and vice-versa in the case of a positive shock. However, in the case of Macroshocks, the described intuition for the occurrence of non-zero capital adjustments does not work. Indeed, the intuition why Corollary 3.1 holds in the presence of Macroshocks is more subtle. Any investment in the BS incorporates an information rent \( b \) per unit of investment, which the investor pays to the banker. If thereby the returns from this investment are high, then the information rent constitutes only a small fraction of the returns from one unit of investment. However, if the returns from this investment are low, then the information rent constitutes a big fraction of the returns from one unit of investment. On the other hand, in the TS there is no information rent that is subtracted from the return per unit investment. Hence, the return per unit of investment for the investors in the BS is more sensitive to the Macroshock than is the return per unit of investment in the TS. This effect feeds into the formation of the competitive equilibrium and leads to non-zero capital adjustments. The core of the above intuition states that the moral hazard causes an asymmetry between the sectors, although both of them are hit by the same shock, and this asymmetry leads to capital adjustments depending on the kind of the shock. Note that this statement is supported by the finding in Section 3.2.2, which states that in the case of Macroshocks and in the absence of moral hazard, there are no capital adjustments indeed.

The above intuition suggests that the capital adjustments in the case of Macroshocks should be less pronounced than in the case of shocks that hit only the BS. Indeed, comparing our function \( G(\sigma) \) with the corresponding function \( H(\sigma) \) in Gersbach and Rochet (2012A), and using inequality (3.10), we find that

\[
G(\sigma) < H(\sigma), \forall \sigma \in (0, \sqrt{1-q}).
\]

Since both functions are decreasing, we obtain that \( \sigma_{EMacroshocks}^E < \sigma_{EBS-shocks}^E \), where the index Macroshocks denotes the results in this paper and the index BS-shocks denotes the results in Gersbach and Rochet (2012A). This implies that \( E[\alpha_{EMacroshocks}^E] > E[\alpha_{EBS-shocks}^E] \). Therefore, \( |E[\alpha_{EMacroshocks}^E]| < |E[\alpha_{EBS-shocks}^E]| \). So in accordance with the above intuition, the absolute value of the mean and the variance of the adjustments are smaller indeed in the case of Macroshocks. In particular, the value of \( \alpha_{EMacroshocks}^E \) is higher in the case of Macroshocks.

Finally, to confirm the above intuition, let us consider a modification of the model with
Macroshocks in which the severity of moral hazard in the BS also is affected by the shock. In what follows, we construct a model in which the relative size of the information rents does not change depending on the shock. If in this model there were non-zero capital adjustments in equilibrium, this would show that our provided intuition is false or incomplete. If, on the other side, there will not occur any capital adjustments in equilibrium in our model, this will confirm the above intuition analytically.

We first describe the modification of the model. Assume that the leisure benefit is $B_l$ in state $l$ and $B_h$ in state $h$. Furthermore, assume that $\mathbb{E}[B_\eta] = B$ and let $b_\eta := \frac{\tau B_\eta}{\Delta}$. Then, the following optimization problem is solved by the banks in equilibrium:

$$\max_{K,\alpha_l,\alpha_l} K\mathbb{E}[b_\eta(1 + \alpha_\eta)],$$

subject to

$$K\mathbb{E}[(1 + \alpha_\eta)(\eta \overline{R} - b_\eta) - \frac{c}{2} \alpha_\eta^2 - \eta p_\eta \alpha_\eta] = K - E.$$ 

Let $\mathcal{L}$ denote the Lagrangian and $\gamma$ denote the multiplier. Hence, the first order conditions read

$$\frac{\partial \mathcal{L}}{\partial K} = \mathbb{E}[b_\eta(1 + \alpha_\eta)] + \gamma [\mathbb{E}[(1 + \alpha_\eta)(\eta \overline{R} - b_\eta) - \frac{c}{2} \alpha_\eta^2 - \eta p_\eta \alpha_\eta] - 1] = 0, \quad (3.27)$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_\eta} = K \cdot Pr(\eta)(b_\eta + \gamma (\eta \overline{R} - b_\eta - c\alpha_\eta - p_\eta)) = 0. \quad (3.28)$$

The second condition implies

$$(b_\eta + \gamma(\eta \overline{R} - b_\eta - c\alpha_\eta - p_\eta)) = 0 \quad (3.29)$$

$$\Rightarrow p_\eta = \frac{b_\eta}{\gamma} + \eta \overline{R} - b_\eta - c\alpha_\eta. \quad (3.30)$$

Inserting this into the first condition yields

$$\frac{\partial \mathcal{L}}{\partial K} = \mathbb{E}[b_\eta(1 + \alpha_\eta)] + \gamma [\mathbb{E}[(1 + \alpha_\eta)(\eta \overline{R} - b_\eta) - \frac{c}{2} \alpha_\eta^2 - (\frac{b_\eta}{\gamma} + \eta \overline{R} - b_\eta - c\alpha_\eta)\alpha_\eta] - 1] = 0.$$ 

Therefore,

$$\mathbb{E}[b_\eta] + \gamma [\mathbb{E}[\eta \overline{R} - b_\eta + \frac{c}{2} \alpha_\eta^2] - 1] = 0$$

$$\Leftrightarrow \frac{b}{\gamma} + \overline{R} - b + \mathbb{E}[\frac{c}{2} \alpha_\eta^2] - 1 = 0$$

$$\Leftrightarrow \frac{b}{\gamma} = 1 - \overline{R} + b - \mathbb{E}[\frac{c}{2} \alpha_\eta^2] \quad (3.31)$$

$$\Leftrightarrow \gamma = \frac{b}{1 - \overline{R} + b - \mathbb{E}[\frac{c}{2} \alpha_\eta^2]}. \quad (3.32)$$
With Assumption 3.1, this implies that $\gamma > 1$ or $\gamma < 0$. Furthermore, with Assumption 3.3, this implies that $\gamma > 1$.

Equation (3.30) and the equilibrium condition $E[p_\eta] = 1$ imply

$$1 = E\left[\frac{b_\eta}{\gamma} + \eta R - b_\eta - c\alpha_\eta\right]$$

$$= \frac{b}{\gamma} + R - b - cE[\alpha_\eta]. \tag{3.33}$$

With equation (3.31), this implies

$$1 = (1 - R + b - E[\frac{c}{2}\alpha^2_\eta]) + R - b - cE[\alpha_\eta]$$

$$= 1 - E[\frac{c}{2}\alpha^2_\eta] - cE[\alpha_\eta].$$

Thus, if we denote the variance of $\alpha_\eta$ by $\sigma^2$, we obtain the following parametrization of the capital adjustments in complete analogy to equation (3.21):

$$\begin{cases}
\alpha_h = \sqrt{1 - \sigma^2} - 1 + \sigma \sqrt{\frac{1-q}{1-q}} \\
\alpha_l = \sqrt{1 - \sigma^2} - 1 - \sigma \sqrt{\frac{1-q}{1-q}}
\end{cases} \tag{3.34}$$

and

$$E[\alpha_\eta] = \sqrt{1 - \sigma^2} - 1, \tag{3.35}$$

$$\frac{1}{2}E[\alpha^2_\eta] = 1 - \sqrt{1 - \sigma^2}. \tag{3.36}$$

Using equations (3.30), (3.32), (3.34) and (3.36), we obtain

$$p_l = \frac{b_\eta}{\gamma} + lR - b_\eta - c\alpha_l$$

$$= \frac{b_\eta}{b} \left(1 - R + b - E[\frac{c}{2}\alpha^2_\eta]\right) + lR - b_\eta - c(\sqrt{1 - \sigma^2} - 1 - \sigma \sqrt{\frac{q}{1-q}})$$

$$= \frac{b_\eta}{b} \left(1 - R + b - c(1 - \sqrt{1 - \sigma^2})\right) + lR - b_\eta - c(\sqrt{1 - \sigma^2} - 1 - \sigma \sqrt{\frac{q}{1-q}}). \tag{3.37}$$

Analogously, we obtain

$$p_h = \frac{b_\eta}{b} \left(1 - R + b - c(1 - \sqrt{1 - \sigma^2})\right) + hR - b_\eta - c(\sqrt{1 - \sigma^2} - 1 + \sigma \sqrt{\frac{q}{1-q}}). \tag{3.38}$$
The investors’ participation constraint reads

$$\frac{E}{K} = 1 - \mathbb{E}[(1 + \alpha_n)(\eta R - b_n) - \frac{c}{2}\alpha_n^2 - p_n\alpha_n]$$

$$\iff \frac{E}{K} = 1 - q((1 + \alpha_h)(h R - b_h) - \frac{c}{2}\alpha_h^2 - p_h\alpha_h) - (1 - q)((1 + \alpha_l)(l R - b_l) - \frac{c}{2}\alpha_l^2 - p_l\alpha_l).$$

Inserting the values of \(p_l\) and \(p_h\) from equations (3.37) and (3.38) and the values of \(\alpha_l\) and \(\alpha_h\) from equation (3.34) yields, after some simplifications:

$$\frac{E}{K} = \sqrt{1 - \sigma^2(1 - \frac{1}{R} + b + c\sqrt{1 - \sigma^2} - c)} + \sigma\sqrt{q(1 - q)}\frac{b_h - b_l}{b}(1 - \frac{1}{R} + b + c\sqrt{1 - \sigma^2} - c).$$

(3.39)

Note that if \(b_h = b_l\), this equation has to coincide with equation (3.23), which it does indeed.

Now, assume, in addition, that \(B_l, B_h\) satisfy

$$\frac{B_l}{l R} = \frac{B_h}{h R}. \quad (3.40)$$

This means that we consider a situation in which for both shocks \(\eta = l, h\), the leisure benefits constitute the same fraction of the returns per unit of investment. Equation (3.40) implies

$$\frac{b_l}{l R} = \frac{b_h}{h R}. \quad (3.41)$$

Hence, for both shocks \(\eta = l, h\), the informational rents constitute the same fraction of the returns per unit of investment. Using \(\mathbb{E}[B_n] = B\) this implies

$$b_l = \frac{b}{q \cdot \frac{h}{l} + (1 - q)} = bl,$$

$$b_h = \frac{b}{q + (1 - q) \cdot \frac{l}{h}} = bh.$$

Inserting those values into equation (3.39) yields

$$\frac{E}{K} = \sqrt{1 - \sigma^2(1 - \frac{1}{R} + b + c\sqrt{1 - \sigma^2} - c)} + \sigma\sqrt{q(1 - q)}(h - l)(1 - \frac{1}{R} + b + c\sqrt{1 - \sigma^2} - c).$$

(3.42)
Finally, note that equation (3.24) from Section 3.2.4 holds in the current setting as well:

\[(\alpha_h - \alpha_l)K = S_h(p_h) - S_l(p_l).\]

Inserting the values of \(\alpha_l, \alpha_h, p_l, p_h\) in this equation yields, together with equation (3.42), the following equation for \(\sigma\):

\[
\frac{1}{\sqrt{q(1-q)}} E\sigma \sqrt{1-\sigma^2(1-\mathcal{R} + b + c\sqrt{1-\sigma^2} - c)} + \sigma\sqrt{q(1-q)(h-l)(1-\mathcal{R} + b + c\sqrt{1-\sigma^2} - c)} = S_h(h-c(1-h)(\sqrt{1-\sigma^2}-1) - c\sigma\sqrt{\frac{1-q}{q}}) - S_l(l-c(1-l)(\sqrt{1-\sigma^2}-1) + c\sigma\sqrt{\frac{q}{1-q}}).
\]

Inserting \(\sigma = 0\) yields

\[0 = S_h(h) - S_l(l).\]

The definition of \(S_\eta(\cdot)\) implies that \(S_h(h) = S_l(l)\). Therefore, we conclude that \(\sigma = 0\) solves equation (3.43). This implies \(\alpha_l = \alpha_l = 0\). Hence, no capital adjustments occur in the modified model. This confirms the correctness of our intuition about the occurrence of non-zero capital adjustments in the presence of Macroshocks.

### 3.2.5 Social Efficiency and Regulation

In this section we explore how and to which extent regulation may increase social welfare. We start from a competitive equilibrium with \(r = 0\). Note that social welfare is equal to

\[
U = K\mathbb{E}[\eta R(1+\alpha_\eta) - \frac{c}{2}\alpha_\eta^2] + \mathbb{E}[\eta F(X - \alpha_\eta K)] + 1 - K - X.
\]

**Two-sided Regulation of Capital Adjustments**

Let us briefly describe what we mean by two-sided regulation of capital adjustments and then show that social welfare can be increased by it. According to their contracts with investors, bankers adjust their capital at \(t = 1\) after observing the value of \(\eta\). By *two-sided regulation of capital adjustments by* \((y, z) \in [-1 - \alpha^E_l, \infty) \times [-1 - \alpha^E_h, \infty)\), we mean regulatory thresholds on short-term debt, that set the following thresholds on capital adjustments at \(t = 1\): In state \(l\), the amount of shifted capital may not be smaller than \(k(\alpha^E_l + y)\). In state \(h\), the amount of shifted capital may not be bigger than \(k(\alpha^E_h + z)\).
As in Gersbach and Rochet (2012A), the implementation of the contracts implies that short-term debt regulation translates into regulation of capital adjustments. Note that after such a regulatory intervention, investors and bankers, in general, will reconsider their investments and contracts, which will lead to a change in capital prices.

Furthermore, note that regulation may increase social welfare even if it does not result in an increase of \( r \). To illustrate this, assume that the rate \( r = 0 \) is given. This means that investors obtain

\[
C_0 + (1 + r)(1 - E - C_0) = 1 - E,
\]

independently of the specific properties of the allocation of capital. However, under an allocation that incorporates \( C_0 = 1 - E \), bankers and entrepreneurs receive less than under an allocation with \( C_0 < 1 - E \). Hence, the value of \( r \) alone does not determine the value of the social welfare.

As the following proposition shows, two-sided regulation may increase social welfare:

**Proposition 3.6**

Assume the economy is in the competitive equilibrium with \( r = 0 \) and the allocation \((K^E, X^E, \alpha^E_H, \alpha^E_l)\). Then, there exist two numbers, denoted by \( \Delta \alpha_l \) and \( \Delta \alpha_h \) with \( \Delta \alpha_l > 0 \), such that two-sided regulation of capital adjustments by \((\Delta \alpha_l, \Delta \alpha_h)\) increases social welfare.

**Proof.** Our proof follows the proof of Proposition 5 in Gersbach and Rochet (2012A) closely. First, note that if \( \Delta \alpha_l > 0 \) and \( \Delta \alpha_h \) are chosen in an appropriate way, investors will continue to provide \( K^E - E \) at \( t = 0 \) to banks. Indeed, investors will do so if and only if

\[
1 - \frac{E}{K^E} = \mathbb{E}[(1 + (\alpha^E_\eta + \Delta \alpha_\eta))(\eta \bar{R} - b) - \frac{C}{2}(\alpha^E_\eta + \Delta \alpha_\eta)^2 - (p^E_\eta + \Delta p_\eta)(\alpha^E_\eta + \Delta \alpha_\eta)].
\] (3.44)

Obviously, for any \( \Delta \alpha_l > 0 \), there exists \( \Delta \alpha_h \) such that the above condition is fulfilled. Thereby \( |\Delta \alpha_l| \) and \( |\Delta \alpha_h| \) may be chosen arbitrarily small.

For \( \eta = h, l \), denote the equilibrium investment in the TS at \( t = 0 \), given adjustments \( \tilde{\alpha}_\eta \), by \( \tilde{X} \), and let \( \Delta X := \tilde{X} - X^E \). Further, let \( \tilde{p}_\eta \) denote the equilibrium capital price, given adjustments \( \tilde{\alpha}_\eta \), and let \( \Delta p_\eta := \tilde{p}_\eta - p^E_\eta \). The interventions \( \Delta \alpha_l \) and \( \Delta \alpha_h \) will impact the prices of capital and investment at \( t = 0 \) in the traditional sector in the following way: On the one hand, the supply of capital to the TS in both states at \( t = 1 \) will change compared to the competitive equilibrium due to the intervention \((\Delta \alpha_l, \Delta \alpha_h)\). Hence, the productivity of the TS will change – since \( F''(\cdot) > 0 \) –, and therefore, the capital prices will change. On the other hand, the expected returns on investment in the TS have to be
equal to 1, due to \( r = 0 \). This means, it has to hold that
\[
\mathbb{E}[\hat{p}_\eta] = 1
\]
\[\Rightarrow \mathbb{E}[p^E_\eta + \Delta p_\eta] = 1.\]

We know that \( \mathbb{E}[p^E_\eta] = 1 \). Hence, \( \mathbb{E}[\Delta p_\eta] = 0 \).

Contrary to Gersbach and Rochet (2012a), it holds that
\[p^E_\eta = \eta F'(X^E - \alpha^E_\eta K^E).\]

Hence, using the Taylor approximation, we obtain
\[
\begin{align*}
\hat{p}_\eta &= p^E_\eta + \Delta p_\eta = \eta F'(X^E - \alpha^E_\eta K^E) + \eta F''(X^E - \alpha^E_\eta K^E)(\Delta X - K^E \Delta \alpha_\eta) \\
\Rightarrow \Delta p_\eta &= \eta F''(X^E - \alpha^E_\eta K^E)(\Delta X - K^E \Delta \alpha_\eta) \\
\Rightarrow 0 &= \mathbb{E}[\Delta p_\eta] = \mathbb{E}[\eta F''(X^E - \alpha^E_\eta K^E)(\Delta X - K^E \Delta \alpha_\eta)].
\end{align*}
\]

Therefore,
\[
\Delta X = K^E \frac{\mathbb{E}[\eta F''(X^E - \alpha^E_\eta K^E)\Delta \alpha_\eta]}{\mathbb{E}[\eta F''(X^E - \alpha^E_\eta K^E)]}
\]
and
\[
\Delta p_\eta = K^E \eta F''(X^E - \alpha^E_\eta K^E)(\frac{\mathbb{E}[\eta F''(X^E - \alpha^E_\eta K^E)\Delta \alpha_\eta]}{\mathbb{E}[\eta F''(X^E - \alpha^E_\eta K^E)]} - \Delta \alpha_\eta). \tag{3.45}
\]

Note that \( \Delta X \) and \( \Delta p_\eta \) both converge to 0 as \( \Delta \alpha_h \) and \( \Delta \alpha_l \) converge to 0. Also note that \( \Delta X \) and \( \Delta p_\eta \) are uniquely determined, given \( \Delta \alpha_\eta \). Finally, note that \( \frac{\partial \Delta p_\eta}{\partial \Delta \alpha_\eta} < 0 \) and \( \frac{\partial \Delta p_\eta}{\partial \Delta \alpha_\eta} > 0 \) for \( \eta \neq \eta' \) for small enough values of \( \Delta \alpha_\eta \).

