Market design for emission trading schemes

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Market Design for Emission Trading Schemes

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to my parents

Sol-Britt & Werner

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Abstract

Emission trading schemes are regulatory frameworks designed to reduce and to control pollution level by creation of economic incentives for responsible emission sources. Currently, there are several systems of this type in operation, with the European Union Emission Trading Scheme (EU ETS) as the largest multi-national mandatory mechanism for trading of carbon dioxide allowances. This thesis is concerned with the mathematical analysis of cap-and-trade schemes and other emissions regulations. We review the existing quantitative analysis on the subject and introduce some of the mathematical challenges posed by the implementation of the new phase of the European Union Emissions Trading Scheme as well as the cap-and-trade schemes touted by the US, Canada, Australia and Japan. From a theoretical point of view, the main thrust of the thesis is the introduction of a new mathematical framework for competitive equilibrium, in which a broad range of emission regulations can be analyzed. From a practical point of view, particular focus is given to the design and numerical analysis of new cap-and-trade schemes for the control and the reduction of atmospheric pollution. We develop tools intended to help policy makers and regulators understand the pros and cons of different regulations.

As some fundamental differences between regulations do not occur in a deterministic setting we propose a stochastic equilibrium model for an economy where firms produce goods to satisfy an inelastic demand and are endowed with permits in order to offset their pollution at compliance time and avoid having to pay a penalty. Thereby we consider risk neutral (as opposed to risk averse) firms with the aim to obtain stronger equivalence results between different emission regulations. Firms that can easily reduce emissions do so, while those for which it is harder buy

permits from firms which anticipate they will not need all their permits, creating a financial market for pollution credits. Our equilibrium model elucidates the joint price formation mechanism for goods and pollution allowances, capturing most of the features of the first phase of the European Union Emissions Trading Scheme. We show existence of an equilibrium and show that its solution reduces to an optimal stochastic control problem. We also prove uniqueness of emission credit prices and characterize the equilibrium prices of goods as well as the optimal production and trading strategies of the firms.

Thereby it is shown that standard cap-and-trade schemes, designed in the spirit of the EU ETS are socially optimal (also in a stochastic setting!). However this does not imply that such schemes are inexpensive for consumers. In particular it is proven that standard cap-and-trade schemes increase consumer costs through a notable price increase for products, whose production causes pollution (for instance electrical power for EU ETS). This observation is evident, since the regulation shifts production towards cleaner, hence more expensive technologies, while the cost of pollution is factored into the prices of those products.

One of the main targets of this thesis is to quantify such effects as well as other qualitative properties of emission regulations in a realistic framework. To this end we use the Japanese electricity market to numerically simulate the equilibrium in different regulatory settings. For this market it turns out that on one hand reduction costs are astonishingly low while on the other hand it is shown that consumers burden exceeds by far the overall reduction costs, giving rise for huge windfall profits for electricity companies, which was also observed in EU ETS. Moreover we use this numerical illustration to show that even tax based regulations and standard cap-and-trade schemes with a 100% auction can fail to reduce windfall profits to a reasonable level.

This fact demonstrates the strong need for an improvement of generic cap-and-trade mechanisms. We show how to adapt the regulatory framework of an emission trading scheme in such a way that the final consumer costs are reasonably low, preserving the social optimality of the mechanism at the same time. The resulting relative and hybrid allocation schemes demonstrate that an apparently innocuous small change in the design can have a dramatic economic impact.

Zusammenfassung

Emissionshandelssysteme sind regulatorische Instrumente, die geschaffen wurden, um Schadstoffemissionen durch ökonomische Anreize für die verantwortlichen Emittenten zu kontrollieren. Derzeit gibt es mehrere Systeme dieser Art. Dabei ist das Europäische Emissionshandelssystem (EU ETS) der grösste verbindliche Mechanismus für den Handel mit Emissionszertifikaten. Diese Arbeit beschäftigt sich mit der mathematischen Analyse von Cap and Trade Systemen und anderen Emissionsregulationen. Wir fassen die bestehenden quantitativen Analysen zu diesem Thema zusammen und stellen einige der mathematischen Herausforderungen bei der Umsetzung der nächsten Phase des Europäischen Emissionshandelssystems und der geplanten Systeme in den USA, Kanada, Australien und Japan vor.

Aus theoretischer Sicht ist das Hauptziel der Arbeit die Einführung eines neuen mathematischen Rahmens für kompetitive Gleichgewichte, in dem ein breites Spektrum von Emissionsregulationen analysiert werden kann. Aus praktischer Sicht wird ein besonderes Augenmerk auf das Design und die numerische Analyse von neuen Emissionshandelssystemen für die Kontrolle und die Verringerung der Luftverschmutzung gerichtet. Wir entwickeln Werkzeuge, die politischen Entscheidungsträgern und Regulierungsbehörden helfen sollen, die Vor- und Nachteile von verschiedenen Emissionsregulationen zu verstehen.

Da einige grundlegende Unterschiede zwischen den Regulationen in einem deterministischen Model nicht auftreten, betrachten wir ein stochastisches Gleichgewichtsmodel für eine Ökonomie, in der Unternehmen Waren produzieren und eine unelastische Nachfrage decken. Dabei betrachten wir risikoneutrale (anstatt risikoscheue) Agenten, um stärkere

Äquivalenzresultate zwischen unterschiedlichen Regulationen zu erhalten. Die einzelnen Firmen sind mit Emissionszertifikaten ausgestattet, mit denen Sie ihre Emissionen kompensieren können. Falls eine Firma mehr ausstösst als sie Zertifikate abrechnet, muss sie eine Strafe bezahlen. Um diese Strafe zu vermeiden, hat das Unternehmen zwei Möglichkeiten. Es kann entweder seinen Ausstoss reduzieren oder Zertifikate am Markt einkaufen. Diejenigen Firmen, die billig Emissionen reduzieren können, werden dies tun und können ihre Zertifikate an Firmen mit teuren Reduktionskosten verkaufen, was zur sozialen Optimalität (d.h. die Treibhausgasemissionen werden unter minimalen volkswirtschaftlichen Kosten gesenkt) des Mechanismus führt. Unser Gleichgewichtsmodell erklärt den Preisbildungsmechanismus für Zertifikate und für Waren, deren Produktion Emissionen verursacht. Ausserdem erfasst es die meisten Funktionen der ersten Phase des EU ETS. Wir leiten die Existenz eines Gleichgewichts her und zeigen, dass dessen Berechnung zu einem optimalen stochastischen Steuerungsproblem reduziert werden kann. Wir beweisen auch, dass die Zertifikatspreisprozesse eindeutig sind und charakterisieren Gleichgewichtspreise von Waren sowie die optimalen Produktions- und Handelsstrategien der Unternehmen.

Dabei wird gezeigt, dass Standard-Emissionshandelssysteme im Sinne des EU-ETS sozial optimal sind (auch in einem stochastischen Model!). Dies bedeutet jedoch nicht, dass solche Systeme für den Endverbraucher billig sind. Insbesondere wird gezeigt, dass Standard-Emissionshandelssysteme zwangsläufig die Verbraucherkosten, durch einen deutlichen Anstieg der Preise für Produkte, deren Herstellung Emissionen verursacht (z. B. elektrische Energie im Falle des EU-ETS), erhöhen. Dies ist offensichtlich, da die Regulation zu einer umweltfreundlicheren Produktion und damit zu höheren Produktionskosten führt, während die Zertifikate in die Produkte eingepreist werden.

Eines der Hauptziele dieser Arbeit ist, diese Effekte sowie weitere qualitative Eigenschaften von Emissionsregulationen in einem realistischen Rahmen zu quantifizieren. Zu diesem Zweck berechnen wir numerisch das Marktgleichgewicht für unterschiedliche Regulationen am Beispiel des japanischen Strommarktes.

Für diesen Markt stellt sich heraus, dass einerseits die Reduktionskosten erstaunlich niedrig sind, während auf der anderen Seite die Kon-

sumentenkosten, die durch den erhöhten Strompreis entstehen, diese bei weitem überschreiten. Für Stromproduzenten führt dies zu hohen extra Gewinnen, so genannten Windfall Profits, welche auch in der ersten Phase des EU ETS beobachtet wurden. Darüber hinaus nutzen wir unsere Japan-Fallstudie, um zu zeigen, dass weder eine Emissionssteuer noch ein Standard-Emissionshandelssystem, in dem die gesamte Anfangsallokation versteigert wird, die Windfall Profits auf ein vernünftiges Niveau reduzieren können.

Diese Tatsache zeigt, dass es notwendig ist die Standard-Emissionshandelssysteme zu verbessern. Wir zeigen, wie man Emissionshandelssysteme in einer Weise ändert, so dass die Verbraucherkosten relativ niedrig ausfallen, während gleichzeitig die soziale Optimalität des Standard-Systems erhalten bleibt. Die resultierenden relativen und hybriden Systeme zeigen, dass eine scheinbar kleine Änderung in der Regulation eine dramatische wirtschaftliche Auswirkung haben kann.

Contents

Chapter 1

Introduction

Chapter Overview

This chapter gives a brief overview about the background, framework and goals of this thesis. In the first section of this chapter the reader is introduced to cap-and-trade schemes and the main topics of this thesis based on a discussion of the EU ETS and lessons learnt from its first phase. We proceed with a short discussion of windfall profits and the role of the initial allocation, and close the section with a literature review and an overview of the structure of the thesis.

1.1. Background and Motivation, EU ETS

The world's changing climate and the pressing need for measures to curtail the man-made effects on ecosystems continue to challenge policy makers. One of the main issues is to establish cost effective emission regulations. The most promising and most popular regulatory instruments are the cap-and-trade schemes. The largest one is the European Union Emission Trading Scheme (EU ETS), designed by the European Union as an instrument to meet targets under the Kyoto Protocol.

In 1997 governments adopted the Kyoto Protocol, which broke new ground with its mandatory requirements to reduce emissions of greenhouse gases (GHG). Each Annex 1 country (industrialized countries and countries with economies in transition) that ratified the Kyoto Protocol is obliged to reach a domestic target for carbon dioxide (CO_2) equivalent emissions, on average 5,2% below 1990 emission levels, by the first commitment period of 2008 to 2012.

To relieve compliance the European Union introduced the EU ETS, launched by the Directive 2003/87/EC of the European Parliament and the Council of October 13, 2003. This scheme is intended to ensure the reduction of carbon dioxide emissions from large industrial sources within the European Union in order to contribute to the EU's targets under the Kyoto Protocol.

The EU ETS imposes mandatory participation of nearly 10,000 installations. These are power plants and industrial users responsible for approximately 44% of the entire EU carbon emission in 2003, see e.g.

PointCarbon (2006). For these installations, the respective governments annually allocate carbon emission allowances (EUAs) according to National Allocation Plans (NAPs), which define an emission limit (cap) for each compliance period. Installations must cover their emissions with these allowances every single year. The precise regulation is as follows: There are two compliance periods 2005-2007 and 2008-2012. Within each period, allowances are valid regardless of the year in which they are actually allocated.

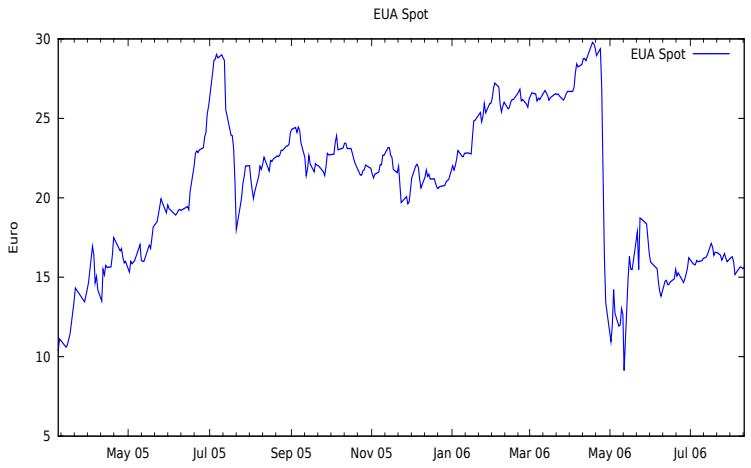

Fig. 1.1: EUA spot price, listed at the European Energy Exchange EEX.

Compliance is to be met yearly. Each operator that does not surrender sufficient allowances by the 30th of April of each year to cover emissions for the preceding year, is liable for a penalty payment for each tonne of excess emissions. Since allowances for the current year are allocated before the 30th of April, and allowances are valid for the whole compliance period permits can be *borrowed* from one year for compliance in the preceding year. Hence in practice the penalty will only be paid at the end of the compliance period. But then the size of the potential payment is considerable: Within the first period, the penalty was 40€, whereas for the second period, agents must pay 100€ per excess tonne of carbon dioxide. Moreover, the penalty payment does not release the operator from the obligation to surrender the missing allowances in the following calendar year. So de facto, the actual cost for each tonne of CO_2

not covered by a surrendered emission permit is equal to the penalty plus the spot price of one certificate from the next period. This fact is incorporated in the models used in Carmona and Fehr (2009b), Cetin and Verschuere (2008) as well as Carmona and Hinz (2009). However, for simplicity's sake, the equilibrium models introduced in this thesis cover only a single compliance period where compliance takes place at the end of the period. Hence we do not include this carry-over feature of the penalty nor the possibility of borrowing.

To avoid these penalties, firms may either reduce their own emissions or purchase allowances from other firms. This transfer of emission credits through trading is considered to be the core principle leading to the minimization of the costs caused by regulation; companies that can easily reduce emissions will do so, while those for which it is harder to make these reductions will buy credits.

Several exchanges in Europe are now committed to the trading of EUAs. The products listed there are spot (see Figure 1.1) and forward contracts with physical delivery. The stochastic nature of this price process is obvious. Consequently, conventional deterministic models are inapplicable to model allowance price formation in EU ETS. In line with this insight, PointCarbon (2006) observe that the carbon market, driven by the power sector (which is the biggest polluter in EU ETS), is to a large extent trading on changes in stochastic fundamentals, such as relative coal/gas prices and temperature/precipitation.

The first part of this thesis proposes a dynamic stochastic equilibrium model that aims to explain not only allowance price formation with respect to these fundamentals, but also the price formation of products (e.g. electricity) at the source of emissions. Figure 1.2 illustrates their dependence. The price drop occurred in April 2006, 9 months before the end of the first phase of the EU ETS, and right before the public announcement of the level of emissions for the year 2005. Installations realized that the level of emissions had been over-estimated and that emission rights were going to be in over supply. Based on this information, the allowance price dropped by 18€, while power futures followed with a drop of 9€. This interdependence of allowance and product prices is explained by the results of Chapter 2 stating that allowances are priced into electricity weighted by the marginal emission factor. In

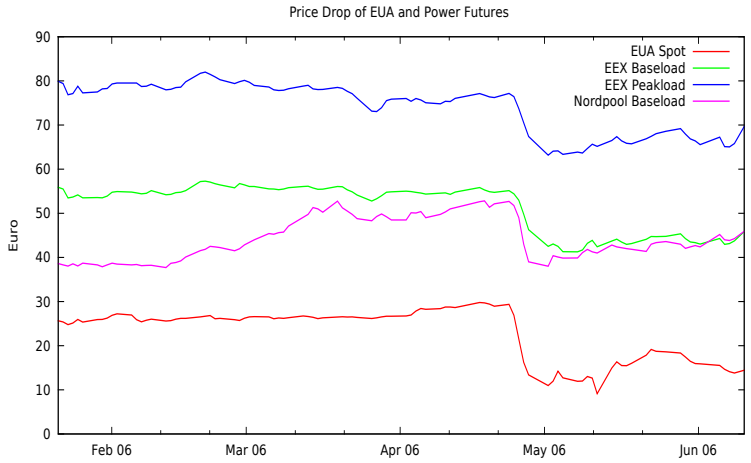

Fig. 1.2: Prices of German (EEX) and Scandinavian (Nordpool) monthly power futures with August 2006 maturity together with EUA spot prices.

other words electricity producers charge consumers for the allowances they have to offset for the produced electricity. In Chapter 2 we show that the resulting increase in consumer costs can exceed the reduction costs by far (a factor 8 for our Japanese Case Study). As observed in EU ETS (see e.g. Sijm et al. (2006)) this gives rise to huge additional profit for power producers, so called windfall profits, which are discussed in detail in Sections 1.2, 1.3, 2.4 and 2.5. Windfall profits have been at the core of the criticism of cap-and-trade schemes.

To reduce windfall profits, several policy alternatives come under consideration. These are emission taxes, standard cap-and-trade schemes where part or all allowances are auctioned and relative cap-and-trade schemes where part of the initial allocation is given relatively to production. E.g. for electricity producers this means that they are endowed with a certain number of allowances for each MWh of electricity that they produce. In Chapter 3 we extend the theoretical framework from Chapter 2 to cover such policies. And use our results to compare the qualitative properties of the schemes at the example of the Japanese electricity market. Due to the great amount of clean production capacity in the Japanese electricity sector we find that the relative scheme (in contrast to the tax and the standard scheme with auction) is the

only scheme that can reduce windfall profits to around zero. However, this comes with a slight efficiency loss compared to the standard scheme. Therefore, in Chapter 4 we propose a new hybrid cap-and-trade scheme that combines the best properties of the relative and the standard scheme and provides a tight control of windfall profits at no efficiency loss, i.e. as the standard cap-and-trade scheme it is socially optimal. In the hybrid scheme, the allocation procedure is a mixture of initial allocation, relative allocation and auctioning. Part of the cap is given as an initial allocation and the rest is put into a 'pot' from which the relative allocation is withdrawn during a fixed time period, while the allowances that remain in the pot at the end of this time interval are auctioned. To omit the situation where the allocation exceeds the cap, we apply a simple rule that prohibits the relative allocation from exceeding the size of the pot. It is intuitively clear that such an allocation rule reduces the marginal production costs and hence the prices of the goods as well as windfall profits since allowances that are relatively allocated do not have to be bought in the auction.

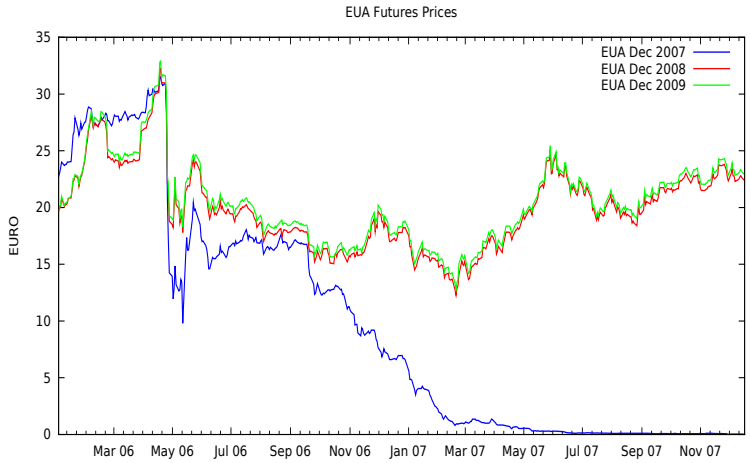

Fig. 1.3: Price drop before the end of the first phase of the EU ETS.

One of the practical questions we revive at several points during this thesis is the calibration of parameters for the emission regulations. For the standard cap-and-trade schemes these are the penalty and the cap. In particular the influence of the cap is illustrated in Figure 1.3 showing what happened after the price drop in April 2006. As due to weather

conditions emissions during the autumn were lower than anticipated, the demand for allowances dropped further. And because first-phase EUAs could not be banked, the price of the first phase allowances did not recover. In fact, these prices ended up essentially converging to 0. Not only is this phenomenon intuitively clear, but we will demonstrate that it happens in great generality. Indeed, our theoretical analysis shows that, in a cap-and-trade scheme without banking like the first phase of the EU ETS, the price of an allowance certificate converges to a two-valued random variable, these two values being zero or the penalty level imposed by the regulator. The price converges toward zero in the case of certificate over supply, and this is exactly what happened. Should this be regarded as a flaw of the scheme, the criticism should be directed toward the regulator who should not have flooded the market with an over supply of certificates. For a numerical analysis on the efficiency of the banking mechanism we refer the reader to Berkes (2007).

1.1.1. CDM/JI

This subsection shows some other interesting features of the EU ETS, which were part of the research that was conducted within the scope of this thesis. However, this research is not a subject of this manuscript as it does not directly connect to market design.

This thesis considers emission trading schemes where no allowances from outside the scheme can be used for compliance. In contrast to this, the EU ETS is coupled with the Clean Development Mechanism (CDM) and Joint Implementation (JI), which allow Annex 1 countries or companies settled in Annex 1 countries to generate allowances through financing emission reduction projects in less developed countries or respectively in other Annex 1 countries. A restricted amount of these allowances, so called Certified Emission Reductions (CERs in the case of CDM) and Emission Reduction Units (ERUs in the case of JI) can be used for compliance in EU ETS. Even though CERs, ERUs and EUAs are used to offset the same amount of CO_2 emissions their prices are not equal. This can be seen in Figure 1.4, which illustrates the historic price relation of EUAs and CERs (prices of ERUs are not shown as today

these are not liquidly traded).

For this more complex setting, an equilibrium theory in the spirit of the theory presented in this thesis is possible: prices of goods, EUAs, ERUs and CERs result jointly from such an equilibrium, and price spreads like the one illustrated in Figure 1.4 can be explained. However, these models are more involved, their notation rather tedious, and in order to avoid distracting the reader from the main thrust of the thesis with mathematical technicalities, we chose not to present these results in this work. Instead, we refer the interested reader to Carmona and Fehr (2009b).

