Doctoral Thesis

Cyclic plastic material behavior leading to crack initiation in stainless steel under complex fatigue loading conditions

Author(s):
Facheris, Giacomo

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Cyclic plastic material behavior leading to crack initiation in stainless steel under complex fatigue loading conditions

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(Dr. sc. ETH Zurich)

presented by

GIACOMO FACHERIS

M. Sc. in Mechanical engineering, Politecnico di Milano, Italy

born on 10.08.1983

citizen of Italy

accepted on the recommendation of

Prof. Dr. Edoardo Mazza (examiner)
Dr. Koenraad Janssens (co-examiner)
Prof. Dr. Lakhdar Taleb (co-examiner)

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Summary

The improvement of the reliability and of the safety in the design of components belonging to the primary cooling circuit of a light water nuclear reactor is nowadays one of the most important research topics in nuclear industry. One of the most important damage mechanisms leading the crack initiation in this class of components is the low cycle fatigue (LCF) driven by thermal strain fluctuations caused by the complex thermo-mechanical loading conditions typical for the primary circuit (e.g. operating thermal transients, thermal stratification, turbulent mixing of cold and hot water flows, etc.). The cyclic application of the resulting plastic deformation to the steel grades commonly used for the fabrication of piping parts (e.g. austenitic stainless steels) is associated with a continuous evolution of the mechanical response of the material. As an additional complication, the cyclic behavior of stainless steels is influenced by temperature, strain amplitude and cyclic accumulation of inelastic strain (i.e. ratcheting). The accurate prediction of the structural response of components belonging to the primary cooling circuit requires the development of a reliable constitutive model that must be characterized by a reduced complexity to allow its application in an industrial context.

In this framework, the main goal of the current dissertation is to formulate, calibrate and implement in a commercial Finite Element code, a constitutive model that is suitable for the stainless stain grade 316L subjected to complex loading conditions.

As a first task, a characterization of the mechanical behavior of 316L subjected to uniaxial and multiaxial strain-controlled conditions (including LCF and ratcheting) is carried out performing several tests in the laboratories of the Paul Scherrer Institute (PSI, Villigen, Switzerland) and of Politecnico di Milano (Italy). The uniaxial experiments demonstrate that, prescribing a strain-controlled ratcheting path, a harder material response is induced with respect to the equivalent uniaxial LCF test. An additional hardening is also noticed in multiaxial tests, when a non-proportional loading history is imposed. The experimental results show that this additional hardening is accompanied by a lifetime reduction.

Further experiments are carried out to investigate the loading-rate influence on the mechanical response of the 316L under strain- and stress-control and to determine the necessity to implement a time-dependent constitutive model.

A set of interrupted tests has been also performed to retrieve samples suitable for the characterization of the microstructural evolution of 316L subjected to ratcheting conditions. The microstructural characterization has been carried out by means of a Transmission Electron Microscope (TEM) in a collaboration with the High Temperature Integrity Group at the Swiss Federal Laboratories for Material Science and Technology (Empa, Dübendorf, Switzerland).

The experimental observations reported in the first part of the current dissertation inspired the formulation of a novel constitutive law consisting in a modification of the well-known Chaboche model. In this formulation named '5DChabEP', the model’s parameters are not constant but are allowed to vary as a function of 5 internal variables. The proposed constitutive model is implemented in the commercial Finite Element code ABAQUS and an
automatized procedure is developed to calibrate the material parameters. The descriptive and predictive capabilities of the constitutive model, coupled with an advanced multiaxial damage criterion, are evaluated under several loading conditions using, as references, experimental data and simulations performed by means of the original Chaboche formulation. In general, the possibility to vary the material parameters as a function of a set of internal variables is found to be an extremely efficient approach to provide accurate stress calculations and lifetime predictions enhancing significantly the performance of the original Chaboche constitutive law.

Finally, a sensitivity analysis is carried out to characterize the confidence bounds of the output of the constitutive model and to identify the factors that are mostly responsible for the uncertainty in the calculations. In this framework, the elementary effects (EE) method is found to be the ideal tool to carry out, with a limited computational cost, a non-local sensitivity analyses on three different case studies.
Il miglioramento della sicurezza e dell’affidabilità nella progettazione di componenti relativi al circuito primario di raffreddamento di un reattore nucleare ad acqua leggera è, ai nostri giorni, uno degli argomenti di ricerca più importanti nel campo della ricerca nucleare.

Il meccanismo di danneggiamento a fatica responsabile della nucleazione della cricca in questa classe di componenti è generato dalle fluttuazioni della deformazione termica causate dalle complesse condizioni di carico termo-mecaniche tipiche del circuito primario (transitori operativi, stratificazione termica, miscelazione turbulenta di correnti d’acqua calde e fredde, ecc.).

L’applicazione ciclica della risultante deformazione plastica alle classi di acciaio comunemente utilizzate per realizzare le tubazioni del circuito primario (acciai inossidabili austenitici) è associata ad una continua evoluzione della risposta meccanica del materiale. Una complicazione addizionale deriva dal fatto che il comportamento ciclico degli acciai austenitici è influenzato dalla temperatura, dall’ampiezza di deformazione e dall’accumulazione ciclica di deformazione (ratcheting). L’accurata previsione della risposta strutturale, in componenti relativi al circuito di raffreddamento primario, richiede lo sviluppo di un modello costitutivo affidabile che abbia allo stesso tempo una complessità limitata in modo da consentirne l’utilizzo in ambito industriale.

In questo contesto, l’obiettivo primario di questa tesi è di formulare, calibrare e implementare in un software commerciale ad elementi finiti, un modello costitutivo che sia appropriato per l’acciaio austenitico di tipo 316L soggetto a complesse condizioni di carico.

Per iniziare, la caratterizzazione del comportamento meccanico dell’acciaio 316L soggetto a condizioni di carico uniassiale e multiassiale (comprendenti fatica oligociclica e ratcheting) è stata portata a termine eseguendo numerosi test nei laboratori del Paul Scherrer Institute (PSI, Villigen, Svizzera) e del Politecnico di Milano (Italia). Gli esperimenti uniassiali mostrano che, imponendo una sollecitazione di ratcheting in controllo di deformazione, viene indotta una risposta del materiale più dura rispetto a quella misurata in un normale test di fatica. Un indurimento addizionale è osservato anche nelle prove multiassiali, nel caso venga imposto una storia di carico non proporzionale. I risultati sperimentali mostrano che questo indurimento addizionale è accompagnato da una riduzione della vita a fatica.

Altri esperimenti sono stati eseguiti con il fine di investigare la dipendenza della risposta meccanica dell’acciaio 316L rispetto alla velocità con cui viene applicato il carico in modo da determinare se vi sia la necessità di implementare un modello costitutivo con dipendenza dal tempo.

Un’ulteriore serie di esperimenti è stata eseguita interrompendo le prove in modo da ottenere campioni adatti per la caratterizzazione dell’evoluzione della microstruttura dell’acciaio 316L soggetto a ratcheting. La caratterizzazione della microstruttura è stata eseguita grazie all’utilizzo di un microscopio elettronico a trasmissione (TEM) in una collaborazione con il gruppo High Temperature Integrity presso i Laboratori Federali Svizzeri per la Scienza e la Tecnologia dei Materiali (Empa, Dübendorf, Svizzera).
Le osservazioni sperimentali riportate nella prima parte della tesi hanno ispirato la formulazione di una legge costitutiva innovativa che consiste in una versione modificata del famoso modello di Chaboche. In questa formulazione chiamata '5DChabEP', i parametri del modello non sono costanti ma possono variare in funzione di 5 variabili interne. Il modello costitutivo qui proposto è stato implementato nel software commerciale ad elementi finiti ABAQUS insieme ad una procedura automatica per la calibrazione dei parametri. La capacità descrittiva e predittiva del modello costitutivo, unitamente ad un criterio avanzato per la determinazione del danno accumulato, è stata valutata considerando svariate condizioni di carico e usando come riferimento dati sperimentali e simulazioni eseguite con la versione originaria del modello di Chaboche.

In generale, la possibilità di variare i parametri del modello in funzione delle variabili interne risulta essere un approccio estremamente efficace al fine di fornire un'accurata valutazione degli sforzi e della vita a fatica migliorando la prestazione del modello di Chaboche nella sua forma originaria.

Infine, un’analisi di sensitività è stata eseguita con il fine di caratterizzare l’intervallo di confidenza degli output del modello costitutivo e in modo da identificare quali siano i fattori responsabili dell’introduzione di una maggiore incertezza nei calcoli. In questo contesto, il metodo degli Effetti Elementari si è rivelato essere lo strumento ideale per portare a termine, con un costo computazionale limitato, una serie di analisi di sensitività su tre diversi casi di studio.
Introduction

Nearly 80% of the energy consumed in the world is generated by means of fossil fuels (i.e. coal, gas and oil). The transformation process of these fuels into energy is responsible to emit every year billions of tons of carbon dioxide in the atmosphere, together with several other air pollutants, leading to irreversible climate and environmental changes. In this framework, several countries subscribed the Kyoto Protocol consisting in a mutual agreement to stabilize the greenhouse gas concentrations in the atmosphere at a level that would stop dangerous anthropogenic interference with the climate system (UN, 2005). The accomplishment of the Kyoto targets requires a strong reduction of the power generated using fossil fuels. The always raising energy demand, together with the increased awareness of the effects of global warming and climate change, were the main driving factors that in the last decade contributed to the so called nuclear renaissance.

1 Framework

The Fukushima accident happened in Japan in 2011 changed the public perception of nuclear safety and raised questions over the future of the atomic power industry revival. At the same time, this disaster underlined the importance to guarantee the reliability, not only of the emergency cooling circuits, but also of other systems having, in case of failure, a serious impact on the safety of the power plant. Among them, a particular attention is dedicated to the components belonging to the primary cooling circuit of light water nuclear reactors (LWR) including steam generators, pressurizers, reactor vessels and piping systems responsible for the transport of the coolant. Depending on the location of the component, different time-dependent degradation phenomena act during operation (e.g. thermo-mechanical fatigue, irradiation embrittlement, stress corrosion, etc.). As reported by Dahlberg et al. (2007), the complex loading conditions taking place in the primary cooling circuit of LWR including

– operating thermal transients,
– thermal stratification,
– turbulent mixing of cold and hot water flows (e.g. in T-junctions),

make thermo-mechanical fatigue one of the most relevant degradation mechanisms in these piping systems.

1.1. PLant Life Management (PLiM) project

In the PLiM project developed at Paul Scherrer Institute (PSI) the influence of thermal fluctuations on fatigue crack initiation and short crack propagation is investigated, with a particular focus on boundary conditions as typically observed in light water nuclear reactor primary circuit pipe systems. A detailed description of the activities carried out in the PLiM project can be found in Niffenegger et al. (2008). An example of thermo-mechanical fatigue
Introduction

(TMF) damage is the infamous Civaux event in which a through-wall crack was observed after only 1500 hours of exercise (see Fig. 1).

In many cases the thermally induced strain causes microcracking on the inner surface of the tubes. The microcracks-pattern resulting in a surface crazing is typical for equibiaxial strain conditions under a cyclic-thermal shock load (see Fig. 2). In some cases a few of these microcracks propagated entirely through the wall of the tube (Faidy, 2000).

In spite of the common agreement on the causes of this class of incidents summarized in a NEA/CSNI study (NEA/CSNI/R, 2005), there is still a lack of tools providing a reliable prediction of fatigue lifetime for components subjected to the operating conditions typical for the primary cooling circuit of a LWR. One of the main objectives pursued in the PLiM project is to determine the boundary conditions at which this phenomenon can occur. Past research part of the PLiM project results (Janssens et al., 2009) has shown that the crack initiation prediction based on the state of art of modeling may deviate from the behavior observed in actual components.

In this framework, the primary objective of this doctoral dissertation is to provide a material description for the cyclic plastic response and damage accumulation in a context of cyclic thermal shock loading. The parameters of this constitutive model must be calibrated against experimental data from previous and own work. This research is focused on stainless steel grade AISI 316L and loading conditions typical for the primary cooling system of a LWR type as operated in Swiss nuclear power plants.

1.2. Coupled methodology

The proposed approach for reliable life estimation is the adoption of a numerical coupled methodology (Hannink et al., 2008) that has been shown to be an efficient tool to deal with the nonlinearities and the complex boundaries typical of piping components subjected to complex thermo mechanical loading conditions (see Fig. 3). The first step of this methodology consists in the calculation of the evolution of the temperature in the flow and in the structure along the time history by a Computational Fluid Dynamics (CFD) simulation.
1. Framework

The output data of the CFD simulation will be the input for the second step of the methodology: the Finite Element Method (FEM) simulation. The reliability of the calculation of stress and strain by FEM code is one of the most critical points of that methodology and it is still a challenge in research (Yang, 2004; Kang, 2005; Takahashi et al., 2008; Taleb and Cailletaud, 2010; Janssens, 2011). The main goal of the current dissertation is to formulate, implement and calibrate a novel robust constitutive law suitable for the commercial code ABAQUS (Abaqus, 2012) with the goal to provide a powerful instrument to calculate the 3D strain and stress field induced by temperature fluctuations in structures.

In the last step, an advanced multi-axial energy based fatigue criterion (Jiang, 2000) is used to provide an estimation of the life of the component.

1.3. Material: stainless steel grade 316L

The focus of the current dissertation is on the stainless steel grade 316L and in particular two different batches are considered in this study.

A first set of samples is machined from a section of a pipe commonly used to fabricate the primary cooling circuit of light water reactor (LWR) and will be referred as 'TP316L' (TP stands for tubular product). Samples are extracted from the tube along the pipe direction.

A second set, which will be referred as '316L', is sampled from a 20 mm thick hot rolled plate. Specimens are extracted along the rolling direction at sufficient distance from the edge of the plate to eliminate data scatter caused by microstructural variations typically observed at the edge of rolled sheet metals.

In Tab.1 the chemical composition is listed for both materials as reference. The manufacturing sequences are the same for both materials and consist of hot working, solution annealing (at 1050-1080 °C), quenching in water, pickling and grinding.
2 Objectives and outline

The main objectives of this dissertation include:

1. the state of the art of the characterization and modeling of the 316L mechanical behavior,
2. the mechanical and microstructural characterization of 316L subjected to complex loading conditions,
3. the formulation, implementation and calibration of a constitutive model providing an accurate stress-strain relation under complex multiaxial loading conditions including cyclic plasticity and ratcheting (e.g. cyclic accumulation of inelastic strain) and temperature variation. At the same time the constitutive model should be characterized by a reduced complexity allowing a relatively easy calibration of its parameter and making this approach suitable for its application in an industrial context,
4. the evaluation of the descriptive and predictive capability of the model,
5. a material parameter sensitivity analysis in order to identify the factors having the main responsibility for the uncertainty in the prediction and to characterize the confidence bounds for the output of the model.

These aspects have been extensively discussed and presented in the following chapters.

Chapter 1 reports the state of the art concerning the characterization and modeling of the fatigue behavior of austenitic stainless steels. The first section is dedicated to the description of the mechanical and microstructural behavior of the investigated material under cyclic loading conditions. In the second section, the most important constitutive models developed in the last decades dealing with cyclic plasticity and ratcheting are presented. The third section consists of a literature review on cumulative damage criteria.

The methods adopted to carry out the experimental characterization are presented in Chapter 2. In the first section, a detailed description of the sample preparation and of
the equipment used to perform the uniaxial isothermal, uniaxial TMF and multiaxial tests is reported. The procedures adopted to prepare TEM specimens and the equipment utilized to characterize the microstructural evolution of the 316L are documented in the second section. In the third section, the methodology used to determine the amount of strain-induced $\alpha'$-martensite formation in fatigued specimens is presented. In the fourth section, a short overview on the Finite Element Method adopted to carry out the numerical simulations is given.

Chapter 3 summarizes the main results concerning the experimental characterization of 316L. In the first section, the mechanical behavior of 316L in uniaxial strain-controlled LCF and ratcheting experiments is investigated. The mechanical response of 316L under multiaxial strain-controlled LCF and ratcheting conditions is reported in the second section. An evaluation of the time-dependency of the room temperature mechanical behavior of the selected stainless steel grade is carried out in the third section. In section 4, the results of the microscopical investigation performed to determine the influence of ratcheting on the microstructural evolution of dislocation structures are described. The fifth section consists of a study on the formation of strain-induced $\alpha'$-martensite in fatigued specimens.

Chapter 4 is completely dedicated to the presentation of the proposed material description. In the first section, the constitutive model is formulated and the procedure adopted to calibrate its parameters is documented. The advanced multiaxial damage criterion used to predict the lifetime is presented in the second section. In the third section, the descriptive and predictive capability of the constitutive model is evaluated with a particular emphasis on the lifetime assessment.

Chapter 5 treats the sensitivity analysis performed on 3 different case studies. In section 1, the common approaches used to carry out a sensitivity analysis are described, focusing the attention on the elementary effects method. The procedure adopted to set up the sensitivity analyses is documented in the second section. The results of the sensitivity analyses performed on 3 different case studies are extensively discussed in the third section.

Chapter 6 summarizes the conclusions of the current dissertation and proposes an outlook for possible future developments.

Additional information not included into these 6 chapters is reported in Appendices A, B and C.

In Appendix A, the complete list of experiments is summarized together with the corresponding testing parameters.

In Appendix B, a detailed description of the procedure developed in order to implement the 5 internal variables dependent Chaboche model (5DChabEP) in the commercial Finite Element code ABAQUS using either the combined hardening model (already available in ABAQUS) or the User MATerial subroutine is documented.

In Appendix C, the major issues linked with the implementation of the 5DChabEP model in a Finite Element code are discussed.
In the following chapter an overview on the state of the art related to the characterization and modeling of fatigue behavior of austenitic stainless steels is reported. In section 1.1, a literature review on the experiments performed to investigate the response of the considered class of materials is presented. In section 1.2, the most important constitutive models developed in the last decades to deal with cyclic plasticity and ratcheting are reported. Finally, in section 1.3, a literature review on cumulative damage models is presented.

1.1 Material cyclic behavior

In order to create a robust and consistent constitutive model suitable for the complex loading conditions typical for TMF, the main task to perform consists in investigating the behavior of the selected material subjected to such boundaries. In this framework, a literature research on the thermo-mechanical behavior of the stainless steel grade 316L is performed and the main findings are reported in this section. In subsection 1.1.1, results of literature experiments performed to characterize the cyclic behavior of the material under uniaxial loading conditions are presented. Subsequently, in subsection 1.1.2, a similar research on multiaxial loading conditions is reported. In addition, this class of materials was found to show a time-dependency even at room temperature and on this topic subsection 1.1.3 is dedicated. Finally, in subsection 1.1.4, the main results of studies linking the material response with the evolution of the microstructure are reported.

1.1.1. Uniaxial loading

In order to allow an easier understanding of the results, this subsection is divided in three paragraphs, where the material behavior under different loading conditions is reported.

**Strain-controlled symmetric cyclic tests (LCF)**

Since the second half of the 20th century, it is well known that stainless steels and more in general FCC (face centered cubic) materials show a particularly complex stress
amplitude response when subjected to repeated uniaxial strain loading. In the last decades the cyclic behavior of 316L has been extensively studied and recently comprehensive reports (Delobelle, 1993; Polak et al., 1994; Alain et al., 1997) showing that strain amplitude and temperature quantitatively and qualitatively affect the cycling hardening-softening under LCF conditions have been presented. Analyzing the stress amplitude response of 316L it is possible to identify 3 stages: primary hardening, softening and stabilization or eventually secondary hardening (see Fig.1.1(a)). One can observe that increasing the strain amplitude, the value of the stabilized stress amplitude increases. The increase of the stabilized stress value is not linear with the increment of strain amplitude but 3 different regimes can be identified (see Fig.1.1(b)). In addition, the number of cycles necessary to reach stabilization (or change of regime) is lower if the strain amplitude is larger (see Fig.1.1(a)).

As previously mentioned, cyclic hardening is strongly influenced also by temperature. In general it is possible to observe that in LCF tests performed at higher temperatures, the softening stage is less pronounced and secondary hardening becomes more evident (see Fig.1.2(a) and (b)). In addition, the relation between the stabilized stress amplitude and temperature is found to be strongly non-linear (see Fig.1.3(a)). Between 20 and 250 °C the stabilized stress decreases with the increase of temperature, between 250 and 550 °C an increment of temperature is responsible for higher stabilized stress because of the occurrence of the phenomenon known as dynamic strain aging (DSA) (Hong and Lee, 2004). Finally, for temperatures higher than 550 °C one can observe a sudden drop of the stabilized stress because of the thermal activation.

Moreover, it is very important to remark that also the temperature history plays a role in LCF. In fact Bouchou and Delobelle (1996) prove that, performing isothermal LCF tests with stepwise increasing and then decreasing temperature levels, the stress response is affected if the maximum temperature overtakes the threshold of 250 °C (see Fig.1.3(b)). The authors write that the physical nature of the temperature history dependency is anyway not yet clear.

Another well-known and strongly non-linear phenomenon is the so called 'strain range memorization' effect. In fact, as reported by several authors (Nouailhas et al., 1985; Chaboche, 1986; Kang, 2005) cyclic hardening is strongly influenced by the previous loading history and in particular by the maximum plastic strain range reached. This phenomenon can be easily observed in Fig.1.4(a), where the stabilized response of the material previously subjected to a LCF test with higher strain amplitude is shown to be harder than the virgin one. This memory shows a partial evanescence if one performs a test with a sequence of LCF tests with decreasing level of strain amplitude. In addition, as reported by Kang (2005), the memorization effect and its evanescence effect are quantitatively affected by temperature. In fact, as
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Figure 1.2. (a) Map of the hardening stages for 316L subjected to LCF with different strain temperatures (Alain et al., 1997), (b) Stress amplitude evolution during cycling for 316L subjected to LCF with different temperatures (Alain et al., 1997).

Figure 1.3. (a) Value of stabilized stress as a function of temperature for a 316L subjected to LCF with four different strain amplitudes (Delobelle, 1993), (b) stepwise isothermal cyclic tests. Evolution of the average peak values as a function of the maximum temperature (Bouchou and Delobelle, 1996).
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Figure 1.4. (a) Evolution of the stress amplitude during decreasing two-level strain amplitude test on 316 at room temperature (Nouailhas et al., 1985), (b) evolution of the stress amplitude during increasing and decreasing strain amplitude tests on 316L at three temperature levels (Kang, 2005).

reported in Fig.1.4(b), it is evident that the memorization effect is less pronounced for the test performed at higher temperature (e.g. 700 °C).

Stress-controlled non symmetric cyclic tests (classic ratcheting experiments)

Prescribing a non symmetric alternate stress history certain materials may show a progressive accumulation of strain, named in jargon *ratcheting*. Several literature references (Kang and Gao, 2002; Vincent et al., 2004; Jiang and Zhang, 2008; Abdel-Karim, 2009) give slightly different definitions for ratcheting but they agree to describe this phenomenon as the cyclic accumulation of inelastic strain under non symmetric stress cycling. A more precise definition provided by Chaboche (2008) helps to dissipate the confusion between ratcheting and cyclic creep designating ratcheting as a rate-independent phenomenon:

... *ratchetting corresponds with a progressive accumulation of plastic strain, cycle-by-cycle, that resembles to a creep effect. For example, maximum strain evolution shows a primary stage (decreasing ratchetting rate) followed by a stationary stage (constant ratchetting rate), like in the creep test. However, such an accumulation must not be confused with creep (induced by time under a constant stress), because due to plasticity mechanisms during the unloading–reloading (not specifically influenced by time).*

In austenitic stainless steels subjected to non zero mean stress loading conditions, this continuous accumulation of the mean plastic strain (or drifting) during cycling is often observed. In certain cases, when the loading condition is not enough severe, after the first few cycles a shakedown in the plastic strain accumulation is noticed and no more ratcheting occurs (see Fig.1.5).

The most influential parameters on the material response under stress-controlled ratcheting (determining if shakedown occurs or not) are mean stress and stress amplitude. As reported in Fig.1.6(a) and (b), if the stress amplitude is kept constant and the mean stress is stepwise increased, the amount of accumulated inelastic strain increases too. In Fig.1.6(a) and (b) it is evident that when the mean stress is low, shakedown occurs after few cycles.

On the other hand, when the mean stress increases over a certain threshold value, ratcheting takes place. If the mean stress is kept constant, and the maximum stress (and consequently the stress amplitude) is stepwise increased, the amount of accumulated inelastic strain increases too (see Fig.1.6 (b) and 1.7(a)).
1.1. Material cyclic behavior

Figure 1.5. Ratcheting and shakedown.

Figure 1.6. (a) Ratcheting strain for experiments performed on 316 at room temperature with increasing mean stress (Chaboche, 1991), (b) ratcheting strain for experiments performed on 316L at room temperature with different mean stress levels or different stress amplitude levels (Kang et al., 2001).

Figure 1.7. (a) Ratcheting evolution as a function of cyclic number for different maximum stress and a constant mean stress, (b) for different mean stress and a constant maximum stress. Stress-controlled ratcheting tests performed on 316L at room temperature (Feaugas and Gaudin, 2004).
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Figure 1.8. (a) Ratcheting rate as a function of mean stress for different maximum stress for 316L at room temperature (Feaugas and Gaudin, 2004). (b) Ratcheting strain with and without previous LCF cycling (100 cycles - 0.5% strain amplitude) for tests performed on 316L at room temperature (Kang et al., 2006b).

Similarly to what reported before for the mean stress, when the maximum stress (and consequently the stress amplitude) is low, shakedown occurs after few cycles. Only when the maximum stress increases over a certain threshold ratcheting takes place. Feaugas and Gaudin (2004) performed similar tests keeping the maximum stress constant and increasing step by step the mean stress (and consequently reducing the stress amplitude) (see Fig.1.7(b)). In this case, increasing the mean stress value, the amount of accumulated inelastic strain first increases and then decreases. Feaugas and Gaudin (2004) demonstrate that ratcheting rate is maximum when the mean stress is about 50 MPa (see Fig.1.8(a)).

Similarly to what previously reported for LCF, the previous cycling loading history strongly influences also the ratcheting behavior of austenitic stainless steels. In Fig.1.8(b) it is evident that when 316L is first subjected to a LCF cycling and then to a uniaxial ratcheting test, its ratcheting rate is strongly reduced with respect to a virgin material. The reason for this ratcheting-rate reduction is the harder material response induced by cyclic loading.

In literature only few references investigating the influence of temperature on stainless steels under uniaxial ratcheting conditions are available. Kang (2005) showed in his study that the effect of temperature on the material response is strongly non-linear. As reported in Fig.1.9, the increment of temperature from room temperature to 400-500 °C generally causes a reduction in the accumulation of ratcheting strain because, in this temperature range, the material shows a strong dynamic strain aging (DSA) (Hong and Lee, 2004), which results in a high deformation resistance. A further increment of temperature over the thermal activation threshold (550-600 °C) causes a sudden increment of the ratcheting rate. The overtaking of this threshold limit could also cause a transition from shakedown to ratcheting.

**Strain-controlled non symmetric cyclic tests (strain-controlled ratcheting experiments)**

As mentioned in the previous paragraph, the majority of literature references about ratcheting tests are based on non symmetric stress-controlled tests. An alternative way to analyze the response of the material subjected to ratcheting is available. In this kind of experiments, named ‘strain-controlled ratcheting’ or ‘cyclic tension’ tests (Coffin, 1970), the strain path consists of a superposition of a constant amplitude ramp waveform and a continuously varying mean strain (see Fig.1.10). The accumulation of the shift of the mean strain within a cycle will be called ‘ratcheting step’ and will be referred as $\xi$. Because of its infrequent application few literature references for this testing procedure are available.

As reported by Ohno et al. (1998) and Mizuno et al. (2000), the material responses under those loading conditions are totally different comparing with the one corresponding to
1.1. Material cyclic behavior

**Figure 1.9.** Ratcheting strain vs. cyclic number at three temperatures for tests performed on 316L with increasing stress amplitude (Kang, 2005).

**Figure 1.10.** Generic strain path for a cyclic tension test where the total strain path is a superposition of mean and alternate strain (Facheris and Janssens, 2013).
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Figure 1.11. (a),(b) and (c) stress versus strain relations in cyclic tension tests performed at room temperature on 316FR with high ratcheting step $\xi = 0.05$ or 0.10\%, (d),(e) and (f) stress versus strain relations in cyclic tension tests performed at room temperature on 316FR with low ratcheting step $\xi = 0.01$\% (Mizuno et al., 2000).

Symmetric cyclic tests (LCF). According to Mizuno et al. (2000), in experiments performed with low strain amplitudes, the drifting of the mean strain causes the maximum value of the stress in each loop to follow the monotonic curve (see Fig.1.11 and 1.12). This statement, however, is no longer valid when the imposed strain amplitude is higher.

In Fig.1.13, Mizuno et al. (2000) reports the stress amplitude curves for ratcheting tests and LCF experiments performed with the same strain amplitude. The overlapping of these curves suggests that neither the ratcheting step nor the drifting of the mean strain influence the stress amplitude cyclic evolution.

This finding is apparently in contradiction with the results reported by Ohno et al. (1998) who performed similar experiments but at higher temperature (300 °C). In fact Ohno et al. (1998) confirm that the ratcheting step does not play a role in the stress amplitude definition but at the same time they report that the isotropic hardening and therefore the stress amplitude are strongly influenced by the value of maximum plastic strain (see Fig.1.14).
1.1. Material cyclic behavior

Figure 1.12. (a) Change of tensile peak stress in cyclic tension tests performed at room temperature on 316FR with high ratcheting step $\xi = 0.05$ or 0.10%, (b) change of tensile peak stress in cyclic tension tests performed at room temperature on 316FR with low ratcheting step $\xi = 0.01\%$ (Mizuno et al., 2000).

Figure 1.13. Variation of stress range as a function of accumulated inelastic strain for LCF and cyclic tension tests performed at room temperature on 316FR (Mizuno et al., 2000).

Figure 1.14. Isotropic hardening (center) and stress amplitude (right) evolution in strain-controlled ratcheting tests performed at 300 °C on 316FR (Ohno et al., 1998).
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1.1.2. Multiaxial loading

When the loading conditions change from uniaxial to multiaxial, the equivalent stress response could be completely different. This subsection is divided in two paragraphs, where the material behavior under different loading conditions is reported.

Multiaxial strain-controlled symmetric cyclic tests (multiaxial LCF)

To compare the hardening behavior in uniaxial and multiaxial loading, it is necessary to use a criterion reducing the tensorial information to an equivalent scalar. For this purpose, the well-known Von Mises criterion (see Eq.1.1) is the most used.

\[
\sigma_{\text{Mises}}^{\text{eq}} = \frac{1}{\sqrt{2}} \sqrt{\left(\sigma_{11} - \sigma_{22}\right)^2 + \left(\sigma_{22} - \sigma_{33}\right)^2 + \left(\sigma_{11} - \sigma_{33}\right)^2 + 6\left(\sigma_{12}^2 + \sigma_{13}^2 + \sigma_{23}^2\right)}
\] (1.1)

The Von Mises criterion can also be expressed as a function of the deviatoric stress tensor \( \mathbf{s} \) as follows:

\[
\sigma_{\text{Mises}}^{\text{eq}} = \sqrt{3/2 \mathbf{s} : \mathbf{s}}
\] (1.2)

A modified version of Von Mises criterion valid only for incompressible deformation is also used in order to deal with plastic strain tensors.

\[
\varepsilon_{\text{Mises}}^{\text{eq pl}} = \frac{\sqrt{2}}{3} \sqrt{\left(\varepsilon_{11}^{\text{pl}} - \varepsilon_{22}^{\text{pl}}\right)^2 + \left(\varepsilon_{22}^{\text{pl}} - \varepsilon_{33}^{\text{pl}}\right)^2 + \left(\varepsilon_{11}^{\text{pl}} - \varepsilon_{33}^{\text{pl}}\right)^2 + 6\left(\varepsilon_{12}^{\text{pl}}^2 + \varepsilon_{13}^{\text{pl}}^2 + \varepsilon_{23}^{\text{pl}}^2\right)}
\] (1.3)

\[
\varepsilon_{\text{Mises}}^{\text{eq pl}} = \frac{2/3}{\mathbf{e} : \mathbf{e}}
\] (1.4)

A further modification of this law is proposed in order to evaluate the equivalent elastic strain (not fulfilling the incompressibility hypothesis). However, for simplicity this equation is usually not adopted.

\[
\varepsilon_{\text{Mises}}^{\text{eq el}} = \frac{1}{\sqrt{2(1+\nu)}} \sqrt{\left(\varepsilon_{11}^{\text{el}} - \varepsilon_{22}^{\text{el}}\right)^2 + \left(\varepsilon_{22}^{\text{el}} - \varepsilon_{33}^{\text{el}}\right)^2 + \left(\varepsilon_{11}^{\text{el}} - \varepsilon_{33}^{\text{el}}\right)^2 + 6\left(\varepsilon_{12}^{\text{el}}^2 + \varepsilon_{13}^{\text{el}}^2 + \varepsilon_{23}^{\text{el}}^2\right)}
\] (1.5)

When multiaxial loading histories are considered, the most important property to consider is the ‘proportionality’. Two loading histories \( \varepsilon \) (strain in normal direction defined in Eq.1.6) and \( \gamma \) (strain in shear direction defined in Eq.1.7) are defined ‘proportional’ when the ratio \( \rho \) (defined in Eq.1.8) is constant and equal to 1 and the phase shift \( \varphi \) is constant and equal to 0.

\[
\varepsilon = \varepsilon_0 \sin \omega t \quad (1.6)
\]

\[
\gamma = \gamma_0 \sin(\omega t + \varphi) \quad \text{with} \quad \gamma_{ij} = 2\varepsilon_{ij} \quad (1.7)
\]
1.1. Material cyclic behavior

In austenitic steels (and more in general in FCC materials), the equivalent stress response measured in multiaxial proportional test is nearly identical to the stress response retrieved from a uniaxial or a torsional test performed with the same equivalent stress amplitude (see Fig. 1.15).

On the other hand, under non-proportional loading conditions, the activation of secondary slip systems and their interaction promote the occurrence of an additional cycling hardening. This phenomenon is not negligible: the additional hardening could reach 50% of the stress amplitude value. Benallal and Marquis (1987) report that phase shift \( \varphi \) and non-proportionality ratio \( \rho \) have a direct influence on the determination of the amount of additional hardening. As depicted in Fig.1.16(a) and (b), independently of the value of \( \rho \), the increment of \( \varphi \) causes additional hardening. On the other hand, if \( \varphi \) is equal to 0, the variation of \( \rho \) will not cause additional hardening. However, when a certain phase shift is present, the amount of the additional hardening is directly influenced by the value of \( \rho \).

A similar phenomenon is observed when two simple loading histories in different directions (e.g. uniaxial and torsional) are imposed in sequence. As soon as the loading direction changes, a sudden increment of the hardening is observed (see Fig.1.17(a)). This phenomenon reported by Benallal et al. (1989) is known as ‘cross hardening’ effect and could be associated with a non-proportional loading condition.

Benallal et al. (1989) further investigated the effect of non proportionality and observed a partial evanescence of the additional hardening when a non-proportional loading stage is followed by a proportional one (see Fig.1.17(b)).

Similarly to what reported for the uniaxial case, even for multiaxial (proportional and non-proportional) loading conditions the material response is strongly influenced by the previous loading history and in particular by the maximum plastic strain range reached. An example of ‘strain range memorization’ effect and partial evanescence of memory is reported by Tanaka et al. (1985) in Fig.1.18(a).

Delobelle (1993) states that the amount of additional hardening due to non-proportionality is influenced by temperature. As reported in Fig.1.18(b), the measured additional hardening is smaller in experiments carried out at higher temperature levels.
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Figure 1.16. (a) additional hardening versus phase shift $\phi$ for a fixed value of ratio $\rho$, (b) additional hardening versus $\rho$, for a fixed value of $\phi$. Experiments performed on 316 at room temperature (Benallal and Marquis, 1987).

Figure 1.17. (a) stress response during a sequence of loading including uniaxial and torsional strain histories showing the cross hardening effect. Experiments performed on Al 2024 at room temperature, (b) stress response during a loading sequence including non-proportional and proportional strain histories showing the partial recovery of additional hardening. Experiments performed on 316 at room temperature (Benallal et al., 1989).
1.1. Material cyclic behavior

Figure 1.18. (a) Cycling hardening behavior under a sequence of multiaxial tests at different strain amplitudes showing the strain range memorization effect. Experiments performed on 316 at room temperature (Tanaka et al., 1985). (b) Evolution of the equivalent cyclic stress on 316L as a function of temperature. 'a': 1st cycle, 'b': stabilized cycle under proportional loading, 'c' stabilized cycle under non-proportional loading (Delobelle, 1993).

Multiaxial non symmetric cyclic tests (multiaxial ratcheting)

Several authors (Chaboche and Nouailhas, 1989; Delobelle, 1993; Portier et al., 2000; Bocher et al., 2001; Hassan et al., 2008) investigated the influence of ratcheting in the multiaxial loading case. It must be pointed out that it is extremely difficult to provide an exhaustive report on this topic since there is no accepted standard to perform those tests. The large amount of degrees of freedom in testing allowing differences in the control strategy do not consent a direct comparison of the experimental results reported in literature. As example, four loading paths typical for performing multiaxial ratcheting tests are reported in Fig. 1.19. Multiaxial ratcheting tests are generally carried out controlling the axis in axial direction in stress and the one in shear direction in strain or stress. The ratcheting consists in the continuous accumulation of inelastic strain in axial direction. To our knowledge, there is no reference available so far in literature on this topic, reporting results on multiaxial ratcheting tests performed controlling both the axes in strain.

It is well known that both the prescribed mean (or steady) stress in axial direction and the strain (or stress) amplitude in shear direction have a strong influence on the determination of the amount of ratcheting. As reported by Portier et al. (2000), an increment of the axial stress and/or of the shear strain amplitude induces an increment of the inelastic strain accumulation in axial direction (see Fig. 1.20(a) and (b)).

Aubin et al. (2003) and Hassan et al. (2008) report that the accumulation of ratcheting is
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Figure 1.20. Evolution of the ratcheting in axial direction versus the number of cycles for multiaxial ratcheting tests performed on 316 imposing a stepwise increasing shear strain amplitude and an axial stress equivalent to 80 MPa (a) or 100 MPa (b) (Portier et al., 2000).

Figure 1.21. Ratcheting strain in each cycle versus the number of cycle from the axial, shear, cross and square ratcheting experiments performed on a duplex steel at room temperature (Aubin et al., 2003; Hassan et al., 2008).

not only affected by the amount of stress or strain. In multiaxial experiments performed with the same mean (or steady) stress and comparable equivalent stress amplitude but different paths (see Fig.1.19), the amount of accumulated ratcheting is totally different (see Fig.1.21). This difference in the ratcheting accumulation is attributed to the different degree of non-proportionality of loading paths. In the shear experiment (see Fig.1.19(b)), ratcheting is found to be considerably smaller than the one in the uniaxial experiment (see Fig.1.19(a)), demonstrating that the cycling in shear direction is not as influent as the cycling in axial direction. In the cross ratcheting experiment (see Fig.1.19(c)) consisting in a sequence of one cycle in uniaxial direction and one in shear direction, it is not surprising that the accumulated amount of ratcheting is lower than that in the uniaxial case but higher than the one measured in the shear experiment. Finally, in the square ratcheting experiment (see Fig.1.19(d)), the ratcheting rate is found to be comparable to the one in the uniaxial test.
1.1. Material cyclic behavior

1.1.3. Time dependency

The mechanical behavior of austenitic stainless steels is time-dependent even at room temperature. In the following section the main results available in literature for the grade 316L are described. With the aim of filling some experimental gaps, references concerning 304L are also reported.

Static loading

In the last decades some authors performed creep and stress relaxation experiments in order to evaluate the time dependency of stainless steels subjected to static loading conditions. As summarized by Chaboche (1989), Nomine et al. (1982) performed a series of creep tests at room temperature and reported that applying constant stress to 316L, the deformation increases as a function of time. This phenomenon is called 'cold creep' since takes place at temperatures that are lower than 0.3 $T_{melt}$. Nomine et al. (1982) underline that cold creep occurs even with very low stress levels (e.g. 105 MPa corresponding to nearly 1/3 of the yield stress $R_{0.02}$) and the total elongation in a given time increases when the applied stress is higher (see Fig.1.22(a)).

In contrast to the classical creep, it was not possible to find a region with a constant creep rate, but the elongation rate is continuously decreasing. Surprisingly, if the temperature increases, the creep elongation decreases. In fact, according to Taleb (2013a), at higher temperatures (e.g. 350 °C) the dynamic strain aging effect becomes dominant and no significant creep is observed (see Fig.1.22(b)). If the temperature increases, overtaking 550 °C, thermal activation occurs and creep becomes again relevant.

Another way to demonstrate the time-dependent behavior of 316L even at room temperature is performing stress relaxation tests. Chaboche (1989) showed that if the selected material is subjected to a constant strain, the stress response tends to relax as a function of time (see Fig.1.23). The relaxation rate is fast in the very first part of the experiment and continuously decreases with time until it reaches a stabilized stress value. The amount of relaxation is larger when higher strain load are imposed.
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Figure 1.23. Relaxation tests on 316L at room temperature (Chaboche, 1989).

Figure 1.24. Monotonic tests on 316L with different strain rates at 25 and 250 °C (Portier et al., 2000).

Monotonic loading

The influence of loading rate on the mechanical response in different kinds of tests has been also investigated. Portier et al. (2000) demonstrated that at room temperature the monotonic stress response of 316L is rate-dependent. The strain rate sensitivity decreases when the temperature increases. At 250 °C the strain rate dependency on the monotonic response is found to be negligible (see Fig.1.24).

When the temperature increases (overtaking 550 °C), the strain rate dependency becomes again strong. According to Portier et al. (2000) at room temperature a variation of the strain rate from $10^{-6}$ to $10^{-3}$ 1/s (factor 1000) induces a difference in the monotonic stress response of 50 MPa (about 20%). Very similar findings are reported by Kang et al. (2006a) on 304L (see Fig.1.25).
1.1. Material cyclic behavior

Figure 1.25. Monotonic tests on 304L with different strain rates at 25 (a) and 350 °C (b) (Kang et al., 2006a).

Figure 1.26. Ratio between stabilized stress in LCF tests performed on 316L with different strain rates. The values of those ratios are reported as a function of temperature (Delobelle, 1993).

Symmetric strain-controlled cycling tests (LCF)

Similarly to what happens in monotonic tests, strain rate affects the cyclic response even at room temperature. Delobelle (1993) performed several LCF tests on 316L and compared the values of the stabilized stress for experiments having different straining rates (see Fig.1.26). In the interval between room temperature and 250 °C, an increment of the strain rate induces harder stabilized stress responses and the material is therefore said to show a ‘positive sensitivity’ to the strain rate.

Delobelle (1993) also reported that between 250 °C and 550 °C a negative sensitivity is observed. Finally, when the temperature overtakes 600 °C, thermal activation occurs and the strain rate sensitivity becomes positive again. According to Delobelle (1993), at room temperature a variation of the strain rate from $10^{-6}$ to $10^{-3}$ 1/s (factor 1000) induces a difference in the stabilized stress response of about 10%-20%. Very similar findings are reported by Kang et al. (2006a) on 304L (see Fig.1.27).
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Figure 1.27. LCF tests on 304L performed at 25 (a) and 350 °C (b) with a strain amplitude of 0.5% and different strain rates (Kang et al., 2006a).

Figure 1.28. Influence of stress rate on uniaxial ratcheting tests performed on 316FR at room temperature with different stress rates ($\sigma_{\text{mean}}=35$ MPa, $\sigma_{\text{alt}}=245$ MPa) (Mizuno et al., 2000).

Non symmetric strain/stress-controlled cycling tests (ratcheting)

A significant loading rate dependency at room temperature was also found in ratcheting tests. In stress-controlled ratcheting tests, the influence of stress rate (instead of strain rate) on the accumulation of inelastic strain (i.e. ratcheting) will be investigated. Mizuno et al. (2000) performed ratcheting tests on 316FR at room temperature at different loading rates and noticed that when the stress rate increases from 1 to 10 MPa/s (factor 10) the difference in ratcheting strain response is noticeable (about -50%) (see Fig.1.28). Similar findings are reported by Kang et al. (2006a) on 304L (see Fig.1.29(a)).

On the other hand, few references concerning strain-controlled ratcheting tests are available in literature. Among them, the only literature reference investigating the role of strain

Figure 1.29. (a) Influence of stress rate on uniaxial ratcheting tests on 304L at room temperature ($\sigma_{\text{mean}}=78$ MPa, $\sigma_{\text{alt}}=234$ MPa) (Kang et al., 2006a), (b) stress response in strain-controlled ratcheting tests on 316FR with two different strain rate ($10^{-6}$ to $10^{-4}$ 1/s) (Ohno et al., 1998).
1.1. Material cyclic behavior

rate is the one of Ohno et al. (1998) concerning experiments on 316FR at 650°C. Unfortunately no data about tests at lower temperature have been reported. According to Ohno et al. (1998), strain rate has a small influence on the strain response. Comparing two experiments performed with different strain rates (respectively $10^{-6}$ and $10^{-4}$ 1/s), the variation measured in the stress response was negligible and equivalent to 5 MPa (Fig.1.29(b)).

Creep or ratcheting?

Observing that the mechanical behavior of austenitic stainless steel is strongly time-dependent, one could argue that the cyclic accumulation of inelastic strain (i.e. ratcheting) is in reality nothing but creep. Analyzing the material behavior of specimens subjected to the 'classic' stress-controlled experiments, the discrimination between ratcheting and creep is however not always a straightforward task. In order to clarify this aspect, Taleb et al. (Taleb and Cailletaud, 2011; Taleb, 2013a,b) suggested to perform an original kind of experiment consisting in a sequence of creep + ratcheting (see Fig.1.30).

In this kind of experiments carried out at room temperature, specimens made of 304L were first subjected to a constant stress ($\sigma_{\text{max}}=250$ MPa) for 2 hours. Subsequently a non symmetric alternate stress history is applied ($\sigma_{\text{mean}}=100$ MPa, $\sigma_{\text{alt}}=150$ MPa) stopping the tests after 100 loading cycles. Taleb and Cailletaud (2011) compared the accumulation of inelastic strain for a simple ratcheting test and for a creep+ratcheting test. In the first test (simple ratcheting) the cyclic accumulation of inelastic strain takes place normally; on the other hand in the second test (creep+ratcheting) the major part of the inelastic strain accumulation occurs in the first step of the experiment (see Fig.1.31(a)). This finding was confirmed repeating the experiments for larger stress amplitudes ($\sigma_{\text{mean}}=50$ MPa, $\sigma_{\text{alt}}=200$ MPa, time$_{\text{creep}}=2$ h) (see Fig.1.31(b)). Taleb and Cailletaud (2011) concluded that after the creep part, no significant cyclic accumulation of the plastic strain is observed which means that most of the cyclic accumulation of the inelastic strain exhibited in the classical ratcheting tests is mainly due to creep. Taleb noticed that contrary to what it is generally admitted in literature, ratcheting seems very small for this material at room temperature.

In a more recent work (Taleb, 2013b), Taleb repeated the same experiments also on specimens made of different materials including an austenitic stainless steel (316L) and two ferritic steels (35NiCrMo16, XC18). While the results previously obtained for 304L were confirmed also for 316L (see Fig.1.32) this was not the case for the ferritic steels showing a noticeable cyclic accumulation of inelastic strain without suffering any significant creep (see Fig.1.33). These controversial results led Taleb to conclude that further investigations, including microscopical analyses, are required to improve the knowledge on this phenomenon.
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Figure 1.31. Ratcheting and Creep + ratcheting test for the small amplitude case (a) and for the large amplitude case (b) (304L, Room temperature) (Taleb and Cailletaud, 2011).

Figure 1.32. Evolution of the axial stress and strain for both ‘ratcheting’ and ‘creep-ratcheting’ tests (316L, room temperature): (a) axial stress vs axial strain; (b) axial strain peaks vs the number of cycles. (Taleb, 2013b).

Figure 1.33. Evolution of the axial stress and strain for ‘creep-ratcheting’ tests (XC18, room temperature): (a) axial stress vs axial strain; (b) axial strain peaks vs the number of cycles. (Taleb, 2013b).
1.1. Material cyclic behavior

Similar experiments were performed by Nomine et al. (1982) on 316L. This author carried out the first part of the experiment (creep test) applying the prescribed load for 1000 hours. The subsequent part (stress-controlled ratcheting) was performed with a constant ratio between the maximum and the minimum stress \( \frac{\sigma_{\text{max}}}{\sigma_{\text{min}}} = 10 \). The accumulation of the inelastic strain was measured after 1000 cycles. Similarly to what reported by Taleb and Cailletaud (2011), the major part of the accumulation of inelastic strain took part in the creep test (see Fig. 1.34(a)). After the creep part, no significant cyclic accumulation of the plastic strain was observed. Nomine et al. (1982) further investigated the phenomenon changing the order of the loading sequences (ratcheting+creep instead of creep+ratcheting). The order in which the different tests are carried out had no influence on the total elongation up to a maximum stress level of 250 MPa. For higher stress the sequence creep+ratcheting produced a total elongation larger than that of the sequence ratcheting+creep (see Fig. 1.34(b)). Nomine et al. (1982) motivated this result observing that, in creep tests, the inelastic strain accumulation at a given instant increases with the applied stress. The behavior in simple ratcheting tests is completely different: the accumulated inelastic strain after 1000 cycles initially increases with the maximum stress, then decreases and becomes very small at high stresses. These findings allowed the author to conclude that creep elongation can not be directly related to the cyclic accumulation of strain.

1.1.4. Microstructural evolution

Thanks to the pioneering work of Ewing and Humfrey (1903), from the start of the 20th century it is well known that the microstructural evolution of the material plays an important role in the mechanism of fatigue failure. Their study in fact demonstrated the local plasticity induced by cyclic loading induces the formation of fatigue cracks. The link between fatigue and microstructural features was therefore an important matter of research over all the 20th century and in this framework, a considerable help was provided by Transmission Electron Microscopy (TEM). In this subsection a short summary of the most important literature references concerning this topic is reported.
Monotonic loading

The microstructural evolution of 316L under monotonic loading has been studied by several authors in the last decades. Among them, Kang et al. (2010) performed TEM observations on specimens subjected to 2 different strain levels (i.e., 3% and 20%). In Fig.1.35 the corresponding micrographs are reported. The author reports that in the beginning of monotonic deformation, dislocation moves by planar slip and some simple dislocation structures (such as dislocation lines and pile-ups) are formed. Increasing the plastic strain, an increment of dislocation density is observed and the interaction between slip systems promotes the formation of more complicated dislocation structures (e.g. dislocation tangles) (see Fig.1.35(a)). At this stage, the different grain crystallographic orientations cause a non uniform plastic distribution and for that reason simple dislocation structures (e.g. dislocation lines and pile-ups) are still present in some regions (see Fig.1.35(b)). A further increment of strain causes the dislocation density to increase and promotes a higher degree of organization of dislocation in structures like cells and heavy dislocation tangles (see Fig.1.35(c)). At this stage some twins are also observed (see Fig.1.35(d)).

Symmetric strain-controlled cyclic tests (LCF)

Kang et al. (2010), Mayama et al. (2008) and Pham et al. (2011) performed TEM observations on fatigued specimens interrupting a series of LCF experiments with constant
1.1. Material cyclic behavior

Figure 1.36. (a) Dislocation structures in the as-received (solution annealed) condition showing planar structures (dislocation pile-ups and stacking faults), (b) dislocation structures at the end of the hardening stage (LCF with 0.7% strain amplitude, 316L, Room temperature) (Pham et al., 2011).

strain amplitudes and different number of cycles. Since TEM images are 2D projections of 3D structures, their interpretation strongly depends on the direction of observation. As a consequence, it is not trivial to reconstruct the 3D arrangement of dislocation structures. In order to fully reveal the 3D dislocation structures, it is possible to take a series of images exploiting the tilting function of TEM. This approach allowed Pham et al. (2011) to provide a significant contribution to the analysis of 316L subjected to cyclic loading proposing a 3D representation of the developing dislocation structure.

In the as-received condition, dislocations are characterized by an extremely low density and are mainly present in the form of planar structures (i.e. stacking faults and pile-ups) (see Fig.1.36(a)). After few cycles, dislocation density strongly increases (see Fig.1.36(b)) causing the macroscopic cyclic hardening reported in Fig.1.1. At the end of the hardening, dislocations are uniformly distributed in a mixture of tangles and planar dislocation structures.

Further cycling promotes the organization of dislocation in structures like veins and walls causing the creation of relatively free dislocation regions (i.e. channels) (see Fig.1.37(a)). The organization of dislocations in lower energy structures is generally associated with the macroscopic cyclic softening observed in Fig.1.1. The process of dislocations reorganization continues until the end of life. At this stage walls and channels are shorter than those observed at the previous stage and the further cycling promotes the formation of more stable cell structures (see Fig.1.37(b)). This microstructural behavior is associated with the macroscopic stress stabilization observed in Fig.1.1.

Non symmetric stress-controlled cyclic tests (ratcheting)

Kang et al. (2010), Song (2009) and Gaudin and Feaugas (2004) performed TEM observations also on ratcheted specimens interrupting a series of stress-controlled ratcheting experiments with constant stress amplitudes and different number of cycles. It must be pointed out that a fair comparison of specimens subjected to LCF and stress-controlled ratcheting loading conditions is not practical since, because of cyclic hardening, the resultant strain amplitude in ratcheting tests is lower than the one prescribed in the strain-controlled LCF
Figure 1.37. (a) Dislocation structures in the at the middle of softening stage, (b) dislocation structures at the end of life (LCF with 0.7% strain amplitude, 316L, Room temperature) (Pham et al., 2011).

experiments. To the author’s knowledge, no reference is available so far in literature on this topic, reporting results on strain-controlled ratcheting tests.

The microstructural evolution mechanism in LCF and ratcheting is in general similar but some particular aspects can be retrieved from the work of Song (2009) and Gaudin and Feaugas (2004). Song (2009) performed LCF and ratcheting tests on 304L and observed higher dislocation density in the channels for ratcheted specimens (see Fig.1.38). The author attributes this difference to the slower condensation of dislocations into walls and cells because of the increase of dislocation mobility promoted by uniaxial ratcheting. Gaudin and Feaugas (2004) performed a similar study on 316L and observed the ‘polarization’ of dislocation structures during ratcheting (see Fig.1.39). This ‘polarization’ consists in the misorientation between channels and in the preferential orientation of dislocations in certain directions.
1.1. Material cyclic behavior

Figure 1.38. Dislocation density in uniaxial force-controlled ratcheting and LCF experiments (304L, Room temperature) (Song, 2009).

Figure 1.39. Polarized dislocations structures in ratcheted specimens: (a) tangle, (b) walls and (c) cells (316L, Room temperature) (Gaudin and Feaugas, 2004).
1.2 Modeling of cyclic behavior

Several constitutive models have been proposed in the 20th century to provide relations between strain and stress with the aim of increasing the accuracy of structural simulations. In the case of elastic analyses this relation is linear, but in the case of plasticity (and in particular of cyclic plasticity) it becomes strongly non-linear and requires more complex approaches. For simplicity it is possible to divide those models in two big families:

– physics-based models,
– phenomenological models.

Since the constitutive law proposed in this dissertation belongs to the second class of models, this review is focused on literature references concerning a phenomenological approach.

The basic concepts of the generalized multidimensional theory of plasticity were developed by Prager (Prager, 1949, 1956) more than 50 years ago. The author introduced basic notions like the separation of strain in an elastic \( \varepsilon^{el} \) and a plastic part \( \varepsilon^{pl} \), the 'yield condition' (Eq.1.9) and the 'consistency condition' (Eq.1.10). In order to describe the 'Bauschinger effect', he formulated the concept of kinematic hardening assuming that the yield surface is allowed to move in the stress space. The position of the center of the yield surface was defined by means of a tensor \( \alpha \) named 'backstress' and its evolution was described by means of a linear hardening law (Eq.1.11). \( s \) represents the deviatoric part of the stress tensor \( \sigma \) and \( Y \) the size of the yield stress surface.

\[
F_y = \sqrt{\frac{3}{2}} (s - \alpha) : (s - \alpha) - Y \leq 0 \quad (1.9)
\]

\[
\dot{F}_y = 0 \quad (1.10)
\]

\[
\dot{\alpha} = \frac{2}{3} C \dot{\varepsilon}^{pl} \quad (1.11)
\]

Another important improvement was achieved by Shield and Ziegler (1958) with the formulation of the first combined isotropic-kinematic hardening model. Shield and Ziegler (1958) also introduced the 'normality rule' (Eq.1.12) for the definition of the direction of the plastic strain increment in a tridimensional space.

\[
\dot{\varepsilon}^{pl} = \lambda \frac{\partial F_y}{\partial \sigma} = \dot{\lambda} \cdot N \quad (1.12)
\]

To improve the accuracy of the hysteresis loop shape description, Armstrong and Frederick (1966) formulated the first non-linear kinematic hardening law in which the incremental rate for backstress is function of the backstress itself (Eq.1.13). As secondary effect of this modification, the Armstrong-Frederick (AF) model was also suitable for the qualitative description of ratcheting.

\[
\dot{\alpha} = \frac{2}{3} C \dot{\varepsilon}^{pl} - \gamma \alpha \dot{\rho} \quad \text{with} \quad \dot{\rho} = \sqrt{\frac{2}{3} \dot{\varepsilon}^{pl} : \dot{\varepsilon}^{pl}} \quad (1.13)
\]

A further improvement towards a quantitative reproduction of the cyclic material behavior was provided by the constitutive model developed by Chaboche (1977) (see Eq.1.14) and ameliorated by Ohno and Wang (1993) (see Eq.1.15). In these descriptions, the response
of the material is decomposed into the superposition of three or more non-linear backstress components $\alpha^{(k)}$ (see Eq.1.16).

$$\dot{\alpha}^{(k)} = \frac{2}{3} C^{(k)} \dot{\varepsilon}^{pl} - \gamma^{(k)} \alpha^{(k)} \dot{p}$$ (1.14)

$$\dot{\alpha}^{(k)} = \frac{2}{3} C^{(k)} \dot{\varepsilon}^{pl} - \gamma^{(k)} \left( \frac{\|\alpha^{(k)}\|}{C^{(k)}/\gamma^{(k)}} \right)^{m^{(k)}} \left< N : \frac{\alpha^{(k)}}{\|\alpha^{(k)}\|} \right> \alpha^{(k)} \dot{p}$$ (1.15)

$$\alpha = \sum_{k=1}^{n} \alpha^{(k)}$$ (1.16)

Chaboche put a lot of effort in enhancing the accuracy of his model and in 1986, he proposed a unified plasticity theory suitable for the description of strongly non-linear features like 'cyclic hardening', 'strain range memorization' effect and 'ratcheting' (Chaboche, 1986). This formulation of the Chaboche model consists of a superposition of 3 kinematic hardening laws and of an isotropic one. The kinematic hardening parameters $C^{(k)}$ and $\gamma^{(k)}$, $k$ being the component index, are constants calibrated to match:
- the stabilized stress-strain loop shape in uniaxial low cycle fatigue (LCF) experiments ($C^{1,2}$, $\gamma^{1,2}$),
- the cyclic accumulation of inelastic strain in uniaxial stress-controlled ratcheting tests ($\gamma^{3}$).

The cyclic hardening is roughly modeled varying the radius $Y$ of the yield surface by means of an exponentially saturating isotropic hardening law $R$ (see Eq.1.17).

$$Y = \sigma_y 0 + R = \sigma_y 0 + Q(1 - e^{-bp})$$ (1.17)

The experimental evidences presented in section 1.1.3 led some authors to propose further modifications of these constitutive models in order to provide more accurate descriptions of the time-dependent mechanical behavior of austenitic stainless steels. In this framework, it is worthwhile to cite the work of Robinson et al. (1976), Contesti and Cailletaud (1989) and Chaboche (1989) who proposed completely different approaches to take into account the effect of the loading rate on the simulated material response.

In the same years, it became evident that the mechanical behavior of stainless steels under multiaxial loading conditions is strongly influenced by the applied stress/strain trajectory (Tanaka et al., 1985; Benallal and Marquis, 1987). With the aim of improving the accuracy of stress calculations of austenitic stainless steels subjected to multiaxial loading histories, Benallal and Marquis (1987) (Eq.1.18) and Tanaka (1994) developed tensorial theories in order to evaluate the degree of non-proportionality $\varsigma$ corresponding to a generic prescribed loading path.

$$\cos^2 \varsigma = \frac{(\alpha : \hat{\alpha})^2}{(\alpha : \hat{\alpha})(\hat{\alpha} : \hat{\alpha})}$$ (1.18)

For stainless steels subjected to complex loading conditions (low cycle fatigue including strain amplitude variation, temperature change and ratcheting), the accuracy of calculations provided by the original Chaboche (Chaboche, 1986) and Ohno (Ohno and Wang, 1993) models is often not sufficient. To capture the non-linear mechanical behavior of the austenitic stainless steels such as 316L and to accurately reproduce the hysteresis loop shape evolution, several different approaches characterized by an increasing complexity were implemented in the last decades.
A first approach consisted in formulating modified versions of the kinematic hardening laws provided by Chaboche and Ohno. Among the numerous examples available in literature, two of the most popular models were formulated by Jiang and Sehitoglu (1996) (see Eq.1.19-1.20) and by Bari and Hassan (2002) (see Eq.1.21-1.22).

\[
\dot{\alpha}^{(k)} = \frac{2}{3} C^{(k)} \dot{\varepsilon}^{pl} - \gamma^{(k)} \left( \frac{f(\alpha^{(k)})}{C^{(k)}} \right) m^{(k)} \alpha^{(k)} \dot{p} 
\]

\[
m^{(k)} = A_0^{(k)} \left( 2 - \frac{\dot{\varepsilon}^{pl}}{\dot{p}} \cdot \frac{\alpha^{(k)}}{f(\alpha^{(k)})} \right) 
\]

\[
\dot{\alpha}^{(k)} = \frac{2}{3} C^{(k)} \dot{\varepsilon}^{pl} - \gamma^{(k)} \left\{ \delta' \alpha^{(k)} + \left( 1 - \delta' \right) \left( \alpha^{(k)} N \right) N \right\} \dot{p} \quad \text{for} \quad k = 1, 2, 3 \tag{1.21}
\]

\[
\dot{\alpha}^{(k)} = \frac{2}{3} C^{(k)} \dot{\varepsilon}^{pl} - \gamma^{(k)} \left\{ \delta' \alpha^{(k)} + \left( 1 - \delta' \right) \left( \alpha^{(k)} N \right) N \right\} \left( 1 - \frac{\alpha^{(k)}}{f(\alpha^{(k)})} \right) \dot{p} \quad \text{for} \quad k = 4 \tag{1.22}
\]

Other researchers (Kurtyka and Zyczkowski, 1996; Vincent et al., 2004) relaxed one of the basic assumptions of the generalized plasticity theory: the isotropy of the yield surface. In this family of models, the description of the hardening and of ratcheting is carried out by means of complicated algorithms governing the distortion of the yield surface (see Fig.1.40).

A completely different approach is represented by the multiple-mechanisms multiple-criteria models proposed by Säi (1993) and Cailletaud and Säi (1995). While the constitutive equations presented so far consider only one inelastic mechanism and a single yield criterion, a generic iMjC model is assumed to depend on \(i\) mechanisms and on \(j\) criteria. An example of constitutive law belonging to this family is the 2M2C model (2 inelastic mechanisms and 2 yield criteria) that can be seen as the collection of two unified models (see Eq.1.23-1.30). The overall stress \(\sigma\) is used to compute the two local stresses \(\sigma^{(i)}\) according to a scale transition rule. For each inelastic mechanism \(i\), a backstress \(\alpha^{(i)}\) is associated to a local kinematic internal variable \(\chi^{(i)}\). For each yield criterion \(j\), a law \(R^{(j)}\) governing the change of the size of the yield surface is associated to a local isotropic internal variable \(r^{(j)}\). Knowing the updated values of \(\sigma^{(i)}, \alpha^{(i)}, R^{(j)}\) and the initial size of the local yield surface \(\sigma_{y0}^{(j)}\), the yield
functions are evaluated. The overall plastic strain rate \( \dot{\varepsilon}_{pl} \) is finally calculated as the average of the plastic deformation of each mechanism. In order to improve the accuracy of this class of models for the description of ratcheting, further modifications of the iMJC constitutive law were recently proposed (Taleb et al., 2006; Taleb and Cailletaud, 2010).

\[
\dot{\varepsilon} = \dot{\varepsilon}_{el} + A_1 \dot{\varepsilon}^{(1)}_{pl} + A_2 \dot{\varepsilon}^{(2)}_{pl} \\
\sigma^{(1)} = A_1 \sigma, \quad \sigma^{(2)} = A_2 \sigma
\]

(1.23) \hspace{1cm} (1.24)

\[
F^{(1)}_y = J_2(s^{(1)} - \alpha^{(1)}) - \sigma^{(1)}_{y(0)} - R^{(1)} \leq 0, \quad F^{(2)}_y = J_2(s^{(2)} - \alpha^{(2)}) - \sigma^{(2)}_{y(0)} - R^{(2)} \leq 0
\]

(1.25)

\[
\chi^{(1)} = \dot{\lambda}^{(1)} \left[ N^{(1)} - \frac{3D_1}{2C_{11}} \alpha^{(1)} \right], \quad \chi^{(2)} = \dot{\lambda}^{(2)} \left[ N^{(2)} - \frac{3D_2}{2C_{22}} \alpha^{(2)} \right]
\]

(1.26) \hspace{1cm} (1.27)

\[
\dot{\varepsilon}^{(1)}_{pl} = \dot{\lambda}^{(1)} N^{(1)} = \dot{\lambda}^{(1)} \frac{\partial F_y^{(1)}}{\partial \sigma^{(1)}}, \quad \dot{\varepsilon}^{(2)}_{pl} = \dot{\lambda}^{(2)} N^{(2)} = \dot{\lambda}^{(2)} \frac{\partial F_y^{(2)}}{\partial \sigma^{(2)}}
\]

(1.28)

\[
\begin{bmatrix} R^{(1)} \\ R^{(2)} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \begin{bmatrix} r^{(1)} \\ r^{(2)} \end{bmatrix}
\]

(1.29)

\[
r^{(1)} = \dot{\lambda}^{(1)} [1 - (b_1/Q_{11})R^{(1)}], \quad r^{(2)} = \dot{\lambda}^{(2)} [1 - (b_2/Q_{22})R^{(2)}]
\]

(1.30)

Another promising approach consists in keeping the constitutive laws as simple as possible and allowing both the isotropic and kinematic hardening parameters to vary with cycling. The first example of an elasto-plastic constitutive model with an internal variable dependency can be found in the endochronic theory of plasticity proposed by Valanis (1978), which is based on the hereditary form of thermodynamics of irreversible processes. This approach was further refined by more recently developed, advanced constitutive models (Kang et al., 2002; Döring et al., 2003; Yang, 2004; Takahashi et al., 2008; Hassan et al., 2008), consisting of modifications of the original formulations of Chaboche and Ohno in which different internal variable dependencies are directly introduced in the hardening laws by means of analytical relations. However, increasing the number of parameters, the strong coupling existing between the loop shape and the cyclic hardening response makes the calibration of such models a difficult task, limiting de facto their use in an industrial context.

A recognized drawback of these phenomenological models is that they lack of physical background. As a consequence, they are not capable to accurately simulate the mechanical behavior of the investigated material once the boundary conditions go beyond the considered mechanism range. An alternative approach to improve the descriptive and predictive capabilities of the internal variables dependent models consists in providing a satisfactory theoretical basis for the evolution of some physically relevant quantities named internal stresses (i.e. effective stress and backstresses). The definition commonly adopted for the internal stresses is the following:

- The effective stress is the loading required to make dislocations overcome the resistance to dislocation movement (including the intrinsic and the extrinsic resistance of the material). For that reason the effective stress is generally associated with the yield stress concept and its evolution is related with the isotropic hardening.

- Backstress is the loading associated with long range interactions of dislocations originated by inhomogeneous plastic deformation leading to microstructural heterogeneities of materials on different scales (inter-granular or intra-granular backstress). The evolution of the backstress is related with the kinematic hardening.
The observation that the movement of dislocations is the main physical basis for plastic deformation led several authors to use the dislocation densities as microstructure-related internal state variables for this class of constitutive models (Estrin et al., 1996; Roters et al., 2000; Arsenlis and Parks, 2002; Sauzay, 2008; Pham et al., 2013). The investigations conducted in the last decades (Cotrell, 1953; Feaugas, 1999; Kocks and Mecking, 2003) on the relationships existing between microstructural conditions and internal stresses made possible the formulation of several complex physically-based evolutionary laws (Estrin et al., 1996; Roters et al., 2000; Arsenlis and Parks, 2002; Sauzay, 2008; Pham et al., 2013). Among them, a significant contribution for the description of the mechanical behavior of AISI 316L subjected to uniaxial LCF conditions similar to the ones investigated in the current dissertation, is provided by the work recently published by Pham et al. (2013). Pham et al. suggest that the cyclic hardening/softening behavior observed in 316L and previously discussed in Section 1.1.1, can be modeled expressing the given internal stress $\sigma_i$ by means of Taylor-type (Taylor, 1934) equations (see Eq.1.31) as a function of the density $\rho_i$ of specific kinds of dislocations (dislocations present close to grain boundaries, interior dislocations, dislocation sessile junctions, wall dislocations and channel dislocations) and of some geometrical parameters (e.g. fraction of walls, wall spacing, wall thickness, cell-to-cell misorientation, etc.).

$$\sigma_i = M\alpha GB \sqrt{\rho_i} \quad (1.31)$$

The evolution of the density $\rho_i$ of each kind of dislocations is then computed by means of a non-linear differential equations based on the model proposed by Mecking et al. (1976) (see Eq.1.32).

$$\frac{d\rho_i}{d\varepsilon_{pl}} = k_1 \frac{1}{L} - k_2 \rho_i \quad (1.32)$$

Through an analytical simplification, Pham et al. (2013) showed that the presented constitutive law has strong analogies with a two-component Chaboche-type model (Chaboche, 1986), underpinning a relatively simple implementation of this material description in a commercial finite element code. Although these promising results, further developments are required for this class of models in order to improve their performance for simulating the material response under more complex boundary conditions including ratcheting and multiaxial loading.

All these different examples highlight that the continued research in this field pushed the authors to develop more and more complicated constitutive models. Today the formulation of constitutive models able to provide accurate stress-strain relation with a reduced complexity is still a challenge. The purpose of the current work is to provide a constitutive law, suitable for an industrial application, which in spite of a drastic reduction of the number of parameters, keeps the capability to quantitatively describe the mechanical behavior with sufficient accuracy for the selected material under the given boundary conditions.

### 1.3 Modeling of damage

The first attempt to discover the reason of the unexpected and systematic failures encountered by components subjected to repeated loading is attributed to Wöhler (1870) who identified repeated stressing as the cause of the failures of railway axles. Wöhler experimentally proved this finding performing several fatigue tests with an appositely designed testing machine. By means of this machine he was able to reproduce on a sample the loading conditions typical for the axles. The data collected by Wöhler allowed engineers to construct the well known S-N (stress versus number of cycles to failure $N_f$) curve and Basquin (1910) to formulate his famous equation (Eq.1.33) that was the fundamental tool for fatigue design for nearly 100 years.
This approach was effective dealing with components subjected to stress levels lower than the elastic limit and characterized by a long-life or high cycle fatigue range (HCF). However, after the middle of the 20th century, with the development of new applications, especially in the aeronautical industry, components were subjected to more severe loading overtaking the elastic limit of the material. It was suddenly evident that the S-N curve was not the ideal tool to deal with such a kind of loading conditions involving short-life fatigue (LCF). In order to overcome this problem some authors (Manson, 1952; Coffin, 1954) and later Morrow (1964) associated fatigue life with strain and more precisely with plastic strain. Their efforts allowed to construct the famous Manson-Coffin-Morrow curve establishing the relation between strain amplitude and fatigue life (Eq.1.34).

\[
\frac{\Delta \varepsilon}{2} = \left( \frac{\sigma_f}{E} \right) \cdot 2N_f^b + \varepsilon'_f \cdot 2N_f^c
\]  

(1.34)

For most materials and simple loading conditions the strain-based approach is a powerful instrument to assess the lifetime of components. However, for a certain class of materials and non-trivial loading conditions, the mechanism leading to fatigue is extremely complex and it is not surprising that life predictions calculated by this simple life curves approach are not sufficiently accurate. For example, as reported in section 1.1.1, the properties of certain austenitic stainless steels are progressively changing during the fatigue experiment and temperature, strain amplitude and mean strain quantitatively and qualitatively affect the cyclic response and therefore the damage accumulation process. In addition it is necessary to underline that the loading conditions typical for real components are not as simple as the ones imposed in common fatigue experiments (e.g. uniaxiality, constant stress/strain amplitude, zero mean stress/strain).

