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Theoretical, numerical and experimental analysis

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Energy-based method for sudden column failure scenarios: theoretical, numerical and experimental analysis
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Summary

Sudden column failure scenarios are commonly proposed by design codes in order to perform homogeneous robustness assessments of buildings, as they allow measurement of their capacity to redistribute loads. However, the analysis of such an event is complex as it includes nonlinearities and dynamics. Simple energy-based approaches can, nevertheless, be used. The proposed method simplifies the dynamic component, reducing the analysis to the calculation of the static response. This approach is based on several simplifications that lead to approximate, yet accurate, results. The theoretical background and physical interpretation of this approach, together with a numerical parameter study are presented. In addition, this methodology has been applied to an experimental campaign carried out at the University of Nottingham. The numerical outcome is compared to the experimental results in order to verify the accuracy and reliability of the approach.

Keywords: structural robustness; alternative load path; sudden column failure; energy conservation; progressive collapse; nonlinear dynamic response.

1. Introduction

Highly dynamic problems have always been a challenge for structural engineers. Energy-based methods are a useful tool for analysing scenarios where a great exchange of energy is produced within a short time period, such as blast or impact events. Time history responses cannot be estimated through these approaches, which allow only calculation of the maximum structural response. In explosions, rockfall hazards, vehicles impact and other similar scenarios this maximum response is the one of interest for structural engineers. Their main objective is to estimate whether or not the structure will withstand this extreme event.

These methods are based on the classical physical law of energy conservation. That is, the energy introduced in a system by an external agent must be the same as the energy stored in the system. In structural engineering, the energy is normally introduced in the system as potential or kinetic energy and stored as strain energy. Newmark was the first to formally describe this approach for blast resistant design during the early 1950’s [1]. The approach was further developed and applied for impact and blast analysis during decades.

The growing awareness about structural robustness and the desire to provide buildings with alternate load paths that enhance their redundancy have made necessary the analysis of structures subjected to sudden column failure scenarios. Such events can also be efficiently analysed through energy conservation. Several authors [2] [3] [4] have developed and adapted the approach of Newmark for sudden column removal scenarios. This new method consists in estimating the static pushover response of the system and simplifying the dynamic component through an energy balance. This approach is a compromise between accuracy and complexity as it includes energy absorption capacity, ductility supply and redundancy in the analysis [4].
2. Physical interpretation and description of the approach

The basic assumption for successfully using this energy balance method for structures subjected to sudden column failure scenarios is that this scenario is similar in effect to applying a step load of the existing forces on the affected assembly. In other words, it is assumed that the sudden impact of the existing loads on the structure yields to a similar maximum dynamic response as the one for the same structure previously loaded subjected to an instantaneous column loss.

This method can be easily described in physical terms. For every step of the deformation process, the energy balance of the system must be fulfilled as it can be seen in Fig. 1 a). This method assumes that, before the column is suddenly removed, both, potential and kinetic energy, are zero. Immediately after the column loss, the potential energy of the gravity loads is suddenly released and equilibrium is not satisfied. Hence, the structure deflects and deforms in order to increase the stored strain energy. The excess of external work is then transformed into kinetic energy, increasing the velocity of the system until a maximum is reached. After this point, as the structures deforms, the kinetic energy decreases, and so does the difference between the external work and the absorbed strain energy. When the kinetic energy is zero and the external work and strain energy are equal, no more energy is available for continuing the deformation process and the maximum dynamic response is reached.

\[ Q_d(u_d) = \frac{1}{u_d} \int_0^{u_d} Q_{LD}(u) du \]  

where \( Q_d \), \( Q_{LD} \), \( u_d \) and \( u \) correspond to the dynamic capacity curve, the static load-deflection curve, the maximum dynamic deflection and the static deflections, respectively. This dynamic capacity curve yields to approximate results, which are very similar to the ones given by more rigorous nonlinear dynamic analyses. Hence, the approximate maximum dynamic response of a system subjected to a sudden column removal for different loading levels can be obtained by calculating its static response, without performing a dynamic analysis. The implicit error in the results when applying the energy balance method arises from several assumptions and simplifications that are described and analysed in the following section.
3. Parameter study

The proposed energy balance method incorporates several simplifications leading to approximate solutions. Therefore, the main interest of this work is to investigate the impact of these assumptions on the accuracy of the results. A preliminary analysis and parameter study was already performed by the authors in [5]. The main conclusions and most relevant results of this study are summarised in the following paragraphs together with new findings and results from further investigations. It must be noted that all the errors referred in this section are relative errors between the results given by this new method and the ones given by the more rigorous method considered for each assumption.