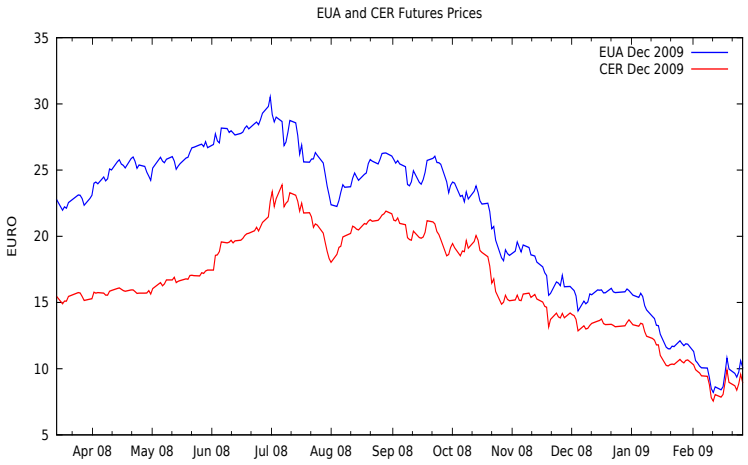

Fig. 1.4: Prices of the December 2009 EUA futures contract (EU ETS second phase), together with the price of the corresponding CER futures contract.

As transparency in price formation increases and carbon price signals become reliable, the market for futures contracts on EUAs and CERs gained in liquidity and options on these futures contracts are already traded in significant numbers. While equilibrium models like those presented in this thesis are of great value to understand the inner workings of these markets, they have little to offer when it comes to pricing and hedging these derivatives. These tasks are better performed with simpler reduced form models designed for their tractability and the ease with which they can be calibrated to price data. Such a model is proposed in Carmona and Hinz (2009) for EUAs. As CERs can be banked

their price processes are connected to those of perfectly storable commodities and can be modeled by the procedures proposed in Fehr and Hinz (2008). We expect that more research in this direction will develop in the near future.

1.2. Windfall Profits

Naturally, any emission reduction policy increases the cost of the goods whose production causes those emissions. These costs are passed on to the consumers and are partly responsible for the occurrence of substantial windfall profits. Based on an empirical analysis of power generation profitability in the context of the first phase of the EU ETS, strong empirical evidence for the existence of such profits is given in Häfliger (2007) and Sijm et al. (2006). The authors of the latter study conclude that power companies realize substantial profits since allowances are received for free while they are always priced into electrical power at a rate that depends upon the emission rate of the marginal production unit: producers seem to take advantage of the trading scheme to make extra profits. We believe that the free distribution of allowances is not the only reason for producers' windfall profits. This phenomenon is more subtle. For the sake of illustration, we present a simple instance of these profits.

Let us consider a set of firms that must satisfy a demand of $D = 1$ MWh of electricity at each time $t = 0, 1, \cdots, T-1$, and let us assume that there are only two possible technologies to produce electricity: gas technology which has unit cost 2\$ and emits 1 tonne of CO_2 per MWh produced, and coal technology which has unit cost 1\$ and emits 2 tonnes of CO_2 per MWh. In this simple model, the total capacity of gas is 1 MWh and the total capacity of coal is also 1 MWh. We also suppose that producers face a penalty of \$ π per tonne of CO_2 not offset by credits, and that a total of $T-1$ credits are distributed to the firms, allowing them to offset altogether $T-1$ tonnes of CO_2. Here π is a number strictly greater than 1. In this situation, we arrive at two conclusions. First, as demand needs to be met, total emissions will be greater than or equal to T tonnes, even if all firms use the clean technology (gas). Secondly, firms are always better off reducing emissions than paying the

penalty. As a consequence, the optimal generation strategy is to only use gas technology and emit T tonnes of CO_2. At least one firm has to pay the penalty and the price of emission credits is necessarily equal to π at each time. The missing credit has a value π for both the buyer and the seller, so the price of electricity is $2 + \pi$ because a marginal decrease in demand will induce a marginal decrease in generation cost and in penalty paid. The total profit for the producers is $\pi(T-1)$, the penalty paid by the producers to the regulator is π, and the total cost for the customers is $(2+\pi)T$. Consider now, still in the competitive equilibrium framework, the Business As Usual (BAU) scenario: the demand is met by using coal technology, the price of electricity is 1, the total profit for producers is 0 and the total cost for the customers is T. In this simple example the producer costs induced by the trading scheme is $T + \pi$; producers must buy more expensive fuel, so a profit T is made by the fuel supplier and the producers have to pay the penalty π. The increase in fuel price, or switching cost, is a marginal cost that must factor into the electricity price. The penalty is a fixed cost paid at the end, but we see that in this trading scheme, this fixed cost is rolled over the entire period and paid by the customers at each time, inducing a windfall profit for the producers. This windfall profit is exactly equal to the market value of the $T-1$ credits. However, notice that if we increase the demand to 2 MWh and cover the extra demand with nuclear power at each time $t = 0, 1, \cdots, T-1$ then the windfall profits exceed the market value of the allowances. The reason is that nuclear production is carbon neutral while its marginal costs are cheaper than the marginal costs of gas and coal. Hence electricity prices are the same and the nuclear plant generates significant windfall profits while the market value of the allowances stays the same. This illustrates that in markets with a lot of clean production capacity, auctioning can fail to reduce windfall profits to a reasonable level. In Chapter 3 we show that this problem does not only occur in the example shown above but also in more realistic settings such as in our case study of the Japanese electricity sector. In particular, this effect will become important in all markets when clean production capacity increases.

In the present work, we give a precise mathematical foundation to the analysis of emission trading schemes and quantitatively investigate the impact of emission regulations on consumer costs and firm profits.

Based on a model for perfect competition, we show that in equilibrium, a standard emission trading scheme combines two contrasting aspects. On the one hand, the system reduces pollution at the lowest cost for the society, as expected. On the other hand, it forces a notable transfer of wealth from consumers to producers, which in general exceeds the social costs of pollution reduction.

In a perfect economy where all customers are shareholders, windfall profits are redistributed, at least partially through dividends payments. However, this situation is not always the case, and the impact of regulation on prices needs to be addressed.

1.3. Reduction of Windfall Profits versus Demand Response

There are several ways to return part of the windfall profits back to the consumers. The most prominent ones are taxation and auctioning of the initial allowance distribution. Beyond the political risks associated with the levy of new taxes, we will show that one of the main disadvantages of this first method is its poor control of the final level of emissions under random demand for goods and stochastic abatement costs while windfall profits are not reduced to a reasonable level.

Auctioning is very appealing: if producers have to pay for their allowances, regulators can return revenues to consumers or invest these revenues in other emission reduction projects. However, we argue that auctions are insufficient because their return is essentially equal to the monetary value of these allowances, which is not enough to match the overall consumer burden since the latter is not directly related to the cap, but instead is related to the quantities consumed within one compliance period. In the case of our Japanese case study, the incomes from the auction can merely cover two thirds of the consumer costs. The reason is the huge amount of clean production capacity in the Japanese portfolio. Today most markets are dirty enough that it is likely that a 100% auction can cover consumer costs. However, the problem we highlight will occur in all markets when the production portfolio becomes cleaner. Furthermore, it involves a significant risk for companies

since the capital invested to procure allowances at the auction may be higher than the income later recovered from allowance prices. For a non-technical discussion of the pros and the cons of auctioning of EU ETS phase II allowances, the interested reader is referred to Hepburn et al. (2006). A quantitative analysis of some of these issues is provided in Carmona and Fehr (2009a) and Carmona et al. (2008) as well as in Chapters 3 and 4.

In this thesis we advocate the relative and the new hybrid allocation mechanisms, which provide a tight control of windfall profits even if the markets production portfolio is very clean. In the case of inelastic demand, this reduction of windfall profits using the hybrid scheme comes at no efficiency loss, as the hybrid scheme possesses the same equilibrium production strategies as the standard scheme. However, in the case of elastic demand this is not true. Both the relative and the hybrid scheme reduce windfall profits at their origin by reducing product prices. Thus the cost of pollution does not fully enter product prices and an efficient demand response is not guaranteed. Hence the relative and the hybrid schemes are unsuitable for markets with a strong demand response. However, they are crucial for electricity markets where the demand response is relatively small, giving only a small improvement in efficiency, that comes at huge consumer costs (which sometimes exceed reduction costs by several factors, as will be shown later). This thesis points out that when designing cap-and-trade schemes, policy makers should weigh consumer costs and windfall profits against efficiency and control these quantities by appropriate schemes. Unfortunately, including demand response into the numerical procedures would go beyond the scope of this thesis. In real life different industry sectors can be covered by the same emission trading scheme, where sectors for which demand response is desirable, are covered by a standard allocation while sectors with a poor demand response could be covered by a hybrid allocation mechanism.

1.4. Initial Allocation

While the total initial allocation depends on the emission target known as the cap, the impact of the specific allocation of these allowances to

the different installations is not as clear. While we do not attempt to describe exhaustively what allocation schemes can or cannot do, we analyze both theoretically and numerically the advantages of a natural extension of some of the schemes already implemented or considered in the scientific literature.

As is well known for cap-and-trade schemes and will be shown for our framework in Subsection 2.4.2 the specific distribution of the initial allocation of emission allowances among the various installations does not influence the overall reduction, but it plays other important roles. One of the goals of a climate policy is to incentivize the use of cleaner technologies and such objectives can be reached by distributing part of the initial allowances for free depending upon the type of power plant (benchmarking), or even as a reward for building clean plants. Because this mechanism is well suited to incentivize clean technologies if applied appropriately, and despite the accusations found in the popular press that this hand-out is no more than a license to pollute, we still believe that it is a tool that regulators should be able to use since it is by far not the only source of the windfall profits identified as the main social inequities of the cap-and-trade schemes. Fueled by populist pressure, auctioning of the allowances has been touted as the solution to this problem. However, it is important to notice that for the second phase of the EU ETS, individual countries did not have to give away the totality of their credit allowances for free. They could choose to auction up to 10% of their total allowances. Strangely enough, with the exception of Denmark, none chose to exercise this option. While the measure, which passed in the US House of Representative this spring included auctioning of a mere 15% of the initial allocation, the original proposal favored by President Obama and ushered in by Reps. Waxman and Markey was for 100% auctioning of the allowances.

Furthermore, a significant reduction of windfall profits through auctioning (if this is possible at all) requires that a huge amount or even the total initial allocation is auctioned. But in this case the regulatory mechanism to control the incentives is lost.

1.5. Literature Review

This thesis is based on the papers Carmona et al. (2009b), Carmona and Fehr (2009a) and Carmona et al. (2008). Other papers that were conducted in the scope of this thesis are Carmona et al. (2009a) on the interconnection of emission trading and stochastic optimal control, Carmona and Fehr (2009b) on allowance price formation in markets which are linked to CDM/JI, Fehr and Hinz (2008) on option pricing when the underlyings are storable commodities and Doege et al. (2009) on risk management in power markets.

We now review the recent literature on *equilibrium modeling* of the emissions markets, and we introduce the main mathematical assumptions of our approach.

The authors Dales (1968) and Montgomery (1972) proposed a market model for the public good *environment* introduced by tradable emission credits. Using a static model for a perfect market with pollution certificates, Montgomery (1972) considers several agents obliged to cover their entire emissions through allowances. To this end, the agents are allocated a certain initial amount of permits, reflecting the overall emission target. Agents face different reduction costs and, based on allowance prices, optimize their emissions and trade allowances. It turns out that, optimal emission rates equate marginal control costs and allowance prices. Montgomery (1972) proves the existence of a market equilibrium, where the emission target is reached at minimal costs. Numerous subsequent studies have addressed environmental trading schemes within a static framework of non-bankable permits. This line of research is not considered in the present work as our focus is on the dynamic aspects of emission trading. Publications Tietenberg (1985), Cronshaw and Kruse (1996) and Rubin (1996) provide a foundation for dynamic permit trading in a deterministic environment. They derive market equilibria, similar to Montgomery (1972), but consider different multi-period settings where banking and borrowing of allowances are allowed. The terminology *banking* stands for a mechanism where credits remain valid for compliance in all future periods, while *borrowing* means that credits from future periods can be used for compliance. The work Tietenberg (1985) considers one dynamic compliance period

with discrete time-steps, where the abatement costs curve is static and reflects only short term abatement. All allowances are allocated at the beginning of the compliance period, while the total emissions have to be covered by allowances only at the end of the period. It is concluded that before the permits are used up, their price must rise at the rate of interest, while also in this dynamic framework, emission levels are chosen such that marginal control costs and allowance price coincide. In Cronshaw and Kruse (1996) a permit system with banking is examined, where permits are allocated to firms in each of T compliance periods. Further Rubin (1996) extends the work of Tietenberg (1985) and Cronshaw and Kruse (1996) by providing a more general treatment of permit trading in continuous time through the use of optimal-control theory. Considering both, borrowing and banking with restrictions on borrowing as a special case, this formulation allows the extension of the results from Tietenberg (1985) and Cronshaw and Kruse (1996). Further generalization is given in Stevens and Rose (2002), where also explicit constraints on permit trading are examined. In the area of stochastic modeling, to the best of our knowledge, only Maeda (2004) takes into account the uncertainty aspects in an equilibrium with banking. In this two-step stochastic equilibrium model, it is assumed that both, emissions and abatement costs are stochastic, although obeying a specific structure. The study Maeda (2004) discusses how uncertainty, technological progress and different types of market participants affect allowance prices. Beyond equilibrium modeling, the issues of uncertainty are addressed in several papers. We single out Schennach (2000), which discusses the optimal decisions of the representative agent under uncertainty. The approach of Schennach (2000) is based on Rubin (1996) and presents a model where both allowance demand and abatement costs are stochastic although in a very restrictive way. However, equilibrium formation is not considered. Finally, we mention the recent works Seifert et al. (2006) and Wagner (2006), which deal with the allowance price formation within EU ETS. Here, rather than modeling equilibrium allowance prices, the EUA's evolution is introduced as the marginal abatement costs when the market follows an overall minimal-cost abatement policy. In Chesney and Taschini (2008) valuation of options on emission allowances is addressed, and Paolella and Taschini (2008) treats econometric aspects of emission allowance prices. Beyond

these examples, there exists a vast literature on several related topics, including equilibrium Bueler (1997), empirical evidence from already existing markets Joskow et al. (1998), Sijm et al. (2006), and uncertainty and risk Feng and Zhao (2006), Lemming (2003), Wirl (2006).

Our contribution to equilibrium modeling of emission markets is twofold. On one hand, we address the need for a model which takes into account the particularities of the EU ETS. Indeed, due to the existence of a penalty payment, the fundamental connection with electricity production, the stochastic nature of demand for power and fuel prices, the abatement costs, etc, there is no obvious way to extend the existing models reviewed above to fit the EU ETS. On the other hand, the theoretical value of our model will go beyond EU ETS since it encompasses most of the stylized facts in a generic cap-and-trade system, and yields a straightforward extension of Tietenberg (1985) (see Tietenberg (2003), Chapter 2) to a stochastic framework. In particular, we expect that our work will be relevant for the cap-and-trade schemes to be implemented in Japan, Canada, Australia and the US.

1.6. Structure of the Thesis

We close this rather lengthy introduction with a quick summary of the contents of the thesis.

Chapter 2 proposes an equilibrium framework for standard emission trading schemes and discusses the effect of such a regulation on the market. Section 2.1 gives the details of the mathematical model used to capture the dynamic features of a cap-and-trade system in an uncertain environment where demand for goods and costs of production are random, modeled by means of exogenously given adapted stochastic processes. We introduce the notation needed to describe the production of goods and the profit mechanisms in a competitive economy. We assume that demand is inelastic in the sense that it has to be met exactly by supply. This assumption could be viewed as restrictive, but we argue that it is quite realistic in the case of the electricity markets. We also introduce the allowance allocations and the rules of trading in these allowances. Section 2.2 defines the notion of competitive equilibrium for

risk neutral firms involved in our cap-and-trade scheme. Preliminary work shows that most of the theoretical results of this thesis still hold for risk averse firms if preferences are modeled with exponential utility. However, in order to avoid *muddying the water* with unnecessary technical issues, which could distract the reader from the important issues of pollution abatement, we restrict ourselves to the less technical case of risk neutral firms. In particular this has the advantage that we obtain stronger theoretical results such as equivalence of standard cap-and-trade schemes and hybrid schemes. For the sake of completeness, we solve the equilibrium problem in the Business As Usual (BAU) case corresponding to the absence of a market for emission permits. In this case, as expected, the prices of goods are given by the standard *merit order* pricing typical of deregulated markets. The section closes with the proof of a couple of enlightening, necessary conditions for the existence of an equilibrium in our framework. These mathematical results show that at compliance time, the equilibrium price of an emission certificate can only be equal to 0 or the penalty level chosen by the regulator. The second important necessary condition proven in this section shows that in equilibrium, the prices of the goods are still given by a merit order pricing, provided that the production costs are adjusted for the cost of emissions. This result is important as it shows exactly how the price of pollution becomes incorporated in the prices of goods in the presence of a cap-and-trade scheme. The following Section 2.3 is devoted to the rigorous proof of the existence of an equilibrium. The proof uses classical functional analysis results from optimization theory in infinite dimensional spaces. It follows the lines of a standard argument based on the analysis of what an informed central planner (*representative agent*) would do in order to minimize the social cost of meeting the demand for goods. Sections 2.4 and 2.5 are devoted to the analysis of the standard cap-and-trade scheme featured in the implementation of the first phase of the EU ETS. We show that properly chosen levels of penalty and allowance allocation lead to desired emission targets. However, our numerical experiments with data from the electricity market in Japan show the existence of excessive *windfall profits*. Our choice of Japan was made for several compelling reasons. It is isolated from the other markets in the sense that there are no electrical interconnections. This electrical isolation, combined with the

information readily available concerning power generation capacity and fuel types, made Japan a natural choice for our case study. It is important to emphasize that the results of our numerical experiments are typical of many electricity markets. The interested reader can consult Carmona et al. (2009b) for a similar case study devoted to the analysis of the potential impacts of cap-and-trade schemes on the Texas electricity sector. As explained earlier in our literature review, windfall profits have been observed in the first phase of EU ETS, giving credibility to the critics of cap-and-trade systems.

Chapter 3 discusses alternative emission regulations such as a carbon tax, a standard scheme with auctioning and a relative cap-and-trade scheme. Section 3.1 gives the theoretical foundation to equilibria under the different regulations while Section 3.2 covers the calibration of the parameters to reach emission targets formulated as quantile measures under the different regulations. Section 3.3 compares the different regulations at the example of the Japanese electricity sector. We demonstrate the shortcomings of the tax systems, which suffer from poor control of the windfall profits and unexpected expensive reduction policies when it comes to emission targets under stochastic abatement costs. Moreover, we show that auctioning cannot lower windfall profits to a reasonable level in markets with relatively clean production portfolios. We show that a relative allocation scheme can resolve most of the issues with the other schemes. Such a relative allocation scheme is easy to describe and implement as pollution allowances are distributed proportionally to production. It is reminiscent of the *uniform generation performance standard* touted in some of the non-technical economic studies of cap-and-trade schemes. Even though the number of permits is random in a relative scheme, and hence cannot be known exactly in advance, its statistical distribution is well understood as it is merely a scaled version of the distribution of the demand for goods. Consequently, setting up caps to meet pollution targets is not much different from the standard cap-and-trade schemes. Indeed, one shows that properly calibrated, the relative schemes reach the same pollution targets as the standard schemes while at the same time, keeping the consumer burden and windfall profits under control. Section 3.4 gathers more mathematical properties of the generalized cap-and-trade schemes introduced in the previous section. Our results demonstrate the versa-

tility and the flexibility of such a generalized framework. They show that regulators can control cap-and-trade schemes in order to reach pre-assigned pollution targets with zero windfall profits and *reasonably small* consumer costs, or even to force equilibrium prices of goods to be equal to target prices. However, because of the level of complexity of their implementations, it is unlikely that policy makers or regulators will make use of the schemes identified there.

Chapter 4 proves social optimality of standard cap-and-trade schemes and proposes a hybrid cap-and-trade scheme, where the allocation procedure is a mixture of relative allocation and auctioning. This scheme reduces prices of goods by relative allocation while, in contrast to relative schemes, social optimality is preserved. Social optimality for cap-and-trade schemes is a well known result for cap-and-trade schemes in the original layout of Montgomery (1972) with a stringent emission target. In Section 4.2 this result is carried over to a stochastic setting for emission trading schemes allowing excess emissions modulo a penalty π, in the spirit of EU ETS. However, the relative scheme introduced in Chapter 3 is not socially optimal. Another drawback of the relative schemes is that the actual amount of allowances injected in the market is not known in advance since it depends upon the demand for goods. Therefore, we propose in Section 4.1 a hybrid cap-and-trade scheme where part of the initial allocation is put into a pot from which the relative allocation is withdrawn during a time period. If the relative allocation exceeds the amount of allowances in the pot, each agent is given a fixed amount of allowances (which is computed using only the production strategies of the other agents). The rest of the allowances are auctioned. If the relative allocation does not exceed the amount of allowances in the pot, the allowances that remain in the pot are auctioned. In Section 4.3 we prove a one-to-one correspondence between equilibria of standard and hybrid cap-and-trade schemes and give explicit formulae for the changes in equilibrium prices. Moreover, we show that both schemes in equilibrium give exactly the same production strategies, proving that hybrid cap-and-trade schemes are socially optimal. Notice that this determines the equilibrium of standard schemes where up to 100% of the cap is auctioned, as these schemes are hybrid schemes with zero relative allocation. In Section 4.4 we show how to calibrate the different parameters of the scheme and compare the qualitative properties

of the hybrid scheme with those of standard schemes with and without auction. In particular this demonstrates that in contrast to standard designs both with and without auction hybrid schemes allow for a tight control of windfall profits.