The investors’ participation constraint in equilibrium without regulation reads
\[
1 - \frac{E}{K^E} = \mathbb{E}[(1 + \alpha^E_\eta)(\eta R - b) - \frac{c}{2}(\alpha^E_\eta)^2 - p^E_\eta \alpha^E_\eta].
\]

Totally differentiating this equation with respect to \( \alpha^E_\eta \) and \( p^E_\eta \) yields
\[
\mathbb{E}[(\eta R - b - c\alpha^E_\eta - p^E_\eta)\Delta \alpha_\eta - \alpha^E_\eta \Delta p_\eta] = 0. \tag{3.46}
\]

Equation (3.19) reads
\[
p^E_\eta = \eta R - b - c\alpha^E_\eta + \frac{b}{\nu} \tag{3.47}
\]
and we have derived that $\nu > 1$. Hence,

$$\eta R - b - c\alpha^E_n - p^E_n = -\frac{b}{\nu} < 0.$$ 

Together with equation (3.46), this implies

$$\mathbb{E}\left[\frac{b}{\nu} \Delta \alpha_n - \alpha^E_n \Delta p_n\right] = 0$$

$$\Leftrightarrow \mathbb{E}[\Delta \alpha_n] = -\frac{\nu}{b} \mathbb{E}[\alpha^E_n \Delta p_n].$$

The impact on social welfare, if we omit terms of high order, is

$$\Delta U = K^E \mathbb{E}\left[\left(\eta R - c\alpha^E_n\right) \Delta \alpha_n\right] + \mathbb{E}[\eta F'(X^E - \alpha^E_n K^E) \cdot (\Delta X - K^E \cdot \Delta \alpha_n)] - \Delta X$$

$$= K^E \mathbb{E}\left[\left(\eta R - c\alpha^E_n - \eta F'(X^E - \alpha^E_n K^E)\right) \Delta \alpha_n\right] + \mathbb{E}[\eta F'(X^E - \alpha^E_n K^E) - 1] \Delta X$$

$$= K^E \mathbb{E}\left[\left(\eta R - c\alpha^E_n - p^E_n\right) \Delta \alpha_n\right] + \mathbb{E}[p^E_n - 1] \Delta X$$

$$= K^E \mathbb{E}\left[\left(\eta R - c\alpha^E_n - p^E_n\right) \Delta \alpha_n\right].$$

Now, equation (3.47) implies

$$\eta R - c\alpha^E_n = p^E_n + \frac{b}{\nu}.$$ 

Thus,

$$\Delta U = K^E b \left(1 - \frac{1}{\nu}\right) \mathbb{E}[\Delta \alpha_n]$$

$$= -K^E b \left(1 - \frac{1}{\nu}\right) \mathbb{E}[\alpha^E_n \Delta p_n]$$

$$= K^E \left(1 - \nu\right) \mathbb{E}[\alpha^E_n \Delta p_n].$$

Since $\nu > 1$, it remains to verify that $\mathbb{E}[\alpha^E_n \Delta p_n] < 0$. This can be done in two steps, which are described in Gersbach and Rochet (2012A) at the end of the proof of Proposition 5. In the first step, it is shown, that $\Delta p_l > 0$ and $\Delta p_h < 0$, by assuming the contrary and deriving a contradiction to equation (3.48). In the second step, using $\alpha^E_l < 0$ and $\alpha^E_h > 0$, it is inferred that $\mathbb{E}[\alpha^E_n \Delta p_n] < 0$. □

**One-sided Regulation of Capital Adjustments**

Let us now briefly describe what we mean by one-sided regulation of capital adjustments and then explore how it may impact social welfare.

According to their contracts with investors, bankers adjust their capital at $t = 1$ af-
ter observing the value of \( \eta \). By one-sided regulation of capital adjustments by \( y \in [-1 - \alpha^E, \infty) \), we mean regulatory thresholds on short-term debt that induce the following threshold on capital adjustments at \( t = 1 \) in state \( l \): the amount of shifted capital may not be smaller than \( k(\alpha^E_l + y) \). As in Gersbach and Rochet (2012A), the implementation of the contracts implies that short-term debt regulation translates into regulation of capital adjustments.

Note that after such a regulatory intervention, investors and bankers will, in general, reconsider their investments and contracts, which includes also a reconsideration of the amount of shifted capital in state \( \eta = h \). This, of course, also leads to changes of capital prices.

The following proposition describes the impact of one-sided regulation on social welfare:

**Proposition 3.7**

*Under one-sided regulation with \( \Delta \alpha > 0 \) it holds that \( \Delta U > 0 \).*

**Proof.** We closely follow the logic of the proof of Proposition 6 in Gersbach and Rochet (2012A).

Increasing \( \alpha^E_l \) by \( \Delta \alpha_l > 0 \) will have an impact on the chosen capital adjustment in the state \( \eta = h \), on capital prices in both states, and on investment decisions at \( t = 0 \).

**Step 1:**

The amount of shifted capital in state \( \eta = l \) is given by \( K \alpha^R_l = K(\alpha^E_l + \Delta \alpha_l) \).

If \( \Delta \alpha_l \) is small and positive, at the aggregate level, the bankers’ optimization problem is

\[
\max_{\alpha_h, K} K \{ q(1 + \alpha_h) + (1 - q)(1 + \alpha^R_l) \},
\]

subject to

\[
K \left( q(1 + \alpha_h)(hR - b) - \frac{c}{2} - p_h \alpha_h \right) + (1 - q) \left( (1 + \alpha_l)(lR - b) - \frac{c}{2} \right) = K - E.
\]

(3.49)

Let \( L \) and \( \lambda \) be the corresponding Lagrangian and multiplier. Then, the first order conditions of the above optimization problem read

\[
\frac{\partial L}{\partial K} = bE[1 + \alpha_n] + \lambda E[(1 + \alpha_n)(\eta R - b) - \frac{c}{2} \alpha^2_n - p_n \alpha_n] - \lambda = 0,
\]

\[
\frac{\partial L}{\partial \alpha_h} = qK[ b + \lambda (hR - b - c\alpha_h - p_h)] = 0.
\]
The second condition implies

\[ p_h = hR - b - c\alpha_h + \frac{b}{\lambda}. \]  
(3.50)

For equilibrium prices \( p_\eta \) it has to hold that

\[ \mathbb{E}[p_\eta] = qp_h + (1 - q)p_l = 1. \]  
(3.51)

Solving for \( p_l \) and inserting it together with equation (3.50) into the first condition yields

\[ 1 - \frac{b}{\lambda}(1 + \alpha) = \mathbb{E}[(\eta R - b)(1 + \alpha_\eta) - \frac{c}{2}\alpha^2_\eta - \alpha_i^R - q(\alpha_h - \alpha_i^R)(hR - b - c\alpha_h + \frac{b}{\lambda})] \]

\[ \Leftrightarrow \frac{b}{\lambda} = 1 + b - R + \frac{c}{2} \left( \frac{(\alpha_i^R)^2 - q(\alpha_h - \alpha_i^R)^2}{1 + \alpha_i^R} \right). \]

Let

\[ \psi(\alpha_i^R, \alpha_h) := \frac{(\alpha_i^R)^2 - q(\alpha_h - \alpha_i^R)^2}{1 + \alpha_i^R}. \]

Then,

\[ p_h = (h - 1)R + 1 + \frac{c}{2}(\psi(\alpha_i^R, \alpha_h) - 2\alpha_h). \]  
(3.52)

\( \alpha_h \) is uniquely determined by this equation, given the values of \( p_h \) and \( \alpha_i^R \). We denote the solution by \( \alpha_h(p_h, \alpha_i^R) \).

**Step 2:**

Given a value of \( \alpha_i^R \), we define an equilibrium as a tuple \( \Sigma = \{C_0, K, X, \alpha_h, p_h, p_l | \alpha_i^R \} \) such that

(i) \( K = \frac{E}{\rho(p_i|\alpha_i^R)} \),

where

\[ \rho(p_i|\alpha_i^R) = 1 - q \left( (1 + \alpha_h(p_h, \alpha_i^R))(hR - b) - \frac{c}{2} \alpha_h^2(p_h, \alpha_i^R) - p_h\alpha_h(p_h, \alpha_i^R) \right) \]

\[ - (1 - q) \left( (1 + \alpha_i^R)(\overline{R} - b) - \frac{c}{2} (\alpha_i^R)^2 - p_l\alpha_i^R \right) \];

(ii) the shares of capital in the BS that are shifted at \( t = 1 \) are \( \alpha_i^R \) in state \( \eta = l \) and \( \alpha_h(p_h, \alpha_i^R) \) in state \( \eta = h \);

(iii) \( p_h = \eta F'(X - \alpha_h(p_h, \alpha_i^R)K) \),

\[ p_l = \eta F'(X - \alpha_i^R K) \];

(iv) \( qp_h + (1 - q)p_l = 1; \)
(v) $K + X + C_0 = 1$;
(vi) $K > 0, X > 0, C_0 > 0$.

**Step 3:**
If we choose $\alpha_l^R = \alpha_l^E$, the above equilibrium is equal to the competitive equilibrium without regulation. Hence, as $\Delta \alpha_l$ converges to 0, due to the continuity of the problem, $\Delta X, \Delta p_\eta, \Delta K$ and $\Delta \alpha_h$ also converge to 0.

**Step 4:**
It holds that $\Delta p_l > 0$ and $\Delta p_h < 0$. The proof of this statement is identical to step 4 of the proof of Proposition 6 in Gersbach and Rochet (2012A), so we omit it here. However, after the end of this proof, we provide the proof of Lemma 3.1, as it slightly differs from the one in Gersbach and Rochet (2012A). This lemma is used by Gersbach and Rochet (2012A) to derive a contradiction from the supposition that $\Delta p_l < 0$ and $\Delta p_h > 0$.

**Step 5:**
We explore now how social welfare changes with the described kind of one-sided regulation. Let $\Delta \alpha_h = \alpha_h(p_h^E + \Delta p_h, \alpha_l^R) - \alpha_h^E$.

First, we express $\Delta X, \Delta p_\eta$ and $\Delta K$ in terms of $\Delta \alpha_l$. Analogously to the proof of Proposition 3.6 we have

\[
\mathbb{E}[\tilde{p}_\eta] = 1 \Rightarrow \mathbb{E}[p_\eta^E + \Delta p_\eta] = 1.
\]

We know that $\mathbb{E}[p_\eta^E] = 1$. Hence, $\mathbb{E}[\Delta p_\eta] = 0$.

It holds that

\[
p_\eta^E = \eta F'(X^E - \alpha_\eta^E K^E).
\]

Hence, using the Taylor approximation and afterwards omitting terms of high order, we obtain

---

8 This step was not provided explicitly in Gersbach and Rochet (2012A).
\[ \tilde{p}_\eta = p^E_n + \Delta p_\eta \]
\[ = \eta F'(X^E - \alpha^E_n K^E) + \eta F''(X^E - \alpha^E_n K^E)(\Delta X - K^E \Delta \alpha_\eta - \Delta K \Delta \alpha_\eta) \]
\[ = p^E_n + \Delta p_\eta = \eta F'(X^E - \alpha^E_n K^E) + \eta F''(X^E - \alpha^E_n K^E)(\Delta X - K^E \Delta \alpha_\eta - \Delta K \alpha_\eta) \]
\[ \Rightarrow \Delta p_\eta = \eta F''(X^E - \alpha^E_n K^E)(\Delta X - K^E \Delta \alpha_\eta - \Delta K \alpha_\eta) \] (3.53)
\[ \Rightarrow 0 = \mathbb{E}[\Delta p_\eta] = \mathbb{E}[\eta F''(X^E - \alpha^E_n K^E)(\Delta X - K^E \Delta \alpha_\eta - \Delta K \alpha_\eta)]. \]

Consequently,
\[ \Delta X = \frac{\mathbb{E}[\eta F''(X^E - \alpha^E_n K^E)(K^E \Delta \alpha_\eta + \Delta K \alpha_\eta)]}{\mathbb{E}[\eta F''(X^E - \alpha^E_n K^E)]} \] (3.54)

and
\[ \Delta p_\eta = \eta F''(X^E - \alpha^E_n K^E)\left(\frac{\mathbb{E}[\eta F''(X^E - \alpha^E_n K^E)(K^E \Delta \alpha_\eta + \Delta K \alpha_\eta)]}{\mathbb{E}[\eta F''(X^E - \alpha^E_n K^E)]} - K^E \Delta \alpha_\eta - \Delta K \alpha_\eta^E \right). \] (3.55)

The investor's participation constraint in the situation with one-sided regulation reads
\[ 1 - \frac{E}{K^E + \Delta K} = \mathbb{E}[(1 + \alpha^E_\eta + \Delta \alpha_\eta)(\eta \bar{R} - b) - \frac{c}{2}(\alpha^E_\eta + \Delta \alpha_\eta)^2 - (p^E_\eta + \Delta p_\eta)(\alpha^E_\eta + \Delta \alpha_\eta)]. \]

Therefore,
\[ \Delta K = \frac{E}{1 - \mathbb{E}[(1 + \alpha^E_\eta + \Delta \alpha_\eta)(\eta \bar{R} - b) - \frac{c}{2}(\alpha^E_\eta + \Delta \alpha_\eta)^2 - (p^E_\eta + \Delta p_\eta)(\alpha^E_\eta + \Delta \alpha_\eta)]} - K^E. \]
We next calculate the change in social welfare, thereby omitting terms of high order:

\[
\Delta U = (K^E + \Delta K)\mathbb{E}[\eta \bar{R}(1 + \alpha_n^E + \Delta \alpha_n) - \frac{c}{2} (\alpha_n^E + \Delta \alpha_n)^2] \\
+ \mathbb{E}[\eta F(X^E + \Delta X - (\alpha_n^E + \Delta \alpha_n)(K^E + \Delta K))] + 1 - (K^E + \Delta K) - (X^E + \Delta X) \\
- K^E \mathbb{E}[\eta \bar{R}(1 + \alpha_n^E) - \frac{c}{2}(\alpha_n^E)^2] - \mathbb{E}[\eta F(X^E - \alpha_n^E K^E)] - 1 + K^E + X^E
\]

\[
= K^E \mathbb{E}[\eta \bar{R} \Delta \alpha_n - \alpha_n^E \Delta \alpha_n - \frac{c}{2}(\Delta \alpha_n)^2] \\
+ \mathbb{E}[\eta F'(X^E - \alpha_n^E K^E) \cdot (\Delta X - K^E \cdot \Delta \alpha_n - \Delta K \alpha_n^E - \Delta K \Delta \alpha_n)] \\
- \Delta K - \Delta X + \Delta K \mathbb{E}[\eta \bar{R}(1 + \alpha_n^E + \Delta \alpha_n) - \frac{c}{2}(\alpha_n^E + \Delta \alpha_n)^2]
\]

\[
= K^E \mathbb{E}[\eta \bar{R} \Delta \alpha_n - \alpha_n^E \Delta \alpha_n] \\
+ \mathbb{E}[\eta F'(X^E - \alpha_n^E K^E) \cdot (\Delta X - K^E \cdot \Delta \alpha_n - \Delta K \alpha_n^E)] \\
- \Delta K - \Delta X + \Delta K \mathbb{E}[\eta \bar{R}(1 + \alpha_n^E) - \frac{c}{2}(\alpha_n^E)^2] - 1 - \mathbb{E}[\alpha_n^E \eta F'(X^E - \alpha_n^E K^E)]
\]

\[
= K^E \mathbb{E}[\eta \bar{R} - \alpha_n^E \eta - \eta \bar{R} F(X^E - \alpha_n^E K^E)] \Delta \alpha_n + \mathbb{E}[\eta F'(X^E - \alpha_n^E K^E) - 1] \Delta X \\
+ \Delta K \mathbb{E}[\eta \bar{R}(1 + \alpha_n^E) - \frac{c}{2}(\alpha_n^E)^2] - 1 - \mathbb{E}[\alpha_n^E \eta F'(X^E - \alpha_n^E K^E)]
\]

\[
= K^E \mathbb{E}[\eta \bar{R} - \alpha_n^E \eta - \eta \bar{R} \alpha_n^E K^E] \Delta \alpha_n + \mathbb{E}[\eta \bar{R}(1 + \alpha_n^E) - \frac{c}{2}(\alpha_n^E)^2] - 1 - \mathbb{E}[\alpha_n^E \eta \bar{R}^E]
\]

Since \(\mathbb{E}[\eta \bar{R}^E] = 1\), we obtain

\[
\Delta U = K^E \mathbb{E}[\eta \bar{R} - \alpha_n^E - \alpha_n^E \eta \bar{R}] \Delta \alpha_n + \Delta K(\mathbb{E}[\eta \bar{R}(1 + \alpha_n^E) - \frac{c}{2}(\alpha_n^E)^2 - \alpha_n^E \eta \bar{R}^E] - 1).
\]

Now, from the investor’s participation constraint we obtain

\[
\mathbb{E}[\eta \bar{R}(1 + \alpha_n^E) - \frac{c}{2}(\alpha_n^E)^2 - \alpha_n^E \eta \bar{R}^E] = 1 - \frac{E}{K^E} + b(1 + \mathbb{E}[\alpha_n^E]).
\]

Hence,

\[
\Delta U = K^E \mathbb{E}[\eta \bar{R} - \alpha_n^E - \alpha_n^E \eta \bar{R}] \Delta \alpha_n + \Delta K(-\frac{E}{K^E} + b(1 + \mathbb{E}[\alpha_n^E])).
\]