Therefore, in the second half of the 20th century, several authors put their efforts in order to develop damage accumulation criteria able to take into account those complexities. The first problem to solve was the determination of the life of the structure subjected to varying loading conditions and the first approach proposed for treating cumulative damage was the linear damage rule (LDR) (Palmgren, 1924; Miner, 1945). The main assumption of the LDR is that, considering a variable amplitude loading history, the damage fraction at any stress level, is proportional to \( n_i \) the ratio of number of cycles imposed at that stress level, to the total number of cycles \( N_i \) leading to failure at that stress level (Eq.1.35).

\[
\sum \frac{n_i}{N_i} = 1
\]  

(1.35)

Further experiments demonstrated that the linearity assumption was not strictly correct since it was not able to explain phenomena like the loading-order and load interactions effect. In order to overcome the LDR’s limitations, several alternative methods have been proposed and they can be divided in the following categories:

- double linear damage rule approaches (DLDR) (Grover, 1960; Manson, 1966)
- nonlinear damage curve approach (DCA) (Manson and Halford, 1986)
- life curve modifications accounting for load interactions (Bui-Quoc, 1982)
- approaches based on crack growth concept (Miller, 1982)
- models based on continuum damage mechanics (CDM) (Chaboche, 1974)
- energy-based methods (Kujawski and Ellyin, 1984)
Due to the complexity of the fatigue mechanism, every damage model belonging to these categories is able to take into account only few phenomenological factors and therefore has not general validity. In this framework, some models were appositely developed with the aim of improving the life predictions of material, such as stainless steels, showing strong deformation history dependence under complex loading conditions including multiaxial loading and ratcheting.

Certain authors recognized that, in order to take into account the effect of cycling hardening on the damage accumulation, it is necessary to consider in the fatigue criterion not only the strain but also the stress. Among them, Smith-Watson-Topper (SWT) proposed an energy based damage model able to significantly enhance the accuracy of lifetime predictions for loading cases with a non zero mean stress (Eq.1.36) (Smith et al., 1970).

\[
\sigma_{\text{max}}\varepsilon_a = \sigma' f' f (2N_f)^{b+c} + \frac{\sigma_f^2}{E}(2N_f)^{2b}
\] (1.36)

Another criterion introducing a considerable improvement for multiaxial (and in particular non-proportional) loading conditions, is the well-known Fatemi-Socie (Fatemi and Socie, 1988) fatigue parameter (FP) (Eq.1.37). This model consists in a critical plane criterion meaning that the terms of the equation are calculated in the critical material plane where the damage is maximum. The Fatemi-Socie criterion is also able to take into account the detrimental effect of non-proportionality in multiaxial fatigue considering the eventual additional hardening.

\[
FP = \frac{\Delta\gamma}{2} \left( 1 + K \frac{\sigma_{\text{max}}}{\sigma_y} \right)
\] (1.37)

In this framework, Jiang (Jiang, 2000) combined the main advantages of the SWT and Fatemi-Socie criteria, proposing another advanced model suitable to deal with damage accumulation under multiaxial cyclic loading including ratcheting (Eq.1.38 and 1.39). Jiang incorporated the critical plane concept in multiaxial fatigue together with plastic strain energy and material memory. Thanks to its formulation, the model is also designed to consider mean stress and loading sequence effects.

\[
dD = \left( \frac{\sigma_{\text{mr}}}{\sigma_0} - 1 \right)^m + \left( 1 + \frac{\sigma}{\sigma_f} \right) dY
\] (1.38)

\[
dY = a\sigma d\varepsilon^{pl} + \frac{1-a}{2}\tau d\gamma^{pl}
\] (1.39)

Owing to its capability to deal with complex multiaxial loading conditions and its relatively simple calibration, this model has been implemented in the current study and a discussion on its performance is reported in section 4.3.
In the following chapter, an overview of the technologies and methods adopted to carry out this study is reported. In section 2.1, a detailed description of the equipment and of the procedures followed to perform the uniaxial and multiaxial tests is presented. In sections 2.2 and 2.3, an overview of the technologies used to characterize the microstructural evolution (respectively microscopy and magnetic measurements) is given. Finally, section 2.4 is dedicated to summarize the basic concepts of finite element method.

2.1 Mechanical tests

In this section, a detailed description of the sample preparation and of the equipment used to perform the uniaxial, uniaxial TMF and multiaxial tests is presented.

2.1.1. Uniaxial tests experimental setup

Sample preparation

Samples for uniaxial isothermal tests are machined from both the 316L material batches already characterized in section 1.3. In particular, for pipe material (TP316L), cylindrical samples with a diameter of 18 mm and a length of 110 mm are extracted from the tube along the pipe direction. From those cylinders, specimens are then manufactured to perform uniaxial tests according with the ASTM standard E606 (ASTM, 2012) and with the geometry reported in Fig.2.1. Plate material is sampled from a 20 mm thick hot rolled plate. Parallelepips with a length of 110 mm along the rolling direction and a 20 mm wide square cross section are extracted from the plate and specimens are machined using the same manufacturing process and geometry as for TP316L. Specimens are extracted at sufficient distance from the edge of the plate to eliminate data scatter caused by microstructural variations typically observed at the edge of rolled sheet metals.
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Figure 2.1. Geometry of specimens for uniaxial isothermal fatigue experiments (PSI, 2008).

Testing equipment

All the uniaxial isothermal experiments are executed using a 250 kN uniaxial Schenk Hydropuls fatigue bench. While LCF tests are carried out according to the ASTM standard E606 (ASTM, 2012), no standard is available for strain controlled ratcheting test. The tests are performed controlling the machine in strain by a clip-on extensometer Sandner having an initial gage of 20 mm and measuring displacement of $\pm 1.25$ mm. The experiments are performed at two constant temperature levels, namely room temperature and $200 \, ^\circ C$. A thermocouple is placed on the specimen surface to check for unwanted heating of the specimen caused by the dissipation of the energy due to plastic deformation. This equipment is used to perform the experiments described in detail in sections 3.1, 3.2, 3.3 and 3.4.

2.1.2. Uniaxial TMF tests experimental setup

Sample preparation

Hollow samples for uniaxial non-isothermal tests are machined from the pipe material batch (TP316L) characterized in section 1.3. The specimens have an outer diameter of 10 mm and an inner diameter of 5 mm (see Fig.2.2). They are machined out of the tube wall along pipe direction according to the ASTM standard E2368 (ASTM, 2010). As recommended by the standard, the bore of the tubular specimens are honed to an average surface roughness of $0.2 \, \mu m$. The resulting wall thickness of 2.50 mm guarantees a quick temperature variation and limits the temperature difference between the specimen and the water flowing inside the hollow sample. The inlet and outlet nozzles are welded by electron beam to the specimen heads. Two additional nozzles and short dead end pipe sections are welded for symmetry reasons in order to guarantee symmetrical heating and straining.

Testing equipment

All the uniaxial TMF experiments are executed using a 100 kN uniaxial Instron 8862 fatigue bench and the TMF facility at PSI according to the ASTM standard E2368 (ASTM, 2010). The tests are performed controlling the machine in strain by an extensometer MTS suitable for high temperatures having an initial gage of 15 mm and measuring displacement of
± 0.3 mm. The temperature of specimens is regulated by means of the fluid (high-pressure water) flowing through the hole of the sample. The water flows in a closed circuit and can be heated up to 340 °C by an induction coil and cooled down to room temperature by a cooling unit. The maximum heating and cooling rate allowed by the facility is 4 °C/s. A digital universal processor controls this system in a closed loop allowing the user to impose the desired temperature profile directly to the specimen. The temperature measurements are performed by means of a ribbon thermocouple wrapped around the sample at the middle of the gage length. A picture and a sketch of the testing facility are reported in Fig. 2.3. This equipment is used to perform the experiment used as benchmark for the constitutive model in section 4.3.

2.1.3. Multiaxial tests experimental setup

Sample preparation

Hollow samples for multiaxial tests are machined from the plate material batch characterized in section 1.3 according to the ASTM standard E2207 (ASTM, 2008). The specimens have an outer diameter of 16 mm and an inner diameter of 13 mm (see Fig. 2.4). The resulting wall thickness of 1.50 mm limits unwanted buckling effects and minimizes the error in the measurement of the shear stress due to the stress gradient effect.
Testing equipment

All the multiaxial experiments are executed in collaboration with Politecnico di Milano using a triaxial fatigue MTS 809 fatigue bench (250 kN, 2500 Nm). Multiaxial LCF and ratcheting tests are carried out controlling in strain the axial and the torsional axis. No internal pressure is imposed. The strain is measured by a biaxial extensometer having an initial gauge of 25 mm and measuring displacement of ± 2.50 mm and a rotation of ± 5.0 degrees. The experiments are performed only at room temperature. This equipment is used to perform the experiments described in detail in section 3.2.

2.2 Microstructural investigation

In this section, a detailed description of the sample preparation and of the equipment used to characterize the microstructural evolution of the material is presented.

2.2.1. Transmission electron microscopy

Transmission electron microscopy (TEM) is a powerful technique to characterize the microstructural evolution of the material at dislocations scale level. The schematics illustrating the typical structure of TEM is shown in Fig.2.5(a). The electron source emits electrons that are collimated by means of two condenser lenses and are projected on the surface of a very thin specimen. The specimen because of its crystal structure diffracts the electron beam along certain orientation according to Bragg’s law. The crystal distorted by the presence of dislocations diffracts the electron beam in a different way allowing the observer to detect the dislocations. An objective lens immediately below the specimen collects the diffracted beam and focuses it in the back focal plane. Using an appropriate set of lenses, the diffracted beams can be captured and imaged in the diffraction mode or in the image mode.

Sample preparation

Uniaxial fatigue specimens from the plate material batch (characterized in section 1.3) are first sectioned along the loading axis by means of a cutting machine. Subsequently,
samples are mechanically ground and polished producing thin plates having a thickness of about 0.1 ± 0.02 mm. These foils are then punched out generating discs with a diameter of 3 mm suitable for the TEM sample holder. It is important to underline that at this stage a mark is produced on the samples in order to indicate the loading direction. The discs are then electrolytically polished by means of the double jet device TenuPol5 using an electrolyte solution of acetic acid and perchloric acid. This procedure guarantees that the area of specimen close to the central hole has the suitable thickness to perform TEM observations (80-150 nm).

Testing equipment

TEM observations were performed in collaboration with the High Temperature Integrity Group at EMPA Dübendorf. The samples are observed by means of a Philips CM30 transmission electron microscope (see Fig.2.5(b)) at 300kV equipped with a double tilt holder. The area close to the thin hole is examined and some grains containing the most representative microstructural structures are selected for the analysis. The mark of the loading direction (LD) is aligned along the axis of the holder in order to retrieve this information in the collected images. Diffraction patterns are used to identify the orientation of the grain. This equipment is used to perform the experimental observations described in detail in section 3.4.

2.3 Magnetic measurements

The main phase of 316L is the paramagnetic FCC austenite and its stability depends on the chemical composition and temperature. Depending on the stability of the austenitic phase, cyclic straining could lead to a different amount of strain-induced ferromagnetic α’-martensite formation (Leber et al., 2007). Talonen et al. (2004) and the ‘GRETE’ project carried out at PSI (Niffenegger et al., 2003) demonstrated that the measurement of the magnetic susceptibility is a simple but effective and reliable technique to detect the amount of
strain-induced $\alpha'$-martensite. In the current study, magnetic measurements are performed exclusively by means of Ferromaster (see Fig. 2.6(a)). This instrument contains a permanent magnet that partially magnetizes the specimen. Two coils, acting as sensors, are responsible to measure the variation of corresponding strain field caused by the change of the applied magnetization (see further details in Niffenegger (2004)). The relation between the volume of magnetic phase $\alpha'_{\text{vol}}$ and the measured susceptibility $\chi$ is nearly linear and is described by a master curve calibrated using reference specimens (see Fig. 2.6(b)). Susceptibility measurements are repeated four times on each sample and the average values is used as input of the following equation in order to calculate the amount of strain induced $\alpha'$-martensite fraction:

$$\alpha'_{\text{vol}}[\%] = \frac{\chi_{\text{ave}}}{0.02733} \quad (2.1)$$

Since specimen geometry affects the susceptibility measurements, only uncut (and unbroken) specimens are suitable for such a technique. This equipment is used to perform the experimental observations described in detail in section 3.5.

### 2.4 Finite element method

In order to predict the response of structures, engineers must deal with problems characterized by several non-linearities (e.g. material, boundary conditions, geometry, etc.). For this class of problems, deriving an analytical solution is often impossible but an acceptable approximation is given by the finite element method (FEM). The first example of FEM dates back to the fifties (Turner et al., 1956). In the following decades FEM was widely used and the first commercial FEM code appeared. Today, the continuous improvement of those commercial codes such as ABAQUS (Abaqus, 2012) gives to the end-user the possibility to utilize several already implemented constitutive models in order to deal with material non-linearities. In this dissertation, the elasto-plastic combined kinematic-isotropic hardening model implemented in ABAQUS has been used as starting point to simulate the material deformation behavior. In order to improve the performance of the model, the potentiality of the commercial code has been exploited defining the parameters of the model as a function of a set of internal variables. In the current work, it was also exploited the possibility offered by ABAQUS to implement an alternative constitutive model providing the corresponding User MATerial (UMAT) subroutine to the solver. A detailed description of the constitutive model formulation and of the corresponding calibration procedure is given in section 4.1. A general overview on FEM is available in Bathe (1996) and Belytschko et al. (1999).
Experimental characterization of 316L

In this chapter, results of the experiments performed to characterize the fatigue behavior of the investigated material under loading conditions relevant for the primary cooling circuit of a light water nuclear reactor are described. In section 3.1, experimental results corresponding to uniaxial isothermal LCF and ratcheting experiments are reported. Multiaxial LCF and ratcheting tests are also performed and the findings are described in section 3.2. Section 3.3 is dedicated to the analysis of the time-dependent behavior of the investigated materials. In section 3.4, the results of the study performed in order to characterize the microstructural evolution on 316L under LCF and ratcheting are reported. Finally, magnetic measurements are conducted on fatigued specimens in order to detect the eventual formation of strain induced martensite (see section 3.5).

3.1 Cyclic plastic mechanical behavior of 316L: uniaxial tests

In this section, the results of the uniaxial isothermal tests performed to analyze the fatigue behavior under loading conditions relevant for the primary cooling circuit of a light water nuclear reactor are reported and discussed.

3.1.1. Introduction

The experiments consist of a set of uniaxial low cycle fatigue and strain-controlled ratcheting tests (also named ‘cyclic tension tests’) carried out at room temperature and at 200 °C on specimens manufactured from the two different batches of stainless steel grade 316L previously characterized (see section 3.1.2). The experiments are repeated varying strain amplitude, cyclic ratcheting rate and ratcheting direction in order to investigate the influence on the cyclic deformation behavior.

As reported in section 1.1.1, a substantial amount of studies treat ratcheting experiments performed under stress control (Kang et al., 2001; Feaugas and Gaudin, 2004). Much less common are strain-controlled tests, performed for the first time by Coffin (1970), who coined them as ‘cyclic tension tests’. Later a few other research groups also published strain-controlled ratcheting experiments (Mizuno et al., 2000; Ohno et al., 1998). Because of its
Table 3.1. Standard designations and chemical composition of the investigated austenitic stainless steels (weight percentages).

<table>
<thead>
<tr>
<th>ASME SA-312/SA312M</th>
<th>SN EN 10216-5</th>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Cr</th>
<th>Ni</th>
<th>Mo</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>TP316L</td>
<td>X2CrNiMo17-12-2</td>
<td>0.021</td>
<td>0.26</td>
<td>1.69</td>
<td>0.033</td>
<td>0.003</td>
<td>17.50</td>
<td>11.14</td>
<td>2.15</td>
<td>0.060</td>
</tr>
<tr>
<td>316L</td>
<td>X2CrNiMo17-3</td>
<td>0.024</td>
<td>0.46</td>
<td>1.59</td>
<td>0.039</td>
<td>0.001</td>
<td>17.51</td>
<td>12.53</td>
<td>2.55</td>
<td>0.085</td>
</tr>
</tbody>
</table>

Infrequent application, a standard for this testing procedure is not yet available. The motivation to perform strain- instead of stress-controlled ratcheting tests is that they can be run at constant ratcheting rate, allowing a more straightforward use of the data for model calibration.

Several authors (Mizuno et al., 2000; Delobelle, 1993) have reported that, even at room temperature, 316L behaves in a visco-plastic manner, meaning that the loading rate influences the mechanical behavior under monotonic, LCF and ratcheting loading conditions. This time (or loading rate) dependency is not the subject of study of the current section (refer instead to section 3.3) and all tests have been performed at a constant strain rate.

3.1.2. Experimental setup

Material: stainless steel grade 316L

Two different batches of 316L are considered in this study. A first set of samples is machined from a section of a pipe commonly used to fabricate the primary cooling circuit of light water reactor (LWR) and will be referred as ‘TP316L’ (TP stands for tubular product). Samples are extracted from the tube along the pipe direction. A second set, which will be referred as ‘316L’, is sampled from a 20 mm thick hot rolled plate. Specimens are extracted along the rolling direction at sufficient distance from the edge of the plate to eliminate data scatter caused by microstructural variations typically observed at the edge of rolled sheet metals. In Tab.3.1 the chemical composition is listed for both materials as reference. The manufacturing sequences are the same for both materials and consist of hot working, solution annealing (at 1050-1080°C), quenching in water, pickling and grinding.

Fig.3.1 shows the optical microscope images of the samples corresponding to the two investigated materials, subjected to a particular etching procedure highlighting the grain boundaries. The pipe and the plate material in the as-received condition show a similar microstructure, which is characterized by a precipitate-free austenitic matrix and annealing twins. Plate material has a smaller average grain size compared with pipe material (respectively 50 μm and 65 μm).

Testing equipment and samples preparation

Testing equipment and samples preparation are described in section 2.1.1.

Loading paths

The LCF tests are performed with three different strain amplitudes (i.e. 0.40, 0.65 and 1.00%) prescribing an alternate strain path with ramp waveform with a cycling period that is changed in order to obtain a constant strain rate over all experiments (equal to 0.32%/s). As can be seen in Fig.1.10, in a ratcheting experiment the imposed strain path is a superposition of a constant amplitude ramp waveform and a continuously varying mean strain. We will characterize the ratcheting rate by means of the ratcheting step, which is the accumulation
3.1. Cyclic plastic mechanical behavior of 316L: uniaxial tests

Figure 3.1. (left) Optical microscopic observation of pipe material (TP316L) in the as-received condition, (right) optical microscopic observation of plate material (316L) in the as-received condition.

Table 3.2. Summary of the testing parameters used to perform the uniaxial strain-controlled LCF and strain controlled ratcheting experiments

<table>
<thead>
<tr>
<th>temp. (°C)</th>
<th>strain ampl. (%)</th>
<th>0.00</th>
<th>+0.10</th>
<th>-0.10</th>
<th>-0.01</th>
<th>+0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>RT (≤40)</td>
<td>0.40</td>
<td>AX-LCF-RT-040</td>
<td>AX-RAT-RT-040-P10</td>
<td>AX-RAT-RT-040-N10</td>
<td>AX-RAT-RT-040-N01</td>
<td>AX-RAT-RT-040-P01</td>
</tr>
<tr>
<td>RT (≤40)</td>
<td>0.65</td>
<td>AX-LCF-RT-065</td>
<td>AX-RAT-RT-065-P10</td>
<td>AX-RAT-RT-065-N10</td>
<td>AX-RAT-RT-065-N01</td>
<td>AX-RAT-RT-065-P01</td>
</tr>
<tr>
<td>RT (≤40)</td>
<td>1.00</td>
<td>AX-LCF-RT-100</td>
<td>AX-RAT-RT-100-P10</td>
<td>AX-RAT-RT-100-N10</td>
<td>AX-RAT-RT-100-N01</td>
<td>AX-RAT-RT-100-P01</td>
</tr>
<tr>
<td>200</td>
<td>0.40</td>
<td>AX-LCF-200-040</td>
<td>AX-RAT-200-040-P10</td>
<td>AX-RAT-200-040-N10</td>
<td>AX-RAT-200-040-N01</td>
<td>AX-RAT-200-040-P01</td>
</tr>
<tr>
<td>200</td>
<td>0.65</td>
<td>AX-LCF-200-065</td>
<td>AX-RAT-200-065-P10</td>
<td>AX-RAT-200-065-N10</td>
<td>AX-RAT-200-065-N01</td>
<td>AX-RAT-200-065-P01</td>
</tr>
<tr>
<td>200</td>
<td>1.00</td>
<td>AX-LCF-200-100</td>
<td>AX-RAT-200-100-P10</td>
<td>AX-RAT-200-100-N10</td>
<td>AX-RAT-200-100-N01</td>
<td>AX-RAT-200-100-P01</td>
</tr>
</tbody>
</table>

of the shift of the mean strain within a cycle. When a maximum strain magnitude of 5% is reached, limited by the extensometer range, the mean strain is kept constant and the experiment is continued until failure. The ratcheting tests have been performed at two constant temperatures, using three strain amplitudes (the same previously used in LCF experiments) and four different ratcheting rates (i.e. +0.10, +0.01, -0.10 and -0.01%/cycle). All the experiments are isothermal and are performed at two constant temperature levels, namely room temperature and 200 °C.

Test designation

The name of each test begins with two strings identifying the typology of the experiment (e.g. uniAXial and ‘LCF’ or ‘RAT’). The information regarding the testing temperature and the imposed strain amplitude is indicated in the next 2 positions of the experiment name. For ratcheting tests, the information about the direction (e.g. ‘P’ stands for positive and ‘N’ for negative) and the ratcheting step is given at the last position of the name. As an example, if the experiment called ‘AX-RAT-200-040-N10’ is considered, it can be easily identified as a uniaxial ratcheting test performed at 200 °C, with a strain amplitude of 0.40% and a negative ratcheting step of 0.10%/cycle. The complete list of experiment typologies with the corresponding testing parameters is for simplicity summarized in Tab.3.2. The complete list of performed experiments is reported in Appendix A.

3.1.3. LCF: Zero mean strain low cycle fatigue experiments

Elastic modulus

A fatigue bench is not the ideal equipment to precisely measure the value of the elastic modulus. Owing to small errors in the determination of the cross sectional area of the sample
and/or small differences in the extensometer positioning procedure, a significant shift in the observed modulus can occur between different experiments, while the precision within the same experiment is much better. To allow a better relative comparison, the measured data is shifted so that the modulus for the undeformed material, observed at the beginning of the first cycle, is the same value for all experiments run at the same temperature. The overall scatter observed is indicated in the plots by a vertical line ended by two asterisk symbols, representing the observed minimum and maximum. For each loading and unloading phase, an evaluation of the elastic modulus along the compressive and tensile trait of the hysteresis loop has been performed and only a negligible difference between those two values has been found during cycling. The per-cycle-mean value is plotted as a function of the accumulated plastic strain (see Fig.3.2) evaluated at the beginning of the corresponding hysteresis loop.

A monotonic reduction of the Young’s modulus is observed during cycling. Moreover, when a higher level of strain amplitude is applied, the measured stiffness is found to decrease faster. As suggested by Polak et al. (2001), the drop of the elastic modulus with cyclic loading corresponds to the additional anelastic strain due to bowing of dislocation segments. For higher strain amplitude levels, this reduction, connected with the production of new dislocation segments due to dislocation multiplication is enhanced. In the same plot one can observe that an increase of temperature induces an expected reduction of the elastic modulus, which is in agreement with the observations of Alain et al. (1997) and with more accurate dynamic methods measurements (e.g. acoustic resonance) (Lee et al., 1998).

**Yield stress**

In austenitic stainless steel grades, as is 316L, the transition between the elastic and plastic regime occurs very gradually. This makes the determination of the proportionality limit somewhat problematic, in the sense that the way one determines the yield stress becomes part of its definition. While the definition of the yield stress using an offset concept is widely used in cyclic plasticity, there is no universal accepted threshold value that must be adopted. The 0.2% offset concept, generally used in monotonic tests, can not be applied...
3.1. Cyclic plastic mechanical behavior of 316L: uniaxial tests

because this strain value is significant in LCF. Jiang and Zhang (2008) reported that the
determination of the proportional limit by an offset smaller than 0.001% is a nearly impossible
task because of practical problems linked with the extensometer measurements reliability. For
these reasons, the offset value is commonly chosen between 0.01% and 0.05%. To illustrate the
importance of the selection of the offset value, two analyses have been performed with different
thresholds: the first similar to the one usually adopted in literature (0.025%) and the second
one ten times smaller (0.0025%). An evaluation of the yield stress along the compressive and
tensile trait of the hysteresis loop has been performed for both the presented offset concepts
and only a negligible difference between those two values has been found during cycling.
The mean yield stress is plotted in Fig.3.3 as a function of the accumulated plastic strain
for the two different yield definitions. Comparing the upper and the lower graph, one can
observe not only a quantitative but also a qualitative difference. In fact, for the highest
offset value (i.e. 0.025%), the calculated yield stress is influenced by the strain amplitude,
and for severe loading conditions, cyclic hardening instead of softening is observed for both
temperatures. This finding is in agreement with the results of Goyal et al. (2011) for 304LN,
who used a similar offset to analyze room temperature LCF experiments. When the lower
offset (i.e. 0.0025%) is adopted, Fig.3.3 illustrates that the tests executed with different strain
amplitudes show a cyclic yield stress evolution that does not depend on the strain amplitude,
at least not in the amplitude range in which we performed the experiments. The conclusion
of this analysis is that the selection of different yield definitions leads to dissimilar results,
and for that reason this aspect must be carefully considered when a constitutive model is
calibrated. In fact, the selection of the offset threshold will determine in which measure the
mechanical behavior must be attributed to isotropic or to kinematic cyclic hardening. Fig.3.3
shows also that, as expected, for higher testing temperature a systematic reduction of the
value of the proportionality limit is observed. Finally, it is mentioned that the yield stress
evolution for TP316L has the same behavior as the plate material, but with systematically
15% lower values.

Figure 3.3. Yield stress versus accumulated plastic strain in plate material (316L) at three different strain
amplitudes and two temperature levels showing the importance of the offset value selection (only the first 500
cycles are represented) (Facheris and Janssens, 2013).
Cyclic hardening/softening: stress amplitude response

As previously reported in section 1.1.1, it is well known that stainless steels and more in general FCC materials, show a particularly complex stress amplitude response when subjected to repeated strain loading. Strain amplitude and temperature can quantitatively and qualitatively affect their cycling hardening-softening under LCF conditions. The stress amplitude evolution for the room temperature LCF tests performed with both materials is shown in Fig. 3.4. One can observe that the measured stress response is 5-15% lower for TP316L when compared with the measurements performed on 316L. This finding is quantitatively in agreement with the Hall-Petch relationship (Hall, 1951; Petch, 1953) since, as reported in section 3.1.2 the average grain size is about 25% smaller in plate material. Both materials show primary hardening followed by softening and then stabilization. Analyzing the experiment with the highest strain amplitude of 1.00%, one also notices a secondary hardening that is more pronounced in the case of 316L. The experiments performed at 200 °C, reveal qualitatively the same behavior at about 15% lower values, and do not show any secondary hardening. An interesting phenomenon that can be observed only for TP316L, is that a small but non-zero mean stress can be observed, which progressively fades out as cycling continues. This phenomenon, which suggests that the material shows a larger deformation resistance under compression than under tension and has also been named ‘tension-compression asymmetry’ or ‘strength differential effect’, has been reported for martensitic (Rauch and Leslie, 1972) and ferritic steels (Yaguchi and Takahashi, 2005) and for copper (Ma et al., 1990).

3.1.4. Strain-controlled ratcheting experiments

Elastic modulus

As for the zero mean strain experiments, the cyclic evolution of the mean-per-cycle elastic modulus has been analyzed for the strain-controlled ratcheting experiments. In Fig. 3.5 the
3.1. Cyclic plastic mechanical behavior of 316L: uniaxial tests

Figure 3.5. Mean elastic modulus versus accumulated plastic strain in pipe material (TP316L) at three different strain amplitude levels and room temperature showing the influence of ratcheting (Facheris and Janssens, 2013).

mean elastic modulus corresponding to TP316L is plotted as a function of the accumulated plastic strain. For clarity, the results are separated according to the three imposed strain amplitudes. To visualize the influence of ratcheting on the elastic modulus, the corresponding LCF tests are included for reference. The figure only shows data corresponding to the hysteresis loops antecedent to the reaching of the maximum strain level, after which the mean strain no longer changes. Under all the considered loading conditions, in the first cycles, when ratcheting is not yet severe, the elastic modulus reasonably fits the curve measured in the corresponding LCF test. However, with the increasing of the mean strain drifting, an incipient buckling takes place and for certain experiments, a deviation of the apparent elastic modulus from the corresponding LCF behavior is observed. As one can straightforwardly estimate using an elastic finite element simulation, this slight buckling can considerably affect the reliability of the strain measurements provided by the extensometer. It can be easily demonstrated by means of a FEM simulation that a misalignment of only 0.05 mm of the specimen from its axis, is sufficient to explain the variance of the apparent elastic modulus noticed in the experimental analysis. We therefore conclude that the deviations observed between the elastic modulus under LCF conditions and strain-controlled ratcheting conditions, cannot be consistently attributed to ratcheting. Analogous results were obtained when repeating the analysis at 200 °C and also for 316L.

Yield stress

As for the LCF experiments previously presented in section 3.1.3, the cyclic evolution of the mean yield stress has been analyzed for strain-controlled ratcheting experiments, showing that the definition of the yield stress affects the interpretation of the raw data for this case too. Fig.3.6 shows the yield stress values determined using two different offset thresholds (0.025% and 0.0025%) for the lowest strain amplitude level equal to 0.40%. A similar material behavior is observed for higher strain amplitudes as well as for TP316L. When the yield stress behavior calculated on the highest offset value is compared to the corresponding LCF test, ratcheting seems to affect the proportionality limit introducing an additional hardening, similarly to
Chapter 3. Experimental characterization of 316L

Figure 3.6. Yield stress versus accumulated plastic strain in plate material (316L) at two different temperatures (room temperature and 200 °C) and strain amplitude 0.40% showing the importance of the offset value selection and the influence of ratcheting (Facheris and Janssens, 2013).

what has been found in the tests performed on 316FR at 650 °C (Ohno et al., 1998) with a very similar offset definition. On the other hand, if the lowest offset value is used to analyze the same dataset, one finds no evidence that ratcheting affects the yield stress significantly. When a constitutive model suitable for ratcheting is formulated and then calibrated, the selection of the yield definition must be carefully evaluated. In this case, since there is no evidence for a dependency of the yield stress on ratcheting when using the lower offset criterion, calibration will be easier.

Cyclic hardening/softening: stress amplitude and mean stress response

The authors that previously performed strain-controlled ratcheting tests (Mizuno et al., 2000; Ohno et al., 1998) refer only to the unloading phase for the definition of the strain and stress range. However, in the case of ratcheting, the tensile stress range differs significantly from the compressive one within one cycle. The used definition of mean stress and of per-cycle stress amplitude must return a value that must be consistent in case these features vary from cycle to cycle and that must not be sensitive to the ratcheting direction. Therefore, the definition adopted in this dissertation considers a cycle as the sequence of a tensile and a compressive trait (or the reverse), computes the range and mean values using the extrema of each of these traits, and calculates the per-cycle values as the average of the compressive and tensile traits; the first cycle is treated as a special case as it is not a complete cycle. The stress amplitude and mean stress for the generic \textit{i}-th hysteresis loop can be summarized by the following equations using Fig.3.7 as reference:

\[
\sigma_{amp}(i) = \frac{1}{2} \left( \frac{\sigma(B) - \sigma(A)}{2} + \frac{\left| \sigma(B) - \sigma(C) \right|}{2} \right) / 2
\]

\[
\sigma_{mean}(i) = \left( \frac{\sigma(B) + \sigma(A)}{2} + \frac{\sigma(B) + \sigma(C)}{2} \right) / 2
\]
3.1. Cyclic plastic mechanical behavior of 316L: uniaxial tests

It must be pointed out that the same approach has been used, later on, to determine the per-cycle mean value of the plastic strain. Comparing the evolution of the stress amplitude in cyclic tension experiments and in the corresponding LCF tests as a function of the accumulated plastic strain (see Fig.3.8), one finds that ratcheting introduces an additional hardening. This result seems to be in contradiction with the data published by Mizuno et al. (2000), corresponding to experiments on 316FR at room temperature, who noticed no additional hardening under strain-controlled ratcheting conditions. However, the apparent difference can be explained with the fact that Mizuno considered lower strain amplitude levels, limited his analysis to lower cycle numbers and adopted a different strain amplitude definition. One can also observe that the stress amplitude responses in experiments performed with the same loading conditions but opposite ratcheting direction, look qualitatively and quantitatively very similar. The initial difference is essentially due to fact that uniaxial ratcheting experiments are carried out controlling the deformation in engineering strain instead of true strain. The imposed constant engineering strain amplitude is calculated to have the correct true strain amplitude when the mean strain limit is reached (i.e. 5% or -5%). Consequently, in the positive ratcheting case, in the very first cycles, the true strain amplitude is higher than the one applied in the corresponding LCF test resulting in a harder stress response. Vice versa, in the negative ratcheting case, in the very first cycles, the true strain amplitude is lower than the one applied in the corresponding LCF test resulting in a softer stress response. Therefore, if the error due to the control strategy is considered, it can be stated that ratcheting direction does not play a role in the stress amplitude determination.

When the mean stress is plotted versus the accumulated plastic strain (see Fig.3.9), it can be observed that, as already anticipated, LCF tests performed with plate material do not show a tension-compression asymmetry. On the other hand, when the data related to cyclic tension tests are analyzed, it is clear that ratcheting introduces a drifting of the mean stress. As expected, when a positive ratcheting test is performed, the mean stress drifting is positive and vice versa. While in this paragraph, only room temperature plate data have been represented, the mechanical behavior at 200 °C was found to be qualitatively similar. The analysis of the stress response corresponding the other material source (i.e. TP316L) confirmed the same findings with in addition the presence of the previously described tension-

![Figure 3.7](image.png)
3.1.5. Discussion and results

In the previous section, the analysis of the stress amplitude evolution revealed that ratcheting introduces an additional hardening compared with LCF conditions. To quantitatively separate the hardening owing to ratcheting from the one owing to the cyclic hardening, we assume that the total hardening is a linear superposition of both.

The first step of the proposed methodology consists of determining approximate curves for the evolution of the stress amplitude for the LCF and ratcheting tests. In order to fit the selected experimental curve \( y \), we use a summation of exponential equations that are function of the accumulated plastic strain \( p \) and few constant parameters (i.e. \( a_i, b_i \)) that can be easily determined (see equation 3.3).

\[
y = \sum_{i=1}^{n} a_i \cdot e^{-b_i \cdot p} \quad \text{with } p \text{ accumulated plastic strain} \quad (3.3)
\]

Using these curves, one can straightforwardly determine the difference between the stress amplitude in a ratcheting and in a LCF test with the same strain amplitude, this difference being equivalent to the additional hardening from ratcheting. This additional hardening can be evaluated for each ratcheting test and plotted against the absolute value of the mean plastic strain as in Fig.3.10. If one only considers the slopes, attributing the vertical shift between the curves to the material heterogeneity and to experimental errors, one can approximate the additional hardening to be linearly related to the absolute value of the mean plastic strain, thus independent from the ratcheting direction. The physical meaning of the slope is that of the rate of the additional hardening induced by ratcheting. This rate \( R_{\text{RiaH}} \) is found to be a function of the strain amplitude, temperature and ratcheting rate.
3.1. Cyclic plastic mechanical behavior of 316L: uniaxial tests

Figure 3.9. Mean stress versus accumulated plastic strain in plate material (316L) at three different strain amplitude levels and room temperature showing the influence of ratcheting (Facheris and Janssens, 2013).

Figure 3.10. Additional hardening versus absolute value of mean plastic strain in plate material (316L) at three different strain amplitude levels and room temperature showing the influence of ratcheting (Facheris and Janssens, 2013).
The value of $R_{\text{RiaH}}$ is identified by a linear regression as a function of the absolute value of the mean plastic strain and it is plotted in Fig.3.11 to show the dependency on testing conditions. One can observe a reduction of the numerical value of $R_{\text{RiaH}}$ for higher strain amplitudes and for lower ratcheting rates. Comparing the value of $R_{\text{RiaH}}$ corresponding to room temperature and 200 °C, it is surprising that it is not possible to notice a significant difference between them. This suggests that, in the investigated range, temperature does not influence the additional hardening due to ratcheting.

The separation of cyclic and ratcheting contributions can be also performed for the mean stress evolution. In the particular case of 316L, no tension-compression asymmetry has been observed and consequently the separation is a straightforward task because the whole mean stress drifting is considered to be a ratcheting effect. In Fig.3.12 the absolute value of the mean stress is plotted versus the magnitude of the mean plastic strain. Comparing experiments with the same loading conditions, but opposite ratcheting directions, it is evident that they have a nearly coincident behavior. On the other hand, the amount of the mean stress is found to be significantly lower for tests performed with lower ratcheting rate and higher strain amplitude. When also experiments performed at 200 °C are considered, one can observe a further reduction of the magnitude of the mean stress. The same analyses have been repeated for TP316L and they show qualitatively the same results with the additional presence of a slight tension-compression asymmetry that progressively fades out during cycling.

Finally, in Fig.3.13, an evaluation of the influence of ratcheting on the life endurance reduction is presented. The criterion adopted to define the failure is a drop of 10% in the maximum stress from the stabilized response.

In Fig.3.13, the information about the observed crack type according with the definition given by Nishimura et al. (2000) is also reported. As depicted in Fig.3.14, a specimen showing a crack completely inside the gage length is classified as 'A'. Type 'C' means that the crack nucleated clearly outside the gage length. Type 'B' crack initiated at the edge of the extensometer gage. Since fatigue life is affected by crack propagation, it is plausible to assume that the more severe loading conditions promoted by the nucleation of cracks 'B' and 'C' lead to a reduction of the lifetime. However, in our experiments it was impossible to notice a systematical relation between the crack typology and the lifetime reduction. This
3.1. Cyclic plastic mechanical behavior of 316L: uniaxial tests

Figure 3.12. Absolute value of mean stress versus absolute value of mean plastic strain in plate material (316L) at three different strain amplitude levels showing the influence of ratcheting (Facheris and Janssens, 2013).

Figure 3.13. Relationship between strain amplitude and number of cycles to failure showing the influence of ratcheting on life reduction for (left) 316L at room temperature, (middle) 316L at 200°C and (right) TP316L at room temperature. The information about the observed crack type according with the definition given by Nishimura et al. (2000) is also reported.

Figure 3.14. Crack types according to the definition given by Nishimura et al. (2000).
observation led us to conclude that the influence of the crack type is smaller than the usual lifetime scatter.

The observation that in every single cyclic tension experiment, a lower number of cycles to failure is noticed comparing with the corresponding LCF test, suggests that both positive and negative ratcheting have a pejorative effect on fatigue life. However, in the worst case, the noticed life reduction was a factor of two, which is only partially in agreement with the work of Date et al. (2008) performed on 316FR at 550 °C, where a much higher endurance drop (a factor of 10) has been observed for positive ratcheting experiments. This discrepancy can be explained with the fact that Date et al. (2008) performed ratcheting tests allowing the mean strain to vary continuously until the specimen failure and not only until a maximum strain value of ± 5%.

In accordance with the only marginal microstructural differences highlighted in section 3.1.2, the lifetime endurance of the two material sources subjected to the same loading conditions was found to be very similar. As a final observation, it must be pointed out that in the current study, the deformation was assumed to be homogeneous inside the gage volume. However, it is plausible to suspect that the occurrence of the buckling reported in section 3.1.4 induces local increments of the values of the strain amplitudes affecting the accuracy of the specimen lifetime measurements. In this context, a dimensional analysis of the fatigued samples allowing to determine if the deformation was homogeneous inside the gage section and to assess if the test was stable, is suggested as an outlook.

3.1.6. Conclusions

A set of uniaxial LCF and strain-controlled ratcheting experiments has been carried out on two different batches of 316L and the influence of temperature, strain amplitude, ratcheting rate and ratcheting direction on the cyclic mechanical behavior has been investigated.

The apparent elastic modulus is observed to monotonically decrease during cycling, and the rate of this decrease is found to depend on the imposed strain amplitude and temperature, but not on the imposed ratcheting. Moreover, if a small enough offset value (e.g. 0.0025%) is adopted for its definition, the yield stress exhibits a monotonic decrease during cycling that is influenced mainly by temperature.

It was shown that, when implementing the selected, small offset threshold, strain amplitude and ratcheting do not affect the evolution of the yield stress.

In the performed experiments it was found that ratcheting has an impact on the stress amplitude and mean stress response. The stress response in cyclic tension tests has been considered as a superposition of two hardening mechanisms: one owing to mean strain drifting (i.e. ratcheting) and another one owing to cyclic loading. A methodology has been developed in order to separate the two contributions. The data analysis revealed that the additional hardening observed in strain-controlled ratcheting experiments is quasi-linearly linked with the absolute value of mean plastic strain, and that it is independent of the ratcheting direction. The additional hardening has been observed to decrease with higher strain amplitude and lower ratcheting rate. On the other hand, in the considered investigation range, temperature did not show to consistently influence the additional hardening. The analysis of the ratcheting tests data also exhibited a drift of the mean stress in the same direction of ratcheting. A higher mean stress magnitude has been measured for lower strain amplitude levels and temperatures.

Comparing the mechanical response of the two material batches, a similar qualitative behavior is observed under all the loading conditions, with the occurrence of a slight
tension-compression asymmetry for TP316L. Finally, under the investigated loading conditions, ratcheting is noticed to cause but a weak reduction (less than a factor 2) of the number of cycles to fatigue failure.
Chapter 3. Experimental characterization of 316L

3.2 Cyclic plastic mechanical behavior of 316L: multiaxial tests

In this section results of uniaxial, torsional and multiaxial LCF and ratcheting tests performed in collaboration with Politecnico di Milano in order to analyze the fatigue behavior of 316L under complex multiaxial loading conditions are reported and discussed.

3.2.1. Introduction

As reported in section 1.1.2, several authors studied the behavior of stainless steels when subjected to multiaxial LCF and ratcheting loading conditions. However, according to the author’s knowledge, no reference is available in literature for multiaxial ratcheting experiments performed controlling at the same time both the axial and torsional axis of the testing machine in strain. Similarly to what previously reported for the uniaxial case (see section 3.1), in this study an alternative way to perform multiaxial ratcheting tests is proposed controlling in strain both axes of the testing machine. This approach allows an easy comparison of ratcheting experiments with data corresponding to multiaxial LCF tests (commonly performed under strain control) and uniaxial strain-controlled tests.

The experimental data considered in this analysis belong to five different subsets:
1. uniaxial LCF
2. uniaxial strain-controlled ratcheting
3. torsional LCF
4. multiaxial LCF
5. multiaxial strain-controlled ratcheting

All the tests are performed at room temperature on specimens manufactured from the plate material batch previously characterized (see section 3.1.2). Several authors (Mizuno et al., 2000; Delobelle, 1993) have reported that, even at room temperature, 316L behaves in a visco-plastic manner, meaning that the loading rate influences the mechanical behavior under monotonic, LCF and ratcheting loading conditions. This time (or loading rate) dependency is not the subject of study of the current section (refer instead to section 3.3) and all tests have been performed with comparable strain rate.

3.2.2. Experimental setup

Testing equipment and samples preparation

Testing equipment and samples preparation are described in section 2.1.1 (for uniaxial experiments) and in section 2.1.3 (for torsional and multiaxial experiments).

Loading paths

The first and the second set of experiments are performed applying the cycling loading only in uniaxial direction (see Fig.3.15(a)). LCF tests are carried out with three different strain amplitudes (i.e. 0.40, 0.65 and 1.00%) imposing a strain path in axial direction with ramp waveform and with a cycling period that is changed in order to obtain a constant equivalent strain rate over all experiments (equal to 0.32%/s). The strain-controlled ratcheting tests are carried out with only one strain amplitude (0.65%) superposing a continuously increasing mean strain in axial direction with a constant ratcheting step (+0.10 %/cycle). The
ratcheting step $\xi$ is defined as the increment of the mean strain per cycle. When a maximum axial strain of 5% is reached, limited by the extensometer range, the mean strain is kept constant and the experiment is continued until failure.

The third set of experiments is performed applying the cycling loading only in shear direction (see Fig.3.15(b)). These torsional LCF tests are carried out with three different equivalent strain amplitudes (i.e. 0.40, 0.65 and 1.00%) imposing a strain path in shear direction with sinusoidal waveform and with the same cycling periods as in the uniaxial experiments. For simplicity, the equivalent strain is calculated by means of Eq.1.3. The relation between the shear stress $\tau$ and the measured torque $\Theta$ is given by Eq.3.4 assuming the stress to be linearly distributed along the specimen thickness. A detailed discussion on the validity of this assumption is reported in the following section. For a hollow sample, the shear stress on the external surface of the specimen is computed by means of Eq.3.5, expressing the polar moment of inertia $J_p$ as a function of the internal $D_{\text{int}}$ and of the external diameter $D_{\text{ext}}$.

$$\Theta = \int_A \tau \cdot r \cdot dA = \tau \int_A r^2 \cdot dA = \frac{\tau \cdot J_p}{r}$$ (3.4)

$$\tau(D_{\text{ext}}) = \frac{\Theta \cdot D_{\text{ext}}/2}{\pi/32(D_{\text{ext}}^4 - D_{\text{int}}^4)}$$ (3.5)

The fourth set of tests consists of two kinds of multiaxial LCF experiments carried out controlling the strain simultaneously in 2 directions: axial and shear. In the first case (see Fig.3.15(c)), the two loading histories are defined as proportional since their equivalent amplitude is the same and there is no phase shift. In the second case (see Fig.3.15(d)), the two loading histories are non-proportional with a phase shift of $\pi/2$ resulting in a circular trajectory. Those multiaxial LCF tests are carried out for only one equivalent strain amplitude level (0.65%) and a constant period (equal to the one previously used in the corresponding uniaxial experiment).

The last two sets of tests consist of two strain-controlled ratcheting multiaxial experiments in which the strain paths already presented for LCF are overlapped to a continuously increasing mean strain in axial direction (see Fig.3.15(e) and Fig.3.15(f)). These tests are carried out for an equivalent strain amplitude level (0.65%) superposing a continuously increasing mean strain in axial direction with a constant ratcheting step (+0.10 %/cycle). As for the uniaxial ratcheting tests, when a maximum axial strain of 5% is reached (technical limit imposed by buckling) the axial mean strain is kept constant and the experiment is continued until failure.

All the experiments are isothermal and are performed at room temperature.

**Test designation**

The name of each test begins with two strings that identify the typology of the experiment (e.g. uniAXial, TOrsional, MultiAXial Proportional or MultiAXial Non-proportional and ‘LCF’ or ‘RAT’). The information regarding the testing temperature and the imposed equivalent strain amplitude is indicated in the next 2 positions of the experiment name. For ratcheting tests, the information about the direction (e.g. ‘P’ stands for positive) and the ratcheting step in axial direction is given at the end of the name. As an example, if the experiment called ‘MN-RAT-RT-065-P10’ is considered, it can be easily identified as a multiaxial non-proportional ratcheting test performed at room temperature, with an equivalent
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Figure 3.15. Imposed strain paths: (a) axial, (b) torsional, (c) multiaxial proportional, (d) multiaxial non-proportional, (e) multiaxial proportional ratcheting, (f) multiaxial non-proportional ratcheting.

Table 3.3. Summary of the testing parameters used to perform the axial, torsional and multiaxial strain-controlled LCF and ratcheting experiments.

<table>
<thead>
<tr>
<th>test designation</th>
<th>temperature</th>
<th>kind</th>
<th>loading path</th>
<th>ε_{amp} (%)</th>
<th>γ_{amp} (%)</th>
<th>γ_{eqamp} (%)</th>
<th>ε_{mean} (%)</th>
<th>ξ (%/cyc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AX-LCF-RT-040</td>
<td>RT</td>
<td>LCF</td>
<td>axial</td>
<td>0.40</td>
<td>0</td>
<td>0.40</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>AX-LCF-RT-065</td>
<td>RT</td>
<td>LCF</td>
<td>axial</td>
<td>0.65</td>
<td>0</td>
<td>0.65</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>AX-LCF-RT-100</td>
<td>RT</td>
<td>LCF</td>
<td>axial</td>
<td>0.65</td>
<td>1.00</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>AX-RAT-RT-065</td>
<td>RT</td>
<td>RAT</td>
<td>axial</td>
<td>0.65</td>
<td>0</td>
<td>0.65</td>
<td>0-5</td>
<td>0.1</td>
</tr>
<tr>
<td>TO-LCF-RT-040</td>
<td>RT</td>
<td>LCF</td>
<td>torsional</td>
<td>0</td>
<td>0.40 √3</td>
<td>0.40</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>TO-LCF-RT-065</td>
<td>RT</td>
<td>LCF</td>
<td>torsional</td>
<td>0</td>
<td>0.65 √3</td>
<td>0.65</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>TO-LCF-RT-100</td>
<td>RT</td>
<td>LCF</td>
<td>torsional</td>
<td>0</td>
<td>1.00 √3</td>
<td>1.00</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>MP-LCF-RT-065</td>
<td>RT</td>
<td>LCF</td>
<td>multiax. prop.</td>
<td>0.65 √2/2</td>
<td>0.65 √6/2</td>
<td>0.65</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>MN-LCF-RT-065</td>
<td>RT</td>
<td>LCF</td>
<td>multiax. non-prop.</td>
<td>0.65</td>
<td>0.65 √2</td>
<td>0.65</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>MP-RAT-RT-065-P10</td>
<td>RT</td>
<td>RAT</td>
<td>multiax. non-prop.</td>
<td>0.65 √2/2</td>
<td>0.65 √6/2</td>
<td>0.65</td>
<td>0-5</td>
<td>0.1</td>
</tr>
<tr>
<td>MN-RAT-RT-065-P10</td>
<td>RT</td>
<td>RAT</td>
<td>multiax. non-prop.</td>
<td>0.65</td>
<td>0.65 √2</td>
<td>0.65</td>
<td>0-5</td>
<td>0.1</td>
</tr>
</tbody>
</table>

strain amplitude of 0.65% and a positive ratcheting step of 0.10%/cycle in axial direction.

The complete list of experiment typologies with the corresponding testing parameters is for simplicity summarized in Tab.3.3. The complete list of performed experiments is reported in Appendix A.

3.2.3. LCF: Zero axial mean strain low cycle fatigue experiments

Data corresponding to uniaxial, torsional and multiaxial LCF tests are analyzed in order to evaluate the deformation response of the material. The cyclic evolution of the elastic stiffness and of the yield point cannot be measured for multiaxial non-proportional experiments since the mechanical response does not switch between elastic and plastic behavior for all the duration of the test (except for the first loading).

Elastic and shear modulus

While a fatigue bench is not the ideal equipment to retrieve accurate stiffness measurements, the methodology proposed in section 3.1 is adopted in the current study to get rid of the systematic errors due to the wrong determination of the cross sectional geometry of the sample and/or to small differences in the extensometer positioning. This approach consists in shifting the measured data so that the elastic modulus for the undeformed material, observed at the beginning of the first cycle, is the same value for all the considered experiments. A vertical line ended by two asterisk symbols, representing the observed minimum and maximum, is used to indicate the overall scatter.
3.2. Cyclic plastic mechanical behavior of 316L: multiaxial tests

The apparent elastic modulus $E$ is evaluated for each cycle considering the loading and the unloading traits of the hysteresis loop $\sigma - \varepsilon$ and only a negligible difference between those two values is found during cycling. In an analogous way, the value of the apparent shear modulus $G$ is estimated using the hysteresis loops $\tau - \gamma$ as input. The per-cycle-mean value for the shear and the elastic modulus is plotted as a function of the accumulated plastic strain (see Fig.3.16) evaluated at the beginning of the corresponding hysteresis loop. For uniaxial tests a monotonic reduction of the apparent Young’s modulus is observed during cycling (see lower graph in Fig.3.16). Moreover, imposing a higher strain amplitude level, a faster decreasing of measured stiffness is noticed. Analyzing torsional data, the same qualitative and quantitative behavior (considering a Poisson coefficient of 0.3) is observed for the apparent shear modulus (see upper graph in Fig.3.16). According to Polak et al. (2001), the stiffness reduction promoted by cyclic loading is associated with the additional anelastic strain due to bowing of dislocation segments. Prescribing a higher strain amplitude level, this reduction is accelerated by the enhanced production of new dislocation segments due to dislocation multiplication.

The same analysis is performed for multiaxial proportional tests considering stress and strain measurements in axial and shear direction. The observed reduction for $E$ and $G$ is qualitatively and quantitatively similar to the one reported for uniaxial and torsional experiments performed with the same equivalent strain amplitude level (0.65%).

Yield stress

Similarly to other austenitic stainless steels, in 316L the transition between the elastic and plastic regime is extremely gradual. For that reason, the determination of the proportionality limit is often problematic since the definition adopted to evaluate the yield stress strongly influences the interpretation of the results. While in cyclic plasticity the offset concept is widely used for the definition of the yield stress, no universally accepted threshold value is defined. As commonly recognized, the adoption of the 0.2% offset concept generally used in monotonic tests, is not applicable to LCF data in which this strain value is
comparable to the imposed strain amplitude. As reported by Jiang and Zhang (2008), the determination of the yield stress using an offset smaller than 0.001% is not possible due to practical limitations linked with the measurements reliability. As a consequence, the offset value is commonly chosen in the range 0.01-0.05%.

Section 3.1.3 highlights the importance of the selection of this offset value in the case of uniaxial tests and the same analysis in now repeated also for torsional experiments. Two analyses are performed with different thresholds with the aim of illustrating the importance of the selection of the offset value: the first similar to the one usually adopted in literature (0.025%) and the second one ten times smaller (0.0025%). To allow a quantitative comparison between data corresponding to uniaxial and torsional experiments, the yield offset thresholds and the yield stress are referred as the equivalent ones. For the torsional tests, the Von Mises criterion (see equation 1.1) can be simplified and the equivalent yield stress is obtained scaling the yield stress in shear direction \( \tau \) by a factor of \( \sqrt{3} \). In the case of multiaxial tests, the equivalent yield stress is calculated considering both the axial and the shear stress contribution \( \sqrt{\sigma^2 + 3\tau^2} \).

For each offset concept, an evaluation of the equivalent yield stress along the loading and unloading trait of the hysteresis loop is performed and only a negligible difference between these two values is found during cycling. The equivalent mean yield stress is then plotted in Fig.3.17 as a function of the accumulated plastic strain for the two different yield definitions (upper and lower graphs) and for the uniaxial, torsional and multiaxial proportional experiments (left, center and right).

Comparing the upper and the lower graphs, one can observe the qualitative difference already reported in section 3.1.3 for uniaxial tests also for data corresponding to torsional experiments. For the highest offset value (i.e. 0.025%), the calculated yield stress is influenced by the strain amplitude and, for the highest amplitude, cyclic hardening is observed instead of softening. On the other hand, when the lower offset (i.e. 0.0025%) is adopted (see lower plots in Fig.3.17), the dependency of the cyclic evolution of the yield stress on the strain amplitude is weaker, though the lower threshold does suffer from more uncertainty. As additional remark, it is found that the equivalent yield stress is consistently 5-10% higher in torsional tests. The reason for this difference is explained in the next paragraph attributing it to a wrong assumption adopted to retrieve the shear stress from the measured torque value.

As already pointed out in section 3.1.3, the current analysis confirms that the selection of different yield definitions leads to dissimilar results. This aspect should be carefully considered when a constitutive model is calibrated since the selection of the offset threshold will determine in which measure the mechanical behavior is attributed to isotropic or to kinematic cyclic hardening.

The analysis of the yielding point in multiaxial proportional tests (see right hand side of Fig.3.17) returns yield stress values that for both the considered offset definitions are qualitatively similar and a bit higher than the ones observed in the torsional experiment performed with the same equivalent strain amplitude.

**Cyclic hardening/softening**

As reported in section 1.1, stainless steels show a particularly complex stress amplitude response when subjected to repeated loading. The imposed strain amplitude and loading trajectory are known to quantitatively and qualitatively affect the cycling hardening-softening behavior of 316L.

An effective approach to compare uniaxial and torsional LCF data is to represent the Von Mises equivalent maximum stress evolution as a function of the number of cycles (see
3.2. Cyclic plastic mechanical behavior of 316L: multiaxial tests

Figure 3.17. Equivalent yield stress versus accumulated plastic strain in uniaxial (left) torsional (center) and multiaxial proportional (right) experiments performed on plate material (316L) at different strain amplitudes and room temperature showing the importance of the offset value selection (only the first 500 cycles are represented).

For both testing typologies, 316L shows primary hardening followed by softening and then stabilization. For the experiments with the highest strain amplitude (1.00%), one notices a secondary hardening that is more pronounced in the uniaxial experiment. A careful analysis reveals that, in the very first cycles, the measured stress response is 10-15% higher in the torsional tests compared with the uniaxial ones. This difference slowly but not completely fades out during cycling with a rate that is faster in the case of experiments performed with higher equivalent strain amplitudes.

This observation is not completely consistent with literature: in the experiments performed by Benallal and Marquis (1987), Tanaka et al. (1985) and Delobelle (1993) the difference between the equivalent response in uniaxial and torsional tests is reported to be negligible. The difference noticed in the current work is mainly attributed to the experimental error caused by assuming a linear shear stress distribution along the radius of the specimen (see left hand side plot in Fig.3.19). It is relatively easy to demonstrate that retrieving the shear stress \( \tau \) from the measured torque \( \Theta \) by means of Eq.3.5 (derived from Eq.3.4) leads to inaccurate results when plasticity occurs. In fact, in the case of inelastic deformation, the shear stress distribution is not linear along the radius of the specimen and the ratio \( \tau/r \) is no longer a constant. To approximately solve this problem, the ASTM norm (ASTM, 2008) suggests to assume the shear stress to be uniformly distributed along the thickness of the specimen (see right hand side plot in Fig.3.19) and to compute \( \tau \) by means of Eq.3.6.

\[
\tau(D_{ext}) = \frac{\Theta}{\pi/16(D_{ext}^2 - D_{int}^2)(D_{ext} + D_{int})} \tag{3.6}
\]

Representing in Fig.3.18 the equivalent value of the experimental shear stress calculated assuming a uniform stress profile, a difference is still noticeable with respect to the uniaxial data. Neither of these assumptions are correct since in reality the shear stress profile is non-linear and changes continuously during loading. To quantify the corresponding error, a set of FEM (Finite Element Method) simulations using a the '5DChabEP' elasto-plastic material description (see section 4.1) is performed for the three torsional experiments TO-LCF-RT-xxx. The simulated torque \( \Theta \) is used as input of Eq.3.5 and Eq.3.6 to compute the
Figure 3.18. Equivalent maximum stress versus number of cycles in uniaxial and torsional LCF experiments performed on plate material (316L) at three different strain equivalent amplitudes and room temperature.

Figure 3.19. Linear (left) and uniform (right) shear stress distribution profiles along the thickness of an hollow specimen subjected to torsion.
3.2. Cyclic plastic mechanical behavior of 316L: multiaxial tests

Figure 3.20. Maximum shear stress versus number of cycles in simulations of torsional experiments. The shear stress is computed deriving it from the torque value or extrapolating it from one node on the external surface of the specimen.

resulting $\tau$. The per-cycle maximum values of $\tau$ are then compared to the maximum shear stress extrapolated from a node on the external surface of the specimen (see Fig. 3.20). As expected, while Eq. 3.5 overestimates the value of the shear stress on the external surface, Eq. 3.6 underestimates it. The shear stress computed assuming a uniform distribution of $\tau$ is generally more precise than the one calculated postulating a linear distribution. However, it is noticed that, because of the inhomogeneous stress field promoted by the cyclic hardening, the assumption of uniform stress distribution becomes less and less accurate with the increasing of the number of cycles. Since, these data could be used as input of damage criteria, in the remainder of the current work we chose to compute $\tau$ by means of Eq. 3.5, hence favoring conservative lifetime predictions.

For the equivalent strain amplitude level 0.65% multiaxial proportional and non-proportional LCF tests are performed and the equivalent maximum stress evolution is reported in Fig. 3.21 together with the corresponding uniaxial and torsional data. According to literature (Benallal and Marquis, 1987; Tanaka et al., 1985; Delobelle, 1993), the equivalent response measured in the proportional test is nearly identical to the torsional one. On the other hand, in the non-proportional experiment, the activation of multiple slip systems causes, as expected, a considerable additional hardening compared to the proportional test (up to 50-60% higher). As reported by Benallal and Marquis (1987) and Tanaka et al. (1985), the majority of the non-proportionality-induced additional hardening is quickly accumulated in the firsts cycles (in the considered case about 10-20).

3.2.4. Strain-controlled ratcheting experiments

Data corresponding to uniaxial and multiaxial LCF and ratcheting tests are analyzed in order to evaluate the deformation response of the material when a drifting of the mean strain in axial direction occurs. As previously reported for LCF tests, the cyclic evolution of elastic stiffness and the yield point cannot be measured for multiaxial non-proportional experiments since the mechanical response does not switch between elastic and plastic behavior for all the
duration of the test (except for the first loading).

**Elastic and shear modulus**

As for the zero mean strain experiments, the cyclic evolution of $E$ and $G$ is analyzed for the uniaxial and multiaxial proportional ratcheting experiments plotting the mean elastic and shear moduli as a function of the accumulated plastic strain (see Fig.3.22). To visualize the influence of ratcheting on the stiffness of the material, the corresponding LCF tests are used as reference. Fig.3.22 shows only ratcheting data corresponding to the hysteresis loops antecedent to the reaching of the maximum strain level, after which the mean strain no longer changes. For uniaxial and multiaxial experiments, only small differences between the elastic moduli under LCF and ratcheting conditions are observed. A larger but limited difference (up to 5%) is observed in the shear modulus for multiaxial proportional data. A more precise determination of the effect of ratcheting on the stiffness of the material requires further testing.

**Yield stress**

As for the LCF experiments, the cyclic evolution of the equivalent yield stress is analyzed for ratcheting experiments adopting two threshold values for the definition of the proportionality limit. The left hand side plots in Fig.3.23 report the yield stress values calculated using two different offset thresholds (0.025% and 0.0025%) for uniaxial tests. For multiaxial proportional tests the yield stresses with the two adopted offsets are computed considering both the axial and the shear stress contribution ($\sqrt{\sigma^2 + 3\tau^2}$) (see right plots in Fig.3.23). Fig.3.23 only shows ratcheting data corresponding to the cycles antecedent to the reaching of the maximum axial strain level, after which the mean axial strain no longer changes.

As reported in section 3.1.4, comparing the yield stress behavior calculated on the highest offset value with the corresponding LCF test, ratcheting affects the yield limit introducing an
3.2. Cyclic plastic mechanical behavior of 316L: multiaxial tests

Figure 3.22. Mean elastic modulus in uniaxial (top) and multiaxial proportional tests (center) and mean shear modulus in multiaxial proportional tests (bottom) versus accumulated plastic strain in plate material (316L) at an equivalent strain amplitude of 0.65% and room temperature showing the influence of ratcheting.

additional hardening. The same effect is reported also for multiaxial proportional experiments in which the amount of accumulated additional hardening measured for axial and shear components is quantitatively similar and consistently higher than the one noticed in axial test. On the other hand, when the lowest offset value is used to analyze the same dataset, no significant evidences of ratcheting effects on the yield stress are observed. This finding confirms the results reported in section 3.1.4 that when a constitutive model suitable for ratcheting is formulated and calibrated, the selection of the yield definition must be carefully evaluated. In this case, the absence of the dependency of the yield stress on ratcheting when using the lower offset criterion, allows a more straightforward calibration of the model.

Cyclic hardening/softening

As for the LCF experiments, the cyclic evolution of the equivalent maximum stress for ratcheting tests performed with the same equivalent strain amplitude and ratcheting step is reported in Fig.3.24. The graph only shows data corresponding to the cycles antecedent to the reaching of the maximum axial strain level, after which the mean axial strain no longer changes. Comparing the maximum equivalent stress amplitude measured in uniaxial LCF and ratcheting experiments (see Fig.3.24) one finds that the increment of the mean strain in axial direction introduces an additional hardening. The initial difference is due to fact that uniaxial ratcheting experiments are carried out controlling the deformation in engineering strain instead of true strain. The imposed constant engineering strain amplitude is calculated to have the correct true strain amplitude when the mean strain limit is reached (i.e. 5%). Consequently, in the very first cycles, the true strain amplitude is higher than the one applied in the corresponding LCF test (0.68 instead of 0.65%) resulting in a harder stress response. A consistent ratcheting-induced hardening is found also analyzing the multiaxial proportional data. On the other hand, in non-proportional experiments the drifting of the mean strain in axial direction does not influence the deformation response of the material.

In the case of ratcheting tests, the representation of the results by means of the maximum equivalent stress does not allow a complete understanding of the effect of ratcheting on the
Chapter 3. Experimental characterization of 316L

Figure 3.23. Equivalent yield stress versus accumulated plastic strain in uniaxial (left) and multiaxial proportional experiments (right) performed on plate material (316L) at an equivalent strain amplitude of 0.65% and room temperature showing the importance of the offset value selection.

Figure 3.24. Equivalent maximum stress versus number of cycles in uniaxial and multiaxial ratcheting experiments performed on plate material (316L) with an equivalent strain amplitude of 0.65%, an axial ratcheting step of +0.1%/cyc and room temperature.
3.2. Cyclic plastic mechanical behavior of 316L: multiaxial tests

cyclic deformation behavior. A promising approach consists in analyzing the mean stress and the stress amplitude evolution instead of the maximum stress. Since it is not meaningful to retrieve the alternate and mean stress value in the equivalent space, an alternative approach consists in analyzing separately the contributions of axial and shear stress components and evaluating the evolution of their amplitude and mean value as a function of cycling. The cyclic evolution of the stress amplitude and mean stress in axial $\sigma$ and shear direction $\tau$ are reported for ratcheting tests in Fig.3.25 and 3.26. It is important to remark that in the case of ratcheting, the stress range during loading differs significantly from the unloading one within one cycle. The definition of mean stress and of per-cycle stress amplitude must return a value that has to be consistent in case these features vary from cycle to cycle and that must not be sensitive to the ratcheting direction. The definition used here is the same adopted in section 3.1.4 considering a cycle as the sequence of a loading and a unloading trait (or the reverse), computing the range and mean values using the extrema of each of these traits and calculating the per-cycle values as the average of the two traits; the first cycle is treated as a special case as it is not a complete cycle. To allow an easier visualization of the influence of ratcheting on deformation response, the corresponding LCF tests are also included for reference. Fig.3.25-3.26 only show data corresponding to the cycles antecedent to the reaching of the maximum axial strain level, after which the mean axial strain no longer changes. To allow a quantitative comparison of the two stress components, the shear stress $\tau$ is scaled by a factor of $\sqrt{3}$.

As reported in section 3.1.4, the drifting of the mean strain in axial direction causes an increment of the axial stress amplitude (see left hand side plot in Fig.3.25). A similar result is observed for multiaxial proportional tests (see middle plot in Fig.3.25), in which the drifting of the mean strain in axial direction causes an additional hardening not only of the axial stress but also of the shear stress component. On the other hand, in non-proportional experiments (see right hand side plot in Fig.3.25), the ratcheting does not affect the stress amplitude neither for the axial nor for the shear component.

In the uniaxial experiment, the drifting of the mean strain in axial direction affects not only the evolution of the stress amplitude but also of the mean stress. While nearly no
tension-compression asymmetry is found in uniaxial LCF experiments, ratcheting introduces a drifting of the mean stress having the same sign of the imposed mean strain (see left hand side plot in Fig.3.26). In multiaxial proportional LCF data, the mean stress evolution (see middle plot in Fig.3.26) shows a slight tension-compression asymmetry that slowly fades away during cycling. As for the uniaxial case, when ratcheting occurs a drifting of the axial and shear mean stress is observed. In order to define the sign of this drifting, axial stress and strain are considered to be positive when a tension is applied. The definition of the sign for shear components is arbitrary, but considering that in proportional tests shear and axial strain histories are in phase, it is convenient to define the shear to be positive when a positive axial strain is applied. Using this convention and imposing a positive axial mean strain, the drifting of a mean stress observed in axial direction is positive and the one in shear direction is found to be negative. Finally, a completely different behavior is observed for multiaxial non-proportional experiments (see right hand side plot in Fig.3.26) in which the continuous increment of the axial mean strain has no influence on the cyclic evolution of the mean stress neither in axial nor shear direction.

3.2.5. Discussion

The analysis of the stress amplitude evolution reveals that ratcheting introduces an additional hardening compared with LCF conditions. The procedure proposed in section 3.1.5 in order to quantitatively separate the hardening owing to ratcheting from the one owing to the cyclic hardening, is adopted in the current work section. The first step of this methodology consists in determining approximate curves describing the evolution of the stress amplitude for the considered dataset. The experimental curves are fitted by means of Eq.3.3 which is a summation of exponential equations that are function of the accumulated plastic strain $\psi$ and few constant parameters (i.e. $a_i$, $b_i$) that can be easily determined. These fitting curves allow a straightforward determination of the difference between the stress amplitude in a ratcheting and in a LCF test with the same strain amplitude. This difference, equivalent to the ratcheting-induced additional hardening, is plotted against the value of the mean strain in
3.2. Cyclic plastic mechanical behavior of 316L: multiaxial tests

Figure 3.27. Additional hardening (left) and mean stress (right) versus mean strain in axial direction in uniaxial and multiaxial ratcheting experiments performed on plate material (316L) with an equivalent strain amplitude of 0.65%, an axial ratcheting step of +0.1%/cyc and room temperature.

axial direction (see left hand side plot in Fig.3.27). If one attributes the vertical shift of the curves to the material heterogeneity and to experimental error and only considers the slopes, one can approximate the additional hardening to be linearly related to the axial mean strain for uniaxial and proportional tests, while no noticeable additional hardening is noticed for the non-proportional case. It is interesting to remark that the amount of additional hardening measured in the proportional test is very similar for the axial and the shear stress component and is larger than the one measured in uniaxial tests (a factor of 3).

The separation of cyclic and ratcheting-induced hardening contributions is also performed for the mean stress case. Considering the tension-compression asymmetry effect to be negligible, this separation is a straightforward task and the whole mean stress drifting is considered to be a ratcheting effect. In the right hand side plot in Fig.3.27 the mean stress is plotted versus the mean strain in axial direction. The ratcheting-induced mean stress evolution is qualitatively similar for uniaxial and proportional tests but in the latter the measured values are considerably higher (a factor of 3). No noticeable effect of the drifting of the axial mean strain is noticed for the non-proportional case. This observation can be explained considering the fact that, as reported by Doong et al. (1990), under non-proportional loading conditions the continuous change of the principal stress and strain directions promotes a stronger interaction between different slip systems compared to the proportional loading case (see Fig.3.28). It is plausible to assume that this interaction prevents the activation of the particular microstructural evolution mechanisms observed under uniaxial ratcheting conditions (described in section 3.4). A very similar dislocation structure for specimens subjected to non-proportional LCF and ratcheting experiments would explain the nearly identical mechanical response noticed in the current work.

In Fig.3.29, an evaluation of the influence of the loading path on the life endurance reduction is presented for zero mean strain LCF experiments. The criterion adopted to define the failure is a drop of 10% in the maximum equivalent stress from the stabilized response.