Chapter 5 is concerned with the computation of the equilibrium by solving the optimal control problem numerically. Besides explaining the exact dynamic programming procedure, we also give error bounds for allowance prices and for the global optimal control problem, using duality results from infinite dimensional convex analysis / linear programming.

The thesis concludes with Chapter 6, which reviews the main results of the thesis, recasting them in the perspective of the challenging public policy issues uncovered by the results of the thesis.

Chapter 2

Standard Cap and Trade Schemes

Chapter Overview

In this chapter we develop a rigorous equilibrium theory for standard cap-and-trade schemes and discuss the effect of such a regulation on the market. Sections 2.1 and 2.2 present the elements of our mathematical analysis. We consider an economy where firms produce and supply goods to consumers over a period $[0, T]$. The production of these goods is a source of pollutant emissions. In order to reduce this externality, a regulator distributes emission allowances to the firms at time 0, allows them to trade the allowances on an organized market between times 0 and T, and at the end of this compliance period, levies penalties proportionally to their net cumulative emissions. As explained in the introduction, our stylized scheme does not allow for banking in the sense that unused allowances cannot be used in future compliance periods. Moreover, we ignore borrowing and assume that allowances are only to be surrendered at the end of the compliance period. As can be seen from the analysis of more realistic models, our quantitative results still hold in more general set-ups without these simplifying assumptions. See for example Cetin and Verschuere (2008); Carmona and Hinz (2009), and Carmona and Fehr (2009b). Section 2.3 is devoted to the rigorous proof of the existence of an equilibrium and characterizes equilibrium price processes as well as trading and production strategies. Moreover, we prove the uniqueness of allowance price processes. Finally Sections 2.4 and 2.5 discuss the effect of standard cap-and-trade schemes on the market at the example of the Japanese electricity sector. In particular we will confirm the occurrence of huge windfall profits that were (as anticipated by economists) observed in the first phase of EU ETS.

2.1. Description of the Model and First Notations

In what follows $(\Omega, \mathcal{F}, \mathbb{F} = \{\mathcal{F}_t,\ t \in \{0, 1, \ldots, T\}\}, \mathbb{P})$ is a filtered probability space. We denote by $\mathbb{E}[.]$ the expectation operator under the probability $\mathbb{P}$ and by $\mathbb{E}_t[.]$ the expectation operator conditional to $\mathcal{F}_t$. The σ-field $\mathcal{F}_t$ represents the information available at time t. We will also make use of the notation $\mathbb{P}_t(.) := \mathbb{E}_t[\mathbf{1}_{\{.\}}]$ for the conditional probability with respect to $\mathcal{F}_t$.

2.1.1. Production of Goods

A finite set I of firms produce and sell a set K of different goods at times $0, 1, \ldots, T-1$. Each firm $i \in I$ has access to a set $J^{i,k}$ of different technologies to produce good $k \in K$, that are sources of emissions (e.g. greenhouse gases). Each technology $j \in J^{i,k}$ is characterized by:

- a marginal cost $\widetilde{C}_t^{i,j,k}$ of producing one unit of good k at time t;
- an emission factor $e^{i,j,k}$ measuring the volume of pollutants emitted per unit of good k produced by firm i with technology j;
- a production capacity $\kappa^{i,j,k}$.

The fact that $\kappa^{i,j,k}$ is not-random and constant over time rules out the possibility to take plants offline for maintenance or emergency repairs. Doing so would significantly increase the complexity of the notation already cumbersome enough, without changing much to the nature of the results, so we decided to refrain from doing it. For the sake of notation we introduce the index sets

$$\begin{aligned} M_i &= \{(j,k) : k \in K, j \in J^{i,k}\}, \qquad i \in I \ , \\ M &= \{(i,j,k) : i \in I, k \in K, j \in J^{i,k}\} \ . \end{aligned}$$

Since electricity production from fossil fuels is the main source of CO_2 emissions, our main example of a produced good is electric power. We make the assumption that the production costs are adapted and integrable processes.

For each $0 \leq t \leq T-1$, firm $i \in I$ produces the amount $\xi_t^{i,j,k}$ of good $k \in K$, throughout the period $[t, t+1)$, using the technology $j \in J^{i,k}$. Since the choice of the production level $\xi_t^{i,j,k}$ is based only on present and past observations, the processes $\xi^{i,j,k}$ are also supposed to be adapted and, since production cannot exceed capacity, we require that the inequalities

$$0 \leq \xi_t^{i,j,k} \leq \kappa^{i,j,k}, \qquad i \in I,\, k \in K,\, j \in J^{i,k},\, t = 0, 1, \cdots, T-1 \quad (2.1)$$

hold almost surely. Our market is driven by an exogenous and *inelastic* demand for goods. Since electricity production is a significant proportion of the emissions covered by the existing schemes, this inelasticity

assumption is reasonable. We denote by D_t^k the demand at time t for good $k \in K$. This demand process is supposed to be adapted to the filtration $\{\mathcal{F}_t\}_t$. For each good $k \in K$, we assume that the demand is always smaller than the total production capacity for this good, namely that:

$$0 \leq D_t^k \leq \sum_{i \in I} \sum_{j \in J^{i,k}} \kappa^{i,j,k} \quad \text{almost surely, } t = 0, \ldots, T-1, k \in K. \quad (2.2)$$

This assumption is a natural extension of the assumption of inelasticity of the demand as it will conveniently discard issues such as blackouts, which would only be a distraction given the purposes of the thesis.

2.1.2. Emission Trading

We denote by $\pi \in [0, \infty)$ the *penalty per unit of pollutant*. As reported in the introduction, in the original design of the European Union Emission Trading Scheme (EU ETS) π was set to 40€ per metric ton of *Carbon Dioxyde equivalent* (tCO_2e). For each firm, the net cumulative emission is the amount of emissions which have not been offset by *allowances* at the end of the compliance period. It is computed at time T as the difference between the total amount of pollutants emitted over the entire period $[0, T]$ minus the number of allowances held by the firm at time T and redeemed for the purpose of emission abatement. The net cumulative emission is this difference whenever positive, and 0 otherwise.

For the sake of simplicity we assume that the entire period $[0, T]$ corresponds to one simple compliance period. In particular, at maturity T, all the firms have to cover their emissions by allowances or pay a penalty. Moreover, certificates become worthless if not used as we do not allow *banking* from one phase to the next. So in this economy, operators of installations that emit pollutants will have two fundamental choices in order to avoid unwanted penalties: *reduce* emissions by producing with cleaner technologies or *buy* allowances.

In the simplest form of allowance allocation mechanism each firm $i \in I$ is endowed with Λ_0^i pollution permits called allowances at time $t = 0$. So if it were to hold on to this initial allowance endowment until the end,

it would be able to offset up to Λ_0^i units of emissions, and start paying only if its actual cumulative emissions exceeded that level. This is the *cap* part of a cap-and-trade scheme. Depending upon their views on the demand for the various products and their risk appetites, firms may choose production schedules leading to cumulative emissions in excess of their caps. In order to offset expected penalties, they may engage in buying allowances from firms which expect to meet demand with less emissions than their own cap. This is the *trade* part of cap-and-trade schemes.

However, there is no real reason to force the allocation of allowances to take place at one single time point $t = 0$. For example, in the 2007-2012 phase of the EU ETS, allowances are allocated in March each year, while the 5 year compliance period starts in January. For later generalizations we deal with a more general set-up and assume that the distribution of pollution permits is given by adapted stochastic processes $\{\Lambda_t^i\}_{t=0,1,\cdots,T-1}$. For the sake of consistency, we will denote by

$$\Lambda^i = \sum_{t=0}^{T-1} \Lambda_t^i \tag{2.3}$$

the total allowance endowment of firm i before the end of the compliance period, and we will use the notation

$$\Lambda = \sum_{i \in I} \Lambda^i \tag{2.4}$$

for the cap (i.e. the total allocation).

Remark 1 *This leads the way to further generalizations of the allowance distribution scheme, that we use in Chapters 3 and 4. There we will consider the following. The regulator rewards the firms with allocations which, at each time $t = 0, 1, \cdots, T-1$, depend in a specific algorithmic way, on their production ξ_t^i. Thereby, we shall consider only cases where the allocation is an affine function of the production.*

However, in the current chapter we assume that the initial allocation is given independent of production at time $t = 0$.

As our analysis will show, existence, uniqueness and characterization of the equilibrium allowance price processes depend only upon the total number of emission permits issued during the compliance period,

not on the way the permits are distributed over time and among the various participating installations. However, as we will demonstrate, the statistical properties of consumer costs and windfall profits depend strongly upon the way permits are allocated. The challenge faced by policy makers is to optimally design these allocation schemes to minimize consumer costs while satisfying emission targets, controlling producers windfall profits and setting incentives for the development of cleaner production technologies. We shall concentrate on these special allocation procedures in Chapters 3 and 4.

Allowances are physical in nature, since they are certificates which can be redeemed at time T to offset measured emissions. But, firms do not have to wait for the time of compliance to buy and sell allowances, and the latter can change hands at each time $t = 0, 1, \cdots, T$. Because compliance takes place at time T, and only at that time, and because entering a forward contract does not require any initial capital (not even owning the commodity), it is natural to allow the trading at time $t = 0, 1, \cdots, T-1$ of emission allowances to be done via forward contracts settled at time T. Under these conditions, and without any loss of generality, one can also restrict trading of the actual allowance certificates to time $t = T$. At times $t = 0, 1, \cdots, T-1$, each firm can take very large long and short positions, irrespective of the actual number of allowance certificates actually existing. However, except for settling financially the outstanding forward contracts, trading at time $t = T$ can only be done with existing allowances that can be delivered, hence the specific set of restrictions, which we will have to impose at that time. Allowing trading in forward contracts provides a more flexible setting: it is more general than considering only spot trading, since it allows for trading pollution permits even before these allowances are issued and allocated. This turns out to be an important feature when dealing with general allocation schemes. For $t < T$ emission permits are considered as financial assets allowing for short and long positions whose cost of carry can be neglected. For standard financial assets, the differences between spot, forward and futures prices can be explained by the term structure of interest rates. So, modeling allowance forward price is sufficient since given forward prices, spot and futures prices can be derived by simple no-arbitrage arguments once a model for the term structure of interest rates is specified. Therefore, under the equilibrium definition

that will be introduced in Section 2.2, considering only forward trading yields no loss of generality.

We denote by A_t the price at time t of a forward contract guaranteeing either physical or financial settlement of one allowance certificate at maturity T. As we just mentioned, A_T is the spot price at time T. The terminology *forward price at time t* is misleading as there is no exchange of funds at time t. A_t is better seen as a *strike* than a price: it is the price (in time T currency) at which the buyer at time t of the forward contract agrees to purchase the allowance certificate at time T.

Each firm can take positions on the forward market, and we denote by θ_t^i the number of forward contracts held by firm i at time t. As usual, $\theta_t^i > 0$ when the firm is long θ_t^i contracts, and $\theta_t^i < 0$ when it is short $|\theta_t^i|$ contracts. A trading strategy of firm i in the forward allowance market is understood as an adapted process $\{\theta_t^i\}_{t=0,\cdots,T-1}$.

The net cash position at time T resulting from trading the forward contracts is given by:

$$\sum_{t=0}^{T-2} \theta_t^i (A_{t+1} - A_t) - \theta_{T-1}^i A_{T-1}. \tag{2.5}$$

Notice that even though firms can take very large long/short positions, each sale must be offset by a purchase and vice-versa so that the clearing constraint

$$\sum_{i \in I} \theta_t^i = 0 \tag{2.6}$$

must hold at each time $t = 0, \ldots, T-1$. Following such a forward trading strategy at time T the agent holds Λ^i allowances while he receives/delivers θ_{T-1}^i allowances either financially or physically. We denote by $\tilde{\theta}_T^i$ the number of allowance certificates actually held by firm i at time T. Only these certificates can be surrendered to the regulator in order to comply with the rules of the trading scheme at time T. Since compliance can only be realized with physical allowances following restrictions must be fulfilled:

$$\sum_{i \in I} \tilde{\theta}_T^i = \sum_{i \in I} \Lambda^i \tag{2.7}$$

and

$$0 \leq \tilde{\theta}^i_T \leq \sum_{i \in I} \Lambda^i \qquad \text{for all } i \in I. \tag{2.8}$$

Clearly, these constraints are redundant. Indeed, if the left inequality in (2.8) is satisfied for every $i \in I$, then (2.7) implies that the right inequality in (2.8) is also satisfied for every $i \in I$. Consequently, beyond the physical trading clearing condition (2.7), we will only enforce the trading constraint

$$0 \leq \tilde{\theta}^i_T, \qquad i \in I \tag{2.9}$$

which merely says that firms cannot surrender a negative amount of allowances to the regulator.

Accumulated costs for adjusting the financial and physical positions at time T are given by $(\tilde{\theta}^i_T - \Lambda^i - \theta^i_{T-1})A_T$. For the ease of notation we introduce $\theta^i_T := \tilde{\theta}^i_T - \Lambda^i$ and define a trading strategy as an adapted stochastic process $\{\theta^i_t\}_{i \in I, t=0,1,\cdots,T}$. With this notation conditions (2.6), (2.7) and (2.9) transfer to the clearing condition

$$\sum_{i \in I} \theta^i_t = 0, \qquad t = 0, 1, \cdots, T \tag{2.10}$$

and the compliance constraints

$$-\Lambda^i \leq \theta^i_T, \qquad i \in I. \tag{2.11}$$

The net cash position at time T resulting from trading the forward contracts and the physical allowances is given by:

$$\begin{aligned} R^A_T(\theta) &= \sum_{t=0}^{T-2} \theta^i_t (A_{t+1} - A_t) - \theta^i_{T-1} A_{T-1} - (\tilde{\theta}^i_T - \Lambda^i - \theta^i_{T-1}) A_T \qquad (2.12) \\ &= \sum_{t=0}^{T-1} \theta^i_t (A_{t+1} - A_t) - \theta^i_{T-1} A_T - (\theta^i_T - \theta^i_{T-1}) A_T \\ &= \sum_{t=0}^{T-1} \theta^i_t (A_{t+1} - A_t) - \theta^i_T A_T. \end{aligned}$$

With this interpretation, $\Lambda^i + \theta^i_T$ is the number of (physical) allowance certificates surrendered by firm i and is used for the computation of the penalty given by the formula (2.16) below.

2.1.3. Profits

As we argued earlier, it is natural to work with T-forward allowance contracts because compliance takes place at time T. By consistency, it is convenient to express all cash flows, position values, firm wealth, and goods values in time T-currency. As a side fringe benefit, this will help us avoid having to discount cash flows in the computations to come. So we use for numéraire the price $B_t(T)$ at time t of a Treasury (i.e. non defaultable) zero coupon bond maturing at T. We denote by $\{\tilde{S}^k_t\}_{t=0,1,\cdots,T}$ the adapted spot price process of good $k \in K$, and according to the convention stated above, we shall find it convenient to work at each time t with the T-forward price

$$S^k_t = \tilde{S}^k_t / B_t(T)$$

and we skip the dependence in T from the notation of the T-forward price as T is the only maturity we are considering.

Hence, a cash flow X_t at time t is equivalently valued as a cash flow $X_t/B_t(T)$ at maturity T. So if firm i follows the production policy $\xi^i = \{(\xi^{i,j,k}_t)_{k\in K, j\in J^{i,k}}\}_{t=0}^{T-1}$ its instantaneous revenues at time t from the production of goods is given by

$$\sum_{(j,k)\in M_i} (\tilde{S}^k_t - \widetilde{C}^{i,j,k}_t)\xi^{i,j,k}_t$$

and its time T-forward value is given by:

$$\sum_{(j,k)\in M_i} (S^k_t - C^{i,j,k}_t)\xi^{i,j,k}_t$$

provided we set $C^{i,j,k}_t = \widetilde{C}^{i,j,k}_t / B_t(T)$. The total net gains from producing and selling goods are thus:

$$\sum_{t=0}^{T-1} \sum_{(j,k)\in M_i} (S^k_t - C^{i,j,k}_t)\xi^{i,j,k}_t \ . \tag{2.13}$$

In order to hedge their production decisions, firms trade on the emissions market by adjusting their forward positions in allowances. In

addition, at maturity T, each firm i surrenders allowances to cover its emissions and/or pay a penalty. Let

$$\Pi^i(\xi^i) := \sum_{t=0}^{T-1} \sum_{(j,k)\in M_i} e^{i,j,k} \xi_t^{i,j,k} \tag{2.14}$$

be the actual cumulative emissions of firm i when it uses production strategy ξ^i. We also suppose that there exist other sources of emissions on which firm i has no control, denoted by Δ^i, being an $\mathcal{F}_T$-measurable random variable.

If we think of electricity as one of the produced goods for example, the presence of this uncontrolled source of emissions can easily be explained. Usually electricity producers are required to hold a reserve margin in order to respond to short time demand changes and to protect against sudden outages or unexpectedly rapid ramps in demand. When scheduling their plants it is not yet known how much of this reserve margin will be used. Therefore, in most markets there is an uncertainty on the exact emission level when a production decision is made.

In a first reading Δ^i can be thought of as being 0 for the sake of simplicity. We shall see later in this chapter that its presence helps characterizing the equilibrium of the economy and that it is a useful tool for modeling several variations of the model. Moreover, we introduce the negative net amount Γ^i of allowances that producer $i \in I$ can use to offset the scheduled emissions by

$$\Gamma^i = \Delta^i - \Lambda^i = \Delta^i - \sum_{t=0}^{T-1} \Lambda_t^i. \tag{2.15}$$

With this notation the total penalty paid by firm i at time T is:

$$\pi(\Gamma^i + \Pi^i(\xi^i) - \theta_T^i)^+ \ . \tag{2.16}$$

Combining (2.13) and (2.16) together with (2.12), we obtain the expression for the terminal wealth (profits and losses at time T) of firm i:

$$L^{A,S,i}(\theta^i,\xi^i) := \sum_{t=0}^{T-1} \sum_{(j,k)\in M_i} (S_t^k - C_t^{i,j,k})\xi_t^{i,j,k} + \sum_{t=0}^{T-1} \theta_t^i (A_{t+1} - A_t) - \theta_T^i A_T - \pi(\Gamma^i + \Pi^i(\xi^i) - \theta_T^i)^+ \ . \tag{2.17}$$

To emphasize the mathematical technicalities of the model, we underline the fact that demand and production costs change with time in a stochastic manner. The statistical properties of these processes are given exogenously, and are known at time 0 by all firms. Moreover, we always assume that these processes satisfy the constraints (2.1) and (2.2) almost surely. Firms adjust their production and trading strategies in a non-anticipative manner depending on their observations of the fluctuations in demand and production costs. In turn, the production and trading strategies ξ^i and θ^i become respectively adapted stochastic processes on the stochastic base of the demand and production costs.

2.2. Market Equilibrium

In this section, we follow the common apprehension that a realistic market state is described by prices which correspond to a so-called market equilibrium, a situation, where the demand for each product is covered, all financial positions are in zero net supply, and each firm is satisfied by its own strategy. We define such an equilibrium and provide necessary conditions for its existence.

2.2.1. Definition of Equilibrium

Before we define the equilibrium we need to fix the spaces for the price processes and the agents strategies. To this end we introduce for any

$1 \leq p \leq \infty$ and for any normed vector space F, the following space of adapted processes:

$$\mathcal{L}_t^p(F) := \left\{(X_s)_{s=0}^t;\ \mathbb{F}\text{-adapted},\ F\text{-valued}, \|X_s\| \in L^p(\mathcal{F}_s), s = 0, \ldots, t\right\}. \tag{2.18}$$

Based on these spaces we choose allowance and product price processes in the space $\mathcal{L}_T^1(\mathbb{R}) \times \mathcal{L}_{T-1}^1(\mathbb{R}^{|K|})$ and introduce the spaces of admissible production strategies:

$$\mathcal{U}^i := \left\{(\xi_t^i)_{t=0}^{T-1} \in \mathcal{L}_{T-1}^\infty(\mathbb{R}^{M_i});\ 0 \leq \xi_t^{i,j,k} \leq \kappa^{i,j,k}\ a.s.\ ,\quad t = 0, \ldots, T-1\right\},$$

$$\mathcal{U} := \left\{\xi \in \prod_{i\in I} \mathcal{U}^i;\ \sum_{i\in I}\sum_{j\in J^{i,k}} \xi_t^{i,j,k} = D_t^k\ a.s.\ ,\ \ k \in K,\ \ t = 0, \cdots, T-1\right\}.$$

As production strategies are bounded the choice of $p = \infty$ is no further restriction and hence the most general setting. Moreover, we define the spaces of admissible trading strategies:

$$\mathcal{V}^i(A, \Lambda^i) := \left\{(\theta_t^i)_{t=0}^T,\ \mathbb{F}\text{-adapted},\ R_T^A(\theta^i) \in L^1(\mathcal{F}_T),\ \theta_T^i \geq -\Lambda^i\ a.s.\right\}$$
$$\mathcal{V}(A, \Lambda) := \prod_{i\in I} \mathcal{V}^i(A, \Lambda^i).$$

For the results stated throughout this thesis we need further following natural assumptions:

Assumption 1 *We do not consider scenarios where the regulator does not give away any permits, i.e.*

$$\mathbb{P}\{\Lambda > 0\} = 1. \tag{2.19}$$

A cap-and-trade scheme does not make sense without allocation of pollution permits!