Using equation (3.47), we obtain that

\[
\Delta U = K^E b(1 - \frac{1}{\nu}) \mathbb{E}[\Delta \alpha_n] + \Delta K(-\frac{E}{K^E} + b(1 + \mathbb{E}[\alpha_n^E])).
\]

The investors’ participation constraint in equilibrium without regulation reads

\[
1 - \frac{E}{K^E} = \mathbb{E}[(1 + \alpha_n^E)(\eta \bar{R} - b) - \frac{c}{2}(\alpha_n^E)^2 - \alpha_n^E \eta \bar{R}^E).
\]
Totally differentiating this equation with respect to $\alpha_E^\eta$ and $p_E^\eta$ yields

$$\mathbb{E}(\eta \overline{R} - b - c\alpha_E^\eta - p_E^\eta)\Delta \alpha_E^\eta - \alpha_E^\eta \Delta p_E^\eta - \frac{E}{(K_E^E)^2} \Delta K = 0.$$  \hspace{1cm} (3.56)

Equation (3.47) reads

$$p_E^\eta = \eta \overline{R} - b - c\alpha_E^\eta + \frac{b}{\nu}.$$  

This implies, together with equation (3.56), that

$$\mathbb{E}[(\eta \overline{R} - b - c\alpha_E^\eta - p_E^\eta)\Delta \alpha_E^\eta - \alpha_E^\eta \Delta p_E^\eta] - \frac{E}{(K_E^E)^2} \Delta K = 0 \Rightarrow \mathbb{E}[\Delta \alpha_E^\eta] = -\frac{\nu}{b} [\mathbb{E}[\alpha_E^\eta \Delta p_E^\eta] + \frac{E}{(K_E^E)^2} \Delta K].$$

Therefore,

$$\Delta U = K_E b(1 - \frac{1}{\nu}) \mathbb{E}[\Delta \alpha_E^\eta] + \Delta K(-\frac{E}{K_E^E} + b(1 + \mathbb{E}[\alpha_E^\eta]))$$

$$= K_E^E b(1 - \frac{1}{\nu}) \frac{-\nu}{b} [\mathbb{E}[\alpha_E^\eta \Delta p_E^\eta] + \frac{E}{(K_E^E)^2} \Delta K] + \Delta K(-\frac{E}{K_E^E} + b(1 + \mathbb{E}[\alpha_E^\eta]))$$

$$= K_E^E (1 - \nu) [\mathbb{E}[\alpha_E^\eta \Delta p_E^\eta] + \frac{E}{(K_E^E)^2} \Delta K] + \Delta K(-\frac{E}{K_E^E} + b(1 + \mathbb{E}[\alpha_E^\eta]))$$

$$= K_E^E (1 - \nu) [\mathbb{E}[\alpha_E^\eta \Delta p_E^\eta]] + \Delta K(b(1 + \mathbb{E}[\alpha_E^\eta]) - \nu \frac{E}{K_E^E})$$

$$= K_E^E (1 - \nu) [\mathbb{E}[\alpha_E^\eta \Delta p_E^\eta]] + \Delta K \nu \frac{b}{\nu} (1 + \mathbb{E}[\alpha_E^\eta]) - \frac{E}{K_E^E}).$$  \hspace{1cm} (3.57)

Using equation (3.20) and the equilibrium values of $\frac{E}{K_E^E}$ and $\alpha_E^\eta$ we obtain

$$\frac{b}{\nu} (1 + \mathbb{E}[\alpha_E^\eta]) - \frac{E}{K_E^E} = (1 - \overline{R} + b - c \mathbb{E}[(\alpha_E^\eta)^2]) (1 + \mathbb{E}[\alpha_E^\eta])$$

$$= \sqrt{1 - (\sigma_E^E)^2} \left(1 - \overline{R} + b - c (1 - \sqrt{1 - (\sigma_E^E)^2})\right)$$

$$= \sqrt{1 - (\sigma_E^E)^2} c \left(-\frac{1}{2} \mathbb{E}[(\alpha_E^\eta)^2] + 1 - \sqrt{1 - (\sigma_E^E)^2}\right)$$

$$= \sqrt{1 - (\sigma_E^E)^2} c \left(-\frac{1}{2} (\sigma_E^E)^2 + (\mathbb{E}[\alpha_E^\eta])^2 + 1 - \sqrt{1 - (\sigma_E^E)^2}\right)$$

$$= \sqrt{1 - (\sigma_E^E)^2} c \left(-\frac{1}{2} (\sigma_E^E)^2 + (\sqrt{1 - (\sigma_E^E)^2} - 1)^2 + 1 - \sqrt{1 - (\sigma_E^E)^2}\right)$$

$$= 0.$$
Thus, equation (3.57) simplifies to

$$
\Delta U = KE(1 - \nu)(\mathbb{E}[\alpha_{\eta}^E \Delta p_{\eta}]).
$$

(3.58)

We know from step 4 that $\Delta p_l > 0$ and $\Delta p_h < 0$. On the other hand, equations (3.6) and (3.7) imply that $\alpha_l^E < 0$ and $\alpha_h^E > 0$. Hence, $\mathbb{E}[\alpha_{\eta}^E \Delta p_{\eta}] < 0$. Together with $\nu > 1$, this implies $KE(1 - \nu)(\mathbb{E}[\alpha_{\eta}^E \Delta p_{\eta}]) > 0$. Thus, we obtain that $\Delta U > 0$. □

With regard to step 4 of the above proof, note that as in Gersbach and Rochet (2012A), the following lemma holds in our set-up:

**Lemma 3.1**

Consider the situation of one-sided regulation with $\Delta \alpha_l > 0$. Then, $\Delta p_l \leq 0$ and $\Delta p_h \geq 0$ imply $\Delta K < 0$.

**Proof.** Assume $\Delta p_l \leq 0$ and $\Delta p_h \geq 0$. With equation (3.53), which has been derived in step 5 without the usage of Lemma 3.1, and omitting the terms $\Delta K \Delta \alpha_{\eta}$, we obtain

$$
\Delta X - KE \Delta \alpha_l - K \alpha_l^E \geq 0,
$$

$$
\Delta X - KE \Delta \alpha_h - K \alpha_h^E \leq 0.
$$

Hence,

$$
KE \Delta \alpha_h + K \alpha_h^E \geq KE \Delta \alpha_l + K \alpha_l^E
\iff KE \Delta \alpha_h \geq KE \Delta \alpha_l + K(\alpha_l^E - \alpha_h^E).
$$

(3.59)

Note that the expected payoff to a banker, given the allocation $(KE + \Delta K, \alpha_l^E + \Delta \alpha_l, \alpha_h^E + \Delta \alpha_h)$ with prices $(p_l^E + \Delta p_l, p_h^E + \Delta p_h)$, has to fulfill the investor’s participation constraint. Therefore, due to $\Delta p_l \leq 0$ and $\Delta p_h \geq 0$ – which means that the prices $p_l^E + \Delta p_l$ make it at least as difficult to fulfill the investor’s participation constraint as prices $(p_l^E, p_h^E)$ do –, the allocation $(KE + \Delta K, \alpha_l^E + \Delta \alpha_l, \alpha_h^E + \Delta \alpha_h)$ with prices $(p_l^E, p_h^E)$ also fulfills the investor’s participation constraint. Thus, if

$$
(KE + \Delta K)b\mathbb{E}[1 + \alpha_{\eta} + \Delta \alpha_{\eta}] \geq KEb\mathbb{E}[1 + \alpha_{\eta}],
$$

this would contradict optimality or uniqueness of $(KE, \alpha_l^E, \alpha_h^E)$. 
Consequently, it holds

\[(K^E + \Delta K)bE[1 + \alpha_\eta + \Delta \alpha_\eta] < K^EbE[1 + \alpha_\eta] \]  \hspace{1cm} (3.60)

\[\Leftrightarrow (K^E + \Delta K)(q[1 + \alpha_h + \Delta \alpha_h] + (1 - q)[1 + \alpha_l + \Delta \alpha_l]) < K^E(q[1 + \alpha_h] + (1 - q)[1 + \alpha_l]) \]

\[\Leftrightarrow \Delta K(1 + \eta^E) + (K^E + \Delta K)(q\Delta \alpha_h + (1 - q)\Delta \alpha_l) < 0.\]

We neglect terms with $\Delta K \Delta \alpha_\eta$ and obtain

\[\Delta K(1 + \alpha^E) + K^E q \Delta \alpha_h + K^E(1 - q)\Delta \alpha_l < 0.\]

With inequality (3.59) this implies

\[\Delta K(1 + \alpha^E + q(K^E \Delta \alpha_l + \Delta K(\alpha_l^E - \alpha_h^E))) + K^E(1 - q)\Delta \alpha_l < 0\]

\[\Leftrightarrow \Delta K(1 + \alpha^E) + K^E \Delta \alpha_l + q\Delta K(\alpha_l^E - \alpha_h^E) < 0\]

\[\Leftrightarrow \Delta K(1 + q\alpha_h^E + (1 - q)\alpha_l^E) + K^E \Delta \alpha_l + q\Delta K(\alpha_l^E - \alpha_h^E) < 0\]

\[\Leftrightarrow \Delta K(1 + q\alpha_h^E + (1 - q)\alpha_l^E + q\alpha_l^E - q\alpha_h^E) + K^E \Delta \alpha_l < 0\]

\[\Leftrightarrow \Delta K(1 + \alpha_l^E) + K^E \Delta \alpha_l < 0.\]

Since $\alpha_l^E \geq -1$ and $\Delta \alpha_l > 0$, this implies

\[\Delta K < 0.\]

\[\Box\]

The intuition why this lemma holds is as follows: In both states, prices $p_l^E + \Delta p_l \leq 0$ and $p_h^E + \Delta p_h \geq 0$ are less favorable for bankers than prices $p_l^E \leq 0$ and $p_h^E \geq 0$. Consequently, investment in the BS declines.

Proposition 3.7 states that social welfare can be increased by one-sided regulation. Thus, we have shown that even in the presence of Macroshocks that hit both sectors, the results of Gersbach and Rochet (2012A) continue to hold. Aggregate investment fluctuates excessively thereby generating a negative pecuniary externality and lowering social welfare. The negative impact of this externality can be moderated by one-sided regulation.

### 3.3 Contracts Written by Investors

In this section we consider another modification of the original model of Gersbach and Rochet (2012A), in which the contract between the banker and the investor is written not
by the banker, but by the investor. We translate this into the model assumption that the banker is kept at his participation constraint. Thereby we return to the model assumption of Gersbach and Rochet (2012A) that the shock $\eta$ hits only the BS, and does not hit the TS. We investigate whether the uniqueness and existence of the competitive equilibrium in Gersbach and Rochet (2012A) are robust with respect to a change of authorship of contracts. Furthermore, we investigate whether the impact of regulation described in Gersbach and Rochet (2012A) is robust to a change of authorship of contracts.

### 3.3.1 Two Simple Benchmarks

#### The Frictionless Economy

The first-best allocation is obviously the one of Gersbach and Rochet (2012A).

#### The Rigid Economy

As in Gersbach and Rochet (2012A), we now consider the benchmark case in which no adjustments of capital are possible at $t = 1$.

The investor chooses his investment in the BS, in the TS, and his consumption in period $t = 0$ by first deciding whether or not to invest some amount of capital in the BS. If the investor does not invest in the BS, his total payoff amounts to $(1 - x) + x(1 + r)$. If $r > 0$, the investors chooses $x = 1 - e$. Otherwise, he is indifferent between choosing $x \in [0, 1 - e]$.\(^9\)

If the investor chooses to invest in the BS, he solves the following constrained maximization problem:

$$
\max_{k \in [1 + \frac{r}{b}e, 1], x \in [0, 1 - e]} k(R - b) + x(1 + r) + (1 - k - x),
$$

subject to the investor’s budget constraint

$$
1 - k - x \geq 0.
$$

The restriction $k \geq \frac{1 - e}{b}$ reflects the banker’s participation constraint $kb \geq e(1 + r)$\(^{10}\).

---

\(^9\) However, the value of the aggregate investment in the TS in equilibrium will be determined by the condition of absence of arbitrage.

\(^{10}\) The bankers’ participation constraint is the one of Gersbach and Rochet (2012A). Note that it does not read $kb \geq eR$, since Gersbach and Rochet (2012A) assume, like Holmström and Tirole (1997), that a banker cannot finance his project in the BS without external funding. Furthermore, note that by assuming that the necessary amount of external funding relative to a bank’s endowment $e$ is sufficiently small – in particular, smaller than $\frac{1}{b}$ –, we do not have to state this constraint on investments in the BS explicitly, as it is automatically fulfilled whenever the banker’s participation constraint is fulfilled.
Note that if \( r = 0 \), the value of \( x \) is not determined by the maximization problem and the solution incorporates \( k = \frac{1}{b} e \), as it holds that \( R - b < 1 \) due to Assumption 3.3. On the other hand, if \( r > 0 \), then the solution is \( x = 1 - k \) and \( k = \frac{1 + r}{b} e \). The investor invests just as much in the BS as necessary to make the banker sign a contract, thereby making the investor benefit from the banker’s capital \( e \). This benefit is the main incentive for the investor to invest in the BS. Any investment of the investor in the BS that exceeds the level necessary to generate this benefit is not profitable for the investor, as it holds that \( \mathbb{E}[\eta R - b] = R - b < 1 \).

The investor decides to invest in the BS if and only if
\[
\frac{1 + r}{b} e (R - b) + (1 - \frac{1 + r}{b} e)(1 + r) \geq (1 - e)(1 + r)
\]
\[
\iff \frac{1 + r}{b} e (R - b) - \frac{1 + r}{b} e (1 + r) \geq -e(1 + r)
\]
\[
\iff \frac{1}{b} (R - b) - \frac{1}{b} (1 + r) \geq -1
\]
\[
\iff (R - b) - (1 + r) \geq -b
\]
\[
\iff R \geq 1 + r.
\]

Note that keeping the banker at his participation constraint yields the following return per unit of investment in the BS to the investor:
\[
\frac{k(R - b)}{k - e} = \frac{1 + r}{b} e (R - b)
\]
\[
= \frac{(1 + r)(R - b)}{1 + r - b} \geq 1 + r,
\]
as \( 1 + r \leq R \) holds in any equilibrium because otherwise, there would be a profitable deviation to invest more in the TS. Thus, the investor decides to invest in the BS and invests the amount \( k = \frac{1 + r}{b} e \) in the BS. On the aggregate level, we obtain \( K_{rig} = \frac{1 + r}{b} E \).

On the aggregate level, the investor’s participation constraint in the BS is
\[
K(R - b) + (1 - K)(1 + r) \geq (1 - E)(1 + r).
\]
This is equivalent to
\[
E \geq \frac{1 - R + b + r}{1 + r} K.
\]
Thus, in equilibrium, an investor is willing to invest
\[ K \leq \frac{E(1 + r)}{1 - \frac{1}{R} + b + r} \]
in the BS. Of course, due to \( 1 + r \leq \frac{1}{R} \), it holds that
\[ K_{\text{rig}} = \frac{1 + r}{b} E \leq \frac{E(1 + r)}{1 - \frac{1}{R} + b + r}. \]
The interdependence between \( K \) and \( r \) in equilibrium is given by
\[ K = S(1 + r). \]
Thereby, in the case of \( C_0 = 0 \), for \( p \in [0, \infty) \), \( S(p) \) is defined by
\[ F'[1 - S(p)] = p. \]
Hence,
\[ S(p) = 1 - (F')^{-1}(p). \]
\( S(p) \) is an increasing function with \( S(0) = 0 \) and \( S(\infty) = 1 \).
Thus,
\[ K_{\text{rig}} = S(1 + r_{\text{rig}}) = 1 - (F')^{-1}(1 + r_{\text{rig}}). \] (3.61)
Hence, if \( F'(1 - K_{\text{rig}}) \geq 1 \), we obtain
\[ r_{\text{rig}} = F'(1 - K_{\text{rig}}) - 1. \] (3.62)
Otherwise it holds that \( r_{\text{rig}} = 0 \) and \( C_{0,\text{rig}} > 0 \), such that \( F'(1 - K_{\text{rig}} - C_{0,\text{rig}}) = 1 \).
We obtain the following proposition:

**Proposition 3.8**
The unique equilibrium in the rigid economy has the following form:

- If \( E < bS(1) =: E_{\text{rig}}^* \), it holds \( K_{\text{rig}} = \frac{1}{b} E, r_{\text{rig}} = 0, C_{0,\text{rig}} > 0 \).
- If \( E_{\text{rig}}^* \leq E \leq \frac{b}{R} S(\overline{R}) =: E_{\text{rig}}^{**} \), it holds \( K_{\text{rig}} = \frac{1 + r_{\text{rig}}}{b} E, C_{0,\text{rig}} = 0 \).
- If \( E > E_{\text{rig}}^{**} \), it holds \( K_{\text{rig}} = S(\overline{R}), C_{0,\text{rig}} = 0. \)
3.3.2 Privately-Optimal Contracts

We leave now the benchmark cases from the previous section and return to the model described in Section 3.3. The structure of a contract between an investor and a banker is the same as in the setting where the banker writes the contract. We have described it in Section 3.2.1. Given that an investor wants to invest in the BS, the optimal banking contract is the solution of the following optimization problem:

\[
\max_{k,\alpha_h,\alpha_l} k \mathbb{E}[(1 + \alpha_\eta)(\eta R - b) - \frac{c}{2} \alpha_\eta^2 - p_\eta \alpha_\eta] + (1 - k)(1 + r),
\]

subject to the banker’s participation constraint

\[
\mathbb{E}[k(1 + \alpha_\eta)b] \geq c(1 + r).
\]

Thereby, \( b = \frac{\tau B}{\Delta} \) guarantees that the banker exerts effort.

Due to the constant returns to scale of the BS, the optimal contract can also be characterized at the aggregate level:

\[
\max_{K,\alpha_h,\alpha_l} KE[(1 + \alpha_\eta)(\eta R - b) - \frac{c}{2} \alpha_\eta^2 - p_\eta \alpha_\eta] + (1 - K)(1 + r),
\]

subject to

\[
\mathbb{E}[K(1 + \alpha_\eta)b] \geq E(1 + r).
\]

Finally, note that the optimal contract is implementable in our set-up exactly as in Gersbach and Rochet (2012A), (Section 4.3), where, in particular, short-term debt is used to force a banker to reduce the size of his bank by the amount \( k\alpha_l \) if state \( l \) occurs.