In Fig.3.29, the information about the observed crack type according with the defini-
Chapter 3. Experimental characterization of 316L

Figure 3.28. Interaction between slip systems due to non-proportional straining. (Itoh and Miyazaki, 2003)

...tion given by Nishimura et al. (2000) is also reported. As depicted in Fig.3.14, a specimen showing a crack completely inside the gage length is classified as 'A'. Type 'C' means that the crack nucleated clearly outside the gage length. Type 'B' crack initiated at the edge of the extensometer gage. Since fatigue life is affected by crack propagation, it is plausible to assume that the more severe loading conditions promoted by the nucleation of cracks 'B' and 'C' lead to a reduction of the lifetime. However, in our experiments it was impossible to notice a systematical relation between the crack typology and the lifetime reduction. This observation led us to conclude that the influence of the crack type is smaller than the usual lifetime scatter.

Comparing data corresponding to uniaxial and torsional LCF tests, it is evident that specimens systematically endure longer (a factor of 2) under torsion for all the considered equivalent strain amplitude levels. This result is in agreement with the observation of Socie (1987), Kim et al. (2000) and Chen et al. (2006) who performed similar experiments on stainless steel grade 304 at room temperature.

They observed that the failure mode of this material depends on the stress state and on the applied cyclic strain amplitude. While under axial loading stainless steel grade 304 shows a normal fracture, under torsion it fails in shear mode in the low-cycle regime. Because of the irregularity of the shape of the crack surface, a mechanical interlocking occurs during pure shear loading causing the development of high frictional forces (see Fig.3.30). The consequent reduction of the stress and strain field close to the crack tip is responsible for a lower crack growth rate in torsional tests. In multiaxial proportional LCF tests the simultaneous application of stress and strain in a direction perpendicular to the crack reduces the friction forces and explains why in this case the observed lifetime is comparable to the one measured in uniaxial experiments performed with the same equivalent strain amplitude. On the other hand, in the non-proportional LCF tests the lifetime is substantially reduced (up to a factor of 10). This finding is in agreement with the work of Socie (1987) who attributes the lifetime reduction to the considerably harder material response induced by non-proportionality.

In Fig.3.31, an evaluation of the influence of ratcheting on the life endurance reduction for uniaxial and multiaxial loading conditions is presented. In uniaxial ratcheting experiments a lower number of cycles to failure is noticed with respect to the corresponding LCF test (a factor of 2) suggesting that the ratcheting-induced additional hardening and the positive
3.2. Cyclic plastic mechanical behavior of 316L: multiaxial tests

Figure 3.29. Relationship between equivalent strain amplitude and number of cycles to failure for axial, torsional and multiaxial LCF experiments performed on the 316L plate material at room temperature. The information about the observed crack type according with the definition given by Nishimura et al. (2000) is also reported.

Figure 3.30. Shear damage model for torsion and tension loading conditions (Socie, 1987).
Chapter 3. Experimental characterization of 316L

Figure 3.31. Relationship between equivalent strain amplitude and number of cycles to failure for axial and multiaxial experiments performed on the 316L plate material at room temperature showing the influence of ratcheting on lifetime. The information about the observed crack type according with the definition given by Nishimura et al. (2000) is also reported.

mean stress have a pejorative effect on the fatigue endurance (see discussion in section 3.1.5). The effect of ratcheting on the lifetime reduction is found to be stronger (a factor of 5) in the case of multiaxial proportional loading. This observation is in agreement with the fact that the hardening behavior has a strong impact on the fatigue endurance. It is therefore logical to expect a stronger lifetime reduction for multiaxial proportional experiments, in which the amount of ratcheting-induced additional hardening is considerably higher (a factor of 3) than in the corresponding uniaxial tests. The influence of (initial) ratcheting (followed by cyclic relaxation) on the fatigue lifetime can change substantially considering different materials. The ratcheting influence will be particularly evident in materials like stainless steel 316L in which the loading sequence “high to low” plays an important role on the lifetime (Wong et al., 2002). Finally, the specimen subjected to multiaxial non-proportional ratcheting does not show any noticeable lifetime reduction comparing with the corresponding LCF condition. This observation is in agreement with the fact that, for this class of experiments, the initial drifting of mean strain in axial direction does not influence the material response.

3.2.6. Conclusions

Uniaxial, torsional and multiaxial LCF and strain-controlled ratcheting experiments have been carried out on a 316L plate material batch and the influence of strain amplitude, loading path and ratcheting on the cyclic mechanical behavior has been investigated. Differently from the existing literature references on multiaxial experiments, controlling at the same both the axial and the torsional axes in strain, it was possible to perform a direct comparison of LCF and ratcheting data and to extrapolate the effect of the mean strain drifting on the material response. The elastic and the shear modulus are observed to monotonically decrease during cycling in LCF tests, and the rate of this decrease is found to depend on the imposed equivalent strain amplitude. In addition, when a small enough offset value (e.g. 0.0025%) is adopted for its definition, the equivalent yield stress exhibits a monotonic decrease during cycling that is not particularly influenced by strain amplitude. This observation is particularly helpful allowing a considerable reduction of complexity in the formulation of the constitutive
In the performed experiments it is found that the maximum equivalent stress response evolution is similar for uniaxial, torsional and multiaxial proportional LCF tests having the same equivalent strain amplitude. A considerably harder cyclic response is measured when a non-proportional loading history is imposed justifying the huge reduction of fatigue life noticed in these tests.

In uniaxial experiments ratcheting is found to have an impact on the cyclic deformation behavior. In fact, the comparison of ratcheting and corresponding LCF experiments, highlights that the continuous increasing of the mean strain in axial direction induces an additional hardening of the stress amplitude and a drifting of the mean stress in the same direction of ratcheting. Those ratcheting-induced differences in the cyclic deformation behavior explain the moderate lifetime reduction measured in uniaxial experiments. Similar observations are also performed for proportional tests, in which the drifting of mean strain in axial direction, influences also the shear stress component response. In addition, for this loading case, the effect of ratcheting on the mechanical response is found to be quantitatively stronger causing a more pronounced drop of fatigue life. On the other hand, ratcheting is found not to influence particularly neither mechanical response nor the lifetime in non-proportional experiments.
3.3 Time dependency

In this section results of the uniaxial isothermal test performed in order to analyze the time-dependent mechanical behavior of 316L under loading conditions relevant for the primary cooling circuit of a light water nuclear reactor are reported and discussed below.

3.3.1. Introduction

The literature review in section 1.1.3 reported that the cyclic material behavior of stainless steels at room temperature is time-dependent. The main observations can be summarized in the following items:

– 316L suffers a substantial amount of cold creep when subjected to a constant load
– the loading rate moderately influences the deformation response under monotonic and cycling straining
– the loading rate strongly influences the cyclic accumulation of inelastic strain under stress controlled ratcheting conditions

The last item led Taleb and Cailletaud (2011) to perform a set of combined creep and ratcheting tests. The authors show that a specimen initially subjected to creep, does not accumulate further inelastic strain when subjected to a stress controlled ratcheting test (with the same maximum stress). This observation led Taleb and Cailletaud (2011) to conclude that most of the cyclic accumulation of the inelastic strain exhibited in the classical ratcheting tests is mainly due to creep. This conclusion must be carefully taken into account in order to propose a constitutive model suitable for complex loading conditions including ratcheting. In order to further investigate this phenomenon and to fill some experimental gaps, not only stress-controlled but also strain-controlled ratcheting tests are performed.

3.3.2. Testing equipment and samples preparation

Testing equipment and samples preparation are already described in section 2.1.1. For the investigation of time-dependency only specimens machined from plate material batch are considered.

3.3.3. Stress controlled tests

Loading paths

Two different typologies of stress controlled experiments are carried out in this dissertation:

– Ratcheting experiments (i.e. PE36 and PE55) consisting in stress controlled tests using a path with a ramp waveform and a cycling period that is changed in order to investigate the influence of the stress rate on the material response,
– sequence of creep + ratcheting tests (i.e. PE38, PE56 and PE64) inspired by the work of Taleb and Cailletaud (2011) previously presented in section 1.1.3.

As can be seen in Fig.1.30, in a creep+ratcheting experiment the sample is first subjected to a constant stress for a certain amount of time. Afterwards, a conventional stress controlled ratcheting tests is imposed in which the maximum stress is the same as the one applied during the creep phase. One should notice that, these experiments are performed applying more severe loading conditions (i.e. higher amplitude) and considering a larger number of
Table 3.4. Summary of the testing parameters used to perform the room temperature stress-controlled ratcheting and creep+ratcheting tests

<table>
<thead>
<tr>
<th>Sample name</th>
<th>(\sigma_{\text{hold}}) (MPa)</th>
<th>time(_{\text{hold}}) (min)</th>
<th>(\dot{\sigma}_{\text{ramp}}) (MPa/s)</th>
<th>(\sigma_{\text{amp}}) (MPa)</th>
<th>(\sigma_{\text{mean}}) (MPa)</th>
<th>(\dot{\sigma}) (MPa/s)</th>
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<tr>
<td>PE36</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>400</td>
<td>30</td>
<td>200</td>
</tr>
<tr>
<td>PE55</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>400</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>PE38</td>
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<td>2</td>
<td>20</td>
<td>400</td>
<td>30</td>
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</tr>
<tr>
<td>PE56</td>
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<td>120</td>
<td>20</td>
<td>400</td>
<td>30</td>
<td>200</td>
</tr>
<tr>
<td>PE64</td>
<td>430</td>
<td>1200</td>
<td>20</td>
<td>400</td>
<td>30</td>
<td>200</td>
</tr>
</tbody>
</table>

Figure 3.32. (left) Stress-strain curves corresponding to the first loading in stress-controlled tests (316L, Room temperature). (right) Inelastic strain accumulation as a function of time for creep tests (316L, Room temperature).

Cycles compared with the ones of Taleb and Cailletaud (2011) and Taleb (2013b). The aim of those experiments is to determine the cause of the accumulation of inelastic strain. The complete list of the stress controlled experiments and the corresponding testing parameters are reported in Tab.3.4 and in Appendix A.

Monotonic response

If only the first loading trait is considered, one can retrieve information about the monotonic response of the material (see left hand side plot in Fig.3.32). In the tests in which the applied force reaches its maximum in about 20 seconds (e.g. PE55, PE56, PE38 and PE64), the stress-strain curves are nearly identical. In agreement with Portier et al. (2000), in the experiment carried out with a higher loading rate (a factor of 10 for PE36) the material response is found to be slightly harder (about 5%).

Creep

In experiments in which the maximum level of force is held (PE38, PE56 and PE64) for a certain time interval, one can observe a cold creep behavior of the material (see right hand side plot in Fig.3.32) that is consistent with literature (Nomine et al., 1982). The three curves do not provide a perfect overlapping but after the first seconds the inelastic strain accumulation rate can be considered to be equivalent.

Stress rate influence

Comparing two stress-controlled ratcheting tests with different stress rates (i.e. PE36 and PE55), the results reported in literature are confirmed: the application of the same load path imposing a lower stress rate leads to a higher cyclic accumulation of inelastic strain (see left hand side plot in Fig.3.33).
Figure 3.33. (left) mean strain evolution as a function of number of cycles for simple ratcheting tests, (right) strain amplitude evolution as a function of number of cycles for simple ratcheting tests (316L, Room temperature).

Figure 3.34. Selected hysteresis loops for stress ratcheting controlled tests having two different stressing rates (PE36 and PE55) (316L, Room temperature).

The first cycles are the main responsible for the measured difference in the cyclic accumulation of inelastic strain. This finding becomes evident if one adequately shifts the hysteresis loops and represents them in the same plot keeping the same scaling factor (see Fig.3.34). In fact, except for the very first cycles, the hysteresis loop shapes and the strain amplitude (see right hand side plot in Fig.3.33) of the experiments carried out with different stress rates look very similar.

A further evidence of this finding is recognizable plotting the ratcheting rate evolution as a function of the number of cycles (see left hand side plot in Fig.3.39). After the first cycles the ratcheting rate measured in PE36 and PE55 are practically equivalent. Moreover, the persistent small difference in ratcheting rate and in the loop shapes between the two experiments is most likely due to the fact that the tests are performed under engineering stress control.

As depicted in Fig.3.35, the drifting of the mean strain causes the true stress loading path to be considerably different compared with the corresponding nominal engineering stress...
3.3. Time dependency

Figure 3.35. (left) true mean stress evolution as a function of number of cycles for simple ratcheting tests, (right) true stress amplitude evolution as a function of number of cycles for simple ratcheting tests (316L, Room temperature).

\( \sigma_{\text{amp}} = 400 \text{ MPa}, \ \sigma_{\text{mean}} = 30 \text{ MPa} \). In the experiment performed with lower stress rate (i.e. PE55), the higher amount of inelastic strain accumulated in the first cycles is responsible for the more severe loading condition and is consistent with the observed differences. One can therefore conclude that the variation of the stress rate causes mostly a rigid shifting of the ratcheting curves along the vertical axis. This finding is consistent with the work of Kang et al. (2006a) but not with that of Mizuno et al. (2000).

Creep or ratcheting?

To further investigate the phenomenon reported by Taleb and Cailletaud (2011), a set of creep+ratcheting experiments on 316L is performed. It must be underlined that the adopted stress level was considerably higher compared with the ones used by Taleb and Cailletaud (2011).

Comparing the material response in the first 50 cycles for a simple ratcheting test and the corresponding creep+ratcheting experiment (respectively PE36 and PE56), the finding of Taleb and Cailletaud (2011) seems to be confirmed. In the initial part of the experiment, the material subjected to a constant stress shows a noticeable inelastic strain accumulation due to creep. The cyclic accumulation of inelastic strain in the subsequent ratcheting test seems to be negligible (see upper plot in Fig.3.36). However, when the complete dataset is taken into account, a different result is evident (see lower plot in Fig.3.36 and left plot in Fig.3.37). As a first consideration, because of more severe loading conditions compared to Taleb’s experiments, the simple ratcheting test does not show any shakedown. Moreover, in creep + ratcheting tests, the inelastic accumulation occurring in the second part of the experiment is negligible only in the first 50-100 cycles. Further cycling slowly reactivates ratcheting and in the end the ratcheting rate is comparable with the one measured in the classical ratcheting test (see left hand side plot in Fig.3.39).

Comparing the strain responses of creep+ratcheting tests with a different initial holding time (respectively 2 minutes, 2 hours and 20 hours), it is evident that when the first loading phase is short, less cycles are necessary to reactivate the cyclic accumulation of inelastic strain. Compared with Taleb and Cailletaud (2011), in our study we additionally analyzed the strain amplitude evolution during cycling (see right plot in Fig.3.37). As expected, except for the first cycles, the strain amplitude evolution is nearly identical in the two simple ratcheting tests with different stressing rate (PE36 and PE55). On the other hand, the specimens previously subjected to creep show a considerably lower strain amplitude. Further cycling seems to soften the material leading to a gradual increment of the strain amplitude until a level similar to the one measured in the simple ratcheting experiments. Similarly to what
Figure 3.36. Ratcheting and creep + ratcheting tests (PE36 and PE56) (316L, Room temperature).

Figure 3.37. (left) mean strain evolution as a function of number of cycles for ratcheting and creep+ratcheting tests, (right) strain amplitude evolution as a function of number of cycles for ratcheting and creep+ratcheting tests (316L, Room temperature).

observed for the mean strain evolution, the cycling-induced strain amplitude evolution is faster when the first loading phase is short.

These observations lead to the conclusion that the cyclic accumulation of the inelastic strain in cyclic tests is only in part due to creep. In fact, cyclic loading was found to play a very important role triggering a softer material response and additional strain drifting. A further evidence of the fact that creep and ratcheting are distinct phenomena can be found plotting the maximum strain versus time (see Fig.3.38). Analyzing the data corresponding to creep + ratcheting experiments (PE38, PE56 and PE64) a sudden variation of the slope of the curves can be noticed when cycling occurs (i.e. beginning of the the second phase of the experiment).

In order to exclude that the cyclic evolution of the inelastic strain is somehow a result of the heating due to deformation-induced energy dissipation, the evolution of the temperature measured on the surface of the specimens as a function of number of cycles is reported (see right hand side plot in Fig.3.39). Looking at the graph it is clear that the temperature
3.3. Time dependency

![Graph showing time dependency](image)

**Figure 3.38.** Maximum strain as a function of time in ratcheting and creep+ratcheting tests (316L, Room temperature).

![Graph showing ratcheting rate](image)

**Figure 3.39.** (left) Ratcheting rate evolution in ratcheting and creep+ratcheting tests (316L, Room temperature), (right) temperature of the specimen in ratcheting and creep+ratcheting tests (316L, Room temperature)

increment promoted by deformation can be considered to be negligible ($\Delta T \leq 6$ °C).

### 3.3.4. Strain controlled tests

**Loading paths**

A further investigation on the time dependency on the cyclic mechanical behavior of 316L is carried out performing 2 sets of LCF and strain-controlled ratcheting experiments at room temperature. In the first set (i.e. PE45, PE50 and PE53) the influence of ratcheting step is investigated. As a first task, a LCF test (i.e. PE45) is performed applying a ramp strain path. Subsequently 2 strain-controlled ratcheting tests (i.e. PE50 and PE53) are carried out superposing the already presented strain path with an increasing mean strain having a different ratcheting step $\xi$ (i.e. cyclic accumulation of strain per cycle) but the same cycling period (and therefore the same strain rate). The testing parameters are listed in Tab.3.5 and a schematic of the loading path is reported in Fig.3.40.
Figure 3.40. Ramp waveform strain paths.

Table 3.5. Summary of the testing parameters used to perform the room temperature strain-controlled ratcheting and LCF experiments

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Waveform</th>
<th>$\varepsilon_{amp}$ (%)</th>
<th>$\dot{\varepsilon}$ (%)</th>
<th>Period (s)</th>
<th>$\varepsilon_{mean}$ (%)</th>
<th>$\varepsilon$ (%)</th>
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<td>0.35</td>
<td>9</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
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<td>Ramp</td>
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<td>0.35</td>
<td>9</td>
<td>0-5</td>
<td>0.1</td>
</tr>
<tr>
<td>PE53</td>
<td>Ramp</td>
<td>0.70</td>
<td>0.35</td>
<td>9</td>
<td>0-5</td>
<td>0.1</td>
</tr>
<tr>
<td>PE42</td>
<td>Saw tooth</td>
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<td>0.5-0.05</td>
<td>44</td>
<td>0</td>
<td>-</td>
</tr>
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<td>-</td>
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<td>PE44</td>
<td>Saw tooth</td>
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<td>0.05-0.005</td>
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<td>0.1</td>
</tr>
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<td>0.5-0.05</td>
<td>17.6</td>
<td>0-5</td>
<td>0.1</td>
</tr>
<tr>
<td>PE59</td>
<td>Saw tooth</td>
<td>0.40</td>
<td>0.05-0.005</td>
<td>176</td>
<td>0-5</td>
<td>0.1</td>
</tr>
</tbody>
</table>

In the second set (i.e. PE42, PE41, PE43, PE44, PE58 and PE59) the focus of the analysis is on the strain rate dependency. In order to mimic the loading profile observed in industrial components subjected to complex loading conditions, the imposed strain waveform is not the classical one but it has a saw tooth shape (see Fig.3.41). Imposing this particular saw tooth strain path, the resulting strain rate will not be constant within a cycle but the load is imposed with 2 different strain rates (having a factor of 10 of difference). Two LCF tests (i.e. PE42 and PE41) are first performed applying the presented strain path with different cycling period (i.e. 44 s and 440 s). Two strain-controlled ratcheting tests (i.e. PE43 and PE44) are then carried out superposing the already presented saw tooth strain path with an increasing mean strain. Moreover, in order to check the influence of strain amplitude, two additional ratcheting experiments (i.e. PE58 and PE59) are performed varying the strain amplitude and the cycling period (but keeping the same strain rates of the previous tests). The testing parameters for all the experiments are listed in Tab.3.5 and in the appendix A.

**Ratcheting step influence**

In Fig.3.42 the evolution of the mean stress and stress amplitude is plotted for the data corresponding to the first set of strain-controlled experiments. As already reported in section 3.1.4, analyzing the stress amplitude response, it is evident that for ratcheting loading conditions, an additional cyclic hardening takes place compared with LCF test. The magnitude of this difference increases as a function of the mean strain until the ratcheting
Figure 3.41. Saw tooth waveform strain paths.

Figure 3.42. Average and alternate stress evolution in strain-controlled test with strain amplitude 0.7% (316L, room temperature).

limit (i.e. 5%) is reached (respectively 50 and 500 cycles). After this point, 316L shows a cyclic softening response (slowly stabilizing) until end of life. For a ratcheting test with a higher ratcheting step (i.e. 0.1%/cycle), the material exhibits higher maximum value of stress amplitude. After a high number of cycles, the stress amplitudes for the two ratcheting tests become nearly equivalent. The mean stress for LCF loading is practically zero for 316L. On the other hand, during ratcheting tests, the asymmetrical nature of loading (tensile straining amplitude is higher than compressive) causes an increment of the mean stress that is function of the value of the mean strain. Once the ratcheting limit is reached and the tensile straining amplitude is equivalent to the compressive, a cyclic relaxation of the mean stress takes place. The maximum value reached by the mean stress is higher for the ratcheting test performed with higher ratcheting step (i.e. 0.1%/cycle). However, after reaching the ratcheting limit, the relaxation of the material results in similar values of mean stress for the two ratcheting tests.
Chapter 3. Experimental characterization of 316L

Figure 3.43. Maximum, minimum, average and stress amplitude evolution in saw tooth LCF tests with $\varepsilon_{amp}=1.00\%$ (316L, room temperature).

**Strain rate influence**

In order to investigate the influence of strain rate, only the second set of strain-controlled experiments is considered. In Fig.3.43 the evolution of maximum, minimum, average and stress amplitude is plotted for both the performed LCF tests. The strain rate dependency on the stress response of the material is evident, in fact, the stress amplitude measured in the ‘fast’ test is about 20 MPa higher (5-7% higher) compared with the ‘slow’ test (having a cycling period 10 times longer). This finding is quantitatively in agreement with literature findings reported in section 1.1.1. Moreover when the in-cycle strain rate suddenly changes (increasing of a factor 10 at the edge of the saw tooth profile), the hysteresis loops clearly show a step in the absolute value of stress of about 20 MPa (5-7%) (see Fig.3.44). This is further evidence of the strain rate dependency of the stress response of the material.

Analyzing the maximum, minimum, average and stress amplitude evolution for ratcheting experiments (see Fig.3.45 and 3.46), the same result reported for LCF test is confirmed: the stress amplitude measured in the ‘fast’ test is about 5-7% higher compared with the ‘slow’ test (having a cycling period 10 times longer). On the other hand, in both cases the strain rate does not affect the evolution of the mean stress.

This conclusion is surprising considering the fact that, as reported in the literature review in section 1.1.1, the stress-controlled ratcheting tests were strongly affected by the loading rate. As stress relaxation is the ‘counterpart effect’ of creep, cyclic mean stress relaxation can be considered the ‘counterpart effect’ of ratcheting (see Fig.3.47(a)). Comparing the mean stress evolution in the experiments performed with different strain amplitudes (see Fig.3.45 and 3.46), another important observation is reported: strain amplitude plays a main role in the cyclic relaxation of mean stress. Applying the same mean strain drifting, the mean stress is considerably lower in the experiments performed with higher strain amplitudes. This result is in agreement with the cyclic relaxation experiments performed by Jhansale and Topper (1973) on a mild steel (grade SAE 1015) at room temperature (see Fig.3.47(b)).
3.3. Time dependency

Figure 3.44. Hysteresis loops at 2 different cycles in saw tooth LCF tests with $\varepsilon_{\text{amp}}=1.00\%$ (316L, room temperature).

Figure 3.45. Maximum, minimum, average and stress amplitude evolution in saw tooth ratcheting tests with $\varepsilon_{\text{amp}}=1.00\%$ (316L, room temperature).
Figure 3.46. Maximum, minimum, average and stress amplitude evolution in saw tooth ratcheting tests with $\varepsilon_{\text{amp}}=0.40\%$ (316L, room temperature).

Figure 3.47. (a) Cyclic mean stress relaxation as a counterpart effect of ratcheting, (b) Cyclic mean stress relaxation as a function of cycle number showing the influence of strain amplitude (SAE 1015, Room Temperature). (Jhansale and Topper, 1973)
3.3.5. Discussion and conclusions

The time-dependent mechanical behavior of 316L and similar stainless steel grades at room temperature are studied in literature (see section 1.1.3). In agreement with literature references, 316L is found to suffer cold creep and to show a particularly strong loading rate influence on the accumulation of inelastic strain for stress-controlled ratcheting tests. It is also observed that the very first cycles are responsible for the major part of the difference in ratcheting accumulation. After the initial cycles, the hysteresis loop shapes and the elongation rates were found to be very similar in experiments performed with different stress rates. This suggests that in the investigated case the time dependent behavior can not be exhaustively explained by creep.

The necessity to further investigate this phenomenon induced the author of this dissertation to perform a set of creep+ratcheting experiments similar to the ones proposed by Taleb and Cailletaud (2011). Applying more severe loading conditions and considering a larger number of cycles, results deviating from the work of Taleb and Cailletaud (2011) are observed. In fact it is demonstrated that, in the performed creep+ratcheting tests, the cyclic accumulation of inelastic strain is only apparently blocked by the occurring of creep. Cyclic loading triggers a different material response and slowly reactivates the strain drifting. This suggests that creep and ratcheting are distinct phenomena and that the accumulation of the inelastic strain in cyclic tests is only in part creep. This conclusion is in contrast with Taleb and Cailletaud (2011) in which most of the cyclic accumulation of the inelastic strain exhibited in their lower amplitude ratcheting tests was found to be mainly due to creep.

In the current work the influence of loading rate in strain controlled ratcheting tests is also analyzed. As reported in section 1.1.1 several references are available in literature for LCF tests but accordingly with our knowledge no author investigated the influence of loading rate for strain-controlled ratcheting experiments. A first set of experiments is performed to investigate the influence of ratcheting step on the strain controlled ratcheting case. No evidence of any ratcheting specific response beyond an increase of the stress amplitude is found. A second set of strain controlled experiments is performed varying the cycling period with the purpose of analyzing the influence of the strain rate. In agreement with literature for LCF tests (see section 1.1.3), the stress amplitude response is influenced by the strain rate. However, the loading rate effect is considerably weaker than the one measured in the stress-controlled case. On the other hand, the strain rate does not significantly affect the evolution of the mean stress. This result is surprising considering that the stress controlled ratcheting tests are strongly affected by the loading rate. In fact, as the stress relaxation is the ‘counterpart effect’ of creep, similarly cyclic mean stress relaxation can be considered the ‘counterpart effect’ of ratcheting.

In a recent work, Taleb (2013a) performs creep and ratcheting tests not only at room temperature but also at higher temperature levels (i.e. 350 °C). The author reports that in 316L both creep and time-dependent accumulation of inelastic strain in ratcheting tests slowly disappears with the temperature increasing. Since the aim of this dissertation is to characterize the material behavior not only at room temperature but also at higher temperature levels, an interesting outlook would be performing further experiments in order to check the time-dependency of the material behavior at higher temperatures.
3.4 Microscopic analysis of the influence of ratcheting on the evolution of dislocation structures during low cycle fatigue

In this section the main results of the analysis on the microstructural evolution of dislocation structures of 316L stainless steel subjected to LCF and ratcheting are reported. In order to accomplish this task a series of testpieces have been fatigued to selected number of cycles, corresponding to different cyclic deformation response stages. The microstructural conditions of fatigued specimens are consequently characterized by transmission electron microscopy (TEM).

3.4.1. Introduction

Thermo-mechanical fatigue can cause crack initiation and propagation in various components of the primary cooling circuit of a light water nuclear reactor, which is partially manufactured using austenitic stainless steels (Dahlberg et al., 2007). The thermo-mechanical loading existing in these components (e.g. thermal transients and coolant stratification) induces conditions that can involve low cycle fatigue with a drifting mean strain (i.e. ratcheting). Several studies have been carried out in the past with the purpose of analyzing the mechanical response of this class of materials, in particular for the stainless steel grade 316L under thermomechanically relevant loading conditions (Polak et al., 1994; Ohno et al., 1998; Mizuno et al., 2000; Facheris and Janssens, 2013). Other authors have investigated the evolution of the microstructure with cycling and correlated their observations with the measured deformation response (Polak et al., 1994; Mayama et al., 2008; Pham et al., 2011; Pham and Holdsworth, 2012). A limited number of publications treat the microstructural evolution under ratcheting conditions (Bocher et al., 2001; Gaudin and Feaugas, 2004; Dutta et al., 2010; Kang et al., 2010; Song, 2009), none of which treat strain controlled ratcheting. The motivation to perform strain instead of stress controlled ratcheting experiments is that the former collects data keeping the strain amplitude constant, allowing a more straightforward use in the calibration of constitutive material descriptions, which in our case use strain amplitude as an internal parameter.

When subjected to controlled cyclic deformation, the response of austenitic stainless steel typically involves primary hardening followed by softening, and eventually cyclic stabilization with or without secondary hardening. If a continuously drifting mean strain is superposed to an alternating strain path (i.e. strain controlled ratcheting), the response in terms of mean stress and strain amplitude is significantly different. A series of low cycle fatigue and ratcheting experiments are performed at room temperature on round specimens extracted from the AISI 316L plate material batch previously characterized (see section 3.1.2). The experiments are interrupted at cycle numbers selected to correspond with the different strain controlled cycle response stages. The mechanical response of the material is presented in section 3.4.3. The as received material and the fatigued specimens are analyzed by means of transmission electron microscopy to characterize the microstructure and its evolution with cyclic loading (see section 3.4.4). Finally, in section 3.4.5, an interpretation of the results is given and a discussion including findings from literature references is included.

3.4.2. Experimental setup

Testing equipment and samples preparation

Testing equipment and samples preparation are described in section 2.1.1.
3.4. Microscopic analysis on the evolution of dislocation structures during LCF

Loading paths

Experiments are performed at room temperature. A thermocouple is placed on the specimen surface to monitor eventual deformation-induced temperature increases in the specimen. All experiments are performed under strain control using a clip-on extensometer. Two kinds of tests are conducted:

1. Low cycle fatigue (LCF) tests at a strain amplitude $\varepsilon_a = 0.70\%$, using a strain path with a ramp waveform and strain rate $\dot{\varepsilon} = 0.31\%/s$.

2. Strain controlled ratcheting tests consisting of the superposition of a ramp waveform with a constant strain amplitude (equivalent to 0.70%) having a strain rate of 0.31%/s and a continuously increasing mean strain as schematically shown in Fig.1.10. The continuous increasing of mean strain per unit of time, i.e. the ratcheting rate $\dot{\varepsilon}_r$, can alternatively be defined per cycle as ratcheting step $\xi$. When the mean strain reaches the value $\varepsilon_r = 5\%$ (imposed by the extensometer range) it is kept constant and the experiment is continued as a cyclic relaxation experiment. The ratcheting tests are carried out for two different ratcheting steps ($+0.10\%$ and $+0.01\%$ per cycle).

A preliminary low cycle fatigue test is performed to identify the cyclic deformation response stages of the material and to determine:

- the number of cycles at which the initial cyclic hardening reaches a maximum: $N_h = 22$,
- the number of cycles at which the cyclic softening phase ends: $N_s = 500$,
- the number of cycles at which the specimen fails: $N_f = 4791$. The criterion adopted to define the failure is a drop of 10% in the maximum stress from the stabilized response (assuming the stress response to be stable after 1000 cycles).

A set of interrupted LCF and ratcheting tests is then performed, the purpose of which is to provide samples for observation the evolution of dislocation microstructure in the transmission electron microscope. In addition to the tests interrupted at $N_h$ and $N_s$, further ratcheting tests are interrupted at the numbers of cycles for which the mean strain reaches its maximum value $N_{\varepsilon_{max}}$, i.e. after 50 cycles for $\xi = +0.10%/cyc$ and after 500 for $\xi = +0.01%/cyc$. The complete list of interrupted experiments is summarized in Tab.3.6.

Test designation

The name of each test begins with two strings that identifies the typology of the experiment (e.g. uniAXial and ‘LCF’ or ‘RAT’). The information regarding the testing temperature and the imposed strain amplitude is indicated in the next 2 positions of the experiment name. For ratcheting tests, the information about the direction (e.g. ‘P’ stands for positive) and the ratcheting step is given at the last position of the name. As an example, if the experiment called ‘AX-RAT-RT-070-P10’ is considered, it can be easily identified as a uniaxial ratcheting test performed at room temperature, with a strain amplitude of 0.70% and a positive ratcheting step of 0.10%/cycle.

3.4.3. Fatigue experiments: deformation response to LCF and strain controlled ratcheting

Fig.3.48 shows the results of the three low cycle fatigue experiments. In Fig.3.49 the stress amplitude and the mean stress responses for the ratcheting tests performed with the
Table 3.6. Summary of low cycle fatigue and strain controlled ratcheting experiments. The cyclic strain amplitude $\varepsilon_a = 0.70\%$ and the cyclic strain rate $\dot{\varepsilon} = 0.31\%/s$ are constant in all experiments. The maximum mean strain after which the ratcheting experiments are continued as cyclic relaxation experiments is 5%.

<table>
<thead>
<tr>
<th>Test N.</th>
<th>test designation</th>
<th>specimen name</th>
<th>temperature (°C)</th>
<th>$\varepsilon_{\text{amp}}$ (%)</th>
<th>$\varepsilon_{\text{mean}}$ (%)</th>
<th>$\varepsilon_{\text{maxr}}$ (%)</th>
<th>Cycle of interruption</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>as received</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>AX-LCF-RT-070</td>
<td>PE45</td>
<td>RT</td>
<td>0.70</td>
<td>0</td>
<td>-</td>
<td>$N_f$ 4791</td>
</tr>
<tr>
<td>3</td>
<td>AX-LCF-RT-070</td>
<td>PE46</td>
<td>RT</td>
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<td>0</td>
<td>-</td>
<td>$N_h$ 22</td>
</tr>
<tr>
<td>4</td>
<td>AX-LCF-RT-070</td>
<td>PE47</td>
<td>RT</td>
<td>0.70</td>
<td>0</td>
<td>-</td>
<td>$N_s$ 500</td>
</tr>
<tr>
<td>5</td>
<td>AX-RAT-RT-070-P10</td>
<td>PE49</td>
<td>RT</td>
<td>0.70</td>
<td>0.5</td>
<td>+0.1</td>
<td>$N_f$ 22</td>
</tr>
<tr>
<td>6</td>
<td>AX-RAT-RT-070-P10</td>
<td>PE48</td>
<td>RT</td>
<td>0.70</td>
<td>0.5</td>
<td>+0.1</td>
<td>$N_{\text{max}}$ 50</td>
</tr>
<tr>
<td>7</td>
<td>AX-RAT-RT-070-P10</td>
<td>PE51</td>
<td>RT</td>
<td>0.70</td>
<td>0.5</td>
<td>+0.1</td>
<td>$N_f$ 500</td>
</tr>
<tr>
<td>8</td>
<td>AX-RAT-RT-070-P10</td>
<td>PE50</td>
<td>RT</td>
<td>0.70</td>
<td>0.5</td>
<td>+0.1</td>
<td>$N_f$ 2873</td>
</tr>
<tr>
<td>9</td>
<td>AX-RAT-RT-070-P01</td>
<td>PE52</td>
<td>RT</td>
<td>0.70</td>
<td>0.5</td>
<td>+0.01</td>
<td>$N_f$ 22</td>
</tr>
<tr>
<td>10</td>
<td>AX-RAT-RT-070-P01</td>
<td>PE54</td>
<td>RT</td>
<td>0.70</td>
<td>0.5</td>
<td>+0.01</td>
<td>$N_{\text{max}}$ 50</td>
</tr>
<tr>
<td>11</td>
<td>AX-RAT-RT-070-P01</td>
<td>PE53</td>
<td>RT</td>
<td>0.70</td>
<td>0.5</td>
<td>+0.01</td>
<td>$N_f$ 2773</td>
</tr>
</tbody>
</table>

Figure 3.48. Stress amplitude $\sigma_a$ versus cycle number for AX-LCF-RT-070 tests. The markers in the plots indicate the number of cycles at which the experiments are interrupted.

Figure 3.49. (top) Stress amplitude $\sigma_a$ and (bottom) mean stress $\sigma_m$ versus cycle number for ratcheting tests AX-RAT-RT-070-P10. The markers in the plots indicate the number of cycles at which the experiments are interrupted.
3.4. Microscopic analysis on the evolution of dislocation structures during LCF

Figure 3.50. (top) Stress amplitude $\sigma_a$ and (bottom) mean stress $\sigma_m$ versus cycle number for low cycle fatigue (AX-LCF-RT-070) and ratcheting tests (AX-RAT-RT-070-P10 and AX-RAT-RT-070-P01). The markers in the plots indicate the number of cycles at which the experiments are interrupted.

higher cyclic ratcheting step (+0.1 %/cycle) are plotted. In Fig.3.50, the stress amplitude response during low cycle fatigue and ratcheting is plotted.

The analysis of the cyclic material response gives results analogous to the ones already presented in section 3.1 for experiments performed at different strain amplitude levels. As a first consideration, an additional hardening can be observed in the ratcheting experiments. The magnitude of this difference increases as a function of the mean strain until the end of the ratcheting phase is reached (respectively 50 and 500 cycles). During the subsequent cyclic relaxation, the material exhibits cyclic softening, and then slowly stabilizes towards the end of life. As can clearly be seen in Fig.3.50, part of the ratcheting induced, additional hardening is not recovered. Furthermore, while the maximum stress amplitude depends on the ratcheting step, the stress level to which it softens during cyclic relaxation does not.

Plotted in the same figure, the mean stress response for low cycle fatigue loading is practically zero. In contrast, the asymmetrical nature of the strain cycle during ratcheting, in which the tensile stretch is larger than the compressive, causes the mean stress to increase with the value of the mean strain. When ratcheting ends and the magnitude of the tensile strain extent of the cycle becomes equal to the compressive one, cyclic relaxation occurs and the mean stress gradually returns to zero. The ratcheting experiment performed with the higher ratcheting step leads to a higher maximum value of the mean stress. However, the cyclic relaxation of the material returns to the same value of mean stress for both the two ratcheting tests.

Regarding fatigue endurance, ratcheting is not observed to cause a significant reduction of the number of cycles to failure. The deviation in endurance observed is not larger than a factor of two as reported in section 3.1.

3.4.4. TEM observations

Tested specimens are sectioned along the loading axis. The samples are ground and polished to produce thin foils of thickness 0.1 mm ($\pm 0.02\text{mm}$). Discs of 3 mm in diameter are punched from these foils and marked to indicate the loading direction (LD). They are subsequently polished electrolytically using a double jet device with an electrolyte solution of acetic acid and perchloric acid to thin the specimens for TEM investigation (final thickness between 80 and 150 $\text{nm}$).
Microstructure in the as received condition

The microstructure of the material in the as received condition is characterized by very planar dislocation structures, with for example stacking faults and regular dislocation arrays as shown in Fig.3.51a. The dislocation density is mostly observed to be higher for regions close to grain boundaries as depicted in Fig.3.51b. This higher density makes the dislocation structures more tangled.

Microstructural evolution during low cycle fatigue (AX-LCF-RT-070)

The evolution of the dislocation structure during zero mean strain low cycle fatigue is characterized to serve as a reference for comparison with the structure evolution during ratcheting. At the end of the cyclic hardening phase at \(N_h = 22\), dislocation structures in the bulk center of the grains remain similar to those in the as-received condition, except for a slight increase of the dislocation density. Stacking faults and planar dislocation arrays are still often observed at this stage, indicating that planar dislocation motion suffices to accommodate the local deformation during cycling in these areas. In regions near the grain boundaries, the dislocation density is observed to increase and to consist of tangled dislocation structures as shown in Fig.3.52a.

In some grains, edge dislocations have started to form stable dipoles resulting in the formation of low density dislocation veins, as shown in Fig.3.52b. Dislocation lines in low density regions are more often observed in zigzag form (Fig.3.52c). Moreover, diffusion rings are seen in selected area diffraction patterns of regions where zigzag dislocation lines were present, indicating the dislocation motion during cyclic plastic deformation is strongly influenced by solute atoms, e.g. C and N, owing to the formation of solute atom clouds around dislocation lines (Pham and Holdsworth, 2012).

Near the end of the cyclic softening phase, after \(N_s = 500\) cycles, dislocation high/low density regions become more organized. The degree of organization in the dislocation structures depends on the characteristics of dislocation conditions in earlier fatigue life. In the bulk center of the grains, vein and channel structures with a channel width between 1 and 2\(\mu m\) and a vein thickness of about 0.5\(\mu m\) are often observed (Fig.3.53a). Closer to the grain...
3.4. Microscopic analysis on the evolution of dislocation structures during LCF

Figure 3.52. Microstructural conditions in low cycle fatigue AX-LCF-RT-070 at the peak of hardening response stage (test 3 - $N_h=22$): (a) dense and tangled dislocation structure close to grain boundaries, (b) vein embryos, (c) a zigzag dislocation line.
boundaries, well organized wall and channel structures with a channel width of about 0.5\(\mu m\) form (Fig.3.53b). Persistent slip bands are also frequently observed, both in vein and channel structures (Fig.3.53c) and in the denser wall and channel structures (Fig.3.53d).

Upon further cycling, the volume fraction of persistent slip bands is expected to increase, which can lead to a transformation from the wall and channel structure into a structure consisting of labyrinth structures or dislocation cells, similar to the observations of Mughrabi (1978) for copper.

The dislocation structures at the end of fatigue life (\(N_f=4791\)) are similar to those at the end of the softening phase. For example, some less organized dislocation configurations (e.g. the vein/channel structure) (Fig.3.54a, 3.54b) and a carpet-like structure (3.54c) are still observed. In addition, dislocation lines within channels of plate material are frequently observed in a zigzag form as well (Fig.3.55b), which can indicate strong interactions exist between dislocations and solute atoms. Dislocation structures are, however, in more well organized forms (e.g. walls, cells and labyrinths) (Fig.3.55a, 3.55c and 3.55d). Since walls consist of very dense dislocation aggregations, it is very difficult to estimate the dislocation...
3.4. Microscopic analysis on the evolution of dislocation structures during LCF

Figure 3.54. Microstructural condition in low cycle fatigue AX-LCF-RT-070 at the end of fatigue life (test 2 - $N_f=4791$): (a-c) ill-organized dislocation walls, (d) wall/channel dislocation structure.

density at this stage. However, one can observe that in the areas where well organized structures are formed, the dislocation density in channels gradually decreases with the number of cycles.

Microstructural condition during strain controlled ratcheting with a ratcheting step of +0.10% per cycle (AX-RAT-RT-070-P10)

At a ratcheting step $\xi = +0.10\%$ per cycle it takes 50 cycles to reach the maximum ratcheting strain $\varepsilon_r = 5\%$. After 22 cycles, when the mean strain reaches a value of 2.2%, the dislocation structures are different compared to the ones observed in zero mean strain low cycle fatigue: the movement of dislocations is less planar as in Fig.3.56, the dislocation density is in general higher than that of the equivalent condition during low cycle fatigue, and embryonic veins begin to form. In certain locations, dislocations are observed to be aligned along preferred directions.

After 50 cycles, when the maximum value of mean strain (equivalent to 5%) is reached, there are some distinct differences compared to after 22 cycles of ratcheting:
Figure 3.55. Microstructural condition in low cycle fatigue AX-LCF-RT-070 at the end of fatigue life (test 2 - $N_f=4791$): (a) well-organized dislocation walls, (b) zigzag dislocation lines within dislocation channels, (c) a mixture of dislocation wall/channel structure and irregular cells, (d) labyrinth structure.
3.4. Microscopic analysis on the evolution of dislocation structures during LCF

Figure 3.56. Microstructural condition after 22 cycles in the ratcheting test with the higher ratcheting step (AX-RAT-RT-070-P10) (test 5 - $N_h=22$): (a) wavy movement of dislocations showing a preferred orientation, (b) dense dislocation tangles and veins.

- Dislocation walls (or veins) are formed as in Fig.3.57, whereas only dislocation tangles and veins were seen after 22 cycles.
- The total dislocation density is considerably higher than after 22 cycles (Fig.3.57b).
- The dislocations in channels tend to align along a preferred orientation more often (Fig.3.57a).
- Persistent slip bands are observed (Fig.3.57c).

After 500 cycles, the dislocation structures are much more developed than those observed after 50 cycles. During 450 cycles of cyclic relaxation, no additional ratcheting occurs and the total strain is kept constant. Elongated dislocation walls (and sometimes dislocation cells) are observed as in Fig.3.58.

The dislocation density inside the channels is lower than that after 50 cycles (corresponding to the beginning of the cyclic relaxation phase). However, compared with the equivalent condition for low cycle fatigue after 500 cycles, the dislocation density in the channels and cells of the ratcheted microstructure is higher. This is in line with the fact that most of the additional hardening for AX-RAT-RT-070-P10 is still maintained, as can be seen in Fig.3.50.

At the end of life ($N_f = 2873$), the degree of organization of dislocation structures is more developed than the one observed in earlier life stages and much higher than the one noticed in the low cycle fatigue specimen. Cell and wall structures are present nearly everywhere and the fraction of tangled dislocation structures has diminished (Fig.3.59).

The dislocation density in the cells and channels has decreased compared with the previous condition of ratcheting (test 7 - $N_s=500$), but it is still noticeably higher than in the zero mean stress low cycle fatigue (test 2 - $N_f=4791$).

In contrast to the minor misorientation between channels in the low cycle fatigue specimens (less than 1 degree), the misorientation between contiguous channels in the ratcheted specimens reaches magnitudes up to 5 degrees, resulting in a clear contrast (Fig.3.60a and b) in the two-beam electron diffraction contract images. Dislocations in the channels (or in low density dislocation regions) of ratcheted specimens are often oriented along a preferred orientation (Fig.3.59b, Fig.3.60b and c).
Figure 3.57. Microstructural condition after 50 cycles in the ratcheting test with the higher ratcheting step (AX-RAT-RT-070-P10) (test 6 - $N_{\text{max}} = 50$): (a) preferred orientation of dislocations, (b) tangled dislocation structure and veins, (c) PSBs.
3.4. Microscopic analysis on the evolution of dislocation structures during LCF

Figure 3.58. Microstructural condition after 500 cycles in the ratcheting test with the higher ratcheting step (AX-RAT-RT-070-P10) (test 7 - \(N_s=500\)): (a) cells and high dislocation density inside cells, (b) walls and high dislocation density inside channels.

Figure 3.59. Microstructural condition at end of life in the ratcheting test with the higher ratcheting step (AX-RAT-RT-070-P10) (test 8 - \(N_f=2873\)): (a) organized cell/wall structure, (b) wall structure and preferred orientation of dislocations.
Figure 3.60. Microstructural condition at end of life in the ratcheting test with the higher ratcheting step (AX-RAT-RT-070-P10) (test 8 - $N_f=2873$): (a) well organized and elongated cell structure, (b) the higher magnification of a region in (a) shows that cells and dislocations are elongated along a preferred direction, (c) preferred orientation of dislocations in a lower density region.
3.4. Microscopic analysis on the evolution of dislocation structures during LCF

Figure 3.61. Microstructural condition at the end of life in the ratcheting test with the lower ratcheting step (AX-RAT-RT-070-P01) (test 11 - \(N_f=2773\)): (a) well organized wall/channel structure, (b) high dislocation density inside cells and channels.

Microstructural condition during strain controlled ratcheting with a ratcheting step of +0.01% per cycle (AX-RAT-RT-070-P01)

The microstructural analysis of the ratcheting experiments run at a lower ratcheting step reaffirms the observations previously reported. The dislocation structures that develop during the ratcheting phase are similar though less clear than for the higher ratcheting step experiments. However, the dislocation density remaining in the channels of the structures at the end of life is also higher than observed in the low cycle fatigue specimens, as one can conclude from a comparison of Fig.3.55, 3.59 and 3.61. This observation is consistent with the left-over hardening observed in ratcheting tests.

3.4.5. Discussion

Strain controlled ratcheting induces a significant increase in dislocation density compared to zero mean strain low cycle fatigue loading. The increase in dislocation density is observed initially during the first few cycles, and after more cycles mostly in the channels of the otherwise typical fatigue dislocation structures. As dislocation motion in these channels is the main mechanism activated during cyclic deformation, the increased density in the channels can be considered responsible for the additional hardening observed during the ratcheting phase of the experiments as shown in Fig.3.50. An increased dislocation density in the channels has been observed before by Song (2009), who puts forward the hypothesis that the higher dislocation density observed in the channels for ratcheted specimens is due to the slower condensation of dislocations into walls and cells because of the increase of dislocation mobility promoted by uniaxial ratcheting. However, the quick development of dislocation substructures noticed in the current study does not confirm the hypothesis of Song.

Another effect of the mean strain accumulation observed in ratcheted specimens, is the less planar character of the dislocation movement compared with zero mean strain low cycle fatigue. Analogous to what Gaudin and Feaugas (2004) reported for stress controlled tests, the increment of mean strain promotes the activation of multiple slip systems. In addition, compared to the zero mean strain low cycle fatigue response, the drifting mean strain in the ratcheting experiments imposes an additional strain to be carried by the microstructure. As
a consequence one finds that the fatigue related dislocation structures develop more quickly and occupy a larger fraction of the microstructure in the ratched specimens. This is only apparently in disagreement with the work of Kang et al. (2010) who observed a quicker dislocation structure organization in zero mean strain low cycle fatigue tests. However only apparent, as Kang et al. (2010) observed, in the stress controlled ratcheting tests, because of cyclic hardening, the resultant strain amplitude is lower than the one prescribed in the strain controlled low cycle fatigue experiments.

Once the maximum level of mean strain is reached, the dislocation density in the channels does not further increase. Similar to what has been observed in low cycle fatigue (Mayama et al., 2008; Pham et al., 2011), dislocations are found to condense into walls and cells and their density in the channels progressively reduces. However, it is noticed that in ratched specimens a higher dislocation density persists through to the end of life. The higher remaining number of dislocations in the channels are potentially the reason for the remaining hardening in comparison to the zero mean strain low cycle fatigue response.

The TEM observations on ratched specimens reveal the presence of polarized dislocation structures. As shown in Fig.3.60a and 3.60b, a relatively strong misorientation between the crystallographic orientation of the channels is measured. A preferential alignment of dislocations in channels on specific glide planes was often observed (Fig.3.57a, 3.59b,3.60b and 3.60c). Gaudin and Feaugas (2004) suggest that the observation of a relatively strong crystallographic misorientation between the channels, and the presence of a preferred orientation of dislocation lines in the channels, indicate the polarization of dislocation structures during ratcheting:

Sample loading for mean stress higher than 50 MPa, presented only polarized structures . . . thus, (for lower mean stress levels) the competition between dipolar walls (typical of symmetric cyclic test) and polarized walls (typical of tensile loading) in the same grain is obvious.

In our experiments, after reaching the maximum mean strain, the dislocation structure relaxes and the dislocation density in the channels decreases. However, a strong misorientation between channels and the presence of preferentially orientated dislocations at the end of fatigue life for ratcheting tests indicate that part of the dislocation polarization remains. This together with the additionally increased dislocation density is at least in part responsible for the additional cyclic hardening during ratcheting. The transmission electron microscopy observations show that ratcheting does not impede the formation of persistent slip bands. On the contrary, ratcheting even seems to induce more frequent persistent slip band activity during ratcheting. Although the images do not provide a significant statistical basis concerning the activity of persistent slip bands, their more frequent observation is potentially a reason why ratcheting induces a small reduction in fatigue life. As a final remark, the fact that nearly no strain-induced α′-martensite is measured in fatigued 316L plate specimens (see section 3.5), suggests that is reasonable to exclude the existence of a relation between cycling hardening and the phase transformation.

3.4.6. Conclusions

Comparing the behavior of stainless steel grade 316L in low cycle fatigue experiments and in strain controlled ratcheting tests, it is concluded that the observed deformation response of the steel is clearly influenced by strain controlled ratcheting. Not only does strain controlled ratcheting lead to an increased mean stress, an additional cyclic hardening is observed, from which the material cannot completely recover if cyclic relaxation follows the ratcheting. For the investigated loading conditions only a small reduction of the number of cycles to failure is observed.
Multiple experiments interrupted at specific cycle numbers corresponding with the different cyclic deformation response stages have been submitted to a detailed analysis of the microstructure evolution using transmission electron microscopy. In these analyses, differences between the specimens subjected to zero mean strain low cycle fatigue and the ones loaded in strain controlled ratcheting are observed. Ratcheted specimens exhibit increased dislocation densities mostly in the channels and a higher fraction of fatigue dislocation structures. In the ratcheted specimens the increased misorientation between contiguous channels and the presence of preferred orientations of dislocations can be considered indicators of increasingly polarized dislocation structures. The polarization of dislocation structures is potentially a significant mechanism involved in the increase of the mean stress measured in the experiments.
3.5 Magnetic measurements: strain-induced $\alpha'$-martensite formation

A substantial amount of studies treat strain-induced $\alpha'$-martensite formation in 316L steels subjected to cyclic loading but the presented results and their interpretation are often controversial (Leber et al., 2007; Jeon et al., 2008; Man et al., 2011). In fact, while Man et al. (2011) notices nearly no martensite formation in specimens fatigued at room temperature, this is not the case for Jeon et al. (2008) reporting an increment of $\alpha'$-martensite up to 6-7 %. The relation between the strain-induced $\alpha'$-martensite formation and cyclic deformation is investigated for two batches of 316L grade stainless steel characterized in section 3.1.2. Room temperature LCF and strain-controlled ratcheting tests are conducted at different strain amplitude levels until end of fatigue life. The amount of strain-induced $\alpha'$-martensite is determined by means of susceptibility measurements on the fatigued samples. The relation between martensite content and cyclic hardening behavior presented in section 3.1 is discussed comparing the experimental results with the work presented in two selected literature references.

3.5.1. Experimental setup

Testing equipment and samples preparation

The testing equipment and samples preparation are described in sections 2.1.1 and 2.3.

Loading paths

The LCF and ratcheting tests considered in this analysis are performed at room temperature imposing the loading conditions already presented in section 3.1.2.

Test designation

The test designation is the same of the one adopted in section 3.1.2.

3.5.2. Experimental results

In the current study, the amount of $\alpha'$-martensite is measured on samples in the as received conditions and on specimens subjected to LCF and ratcheting loading until the end of life. As observed in 'GRETE' project (Niffenegger et al., 2003), the measured scattering can be caused by material inhomogeneities and by the presence of macro cracks. However, in the current study the small amount of scattering in the data do not require a more detailed investigation on the effect of cracks.

In Fig.3.62 the measured volume fraction of $\alpha'$-martensite for the pipe and the plate material batch is reported as a function of the imposed strain amplitude. In pipe material, cycling loading is observed to originate a small but not negligible (always < 1.5 %) amount of strain-induced $\alpha'$-martensite. Considering the experiments carried out with similar strain amplitude (i.e. 0.40%), the results are quantitatively in agreement with a previous study carried out at PSI (Niffenegger et al., 2008) on the same batch. In addition, similarly to what observed by Jeon et al. (2008) (see left hand side plot in Fig.3.63), the increase of strain amplitude enhances the phase-transformation rate. In any of the considered loading cases...
3.5. Magnetic measurements: strain-induced $\alpha'$-martensite formation

The discrepancy between measurements of strain-induced $\alpha'$-martensite performed in the current study and the ones reported in the work of Jeon et al. (2008) and of Man et al. (2011) can be explained analyzing the differences in the chemical composition (see Tab.3.1) and relating them with the stability of the austenitic phase. An empirical approximation of the stability of the austenitic phase is available representing the four investigated steel batches in the Schaeffler diagram (see Fig.3.64). The Schaeffler diagram provides information on kind of microstructural phases present in an iron alloy, as a function of the alloying elements.

The data corresponding to the pipe material batch can be reorganized representing the measured volume fraction of $\alpha'$-martensite as a function of the experiment typology (see right hand side plot of Fig.3.63). In this graph no systematic formation of additional $\alpha'$-martensite is noticed in ratcheting tests and therefore it is concluded that the drifting of mean strain (i.e. ratcheting) does not play an evident role in the phase transformation. It is not meaningful to perform the same analysis on specimens belonging to the plate material batch because the measured susceptibility is extremely small and can be hardly detected by the instrument.

3.5.3. Discussion

The discrepancy between measurements of strain-induced $\alpha'$-martensite performed in the current study and the ones reported in the work of Jeon et al. (2008) and of Man et al. (2011) can be explained analyzing the differences in the chemical composition (see Tab.3.1) and relating them with the stability of the austenitic phase. An empirical approximation of the stability of the austenitic phase is available representing the four investigated steel batches in the Schaeffler diagram (see Fig.3.64). The Schaeffler diagram provides information on kind of microstructural phases present in an iron alloy, as a function of the alloying elements.
Chapter 3. Experimental characterization of 316L

Figure 3.64. Position of the investigated austenitic steel grades in the Schaeffler diagram.

Table 3.7. Estimation of Ni\textsubscript{eq} and Cr\textsubscript{eq} for the 316L grades investigated in the current study and in two literature references.

<table>
<thead>
<tr>
<th>Material</th>
<th>Ni\textsubscript{eq}</th>
<th>Cr\textsubscript{eq}</th>
</tr>
</thead>
<tbody>
<tr>
<td>TP316L pipe</td>
<td>14.9</td>
<td>19.1</td>
</tr>
<tr>
<td>316L plate</td>
<td>16.6</td>
<td>20.7</td>
</tr>
<tr>
<td>316L (Jeon et al., 2008)</td>
<td>10.8</td>
<td>20.0</td>
</tr>
<tr>
<td>316L (Man et al., 2011)</td>
<td>18.9</td>
<td>20.8</td>
</tr>
</tbody>
</table>

The equivalent Chromium Cr\textsubscript{eq} is calculated using the weight percentage of ferrite stabilising elements and the equivalent Nickel Ni\textsubscript{eq} using the weight percentage of austenite stabilising elements.

\[
\text{Cr}^{\text{eq}}[\%] = 1.5 \cdot \text{Si} + \text{Cr} + \text{Mo} + 2 \cdot \text{Ti} + 0.5 \cdot \text{Nb} \quad (3.7)
\]

\[
\text{Ni}^{\text{eq}}[\%] = 30 \cdot \text{C} + 0.5 \cdot \text{Mn} + \text{Ni} \quad (3.8)
\]

The knowledge of the concentration of most of those chemical components allows a rough estimation of Ni\textsubscript{eq} and Cr\textsubscript{eq} not only for the pipe and plate material batches but also for the 316L grades investigated by Jeon et al. (2008) and Man et al. (2011) (see Tab.3.7). Analyzing the position of the 4 steel grades into the Schaeffler diagram (see Fig.3.64) it is evident that the closer is the selected material to the martensite+austenite region, the higher is the \(\alpha^{\prime}\)-martensite transformation rate measured in the experiments. As reported by Leber et al. (2007), the stability of the austenitic phase is promoted by a high value of the stacking fault energy that is strongly related with the Ni\textsubscript{eq} content:

\text{higher stacking fault energy values may allow the required cross slip for a cellular arrangement of the dislocations, whereas the lower values favor a planar arrangement. In the latter case cross slip becomes more difficult, which is a requirement for the strain-induced martensitic deformation.}

Another controversial aspect is the relation between the martensite formation and the cyclic hardening behavior (extensively described in section 3.1). As depicted in Fig.3.4, when
subjected to cyclic loading, 316L shows primary hardening followed by softening and then stabilization. Analyzing the experiment with the highest strain amplitude of 1.00%, one also notices a secondary hardening that is more pronounced in the case of plate material. Qualitatively similar results are observed by Jeon et al. (2008) and Man et al. (2011). As reported in section 3.4, it is nowadays well accepted that the first hardening stage is due to the increase of dislocation density and that the following softening stage is linked with the reorganization of dislocations in substructures characterized by lower energy. However, the reason for the secondary hardening is still uncertain. While Jeon et al. (2008) associates the secondary hardening with the generation of martensite, this hypothesis is clearly excluded by Man et al. (2011) attributing it to the formation of specific dislocations structures involving activation of secondary slip systems. Leber et al. (2007) concludes that because small variations of martensite of the order of 0.1% have a critical effect on the hardening stress, the role of martensite as the sole hardening source affecting the dislocation mobility is proposed. Surprisingly, comparing experiments performed with similar strain amplitude levels (see Fig.3.4 and 3.65), the secondary hardening stage is found to be more pronounced in 316L grades characterized by higher austenitic phase stability. This finding seems to confirm the thesis of Man et al. (2011). Another evidence that leads the author to prefer to associate the cyclic hardening behavior with the organization of dislocations in particular structures comes from the analysis of ratcheted specimens. In fact, while it is possible to find a reasonable relation between the ratcheting-induced hardening and the formation of particular dislocation structures (see section 3.4), no additional amounts of $\alpha'$-martensite are measured on ratcheted specimens in the current study. As a final remark, it must be pointed out that a more detailed study should be carried out in order to exclude that the room temperature cyclic response of 316L is related with the formation of martensite. In fact, the technique adopted in the current study cannot detect the eventual formation of $\varepsilon$-martensite since this phase is paramagnetic.

### 3.5.4. Conclusions

A set of room temperature uniaxial LCF and strain-controlled ratcheting experiments has been carried out on two different batches of 316L until the end of life. The amount of strain-induced $\alpha'$-martensite is determined by means of susceptibility measurements on the fatigued samples. The differences noticed in the austenitic phase stability for the investigated 316L grades is found to be strongly related with the chemical composition and in particular with the equivalent Nickel $N_{eq}$ content. In fact, considering the Schaeffler diagram, the austenitic steels closer to the martensite+austenite region show a higher $\alpha'$-martensite transformation.
rate. In the current study it is not possible to find a relation between the formation of martensite and the secondary hardening noticed in LCF experiments performed with higher strain amplitudes. In fact, the secondary hardening stage is observed to be generally more pronounced in 316L grades characterized by higher austenitic phase stability. In addition, the fact that no additional amounts of $\alpha'$-martensite are measured in ratcheting experiments, suggest that is more reasonable to relate cycling hardening with the formation of specific dislocation structures.
As reported in section 1.2, several constitutive models consisting in appropriate combinations of isotropic and kinematic hardening laws have been proposed in the last decades to describe the cyclic behavior of materials. An alternative approach characterized by reduced complexity is proposed in this chapter to provide an accurate material description of the stress-strain relation for stainless steels subjected to cyclic loading and ratcheting. In section 4.1, this innovative constitutive model is formulated and the procedures responsible for the material parameters calibration and the internal variables updating are presented. The constitutive model has been coupled with an advanced multiaxial damage criterion described in section 4.2. Finally, the evaluation of the descriptive and predictive capability of the material description and of the damage model under several loading conditions is reported in section 4.3.

4.1 Constitutive model

Several elasto- and visco-plastic constitutive models have been formulated in the last decades in the framework of the generalized plasticity theory to describe the stress-strain relation for ductile materials. As a first task, a discussion on the reasons that lead the author of the current work to implement a time-independent material description is proposed. To allow an easier understanding of this section, the fundamentals concerning the modeling of elasto-plasticity are then reported. The constitutive model is then formulated and the calibration methodology is described in detail. Finally, an innovative procedure responsible to trace the plastic strain tensor and to return the updated value of the internal variables is presented.

4.1.1. Elasto-plastic or visco-plastic model?

The experimental results summarized in section 3.3, reporting that austenitic stainless steels show a strong loading rate-dependent material response when subjected to stress-controlled loading conditions, seem to suggest that the implementation of a time-dependent constitutive model is the most desirable approach to describe the mechanical behavior of
AISI 316L. While the calibration of the visco-plastic version of the original Chaboche model (Chaboche, 1986) is a relatively straightforward task, this is not the case for more recent Chaboche-type constitutive laws (Kang and Gao, 2002; Döring et al., 2003; Yang, 2004; Takahashi et al., 2008; Hassan et al., 2008). In fact, in these internal variable dependent models, increasing the number of parameters, the strong coupling existing between the hardening and the cyclic accumulation of inelastic strain makes their calibration an extremely difficult task. The calibration of a visco-plastic version of the 5 internal variables dependent model proposed in the current manuscript would require a huge amount of work, going beyond what one can do within the limited time-frame of a single PhD. In addition, the intrinsic complexity of the time-dependent version of the formulated material description together with a significant increase of the cpu-time would reduce the effectiveness of its application in an industrial context.

These reasons induced the author of the current work to opt for a time-independent constitutive model, suggesting to dedicate a future PhD project to adress the aspects related to the observed rate dependency. It must be pointed out that, while the lack of performance of such an elasto-plastic approach would lead to totally wrong results for the simulation of parts subjected to creep, its accuracy could be anyway acceptable considering ratcheting conditions. In fact, as reported in section 3.3, the comparison of stress-controlled ratcheting tests performed with different stress rates shows that the first cycles are the main responsible for the measured difference in the cyclic accumulation of inelastic strain and that after further cycling the ratcheting rates becomes practically equivalent. An additional confirmation of the effectiveness of the selected approach is available observing that under strain-controlled ratcheting conditions, mimicking the ones occurring in the considered industrial components, the influence of the loading rate on the cyclic behavior of 316L becomes considerably weaker.

4.1.2. Basic notions on generalized plasticity theory

In Fig.4.1 the idealized stress-strain behavior of a ductile material undergoing a uniaxial tensile test is reported. In the first loading trait the material behaves elastically meaning that an increment of strain $\Delta \varepsilon$ causes a proportional increment of stress $\Delta \sigma$ where the proportionality constant is defined as the Elastic modulus $E$. When the stress reaches a certain threshold limit $\sigma_y$ (named 'yield stress'), plasticity occurs. If a further increment of strain induces an increment of stress compared to the perfect plastic behavior, the material is said to harden. When the loading is reversed, the material stops to deform plastically and shows a linear strain recovery with a slope $\sigma - \varepsilon$ equivalent to $E$. Once the stress reaches the zero value, the strain that has not been recovered is the plastic strain $\varepsilon^{pl}$ and the recovered one is the elastic strain $\varepsilon^{el}$. The total strain can be therefore decomposed in the two strain components as follows:

$$\varepsilon = \varepsilon^{el} + \varepsilon^{pl}$$

with

$$\varepsilon^{el} = \frac{\sigma}{E}$$

Since stresses and strains take place in three dimensions, it is convenient to express them by means of second order tensors:

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$
4.1. Constitutive model

Figure 4.1. Strain decomposition into elastic and plastic parts (Dunne and Petrinic, 2005).

\[
\mathbf{\varepsilon} = \begin{bmatrix}
\varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\
\varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\
\varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33}
\end{bmatrix}
\] (4.4)

The plastic deformation is generally assumed to satisfy the incompressibility condition and for that reason the sum of the diagonal components of the plastic strain rate is necessarily equal to zero:

\[
\dot{\varepsilon}_{p}^{11} + \dot{\varepsilon}_{p}^{22} + \dot{\varepsilon}_{p}^{33} = 0
\] (4.5)

Another important concept widely used in plasticity is the so called 'yield function’. Once a stress field is defined, the yield function \( \mathcal{F}_y \) determines if the material response is completely elastic or if plasticity occurs. For isotropic ductile materials, a good approximation of the yield condition is provided by the following equation:

\[
\mathcal{F}_y = \mathcal{J}_2(s - \alpha) - Y \leq 0
\] (4.6)

where \( Y \) is the size of the yield surface, \( \mathcal{J}_2 \) is the second invariant of a generic second order tensor, \( s \) is the deviatoric stress tensor and \( \alpha \) is the backstress tensor (as defined later).

\[
\mathcal{J}_2(s - \alpha) = \sqrt{\frac{3}{2}}(s - \alpha) : (s - \alpha)
\] (4.7)

\[
s = \text{dev}(\mathbf{\sigma}) = \mathbf{\sigma} - \frac{1}{3}tr(\mathbf{\sigma})
\] (4.8)

In the majority of the constitutive models, when plasticity takes place, the increment in the plastic strain tensor is assumed to occur in the direction normal to the tangent to the yield surface. This hypothesis is named 'normality rule' and can be expressed as follows:

\[
\dot{\varepsilon}^{pl} = \dot{\lambda} \frac{\partial \mathcal{F}_y}{\partial \mathbf{\sigma}} = \dot{\lambda} \cdot \mathbf{N}
\] (4.9)

where \( \mathbf{N} \) is the flow tensor defining the direction of the plastic strain increment and \( \dot{\lambda} \) is the plastic multiplier defining its magnitude.
In any case the yield condition can be violated (i.e. $F_y > 0$) and the condition guaranteeing that this rule is respected is the so called ‘consistency condition’: 

$$\frac{\partial F_y}{\partial \sigma} : d\sigma = 0 \quad (4.10)$$

As previously reported, when a plastic deformation occurs, materials are generally found to harden. The size of the yield surface $Y$ is therefore not constant and its expansion (or contraction) in all the directions of the stress space consists in the isotropic hardening (see Fig.4.2) that is described by the hardening law $R$. 

$$Y = \sigma_{y0} + R \quad (4.11)$$

For the monotonic loading case, it is generally reasonable representing the total hardening by means of an isotropic hardening law. However, the Bauschinger effect taking place for cyclic loading can not be described by changing the yield surface size but only allowing its translation. The hardening law defining the translation of the center of the yield surface is the so called ‘kinematic hardening’ (see Fig.4.3) and the coordinates of the center are represented by the backstress tensor $\alpha$. 

---

**Figure 4.2.** Isotropic hardening (Dunne and Petrinic, 2005).

**Figure 4.3.** Kinematic hardening (Dunne and Petrinic, 2005).
4.1. Constitutive model

4.1.3. Constitutive law formulation

The constitutive elasto-plastic model proposed in this dissertation consists in a superposition of kinematic and isotropic hardening laws in the framework of the generalized plasticity theory (e.g. normality rule, yield condition, etc.) including temperature variations.

\[ \mathbf{\varepsilon} = \mathbf{\varepsilon}^{el} + \mathbf{\varepsilon}^{pl} + \mathbf{\varepsilon}^{th} \]  (4.12)

The thermal strain tensor is defined as:

\[ \mathbf{\varepsilon}^{th} = \kappa \cdot (T - T_{ref}) \cdot \mathbf{1} \]  (4.13)

where \( \mathbf{1} \) is the second order identity tensor and \( \kappa \) is the mean (or secant) linear thermal expansion coefficient. The coefficient \( \kappa \) is dependent on the actual temperature \( T \) and on the reference temperature \( T_{ref} \) and can be retrieved by means of Eq.4.14 following the procedure explained in detail by Niffenegger and Reichlin (2012).

\[ \kappa = \kappa(T, T_{ref}) = \frac{L_T - L_{T_{ref}}}{L_{T_{ref}}(T - T_{ref})} \]  (4.14)

As reported in section 1.2, the possibility to qualitatively describe the hysteresis loop shape and ratcheting is given by non linear kinematic hardening laws like the one proposed by Armstrong-Frederick Armstrong and Frederick (1966):

\[ \dot{\alpha} = \frac{2}{3} C \dot{\varepsilon}^{pl} - \gamma \alpha \dot{p} \]  (4.15)
\[ \dot{p} = \sqrt{\frac{2}{3} \varepsilon^{pl} : \dot{\varepsilon}^{pl}} \]  (4.16)

An acceptable accuracy in the reproduction of the cyclic material behavior is achievable only superposing several non-linear kinematic hardening components (e.g. 3 components for the Chaboche model (Chaboche, 1986)).

\[ \alpha = \sum_{k=1}^{3} \alpha^{(k)} \]  (4.17)
\[ \dot{\alpha}^{(k)} = \frac{2}{3} C^{(k)} \dot{\varepsilon}^{pl} - \gamma^{(k)} \alpha^{(k)} \dot{p} \]  (4.18)

In the original formulation of the Chaboche model the kinematic hardening parameters \( C^{(k)} \) and \( \gamma^{(k)} \) are constants calibrated on the stabilized behavior of the material and the cyclic hardening is modeled by means of an exponentially saturating isotropic hardening law:

\[ Y = \sigma_{y0} + R = \sigma_{y0} + Q(1 - e^{-bp}) \]  (4.19)

For stainless steels subjected to complex loading conditions (LCF including strain amplitude variation, temperature change and ratcheting), the accuracy of calculations provided by the original Chaboche model is often not sufficient. As reported in section 1.2, a promising approach to improve the performance of a Chaboche-type model consists in allowing both the isotropic and kinematic hardening parameters to vary with cycling. The first example of an elasto-plastic constitutive model with an internal variable dependency can be found in the
Table 4.1. Summary of the internal variables used in the 5DChabEP model.