Moreover, we assume that:

Assumption 2 *With probability one, the net emission exceeds the trading bound, i.e.*

$$\Gamma^i + \Pi^i(\xi^i) \geq -\Lambda^i \tag{2.20}$$

almost surely for all $\xi^i \in \mathcal{U}^i$ and $i \in I$.

This is very natural for standard emission trading schemes where besides (2.15) also $\Delta^i \geq 0$ a.s. and $e^{i,j,k} \geq 0$ hold. In this case (2.20) translates to $\Delta^i + \Pi^i(\xi^i) \geq 0$ a.s. meaning that the emissions are almost surely nonnegative. In Assumption 2 we use rather the form (2.20) since it allows for more general market designs where (2.15) and $e^{i,j,k} \geq 0$ does not hold.

Furthermore in order to avoid possible problems with existence of the expected values of the random variables appearing in (2.17), we assume that:

Assumption 3 *The uncontrolled emissions, the allocation and the production costs are integrable, i.e.*

$$\Delta^i \in L^1(\mathcal{F}_T), \quad \Lambda^i \in L^1(\mathcal{F}_T), \quad \{C_t^i = (C_t^{i,j,k})_{(j,k)\in M_i}\}_{t=0}^{T-1} \in \mathcal{L}_{T-1}^1(\mathbb{R}^{M_i})$$

for all $i \in I$.

In what follows, we also use a technical assumption on the nature of the uncontrolled emissions. Even though this assumption is not needed for most of the results, it will help us characterize the prices in equilibrium by ruling out pathological situations. This technical assumption states that up until the end of the compliance period, there is always uncertainty about the expected pollution level due to unpredictable events as described in Section 2.1.3 in the sense that, conditionally on the information available at time $T-1$, the sum of all the Γ^is has a continuous distribution. More precisely, we shall assume that:

Assumption 4 *The $\mathcal{F}_{T-1}$-conditional distribution of the total uncontrolled emissions $\Delta := \sum_{i\in I} \Delta^i$ possesses almost surely no point mass, or equivalently, for all $\mathcal{F}_{T-1}$-measurable random variables Z*

$$\mathbb{P}\{\Delta = Z\} = 0. \tag{2.21}$$

As we already pointed out, this technical assumption will help us refine the statements of some of the equilibrium results. Throughout this thesis we make also the assumption that the sets I and K are nonempty and finite. Moreover, we assume that for all agents $i \in I$ the set $J^{i,k}$ of different technologies to produce good $k \in K$ is finite and that for all $k \in K$ there is at least one $i \in I$ with $J^{i,k}$ nonempty.

Following the intuition that given price processes $A = \{A_t\}_{t=0}^T$ and $S = \{(S_t^k)_{k\in K}\}_{t=0}^{T-1}$ each firm aims at increasing its own wealth by maximizing

$$(\theta^i, \xi^i) \mapsto \mathbb{E}[L^{A,S,i}(\theta^i, \xi^i)], \tag{2.22}$$

over its admissible investment and production strategies, we are led to define equilibrium in the following way:

Definition 1 *The pair of price processes* $(A^*, S^*) \in \mathcal{L}_T^1(\mathbb{R}) \times \mathcal{L}_{T-1}^1(\mathbb{R}^{|K|})$ *form an equilibrium of the market if for each* $i \in I$ *there exists* $(\theta^{*i}, \xi^{*i}) \in \mathcal{V}^i(A^*, \Lambda^i) \times \mathcal{U}^i$ *such that:*
(i) All financial positions are in zero net supply, i.e.

$$\sum_{i\in I} \theta_t^{*i} = 0 \ \text{a.s. for all } t = 0, \dots, T \tag{2.23}$$

(ii) Supply meets demand for each good

$$\sum_{i\in I} \sum_{j\in J^{i,k}} \xi_t^{*i,j,k} = D_t^k \ \text{a.s. for all } k \in K, \ \ t = 0, \dots, T-1 \tag{2.24}$$

(iii) Each firm $i \in I$ *is satisfied with its own strategies in the sense that*

$$\mathbb{E}[L^{A^*,S^*,i}(\theta^{*i}, \xi^{*i})] \geq \mathbb{E}[L^{A^*,S^*,i}(\theta^i, \xi^i)] \quad \text{for all } (\theta^i, \xi^i) \in \mathcal{V}^i(A^*, \Lambda^i) \times \mathcal{U}^i. \tag{2.25}$$

Later in this chapter it will turn out that the equilibrium allowance price process A^* is in $\mathcal{L}_T^\infty(\mathbb{R})$. However, the above definition allows in principle for a rather general set $\mathcal{L}_T^1(\mathbb{R})$ of equilibrium prices. As we will prove uniqueness of the allowance price process, this leads to stronger results than if we would have restricted A^* to $\mathcal{L}_T^\infty(\mathbb{R})$.

2.2.2. Equilibrium in the Business-As-Usual Scenario

When the penalty π is equal to zero, in other words in the absence of a climate policy and an emission market, our equilibrium theory should recover what is known as the Business-As-Usual (BAU for short) scenario. As we explain below, it is characterized by the classical merit order production strategy, which can be described in the following way.

At each time t, and for each good k, all the production means of the economy are ranked by increasing production costs $C_t^{i,j,k}$. Demand is met by producing from the cheapest production means and good k's equilibrium spot price is the marginal cost of production of the most expensive production means used to meet demand D_t^k. In order to prove this claim we notice that, if (A^*, S^*) is an equilibrium when $\pi = 0$, the optimization problem (2.22) of firm i becomes

$$\sup_{(\theta^i,\xi^i)\in\mathcal{V}^i(A^*,\Lambda^i)\times\mathcal{U}^i} \mathbb{E}\left[\sum_{t=0}^{T-1}\sum_{(j,k)\in M_i}(S_t^{*k}-C_t^{i,j,k})\xi_t^{i,j,k} + \sum_{t=0}^{T-1}\theta_t^i(A_{t+1}^*-A_t^*)-\theta_T^i A_T^*\right].$$

The objective function to maximize is the sum of a function of the production strategies and a function of the investment strategy. Moreover, trading and production strategies are *decoupled* and we can maximize the two terms separately. Production strategies and the prices of the goods are determined by the first (and only meaningful) optimization problem, which reads like a classical competitive equilibrium problem where each firm maximizes

$$\sup_{\xi^i\in\mathcal{U}^i}\mathbb{E}\left[\sum_{t=0}^{T-1}\sum_{(j,k)\in M_i}(S_t^{*k}-C_t^{i,j,k})\xi_t^{i,j,k}\right], \tag{2.26}$$

and the equilibrium prices S^* are set so that supply meets demand. The solution of this equilibrium problem is given by the following pointwise linear optimization problem. For each good $k \in K$:

$$\begin{aligned}&((\xi_t^{*i,j,k})_{j\in J^{i,k}})_{i\in I} \\ &\quad\in \operatorname{argmax}_{((\xi_t^{i,j,k})_{j\in J^{i,k}})_{i\in I}} \sum_{i\in I}\sum_{j\in J^{i,k}} -C_t^{i,j,k}\xi_t^{i,j,k} \\ &\qquad\text{s.t. } \sum_{i\in I}\sum_{j\in J^{i,k}}\xi_t^{i,j,k} = D_t^k \\ &\qquad\quad 0\le \xi_t^{i,j,k}\le \kappa^{i,j,k} \quad \text{for } i\in I, j\in J^{i,k}\end{aligned} \tag{2.27}$$

for all times t. The associated equilibrium prices are

$$S_t^{*k} = \max_{i\in I,\, j\in J^{i,k}} C_t^{i,j,k}\mathbf{1}_{\{\xi_t^{*i,j,k}>0\}}, \tag{2.28}$$

This is exactly the merit order pricing mechanism described earlier. In the case of electricity, it can be observed in most deregulated electricity markets without emission trading scheme. Conversely, it is easily seen that the above prices together with the above strategies define an equilibrium. In Section 2.3 we will see that even under an emission trading scheme, the dispatching of production among producers is still a merit order-like dispatching with costs adjusted to take into account the mark-to-market value of emissions.

2.2.3. Necessary Conditions for the Existence of an Equilibrium

Before turning to the full characterization of the equilibria, we present some necessary conditions that will provide interesting insight.

Proposition 1 (Necessary Conditions) *Let (A^*, S^*) be an equilibrium and (θ^*, ξ^*) an associated set of optimal strategies. Under Assumptions 1 and 3 the allowance price process A^* is a bounded martingale with values in $[0, \pi]$. If moreover Assumption 2 is satisfied, then the following conditions hold:*
(i) The allowance price process A^* fulfills

$$\{A_T^* = 0\} := \{\omega \in \Omega | A_T^*(\omega) = 0\} \supseteq \{\sum_{i \in I} (\Gamma^i + \Pi^i(\xi^{*i})) < 0\} \tag{2.29}$$

$$\{A_T^* = \pi\} := \{\omega \in \Omega | A_T^*(\omega) = \pi\} \supseteq \{\sum_{i \in I} (\Gamma^i + \Pi^i(\xi^{*i}) > 0\} \tag{2.30}$$

up to sets of probability zero.
(ii) If in addition Assumption 4 holds, then the price process A^* is almost surely given by

$$A_t^* = \pi \mathbb{P}\Big[\sum_{i \in I} (\Gamma^i + \Pi^i(\xi^{*i})) \geq 0 \Big| \mathcal{F}_t\Big] \tag{2.31}$$

for all $t = 0, \ldots, T$.
(iii) The spot prices S^{*k} and the optimal production strategy ξ^{*i} correspond to a merit order-type equilibrium with adjusted costs $C_t^{i,j,k} + e^{i,j,k} A_t^*$.

Proof. We first show that A^* has to be a martingale. This is seen as follows: if not, there exists a time $t \in \{0, \dots, T-1\}$ such that the set $\mathcal{A} := \{\mathbb{E}[A^*_{t+1}|\mathcal{F}_t] > A^*_t\}$ (resp. $<$) is of non-zero probability and $\mathcal{A} \in \mathcal{F}_t$. Then for each agent $i \in I$ the trading strategy given by $\bar{\theta}^i_s = \theta^{*i}_s$ for all $s \neq t$ and $\bar{\theta}^i_t = \theta^{*i}_t + 1_{\mathcal{A}}$ (resp $\bar{\theta}^i_t = \theta^{*i}_t - 1_{\mathcal{A}}$) outperforms the strategy θ^{*i}, contradicting the third property of an equilibrium.
To prove that $A^*_T \in [0, \pi]$ holds almost surely, we assume on the contrary that $\mathbb{P}[A^*_T < 0] > 0$. Then for all $i \in I$ the strategy given by $\bar{\theta}^i_s = \theta^{*i}_s$ for all $s < T$ and $\bar{\theta}^i_T = \theta^{*i}_T + 1_{\{A^*_T < 0\}}$ outperforms the equilibrium strategy, fulfills $\bar{\theta}^i \in \mathcal{V}^i(A^*, \Lambda^i)$ and hence contradicts the third property of an equilibrium, proving that $A^*_T \geq 0$ almost surely. Similarly if $\mathbb{P}[A^*_T > \pi] > 0$ then for some $i \in I$ the equilibrium strategy θ^{*i} is outperformed by the strategy $\bar{\theta}^i$ which is defined as $\bar{\theta}^i_s = \theta^{*i}_s$ for all $s < T$ and $\bar{\theta}^i_T = (\theta^{*i}_T - 1_{\{A^*_T > \pi\}}) \vee -\Lambda^i$. Since again it holds that $\bar{\theta}^i \in \mathcal{V}^i(A^*, \Lambda^i)$ this contradicts the third property of an equilibrium. Thus we have $A^*_T \leq \pi$ almost surely.

To prove (2.29) and (2.30), notice that according to the definition of the equilibrium and the fact that $A^*_T \in [0, \pi]$ almost surely, $\theta^{*i}_T(\omega)$ coincides for almost all $\omega \in \Omega$ with the maximizer of

$$z \to \varphi^{*i}(z) = -A^*_T(\omega)z - \pi(\Gamma^i(\omega) + \Pi^i(\xi^{*i})(\omega) - z)^+ \tag{2.32}$$

over the interval $[-\Lambda^i(\omega), \infty)$. Further, observe that due to Assumption 2 it holds that $\Gamma^i + \Pi^i(\xi^{*i}) \geq -\Lambda^i$ almost surely. Hence, if $A^*_T(\omega) \in (0, \pi]$ the maximizer is attained somewhere on $[-\Lambda^i(\omega), \Gamma^i(\omega) + \Pi^i(\xi^{*i})(\omega)]$. Since this holds almost surely for each $i \in I$, the following inclusions are satisfied almost surely as well:

$$\{A^*_T \in (0, \pi]\} \subseteq \cap_{i \in I}\{\theta^{*i}_T \leq \Gamma^i + \Pi^i(\xi^{*i})\} \subseteq \{\sum_{i \in I} \theta^{*i}_T \leq \sum_{i \in I}(\Gamma^i + \Pi^i(\xi^{*i}))\} \tag{2.33}$$

and consequently

$$\{A^*_T \in (0, \pi]\} \cap \{\sum_{i \in I}(\Gamma^i + \Pi^i(\xi^{*i})) < 0\} \subseteq \left\{\sum_{i \in I} \theta^{*i}_T < 0\right\} \tag{2.34}$$

Since $\sum_{i \in I} \theta^{*i}_T = 0$ because of the first equilibrium condition, the event on the right hand sides of (2.34) is a set of probability zero. This implies inclusion (2.29).

Now if $A_T^*(\omega) \in [0,\pi)$, then the maximizer is greater than or equal to $\Gamma^i(\omega) + \Pi^i(\xi^{*i})(\omega)$. Consequently we have

$$\{A_T^* \in [0,\pi)\} \subseteq \cap_{i\in I}\{\theta_T^{*i} \geq \Gamma^i + \Pi^i(\xi^{*i}) \subseteq \{\sum_{i\in I}\theta_T^{*i} \geq \sum_{i\in I}(\Gamma^i + \Pi^i(\xi^{*i}))\} \tag{2.35}$$

Consequently, using again the fact that $\sum_{i\in I}\theta_T^{*i} = 0$, we get the desired inclusion (2.30)

$$\{A_T^* \in [0,\pi)\} \subseteq \{\sum_{i\in I}(\Gamma^i + \Pi^i(\xi^{*i})) \leq 0\}. \tag{2.36}$$

Condition (ii) is a direct consequence of (i) and Assumption 4.

In order to prove (iii), we first notice that the optimization problem for firm i reads

$$\sup_{\xi^i\in\mathcal{U}^i}\sup_{\theta_T^i\in\mathcal{V}^i(A^*,\Lambda^i)} \mathbb{E}[L^{A^*,S^*,i}(\theta^i,\xi^i)] \tag{2.37}$$

and we argue as before that for every $\xi^i \in \mathcal{U}^i$ the inner supremum can be attained by maximizing pointwise inside the expectation. This inner supremum is attained at $\theta^i(\xi^i)$ with $\theta_t^i(\xi^i) = 0$ for all $t = 0,\dots,T-1$ and a $\theta_T^i(\xi^i)$ which is up to null sets the maximizer of

$$z \hookrightarrow \varphi^i(z) = -A_T^*(\omega)z - \pi(\Gamma^i(\omega) + \Pi^i(\xi^i)(\omega) - z)^+ \tag{2.38}$$

over $[-\Lambda^i(\omega),\infty)$ for all $\omega \in \Omega$. We use essentially the same argument as before together with the fact that we now know that A_T^* takes values in $\{0,\pi\}$ almost surely. Fix any $\xi^i \in \mathcal{U}^i$. On $\{A_T^* = 0\}$ the optimal value of (2.38) is 0. On the other hand, on $\{A_T^* = \pi\}$, using the same argument as in (i) it follows from Assumption 2 that the optimal value of (2.38) is $-(\Gamma^i + \Pi^i(\xi^i))\pi$. In both cases, the optimal value of (2.38) is almost surely $-(\Gamma^i + \Pi^i(\xi^i))A_T^*$.
Hence the outer optimization problem reduces to

$$\begin{aligned}\sup_{\xi^i\in\mathcal{U}^i}\mathbb{E}\left[\sum_{t=0}^{T-1}\sum_{(j,k)\in M_i}(S_t^{*k} - C_t^{i,j,k})\xi_t^{i,j,k} - (\Gamma^i + \Pi^i(\xi^i))A_T^*\right] \\ = \sup_{\xi^i\in\mathcal{U}^i}\mathbb{E}\left[\sum_{t=0}^{T-1}\sum_{(j,k)\in M_i}(S_t^{*k} - C_t^{i,j,k} - e^{i,j,k}A_t^*)\xi_t^{i,j,k} - \Gamma^i A_T^*\right].\end{aligned} \tag{2.39}$$

where we used again the martingale property of A^*. Comparing the above optimization problem with (2.26), we observe that the equilibrium can be seen as a competitive production equilibrium with adjusted costs $C_t^{i,j,k} + e^{i,j,k} A_t^*$. This concludes the proof. □

Remark 2 *The proof of inclusions (i)-(ii) was carried out under fixed production strategies ξ^*. Hence it is straightforward to extend Proposition 1 (i)-(ii) to the case where the allocation depends on production strategies. Moreover, (iii) holds if the trading bound depends on production strategies, as long as Assumption 2 is fulfilled.*

The above results provide a better understanding of what a potential equilibrium should be. The allowance price must always be in $[0, \pi]$, which is very intuitive since buying an extra allowance at time t will result in a gain of at most π at time T. As highlighted in the previous subsection, the equilibrium in the BAU scenario can be related to a global cost minimization problem. We shall see in the next section that the equilibrium in the presence of a trading scheme enjoys the property of social optimality in the sense that any equilibrium corresponds to the solution of a certain global stochastic optimal control problem, where the total pollution is reduced at minimal overall costs. We call this optimization problem the *representative agent* problem. Note that it is sometimes called the *informed central planner* problem. Beyond the economic interpretation of social-optimality, the importance of the global optimal control problem is that its solution helps to calculate the allowance prices in equilibrium. We now explore this connection in detail.

2.3. Equilibrium and Global Optimality

In this section, we show rigorously the existence of an equilibrium as defined in Definition 1. We do so by framing the problem as an equivalent global optimal control problem involving a hypothetical *informed central planner* whom we call the representative agent. We prove the equivalence of the two approaches, and as a by-product of the necessary condition proven in the previous section, we derive the uniqueness of the allowance price process.

2.3.1. The Representative Agent Problem

For each admissible production strategy $\xi = \{\xi^i\}_{i\in I} \in \mathcal{U}$, the overall production costs are defined as

$$C(\xi) := \sum_{t=0}^{T-1} \sum_{(i,j,k)\in M} \xi_t^{i,j,k} C_t^{i,j,k},$$

and the overall cumulated emissions as

$$\Pi(\xi) \;:=\; \sum_{t=0}^{T-1} \sum_{(i,j,k)\in M} e^{i,j,k} \xi_t^{i,j,k} = \sum_{i\in I} \Pi^i(\xi^i). \tag{2.40}$$

Using the notation

$$\Gamma := \Delta - \Lambda = \sum_{i\in I} (\Delta^i - \Lambda^i) = \sum_{i\in I} \Gamma^i \tag{2.41}$$

for the aggregate uncontrolled emissions minus the allowance endowments, the societal costs from production and penalty payments can be defined as

$$G(\xi) := C(\xi) + \pi(\Gamma + \Pi(\xi))^+, \qquad \xi \in \mathcal{U}\ . \tag{2.42}$$

We introduce the global optimal control problem

$$\inf_{\xi\in\mathcal{U}} \mathbb{E}[G(\xi)] \tag{2.43}$$

which corresponds to the objective of an informed central planner trying to minimize overall expected societal costs. Recall that ξ is admissible if $\xi \in \mathcal{U}$, i.e. if the demand is met and the capacity constraints are satisfied. Notice that this is a convex optimization problem. The reason for the introduction of this global optimal control problem is contained in the second necessary condition for the existence of equilibrium.