3.3.3 Competitive Equilibrium

Properties

As in Gersbach and Rochet (2012A), a competitive equilibrium in the economy with capital reallocation and moral hazard is an array \( \Sigma = \{C_0, C_{2h}, C_{2l}, K, X, \alpha_h, \alpha_l, p_h, p_l\} \), such that

(i) the contracts between bankers and investors as well as the investment on the aggregate level are according to Section 3.3.2;

(ii) the price of capital at \( t = 1 \) in state \( \eta \) is equal to \( p_\eta = F'(X - \alpha_\eta(p)K), \eta \in \{h, l\} \);

(iii) \( \mathbb{E}[p_\eta] = qp_h + (1 - q)p_l = 1 + r \);
(iv) \( K + X + C_0 = 1; \)
(v) \( C_2 = K((1 + \alpha_\eta)\eta R - \frac{c}{2}\alpha_\eta^2) + F(X - \alpha_\eta K); \)
(vi) \( C_0 \geq 0, K \geq 0, X \geq 0; \)
(vii) \( rC_0 = 0. \)

Existence and Uniqueness

We want to explore whether a competitive equilibrium with an interior solution exists, in which it holds that \( r = 0. \)

Proposition 3.9

The set of interior competitive equilibria which incorporate \( r = 0 \) is a singleton. The variance of credit growth \( \sigma^2 = q(1 - q)(\alpha_h - \alpha_l)^2 \) may be used to parametrize it. The only such competitive equilibrium is characterized by:

- \( \sigma^E \) is the unique solution of \( J(\sigma) = E; \)
- \( \alpha^E = \sqrt{1 - (\sigma^E)^2} - 1; \)
- \( p_h^E = (h - 1)R + (1 + c\sigma^E)\sqrt{1-q}, p_l^E = (l - 1)R + (1 - c\sigma^E)\sqrt{1-q}; \)
- \( \frac{E}{K(1+\alpha^E)} = b. \)

Thereby, \( J(\sigma) \) is a function given by

\[
J(\sigma) := b\sqrt{q(1-q)(\frac{1}{\sigma^2} - 1)} \cdot \left( S((h - 1)R + 1 - c\sigma)\sqrt{\frac{1-q}{q}} - S((l - 1)R + 1 + c\sigma)\sqrt{\frac{q}{1-q}} \right).
\]

Proof. We have to solve the following maximization problem of the investor on the aggregate level:

\[
\max_{K,\alpha_h,\alpha_l} K\mathbb{E}[(1 + \alpha_\eta)(\eta R - b) - \frac{c}{2}\alpha_\eta^2 - p_\eta^2 \alpha_\eta] + (1 - K)(1 + r),
\]
subject to
\[
\mathbb{E}[K(1 + \alpha_\eta)b] \geq E(1 + r).
\]

Let \( \mathcal{L} \) denote the Lagrangian and \( \mu \) denote the multiplier.

\[
\mathcal{L} = K\mathbb{E}[(1 + \alpha_\eta)(\eta R - b) - \frac{c}{2}\alpha_\eta^2 - p_\eta\alpha_\eta] + (1 - K)(1 + r) + \mu(\mathbb{E}[K(1 + \alpha_\eta)b] - E(1 + r)).
\]
The first order conditions read

\[ \frac{\partial L}{\partial K} = \mathbb{E}[(1 + \alpha_\eta)(\eta R - b) - \frac{c}{2} \alpha_\eta^2 - p_\eta \alpha_\eta] - (1 + r) + \mu \mathbb{E}[(1 + \alpha_\eta)b] = 0, \]
\[ \frac{\partial L}{\partial \alpha_\eta} = KP_r(\eta)[\eta R - b - c \alpha_\eta - p_\eta + \mu b] = 0. \]

This is equivalent to

\[ \frac{\partial L}{\partial K} = \mathbb{E}[(1 + \alpha_\eta)(\eta R - (1 - \mu)b) - \frac{c}{2} \alpha_\eta^2 - p_\eta \alpha_\eta] - (1 + r) = 0, \quad (3.63) \]
\[ \frac{\partial L}{\partial \alpha_\eta} = \eta R - (1 - \mu)b - c \alpha_\eta - p_\eta = 0. \quad (3.64) \]

The second condition implies

\[ p_\eta = \eta R - (1 - \mu)b - c \alpha_\eta. \quad (3.65) \]

Plugging this into the first condition yields

\[ \mathbb{E}[(1 + \alpha_\eta)(\eta R - (1 - \mu)b) - \frac{c}{2} \alpha_\eta^2 - (\eta R - (1 - \mu)b - c \alpha_\eta)\alpha_\eta] - (1 + r) = 0 \quad (3.66) \]
\[ \iff \mathbb{E}[\eta R - (1 - \mu)b + \frac{c}{2} \alpha_\eta^2] - (1 + r) = 0 \]
\[ \iff \eta R - (1 - \mu)b + \frac{c}{2} \mathbb{E}[\alpha_\eta^2] - (1 + r) = 0 \]
\[ \iff \eta R + \frac{c}{2} \mathbb{E}[\alpha_\eta^2] - (1 + r) = (1 - \mu)b. \quad (3.67) \]

In analogy with Section 3.2.4, we conclude that \( 0 < \mu < 1. \)

Hence, the banker’s participation constraint holds tight and we have

\[ K = \frac{E(1 + r)}{b \mathbb{E}[1 + \alpha_\eta]}. \]

Equations (3.67) and (3.65) imply

\[ p_\eta = (\eta - 1)\bar{R} - \frac{c}{2} \mathbb{E}[\alpha_\eta^2] - (1 + r) - c \alpha_\eta. \]

In equilibrium, it holds that \( \mathbb{E}[p_\eta] = 1 + r. \) Hence,

\[ 1 + r = -\frac{c}{2} \mathbb{E}[\alpha_\eta^2] - (1 + r) - c \mathbb{E}[\alpha_\eta] \]
\[ \iff \mathbb{E}[\alpha_\eta^2] + 2 \mathbb{E}[\alpha_\eta] = 0. \quad (3.68) \]
We obtain a parametrization of the capital adjustments by their variance $\sigma^2 = q(1 - q)(\alpha_h - \alpha_l)^2$:

$$
\begin{align*}
\alpha_h^E &= \sqrt{1 - \sigma^2} - \sigma \sqrt{\frac{1 - q}{q}} \\
\alpha_l^E &= \sqrt{1 - \sigma^2} - 1 - \sigma \sqrt{\frac{q}{1 - q}}
\end{align*}
$$

(3.69)

and

$$
\begin{align*}
p_h^E &= (h - 1)R + 1 - c\sigma \sqrt{\frac{1 - q}{q}} \\
p_l^E &= (l - 1)R + 1 + c\sigma \sqrt{\frac{q}{1 - q}}
\end{align*}
$$

(3.70)

We focus now on the case $r = 0$. The capital supply to the BS in the case of $C_0 \geq 0$ is defined as

$$F'(1 - C_0 - S(p)) = p.$$ 

Then, as in Gersbach and Rochet (2012A), using the equilibrium condition

$$p_\eta = F'(X - \alpha_\eta K)$$

and the strict monotonicity of $F'(\cdot)$, we obtain

$$1 - C_0 - S(p_\eta) = X - \alpha_\eta K.$$ 

Eliminating $X + C_0$ for $\eta = l, h$ yields

$$1 - S(p_h) + K\alpha_h = 1 - S(p_l) + K\alpha_l$$

$$\iff (\alpha_h - \alpha_l)K = S(p_h) - S(p_l).$$

Inserting the expressions for $K, \alpha_h, \alpha_l, p_l, p_h$ yields

$$\frac{\sigma}{\sqrt{q(1 - q)}} \frac{E}{b\sqrt{1 - \sigma^2}} = S((h - 1)R + 1 - c\sigma \sqrt{\frac{1 - q}{q}}) - S((l - 1)R + 1 + c\sigma \sqrt{\frac{q}{1 - q}}).$$

This is equivalent to

$$E = b\sqrt{q(1 - q)} \left( \frac{1}{\sigma^2} - 1 \right) \cdot \left( S((h - 1)R + 1 - c\sigma \sqrt{\frac{1 - q}{q}}) - S((l - 1)R + 1 + c\sigma \sqrt{\frac{q}{1 - q}}) \right).$$

(3.71)
Denote the right-hand side of this equation as a function $J(\sigma)$. Note that $J(\sigma)$ is continuous and strictly decreasing on $(0, 1)$ with $\lim_{\sigma \to 0} J(\sigma) = \infty$ and $J(1) = 0$. Thus, if $E \geq J(\sqrt{1 - q}) =: E^*_i$, then the equation $E = J(\sigma)$ has a unique solution. Furthermore, as in Gersbach and Rochet (2012A), there exist thresholds $E^{**}_i$ and $E^{***}_i$ such that if, on the one hand, $E \leq E^{**}_i$, we obtain a boundary solution $\alpha_l = -1$. On the other hand, if $E^{**}_i < E < E^{***}_i$, then $0 < r = 0$. Finally, note that if $E \geq E^{***}_i$, then moral hazard is not binding and it holds that $r = R - 1$.

We have shown in the proof of Proposition 3.9 that the results of Gersbach and Rochet (2012A) about the existence and uniqueness of a competitive equilibrium with $r = 0$, as well as the structure of equilibria described in Corollary 1 in Gersbach and Rochet (2012A) are robust to a change of the assumption on who – bankers or investors – writes the contracts. It is straightforward to see that Corollaries 2 and 3 of Gersbach and Rochet (2012A) also hold in our model.

### 3.3.4 Social Efficiency and Regulation

Let us now explore whether the regulation of capital adjustments described in Section 3.2.5 may increase social welfare in a competitive equilibrium as described in Proposition 3.9. Note that social welfare is equal to

$$U = KE[\eta R (1 + \alpha)] - \frac{c}{2} \alpha^2 + \mathbb{E}[F(X - \alpha K)] + 1 - K - X.$$

We start by proving the following lemma:

**Lemma 3.2**

Assume the economy is in the competitive equilibrium with $r = 0$ and the allocation $(K^E, X^E, \alpha^E_l, \alpha^E_h)$. Then, there exists $\epsilon > 0$, such that for all $\Delta \alpha_l$ and $\Delta \alpha_h$ with $\max(|\Delta \alpha_l|, |\Delta \alpha_h|) < \epsilon$, under two-sided regulation by $(\Delta \alpha_l, \Delta \alpha_h)$, it holds that

$$ (K^E + \Delta K)E[1 + \alpha^E] = \frac{1}{b} E = K^E E[1 + \alpha^E]. $$

**Proof.** Equation (3.63), $\mu > 0$ and $r = 0$ imply that

$$ \mathbb{E}[(\eta R - b)(1 + \alpha^E) - \frac{c}{2} \alpha^2 - \alpha^E p] < 1. $$

Hence, when $\epsilon$ is chosen small enough, it also holds that

$$ \mathbb{E}[(\eta R - b)(1 + (\alpha^E + \Delta \alpha)) - \frac{c}{2} (\alpha^E + \Delta \alpha)^2 - (\alpha^E + \Delta \alpha)(p + \Delta p)] < 1. $$
This implies that under two-sided regulation, the bankers’ participation constraint binds in the investors’ constrained maximization problem, as is does in the competitive equilibrium. Thus,

$$\left( K^E + \Delta K \right) E [1 + \alpha_{\eta}^E + \Delta \alpha_{\eta}] b = E \cdot 1$$

$$\Leftrightarrow \left( K^E + \Delta K \right) E [1 + \alpha_{\eta}^E + \Delta \alpha_{\eta}] = \frac{1}{b} E.$$ 

The second equality in the claim of Lemma 3.2 was shown in the proof of Proposition 3.9. □

Lemma 3.2 states that the two-sided regulation of capital adjustments by small amounts cannot change the expected amount of capital invested in the BS. Lemma 3.2 reflects the straightforward fact that investors are interested in keeping the amount of capital invested in the joint project in the BS just as low as is necessary to make the banker invest in the joint project. This allows the investor to benefit from the banker’s capital. As, due to the banker’s moral hazard, the return to the investor per unit of joint investment is lower than 1 in the BS, the investor is not interested in investing a higher amount of capital in the BS. The regulation of capital adjustments by values of the shifting additions that are small enough cannot undo this effect, as thereby the return to the investor per unit of joint investment in the BS is still lower than 1.

Now we state our main result about the efficacy of the regulation of capital adjustments:

**Proposition 3.10**

Assume the economy is in the competitive equilibrium with $r = 0$ and with the allocation $(K^E, X^E, \alpha_{h}^E, \alpha_{l}^E)$. Then, there exists an $\epsilon > 0$, such that for no $\Delta \alpha_l$ and $\Delta \alpha_h$ with $\max(|\Delta \alpha_l|, |\Delta \alpha_h|) < \epsilon$, two-sided regulation of capital adjustments by $(\Delta \alpha_l, \Delta \alpha_h)$ increases social welfare.

**Proof.** Choose $\epsilon > 0$ arbitrary small, and small enough for Lemma 3.2 to apply. Choose $\Delta \alpha_l, \Delta \alpha_h$ such that $\max(|\Delta \alpha_l|, |\Delta \alpha_h|) < \epsilon$.

It holds that

$$\Delta U = (K^E + \Delta K)E[\eta R (1 + \alpha_{\eta}^E + \Delta \alpha_{\eta}) - \frac{c}{2} (\alpha_{\eta}^E + \Delta \alpha_{\eta})^2]$$

$$-K^E E[\eta R (1 + \alpha_{\eta}^E) - \frac{c}{2} (\alpha_{\eta}^E)^2]$$

$$+E[F(X^E + \Delta X - (K^E + \Delta K)(\alpha_{\eta}^E + \Delta \alpha_{\eta}))]$$

$$-E[F(X^E - K^E \alpha_{\eta}^E)]$$

$$+(1 - (K^E + \Delta K) - (X^E + \Delta X))$$

$$-(1 - K^E - X^E).$$
3.3 Contracts Written by Investors

Simplifying and using the second order approximation of $F(\cdot)$, we obtain

$$
\Delta U = K^E \mathbb{E}[\eta R \Delta \alpha_{\eta} - c \alpha_{\eta}^E \Delta \alpha_{\eta} - \frac{c}{2} (\Delta \alpha_{\eta})^2] \\
+ \Delta K \mathbb{E}[\eta R (1 + \alpha_{\eta}^E + \Delta \alpha_{\eta}) - \frac{c}{2} (\alpha_{\eta}^E + \Delta \alpha_{\eta})^2] \\
+ \mathbb{E}[F'(X^E - K^E \alpha_{\eta}^E) + F''(X^E - K^E \alpha_{\eta}^E)(\Delta X - K^E \Delta \alpha_{\eta} - \Delta K \alpha_{\eta}^E - \Delta K \Delta \alpha_{\eta})] \\
+ \frac{1}{2} F''(X^E - K^E \alpha_{\eta}^E)(\Delta X - K^E \Delta \alpha_{\eta} - \Delta K \alpha_{\eta}^E - \Delta K \Delta \alpha_{\eta})^2] \\
- \mathbb{E}[F(X^E - K^E \alpha_{\eta}^E)] \\
- \Delta K - \Delta X.
$$

The equilibrium condition $\mathbb{E}[F'(X^E - K^E \alpha_{\eta}^E)] = 1$ implies that the terms $\Delta X$ and $\mathbb{E}[F'(X^E - K^E \alpha_{\eta}^E) \Delta X]$ cancel out. Thus, we obtain

$$
\Delta U = K^E \mathbb{E}[\eta R \Delta \alpha_{\eta} - c \alpha_{\eta}^E \Delta \alpha_{\eta} - \frac{c}{2} (\Delta \alpha_{\eta})^2] \\
+ \Delta K \mathbb{E}[\eta R (1 + \alpha_{\eta}^E + \Delta \alpha_{\eta}) - \frac{c}{2} (\alpha_{\eta}^E + \Delta \alpha_{\eta})^2 - 1] \\
+ \mathbb{E}[p_{\eta}^E (- \Delta K \alpha_{\eta}^E - \Delta K \Delta \alpha_{\eta})] \\
+ \frac{1}{2} F''(X^E - K^E \alpha_{\eta}^E)(\Delta X - K^E \Delta \alpha_{\eta} - \Delta K \alpha_{\eta}^E - \Delta K \Delta \alpha_{\eta})^2].
$$