<table>
<thead>
<tr>
<th>internal variable</th>
<th>symbol</th>
<th>features described</th>
</tr>
</thead>
<tbody>
<tr>
<td>accumulated equivalent plastic strain</td>
<td>$p$</td>
<td>cyclic hardening/softening</td>
</tr>
<tr>
<td>equivalent plastic strain amplitude</td>
<td>$\varepsilon_{amp}$</td>
<td>strain amplitude dependency</td>
</tr>
<tr>
<td>equivalent mean plastic strain</td>
<td>$\varepsilon_{mean}$</td>
<td>ratcheting-induced hardening</td>
</tr>
<tr>
<td>equivalent ratcheting rate</td>
<td>$\xi$</td>
<td>ratcheting-induced hardening</td>
</tr>
<tr>
<td>temperature</td>
<td>$T$</td>
<td>temperature dependency</td>
</tr>
</tbody>
</table>

endochronic theory of plasticity proposed by Valanis (1978), which is based on the hereditary form of thermodynamics of irreversible processes. This approach was further refined by more recently developed, advanced constitutive models (Kang and Gao, 2002; Döring et al., 2003; Yang, 2004; Takahashi et al., 2008; Hassan et al., 2008), consisting of modifications of the original formulations of Chaboche and Ohno in which different internal variable dependencies are directly introduced in the hardening laws by means of analytical relations. However, increasing the number of parameters, the strong coupling existing between the loop shape and the cyclic hardening response makes the calibration of such models a difficult task, limiting de facto their use in an industrial context.

An alternative approach allows to simplify the calibration of this class of models, keeping at the same time the internal variables dependency. This solution consists in maintaining the original Chaboche formulation and in evaluating, at the beginning of each integration step, the updated values of the model’s parameters using a multi-dimensional interpolation. This ‘evolutionary approach’ was evaluated at PSI using a 2-component Chaboche material description using a single internal variable (Janssens, 2011) and is further enhanced in the current work with the development of a much more detailed 5 internal variables dependent model named ‘5DChabEP’.

The 5 internal variables selected in order to reproduce the dependency on temperature, strain amplitude and ratcheting are reported in Tab.4.1.

The experimental results reported in section 3.1 allowing to simplify as much as possible the constitutive model are summarized below:

1. the elastic modulus $E$ is supposed to vary only as a function of the accumulated plastic strain $p$ and temperature $T$. For simplicity, the influence of plastic strain amplitude on $E$ is neglected since it is easy to demonstrate that this dependency has no significant influence on the stress response computed by the constitutive model. Ratcheting is found to have no influence on $E$.

2. The isotropic hardening parameter defining the yield surface size $Y$ is calculated adopting the lower threshold value (0.0025%). In this way the isotropic hardening is found to be dependent only on the accumulated plastic strain $p$ and temperature $T$.

3. The cycling hardening cannot be modeled by means of the isotropic hardening law but only by means of kinematic hardening. Cycling hardening is dependent on plastic strain amplitude $e_{amp}^{pl}$, accumulated plastic strain $p$ and temperature $T$.

4. The material response in ratcheting tests is considered to be a linear superposition of cyclic hardening and ratcheting-induced hardening. The ratcheting-induced hardening and mean stress drifting cannot be modeled by means of the isotropic hardening law but only by means of kinematic hardening. These features are dependent on mean plastic strain $\varepsilon_{mean}^{pl}$, ratcheting rate $\xi$, plastic strain amplitude $\varepsilon_{amp}^{pl}$ and temperature $T$.

These assumptions lead to a sensible reduction of the number of dependencies for $Y$ and $E$. 

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\[ C^{(k)}, \gamma^{(k)} = f(p, \varepsilon_{pl}^{mean}, \varepsilon_{ampl}, \xi, T) \]  
\[ Y, E = f(p, T) \]

The parameters \( E, Y \), \( C^{(k)} \) and \( \gamma^{(k)} \) are fitted to the experimental data and collected in a table as a function of the 5 internal variables by means of a calibration procedure (see section 4.1.4). A subroutine has been implemented in ABAQUS (Abaqus, 2012) to analyze the previous loading history (plastic strain tensor) and to update the values of the internal variables at the beginning of each integration step (see section 4.1.5). Finally, the updated internal variables are used as input of a multi-dimensional interpolation procedure to extrapolate the updated values of the hardening parameters \( E, Y \), \( C^{(k)} \) and \( \gamma^{(k)} \) and to evaluate the stress response at the end of the increment.

As pointed out by several authors (Chaboche, 1977; Walker, 1981; Hartmann, 1990; Ohno and Wang, 1992), a further modification of the Eq.4.18 is required to ensure a sufficient accuracy of calculations for applications involving thermal loading. In order to take into account the effect of the variation of the temperature on the computed stress response, (Chaboche, 1977) added a third term to Eq.4.18 leading to:

\[ \dot{\alpha}^{(k)} = \frac{2}{3} C^{(k)} \dot{\varepsilon}^{pl} - \gamma^{(k)} \alpha^{(k)} \dot{p} + \frac{1}{C^{(k)}} \frac{\partial C^{(k)}}{\partial T} \alpha^{(k)} \dot{T} \]  

A simplified version of the derivation of Eq.4.22 is reported by Chaboche (2008), starting from the definition of the state potential (Helmholz free energy) in a consistent thermodynamic framework

\[ \psi = \psi^{el}(\varepsilon^{el}, T) + \psi^{pl}(\omega, T) \]

where \( \omega \) is a truly independent 'strain like' state variable.

The derivative of the Helmholz free energy with respect to elastic strain returns the stress tensor (Hooke’s law).

\[ \sigma = \frac{\partial \psi}{\partial \varepsilon^{el}} \]

Considering for simplicity a single component Chaboche-type law, the energy \( \psi^{pl} \) stored in material by kinematic hardening can be expressed, according to Chaboche (2008), as a function of \( \omega \) using the following quadratic form.

\[ \psi^{pl}(\omega, T) = \frac{1}{3} C(T) \omega \cdot \omega \]

The corresponding backstress is obtained from the derivative of the free energy with respect to the variable \( \omega \).

\[ \alpha = \frac{\partial \psi}{\partial \omega} = \frac{2}{3} C(T) \omega \]

Since \( \omega \) is a truly independent variable, the derivation of \( \alpha \) with respect to the time leads to:

\[ \dot{\alpha} = \frac{2}{3} C \dot{\omega} + \frac{2}{3} \frac{\partial C}{\partial T} \omega \dot{T} \]  

Substituting \( \omega \) computed using Eq.4.26 into Eq.4.27 the resulting equation is:

\[ \dot{\alpha} = \frac{2}{3} C \dot{\omega} + \frac{1}{C} \frac{\partial C}{\partial T} \alpha \dot{T} \]

According to Chaboche (1993), the relationship between \( \dot{\omega} \) and the time derivatives of the other 'strain like' variables is given by

\[ \dot{\omega} = \dot{\varepsilon}^{pl} - \gamma \dot{p} \]
leading to

\[ \dot{\omega} = \dot{\varepsilon}^{pl} - \frac{3}{2} \gamma C \dot{\alpha} \hat{p} \]  

(4.30)

Substituting Eq.4.28 in Eq.4.30 we obtain

\[ \dot{\alpha} = \frac{2}{3} C \dot{\varepsilon}^{pl} - \gamma \alpha \dot{\alpha} \hat{p} + \frac{1}{C} \frac{\partial C}{\partial T} \dot{\alpha} \dot{T} \]  

(4.31)

that for can be easily extended to Eq.4.22 when a multiple component Chaboche-type model is considered.

Since, in the proposed constitutive model formulation, the parameters are not dependent exclusively on \( T \), Eq.4.22 was further modified in order to consider the effect of the rate of change of \( C^{(k)} \) also with respect to the other internal variables. Calling \( x_j \) the generic internal variable, Eq.4.22 can be extended as follows:

\[ \dot{\alpha}^{(k)} = \frac{2}{3} C^{(k)} \dot{\varepsilon}^{pl} - \gamma^{(k)} \alpha^{(k)} \dot{\alpha} \hat{p} + \sum_j \frac{1}{C^{(k)}} \frac{\partial C^{(k)}}{\partial x_j} \alpha^{(k)} \dot{x}_j \]  

(4.32)

It must be pointed out that, in the current work, the effect of the rate of change of \( \gamma^{(k)} \) with respect to the internal variables is not taken into account in the evolution law for \( \alpha^{(k)} \). While this dependency is commonly considered to be small and therefore neglected (Chaboche, 2008; Abaqus, 2012), the study presented by Kühner et al. (2000) and by Kühner (2000) seems to suggest that it is not the case for ratcheting conditions. This observation led Kühner to propose a further modification of Eq.4.22 including the temperature rate dependency also on \( \gamma^{(k)} \):

\[ \dot{\alpha}^{(k)} = \frac{2}{3} C^{(k)} \dot{\varepsilon}^{pl} - \gamma^{(k)} \alpha^{(k)} \dot{\alpha} \hat{p} + \frac{1}{C^{(k)}} \frac{\partial C^{(k)}}{\partial T} \alpha^{(k)} \dot{T} + \frac{\partial \gamma^{(k)}}{\partial T} \alpha^{(k)} \dot{T} \]  

(4.33)

Completing the material description considering the effect of the rate of change of \( \gamma^{(k)} \) with respect to the internal variables requires a substantial amount of additional work, which is suggested as future research project.

The implementation of the proposed constitutive law in the commercial finite element code ABAQUS is straightforward since the Chaboche model is already available (i.e. combined kinematic/isotropic hardening) and the dependency of parameters on the internal variables can be easily provided as input. The necessity to have a flexible code allowing future modifications led the author of this dissertation to implement the same constitutive law also by means of User MATerial subroutine. A detailed description of the implementation of the UMAT subroutine is reported in Appendix B. The major issues linked with the implementation of the user defined material model in a FE is discussed in Appendix C.

4.1.4. Parameters calibration

A calibration toolkit is developed in MATLAB (MATLAB, 2012) with the aim of analyzing the uniaxial LCF and strain-controlled ratcheting tests presented in section 3.1 and providing a set of calibrated parameters suitable for the constitutive model formulated in section 4.1.3. The list of the 30 experiment typologies required to calibrate the constitutive law is summarized in Tab.3.2. The complete list of tests has been performed only for the plate material batch as reported in Appendix A.

In order to correctly feed the calibration procedure, the experimental data must respect the following conditions:
4.1. Constitutive model

Figure 4.4. Flowchart illustrating the logical structure of the calibration procedure.

- reporting information about the cycle number, engineering stress and strain (the information about temperature is optional but it is useful to check the correctness of testing conditions),
- having a temporal resolution of at least 800 points per cycle (in order to guarantee an accurate and consistent evaluation of the yield stress with the selected offset criterion),
- including data for all the first 5000 cycles (and then 3 consecutive cycles each 100).

For simplicity the toolkit is subdivided in 6 sequential logical steps acting as summarized in Fig.4.4 and including:

1. evaluation of the isotropic hardening in LCF tests
2. evaluation of the monotonic hardening in a LCF test with large strain amplitude (i.e. 1.00%)
3. evaluation of the cyclic hardening in LCF tests
4. simulation of ratcheting tests with parameters calibrated on LCF tests (evaluation of the ratcheting-induced hardening)
5. evaluation of the relation between ratcheting-induced hardening and internal variables
6. creation of the calibrated set of parameters

Isotropic hardening

As previously demonstrated in section 3.1, if an appropriate yield definition is chosen (i.e. lower offset value), the isotropic hardening is not dependent on strain amplitude and ratcheting. In this first step, the toolkit analyzes data corresponding to LCF tests belonging to a certain temperature level and determines a unique reference curve describing the cyclic evolution of isotropic hardening (i.e. yield stress evolution).
Figure 4.5. (a) Definition of a stress-based defined searching interval inside which the elastic modulus is evaluated, (b) linear regression (red) of the experimental data (blue) inside the defined stress-based searching interval.

As a first task, the engineering stress and strain are computed from the measured force $F$ and elongation $\Delta L$, knowing the initial gage length $L_0$ and the initial cross-sectional area $A_0$ of the specimen.

\[ \varepsilon_{\text{eng}} = \frac{\Delta L}{L_0} \]  
\[ \sigma_{\text{eng}} = \frac{F}{A_0} \]

It is easy to demonstrate that the drifting of the mean strain occurring in ratcheting experiments causes a significant deviation of the engineering stress and strain from the corresponding true values. It can be shown that, in the strain range considered in the current dissertation ($\pm 5 - 6\%$), a sufficient approximation of the true strain is given by the logarithmic strain defined as follows:

\[ \varepsilon_{\log} = \ln(1 + \varepsilon_{\text{eng}}) \]

Under the assumption that gage volume remains constant during the deformation, one can use the following equation to compute the logarithmic stress:

\[ \sigma_{\log} = \sigma_{\text{eng}} \cdot (1 + \varepsilon_{\text{eng}}) \]

Subsequently, in order to simplify the following steps, the experimental data is split into collecting the information corresponding to each cycle in a corresponding array. Then the routine analyzes every single hysteresis loop and evaluates the compressive $E_c$ and the tensile $E_t$ elastic modulus retrieving the slope of the elastic trait of the stress-strain curve by means of a linear regression (see Fig.4.5(b)). The robustness of the procedure is guaranteed defining a stress-based searching interval (see 'b-c' trait in Fig.4.5(a)) inside which the elastic modulus is calculated. $E_c$ and $E_t$ are evaluated for each hysteresis loop and their difference is found to be negligible.

The per-cycle mean value of the elastic modulus $E_m$ is retrieved and is used to decompose the total strain in the elastic and plastic components. The knowledge of $\varepsilon^{pl}$ allows a straightforward calculation of the accumulated plastic strain $p$.

\[ \varepsilon^{el} = \frac{\sigma_{\log}}{E_m} \]  
\[ \varepsilon^{pl} = \varepsilon_{\log} - \varepsilon^{el} \]  
\[ p = \sum |\varepsilon^{pl}| \]
4.1. Constitutive model

Figure 4.6. Fitted yield stress vs. accumulated plastic strain in 316L plate material for 3 different strain amplitudes and at room temperature. The average yield stress curve $Y_{\text{ave}}^{\text{fit}}$ is also reported.

After a necessary smoothing of the stress-strain curve, $E_m$ is used to evaluate the compressive $Y_c$ and the tensile yield stress $Y_t$ for each hysteresis loop by means of the offset definition choosing a threshold equal to 0.0025% (see discussion in section 3.1). The difference between $Y_c$ and $Y_t$ is negligible and the average yield stress $Y_m$ is straightforwardly evaluated for each hysteresis loop. An approximate curve is used to fit the evolution of $Y_m$ as a function of the accumulated plastic strain evaluated at the beginning of the corresponding cycle:

$$Y_{fit}(p) = \sum_{i=1}^{n} a_i \cdot e^{-b_i \cdot p}$$

Repeating this operation for all the LCF tests performed at a certain temperature level and plotting the yield stress curves versus the accumulated plastic strain, it is easy to verify that, except for small differences, the initial assumption is fulfilled: the isotropic hardening is not strongly influenced by the imposed strain amplitude (see Fig.4.6). To improve the accuracy of the following steps of the calibration procedure, it is necessary to overcome the small differences between the yield curves evaluating a unique reference average curve $Y_{\text{ave}}^{\text{fit}}$. This procedure is repeated twice in order to evaluate the isotropic hardening reference curves ($Y_{\text{ave},250}^{\text{fit}}$ and $Y_{\text{ave},200}^{\text{fit}}$) for the two considered temperature levels.

**Monotonic hardening**

As reported in section 1.1.1, in austenitic stainless steels the monotonic curve differs considerably from the cyclic curve. Therefore in the second step of the calibration procedure, the toolkit analyzes data corresponding only to the first loading trait of the LCF test with the largest strain amplitude (i.e. 1.00%) for a certain temperature level and determines the hardening behavior under monotonic loading.

Once the logarithmic stresses and strains are retrieved and the data corresponding to the first loading are isolated, the routine utilizes the reference isotropic hardening curve $Y_{\text{ave}}^{\text{fit}}$ to determine the first yield point and to extrapolate the plastic strain-stress curve. A Levenberg–Marquardt algorithm is used to evaluate the Chaboche parameters ($C^{(k)}$ and $\gamma^{(k)}$) corresponding to the first and second backstress components that best fit the first part of the plastic strain-stress curve (just after yield). The same algorithm is used to evaluate the
hardening parameters $C^{(3)}$ and $\gamma^{(3)}$ providing a good fit of the second part of the plastic strain-stress curve (see Fig. 4.7). This procedure is repeated twice in order to evaluate the monotonic hardening parameters $(C^{(1,2,3)}_{\text{monotonic}}$ and $\gamma^{(1,2,3)}_{\text{monotonic}})$ for the two considered temperature levels.

**Cyclic hardening**

Once the monotonic and the isotropic hardening behavior have been evaluated, the toolkit analyzes data corresponding to LCF tests to determine the kinematic hardening parameters responsible for the description of the cyclic hardening. As previously reported, the cyclic evolution of isotropic hardening is not influenced by strain amplitude and for that reason the observed strain-range dependent cycling hardening must be necessarily modeled by means of kinematic hardening laws.

As a first task, logarithmic stresses and strains are retrieved and the experimental data are splitted collecting the information related to each cycle in a corresponding array. The maximum, minimum, amplitude and mean stress and strain values are then evaluated for each hysteresis loop. The compressive $E_c$, tensile $E_t$ and mean $E_m$ elastic moduli are retrieved for each hysteresis loop as previously presented. The information on the stiffness evolution allows the decomposition of the total strain in the elastic and plastic components and the calculation of the accumulated plastic strain $p$. An approximate curve is used to fit the evolution of $E_m$ as a function of the accumulated plastic strain evaluated at the beginning of the corresponding cycle:

$$E_{fit}(p) = \sum_{i=1}^{n} a_i \cdot e^{-b_i \cdot p} \quad (4.42)$$

The isotropic hardening reference curve $Y_{ave}^{fit}$ is then used to retrieve the total backstress $\alpha$:

$$\alpha = \sigma - Y_{fit}^{ave}(p) \quad if \ \dot{\varepsilon}^{pl} > 0 \quad (4.43)$$

$$\alpha = \sigma + Y_{fit}^{ave}(p) \quad if \ \dot{\varepsilon}^{pl} < 0 \quad (4.44)$$

While it is straightforward to evaluate the maximum and minimum values of the total backstress for each hysteresis loop, it is not trivial to determine the upper and lower limits for

---

*Figure 4.7. Stress vs. plastic strain in the very first loading. Data corresponding to the experiments and to the calibrated Chaboche model are reported.*
the 3 backstress components. A consistent approach consists in assuming the first backstress component to saturate (and therefore to reach its extrema) when the slope of the stress-plastic strain curve drops below a certain threshold value. The third backstress component is not supposed to saturate and its extrema are evaluated analytically knowing $\varepsilon_{pl}$ and using $C^{(3)}$ and $\gamma^{(3)}$ previously calibrated on the monotonic behavior.

$$C^{(3)} = C^{(3)}_{\text{monotonic}}$$ (4.45)

$$\gamma^{(3)} = \gamma^{(3)}_{\text{monotonic}}$$ (4.46)

$$\alpha^{(3)}_{\text{max}} = 2C^{(3)}/\gamma^{(3)} (1 - e^{-\gamma^{(3)}\varepsilon_{pl}^{\text{max}}})$$ (4.47)

$$\alpha^{(3)}_{\text{min}} = -2C^{(3)}/\gamma^{(3)} (1 + e^{\gamma^{(3)}\varepsilon_{pl}^{\text{min}}})$$ (4.48)

The extrema of the second backstress component are easily computed knowing the total backstress and upper and lower limits of $\alpha^{(1)}$ and $\alpha^{(3)}$.

$$\alpha^{(2)}_{\text{max}} = \alpha_{\text{max}} - \alpha^{(1)}_{\text{max}} - \alpha^{(3)}_{\text{max}}$$ (4.49)

$$\alpha^{(2)}_{\text{min}} = \alpha_{\text{min}} - \alpha^{(1)}_{\text{min}} - \alpha^{(3)}_{\text{min}}$$ (4.50)

Once the information about the extrema of the backstress components is available, it is possible to estimate the kinematic hardening parameters. Assuming the first backstress component to reach saturation each loading reversal, that the ratio $C^{(1)}/\gamma^{(1)}$ is found to be equivalent to $\alpha^{(1)}_{\text{extr}}$. This ratio is evaluated separately for the tensile and for the compressive trait of the stress-strain curve.

$$\alpha^{(1)}_{\text{max}} = C^{(1)}/\gamma^{(1)}_{t} = r_{t}^{(1)}$$ (4.51)

$$-\alpha^{(1)}_{\text{min}} = C^{(1)}/\gamma^{(1)}_{c} = r_{c}^{(1)}$$ (4.52)

Once $r_{c}^{(1)}$ and $r_{t}^{(1)}$ have been defined, the Levenberg–Marquardt algorithm is used to evaluate the parameters $\gamma^{(1)}_{t}$ and $\gamma^{(1)}_{c}$ that guarantee the best fit between the equations 4.53-4.54 and the first part of the tensile and compressive backstress-plastic strain curves for each hysteresis loop (see Fig.4.8).

$$r_{t}^{(1)} + (\alpha^{(1)}_{\text{min}} - r_{t}^{(1)})e^{-\gamma^{(1)}_{t}(\varepsilon_{pl}^{t} - \varepsilon_{pl}^{\text{min}})} + \alpha^{(2)}_{\text{max}} + \alpha^{(3)}_{\text{min}}$$ (4.53)

$$-r_{c}^{(1)} + (\alpha^{(1)}_{\text{max}} + r_{c}^{(1)})e^{\gamma^{(1)}_{c}(\varepsilon_{pl}^{c} - \varepsilon_{pl}^{\max})} + \alpha^{(2)}_{\text{min}} + \alpha^{(3)}_{\text{max}}$$ (4.54)

Finally the per-cycle mean values of $C^{(1)}$ and $\gamma^{(1)}$ are evaluated averaging the corresponding compressive and tensile values.

$$\gamma^{(1)} = \frac{\gamma^{(1)}_{t} + \gamma^{(1)}_{c}}{2}$$ (4.55)

$$C^{(1)} = \frac{r_{t}^{(1)} \cdot \gamma^{(1)}_{t} + r_{c}^{(1)} \cdot \gamma^{(1)}_{c}}{2}$$ (4.56)
The knowledge of $C^{(1)}$, $\gamma^{(1)}$ together with $C^{(3)}$ and $\gamma^{(3)}$ previously calibrated on the monotonic behavior allows an easy calculation of the compressive and tensile hardening parameters for the second backstress component. For this purpose, the Levenberg–Marquardt algorithm is used once again to estimate the parameters $C^{(2)}$ and $\gamma^{(2)}$ that guarantee the best fit between the equations 4.57-4.58 and the second part of the tensile and compressive backstress-plastic strain curves for each hysteresis loop (see Fig. 4.9). Similarly to what previously presented, the per-cycle mean values of $C^{(2)}$ and $\gamma^{(2)}$ are evaluated averaging the compressive and tensile parameters.

$$
C_t^{(1)} / \gamma_t^{(1)} + (\alpha_{\text{min}}^{(1)} - C_t^{(1)} / \gamma_t^{(1)}) \cdot e^{-\gamma_t^{(1)}(\varepsilon_{\text{pl}} - \varepsilon_{\text{pl,max}})} + ...
$$

$$
- C_c^{(1)} / \gamma_c^{(1)} + (\alpha_{\text{max}}^{(1)} + C_c^{(1)} / \gamma_c^{(1)}) \cdot e^{\gamma_c^{(1)}(\varepsilon_{\text{pl}} - \varepsilon_{\text{pl,max}})} + ...
$$

$$
+C_t^{(2)} / \gamma_t^{(2)} + (\alpha_{\text{min}}^{(2)} - C_t^{(2)} / \gamma_t^{(2)}) \cdot e^{-\gamma_t^{(2)}(\varepsilon_{\text{pl}} - \varepsilon_{\text{pl,min}})} + ...
$$

$$
+C_c^{(2)} / \gamma_c^{(2)} + (\alpha_{\text{max}}^{(2)} + C_c^{(2)} / \gamma_c^{(2)}) \cdot e^{\gamma_c^{(2)}(\varepsilon_{\text{pl}} - \varepsilon_{\text{pl,max}})} + ...
$$

$$
+C^{(3)} / \gamma^{(3)} + (\alpha_{\text{min}}^{(3)} - C^{(3)} / \gamma^{(3)}) \cdot e^{-\gamma^{(3)}(\varepsilon_{\text{pl}} - \varepsilon_{\text{pl,min}})}
$$

$$
- C_c^{(3)} / \gamma_c^{(3)} + (\alpha_{\text{max}}^{(3)} + C_c^{(3)} / \gamma_c^{(3)}) \cdot e^{\gamma_c^{(3)}(\varepsilon_{\text{pl}} - \varepsilon_{\text{pl,max}})}
$$

It is then convenient to express the evolution of $C^{(k)}$ and $\gamma^{(k)}$ as a function of the accumulated plastic strain by means of approximate fitting curves. An example of the performance of the fitting procedure for $C^{(2)}$ and $\gamma^{(2)}$ is reported in Fig.4.10.
4.1. Constitutive model

Figure 4.9. Calibrating \( C^{(2)} \) and \( \gamma^{(2)} \) in order to best fit the second part of the tensile and compressive backstress-plastic strain curves.

\[
C^{(1,2,3)}_{fit}(p), \gamma^{(1,2,3)}_{fit}(p) = \sum_{i=1}^{n} a_i \cdot e^{-b_i \cdot p} \tag{4.59}
\]

The same procedure is repeated for all the LCF tests and the approximate fitting curves for \( E, Y, C^{(k)} \) and \( \gamma^{(k)} \) are used to create a preliminary set of parameters suitable for the calibration of a 1 internal variable dependent Chaboche model (the dependency is on \( p \)).

For each dataset is then created a calibration file respecting a fixed tabular format (see Tab. 4.2) containing:

1. the number of backstress components,
2. the value of the Poisson ratio \( \nu \) (assumed for simplicity equivalent to 0.3),
3. a selected number of entries for \( p \). These entries (named \( x_1 \)) are not equispaced but they are biased in order to provide an accurate description of the quick parameters’ variations observed in the first cycles,
4. the total number of entries \( x_1 \) (named \( n_{r1} \)) determining the dimension of the calibration table,
5. the values of the approximate fitting curves for \( C^{(i)}, \gamma^{(i)}, E \) and \( Y \) evaluated for every single entry \( x_1 \).

Extrapolation of the ratcheting-induced hardening contribution

As reported in section 3.1, the drifting of the mean strain influences the cyclic response of the material. The hardening occurring in ratcheting tests is considered as a superposition of the LCF cyclic hardening and of an additional term. In order to quantify the contribution of this additional ratcheting-induced term a simple approach is suggested:
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Figure 4.10. Compressive, tensile, average and fitted values for $C^{(2)}$ and $\gamma^{(2)}$ as a function of the accumulated plastic strain.

Table 4.2. Format of the preliminary calibration file suitable for the 1 internal variable dependent Chaboche model.

<table>
<thead>
<tr>
<th>rows/columns</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>number of $p$ entries (nr1)</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>number backstress comp.</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>poisson ratio</td>
<td>-</td>
</tr>
<tr>
<td>3+1</td>
<td>$Y(1)$</td>
<td>$x_1(1)$</td>
</tr>
<tr>
<td>...</td>
<td>$Y(...)$</td>
<td>$x_1(...) $</td>
</tr>
<tr>
<td>3+nr1</td>
<td>$Y(nr1)$</td>
<td>$x_1(nr1)$</td>
</tr>
<tr>
<td>3+1+nr1</td>
<td>$E(1)$</td>
<td>$x_1(1)$</td>
</tr>
<tr>
<td>...</td>
<td>$E(...)$</td>
<td>$x_1(...) $</td>
</tr>
<tr>
<td>3+2*nr1</td>
<td>$E(nr1)$</td>
<td>$x_1(nr1)$</td>
</tr>
<tr>
<td>3+1+2*nr1</td>
<td>$C^{(1)}(1)$</td>
<td>$x_1(1)$</td>
</tr>
<tr>
<td>...</td>
<td>$C^{(1)}(...)$</td>
<td>$x_1(...) $</td>
</tr>
<tr>
<td>3+3*nr1</td>
<td>$C^{(1)}(nr1)$</td>
<td>$x_1(nr1)$</td>
</tr>
<tr>
<td>3+1+3*nr1</td>
<td>$\gamma^{(1)}(1)$</td>
<td>$x_1(1)$</td>
</tr>
<tr>
<td>...</td>
<td>$\gamma^{(1)}(...) $</td>
<td>$x_1(...) $</td>
</tr>
<tr>
<td>3+4*nr1</td>
<td>$\gamma^{(1)}(nr1)$</td>
<td>$x_1(nr1)$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>3+8*nr1</td>
<td>$\gamma^{(3)}(nr1)$</td>
<td>$x_1(nr1)$</td>
</tr>
</tbody>
</table>
1. The preliminary calibration sets previously created for all the LCF tests are used to simulate the material response when the ratcheting strain paths imposed in the experiments are applied. For each ratcheting path the simulation is performed using the parameters set calibrated on the LCF test with corresponding strain amplitude and temperature level.

2. The toolkit analyzes the ratcheting experimental data and determines the differences between tests and simulations concerning the per-cycle mean and stress amplitude values.

As a first task, logarithmic stresses and strains are retrieved from ratcheting experiments and the data is splitted collecting the information related to each cycle in a corresponding array. The maximum, minimum, amplitude and mean stress and strain values are then evaluated for each hysteresis loop. One should notice that the adopted definition of per-cycle stress amplitude and mean stress must return a value that must be not sensitive to the ratcheting direction. Therefore this definition considers a cycle as the sequence of a tensile and a compressive trait, computes the range and mean values using the extrema of each of these traits, and calculates the per-cycle values as the average of the compressive and tensile traits (see Fig.3.7). The imposed strain path is then extrapolated and processed in order to create a denoised equivalent loading history.

The reconstructed strain path and the preliminary calibration set corresponding to the same strain amplitude and temperature level are then used as input of a 1 internal variable dependent Chaboche model in order to simulate the stress response of the material. The constitutive model is implemented adapting for one dimension the integration algorithm proposed by Kobayashi and Ohno (2002) consisting in the following steps:

1. At the beginning of each integration step \(n + 1\), the value of the accumulated plastic strain \(p\) at the end of the previous step \(n\) is used to calculate the updated values of \(C^{(k)}\), \(\gamma^{(k)}\), \(E\) and \(Y\). This operation is performed by a linear interpolation of the parameters value reported in the calibration table. In the case of \(Y\) also the derivative of the yield as a function of the internal variable \(\frac{dy}{dp}\) is computed. The one dimensional linear interpolation procedure is a modification of the routine proposed by Wu (2010).

2. A trial stress \(\sigma^*_{n+1}\) is calculated assuming the behavior of the material to be purely elastic.

\[
\sigma^*_{n+1} = E \cdot (\varepsilon^e_n + d\varepsilon_{n+1})
\]  

(4.60)

3. If the yield condition \(F_y\) is fulfilled, the material is confirmed to behave elastically and the increment is concluded.

\[
F_y = |\sigma^*_{n+1} - \alpha_n| - Y \leq 0
\]  

(4.61)

4. Otherwise the material response is plastic and an iterative method computes the plastic strain increment and to correct the stress response. For each iteration, the following tasks are sequentially performed:

- evaluation of the increment of the accumulated plastic strain \(dp\)

\[
dp_{n+1} = \frac{|\sigma^*_{n+1} - \sum_k \alpha^{(k)}_n \cdot \theta^{(k)}_{n+1}| - Y_{n+1}}{E + H}
\]  

(4.62)

- calculation of the difference \(\beta\) between the stress and the backstress

\[
\beta_{n+1} = \frac{(\sigma^*_{n+1} - \sum_k \alpha^{(k)}_n \cdot \theta^{(k)}_{n+1}) \cdot Y_{n+1}}{Y_{n+1} + dp_{n+1} \cdot (H + E)}
\]  

(4.63)
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Figure 4.11. Experimental and simulated stress amplitude (left) and mean stress response (right) for the ratcheting test RAT-RT-040-P10 performed with plate material.

- updating the flow tensor $N$
  \[ N_{n+1} = \frac{\beta_{n+1}}{Y_{n+1}} \]  
  (4.64)

- updating of the isotropic hardening
  \[ Y_{n+1} = Y + \frac{dY}{dp} dp_{n+1} \]  
  (4.65)

- updating of the kinematic hardening
  \[ \theta_{n+1}^{(k)} = \frac{1}{1 + \gamma^{(k)} dp_{n+1}} \]  
  (4.66)

  \[ H = \sum_k C^{(k)} \theta_{n+1}^{(k)} \]  
  (4.67)

The iterative method reaches convergence when the ratio between the increments of the accumulated plastic strain calculated in two consecutive iterations $i$ and $i+1$ is close enough to 1

\[ \left| 1 - \frac{dp_{n+1}^{(i+1)}}{dp_{n+1}^{(i)}} \right| \leq \text{Threshold} \]  
(4.68)

5. Once the iterative method has converged, the plastic strain, elastic strain, stress and backstress are updated

\[ \varepsilon_{n+1}^{pl} = \varepsilon_{n}^{pl} + N_{n+1} \cdot dp_{n+1} \]  
(4.69)

\[ \varepsilon_{n+1}^{el} = \varepsilon_{n+1} - \varepsilon_{n+1}^{pl} \]  
(4.70)

\[ \alpha_{n+1}^{(k)} = (\alpha_n^{(k)} + C^{(k)} \cdot d\varepsilon_{n+1}^{pl}) \cdot \theta_{n+1}^{(k)} \]  
(4.71)

\[ \alpha_{n+1} = \sum_k \alpha_{n+1}^{(k)} \]  
(4.72)

\[ \sigma_{n+1} = E \cdot \varepsilon_{n+1}^{el} \]  
(4.73)

A more detailed description of the integration scheme adopted for the implementation of the constitutive model can be found in Kobayashi and Ohno (2002) and in appendix B.

For reference the simulated and the experimental material stress responses are finally compared evaluating their differences. As expected the model calibrated on LCF data is not able to correctly catch the ratcheting-induced additional hardening and mean stress drifting (see Fig.4.11 and 4.12).
4.1. Constitutive model

Figure 4.12. Experimental and simulated hysteresis loops for the ratcheting test RAT-RT-040-P10 performed with plate material (only 1 loop every 10 is plotted).

Figure 4.13. Per-cycle, test average and common average values for the plastic strain amplitude in the set of ratcheting tests RAT-RT-100-xxx.

Linking of ratcheting-induced hardening with internal variables

Once the contribution of ratcheting on the mechanical behavior has been extrapolated for all the experiments performed at a certain temperature level, a methodology is proposed to establish the relation between the ratcheting-induced material response and the internal variables $\varepsilon^{pl}_{\text{mean}}$, $\varepsilon^{pl}_{\text{ampl}}$ and $\xi$.

As a first consideration, it must be pointed out that the values of $\varepsilon^{pl}_{\text{ampl}}$ and $\xi$ are not constant during the tests execution because of the adopted engineering strain control strategy and of the cyclic hardening. This finding becomes evident plotting as example the per-cycle plastic strain amplitude versus the number of cycles (see blue curves in Fig.4.13). In addition, comparing the average value of the plastic strain amplitude for tests executed with the same nominal strain amplitude (see red curves in Fig.4.13), it is evident that they are not coincident.

To define a unique reference value (named ‘common average’) for tests carried out with the same nominal strain amplitude, it is therefore necessary to perform a further averaging (see green curve in Fig.4.13). The same approach is adopted to compute the ‘common average’ ratcheting steps assuming negative and positive ratcheting tests to be part of the same nominal category. This assumption is motivated by the observation that changing the ratcheting direction the material response is symmetrical, leading to the conclusion that the
material behavior does not depend on the sign of the mean strain drifting (see discussion in section 3.1).

The quantification of the ratcheting contributions starts from the analysis of the stress amplitude. The difference between the experimental and the simulated stress amplitude is found to be quasi-linearly related with the absolute value of mean plastic strain (see Fig.4.14). The physical meaning of the slope is the rate of the additional hardening induced by ratcheting. One can estimate the value of this slope $R_{\text{RiaH}}$ for each one of the ratcheting tests and represent it as a function 'common average' values of the internal variables (see Fig.4.15).

$$R_{\text{RiaH}} = f(\xi, \varepsilon_{\text{ampl}}^{pl})$$ (4.74)

The following step consists in the quantification of the effect of ratcheting on the mean stress drifting. In this case the separation of cyclic and ratcheting contributions is considerably easier since nearly no tension-compression asymmetry has been observed in LCF tests and consequently the whole mean stress drifting is considered to be a ratcheting effect. On the other hand, because of the strong non-linearity of this phenomenon, it is not trivial to find relation between the internal variables and the hardening parameters. In order to overcome this problem, the first and second backstress components are assumed to saturate each loading reversal, meaning that $\alpha^{(1)}$ and $\alpha^{(2)}$ cannot describe any drifting of the mean stress.

$$\alpha^{(1)}_{\text{max}} = -\alpha^{(1)}_{\text{min}} = C^{(1)}/\gamma^{(1)}$$ (4.75)

$$\alpha^{(1)}_{\text{mean}} = 0$$ (4.76)

The whole mean stress drifting must be therefore modeled by means of the third backstress component.

$$\alpha^{(3)}_{\text{mean}} = \sigma_{\text{mean}}$$ (4.77)

Assuming $C^{(3)}$ to be constant and equivalent to the value previously evaluated for the monotonic behavior, an automated routine responsible for the estimation of $\gamma^{(3)}_{\text{RimS}}$ guaranteeing the best fit between $\alpha^{(3)}_{\text{mean}}$ and $\sigma_{\text{mean}}$ is implemented (see Fig.4.16). This task is accomplished:

1. expressing explicitly for each hysteresis loop $i$, $\alpha^{(3)}_{\text{max}}$ and $\alpha^{(3)}_{\text{min}}$ as function of the unknown parameter $\gamma^{(3)}_{\text{RimS}}$,

$$\alpha^{(3)}_{\text{max}}(i) = C^{(3)}/\gamma^{(3)}_{\text{RimS}} + [\alpha^{(3)}_{\text{min}}(i - 1) - C^{(3)}/\gamma^{(3)}_{\text{RimS}}] \cdot e^{-\gamma^{(3)}_{\text{RimS}}[\varepsilon_{\text{pl}}^{\text{max}}(i) - \varepsilon_{\text{pl}}^{\text{min}}(i-1)]}$$

$$\alpha^{(3)}_{\text{min}}(i) = -C^{(3)}/\gamma^{(3)}_{\text{RimS}} + [\alpha^{(3)}_{\text{max}}(i) + C^{(3)}/\gamma^{(3)}_{\text{RimS}}] \cdot e^{\gamma^{(3)}_{\text{RimS}}[\varepsilon_{\text{pl}}^{\text{max}}(i) - \varepsilon_{\text{pl}}^{\text{min}}(i)]}$$ (4.78)

2. evaluating $\alpha^{(3)}_{\text{mean}}$ for each loading cycle,

3. finding the value of $\gamma^{(3)}_{\text{RimS}}$ that minimizes the following residual function

$$\text{residual function} = \sum_{i=1}^{n_{\text{cyc rat}}} \left[\alpha^{(3)}_{\text{mean}}(i) - \sigma_{\text{mean}}(i)\right]^2$$ (4.80)

where $n_{\text{cyc rat}}$ is the number of cycles imposed before reaching the ratcheting limit (i.e. ± 5%).

One can estimate $\gamma^{(3)}_{\text{RimS}}$ for each ratcheting test and represent it as a function of the 'common average' values of the internal variables (see Fig.4.17).

$$\gamma^{(3)}_{\text{RimS}} = f(\xi, \varepsilon_{\text{ampl}}^{pl})$$ (4.81)
Figure 4.14. Difference between experimental and simulated stress amplitude vs. abs(mean plastic strain) in a complete set of ratcheting tests. The value for the slope $R_{\text{ialr}}$ corresponding to each experiment is also reported.
Creation of the set of calibrated parameters

Once all the previous steps have been accomplished, the complete information on the material behavior is available. The next task to perform is merging the information concerning monotonic, cyclic and ratcheting material response provided by the previous steps of the calibration procedure. Then this data must be packed in a convenient format that is suitable for the calibration of the constitutive model formulated in section 4.1.3. As already pointed out, two different implementations for the 5DChabEP model are provided (ABAQUS combined hardening and UMAT) requiring the calibration files to have different formats.

The flowchart of the procedure responsible for the merging of the information is summarized in Fig.4.18 and consists of:

1. Loading of the output of the previous calibration steps.
2. Definition of the values for the internal variables used as entries of the calibration table (see Tab.4.3). The entries $x_1$ for $p$ are not equispaced but they are biased in order to provide an accurate description of the quick parameter variations noticed in the first cycles. The entries $x_4$ for $\xi$ are retrieved from the common average values (see the previous subsection). The entries $x_2$ for $\varepsilon^{pl}_{\text{ampl}}$ are also in part retrieved from the common average values but in this case two additional entries are artificially created. The definition of the entry $x_2(1)$ with $\varepsilon^{pl}_{\text{ampl}} = 0\%$ is necessary to store the parameters responsible for the monotonic behavior. Moreover, in order to better reproduce the transition between monotonic ($x_2(1)$) and cyclic behavior ($x_2(3, 4, 5)$) an additional intermediate entry ($x_2(2)$) is created. The entries $x_3$ for $\varepsilon^{pl}_{\text{mean}}$ are respectively defined as 0 and as the maximum equivalent value measured for the mean plastic strain. The entries $x_5$ for $T$ are respectively defined as 25 °C (equivalent to room temperature) and 200 °C. $nr_1$, $nr_2$, $nr_3$, $nr_4$ and $nr_5$ are the number of entries for $x_1$, $x_2$, $x_3$, $x_4$ and $x_5$.
3. Calculation of the tabular entries for $E$ evaluating the fit functions $E_{fit}$ retrieved from the LCF tests performed with the lower strain amplitude (i.e. 0.40%) using $x_1$ as input. $E$ is dependent on $p$ (i.e. $x_1$) and on $T$ (i.e. $x_5$).

$$E[i, 1] = E_{fit}^{LCF0.40\%,25C}(x_1(i)) \quad \text{with} \quad i = 1, ..., nr_1$$  \hspace{1cm} (4.82)
4.1. Constitutive model

Figure 4.16. \( \alpha_{\text{mean}} \) and \( \sigma_{\text{mean}} \) evolution as a function of number of cycles. In each plot the value for \( \gamma_{\text{Rim,S}}^{(3)} \) that guarantees the best fit between \( \alpha_{\text{mean}}^{(3)} \) and \( \sigma_{\text{mean}} \) is reported.
Chapter 4. Material description

Figure 4.17. Relation between $\gamma^{(3)}_{\text{RimS}}$ and the internal variables.

4. Calculation of the tabular entries for $Y$ evaluating the fit functions $Y_{\text{ave}}$ (previously retrieved from the analysis of LCF tests) using $x_1$ as input. $Y$ is dependent on $p$ (i.e. $x_1$) and on $T$ (i.e. $x_5$).

\[ E[i,2] = E_{\text{fit}}^{LCF0.40\%_0,200C}(x_1(i)) \text{ with } i = 1, \ldots, nr1 \]  

(4.83)

\[ Y[i,1] = Y_{\text{ave},25C}^{\text{fit}}(x_1(i)) \text{ with } i = 1, \ldots, nr1 \]  

(4.84)

\[ Y[i,2] = Y_{\text{ave},200C}^{\text{fit}}(x_1(i)) \text{ with } i = 1, \ldots, nr1 \]  

(4.85)

5. Calculation of the tabular entries for $C^{(k)}$ consisting in a superposition of several contributions. $C^{(k)}$ is dependent on all the 5 internal variables. For the first and second entry of $x_2$ (i.e. $\varepsilon_{\text{ampd}}^p$) the determination of the corresponding parameters is discussed later. On the other hand, if $j \geq 3$, $C^{(k)}$ consists in a superposition of the LCF ($C_{\text{LF}}^{(k)}$) and of the ratcheting ($C_{\text{Ratch}}^{(k)}$) contributions:

\[ C^{(1,2,3)}[i,j,k,l,1] = C_{\text{LCF},25C}^{(1,2,3)}[i,j] + C_{\text{Ratch},25C}^{(1,2,3)}(x_1(i), x_2(j), x_3(k), x_4(l)) \]  

(4.86)

\[ C^{(1,2,3)}[i,j,k,l,2] = C_{\text{LCF},200C}^{(1,2,3)}[i,j] + C_{\text{Ratch},200C}^{(1,2,3)}(x_1(i), x_2(j), x_3(k), x_4(l)) \]  

(4.87)

\[ \text{with } i = 1, \ldots, nr1, \quad j=3,\ldots, nr2, \quad k=1,\ldots, nr3 \text{ and } l=1,\ldots, nr4 \]

$C_{\text{LF}}^{(k)}$ is calculated evaluating the fit functions $C_{\text{fit}}^{(k)}(x_1)$ using $x_1$ as input.

\[ C_{\text{LCF},25C}^{(1,2,3)}[i,j,3] = C_{\text{fit},25C,0.40\%}^{(1,2,3)}(x_1(i)) \]  

(4.88)

\[ C_{\text{LCF},200C}^{(1,2,3)}[i,j,3] = C_{\text{fit},200C,0.40\%}^{(1,2,3)}(x_1(i)) \]  

(4.89)

\[ C_{\text{LCF},25C}^{(1,2,3)}[i,j,4] = C_{\text{fit},25C,0.65\%}^{(1,2,3)}(x_1(i)) \]  

(4.90)
4.1. Constitutive model

Figure 4.18. Flowchart illustrating the logical structure of the calibration set creation.

... with \( i = 1, \ldots, nr_1 \) and \( j = 3, \ldots, nr_2 \)

The ratcheting contribution \( C_{RiaH}^{(k)} \) can be implemented recalling the fact that the ratcheting-induced additional hardening is linearly linked with the absolute value of the mean plastic strain.

\[
R_{iaH} = R_{RiaH} \cdot |\varepsilon_{pl}^{amp}| \tag{4.91}
\]

The additional hardening can be modeled splitting its total amount into the two backstress components assumed to saturate each reversal (\( \alpha^{(1)} \) and \( \alpha^{(2)} \)).

\[
C_{RiaH,25C}^{(k)}(x_1(i), x_2(j), x_3(k), x_4(l)) = R_{RiaH,25C}(x_2(j), x_4(l)) \cdot x_3(k) \cdot \frac{r_{LCF,25C}^{(k)}}{r_{LCF,25C}^{(k)} + r_{LCF,25C}^{(k)}} \tag{4.92}
\]

\[
C_{RiaH,25C}^{(2)}(x_1(i), x_2(j), x_3(k), x_4(l)) = R_{RiaH,25C}(x_2(j), x_4(l)) \cdot x_3(k) \cdot \frac{r_{LCF,25C}^{(2)}}{r_{LCF,25C}^{(2)} + r_{LCF,25C}^{(2)}} \tag{4.93}
\]

\[
C_{RiaH,25C}^{(3)}(x_1(i), x_2(j), x_3(k), x_4(l)) = 0 \tag{4.94}
\]

... with \( i = 1, \ldots, nr_1, \ j = 3, \ldots, nr_2, \ k = 1, \ldots, nr_3 \) and \( l = 1, \ldots, nr_4 \)

where \( \gamma_{LCF}^{(k)} \) and \( r_{LCF}^{(k)} \) are

\[
\gamma_{LCF,25C}[i, j = 3] = \gamma_{fit,25C,0.40%}(x_1(i)) \tag{4.95}
\]
Table 4.3. Schematics for the definition of the internal variables entry.

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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>nr1</th>
</tr>
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<tr>
<td>x1(p)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x2(\varepsilon_{ampl}^p)</td>
<td>0</td>
<td>0.2 \text{com}<em>{\varepsilon</em>{ampl}}^p(1)</td>
<td>\text{com}<em>{\varepsilon</em>{ampl}}^p(1)</td>
<td>\text{com}<em>{\varepsilon</em>{ampl}}^p(2)</td>
<td>\text{com}<em>{\varepsilon</em>{ampl}}^p(3)</td>
<td>-</td>
</tr>
<tr>
<td>x3(\varepsilon_{ampl})</td>
<td>0</td>
<td>\text{max}(\varepsilon_{ampl})</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>x4(x)</td>
<td>0</td>
<td>\text{com}<em>{\varepsilon</em>{pl, ampl}}(1)</td>
<td>\text{com}<em>{\varepsilon</em>{pl, ampl}}(2)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>x5(T)</td>
<td>25</td>
<td>200</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

\[
r^{(1,2,3)}_{LCF,25C}[i,j] = C^{(1,2,3)}_{LCF,25C}[i,j] / \gamma^{(1,2,3)}_{LCF,25C}[i,j] \tag{4.96}
\]

with \( i = 1, \ldots, nr1 \) and \( j=3,\ldots,nr2 \)

6. Calculation of the tabular entries for \( \gamma^{(k)} \) consisting in a superposition of several contributions. \( \gamma^{(k)} \) is dependent on all the 5 internal variables. For the first and second entry of \( x^2 \) (i.e. \( \varepsilon_{pl, ampl}^p \)) the determination of the corresponding parameters is discussed later. On the other hand, if \( j \geq 3 \), \( \gamma^{(k)} \) corresponds to:

\[
\gamma^{(1,2)}[i,j,k,l,1] = \gamma^{(1,2)}_{LCF,25C}[i,j] \tag{4.97}
\]

\[
\gamma^{(3)}[i,j,k,l,1] = \gamma^{(3)}_{RimS,25C}(x2(j), x4(l)) \tag{4.98}
\]

\[
\gamma^{(1,2)}[i,j,k,l,2] = \gamma^{(1,2)}_{LCF,200C}[i,j] \tag{4.99}
\]

with \( i = 1, \ldots, nr1 \), \( j=3,\ldots,nr2 \), \( k = 1, \ldots, nr3 \) and \( l = 1, \ldots, nr4 \)

7. For \( x^2(j = 1) \) (i.e. \( \varepsilon_{pl}^{pl} = 0 \)), the values of \( C^{(k)} \) and \( \gamma^{(k)} \) are easily retrieved from the monotonic calibration step.

\[
C^{(1,2,3)}[i,1,k,l,1] = C^{(1,2,3)}_{monotonic,25C} \tag{4.100}
\]

\[
\gamma^{(1,2,3)}[i,1,k,l,1] = \gamma^{(1,2,3)}_{monotonic,25C} \tag{4.101}
\]

\[
C^{(1,2,3)}[i,1,k,l,2] = C^{(1,2,3)}_{monotonic,200C} \tag{4.102}
\]

with \( i = 1, \ldots, nr1 \), \( k = 1, \ldots, nr3 \) and \( l = 1, \ldots, nr4 \)

8. When \( x^2(j = 2) \), the values of \( C^{(k)} \) and \( \gamma^{(k)} \) are assumed to be equivalent to the ones already calculated for \( x^2(j = 3) \). The artificial creation of an intermediate entry for \( \varepsilon_{pl, ampl} \) is necessary to improve the description of the transition between monotonic and cyclic response.

\[
C^{(1,2,3)}[i,2,k,l,m] = C^{(1,2,3)}[i,3,k,l,m] \tag{4.103}
\]

\[
\gamma^{(1,2,3)}[i,2,k,l,m] = \gamma^{(1,2,3)}[i,3,k,l,m] \tag{4.104}
\]

with \( i = 1, \ldots, nr1 \), \( k = 1, \ldots, nr3 \), \( l = 1, \ldots, nr4 \) and \( m = 1, \ldots, nr5 \)
Table 4.4. Format of the calibration file suitable for 5DChabEP model for the UMAT implementation.

<table>
<thead>
<tr>
<th>rows/columns</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<tr>
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<td>-</td>
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<tr>
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<td>number $c_{ij}^{(nr2)}$ entries (nr3)</td>
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</tr>
<tr>
<td>4</td>
<td>number $c_{ij}^{(nr3)}$ entries (nr4)</td>
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<td>-</td>
<td>-</td>
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<td>-</td>
</tr>
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<td>number T entries (nr5)</td>
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</tr>
<tr>
<td>6</td>
<td>number poisson coeff.</td>
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<td>-</td>
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</tr>
<tr>
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<td>poisson coefficient $v$</td>
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<td>-</td>
<td>-</td>
<td>-</td>
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</tr>
<tr>
<td>7+1</td>
<td>$E_s(1)$</td>
<td>$x_1(1)$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$x_5(1)$</td>
</tr>
<tr>
<td>...</td>
<td>$E_s(...)$</td>
<td>$x_1(...)$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$x_5(1)$</td>
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<td>-</td>
<td>$x_5(nr5)$</td>
</tr>
<tr>
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<td>$E_s(...)$</td>
<td>$x_1(1)$</td>
<td>-</td>
<td>-</td>
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<td>$x_5(nr5)$</td>
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<tr>
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<td>$E_s(nr1*nr5)$</td>
<td>$x_1(nr1)$</td>
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<td>-</td>
<td>-</td>
<td>$x_5(nr5)$</td>
</tr>
<tr>
<td>7+nr1*nr5+1</td>
<td>$Y_s(1)$</td>
<td>$x_1(1)$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$x_5(1)$</td>
</tr>
<tr>
<td>...</td>
<td>$Y_s(...)$</td>
<td>$x_1(...)$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$x_5(...)</td>
</tr>
<tr>
<td>7+2<em>nr1</em>nr5</td>
<td>$Y_s(nr1*nr5)$</td>
<td>$x_1(nr1)$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$x_5(nr5)$</td>
</tr>
<tr>
<td>7+2<em>nr1</em>nr5+1</td>
<td>$C_s^{(1)}(1)$</td>
<td>$x_1(1)$</td>
<td>$x_2(1)$</td>
<td>$x_3(1)$</td>
<td>$x_4(1)$</td>
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</tr>
<tr>
<td>...</td>
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<td>$x_2(nr2)$</td>
<td>$x_3(nr3)$</td>
<td>$x_4(nr4)$</td>
<td>$x_5(nr5)$</td>
</tr>
</tbody>
</table>

Once the complete set of parameters has been defined, it is necessary to properly structure this information to feed the two implemented versions of the constitutive model. For that reason it is convenient to transform the multidimensional information about $E, Y, C^{(k)}$ and $\gamma^{(k)}$ into a one dimensional set of arrays $E_s, Y_s, C_s^{(k)}$ and $\gamma_s^{(k)}$ by means of the following simple algorithm:

$$E_s[i \cdot m] = E[i, m] \quad (4.105)$$
$$Y_s[i \cdot m] = Y[i, m] \quad (4.106)$$
$$C_s^{(1,2,3)}[i \cdot j \cdot k \cdot l \cdot m] = C^{(1,2,3)}[i, j, k, l, m] \quad (4.107)$$
$$\gamma_s^{(1,2,3)}[i \cdot j \cdot k \cdot l \cdot m] = \gamma^{(1,2,3)}[i, j, k, l, m] \quad (4.108)$$

with $i = 1, ..., nr1$, $j = 1, ..., nr2$, $k = 1, ..., nr3$, $l = 1, ..., nr4$ and $m = 1, ..., nr5$

In the case of UMAT, all the parameters are then stored in a single file respecting the format reported in Tab.4.4.

On the other hand, in the case of the combined hardening material model implemented in ABAQUS, it is required to split the parameters in 3 different files responsible for the description of:

1. stiffness (see format in Tab.4.5)
2. isotropic hardening (see format in Tab.4.6)
3. kinematic hardening (see format in Tab.4.7)

It is worthwhile to mention that a meaningful utilization of the constitutive model requires that the internal variables do not overtake the calibration limits reported in Tab.4.8. In order to extend the calibration range it is necessary to perform further experiments with higher temperature, ratcheting rate and/or strain amplitude levels.
Table 4.5. Format of the stiffness calibration file suitable for 5DChabEP model for the ABAQUS combined hardening implementation.

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</thead>
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<td>$\varepsilon_5(1)$</td>
<td>$x_{1}(1)$</td>
</tr>
<tr>
<td>...</td>
<td>$E_1(...)$</td>
<td>$\nu$</td>
<td>$\varepsilon_5(1)$</td>
<td>$x_{1}(...)$</td>
</tr>
<tr>
<td>nr1</td>
<td>$E_1(nr1)$</td>
<td>$\nu$</td>
<td>$\varepsilon_5(1)$</td>
<td>$x_{1}(nr1)$</td>
</tr>
<tr>
<td>...</td>
<td>$E_1(...)$</td>
<td>$\nu$</td>
<td>$\varepsilon_5(nr5)$</td>
<td>$x_{1}(1)$</td>
</tr>
<tr>
<td>nr1*nr5</td>
<td>$E_1(nr1*nr5)$</td>
<td>$\nu$</td>
<td>$\varepsilon_5(nr5)$</td>
<td>$x_{1}(nr1)$</td>
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</tbody>
</table>

Table 4.6. Format of the isotropic hardening calibration file suitable for 5DChabEP model for the ABAQUS combined hardening implementation.

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</tr>
</thead>
<tbody>
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<td>0</td>
<td>$\varepsilon_5(1)$</td>
<td>$x_{1}(1)$</td>
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<td>$x_{2}(1)$</td>
<td>$x_{4}(1)$</td>
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<td>0</td>
<td>$\varepsilon_5(1)$</td>
<td>$x_{1}(...)$</td>
<td>0</td>
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<td>$x_{2}(1)$</td>
<td>$x_{4}(1)$</td>
</tr>
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<td>0</td>
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<td>$Y_1(...)$</td>
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<td>$\varepsilon_5(nr5)$</td>
<td>$x_{1}(1)$</td>
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<td>$x_{2}(1)$</td>
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<tr>
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<td>$Y_1(nr1*nr5)$</td>
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<td>$\varepsilon_5(nr5)$</td>
<td>$x_{1}(nr1)$</td>
<td>0</td>
<td>$x_{3}(nr3)$</td>
<td>$x_{2}(nr2)$</td>
<td>$x_{4}(nr4)$</td>
</tr>
</tbody>
</table>

Table 4.7. Format of the kinematic hardening calibration file suitable for 5DChabEP model for the ABAQUS combined hardening implementation.

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<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
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</thead>
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<td>$C^{\varepsilon_5}(1)$</td>
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<td>$\gamma_{5}(1)$</td>
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<td>$x_{1}(1)$</td>
<td>0</td>
<td>$x_{3}(1)$</td>
</tr>
<tr>
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<td>$C^{\varepsilon_1}(...)$</td>
<td>$C^{\varepsilon_5}(...)$</td>
<td>$\gamma_{1}(...)$</td>
<td>$\gamma_{5}(...)$</td>
<td>$x_{5}(1)$</td>
<td>$x_{1}(...)$</td>
<td>0</td>
<td>$x_{3}(1)$</td>
</tr>
<tr>
<td>nr1</td>
<td>$C^{\varepsilon_1}(nr1)$</td>
<td>$C^{\varepsilon_5}(nr1)$</td>
<td>$\gamma_{1}(nr1)$</td>
<td>$\gamma_{5}(nr1)$</td>
<td>$x_{5}(1)$</td>
<td>$x_{1}(nr1)$</td>
<td>0</td>
<td>$x_{3}(1)$</td>
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<tr>
<td>...</td>
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<td>$C^{\varepsilon_5}(...)$</td>
<td>$\gamma_{1}(...)$</td>
<td>$\gamma_{5}(...)$</td>
<td>$x_{5}(1)$</td>
<td>$x_{1}(...)$</td>
<td>0</td>
<td>$x_{3}(1)$</td>
</tr>
<tr>
<td>nr1*nr5</td>
<td>$C^{\varepsilon_1}(nr1*nr5)$</td>
<td>$C^{\varepsilon_5}(nr1*nr5)$</td>
<td>$\gamma_{1}(nr1*nr5)$</td>
<td>$\gamma_{5}(nr1*nr5)$</td>
<td>$x_{5}(nr5)$</td>
<td>$x_{1}(nr5)$</td>
<td>0</td>
<td>$x_{3}(nr3)$</td>
</tr>
<tr>
<td>...</td>
<td>$C^{\varepsilon_1}(...)$</td>
<td>$C^{\varepsilon_5}(...)$</td>
<td>$\gamma_{1}(...)$</td>
<td>$\gamma_{5}(...)$</td>
<td>$x_{5}(nr5)$</td>
<td>$x_{1}(...)$</td>
<td>0</td>
<td>$x_{3}(nr3)$</td>
</tr>
</tbody>
</table>

Table 4.8. Calibration limits for the internal variables.

<table>
<thead>
<tr>
<th>internal variable / boundary</th>
<th>lower</th>
<th>upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1(p)$ [-]</td>
<td>0</td>
<td>41.07995</td>
</tr>
<tr>
<td>$x_2(\varepsilon_{amp})$ [-]</td>
<td>0</td>
<td>0.00784</td>
</tr>
<tr>
<td>$x_3(\varepsilon_{mean})$ [-]</td>
<td>0</td>
<td>0.04982</td>
</tr>
<tr>
<td>$x_4(\xi)$ [-/cyc]</td>
<td>0</td>
<td>0.00094</td>
</tr>
<tr>
<td>$x_5(T)$ [°C]</td>
<td>25</td>
<td>200</td>
</tr>
</tbody>
</table>
4.1. Constitutive model

4.1.5. Internal variables updating procedure

To evaluate the updated value of the hardening parameters at the beginning of each integration step, the corresponding internal variables must be first retrieved. In this framework a performant tracer is essential because during a FE simulation one must be able to compute the values of these internal variables using the values of previously computed internal variables only. This task is straightforward for $p$ and $T$, while the estimation of $\varepsilon_{\text{mean}}^p$, $\varepsilon_{\text{ampl}}^p$, and $\xi$ is more complicated. In this section the common approach usually adopted in literature to trace the mean plastic strain and the plastic strain amplitude is summarized. In order to improve the performance of the 'memory surface models' another algorithm is proposed and implemented in ABAQUS.

Basic notions on memory surface models

The easiest way to track the strain amplitude in the loading history is to adopt the 'memory surface model' proposed by Chaboche (1987):

$$ F = \sqrt{\frac{2}{3}}(\varepsilon^p - \psi) : (\varepsilon^p - \psi) - q \leq 0 \quad (4.109) $$

where $q$ represents the radius of the memory surface (corresponding to $\varepsilon_{\text{ampl}}^p$) and $\psi$ the coordinates of its center. For deformations within the memory surface (i.e. $F \leq 0$), no translation or size change of the strain memory surface occurs. On the other hand, for larger deformations, the differential equations describing the movement of the center and the size of the yield surface correspond to:

$$ \dot{\psi} = \sqrt{\frac{3}{2}}(1 - \eta)H(F)(N : N^*)N^* \dot{p} \quad (4.110) $$

$$ \dot{q} = \eta H(F)(N : N^*)N^* \dot{p} \quad (4.111) $$

where $H$ is the Heaviside function, $\eta$ is a material parameter, $N$ is the tensor defining the normal direction with respect to the yield surface (see eq.4.9) and $N^*$ is the normal to the memory surface.

$$ N^* = \frac{\varepsilon^p - \psi}{q} \quad (4.112) $$

This model works well only under simple loading conditions and in order to improve its performance some modifications have been proposed in the last decades (Nouailhas et al., 1985; Yang, 2004). However, as demonstrated by Krishna et al. (2009), the accurate determination of $\varepsilon_{\text{ampl}}^p$ and $\varepsilon_{\text{mean}}^p$ under complex loading conditions including ratcheting and multiaxial loading is still a challenge.

Plastic strain tracing procedure

To overcome the main drawbacks of the commonly used memory surface models, a tracer is implemented such that each component of the strain is analyzed separately. Only in this way the sign of the variables can be traced consistently. Any equivalent value is then computed based on the trace of the complete tensor.

The following variables are part of the tracer code:
\( \varepsilon \) the plastic strain tensor

\( d\varepsilon \) the plastic strain tensor increment

\( \Delta \varepsilon \) the cyclic plastic strain range tensor observed at the last sign change of \( d\varepsilon \) (change of direction in the evolution of \( \varepsilon \)), tracing each tensor component \( \varepsilon \) independently

\( \tilde{\varepsilon} \) ditto for the extremum value observed at the last change of direction

\( n_{dc} \) the accumulated number of direction changes counted so far

\( \varepsilon_{\text{mean}} \) the cyclic plastic mean strain tensor

\( \varepsilon_{\text{ampl}} \) the average cyclic plastic strain amplitude tensor based on the plastic strain ranges observed in the last two half cycles

\( i \) the value of any of the above variables at increment \( i \)

\( I \) the value of any of the above variables at the consecutively numbered direction changes (i.e. a short notation for \( n_{dc} \)). Note that this is also done separately for each tensor component.

The tracer procedure, evaluating separately each component of the plastic strain tensor, consists of the following steps:

- The values of all the tracer tensor components are initialized to zero at the beginning of the first increment; \( I = 0 \).
- For each integration increment \( i \):
  - Each time a sign change is observed for \( d\varepsilon \):
    - The average cyclic plastic strain amplitude is computed as
      \[
      I\varepsilon_{\text{ampl}} = \frac{|I\Delta\varepsilon| + |i\varepsilon - I\tilde{\varepsilon}|}{4}
      \]  
      (4.113)
    - The internal variables are updated:
      \[
      I = I + 1
      \]  
      (4.114)
      \[
      I\Delta\varepsilon = i\varepsilon - I-1\tilde{\varepsilon}
      \]  
      (4.115)
      \[
      I\tilde{\varepsilon} = i\varepsilon
      \]  
      (4.116)
  - If \( n_{dc} \geq 2 \) (ignoring the first cycle) and if \( |i\varepsilon - I\tilde{\varepsilon}| > |I\Delta\varepsilon| \) (current plastic strain outside of the plastic strain range of the previous half cycle) then the mean plastic strain follows the increment of the plastic strain as it extends beyond the plastic strain range of the previous half cycle:
    \[
    i\varepsilon_{\text{mean}} = \text{sign}(d\varepsilon) \left( |i-1\varepsilon_{\text{mean}} - I\tilde{\varepsilon}| - |I\Delta\varepsilon| \right) + i-1\varepsilon_{\text{mean}}
    \]  
    (4.117)

Note that this procedure does not capture ratcheting that would occur combined with a too rapidly decreasing cyclic strain amplitude. However, it will catch up once the strain amplitudes increases again after a decreasing phase.

Once the tensors \( \varepsilon_{\text{ampl}} \) and \( \varepsilon_{\text{mean}} \) have been evaluated, the von Mises equivalent values are computed:

\[
\varepsilon_{\text{ampl}} = \sqrt{\frac{2}{3} \varepsilon_{\text{ampl}} : \varepsilon_{\text{ampl}}}
\]  
(4.118)

\[
\varepsilon_{\text{mean}} = \sqrt{\frac{2}{3} \varepsilon_{\text{mean}} : \varepsilon_{\text{mean}}}
\]  
(4.119)

As final task, the equivalent ratcheting step \( \xi \) must be evaluated. For constant ratcheting rate, as in our experiments, the equivalent ratcheting step per cycle can be computed as
4.1. Constitutive model

Figure 4.19. Illustration of the ratcheting tracer procedure for an artificial cyclic plastic strain component following the function \(2 \sin(p/50) + (1 + 0.25 \cos(p/10)) \sin(2p\pi).\)

\[\xi = \frac{\varepsilon_{pl}^{mean}}{N}.\] If the strain amplitude is constant \(N \approx \frac{p}{(4\varepsilon_{pl}^{ampl})},\) which leads to the following equation:

\[
\xi = 4\frac{\varepsilon_{mean}^{pl} \varepsilon_{ampl}^{pl}}{p} \quad (4.120)
\]

For the general case one needs to revert to a numerically smoothened definition of \(\xi,\) as sudden changes in its value may lead to divergence of a finite element integration.

It must be pointed out that tracer variables are only updated if the difference between their values at the current increment \(i\) and last updated value at increment \(j\) of the plastic strain component is larger than a minimal step \(\varepsilon_c = 0.0001: |\varepsilon_{pl}^i - \varepsilon_{pl}^j| \geq \varepsilon_c.\) This limitation is useful when the procedure is implemented to analyze noise-prone experimental data and to reduce computing time when the integration increment is small, e.g. owing to strain localization in a mesh.

Inherent to its definition, the computed increment for the tracer variable \(d\varepsilon_{pl}^{mean}\) is automatically limited by the plastic strain increment \(d\varepsilon_{pl}\) observed in the same integration increment. This limitation prevents time-step independent jumps in the variable value and guarantees that convergence can be controlled by the size of the integration increment at all times.

Fig.4.19 illustrates the performance of the plastic strain tracing procedure for a generic plastic strain component.
4.2 Damage accumulation model

The stress and plastic strain tensorial information at the end of each integration step is used as input for the Jiang incremental fatigue damage model (Jiang, 2000) responsible for the evaluation of the damage accumulating during cycling. The damage criterion consists in a critical plane approach that combines an energy concept and the material memory. Thanks to its incremental form, the model does not require a cycle counting method to deal with complex loading histories characterized by variable amplitude. Because of its features, this fatigue criterion is attractive for the lifetime assessment of components subjected to multiaxial loading and it is able to take into account the effect of a non zero mean stress and of loading sequences.

In its final version (Jiang et al., 2007) the increment of damage \(dD\) is calculated by means of the following equation:

\[
dD = \left( \frac{\sigma_{mr}}{\sigma_0} - 1 \right)^m + \left( 1 + \frac{\sigma}{\sigma_f} \right)dY \tag{4.121}
\]

where

\[
dY = a\sigma d\varepsilon^p + \frac{1}{2} - \frac{a}{2} d\gamma
\]

\(\sigma_0\) is the fatigue endurance limit

The critical plane is defined as the material plane in which the fatigue damage accumulation first reaches the critical value \(D_0\). The first term on the right side of equation 4.121 is used to consider the loading sequence effect. This feature is necessary to reproduce the well-known lifetime reduction effect occurring when a sequence of high-low loading is applied instead of an equivalent low-high sequence. To take into account the loading magnitude effect, a memory surface concept is introduced. This memory surface expands and contracts in a variable amplitude loading history and the evolution for the memory stress \(\sigma_{mr}\) is defined by the following equation:

\[
d\sigma_{mr} = \sqrt{\frac{3}{2}}H(g)\left\langle \frac{s}{||s||} : ds \right\rangle - c[1 - H(g)](\sigma_{mr} - \sigma_0)dp \tag{4.123}
\]

with \(g\) the memory surface for fatigue defined as

\[
g = ||s|| - \sqrt{\frac{2}{3}}\sigma_{mr} \leq 0 \tag{4.124}
\]

where

- \(\sigma\) is the normal stress on a material plane
- \(\tau\) is the shear stress on a material plane
- \(\varepsilon^p\) is the plastic strain corresponding to \(\sigma\)
- \(\gamma^p\) is the plastic strain corresponding to \(\tau\)
- \(a\) is the material constant used to consider the material fatigue cracking behavior
- \(m\) is a material constant
- \(\sigma_f\) is the true fracture stress of the material
- \(\sigma_{mr}\) is a material memory parameter
- \(\sigma_0\) is the fatigue endurance limit
- \(D\) is the fatigue damage
- \(D_0\) is the critical fatigue damage
– $s$ is the deviatoric stress tensor
– $H(x)$ is the Heaviside function
– $c$ is a material constant governing the contraction of the memory surface
– $p$ is the accumulated plastic strain

Thanks to this characteristic of the memory surface, the model is able to consider the well-documented loading sequence effect.

The second term on the right side of equation 4.121 accounts for the mean stress effect. The model implies that the normal stress on the critical plane is responsible for the mean stress effect. A more detailed description of the procedure adopted to implement and calibrate the Jiang model is available in Janssens and Facheris (2013).
4.3 Benchmark

In this section the performance of the constitutive model 5DChabEP coupled with the Jiang damage criterion is evaluated under several loading conditions. The results of this analysis are regrouped in two separate subsections in which the descriptive and the predictive capabilities of the model are presented.