Proposition 2 *If (A^*, S^*) is an equilibrium with associated strategies (θ^*, ξ^*) and Assumptions 1, 2 and 3 are satisfied, then ξ^* is a solution of the global optimal control problem (2.43) .*

Proof. Obviously, it suffices to show that

$$\mathbb{E}[G(\xi^*)] \le \mathbb{E}[G(\xi)] \quad \text{for all } \xi \in \mathcal{U}. \tag{2.44}$$

In order to do so we notice that:

$$\begin{aligned}\sum_{i\in I}\mathbb{E}[L^{A^*,S^*,i}(\theta^{*i},\xi^{*i})] &= \mathbb{E}\Big[\sum_{t=0}^{T-1}\sum_{i,j,k\in M}(S_t^{*k}-C_t^{i,j,k})\xi_t^{*i,j,k}\\ &\quad+\sum_{t=0}^{T-1}\Big(\sum_{i\in I}\theta_t^{*i}\Big)(A_{t+1}^*-A_t^*)-\Big(\sum_{i\in I}\theta_T^{*i}\Big)A_T^*\\ &\quad-\pi\sum_{i\in I}(\Gamma^i+\Pi^i(\xi^{*i})-\theta_T^{*i})^+\Big]\\ &= \mathbb{E}\Big[\sum_{t=0}^{T-1}\sum_{k\in K}S_t^{*k}\big(\sum_{i\in I}\sum_{j\in J^{i,k}}\xi_t^{*i,j,k}\big)-C(\xi^*)\\ &\quad-\pi\sum_{i\in I}(\Gamma^i+\Pi^i(\xi^{*i})-\theta_T^{*i})^+\Big]\end{aligned}$$

where we used the fact that in equilibrium, $\sum_{i\in I}\theta_t^{*i}=0$ holds for all $t=0,\dots,T$ due to the clearing condition (i) of Definition 1. Next we use the convexity inequality

$$\sum_{i\in I}x_i^+\geq\big(\sum_{i\in I}x_i\big)^+$$

and once more the fact that the financial positions are in zero net supply to conclude that

$$\begin{aligned}\sum_{i\in I}\mathbb{E}[L^{A^*,S^*,i}(\theta^{*i},\xi^{*i})] &\leq \sum_{t=0}^{T-1}\sum_{k\in K}\mathbb{E}[S_t^{*k}D_t^k]-\mathbb{E}[C(\xi^*)]\\ &\quad-\pi\mathbb{E}\Big[\big(\sum_{i\in I}\Gamma^i+\sum_{i\in I}\Pi^i(\xi^{*i})\big)^+\Big]\\ &= \sum_{t=0}^{T-1}\sum_{k\in K}\mathbb{E}[S_t^{*k}D_t^k]-\mathbb{E}[C(\xi^*)]-\pi\mathbb{E}[(\Gamma+\Pi(\xi^*))^+]\\ &= \sum_{t=0}^{T-1}\sum_{k\in K}\mathbb{E}[S_t^{*k}D_t^k]-\mathbb{E}[G(\xi^*)].\end{aligned}$$

Now, for each $\xi \in \mathcal{U}$ we define $\theta(\xi)$ as

$$\theta_t^i(\xi) = 0 \quad \text{for all } i = 1, \dots, N,\ t = 0, \dots, T-1,$$
$$\theta_T^i(\xi) = \Gamma^i + \Pi^i(\xi^i) - \frac{\sum_{i \in I}(\Gamma^i + \Pi^i(\xi^i))}{\sum_{i \in I}(\Gamma^i + \Lambda^i + \Pi^i(\xi^i))}(\Gamma^i + \Lambda^i + \Pi^i(\xi^i)). \quad (2.45)$$

By Assumptions 1 and 2 it follows that $\theta_T^i(\xi)$ satisfies $-\Lambda^i \leq \theta_T^i(\xi)$. Due to Assumptions 1 and 3 it holds also that $\theta(\xi) \in \mathcal{V}(A^*, \Lambda)$. Moreover, repeating the above argument for $(\theta(\xi), \xi)$ yields

$$\sum_{i \in I} \mathbb{E}[L^{A^*, S^*, i}(\theta^i(\xi), \xi^i)] = \sum_{t,k} \mathbb{E}[S^{*k}_t D^k_t] - \mathbb{E}[G(\xi)]. \quad (2.46)$$

Applying the third property (*each agent is satisfied with its own strategies*) of the (A^*, S^*) equilibrium to the optimal investment and production strategies (θ^{*i}, ξ^{*i}) and $(\theta^i(\xi), \xi^i)$ yields

$$\begin{aligned} \mathbb{E}[G(\xi^*)] &\leq \sum_{t,k} \mathbb{E}[S^{*k}_t D^k_t] - \sum_{i \in I} \mathbb{E}[L^{A^*, S^*, i}(\theta^{*i}, \xi^{*i})] \\ &\leq \sum_{t,k} \mathbb{E}[S^{*k}_t D^k_t] - \sum_{i \in I} \mathbb{E}[L^{A^*, S^*, i}(\theta^i(\xi), \xi)] = \mathbb{E}[G(\xi)]. \end{aligned}$$

This holds for all $\xi \in \mathcal{U}$ completing the proof. □

The existence of an optimal $\overline{\xi}$ for the global optimal control problem (2.43) follows from standard functional analytic arguments.

Proposition 3 *Under Assumption 3, there exists a solution $\overline{\xi} \in \mathcal{U}$ of the global optimal control problem* (2.43)*.*

Our proof relies on two simple properties, which we state and prove as lemmas for the sake of clarity. First, we note that $\mathcal{L}^1 := \mathcal{L}^1_{T-1}(\mathbb{R}^M)$, equipped with the norm

$$\|X\| = \sum_{t=0}^{T-1} \sum_{(i,j,k) \in M} \mathbb{E}[|X^{i,j,k}_t|]$$

is a Banach space with dual $\mathcal{L}^\infty := \mathcal{L}^\infty_{T-1}(\mathbb{R}^M)$, the duality form being given by

$$\langle X, \xi \rangle := \sum_{t=0}^{T-1} \sum_{(i,j,k) \in M} \mathbb{E}[X^{i,j,k}_t \xi^{i,j,k}_t], \qquad X \in \mathcal{L}^1, \xi \in \mathcal{L}^\infty.$$

Next, we consider the weak* topology $\sigma(\mathcal{L}^\infty, \mathcal{L}^1)$ on $\mathcal{L}^\infty$ (see Robertson and Robertson (1973) or Megginson (1998)), namely the weakest topology for which all the linear forms

$$\mathcal{L}^\infty \ni \xi \longmapsto \langle X, \xi \rangle \in \mathbb{R}, \tag{2.47}$$

for $X \in \mathcal{L}^1$ are continuous.

Lemma 1 *The real valued function*

$$\mathcal{L}^\infty \ni \xi \longmapsto \mathbb{E}[G(\xi)] = \mathbb{E}[C(\xi)] + \pi\mathbb{E}[(\Gamma + \Pi(\xi))^+] \tag{2.48}$$

is lower semi-continuous for the weak topology.*

Proof. Obviously, the real valued function

$$\mathcal{L}^\infty \ni \xi \longmapsto \mathbb{E}[C(\xi)]$$

is continuous for the weak* topology since it is of the form $\xi \longmapsto \langle X, \xi \rangle$ for some $X \in \mathcal{L}^1$ since $X = C = \{C_t^{i,j,k}\}$ is a fixed element in $\mathcal{L}^1$ by assumption. So we only need to prove that the real valued function

$$\mathcal{L}^\infty \ni \xi \longmapsto \mathbb{E}[(\Gamma + \Pi(\xi))^+] \tag{2.49}$$

is lower semi-continuous. Using the fact that for any integrable random variable X one has

$$\mathbb{E}[X^+] = \sup_{0 \le Y \le 1} \mathbb{E}[XY]$$

one sees that

$$\mathbb{E}[(\Gamma + \Pi(\xi))^+] = \sup_{0 \le Y \le 1} (\mathbb{E}[\Gamma Y] + \mathbb{E}[Y\Pi(\xi)])$$

and hence that the function (2.49) is the supremum of a family of continuous function since for fixed Y, $\mathbb{E}[\Gamma Y]$ is a constant and $\xi \longmapsto \mathbb{E}[Y\Pi(\xi)]$ is continuous for the weak* topology by the very definition of this topology. Since the supremum of any family of continuous functions is lower semi-continuous, this concludes the proof that (2.48) is lower semi-continuous. □

Lemma 2 *For the convex subset $\mathcal{U}$ of $\mathcal{L}^\infty$ it holds that:*

(i) $\mathcal{U}$ is norm-closed in $\mathcal{L}^1$

(ii) $\mathcal{U}$ is weakly closed in $\mathcal{L}^\infty$.*

Proof. (i)If $(\xi_n)_{n\in\mathbb{N}}$ is a sequence in $\mathcal{U}$ converging in $\mathcal{L}^1$ to an element $\xi \in \mathcal{L}^1$, then $(\xi_{t,n}^{i,j,k})_{n\in\mathbb{N}}$ converges in mean for each $(i,j,k) \in M$ and $t = 0,\ldots,T-1$, and extracting a subsequence if necessary, one concludes that $(\xi_{t,n}^{i,j,k})_{n\in\mathbb{N}}$ and $(\sum_{i\in I}\sum_{j\in J^{i,k}} \xi_{t,n}^{i,j,k})_{n\in\mathbb{N}}$ converge almost surely to $\xi_t^{i,j,k}$ and $\sum_{i\in I}\sum_{j\in J^{i,k}} \xi_t^{i,j,k}$ respectively, showing that the constraints defining $\mathcal{U}$ are satisfied in the limit, implying that $\xi \in \mathcal{U}$.

(ii) Since $\mathcal{U}$ is a convex norm-closed subset of $\mathcal{L}^1$, it follows from the Hahn-Banach Theorem that $\mathcal{U}$ is the intersection of halfspaces containing $\mathcal{U}$, i.e. of the form $H_{\xi,c} = \{X \in \mathcal{L}^1 | \langle X, \xi\rangle \leq c\}$ with $\xi \in \mathcal{L}^\infty$ and $c \in \mathbb{R}$ such that $\mathcal{U} \subseteq H$. Since $\mathcal{L}^\infty \subseteq \mathcal{L}^1$ it holds for each of these halfspaces $H_{\xi,c}$ that $\xi \in \mathcal{L}^1$. Thus we conclude that $H_{\xi,c} \cap \mathcal{L}^\infty = \{X \in \mathcal{L}^\infty | \langle X, \xi\rangle \leq c\}$ is closed in $(\mathcal{L}^\infty, \sigma(\mathcal{L}^\infty, \mathcal{L}^1))$. Since by definition it holds that $\mathcal{U} \subseteq \mathcal{L}^\infty$, it follows that $\mathcal{U}$ is given by the intersection of the sets $H_{\xi,c} \cap \mathcal{L}^\infty$. Since any intersection of closed sets is closed we conclude that $\mathcal{U}$ is weakly* closed in $\mathcal{L}^\infty$. □

Proof of Proposition 3. Since $\mathcal{U}$ is bounded and weakly* closed due to Lemma 2, it follows from the theorem of Banach-Alaoglu that $\mathcal{U}$ is weakly* compact. We then conclude using Lemma 1 and the fact that any lower semi-continuous function attains its minimum on a compact set. □

2.3.2. Relation with the Original Equilibrium Problem

As a consequence of Assumption 4, for each production policy $\xi \in \mathcal{U}$, no point masses occur in the $\mathcal{F}_{T-1}$-conditional distribution of $\Gamma + \Pi(\xi)$, since $\Pi(\xi)$ is $\mathcal{F}_{T-1}$-measurable. Hence, for all $t = 0,\ldots,T-1$ we have:

$$\mathbb{P}_t(\Gamma + \Pi(\xi) \geq 0) = \mathbb{P}_t(\Gamma + \Pi(\xi) > 0). \tag{2.50}$$

In the next theorem, we show that the value of the conditional probability in (2.50) characterizes the equilibrium allowance price at time t. To prepare for the proof of this important result, we first prove a technical lemma.

Lemma 3 *Let $\overline{\xi}$ be a solution of the global optimal control problem (2.43) whose existence is guaranteed by Proposition 3. It holds:*
(i) For fixed $t \in \{0, \dots, T-1\}$ and any $\xi \in \mathcal{U}$ with $\xi_s = \overline{\xi}_s$ for all $s = 0, \dots, t-1$

$$\mathbb{E}_t[G(\xi)] \geq \mathbb{E}_t[G(\overline{\xi})] \tag{2.51}$$

holds almost surely.
(ii) If Assumption 4 is satisfied, then for each $k \in K$ and $i, i' \in I$, $j \in J^{i,k}, j' \in J^{i',k}$ it holds that

$$\begin{aligned} &\{\overline{\xi}_t^{i,j,k} \in [0, \kappa^{i,j,k})\} \cap \{\overline{\xi}_t^{i',j',k} \in (0, \kappa^{i',j',k}]\} \\ &\quad \subseteq \ \{C_t^{i,j,k} + e^{i,j,k}\overline{A}_t \geq C_t^{i',j',k} + e^{i',j',k}\overline{A}_t\} \end{aligned} \tag{2.52}$$

for all $t = 0, \dots, T-1$ where $\overline{A}_t = \pi \mathbb{P}_t(\Gamma + \Pi(\overline{\xi}) \geq 0)$.

Proof. (i) Assertion (2.51) is proven in the following way. If it does not hold, the $\mathcal{F}_t$–measurable set

$$\mathcal{O} := \{\mathbb{E}_t(G(\xi)) < \mathbb{E}_t(G(\overline{\xi})\},$$

has positive measure, i.e. $\mathbb{P}(\mathcal{O}) > 0$, and can be used to outperform $\overline{\xi}$ by ξ' defined as

$$\xi'_s = 1_{\mathcal{O}}\xi_s + 1_{\Omega\setminus\mathcal{O}}\overline{\xi}_s \text{ for all } s = 0, \dots, T-1. \tag{2.53}$$

Note that since ξ and ξ' coincide at times $0, \dots, t-1$, this definition yields an adapted process $\xi' \in \mathcal{U}$. With (2.53), we have the decomposition

$$G(\xi') = \mathbf{1}_{\mathcal{O}} G(\xi) + \mathbf{1}_{\Omega\setminus\mathcal{O}} G(\overline{\xi}),$$

which gives a contradiction to the optimality of $\overline{\xi}$ because:

$$\begin{aligned} \mathbb{E}[G(\xi')] &= \mathbb{E}[\mathbb{E}_t[1_{\mathcal{O}}G(\xi) + \mathbf{1}_{\Omega\setminus\mathcal{O}}G(\overline{\xi})]] \\ &= \mathbb{E}[\mathbf{1}_{\mathcal{O}}\mathbb{E}_t[G(\xi)] + \mathbf{1}_{\Omega\setminus\mathcal{O}}\mathbb{E}_t[G(\overline{\xi})] \\ &< \mathbb{E}[\mathbf{1}_{\mathcal{O}}\mathbb{E}_t[G(\overline{\xi})] + \mathbf{1}_{\Omega\setminus\mathcal{O}}\mathbb{E}_t[G(\overline{\xi}))] = \mathbb{E}[G(\overline{\xi})]. \end{aligned}$$

(ii) So let us now assume that $\overline{\xi}_t^{i,j,k} \in [0, \kappa^{i,j,k})$ and $\overline{\xi}_t^{i',j',k} \in (0, \kappa^{i',j',k}]$, and let us consider at time t, a small deviation from the global optimal strategy $\overline{\xi}$ consisting in a shift in production of $h_t > 0$ units of the good $k \in K$ whereby the firms $i \in I$ and $i' \in I$ increase and decrease their

output from technologies $j \in \mathcal{J}^{i,k}$ and $j' \in \mathcal{J}^{i',k}$ respectively, in such a way that the new policy $\overline{\xi} + \chi$ is still in $\mathcal{U}$. To this end we let χ be almost surely zero with exception of

$$\chi_t^{i,j,k} = h_t, \quad \chi_t^{i',j',k} = -h_t$$

and define $h_t = \min\{\kappa^{i,j,k} - \overline{\xi}_t^{i,j,k}, \overline{\xi}_t^{i',j',k}\}$. Consider

$$D(\overline{\xi}, \lambda) = \frac{\mathbb{E}_t[G(\overline{\xi} + \lambda\chi)] - \mathbb{E}_t[G(\overline{\xi})]}{\lambda}, \quad \text{for} \quad \lambda \in (0, 1],$$

and let $\lambda \searrow 0$ along a countable set $(0, 1] \cap \mathbb{Q}$. By dominated convergence, we have

$$\lim_{\lambda \to 0} D(\overline{\xi}, \lambda) = -\Big((C_t^{i,j,k} - C_t^{i',j',k}) - \pi\mathbb{P}_t[\Gamma + \Pi(\overline{\xi}) > 0](e^{i,j,k} - e^{i',j',k})\Big) h_t$$

if $e^{i,j,k} \leq e^{i',j',k'}$ and

$$\lim_{\lambda \to 0} D(\overline{\xi}, \lambda) = -\Big((C_t^{i,j,k} - C_t^{i',j',k}) - \pi\mathbb{P}_t[\Gamma + \Pi(\overline{\xi}) \geq 0](e^{i,j,k} - e^{i',j',k})\Big) h_t$$

if $e^{i,j,k} \geq e^{i',j',k'}$. So if Assumption 4 holds, both limits are given by

$$\lim_{\lambda \to 0} D(\overline{\xi}, \lambda) = -\Big((C_t^{i,j,k} - C_t^{i',j',k}) - \overline{A}_t(e^{i,j,k} - e^{i',j',k})\Big) h_t, \tag{2.54}$$

and since both limits should be non-positive by part (i), we conclude that

$$-\Big((C_t^{i,j,k} - C_t^{i',j',k}) - \overline{A}_t(e^{i,j,k} - e^{i',j',k})\Big) h_t \leq 0$$

almost surely. Hence the inclusion

$$\{h_t > 0\} \subseteq \{-\Big((C_t^{i,j,k} - C_t^{i',j',k}) - \overline{A}_t(e^{i,j,k} - e^{i',j',k})\Big) \leq 0\}$$

holds almost surely, which is equivalent to (2.52). □

We can now turn to the main result of this section.

Theorem 1 *If Assumptions 1-4 hold and $\overline{\xi}$ is a solution of the global optimal control problem (2.43) whose existence is guaranteed by Proposition 3, we have:*
(i) The processes $(\overline{A}, \overline{S})$ defined by

$$\overline{A}_t = \pi \mathbb{P}_t[\Gamma + \Pi(\overline{\xi}) \geq 0], \qquad t = 0, \ldots, T \tag{2.55}$$

and

$$\overline{S}^k_t = \max_{i \in I,\, j \in J^{i,k}} (C^{i,j,k}_t + e^{i,j,k} \overline{A}_t) 1_{\{\overline{\xi}^{i,j,k}_t > 0\}}, \qquad t = 0, \ldots, T-1 \;\; k \in K, \tag{2.56}$$

form a market equilibrium in the sense of Definition 1, for which the associated production strategy is $\overline{\xi}$.
(ii) The equilibrium allowance price process is unique almost surely.
(iii) For each good $k \in K$, the price $\overline{S}^k$ is the smallest equilibrium price for good k in the sense that for any other equilibrium price process S^{*k}, we have $\overline{S}^k \leq S^{*k}$ almost surely.

Proof. (i) We show that $\overline{A}$ and $\overline{S}$ so defined form an equilibrium by an explicit construction of firms' investment strategies $\overline{\theta} = \theta(\overline{\xi})$ such that $(\overline{\theta}^i, \overline{\xi}^i)$ satisfies (2.23), (2.24) and (2.25). For the sake of convenience we recall the definition of $\overline{\theta} = \theta(\overline{\xi})$ from (2.45).

$$\begin{aligned} &\overline{\theta}^i_t = 0 \quad \text{for all } i = 1, \ldots, N,\ t = 1, \ldots, T-1, \\ &\overline{\theta}^i_T = \Gamma^i + \Pi^i(\overline{\xi}^i) - \frac{\sum_{i \in I} (\Gamma^i + \Pi^i(\overline{\xi}^i))}{\sum_{i \in I} (\Gamma^i + \Lambda^i + \Pi^i(\overline{\xi}^i))} (\Gamma^i + \Lambda^i + \Pi^i(\overline{\xi}^i)) \ . \end{aligned} \tag{2.57}$$

Since conditions (2.23) and (2.24) are obviously fulfilled, we focus on (2.25). We first show that $\mathbb{E}[L^{\overline{A},\overline{S},i}(\overline{\theta}^i(\xi^i), \xi^i)] \geq \mathbb{E}[L^{\overline{A},\overline{S},i}(\theta^i, \xi^i)]$ for all $(\theta^i, \xi^i) \in \mathcal{V}^i(\overline{A}, \Lambda^i) \times \mathcal{U}^i$, where $\overline{\theta}^i(\xi^i)$ is constant equal to 0 until time $T-1$ while at time T it is given by

$$\overline{\theta}^i_T(\xi^i) := \Gamma^i + \Pi^i(\xi^i) - \frac{\sum_{i \in I} (\Gamma^i + \Pi^i(\overline{\xi}^i))}{\sum_{i \in I} (\Gamma^i + \Lambda^i + \Pi^i(\overline{\xi}^i))} (\Gamma^i + \Lambda^i + \Pi^i(\xi^i)) \tag{2.58}$$

which differs from (2.45). Notice that $\overline{\theta}^i_T(\xi^i) \leq \Gamma^i + \Pi^i(\xi^i)$ when $\Gamma + \Pi(\overline{\xi}) \geq 0$ and $\overline{\theta}^i_T(\xi^i) > \Gamma^i + \Pi^i(\xi^i)$ otherwise. We have:

$$\begin{aligned}
&\mathbb{E}[L^{\overline{A},\overline{S},i}(\theta^i, \xi^i)] \\
&= \mathbb{E}\left[\sum_{t=0}^{T-1} \sum_{(j,k)\in M_i} (\overline{S}^k_t - C^{i,j,k}_t)\xi^{i,j,k}_t - \theta^i_T \overline{A}_T - \pi(\Gamma^i + \Pi^i(\xi^i) - \theta^i_T)^+\right]
\end{aligned}$$

since $\overline{A}$ defined by (2.55) is a bounded martingale. For each $\xi^i \in \mathcal{U}^i$, we show that we can maximize the above quantity by computing the maximum pointwise in θ^i inside the expectation. In view of (2.55), when $\omega \in \{\Gamma + \Pi(\overline{\xi}) < 0\}$ we have $\overline{A}_T(\omega) = 0$ and the maximum of the function

$$z \hookrightarrow \overline{\varphi}^i(z) = -z\overline{A}_T(\omega) - \pi(\Gamma^i(\omega) + \Pi^i(\xi^i)(\omega) - z)^+ \quad z \geq -\Lambda^i(\omega) \tag{2.59}$$

is attained at any point $z \in [\Gamma^i(\omega) + \Pi^i(\xi^i)(\omega), \infty)$ showing that $\overline{\theta}^i_T(\xi^i)(\omega)$ is a maximizer. On the other hand, when $\omega \in \{\Gamma + \Pi(\overline{\xi}) \geq 0\}$, we have $\overline{A}_T(\omega) = \pi$, the maximum of (2.59) is attained at any point $z \in [-\Lambda^i(\omega), \Gamma^i(\omega) + \Pi^i(\xi^i)(\omega)]$ and once again, $\overline{\theta}^i(\xi^i)$ is a maximizer. Notice for later reference that in both cases, the value of the maximum of (2.59) is almost surely $-(\Gamma^i + \Pi^i(\xi^i))\overline{A}_T$.