3.1 Sudden column failure as a sudden impact of loads

As it has been previously mentioned, the main simplification of this approach is applying suddenly the existing loads on the substructure instead of considering a sudden support loss. The error of this assumption comes from the fact that all the work done by the loads is considered to have a dynamic nature. The system, however, is originally statically deformed by the existing loads before the column is removed. Thus, for a sudden column loss scenario a portion of the external work has a static nature, which is ignored by this energy-based approach. This simplification produces overestimations in the maximum dynamic response when applying the energy balance method. Moreover, structures subjected to a uniform step load generally have their maximum deflection at a different point as structures subjected to sudden column failure scenarios [5].

A parameter study was performed in [5] for simply supported beams and slabs in order to better estimate the error due to this simplification. The results for beams showed that the error for the maximum response remains lower than 5% for most usual configurations and depends only in the position along the beam of the removed support. Slabs are more sensible to this assumption, leading to larger errors that remain lower than 10% for typical geometries. The error in slabs depends not only in the position of the removed support but also in the slab’s aspect ratio (length/width). This study also showed that this error correlates very well with the so-called dynamic factor [5]. This factor represents the ratio of the maximum dynamic displacement originated by the sudden applied loads without support to the maximum static displacement caused by the same loads before the support is removed. The larger the value of this factor, the smaller the error when using this approach, as a higher factor means a higher proportion of the deformation is due to the removal effect, compared to the pre-existing state. Finally, a last rule extracted from this previous investigation is that, the more symmetric the structural configuration is before the support removal, the smaller the error of the energy method will be.

3.2 The affected substructure behaves like a SDOF system

The second assumption of this method is that the response of a structure subjected to a column failure is controlled by a single deformation mode and that the shape of this mode remains constant all during the dynamic response. In other words, the method assumes that the structure behaves like a single degree of freedom (SDOF) system. This simplification allows linking the energy of the whole substructure to the energy of a reference point of the substructure. The implicit error arises from assuming that the maximum response occurs when the kinetic energy of the whole system is zero at some specific time. In real structures, this never happens, as each of the existing infinite number of deformation modes will reach its peak response at different time moments. Hence, when using the energy balance method, the stored strain energy of the system is overestimated and so are the maximum deflections.

The implicit error due to this simplification is, nevertheless, relatively small. For linear elastic structures, the error in assuming a dynamic behaviour like a SDOF system is zero, as it was proven in [5]. In addition, a parameter study was performed for a simple case by comparing the results given by the energy balance method and a nonlinear dynamic analysis calculated with a finite element software. The study showed that during the elastic stage the dynamic capacity curve is equal to the nonlinear dynamic response, while the largest error (8% in this case) is produced right after yielding. Although the study was performed for a simple example, the similarity between the nonlinear dynamic response and the dynamic capacity curve allows expecting also small errors for real structures subjected to a step load. Hence, they can be accurately modelled as SDOF systems.
3.3 Damping and energy dissipation mechanisms

A further simplification of the proposed approach is to consider that all the energy introduced in the system by the loads is transformed into pure strain energy. That is, the method neglects the energy dissipated by damping or other mechanisms, and therefore overestimates the maximum responses. This new approach also does not consider the fracture energy during concrete cracking. Hence, the possible differences between fracture energies for static and dynamic excitations are not relevant for this method. Often, this fracture energy is just considered in the analysis as a part of the energy dissipated by damping. Since the maximum dynamic response for a sudden column failure scenario is normally reached close to the half of the first oscillation, it is reasonable to assume that the energy dissipated through damping and cracking during this short period is little in comparison to the total absorbed strain energy. A study has been performed in order to substantiate this assumption.

An important shortcoming when investigating the effects of damping for sudden column failure scenarios is to decide which kind of damping simulates better the dissipation mechanism. Structural engineers commonly model the dissipated energy by means of viscous damping, which is linked to the velocity of the system. There are, however, other approaches such as Coulomb damping, which is linked to friction. It is difficult to assert which model suits better for these scenarios. For the presented simplified parameter study, viscous damping has been used. In case the dissipated energy has a large influence on the maximum response of the system, further investigations into the nature of the dissipation mechanisms are required in order to obtain a better-suited model.

In the previous section, it was described that structures subjected to a uniform step load can be suitably represented by an equivalent SDOF system. Hence, the simple model used for the parameter study consists in a bilinear SDOF system subjected to a step load for different constant damping ratios along the response as it can be seen in Fig. 2 a), b) and c). The considered variables are the damping ratios (\( \xi \)) and the ratio of the post-yielding (\( k_2 \)) to elastic (\( k_1 \)) stiffness. For the stiffness ratios, an upper and lower boundary of 10% and 0%, respectively, were set. Different damping ratios up to 10% were considered in this parameter study as it can be seen in Fig. 2 d).