Hence,

$$
\Delta U = K^E \mathbb{E}[(\eta R - c \alpha_{\eta}^E - p_{\eta}^E) \Delta \alpha_{\eta}] \\
+ \Delta K \mathbb{E}[\eta R (1 + \alpha_{\eta}^E + \Delta \alpha_{\eta}) - \frac{c}{2} (\alpha_{\eta}^E + \Delta \alpha_{\eta})^2 - 1 - \alpha_{\eta}^E \alpha_{\eta}^E] \\
- K^E \mathbb{E}[\frac{c}{2} (\Delta \alpha_{\eta})^2] - \mathbb{E}[p_{\eta}^E \Delta K \Delta \alpha_{\eta}] \\
+ \frac{1}{2} F''(X^E - K^E \alpha_{\eta}^E)(\Delta X - K^E \Delta \alpha_{\eta} - \Delta K \alpha_{\eta}^E - \Delta K \Delta \alpha_{\eta})^2] \\
= K^E \mathbb{E}[(\eta R - c \alpha_{\eta}^E - p_{\eta}^E) \Delta \alpha_{\eta}] \\
+ \Delta K \mathbb{E}[\eta R (1 + \alpha_{\eta}^E) - \frac{c}{2} (\alpha_{\eta}^E)^2 - 1 - \alpha_{\eta}^E \alpha_{\eta}^E] \\
- K^E \mathbb{E}[\frac{c}{2} (\Delta \alpha_{\eta})^2] + \Delta K \mathbb{E}[-p_{\eta}^E \Delta \alpha_{\eta} + \eta R \Delta \alpha_{\eta} - c \alpha_{\eta} \Delta \alpha_{\eta} - \frac{c}{2} (\Delta \alpha_{\eta})^2] \\
+ \frac{1}{2} F''(X^E - K^E \alpha_{\eta}^E)(\Delta X - K^E \Delta \alpha_{\eta} - \Delta K \alpha_{\eta}^E - \Delta K \Delta \alpha_{\eta})^2].
$$
It holds that
\[ \mathbb{E}[\eta T (1 + \alpha_{\eta}^E) - \frac{C}{2} (\alpha_{\eta}^E)^2 - \alpha_{\eta}^E p_{\eta}^E - 1] = (1 - \mu) b \mathbb{E}[1 + \alpha_{\eta}^E], \]  
(3.72)
which follows from the first first-order condition in the competitive equilibrium. Inserting this, as well as the expression for \( p_{\eta}^E \) from equation (3.65), yields
\[
\Delta U = K^E (1 - \mu) b \mathbb{E}[\Delta \alpha_{\eta}]
+ \Delta K (1 - \mu) b \mathbb{E}[1 + \alpha_{\eta}^E]
- K^E \mathbb{E}[\frac{C}{2} (\Delta \alpha_{\eta})^2] + \Delta K \mathbb{E}[-p_{\eta}^E \Delta \alpha_{\eta} + \eta \bar{R} \Delta \alpha_{\eta} - c \alpha_{\eta}^E \Delta \alpha_{\eta} - \frac{C}{2} (\Delta \alpha_{\eta})^2]
+ \mathbb{E}[\frac{1}{2} F''(X^E - K^E \alpha_{\eta}^E) (\Delta X - K^E \Delta \alpha_{\eta} - \Delta K \alpha_{\eta}^E - \Delta K \Delta \alpha_{\eta})^2].
\]

Lemma 3.2 implies
\[ K^E \mathbb{E}[\Delta \alpha_{\eta}] + \Delta K \mathbb{E}[1 + \alpha_{\eta}^E] = - \Delta K \mathbb{E}[\Delta \alpha_{\eta}]. \]

Hence,
\[
\Delta U = -(1 - \mu) b \Delta K \mathbb{E}[\Delta \alpha_{\eta}]
- (K^E + \Delta K) \mathbb{E}[\frac{C}{2} (\Delta \alpha_{\eta})^2] + \Delta K \mathbb{E}[\Delta \alpha_{\eta} (-p_{\eta}^E + \eta \bar{R} - c \alpha_{\eta}^E)]
+ \mathbb{E}[\frac{1}{2} F''(X^E - K^E \alpha_{\eta}^E) (\Delta X - K^E \Delta \alpha_{\eta} - \Delta K \alpha_{\eta}^E - \Delta K \Delta \alpha_{\eta})^2].
\]
Together with equation (3.65), this implies
\[
\Delta U = -(1 - \mu) b \Delta K \mathbb{E}[\Delta \alpha_{\eta}]
- (K^E + \Delta K) \mathbb{E}[\frac{C}{2} (\Delta \alpha_{\eta})^2] + \mathbb{E}[\frac{1}{2} F''(X^E - K^E \alpha_{\eta}^E) (\Delta X - K^E \Delta \alpha_{\eta} - \Delta K \alpha_{\eta}^E - \Delta K \Delta \alpha_{\eta})^2].
\]
When \( \epsilon \) is sufficiently small, it holds that \( |\Delta K| < K^E \). Thus, \( K^E + \Delta K > 0 \) and hence, the first summand is negative. The second summand is also negative, as \( F(\cdot) \) is strictly
concave. Thus, we conclude that

\[ \Delta U < 0. \]

\[ \square \]

Of course, this proposition implies the following corollary:

**Corollary 3.2**

Assume the economy is in the competitive equilibrium with \( r = 0 \) and with the allocation \((K^E, X^E, \alpha^E_h, \alpha^E_l)\). Then, there exists an \( \epsilon > 0 \), such that for no \( \Delta \alpha_l > 0 \) with \( \Delta \alpha_l < \epsilon \), the one-sided regulation of capital adjustments by \( \Delta \alpha_l \) – as described in Section 3.2.5 – increases social welfare.

From different perspectives, we now want to explain why in our model, and conversely to Gersbach and Rochet (2012A), the moderation of price fluctuations – via the regulation of capital adjustments – does not improve welfare. The returns to an investor per unit of investment in the BS depend on two components: First, they depend on the returns per unit of the joint investment in the BS, which amount to

\[ \mathbb{E}\left[(1 + \alpha_\eta)\left(\eta R - b - \frac{\epsilon}{2} \alpha_\eta^2 - p_\eta \alpha_\eta\right)\right]. \]

Second, they depend on the expected relative size of the banker’s contribution to the joint investment, which amounts to \( \frac{E}{K \mathbb{E}[1 + \alpha_\eta]} \). Note that price fluctuations decrease the first component.

In Gersbach and Rochet (2012A), investors are kept tight at their participation constraint. Thus, when the first component is decreased due to price fluctuations, the second component has to be adjusted in the investors’ favor in order to keep the investors’ participation constraint fulfilled – i.e., to prevent the BS to become unprofitable for investors. This means a decrease of the expected capital investment in the BS by the investors. This is the negative externality on aggregate investment in the BS by the investors. This is the negative externality on aggregate investment in the BS which arises due to price fluctuations. Thus, moderating price fluctuations via the regulation of capital adjustments decreases this negative externality and increases social welfare.

In our model, investors are not kept at their participation constraint. Moreover, the second component is chosen by the investors at its most favorable value for them, namely \( \frac{E}{K \mathbb{E}[1 + \alpha_\eta]} = b \) (see Lemma 3.2). Thus, a slight moderation of capital price fluctuations will not affect the investors’ investment decisions, but will merely lead to a redistribution of the output between the investors and the entrepreneurs and will be in favor of the investors, as entrepreneurs benefit from high price fluctuations.

The following structural difference between the model in Gersbach and Rochet (2012A) and our model leads to the difference in the efficiency of regulation. In Gersbach and
Rochet (2012A), investors are kept at their participation constraint which reads
\[ K\mathbb{E}[(1 + \alpha\eta)(\eta R - b) - \frac{c}{2}\alpha^2\eta - p\eta\alpha\eta] = K - E. \]

In our model, bankers are those who are kept at their participation constraint, which reads
\[ bK\mathbb{E}[1 + \alpha\eta] = E. \] (3.73)

We see that the investors’ participation constraint contains the capital prices, while the bankers’ participation constraint does not. This explains why controlling the capital prices in Gersbach and Rochet (2012A) affects the investment decisions, while it does not affect them in our model. The mentioned structural difference is due to the bankers’ moral hazard, which makes them care only about the expected size of the investment, but not about the actual returns from the investment, which depend on the capital prices, in particular.

### 3.4 Moral Hazard: Effort Cost versus Leisure Benefit

In Gersbach and Rochet (2012A), as well as in the extensions considered so far, a banker’s moral hazard is modeled by assuming that the banker obtains a leisure benefit \( B \) if he does not exert effort. This *benefit approach* is common to many models of principal-agent relations with agent-moral-hazard. However, there is another approach, which is also common in the literature. Namely, one models principal agent relations with agent-moral-hazard assuming a *cost*, which the agent incurs, if he exerts effort. At first glance, it may seem as if these approaches were equivalent. However, in this section we show how the approach choice influences the results of Gersbach and Rochet (2012A).

We now continue our robustness analysis of the paper by Gersbach and Rochet (2012A) by showing how the results will change if we incorporate a moral hazard problem based on effort cost instead of leisure benefit into the model. In particular, we will explore how the structure of equilibria is affected if we use the *cost approach* instead of the *benefit approach*.

As in Gersbach and Rochet (2012A), in the optimal contract, the banker is kept tightly at his incentive compatibility constraint. Intuitively, the *cost approach* implies that the BS becomes less productive, as production costs increase. Hence, investment in the BS should decrease compared to the case when the *benefit approach* is used. We will now show that this intuition is right for high values of bank equity. However, for intermediate values of bank equity, this intuition has to be completed by accounting for general
equilibrium effects, as we will see.

We consider the basic model of Gersbach and Rochet (2012A), but assume that if the banker exerts effort, he incurs a cost $B_{k}$, and assume that the banker does not enjoy any leisure benefit in the no-effort case.\footnote{We chose the value of the effort cost to be equal to the value of the leisure benefit in Gersbach and Rochet (2012A) to ease the comparison of the two modeling approaches.} The effort cost, of course, enters the productivity of the BS. Namely the expected return in the BS in the effort case then amounts to $\bar{R} - B$ per unit of investment. On the one hand, affects the level of the social welfare in the model. On the other hand, if $\bar{R} - B \leq 1$, then the BS will be shut down completely, independently of the value of $E$, as consumption at $t = 0$ yields a higher utility. To avoid this case, the cost approach needs the following additional assumption:

**Assumption 3.4**

$\bar{R} - B > 1$.

Note that together with Assumption 3 of Gersbach and Rochet (2012A), which states that $\bar{R} - b < 1$, the above assumption restricts the range of admissible values of $B$ from below and from above.

The banker’s incentive-compatibility constraint under the cost approach reads

$$k_{\eta}((\tau - \Delta)_{b}^{S} + (1 - \tau + \Delta)_{b}^{F}) \leq k_{\eta}(\tau_{b}^{S} + (1 - \tau)_{b}^{F}) - k_{\eta}B.$$ 

This is equivalent to

$$k_{\eta}((\tau - \Delta)_{b}^{S} + (1 - \tau + \Delta)_{b}^{F}) + k_{\eta}B \leq k_{\eta}(\tau_{b}^{S} + (1 - \tau)_{b}^{F}).$$

which is the incentive-compatibility constraint under the benefit approach. Thus, we see that the incentive-compatibility constraint of the banker does not change if we use the cost approach.

However, under effort the banker’s participation constraint changes. Namely under the cost approach, it reads as follows at the aggregate level:

$$K(bE[1 + \alpha_{\eta}] - B) > E(1 + r).$$

This can be rewritten as

$$KbE[1 + \alpha_{\eta}] > E(1 + r) + KB.$$
Under effort the banker’s participation constraint under the \textit{benefit approach} reads

\[ KbE[1 + \alpha_\eta] > E(1 + r). \]

Therefore, we see that under the \textit{cost approach}, the upper bound on \( E \) that guarantees that the banker’s participation constraint is fulfilled, and thus, that there exists a competitive equilibrium with \( r = 0 \), in which \( \alpha_I > -1 \), is lower than under the \textit{benefit approach}. In Gersbach and Rochet (2012A) (in Corollary 1), this bound on \( E \) is denoted by \( E_{1^*} \). Under the \textit{cost approach}, we denote this bound by \( E_{c^*} \). Hence, we see that under the \textit{cost approach}, the range of values of \( E \), for which an equilibrium with \( r > 0 \) arises is larger than under the \textit{benefit approach}. The intuition behind this is as follows: Under the \textit{cost approach}, the banker’s participation constraint starts binding for lower levels of \( E \) than it does under the \textit{benefit approach}. Thus, to fulfill this constraint under the \textit{cost approach}, the value of \( K \) has to be higher in equilibrium. But then, less capital is invested in the TS, which thus becomes more productive. Consequently, an equilibrium with \( r = 0 \) cannot arise for a range of values of \( E \) for which it can arise under the \textit{benefit approach}.

Furthermore, note that in Gersbach and Rochet (2012A), the frictionless allocation with \( 1 + r = \overline{R} \) is realized if \( E \) exceeds the bound \( E_{1^*} \). With the \textit{cost approach}, however, the first-best allocation is never realized, as \( 1 + r \) can never exceed the value \( \overline{R} - B \). Hence, under the \textit{cost approach}, there is a threshold \( E_{c^*} \) which is smaller than \( E_{1^*} \), such that for \( E \geq E_{c^*} \) it holds that \( 1 + r = \overline{R} - B \). Thereby, the returns from the BS, as well as the effort costs, are divided between investors and bankers proportional to the capital invested by each of them. Namely the banker receives share \( e \) of the output and the investor receives share \( k - e \).

Finally, the bound \( E_{1^*} \) on \( E \) from Corollary 1 in Gersbach and Rochet (2012A), below which the equilibrium with \( r = 0 \) and \( \alpha_I = -1 \) arises, does not change with the introduction of the \textit{cost approach}. The reason is that this bound was constructed using only the incentive-compatibility constraint, but not the participation constraint of the banker.

Thus, the structure of the equilibria under the \textit{cost approach} may be summarized as follows, in analogy to Corollary 1 from Gersbach and Rochet (2012A):

\begin{proposition}
There exist three threshold values \( E_{c^{***}} > E_{c^{**}} > E_{c^*} = H(\sqrt{1-q}) \) which induce the following types of equilibria:

(i) If \( E \geq E_{c^{***}} \), then \( r = \overline{R} - B - 1 \) and effort costs and returns are shared by investors and bankers proportionally to their invested capital;

(ii) if \( E_{c^{**}} > E > E_{c^*} \), then \( 0 < r < \overline{R} - B - 1 \);
\end{proposition}
(iii) if $E^{* *}_c \geq E > E^{*}_c$, then $r = 0$ and $\alpha_l > -1$;

(iv) if $E^{*}_c \geq E$, then $r = 0$ and $\alpha_l = -1$; and

thereby, it holds that $E^{***}_c < E^{***}_1$, $E^{**}_c < E^{**}_1$ and $E^{*}_c = E^{*}_1$.

We see that compared to Corollary 1 in Gersbach and Rochet (2012A) the differences in our result are mainly of quantitative character. Especially the competitive equilibrium with $r = 0$, which is the most interesting for the exploration of the impact of regulation, also exists under the cost approach.

Finally, note that it is straightforward that the results in Gersbach and Rochet (2012A) on social efficiency and regulation departing from the competitive equilibrium with $r = 0$ and $\alpha_l > -1$ apply analogously to the equilibrium described in Proposition 3.11, (iii).

3.5 Conclusion

We have shown that the structure of equilibria in Gersbach and Rochet (2012A) is robust with respect to the introduction of Macroshocks. Especially, non-zero capital adjustments in the first period do occur, although the shock hits both sectors. The reason for this lies in the relative change of the informational rents compared to the outputs in the different states. Moreover, we have demonstrated that regulation of short-term debt in the presence of Macroshocks can increase social welfare and thus act as a macroprudential tool for the moderation of credit cycles.

In the modification of the model of Gersbach and Rochet (2012A) in which the investors rather than the bankers write the contracts, the results of Gersbach and Rochet (2012A) on the existence and uniqueness of an equilibrium with $r = 0$ and on an inner solution for capital adjustments also do hold. Moreover, the sensitivity analysis of the equilibria arising for different levels of aggregate bankers’ equity and for different levels of the severity of moral hazard studied in Gersbach and Rochet (2012A) still hold qualitatively in our current setting. However, unlike in Gersbach and Rochet (2012A), in this setting, the investors always keep the bankers at their participation constraint to minimize informational rent payments to the bankers and to thereby still benefit from the bankers’ capital endowments. Thus, the expected amount of capital invested in the BS is determined by the bankers’ aggregate endowment and by the severity of moral hazard. Regulation of short-term debt cannot increase social welfare, as it cannot undo the effect described above.

In a further modification of the model, we have shown that the qualitative results of Gersbach and Rochet (2012A) are robust with respect to a change of the approach to modeling moral hazard.
In general, note that the regulator faces further substantial constraints which are not captured by our model. As was shown by Gala and Volpin (2012), for instance, public information can have a strong impact on allocation efficiency in a context similar to ours. In this context, the idea for a rationale for the type of regulation that we investigated, still remains valid, as the use of regulation is measured by its aggregate impact, in which welfare losses and gains stemming from different levels of public information cancel out. In general, we should keep in mind that the world is much more complex than our model, and future research should in particular focus on the analysis of the interactions of effects that have been discovered and explained so far. Furthermore, note that as in Gersbach and Rochet (2012A), one-sided regulation of capital adjustments means the imposition of a regulatory upper bound on short-term debt. A regulatory liquidity ratio can implement such an upper bound. Thus, our results can be used to assess the macroeconomic impact of the introduction of the Net Stable Funding Ratio of Basel III.
## 3.6 Notation List