To finish the proof, we prove that $\mathbb{E}[L^{\overline{A},\overline{S},i}(\overline{\theta}^i, \overline{\xi}^i)] \geq \mathbb{E}[L^{\overline{A},\overline{S},i}(\overline{\theta}^i(\xi^i), \xi^i)]$ for all $\xi^i \in \mathcal{U}^i$. According to the above computation, we have:

$$\begin{aligned}
&\mathbb{E}[L^{\overline{A},\overline{S},i}(\overline{\theta}^i(\xi^i), \xi^i)] \\
&\quad = \mathbb{E}\left[\sum_{t=0}^{T-1} \sum_{(j,k)\in M_i} (\overline{S}^k_t - C^{i,j,k}_t)\xi^{i,j,k}_t - (\Gamma^i + \Pi^i(\xi^i))\overline{A}_T\right] \\
&\quad = \mathbb{E}\left[\sum_{t=0}^{T-1} \sum_{(j,k)\in M_i} (\overline{S}^k_t - C^{i,j,k}_t - e^{i,j,k}\overline{A}_T)\xi^{i,j,k}_t - \Gamma^i \overline{A}_T\right] \\
&\quad = \mathbb{E}\left[\sum_{t=0}^{T-1} \sum_{(j,k)\in M_i} (\overline{S}^k_t - C^{i,j,k}_t - e^{i,j,k}\overline{A}_t)\xi^{i,j,k}_t - \Gamma^i \overline{A}_T\right] .
\end{aligned} \tag{2.60}$$

We now show that the following inclusions hold almost surely:

$$\{\overline{S}^k_t - C^{i,j,k}_t - e^{i,j,k}\overline{A}_t > 0\} \subseteq \{\overline{\xi}^{i,j,k}_t = \kappa^{i,j,k}\}, \tag{2.61}$$

$$\{\overline{S}^k_t - C^{i,j,k}_t - e^{i,j,k}\overline{A}_t < 0\} \subseteq \{\overline{\xi}^{i,j,k}_t = 0\}. \tag{2.62}$$

Inclusion (2.62) is a direct consequence of Definition (2.56) of the price process $\overline{S}$. Using this same Definition (2.56) and Lemma 3 we see that:

$$\begin{aligned}
&\{\overline{S}_t^k > C_t^{i,j,k} + e^{i,j,k}\overline{A}_t\} \\
&\quad\subseteq \bigcup_{i'\in I, j'\in J^{i',k}} \{C_t^{i',j',k} + e^{i',j',k}\overline{A}_t > C_t^{i,j,k} + e^{i,j,k}\overline{A}_t\} \cap \{\overline{\xi}_t^{i',j',k} > 0\} \\
&\quad\subseteq \bigcup_{i'\in I, j'\in J^{i',k}} \Big(\{\overline{\xi}_t^{i,j,k} = \kappa^{i,j,k}\} \cup \{\overline{\xi}_t^{i',j',k} = 0\}\Big) \cap \{\overline{\xi}_t^{i',j',k} > 0\} \\
&\quad\subseteq \{\overline{\xi}_t^{i,j,k} = \kappa^{i,j,k}\}\ .
\end{aligned}$$

These inclusions allow us to show that $\mathbb{E}[L^{\overline{A},\overline{S},i}(\overline{\theta}^i(\xi^i), \xi^i)] \leq \mathbb{E}[L^{\overline{A},\overline{S},i}(\overline{\theta}^i, \overline{\xi}^i)]$, thus completing the proof of (i).

(ii) Proposition 1 gives the form of an equilibrium price. Due to part (i) of Proposition 1 and Proposition 2 to prove almost sure uniqueness of the allowance price process, it is sufficient to prove that for any two solutions $\hat{\xi}$, $\tilde{\xi}$ of the global optimal control problem (2.43) we have:

$$\begin{aligned}
\mathbb{P}\Bigg[\Big(\{\Gamma + \Pi(\hat{\xi}) > 0\} \cap \{\Gamma + \Pi(\tilde{\xi}) > 0\}\Big) \qquad (2.63)\\
\bigcup \Big(\{\Gamma + \Pi(\hat{\xi}) < 0\} \cap \{\Gamma + \Pi(\tilde{\xi}) < 0\}\Big)\Bigg] = 1
\end{aligned}$$

We know that these production strategies are a solution of the global problem (2.43), that we rewrite as a linear programming problem:

$$\inf_{\substack{\xi\in\mathcal{U},\ Z\in L^1(\mathcal{F}_T)\\ Z\geq\Gamma+\Pi(\xi),\ Z\geq 0}} \mathbb{E}[C(\xi) + \pi Z], \qquad (2.64)$$

i.e. each solution (ξ^*, Z^*) of (2.64) satisfies

$$Z^* = (\Gamma + \Pi(\xi^*))^+ \qquad (2.65)$$

almost surely. Assume now that there are two optimal solutions $(\hat{\xi}, \hat{Z})$ and $(\tilde{\xi}, \tilde{Z})$ of the above linear programming problem. Due to the linearity of (2.64) it follows that any convex linear combination

$$(\lambda\hat{\xi} + (1-\lambda)\tilde{\xi}, \lambda\hat{Z} + (1-\lambda)\tilde{Z}) \qquad (2.66)$$

is also a solution to (2.64) for all $\lambda \in [0,1]$. In view of (2.65), we conclude that for each $\lambda \in [0,1]$

$$\lambda(\Gamma+\Pi(\hat{\xi}))^+ + (1-\lambda)(\Gamma+\Pi(\tilde{\xi}))^+ = \Big(\lambda(\Gamma+\Pi(\hat{\xi})) + (1-\lambda)(\Gamma+\Pi(\tilde{\xi}))\Big)^+$$

holds almost surely. Since the above assertion is obviously violated on

$$\{\Gamma+\Pi(\hat{\xi}) < 0 < \Gamma+\Pi(\tilde{\xi})\} \cup \{\Gamma+\Pi(\hat{\xi}) > 0 > \Gamma+\Pi(\tilde{\xi})\}$$

this union must have a probability 0, which together with Assumption 4, yields (2.63).

(iii) Assume on the contrary that there exists an equilibrium price process S^* with

$$S_t^{*k}(\omega) < \bar{S}_t^k(\omega) \text{ for all } \omega \in B \tag{2.67}$$

for some $t \in \{0,1,\dots,T-1\}$, $B \in \mathcal{F}_t$, $\mathbb{P}(B) > 0$ and $k \in K$. Let ξ^* be the corresponding equilibrium strategies. Since the equilibrium allowance price $\bar{A}$ is unique, it follows from (2.39) that

$$\{S_t^{*k} - C_t^{i,j,k} - e^{i,j,k}\bar{A}_t < 0\} \subseteq \{\xi_t^{*i,j,k} = 0\}$$

up to sets of probability zero. Consequently we obtain

$$\begin{aligned} \sum_{i\in I}\sum_{j\in J^{i,k}} \xi_t^{*i,j,k} &= \sum_{i\in I}\sum_{j\in J^{i,k}} \xi_t^{*i,j,k} 1_{\{S_t^{*k} \geq C_t^{i,j,k} + e^{i,j,k}\bar{A}_t\}} \\ &\leq \sum_{i\in I}\sum_{j\in J^{i,k}} \kappa^{i,j,k} 1_{\{S_t^{*k} \geq C_t^{i,j,k} + e^{i,j,k}\bar{A}_t\}} \end{aligned} \tag{2.68}$$

almost surely. Moreover, it follows from (2.62) and (2.61) that

$$\begin{aligned} &\sum_{i\in I}\sum_{j\in J^{i,k}} \kappa^{i,j,k} 1_{\{\bar{S}_t > C_t^{i,j,k} + e^{i,j,k}\bar{A}_t\}} \\ &\quad = \sum_{i\in I}\sum_{j\in J^{i,k}} \bar{\xi}_t^{i,j,k} - \sum_{i\in I}\sum_{j\in J^{i,k}} \bar{\xi}_t^{i,j,k} 1_{\{\bar{S}_t^k = C_t^{i,j,k} + e^{i,j,k}\bar{A}_t\}} \\ &\quad < \sum_{i\in I}\sum_{j\in J^{i,k}} \bar{\xi}_t^{i,j,k} \end{aligned} \tag{2.69}$$

holds almost surely. In the last equality we used the fact that

$$\sum_{i\in I}\sum_{j\in J^{i,k}} \bar{\xi}_t^{i,j,k} 1_{\{\bar{S}_t^k = C_t^{i,j,k} + e^{i,j,k}\bar{A}_t\}}(\omega) > 0 \text{ for all } \omega \in \Omega$$

which follows from the definition of $\bar{S}$. Moreover, (2.67) implies that

$$\sum_{i\in I}\sum_{j\in J^{i,k}} \kappa^{i,j,k} 1_{\{S_t^{*k} \geq C_t^{i,j,k} + e^{i,j,k}\bar{A}_t\}}(\omega) \tag{2.70}$$
$$\leq \sum_{i\in I}\sum_{j\in J^{i,k}} \kappa^{i,j,k} 1_{\{\bar{S}_t^{k} > C_t^{i,j,k} + e^{i,j,k}\bar{A}_t\}}(\omega) \text{ for all } \omega \in B.$$

From (2.68), (2.69) and (2.70) we conclude that there exists $C \subseteq B$ with $\mathbb{P}(C) > 0$ such that

$$\sum_{i\in I}\sum_{j\in J^{i,k}} \xi_t^{*i,j,k}(\omega) < \sum_{i\in I}\sum_{j\in J^{i,k}} \bar{\xi}_t^{i,j,k}(\omega) = D_t(\omega)$$

holds for all $\omega \in C$, which implies that S^* cannot be a set of equilibrium prices for the goods. □

Remark 3 *The above proof shows that because we are using a linear utility function (due to the fact that our firms are risk neutral), waiting* until the last minute *and purchasing the required amount of allowances on the last time trading is possible, constitutes an optimal trading strategy. However, trading is possible at all times and since there is no reason to believe that the optimal trading strategy is unique, one can imagine that there are less extreme strategies leading to the optimum.*

In the introduction, we referred to ***reduction costs*** or ***abatement costs*** as the costs of regulation, i.e. the pollution reduction costs. We now give a formal definition of what we mean by reduction costs. For all regulatory parameters π and $\{(\Lambda_t^i)_{i\in I}\}_{t=0}^{T-1}$, and for any choice of an equilibrium production schedule $\xi^* \in \mathcal{U}$, we define the reduction costs RC as the random variable given by the difference between the production costs $C(\xi^*)$ under this production schedule and the production costs incurred in the same random scenarios had we used the BAU equilibrium production schedule ξ^{BAU}. In other words, the reduction costs are given by the random variable:

$$RC = C(\xi^*) - C(\xi^{BAU}) = \sum_{t=0}^{T-1}\sum_{(i,j,k)\in M} (\xi_t^{*i,j,k} - \xi_t^{BAU,i,j,k}) C_t^{i,j,k}. \tag{2.71}$$

Notice also that as defined, the reduction costs do not depend upon the trading strategies of the individual firms in the emissions market.

2.4. Prices and Windfall Profits in the Standard Scheme

The previous sections were devoted to the introduction and the mathematical analysis of what we called the standard emission trading scheme. This cap-and-trade scheme was chosen because it is representative of the implementation of the first phase of the EU ETS. In this section, we discuss the theoretical results of the previous sections. In particular we focus on the interpretation of product prices and windfall profits in Subsection 2.4.1 and on the role of the initial allocation in Subsection 2.4.2.

2.4.1. Windfall Profits and Penalty Under the Standard Scheme

We already emphasized that uniqueness of prices of goods was not guaranteed. However, we identified the minimal price among all the possible equilibrium prices in Equation (2.56). In what follows, we focus on this price at the example of power markets.

Equation (2.56) shows two sources of change in the spot price compared to business as usual. First, the marginal technology may be different: this induces a variation in marginal cost. This variation is likely to be positive but a negative variation is possible. Suppose for example that in a BAU scenario, coal is started first but that demand is high enough so that gas is the marginal technology. Suppose that in the presence of the trading scheme, allowance price is high enough to induce a fuel switch, so that gas is started first. Assume also that demand is high so that coal is the marginal technology. In this case, the variation in marginal cost can be negative. The second source of variation is the price of pollution $e^{i,j,k}A_t^*$ for the marginal technology. The producers pass through the cost of expected penalties to consumers. This second contribution is always positive and is such that the spot price under the trading scheme is always greater than the spot price in BAU.

A possible interpretation of formula (2.56) is that the allowance price enters the electricity price as the price of an additional commodity that

is used for power generation besides fuels. Producing the last infinitesimal unit of electricity at time t induces not only costs due to extra fuel consumption, but also increases the emissions by $e^{i,j,k}$ and hence also the expected penalty at time T by $e^{i,j,k}A_t^*$. Consequently these costs have to be covered by the end-consumers, for the marginal production of product k to be profitable. Since this amount is passed on to the end-consumer in each time step the *consumer costs* defined as

$$\sum_{t=0}^{T-1}(S_t^* - S_t^{BAU})D_t \tag{2.72}$$

are usually much bigger than the penalty that is only paid for excess emissions. As we will see in the following the consumer costs exceed also by far the reduction cost of the scheme, which gives rise for significant windfall profits.

In Section 2.5 we will quantify this effect at a realistic case study. To this end we need to define what exactly we mean by windfall profits. This is done as follows in the general framework of a standard cap-and-trade system with multiple goods. If ξ^* is an optimal production strategy associated with the equilibrium (A^*, S^*), we define the *target price* $\hat{S}_t^k$ of good k as:

$$\hat{S}_t^k := \max_{i\in I, j\in J^{i,k}} C_t^{i,j,k}\mathbf{1}_{\{\xi_t^{*i,j,k}>0\}}. \tag{2.73}$$

This price is the marginal cost under the optimal production schedule without taking into account the cost of pollution. We then define the windfall profits of firm i as:

$$WP^i = \sum_{t=0}^{T-1}\sum_{(j,k)\in M_i}(S_t^{*k} - \hat{S}_t^k)\xi_t^{*i,j,k},$$

and the overall windfall profits as

$$WP = \sum_{i\in I} WP^i = \sum_{t=0}^{T-1}\sum_{k\in K}(S_t^{*k} - \hat{S}_t^k)D_t^k. \tag{2.74}$$

These windfall profits measure the profits for the production of goods in excess over what the profits would have been, had the same dispatching

schedule been used, and had the target prices (e.g. the marginal fuel costs) been charged to the consumers without the cost of pollution.

Remark 4 *Another reasonable definition of the windfall profits of firm i could have been*

$$\sum_{t=0}^{T-1} \sum_{(j,k)\in M_i} (S_t^{*k} - \hat{S}_t^k)\xi_t^{*i,j,k} - \pi \left(\Gamma^i - \Pi^i(\xi^{*i})\right)^+ \tag{2.75}$$

in which the penalty payments due to the scheme are withdrawn from the extra profits. Since producers decide upon their production strategies and therewith the risk to pay the penalty, we take the point of view that they should pay the penalty and not the end-consumer. However, as can be seen in Figure 2.2 the penalty payments vanish in comparison to the windfall profits as defined in (2.74). Hence in practical applications, both definitions give similar results.

2.4.2. Incentives for the Use of Cleaner Technologies

Using (2.39) we see that the expected profits and losses of firm $i \in I$ in an equilibrium (A^*, S^*) with associated production schedules ξ^* are given by

$$\mathbb{E}[L^{A^*,S^*,i}(\theta^{*i}, \xi^{*i})] = \mathbb{E}\left[(-\Delta^i + \sum_{t=0}^{T-1} \Lambda_t^i) A_T^*\right] + \mathbb{E}\left[\sum_{t=0}^{T-1} \sum_{(j,k)\in M_i} (S_t^{*k} - C_t^{i,j,k} - e^{i,j,k} A_t^*)\xi_t^{*i,j,k}\right].$$

As shown in Theorem 1, both the equilibrium price processes (A^*, S^*) and the production strategies ξ^* are preserved under a change of the regulatory allocation from $((\Lambda_t^i)_{t=0}^{T-1})_{i\in I}$ to $((\Lambda'^i_t)_{t=0}^{T-1})_{i\in I}$ as long as

$$\sum_{i\in I} \sum_{t=0}^{T-1} \Lambda_t^i = \sum_{i\in I} \sum_{t=0}^{T-1} \Lambda'^i_t$$

holds almost surely. However, such an adjustment of the allocation changes the expected profits and losses of producer $i \in I$ by the amount:

$$\mathbb{E}\left[(\Lambda'^i - \Lambda^i)A_T^*\right]. \tag{2.76}$$

Obviously this creates a money transfer from producers with $\Lambda'^i - \Lambda^i < 0$ to producers with $\Lambda'^i - \Lambda^i > 0$. If the initial allocation can vary with the type of production plant, it is possible to set it up in order to increase or decrease the income of clean or dirty plants respectively. In other words, the initial allocation can be used to adjust incentives to build cleaner plants.

Even though our model concerns only the management of existing technologies, and does not address the important issue of investment in the development of cleaner technologies, the mechanism highlighted above is one of the most powerful tools put in the hands of policy makers by a cap-and-trade scheme. Clearly, this leverage will disappear if auctioning of the allowances is used.

2.5. Case Study: The Japanese Electricity Market

Our analysis of cap-and-trade schemes would not be complete without numerical evidence from realistic examples based on market data, we spend this section introducing a case study, which we use for illustration purposes throughout this thesis. For the purpose of numerical computations, we restrict our attention to an economy where one type of goods is produced. We choose the example of *electricity* because the power sector is worldwide one of the most important sources of green house gas emissions. In order to provide specific insight on the effects of the cap-and-trade scheme on the market we choose to perform numerical simulations of the Japanese electricity sector with data expected for 2012. At the core of our analysis is the main abatement mechanism in the electricity production, the fuel switch from coal to gas (i.e. when for some period of time, coal plants are turned off in favor of cleaner gas plants). This was the main abatement mechanism in EU ETS Reinaud (2007). As explained in the governmental task force report Kiko-Network (2000), this is also the abatement mechanism with the largest abatement potential in the Japanese electricity sector.

The Japanese electricity market is divided into an Eastern and a Western part, with only 1 Giga Watt interconnection capacity. Hence, for the purpose of this case study, we can safely assume that we are dealing with two separated electricity markets sharing a joint emission cap. We use the following production capacities.

Production Capacity in GW		
Type	East	West
Nuclear	27	26
Coal	31	11
LNG	35	33
Oil	27	14

Assuming further that the Japanese electricity market is totally deregulated we can apply Theorem 1 with $|K| = 2$ to this situation.

The numerical results reported below are based on the following assumptions:

Goal of the regulator: to reduce emissions by 20% of what the emissions level would have been in 2012 under the Business-As-Usual (BAU) scenario. This implies an emission target of less than or equal to 300Mt of carbon dioxide. Since we are working in a non-deterministic framework, this target must be interpreted accordingly. We agree to consider reduction schemes whose compliance period equals to the year 2012 with total emissions less than or equal to the cap of 300Mt of carbon dioxide, emitted with probability of 95%. We choose the parameters in the following way. For the standard scheme, the penalty is set to 100$ and the initial allocation to 300 million allowances, each of which represents the right to emit 1 tonne of carbon dioxide. Details on the calibration of the parameters of the scheme are shown for the Texas electricity sector in Section 3.2. The calibration of the fuel switch price process and the demand processes is given in the appendix. The numerical procedure used and its errors are discussed in Chapter 5.