![Fig. 2](image)

**Fig. 2.** a) Equivalent SDOF system; b) Applied step load; c) Resistance function of the structure; d) Results of the parameter study for different damping and stiffness ratios

In Fig. 2 d), the overestimations of the maximum displacements when using the energy balance method (no damping) in comparison with the ones considering damping for increasing ductility values are plotted. Different conclusions can be extracted from these results. Basically, the error grows exponentially with increasing damping ratios and decreases for larger ductility. Damping ratios up to 5% lead to moderate overestimations, below 10%. In addition, the stiffness ratios do not have a large influence in the results for moderate damping ratios. On the other hand, damping ratios above 5%, produce large overestimations, which would make the energy balance not reliable. Moreover, the stiffness ratio does play an important role in these cases, for which larger post-yielding stiffness lead to significant smaller overestimations.

Damping ratios in structural engineering are rarely larger than 5%. Hence, the proposed approach should lead to accurate results. In any case, the results of this parameter study should be carefully
interpreted as several simplifications have been made. Additionally, this investigation is based in the assumption of pure viscous damping, which almost certainly is not the case for these scenarios.

### 3.4 Static and dynamic strain energy storage capacity are equal

The last assumption of the energy balance method is that the strain energy storage capacity of a system for a given displacement is equal when subjected to static and dynamic excitations. In reality, the material properties of a system vary by the rate they are deformed, they are strain rate dependent. Construction materials are differently influenced by strain rates. Moreover, strain rates are not constant along the deformation response, neither in time nor in space and are influenced by a large number of parameters, making it difficult to assess their impact for a general scenario. Higher strain rates commonly enhanced the resistance and stiffness properties of materials. The new proposed method ignores their influence and therefore overestimates the maximum responses of the system.

It was already showed in [5] that the strain rates of systems subjected to a sudden application of the load are directly proportional to its natural period. A simple parameter study has been performed in order to obtain an approximate value of the strain rates of a simply supported reinforced concrete slab subjected to a step load. The dynamic response has been again modelled through a bilinear SDOF system described in Fig. 2 a), b) and c). A value for the concrete’s $E$-modulus of 30 GPa and a stiffness ratio $(k_2/k_1)$ of 5% were used for the study. In addition, a span-thickness ratio of 50, which is a typical value for slabs subjected to a column loss, has been employed.

![Fig. 3. a) Maximum values of the strain rates for increasing natural periods $(T_1)$ of the system and different slab thickness $(h)$; b) Normalized displacements and strain rates along the response](image)

This study confirms that strain rates depend strongly on the natural periods. In Fig. 3 a), the obtained maximum strain rates are relative low for influencing the results of the maximum response. It must be noted that these are the maximum strain rates in time and space. The variation of maximum strain rates along the dynamic response are plotted in Fig. 3 b). The peak rate occurs before the maximum displacement response is reached. As the maximum strain rates are moderate and they occur in a small area of the slab during a short period, it is reasonable to assume that their impact in the results is small and that the error of the energy balance method due to strain rates is acceptable.

### 4. Experimental analysis

The described energy balance method has been applied to an experimental campaign performed at the University of Nottingham [6] as a last step in evaluating its accuracy and reliability. It must be noted, however, that this campaign was not designed for this specific investigation and therefore can show large deviations. Hence, the results and conclusions of this experimental analysis must be carefully interpreted and can be used as an example or reference, but not as a rule.

#### 4.1 Methodology

Two series of 1/3 scale flat slabs were built at the University of Nottingham. These allowed simulation of the removal of a corner or penultimate edge column. For each of the two series, two similar slabs were casted. One was tested quasi-statically while the other one was tested dynamically simulating a sudden column failure for different loading levels.
In the static case, the slab was placed on permanent supports and the column under investigation was removed, allowing the slab to deflect. An increasing uniform load was then imposed on the specimen surface by means of sand bags and the complete load-deflection relationship was recorded.

For the dynamic test, a slab was loaded also with sand bags whilst supported by the permanent and temporary supports. Once the required uniformly distributed load was reached, the respective support was removed and the response recorded with a high speed camera. Before conducting the next dynamic test, the specimen was restored to the original position in order to reduce the effect of previous cracking, yielding or damage. The energy balance method assumes, however, that for every different loading level the slab has not suffered any damage and the energy absorption capacity is the same. This can lead to inaccuracies when comparing experimental and numerical results.

4.2 Description of the test specimens

The series consist of a 2x1 bay flat slab substructure. The main dimensions of the specimens can be seen in Fig. 4. This assembly was used to simulate both, corner and penultimate column loss.