The improvement introduced by the 5DChabEP constitutive law calibrated on the plate material batch is discussed using as reference the calculations provided by:

- the original Chaboche model (Chaboche, 1986) (for most of the cases),
- a modification of the 5 internal variables dependent model not taking into account the temperature dependency (i.e. 4DChabEP) (for isothermal experiments carried out at 200 °C and for the anisothermal TMF test). The parameters of the constitutive law are calibrated exclusively on room temperature experiments.

This comparison is considered to be fair calibrating the parameters of original Chaboche constitutive law for the 316L plate material as follows:

- \( C^{(1)}, \gamma^{(1)}, C^{(2)}, \gamma^{(2)} \) on the stabilized behavior of the investigated batch subjected to a room temperature LCF test with a strain amplitude of 0.65%,
- \( C^{(3)} \) on the quasi asymptotic tangent modulus of the second trait of the room temperature monotonic curve,
- \( \gamma^{(3)} \) to fit the mean stress evolution measured in room temperature strain-controlled ratcheting test with a strain amplitude of 0.65% and a ratcheting step of +0.10%/cyc,
- \( Q \) and \( b \) defining the isotropic hardening (see equation 4.19) to fit the average yield stress curve measured in room temperature LCF tests.

Finally, the parameters of the Jiang damage criterion have been calibrated on room temperature LCF datasets corresponding to 316L plate material.

4.3.1. Descriptive capability

Descriptive capability refers to the potentiality of a model to describe the relationship between output (e.g. cyclic stress evolution) and input (e.g. strain path, temperature) for datasets used to calibrate the model itself. The capability of the 5DChabEP constitutive law of describing the cyclic behavior of 316L including strain amplitude, temperature and ratcheting dependencies is discussed in the following section. In addition, an evaluation of the accuracy of the fatigue life predictions provided by the adopted damage criterion is presented. Since the material description is not time dependent, the imposed cyclic period and the strain waveform shape do not have any effect on the output of the simulations. For simplicity, the following simulations are carried out applying the prescribed loading histories in axial direction to a single 3D hexaedral quadratic element (i.e. ABAQUS C3D20) (see left hand side plot in Fig.4.20).

Strain amplitude dependent cyclic hardening

To investigate the capability of the models to describe the strain amplitude dependent cyclic hardening, a set of simulations is carried out applying the loading conditions imposed in uniaxial LCF tests presented in section 3.1 and consisting in cyclic straining with 3 different strain amplitude levels (i.e. 0.40%, 0.65% and 1.00%) and a constant temperature (i.e. room temperature). Only the first 300 cycles are considered in this analysis.
4.3. Benchmark

Figure 4.20. (left) loading scheme of a single 3D hexaedral element for uniaxial LCF and ratcheting tests, (right) stress amplitude vs. cycle number for experimental and simulated data corresponding to 316L plate material subjected to LCF loading conditions with 3 different strain amplitude levels at room temperature (only the first 300 cycles are reported).

On the right hand side plot of Fig.4.20, the stress amplitude evolution is plotted as a function of the number of cycles for the experiments and simulations. As already reported in section 3.1, the experimental cyclic hardening is non-monotonic and is qualitatively and quantitatively affected by the strain amplitude. As expected, the original Chaboche model cannot describe the observed non-monotonic shape of the stress amplitude curve since the isotropic hardening law responsible for the cyclic hardening description consists of an exponentially saturating function (see Eq.4.19). In addition, the original Chaboche constitutive law cannot take into account the strain amplitude dependency, explaining why it is able to provide accurate stress calculations only for the calibration condition (i.e. stabilized behavior for the LCF test performed with a strain amplitude of 0.65%). The proposed 5DChabEP model performs much better, providing accurate descriptions of the cyclic hardening for all the considered strain amplitude levels, with a nearly negligible overestimation of $\sigma_{\text{amplitude}}$. This slight overestimation is attributed to some practical issues linked with the updating procedure of the model’s parameters that makes the reproduction of the transition between monotonic and cyclic a non-trivial task.

In Tab.4.9, a summary of the average error $e_{\text{ave}}$ occurred in the simulated stress amplitude is reported. The average error is defined as:

$$e_{\text{ave}}(x) = \sum_{i=1}^{n \text{ cyc}} \frac{100\%}{n \text{ cyc}} \cdot \left| \frac{x^i_{\text{model}} - x^i_{\text{exp}}}{x^i_{\text{exp}}} \right|$$  \hspace{1cm} (4.125)

A more detailed information on the performance of the models can be retrieved plotting the first and the last considered (e.g 300th) hysteresis loops for the experiments and simulations (see Fig.4.21). The description of the loop shape provided by the original Chaboche constitutive law is acceptable only for the stabilized behavior of LCF experiments performed with a strain amplitude of 0.65% and 0.40% but not of 1.00%. In none of the analyzed cases, the Chaboche model precisely reproduces the first hysteresis loop shape. On the other hand, the possibility to vary the values of the parameters as a function of some internal variables allows the 5DChabEP model to accurately describe the cyclic evolution of the shape of the hysteresis loops and to provide an improved representation of the transition between monotonic and cycling behavior for all the considered loading cases.

Analyzing equations 4.121-4.122 responsible for the definition of the accumulated damage, it is evident that an accurate evaluation of the plastic strain and a correct representa-
Chapter 4. Material description

Figure 4.21. 1st (left) and 300th (right) hysteresis loops for experimental and simulated data corresponding to 316L plate material subjected to LCF loading conditions with 3 different strain amplitude levels at room temperature.

Table 4.9. Summary of the average errors $e_{ave}$ occurred in the simulations performed by the 5DChabEP and the original Chaboche models taking as reference the experiments. Uniaxial strain-controlled LCF experiments on 316L plate material at room temperature.

<table>
<thead>
<tr>
<th>Test Name</th>
<th>Specimen Name</th>
<th>$e_{ave} \sigma_{amp}$ [%]</th>
<th>$e_{ave} \Delta \sigma$ [%]</th>
<th>$e_{ave} \Delta W_p$ [%]</th>
<th>$e_{ave} \sigma_{amp}$ [%]</th>
<th>$e_{ave} \Delta \sigma$ [%]</th>
<th>$e_{ave} \Delta W_p$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>AX-LCF-RT-040</td>
<td>PE01</td>
<td>0.2</td>
<td>0.1</td>
<td>0.2</td>
<td>1.8</td>
<td>1.3</td>
<td>1.5</td>
</tr>
<tr>
<td>AX-LCF-RT-065</td>
<td>PE07</td>
<td>0.6</td>
<td>0.8</td>
<td>0.8</td>
<td>1.4</td>
<td>1.2</td>
<td>1.7</td>
</tr>
<tr>
<td>AX-LCF-RT-100</td>
<td>PE13</td>
<td>0.2</td>
<td>0.8</td>
<td>0.2</td>
<td>12.5</td>
<td>4.2</td>
<td>9.8</td>
</tr>
</tbody>
</table>

...tion of the hysteresis loop shape are crucial aspects to be considered in order to guarantee reliable lifetime predictions. For this reason, a preliminary but significant information on the performance of the coupled methodology (constitutive model + damage criterion) can be retrieved plotting the per-cycle increment of accumulated plastic strain $\Delta \sigma$ and plastic work $\Delta W_p$ as a function of the number of cycles for the experiments and simulations (see Fig.4.22). The original Chaboche description performs well only for the stabilized condition corresponding to strain amplitude 0.40% and 0.65%. The considerable underestimation of the plastic work for the test AX-LCF-RT-100 is responsible for the overestimation of the fatigue lifetime reported later in this dissertation (see right hand side plot in Fig.4.31). The 5DChabEP material description provides an extremely accurate calculation of $\Delta \sigma$ for all the considered loading cases with an average error that is always lower than 1%. The 5DChabEP constitutive model introduces an improvement in the calculations of $\Delta W_p$ but, in this case, a small deviation between experimental data and simulations is observed in the first cycles (see lower plots in Fig.4.22). This deviation is strictly linked with the previously reported imperfect reproduction of the transition between monotonic and cyclic response due to some practical issues occurring in the model’s parameters updating. For each experiment, the average errors occurred in the calculation of the per-cycle increment of accumulated plastic strain and plastic work are reported in Tab.4.9...
4.3. Benchmark

Figure 4.22. Per-cycle increment of accumulated plastic strain and plastic work for experimental and simulated data corresponding to 316L plate material subjected to LCF loading conditions with 3 different strain amplitude levels at room temperature (only the first 300 cycles are reported).

Ratcheting-induced response

The capability of the models to describe the ratcheting-induced material response is investigated carrying out a set of simulations imposing the same loading conditions applied in uniaxial strain-controlled ratcheting tests presented in section 3.1 and consisting in cyclic straining with 3 different strain amplitude nominal levels (i.e. 0.40%, 0.65% and 1.00%), 4 ratcheting steps (i.e. +0.10%/cyc, -0.10%/cyc, +0.01%/cyc and -0.01%/cyc) and a constant temperature (i.e. room temperature). Since these experiments are performed controlling the deformation in engineering strain, the true strain amplitude cannot be constant during cycling. The imposed engineering strain amplitude is calculated to have the intended true strain amplitude when the mean strain limit is reached (i.e. 5% or -5%). Consequently in the positive ratcheting case, in the first cycles, the true strain amplitude is higher than the one applied in the corresponding LCF test resulting in a harder stress response. Vice versa in the negative ratcheting case, in the first cycles, the true strain amplitude is lower than the one applied in the corresponding LCF test resulting in a softer stress response. Once the engineering strain path has been defined, it is straightforward to extrapolate the corresponding true strain trajectory using it as input for the simulations. Only the hysteresis loops antecedent to the reaching of the maximum strain level, after which the mean strain no longer changes, are considered in this analysis.

In Fig.4.23 the stress amplitude and the mean stress evolution is plotted as a function of the number of cycles for the experiments and simulations. As already reported in section 3.1, the drifting of the mean strain is responsible for:
- an additional hardening,
- a mean stress drifting in the same direction of ratcheting.

The initial difference in the stress amplitude between positive and negative ratcheting tests is due to the control strategy adopted to carry out the experiments. As expected, the original Chaboche constitutive law cannot describe the observed ratcheting-induced additional
Chapter 4. Material description

Figure 4.23. Stress amplitude and mean stress vs. cycle number for experimental and simulated ratcheting loading conditions with 3 different strain amplitude levels at room temperature (316L, plate material).

hardening since the dependency of the model’s parameters on the mean strain is not implemented. The selected $\gamma^{(3)}$ allows the original Chaboche model to provide an accurate description of the mean stress evolution for the experiments performed with the higher ratcheting step (i.e. $\pm 0.10 \%$/cyc). On the other hand, the missing dependency of $\gamma^{(3)}$ on the internal variables is responsible for the underestimation of the mean stress observed for the lower ratcheting step loading cases. Implementing the dependency between the selected set of internal variables and the hardening parameters ($C^{(1)}/\gamma^{(1)}$, $C^{(2)}/\gamma^{(2)}$ and $\gamma^{(3)}$), the 5DChabEP model achieves a considerable improvement in the reproduction of the material response under ratcheting conditions for all the investigated loading cases. In Tab.4.10, a summary of the average error $e_{ave}$ occurred in the calculation of stress amplitude in simulations is reported. The average error for mean stress is not reported since in the case of low stress values a non meaningful error is computed.

The performance of the models are further investigated plotting a selected set of hysteresis loops for the experiments and simulations (see Fig.4.24 and 4.25). In these graphs only ratcheting experiments performed with a strain amplitude of 0.65% are reported. The description of the loop shape provided by the original Chaboche constitutive law is acceptable for the first cycles when only a small amount of ratcheting has taken place. A noticeable deviation between experimental results and calculations is observed when a further drifting of the mean strain occurs. On the other hand, for all the considered loading cases, the hysteresis loop shapes are consistently well reproduced by the 5DChabEP model.

As previously done for LCF tests, the per-cycle increment of accumulated plastic strain and plastic work is plotted as a function of the number of cycles for the experiments and simulations (see Fig.4.26). The initial difference in $\Delta p$ and $\Delta W_p$ between positive and negative ratcheting tests is due to the control strategy adopted to carry out the experiments. Because of the impossibility to reproduce the ratcheting-induced additional hardening, the original Chaboche description provide a systematic overestimation of $\Delta p$ and an underestimation of $\Delta W_p$ compared to the experimental data. As explained later, these observations are strictly linked with the overestimation of the lifetime predictions. On the other hand, 5DChabEP
Figure 4.24. Selected hysteresis loops for experimental and simulated ratcheting loading conditions with 0.65% strain amplitude level and a ratcheting step of -0.01%/cyc (top) and +0.01%/cyc (bottom) at room temperature (316L, plate material).

Figure 4.25. Selected hysteresis loops for experimental and simulated ratcheting loading conditions with 0.65% strain amplitude level and a ratcheting step of +0.1%/cyc (top) and -0.1%/cyc (bottom) at room temperature (316L, plate material).
improves considerably the reproduction of the plastic strain and plastic work accumulation during cycling for all the considered loading cases. For each experiment, the average errors corresponding to simulated $\Delta p$ and $\Delta W_p$ are reported in Tab.4.10.

Temperature dependency

To investigate the capability of the models to describe the temperature dependency, another set of simulations is carried out applying the same LCF and ratcheting strain paths imposed in the previous analysis and changing the temperature (i.e. 200 ºC). It must be recalled that, for these loading cases, instead of the original Chaboche material description, a modification of the 5 internal variables dependent constitutive law not taking into account the temperature dependency (i.e. 4DChabEP) is used as reference model. Using the data simulated by means of the 4DChabEP constitutive law as reference, a meaningful evaluation

Table 4.10. Summary of the average errors $e_{ave}$ occurring in the simulations performed by the 5DChabEP and the original Chaboche model taking as reference the experiments. Uniaxial strain-controlled ratcheting experiments on 316L plate material at room temperature.

<table>
<thead>
<tr>
<th>test name</th>
<th>specimen name</th>
<th>5DChabEP</th>
<th>Chaboche</th>
</tr>
</thead>
<tbody>
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<td>AX-RAT-RT-040-N01 PE05</td>
<td>2.0</td>
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</tr>
<tr>
<td>AX-RAT-RT-040-N01 PE39</td>
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</tr>
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</tr>
<tr>
<td>AX-RAT-RT-065-N01 PE12</td>
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<td>1.1</td>
<td>2.2</td>
</tr>
<tr>
<td>AX-RAT-RT-065-N10 PE08</td>
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<td>3.1</td>
<td>3.3</td>
</tr>
<tr>
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<tr>
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<td>1.2</td>
<td>0.4</td>
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<td>AX-RAT-RT-100-N10 PE15</td>
<td>2.9</td>
<td>0.4</td>
<td>1.5</td>
</tr>
</tbody>
</table>
4.3. Benchmark

Figure 4.27. Stress amplitude vs. cycle number for experimental and simulated data corresponding to 316L plate material subjected to LCF loading conditions with 3 different strain amplitude levels at 200 °C (only the first 300 cycles are reported).

of the sole effect of the missing implementation of temperature dependency on the outputs is possible.

In Fig. 4.27 and 4.28 the stress amplitude evolution for LCF and ratcheting conditions is plotted as a function of the number of cycles for the experiments and simulations. It is not surprising that the 4DChabEP constitutive law calibrated on room temperature data-sets cannot describe the softer material response promoted by the higher temperature. The missing implementation of temperature dependency is responsible for the introduction of an average error in the stress amplitude calculation up to 23% (see Tab. 4.11). On the other hand, also in this case the proposed 5DChabEP model performs much better providing accurate descriptions of the cyclic hardening for all the considered strain paths.

The inaccurate estimation of the cyclic hardening does not allow the 4DChabEP material description to precisely reproduce the observed per-cycle increment of plastic strain and plastic work (see Fig. 4.29 and 4.30). In general the 4DChabEP model is observed to underestimate the $\Delta p$ and to overestimate the $\Delta W_p$. The implementation of the temperature dependency allows the 5DChabEP constitutive law to perform much better. The $\Delta p$ and $\Delta W_p$ computed by 5DChabEP qualitatively follow the experimental behavior with an average error always smaller than 5% (see Tab. 4.11).

Lifetime assessment

The simulated stress-strain paths are used as input of the incremental damage criterion to predict the fatigue lifetime. Since the simulations have been performed only for a limited number of hysteresis loops (i.e. 300 for LCF tests), a simple approach is proposed to determine the number of cycles to failure $N_f$ for each loading condition. Considering a LCF test and assuming that after 200 cycles the stabilization is occurred, it is possible to estimate the increment of damage per-cycle $\Delta D$ performing a linear regression on the damage accumulated in the last 100 of 300 hysteresis loops (see left hand side plot in Fig. 4.31).
Chapter 4. Material description

Figure 4.28. Stress amplitude vs. cycle number for experimental and simulated data corresponding to 316L plate material subjected to ratcheting loading conditions with 3 different strain amplitude levels at 200 °C.

Figure 4.29. Per-cycle increment of accumulated plastic strain and plastic work for experimental and simulated data corresponding to 316L plate material subjected to LCF loading conditions with 3 different strain amplitude levels at 200 °C (only the first 300 cycles are reported).
4.3. Benchmark

Figure 4.30. Per-cycle increment of accumulated plastic strain and plastic work for experimental and simulated data corresponding to 316L plate material subjected to ratcheting loading conditions with 3 different strain amplitude levels at 200 °C.

Table 4.11. Summary of the average errors $\varepsilon_{ave}$ occurred in the simulations performed by the 5DChabEP and the 4DChabEP models taking as reference the experiments. Uniaxial strain-controlled LCF and ratcheting experiments on 316L plate material at 200 °C.

<table>
<thead>
<tr>
<th>Test name</th>
<th>Specimen name</th>
<th>$\varepsilon_{ave} \sigma_{amp}$ [%]</th>
<th>$\varepsilon_{ave} \Delta p$ [%]</th>
<th>$\varepsilon_{ave} \Delta W_p$ [%]</th>
<th>$\varepsilon_{ave} \sigma_{amp}$ [%]</th>
<th>$\varepsilon_{ave} \Delta p$ [%]</th>
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Figure 4.31. (left) Linear regression performed in order to evaluate the slope of the damage accumulation curve after stabilization, (right) experimental and simulated fatigue lives plotted as a function of the imposed strain amplitude for LCF tests performed on 316L plate material at room temperature; the dashed lines define the twice error band.

Table 4.12. Summary of the number of cycles to failure $N_f$ and of the corresponding error in simulations performed by the 5DChabEP and the original Chaboche models taking as reference the experiments. Uniaxial strain-controlled LCF and ratcheting experiments on 316L plate material at room temperature.

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<td></td>
<td></td>
<td>$\epsilon$%</td>
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<td>2247</td>
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Knowing $\Delta D$, the determination of the number of cycles $N_f$ necessary to reach the critical damage value $D_0$ equivalent to 12 GPa (calibrated on 316L plate material at room temperature) is a straightforward task. An analogous approach is used for ratcheting tests assuming that the stabilization of the cyclic response occurs 50 cycles after reaching the ratcheting limit. The predicted $N_f$ for room temperature tests are reported in Tab.4.12 together with the errors estimated using the corresponding experimental fatigue lives as reference. Finally, the average and the maximum errors for the three typologies of loading conditions including LCF, positive ratcheting and negative ratcheting are computed and reported in Tab.4.13.

Experimental and predicted $N_f$ are represented for LCF tests on the right hand side plot of Fig.4.31 as a function of the imposed strain amplitude. In this graph it is evident that, because of the underestimation of the plastic work previously reported, the $N_f$ predicted by means of the Chaboche model are always higher than the ones computed by 5DChabEP. As expected, this difference becomes important (nearly a factor of 2) for the strain amplitude
level 1.00\% because of the considerable error occurred in the computation of the plastic work (i.e. about 10\%) (see Tab.4.9). As reported in Tab.4.13, the enhanced reproduction of the material response provided by 5DChabEP generally improves the accuracy of the predicted $N_f$. For all the considered LCF tests, the predictions provided by the Chaboche and the 5DChabEP models are within a factor of two comparing with the experimental data.

A similar analysis is carried out for strain-controlled ratcheting experiments. In Fig.4.32 the predicted fatigue lives are represented versus the observed ones identifying with different symbols the results belonging to the various typologies of loading conditions. As reported in section 3.1, a lower number of cycles to failure is noticed in ratcheting experiments compared to the corresponding LCF test. In general, the life reduction observed in the experiments is within a factor of two. The fact that both positive and negative ratcheting have an adverse effect on fatigue life is in agreement with the more severe loading conditions promoted by the drifting of the mean strain (i.e. ratcheting-induced additional hardening). This observation could be surprising recalling that the fatigue assessment criteria commonly used in literature (Goodman, 1899; Smith et al., 1970) attribute longer lifetimes to specimens subjected to stress-controlled loading histories characterized by negative mean stresses. On the other hand, it must be pointed out that the general validity of these criteria can not be directly extended to the experiments carried out in this dissertation performed adopting a different control strategy (i.e. strain-control).

As previously presented for ratcheting tests, the systematic underestimation of the plastic work causes an overestimation of the $N_f$ predicted by means of the original Chaboche model comparing with the ones computed by 5DChabEP. This effect is particularly evident (up to a factor of 3) for the experiments performed with a strain amplitude equivalent to 1.00\% in which the resulting error $e_{ave}\Delta W_p$ for the original Chaboche constitutive law was larger (i.e. up to 13.8\%). As reported in Tab.4.12 and 4.13, the possibility to precisely reproduce the additional hardening by the 5DChabEP constitutive law promotes a general improvement of the $N_f$ predictions for ratcheting tests. For the majority of the considered cases the predictions provided by the 5DChabEP model are within a factor of two compared to the experimental data. The only exceptions are three ratcheting experiments in which the experimental lifetimes are underestimated (about a factor of 2).

The reason for this underestimation becomes evident analyzing the performance of the 5DChabEP constitutive model for the description of the cycles subsequent to the reaching of the maximum strain level, after which the mean strain no longer changes. In ratcheting tests performed with the highest strain amplitude level (i.e. AX-RAT-RT-100-xxx), the simulated mechanical response is in good agreement with the experimental data even for the second phase of the test (see Fig.4.33). The accurate reproduction of the stress amplitude and of $\Delta W_p$ evolution provided by the 5DChabEP allows a precise estimation of the fatigue lifetimes. On the other hand, in ratcheting tests performed with the lowest strain amplitude level (i.e. AX-RAT-RT-040-xxx), the constitutive model is not able to catch the noticeable stress amplitude bump occurring after the reaching of the maximum strain level (see Fig.4.34). The underestimation of the stress amplitude is responsible for the overestimation of $\Delta W_p$ (about 10\%) and consequently of the per-cycle damage increment damage. Since the stabilized value

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**Table 4.13.** Summary of the maximum $\epsilon_{max}$ and average errors $\epsilon_{ave}$ in the lifetime assessment performed by the 5DChabEP and the original Chaboche models taking as reference the experiments. Uniaxial strain-controlled LCF experiments on 316L plate material at room temperature.

<table>
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<tr>
<th>Test typology</th>
<th>5DChabEP $\epsilon_{ave}$ $N_f$ [%]</th>
<th>5DChabEP $\epsilon_{max}$ $N_f$ [%]</th>
<th>Chaboche $\epsilon_{ave}$ $N_f$ [%]</th>
<th>Chaboche $\epsilon_{max}$ $N_f$ [%]</th>
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Comparison of predictions with experimental fatigue lives using 5DChabEP and Chaboche constitutive model for LCF and strain controlled ratcheting tests performed on 316L plate material at room temperature; the dashed lines define the twice error band.

Although the damage criterion has been calibrated exclusively on room temperature data, a similar analysis can be performed on LCF and ratcheting experiments carried out at 200 °C in order to explore the potentiality of the Jiang model. It must be recalled that, for these loading cases, instead of the original Chaboche material description, a modification of the 5 internal variables dependent constitutive law not taking into account the temperature dependency (i.e. 4DChabEP) is used as reference model. Using the data simulated by means of the 4DChabEP constitutive law as reference, a meaningful evaluation of the sole effect of the missing implementation of temperature dependency on the outputs is possible.

Experimental and predicted $N_f$ are represented for LCF tests on the right hand side of Figure 4.32.

**Figure 4.32.** Comparison of predictions with experimental fatigue lives using 5DChabEP and Chaboche constitutive model for LCF and strain controlled ratcheting tests performed on 316L plate material at room temperature; the dashed lines define the twice error band.

**Figure 4.33.** Experimental and simulated stress amplitude (left) and $\Delta W_p$ (right) evolution versus cycle numbers for test AX-RAT-100-P10 performed on 316L plate material.
4.3. Benchmark

Figure 4.34. Experimental and simulated stress amplitude (left) and $\Delta W_p$ (right) evolution versus cycle numbers for test AX-RAT-040-P10 performed on 316L plate material.

Figure 4.35. (left) experimental and simulated fatigue lives plotted as a function of the imposed strain amplitude for LCF tests performed on 316L plate material at 200 $^\circ$C; the dashed lines define the twice error band, (right) comparison of predictions with experimental fatigue lives using 5DChabEP and 4DChabEP constitutive models for LCF and strain controlled ratcheting tests performed on 316L plate material at 200 $^\circ$C; the dashed lines define the twice error band.

plot of Fig.4.35 as a function of the imposed strain amplitude. The predicted $N_f$ for room temperature tests are reported in Tab.4.14 together with the errors estimated using the corresponding experimental fatigue lives as reference. Surprisingly, the lifetime predictions computed by the inaccurate 4DChabEP material description are more precise than the ones calculated by 5DChabEP. This observation can be justified assuming that the error linked with the large overestimation of plastic work (up to 14.8%) is somehow compensated by the fact that at 200 $^\circ$C the failure of the specimen is associated with a lower critical damage $D_0$. This hypothesis is corroborated observing that, differently from what observed for the 4DChabEP constitutive law, the fatigue curve computed by means of the 5DChabEP model is nearly parallel to the experimental one meaning that the error occurring in the $N_f$ predicted by 5DChabEP is systematic (see left hand side plot in Fig.4.35).

Similar results are confirmed representing the predicted fatigue lives versus the observed ones for ratcheting tests (see right hand side plot in Fig.4.35). The lifetime predictions provided by the inaccurate 4DChabEP constitutive model are generally more precise than the ones by 5DChabEP (see Tab.4.14). As previously reported for LCF, most of the points corresponding to the 5DChabEP model are systematically aligned with the dashed line describing an overestimation of a factor of 2 with respect to the experimental data.
Table 4.14. Summary of the number of cycles to failure $N_f$ and of the corresponding error in simulations performed by the 5DChabEP and the 4DChabEP models taking as reference the experiments. Uniaxial strain-controlled LCF and ratcheting experiments on 316L plate material at 200 °C.

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</table>

Summary of the descriptive capability

Allowing the parameters to vary as a function of a set of internal variables, the 5DChabEP model provides a consistent improvement in the calculation of the cyclic material response compared to the original Chaboche formulation. The proposed constitutive law is able to quantitatively and qualitatively describe the complex cyclic behavior of 316L including the strain amplitude dependency and the effect of ratcheting. The comparison with simulations carried out by means of the 4DChabEP model shows that the implementation of the temperature dependency in the material description is a necessary requirement to improve the accuracy of simulations. Thanks to the improved description of the hysteresis loop shapes, the 5DChabEP model allows a more precise estimation of the per-cycle increment of plastic strain and plastic work that is a crucial aspect to be considered in order to obtain reliable lifetime prediction with the adopted damage criterion. For the majority of the considered room temperature experiments, the lifetime predictions provided by the Jiang criterion are acceptable and the improvement in the material description provided by 5DChabEP generally enhances the precision of the computed $N_f$. The Jiang criterion demonstrates a promising potentiality in the lifetime assessment also for experiments performed at 200 °C but in this case a specific calibration for higher temperatures is required to obtain reasonable results.

4.3.2. Predictive capability

Predictive capability refers to the potentiality of a model to predict the relationship between output (e.g. cyclic stress evolution) and input (e.g. strain path, temperature) for datasets that are not used to calibrate the model itself. The capability of the 5DChabEP constitutive law in describing the cyclic behavior of 316L under complex loading conditions including
- strain-controlled torsional LCF
- strain-controlled multiaxial LCF and ratcheting
- stress-controlled ratcheting
- anisothermal fatigue

is discussed in the following section. For all the considered cases, an evaluation of the performance of the damage criterion is presented. Since the material description is not time
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Figure 4.36. Schematics of the simulation setup used to reproduce the torsional and multiaxial LCF and ratcheting tests.

dependent, the imposed cyclic period does not have any effect on the output of the simulations.

Torsional tests

The analysis of the performance of the constitutive models to describe the material behavior under complex loading conditions starts from the alternate torsion case. A set of simulations is carried out applying the strain paths imposed in torsional LCF tests presented in section 3.2 and consisting in cyclic straining with 3 different equivalent strain amplitude levels (i.e. 0.40%, 0.65% and 1.00%) and a constant temperature (i.e. room temperature). As reported on the left hand side plot in Fig.4.36, the considered geometry consists exclusively of the gage of the specimen ($L=25$ mm, $R_{ext}=8.00$ mm and $R_{int}=6.50$ mm) and is meshed by means of 280 3D hexahedral quadratic elements (i.e. ABAQUS C3D20). In simulations, the prescribed shear strain path is imposed applying an adequate rotation into the plane $xz$ to one edge of the part. Only the first 100 loading cycles are considered in this analysis.

The resulting shear stress on the external surface of the virtual specimen is calculated by means of two definitions:

1. $\tau$ is the shear stress value extracted in correspondence of a node on the external surface of the specimen (see Fig.4.36),

2. $\tau^*$ is the shear stress value computed from the torque by means of the equation 4.126 assuming the shear stress to be linearly distributed along the radius of the specimen.

$$
\tau^* = \frac{\Theta \cdot R_{ext}}{J_p}
$$

(4.126)

where $\Theta$ is the torque, $R_{ext}$ is the external radius of the specimen and $J_p$ is the polar moment of inertia. As previously reported in Fig.3.20, the amplitude of the shear stresses $\tau$ and $\tau^*$ is systematically different (about 10%). This result is justified recalling that the linear stress distribution assumption is not anymore valid when plasticity occurs. It must be
pointed out that, although $\tau^*$ is an overestimation of the exact stress value $\tau$, at the same time it allows a direct comparison with the experimental shear stress value computed in the same way. In fact, as reported in section 3.2, the experimental shear stress values used here as reference are derived from the measured torque by means of Eq.4.126 assuming a linear shear stress distribution.

In Fig.4.37, the shear stress amplitude evolution $\tau_{amp}$ and $\tau^*_{amp}$ is plotted as a function of the number of cycles for the experiments and simulations. Similarly to uniaxial LCF tests, the cyclic hardening is non-monotonic and is qualitatively and quantitatively affected by the strain amplitude. As expected, the original Chaboche model is not able to describe the observed non-monotonic shape of the shear stress amplitude curve since the isotropic hardening law responsible for the cyclic hardening description consists of an exponentially saturating function (see Eq.4.19). In addition, the original Chaboche constitutive law cannot take into account the strain amplitude dependency, explaining why it is able to provide accurate stress calculations only for a loading situation similar to the calibration condition (i.e. stabilized behavior for the uniaxial LCF test performed with a strain amplitude of 0.65%). The proposed 5DChabEP model performs better, improving the description of the cyclic hardening for all the considered strain amplitude levels. It is plausible to assume that the small deviation (about 5%) observed in the first cycles between the experimental shear stress amplitude and the $\tau^*_{amp}$ simulated by 5DChabEP is due to a slight anisotropy of the material. This hypothesis is confirmed observing that after few cycles the slight anisotropy vanishes and that the 5DChabEP constitutive model calibrated exclusively on uniaxial tests shows excellent performance also under torsional loading conditions being able to qualitatively and quantitatively reproduce the strain amplitude dependent cyclic response of 316L. A summary of the average error $e_{ave}$ occurred in calculation of $\tau^*_{amp}$ is reported in Tab.4.15.

A more detailed information on the performance of the constitutive model is available analyzing the hysteresis loop shape for experiments and simulations. In Fig.4.38 the experimental and simulated data corresponding to last considered hysteresis loop (i.e. 100$^{th}$) are plotted. This investigation is limited to the loading case in which the stress calculations provided by the constitutive models are less accurate (i.e. TO-LCF-RT-100). The violation of the linear stress distribution assumption does not allow a direct comparison between the experimental hysteresis loop and the simulated one constructed using $\tau$ and $\varepsilon^{sh}_{pl}$ extracted in correspondence of a node on the external surface of the specimen (see Fig.4.36). Nevertheless a direct comparison with the experimental data is available constructing the hysteresis loop using $\tau^*$ and the plastic strain in shear direction $\varepsilon^{sh*}_{pl}$ calculated by means of the equations 4.127-4.128

$$\varepsilon^{sh*} = \frac{R_{ext} \cdot \dot{\vartheta}}{2 \cdot L} \quad (4.127)$$

$$\varepsilon^{sh*}_{pl} = \varepsilon^{sh*} - \frac{\tau^*}{G} \quad (4.128)$$

where $\dot{\vartheta}$ is the rotation applied to the specimen, $L$ is the gage length and $G$ is a plausible value for the shear modulus $G$.

Analyzing the plot in Fig.4.38 it is evident that the simulated hysteresis loop $\tau - \varepsilon^{sh}$ computed by the original Chaboche model does not provide an accurate reproduction of the experimental one. Thanks to the possibility to correctly reproduce the cyclic hardening and the strain amplitude dependency, the 5DChabEP model considerably improves the hysteresis loop description although without reaching the precision level shown for the uniaxial LCF loading case.
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Figure 4.37. Experimental and simulated shear stress amplitude plotted as a function of the number of cycles for torsional LCF tests performed on 316L plate material at room temperature with 3 different strain amplitude levels.

Figure 4.38. Experimental and simulated 100th hysteresis loop for torsional LCF test performed on 316L plate material at room temperature with an equivalent strain amplitude of 1.00%.
A further analysis is conducted plotting the per-cycle increment of plastic strain $\Delta p$ and plastic work $\Delta W_p^*$ for experiment and simulations (see Fig.4.39 and 4.40). A direct comparison between simulated and experimental data is available reporting $\Delta p^*$ and $\Delta W_p^*$ computed using $\tau^*$ and $\varepsilon^{sh*}$. The original Chaboche description is not able to catch the qualitative behavior of $\Delta p^*$ and $\Delta W_p^*$ for any of the investigated loading case. The $\Delta W_p^*$ values computed by the original Chaboche constitutive law are particularly inaccurate for the test performed with the higher strain amplitude (error up to 13.1%). The systematic underestimation of the plastic work comparing with 5DChabEP is responsible for the overestimation of the fatigue lifetime reported later (see Fig.4.41). The 5DChabEP material description provides a general improvement in the calculation of $\Delta W_p^*$ but the corresponding average error is considerably larger than the one experienced for the uniaxial LCF cases. The reason for this precision loss is strictly linked with the error occurred in the hysteresis loop description (see Fig.4.38). For each experiment, the average errors corresponding to simulated $\Delta p^*$ and $\Delta W_p^*$ are reported in Tab.4.15.

The simulated stress-strain paths are used as input of the incremental damage criterion to predict the fatigue lifetime. The procedure previously illustrated (see section 4.3.2) is
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Figure 4.40. Experimental and simulated per-cycle increment of plastic work plotted as a function of the number of cycles for torsional LCF tests performed on 316L plate material at room temperature with 3 different strain amplitude levels.

adopted to compute the number of cycles $N_f$ necessary to reach the critical damage value $D_0$ in simulations. The predicted $N_f$ for room temperature torsional tests are reported in Tab.4.16 together with the errors estimated using the corresponding average experimental fatigue lives as reference. Experimental and predicted $N_f$ are represented in Fig.4.41 as a function of the imposed equivalent strain amplitude.

In this graph it is evident that, because of the underestimation of the plastic work previously reported, the $N_f$ predicted by means of the Chaboche model are always higher than the ones computed by 5DChabEP. As expected, this difference becomes important (nearly a factor of 2) for the test performed with the strain amplitude level 1.00% in which the underestimation of $\Delta W_p$ is considerable (average error about 13.1%) (see Tab.4.15). The enhanced reproduction of the material response provided by 5DChabEP generally improves the accuracy of the predicted $N_f$. For all the considered cases the predictions provided by the Chaboche and the 5DChabEP model are within a factor of two compared to the experimental data. The Jiang model demonstrates to have the potentiality to qualitatively and quantitatively describe the longer lifetime observed under torsional LCF conditions comparing with uniaxial experiments performed with equivalent strain amplitude levels.
Figure 4.41. Experimental and simulated fatigue lives plotted as a function of the imposed equivalent strain amplitude for torsional LCF tests performed on 316L plate material at room temperature; the dotted line represents the average $N_f$ measured in the experiments and the dashed lines define the twice error band.

Table 4.16. Summary of the number of cycles to failure $N_f$ and of the corresponding error in simulations performed by the 5DChabEP and the original Chaboche models taking as reference the average $N_f$ measured in the experiments. Torsional strain-controlled LCF experiments on 316L plate material at room temperature.

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<th>Chaboche</th>
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Multiaxial tests

The performance of the constitutive models under multiaxial LCF loading conditions is here investigated. A set of simulations is carried out applying the strain paths imposed in multiaxial room temperature LCF and ratcheting tests presented in section 3.2. The experiments are carried out controlling in strain both the axial and the torsional axes applying proportional and non-proportional loading histories having an equivalent strain amplitude level of 0.65%. In ratcheting tests a continuous drifting of the mean strain in axial direction is superposed to the alternate loading. The per-cycle increment of mean axial strain (i.e. ratcheting step) is equivalent to +0.10 %/cycle until the ratcheting limit (i.e. 5%) is reached. Afterwards the axial mean strain is kept constant and the experiment is continued until failure. As reported on the left hand side plot of Fig.4.36 the considered geometry consists exclusively of the gage of the specimen (\(L=25\) mm, \(R_{\text{ext}}=8.00\) mm and \(R_{\text{int}}=6.50\) mm) and is meshed by means of 280 3D hexaedral quadratic elements (i.e. ABAQUS C3D20). In simulations the prescribed shear and axial strain paths are imposed respectively applying an adequate rotation into the plane \(xz\) and a displacement into \(y\) direction to one edge of the part. Only the first 100 loading cycles are considered for LCF tests and only 50 for ratcheting experiments (i.e. number of cycles necessary to reach the ratcheting limit equivalent to 5%).

Similarly to what previously done for \(\tau\), the resulting stress in axial direction on the external surface of the specimen is calculated by means of two definitions:

1. \(\sigma\) is the axial stress value extracted in correspondence of a node on the external surface of the specimen (see Fig.4.36),

2. \(\sigma^*\) is the axial stress value computed from the force \(F\) by means of the equation 4.129 assuming the axial stress to be uniformly distributed along the cross section of the specimen.

\[
\sigma^* = \frac{F}{A} \quad (4.129)
\]

Similarly to what reported for torsional tests, the shear stresses \(\tau\) and \(\tau^*\) are found to be systematically different for all the investigated loading cases. An analogous phenomenon takes place in the case of proportional loading, leading to a non uniform axial stress distribution and to a consequent difference between \(\sigma\) and \(\sigma^*\). \(\tau^*\) and \(\sigma^*\) overestimate the exact stress values \(\tau\) and \(\sigma\) but at the same time they allow a direct comparison with the experimental shear and axial stress computed in the same way. In Fig.4.42, the axial and shear stress amplitude evolution is plotted as a function of the number of cycles for LCF experiments and simulations. In the same figure the maximum Von Mises stress is also reported.

For the proportional LCF loading case (see left hand side plots in Fig.4.42) the equivalent experimental and simulated responses are qualitatively and quantitatively similar to the one previously presented for the corresponding torsional test (see Fig.4.37). The original Chaboche model is not able to reproduce the observed non-monotonic shape of the stress amplitude curves since the isotropic hardening law responsible for the cyclic hardening description consists of an exponentially saturating function (see Eq.4.19). The proposed 5DChabEP model performs better, improving the description of the cyclic hardening but a systematic underestimation of shear and axial stress amplitude within the spread caused by material inhomogeneities (about 5%) is noticed.

Since part of the difference between experimental stress amplitudes and \(\tau_{\text{amp}},\sigma_{\text{amp}}\) simulated by 5DChabEP vanishes after few cycles it is plausible to assume that it is due to the slight anisotropy of the material reported in section 3.2.
Figure 4.42. Experimental and simulated axial stress amplitude (top), shear stress amplitude (middle) and maximum von Mises stress (bottom) plotted as a function of the number of cycles for multiaxial proportional (left) and non-proportional (right) LCF tests performed on 316L plate material at room temperature.
When non-proportional experiments are considered (see right hand plots in Fig.4.42) the performance of both constitutive models is not satisfactory. This result was expected since none of the considered formulations is able to estimate the non-proportionality degree and consequently the constitutive laws have no possibility to take into account the corresponding additional hardening.

In Fig.4.43-4.44, the axial and shear stress amplitude and mean stress evolution is plotted as a function of the number of cycles for ratcheting experiments and simulations. In Fig.4.43 the maximum Von Mises stress is also reported.

For the proportional ratcheting loading case (see left hand side plots in Fig.4.43-4.44), the experimental and simulated responses have some analogies with the mechanical behavior previously presented for the corresponding uniaxial test (see Fig.4.23). The drifting of the mean strain in axial direction causes a ratcheting-induced additional hardening in the shear and the axial stress amplitude curves that is not reproduced by the original Chaboche constitutive law. The 5DChabEP material description qualitatively reproduces the effect of ratcheting on the stress amplitude. However, since its parameters are calibrated on the uniaxial case in which the influence of ratcheting is found to be weaker (see section 3.2.4), 5DChabEP underestimates the amount of additional hardening. The drifting of the axial mean strain is also responsible to induce a non-zero mean stress in both axial and shear direction (see left hand side plots in Fig.4.44). Both the 5DChabEP and the original Chaboche models are able to qualitatively catch the mean stress drifting in axial direction although the simulated values are not accurate. This result can be explained recalling that the parameters of the constitutive models are calibrated on the uniaxial case in which the influence of ratcheting on the mean stress is found to be weaker (see section 3.2.4). None of the constitutive laws has the potentiality to reproduce the mean stress drifting in shear direction observed in the experiment (see Fig.4.44).

For non-proportional ratcheting conditions (see right hand side plots in Fig.4.43-4.44) the experimental mechanical behavior is nearly equivalent to the corresponding LCF test. As reported in section 3.2.4, the ratcheting in axial direction is found not to influence the mechanical response: neither the stress amplitude nor the mean stress evolution. For this loading case the performance of the investigated constitutive models is not satisfactory. Both constitutive laws are not able to reproduce the additional hardening due to non-proportionality. As expected, the 5DChabEP material description introduces a ratcheting-induced hardening on the stress amplitude but this is in contradiction with the experimental observations. In addition, both investigated constitutive laws introduce a ratcheting-induced mean stress drifting in axial direction but also in this case this result is not supported by measurements. A summary of the average errors $\epsilon_{ave}$ occurred in calculation of $\tau_{amp}^*$ and $\sigma_{amp}^*$ is reported in Tab.4.17. $\epsilon_{ave}$ for mean stresses are not reported since in the case of low mean stress values a non meaningful error is computed.

The per-cycle increment of accumulated plastic strain and plastic work is reported in Fig.4.45 and 4.46 for experiments and simulations. For the LCF proportional loading case (see upper left hand side plots in Fig.4.45-4.46), the investigated constitutive models provide calculations for $\Delta p^*$ and $\Delta W_p^*$ with an acceptable accuracy (average error smaller than 9%). Thanks to the possibility to reproduce cyclic hardening, the simulated $\Delta p^*$ and $\Delta W_p^*$ provided by 5DChabEP are more accurate than the ones computed by means of the original Chaboche model. As expected, the performance of the constitutive models is not satisfactory:

- for the proportional ratcheting case because the models underestimate the amount of ratcheting-induced additional hardening and of mean stress drifting (see lower left hand side plots in Fig.4.45-4.46),
- for both non-proportional loading conditions because the models cannot take into account the additional hardening (see right hand side plots in Fig.4.45-4.46).
Figure 4.43. Experimental and simulated axial stress amplitude (top), shear stress amplitude (middle) and maximum von Mises stress (bottom) plotted as a function of the number of cycles for multiaxial proportional (left) and non-proportional (right) ratcheting tests performed on 316L plate material at room temperature.
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Figure 4.44. Experimental and simulated mean axial stress (top) and mean shear stress (bottom) plotted as a function of the number of cycles for multiaxial proportional (left) and non-proportional (right) ratcheting tests performed on 316L plate material at room temperature.

Table 4.17. Summary of the average errors $e_{ave}$ occurred in the simulations performed by the 5DChabEP and the original Chaboche models taking as reference the experiments. Multiaxial strain-controlled LCF and ratcheting experiments on 316L plate material at room temperature.

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A summary of the average errors $e_{ave}$ occurred in calculation of $\Delta p^*$ and $\Delta W^*_p$ is reported in Tab.4.17.

The simulated stress-strain paths are used as input of the incremental damage criterion to predict the fatigue lifetime. The procedure previously illustrated (see section 4.3.2) is adopted to compute the number of cycles $N_f$ necessary to reach the critical damage value $D_0$ in simulations. The predicted $N_f$ for room temperature multiaxial tests are reported in Tab.4.18 together with the errors estimated using the corresponding average experimental fatigue lives as reference. In Fig.4.47, the predicted fatigue lives are represented versus the observed ones identifying with different symbols the results belonging to the various typologies of loading conditions. As expected the accuracy of the predicted $N_f$ is strictly linked with the performance of the constitutive models. The precise stress-strain relation provided by 5DChabEP for the MP-LCF-RT-065 loading case allows an extremely accurate prediction of $N_f$. On the other hand, the poor accuracy in the lifetime assessment noticed for the remaining loading cases is due to:

- the underestimation of the effect of ratcheting on stress amplitude and mean stress
Figure 4.45. Experimental and simulated per-cycle increment of accumulated plastic strain plotted as a function of the number of cycles for multiaxial LCF and ratcheting tests performed on 316L plate material at room temperature.

- the incapability to reproduce the effect of non-proportionality on the mechanical response (for both the non-proportional cases).

For all the investigated conditions the enhanced material description provided by 5DChabEP is responsible for the improvement of accuracy of $N_f$ predictions comparing with the original Chaboche model (error up to a factor of 3 instead of a factor of 4/5). The consistent difference observed between predicted $N_f$ in proportional and non-proportional LCF conditions suggests that the Jiang model has the potentiality to qualitatively describe the detrimental effect of non-proportionality on fatigue. To confirm this result it is necessary to repeat the current investigation using an improved constitutive model having the capability to reproduce more accurately the mechanical behavior of 316L under multiaxial loading conditions including ratcheting and non-proportionality.

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<th>$\epsilon$ [%]</th>
<th>Chaboche $N_f$ [-]</th>
<th>$\epsilon$ [%]</th>
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4.3. Benchmark

Figure 4.46. Experimental and simulated per-cycle increment of accumulated plastic work plotted as a function of the number of cycles for multiaxial LCF and ratcheting tests performed on 316L plate material at room temperature.

Figure 4.47. Comparison of predictions with experimental fatigue lives using 5DChabEP and Chaboche constitutive model for LCF and strain-controlled ratcheting multiaxial tests performed on 316L plate material at room temperature; the dashed lines define the twice error band.
In this section, the performance of the constitutive models under uniaxial stress-controlled ratcheting conditions is investigated. A set of simulation is carried out applying the stress path imposed in uniaxial room temperature ratcheting tests presented in section 3.3. In the experiments the prescribed stress path has a ramp waveform with an amplitude of 400 MPa and a mean stress of 30 MPa. The tests are carried out with two different stress rates (i.e. \( \dot{\sigma} = 200 \text{ MPa/s} \) for PE36 and \( \dot{\sigma} = 20 \text{ MPa/s} \) for PE55). As reported in section 3.3, under stress-control the mechanical response of 316L shows a non-negligible time-dependency even at room temperature. Anyway, the first cycles are responsible for the major part of the difference in ratcheting accumulation. After the first cycles the hysteresis loop shapes and the elongation rates are found to be very similar in tests performed with different stress rates. This result suggests that the observed time-dependent behavior can not be exhaustively explained by creep. The performance of the proposed time-independent constitutive models under stress-controlled ratcheting conditions is investigated to determine which is the most important aspect to be considered in order to accurately simulate the material behavior: the description of the time-dependency or the reproduction of the cyclic and ratcheting-induced hardening. It is worthwhile to mention that because of the time-independent formulation of the constitutive models, the results of the simulations are not affected by the loading rate. For simplicity, the simulations are carried out applying the prescribed loading history in axial direction to a single 3D hexaedral quadratic element (i.e. ABAQUS C3D20) (see left hand side plot in Fig.4.20). A preliminary analysis shows that to avoid the overtaking of the calibration limit for the internal variable \( \varepsilon^{pl}_{\text{mean}} \) it is necessary to limit this analysis only to the first 50 cycles (see Tab.4.8). The necessity to perform the experiments under engineering stress-control does not allow to impose a constant true stress amplitude (see Fig.3.35). This aspect is not taken into account in the simulations in which the true stress amplitude is assumed to be constant.

In Fig.4.48 the mean strain and the strain amplitude evolution is plotted as a function of cycles for experiments and simulations. In the experiments the application of a stress path with a non-zero mean stress leads to a drifting of the mean strain (i.e. ratcheting). The original Chaboche model is qualitatively able to reproduce the cyclic accumulation of mean strain but greatly overestimates the amount of ratcheting after 50 cycles (21% instead of 5-6%). Making the parameter \( \gamma^{(3)} \) dependent on the internal variables, the 5DChabEP model is able to quantitatively describe the mean strain evolution catching the slope of the second part of the curve (see left hand side plot in Fig.4.48). The accurate estimation of the mean strain allows the 5DChabEP model to correctly evaluate the additional hardening linked with ratcheting and the consequent reduction of strain amplitude (see right hand side plot in Fig.4.48). On the other hand, because of the impossibility to take into account the cyclic hardening induced by ratcheting, the material response computed by the original Chaboche model is considerably softer than the measured one, leading to an overestimation of the strain amplitude of a factor of 4.

The correct estimation of the internal variables is not a trivial task and the half-cycle delay in the hardening parameters updating procedure can considerably affect the reproduction of the mechanical behavior. This aspect is particularly evident plotting a selected set of experimental and simulated hysteresis loops (see Fig.4.49). Thanks to its formulation, including the possibility to describe the monotonic response, the 5DChabEP model provides a reasonable description of the very first loading trait for the PE55 experiment (performed with a loading rate comparable to the one used to calibrate the parameters of the constitutive law). The extremely soft material response noticed in the following cycles induces the internal variable \( \xi \) (i.e. ratcheting step) to overtake the calibration range of the model (see Tab.4.8). The overtaking of the \( \xi \) calibration range and the sudden variation of the internal variables...
is responsible for the wrong estimation of the hardening parameters. The consequent error together with the impossibility to reproduce the time-dependency of the material leads to the inaccurate reproduction of the mean strain and strain amplitude evolution in the first part of the experiment (see 1st and 10th loops in Fig.4.49). Upon further cycling the slower internal variables variation makes their updating an easier task. The more precise estimation of the hardening parameters together with the weakening of time-dependency leads to the improvement of the performance of the 5DChabEP model (see 25th and 50th loops in Fig.4.49). On the other hand, the hysteresis loops simulated by the original Chaboche material description are totally different compared to the experimental ones. The impossibility to vary the kinematic hardening parameters as a function of the internal variables does not allow the original Chaboche model to catch any noticeable variation of the hysteresis loop shape.

As a consequence, the calculations of the per-cycle increment of accumulated plastic strain and plastic work provided by the original Chaboche material description are extremely poor. The impossibility to describe the ratcheting-induced hardening leads to an overestimation of a factor of 3/4 of $\Delta p$ and $\Delta W_p$ (see Fig.4.50). The enhanced reproduction of the cyclic hardening, allows the 5DChabEP model to provide enhanced predictions for $\Delta p$ and $\Delta W_p$ with a precision that improves considering the second part of the experiment.

As previously pointed out, the accurate determination of the plastic work accumulation is a very important aspect to consider in order to guarantee consistent lifetime predictions using the Jiang criterion. The correlation between plastic work and damage is particularly evident plotting the per-cycle increment of damage $\Delta D$ (see Fig.4.51). The wrong estimation of $\Delta W_p$ occurring in the simulation performed by the original Chaboche model is responsible for the overestimation of a factor of 4 of the damage accumulation. The procedure previously illustrated (see section 4.3.2) is adopted to compute the number of cycles $N_f$ necessary to reach the critical damage value $D_0$ in simulations assuming $\Delta D$ to be constant after the first 50 cycles. For the considered loading case, the $N_f$ computed using the stress-strain path provided by the constitutive models are respectively 2423 cycles for 5DChabEP and 526 cycles for the original Chaboche formulation.

Unfortunately it is not possible to directly compare these values with the experimental fatigue lifetime since both ratcheting tests were interrupted before failure because of the reaching of the extensometer range. On the other hand, some criteria are available in literature to obtain accurate lifetime estimations when a non-zero mean stress loading history is imposed. Among them, the Smith-Watson-Topper (SWT) criterion (Smith et al., 1970) demonstrated its reliability in a previous study carried out in the framework of the PLiM project (Janssens et al., 2009).
1st hysteresis loop

10th hysteresis loop

25th hysteresis loop

50th hysteresis loop

$\sigma_{\text{max}} \varepsilon_a = \sigma' f \varepsilon' f (2N_f)^{b+c} + \frac{\sigma'^2}{E} (2N_f)^{2b}$

Figure 4.49. 1$^{st}$, 10$^{th}$, 25$^{th}$ and 50$^{th}$ hysteresis loops in stress-controlled ratcheting experiments and simulations.

The parameters are calibrated to best fit the ASME-III fatigue curve (Chopra and Shack, 2003) for 316L are $(\sigma' f, \varepsilon' f, b, c, E) = (302, 0.3054, -0.0004997, -0.500028, 185000)$. Assuming the strain amplitude $\varepsilon_a$ to be constant and equivalent to the asymptotic value (i.e. 0.0053) measured for both the ratcheting experiments (see right hand plot in Fig.4.48) and recalling that the maximum stress $\sigma_{\text{max}}$ is prescribed and equal to 430 MPa, the $N_f$ computed by the SWT criterion are 2630 cycles. A good agreement between the lifetime prediction given by the SWT criterion and the one provided by the Jiang model is observed only when the most accurate material description (i.e. 5DChabEP) is considered. This result suggests that the Jiang criterion has a promising potentiality in the lifetime assessment also for the stress-controlled loading cases and that the reliability of the $N_f$ predictions is subordinated to the utilization of an accurate constitutive model.

It is plausible to assume that, introducing a time-dependency, the 5DChabEP model would improve its accuracy in the first cycles. However, for the investigated case, this slight precision improvement would probably not justify the adoption of a visco-plastic model characterized by a more complex implementation and calibration. The poor performance of the original Chaboche model underpins that a careful determination of the parameters of the constitutive law should be preferred to the implementation of the time-dependency. As demonstrated in section 3.3, the accumulation rate of inelastic strain is found to be independent on the loading rate after the first cycles. For that reason, the accurate determination of the mean strain drifting after the first cycles can be achieved selecting a convenient value for $\gamma^{(3)}$ rather than implementing a time-dependent material law. In addition, a visco-plastic version of the
4.3. Benchmark

Figure 4.50. Per-cycle increment of accumulated plastic strain and plastic work in stress-controlled ratcheting experiments and simulations.

Figure 4.51. Per-cycle increment of damage in stress-controlled ratcheting experiments and simulations.
original Chaboche model would not be able to reproduce the observed ratcheting-induced hardening and the resulting shrinkage of the hysteresis loops. In conclusion, to achieve an accurate estimation of the per-cycle increment of plastic work allowing precise lifetime predictions, the capability of a constitutive model to correctly reproduce the cyclic hardening is far more important than the possibility to describe its time-dependency.

**Cyclic thermal shocks on a notched ring**

In this section, the proposed constitutive models are used to simulate the stress-strain relation in a study case developed in the framework of the PLiM project to investigate the crack initiation and propagation process driven by thermal fluctuations. The considered study case consists of a ring shaped specimen subjected to cyclic thermal shocks mimicking the loading conditions existing in the primary cooling circuit of a LWR during startup transients. The aim of this analysis is to highlight the differences between simulations performed by the original Chaboche model and by the more advanced 5DChabEP material description. In the PLiM project, a thermo-shock facility has been created to reproduce on specimens having the shape of a notched ring the loading conditions that may occur due to thermal fluctuations of water in piping. Varying the temperature of the fluid between 50 and 250 °C, cyclic thermal shocks are applied on the inner wall of the samples. The utilization of oil instead of water allows to reduce the complexity of the experimental equipment since oil must not be pressurized in this temperature range. As shown on the left hand side plot in Fig.4.52, the ring specimen is fixed between two flanges and the oil flows within this circuit. An extrusion body is placed in the flow with the aim of increasing the flow velocity and amplifying the heat exchange at the inner wall of the specimen. The samples are insulated by a sealing and the heat conduction in axial direction is assumed to be negligible. The outer wall of the specimen is in contact with air at ambient temperature (i.e. about 40 °C). Four springs ensure the free thermal expansion leading to a negligible force in axial direction. The geometry of the ring specimen is given on the right hand side plot in Fig.4.52. The samples are manufactured wire-cutting a notch with a depth of 1 mm and a width of 0.1 mm at the inner surface. The role of the notch is to induce a stress concentration defining the location for the crack initiation.

The thermal loading applied on the inner surface of the specimen is chosen to allow a crack initiation in a reasonable time limit. In the original study case used for the PLiM project, the experiments have been carried out imposing the temperature profile represented in blue in Fig.4.53. The original triangular temperature profile consists of a slow (i.e. 3600 s) heating up from 50 to 250 °C followed by a fast cooling (i.e. 10 s) down to 50 °C. A preliminary simulation of the mechanical behavior of the notched ring performed by means of the 5DChabEP model shows the overtaking of the temperature and ratcheting step calibration limits reported in Tab.4.8.

With the objective of extending in the next future the calibration range of the constitutive model, the intention of the author of the current dissertation was to have an approximate idea of the differences between simulations performed by the original Chaboche model and by the 5DChabEP material description for components subjected to complex TMF conditions. Since a meaningful comparison requires the respect of the calibration limits of the constitutive models involved in the analysis, it was necessary to modify the prescribed temperature profile reducing the maximum temperature to 150 °C and adopting a slower cooling rate (i.e. 30 s) (see red curve in Fig.4.53).

Once the boundaries of the problem have been defined, the procedure adopted to set up a FE simulation of these thermo-shocks is presented. As reported in the introduction, the adoption of a *numerical coupled methodology* is found to be an efficient tool to deal with
Figure 4.52. (left) Schematics of the ring mounted in the sample holder, (right) geometry of the ring specimen (Janssens et al., 2009).

Figure 4.53. Original and modified temperature profiles for triangular cold thermal shock.
nonlinearities and complex boundaries (Hannink et al., 2008). The first step of this methodology consists in the calculation of the interaction between fluid and structure in order to evaluate the temperature distribution in the component along the considered time history. A detailed description of the modeling approach used to simulate the temperature transients in the notched ring can be found in Niffenegger et al. (2008). The comparison between simulated and measured temperature profiles demonstrated a very good performance of this first step of the coupled methodology. The output of the thermal simulation is the input for the second step of the methodology: the thermal stress FE simulation. The structural simulation is performed twice by means of the two considered material descriptions: the original Chaboche model and the more advanced 5DChabEP constitutive law. The main goal of the current investigation is to highlight the differences between stress calculations provided by the two constitutive models and to analyze their effect on the lifetime predictions computed in the third step of the methodology using the Jiang fatigue criterion. Only the first 50 thermal shocks are considered in this analysis. The notched ring specimen has been modeled by a 2D mesh, consisting of 3288 plane strain elements showing a refinement in correspondence of the notch in order to capture the local distribution of the stress gradients (see Fig. 4.54). Niffenegger et al. (2008) demonstrates that the adopted mesh size and the plane strain assumption provide a reasonable approximation of the stress and strain distribution for the considered case study. A practical analysis of the simulations’ output is possible focusing the investigation on stresses and strains computed in correspondence of the centroid of the element 640 (see right hand plot in Fig. 4.58) positioned at the notch tip considering only the components perpendicular to the plane of the notch slot (i.e. direction y in Fig. 4.54).

In Fig. 4.55 the evolution of the simulated mean strain and strain amplitude is plotted versus the number of applied thermal shocks. The analysis of the mean strain computed by both the considered models shows that the repeated application of triangular thermal cold shocks leads to a positive cyclic accumulation of inelastic strain in y direction. These observations confirm the results reported by Janssens et al. (2009) obtained using a single component Chaboche model. The ratcheting is induced by the asymmetrical shape of the thermal shocks: the considerable temperature gradient taking place in cold shocks is respons-
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Figure 4.55. Evolution of the strain amplitude and mean strain in $y$ direction vs. cycle number for simulations performed by the 5DChabEP and the original Chaboche model.

Figure 4.56. Evolution of the stress amplitude and mean stress in $y$ direction vs. cycle number for simulations performed by the 5DChabEP and the original Chaboche model.

able to impose a positive deformation on the inner surface having a magnitude that is larger than the negative deformation occurring during the slow heating up. The amount of accumulated ratcheting after 50 cycles computed by the 5DChabEP model is about 20% higher compared to the calculation provided by the original Chaboche material description. This observation is surprising recalling the great overestimation of the mean strain drifting reported for the original Chaboche model under stress-controlled ratcheting conditions. However, this result is justified noticing the higher value of mean stress computed by the 5DChabEP constitutive law (see right hand side plot in Fig.4.56). Thanks to its capability to reproduce the transition between monotonic and cycling behavior, the 5DChabEP model computes a mechanical response that, in the first cycles, is softer than the one observed for the original Chaboche constitutive law.

Upon further cycling, the occurring mean strain drifting leads the 5DChabEP model to compute a ratcheting-induced hardening (see left hand side plot in Fig.4.56) and a consequent strain amplitude reduction (see left hand side plot in Fig.4.55). As expected, a different behavior is observed for the original Chaboche material description that is not able to reproduce the additional hardening due to ratcheting.

In Fig.4.57 the simulated per-cycle increment of the accumulated plastic strain and plastic work is plotted versus the number of applied thermal shocks. While $\Delta p$ and $\Delta W_p$ are nearly constant for the simulation performed by the original Chaboche model, the possibility to vary the hardening parameters as a function of the internal variables allows 5DChabEP to reproduce the ratcheting-induced hardening and the consequent shrinking of the hysteresis
loops leading to a continuous reduction of the accumulation rate of $p$ and of $W_p$ with cycling.

The simulated stress-strain paths are used as input of the incremental damage criterion to predict the fatigue lifetime. The computed per-cycle increment of damage $\Delta D$ is plotted as a function of the number of applied thermal shocks (see left hand plot in Fig.4.58). While $\Delta D$ is nearly constant for the simulation performed by the original Chaboche model, this is not the case for the 5DChabEP material description showing a continuous increasing of the damage increment per-cycle. The continuous increasing of $\Delta D$ and the simultaneous reduction of $\Delta W_p$ highlights that, differently from what observed so far, the relation $\Delta D$-$\Delta W_p$ is not linear and that other factors (e.g. mean stress, size of the stress surface, etc..) play a role in the determination of the damage.

The procedure previously illustrated (see section 4.3.2) is adopted to compute the number of cycles $N_f$ necessary to reach the critical damage value $D_0$ in simulations, assuming $\Delta D$ to be constant after the first 50 cycles. The $N_f$ computed using the stress-strain path provided by the constitutive models are respectively 2811 cycles for 5DChabEP and 3388 cycles for the original Chaboche formulation. Unfortunately it is not possible to compare these values with the experimental fatigue lifetime since a different temperature profile was prescribed in the thermo-shock tests performed in the PLiM project. Although the stress-strain paths computed by the 2 investigated constitutive models show considerable qualitative and quantitative differences, the resulting $N_f$ predicted by means of the Jiang damage criterion are surprisingly quite similar. A further investigation should be performed in order to confirm this observation imposing different temperature profiles.

Finally, it can be easily demonstrated that moving away from the notch tip, the loading path becomes more complex. While for the element 640 the resulting strain path is quite similar to the one imposed in a uniaxial ratcheting experiment (see left hand side plot in Fig.4.59), this is not the case for the element 565 where a non-negligible strain takes place also in shear direction (i.e. $xy$) (see right hand side plot in Fig.4.59). The strain path corresponding to the element 565 (see position of the element on the right hand side plot in Fig.4.58), underpins the occurrence of multiaxial ratcheting with a small but not negligible degree of non-proportionality (see right hand side plot in Fig.4.59). The strong lifetime reduction previously observed for multiaxial loading conditions suggests to compare the results presented in this section with simulations performed by a constitutive model having an improved performance under multiaxial ratcheting and non-proportional strain paths.
4.3. Benchmark

Figure 4.58. (left) Per-cycle increment of damage vs. cycle number for simulations performed by the 5DChabEP and the original Chaboche model, (right) zoom on the mesh used to discretize the specimen region close to the notch tip showing the location of the elements 640 and 565.

Figure 4.59. (left) Simulated strain path in correspondence of the element 640, (right) simulated strain path in correspondence of the element 565.
Chapter 4. Material description

Anisothermal LCF

In this section, the capability of the proposed constitutive models to describe the stress-strain relation under anisothermal LCF conditions is evaluated. The main goal of this investigation is to check whether the material model 5DChabEP calibrated versus isothermal experiments has the potentiality to reproduce the mechanical behavior of 316L when temperature changes during cycling. This objective can be achieved comparing simulations with the experimental data obtained performing a test by means of the TMF facility described in section 2.1.1.

The prescribed strain and temperature paths are defined in order to mimic the loading conditions experienced in the notched ring study case investigated in the framework of the PLiM project. The procedure adopted to identify the suitable strain-temperature path consists of:

1. performing a simulation to evaluate the effect of the thermo-shocks on the ring specimen,
2. extrapolation of the strain-temperature path in a convenient location,
3. modification of the extrapolated strain-temperature path to make it suitable for the execution of a TMF experiment.

It should be noticed that the simultaneous variation of $T$ and $\varepsilon$ requires a distinction of the total strain from the mechanical one. The total strain is intended as the sum of thermal and mechanical strain.

$$\varepsilon^{tot} = \varepsilon^{mech} + \varepsilon^{th}$$ (4.131)

The first step of this procedure is carried out performing several simulations by means of the 5DChabEP model imposing different temperature thermo-shock profiles. The geometry and the mesh used to carry out these simulations is identical to the one described in the previous section (see Fig.4.52 and 4.54). As already pointed out, the application of an asymmetric temperature profile (e.g. triangular) leads to ratcheting. The impossibility to perform strain-controlled ratcheting experiments by the TMF testing machine enforces the selection of a symmetric temperature profile. The adopted temperature profile has a rectangular shape and consists of a rapid (i.e. 10 s) heating up from 50 to 200 °C followed by a fast cooling (i.e. 10 s) down to 50 °C (see Fig.4.60).

Simulating the effect of a series of rectangular thermo-shocks on the specimen, a particularly severe loading state is observed at the notch tip leading to an alternate mechanical strain in $y$ direction having an amplitude in the order of magnitude of 2.5%. Since it is impossible to subject the hollow specimens designed for TMF tests (see section 2.1.1) to such a severe straining without having buckling, it is therefore necessary to select another region for the extrapolation of the strain path. Moving from the inner surface of the notch towards the outer surface of the ring, the loading conditions are found to be milder. A strain path suitable for the execution of a uniaxial LCF test, is extracted in correspondence of the element 3286 placed at a distance of 50 $\mu$m from the notch tip. The temporal evolution of the $\varepsilon^{mech}$ components in $x$ and $y$ directions is represented on the left hand side plot in Fig.4.61 for the first 5 thermal shocks. On the right hand side plot in Fig.4.61 the simulated temperature profile versus time is reported for the same element.

The extrapolated mechanical strain and temperature profiles are then appositely modified to be suitable for the TMF experiment following the guidelines reported below:

- the maximum temperature rate allowed by the TMF facility is $\pm 4$ °C/s,
4.3. Benchmark

Figure 4.60. Temperature profile for rectangular thermal shock.

Figure 4.61. (left) simulated mechanical strain history in $x$ and $y$ direction extrapolated in correspondence of the element 3286, (right) simulated temperature extrapolated in correspondence of the element 3286.
in order to limit the testing time, the cycling period is reduced as much as possible, excluding from the experiments the time intervals in which temperature and mechanical strain are nearly constant,

- the cyclic application of the loading path requires the initial and the final mechanical strain to be equal to zero.

Adequately trimming, scaling and shifting the simulated mechanical strain and temperature profiles, it is possible to transform the original loading paths reported in the upper plots of Fig. 4.62 into the modified ones represented in the lower graphs of Fig. 4.62.

Subsequently, the cyclic application of the identified mechanical strain and temperature paths allows to carry out an anisothermal LCF tests with the TMF facility presented in section 2.1.1. In order to guarantee that the stress response is due exclusively to the mechanical strain it is necessary to evaluate the contributions of the thermal strain. For that reason, before running the experiment, the prescribed temperature profile is imposed under zero-load control and the thermal strain corresponding to the thermal expansion and contraction of the sample is measured by the testing machine. In the subsequent verification phase, in addition to the temperature profile the captured thermal strain profile is actively applied in strain-control in order to verify that the resulting measured stress is zero. Afterwards, the experiment is carried out superposing the prescribed mechanical strain path to the thermal strain profile.

Finally, a set of simulations is carried out applying the mechanical strain and temperature path imposed in the TMF test. The resulting stress response computed by means of the 5DChabEP, the 4DChabEP (not accounting for temperature dependency) and the original Chaboche model are compared to the experimental data to evaluate the performance of the proposed material descriptions. For simplicity, the simulations are carried out applying the prescribed mechanical strain history in axial direction to a single 3D hexaedral quadratic element (i.e. ABAQUS C3D20) (see left hand side plot in Fig. 4.20). Only the first 50 cycles are considered in this analysis.

In Fig. 4.63 the evolution of maximum, minimum, mean and stress amplitude is plotted.
as a function of the number of cycles for the experiments and simulations. As expected, the original Chaboche model cannot describe the observed non-monotonic shape of the stress amplitude curve since the isotropic hardening law responsible for the cyclic hardening description consists of an exponentially saturating function (see Eq.4.19). The 4DChabEP constitutive law qualitatively reproduces the cyclic hardening, but because of the missing implementation of the dependency on $T$, it is not able to compute the asymmetry between maximum and minimum stress due to the temperature variation during cycling. As a consequence, the original 4DChabEP model cannot describe the resulting negative drifting of the mean stress and overestimates the stress amplitude. Thanks to the dependency on the 5 internal variables, the proposed 5DChabEP model has the capability to reproduce the stress asymmetry resulting in a more precise computation of the stress amplitude. The average errors occurred in the calculation of the stress amplitude are reported in Tab.4.19.

A more detailed information on the performance of the models can be retrieved plotting the first and the last considered (e.g 50th) hysteresis loops for the experiments and simulations (see Fig.4.64). The missing dependency of the hardening parameters on the internal variables does not allow the original Chaboche model to reproduce the hysteresis loop shape variation observed during cycling. On the other hand, the possibility to vary the values of the parameters as a function of some internal variables allows the 4DChabEP and the 5DChabEP model to improve the description of the cyclic evolution of the shape of the hysteresis loops and to provide an enhanced representation of the transition between monotonic and cycling behavior. In addition, the implementation of the temperature dependency leads the 5DChabEP constitutive law to catch the asymmetric shape of the hysteresis loops.

The per-cycle increment of accumulated plastic strain and plastic work is plotted as a function of the number of cycles for the experiments and simulations (see Fig.4.65). The original Chaboche description calibrated on the stabilized condition cannot describe the qual-
Chapter 4. Material description

Figure 4.64. 1st and 50th hysteresis loops in TMF experiment and simulations.