2.5.1. Can a Standard Cap-and-Trade Scheme Meet Emissions Targets?

As observed in the SOx and NOx RECLAIM program and after the first phase of the EU ETS, cap-and-trade systems can fail to reach their emission targets as too generous an allocation of pollution permits serve as a disincentive for emission reductions and deflate pollution prices. However, the same is also true for a regulation with too low a penalty in the case of non compliance. This effect is illustrated in Figure 2.1, which depicts the histograms of the total emissions computed for each of the equilibrium Monte Carlo scenarios generated for the purpose of our case study. For the sake of definiteness, we consider the standard cap-and-trade scheme with different penalties, and with an initial allocation of 300 million allowances, which corresponds, in the case of Japan's electricity market, to a 20% reduction target. See details given below.

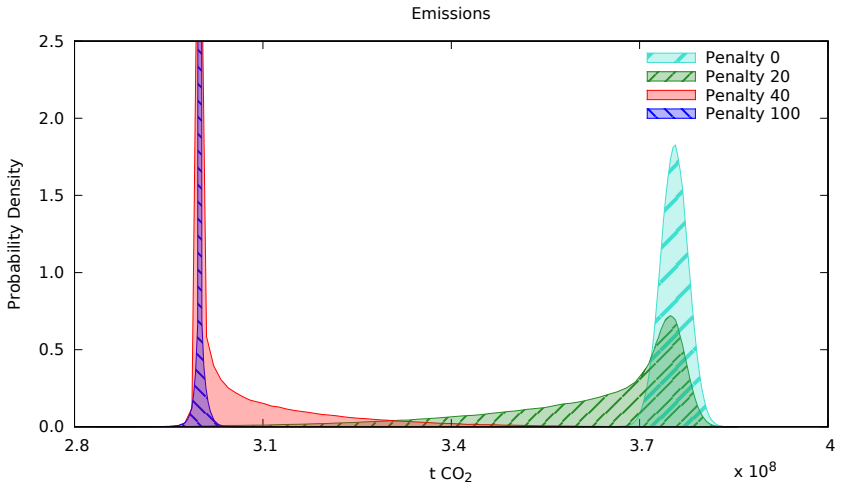

Fig. 2.1: Effect of the level of the penalty on the statistical distribution of the actual emissions at the end of a period of implementation of a standard cap-and-trade scheme.

The case with zero penalty corresponds to *Business As Usual* (BAU): obviously, it misses the reduction target by far. When the penalty is set at 20$ per tonne, the emission target is missed by the same amount with a very large probability. With a 40$ penalty, there is a significant probability to meet the target, but the distribution of the emissions has

a long tail overshooting the target. On the other hand in the case of a 100$ penalty, the probability to reach the reduction target is 95%. This shows that the probability for a cap-and-trade scheme to reach a given emission target depends not only significantly on the initial allocation of allowances, but also on the penalty applied in case of non compliance, though the importance of the penalty is often underestimated.

2.5.2. Electricity Price Impact Under the Standard Scheme

In this subsection, we discuss the impact of the regulation on electricity prices. Figure 2.2 quantifies both the penalty payments and the consumer costs, and compares them to reduction costs and windfall profits (as defined in the next section) under a standard trading scheme for the Japanese electricity sector. For these computations, we use a 100$ penalty and an initial allocation of 300 million allowances.

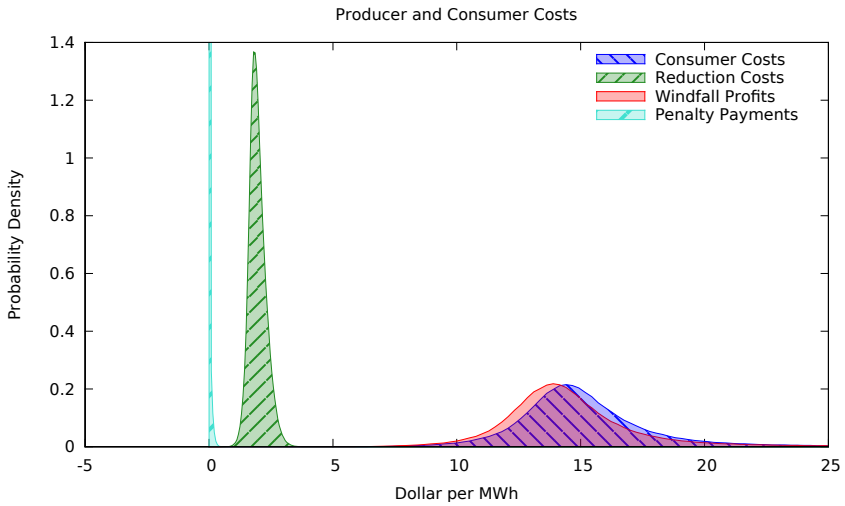

Fig. 2.2: Histograms of the differences between the consumer costs, reduction costs, windfall profits and penalty payments of a standard cap-and-trade scheme and the corresponding quantities for BAU.

It turns out that consumer costs (in average 15.13$ per MWh) exceed by far the overall reduction costs (in average 1.96$ per MWh), which as it was the case in the first phase of EU ETS, gives rise to large windfall

profits for the producers. Obviously, costs of production are higher in the presence of a cap-and-trade scheme. This is because, due to the emission constraints, producers switch to cleaner and more expensive technologies to avoid paying the penalty. However, for a 20% reduction target, average abatement costs are only 1.96$ per MWh of produced electricity! Though as observed in EU ETS consumer costs exceed the overall reduction costs by far (a factor of 8 in the present case). This is one of the main reasons for the large windfall profits, which have been the core of the main criticism of cap-and-trade schemes by consumer advocates.

Chapter 3

Tax and Relative Allocation Schemes

Chapter Overview

In view of the shortcomings of the standard cap-and-trade schemes presented in Chapter 2 we discuss in this chapter alternative emission regulations such as a carbon tax, a standard scheme with auctioning as well as relative cap-and-trade schemes, where allowances are allocated relatively to production. In Section 3.1 we introduce a general (and fairly complex) cap-and-trade scheme covering such emission regulations. We prove that the equilibrium framework in Chapter 2 is general enough such that the theoretical results derived earlier for standard schemes, can be transferred to this more general framework. Section 3.2 covers the calibration of the parameters to reach emission targets formulated as quantile measures under the different regulations.

Section 3.3 compares the different regulations at the example of the Japanese electricity sector. We demonstrate the shortcomings of the tax systems, which suffer from poor control of the windfall profits and unexpected expensive reduction policies when it comes to emission targets under stochastic abatement costs. Moreover, we show that auctioning cannot lower windfall profits to a reasonable level. In the case of our Japanese case study, the incomes from the auction can merely cover two thirds of the consumer costs. The reason is the huge amount of clean production capacity in the Japanese portfolio. Today most markets are dirty enough that it is likely that a 100% auction can cover consumer costs. However, the problem we highlight will occur in all markets when the production portfolio becomes cleaner. We show that a relative allocation scheme can resolve most of the issues with the other schemes. Even though the number of permits is random in a relative scheme, and hence cannot be known exactly in advance, its statistical distribution is well understood and setting its parameters to meet pollution targets is not much different from the standard cap-and-trade schemes. Indeed, we show that properly calibrated, the relative schemes reach the same pollution target as the standard schemes at low cost while at the same time, keeping consumer burden and windfall profits in control.

Section 3.4 gathers more mathematical properties of the generalized cap-and-trade schemes introduced in the previous section. Our results demonstrate the versatility and the flexibility of such a gener-

alized framework. They show that regulators can control cap-and-trade schemes in order to reach pre-assigned pollution targets with zero windfall profits and *reasonably small* consumer costs, or even to force equilibrium prices of goods to be equal to target prices. However, because of the level of complexity of their implementations, it is unlikely that policy makers or regulators will make use of the schemes identified in this section.

Another important mechanism of emission regulations is to set and control incentives to develop and build cleaner production facilities. As explained in the introduction, this last objective cannot be fully investigated in the framework of our equilibrium models since they are limited to the optimization of the production from the already existing technologies. Despite this limitation, we claimed in the introduction the importance of the initial allocation to control such incentives. In this section we show that besides the standard cap-and-trade scheme only the relative scheme provides a significant initial allocation that can be used to control these incentives.

3.1. General Market Designs for Emission Trading Schemes

We first generalize the allocation procedure. Beyond the standard allocation, the regulator is now allowed to distribute credits dynamically and proportionally to production. To be more specific, at each time $0 \leq t < T$, firm i is provided with an allocation

$$\hat{\Lambda}_t^i(\xi^i) = X_t^i + \sum_{(j,k)\in M_i} Y_t^{i,j,k} \xi_t^{i,j,k}, \tag{3.1}$$

where X^i and $Y^{i,j,k}$ are adapted processes in $\mathcal{L}^1_{T-1}(\mathbb{R})$.

In addition, the regulator can also *tax* or *subsidize* the various firms by means of financial incentives or disincentives similar to the credit endowments described above. In this case, the profits of the firms are

lowered at time t by an amount

$$V_t^i + \sum_{(j,k)\in M_i} Z_t^{i,j,k}\xi_t^{i,j,k}, \tag{3.2}$$

where V^i and $Z^{i,j,k}$ are also adapted processes in $\mathcal{L}^1_{T-1}(\mathbb{R})$. Notice that V^i and $Z^{i,j,k}$ represent a tax when positive and a subsidy when negative. Examples of positive $Z^{i,j,k}$ include fuel and carbon taxes. Under such a generalized cap-and-trade scheme, the terminal wealth (or profits and losses) of firm $i \in I$ reads:

$$\begin{aligned} H^{A,S,i}(\theta^i,\xi^i) := &-\sum_{t=0}^{T-1} V_t^i + \sum_{t=0}^{T-1}\sum_{(j,k)\in M_i} (S_t^k - C_t^{i,j,k} - Z_t^{i,j,k})\xi_t^{i,j,k} \\ &+ \sum_{t=0}^{T-1} \theta_t^i(A_{t+1} - A_t) - \theta_T^i A_T \\ &- \pi\left(\Delta^i + \Pi^i(\xi^i) - \sum_{t=0}^{T-1}\left(X_t^i + \sum_{(j,k)\in M_i} Y_t^{i,j,k}\xi_t^{i,j,k}\right) - \theta_T^i\right)^+. \end{aligned} \tag{3.3}$$

For such a regulation Definition 1 carries over by replacing $L^{A,S,i}(\theta^i,\xi^i)$ by $H^{A,S,i}(\theta^i,\xi^i)$ and Λ^i by $\hat{\Lambda}^i(\xi^i) := \sum_{t=0}^{T-1}\hat{\Lambda}_t^i(\xi^i)$. Moreover condition (2.19) of Assumption 1 has to be rephrased as

$$\mathbb{P}(\sum_{i\in I}\hat{\Lambda}^i(\xi^i) > 0) = 1 \tag{3.4}$$

for all $\xi \in \mathcal{U}$.

Despite the obviously greater generality of the present framework, the proofs of the results of Theorem 1 are sufficient to cover the analysis of this broader class of trading schemes:

Proposition 4 *Let $\Delta^i \geq 0$ a.s. and $e^{i,j,k} \geq 0$ for all $(i,j,k) \in M$. Then the conclusions of Theorem 1 still hold true in the case of the generalized cap-and-trade scheme of this section provided we use the* adjusted parameters*:*

$$\hat{C}_t^{i,j,k} := C_t^{i,j,k} + Z_t^{i,j,k} \quad \textit{and} \quad \hat{e}_t^{i,j,k} := e^{i,j,k} - Y_t^{i,j,k} \quad \textit{and} \tag{3.5}$$

an adjusted lower bound

$$\hat{\Lambda}^i(\xi^i) = \sum_{t=0}^{T-1} \hat{\Lambda}_t^i(\xi^i) = \sum_{t=0}^{T-1} X_t^i + \sum_{t=0}^{T-1} \sum_{(j,k)\in M_i} Y_t^{i,j,k} \xi_t^{i,j,k} \tag{3.6}$$

and adjusted uncontrolled emissions

$$\hat{\Gamma}^i = \Delta^i - \sum_{t=0}^{T-1} X_t^i \tag{3.7}$$

instead of the original parameters of the model.

Notice that $\hat{\Gamma}^i$ and $\hat{\Lambda}^i(\xi^i)$ do not fulfill equation (2.15) as was the case for the standard scheme. Hence $\hat{\Gamma}^i$ cannot anymore be interpreted as the negative net amount of allowances, which is rather given by $\Delta^i - \hat{\Lambda}^i(\xi^i)$. However, as we never used (2.15) in the proof of Theorem 1 it still holds true for the more general setting where the trading bound Γ^i and Λ^i are not related by equation (2.15). Though we still need to prove that Theorem 1 holds true in the case of the strategy dependent lower bound $\hat{\Lambda}^i(\xi^i)$.

Proof. First we show that Theorem 1 is true for a strategy dependent lower bound as long as the reparametrized counterparts of Assumptions 1-4 are fulfilled. In particular it is required that

$$\hat{\Gamma}^i + \hat{\Lambda}^i(\xi^i) + \Pi^i(\xi^i) \geq 0 \tag{3.8}$$

holds almost surely for all $\xi^i \in \mathcal{U}^i$. We only have to check that the proofs of Theorem 1 part (i), Proposition 1 part (i) and (iii) as well as Proposition 2, the only proofs where Λ^i occurs explicitly, are still valid. Here (3.4) guaranties that A_T^* is bounded from above by π, which is needed for Proposition 1. Moreover (3.4) together with (3.8) imply that the strategies (2.57) and (2.58) in Theorem 1 as well as the strategies

(2.45) in Proposition 2 remain feasible. Furthermore, (3.8) implies that $[-\hat{\Lambda}^i(\xi^i), \hat{\Gamma}^i + \Pi^i(\xi^i)]$ is a.s. nonempty for all $\xi^i \in \mathcal{U}^i$ which is needed to prove Theorem 1 part (i) and Proposition 1 part (i) and (iii).

The results of Theorem 1 were derived under the assumption that the emission coefficients $e^{i,j,k}$ were constant. However, by mere inspection of the proofs, the reader will easily be convinced that all the results remain true if these emission coefficients are instead adapted stochastic processes in $\mathcal{L}^1_{T-1}(\mathbb{R})$.

It only remains to check that the condition (3.8) is fulfilled for this reparameterization. This is seen as

$$\begin{aligned}
&\hat{\Gamma}^i + \hat{\Lambda}^i(\xi^i) + \sum_{t=0}^{T-1} \sum_{(j,k)\in M_i} (e^{i,j,k} - Y_t^{i,j,k})\xi_t^{i,j,k} \\
&= \Big(\Delta^i - \sum_{t=0}^{T-1} X_t^i\Big) + \Big(\sum_{t=0}^{T-1} X_t^i + \sum_{t=0}^{T-1} \sum_{(j,k)\in M_i} Y_t^{i,j,k}\xi_t^{i,j,k}\Big) \\
&\qquad\qquad + \sum_{t=0}^{T-1} \sum_{(j,k)\in M_i} (e^{i,j,k} - Y_t^{i,j,k})\xi_t^{i,j,k} \\
&= \Delta^i + \sum_{t=0}^{T-1} \sum_{(j,k)\in M_i} e^{i,j,k}\xi_t^{i,j,k} \geq 0
\end{aligned}$$

holds almost surely since by assumption the original uncontrolled emissions and the original emission factors fulfill $\Delta^i \geq 0$ a.s. and $e^{i,j,k} \geq 0$ for all $(i,j,k) \in M$. □

So if the reparametrized counterparts of Assumptions 1-4 hold and if $\overline{\xi} \in \mathcal{U}$ is a solution of the global optimal control problem (2.43) with adjusted uncontrolled emissions $\hat{\Gamma}^i$, adjusted emission factors $\hat{e}_t^{i,j,k}$ and adjusted production costs $\hat{C}_t^{i,j,k}$, then the couple of processes $(\overline{A}, \overline{S})$ defined by

$$\overline{A}_t = \pi \mathbb{P}_t \Big[\sum_{i\in I} \hat{\Gamma}^i + \sum_{t=0}^{T-1} \sum_{(i,j,k)\in M} \hat{e}_t^{i,j,k} \overline{\xi}_t^{i,j,k} \geq 0 \Big] \tag{3.9}$$

for all $t = 0, \dots, T$ and

$$\begin{aligned} \overline{S}_t^k &= \max_{i \in I,\, j \in J^{i,k}} (\hat{C}_t^{i,j,k} + \hat{e}_t^{i,j,k} \overline{A}_t) \mathbf{1}_{\{\overline{\xi}_t^{i,j,k} > 0\}} \\ &= \max_{i \in I,\, j \in J^{i,k}} (C_t^{i,j,k} + Z_t^{i,j,k} + (e^{i,j,k} - Y_t^{i,j,k}) \overline{A}_t) \mathbf{1}_{\{\overline{\xi}_t^{i,j,k} > 0\}} \end{aligned} \tag{3.10}$$

for all $t = 0, \dots, T$, $k \in K$ form a market equilibrium. Moreover, the equilibrium allowance price process is almost surely unique, while the process $\overline{S}$ is the smallest equilibrium price in the sense of Theorem 1.

The present formulation gives a general framework for the analysis of a broader class of cap-and-trade schemes. We mostly focus on two important particular cases: 1) the case where $Z_t^{i,j,k} \geq 0$ varies with i, j and k, which leads to a scheme based on a fuel or emission tax scheme, and 2) the case where $Y_t^{i,j,k} \geq 0$ only depends on k, which corresponds to a relative allocation or a subsidy for the production of good k given as emission allowances. From the form of (3.10) we anticipate that the emission tax increases product prices while the relative allocation decreases product prices.

For an equilibrium (A^*, S^*) of the generalized scheme with associated strategies (θ^*, ξ^*), it is straightforward to extend the definition of windfall profits of firm i as:

$$GWP^i = \sum_{t=0}^{T-1} \sum_{(j,k) \in M_i} (S_t^{*k} - \hat{S}_t^k) \xi_t^{*i,j,k} - \sum_{t=0}^{T-1} \left(V_t^i + \sum_{(j,k) \in M_i} Z_t^{i,j,k} \xi_t^{*i,j,k} \right),$$

the overall windfall profits being defined as

$$\begin{aligned} GWP &= \sum_{i \in I} GWP^i \\ &= \sum_{t=0}^{T-1} \sum_{k \in K} (S_t^{*k} - \hat{S}_t^k) D_t^k - \sum_{t=0}^{T-1} \left(\sum_{i \in I} V_t^i + \sum_{(i,j,k) \in M} Z_t^{i,j,k} \xi_t^{*i,j,k} \right), \end{aligned} \tag{3.11}$$

and used in this form in the case study of the Japanese electricity market.

3.1.1. Cap-and-Trade Schemes with Relative Allowance Allocation

A positive relative allocation for some product $Y_t^{i,j,k} = y^k < 0$ for some $k \in K$ all $i \in I$ and those $j \in J^{i,k}$ can be seen as a subsidy for good k that is given in the form of allowances rather than in cash. When producing one unit of good k, the marginal penalty increases only by $(e^{i,j,k} - y^k)A_t^*$ rather than by $e^{i,j,k}A_t^*$ as in a standard scheme. Thus the net marginal overall production costs of the firms are lower when compared to the standard scheme. This should result in a decrease of the price of good k. In the present subsection, we study the simplest generalized cap-and-trade scheme taking advantage of this mechanism. It applies this mechanism only for production means $J_{\text{marg}}^{i,k} \subseteq J^{i,k}$ which can be marginal (e.g. in the case of the electricity markets, this excludes nuclear plants) and is obtained by setting:

$$\begin{aligned} Y_t^{i,j,k} &= y^k 1_{\{j \in J_{\text{marg}}^{i,k}\}} \in \mathbb{R} \quad \text{for all } t = 0, \ldots, T-1 \\ X_0^i &= x^i \in \mathbb{R}, \qquad X_t^i = 0 \quad \text{for all } t = 1, \ldots, T-1 \\ V_t^i &= 0, \quad Z_t^{i,j,k} = 0 \quad \text{for all } t = 0, \ldots, T-1 \end{aligned}$$

for all $(i, j, k) \in M$.

In what follows, not only do we discuss this relative cap-and-trade scheme, but we also gain new insight into the standard cap-and-trade scheme by treating it as a relative cap-and-trade scheme with $y^k = 0$ for all $k \in K$.

3.1.2. Auction

The most frequently advocated approach to reduce windfall profits is to replace free allocation of allowances (which was common practice in the first phase of EU ETS) by an auction-like procurement. In the following case study, we assume that prices are not changed by the auction and the price settled in the auction corresponds to the equilibrium price settled at the beginning of the trading period under free allocation. Even though the standard cap-and-trade scheme with auction is not

covered by our general market design we treat it as generalized cap-and-trade scheme with

$$\sum_{i\in I} V_0^i = \sum_{i\in I} \Lambda_0^i \overline{A}_0, \qquad \sum_{i\in I} X_0^i = \sum_{i\in I} \Lambda_0^i, \tag{3.12}$$

the other parameters being zero, when it comes to the computation of windfall profits. The theoretical framework for these assumptions will be given in Chapter 4.