![Fig. 4. Geometry and details for corner and penultimate column removal tests](image)

The substructure was quasi-laterally and rotationally unrestrained. Columns were replicated by square steel plates on bearings and horizontal displacements were only restrained through friction.

4.3 Experimental results and application of the energy-based approach

4.3.1 Corner column removal

The results for the static and dynamic tests as well as the dynamic capacity curve can be seen in Fig. 5. The static response is linear until yielding with no compressive membrane action. After yielding, the slab’s capacity moderately grows, probably due to strain hardening and tensile membrane action.

![Fig. 5. Load-deflection relationship of different responses for corner column removal](image)
The maximum dynamic responses are very similar to the static ones for the same loading levels, excluding the largest one. Further results of the tests show that there is a clear dominant frequency for each loading level. Finally, the viscous damping ratios for the residual vibration for the elastic range are lower than 1%, while for the inelastic range are much larger reaching values up to 20% [6].

For this test, the dynamic capacity curve shows a significant deviation from the dynamic tests results. The difference between the numerical and experimental results is very large and it is difficult to confirm a certain reason. The large damping ratios may be a possible cause but, in the authors’ opinion, such great errors are not likely to be due to the previously described and analyzed simplifications and assumptions made by the energy balance method.

Specimens displayed a dominant deformation mode during the tests and therefore the error due to reducing the system to a SDOF system is expected to be negligible. In addition, strain rates also remained too low and localized to have a significant influence in the maximum dynamic response.

Damping is the most controversial point of this analysis. The experimental viscous damping ratios have been extracted from the specimen’s residual oscillation. It is not clear, however, if this damping ratio is similar to the mean value before the peak response is reached. This point is difficult to evaluate even with experimental results. Only complex numerical simulations could somehow answer this question, but due to the nonlinearities the results would not be reliable. Moreover, viscous damping ratios above 20% are extremely high for structural engineering and this could underline the fact that, using viscous damping for sudden column loss scenarios is a false assumption.

Observing the resemblance between the experimental maximum dynamic responses and the static ones, can lead to the assumption that some errors may have occurred during the performance or measuring of the dynamic tests, such as incomplete similarity of the specimens, loss of dynamic effects due to excessive support release times, wrong calibration of the equipment, etc.

4.3.2 Penultimate column removal

The results for the static and dynamic tests as well as the dynamic capacity curve can be seen in Fig. 6. The static response is linear until yielding without compressive membrane action. Afterwards, the slab can still bear increasing loads, probably due to strain hardening and tensile membrane action.

The two available readings for the maximum dynamic response are significantly larger than the static responses for same loading levels, especially for the highest one. Further results of these tests are only available for the lower loading level. These results [6] show again that there is a clear dominant frequency for this loading level and that the damping ratio is low during the elastic range.

![Fig. 6. Load-deflection relationship of different responses for penultimate column removal](image-url)
In this case, the extended dynamic capacity curve yields to very similar results as the ones of the dynamic tests. Unfortunately, only two dynamic tests were performed, making it difficult to assess how well the curve would have correlated for intermediate loading levels. Errors due to the simplifications of the proposed energy-based approach, however, do not largely differ from the ones of the corner column removal as the specimens also displayed a dominant deformation mode during the tests and strain rates were of the same order of magnitude.

Moreover, damping ratios were also very similar for corner and penultimate column removal tests in the elastic range. Even though the inherent error is almost the same for both scenarios, the experimental maximum dynamic responses for the elastic range correlate completely different with the numerical results given by the approach. This point also suggests that further errors not considered in this analysis may have happened during the corner column tests.

The energy balance method leads to very satisfying results for this penultimate column series.

5. Conclusions

Simple energy-based methods are suitable for analyzing sudden column removal scenarios. The proposed energy balance method yields to approximate, but still fairly accurate results and does not require complex calculations. The exact errors when applying this method are difficult to estimate and the results from the parameter study presented in this paper can be used as a reference. Based on these results, these implicit errors are not large enough to have a great impact on the results.

Under consideration of all the described errors and simplifications of this new approach, the dynamic structural capacity of the system would be larger. This means that the energy balance method underestimates the strength of the structure and overestimates the dynamic component of the loads, leading always to conservative results. A further advantage of this approach is that it requires few parameters for the analysis, reducing the uncertainties and helping obtaining more homogeneous robustness assessments.

The results from applying the energy balance method in both experimental tests are very different and therefore it is not possible to state a well-founded judgment about its accuracy. While for the corner column test the energy balance method leads to large underestimations of the slab capacity, for the penultimate column test it correlates quite well. Further experimental results are needed in order to be able to better estimate the validity of this approach. The fact that the energy balance method always leads to conservative results has also been substantiated by the experimental results.

6. References