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>BS</td>
<td>Banking sector</td>
</tr>
<tr>
<td>TS</td>
<td>Traditional sector</td>
</tr>
<tr>
<td>$t$</td>
<td>Index of a period, $t \in {0, 1, 2}$</td>
</tr>
<tr>
<td>$i$</td>
<td>Index of a banker</td>
</tr>
<tr>
<td>$e_i$</td>
<td>Initial endowment of banker $i$</td>
</tr>
<tr>
<td>$e$</td>
<td>Initial endowment of a typical banker</td>
</tr>
<tr>
<td>$E$</td>
<td>Aggregate bankers’ endowment, $0 &lt; E &lt; 1$</td>
</tr>
<tr>
<td>$U$</td>
<td>Expected aggregate consumption</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Economic shock</td>
</tr>
<tr>
<td>$h$</td>
<td>High value of $\eta$</td>
</tr>
<tr>
<td>$l$</td>
<td>Low value of $\eta$, $0 &lt; l &lt; 1 &lt; h$</td>
</tr>
<tr>
<td>$q$</td>
<td>Probability of the event that $\eta = h$</td>
</tr>
<tr>
<td>$Pr(\eta)$</td>
<td>Probability of the event that $\eta$ occurs</td>
</tr>
<tr>
<td>$m$</td>
<td>Expectation of $\eta$, normalized such that $m = 1$</td>
</tr>
<tr>
<td>$C_0$</td>
<td>Aggregate consumption in period 0</td>
</tr>
<tr>
<td>$C_{2\eta}$</td>
<td>Aggregate consumption in period 2 depending on the value of $\eta$</td>
</tr>
<tr>
<td>$X$</td>
<td>Aggregate investment in the TS at $t = 0$</td>
</tr>
<tr>
<td>$X^E$</td>
<td>Aggregate investment in the TS at $t = 0$ in the competitive equilibrium</td>
</tr>
<tr>
<td>$F(X)$</td>
<td>Function describing the aggregate output of the TS in the baseline model</td>
</tr>
<tr>
<td>$k$</td>
<td>Amount invested in the BS at $t = 0$ by a typical investor and a typical banker</td>
</tr>
<tr>
<td>$K$</td>
<td>Aggregate investment in the BS at $t = 0$</td>
</tr>
<tr>
<td>$K^E$</td>
<td>Aggregate investment in the BS at $t = 0$ in the competitive equilibrium</td>
</tr>
<tr>
<td>$p_\eta$</td>
<td>Capital price at $t = 1$ depending on the value of $\eta$</td>
</tr>
<tr>
<td>$p^E_\eta$</td>
<td>Capital price at $t = 1$ in the competitive equilibrium depending on the value of $\eta$</td>
</tr>
<tr>
<td>$r$</td>
<td>Interest rate, $r \geq 0$</td>
</tr>
<tr>
<td>$\alpha_\eta$</td>
<td>Rate of credit growth in the BS at $t = 1$ depending on the value of $\eta$</td>
</tr>
<tr>
<td>$\alpha^E_\eta$</td>
<td>Rate of credit growth in the BS at $t = 1$ in the competitive equilibrium depending on the value of $\eta$</td>
</tr>
<tr>
<td>$\bar{x}$</td>
<td>Expectation of $\alpha_\eta$</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>Variance of $\alpha_\eta$</td>
</tr>
<tr>
<td>$k_\eta$</td>
<td>Capital invested in the BS by a typical bank after the adjustment at $t = 1$</td>
</tr>
<tr>
<td>$c$</td>
<td>Adjustment cost parameter, $c &gt; 0$</td>
</tr>
<tr>
<td>$o$</td>
<td>Index of the project outcome in the BS, $o \in {S, F}$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Meaning</td>
</tr>
<tr>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>$\bar{R}^o$</td>
<td>Value of $\tilde{R}_i$ depending on the project outcome, $R^S &gt; R^F$</td>
</tr>
<tr>
<td>$\tilde{R}$</td>
<td>Idiosyncratic return of the project of a typical banker, $\tilde{R} \in {R^S, R^F}$</td>
</tr>
<tr>
<td>$\bar{R}$</td>
<td>Expectation of $\tilde{R}_i$ in the case that banker $i$ exerts effort</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Project’s success probability, if effort is exerted</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Decrease of success probability, if no effort is exerted</td>
</tr>
<tr>
<td>$B$</td>
<td>Private leisure benefit per unit of investment, $B &gt; 0$</td>
</tr>
<tr>
<td>$b^\eta_i$</td>
<td>Payment to the banker by the investor per unit of investment, depending on $\sigma$ and $\eta$</td>
</tr>
<tr>
<td>$C(k, \alpha^\eta, b^\eta_i)$</td>
<td>Contract between a banker and an investor</td>
</tr>
<tr>
<td>$S^\eta(p_\eta, p_h)$</td>
<td>Supply of capital in the model where the shock $\eta$ hits only the BS</td>
</tr>
<tr>
<td>$S^\eta(p_\eta)$</td>
<td>Supply of capital in the model where the shock $\eta$ hits both, the BS and the TS</td>
</tr>
<tr>
<td>$b$</td>
<td>Expected payment to the banker per unit of investment under an optimal contract</td>
</tr>
<tr>
<td>$\rho_0(r)$</td>
<td>Market imposed capital ratio in the rigid economy</td>
</tr>
<tr>
<td>$\rho(r)$</td>
<td>Market imposed capital ratio in the baseline model</td>
</tr>
<tr>
<td>$G(\sigma)$</td>
<td>Function that determines a unique inner competitive equilibrium with $r = 0$ in the model with macroshocks</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Lagrange multiplier in the competitive equilibrium in the model where $\eta$ hits both sectors</td>
</tr>
<tr>
<td>$B^\eta$</td>
<td>Private leisure benefit per unit investment depending on $\eta$</td>
</tr>
<tr>
<td>$b^\eta_i$</td>
<td>Expected payment to the banker per unit of investment under an optimal contract in the model where the leisure benefit depends on $\eta$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Lagrange multiplier in the competitive equilibrium in the model where $\eta$ hits both sectors and the leisure benefit depends on $\eta$</td>
</tr>
<tr>
<td>$\Delta \alpha^\eta$</td>
<td>Change of $\alpha^\eta$ under regulation</td>
</tr>
<tr>
<td>$\Delta X$</td>
<td>Change of $X$ under regulation</td>
</tr>
<tr>
<td>$\Delta K$</td>
<td>Change of $K$ under regulation</td>
</tr>
<tr>
<td>$\Delta p^\eta$</td>
<td>Change of $p^\eta$ under regulation</td>
</tr>
<tr>
<td>$\hat{\alpha}^\eta$</td>
<td>$\alpha^E + \Delta \alpha^\eta$</td>
</tr>
<tr>
<td>$\hat{X}$</td>
<td>$X^E + \Delta X$</td>
</tr>
<tr>
<td>$\tilde{p}^\eta$</td>
<td>$p^E + \Delta p^\eta$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Lagrange multiplier under one-sided regulation in the model where $\eta$ hits both sectors</td>
</tr>
<tr>
<td>$J(\sigma)$</td>
<td>Function that determines a unique inner competitive equilibrium with $r = 0$ in the model where investors write the contracts</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Lagrange multiplier under one-sided regulation in the model where investors write the contracts</td>
</tr>
</tbody>
</table>
4 On Moral Hazard: Costly Effort or Enjoyable Leisure – Does it Matter?

4.1 Introduction

Approaches to model moral hazard

Moral hazard is a crucial ingredient in many micro- and macroeconomic models, where it is used to explain arising inefficiencies. In the literature on financial intermediation, in particular, moral hazard plays an important role (see, for instance, Freixas and Rochet (2008)).

The idea of modeling the subjection of an agent to moral hazard relies on three main assumptions. First, it is assumed that the agent’s effort choice is not observable by the principal. Second, it is assumed that the agent’s effort choice has an impact on the produced output: The higher the effort, the higher the expected output. Third, it is assumed that ceteris paribus, the agent’s utility is decreasing with his effort. This last assumption is often formulated either as an assumption of the occurrence of an effort cost for the agent if he does exert effort (cost approach) or as an assumption of the occurrence of a private leisure benefit for the agent if he does not exert effort (benefit approach).¹ The focus of our paper will lie on this third assumption.

There are many papers that use one of the above-mentioned approaches to moral hazard. We name just a few of them to give an idea of some applications. A much-quoted paper that incorporates the benefit approach is Holmström and Tirole (1997). In their work, financial intermediaries monitor entrepreneurs who are subject to moral hazard. Gersbach and Rochet (2012A), to whose work we will come back later, use the benefit approach in a macroeconomic model to model bankers’ moral hazard in their interaction with investors. In their famous investigation of the causes of unemployment, Shapiro and

¹ However, there are also models that keep this assumption on an abstract level, without specifying a leisure benefit or an effort cost and just stating that the agent’s utility decreases in effort (see Holmström (1979), for instance). Keuschnigg and Nielsen (2004) consider a further way to model moral hazard involving the agent’s outside option.
Stiglitz (1984) use the cost approach to model workers’ moral hazard. This approach is also used by Emons (1988) who models a consumer’s treatment of a purchased product in the presence of a warranty for this product. Fuchs (2007) uses the cost approach to model agent moral hazard in combination with a principal’s private evaluation of the output. In their experimental study on reciprocity in the context of contract enforcement, Fehr et al. (1997) use the cost approach for the design of their experiment. Köszegi (2013) provides an overview of current behavioral economics research on moral hazard.

Most of these papers do not explain why they prefer one approach to the other. In fact, these approaches seem to be regarded as economically equivalent. This view might be correct in many cases, where the choice of an approach merely changes some technical properties of the model, but not its qualitative implications. However, when moral hazard occurs in a bargaining situation – as opposed to a take-it-or-leave-it offer – there may occur qualitative and quantitative differences of the model’s results, depending on the approach to agent moral hazard that is used. This is what we will show in this chapter. This idea is related to Keuschnigg and Nielsen (2004). They conduct a sensitivity analysis of the welfare effects of taxation by considering two different ways to model moral hazard, namely the benefit approach and the approach in which shirking is realized by withholding some effort and using it for the earning of additional income in some outside projects. They find that in a general equilibrium framework with risk-averse agents, the welfare effects of taxes might crucially depend on the way to modeling moral hazard. Contrary to Keuschnigg and Nielsen (2004), we consider a situation where in addition to the moral hazard, there is bargaining between the principal and the agent. Furthermore, we keep our microeconomic model deliberately as simple as possible to focus on the distinction of the approaches to model moral hazard.

**Moral hazard and bargaining power**

As mentioned above, our paper considers a situation in which both moral hazard and bargaining occur between the players – as opposed to a take-it-or-leave-it offer made either by the agent or by the principal. This situation was considered in various papers. Balkenborg (2001) provides an impressive example of the importance of paying attention to the bargaining power distribution when modeling moral hazard problems, by comparing two groups of papers, that use very similar models while yielding opposite results. In his model, Balkenborg (2001) argues that such a difference in the results arises only from differing assumptions on the bargaining power distribution between the principal and the agent. Pitchford (1998) finds that the distribution of the bargaining power has an impact on social welfare in the presence of moral hazard – modeled by the cost approach – when

---

2 For an overview over different results about reciprocity and moral hazard modeled via the cost approach see Fehr and Gächter (2000).
the agent has limited liability and when the effort choice is continuous.

In our paper we assume the bargaining outcome to be the Generalized Nash Bargaining Solution. The same method to model bargaining power was chosen by Balkenborg (2001). Some articles from the labor economics literature also incorporate the Generalized Nash Bargaining Solution to model the bargaining between workers and employees, and at the same time consider the workers’ moral hazard. Rocheteau (2001) investigates equilibrium unemployment and wage formation, and uses the \textit{cost approach} to model moral hazard. Costain and Jansen (2010) use the \textit{benefit approach} and study the fluctuations of the emergence and disappearance of jobs. Mavroeidis and Malcolmson (2011) use the \textit{cost approach} in a quantitative model to investigate how well different theoretical approaches to modeling wage determination fit US data.

\textbf{Main results}

The main idea in our paper is complementary to Demougin and Helm (2006). They model moral hazard via the \textit{cost approach}, and compare the resulting allocations between the principal and the agent, depending on the method used to model the bargaining situation. They consider three methods: a \textit{take-it-or-leave-it} offer by the principal given the agent’s outside option, an alternating offer game as in Rubinstein (1982), and the Generalized Nash Bargaining Solution. They find that for sufficiently high levels of the principal’s bargaining power, a marginal change of the bargaining power distribution has differing impacts on the resulting allocations, depending on the choice of method. Our paper is complementary to Demougin and Helm (2006) in the sense that we set one method to model the bargaining situation and vary the approaches to model moral hazard.

Our model is a simple two-player game between a principal and an agent who may sign a contract on taking part in a joint project and being paid after the project has ended. The agent is subject to moral hazard with regard to exerting effort or not, so that we consider a simple binary effort choice. The effort choice influences the expectation of the outcome of the players’ joint project. We consider two variants of the model, one with the \textit{benefit approach} and one with the \textit{cost approach} to model moral hazard. The outcome of the project is divided between the agent and the principal according to the contract signed before the start of the project. The agent and principal are both risk-neutral and may both have non-zero outside options. As a solution of the contract bargaining, we assume Generalized Nash Bargaining.

Our main contribution is the derivation of some qualitative and quantitative differences in the allocations resulting from our model, depending on which of the two approaches to model moral hazard is used. In particular, we explore the conditions for the bindingness (tightness) of the incentive compatibility constraint under both approaches, and point out some biases in these approaches’ allocations. The bargaining power distribution between
the players will turn out to be a crucial factor for the occurrence of qualitative and quantitative differences of the results of the model depending on the approach chosen. We will also discuss some limitations of our results. We will discuss, in particular, why our results do not apply to the model of Gersbach and Rochet (2012A), as was shown in Section 3.4 in Chapter 3 of this dissertation.

This chapter is organized as follows: In the next section we will describe our model. In Section 4.3 we calculate the allocations resulting from the two approaches to modeling moral hazard. We analyze qualitative and quantitative differences of these allocations in Section 4.4. Section 4.5 contains a summary and a discussion.

4.2 The Model

We consider a simple principal-agent problem with an agent subject to moral hazard and a cake to be shared between the principal and the agent. We assume that both players are risk-neutral.

First, we describe the multi-stage structure of the game. Then, we will specify the stages of the game.

Stage 1: The principal and the agent may agree to sign a contract about a joint project. If they do not agree, the game ends and the players obtain their outside options — namely $e_p$ for the principal and $e_a$ for the agent — as payoffs. Otherwise, the game moves on to the second stage.

Stage 2: The agent decides whether to exert effort or not. Depending on the model specifications described later, the agent either enjoys a leisure benefit in the case of no-effort and does not incur effort costs in the case of effort (benefit approach), or he does not enjoy a leisure benefit in the case of no-effort and incurs effort costs in the case of effort (cost approach). The principal knows the kind of moral hazard – benefit or cost – faced by the agent, but the principal cannot observe whether the agent exerts effort or not.

Stage 3: The project outcome $o$ is observed by both players. It can take two values: $o = S$ (success) or $o = F$ (failure). Depending on the contract and on the project outcome, both players obtain their payoffs, which will be described later.
4.2 The Model

We assume

\[ \text{prob}[o = S|\text{effort}] = \tau \]

and

\[ \text{prob}[o = S|\text{no-effort}] = \tau - \Delta, \]

where \( \tau, \Delta \in (0, 1) \) and \( \tau > \Delta \).

We assume that the project output is \( R_S \) in the case of success and \( R_F \) in the case of failure, with \( R_S > R_F > 0 \).

A contract between an agent and a principal specifies the shares of \( R_S \) and \( R_F \) which the agent obtains in the case of success and of failure, respectively. Thus, a contract between an agent and a principal is a pair \((b_S, b_F)\) where for \( o = S, F \), \( b_o \in [0, R_o] \). \( b_o \) denotes the share the agent receives if the outcome is \( o \). Accordingly, \( 1 - b_o \) is the share of the principal.

Let \( \bar{R} := \tau R_S + (1 - \tau)R_F \) denote the expected project output under effort, and let \( \bar{R} := (\tau - \Delta)R_S + (1 - \tau + \Delta)R_F \) denote the expected project output under no-effort.

We make the following assumption on the payoff structure:

Assumption 4.1

\( \bar{R} < e_p \).

This assumption states that the principal will not sign a contract if he expects that the agent is not going to exert effort under it. Thus, without loss of generality, it suffices to restrict the set of possible contracts to those under which the agent exerts effort.

4.2.1 Modeling Moral Hazard with the Benefit Approach

Under the benefit approach, the agent enjoys a leisure benefit of \( B \), with \( B \geq 0 \) in the case of no-effort. In the case of effort, he does not enjoy this benefit, but does not incur effort costs either. Then the agent exerts effort if and only if his incentive compatibility constraint is fulfilled. It reads as follows:

\[ \tau b_S + (1 - \tau)b_F \geq (\tau - \Delta)b_S + (1 - \tau + \Delta)b_F + B \]

\[ \Leftrightarrow \quad b_S - b_F \geq \frac{B}{\Delta}. \]

Note that in Gersbach and Rochet (2012A) the agent, i.e. the banker, choses the values of his share of the project output to be \( b_S = \frac{\bar{R}}{\Delta} \) and \( b_F = 0 \), which make his incentive compatibility constraint hold tight. He does this, as it allows to increase the size of the investment and thus increases the size of the project output, which subsequently increases
the banker’s payoff. In our current setting, the size of the cake is given. More precisely, its expected value only depends on the agent’s effort choice. Thus, the agent always wishes to increase the expected value of his share of this cake as long as thereby the principal’s share is still big enough to sign the contract. To keep things simple, we will also assume that \( b^F = 0 \). Hence, a contract may be represented as \( b^S \in [0, R^S] \), which denotes the share that the agent obtains if \( o = S \).

Then, the incentive compatibility constraint reads

\[
b^S \geq \frac{B}{\Delta},
\]

We make the following four assumptions about the severity of moral hazard and the payoff structure. They will hold throughout the remaining part of the paper whenever the benefit approach is used.

**Assumption 4.2**

\[
\bar{R} > e_p + e_a.
\]

On the one hand, this assumption implies that there exists a contract that the principal would sign if he expected the agent to exert effort under it. On the other hand, it implies that in the effort case, the project output \( \bar{R} \) suffices to deliver payoffs that are above the principal’s and agent’s participation constraints to both of them simultaneously. Note that \( B \) does not enter Assumption 4.2, as in the effort case the agent does not obtain the leisure benefit.

**Assumption 4.3**

\[
\frac{\bar{R}}{\Delta} \leq R^S.
\]

This inequality states that there exist feasible contracts that satisfy the incentive compatibility constraint.

**Assumption 4.4**

\[
e_p < \bar{R} - \tau \frac{B}{\Delta}.
\]

This inequality states that there exists a contract under which the agent — given his participation — would exert effort and which the principal would sign.

Given a contract \( b^S \), the agent’s participation constraint if he will exert effort is given by

\[
e_a \leq \tau b^S
\]

\(^3\text{We could also derive this assumption from more basic assumptions on the parameters.}\)
and is given by

\[ e_a \leq (\tau - \Delta)b^S + B \]

if the agent will not exert effort.

**Assumption 4.5**

\[ \tau R^S > e_a. \]

This assumption states that there exist contracts, e.g. \( b^S = R^S \), which the agent would sign and be better off with if he decided to exert effort, than he would be with his outside option.

We assume that the solution of the contract bargaining is the Generalized Nash Bargaining Solution. Let \( \beta \) denote the bargaining power of the principal. For the rest of our paper, we will assume a non-polar bargaining power distribution, i.e. \( \beta \in (0, 1) \).\(^4\) In the absence of moral hazard, the corresponding maximization problem reads as follows

\[
\max_{b^S \in [0, R^S]} (R - b^S \tau - e_p)^{\beta} \cdot (b^S \tau - e_a)^{1-\beta}.
\]

We denote the solution of this maximization problem by \( b^S_{\beta} \). Note that this frictionless problem is identical for both approaches to modeling moral hazard.