The simulated stress-strain paths are used as input of the incremental damage criterion to predict the fatigue lifetime. Observing the per-cycle increment of damage as a function of the number of cycles (see Fig.4.66), a close correlation between $\Delta W_p$ and $\Delta D$ becomes evident. The procedure previously illustrated (see section 4.3.2) is adopted to compute the number of cycles $N_f$ necessary to reach the critical damage value $D_0$ in simulations, assuming $\Delta D$ to be constant after the first 50 cycles.

While the experimental lifetime is 716 cycles, the $N_f$ computed using the stress-strain path provided by the constitutive models are respectively 2199 cycles for 5DChabEP, 1934 cycles for 4DChabEP and 3792 cycles for the original Chaboche formulation. The overestimation of about a factor of 3 computed using the 5DChabEP model is partially explained recalling that, as previously reported in section 4.3.1, the Jiang model is calibrated on room temperature data and that at higher temperatures the failure of the specimen is associated with a lower critical damage $D_0$. The similar result obtained by means of the 4DChabEP constitutive law suggests that, in the current case, a good reproduction of the cyclic hardening behavior should be preferred to the implementation of the temperature dependency in order to improve the accuracy of the lifetime predictions. The underestimation of $\Delta W_p$ and consequently of $\Delta D$ leads the original Chaboche model to introduce an additional overestimation.
Figure 4.65. Per-cycle increment of accumulated plastic strain and plastic work in TMF experiment and simulations.

Figure 4.66. Per-cycle increment of damage in TMF experiment and simulations.
Table 4.19. Summary of the average errors $e_{ave}$ occurred in the simulations performed by the 5DChabEP, the 4DChabEP and the original Chaboche models taking as reference the TMF experiment.

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<td>3.0</td>
<td>9.9</td>
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</table>

of the lifetime (up to a factor of 5) compared to 5DChabEP. In this framework, a modification of the Jiang criterion considering the role of temperature in damage accumulation could be a promising approach to improve the accuracy of lifetime assessment.

Summary of the predictive capability

The possibility to vary the parameters as a function of a set of internal variables leads the 5DChabEP model to provide a consistent improvement in the calculation of the cyclic material response compared to the original Chaboche formulation. For most of the considered cases, the lifetime predictions provided by the Jiang criterion are acceptable and in general the improvement in the material description provided by 5DChabEP enhances the precision of the computed $N_f$.

The predictive capability of the 5DChabEP material description for torsional tests is found to be extremely satisfactory for all the considered strain amplitude levels. The accurate reproduction of the cyclic hardening and the precise calculations of the per-cycle increment of plastic strain and plastic work obtained by means of the 5DChabEP model allows the Jiang criterion to reliably estimate the $N_f$.

The multiaxial experiments shows an acceptable performance of the 5 internal variables dependent model only for the proportional LCF case. All the other tension-torsion tests, demonstrate that a further modification of the 5DChabEP constitutive law is required to improve the description of the material behavior under multiaxial loading conditions including ratcheting and non-proportionality. In this case a repetition of the analysis with an enhanced constitutive model is necessary to determine whether the Jiang criterion has the potentiality to take into account the detrimental effect of non-proportionality on fatigue.

A considerable improvement in the description of the cyclic material behavior is introduced by 5DChabEP model for the stress-controlled ratcheting case. The possibility to correctly take into account the effect of the drifting of the mean strain on the hardening, causing the consequent plastic strain amplitude reduction, allows the 5DChabEP constitutive law to accurately reproduce the per-cycle accumulation of plastic work. A good agreement between the lifetime prediction given by the SWT criterion and the one provided by the Jiang model is observed for the 5DChabEP material description but not for the original Chaboche one. This result suggests that the Jiang criterion has a promising potentiality in the lifetime assessment also for the stress-controlled loading cases and that the reliability of the $N_f$ predictions is subordinated to the utilization of an accurate constitutive model. The current analysis pointed out that, introducing a visco-plastic version of the 5DChabEP model, the computations accuracy would be improved in the first cycles in which the material shows an evident time-dependency. However, for the investigated case, this slight precision improvement would probably not justify the adoption of a time-dependent model characterized by a more complex implementation and calibration. The poor performance of the original Chaboche model underpins that a careful determination of the parameters of the constitutive law should be preferred to the implementation of the time-dependency.
The analysis of the notched ring study case confirms that the two considered constitutive models give results with substantial qualitative and quantitative differences. However, this example shows that, when stresses and strains are not directly prescribed, the additional accumulation of damage due to the occurrence of the hardening is somehow compensated by the consequent reduction of the plastic strain amplitude. As a consequence, the resulting $N_f$ returned by the Jiang criterion for simulations performed the original Chaboche model and the 5DChabEP material description can be surprisingly similar.

Finally, the 5DChabEP model calibrated on isothermal experiments shows excellent performances even under anisothermal loading conditions. The relatively accurate results obtained by means of the 4DChabEP constitutive law with respect to the original Chaboche model suggest that, in the current case, a reliable reproduction of the cyclic hardening behavior should be preferred to the implementation of the temperature dependency in order to improve the accuracy of the mechanical response and consequently of the lifetime predictions. The poor accuracy generally observed in the predictions of the $N_f$ is attributed to the fact that the Jiang criterion is calibrated exclusively on room temperature experiments. In this framework, a modification of the Jiang criterion considering the role of temperature in damage accumulation could be a promising approach to improve the accuracy of lifetime assessment.
5

Batch variability analysis

In this chapter a sensitivity analysis is carried out in order to characterize the confidence bounds of the output of the constitutive model and to identify the factors that are mostly responsible for the uncertainty in the calculations. In section 5.1 the basic concepts of the sensitivity analysis are presented with a particular focus on the selected approach (i.e. elementary effects method). In order to summarize the main concepts of the elementary effects method, the author of this dissertation decided to report an extract from the third chapter of the book *Global sensitivity analysis: the primer* published by Wiley-Interscience in 2008 (Saltelli et al., 2008). Then in section 5.2 the procedure adopted to set up and carry out the design of experiment is described in detail. Finally, in section 5.3 the results extrapolated from the sensitivity analyses carried out on 3 different case studies are reported and discussed.

5.1 Basic concepts on sensitivity analysis

5.1.1. State of the art

Disparate approaches are available in literature to carry out a sensitivity analysis. Several authors perform sensitivity analysis by changing the value of selected factors one-at-a-time (OAT) while keeping the others constant. In OAT, the analysis is performed moving one factor at a time from the baseline value and measuring the change on the output. At the baseline point all the selected factors correspond to the best estimated value (i.e. calibrated set of parameters). All factors considered in these OAT analyses are assumed to be independent from one another. As demonstrated by Saltelli and Annoni (2010), such a kind of analyses can provide meaningful results only when the model is proven to be linear or at least additive. If those assumptions are not fulfilled, OAT is inadequate since it can not detect interactions among factors. Another drawback of the OAT approach is the so called curse of dimensionality. In OAT all the points of the design are contained in a hypersphere that is included and tangent to the unit hypercube representing the domain of the problem (see Fig.5.1). It can be easily demonstrated that, increasing the number of factors $k$, the ratio between the volume of the sphere and of the cube representing the fraction explored variable space rapidly decreases. For example, assuming to carry out a OAT sensitivity analysis with
5.1. Basic concepts on sensitivity analysis

Figure 5.1. The curse of dimensionality. The ratio $r$ representing the fraction of the explored variable space domain rapidly decreases with the increasing of the number of factors $k$. (Saltelli and Annoni, 2010)

the same number of factors used in this dissertation (i.e. $k = 8$), only 15% of the variable space domain would be explored.

To overcome the drawbacks of OAT, several authors proposed alternative methods. Among them, a particular mention must be dedicated to the so called variance based methods in which the relevance of the factors is assessed computing variance based sensitivity indexes. The set of input parameters is generated by means of random techniques (e.g. Monte Carlo or Latin Hypercube Sampling (Blower et al., 1991; McKay et al., 1979)) ensuring a uniform exploration of the variable space. The variance based methods are generally used when each run of the design of experiment (DOE) is carried out in a limited CPU-time allowing the execution of several simulations (e.g. thousands).

When the CPU-time becomes larger, different approaches are more attractive. As depicted by Saltelli and Annoni (2010):

the practice of reverting to the baseline point in order to compute any new effect is what gives to OAT its appealing symmetry as well as its poor efficiency. A good OAT would be one where, after having moved of one step in one direction, say along $X_1$, one would straightway move of another step along $X_2$ and so on till all factors up to $X_k$ have been moved of one step each.

Following this concept, Morris (1991) proposed an innovative approach named Elementary Effects (EE) method that has been later improved by Campolongo et al. (2007). The EE method shares many of the positive features of the variance based methods with a considerably reduced number of simulations. Its application is ideal in the framework of structural FEM simulations where generally the maximum number of runs is limited for practical reasons.

5.1.2. Elementary effects method

The EE method can be conceptually associated to the class of OAT designs but partially overcomes their main limitations. Although it respects the main concept of local variation around a base point, this method introduces wider ranges of variations for the inputs removing the dependence on a single sample point. In order to summarize the main concepts of this method, the author of this dissertation decided to report here below an extract from the third chapter of the book *Global sensitivity analysis: the primer* published by Wiley-Interscience in 2008 (Saltelli et al., 2008).
Chapter 5. Batch variability analysis

Figure 5.2. Representation of the four-level grid \((q = 4)\) in the two-dimensional input space \((k = 2)\). The value of \(\Delta\) is 2/3. The arrows identify the eight points needed to estimate the elementary effects relative to factor \(X_i\). (Saltelli et al., 2008)

An elementary effect is defined as follows. Consider a model with \(k\) independent inputs \(X_i, i = 1, \ldots, k\) which varies in the \(k\)-dimensional unit cube across \(q\) selected levels. In other words, the input space is discretized into a \(q\)-level grid \(\Omega\). For a given value of \(X\), the elementary effect of the \(i\)-th input factor is defined as:

\[
EE_i = \frac{Y(X_1, X_2, ..., X_{i-1}, X_i + \Delta, ... X_k) - Y(X_1, X_2, ..., X_k)}{\Delta}
\]  

(5.1)

where \(q\) is the number of levels, \(\Delta\) is a value in \(\{1/q - 1, ..., 1 - 1/(q-1)\}\), \(X = (X_1, X_2, ..., X_k)\) is any selected value in \(\Omega\) such that the transformed point \((X + e_i \Delta)\) is still in \(\Omega\) for each index \(i = 1, ..., k\) and \(e_i\) is a vector of zeros but with a unit as its \(i\)th component.

The distribution of elementary effects associated with the \(i\)th input factor is obtained by randomly sampling different \(X\) from \(\Omega\), and is denoted by \(F_i\), i.e. \(EE_i \sim F_i\). The \(F_i\) distribution is finite and, if \(q\) is even and \(\Delta\) is chosen to be equal to \(q/(2(q-1))\), the number of elements of \(F_i\) is \(q^{k-1}[q-\Delta(q-1)]\). Assume, for instance, that \(k = 2\), \(q = 4\) and \(\Delta = 2/3\), for a total number of eight elements for each \(F_i\). The four-level grid in the input space is represented in Fig.5.2. The total number of elementary effects can be counted from the grid by simply keeping in mind that each elementary effect relative to a factor \(i\) is computed by using two points whose relative distance in the coordinate \(X_i\) is \(\Delta\), and zero in any other coordinate.

The sensitivity measures \(\mu\) and \(\sigma\) proposed by Morris (1991) are respectively the estimates of the mean and the standard deviation of the distribution \(F_i\):

\[
\mu_i = \frac{1}{r} \sum_{j=1}^{r} EE_i^j
\]  

(5.2)

\[
\sigma_i^2 = \frac{1}{r-1} \sum_{j=1}^{r} (EE_i^j - \mu)^2
\]  

(5.3)

where \(r\) is the number of trajectories (see explanation later). The mean \(\mu\) assesses the overall influence of the factor on the output (see Fig.5.3). The stand-
5.1. Basic concepts on sensitivity analysis

Figure 5.3. Plot of $\sigma$ versus $\mu^*$ for the $W_p$ output in the case study A. The scatterplots for 2 influential factors are also reported in order to highlight the physical meaning of $\mu^*$ and $\sigma$.

The standard deviation $\sigma$ estimates the ensemble of the factor’s effects, whether nonlinear and/or due to interactions with other factors. An intuitive explanation of the meaning of $\sigma$ is the following. Assume that for factor $X_i$ we obtain a high value of $\sigma$ (e.g. consider as example the factor $\phi_R T$ in the case study (see Fig.5.3)). The elementary effects relative to this factor thus differ notably from one another, implying that the value of an elementary effect is strongly affected by the choice of the sample point at which it is computed, i.e. by the choice of the other factors’ values. By contrast, a low value of $\sigma$ indicates very similar values among the elementary effects, implying that the effect of $X_i$ is almost independent of the values taken by the other factors (e.g. consider as example the factor $E_R T$ in the case study (see Fig.5.3)).

Campolongo et al. (2007) proposed to replace the use of the mean $\mu$ with $\mu^*$, which is defined as the estimate of the mean of the distribution of the absolute values of the elementary effects that we denote with $G_i$, i.e. $|EE_i| \sim G_i$.

$$\mu^*_i = \frac{1}{r} \sum_{j=1}^{r} |EE^j_i| \quad (5.4)$$

The use of $\mu^*$ is convenient as it solves the problem of type II errors (failing to identify a factor with considerable influence on the model), to which the original measure $\mu$ is vulnerable. Type II errors might occur when the distribution $F_i$ contains both positive and negative elements, i.e. when the model is non monotonic or has interaction effects.

In the book *Global sensitivity analysis: the primer* published by Wiley-Interscience in 2008 (Saltelli et al., 2008) is also described in detail the procedure followed in the EE method to sample the variable space domain:

In order to estimate the sensitivity measures (i.e. the statistics of the distributions $F_i$ and $G_i$), the design focuses on the problem of sampling a number $r$ of elementary effects from each $F_i$. As the computation of each elementary effect requires two sample points, the simplest design would require $2r$ sample points for each input, for a total of $2rk$, where $k$ is the number of input factors. Morris (1991) suggested a more efficient design that builds $r$ trajectories of $(k+1)$ points in the input space, each providing $k$ elementary effects, one per input factor, for a total of $r(k+1)$ sample points. The trajectories are generated in the following manner. A base value $x^*$ for the vector $X$ is randomly selected in the q-level grid $\Omega$. $x^*$ is not part of the trajectory but is used to generate all the trajectory
points, which are obtained from \( \mathbf{x}^* \) by increasing one or more of its \( k \) components by \( \Delta \). The first trajectory point, \( \mathbf{x}^{(1)} \), is obtained by increasing one or more components of \( \mathbf{x}^* \) by \( \Delta \), in such a way that \( \mathbf{x}^{(1)} \) is still in \( \Omega \). The second trajectory point, \( \mathbf{x}^{(2)} \), is generated from \( \mathbf{x}^* \) with the requirement that it differs from \( \mathbf{x}^{(1)} \) in its \( i \)th component, which has been either increased or decreased by \( \Delta \), i.e. \( \mathbf{x}^{(2)} = \mathbf{x}^{(1)} + e_i \Delta \) or \( \mathbf{x}^{(2)} = \mathbf{x}^{(1)} - e_i \Delta \). The index \( i \) is randomly selected in the set \( \{1, 2, \ldots, k\} \). The third sampling point, \( \mathbf{x}^{(3)} \), is generated from \( \mathbf{x}^* \) with the property that \( \mathbf{x}^{(3)} \) differs from \( \mathbf{x}^{(2)} \) for only one component \( j \), for any \( j \neq i \). It can be either \( \mathbf{x}^{(3)} = \mathbf{x}^{(2)} + e_i \Delta \) or \( \mathbf{x}^{(3)} = \mathbf{x}^{(2)} - e_i \Delta \). And so on until \( \mathbf{x}^{(k+1)} \), which closes the trajectory. The design produces a trajectory of \( (k+1) \) sampling points \( \mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \ldots, \mathbf{x}^{(k+1)} \) with the key properties that two consecutive points differ in only one component and that any value of the base vector \( \mathbf{x}^* \) has been selected at least once to be increased by \( \Delta \). An example of a trajectory for \( k = 3 \) is illustrated in Fig. 5.2.

A technical scheme to generate trajectories with the required properties is as follows. A trajectory can be seen in the form of a matrix \( \mathbf{B}^* \), with dimension \( (k+1) \times k \), whose rows are the vectors \( \mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \ldots, \mathbf{x}^{(k+1)} \). To build \( \mathbf{B}^* \), the first step is the selection of a matrix \( \mathbf{B} \), whose dimensions are \( (k+1) \times k \), with elements that are 0’s and 1’s and the key property that for every column index \( j \), \( j = 1, \ldots, k \), there are two rows of \( \mathbf{B} \) that differ only in the \( j \)th entry. A convenient choice for \( \mathbf{B} \) is a strictly lower triangular matrix of 1’s:

\[
\mathbf{B} = \begin{bmatrix}
0 & 0 & 0 & \ldots & 0 \\
1 & 0 & 0 & \ldots & 0 \\
1 & 1 & 0 & \ldots & 0 \\
1 & 1 & 1 & 0 & \ldots \\
\vdots & \vdots & \vdots & \ddots & \vdots 
\end{bmatrix} \quad (5.5)
\]

The matrix \( \mathbf{B}' \), given by

\[
\mathbf{B}' = \mathbf{J}_{k+1,k} \mathbf{x}^* + \Delta \mathbf{B} \quad (5.6)
\]

where \( \mathbf{J}_{k+1,k} \) is a \( (k+1) \times k \) matrix of 1’s and \( \mathbf{x}^* \) is a randomly chosen base value of \( \mathbf{X} \), is a potential candidate for the desired design matrix, but it has the limitation that the \( k \)th elementary effect it produces would not be randomly selected. A randomized version of the sampling matrix is given by

\[
\mathbf{B}^* = (\mathbf{J}_{k+1,k} \mathbf{x}^* + (\Delta/2)[(2 \mathbf{B} - \mathbf{J}_{k+1,k}) \mathbf{D}^* + \mathbf{J}_{k+1,k}]) \mathbf{P}^* \quad (5.7)
\]

where \( \mathbf{D}^* \) is a \( k \)-dimensional diagonal matrix in which each element is either +1 or -1 with equal probability, and \( \mathbf{P}^* \) is a \( k \)-by-\( k \) random permutation matrix in which each row contains one element equal to 1, all others are 0, and no two columns have 1’s in the same position. Read row by row, \( \mathbf{P}^* \) gives the order in which factors are moved; \( \mathbf{D}^* \) states whether the factors will increase or decrease their value along the trajectory. \( \mathbf{B}^* \) provides one elementary effect per input, which is randomly selected.

As additional feature, the EE method has the possibility to group the input variables and to assess the effect of the whole variables cluster on the output of the model. The following extract taken from the book *Global sensitivity analysis: the primer* published by Wiley-Interscience in 2008 (Saltelli et al., 2008) describes how the elementary effects method deals with groups of factors:
5.2 Setting up the sensitivity analysis

When working with groups, the idea is to move all factors of the same group simultaneously. In the original definition given by Morris (1991), the elementary effect is obtained by subtracting the function evaluated at \( \mathbf{X} \) from that evaluated after incrementing one factor (see Eq.5.1). This definition cannot be extended straightforwardly to cases in which more than one factor is moved at the same time, as two factors may have been changed in opposite directions, i.e. one increased and one decreased by \( \Delta \). By contrast, using \( \mu^* \) overcomes this problem, as the focus is not on the elementary effect itself but on its absolute value, i.e. the elementary effect is always positive, regardless of the displacement of the factors.

For a two-factor group \( \mathbf{u} = (X_{i_1}, X_{i_2}) \), the absolute elementary effect in point \( \mathbf{X} \) is

$$|EE_u(\mathbf{X})| = \frac{|y(\mathbf{X}) - y(\tilde{\mathbf{X}})|}{\Delta} \quad (5.8)$$

where \( \mathbf{X} \) is any selected value in \( \Omega \) such that the transformed point \( \tilde{\mathbf{X}} \) is still in \( \Omega \), and each of the components \( (\tilde{X}_{i_1}, \tilde{X}_{i_2}) \) has been either increased or decreased by \( \Delta \) with respect to \( (X_{i_1}, X_{i_2}) \).

### 5.2 Setting up the sensitivity analysis

In this section, the procedure adopted in order to carry out the sensitivity analyses for 3 case studies by means of the EE method studies is described in detail. As a first task, the influential factors of the constitutive model are defined. Subsequently, for each factor, four different levels are identified in order to reproduce the effect of the batch variability. The design of experiment is then generated defining the complete list of simulations and creating the corresponding parameter sets. Finally, the simulations are performed and the EE method is used to assess the influence of the factors on the model output.

#### 5.2.1. Definition of the factors

As reported in section 4.1.3, the parameters driving the constitutive model 5DChabEP are 8: \( Y, E, C^{(k)} \) and \( \gamma^{(k)} \) with \( k = 1, ..., 3 \). It is important to recall that these parameters are
not constant values but are allowed to vary as a function of 5 internal variables. The relation
between the parameters and the internal variables is not analytical since the constitutive
model extrapolates their updated values by means of an interpolation procedure. In this
framework, a valuable help to treat the complexity of the problem comes from the capability
of the EE method to deal with groups of factors.

A reasonable approach consists in assuming the groups of parameters (i.e. \( Y, E, C^{(k)} \)
and \( \gamma^{(k)} \)) to vary simultaneously with respect to the internal variables \( p, \varepsilon^{pl}_{\text{mean}}, \varepsilon^{pl}_{\text{ampl}} \) and \( \xi \). On the other hand, the non-linear effect of temperature on the mechanical behavior of
316L (see section 1.1.1) together with the necessity to investigate the model performance
under anisothermal conditions suggests not to consider these groups of parameters to vary
simultaneously with respect to the fifth internal variable \( T \). As a first approximation, 2
temperature levels are considered (i.e. RT and 200 \(^\circ\)C) and 16 groups of influential factors
are identified:

\[
Y_{RT}, E_{RT}, C^{(k)}_{RT}, Y_{200C}, E_{200C}, C^{(k)}_{200C}, \gamma^{(k)}_{200C}.
\]

A reduction of the number of groups of factors is possible recalling that during the
calibration of the 5DChabEP model (see section 4.1.4):

- the first and the second backstress components are supposed to saturate each reversal
  and therefore they are used to describe the stress amplitude evolution (i.e. cycling
  hardening). An efficient way to reproduce the effect of the batch variability on the
cyclic hardening behavior is to vary simultaneously \( C^{(1)} \) and \( C^{(2)} \), assuming the values
  of \( \gamma^{(1)} \) and \( \gamma^{(2)} \) to be constant and equivalent to the ones calibrated on the LCF
  experiments. As a consequence, it is possible to combine all these groups of factors in
  a single factor cluster named \( \phi \) (i.e. \( \phi = C^{(1)} / \gamma^{(1)} + C^{(2)} / \gamma^{(2)} \)).
- the third backstress component is supposed to never reach saturation and therefore is
  used to describe the mean stress evolution. An efficient way to reproduce the effect of
  the batch variability on the mean stress behavior is therefore to vary \( \gamma^{(3)} \), assuming the
  values of \( C^{(3)} \) to be constant and equivalent to the ones calibrated on the LCF
  experiments.

As a consequence of these observations, the numbers of the groups of influential factors are
reduced to 8:

\[
Y_{RT}, E_{RT}, \phi_{RT}, \gamma^{(3)}_{RT}, Y_{200C}, E_{200C}, \phi_{200C}, \gamma^{(3)}_{200C}.
\]

The assumption previously performed on the simultaneous variation of the groups of factors with respect to the internal variables \( p, \varepsilon^{pl}_{\text{mean}}, \varepsilon^{pl}_{\text{ampl}} \) and \( \xi \) is justified observing that in Fig.5.5-5.7 the curves
corresponding to the plate material are qualitatively similar to the ones corresponding to the pipe.

### 5.2.2. Definition of the levels

Once the groups of factors have been identified, their levels must be defined. Since
the EE method must deal with groups of factors instead of scalar factors, it is necessary to
slightly modify the sampling strategy presented in section 5.1.2. The assumption previously
performed on the simultaneous variation of the groups of factors with respect to the internal
variables \( p, \varepsilon^{pl}_{\text{mean}}, \varepsilon^{pl}_{\text{ampl}} \) and \( \xi \) makes the creation of the levels a straightforward task. The

The group of curves \( x_i^{(L)} \) defining a certain level \( L \) for the group of factors \( i \) can be easily retrieved
scaling by means of the coefficients \( Z_i^{(L)} \) (see Tab.5.1) the baseline curve \( x_i^{\text{base}} \) (extrapolated
from the calibration set for the plate material):

\[
x_i^{(L)} = x_i^{\text{base}} \cdot \left( 100\% + Z_i^{(L)} \right) \quad (5.9)
\]

The scaling coefficients \( Z \) are carefully selected:
5.2 Setting up the sensitivity analysis

Table 5.1. Scaling coefficients $Z$ used to retrieve the values of the 8 groups of factors for the 4 levels $L$. The upper and lower boundaries are selected in order to enclose the mechanical behavior of 316L considering a realistic variability of the material batches.

<table>
<thead>
<tr>
<th>Level</th>
<th>$Z_{Y_{RT}}^{(L)}$</th>
<th>$Z_{E_{RT}}^{(L)}$</th>
<th>$Z_{\phi_{RT}}^{(L)}$</th>
<th>$Z_{\gamma_{200C}}^{(L)}$</th>
<th>$Z_{E_{200C}}^{(L)}$</th>
<th>$Z_{\phi_{200C}}^{(L)}$</th>
<th>$Z_{\gamma_{200C}}^{(L)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 1 (upper boundary)</td>
<td>+20%</td>
<td>+10%</td>
<td>+30%</td>
<td>+20%</td>
<td>+20%</td>
<td>+10%</td>
<td>+30%</td>
</tr>
<tr>
<td>Level 2</td>
<td>+6.7%</td>
<td>+3.3%</td>
<td>+10%</td>
<td>+6.7%</td>
<td>+6.7%</td>
<td>+3.3%</td>
<td>+10%</td>
</tr>
<tr>
<td>Level 3</td>
<td>-6.7%</td>
<td>-3.3%</td>
<td>-10%</td>
<td>-6.7%</td>
<td>-6.7%</td>
<td>-3.3%</td>
<td>-10%</td>
</tr>
<tr>
<td>Level 4 (lower boundary)</td>
<td>-20%</td>
<td>-10%</td>
<td>-30%</td>
<td>-20%</td>
<td>-20%</td>
<td>-10%</td>
<td>-30%</td>
</tr>
</tbody>
</table>

Figure 5.5. $Y_{RT}$ evolution for the baseline curve (plate material) and for the level curves used to perform the sensitivity analysis. Also the curve corresponding to another material batch (pipe material) is reported as reference.

- to determine four equally spaced levels as suggested in literature (Saltelli et al., 2008),
- in a way that the resulting upper and lower boundary level curves enclose the mechanical behavior of 316L considering a realistic variability of the material batches.

It is important to underline that the scaling coefficient used to create the levels’ curves are not identical for all the groups of factors. An adequate definition of the upper and lower level is crucial to obtain meaningful results from the sensitivity analysis, since as depicted by Saltelli et al. (2008), the resulting influence of a certain factor can be totally different contracting or expanding the boundary range.

5.2.3 Design of experiment

Once the groups of factors and their levels have been identified, the following task is the creation of the complete list of the simulations to carry out using the corresponding parameter sets (i.e. design of experiment). The generation of this list is performed following the methodology proposed by Morris (1991) and described in detail in section 5.1.2 using an algorithm consisting of a modification of the MATLAB code developed by Eikos (2013).

The efficient design proposed by Morris (1991) creates $r$ trajectories of $(k + 1)$ points
Figure 5.6. $E_{RT}$ evolution for the baseline curve (plate material) and for the level curves used to perform the sensitivity analysis. Also the curve corresponding to another material batch (pipe material) is reported as reference.

in the variable space domain, each providing $k$ elementary effects, one per input factor, for a total of $r(k + 1)$ sample points. It is well known that a drawback of the original Morris method is that, although randomized, the sampling strategy may not provide sufficient coverage of the variable space, especially when dealing with a large number of input variables (Campolongo et al., 2007). To overcome this problem (Campolongo et al., 2007) suggested an improved sampling strategy creating a large number of trajectories, and selecting only a set of trajectories that results in the greatest spread throughout $\mathbf{X}$. Campolongo et al. (2007) demonstrates that this approach improves considerably the distribution of the sampled points and consequently the reliability of the EE method. However, since in the investigated case the distribution of points is not considered to be an issue due to the small number of factors involved, this approach is not employed in the current dissertation.

Before generating the list of simulations, a critical choice must be taken: the selection of the number of trajectories $r$ to be created (that is strictly linked with the number of levels $q$). Increasing the number of levels to be explored, the accuracy of sampling is only apparently enhanced. If the increment of $q$ is not coupled with the choice of an adequate value for $r$, many possible levels will not be explored wasting the computational effort. Saltelli and Annoni (2010) reports an empirical law for the determination of the minimum value of $r$ that guarantees the reliability of the elementary effects method (i.e. $r = round\{(1 + 4k)/(k + 1)\}$ for $q = 4$). In the current study (i.e. $k = 8$ and $q = 4$) the suggested value for $r$ would be 4. An acceptable compromise between accuracy and computational costs is achievable selecting $r = 5$ requiring the execution of $r(k + 1) = 45$ simulations. Following this sampling strategy, the complete list of simulations to be performed in the DOE is generated together with the corresponding list of scaling coefficients $Z$ (see Tab.5.2) that will be later used to compute the level curves.

Once the complete DOE list has been defined, the corresponding sets of parameters are created in a format that is suitable for the FE code ABAQUS. This task is easily accomplished by means of a MATLAB routine that performs the following tasks:
5.2. Setting up the sensitivity analysis

Figure 5.7. $\phi_{RT}$ evolution for the baseline curve (plate material) and for the level curves used to perform the sensitivity analysis. Also the curve corresponding to another material batch (pipe material) is reported as reference. $PSA$ is the equivalent plastic strain amplitude, $MPS$ is the equivalent mean plastic strain and $RR$ is the equivalent ratcheting rate.
Table 5.2. Scaling coefficients $Z$ used to compute the level curves for each group of factors for each simulation run.

<table>
<thead>
<tr>
<th>run</th>
<th>$Y_{RT}$</th>
<th>$E_{RT}$</th>
<th>$\phi_{RT}$</th>
<th>$\gamma_{RT}$</th>
<th>$Y_{200C}$</th>
<th>$E_{200C}$</th>
<th>$\phi_{200C}$</th>
<th>$\gamma_{200C}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$Z_{Y_{RT}}^{(3)}$</td>
<td>$Z_{E_{RT}}^{(3)}$</td>
<td>$Z_{\phi_{RT}}^{(3)}$</td>
<td>$Z_{\gamma_{RT}}^{(3)}$</td>
<td>$Z_{Y_{200C}}^{(3)}$</td>
<td>$Z_{E_{200C}}^{(3)}$</td>
<td>$Z_{\phi_{200C}}^{(3)}$</td>
<td>$Z_{\gamma_{200C}}^{(3)}$</td>
</tr>
<tr>
<td>2</td>
<td>$Z_{Y_{RT}}^{(3)}$</td>
<td>$Z_{E_{RT}}^{(1)}$</td>
<td>$Z_{\phi_{RT}}^{(3)}$</td>
<td>$Z_{\gamma_{RT}}^{(3)}$</td>
<td>$Z_{Y_{200C}}^{(3)}$</td>
<td>$Z_{E_{200C}}^{(3)}$</td>
<td>$Z_{\phi_{200C}}^{(3)}$</td>
<td>$Z_{\gamma_{200C}}^{(3)}$</td>
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<tr>
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<td>$Z_{Y_{RT}}^{(3)}$</td>
<td>$Z_{E_{RT}}^{(1)}$</td>
<td>$Z_{\phi_{RT}}^{(3)}$</td>
<td>$Z_{\gamma_{RT}}^{(3)}$</td>
<td>$Z_{Y_{200C}}^{(1)}$</td>
<td>$Z_{E_{200C}}^{(3)}$</td>
<td>$Z_{\phi_{200C}}^{(3)}$</td>
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<td>$Z_{\phi_{RT}}^{(2)}$</td>
<td>$Z_{\gamma_{RT}}^{(3)}$</td>
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<td>$Z_{E_{200C}}^{(3)}$</td>
<td>$Z_{\phi_{200C}}^{(3)}$</td>
<td>$Z_{\gamma_{200C}}^{(3)}$</td>
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<td>$Z_{E_{RT}}^{(1)}$</td>
<td>$Z_{\phi_{RT}}^{(1)}$</td>
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<td>$Z_{E_{RT}}^{(3)}$</td>
<td>$Z_{\phi_{RT}}^{(4)}$</td>
<td>$Z_{\gamma_{RT}}^{(3)}$</td>
<td>$Z_{Y_{200C}}^{(4)}$</td>
<td>$Z_{E_{200C}}^{(3)}$</td>
<td>$Z_{\phi_{200C}}^{(3)}$</td>
<td>$Z_{\gamma_{200C}}^{(3)}$</td>
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</table>

- loading the file containing the calibrated set of parameters (i.e. baseline curves),
- loading the complete list of scaling coefficients $Z$ (see Tab.5.2),
- scaling the baseline curves by means of the coefficients $Z$ (see Eq.5.9) in order to retrieve the desired level curves,
- saving the 45 parameters sets in calibration files suitable for the execution of the simulations by the FE code ABAQUS.

5.2.4. Simulations: selection and generation of the output

All the 45 simulations are performed for the 3 selected case studies feeding the 5DChabEP constitutive model with the corresponding calibration files. While a structural simulation generally gives as results a large number of outputs (displacement, strain and stress fields over the entire structure along the time history) consisting in millions of numbers, in practice only few of them are used. To perform a meaningful sensitivity analysis, it is then important to select and to extrapolate few representative outputs. The list of the selected outputs for each case study is reported later in the corresponding section.

5.2.5. Estimation of the elementary effects

For all the investigated outputs, the effect of each influential factor is estimated calculating the elementary effects by means of Eq.5.1. As shown in the next section, the analysis of the mean value and of the standard deviation of the elementary effects (see Eq.5.2-5.4) provides a powerful tool to characterize the confidence bounds of the output of the constitutive model and to identify the factors that are mostly responsible for the uncertainty in the calculations.

5.3 Application to 3 case studies

As pointed out by Asserin et al. (2011), the observations performed in a sensitivity analysis have not general validity but they must be considered connected to the corresponding
5.3. Application to 3 case studies

For that reason, general conclusions on the influence of a certain parameter on the output of the constitutive model 5DChabEP must be carefully verified on different case studies. In the current dissertation, three applicative studies are presented. The first two case studies consist of a single hexaedral element subjected to uniaxial ratcheting conditions. Since, for simple loading cases, the role played by factors can be reasonably estimated a priori, the aim of this preliminary study is to validate the entire sensitivity analysis procedure. In the third case study, more complex and computationally heavier, the same methodology is applied to the notched ring specimen subjected to complex loading conditions mimicking the boundaries typical for the primary cooling circuit of a LWR.

5.3.1. Case study A: isothermal ratcheting on a single element

In the first case study the geometry consists of a single 3D hexaedral quadratic element (i.e. ABAQUS C3D20) (see left hand side plot in Fig.4.20) subjected to uniaxial strain-controlled ratcheting with a constant temperature equivalent to 25 °C. The prescribed strain path is a superposition of an alternate strain history with an amplitude of 0.40% and a continuously increasing mean strain. The mean strain increases from 0 to 5% with a constant ratcheting step of 0.1%/cyc. For this investigation only the first 50 cycles are considered.

The outputs selected for the sensitivity analysis are:
- \(D\): accumulated damage (according to the Jiang (2000) model) after 50 cycles,
- \(p\): accumulated equivalent plastic strain after 50 cycles,
- \(W_p\): accumulated plastic work after 50 cycles,
- \(\sigma^{ampl}\): stress amplitude after 50 cycles,
- \(\sigma^{mean}\): mean stress after 50 cycles.

The mean values \(\mu^*\), \(\mu\) and the standard deviation \(\sigma\) of the distributions of the elementary effects are reported for each influential factor in Fig.5.8-5.12. For the mean statistical operators (i.e. \(\mu^*\) and \(\mu\)), on the right vertical axis of the bar plots are also represented their corresponding normalized value (i.e. \(\mu_N^*\) and \(\mu_N\)) calculated as shown in Eq.5.10:

\[
\mu_N^*[\%] = \frac{\mu^*}{\sum_{j=1}^{r(k+1)} y_j} \cdot 100\%
\]

where \(y_j\) is the output of the model corresponding to the \(j\)th run. The physical meaning of \(\mu_N^*\) is the overall influence of a factor on the selected output expressed in percentage using the average value of the output as reference. The goal of the proposed normalization of the statistical operators is to allow a direct comparison of sensitivity results corresponding to different outputs. In this way it is possible to recognize immediately which outputs are mostly affected by the factors variability. A convenient representation of the results is available ranking the factors according to the descending value of \(\mu^*\) and reporting them in Tab.5.3. As an additional information, the cells of the table are filled with a color symbolizing the relevance of each factor on the corresponding output:
- white: negligible influence on the output (i.e. \(\mu_N^* < 5\%\))
- yellow: small influence on the output (i.e. \(5\% < \mu_N^* < 10\%\))
- orange: moderate influence on the output (i.e. \(10\% < \mu_N^* < 25\%\))
- red: strong influence on the output (i.e. \(25\% < \mu_N^* < 50\%\))
- purple: huge influence on the output (i.e. \(\mu_N^* > 50\%\))

Analyzing Tab.5.3, it is evident that none of the factors corresponding to \(T=200\ °C\) are influential in this analysis. This result was expected since, in this case study, the deformation is applied keeping \(T\) constant and equivalent to 25 °C. A detailed discussion on the results of the sensitivity analysis is reported below for each output.
Chapter 5. Batch variability analysis

Figure 5.8. $\mu^*$, $\sigma$ and $\mu$ of the distribution of the EE for the output $D$ in the case study A.

Damage

Among the considered outputs, $D$ is the one exhibiting the strongest sensitivity to the input variability (see Tab.5.3). The factor having the major influence on the damage calculation is $\phi_{RT}$ (see Fig.5.8) with $\mu_N^* \sim 105\%$. The considerably high value of $\mu_N^*$ implies that the variation of $\phi_{RT}$ from the lower to its upper boundary would cause a dramatical increasing of $D$ (in the order of a factor of 3). The parameter responsible for the isotropic hardening description (i.e. $Y_{RT}$) is found to have a smaller but still strong effect on the damage accumulation ($\mu_N^* \sim 40\%$). These observations are not surprising considering that $\phi_{RT}$ and $Y_{RT}$ are the parameters responsible for the definition of the hardening behavior influencing the calculation of $D$. Although the stiffness (i.e. $E_{RT}$) has a very limited influence on the hardening behavior of the material (see Fig.5.11), this factor is observed to have a moderate impact on $D$ ($\mu_N^* \sim 15\%$). This unexpected result is explained noticing the important role played by $E_{RT}$ in the definition of the plastic work (see Fig.5.10) that, as demonstrated in section 4.3, is strictly correlated with damage for uniaxial loading conditions. Since the factor $\gamma_{RT}^{(3)}$ influences the evolution of the mean stress (see Fig.5.12), it is logical to expect that it also affects the output $D$. Nevertheless, under the prescribed loading conditions, the role played by $\gamma_{RT}^{(3)}$ in the damage accumulation is secondary ($\mu_N^* < 10\%$). Further analyses, performed imposing strain paths characterized by different ratcheting steps, are required in order to confirm the general validity of this conclusion.

Accumulated plastic strain

$\phi_{RT}$ is the factor having the major influence on the output $p$ (see Fig.5.9). The negative value observed for $\mu_N$ signifies that increasing $\phi_{RT}$ from the lower to its upper boundary, $p$ would decrease moderately ($\mu_N^* \sim 25\%$). $Y_{RT}$ and $E_{RT}$ are found to have a smaller but not negligible effect on the equivalent plastic strain accumulation ($\mu_N^* \sim 10\%$). These results are not surprising, recalling that both the stress and the stiffness have a role in the decomposition of the strain in the elastic and plastic part (see Eq.4.1-4.2). The fact that $\gamma_{RT}^{(3)}$ has no influence on the output $p$ is consistent with the expectations.
Accumulated plastic work

Among the considered outputs, $W_p$ is the one exhibiting the weakest sensitivity to the input variability (see Tab.5.3). $E_{RT}$ is the factor having the major influence on the output $W_p$ (see Fig.5.10) ($\mu^*_{N} \sim 15\%$). This result could be surprising considering that $E_{RT}$ only slightly affects the hardening behavior of the material (see Fig.5.11), but is explained recalling that the stiffness has direct role in the definition of the plastic strain. On the other hand, the effect of $Y_{RT}$ and $\phi_{RT}$ on the equivalent plastic work accumulation is secondary ($\mu^*_{N} < 8\%$). This observation is justified noticing that the expansion of the hysteresis loop area linked with the hardening is in part compensated by its contraction due to the consequent reduction of the plastic strain amplitude. As expected, the influence of $\gamma^{(3)}_{RT}$ on the output $W_p$ is negligible.

Stress amplitude

Consistently with the expectations, the factors having the strongest influence on the output $\sigma^{ampl}$ are the parameters that directly determine the amount of isotropic and kinematic hardening (i.e. $Y_{RT}$ and $\phi_{RT}$) (see Fig.5.11). Increasing the values of the factors responsible for the hardening definition from the lower to their upper boundaries, a moderate increment of $\sigma^{ampl}$ is noticed (respectively $\mu^*_{N} \sim 10\%$ and $35\%$). As anticipated, the stiffness (i.e. $E_{RT}$) does not have a direct role in the definition of the hardening and consequently its effect on the output $\sigma^{ampl}$ is found to be negligible. Finally, the evidence that $\gamma^{(3)}_{RT}$ has no influence on the output $\sigma^{ampl}$ is in agreement with the hypothesis performed during the calibration of the 5DChabEP model (i.e. the third backstress component was assumed to have a minor role in the determination of the cyclic hardening (see section 4.1.4)).

Mean stress

Differently from what expected, the responsibility of the definition of the output $\sigma^{mean}$ does not belong exclusively to $\gamma^{(3)}_{RT}$ ($\mu^*_{N} \sim 20\%$) (see Fig.5.12). $Y_{RT}$, $E_{RT}$ and above all $\phi_{RT}$ are found to affect significantly the evolution of the mean stress under ratcheting conditions
Chapter 5. Batch variability analysis

Figure 5.10. \( \mu^*, \sigma \) and \( \mu \) of the distribution of the EE for the output \( W_p \) in the case study A.

Figure 5.11. \( \mu^*, \sigma \) and \( \mu \) of the distribution of the EE for the output \( \sigma_{\text{ampl}} \) in the case study A.
5.3. Application to 3 case studies

Figure 5.12. \( \mu^*, \sigma \) and \( \mu \) of the distribution of the EE for the output \( \sigma^{mean} \) in the case study A.

(respectively \( \mu_N^* \sim 10\%, 8\% \) and 35\%). This surprising result is explained considering the strong coupling between \( \gamma^{(3)} \) and these factors having an important role in the determination the values of the internal variables. This observation underpins that an accurate calibration of \( \gamma^{(3)} \) is not sufficient to precisely determine the mean stress evolution. To guarantee a reliable prediction of \( \sigma^{mean} \), a correct reproduction of the cyclic hardening behavior is therefore required.

Summary of the sensitivity analysis

Among the considered outputs, \( D \) is the one exhibiting the strongest sensitivity to the input variability. This result confirms that, as already pointed out in the benchmark (see section 4.3), limited variations in the mechanical behavior could be responsible for big differences in the damage accumulation, motivating the development of advanced constitutive models like the one proposed in the current dissertation.

As reported in Tab.5.3, none of the factors corresponding to \( T = 200 \, ^{\circ}C \) are influential in this analysis since the deformation is applied keeping \( T \) constant and equivalent to \( 25^{\circ}C \). In general, the factor having the strongest impact on the considered outputs is \( \phi_{RT} \). This result was expected since \( \phi_{RT} \) is directly responsible for the determination of the cyclic and ratcheting-induced hardening. The only exception is the output \( W_p \): in this case the expansion of the hysteresis loop area linked with the hardening (and therefore with \( \phi_{RT} \)) is in part compensated by its contraction due to the consequent reduction of the plastic strain amplitude. While the stiffness (i.e. \( E_{RT} \)) is found to have a limited influence on the cyclic hardening response calculation, this factor influences considerably all the other outputs. This observation underpins the importance to precisely determine \( E_{RT} \) to guarantee an acceptable accuracy in lifetime predictions. Finally, \( \gamma_{RT}^{(3)} \) shows a limited influence on all the outputs except for \( \sigma^{mean} \). This result was expected, since in the formulation of the 5DChabEP model, \( \gamma^{(3)} \) is the parameter responsible to describe the mean stress drifting under ratcheting conditions without affecting the cyclic hardening behavior.

The fact that most of the results are in agreement with the expectations confirms the
reliability of the elementary effects method used to carry out the sensitivity analysis. Adopting the EE method, instead of the most popular variance based approach (i.e. Monte Carlo), it is possible to considerably reduce the computational cost. While the CPU-time necessary for the execution of these 45 simulations on an Intel® Xeon® E5-2643 0 is about 1 hour, the computational cost increases of a factor of 20 considering the 900 simulations required by a Monte Carlo experiment performed using Sobol’ sequence of quasi random numbers (Sobol’, 2001) (i.e. number of runs = $W(k + 1)$ with $W$ the sample size to be larger than 100 (Campolongo et al., 2007)).

### 5.3.2. Case study B: anisothermal ratcheting on a single element

In the second case study the geometry consists of a single 3D hexahedral quadratic element (i.e. ABAQUS C3D20) (see left hand side plot in Fig.4.20) subjected to uniaxial strain-controlled ratcheting varying the temperature between 25 °C and 200 °C (see Fig.5.13). The prescribed strain path is a superposition of an alternate strain history with an amplitude of 0.40% and a continuously increasing mean strain. The mean strain increases from 0 to 5% with a constant ratcheting step of 0.1%/cyc. The prescribed temperature path has the same period of the alternate strain history but shows a phase delay of 90 degrees. For this investigation only the first 50 cycles are considered.

The outputs selected for this sensitivity analysis are the same previously defined for the case study A. The statistical operators $\mu^*$, $\mu$ and $\sigma$ of the distributions of the elementary effects are reported for each influential factor in Fig.5.14-5.19 together with the corresponding normalized values (i.e. $\mu^*_N$ and $\mu_N$). A convenient representation of the results is available ranking the factors according to the descending value of $\mu^*$ and reporting them in Tab.5.4. As an additional information, the cells of the table are filled with a color symbolizing the relevance of each factor on the corresponding output (the color scale is the same of the one adopted for case study A). A detailed discussion on the sensitivity analysis results is reported below for each output.

![Figure 5.13. Strain and temperature paths applied in the case study B.](image)
5.3. Application to 3 case studies

Similarly to what reported for the case study A, among the considered outputs, $D$ is the one exhibiting the strongest sensitivity to the input variability (see Tab.5.4). The elementary effects distributions for room temperature factors are very similar to the ones presented for the case study A (compare Fig.5.8 and Fig.5.14). The factor having the major influence on the output $D$ is again $\varphi_{RT}$ followed by $Y_{RT}$ (respectively $\mu_N^* \sim 105\%$ and 45%). The considerably high value of $\mu_N^*$ implies that the variation of $\varphi_{RT}$ from the lower to its upper boundary would cause a dramatical increasing of $D$ (in the order of a factor of 3).

The other room temperature factors (i.e. $E_{RT}$ and $\gamma_{RT}^{(3)}$) have a small effect on the damage accumulation ($\mu_N^* < 8\%$). Surprisingly the effect of all the 200 $^\circ$C factors on the output $D$ is limited ($\mu_N^* < 15\%$). This result is an indirect consequence of the phase shift between strain and temperature path. Because of the asymmetry of the ratcheting strain path, the larger stresses taking place during tensile loading compared to the compressive one are responsible of the faster accumulation of damage under tension (see Fig.5.15). In the same graph it is evident that, as a result of the phase shift between temperature and strain, the major part of the damage accumulated during tensile loading occurs at a low temperatures. It is therefore logical to expect that the influence of room temperature factors on $D$ is much stronger compared to the 200 $^\circ$C parameters. It is plausible to assume that further analyses, performed changing the phase shift between the strain and the temperature paths, would lead to different results affecting the ranking of the factors.

Accumulated plastic strain

The distributions of the elementary effects for room temperature factors for the output $p$ are very similar to the ones presented for the case study A (compare Fig.5.9 and Fig.5.16). In addition, in this case study, qualitatively comparable distributions are also reported for 200 $^\circ$C factors. The hardening parameters $\varphi_{RT}$ and $\varphi_{200C}$ are the factors having the major influence on the output $p$ (see Fig.5.16) with $\mu_N^* \sim 11\%$. The relevance of the factors $E_{RT}$, $Y_{RT}$, $E_{200C}$ and $Y_{200C}$ on the equivalent plastic strain accumulation is found to be smaller.
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Figure 5.15. Strain, temperature and damage versus time for the first 3 cycles in the case study B. The graph shows that the damage accumulated during the tensile loading is higher than during the compressive phase.

\((\mu_N^* < 6\%)\) and it can be considered negligible for \(\gamma^{(3)}_{RT}\) and \(\gamma^{(3)}_{200C}\).

Accumulated plastic work

In agreement with the previous sensitivity analysis, among the considered outputs, \(W_p\) is the one exhibiting the weakest sensitivity to the input variability (see Tab.5.4). As reported for the case study A, the stiffness (defined by means of the parameters \(E_{RT}\) and \(E_{200C}\)) plays the major role in the definition of the plastic work accumulation \((\mu_N^* \sim 5-7\%)\), taking part to the definition of the plastic strain. The factors responsible for the hardening behavior (i.e. \(\phi_{RT}, \phi_{200C}, Y_{RT}\) and \(Y_{200C}\)) are also found to have a certain but generally smaller influence on the output \(W_p\). This observation is explained noticing that the expansion of the hysteresis loop area linked with the hardening is in part compensated by its contraction due to the consequent reduction of the plastic strain amplitude. As expected, the effect of \(\gamma^{(3)}_{RT}\) and \(\gamma^{(3)}_{200C}\) on the plastic work accumulation is negligible.

Stress amplitude

The distributions of the elementary effects for room temperature factors for the output \(p\) are very similar to the ones presented for the case study A (compare Fig.5.11 and Fig.5.18). In addition, in this case study, qualitatively comparable distributions are also reported for 200 °C factors. The factors having the strongest influence on the output \(\sigma^{ampl}\) are \(\phi_{RT}\) and \(\phi_{200C}\) \((\mu_N^* \sim 15-20\%)\) (see Fig.5.18) and a smaller but not negligible role is played by \(Y_{RT}\) and \(Y_{200C}\) \((\mu_N^* \sim 4-6\%)\). These results confirm the expectations since these parameters are directly responsible for the determination of the isotropic and kinematic hardening. Consistently with what previously presented for the case study A, the effect of the remaining parameters (i.e. \(E_{RT}, E_{200C}, \gamma^{(3)}_{RT}\) and \(\gamma^{(3)}_{200C}\)) is found to be negligible.
5.3. Application to 3 case studies

Figure 5.16. $\mu^*$, $\sigma$ and $\mu$ of the distribution of the EE for the output $p$ in the case study B.

Figure 5.17. $\mu^*$, $\sigma$ and $\mu$ of the distribution of the EE for the output $W_p$ in the case study B.
Mean stress

As previously presented for the case study A, the responsibility of the definition of the output $\sigma^{\text{mean}}$ does not belong exclusively to $\gamma^{(3)}_{RT}$ or to $\gamma^{(3)}_{200C}$ ($\mu^*_N \sim 8 - 12\%$) (see Fig.5.19). All the factors are found to affect in a certain measure the evolution of the mean stress under ratcheting conditions and in particular the most influential one is $\phi_{RT}$ ($\mu_N^* \sim 65\%$) followed by $\phi_{200C}$ ($\mu_N^* \sim 30\%$). This surprising result is explained considering the strong coupling between $\gamma^{(3)}$ and these factors having an important role in the determination the values of the internal variables. This observation underpins that an accurate calibration of $\gamma^{(3)}$ is not sufficient to precisely determine the mean stress evolution. To guarantee a reliable prediction of $\sigma^{\text{mean}}$, a correct reproduction of the cyclic hardening behavior is therefore required.

Summary of the sensitivity analysis

Most of the results previously presented for the case study A are confirmed by the sensitivity analysis carried out on the single element subjected to anisothermal ratcheting. The most significant difference consists in the fact that, in the case study B, the temperature variation implies that the $200\, ^\circ C$ factors have an influence on the outputs (see Tab.5.4). Also for this study case, $D$ exhibits the strongest sensitivity to the input variability, confirming the importance to develop accurate constitutive models in order provide precise lifetime predictions.

In general, the factors having the strongest impact on the considered outputs are $\phi_{RT}$ and $\phi_{200C}$. This result was expected since these parameters are directly responsible for the determination of the cyclic and ratcheting-induced hardening. The only exception is the output $W_p$: in this case the role played by the factors responsible for the definition of the stiffness (i.e. $E_{RT}$ and $E_{200C}$) is predominant. This observation underpins that a precise determination of $E_{RT}$ and $E_{200C}$ could improve the accuracy of the constitutive model and consequently of the lifetime predictions. Finally, $\gamma^{(3)}_{RT}$ and $\gamma^{(3)}_{200C}$ show a limited influence on all the outputs except for $\sigma^{\text{mean}}$. This observation was expected, since in the formulation of
5.3. Application to 3 case studies

Figure 5.19. $\mu^*$, $\sigma$ and $\mu$ of the distribution of the EE for the output $\sigma^\text{mean}$ in the case study B.

Table 5.4. Factors ranking for the study case B.

<table>
<thead>
<tr>
<th>$E_{RT}$</th>
<th>$p$</th>
<th>$W_p$</th>
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<th>$\sigma^\text{mean}$</th>
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</table>

The 5DChabEP model $\gamma^{(3)}$ is the parameter responsible to describe the mean stress drifting under ratcheting conditions without affecting the cyclic hardening behavior.

The good agreement between results and expectations confirms the reliability of the elementary effects method used to carry out the sensitivity analysis. A corroboration of the presented conclusions requires anyway further analyses to be performed changing the phase shift between the strain and the temperature paths in order to check whether the influence attributed to the each factor remains similar or varies. The 45 simulations performed for this sensitivity analysis required a total CPU-time of about 1 hour on a Intel® Xeon® E5-2643 0 allowing a reduction of the computational cost of about a factor of 20 with respect to an equivalent Monte Carlo experiment carried out using Sobol’ sequence of quasi random numbers (Sobol’, 2001).

5.3.3. Case study C: notched ring subjected to thermal shocks

The third case study consists of the notched ring specimen subjected to a sequence of triangular thermal shocks (a detailed description is given in section 4.3.2). The thermal shocks are applied directly on the component varying the temperature of the fluid that is in contact with the inner walls of the specimen. The triangular temperature profile imposed
to the fluid consists of a slow (i.e. 3600 s) heating up from 50 to 150 °C followed by a fast cooling (i.e. 30 s) down to 50 °C (see red curve in Fig.4.53). The notched ring specimen has been modeled by a 2D mesh, consisting of 3288 plane strain elements showing a refinement in correspondence of the notch in order to capture the local distribution of the stress gradients (see Fig.4.54). The large amount of CPU-time required by each FE simulation, limits for practical reasons the current investigation to the first 30 thermo shocks.

The analysis of the simulated data output is carried out computing the following outputs in correspondence of the centroid of the element 640 (see Fig.5.28) positioned at the notch tip:

- $D$: accumulated damage (according to the Jiang (2000) model) after 30 thermo shocks,
- $p$: accumulated equivalent plastic strain after 30 thermo shocks,
- $W_p$: accumulated plastic work after 30 thermo shocks,
- $\sigma^{\text{ampl}}_{yy}$: stress amplitude in $y$ direction after 30 thermo shocks,
- $\sigma^{\text{mean}}_{yy}$: mean stress in $y$ direction after 30 thermo shocks,
- $\varepsilon^{\text{pl,ampl}}_{yy}$: plastic strain amplitude in $y$ direction after 30 thermo shocks,
- $\varepsilon^{\text{pl,mean}}_{yy}$: mean plastic strain in $y$ direction after 30 thermo shocks.

where the direction $y$ is perpendicular to the plane of the notch slot (see Fig.5.28).

The statistical operators $\mu^*$, $\mu$ and $\sigma$ of the distributions of the elementary effects are reported for each influential factor in Fig.5.20-5.27 together with the corresponding normalized values (i.e. $\mu^*_N$ and $\mu_N$). A convenient representation of the results is available ranking the factors according to the descending value of $\mu^*$ and reporting them in Tab.5.5. As an additional information, the cells of the table are filled with a color symbolizing the relevance of each factor on the corresponding output (the color scale is the same of the one adopted for case studies A and B). A detailed discussion on the sensitivity analysis results is reported below for each one of the considered outputs.

### Damage

Similarly to what reported for the case studies A and B, the accumulated damage is the output exhibiting the strongest sensitivity to the input variability (see Tab.5.5). The factor having the major influence on the output $D$ is $\phi_{RT}$ ($\mu^*_N \sim 55\%$). The high value of $\mu^*_N$ implies that the variation of $\phi_{RT}$ from the lower to its upper boundary would cause a substantial increasing of $D$ (less than a factor of 2). This result is supported by the observation that $\phi_{RT}$ is the factor having the strongest effect on the mechanical behavior of the material (see Fig.5.24-5.27). While the effect of $E_{RT}$ on the damage accumulation is moderate for the case studies A and B, the stiffness plays a very important role in the definition of $D$ for the notched ring ($\mu^*_N \sim 25\%$). This result is explained noticing that, in the first 2 cases, the effect of $E_{RT}$ on the hardening behavior of the material is limited by the fact that the strain path is prescribed (see Fig.5.11 and 5.18). On the other hand, in the case study C, the strain path is not prescribed and the factor $E_{RT}$ has a more important part in the definition of the mean plastic strain and of the plastic strain amplitude (see Fig.5.26-5.27) that are strictly correlated with the material hardening. As expected, the role played by the factors $\phi_{200^\circ}$ and $E_{200^\circ}$ on the determination of $D$ has some analogies with the one shown by the corresponding room temperature parameters. The influence of the 200 °C factors on $D$ is found to be much stronger in the case study C with respect to the case study B. This observation can be easily explained comparing Fig.5.21 and Fig.5.15. While for the case study B, the majority of the damage is accumulated at low temperatures, in the graph in Fig.5.21 it is evident that the accumulation of $D$ is simultaneous with the variation of the temperature from 140 to 80 °C. The other factors (i.e. $Y_{RT}$, $Y_{200^\circ}$, $\gamma_{RT}^{(3)}$ and $\gamma_{200^\circ}^{(3)}$) are found to have a secondary effect on the damage accumulation.
5.3. Application to 3 case studies

Figure 5.20. $\mu^*$, $\sigma$ and $\mu$ of the distribution of the EE for the output $D$ in the case study C.

Figure 5.21. Strain, temperature and damage versus time for the 30$^{th}$ thermo shock in the case study C. The plot highlights that the accumulation of $D$ is simultaneous with the variation of the temperature from 140 to 80 °C.
Accumulated plastic strain

The elementary effects distributions for the output $p$ are analogous to the ones presented for the anisothermal case study B (compare Fig.5.16 and Fig.5.22). In both case studies the effect of the 200 °C factors on the equivalent plastic strain accumulation is found to be qualitatively and quantitatively very similar to the one measured for the corresponding room temperature parameters. $\phi_{RT}$ and $\phi_{200C}$ are the factors having the major influence on the output $p$ (see Fig.5.22) ($\mu_N^* \sim 17 - 21\%$). The sensitivity of the equivalent plastic strain accumulation is found to be smaller for the factors $E_{RT}$, $Y_{RT}$, $E_{200C}$ and $Y_{200C}$ ($\mu_N^* < 8\%$) and it can be considered negligible for $\gamma_{RT}^{(3)}$ and $\gamma_{200C}^{(3)}$.

Accumulated plastic work

The sensitivity analysis on the output $W_p$ shows some analogies with the observations previously performed for the case study B (compare Fig.5.17 and Fig.5.23). First of all, $W_p$ is one of the outputs exhibiting the weakest sensitivity to the input variability (showing a $\mu_N^* \leq 12\%$) (see Tab.5.5). In addition, the sensitivity analysis confirms that also for this case study, the stiffness (defined by means of the parameters $E_{RT}$ and $E_{200C}$) plays the major role in the definition of the plastic work accumulation ($\mu_N^* \sim 11 - 12\%$). $\phi_{200C}$ is also found to affect moderately the plastic work accumulation ($\mu_N^* \sim 10\%$) but all the other factors responsible for the hardening behavior (i.e. $\phi_{RT}$, $Y_{RT}$ and $Y_{200C}$) have a much smaller influence on the output $W_p$ ($\mu_N^* < 4\%$). This observation is explained noticing that the expansion of the hysteresis loop area linked with the hardening is in part compensated by its contraction due to the consequent reduction of the plastic strain amplitude. As expected, the effect of $\gamma_{RT}^{(3)}$ and $\gamma_{200C}^{(3)}$ on the plastic work accumulation is negligible.

Stress amplitude in $y$ direction

As first consideration, it is noticed that the distributions of the elementary effects for the stress amplitude for the 200 °C factors are qualitatively and quantitatively very similar to the
one measured for the corresponding room temperature parameters (see Fig.5.24). The factors having the stronger influence on the output $\sigma_{yy}^{ampl}$ are $\phi_{RT}$ ($\mu_N^* \sim 17\%$) and $\phi_{200C}$ ($\mu_N^* \sim 8\%$) and a smaller but non negligible role is played by $Y_{RT}$ and $Y_{200C}$. This result confirms the expectations since these parameters are directly responsible for the determination of the isotropic and kinematic hardening. The influence of stiffness (defined by means of the parameters $E_{RT}$ and $E_{200C}$) $\sigma_{yy}^{ampl}$ is found to be small ($\mu_N^* \sim 4-5\%$) but it is anyway higher than expected. This result is explained observing that, in the case study C, the strain path is not prescribed and consequently the factors $E_{RT}$ and $E_{200C}$ have a more important part in the definition of the mean plastic strain and of the plastic strain amplitude (see Fig.5.26-5.27) that are strictly correlated with the material hardening. Consistently with what previously presented for the case studies A and B, the effect of the remaining parameters (i.e. $\gamma_{RT}^{(3)}$ and $\gamma_{200C}^{(3)}$) is found to be negligible.

**Mean stress in y direction**

As previously presented for the case studies A and B, the responsibility of the definition of the output $\sigma_{yy}^{mean}$ does not belong exclusively to $\gamma_{RT}^{(3)}$ or to $\gamma_{200C}^{(3)}$ ($\mu_N^* \sim 11-13\%$) (see Fig.5.25). All the factors are found to affect in a certain measure the evolution of the mean stress under ratcheting conditions and in particular the most influential one is $\phi_{200C}$ ($\mu_N^* \sim 34\%$) followed by $\phi_{RT}$ ($\mu_N^* \sim 29\%$). This surprising result, but consistent with the case studies A and B, can be explained considering the strong coupling between $\gamma^{(3)}$ and these factors having an important role in the determination the values of the internal variables. This observation underpins that an accurate calibration of $\gamma^{(3)}$ is not sufficient to precisely determine the mean stress evolution. To guarantee a reliable prediction of $\sigma_{yy}^{mean}$, a correct reproduction of the cyclic hardening behavior is therefore required.
Chapter 5. Batch variability analysis

Figure 5.24. $\mu^*$, $\sigma$ and $\mu$ of the distribution of the EE for the output $\sigma_{yy}^{\text{amp}}$ in the case study C.

Figure 5.25. $\mu^*$, $\sigma$ and $\mu$ of the distribution of the EE for the output $\sigma_{yy}^{\text{mean}}$ in the case study C.
5.3. Application to 3 case studies

Figure 5.26. $\mu^*$, $\sigma$ and $\mu$ of the distribution of the EE for the output $\varepsilon_{yy}^{pl \; ampl}$ in the case study C.

**Plastic strain amplitude in $y$ direction**

Since in the case study C the strain path is not prescribed but it is coupled to the material description, it is meaningful to carry out the sensitivity analysis considering the plastic strain amplitude in $y$ direction as output. Because of the strong coupling between plastic strain amplitude and cyclic hardening, it is not surprising that the distributions of the elementary effects for the output $\varepsilon_{yy}^{pl \; ampl}$ are analogous to the ones presented for the output $\sigma_{yy}^{ampl}$ (compare Fig.5.26 and Fig.5.24). The distributions of the elementary effects for the output $\varepsilon_{yy}^{pl \; ampl}$ for the 200 $^\circ$C factors are found to be qualitatively and quantitatively very similar to the ones measured for the corresponding room temperature parameters. The factors having the stronger influence on the output $\varepsilon_{yy}^{pl \; ampl}$ are $\phi_{RT}$ and $\phi_{200C}$ (see Fig.5.26) ($\mu^*_N \sim 17 - 22\%$) and a smaller but non negligible role is played by $Y_{RT}$ and $Y_{200C}$ ($\mu^*_N \sim 5 - 6\%$). This result confirms the expectations since these parameters are directly responsible for the determination of the hardening. The influence of stiffness (defined by means of the parameters $E_{RT}$ and $E_{200C}$) on $\varepsilon_{yy}^{pl \; ampl}$ is found to be small but it is anyway higher than expected. This result is explained observing that, in the case study C, the strain path is not prescribed and consequently the factors $E_{RT}$ and $E_{200C}$ have a more important part in the definition of the plastic strain. Consistently with what previously presented for the case studies A and B, the effect of the remaining parameters (i.e. $\gamma_{RT}^{(3)}$ and $\gamma_{200C}^{(3)}$) is found to be negligible.

**Mean plastic strain in $y$ direction**

Since in the case study C the strain path is not prescribed but it is coupled to the material description, it is meaningful to carry out the sensitivity analysis considering the amount of ratcheting (i.e. cyclic accumulation of mean plastic strain) in $y$ direction as an output. The distributions of the elementary effects for the output $\varepsilon_{yy}^{pl \; ampl}$ for the 200 $^\circ$C factors are found to be qualitatively similar to the ones measured for the corresponding room temperature parameters. Because of the strong coupling between ratcheting and hardening,
it is not surprising that the factors responsible for the hardening definition (i.e. $\phi_{RT}$, $\phi_{200C}$, $Y_{RT}$ and $Y_{200C}$) have a strong effect ($\mu^*_N \sim 18 - 37\%$) on the output $\varepsilon^{pl \; mean}_{yy}$ (see Fig.5.27). The influence of stiffness factors (i.e. $E_{RT}$ and $E_{200C}$) is also found to be important and much higher than expected ($\mu^*_N \sim 12 - 32\%$). This result is explained observing that, in the case study C, the strain path is not imposed giving to the factors $E_{RT}$ and $E_{200C}$ a more important role in the definition of the plastic strain. Finally, the effect of the remaining parameters (i.e. $\gamma^{(3)}_{RT}$ and $\gamma^{(3)}_{200C}$) on $\varepsilon^{pl \; ampl}_{yy}$ is found to be negligible.

Summary of the sensitivity analysis

Most of the results previously presented for the case studies A and B are confirmed for the sensitivity analysis carried out on the notched ring subjected to thermo shocks. The most significant differences are linked with the fact that, in the case study C, the strain path is not prescribed but it is coupled to the material description. Also for this study case, $D$ exhibits the strongest sensitivity to the input variability (see Tab.5.5) confirming the importance to develop accurate constitutive models in order provide precise lifetime predictions. In general, the factors having the strongest impact on the considered outputs are $\phi_{RT}$ and $\phi_{200C}$. This result was expected since these parameters are directly responsible for the determination of the cyclic and ratcheting-induced hardening.

The only exception is the output $W_p$: in this case the role played by the factors responsible for the definition of the stiffness (i.e. $E_{RT}$ and $E_{200C}$) is predominant. As a general remark, in the current analysis the effect of the stiffness factors on the outputs is found to be significantly stronger compared to the case studies A and B. This result is explained recalling that, in the case study C, the strain path is not prescribed and consequently the factors $E_{RT}$ and $E_{200C}$ have a more important part in the definition of the mean plastic strain and of the plastic strain amplitude (see Fig.5.26-5.27) that are strictly correlated with the material hardening. This observation underpins that a precise determination of $E_{RT}$ and $E_{200C}$ could improve the accuracy of the constitutive model and consequently of the lifetime predictions. Finally, $\gamma^{(3)}_{RT}$ and $\gamma^{(3)}_{200C}$ show a very limited influence on all the outputs except for $\sigma^{mean}$. This
5.3. Application to 3 case studies

Table 5.5. Factors ranking for the study case C.

<table>
<thead>
<tr>
<th></th>
<th>D</th>
<th>p</th>
<th>W_p</th>
<th>$\sigma_{yy}^{ampl}$</th>
<th>$\sigma_{yy}^{mean}$</th>
<th>$\varepsilon_{yy}^{pl,ampl}$</th>
<th>$\varepsilon_{yy}^{pl,mean}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{RT}$</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$Y_{RT}$</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>$\phi_{RT}$</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
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<tr>
<td>$\gamma^{(3)}_{RT}$</td>
<td>8</td>
<td>7</td>
<td>7</td>
<td>8</td>
<td>6</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>$E_{200C}$</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>$Y_{200C}$</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>7</td>
<td>6</td>
<td>5</td>
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<tr>
<td>$\phi_{200C}$</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>$\gamma^{(3)}_{200C}$</td>
<td>7</td>
<td>8</td>
<td>8</td>
<td>7</td>
<td>4</td>
<td>7</td>
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</tr>
</tbody>
</table>

observation was expected, since in the formulation of the 5DChabEP model $\gamma^{(3)}$ is the parameter responsible to describe the mean stress drifting under ratcheting conditions without affecting the cyclic hardening behavior.

The good agreement between results and expectations confirms the reliability of the elementary effects method used to carry out the sensitivity analysis. The corroboration of the presented conclusions requires anyway further analyses, to be performed changing the shape of the thermo shock profile, in order to check whether the influence attributed to the each factor remains similar or varies.

In order to confirm these conclusions, the same analysis has been carried out on an additional element (i.e. element 988) positioned at a distance of 7.5 $\mu$m from the notch tip (see Fig.5.28). The distributions of the elementary effects for the two elements are found to be qualitatively similar for all the considered outputs (compare Fig.5.29 and 5.30). As expected, moving away from the notch tip, the values of $\mu^*$ decrease of about a factor of 2, meaning that the resulting absolute variation of the outputs measured for the element 988 is half of the one observed for the element 640 (compare scatterplots on the right hand side of Fig.5.29 and 5.30). On the other hand, the very similar values measured for $\mu^*_N$ in the two elements underpin that the relative influence of the factors is comparable and confirm the validity of the conclusions previously reported for the element 640.

Also in this case the adoption of the EE method for screening purposes allows a reduction of the computational cost of a factor of 20 compared to an equivalent Monte Carlo experiment performed using Sobol’ sequence of quasi random numbers (Sobol’, 2001). While the CPU-time necessary for the execution of these 45 simulations on a Intel® Xeon® E5-2643 0 is about 135 hours, the computational cost of the corresponding Monte Carlo experiment (2700 hours) would be practically not affordable in the framework of this dissertation.
Chapter 5. Batch variability analysis

Figure 5.28. Zoom on the mesh used to discretize the ring specimen region close to the notch tip showing the location of the elements 640 and 988.

Figure 5.29. (left) $\mu^*$ distribution of the EE for the output $D$ and (right) scatterplot of the output $D$ for the factor $\phi_{RT}$ in the case study C for the element 640.