In the presence of moral hazard, the maximization problem has to account for the incentive compatibility constraint of the agent. The problem reads:

\[
\max_{b^S \in [B^S \Delta, R^S]} (R - b^S \tau - e_p)^{\beta} \cdot (b^S \tau - e_a)^{1-\beta}.
\]

We denote the solution of this maximization problem by \( b^S_{b,mh} \). In the lower index, \( b \) stands for the benefit approach and \( mh \) stands for “moral hazard”, in contrary to \( * \), which stands for the frictionless case.

### 4.2.2 Modeling Moral Hazard with the Cost Approach

In this subsection, in the no-effort case, the agent does not enjoy a leisure benefit. In the effort case, however, he has to bear an effort cost of \( C \), with \( C \geq 0 \). Again, the agent exerts effort if and only if his incentive compatibility constraint is fulfilled. It reads as

---

\(^4\) The assumption that \( 0 < \beta < 1 \) is crucial for our analysis. In the polar cases \( \beta = 0 \) and \( \beta = 1 \), our results do not hold, as those cases essentially differ from the bargaining situation considered in our model.
follows:

\[ \tau b^S + (1 - \tau) b^F - C \geq (\tau - \Delta) b^S + (1 - \tau + \Delta) b^F \]
\[ \Leftrightarrow b^S - b^F \geq \frac{C}{\Delta}. \]

Again, to keep things simple, we assume that \( b^F = 0 \). Hence, a contract may be represented as \( b^S \in [0, R^S] \), which denotes the share that the agent obtains if \( o = S \).

Then, the incentive compatibility constraint reads

\[ b^S \geq \frac{C}{\Delta}. \]

We make the following four assumptions about the severity of moral hazard and the payoff structure, which will hold throughout the remaining part of the paper whenever the cost approach is used.

**Assumption 4.6**
\[ R > e_p + e_a + C. \]

On the one hand, this assumption implies that there exists a contract that the principal would sign if he expected the agent to exert effort under it. On the other hand, it implies that in the effort case, the project output \( R \) suffices to deliver payoffs that are above the principal’s and agent’s participation constraints to both of them simultaneously. Note that the effort cost \( C \) enters Assumption 4.6, as under effort the agent loses \( C \). This is an important difference to Assumption 4.2, which does not contain the leisure benefit \( B \), as in the benefit approach under effort the agent does not obtain \( B \).

**Assumption 4.7**
\[ \frac{C}{\Delta} \leq R^S. \]

This inequality states that there exist feasible contracts that satisfy the incentive compatibility constraint.

**Assumption 4.8**
\[ e_p < \frac{R}{\Delta} - \tau \frac{C}{\Delta}. \]

This inequality states that there exists a contract under which the agent — given his participation — would exert effort and which the principal would sign.

Given a contract \( b^S \), the agent’s participation constraint if he will exert effort is given
by

\[ e_a \leq \tau b^S - C \]

and is given by

\[ e_a \leq (\tau - \Delta)b^S \]

if the agent will not exert effort.

**Assumption 4.9**

\[ \tau R^S - C > e_a. \]

This assumption states that there exist contracts (e.g. \( b^S = 1 \)) which the agent would sign and with which – did he decide to exert effort – he would be better off than with his outside option.

Again we assume that the solution of the contract bargaining is the Generalized Nash Bargaining Solution. As in the benefit approach, \( \beta \) denotes the bargaining power of the principal.

In the presence of moral hazard, the maximization problem has to account for the agent’s incentive compatibility constraint. The problem reads

\[
\max_{b^S \in [C, R^S]} (R - b^S \tau - e_p)^\beta \cdot (b^S \tau - e_a)^{1-\beta}.
\]

We denote the solution of this maximization problem by \( b^S_{c,mh} \). In the lower index, \( c \) stands for cost approach and \( mh \) stands for moral hazard.

### 4.3 Resulting Allocations

First, we calculate the allocation resulting from the absence of moral hazard. The first order condition of the concave maximization problem

\[
\max_{b^S \in [0, R^S]} (R - b^S \tau - e_p)^\beta \cdot (b^S \tau - e_a)^{1-\beta}
\]

reads

\[
-\beta \tau (R - b^S \tau - e_p)^{\beta-1} \cdot (b^S \tau - e_a)^{1-\beta} + (1 - \beta) \tau (R - b^S \tau - e_p)^\beta \cdot (b^S \tau - e_a)^{-\beta} = 0.
\]
This is equivalent to

\[-\beta(R - b^S\tau - e_p)^{\beta-1} \cdot (b^S\tau - e_a)^{1-\beta} + (1 - \beta)(R - b^S\tau - e_p)^\beta \cdot (b^S\tau - e_a)^{-\beta} = 0\]

\[\Leftrightarrow -\beta(R - b^S\tau - e_p)^{\beta-1} \cdot (b^S\tau - e_a) + (1 - \beta) = 0\]

\[\Leftrightarrow b^S\tau - e_a - \frac{1 - \beta}{\beta}(R - b^S\tau - e_p) = 0\]

\[\Leftrightarrow b^S\tau(1 + \frac{1 - \beta}{\beta}) - e_a - \frac{1 - \beta}{\beta}(R - e_p) = 0\]

\[\Leftrightarrow b^S\tau\frac{1}{\beta} - e_a - \frac{1 - \beta}{\beta}(R - e_p) = 0\]

\[\Leftrightarrow b^S\tau - \beta e_a - (1 - \beta)(R - e_p) = 0\]

\[\Leftrightarrow b^*_\tau = \frac{\beta e_a + (1 - \beta)(R - e_p)}{\tau}.

### 4.3.1 Allocations under the Benefit Approach

The following lemma characterizes the allocation resulting from the benefit approach:

**Lemma 4.1**

If \( b^*_\tau \geq \frac{B}{\Delta} \), then \( b^*_{b,mh} = b^*_\tau \). Else, \( b^*_{b,mh} = \frac{B}{\Delta} \).

**Proof.** The first part follows immediately from the fact that if the solution of the unconstrained problem satisfies a constraint, then it is also the solution of the constrained problem. The second part follows immediately from the concavity of the objective function. □

Of course, \( b^*_\tau \), viewed as a function of \( \beta \), is strictly decreasing due to Assumption 4.2. Moreover, \( \lim_{\beta \to 1} b^*_\tau(\beta) = \frac{e_a}{\tau} \) and \( \lim_{\beta \to 0} b^*_\tau(\beta) = \frac{R - e_p}{\tau} \) due to Assumption 4.4.

Thus, if

\[\frac{e_a}{\tau} \leq \frac{B}{\Delta},\]

there exists \( \beta_{b,mh} \in (0, 1) \), such that for all \( \beta \geq \beta_{b,mh} \), \( b^*_\tau(\beta) = \frac{B}{\Delta} \) holds, and for all other values of \( \beta \), \( b^*_\tau(\beta) > \frac{B}{\Delta} \) holds.

Thus \( \beta_{b,mh} \) is the value of \( \beta \) above which the agent’s incentive compatibility constraint starts to bind.\(^5\)

---

\(^5\) When we say that a non-strict inequality constraint binds, we mean that it holds with equality.
4.3 Resulting Allocations

To calculate $\beta_{b,mh}$, we solve the equation

$$\frac{B}{\Delta} = \frac{\beta e_a + (1 - \beta)(R - e_p)}{\tau}.$$  

This yields

$$\beta_{b,mh} = \frac{R - \tau \frac{B}{\Delta} - e_p}{R - e_a - e_p}.$$  

Note that due to $\frac{e_a \tau}{\tau} \leq \frac{B}{\Delta}$, it holds that $\beta_{b,mh} \in [0, 1]$. Thereby, a polar value of $\beta_{b,mh}$ does not contradict our model assumptions, as $\beta_{b,mh} = 0$ only means that for all admissible values $\beta \in (0, 1)$, the incentive compatibility constraint binds, while $\beta_{b,mh} = 1$ only means that for no admissible value $\beta \in (0, 1)$ the incentive compatibility constraint binds.

Hence, if it holds that $\frac{e_a \tau}{\tau} \leq \frac{B}{\Delta}$, then incentive compatibility binds exactly for $\beta \geq \frac{R - \tau \frac{B}{\Delta} - e_p}{R - e_a - e_p}$, which means that $b_{b,mh}^S = \frac{B}{\Delta}$.

If, on the contrary, it holds that $\frac{e_a \tau}{\tau} > \frac{B}{\Delta}$, then the incentive compatibility constraint does not bind for any value of $\beta$.

We summarize these findings in the following lemma:

**Lemma 4.2**

If $\frac{e_a \tau}{\tau} > \frac{B}{\Delta}$, then the incentive compatibility constraint does not bind for any bargaining power distribution.

If $\frac{e_a \tau}{\tau} \leq \frac{B}{\Delta}$, then the incentive compatibility constraint binds if and only if $\beta \geq \beta_{b,mh} = \frac{R - \tau \frac{B}{\Delta} - e_p}{R - e_a - e_p}$.

Note that $\frac{R - \tau \frac{B}{\Delta}}{R - e_a - e_p} =: \pi_{b,mh}$ is the principal’s expected payoff when incentive compatibility binds, and $\frac{R - e_a}{}=: \pi$ is the principal’s expected payoff in the absence of moral hazard if the agent is kept at his participation constraint. Thus, we obtain the following representation of $\beta_{b,mh}$:

$$\beta_{b,mh} = \frac{\pi_{b,mh} - e_p}{\pi - e_p}.$$  

If the inequality $\frac{e_a \tau}{\tau} > \frac{B}{\Delta}$ holds, the graph of $b_{b,mh}^S(\beta)$ has a form as depicted in Figure 4.1. If the inequality $\frac{e_a \tau}{\tau} < \frac{B}{\Delta}$ holds, the graph of $b_{b,mh}^S(\beta)$ has a form as depicted in Figure 4.2. The kink means that the incentive compatibility constraint binds.
4.3.2 Allocations under the Cost Approach

The maximization problem under the cost approach reads as follows:

\[ \max_{b_S \in [\frac{C}{\Delta}, R_S]} (\bar{R} - b_S^S \tau - e_p)^\beta \cdot (b_S^S \tau - C - e_a)^{1-\beta}. \]

First, let us solve the following unconstrained maximization problem:\(^6\)

\[ \max_{b^s \in [0, R_S]} (\bar{R} - b^S \tau - e_p)^\beta \cdot (b^S \tau - C - e_a)^{1-\beta}. \]

The solution is

\[ b^S = \frac{\beta (e_a + C) + (1 - \beta) (\bar{R} - e_p)}{\tau}. \]

A similar lemma to Lemma 4.1 holds.

**Lemma 4.3**

*If \( b^S \geq \frac{C}{\Delta} \), then \( b_{c,mh}^S = b^S \). Else, \( b_{c,mh}^S = \frac{C}{\Delta} \).*

The proof is analogous to the proof of Lemma 4.1.

---

\(^6\) This maximization problem should not be confused with the maximization problem in the absence of moral hazard, which is also unconstrained.
Figure 4.2: Share $b_{b,mh}^S$ of the agent, depending on the principal’s bargaining power under the benefit approach in the case of $\frac{e_a}{\tau} < \frac{B}{\Delta}$.

Of course, $b^S$, viewed as a function of $\beta$, is strictly decreasing due to Assumption 4.6. Moreover, $\lim_{\beta \to 1} b^S(\beta) = \frac{e_a + C}{\tau}$ and $\lim_{\beta \to 0} b^S(\beta) = \frac{\bar{R} - e_p}{\tau} > \frac{C}{\Delta}$ due to Assumption 4.8. Hence, if

$$\frac{e_a + C}{\tau} \leq \frac{C'}{\Delta},$$

there exists $\beta_{c,mh} \in (0, 1)$, such that for all $\beta \geq \beta_{c,mh}$ it holds that $b^S(\beta) = \frac{C}{\Delta}$ and for all other values of $\beta$, it holds that $b^S(\beta) > \frac{C}{\Delta}$.

Thus $\beta_{c,mh}$ is the value of $\beta$ above which the agent’s incentive compatibility constraint starts to bind.

To calculate $\beta_{c,mh}$, we solve the equation

$$\frac{C}{\Delta} = \beta (e_a + C) + (1 - \beta) (\bar{R} - e_p) \frac{\tau}{\tau}.$$ 

This yields

$$\beta_{c,mh} = \frac{\bar{R} - \tau \frac{C}{\Delta} - e_p}{\bar{R} - e_a - C - e_p}.$$

Note that due to $\frac{e_a + C}{\tau} \leq \frac{C}{\Delta}$, it holds that $\beta_{c,mh} \in [0, 1]$. Thereby, a polar value of $\beta_{c,mh}$ does not contradict our model assumptions, as $\beta_{c,mh} = 0$ merely means that for all admissible values $\beta \in (0, 1)$ the incentive compatibility constraint binds, while $\beta_{c,mh} = 1$
merely means that for no admissible value \( \beta \in (0, 1) \) the incentive compatibility constraint binds.

Hence, if it holds that \( \frac{e_a + C}{\tau} \leq \frac{C}{\Delta} \), then incentive compatibility binds exactly for \( \beta \geq \frac{\pi - \frac{e_a}{\tau} - e_p}{R - e_a - C - e_p} \), which means that \( b_{c, mh}^S = \frac{C}{\Delta} \).

If, on the contrary, it holds that \( \frac{e_a + C}{\tau} > \frac{C}{\Delta} \), then the incentive compatibility constraint does not bind for any value of \( \beta \).

We summarize these findings in the following lemma:

**Lemma 4.4**

If \( \frac{e_a + C}{\tau} > \frac{C}{\Delta} \), then the incentive compatibility constraint does not bind for any bargaining power distribution.

If \( \frac{e_a + C}{\tau} \leq \frac{C}{\Delta} \), then the incentive compatibility constraint binds if and only if \( \beta \geq \beta_{c, mh} = \frac{\pi_{c, mh} - e_p}{\pi_{c, h} - e_p} \).

Note that \( \frac{\pi - \frac{e_a}{\tau} - e_p}{R - e_a - C - e_p} \) is the principal’s expected payoff when incentive compatibility binds and \( \frac{\pi - \frac{e_a}{\tau} - e_p}{R - e_a - C - e_p} \) may be viewed as the principal’s expected payoff in the absence of moral hazard but in the presence of the effort cost, whereby the agent is kept at his participation constraint, i.e. the “honest agent” denoted by \( h \) in the lower index.\(^7\)

Therefore, we obtain the following representation of \( \beta_{c, mh} \):

\[
\beta_{c, mh} = \frac{\pi_{c, mh} - e_p}{\pi_{c, h} - e_p}.
\]

If the inequality \( \frac{e_a + C}{\tau} > \frac{C}{\Delta} \) holds, the graph of \( b_{c, mh}^S(\beta) \) has a form as depicted in Figure 4.3. If the inequality \( \frac{e_a + C}{\tau} < \frac{C}{\Delta} \) holds, the graph of \( b_{c, mh}^S(\beta) \) has a form as depicted in Figure 4.4. Again, the kink shows the bindingness of the incentive compatibility constraint.

\(^7\) For example, \( \pi_{c, h} \) may be seen as the principal’s expected payoff in the presence of effort cost in a setting where the agent’s effort choice is observable by the principal.
4.3 Resulting Allocations

**Figure 4.3:** Share $b_{c,mh}^S$ of the agent, depending on the principal’s bargaining power under the *cost approach* in the case of $c_a + C_{\Delta} > \frac{C}{\Delta}$.

![Figure 4.3](image)

**Figure 4.4:** Share $b_{c,mh}^S$ of the agent, depending on the principal’s bargaining power under the *cost approach* in the case of $c_a + C_{\Delta} < \frac{C}{\Delta}$.

![Figure 4.4](image)
4.4 Differences Resulting from the Different Approaches

4.4.1 Differences in Assumptions

Consider the differences in the assumptions required by the two approaches, i.e. compare Assumptions 4.2 and 4.5 to Assumptions 4.6 and 4.9. The *benefit approach* merely requires $R > e_p + e_a$ and $\tau R^S > e_a$, while the *cost approach* requires $R > e_p + e_a + C$ and $\tau R^S > e_a + C$. We formulate this as a proposition.

**Proposition 4.1**

*To allow for contracts that may be signed by rational players and that imply the exertion of effort, the cost approach requires larger values of $R - e_a - e_p$ and of $\tau R^S - e_a$ than the benefit approach.*

This proposition formalizes the intuition that under the *cost approach*, the returns from the successful project outcome must be assumed to be higher than those under the benefit approach, as under the *cost approach* those returns have to cover the effort costs, while under the *benefit approach*, the leisure benefit does not have to be covered by the returns from the project outcome.

**Corollary 4.1**

*Consider the necessary assumptions of the cost approach needed to allow for contracts that may be signed by rational players and under which the agent exerts effort. These assumptions, together with the assumption $B = C$, imply the corresponding necessary assumptions of the benefit approach.*

The reason for this is that all such assumptions are identical under both approaches, except for stronger necessary assumptions on the returns from the project under the *cost approach*. Those assumptions have been discussed in the explanation of Proposition 4.1. So, the *benefit approach* can be viewed as more general than the *cost approach* due to the less restrictive assumptions needed to model the relevant situations.

4.4.2 Differences in Allocations

In this section, we assume that $B = C$ and that for both approaches all assumptions are fulfilled. We consider the differences in allocations that result from the two approaches. They are depicted in the graphs in Figures 4.5 to 4.7.
4.4 Differences Resulting from the Different Approaches

Figure 4.5: Share of the agent under both approaches, depending on the principal’s bargaining power in the case of $\frac{e_a}{\tau} > \frac{C}{\Delta}$.

Figure 4.6: Share of the agent under both approaches, depending on the principal’s bargaining power in the case of $\frac{e_a + C}{\tau} < \frac{C}{\Delta}$.
Figure 4.7: Share of the agent under both approaches, depending on the principal’s bargaining power in the case of $\frac{e_a + C}{\tau} > \frac{C}{\Delta} > \frac{e_p}{\tau}$.