Figure 5.30. (left) $\mu^*$ distribution of the EE for the output $D$ and (right) scatterplot of the output $D$ for the factor $\phi_{RT}$ in the case study C for the element 988.
6 Conclusions and outlook

6.1 Conclusions

The current dissertation aims to characterize and to model the cyclic deformation response of the stainless steel grade 316L subjected to complex loading conditions. For this purpose, several experiments are performed to characterize the mechanical evolution of two batches of 316L under uniaxial and multiaxial conditions (including LCF and ratcheting). Subsequently, a constitutive law able to provide an accurate stress-strain relation taking into account the evolution of the hardening characteristics of the material during cycling and its dependency on strain amplitude, ratcheting and temperature is proposed. This constitutive model is characterized by a reduced complexity allowing a relatively easy calibration of its parameter and making this approach suitable for its application in an industrial context. Part of the experimental and modeling results have been published on scientific journals (Facheris and Janssens, 2013; Facheris et al., 2013; Facheris and Janssens, 2014) and have been presented in 2 international conferences. An additional scientific papers has been prepared and it is in the process to be submitted to a scientific journal (Facheris et al., -). A detailed summary of the findings documented in this dissertation is available below.

6.1.1. Uniaxial and multiaxial deformation behavior

Differently from the existing literature references on uniaxial and multiaxial tests, the ratcheting experiments carried out in this dissertation are performed controlling the axis (or the axes) in strain. This approach allows to perform a direct comparison of LCF and ratcheting data and to easily extrapolate the effect of the mean strain drifting on the material response.

Both in uniaxial and multiaxial experiments, the apparent stiffness of 316L is observed to monotonically decrease during cycling with a rate that is dependent on the prescribed strain amplitude but not on the imposed ratcheting. In addition, the equivalent yield stress exhibits a monotonic decrease during cycling when a small enough offset value (e.g. 0.0025%) is adopted for its definition. The observed decreasing rate is not particularly influenced by strain amplitude or ratcheting. These results are particularly helpful allowing a considerable reduction of complexity in the formulation of the constitutive model.
In the LCF experiments, the stress response evolution is found to be strain amplitude-dependent. Comparing uniaxial, torsional and multiaxial proportional LCF tests, the maximum equivalent stress response evolution is very similar considering experiments performed with the same equivalent strain amplitude. A significantly harder response is measured when a non-proportional loading history is imposed justifying the huge reduction of fatigue life noticed in these tests.

In uniaxial tests, ratcheting is found to have an impact both on the stress amplitude and mean stress response. The comparison of ratcheting and corresponding LCF experiments highlights that the continuous increasing of the mean strain induces an additional hardening of the stress amplitude and a drifting of the mean stress in the same direction of ratcheting. The stress response in ratcheting tests can be considered as a superposition of two hardening mechanisms: one owing to mean strain drifting (i.e. ratcheting) and another one owing to cyclic loading. A methodology is developed in order to separate the two contributions. The data analysis revealed that the additional hardening observed in strain-controlled ratcheting experiments is quasi-linearly linked with the absolute value of mean plastic strain, and that it is independent of the ratcheting direction. The analysis of the ratcheting tests data also exhibits a drift of the mean stress in the same direction of ratcheting. A higher mean stress magnitude is measured for lower strain amplitude levels.

Similar observations are also performed for multiaxial proportional tests, in which the drifting of mean strain in axial direction influences also the shear stress component response. In addition, for this loading case, the effect of ratcheting on the mechanical response is found to be quantitatively stronger. Those ratcheting-induced differences in the cyclic deformation behavior explain the lifetime reduction measured in uniaxial and multiaxial proportional experiments. On the other hand, ratcheting is found not to influence particularly neither mechanical response nor the lifetime in non-proportional experiments.

6.1.2. Time-dependent deformation behavior

In agreement with literature references, 316L is found to suffer cold creep and to show a particularly strong loading rate influence on the accumulation of inelastic strain for stress-controlled ratcheting tests. It is also observed that the very first cycles are responsible for the major part of the difference in ratcheting accumulation. After the initial cycles, the hysteresis loop shapes and the elongation rates are found to be very similar in experiments performed with different stress rates. This suggests that, in the investigated case, the time-dependent behavior can not be exhaustively explained by creep.

The necessity to further investigate this phenomenon induced the author of this dissertation to perform a set of creep+ratcheting experiments similar to the ones proposed by Taleb and Cailletaud (2011). Applying more severe loading conditions and considering a larger number of cycles, results deviating from the work of Taleb and Cailletaud (2011) are observed. In fact, it is demonstrated that, in the performed creep+ratcheting tests, the cyclic accumulation of inelastic strain is only apparently blocked by the occurring of creep. Cyclic loading triggers a different material response and slowly reactivates the strain drifting. This result suggests that creep and ratcheting are distinct phenomena and that the accumulation of the inelastic strain in cyclic tests is only in part creep. This conclusion is in contrast with the work of Taleb and Cailletaud (2011) in which most of the cyclic accumulation of the inelastic strain exhibited in their lower amplitude ratcheting tests was found to be mainly due to creep.

In the current work, the influence of loading rate in strain-controlled ratcheting tests is also analyzed. In agreement with literature, in LCF tests the stress amplitude response
is influenced by the strain rate but the loading rate effect is considerably weaker than the one measured in the stress-controlled case. The strain rate does not significantly affect the evolution of the mean stress. This result is surprising considering that the stress controlled ratcheting tests are strongly affected by the loading rate. In fact, as the stress relaxation is the ‘counterpart effect’ of creep, similarly cyclic mean stress relaxation can be considered the ‘counterpart effect’ of ratcheting. These observations are particularly helpful allowing a considerable reduction of complexity in the formulation of the constitutive model.

6.1.3. Microstructural behavior

Multiple LCF and ratcheting experiments interrupted at specific cycle numbers corresponding with the different cyclic deformation response stages have been submitted to a detailed analysis of the microstructure evolution using transmission electron microscopy (TEM).

While the evolution of the microstructure of 316L noticed in LCF experiments is in agreement with the results reported in literature, ratcheting is found to promote the formation of different dislocation substructures. The main microstructural differences observed in specimens subjected to LCF and ratcheting are reported below.

Ratcheted specimens exhibit increased dislocation densities mostly in the channels and a higher fraction of fatigue dislocation structures. As dislocation motion in these channels is the main mechanism activated during cyclic deformation, the increased density in the channels can be considered responsible for the additional hardening observed during the ratcheting phase of the experiments.

Another effect of the mean strain accumulation observed in ratcheted specimens is the less planar character of the dislocation movement compared with zero mean strain low cycle fatigue. Similarly to what Gaudin and Feaugas (2004) reported for stress controlled tests, the increment of mean strain promotes the activation of multiple slip systems. In addition, compared to the zero-mean strain LCF response, the drifting mean strain in the ratcheting experiments imposes an additional strain to be carried by the microstructure. As a consequence, one finds that the fatigue related dislocation structures develop more quickly and occupy a larger fraction of the microstructure in the ratcheted specimens.

Once the maximum level of mean strain is reached, the dislocation density in the channels does not further increase; dislocations are found to condense into walls and cells and their density in the channels progressively reduces. However, it is noticed that in ratcheted specimens a higher dislocation density persists through to the end of life. The higher remaining number of dislocations in the channels are potentially the reason for the remaining hardening in comparison to the zero-mean strain low cycle fatigue response.

The TEM observations on ratcheted specimens reveal the presence of polarized dislocation structures consisting of a relatively strong misorientation between the crystallographic orientation of the channels and of a preferential alignment of dislocations in channels on specific glide planes. In the experiments carried out in the current dissertation, after reaching the maximum mean strain, the dislocation structure relaxes and the dislocation density in the channels decreases. However, a strong misorientation between channels and the presence of preferentially orientated dislocations at the end of fatigue life for ratcheting tests indicate that part of the dislocation polarization remains. This observation, together with the additionally increased dislocation density, is at least in part responsible for the additional cyclic hardening during ratcheting.
As a final remark, the fact that nearly no strain-induced $\alpha'$-martensite is measured in fatigued 316L plate specimens suggests that it is reasonable to exclude the existence of a relation between cycling hardening and the phase transformation.

### 6.1.4. Constitutive model

An innovative elasto-plastic material description is proposed in this dissertation. The constitutive law is named '5DChabEP' and consists of a modification of the well-known Chaboche model (Chaboche, 1986) in which the material parameters are allowed to vary as a function of 5 internal variables. The 5 internal variables are selected in order to reproduce the dependency on temperature, strain amplitude and ratcheting.

The proposed constitutive is implemented in the commercial Finite Element code ABAQUS exploiting the already available combined kinematic/isotropic hardening material model and providing the dependencies of parameters on the internal variables as input. In order to compute the updated value of the internal variables, an innovative memory surface model, analyzing and tracing each component of the strain separately, is proposed. The necessity to have a flexible code, allowing future modifications, led the author of this dissertation to implement the same constitutive law also by means of a User MATerial subroutine.

A calibration toolkit is developed in MATLAB with the aim of analyzing the uniaxial LCF and strain-controlled ratcheting tests performed on 316L providing a set of calibrated parameters suitable for the constitutive model 5DChabEP.

The advanced multiaxial damage criterion model proposed by Jiang (2000) and responsible for the evaluation of the damage accumulating during cycling is also implemented. The damage criterion consists in a critical plane approach that combines an energy concept and the material memory.

The descriptive and predictive capabilities of the constitutive model, coupled with the damage criterion, are evaluated under several loading conditions using as references:

- experimental data,
- the original Chaboche model,
- a modification of the 5 internal variables dependent model not taking into account the temperature dependency (i.e. 4DChabEP).

Considering uniaxial isothermal loading conditions, the 5DChabEP model provides a consistent improvement in the calculation of the cyclic material response compared to the original Chaboche formulation, thanks to the possibility of the parameters to vary as a function of a set of internal variables. The proposed constitutive law is able to quantitatively and qualitatively describe the complex cyclic behavior of 316L including the strain amplitude dependency and the effect of ratcheting. The comparison with simulations carried out by means of the 4DChabEP model shows that the implementation of the temperature dependency in the material description is a necessary requirement to improve the accuracy of simulations. Thanks to the improved description of the hysteresis loop shapes, the 5DChabEP model allows a more precise estimation of the per-cycle increment of plastic strain and plastic work that is a crucial aspect to be considered in order to obtain reliable lifetime prediction with the adopted damage criterion. For the majority of the considered room temperature experiments, the lifetime predictions provided by the Jiang criterion are acceptable and the improvement in the material description provided by 5DChabEP generally enhances the precision of the computed $N_f$ (i.e. number of cycles to failure). The Jiang criterion demonstrates a promising potentiality in the lifetime assessment also for experiments performed at 200 °C but in this case a specific calibration for higher temperatures is required to obtain reasonable results.
The predictive capability of the 5DChabEP material description is found to be extremely satisfactory also for torsional experiments for all the considered strain amplitude levels. The accurate reproduction of the cyclic hardening and the precise calculations of the per-cycle increment of plastic strain and plastic work obtained by means of the 5DChabEP model allows the Jiang criterion to reliably estimate the $N_f$.

The multiaxial experiments show an acceptable performance of the 5 internal variables dependent model only for the proportional LCF case. All the other tension-torsion tests demonstrate that a further modification of the 5DChabEP constitutive law is required to improve the description of the material behavior under multiaxial loading conditions including ratcheting and non-proportionality.

A considerable improvement in the description of the cyclic material behavior is introduced by 5DChabEP model for the stress-controlled ratcheting case. The possibility to correctly take into account the effect of the drifting of the mean strain on the hardening, causing the consequent plastic strain amplitude reduction, allows the 5DChabEP constitutive law to accurately reproduce the per-cycle accumulation of plastic work. A good agreement between the lifetime prediction given by the Smith Watson Topper criterion (Smith et al., 1970) and the one provided by the Jiang model is observed for the 5DChabEP material description but not for the original Chaboche one. This result suggests that the Jiang criterion has a promising potentiality in the lifetime assessment also for the stress-controlled loading cases and that the reliability of the $N_f$ predictions is subordinated to the utilization of an accurate constitutive model. The current analysis pointed out that, introducing a visco-plastic version of the 5DChabEP model, the computations accuracy would be improved in the first cycles in which the material shows an evident time-dependency. However, for the investigated case, this slight precision improvement would probably not justify the adoption of a time-dependent model characterized by a more complex implementation and calibration. The poor performance of the original Chaboche model underpins that a careful determination of the parameters of the constitutive law should be preferred to the implementation of the time-dependency.

The analysis of the notched ring study case confirms that the two considered constitutive models give results with substantial qualitative and quantitative differences. However, this example shows that, when stresses and strains are not directly prescribed, the additional accumulation of damage due to the occurrence of the hardening is somehow compensated by the consequent reduction of the plastic strain amplitude. As a consequence, the resulting $N_f$ returned by the Jiang criterion for simulations performed the original Chaboche model and the 5DChabEP material description can be surprisingly similar.

Finally, the 5DChabEP model calibrated on isothermal experiments shows excellent performances even under anisothermal loading conditions. The relatively accurate results obtained by means of the 4DChabEP constitutive law with respect to the original Chaboche model suggest that, in the current case, a reliable reproduction of the cyclic hardening behavior should be preferred to the implementation of the temperature dependency in order to improve the accuracy of the mechanical response and consequently of the lifetime predictions. The poor accuracy generally observed in the predictions of the $N_f$ is attributed to the fact that the Jiang criterion is calibrated exclusively on room temperature experiments.

### 6.1.5. Sensitivity analysis

A sensitivity analysis is performed to characterize the confidence bounds of the output of the constitutive model and to identify the factors that are mostly responsible for the uncertainty in the calculations. The elementary effects method is found to provide a powerful
tool to carry out a non-local sensitivity analysis with a limited computational cost compared to the most used variance based methods (e.g. Monte Carlo). The procedure adopted to set up the design of experiment by means of the elementary effects method is described in detail. The variability of the input factors is defined in a way that the resulting upper and lower boundary levels enclose the mechanical behavior of 316L considering a realistic variability of the material batches.

It must be pointed out that the observations performed in a sensitivity analysis have not general validity but they must be considered connected to the corresponding system and/or model. For that reason, general conclusions on the influence of a certain parameter on the output of the constitutive model 5DChabEP must be carefully verified on different case studies. In the current dissertation, three applicative studies are presented. The first two case studies consist of a single hexaedral element subjected to uniaxial ratcheting conditions (respectively isothermal and anisothermal). Since, for simple loading cases, the role played by factors can be reasonably estimated a priori, the aim of this preliminary study is to validate the entire sensitivity analysis procedure. In the third case study, more complex and computationally heavier, the same methodology is applied to the notched ring specimen subjected to complex loading conditions mimicking the boundaries typical for the primary cooling circuit of a light water nuclear reactor (LWR).

In the first two case studies, most of the results are in agreement with the expectations attesting the reliability of the elementary effects method used to perform the sensitivity analysis. The same qualitative observations are confirmed for the sensitivity analysis carried out on the notched ring subjected to thermo shocks. For all the case studies, among the considered outputs, the accumulated damage \( D \) is the one exhibiting the strongest sensitivity to the input variability, highlighting that limited variations in the mechanical behavior could be responsible for big differences in the damage accumulation. This observation motivates the development of advanced constitutive models like the one proposed in the current dissertation. In general, the groups of factors having the strongest impact on the considered outputs are \( \phi_{RT} \) and \( \phi_{200C} \) collecting the hardening parameters responsible for the determination of the cyclic and ratcheting-induced hardening. The only exception is the output \( W_p \) (i.e. accumulated plastic work): in this case the expansion of the hysteresis loop area linked with the hardening (and therefore with \( \phi_{RT} \) and \( \phi_{200C} \)) is in part compensated by its contraction due to the consequent reduction of the plastic strain amplitude. While the stiffness (i.e. \( E_{RT} \) and \( E_{200C} \)) is found to have a limited influence on the cyclic hardening response calculation, these groups of factors influence considerably all the other outputs. This observation underpins the importance to precisely determine the stiffness to guarantee an acceptable accuracy in lifetime predictions. Finally, the kinematic hardening parameters \( \gamma_{RT}^{(3)} \) and \( \gamma_{200C}^{(3)} \) show a limited influence on all the outputs except for the value of \( \sigma_{\text{mean}} \). This result was expected since, in the formulation of the 5DChabEP model, \( \gamma^{(3)} \) is the parameter responsible to describe the mean stress drifting under ratcheting conditions without affecting the cyclic hardening behavior.

6.2 Outlook

The results presented in this dissertation show that the proposed 5DChabEP constitutive model has in general a very good descriptive and predictive potential despite its limited complexity. On the other hand, some of the aspects and issues discussed in the current dissertation are suggested to be investigated in more detail performing the following activities.
6.2. Outlook

6.2.1. Creep or ratcheting?

Further tests should be executed applying different stress amplitude and temperature levels in order to confirm the observation reported for creep+ratcheting experiments. A more detailed investigation is required to prove that creep and ratcheting are distinct phenomena and to explain the deviations of the results presented in the current dissertation from the work of Taleb and Cailletaud (2011). The evaluation of the entity of the time-dependency of the mechanical response under different conditions provides a useful information in order to determine whether the implementation of a visco-plastic model is necessary or not.

6.2.2. Extension of the calibration range of the constitutive model

As already pointed out in the dissertation, a meaningful utilization of the constitutive model requires that the internal variables do not overtake the calibration limits reported in Tab.4.8. These limits were consciously selected in the phase of definition of the design of experiment and were imposed by some technical limitations (i.e. overtaking of the extensometer range, buckling of the specimen and reaching the maximum working temperature for the instrumentation). In order to allow a wider utilization of the material description, it is suggested to extend the calibration range of the 5DChabEP model carrying out further uniaxial LCF and ratcheting tests.

6.2.3. Improvement of the performance of the constitutive model under multiaxial loading conditions

As reported in section 4.3.1, the performance of the 5DChabEP model under multiaxial non-proportional conditions is not satisfactory. This result is due to the fact that the proposed formulation is not capable to estimate the non-proportionality degree and consequently the constitutive law has no possibility to take into account the corresponding additional hardening. A significant improvement could be achieved modifying the 5DChabEP model following the suggestions of Benallal and Marquis (1987) and Tanaka (1994) who developed tensorial theories able to evaluate the degree of non-proportionality in complex multiaxial loading histories.

6.2.4. Modification of the hardening law in order to take into account the effect of the variation of the internal variables

As reported in section 4.1.3, the constitutive model proposed in this dissertation neglects the effect of the rate of change of $\gamma(k)$ with respect to the internal variables on the evolution law for $\alpha(k)$. Since the study presented by Kühner et al. (2000) and by Kühner (2000) reports that this simplification may lead to inaccurate results for ratcheting conditions, a further development of the material description considering the effect of the rate of change of $\gamma(k)$ with respect to the internal variables is suggested as future research project.

6.2.5. Implementation of the temperature-dependency in the damage criterion

The analysis of the descriptive and predictive capability of the damage criterion reveals that the lifetime predictions returned by the Jiang model are accurate only considering experiments performed at room temperature. For tests performed at higher temperature levels, in spite of an excellent performance of the constitutive model, the damage criterion, calibrated
exclusively on room temperature experiments, systematically overestimates the number of cycles to failure. In this framework, a modification of the Jiang criterion considering the role of temperature in damage accumulation could be a promising approach to improve the accuracy of lifetime assessment.
A

Complete list of the experiments

Table A.1. Summary of the specimens used to perform uniaxial creep and creep + ratcheting experiments on 316L plate material at room temperature.

<table>
<thead>
<tr>
<th>kind of test</th>
<th>specimen name</th>
<th>temperature (°C)</th>
<th>$\sigma_{\text{hold}}$ (MPa)</th>
<th>$\sigma_{\text{amp}}$ (MPa)</th>
<th>$\sigma_{\text{mean}}$ (MPa)</th>
<th>$\dot{\sigma}$ (MPa/s)</th>
<th>notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratcheting</td>
<td>PE36</td>
<td>RT</td>
<td>-</td>
<td>-</td>
<td>400</td>
<td>30</td>
<td>200</td>
</tr>
<tr>
<td>Ratcheting</td>
<td>PE55</td>
<td>RT</td>
<td>-</td>
<td>-</td>
<td>400</td>
<td>30</td>
<td>200</td>
</tr>
<tr>
<td>Creep+Ratcheting</td>
<td>PE38</td>
<td>RT</td>
<td>400</td>
<td>2 mm</td>
<td>400</td>
<td>30</td>
<td>200</td>
</tr>
<tr>
<td>Creep+Ratcheting</td>
<td>PE37</td>
<td>RT</td>
<td>430</td>
<td>2 h</td>
<td>400</td>
<td>30</td>
<td>-</td>
</tr>
<tr>
<td>Creep+Ratcheting</td>
<td>PE38</td>
<td>RT</td>
<td>430</td>
<td>2 h</td>
<td>400</td>
<td>30</td>
<td>-</td>
</tr>
<tr>
<td>Creep+Ratcheting</td>
<td>PE56</td>
<td>RT</td>
<td>430</td>
<td>2 h</td>
<td>400</td>
<td>30</td>
<td>-</td>
</tr>
<tr>
<td>Creep+Ratcheting</td>
<td>PE64</td>
<td>RT</td>
<td>430</td>
<td>20 h</td>
<td>400</td>
<td>30</td>
<td>-</td>
</tr>
</tbody>
</table>

Table A.2. Summary of the specimens used to perform uniaxial saw tooth LCF and ratcheting experiments on 316L plate material at room temperature. The information about the observed crack type according with the definition given by Nishimura et al. (2000) is reported.

<table>
<thead>
<tr>
<th>specimen name</th>
<th>temperature (°C)</th>
<th>waveform</th>
<th>$\epsilon_{\text{amp}}$ (%)</th>
<th>$\epsilon$ (%/s)</th>
<th>cyclic period (s)</th>
<th>$\epsilon_{\text{mean}}$ (%)</th>
<th>$\xi$ (%/cycle)</th>
<th>Nr.</th>
<th>crack type</th>
<th>notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>PE42</td>
<td>RT</td>
<td>saw tooth</td>
<td>1.00</td>
<td>0.5-0.05</td>
<td>44</td>
<td>0</td>
<td>-</td>
<td>1378</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>PE43</td>
<td>RT</td>
<td>saw tooth</td>
<td>1.00</td>
<td>0.5-0.05</td>
<td>44</td>
<td>0.5</td>
<td>0.1</td>
<td>1527</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>PE44</td>
<td>RT</td>
<td>saw tooth</td>
<td>1.00</td>
<td>0.5-0.05</td>
<td>44</td>
<td>0.5</td>
<td>0.1</td>
<td>1267</td>
<td>C</td>
<td></td>
</tr>
<tr>
<td>PE58</td>
<td>RT</td>
<td>saw tooth</td>
<td>0.40</td>
<td>0.5-0.05</td>
<td>17.6</td>
<td>0.5</td>
<td>0.1</td>
<td>-</td>
<td>-</td>
<td>interrupted</td>
</tr>
<tr>
<td>PE59</td>
<td>RT</td>
<td>saw tooth</td>
<td>0.40</td>
<td>0.05-0.005</td>
<td>176</td>
<td>0.5</td>
<td>0.1</td>
<td>-</td>
<td>-</td>
<td>interrupted</td>
</tr>
</tbody>
</table>

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Table A.3. Summary of the specimens used to perform uniaxial interrupted LCF and ratcheting experiments on 316L plate material at room temperature.

<table>
<thead>
<tr>
<th>test designation</th>
<th>specimen name</th>
<th>temperature (°C)</th>
<th>(\varepsilon_{\text{amp}}) (%)</th>
<th>(\varepsilon_{\text{mean}}) (%)</th>
<th>(\Delta\varepsilon) (%/cycle)</th>
<th>Nf</th>
<th>crack type notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>AX-LCF-RT-040</td>
<td>PE01</td>
<td>RT</td>
<td>0.40</td>
<td>0</td>
<td>-</td>
<td>21730 A</td>
<td>-</td>
</tr>
<tr>
<td>AX-LCF-RT-040</td>
<td>PE02</td>
<td>RT</td>
<td>0.40</td>
<td>0</td>
<td>-</td>
<td>21514 A</td>
<td>-</td>
</tr>
<tr>
<td>AX-LCF-RT-065</td>
<td>PE03</td>
<td>RT</td>
<td>0.65</td>
<td>0.5</td>
<td>-0.01</td>
<td>19683 B</td>
<td>-</td>
</tr>
<tr>
<td>AX-LCF-RT-070</td>
<td>PE04</td>
<td>RT</td>
<td>0.70</td>
<td>0.5</td>
<td>+0.1</td>
<td>14192 A, anomalous hardening behavior</td>
<td></td>
</tr>
<tr>
<td>AX-LCF-RT-100</td>
<td>PE05</td>
<td>RT</td>
<td>1.00</td>
<td>0.5</td>
<td>+0.1</td>
<td>13852 B</td>
<td>-</td>
</tr>
</tbody>
</table>

Table A.4. Summary of the specimens used to perform uniaxial strain-controlled LCF and ratcheting experiments on 316L plate material at room temperature. The information about the observed crack type according with the definition given by Nishimura et al. (2000) is reported.

<table>
<thead>
<tr>
<th>test name</th>
<th>specimen name</th>
<th>temperature (°C)</th>
<th>(\varepsilon_{\text{amp}}) (%)</th>
<th>(\varepsilon_{\text{mean}}) (%)</th>
<th>(\Delta\varepsilon) (%/cycle)</th>
<th>Nf</th>
<th>cycle of interruption</th>
<th>crack type notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>AX-LCF-RT-040</td>
<td>PE01</td>
<td>RT</td>
<td>0.40</td>
<td>0</td>
<td>-</td>
<td>21730 A</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>AX-LCF-RT-065</td>
<td>PE02</td>
<td>RT</td>
<td>0.40</td>
<td>0</td>
<td>-</td>
<td>21514 A</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>AX-LCF-RT-070</td>
<td>PE03</td>
<td>RT</td>
<td>0.40</td>
<td>0</td>
<td>-</td>
<td>19683 B</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>AX-LCF-RT-085</td>
<td>PE04</td>
<td>RT</td>
<td>0.40</td>
<td>0</td>
<td>-</td>
<td>14192 A, anomalous hardening behavior</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AX-LCF-RT-100</td>
<td>PE05</td>
<td>RT</td>
<td>0.40</td>
<td>0</td>
<td>-</td>
<td>13852 B</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>AX-LCF-RT-110</td>
<td>PE06</td>
<td>RT</td>
<td>0.40</td>
<td>0</td>
<td>-</td>
<td>1336 B</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>AX-LCF-RT-120</td>
<td>PE07</td>
<td>RT</td>
<td>0.40</td>
<td>0</td>
<td>-</td>
<td>1208 C</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

Table A.5. Summary of the specimens used to perform uniaxial strain-controlled LCF and ratcheting experiments on 316L plate material at 200°C. The information about the observed crack type according with the definition given by Nishimura et al. (2000) is reported.

<table>
<thead>
<tr>
<th>specimen name</th>
<th>temperature (°C)</th>
<th>(\varepsilon_{\text{amp}}) (%)</th>
<th>(\varepsilon_{\text{mean}}) (%)</th>
<th>(\Delta\varepsilon) (%/cycle)</th>
<th>Nf</th>
<th>cycle of interruption</th>
<th>crack type notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>AX-LCF-200-040</td>
<td>PE01</td>
<td>200</td>
<td>0.40</td>
<td>0</td>
<td>-</td>
<td>20000 A</td>
<td></td>
</tr>
<tr>
<td>AX-LCF-200-040</td>
<td>PE02</td>
<td>200</td>
<td>0.40</td>
<td>0</td>
<td>-</td>
<td>17031 A</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-200-040</td>
<td>PE24</td>
<td>200</td>
<td>0.40</td>
<td>0.5</td>
<td>-0.01</td>
<td>9691 C</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-200-065</td>
<td>PE25</td>
<td>200</td>
<td>0.65</td>
<td>0</td>
<td>-</td>
<td>4375 A</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-200-070</td>
<td>PE26</td>
<td>200</td>
<td>0.70</td>
<td>0.5</td>
<td>+0.1</td>
<td>14410 B</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-200-085</td>
<td>PE27</td>
<td>200</td>
<td>0.85</td>
<td>0.5</td>
<td>+0.1</td>
<td>3090 B</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-200-090</td>
<td>PE28</td>
<td>200</td>
<td>0.95</td>
<td>0.5</td>
<td>+0.1</td>
<td>701 B</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-200-100</td>
<td>PE29</td>
<td>200</td>
<td>1.00</td>
<td>0.5</td>
<td>+0.1</td>
<td>740 B</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-200-105</td>
<td>PE30</td>
<td>200</td>
<td>1.05</td>
<td>0.5</td>
<td>+0.1</td>
<td>408 C</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-200-110</td>
<td>PE31</td>
<td>200</td>
<td>1.10</td>
<td>0.5</td>
<td>+0.1</td>
<td>313 C</td>
<td></td>
</tr>
</tbody>
</table>

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Table A.6. Summary of the specimens used to perform uniaxial strain-controlled LCF and ratcheting experiments on 316L pipe material at room temperature. The information about the observed crack type according to the definition given by Nishimura et al. (2000) is reported.

<table>
<thead>
<tr>
<th>test name</th>
<th>specimen name</th>
<th>temperature (°C)</th>
<th>$\varepsilon_{\text{amp}}$ (%)</th>
<th>$\varepsilon_{\text{mean}}$ (%)</th>
<th>$\xi$ (%)</th>
<th>$N_f$</th>
<th>crack type</th>
<th>notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>AX-LCF-RT-040</td>
<td>BE223</td>
<td>RT</td>
<td>0.40</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>AX-LCF-RT-040</td>
<td>BE213</td>
<td>RT</td>
<td>0.40</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>AX-LCF-RT-040</td>
<td>BE216</td>
<td>RT</td>
<td>0.40</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-RT-040</td>
<td>BE226</td>
<td>RT</td>
<td>0.40</td>
<td>0.5</td>
<td>-0.01</td>
<td>15415</td>
<td>C</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-RT-040</td>
<td>BE128</td>
<td>RT</td>
<td>0.40</td>
<td>0.5</td>
<td>-0.01</td>
<td>11510</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-RT-040</td>
<td>BE225</td>
<td>RT</td>
<td>0.40</td>
<td>0.5</td>
<td>+0.01</td>
<td>6051</td>
<td>A</td>
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</tr>
<tr>
<td>AX-RAT-RT-040</td>
<td>BE116</td>
<td>RT</td>
<td>0.40</td>
<td>0.5</td>
<td>+0.01</td>
<td>11362</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-RT-040</td>
<td>BE220</td>
<td>RT</td>
<td>0.40</td>
<td>0.5</td>
<td>+0.01</td>
<td>2304</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-RT-040</td>
<td>BE33</td>
<td>RT</td>
<td>0.65</td>
<td>0.5</td>
<td>+0.01</td>
<td>2304</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td>AX-LCF-RT-065</td>
<td>BE125</td>
<td>RT</td>
<td>0.65</td>
<td>0</td>
<td>-</td>
<td>4328</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>AX-LCF-RT-065</td>
<td>BE211</td>
<td>RT</td>
<td>0.65</td>
<td>0</td>
<td>-</td>
<td>3137</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-RT-065</td>
<td>BE208</td>
<td>RT</td>
<td>0.65</td>
<td>0.5</td>
<td>-0.01</td>
<td>2658</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-RT-065</td>
<td>BE221</td>
<td>RT</td>
<td>0.65</td>
<td>0.5</td>
<td>-0.01</td>
<td>2133</td>
<td>C</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-RT-065</td>
<td>BE207</td>
<td>RT</td>
<td>0.65</td>
<td>0.5</td>
<td>+0.01</td>
<td>2886</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-RT-065</td>
<td>BE210</td>
<td>RT</td>
<td>0.65</td>
<td>0.5</td>
<td>+0.01</td>
<td>2192</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-RT-065</td>
<td>BE213</td>
<td>RT</td>
<td>0.65</td>
<td>0.5</td>
<td>+0.01</td>
<td>2594</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-RT-065</td>
<td>BE217</td>
<td>RT</td>
<td>0.65</td>
<td>0.5</td>
<td>+0.01</td>
<td>2541</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-RT-065</td>
<td>BE209</td>
<td>RT</td>
<td>0.65</td>
<td>0.5</td>
<td>-0.01</td>
<td>3090</td>
<td>C</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-RT-065</td>
<td>BE218</td>
<td>RT</td>
<td>0.65</td>
<td>0.5</td>
<td>-0.01</td>
<td>3060</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td>AX-LCF-RT-100</td>
<td>BE212</td>
<td>RT</td>
<td>1.00</td>
<td>0</td>
<td>-</td>
<td>1062</td>
<td>C</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-RT-065</td>
<td>BE206</td>
<td>RT</td>
<td>1.00</td>
<td>0.5</td>
<td>-0.01</td>
<td>385</td>
<td>C</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-RT-100</td>
<td>BE202</td>
<td>RT</td>
<td>1.00</td>
<td>0.5</td>
<td>+0.01</td>
<td>992</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-RT-100</td>
<td>BE204</td>
<td>RT</td>
<td>1.00</td>
<td>0.5</td>
<td>+0.01</td>
<td>1068</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>AX-RAT-RT-100</td>
<td>BE205</td>
<td>RT</td>
<td>1.00</td>
<td>0.5</td>
<td>-0.01</td>
<td>1067</td>
<td>C</td>
<td></td>
</tr>
</tbody>
</table>

Table A.7. Summary of the testing parameters used to perform the torsional and multiaxial strain-controlled LCF and ratcheting experiments on 316L pipe material at 200°C. The information about the observed crack type according with the definition given by Nishimura et al. (2000) is reported.

<table>
<thead>
<tr>
<th>test designation</th>
<th>specimen name</th>
<th>temperature (°C)</th>
<th>$\varepsilon_{\text{amp}}$ (%)</th>
<th>$\varepsilon_{\text{mean}}$ (%)</th>
<th>$\gamma$ (%)</th>
<th>$N_f$</th>
<th>crack type</th>
<th>notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>TO-LCF-RT-040</td>
<td>PM30</td>
<td>RT</td>
<td>0.40</td>
<td>0.65</td>
<td>-</td>
<td>31400</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>TO-LCF-RT-040</td>
<td>PM31</td>
<td>RT</td>
<td>0.40</td>
<td>0.65</td>
<td>-</td>
<td>31400</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>TO-LCF-RT-040</td>
<td>PM32</td>
<td>RT</td>
<td>0.40</td>
<td>0.65</td>
<td>-</td>
<td>31400</td>
<td>A</td>
<td></td>
</tr>
<tr>
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<td>RT</td>
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<td>0.65</td>
<td>-</td>
<td>31400</td>
<td>A</td>
<td></td>
</tr>
<tr>
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<td>PM37</td>
<td>RT</td>
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<td>-</td>
<td>31400</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>TO-LCF-RT-100</td>
<td>PM38</td>
<td>RT</td>
<td>1.00</td>
<td>0.65</td>
<td>-</td>
<td>31400</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>TO-LCF-RT-100</td>
<td>PM39</td>
<td>RT</td>
<td>1.00</td>
<td>0.65</td>
<td>-</td>
<td>31400</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>TO-LCF-RT-100</td>
<td>PM40</td>
<td>RT</td>
<td>1.00</td>
<td>0.65</td>
<td>-</td>
<td>31400</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>TO-LCF-RT-065</td>
<td>PM41</td>
<td>RT</td>
<td>0.65</td>
<td>0.65</td>
<td>-</td>
<td>31400</td>
<td>A</td>
<td></td>
</tr>
<tr>
<td>TO-LCF-RT-065</td>
<td>PM42</td>
<td>RT</td>
<td>0.65</td>
<td>0.65</td>
<td>-</td>
<td>31400</td>
<td>A</td>
<td></td>
</tr>
</tbody>
</table>

Table A.8. Summary of the testing parameters used to perform the torsional and multiaxial strain-controlled LCF and ratcheting experiments. The information about the observed crack type according with the definition given by Nishimura et al. (2000) is reported.

<table>
<thead>
<tr>
<th>test designation</th>
<th>specimen name</th>
<th>temperature (°C)</th>
<th>$\varepsilon_{\text{amp}}$ (%)</th>
<th>$\varepsilon_{\text{mean}}$ (%)</th>
<th>$\gamma$ (%)</th>
<th>$N_f$</th>
<th>crack type</th>
<th>notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>TO-LCF-RT-040</td>
<td>PM30</td>
<td>RT</td>
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<td>-</td>
<td>0</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>TO-LCF-RT-040</td>
<td>PM31</td>
<td>RT</td>
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<td>0.40</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>TO-LCF-RT-040</td>
<td>PM32</td>
<td>RT</td>
<td>0.40</td>
<td>0.40</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>TO-LCF-RT-065</td>
<td>PM36</td>
<td>RT</td>
<td>0.40</td>
<td>0.40</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>TO-LCF-RT-065</td>
<td>PM37</td>
<td>RT</td>
<td>0.65</td>
<td>0.40</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>TO-LCF-RT-100</td>
<td>PM38</td>
<td>RT</td>
<td>1.00</td>
<td>0.65</td>
<td>-</td>
<td>0</td>
<td>-</td>
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</tr>
<tr>
<td>TO-LCF-RT-100</td>
<td>PM39</td>
<td>RT</td>
<td>1.00</td>
<td>0.65</td>
<td>-</td>
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</tr>
<tr>
<td>TO-LCF-RT-100</td>
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<td>RT</td>
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<td>-</td>
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<tr>
<td>TO-LCF-RT-065</td>
<td>PM41</td>
<td>RT</td>
<td>0.65</td>
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<td>-</td>
<td>0</td>
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<td></td>
</tr>
<tr>
<td>TO-LCF-RT-065</td>
<td>PM42</td>
<td>RT</td>
<td>0.65</td>
<td>0.65</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

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Implementation of the constitutive and damage model in a finite element code

In this appendix the detailed description of the procedure developed in order to implement in the commercial FE code ABAQUS the 5 internal variables dependent Chaboche model (5DChabEP) using the combined hardening plasticity model (already available in ABAQUS) and the User MATerial subroutine is documented.

B.1 ABAQUS combined hardening plasticity model

In this section the procedure developed in order to implement in ABAQUS the 5 internal variables dependent Chaboche model (5DChabEP) is documented. The implementation is quite straightforward using the combined hardening plasticity material model already available in the FE code ABAQUS.

B.1.1. How to use it?

The calibrated material description can be easily imported in ABAQUS as follows:
B.1. ABAQUS combined hardening plasticity model

1. Copying the folder ’abaqus_plugins/’ and its content in the working directory.
2. Opening the .cae model file.

B.1.2. ’FG_cycplast.lib’

The settings and the calibration parameters of the combined hardening material model are stored in the file ’FG_cycplast.lib’. The material model named ‘AISI 316L PLATE-5DChabEP-FG’ is suitable for plate material and ’AISI 316L PIPE-5DChabEP-FG’ for pipe. Both the material models contain the following keywords.

*DEPVAR

The keyword *DEPVAR defines the number of STATEV to be used (see Fig. B.1).

*USDFLD

Activating the keyword *USDFLD (USer Defined FieLD) the FE code allows the possibility to use a USDFLD subroutine (see Fig.B.2).

*EXPANSION

The keyword *EXPANSION (see Fig.B.3) defines the values of the thermal expansion coefficient as a function of the temperature.
Appendix B. Implementation of the constitutive and damage model in a finite element code

The keyword *ELASTIC (see Fig.B.4) is responsible for the definition of the elastic behavior of the material. The Young modulus $E$ and the Poisson coefficient $\nu$ are listed as a function of 2 internal variables (the accumulated plastic strain $p$ and temperature $T$). The tabular entries are provided by the calibration routine presented in section 4.1.4 and can be copied from the file 'ABA_el_parameters_316L_plate.xls' respecting the format reported in Tab.4.5.

The keyword *PLASTIC (see Fig.B.5 and B.6) is responsible for the definition of the kinematic and isotropic plastic behavior of the material according to the Chaboche model.

The Chaboche parameters $C^{(k)}$ and $\gamma^{(k)}$ are listed as a function of 5 internal variables (the accumulated plastic strain $p$, the equivalent plastic strain amplitude $\varepsilon^\text{pl}_\text{ampl}$, the equivalent mean plastic strain $\varepsilon^\text{pl}_\text{mean}$, the equivalent ratcheting step $\xi$ and the temperature $T$). The entries for the kinematic hardening parameters’ table are provided by the calibration routine presented in section 4.1.4 and can be copied from the file 'ABA_pl_kin_parameters_316L_plate.xls' (see Fig.B.5) respecting the format reported in Tab.4.7.

While, in the proposed formulation of the 5DChabEP model, the isotropic hardening parameter $Y$ is defined as a function of 2 internal variables (the accumulated plastic strain $p$ and the temperature $T$), ABAQUS requires to define it as a function of all the 5 internal variables as previously did for the kinematic hardening parameters. The entries for the isotropic hardening parameters’ table is provided by the calibration routine presented in section 4.1.4 and can be copied from the file 'ABA_pl_iso_parameters_316L_plate.xls' (see Fig.B.6) respecting the format reported in Tab.4.6.

B.1.3. ‘userSubroutines.f’

This is the main routine to call when the job is executed. This routine includes the material related subroutine ‘userSubroutines-UMAT.f’ and any other accessory routine (e.g. DISP, DLOAD, etc..)

include ‘userSubroutines-UMAT.f’
B.1. ABAQUS combined hardening plasticity model

Figure B.4. *ELASTIC keyword

Figure B.5. *PLASTIC keyword (kinematic hardening)
Appendix B. Implementation of the constitutive and damage model in a finite element code

Figure B.6. *PLASTIC keyword (isotropic hardening)

```plaintext
include 'DISP.f'
include 'DLOAD.f'
```

B.1.4. 'userSubroutines-UMAT.f'

This routine includes all the material related subroutines (in this case the routine responsible to update the internal variables and to compute the damage accumulation).

```plaintext
include 'AISI316LNLIK3CompJiangDamageTracer-ABAQUS-3D.f'
```

B.1.5. 'parameters.f'

In this file all the values of the thresholds and of the parameters used in the other routines are defined. To avoid compatibility problems when the code is run on different hardware architectures, the real variables are declared as `real(KIND = 8)` instead of `real`.

```plaintext
! NUMBERS AND CONSTANTS
real(KIND=8),parameter :: ZERO=0.d0,ONE=1.d0,TWO=2.d0,THREE=3.d0,FOUR=4.d0,SIX=6.d0,PI=3.141592653589793d0
```

B.1.6. 'AISI316LNLIK3CompJiangDamageTracer-ABAQUS-3D.f'

This USDFLD subroutine (USeR Defined FieLD) accomplishes 2 different tasks:

1. Calculating the accumulated damage according to the Jiang model (Jiang, 2000) (see detailed explanation in section 4.2).
Evaluation of the damage accumulated in the increment

The first part of the routine is dedicated to the calculation of the amount of damage accumulated at the end of the current increment. As a first task, the evolution of the memory stress \( \sigma_{mr} \) is evaluated

\[
\frac{d\sigma_{mr}}{dp} = \sqrt{\frac{3}{2} H(g)} \left( \frac{\mathbf{s}}{||\mathbf{s}||} : \mathbf{d}s \right) - c[1 - H(g)](\sigma_{mr} - \sigma_0)
\]  

with \( g = ||\mathbf{s}|| - \sqrt{\frac{3}{2}} \sigma_{mr} \leq 0 \)  

For this purpose the stress \( \sigma_{n+1} \), the deviatoric stress \( s_{n+1} \) and the deviatoric stress increment tensors \( \Delta s_{n+1} \) are evaluated.

\[
s_{n+1} = \text{dev}(\sigma_{n+1}) = \sigma_{n+1} - \frac{1}{3} \text{tr}(\sigma_{n+1})
\]

\[
\Delta s_{n+1} = s_{n+1} - s_n
\]
The norm of the deviatoric stress tensor $||\mathbf{s}||$ is computed as follows:

$$||\mathbf{s}|| = \sqrt{s:s} \quad (B.5)$$

Finally, once the plastic strain tensor information is retrieved, the evolution of the memory stress evolution $d\sigma_{mr}$ can be easily evaluated.

The damage increment $dD$ is then computed. Since Jiang is a critical plane criterion, the trial increment of damage is computed for all the defined material planes saving this information in a set of STATEV. At the end of each loading cycle, the critical plane is selected as the one causing the largest damage accumulation within that cycle. 73 material planes have been defined in order to scan the complete solid angle with an interval of 15 degrees.

$$dD = \left( \frac{\sigma_{mr}}{\sigma_0} - 1 \right)^{m} + \left( 1 + \frac{\sigma}{\sigma_f} \right) dY \quad (B.6)$$

where

$$dY = a \sigma d\varepsilon + \frac{1 - a}{2} \tau d\gamma pl \quad (B.7)$$
B1. ABAQUS combined hardening plasticity model

- compute the normal stress and shear stress on the selected plane
  \[ \sigma = n_{me}(1)^2 \cdot s_{11} + n_{me}(2)^2 \cdot s_{22} + n_{me}(3)^2 \cdot s_{33} + 2.0 \cdot n_{me}(1) \cdot n_{me}(2) \cdot s_{12} + 2.0 \cdot n_{me}(1) \cdot n_{me}(3) \cdot s_{13} + 2.0 \cdot n_{me}(2) \cdot n_{me}(3) \cdot s_{23} \]
  \[ \tau = n_{me}(1)^2 \cdot s_{13} + n_{me}(2)^2 \cdot s_{12} + n_{me}(3)^2 \cdot s_{33} + 2.0 \cdot n_{me}(1) \cdot n_{me}(2) \cdot s_{12} + 2.0 \cdot n_{me}(1) \cdot n_{me}(3) \cdot s_{13} + 2.0 \cdot n_{me}(2) \cdot n_{me}(3) \cdot s_{23} \]  
  \[ \tau = SQRT(ABS(\tau)) \]

- compute the normal strain and shear strain on the selected plane
  \[ \varepsilon_p = n_{me}(1)^2 \cdot \varepsilon_{p11} + n_{me}(2)^2 \cdot \varepsilon_{p22} + n_{me}(3)^2 \cdot \varepsilon_{p33} + n_{me}(1) \cdot n_{me}(2) \cdot \varepsilon_{p12} + n_{me}(1) \cdot n_{me}(3) \cdot \varepsilon_{p13} + n_{me}(2) \cdot n_{me}(3) \cdot \varepsilon_{p23} \]
  \[ \gamma_p = (n_{me}(1) \cdot \varepsilon_{p11} + n_{me}(2) \cdot \varepsilon_{p12} + n_{me}(3) \cdot \varepsilon_{p13})^2 + (n_{me}(1) \cdot \varepsilon_{p12} + n_{me}(2) \cdot \varepsilon_{p22} + n_{me}(3) \cdot \varepsilon_{p23})^2 + (n_{me}(1) \cdot \varepsilon_{p13} + n_{me}(2) \cdot \varepsilon_{p23} + n_{me}(3) \cdot \varepsilon_{p33})^2 - \varepsilon_p^2 \]
  \[ \gamma_p = SQRT(ABS(\gamma_p)) \cdot 2 \]

- compute \( dY \)
  \[ dY = a \cdot ABS(\sigma) \cdot ABS(\varepsilon_p) + ((1.0-a)/2) \cdot ABS(\tau) \cdot ABS(\gamma_p) \]

- remember each plane separately
  \[ statev(FIRST_FREE_SDV+17+(iDplane-1)*4+1) = \varepsilon_p \]
  \[ statev(FIRST_FREE_SDV+17+(iDplane-1)*4+2) = \gamma_p \]
  \[ statev(FIRST_FREE_SDV+17+(iDplane-1)*4+3) = \sigma \]
  \[ statev(FIRST_FREE_SDV+17+(iDplane-1)*4+4) = \tau \]

- \( \sigma \) is signed! \( \rightarrow \) it introduces a mean stress effect
  \[ dD = (smrs0^m) \cdot (1.0+\sigma/sf) \cdot dY \]

- remember the maximum damage increment
  \[ if ( dD.gt.dDmax ) then \]
  \[ deps_pMax = deps_p \]
  \[ dgamma_pMax = dgamma_p \]
  \[ dYmax = dY \]
  \[ dDmax = dD \]
  \[ iDplaneMaxD = iDplane \]

- end if

Updating of the internal variables

In this second part, the trajectory of every single component of the plastic strain tensor is analyzed and 2 different kinds of tracers are evaluated: the plastic strain amplitude and the mean plastic strain tracer. A detailed explanation of the logical scheme of the updating procedure is reported in section 4.1.5.

- code for mean plastic strain and cyclic amplitude tracer

- PC from UMAT
  \[ do i = 1,6 \]
  \[ dummy_array2(i) = STATEV(6+i) \]
  \[ end do \]

- do i = 1,6
  \[ epspTracer(i) = dummy_array2(i) \]
  \[ prevExtremumEpsp = STATEV((i-1)*9+FIRST_FREE_TRACER_SDV) \]
  \[ prevDeltaEpsp = STATEV((i-1)*9+FIRST_FREE_TRACER_SDV+3) \]
  \[ directionChangesCounted = STATEV((i-1)*9+FIRST_FREE_TRACER_SDV+2) \]
  \[ absRatchetFilteredEpsp = STATEV((i-1)*9+FIRST_FREE_TRACER_SDV+5) \]
  \[ traceTriggered = STATEV((i-1)*9+FIRST_FREE_TRACER_SDV+6) \]
  \[ avrTracerEpsp = STATEV((i-1)*9+FIRST_FREE_TRACER_SDV+7) \]
  \[ avrTracerEpapa = STATEV((i-1)*9+FIRST_FREE_TRACER_SDV+8) \]
  \[ end do \]

- if check if the epsp step is large enough to be considered
  \[ epspStep = epspTracer(i) - STATEV((i-1)*9+FIRST_FREE_TRACER_SDV+3) \]
  \[ if (ABS(epspStep) .gt. minEpspStep) then \]
  \[ currentSignEpspStep = SIGN(1.D0, epspStep) \]
  \[ if (prevSignEpspStep .eq. 0.D0) then! initialization only \]
  \[ prevSignEpspStep = currentSignEpspStep \]
  \[ traceTriggered = 1 \]
  \[ end if \]
  \[ if (currentSignEpspStep + prevSignEpspStep .lt. 0.D0) .AND. (currentSignEpspStep + prevSignEpspStep .gt. 0.D0) then! opposite direction means product is -1 \]
  \[ directionChangesCounted = directionChangesCounted + 1 \]
  \[ compute strain amplitude \]
  \[ tracremp = (ABS(prevDeltaEpsp)+ABS(STATEV((i-1)*9+FIRST_FREE_TRACER_SDV+3)) + \]
  \[ prevExtremumEpsp)/4.D0 - STATEV((i-1)*9+FIRST_FREE_TRACER_SDV+4) \]
  \[ avrTracerEpsp = STATEV((i-1)*9+FIRST_FREE_TRACER_SDV+8) + tracremp \]
  \[ update Delta and Extremum values \]
  \[ prevDeltaEpsp = STATEV((i-1)*9+FIRST_FREE_TRACER_SDV+3) - prevExtremumEpsp \]
  \[ prevExtremumEpsp = STATEV((i-1)*9+FIRST_FREE_TRACER_SDV+3) \]
  \[ prevSignEpspStep = currentSignEpspStep \]
  \[ end if \]
  \[ if (directionChangesCounted .gt. 1) then! ignore the first cycle \]
  \[ if (ABS(epspTracer(i) - prevExtremumEpsp) .gt. ABS(STATEV((i-1)*9+FIRST_FREE_TRACER_SDV+3)) \]
  \[ traceTriggered = 1 \]
  \[ avrTracerEpsp = prevExtremumEpsp + (prevExtremumEpsp)/4.D0 - avrTracerEpsp \]
  \[ end if \]
  \[ if (traceTriggered .eq. 1) then \]
  \[ currentSignEpspStep = epspStep \]
  \[ else \]
  \[ currentSignEpspStep = (ABS(epspTracer(i)-prevExtremumEpsp)+ABS(prevDeltaEpsp)) \]
  \[ traceTriggered = 1 \]
  \[ end if \]
  \[ avrTracerEpsp = avrTracerEpsp + tracremp \]
  \[ end if \]
Appendix B. Implementation of the constitutive and damage model in a finite element code

```fortran
else
  traceTriggered = 0
end if
do i = 1, 6
  dummy_array(i) = STATEV((i-1)*9+FIRST_FREE_TRACER_SDV+8)
end do
aveTracerEpspa = vMisesEqStrain(dummy_array,NDI,NSHR)
FIELD(FIRST_FREE_TRACER_FLD+1) = aveTracerEpspa
end if
if ( peeq .eq. 0.d0 ) then
  RatStep = 0.d0
else
  RatStep = aveTracerEpsp / peeq * 4.d0 * aveTracerEpspa
end if
FIELD(FIRST_FREE_TRACER_FLD+2) = RatStep
```

When the tracers corresponding to the 6 plastic strain components have been computed, an equivalent tracer is evaluated by the mean of the function 'vMisesEqStrain'. The equivalent mean plastic strain $\varepsilon_{pl}^{\text{mean}}$ is then saved in FV3 and the equivalent plastic strain amplitude $\varepsilon_{pl}^{\text{ampl}}$ is saved in FV4. Finally the third equivalent tracer $\xi$ (equivalent ratcheting step) is evaluated and its value is saved in FV5.

$$\xi = \frac{\varepsilon_{pl}^{\text{mean}}}{p} \cdot 4\varepsilon_{pl}^{\text{ampl}}$$  \hspace{1cm} (B.8)

The function 'vMisesEqStrain' evaluates the second invariant of a strain tensor.

$$\varepsilon_{eq} = \sqrt{2/3 \cdot \varepsilon : \varepsilon}$$  \hspace{1cm} (B.9)

```fortran
real(KIND=8) function vMisesEqStrain(input_array,NDI,NSHR)
  ! remember that ABAQUS provides strains in Voigt notation. 
  res = 0.d0
  do i=1,NDI
    res = res + input_array(i)*input_array(i)
  end do
  do i=NDI+1,NSHR + NDI
    res = res + input_array(i)*input_array(i)/2.d0
  end do
  vMisesEqStrain = dsqrt(2.D0*res/3.d0)
end function vMisesEqStrain
```
B.2 UMAT model

In this section the procedure developed in order to implement in the FE code ABAQUS the 5 internal variables dependent Chaboche model (5DChabEP) is documented. The implementation is carried out by means of the User MATerial subroutine (UMAT) following the modification of the Kobayashi integration scheme (Kobayashi and Ohno, 2002) suggested by Akamatsu et al. (2008).

B.2.1. How to use it?

The material description can be easily imported in ABAQUS as follows:

1. Copying the folder 'abaqus_plugins/' and its content in the working directory.
2. Opening the .cae model file.
3. Importing the material model 'AISI 316L-UMAT-5DChabEP-FG' contained in the material library 'FG_cycplast.lib'.

B.2.2. 'FG_cycplast.lib'

The settings of the UMAT material model are stored in 'FG_cycplast.lib'. The UMAT material model contains the following keywords.

*DEPVAR

The keyword *DEPVAR defines the number of STATEV to be used (see Fig.B.1).
Appendix B. Implementation of the constitutive and damage model in a finite element code

*USDFLD

Activating the keyword *USDFLD (USer Defined FieLD), the FE code allows the possibility to use a USDFLD subroutine (see Fig.B.2).

*EXPANSION

The keyword *EXPANSION (see Fig.B.3) defines the values of the thermal expansion coefficient as a function of the temperature.

*UMAT

Activating the keyword *UMAT (User MATerial), the FE code allows the possibility to use a UMAT subroutine (see figure B.7). No mechanical properties must be defined here since the calibration file is loaded by the UEXTERNALDB subroutine (see section B.2.8).

B.2.3. ‘userSubroutines.f’

This is the main routine to call when the job is executed. This routine includes the material related subroutine ‘userSubroutines-UMAT.f’ and any other accessory routine (e.g. DISP, DLOAD, etc.).

```haskell
include 'userSubroutines-UMAT.f'
... include 'DISP.f'
include 'DLOAD.f'
...```

B.2.4. ‘userSubroutines-UMAT.f’

This routine includes all the materialrelated subroutines.

```haskell
include ’AISI316LBL183compJiangDamageTracer-UMAT-3D.f’
include ’userSubroutines-UMAT-2013.f’
```

B.2.5. ‘userSubroutines-UMAT-2013.f’

This routine includes all the UMAT related subroutines.

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B.2. UMAT model

include 'auxiliaryfunctions.f'
include 'interpolationroutines.f'
include 'UMAT-kobayashi-5DChabEP-anisothermal.f'

B.2.6. 'parameters.f'

All the values of the thresholds and of the parameters used in the other routines are defined in this file. The user can edit these values in order to modify the settings of the constitutive model. The meaning of every single parameter will be explained in the following sections. To avoid compatibility problems when the code is run on different hardware architectures, the real variables are declared as real(KIND = 8) instead of real.

! NUMBERS AND CONSTANTS
real(KIND=8), parameter :: ZERO=0.d0, ONE=1.d0, TWO=2.d0, THREE=3.d0, FOUR=4.d0, SIX=6.d0, PI=3.141592653589793d0
!
! PARAMETERS OF THE PLASTICITY MODEL
real(KIND=8), parameter :: tolerance_1=1.d-5, & ! iterative model tolerance
yield_tolerance=1.0001d0, & ! yield stress tolerance
interpolation_threshold=1.d-12 & ! interpolation threshold
integer, parameter :: n_max_iterations=100, & ! max number of iterations allowed
n_component=3, & ! number of backstress components
jacobian_selection=2 ! kind of jacobian selected (1=sawyer, 2=kobayashi)
integer, parameter :: cmnblck_x1_dim=100, & ! max dimension of the common blocks x1(To increase if necessary)
cmnblck_x2_dim=6, & ! max dimension of the common blocks x2(To increase if necessary)
cmnblck_x3_dim=2, & ! max dimension of the common blocks x3(To increase if necessary)
cmnblck_x4_dim=3, & ! max dimension of the common blocks x4(To increase if necessary)
cmnblck_x5_dim=2 ! max dimension of the common blocks x5(To increase if necessary)
real(KIND=8) max_lin_error(3)
data max_lin_error(1) /0.50d0/ !VERY IMPORTANT PARAMETER - max lin error backstress component 1
data max_lin_error(2) /0.50d0/ !VERY IMPORTANT PARAMETER - max lin error backstress component 2
data max_lin_error(3) /0.50d0/ !VERY IMPORTANT PARAMETER - max lin error backstress component 3

B.2.7. 'UMAT-kobayashi-5DChabEP-anisothermal.f'

In this section the core of the code developed in order to implement a User MATerial model for the 5DChabEP model is presented. The UMAT subroutine is suitable for 3D hexaedral and 2D plane strain elements (but not for 2D plane stress elements).

To allow an easier understanding, the source code is divided in 6 logical steps:
1. Variables declaration.
2. Loading of the information corresponding to the previous increment.
3. Updating of the model parameters.
5. Evaluation of the elasto-plastic Jacobian.

The flowchart of UMAT is reported in Fig.B.8.

Variables declaration

After the standard header of the UMAT subroutine, two files necessary for the execution of the subroutine are loaded and all the variables and common blocks are declared.
Appendix B. Implementation of the constitutive and damage model in a finite element code

DIMENSION STRESS(NTENS),STATEV(NSTATV),DDSDDE(NTENS,NTENS),&
DDSDDT(NTENS),DRPLDE(NTENS),STRAN(NTENS),DSTRAN(NTENS),TIME(2),&
PREDEF(1),DPRED(1),PROPS(NPROPS),COORDS(3),DFGRD0(3,3),&
DFGRD1(3,3)

... integer nrows1,nrows2,nrows3,nrows4,nrows5,ncomponentdata
...

Loading of the information corresponding to the previous increment

The values of the variables calculated at the previous increment are retrieved from STATEV. The adopted storage criterion is the following:

- STATEV 1-6 = elastic strain tensor $\varepsilon^{el}$ (where STATEV 1 = $\varepsilon_{11}^{el}$, ..., STATEV NDI = $\varepsilon_{33}^{el}$, STATEV NDI+1 = $\varepsilon_{12}^{el}$, etc...)
- STATEV 7-12 = plastic strain tensor $\varepsilon^{pl}$
- STATEV 13-18 = mechanical strain tensor ($\varepsilon^{mech} = \varepsilon^{el} + \varepsilon^{pl}$)
- STATEV 19 = accumulated plastic strain $p$
- STATEV 20 = yield stress $Y$
- STATEV 21 = calibration limit flag
- STATEV 22-27 = tensor of the first backstress component $\alpha^{(1)}$
- STATEV 28-33 = tensor of the second backstress component $\alpha^{(2)}$
- STATEV 34-39 = tensor of the third backstress component $\alpha^{(3)}$

As a convention, in ABAQUS all the strain tensors are expressed in Voigt notation instead of the standard one.

\[
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
2 \cdot \varepsilon_{12} \\
2 \cdot \varepsilon_{23} \\
2 \cdot \varepsilon_{31}
\end{bmatrix}
\]

As pointed out in the software manual (Abaqus, 2012), the FE code does not change the basis of the tensorial STATEV variables (e.g. plastic strain, elastic strain and backstress) from the global orientation system to the co-rotational system. Therefore, to guarantee consistent results also under large deformations and rigid body rotations, it is necessary to perform manually the basis changing. This task is accomplished imposing a rotation by means of the rotation matrix DROT and the function ROTSIG specifying if the input tensor is a strain or a stress tensor.

\[
\text{CALL ROTSIG(STATEV(1:NTENS),DROT,old_el_strain,2,NDI,NSHR)}
\]
\[
\text{CALL ROTSIG(STATEV(6+1:6+NTENS),DROT,old_pl_strain,2,NDI,NSHR)}
\]
\[
\text{old_PEEQ_strain=STATEV((3*6)+1)}
\]
\[
\text{DO K3=1,n_component}
\]
\[
\text{CALL ROTSIG(STATEV((3+K3-1)*6+3+1:(3+K3-1)*6+3+NTENS),DROT,old_backstress_component(:,K3,1,NDI,NSHR)}
\]
\[
\text{END DO}
\]

It is not necessary to change the basis of the tensors $\sigma$, $\varepsilon$ and $\Delta\varepsilon$ because this task is automatically performed by the FE code.

Updating of the model parameters

The parameters defining the mechanical behavior of the material ($Y$, $E$, $C^{(k)}$, $\gamma^{(k)}$) are not fixed constants but they are allowed to vary as a function of 5 selected internal variables.
B.2. UMAT model

Therefore their numerical values must be updated at the beginning of each integration step:

\[ C^{(k)}, \gamma^{(k)} = f(p, \varepsilon_{\text{mean}}^{pl}, \varepsilon_{\text{ampl}}^{pl}, \xi, T) \]  
\[ Y, E = f(p, T) \]  

In order to retrieve the updated parameters (and their derivatives), a linear interpolation is performed by means of two functions called ‘INTERPOLATION2DDER’ and ‘INTERPOLATION5DDER’ (see section B.2.9) using as input the calibrated parameters and the updated values of the internal variables. The index \( n \) corresponds to the start of the increment and \( n + 1 \) to the end of the increment.

\[ E = \text{INTERPOLATION2DDER}(p_n, T_{n+1}) \]  
\[ Y, \frac{dY}{dp} = \text{INTERPOLATION2DDER}(p_n, T_{n+1}) \]  
\[ C^{(k)}, \gamma^{(k)} = \text{INTERPOLATION5DDER}(p_n, \varepsilon_{\text{mean}}^{pl}, \varepsilon_{\text{ampl}}^{pl}, \xi, T_{n+1}) \]  
\[ C_{old} = \text{INTERPOLATION5DDER}(p_n, \varepsilon_{\text{mean}}^{pl}, \varepsilon_{\text{ampl}}^{pl}, \xi, T_n) \]

\[ \]  

The routine responsible for providing the updated values of the internal variables is ‘AISI316NLNIK3CompJiangDamageTracer-UMAT.f’ (see section B.2.10). The updated values of the 5 internal variables are stored in the following variables:

- STATEV19 = accumulated plastic strain \( p \)
- FV3 = mean equivalent plastic strain \( \varepsilon_{\text{mean}}^{pl} \)
- FV4 = equivalent plastic strain amplitude \( \varepsilon_{\text{ampl}}^{pl} \)
- FV5 = equivalent ratcheting step \( \xi \)
- TEMP = temperature \( T \)

The interpolation algorithm checks whether the calibration limits have been overtaken. In the case of overtaking, the variable ‘total_calib_limit_flag’ is set to 1 and a warning message is reported in the .msg file

\[ \]  

\[ \text{DO K1=1,5} \]  
\[ \text{total_calib_limit_flag=0} \]  
\[ \text{IF (calib_limit_flag(K1).EQ.1) THEN} \]  
\[ \text{total_calib_limit_flag=1} \]  
\[ \text{WRITE(7,*) 'WARNING - INTERPOLATION PERFORMED OUTSIDE CALIBRATION LIMIT'} \]  
\[ \text{WRITE(7,*) K1} \]  
\[ \text{END IF} \]  
\[ \text{END DO} \]  

\[ \]  

\[ \]  

Calculation of the stress response

Once all the material parameters have been updated, the stress response is evaluated adopting the following solution scheme:
1. Initially, the mechanical response is assumed to be completely elastic. In this case no plastic strain occurs and the trial stress tensor can be easily calculated by means of the elastic stiffness matrix.

2. The elastic assumption is checked.

3. If the trial stress does not fulfill the yield condition, the elastic assumption is correct and the trial stress is equivalent to the stress response.

4. If the trial stress fulfills the yield condition, the elastic assumption is wrong and the material behaves elasto-plastically. In this case, an iterative method is used to evaluate the plastic strain increment that fulfills the consistency condition. This procedure is named 'plastic correction'.

A more detailed explanation of these solution scheme is reported below.

As a first task, the strain tensor is decomposed in elastic and plastic strain assuming the increment of strain to be completely elastic.

\[ \Delta \varepsilon^{n+1} = \varepsilon^{n+1} - \varepsilon^n \] (B.16)

\[ \Delta \varepsilon^{el,n+1} = \Delta \varepsilon^{n+1} \] (B.17)

\[ \Delta \varepsilon^{pl,n+1} = 0 \] (B.18)

\[ \varepsilon^{pl,n+1} = \varepsilon^{pl,n} \] (B.19)

\[ \Delta p^{n+1} = 0 \] (B.20)

\[ p^{n+1} = p^n \] (B.21)

In this case, it is relatively easy to evaluate the stress response by the means of the elastic stiffness matrix \( D^{el} \) (or 'elastic Jacobian') provided by the subroutine JACOBIANELASTIC (see section B.2.8). The product between the 4th order Elastic stiffness tensor \( D^{el} \) and the 2nd order elastic strain tensor \( \varepsilon^{el,n+1} \) provides the 2nd order trial stress tensor \( \sigma^{*,n+1} \).

\[ \sigma^{*,n+1} = D^{el} : \varepsilon^{el,n+1} \] (B.22)

The deviatoric elastic stiffness matrix (named here as \( D^{dev,el} \)) is created by means of the subroutine JACOBIANDEVELASTIC (see section B.2.8). The product between the the 4th order deviatoric elastic stiffness tensor \( D^{dev,el} \) and the 2nd order elastic strain tensor \( \varepsilon^{el,n+1} \) provides the 2nd order deviatoric trial stress tensor \( s^{*,n+1} \).

\[ s^{*,n+1} = D^{dev,el} : \varepsilon^{el,n+1} \] (B.23)
B.2. UMAT model

! definition of the deviatoric elastic stiffness tensor
CALL JACOBIANDEVELASTIC(dev_el_mat_stiffness,k_mu,ndi,nshr)

! calculation of the deviatoric stress trial tensor
DO K1=1, NTENS
  trial_dev_stress(K1)=0.d0
  DO K2=1, NTENS
    trial_dev_stress(K1)=trial_dev_stress(K1)+dev_el_mat_stiffness(K2,K1)*(STRAN(K2)+DSTRAN(K2)-pl_strain(K2))
  END DO
END DO

The trial backstress tensor $\alpha^{(k)}_{n+1}$ is then evaluated for each backstress component. Under anisothermal loading conditions, the trial backstress tensor cannot be equivalent to the backstress tensor at the previous increment but the following correction must be performed.

$$\alpha^{(k)}_{n+1} = \alpha^{(k)}_{n} \cdot C^{(k)} / C^{(k)}_{old} \quad (B.24)$$

The total trial backstress $\alpha^{*}_{n+1}$ tensor can be easily calculated as the sum of all the trial backstress components' tensors.

$$\alpha^{*}_{n+1} = \sum_k \alpha^{(k)}_{n+1} \quad (B.25)$$

The difference between the deviatoric trial stress tensor and the trial backstress tensor $\alpha^{*}_{n+1}$ is evaluated and saved in the tensor $\beta^{*}_{n+1}$.

$$\beta^{*}_{n+1} = s^{*}_{n+1} - \alpha^{*}_{n+1} \quad (B.26)$$

The second invariant of the tensor $\beta^{*}_{n+1}$ is then computed in order to determine if the yield condition is fulfilled. The evaluation of the second invariant of a generic 2$\text{nd}$ order stress tensor is performed by means of the subroutine J2CALC (see section B.2.8).

CALL J2CALC(J_2,beta_fun,NDI,NSHR)

The yield condition is checked and
- if the yield condition is not fulfilled the material response is confirmed to be elastic and the time step increment is already solved,
- if the yield condition is fulfilled, the correct plastic response must be evaluated by means of an iterative method.

$$J_2(\beta^{*}_{n+1}) \geq Y \quad (B.27)$$

Since ABAQUS is not an analytical solver, it can happen that the increment of the accumulated plastic strain is so small to cause the algorithm to suffer numerical problems (division by a value close to zero). To avoid this event, the yield stress is multiplied by a convenient parameter named 'yield_tolerance' defined in 'parameters.f' (see section B.2.6).
Appendix B. Implementation of the constitutive and damage model in a finite element code

The value for 'yield_tolerance’ is adequately selected to introduce a negligible error in the yield condition evaluation.

\[ J_2(B_{n+1}^*) \geq Y \cdot Y_{tolerance} \]  

(B.28)

IF(J_2.GT.yield_stress*yield_tolerance)THEN
...  
!!! SOURCE CODE FOR PLASTIC CORRECTION  
...  
END IF

Before starting the iterative solution scheme, a set of variables is initialized.

! initial values
n_iterations=ONE
d_PEEQ_strain=0.0001d0
error_1=ONE
fakn=0
DO K1=1,NTENS
  kN_normal(K1)=dsqrt(THREE/TWO)*beta_fun(K1)/yield_stress
END DO
DO K3=1,n_component
  theta(K3)=ONE
END DO

The iterative solution scheme is implemented by means of a WHILE statement. The WHILE loop can be terminated only if one of the following conditions are fulfilled:

- The method reaches the convergence (i.e. \( error_1 \leq tolerance_1 \) AND \( \Delta p_{n+1} \geq 0 \)). The definition of the threshold value \( tolerance_1 \) is defined in ’parameters.f’ (see section B.2.6).

- The number of iterations \( z \) overtakes the limit defined by means of the parameter ’n_max_iterations in ’parameters.f’ (see section B.2.6).

Inside the WHILE loop the following tasks are performed:

- Updating the number of iterations \( z \).

- Saving the reference value of \( \Delta p^{(z)}_{n+1} \) at the previous iteration \( z \).

- Calling the subroutine ’RETURNMAPPINGKOBAYASHIISOTHERMAL’ that evaluates the updated value of accumulated plastic strain increments \( \Delta p^{(z+1)}_{n+1} \) fulfilling the consistency condition (see section B.2.8).

- Calling the subroutine ’AITKENACCEL’ (see section B.2.8) responsible to enhance the convergence rate of the iterative method.

- Evaluating the updated value of the error estimator ’error_1’ and checking if convergence occurs.

DO WHILE (( (error_1.GT.tolerance_1).OR.(d_PEEQ_strain.LT.ZERO)).AND.(n_iterations.LT.n_max_iterations))
  ! storing reference value for the evaluation of convergence
  old_d_PEEQ_strain=d_PEEQ_strain
  ! increase iteration counter
  n_iterations=n_iterations+1
  ! CALL return mapping algorithm Kobayashi
  CALL RETURNMAPPINGKOBAYASHIISOTHERMAL(beta_fun,d_PEEQ_strain,d_pl_strain, dYield_dx_2D,k_yield,k_c,k_c_old, k_g,k_mu,kN_normal,J_2,old_backstress_component,trial_dev_stress,theta,yield_stress,NDI,NTENS)
  ! CALL Aitken’s acceleration procedure
  CALL AITKENACCEL(d_PEEQ_strain,xakn,fakn,NTENS)
  ! calculation of the error estimator
  error_1=dabs(ONE-d_PEEQ_strain/old_d_PEEQ_strain)
END DO

If the maximum number of iterations is reached without convergence, the algorithm requires a time step reduction using the ABAQUS command PNEWDT and a warning message is reported in the .msg file.

IF(n_iterations.GE.n_max_iterations)THEN
  write(7,*) 'warning UMAT subroutine - max number of iteration reached -reduction of time increment needed'
  PNEWDT=0.75d0
END IF
On the other hand, if the convergence is correctly achieved, the updated backstress tensor is evaluated using the plastic strain increment returned by the integrations scheme.

\[
\alpha^{(k)}_{n+1} = \theta^{(k)}_{n+1} \cdot C^{(k)} \cdot (\alpha^{(k)}_n / C^{(k)}_{old} + 2/3 \cdot \Delta \varepsilon^{pl}_{n+1}) \quad (B.29)
\]

\[
\theta^{(k)}_{n+1} = \frac{1}{1 + \gamma^{(k)}_n \cdot \Delta p_{n+1}} \quad (B.30)
\]

It must be noticed that, while the ABAQUS combined hardening model takes into account the effect of the variation of the generic internal variable \( x_j \) on the computed stress response (see Eq.B.31), this is not the case for the UMAT version of the model.

\[
\dot{\alpha}^{(k)}_n = \frac{2}{3} C^{(k)}_n \dot{\varepsilon}^{pl} - \gamma^{(k)}_n \alpha^{(k)}_n \dot{p} + \sum_j \frac{1}{C^{(k)}_n} \frac{\partial C^{(k)}_n}{\partial x_j} \alpha^{(k)}_n \dot{x}_j \quad (B.31)
\]

Observing that only the internal variable \( T \) is responsible for a rapid evolution of the hardening parameters, it was decided to modify Eq.B.31 taking into account only the effect of the variation of temperature on the computed stress response. By means of this simplification, it is possible to drastically reduce the computational time without losing accuracy. The modification of the (Kobayashi and Ohno, 2002) integration scheme suggested by Akamatsu et al. (2008) allows the explicitation of Eq.B.31 for temperature and leads to Eq.B.32-B.33 and then to Eq.B.29.

\[
\alpha^{(k)}_{n+1} = \alpha^{(k)}_n + \frac{2}{3} C^{(k)}_n \Delta \varepsilon^{pl}_{n+1} - \gamma^{(k)}_n \alpha^{(k)}_n \Delta p_{n+1} + \frac{C^{(k)}_n - C^{(k)}_{old}}{C^{(k)}_{old}} \alpha^{(k)}_n \quad (B.32)
\]

\[
\alpha^{(k)}_{n+1} = \theta^{(k)}_{n+1} \cdot \left( \alpha^{(k)}_n + \frac{2}{3} C^{(k)}_n \Delta \varepsilon^{pl}_{n+1} + \frac{C^{(k)}_n - C^{(k)}_{old}}{C^{(k)}_{old}} \alpha^{(k)}_n \right) \quad (B.33)
\]

The fact that the accuracy of the stress computation is affected by the increment step size, requires to perform an additional check before accepting the solution. The reason for this increment size–dependency is due to the nature of the adopted integration algorithm linearizing the non–linear kinematic hardening laws. The following procedure is adopted to control the accuracy of the solution:

- Evaluating the value of every single updated backstress component.
- Checking if the linearization error is acceptable (see Eq.B.35).
\[
\Delta \alpha_{n+1}^{(k)} = | \alpha_{n+1}^{(k)} - \alpha_{n}^{(k)} \cdot C^{(k)} / C_{\text{old}}^{(k)} | \tag{B.34}
\]
\[
\frac{\Delta \alpha_{n+1}^{(k)}}{2/3 \cdot C^{(k)} / \gamma^{(k)}} \leq \text{max lin error}^{(k)} \tag{B.35}
\]

If the linearization error check is not fulfilled, the algorithm requires a time step reduction to improve the accuracy of the constitutive model and a warning message is reported in the .msg file. The values of the parameters 'max lin error' are defined in the 'parameters.f' file (see section B.2.6). The user is allowed to set lower values in order to enhance the model accuracy. A series of simulations showed that, setting 'max lin error' equal to 0.5, the error performed in the stress calculations is acceptable (≤ 3 %) under all the considered strain–controlled loading conditions.

The stress and strain tensors are finally updated using the plastic strain increment provided by the integration scheme.

\[
\varepsilon_{pl}^{n+1} = \varepsilon_{pl}^{n} + \Delta \varepsilon_{pl}^{n+1} \tag{B.36}
\]
\[
\varepsilon_{el}^{n+1} = \varepsilon_{n}^{n+1} - \varepsilon_{pl}^{n+1} \tag{B.37}
\]
\[
p_{n+1} = p_{n} + \Delta p_{n+1} \tag{B.38}
\]
\[
\sigma_{n+1} = D^{el} : \varepsilon_{n+1}^{el} \tag{B.39}
\]

Elasto–plastic Jacobian evaluation

To guarantee the achievement of the equilibrium, the FE code requires as input the consistent Jacobian matrix. In the case of elastic deformation, the Jacobian corresponds to the elastic stiffness matrix \( D^{el} \). On the other hand, when plastic deformation occurs, the elasto-plastic Jacobian \( D^{ep} \) must be provided. A detailed description of the subroutine responsible for the evaluation of the elasto-plastic Jacobian is available in section B.2.8.
Storage of updated values

The last task of the UMAT subroutine consists in storing the information necessary for the subsequent integration step in the STATEV variables using the convention previously presented.

```plaintext
DO K1=1,NTENS
    STATEV(K1)=el_strain(K1)
    STATEV(K1+6)=pl_strain(K1)
    STATEV(K1+(6*2))=el_strain(K1)+pl_strain(K1)
END DO
STATEV((3*6)+1)=PEEQ_strain
STATEV((3*6)+2)=yield_stress
STATEV((3*6)+3)=total_calib_limit_flag
DO K1=1,NTENS
    DO K3=1,n_component
        STATEV(K1+(6*(3+K3-1)+3))=backstress_component(K1,K3)
    END DO
END DO
```
Appendix B. Implementation of the constitutive and damage model in a finite element code

Loading initial condition from STATEV

Update parameters defining the mechanical properties
CALL INTERPOLATION2DDER and 5DDER

Evaluation of elastic stiffness and deviatoric elastic stiffness matrix
CALL JACOBIAN ELASTIC
CALL JACOBIAN DEV ELASTIC

Evaluation of trial stress and trial deviatoric stress

Verification of the yield condition

Return mapping algorithm
CALL RETURNMAPPINGKOBAANISOTHERMAL

Update Δp

Check convergence

NOT FULLFILLED

FULLFILLED

Evaluation of the updated backstress, stress and strains

Check linearization error

NOT FULLFILLED

FULLFILLED

Evaluation of the elasto-plastic Jacobian
CALL JACOBIANKOBA

Saving updated values in STATEV

Figure B.8. Flowchart of the UMAT subroutine.
B.2.8. 'auxiliaryfunctions.f'

The implemented UMAT material model requires several other subroutines contained in 'auxiliaryfunctions.f' and listed below:

1. UEXTERNALDB
2. J2CALC
3. RETURNMAPPINGKOBAANISOTHERMAL
4. AITKENACCEL
5. JACOBIANKOBA
6. JACOBIANELASTIC
7. JACOBIANDEVELASTIC
8. DYADPRODUCT
9. MIGS
10. ELGS

**UEXTERNALDB**

The purpose of this routine is to retrieve the data collected in the calibration file and to transform this information in a format that is suitable for the UMAT subroutine. UEXTERNALDB is executed only at the very first increment.

```fortran
IF(LOP.EQ.0)THEN
  ! if lop = 0 --> the if statement is fulfilled only at the 1st increment
  As a first task, the routine checks whether the calibration file exists. If the calibration
  file is not present in the subdirectory ./files, an error message is written in the .msg file.
  The user can select to load which calibration file must be loaded referring to 'parameters_316L_plate.dat' for plate material or to 'parameters_316L_pipe.dat' for pipe. Since Linux and Windows adopts different symbols to define the path (Windows uses \ and Linux uses /) this procedure is able to recognize the operating system.

  ! detect the working directory
  call getoutdir(xoutdir,lxoutdir)
  ! check if the file exist
  if ( xoutdir(1:1) .eq. '/' ) then ! linux format
    inquire (file=xoutdir(1:lxoutdir)//'/files/parameters_316L_plate.dat',exist=lexist)
    if(lexist) then
      else
        write(7,*) 'ERROR UEXTERNALDB routine:The calibration file does not exist!'
        write(7,*) 'Check that the name of the file and of the directory are correct!'
        call XIT
    end if
  else ! windows format
    inquire (file=xoutdir(1:lxoutdir)//'iles\parameters_316L_plate.dat',exist=lexist)
    if(lexist) then
      else
        write(7,*) 'ERROR UEXTERNALDB routine:The calibration file does not exist!'
        write(7,*) 'Check that the name of the file and of the directory are correct!'
        call XIT
    end if
  end if

  If the calibration file exists, the routine is allowed to open it.

  ! load calibration file
  if ( xoutdir(1:1) .eq. '/' ) then ! linux format
    open (unit=101,file=xoutdir(1:lxoutdir)//'/files/parameters_316L_plate.dat',access='sequential',status='old')
  else ! windows format
    open (unit=101,file=xoutdir(1:lxoutdir)//'iles\parameters_316L_plate.dat', access='sequential',status='old')
  endif
```

If the calibration file exists, the routine is allowed to open it.
To allow the correct loading of the data, the calibration file must respect the format reported in Tab. 4.4.

The scalar information corresponding to the number of entries for each internal variable, number of backstress components and Poisson coefficient are easily retrieved.

In order to retrieve the model parameters that are dependent on 2 or 5 internal variables, the loading procedure is more complex. The information must be reorganized in a format that is suitable for the interpolation routines. The one dimensional set of arrays $E_*, Y_*$, $C^{(k)}_*$ and $\gamma^{(k)}_*$ are transformed into a more convenient multidimensional format.

\[
E[i, m] = E_*[i \cdot m] \\
Y[i, m] = Y_*[i \cdot m] \\
C^{(k)}[i, j, k, l, m] = C^{(k)}_*[i \cdot j \cdot k \cdot l \cdot m] \\
\gamma^{(k)}[i, j, j, k, l, m] = \gamma^{(k)}_*[i \cdot j \cdot k \cdot l \cdot m]
\]

with $i = 1, ..., nr1$, $j = 1, ..., nr2$, $k = 1, ..., nr3$, $l = 1, ..., nr4$, $m = 1, ..., nr5$

Once the calibrated parameters values have been loaded, the UEXTERNALDB routine shares this information with UMAT by means of 2 common blocks (one suitable for ‘integer’ and the other one for ‘real’ variables). The common blocks have the structure reported here below:
B.2. UMAT model

Fortran requires the memory blocks used to store the ‘common blocks’ to be allocated before knowing their exact size. It is therefore necessary to postulate their dimension by means of the parameters ‘cmnblck_x1_dim’, ‘cmnblck_x2_dim’, ‘cmnblck_x3_dim’, ‘cmnblck_x4_dim’, ‘cmnblck_x5_dim’ defined in the file ‘parameters.f’ (see section B.2.6). Changing the calibration file, it could be necessary to increase the value of these parameters in order to allocate more memory.

**J2CALC**

The purpose of this routine is to evaluate the second invariant of a 2nd order stress tensor. This routine must not be used to calculate the second invariant of strain tensors (because they are expressed in Voigt notation).

\[
J_2(\boldsymbol{\beta}) = \sqrt{\frac{3}{2}} \cdot \boldsymbol{\beta} : \boldsymbol{\beta}
\]  

(B.44)

```
second_invariant=ZERO
DO K4=1,NDI
    second_invariant=second_invariant+beta_fun(K4)*beta_fun(K4)
END DO
DO K4=NDI+1,NDI+NSHR
    second_invariant=second_invariant+TWO*beta_fun(K4)*beta_fun(K4)
END DO
second_invariant=dsqrt(THREE/TWO*second_invariant)
RETURN
```

**RETURNMAPPINGKOBAAANISOTHERMAL**

The modified version of the Kobayashi and Ohno (2002) return mapping algorithm proposed by Akamatsu et al. (2008) suitable for the anisothermal case is implemented in this subroutine. For each iteration, the following tasks are sequentially performed:

1. Evaluation of the increment of accumulated plastic strain

\[
\Delta p_{n+1} = \frac{J_2(s^*_n + \Delta \theta_n, \alpha_n, C^{(k)}_{old}) - Y_{n+1}}{3G + \sum_k \theta_{n+1, k} \cdot C^{(k)}}
\]  

(B.45)
2. Update the tensor $\beta_{n+1}$

$$\beta_{n+1} = \frac{Y_{n+1}(s_{n+1}^* - \sum_k \theta^{(k)}_{n+1} \cdot \alpha^{(k)}_n)}{(3G + \sum_k \theta^{(k)}_{n+1} \cdot C^{(k)}) \cdot \Delta p_{n+1} + Y_{n+1}} \quad (B.46)$$

```
DO K1=1,NTENS
  beta_fun(K1)=(yield_stress*product_1(K1))/(yield_stress+d_PEEQ_strain*(THREE*k_mu*product_3))
END DO
CALL J2CALC(J_2,beta_fun,NDI,NTENS-NDI)
```

3. Update the flow tensor $N_{n+1}$

$$N_{n+1} = \frac{\beta_{n+1}}{J_2(\beta_{n+1})} \cdot \sqrt{3/2} \quad (B.47)$$

```
DO K1=1,NTENS
  kN_normal(K1)=dsqrt(THREE/TWO)*beta_fun(K1)/J_2
END DO
```

4. Update the plastic strain increment

$$\Delta \varepsilon_{pl}^{n+1} = \sqrt{3/2} \cdot N_{n+1} \cdot \Delta p_{n+1} \quad (B.48)$$

```
! update the plastic strain (NB: remember the voigt notation)
DO K1=1,NDI
  d_pl_strain(K1)=dsqrt(THREE/TWO)*kN_normal(K1)*d_PEEQ_strain
END DO
DO K1=NDI+1,NTENS
  d_pl_strain(K1)=dsqrt(THREE/TWO)*kN_normal(K1)*d_PEEQ_strain*TWO
END DO
```

5. Update the isotropic hardening

$$Y_{n+1} = Y + \left( \frac{dY}{dp} \right)_{n+1} \cdot \Delta p_{n+1} \quad (B.49)$$

```
yield_stress=k_yield+dYield_dx_2D(1)*d_PEEQ_strain
```

6. Update the kinematic hardening

$$\theta^{(k)}_{n+1} = \frac{1}{1 + \gamma^{(k)} \cdot \Delta p_{n+1}} \quad (B.50)$$

```
DO K3=1,n_component
  theta(K3)=ONE/(ONE+k_g(K3)*d_PEEQ_strain)
END DO
```

**AITKENACCEL**

The Aitken’s acceleration procedure (Aitken, 1926) is implemented in this subroutine in order to provide a smart value of $\Delta p^*_n$ to enhance the convergence rate of the iterative method. The smart value of $\Delta p^*_n$ is calculated after 3 consecutive iterations using the following equation:

$$\Delta p^*_n = \Delta p_{n+1}(k) - \frac{[\Delta p_{n+1}(k) - \Delta p_{n+1}(k-1)]^2}{\Delta p_{n+1}(k) - 2 \cdot \Delta p_{n+1}(k-1) + \Delta p_{n+1}(k-2)} \quad k = 3, 6, 9, ... \quad (B.51)$$
B.2. UMAT model

\[ x_{akn}(1) = x_{akn}(2) \]
\[ x_{akn}(2) = x_{akn}(3) \]
\[ x_{akn}(3) = d_{PEEQ \_strain} \]

if (fakn .ne. 3) then
  fakn = fakn + 1
elseif (dabs( x_{akn}(3) - x_{akn}(2) ) .gt. ONE - 10.d0 * d_{PEEQ \_strain}) then
  d_{PEEQ \_strain} = x_{akn}(3) - (x_{akn}(3) - x_{akn}(2)) / (ONE - (x_{akn}(2) - x_{akn}(1)) / (x_{akn}(3) - x_{akn}(2)))
  if (d_{PEEQ \_strain} .lt. ZERO) then
    d_{PEEQ \_strain} = x_{akn}(3)
  else
    x_{akn}(3) = d_{PEEQ \_strain}
  end if
end if

JACOBIANKOBA

The purpose of this subroutine is to provide the consistent elasto-plastic Jacobian according to Kobayashi and Ohno (2002). These authors provided the Jacobian formulation in the standard notation but ABAQUS requires it to be expressed in Voigt notation.

While the relation \( \sigma - \varepsilon \) in the standard notation corresponds to

\[
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{12} \\
\sigma_{23} \\
\sigma_{31}
\end{bmatrix} =
\begin{bmatrix}
D_{1111} & D_{1122} & D_{1133} & D_{1112} & D_{1123} & D_{1131} \\
D_{2211} & D_{2222} & D_{2233} & D_{2212} & D_{2223} & D_{2231} \\
D_{3311} & D_{3322} & D_{3333} & D_{3312} & D_{3323} & D_{3331} \\
D_{1211} & D_{1222} & D_{1233} & D_{1212} & D_{1223} & D_{1231} \\
D_{2311} & D_{2322} & D_{2333} & D_{2312} & D_{2323} & D_{2331} \\
D_{3111} & D_{3122} & D_{3133} & D_{3112} & D_{3123} & D_{3131}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
\varepsilon_{12} \\
\varepsilon_{23} \\
\varepsilon_{31}
\end{bmatrix}
\]

the same relation in Voigt notation can be expressed as follows

\[
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
\varepsilon_{12} \\
\varepsilon_{23} \\
\varepsilon_{31}
\end{bmatrix} = \begin{bmatrix}
D_{1111} & D_{1122} & D_{1133} & 1/2 \cdot D_{1112} & 1/2 \cdot D_{1123} & 1/2 \cdot D_{1131} \\
D_{2211} & D_{2222} & D_{2233} & 1/2 \cdot D_{2212} & 1/2 \cdot D_{2223} & 1/2 \cdot D_{2231} \\
D_{3311} & D_{3322} & D_{3333} & 1/2 \cdot D_{3312} & 1/2 \cdot D_{3323} & 1/2 \cdot D_{3331} \\
D_{1211} & D_{1222} & D_{1233} & 1/2 \cdot D_{1212} & 1/2 \cdot D_{1223} & 1/2 \cdot D_{1231} \\
D_{2311} & D_{2322} & D_{2333} & 1/2 \cdot D_{2312} & 1/2 \cdot D_{2323} & 1/2 \cdot D_{2331} \\
D_{3111} & D_{3122} & D_{3133} & 1/2 \cdot D_{3112} & 1/2 \cdot D_{3123} & 1/2 \cdot D_{3131}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
2 \cdot \varepsilon_{12} \\
2 \cdot \varepsilon_{23} \\
2 \cdot \varepsilon_{31}
\end{bmatrix}
\]

To compute the elasto-plastic Jacobian formulated by Kobayashi and Ohno (2002) using strain tensors expressed in Voigt notation it is often necessary to transform tensors from standard notation into Voigt notation and vice versa. The generic 4th order tensor \( \mathbf{M}_{std} \) expressed in standard notation can be transformed in Voigt notation performing a transposition by means of a convenient tensor \( \mathbf{R}_B \).

\[
\mathbf{M} = \mathbf{M}_{std} \mathbf{R}_B = \mathbf{M}_{std}
\]

Vice versa it is possible to easily transform the generic 4th order tensor \( \mathbf{M} \) expressed in Voigt notation into a tensor expressed in standard notation performing a transposition by means of a convenient tensor \( \mathbf{R}_A \).
Appendix B. Implementation of the constitutive and damage model in a finite element code

\[ \mathbf{M}_{std} = \mathbf{M} \mathbf{R}_A = \mathbf{M} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix} \] (B.55)

Transforming and back-transforming the 4th order tensors, as shown by Halama and Poruba (2009), it is possible to adapt the elasto–plastic Jacobian formulation proposed by Kobayashi and Ohno (2002) in a way that it is suitable for the implementation in the FE code ABAQUS.

The consistent Jacobian \( \mathbf{D}^{ep} \) is equivalent can be computed as follows:

\[ \mathbf{D}^{ep} = \mathbf{D}^{el} - 4G^2 \begin{bmatrix} \mathbf{L}^{-1}_{n+1} : \mathbf{I}_d \end{bmatrix} \mathbf{R}_B \] (B.56)

where \( \mathbf{I}_d \) indicates the following deviatoric operator defined in terms of the 2nd and 4th rank unit tensors, \( \mathbf{i} \) and \( \mathbf{I} \)

\[ \mathbf{I}_d = \mathbf{I} - 1/3 \cdot (\mathbf{i} \otimes \mathbf{i}) \] (B.57)

The total hardening matrix \( \mathbf{L}_{n+1} \) is computed as follows:

\[ \mathbf{L}_{n+1} = 2GI + \left[ \sum_k \mathbf{H}^{(k)}_{n+1} \right] \mathbf{R}_A + \frac{2}{3} \left( \frac{dY}{dp} \right)_{n+1} \mathbf{N}_{n+1} \mathbf{N}_{n+1} \mathbf{R}_A + \frac{2Y_{n+1}}{3 \Delta p_{n+1}} \left( \mathbf{I} - \mathbf{N}_{n+1} \mathbf{N}_{n+1} \right) \] (B.58)

with the kinematic hardening matrix \( \mathbf{H}^{(k)}_{n+1} \) equivalent to

\[ \mathbf{H}^{(k)}_{n+1} = \theta^{(k)}_{n+1} \left( \frac{2}{3} \mathbf{C}^{(k)} \mathbf{I} \right) \mathbf{R}_B - \sqrt{\frac{2}{3} \gamma^{(k)}_{n+1} \mathbf{\alpha}^{(k)}_{n+1} \otimes \mathbf{N}_{n+1} \mathbf{N}_{n+1}} \] (B.59)

The calculation of the elasto–plastic Jacobian has been implemented in the following source code.

```fortran
! definition of theta
DO K3=1,n_component
  ktheta(K3)=ONE/(ONE+k_g(K3)*d_PEEQ_strain)
END DO

! definition of the transformation Matrix R_A (from Voigt to standard notation)
DO K1=1,NTENS
  DO K2=1,NTENS
    kmatrix_R_A(K1,K2)=ZERO
  END DO
END DO

DO K1=1,NDI
  kmatrix_R_A(K1,K1)=ONE
END DO

DO K1=NDI+1,NTENS
  kmatrix_R_A(K1,K1)=TWO
END DO

! definition of the transformation Matrix R_B (from standard to Voigt notation)
DO K1=1,NTENS
  DO K2=1,NTENS
    kmatrix_R_B(K1,K2)=ZERO
  END DO
END DO

DO K1=1,NDI
  kmatrix_R_B(K1,K1)=ONE
END DO

DO K1=NDI+1,NTENS
  kmatrix_R_B(K1,K1)=ONE/TWO
END DO
```
B.2. UMAT model

DO K1=1, NTENS
  DO K2=1, NTENS
    kmatrix_I_dev(K1,K2)=ZERO
  END DO
END DO

DO K1=1, NTENS
  kmatrix_I_dev(K1,K1)=ONE
END DO

DO K1=1, NDI
  DO K2=1, NDI
    kmatrix_I_dev(K1,K2)=kmatrix_I_dev(K1,K2)-(ONE/THREE)
  END DO
END DO

! definition of the identity matrix
DO K1=1,NTENS
  DO K2=1,NTENS
    kmatrix_I(K1,K2)=ZERO
  END DO
END DO

DO K1=1,NTENS
  kmatrix_I(K1,K1)=ONE
END DO

! definition of the kinematic hardening matrix for each component
DO K3=1,n_component
  CALL DYADPRODUCT(kproduct_4(:,:,K3),backstress_component(:,K3),kN_normal(:),NTENS)
END DO

DO K3=1,n_component
  DO K1=1,NTENS
    DO K2=1,NTENS
      kmatrix_H_component(K1,K2,K3)=ktheta(K3)*(TWO/THREE*k_c(K3)*kmatrix_R_B(K1,K2)- dsqrt(TWO/THREE)*k_g(K3)*kproduct_4(K1,K2,K3))
    END DO
  END DO
END DO

! definition of the total kinematic hardening matrix
DO K1=1,NTENS
  DO K2=1,NTENS
    kmatrix_H(K1,K2)=ZERO
  END DO
END DO

DO K3=1,n_component
  DO K1=1,NTENS
    DO K2=1,NTENS
      kmatrix_H(K1,K2)=kmatrix_H(K1,K2)+kmatrix_H_component(K1,K2,K3)
    END DO
  END DO
END DO

kmatrix_H=matmul(kmatrix_H,kmatrix_R_A)

! definition of the isotropic hardening matrix
CALL DYADPRODUCT(kproduct_5,kN_normal,kN_normal,NTENS)

DO K1=1,NTENS
  DO K2=1,NTENS
    kmatrix_ISO(K1,K2) = TWO/THREE*(dYield_dx_2D(1) - J_2/d_PEEQ_strain) * kproduct_5(K1,K2)
  END DO
END DO

kmatrix_ISO=matmul(kmatrix_ISO,kmatrix_R_A)

! building the total hardening matrix
DO K1=1,NTENS
  DO K2=1,NTENS
    kmatrix_L(K1,K2)=(TWO*k_mu + TWO/THREE*J_2/d_PEEQ_strain) * kmatrix_I(K1,K2)+&
                    kmatrix_H(K1,K2)+kmatrix_ISO(K1,K2)
  END DO
END DO

! definition of the inverse of the total hardening matrix
call MIGS(kmatrix_L,NTENS,kinv_matrix_L,kpivot)

kproduct_6=matmul(kmatrix_I_dev,kinv_matrix_L)
kproduct_7=matmul(kproduct_6,kmatrix_R_B)

! definition of the consistent Jacobian
DO K1=1,NTENS
  DO K2=1,NTENS
    DDSDDE(K1,K2)=DDSDDE(K1,K2)-(FOUR*k_mu*k_mu)*kproduct_7(K1,K2)
  END DO
END DO

JACOBIANELASTIC

This subroutine has been developed in order to provide the elastic stiffness tensor $D^{el}$ directly in Voigt notation. A convenient notation is available adopting the Lame' parameter $\delta$ defined as:

$$\delta = \frac{2 \cdot G \cdot \nu}{1 - 2 \cdot \nu}$$

(B.60)
Appendix B. Implementation of the constitutive and damage model in a finite element code

\[
\mathbf{D}^\text{el} = \begin{bmatrix}
\delta + 2G & \delta & \delta & 0 & 0 & 0 \\
\delta & \delta + 2G & \delta & 0 & 0 & 0 \\
\delta & \delta & \delta + 2G & 0 & 0 & 0 \\
0 & 0 & 0 & G & 0 & 0 \\
0 & 0 & 0 & 0 & G & 0 \\
0 & 0 & 0 & 0 & 0 & G \\
\end{bmatrix}
\] (B.61)

Here below the corresponding source code is reported:

```fortran
k_lambda = 2 * k_mu / (1 - 2 * k_v)

! reset the matrix
do i=1,NDI+NSHR
   do j=1,NDI+NSHR
      el_mat_stiffness(i,j) = ZERO
   end do
end do

! fill the axial quadrant
do i=1,NDI
   do j=1,NDI
      el_mat_stiffness(i,j) = k_lambda
   end do
   el_mat_stiffness(i,i) = k_lambda + (2 * k_mu)
end do

! fill the shear quadrant
do i=NDI+1,NDI+NSHR
   el_mat_stiffness(i,i) = k_mu
end do
```

JACOBIANDEVELASTIC

This subroutine has been developed in order to provide the deviatoric elastic stiffness tensor \( \mathbf{D}^{\text{dev,el}} \) directly in Voigt notation.

\[
\mathbf{D}^{\text{dev,el}} = \begin{bmatrix}
4/3G & -2/3G & -2/3G & 0 & 0 & 0 \\
-2/3G & 4/3G & -2/3G & 0 & 0 & 0 \\
-2/3G & -2/3G & 4/3G & 0 & 0 & 0 \\
0 & 0 & 0 & G & 0 & 0 \\
0 & 0 & 0 & 0 & G & 0 \\
0 & 0 & 0 & 0 & 0 & G \\
\end{bmatrix}
\] (B.62)

Here below the corresponding source code is reported:

```fortran
! reset the matrix
do j=1,NDI+NSHR
   do k=1,NDI+NSHR
      dev_el_mat_stiffness(j,k) = ZERO
   end do
end do

! fill the axial quadrant
do j=1,NDI
   do k=1,NDI
      dev_el_mat_stiffness(j,k) = -2 * k_mu / 3
   end do
   dev_el_mat_stiffness(j,j) = 2 * k_mu - 2 * k_mu / 3
end do

! fill the shear quadrant
do j=NDI+1,NDI+NSHR
   dev_el_mat_stiffness(j,j) = k_mu
end do
```

DYADPRODUCT

This routine is responsible to compute the dyadic product of 2 vectors.

\[
\mathbf{u} \otimes \mathbf{v} = \begin{bmatrix}
u_1 \\
v_1 \\
u_2 \\
v_2 \\
u_3 \\
v_3
\end{bmatrix}
\begin{bmatrix}
u_1 & v_1 & v_3 \\
u_1 & v_2 & v_3 \\
u_3 & v_2 & v_3
\end{bmatrix}
= \begin{bmatrix}
u_1 v_1 & u_1 v_2 & u_1 v_3 \\
u_2 v_1 & u_2 v_2 & u_2 v_3 \\
u_3 v_1 & u_3 v_2 & u_3 v_3
\end{bmatrix}
\] (B.63)
Here below the corresponding source code is reported:

```fortran
  DO K4=1,kNTENS
    DO K5=1,kNTENS
      dyadic_product_matrix(K4,K5)=vector_1(K4)*vector_2(K5)
    END DO
  END DO
```

**MIGS**

MIGS is a subroutine published by Pang (2006) that is used to perform an efficient inversion of a generic square matrix using the Gaussian elimination method.

**ELGS**

ELGS is a subroutine published by Pang (2006) required by MIGS subroutine in order to perform the partial-pivoting Gaussian elimination.

**B.2.9. 'interpolationroutines.f'**

The routines responsible for the multidimensional linear interpolation are collected in the file 'interpolationroutines.f' and are listed below:

1. INTERPOLATION1D
2. INTERPOLATION2D
3. INTERPOLATION3D
4. INTERPOLATION4D
5. INTERPOLATION5D
6. INTERPOLATION2DDER
7. INTERPOLATION5DDER

The routines 'INTERPOLATIONxD' perform a linear interpolation in $x$ dimensions. In addition, the routines 'INTERPOLATIONxDDER' provide the first order derivatives in $x$ dimensions.

**INTERPOLATION1D**

This routine inspired on the work of Wu (2010) is used to perform a linear interpolation in one variable in order to extrapolate, from a table $X$–$Y$, the value $Y_0$ corresponding to a certain input $X_0$. In $Y$ is stored the information of the evolution of the selected parameter as a function of the fixed entries of the internal variable stored in $X$. If the input $X_0$ overtakes the upper and the lower boundaries of the calibration limit, the value corresponding to the overtaken boundary of $X$ is used to compute $Y_0$ and the variable named ‘calib_limit’ is set to 1 in order to signal this unexpected event. In order to avoid numerical problems linked with the precision of the variables, it is necessary to correct the input value for $X_0$ when it is closer than a certain value (named ‘interpolation_threshold’) to the boundaries of $X$. The value for ‘interpolation_threshold’ can be set in the file 'parameters.f' (see section B.2.6).

Here below the corresponding source code is reported:
Appendix B. Implementation of the constitutive and damage model in a finite element code

INTERPOLATION2D, 3D, 4D and 5D

These routines consist of an extension of INTERPOLATION1D capable to perform the linear interpolation in more than one dimension. For simplicity only the case of INTERPOLATION5D will be here discussed.

The routine INTERPOLATION5D is a modified version of the one proposed by (Wu, 2010) and performs a linear interpolation in 5 variables in order to extrapolate, from a multidimensional table $X_1$–...–$X_5$–$Y$, the value $Y_0$ corresponding to the input $X_{1,0}$, ..., $X_{5,0}$. In $Y$ is stored the information of the evolution of the selected parameter as a function of the fixed entries of the 5 internal variables stored in $X_1$, ..., $X_5$. If the input $X_{n,0}$ overtakes the upper and the lower boundaries of the calibration limit, the value corresponding to the overaken boundary of $X_n$ is used to compute $Y_0$ and the variable named `calib_limit` is set to 1 in order to signal this unexpected event. In order to avoid numerical problems linked with the precision of the variables, it is necessary to correct the input value for $X_{n,0}$ when it is closer than a certain value (named `interpolation_threshold`) to the boundaries of $X_n$. The value for `interpolation_threshold` can be set in the file `parameters.f` (see section B.2.6).

Here below the corresponding source code is reported:

```fortran
INTERPOLATION5D, 3D, 4D and 5D

These routines consist of an extension of INTERPOLATION1D capable to perform the linear interpolation in more than one dimension. For simplicity only the case of INTERPOLATION5D will be here discussed.

The routine INTERPOLATION5D is a modified version of the one proposed by (Wu, 2010) and performs a linear interpolation in 5 variables in order to extrapolate, from a multidimensional table $X_1$–...–$X_5$–$Y$, the value $Y_0$ corresponding to the input $X_{1,0}$, ..., $X_{5,0}$. In $Y$ is stored the information of the evolution of the selected parameter as a function of the fixed entries of the 5 internal variables stored in $X_1$, ..., $X_5$. If the input $X_{n,0}$ overtakes the upper and the lower boundaries of the calibration limit, the value corresponding to the overaken boundary of $X_n$ is used to compute $Y_0$ and the variable named `calib_limit` is set to 1 in order to signal this unexpected event. In order to avoid numerical problems linked with the precision of the variables, it is necessary to correct the input value for $X_{n,0}$ when it is closer than a certain value (named `interpolation_threshold`) to the boundaries of $X_n$. The value for `interpolation_threshold` can be set in the file `parameters.f` (see section B.2.6).

Here below the corresponding source code is reported:

```
B.2. UMAT model

```fortran
kX5_0=kX5(number_rows5)
END IF

! 1st variable dependency (X1)
! check if the interpolation is performed inside the limits
IF (kX1_0.LT.kX1(1)) THEN
  slope_X(1)=ZERO
  oindex(1)= 1
  pindex(1)= 1
  calib_limit=1
  WRITE(7,*,'(A,1x,F10.4)') 'WARNING: INTERPOLATIONSD ROUTINE - x1 < MIN x1'
ELSEIF (kX1_0.GT.kX1(number_rows1)) THEN
  slope_X(1)=ZERO
  oindex(1)= number_rows1
  pindex(1)= number_rows1
  calib_limit=1
  WRITE(7,*,'(A,1x,F10.4)') 'WARNING: INTERPOLATIONSD ROUTINE - x1 > MAX x1'
ELSEIF (kX1_0.EQ.kX1(1)) THEN
  ! exceptional case: the input is equivalent to the lower calibration limit
  slope_X(1)=ZERO
  oindex(1)= 1
  pindex(1)= 1
ELSEIF (kX1_0.EQ.kX1(number_rows1)) THEN
  ! exceptional case: the input is equivalent to the upper calibration limit
  slope_X(1)=ZERO
  oindex(1)= number_rows1
  pindex(1)= number_rows1
ELSE
  ! otherwise calculate the slope_X1
  K_counter=1
  DO WHILE (kX1_0.GE.kX1(K_counter))
    K_counter=K_counter+1
  END DO
  oindex(1)= K_counter
  pindex(1)= K_counter-1
  slope_X(1)= (kX1_0 - kX1(pindex(1))) / (kX1(oindex(1)) - kX1(pindex(1)))
END IF
```

! Finally Perform the interpolation in 5 dimensions

```fortran
kY_0=ZERO
DO bin=1,32
  indexgetter = bin
  multiplier = ONE
  DO i = 1,5
    index = indexgetter - TWO * floor(indexgetter/TWO)
    indexgetter = (indexgetter - index)/TWO
    IF (index.EQ.0) THEN
      indices(i) = pindex(i)
      multiplier = multiplier * (ONE - slope_X(i))
    ELSE
      indices(i) = oindex(i)
      multiplier = multiplier * slope_X(i)
    END IF
  END DO
  kY_0 = kY_0 + kY(indices(1),indices(2),indices(3),indices(4),indices(5)) * multiplier
END DO
```

INTERPOLATION2DDER and 5DDER

These routines consist in an extension of INTERPOLATION2D and 5D previously described, having the capability to compute not only the interpolated value \( Y_0 \) but also the vector of derivatives \( \left( \frac{dY}{dX_n} \right)_{X_n=0} \) (if the input variable 'derivative_flag' is set to 1). For simplicity only the case of INTERPOLATION5DDER will be here discussed.

The routine INTERPOLATION5DDER is a modified version of the one proposed by (Wu, 2010) and performs a linear interpolation in 5 variables in order to extrapolate, from a multidimensional table \( X_1 \ldots X_5 \cdot Y \), the value of \( Y_0 \) and of the derivatives \( \left( \frac{dY}{dX_1} \right)_{X_1=0}, \ldots, \left( \frac{dY}{dX_5} \right)_{X_5=0} \) corresponding to the input \( X_1=0, \ldots, X_5=0 \). In \( Y \) is stored the information of the evolution of the selected parameter as a function of the fixed entries of the 5 internal variables stored in \( X_1, \ldots, X_5 \). If the input \( X_n=0 \) oversteps the upper and the lower boundaries of the calibration limit, the value corresponding to the overtaken boundary of \( X_n \) is used to compute \( Y_0 \) and the variable named 'der_calib_limit' is set to 1 in order to signal this unexpected event. In order to avoid numerical problems linked with the precision of the variables, it is necessary to correct the input value for \( X_n=0 \) when it is closer than a certain value (named 'interpolation_threshold') to the boundaries of \( X_n \). The value for 'interpolation_threshold' can be set in the file 'parameters.f' (see section B.2.6).
Here below the corresponding source code is reported:

```fortran
! in order to avoid problems due to the precision of the variables it is necessary to correct them when
! they are close to the boundaries of Xn
IF (dabs(kX1(1)-kX1_0).LT.interpolation_threshold) THEN
  kX1_0=kX1(1)
END IF
...
IF (dabs(kX5(1)-kX5_0).LT.interpolation_threshold) THEN
  kX5_0=kX5(1)
END IF
IF (dabs(kX1(number_rows1)-kX1_0).LT.interpolation_threshold) THEN
  kX1_0=kX1(number_rows1)
END IF
...
IF (dabs(kX5(number_rows5)-kX5_0).LT.interpolation_threshold) THEN
  kX5_0=kX5(number_rows5)
END IF

! A. SIMPLE INTERPOLATION

! see subroutine INTERPOLATION5D

! B. DERIVATIVES EVALUATION
IF (derivative_flag.EQ.0) THEN
  dY_dx(1)= 0
  dY_dx(2)= 0
  dY_dx(3)= 0
  dY_dx(4)= 0
  dY_dx(5)= 0
ELSE
  !evaluation of the derivative df_dx1
  !definition of the range where the derivative must be evaluated
  IF (pindex(1).EQ.1) THEN
    der_oindex(1)=2
    der_pindex(1)=1
  ELSEIF (oindex(1).EQ.number_rows1) THEN
    der_oindex(1)=number_rows1
    der_pindex(1)=number_rows1-1
  ELSE
    IF (pindex(1).EQ.oindex(1)) THEN
      der_oindex(1)=oindex(1)+1
      der_pindex(1)=oindex(1)-1
    ELSE
      der_oindex(1)=oindex(1)
      der_pindex(1)=pindex(1)
    END IF
  END IF
  !evaluation of the derivative df_dx1
  CALL INTERPOLATION5D(kX1(1:number_rows1),kX2(1:number_rows2),kX3(1:number_rows3),kX4(1:number_rows4),
          kX5(1:number_rows5),
          kY(1:number_rows1,1:number_rows2,1:number_rows3,1:number_rows4,1:number_rows5),
          kX1(der_oindex(1)),kX2_0,kX3_0,kX4_0,kX5_0,kf1,
          number_rows1,number_rows2,number_rows3,number_rows4,number_rows5,calib_limit)
  CALL INTERPOLATION5D(kX1(1:number_rows1),kX2(1:number_rows2),kX3(1:number_rows3),kX4(1:number_rows4),
          kX5(1:number_rows5),
          kY(1:number_rows1,1:number_rows2,1:number_rows3,1:number_rows4,1:number_rows5),
          kX1(der_pindex(1)),kX2_0,kX3_0,kX4_0,kX5_0,kf0,
          number_rows1,number_rows2,number_rows3,number_rows4,number_rows5,calib_limit)
  kdf = kf1-kf0
  kdx = kX1(der_oindex(1))-kX1(der_pindex(1))
  dY_dx(1)= kdf/kdx
  ...

B.2.10. 'AISI316LNLIK3CompJiangDamageTracer-UMAT-3D.f'

This routine is responsible to evaluate the damage accumulation and to update the internal variables. Its implementation for UMAT is nearly identical to the version developed for the combined hardening ABAQUS material model (see section B.1.6). The only differences consist in the fact that a higher number of STATEV is required (404 instead of 365).
```
C

Issues linked with the implementation of a User MATerial model in a FE code

The implementation in the FE code ABAQUS of the 5 internal variables dependent constitutive model 5DChabEP by means of a User MATerial subroutine provides a powerful tool to analyze the elastic-plastic response of structures with a considerable reduction of simulation time with respect to the combined hardening material description available in ABAQUS. The major issues linked with the implementation of the elasto-plastic constitutive model by means of UMAT are discussed in this appendix.

C.1 Increment size-dependency

A successful application of the proposed elasto-plastic UMAT model for cyclic structural analysis for large number of cycles requires the development of an efficient numerical solution technique capable to maintain an acceptable accuracy for large increment steps. To investigate the effect of the increment size on the output, two sets of simulations are carried out imposing on a single hexaedral 3D quadratic element (i.e. ABAQUS C3D20):

- an alternate strain-controlled path with a strain amplitude of 0.40% and zero mean strain (i.e. AX-LCF-040-RT),
- an alternate stress-controlled path with a stress amplitude of 275 MPa and a mean stress of 25 MPa.

These simulations are repeated changing the maximum allowable time increment size in order to discretize the hysteresis loops with a different number of points. While the cycling period is equivalent to 1 second, the maximum time increment sizes defined for this investigation are: 0.25, 0.125, 0.05, 0.025 and 0.0125 seconds. The performance of the UMAT implementation is discussed using as reference the calculations provided by the combined hardening material description available in ABAQUS. Only the first 50 cycles are considered in this analysis.
Appendix C. Issues linked with the implementation of a User MATerial model in a FE code

Figure C.1. (left) Stress amplitude and (right) mean stress evolution as a function of number of cycles for simulations performed by means of the ABAQUS combined hardening model showing the influence of the increment size (experiment AX-LCF-040-RT).

C.1.1. Strain-controlled simulations

The evolution of the stress amplitude and of the mean stress as a function of number of cycles is represented in Fig. C.1 for the ABAQUS combined hardening model and in Fig. C.2 for the UMAT implementation. While the stress response computed by the ABAQUS combined hardening model is apparently not influenced by the increment size, this is not the case for the User MATerial subroutine. In UMAT, reducing the maximum time increment size from 0.125 to 0.0125 seconds, the computed stress amplitude becomes more accurate getting closer to the reference value given by the ABAQUS combined hardening model. As previously reported in Appendix B, this increment size-dependency is due to the nature of the adopted return mapping algorithm (Kobayashi and Ohno, 2002) linearizing the non-linear kinematic hardening laws. As this integration scheme considers the normal direction and position of the yield surface at the end of the increment step, it tends to compute lower plastic modulus and consequently a lower stress. This result is in agreement with the work of Rahman (2006) who carried out a similar investigation on the performance of the integration scheme proposed by Kobayashi and Ohno (2002).

A procedure has been implemented in UMAT in order to measure and to control this linearization error (see section B.2.7) requiring, where necessary, the repetition of the step using a smaller increment size. In this way it is possible to guarantee that the stress value returned by UMAT is sufficiently accurate under all the considered strain-controlled loading conditions (setting ‘max_lin_error’ equal to 0.5 the error is smaller than 3%). The user is allowed to choose lower values for ‘max_lin_error’ in order to further enhance the accuracy of the constitutive model. It is important to notice that, for the UMAT implementation, results computed with a maximum time increment size of 0.25 seconds are not reported. This decision is motivated by the fact that, in this case, the time increment reduction required by the User MATerial subroutine in order to control the linearization error would cause a significant decreasing of the average size of the increment and would not allow a fair comparison with the results computed by means of the ABAQUS material model.

The capability of the combined hardening ABAQUS and of the UMAT model to reproduce the hysteresis loop is documented in Fig. C.3, highlighting the influence of the increment size. The combined hardening ABAQUS material model provides a satisfactory description of the hysteresis loop only when the maximum time increment is smaller than 0.05 seconds (equivalent to at least 20 points per cycle). For a larger increment size, the low number of points is not sufficient to adequately represent the loop shape (see left hand side plot in Fig. C.3). On the other hand, the time increment reduction, required by the User MATerial subroutine to control the linearization error, automatically leads to an acceptable reproduction of the hysteresis loop for all the considered cases (see right hand side plot in Fig. C.3).
C.1. Increment size-dependency

Figure C.2. (left) Stress amplitude and (right) mean stress evolution as a function of number of cycles for simulations performed by means of the UMAT model showing the influence of the increment size (experiment AX-LCF-040-RT).

Figure C.3. Hysteresis loop simulated by means of the combined hardening ABAQUS (left) and of the UMAT model (right) showing the influence of the increment size (experiment AX-LCF-040-RT).

The poor description of the hysteresis loop shape given by the combined hardening ABAQUS model for large increment size is responsible for the significant error observed in the calculations of the accumulated damage and plastic work (see Fig.C.4). In the case of UMAT, the time increment reduction required by the User MATerial subroutine, leading to an acceptable reproduction of the hysteresis loop for all the considered cases, is responsible to weaken the effect of the increment size effect on $D$ and on $W_p$ (see Fig.C.5).

The same observations can be performed observing the values of the maximum stress amplitude, damage, accumulated plastic strain and plastic work represented as a function of the average number of points used to describe the hysteresis loop (see Fig.C.6). These results highlight that, while the stress computations provided by the ABAQUS combined hardening model are not influenced by the increment size, this observation is not confirmed considering

Figure C.4. (left) Accumulated plastic strain $\varepsilon_{pl}$, (middle) plastic work $W_p$ and (right) damage $D$ evolution as a function of number of cycles for simulations performed by means of the ABAQUS combined hardening model showing the influence of the increment size (experiment AX-LCF-040-RT).
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Figure C.5. (left) Accumulated plastic strain $p$, (middle) plastic work $W_p$, and (right) damage $D$ evolution as a function of number of cycles for simulations performed by means of the UMAT model showing the influence of the increment size (experiment AX-LCF-040-RT).

Figure C.6. Maximum stress amplitude, damage, accumulated plastic strain and plastic work computed by means of the combined hardening ABAQUS and of the UMAT model as a function of the average number of points used to describe the hysteresis loop (experiment AX-LCF-040-RT).

... the outputs $D$ and $W_p$. As a consequence, in order to obtain accurate lifetime predictions, the user must carefully select the maximum allowable time increment size, checking that the hysteresis loop is reproduced by means of a sufficient number of points. On the other hand, controlling the linearization error by means of the proposed strategy, it is possible to limit the influence of the increment size dependency on all the considered outputs allowing the UMAT subroutine to return results with an acceptable accuracy.

These conclusions are confirmed repeating this analysis for different strain-controlled loading conditions (including ratcheting and loading in shear direction).

C.1.2. Stress-controlled simulations

The evolution of the strain amplitude and of the mean strain as a function of number of cycles is represented in Fig.C.7 for the ABAQUS combined hardening model and in Fig.C.8 for the UMAT implementation. While the strain response computed by the ABAQUS combined...
C.1. Increment size-dependency

Figure C.7. (left) Strain amplitude and (right) mean strain evolution as a function of number of cycles for simulations performed by means of the ABAQUS combined hardening model showing the influence of the increment size ($\sigma_{\text{ampl}}$=275 MPa and $\sigma_{\text{mean}}$=25 MPa).

Figure C.8. (left) Strain amplitude and (right) mean strain evolution as a function of number of cycles for simulations performed by means of the UMAT model showing the influence of the increment size ($\sigma_{\text{ampl}}$=275 MPa and $\sigma_{\text{mean}}$=25 MPa).

hardening model is nearly not influenced by the increment size, this is not the case for the User MATerial subroutine. In UMAT, reducing the maximum time increment size from 0.125 to 0.0125 seconds, the calculations of the strain amplitude become more accurate getting closer to the reference values returned by the ABAQUS combined hardening model. The effect of the increment size on the outputs observed under stress-control is considerably stronger (error up to 10-15%) than the one noticed for strain-controlled simulations (error up to 3%). This result is in agreement with the work of Rahman (2006) who carried out a similar investigation on the performance of the integration scheme proposed by Kobayashi and Ohno (2002). As this integration scheme considers the normal direction and position of the yield surface at the end of the increment step, it tends to compute lower plastic modulus and consequently to overpredict the strain.

The procedure implemented in UMAT, in order to measure and to control the linearization error (see section B.2.7), is found to be less effective under stress-control comparing with strain-controlled simulations. For that reason, a further modification of the strategy used to control the linearization error is suggested. It is important to notice that, for the UMAT implementation, results computed with a maximum time increment size of 0.25 seconds are not reported. This decision is motivated by the fact that, in this case, the time increment reduction required by the User MATerial subroutine in order to control the linearization error would cause a significant decreasing of the average size of the increment and would not not allow a fair comparison with the results computed by means of the ABAQUS material model.

The capability of the combined hardening ABAQUS and of the UMAT model to reproduce the hysteresis loop is documented in Fig.C.9, highlighting the influence of the increment size. Similarly to what reported for the strain-controlled case, the combined hardening
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Figure C.9. Hysteresis loop simulated by means of the combined hardening ABAQUS (left) and of the UMAT model (right) showing the influence of the increment size ($\sigma_{\text{ampl}}=275$ MPa and $\sigma_{\text{mean}}=25$ MPa).

Figure C.10. (left) Accumulated plastic strain $p$, (middle) plastic work $W_p$ and (right) damage $D$ evolution as a function of number of cycles for simulations performed by means of the ABAQUS combined hardening model showing the influence of the increment size ($\sigma_{\text{ampl}}=275$ MPa and $\sigma_{\text{mean}}=25$ MPa).

ABAQUS material model provides a satisfactory description of the hysteresis loop only when the maximum time increment is smaller than 0.05 seconds (equivalent to at least 20 points per cycle). For a larger increment size, the low number of points is not sufficient to adequately represent the loop shape (see left hand side plot in Fig.C.9). Differently from what reported for the strain-controlled case, the time increment reduction required by the User MATerial subroutine to control the linearization error is not sufficient to obtain an acceptable reproduction of the hysteresis loop for all the considered cases (see right hand side plot in Fig.C.9).

The poor description of the hysteresis loop shape observed for the combined hardening ABAQUS model and for the UMAT implementation in the case of large increment size is responsible for the significant error observed in the calculations of the accumulated damage and of the plastic work (see Fig.C.10 and C.11).

Figure C.11. (left) Accumulated plastic strain $p$, (middle) plastic work $W_p$ and (right) damage $D$ evolution as a function of number of cycles for simulations performed by means of the UMAT model showing the influence of the increment size ($\sigma_{\text{ampl}}=275$ MPa and $\sigma_{\text{mean}}=25$ MPa).
C.2 Robustness versus rigid body rotation

Another issue to consider in the implementation of a User MATerial subroutine is its robustness when a rigid body rotation is applied. To investigate the effect of the rigid body rotation on the output, two kinds of simulations are carried out, imposing on a single hexaedral 3D quadratic element (i.e. ABAQUS C3D20):

- an alternate stress-controlled path ($\sigma_{ampl}=250$ MPa and $\sigma_{mean}=0$ MPa) and no rigid rotation (i.e. static),
- an alternate stress-controlled path ($\sigma_{ampl}=250$ MPa and $\sigma_{mean}=0$ MPa) and a rigid rotation having a rate of 9 deg/cycle (see Fig.C.13).

The performance of the UMAT implementation is evaluated, using as reference the calculations provided by the combined hardening material description available in ABAQUS. Only
Appendix C. Issues linked with the implementation of a User MATerial model in a FE code

Figure C.13. Single hexaedral element subjected to an alternate loading and a simultaneous rigid body rotation.

Figure C.14. Evolution of the maximum principal strain as a function of time. Only data corresponding to the first 2 cycles are plotted.

The first 10 cycles are considered in this analysis.

A meaningful comparison of the static simulations and of the ones performed imposing a rigid rotation is not possible considering the tensorial components of the strain separately. For that reason, in Fig. C.14, the maximum principal strain evolution is plotted as a function of time. The qualitatively similar evolution of the maximum principal strain for the considered curves suggests that the UMAT implementation is robust when a rigid body rotation is applied.

A more precise evaluation of this robustness is available representing the maximum value of the maximum principal strain as a function of the number of cycles (see left hand side plot in Fig. C.15). In this graph it is evident that the maximum value of the maximum principal strain calculated in the static simulation is nearly equivalent to the one computed applying a rigid body rotation. The quantitative difference (about 5%) noticed between the strain computed by means of the ABAQUS combined hardening model and by the UMAT implementation must be attributed to the increment size-dependency previously reported.

A further evidence of this robustness is obtained observing that the damage accumulated in static loading conditions is nearly identical to the one computed when a rigid rotation is applied (see right hand side plot in Fig. C.15).

The capability of the UMAT model to provide accurate results, when a rigid rotation takes place, is due to the change of the basis of the tensorial state variables from the global orientation system to the co-rotational system (see section B.2.7).
C.3 Effect of the internal variables variation within the increment step

In this section, the motivations of considering the effect of the variation of the internal variables within the increment step are discussed.

In the combined hardening ABAQUS model, the effect of the variation of the parameters as a function of the internal variables is taken into account by the modification of the kinematic hardening law proposed in section 4.1.3:

\[
\dot{\alpha}^{(k)} = \frac{2}{3} C^{(k)} \dot{\varepsilon}^{pl} - \gamma^{(k)} \alpha^{(k)} \dot{p} + \sum_{j} \frac{1}{C^{(k)}} \frac{\partial C^{(k)}}{\partial x_j} \alpha^{(k)} \dot{x}_j 
\]

(C.1)

where \( x_j \) is the generic internal variable.

A simplification of Eq.C.1 is available observing that the sole internal variable \( T \) (i.e. temperature) is responsible for a rapid evolution of the hardening parameters. As already pointed out in section 4.1.3, the possibility to drastically reduce the computational time without losing accuracy, led the author of the current dissertation to implement the User MATerial subroutine taking into account only the effect of the variation of temperature on the computed stress response.

A third version of the constitutive model (named "UMAT B"), not accounting for the effect of the variation of the internal variables, has been developed and has been used in this investigation as reference.

In order to investigate the capability of the different implementations to consider the effect of the variation of the internal variables, two sets of simulations are carried by means of a single hexaedral 3D quadratic element (i.e. ABAQUS C3D20) constraining the strain in axial direction and imposing:

- a temperature profile with a cycling period of 1 second and a rapid (i.e. 0.1 seconds) variation of the temperature from 25°C to 200°C and from 200°C to 25°C (i.e. square profile),
- a temperature profile with a cycling period of 1 second with a slow variation of the temperature between 25°C and 200°C (i.e. sinusoidal profile).

These simulations are performed keeping the same maximum allowable time increment size in order to discretize the loading cycle with a similar number of points. Imposing the two

Figure C.15. (left) Evolution of the maximum value of the maximum principal strain as a function of cycle number. (right) Evolution of the accumulated damage as a function of time.
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Figure C.16. Square and sinusoidal temperature profile imposed in the simulations.

Figure C.17. Evolution of the maximum and minimum stress and plastic strain as a function of the number of cycles.

temperature profiles reported in Fig.C.16 and using a similar number of points to discretize a single loading cycle, it is possible to impose different rates of change of the hardening parameter $C^{(k)}$ with respect to the internal variable $T$.

A robust implementation capable to consider the effect of the variation of the internal variables, should return very similar stress calculations for simulations characterized by different rates of change of $C^{(k)}$ within an incrementation step. Only the first 10 cycles are considered in this analysis.

In Fig.C.17 the evolution of the maximum and minimum stress and plastic strain is represented as a function of the number of cycles. The ABAQUS combined hardening model and the UMAT implementation, thanks to their capability to consider the effect of the variation of the internal variables, return very similar stress and plastic strain calculations for simulations performed with different temperature profiles. On the other hand, the UMAT B implementation is not able to consider the effect of the sudden variation of the hardening parameters within the integration step and gives different results when different temperature profiles are imposed.

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C.4 Overtaking of the calibration limit

As already pointed out in the current dissertation, a meaningful utilization of the constitutive model requires that the internal variables do not overtake the calibration limits reported in Tab.4.8. Both in the combined hardening ABAQUS material and in the UMAT implementation, the accidental overtaking of the calibration limits does not cause the termination of the simulation but leads to the wrong estimation of the hardening parameters and consequently to inaccurate stress calculations.

As example, it is reported the case of a single hexaedral 3D quadratic element (i.e. ABAQUS C3D20) subjected to a strain-controlled ratcheting path in which the mean strain is allowed to continuously increase and to overtake the calibration limit for the internal variable $\varepsilon_{p}^{\text{mean}}$ (see left hand side plot in Fig.C.18). The calculated stress amplitude evolution, reported on the right hand side plot in Fig.C.18, shows that the constitutive model is not able to correctly reproduce the ratcheting-induced hardening once the calibration limit has been overtaken.

Performing simulations by means of the combined hardening ABAQUS model, the user could not be aware of the overtaking of the calibration limit since this unexpected event is not evidenced by any particular warning message. In order to avoid this critical situation, the UMAT implementation uses the state variable 21 as flag to report the eventual overtaking of the calibration range. The visualization of the value of the SDV21 in the CAE environment makes checking the occurrence of this unwanted event a straightforward task (see Fig.C.19).
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Figure C.20. Evolution of $\varepsilon_{pl}^{yy}$ and of the tracers $\varepsilon_{ampl}^{pl}$ and $\varepsilon_{mean}^{pl}$ as a function of time for a single 3D hexaedral element subjected to an alternate strain path (experiment AX-LCF-100-RT) in axial direction $y$.

C.5 Performance of the procedure responsible for the variables updating

As reported in chapter 4, in order to obtain reliable results from the proposed constitutive model, it is necessary to feed it with accurate internal variables values. In this framework, a performant tracer is essential because during a FE simulation one must be able to compute the updated internal variables using the values of previously computed internal variables only. This task is straightforward for $p$ and $T$, while the estimation of $\varepsilon_{mean}^{pl}$, $\varepsilon_{ampl}^{pl}$ and $\xi$ is more complicated. In this appendix, the performance of the routine responsible for tracing and updating of the internal variables proposed in section 4.1.5 is evaluated considering different loading conditions.

C.5.1. LCF

Imposing an alternate LCF strain path (i.e. AX-LCF-100-RT) in axial direction on a single 3D hexaedral element (i.e. ABAQUS C3D20), the plastic strain evolution computed by the 5DChabEP constitutive model is the one represented in Fig.C.20. In the same graph, the values of the updated internal variables $\varepsilon_{mean}^{pl}$ and $\varepsilon_{ampl}^{pl}$ provided by the tracer routine are also reported. As expected, the updating of the internal variable is not immediate but, after the third reversal, the tracer routine has all the necessary information to estimate with an excellent precision the amplitude of the plastic strain path.

A confirmation of the accuracy of the tracer routine is evident observing the plot reported in Fig.C.21, representing the plastic strain evolution computed by the 5DChabEP constitutive model for a single 3D hexaedral element subjected to a LCF strain path in shear direction. It must be pointed out that in this case, in order to allow an easy comparison with the tracers (expressed in the equivalent space), the plastic strain path has been scaled by a factor $\frac{2}{\sqrt{3}}$.

C.5.2. Strain-controlled ratcheting

Imposing a ratcheting strain path (i.e. AX-RAT-040-RT-P10) in axial direction on a single 3D hexaedral element (i.e. ABAQUS C3D20), the plastic strain evolution computed
C.5. Performance of the procedure responsible for the variables updating

Figure C.21. Evolution of $\varepsilon_{xy}$, $\frac{2}{\sqrt{3}}$ and of the tracers $\varepsilon_{\text{pl,ampl}}$ and $\varepsilon_{\text{pl,mean}}$ as a function of time for a single 3D hexaedral element subjected to an alternate strain path in shear direction $xy$.

by the 5DChabEP constitutive model is the one represented in Fig.C.22. In the same graph, the values of the updated internal variables $\varepsilon_{\text{pl,mean}}$, $\varepsilon_{\text{pl,ampl}}$ and $\xi$ provided by the tracer routine are also reported. The updating routine is found to trace with an excellent precision, not only the plastic strain amplitude, but also the mean strain and the ratcheting rate.

C.5.3. Multiaxial LCF

The performance of the tracer routine is also investigated for multiaxial loading conditions. In this case, the considered geometry consists of the gage of the multiaxial specimen represented on the left hand side plot in Fig.4.36 and meshed by means of 280 3D hexaedral quadratic elements (i.e. ABAQUS C3D20). The simulations are carried out prescribing the displacement into $y$ direction and the rotation into the plane $xz$ in order to apply the proportional and non-proportional LCF loading histories presented in the dissertation (i.e. MP-LCF-065-RT and MN-LCF-065-RT).

Proportional loading

For the multiaxial proportional LCF strain path (i.e. MP-LCF-065-RT), the plastic strain evolution computed by the 5DChabEP constitutive model and the corresponding tracers extracted from a node on the external surface of the hollow specimen are reported in Fig.C.23. The routine responsible for the updating of the internal variables demonstrates an excellent performance requiring only few reversals to correctly catch the equivalent strain amplitude.

Non-proportional loading

For the multiaxial non-proportional LCF strain path (i.e. MN-LCF-065-RT), the plastic strain evolution computed by the 5DChabEP constitutive model and the corresponding tracers extracted from a node on the external surface of the hollow specimen are reported in Fig.C.24.

While, for the proportional loading case, the traditional concepts used to identify the
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Figure C.22. Evolution of \( \varepsilon_{yy}^{pl} \) and of the tracers \( \varepsilon_{amp}^{pl} \), \( \varepsilon_{mean}^{pl} \) and \( \xi \) as a function of time for a single 3D hexahedral element subjected to a ratcheting strain path (experiment AX-RAT-040-RT-P10) in axial direction \( y \).

Figure C.23. Evolution of \( \varepsilon_{yy}^{pl} \), \( \varepsilon_{eq}^{pl} \), \( \varepsilon_{amp}^{pl} \) and of the tracers \( \varepsilon_{amp}^{pl} \) and \( \varepsilon_{mean}^{pl} \) as a function of time extracted from a node on the external surface of the hollow specimens meshed by means of 3D hexahedral elements subjected to a multiaxial proportional LCF path (experiment MP-LCF-065-RT).
C.5. Performance of the procedure responsible for the variables updating

Figure C.24. Evolution of $\varepsilon_{pl}$, $\varepsilon_{eq}$ and of the tracers $\varepsilon_{ampl}$ and $\varepsilon_{mean}$ as a function of time extracted from a node on the external surface of the hollow specimens meshed by means of 3D hexaedral elements subjected to a multiaxial non-proportional LCF path (experiment MN-LCF-065-RT).

plastic strain range are easily applicable, in presence of out-of-phase loading conditions, the definition and of the equivalent strain amplitude is not straightforward.

The commonly used designation indicates the equivalent plastic strain magnitude as the radius of the minimum circle that circumscribes the loading path in the plastic strain space (see Fig.C.25). This definition has been adopted also in the current dissertation in order to define the nominal equivalent strain amplitude for the multiaxial experiments described in section 3.2.

In the last decades, several methods have been developed in order to evaluate the plastic strain magnitude, providing consistent results also in the case of particularly complex loading paths. Among them, one of the most used approach consists in the procedure reported in the ASME Boiler and Pressure Vessel code Procedure (ASME, 1988) employing the strain difference $\Delta\varepsilon_{ij}$ between a fixed time point $t_0$ and an arbitrary one $t_1$:

$$
\Delta\varepsilon_{eq}^{ampl} = \frac{\sqrt{2}}{3} \left\{ \left[ \Delta\varepsilon_{11}^{pl} - \Delta\varepsilon_{22}^{pl} \right]^2 + \left[ \Delta\varepsilon_{22}^{pl} - \Delta\varepsilon_{33}^{pl} \right]^2 + \left[ \Delta\varepsilon_{11}^{pl} - \Delta\varepsilon_{33}^{pl} \right]^2 + \ldots 
+ 6\left[ (\Delta\varepsilon_{12}^{pl})^2 + (\Delta\varepsilon_{13}^{pl})^2 + (\Delta\varepsilon_{23}^{pl})^2 \right] \right\}^{1/2}
$$

(C.2)

in which the equivalent strain $\Delta\varepsilon_{eq}^{ampl}$ is maximized with respect to time. It can be demonstrated that this equation computes identical equivalent strain ranges for out-of-phase and for in-phase loading having the same nominal amplitude.

The definition for $\Delta\varepsilon_{eq}^{ampl}$ utilized in the tracer routine presented in section 4.1.5 is different. The routine responsible to update the internal variables traces the plastic strain components separately and use their amplitude values to compute the equivalent plastic strain range:
Appendix C. Issues linked with the implementation of a User MATerial model in a FE code

Figure C.25. Definition of the equivalent plastic strain amplitude proposed by Jiang and Kurath (1997) for a complex loading trajectory.

\[
\Delta \varepsilon_{eq}^{pl \text{ ampl}} = \frac{\sqrt{3}}{3} \left\{ \left[ \Delta \varepsilon_{11}^{pl \text{ ampl}} - \Delta \varepsilon_{22}^{pl \text{ ampl}} \right]^2 + \left[ \Delta \varepsilon_{22}^{pl \text{ ampl}} - \Delta \varepsilon_{33}^{pl \text{ ampl}} \right]^2 + \ldots \\
\ldots + \left[ \Delta \varepsilon_{11}^{pl \text{ ampl}} - \Delta \varepsilon_{33}^{pl \text{ ampl}} \right]^2 + 6\left[ \left( \Delta \varepsilon_{12}^{pl \text{ ampl}} \right)^2 + \left( \Delta \varepsilon_{13}^{pl \text{ ampl}} \right)^2 + \left( \Delta \varepsilon_{23}^{pl \text{ ampl}} \right)^2 \right] \right\}^{1/2} \quad (C.3)
\]

This equation computes a higher equivalent strain range for out-of-phase than for in-phase loading having the same nominal amplitude. Since, under complex loading conditions, the definition of the equivalent strain amplitude could be questionable from a theoretical point of view, a more detailed analysis is necessary to define which is the most appropriate approach to evaluate the strain range in the equivalent space.


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<td>One At a Time</td>
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Nomenclature

\( \alpha \) total backstress tensor
\( \alpha^{(k)} \) backstress tensor for the \( k \) component
\( \beta \) difference between deviatoric stress and backstress tensor
\( \gamma^{(k)} \) kinematic hardening parameter for the \( k \) backstress component
\( \gamma_{RT} \) group of factors used in the sensitivity analysis to describe the influence of the third backstress component at room temperature
\( \gamma_{200C} \) group of factors used in the sensitivity analysis to describe the influence of the third backstress component at 200°C
\( \epsilon \) strain tensor
\( \epsilon^{el} \) elastic strain tensor
\( \epsilon^{mech} \) mechanical strain tensor
\( \epsilon^{pl} \) plastic strain tensor
\( \epsilon^{th} \) thermal strain tensor
\( \epsilon^{pl}_{mean} \) equivalent mean plastic strain
\( \epsilon^{pl}_{ampl} \) equivalent plastic strain amplitude
\( \gamma \) rotation
\( \theta^{(k)} \) hardening parameter for the \( k \) backstress component used in the implementation of the constitutive model
\( \Theta \) torque
\( \kappa \) coefficient of linear thermal expansion
\( \nu \) Poisson ratio
\( \xi \) equivalent ratcheting step
\( \sigma \) stress tensor
\( \tau \) shear stress
\( \phi_{RT} \) group of factors used in the sensitivity analysis to describe the cyclic and ratcheting–induced hardening at room temperature
\( \phi_{200C} \) group of factors used in the sensitivity analysis to describe the cyclic and ratcheting–induced hardening at 200°C

\( C^{(k)} \) kinematic hardening parameter for the \( k \) backstress component
\( D \) damage
\( \epsilon_{ave} \) average error
\( \epsilon_{max} \) maximum error
\( D^{el} \) elastic stiffness tensor
\( D^{dev,el} \) deviatoric elastic stiffness tensor
\( D^{ep} \) deviatoric elastic stiffness tensor
\( E \) elastic modulus
\( E_{RT} \) group of factors used in the sensitivity analysis to describe the stiffness at room temperature
$E_{200C}$ group of factors used in the sensitivity analysis to describe the stiffness at 200°C

$F_y$ yield condition

$G$ shear modulus

$J_2$ second invariant of a generic tensor

$J_p$ polar moment of inertia

$L$ length

$N_f$ number of cycles to failure

$N$ flow tensor

$p$ accumulated plastic strain

$R$ isotropic hardening

$R_{ext}$ external radius

$R_{int}$ internal radius

$s$ deviatoric stress tensor

$T$ temperature

$Y$ yield stress

$Y_{RT}$ group of factors used in the sensitivity analysis to describe the isotropic hardening at room temperature

$Y_{200C}$ group of factors used in the sensitivity analysis to describe the isotropic hardening at 200°C

$W_p$ accumulated plastic work
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Facheris Giacomo,
born on August 10, 1983
in Bergamo, Italy

**Education**

09/2010–02/2014  PhD, Laboratory of Nuclear Energy and Safety (NES), PSI, Villigen, Switzerland and Department of Mechanical and Process Engineering (D-MAVT), ETH Zürich, Switzerland. Thesis: 'Cyclic plastic material behavior leading to crack initiation in stainless steel under complex fatigue loading conditions'. Thesis supervisors: Prof. Edoardo Mazza (ETH), Dr. Koenraad Janssens (PSI).

02/2008  Vocational certificate of engineer regulated profession (Recognition under directive 2005/36/EC), Politecnico di Milano, Italy.


**Working experience**

02/2014–Present  Fatigue analyst at Morson Wynn with working for Agusta Westland (Cascina Costa di Samarate, Italy). Fatigue assessment of the Primary Structural Elements of helicopters according to international standards.
FAR and CS.

02/2008–07/2010 Research and development at Nuova Termostampi (Bergamo, Italy). Production monitoring finalized to productive process improvement. Mould construction and injection moulding of thermosetting (BMC and SMC) and thermoplastic materials.

Publications


Oral presentations

Facheris, G.: Cyclic plasticity of 316L stainless steel under complex loading conditions, Nuclear energy and safety department, PhD. students Day, Villigen (Switzerland), 2013.


Facheris, G., Janssens, K.G.F.: Calibrated Modeling and Simulation of Cyclic Thermal Stress Induced Fatigue in AISI 316L Stainless Steel, 10th International Conference on Multiaxial Fatigue and Fracture (ICMFF10), Kyoto (Japan), 2013.

Janssens, K.G.F., Facheris, G.: Extended finite element model prediction of cyclic thermal shock induced crack initiation and short crack
propagation using a cyclic damage model, 8th European Solid Mechanics Conference (ESMC2012), Graz (Austria), 2012.


**Facheris, G.**: Ratcheting plasticity of 316L stainless steel in low cycle fatigue, Nuclear energy and safety department, PhD. students Day, Villigen (Switzerland), 2012.

**Awards**

Best Paper Award, NES PhD Day 2013 für den besten Beitrag des 3 Doktoranden-Jahres.

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