Figure 4.5 shows the allocations $b_{b,mh}^S$ and $b_{c,mh}^S$ in the case of $\frac{e_a}{\tau} > \frac{C}{\Delta}$ (and thus $\frac{e_a + C}{\tau} > \frac{C}{\Delta}$). We see that the cost approach and the benefit approach both yield straight lines going through $\frac{R - e_p}{\tau}$. As expected, the cost approach line lies above the benefit approach line.

Figure 4.6 shows the allocations $b_{b,mh}^S$ and $b_{c,mh}^S$ in the case of $\frac{e_a + C}{\tau} < \frac{C}{\Delta}$ (and thus $\frac{e_a}{\tau} < \frac{C}{\Delta}$). We see that under the benefit approach, the incentive compatibility constraint starts binding for lower values of $\beta$ than it does under the cost approach.

Figure 4.7 shows the allocations $b_{b,mh}^S$ and $b_{c,mh}^S$ in the case of $\frac{e_a + C}{\tau} > \frac{C}{\Delta} > \frac{e_p}{\tau}$. We see that the incentive compatibility constraint binds from a certain level of $\beta$ under the benefit approach, while it does not bind for any value of $\beta$ under the cost approach. Hence, under this parameter constellation, there is an important qualitative difference in the resulting allocation, depending on the approach choice.

We summarize these findings in the following proposition:

**Proposition 4.2**

Assume that the values of the parameters $e_a, e_p, \tau, \Delta, R^S, R^F$ are given, that the value $C = B$ is given, and that all assumptions of the cost approach are fulfilled. Then the following holds:

(i) $b_{b,mh}^S(\beta) \leq b_{c,mh}^S(\beta)$ for all $\beta \in (0, 1)$. Moreover, for all $\beta \in (0, 1)$ for which the incentive compatibility constraint does not bind under the cost approach, it holds that $b_{b,mh}^S(\beta) < b_{c,mh}^S(\beta)$. 

(ii) If $\frac{C}{\Delta} \leq \frac{e^{a}}{r}$, then the incentive compatibility constraint does not bind under both approaches for any value of $\beta \in (0, 1)$.

(iii) If $\frac{C}{\Delta} > \frac{e^{a} + C}{r}$, then the incentive compatibility constraint binds under both approaches and it holds that $\beta_{b,mh} < \beta_{c,mh}$.

(iv) If $\frac{e^{a} + C}{r} \geq \frac{C}{\Delta} > \frac{e^{a}}{r}$, then the incentive compatibility constraint binds for some values of $\beta$ under the benefit approach, but does not bind for any value of $\beta \in (0, 1)$ under the cost approach.

**Proof.** Note that due to Corollary 4.1, the supposition that all assumptions of the cost approach are fulfilled implies also that all assumptions of the benefit approach are fulfilled.

The rest of the proposition follows straightforward from the comparison of the results for the benefit approach and the cost approach derived in Section 4.3. □

Note that $\frac{C}{\Delta}$ is a measure for the severity of moral hazard from the agent’s perspective. The bigger $C$ and the lower $\Delta$, the more the agent is tempted to refuse exerting effort.

The first item in the proposition states the intuitive result that under the cost approach, the agent will obtain a higher share than under the benefit approach for any fixed non-polar bargaining power distribution. Under the cost approach, indeed, the agent has to obtain a higher share, as this share should compensate him for his effort costs in comparison to his outside option. This does not happen under the benefit approach, as under this approach, choosing the outside option does not yield the leisure benefit. Thus, the first item indicates that the benefit approach is biased towards an allocation that is more favorable for the principal, while the cost approach is biased towards an allocation that is less favorable for the principal.

The second item states that if the increase in the probability of a successful project outcome due to effort is large enough compared to the leisure benefit – effort cost, respectively – then under both approaches, the agent’s incentive to refuse exerting effort will be weaker than his incentive to exert effort. This is intuitive, as the agent is paid only in case of success, and the probability of success increases with effort.

The third item implies that if the expected output loss in the no-effort case is low enough, then under both approaches, there exist bargaining power distributions under which the agent’s incentive compatibility constraint will bind. Thereby, under the benefit approach the incentive compatibility constraint starts binding for lower levels of the principal’s bargaining power than it does under the cost approach. To understand the intuition behind this result, suppose that $\frac{C}{\Delta} > \frac{e^{a} + C}{r}$ and that $\beta_{b,mh} < \beta < \beta_{c,mh}$. Then the incentive compatibility constraint will bind under the benefit approach and will not
bind under the cost approach. The reason for this is that on the one hand, according to the first item of the above proposition, the agent obtains a higher share under the cost approach than under the benefit approach. Thus, the agent’s share under the cost approach is further away from the incentive-compatibility-binding share $\frac{C}{\tau}$ than is the agent’s share under the benefit approach. On the other hand, this difference in agent’s shares under different approaches decreases when $\beta$ increases. Thus, for high values of $\beta$, the allocation approaches the bindingness of the agent’s incentive compatibility constraint under both approaches. Hence, if $\beta$ is within a certain intermediate range, the agent’s share is still high enough under the cost approach to prevent the incentive compatibility constraint from binding, while under the benefit approach, the agent’s share is already determined by his binding incentive compatibility constraint.

The fourth item states that for certain combinations of the parameter values, the two approaches may yield results with important qualitative differences. Namely, it may be that the benefit approach incorporates the bindingness of the incentive compatibility constraint for values of the principal’s bargaining power that are large enough, while under the cost approach, the incentive compatibility constraint does not bind for any bargaining power distribution. The reason for this is that the moral-hazard parameter $C$ plays different roles under the two approaches. While under the benefit approach, $C$ enters only the incentive compatibility constraint, under the cost approach, $C$ also enters the Nash Bargaining Product. Hence, according to the first item of the above proposition, the agent obtains a higher share under the cost approach than under the benefit approach. If thereby the severity of moral hazard is at an intermediate level – namely if $\frac{e + C}{\tau} \geq \frac{C}{\tau} > \frac{e}{\tau}$ –, then this level suffices to keep the agent’s payoff above his incentive compatibility constraint under the cost approach, even when the principal has a high level of bargaining power. Yet it does not suffice to keep the agent’s payoff above his incentive compatibility constraint under the benefit approach if the principal has a high level of bargaining power.

Note that parameter values that are fixed, as the ones demanded in Proposition 4.2, may occur in situations when the parameter values are given either exogenously – for instance, when they are estimated from a data set –, or are given endogenously by a bigger model in which the small two-player game is embedded, as it is the case in Gersbach and Rochet (2012A), for instance.

Now we turn to another important qualitative difference between the allocations under the two approaches. First note that Lemma 4.1 and Lemma 4.2 imply that under the benefit approach, the allocation is not affected by the presence of moral hazard if the incentive compatibility constraint does not bind. This case occurs when the principal has a sufficiently low level of bargaining power. On the contrary, Lemma 4.3 shows that under the cost approach, the presence of moral hazard affects the resulting allocation
4.4 Differences Resulting from the Different Approaches

Independently of the incentive compatibility constraint’s bindingness. We state this result in the following proposition:

**Proposition 4.3**

*Under the benefit approach, the allocation is affected by moral hazard if and only if the incentive compatibility constraint binds. Under the cost approach, the allocation is shifted in favor of the agent, even if the incentive compatibility constraint does not bind.*

As a side remark, note that Proposition 4.3 implies that under the cost approach, the notion of *bindingness of moral hazard* — not being equivalent to the notion of the bindingness of the incentive compatibility constraint — may be somewhat misleading. Under the benefit approach, the bindingness of moral hazard only means that the incentive compatibility constraint binds. Especially if the incentive compatibility constraint does not bind, then the resulting allocation is not affected by the presence of moral hazard. On the contrary, under the cost approach, moral hazard has two impacts on the allocation: one through the incentive compatibility constraint and one through the shift of the allocation in favor of the agent when the incentive compatibility constraint does not bind. The latter impact does not exist under the benefit approach. Thus, the notion of *bindingness of moral hazard* may be misleading under the cost approach. Under that approach, one should rather use the notion of *bindingness of the incentive compatibility constraint* on the one hand, and on the other hand, one should be aware that under the cost approach, the presence of moral hazard has an impact on the allocation in both cases, i.e. when the incentive compatibility constraint does bind and when it does not bind.

### 4.4.3 Translation between Equivalent Maximization Problems

In this subsection, we point out the conditions under which a cost approach maximization problem can be reformulated as an equivalent benefit approach maximization problem, and vice versa. Equivalence is considered to be the identity of the solutions of both problems as functions of the parameter values.

In the following, the lower index $b$ ($c$, respectively) denotes the benefit approach (cost approach, respectively). The benefit approach solves the following maximization problem:

$$\max_{b^S \in \left[ \frac{S^b}{L_b} \right]} \left( R_b - b^S \tau_b - e_{p,b} \right)^{\beta_b} \cdot \left( b^S \tau_b - e_{a,b} \right)^{1-\beta_b}.$$
The *cost approach* solves the following maximization problem:

\[
\max_{b \in [\frac{C - \Delta c}{R_c}, R_c^b]} (R_c^b - b^S \tau_c - e_{p,c})^{\beta_c} \cdot (b^S \tau_c - e_{a,c} - C)^{1-\beta_c}.
\]

To obtain equivalent problems, one obviously has to set \(C = B\), \(\Delta c = \Delta b\), \(\tau_c = \tau_b\), \(e_{p,c} = e_{p,b}\), \(\beta_c = \beta_b\), \(R_c^b = R_c^b\), \(R_F^b = R_F^b\) and \(e_{a,c} = e_{a,b} - C\).

However, note that this is possible without obtaining a negative outside option value \(e_{a,c}\) only if \(e_{a,b} - C \geq 0\). Otherwise, although it is possible to translate the *cost approach* problem into an equivalent *benefit approach* problem by setting \(e_{a,b} = e_{a,c} + C\), it is not possible to translate the *benefit approach* problem into a *cost approach* problem. We formulate this finding as a proposition.

**Proposition 4.4**

For any feasible cost approach problem, there exists an equivalent benefit approach problem. For a benefit approach problem with a non-negative outside option of the agent, there exists an equivalent cost approach problem that also incorporates a non-negative outside option for the agent if and only if it holds that \(e_a \geq B\).

Hence, in addition to the insights from Section 4.4.1, we see that the *benefit approach* is more general than the *cost approach* in the sense that it always may generate the same allocations as the *cost approach* without violating a non-negativity condition on the agent’s outside option, whereas the opposite does not hold.

### 4.5 Conclusion

To our knowledge, this paper is the first paper that systematically explores the qualitative and quantitative differences between the outcome of a principal-agent game with moral hazard and Generalized Nash Bargaining when moral hazard is modeled with the *benefit approach*, and the corresponding outcome when the *cost approach* is used. Our findings may be summarized as follows.

The *cost approach* requires stronger assumptions on the parameters than the *benefit approach* to model situations in which the agent decides to exert effort, as under the *cost approach*, the returns from investment have to cover the effort costs.

Given the same parameter values under both approaches, the *cost approach* never yields allocations which the principal would prefer to the allocations from the *benefit approach*. Moreover, under the *cost approach*, the allocation is shifted in favor of the agent compared to the allocation in the absence of moral hazard, even if the incentive compatibility constraint does not bind. This does not happen under the *benefit approach*. Thus, we see
4.5 Conclusion

biases towards opposite directions in the allocations, resulting from the two approaches.

The conditions under which the incentive compatibility constraint binds also change with the choice of approach. Especially under the benefit approach, the incentive compatibility constraint may start binding for lower values of the principal’s bargaining power than it would do under the cost approach. In particular, it may happen that the incentive compatibility constraint binds for some bargaining power distributions under the benefit approach, while it does not bind for any bargaining power distribution under the cost approach.

Any cost approach problem can be transformed into a benefit approach problem with the same solution function. Under the constraint of a non-negative value of the agent’s outside option, however, only a benefit approach problem with an agent’s outside option that is sufficiently large compared to the value of the leisure benefit can be transformed into a cost approach problem with the same solution function.

Note that our results are subject to the following limitations: The first limitation is the fact that the results of this paper only hold in situations with non-polar bargaining power distributions. Under a polar bargaining power distribution – which is equivalent to a take-it-or-leave-it offer – one of the players is always kept tight at his relevant constraint (incentive compatibility or participation), and thus, most of our results do not apply.

The second limitation is more subtle. Note that our model may be viewed as a cake-division problem in which the size of the cake is assumed to be fixed. This is a crucial assumption, since if the size of the cake can also be varied by the players, the agent might not want to exceed his incentive compatibility constraint by asking for a higher share, i.e. a higher fraction of the output. Instead, the agent might want to increase the size of the cake. In that case, the effects explored in the current paper will not occur.

This is why the results of the current chapter do not apply to Chapter 3 of this dissertation where in Section 3.4 the setting of Gersbach and Rochet (2012A) is modified by replacing the benefit approach by the cost approach. Thus, we see that if bargaining between the players goes beyond the assignment of the shares of a cake with fixed size, the results from the current paper do not necessarily carry over. Third, note that in papers that address the question of a socially optimal allocation, as, for instance, Holmström (1979), our results do not apply as there is no bargaining involved.

In some situations in which bargaining is involved, our results might qualitatively hold, even when the considered bargaining solution is not the Generalized Nash Bargaining Solution. The exploration of the transfer of our results to such models is left for future research. Another possible direction for future research is the generalization of our results to a model with continuous effort choice.

Generally speaking, our current model is a two-player-game that illustrates some effects
that might occur in different economic models. Yet, especially in the case of macroeconomic models involving decisions about the amounts of resources to be invested, only a thorough check in the model will show whether our findings apply there. However, our results may provide a guidance for the appraisal and testing of possible biases in the results of existing moral hazard models. For instance, if a central result of a quantitative model that incorporates the cost approach states that the agent obtains a high share from the project output compared to other numerical values in the model, this might be an indicator for a possible bias of the model due to the approach to model moral hazard that was chosen, as under the benefit approach, the agent would obtain a lower share. On the other hand, if a cost-approach model yields the result that the agent obtains a low share from the project output compared to other numerical values in the model, our findings can be viewed as an additional argument in support of such a result, since, as we have shown, a cost approach model is, by its nature, biased towards the opposite result.

A model builder might use our insights when choosing the approach that is suitable for his purposes. For example, if a model builder wants the allocation resulting from his model to be shifted due to moral hazard even if the incentive compatibility constraint does not hold tight, he should use the cost approach. Conversely, if he wants to examine a situation in which the incentive compatibility constraint binds, and thereby he wants – or needs, due to exogenously-given parameter values for instance –, to keep the principal’s bargaining power on a low level, he might find it advantageous to choose the benefit approach. Thus, our findings might be useful for appropriately designing models that include moral hazard and non-polar bargaining power distributions.

Finally, note that the differences between distinct types of moral hazard suggest that the investigation of moral hazard from a behavioral point of view should be conducted with great attention for the type of moral hazard that is examined. Especially the behavioral effect of loss aversion in contract theory (see Köszegi (2013)) might impact the resulting allocation in different ways, depending on the type of moral hazard. We provide an analysis of the non-behavioral differences in the allocations depending on the type of moral hazard. Thus, our results can help behavioral economists to distinguish behavioral from non-behavioral differences of the two types of moral hazard.
## 4.6 Notation List

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_p$</td>
<td>Outside payoff of the principal</td>
</tr>
<tr>
<td>$e_a$</td>
<td>Outside payoff of the agent</td>
</tr>
<tr>
<td>$o$</td>
<td>Index of the project outcome, $o \in {S, F}$</td>
</tr>
<tr>
<td>$R^o$</td>
<td>Value of project output depending on the project outcome, $R^S &gt; R^F$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Project’s success probability, if effort is exerted</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Decrease of success probability, if no effort is exerted</td>
</tr>
<tr>
<td>$b^o$</td>
<td>Payment of the principal to the agent depending on the project outcome</td>
</tr>
<tr>
<td>$\bar{R}$</td>
<td>Expected project output in the case that the agent exerts effort</td>
</tr>
<tr>
<td>$\bar{R}$</td>
<td>Expected project output in the case that the agent does not exert effort</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Bargaining power of the principal, $0 &lt; \beta &lt; 1$</td>
</tr>
<tr>
<td>$b^S_\beta$</td>
<td>Resulting payment to the agent in the absence of moral hazard if $o = S$</td>
</tr>
<tr>
<td>$B$</td>
<td>Leisure benefit, $B \geq 0$</td>
</tr>
<tr>
<td>$C$</td>
<td>Effort cost, $C \geq 0$</td>
</tr>
<tr>
<td>$b^S_{\beta, mh}$</td>
<td>Resulting payment to the agent under the benefit approach if $o = S$</td>
</tr>
<tr>
<td>$b^S_{c, mh}$</td>
<td>Resulting payment to the agent under the cost approach if $o = S$</td>
</tr>
<tr>
<td>$\beta_{b, mh}$</td>
<td>Value of $\beta$ above which the agent’s incentive compatibility constraint starts to bind under the benefit approach</td>
</tr>
<tr>
<td>$\beta_{c, mh}$</td>
<td>Value of $\beta$ above which the agent’s incentive compatibility constraint starts to bind under the cost approach</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Principal’s expected payoff if the agent is kept at his participation constraint in the absence of moral hazard</td>
</tr>
<tr>
<td>$\pi_{b, mh}$</td>
<td>Principal’s expected payoff when the incentive compatibility constraint binds under the benefit approach</td>
</tr>
<tr>
<td>$\pi_{c, mh}$</td>
<td>Principal’s expected payoff when the incentive compatibility constraint binds under the cost approach</td>
</tr>
<tr>
<td>$\pi_{c, h}$</td>
<td>Principal’s expected payoff in the absence of moral hazard but in the presence of the effort cost, whereby the agent is kept at his participation constraint</td>
</tr>
</tbody>
</table>


Curriculum Vitae

**Elias Aptus**, born on 2 July 1987 in St. Petersburg, Russia

2011 - 2014  *Ph.D. in Economics*
ETH Zurich, Switzerland

2010 - 2011  *M.Sc. in Applied Mathematics*
ETH Zurich, Switzerland

2006 - 2010  *B.Sc. in Mathematics*
ETH Zurich, Switzerland

1998 - 2006  *Abitur (General University Entrance Qualification)*
Friedrich-Eugens-Gymnasium Stuttgart, Germany