3.1.3. Emission Taxes

A carbon tax is a regulation that penalizes the emission of each tonne of carbon by a fixed amount, say $z > 0$. Formally, it can be viewed as a generalized scheme for which $\pi = 0$ and

$$\begin{aligned} Z_t^{i,j,k} &= e^{i,j,k} z, & V_t^i &= 0 \\ Y_t^k &= 0, & X_t^i &> 0 \end{aligned}$$

for all $(i,j,k) \in M$ and $t = 0,\dots,T-1$. Here we choose $X_t^i > 0$ to be in the realm of Proposition 4. As the penalty and hence also the allowance price is zero this corresponds to the case where there is no trading in allowances. Using the results of Proposition 4, we see that in such a tax scheme the prices of goods follow a merit order pricing rule with effective production costs given by $C_t^{i,j,k} + e^{i,j,k} z$ for all $(i,j,k) \in M$ and $t = 0,\dots,T-1$. The earnings under a tax scheme are based on the spread of these effective production costs. Since this spread does not depend solely on the original fuel spread, it is in general not clear what windfall profits will be. It is not even clear if they are negative or positive. To gain some insight on this issue consider a tax of $z = 40$\$ per tonne of carbon dioxide (which is realistic for 20% reduction target as will be seen below), and assume that at some point in time, the marginal production costs of coal and gas are the same while all plants have to run to satisfy the demand. In this case the spread in effective production cost is $(e_c - e_g)z = 17.2$\$ and will be earned for each MWh that is produced with gas. However, if the target price was charged, the earnings would be zero. Hence the windfall profits are 17.2\$ per MWh of electricity produced with gas.

But for any comparison of different cap-and-trade schemes to be meaningful, we need to calibrate their respective parameters to common characteristics. We proceed to the discussion of such a calibration.

3.2. Calibration of the Parameters

In this section we illustrate how to calibrate the parameters of the different schemes at the example of the Texas electricity market. This region known as ERCOT, is one of three electrical interconnections in the United States. It is essentially isolated from the others in the sense that there are no synchronous electrical interconnections between ERCOT and the rest of the US or Mexico, and only one direct high voltage connection. Details on this case study can be found in Carmona et al. (2009b).

3.2.1. Case Study: the Texas Electricity Market

The relative scheme introduced in this section has three regulatory parameters. Using the notation of this section, they are:

i) The penalty π.
ii) The relative allocation coefficient y^e, where e stands for electricity, the only good produced in this market.
iii) The total initial allocation $x = \sum_{i \in I} x^i$ given to the firms $i \in I$.

In this subsection we show, using the example of the Texas electricity market, how one should choose these parameters in order to guarantee an emissions reduction target with given probability while keeping the expected windfall profits near zero and controlling the reduction costs to keep them as low as possible. In the particular simulation used to illustrate the strategy, we choose an emissions reduction target of $1.827 \times 10^8 \mathrm{t}_{\mathrm{CO}_2}$ (i.e. a 10% reduction compared to BAU) to be reached with probability 95%.

To gain a first insight into the numerics, we fix the penalty π at 100\$. Figure 3.1 gives the expected windfall profits while Figure 3.2 gives the

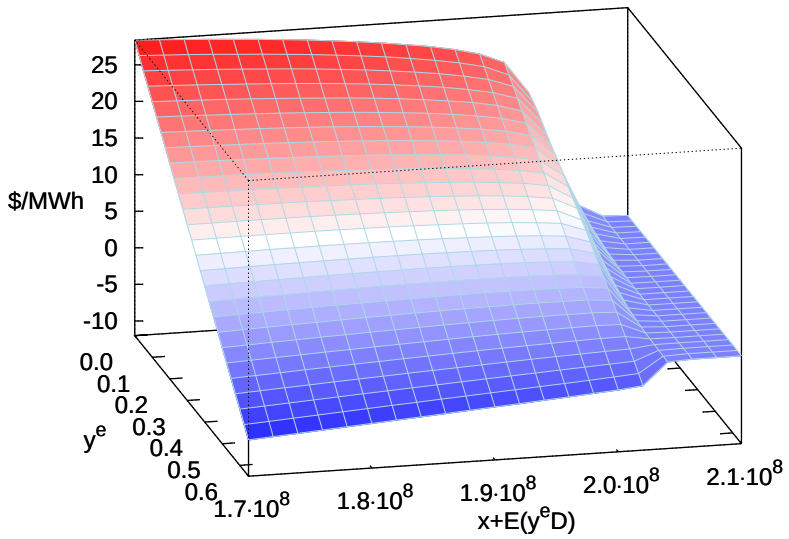

Fig. 3.1: Windfall profits as functions of the relative allocation parameter and the expected allocation. Here $D = \sum_{t=0}^{T-1} D_t^e$ denotes the total electricity demand over one compliance period.

95% percentile of the total emissions for different values of the relative allocation coefficient (y^e) and the expected total allocation. It appears that the expected allocation controls the amount by which carbon emissions are reduced, while the relative allocation coefficient y^e controls the windfall profits. Designing a cap-and-trade scheme with zero windfall profits and pre-assigned emission target can be done by choosing the parameters of our relative scheme at the intersection of the zero windfall profit level set with the $1.827 \times 10^8 \mathrm{t}_{\mathrm{CO}_2}$ emission percentile level set. This procedure is depicted in Figure 3.3. We find $y^e = 0.54$ and $\sum_{i \in I} x^i = 5.4 \times 10^7$.

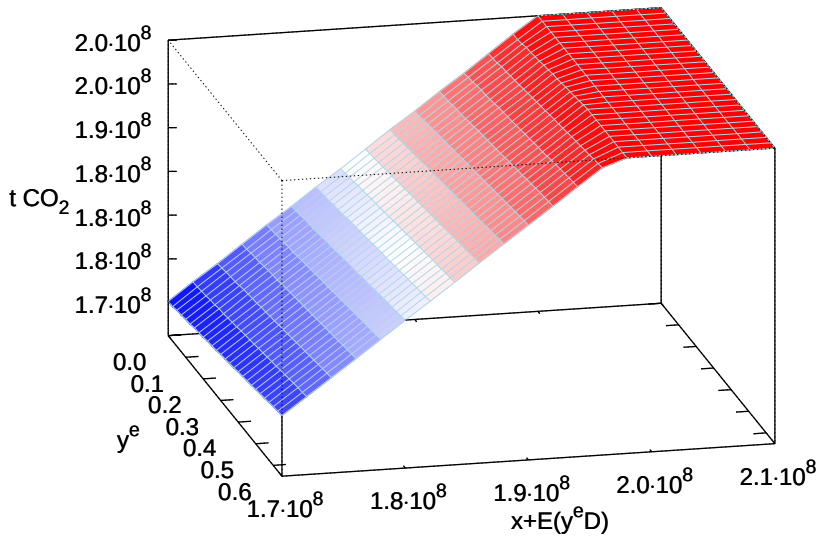

Fig. 3.2: This figure shows the 95% percentile of total emissions (right) as functions of the relative allocation parameter and the expected allocation. As in Figure 3.1 $D = \sum_{t=0}^{T-1} D_t^e$ denotes the total electricity demand over one compliance period.

Since for the standard cap-and-trade scheme the parameter is zero (i.e. $y^e = 0$), we have one less regulatory parameter to calibrate. And since controlling the emissions level is of the utmost importance, it takes precedence over the control of the windfall profits. So in a standard cap-and-trade scheme, the initial allocation is chosen to reach the emission target and the windfall profits follow without being controlled. Hence the desired parameter values are obtained at the intersection of the $1.827 \times 10^8 \mathrm{t}_{\mathrm{CO}_2}$ emission percentile level set with $y^e = 0$. Giving the initial allocation $\sum_{i \in I} x^i = 1.826 \times 10^8$.

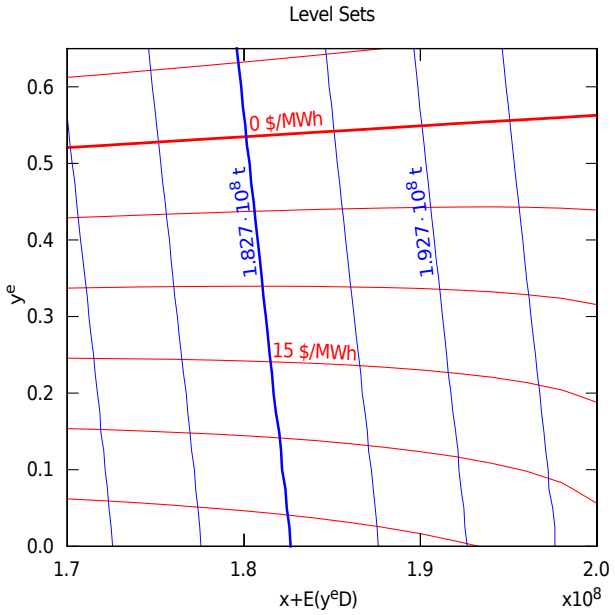

Fig. 3.3: This Figure shows the level sets of the two plots of Figures 3.1 and 3.2. The blue and the red lines indicate level sets of the windfall profits and the 95% quantile of emissions respectively. The free regulatory parameters are chosen to guarantee the desired emissions percentile, and in the case of the relative scheme, such that the windfall profits are zero.

Repeating the above procedure for different penalty levels gives regulatory settings with different production costs for the relative and the standard scheme in Figure 3.4. Obviously the reduction costs are lowered by increasing penalty for both schemes. As shown in Figure 3.4, this decrease in reduction costs is significant until the penalty reaches the level $\pi = 100\$$. After that, the reduction costs stay nearly the same, becoming independent of π for larger values of π. Hence, we conclude that in this setting a penalty of 100\$ is a reasonable choice for both the relative and the standard scheme.

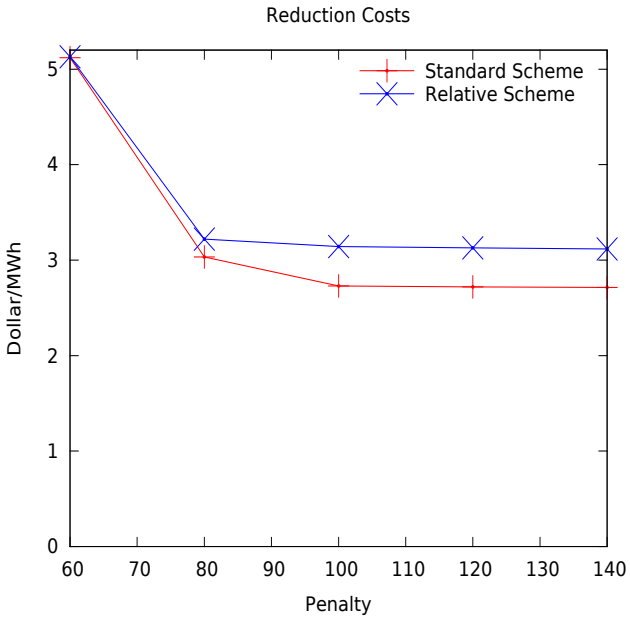

Fig. 3.4: Plots of the overall production costs for electricity for one year as function of the penalty level for both the standard and relative schemes.

Like a standard cap-and-trade scheme, a tax scheme has only one regulatory control parameter: the tax level z. As explained above, this single parameter should be calibrated in order to guarantee a specified reduction target. Thus the windfall profits are automatically given by the reduction target and cannot be controlled.

3.3. Comparison of Standard, Tax and Relative Schemes

This section compares emissions, windfall profits, consumer costs and reduction costs for the different market designs using the example of the Japanese electricity sector introduced in Section 2.5.

3.3.1. Yearly Emissions

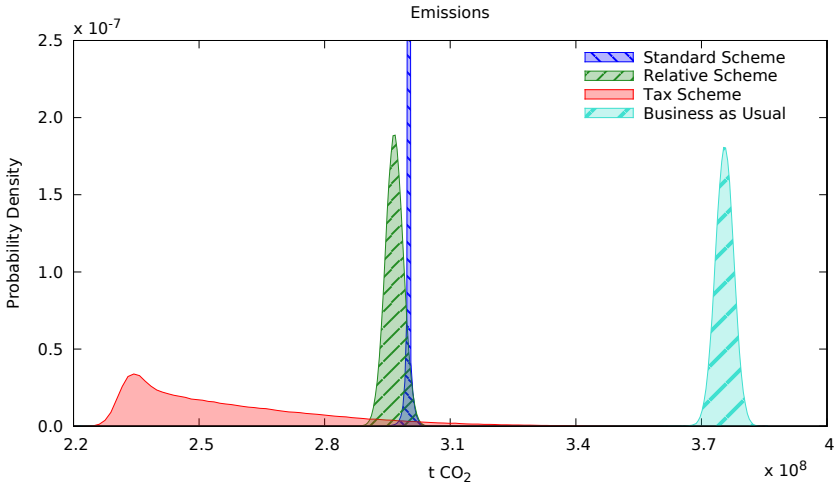

Fig. 3.5: Yearly emissions from electricity production for the standard cap-and-trade scheme, the relative cap-and-trade scheme, a tax scheme and BAU.

Figure 3.5 gives the plots of the statistical distributions of the yearly emissions for the various production schemes considered in this study. As explained above, these schemes were calibrated to the yearly emission target (cap) of 300Mt carbon dioxide, so that only 5% of the scenarios would give a yearly emissions level above target. Obviously, this does not apply to BAU scenarios. They ignore the cap and miss the target because of the lack of penalty. For standard and relative cap-and-trade schemes, producers' emissions are tightly concentrated just below the cap, showing that in both cases producers emit as much as possible while remaining under the cap. Because of the randomness inherent in the relative scheme, producers end up emitting less.

However, the striking fact illustrated by this figure is the width of the histogram of the yearly emissions under an emission tax. A tax of 39.5$ per tonne of carbon dioxide guarantees that the yearly emissions will not be greater than the cap more than 5% of the time. However, this can only be done at the cost of widely fluctuating yearly emissions due to the stochasticity of abatement costs (fuel switching). Since these fluctuations remain below the cap, they could be viewed as a sign that

properly calibrated, the tax scheme is better than the cap-and-trade schemes for emissions reduction. However, clearly the shape of the histogram shows a lack of efficiency in the abatement, and as we are about to see, this abatement happens at a much higher cost.

3.3.2. Windfall Profits

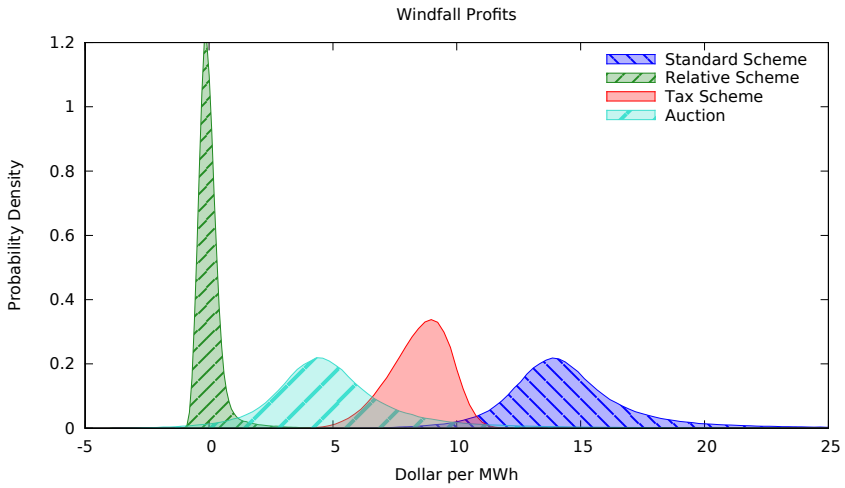

Fig. 3.6: Histograms of the yearly distibution of windfall profits for the four schemes considered in our case study. Note that the only scheme with close to zero windfall profits is the Relative Scheme.

Windfall profits were defined in Section 3.1. Figure 3.6 shows the statistical distributions of the windfall profits. Neither the tax scheme nor the standard scheme with full initial auctioning of the allowances is able to reduce the windfall profits to a satisfactory level. The reason is that due to increased electricity prices, profits are made in all cases with electricity produced from nuclear power plants while at the same time, neither the auctions nor the tax can reduce the earnings of the nuclear plants. The only scheme with close to zero windfall profits is the relative scheme. As explained above, the proportional allocation reduces electricity prices and therefore combats windfall profits at their origin. This works very well for markets with inelastic demand.

3.3.3. Consumer Burden

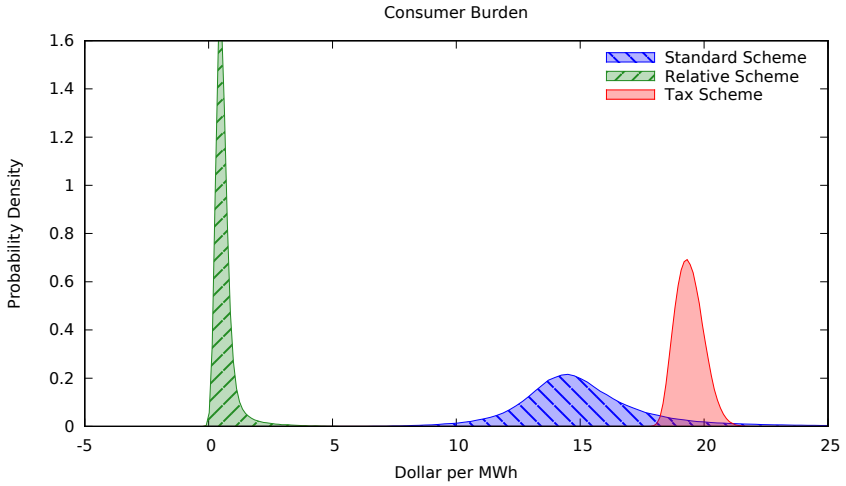

Fig. 3.7: Histogram of the yearly distribution of consumer costs for the Standard Scheme, a Relative Scheme and a Tax Scheme. Notice that the Standard Scheme with Auction possesses the same consumer costs as the Standard Scheme without auction, so the corresponding histogram is not plotted.

As explained in the appendix, the consumer burden, whose histograms are displayed in Figure 3.7 for the different schemes, captures the difference in total consumer costs with and without regulation. The high consumer burden for the standard scheme is in line with its high windfall profits. For the tax scheme the histogram indicates that in average, electricity prices would increase by approximately 20$ per MWh! However, the regulator could give back part of this amount to the end consumer, e.g. by reducing other taxes. Though the income from taxes is only 9.91$ per MWh, which comes nowhere near covering the consumer burden. Of course this is in line with the huge windfall profits. In case of the standard cap-and-trade scheme with auction, the average consumer costs are 15.13$ per MWh while the average auction revenue corresponds to 9.5$ per MWh. Hence consumer costs far exceed the revenue from the auction. Since this amount can only cover about two thirds of the consumer costs, the commonly believed argument that auction revenues can be used to cover costs of consumers is wrong and

there is still significant wealth transfer from consumer to producer. On the other hand, in the case of a relative scheme, the consumer burden is tightly concentrated around zero. This means that even for a reduction target of 20%, electricity costs are nearly the same as in BAU!

3.3.4. Abatement Costs

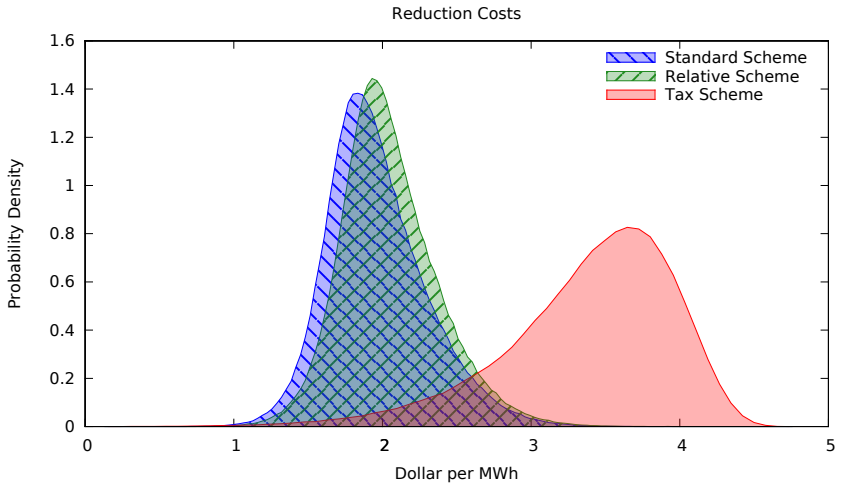

Fig. 3.8: Yearly abatement costs for the Standard Scheme, the Relative Scheme and a Tax Scheme.

Figure 3.8 illustrates the fact that the reduction costs of the relative cap-and-trade scheme are insignificantly higher than the reduction costs of the standard cap-and-trade scheme. However, those higher production costs are not just wasted money, they are paid for higher emission reduction in many scenarios as can be seen in Figure 3.5. In particular, the relative scheme takes advantage of cheap fuel switches when the standard scheme cannot reduce emissions anymore. Moreover, the relative scheme is less sensitive to weather, since in warm winters less allowances are allocated pushing the price up. This in turn is responsible for higher emission abatements and consequently higher abatement costs.

In this example of a relative scheme, approximately 30% of the allowances are given as initial allocation, by allocating these to clean

plants, further incentives can be set to build cleaner plants. This seems to be an important advantage of the relative scheme over other mechanisms such as auctioning and tax.

Moreover, both the standard and the relative cap-and-trade schemes are more economical and emissions control and reduction are easier than with an emission tax. We see that the cost of emission reduction is nearly twice as high in the case of the tax than for the cap-and-trade schemes. Again, these extra costs are paid for extra emission reductions. However, in contrast with the results in the case of the relative scheme, the average cost increase per reduced tonne of carbon is considerable when we compare it to the case of the standard scheme. This shows that the lack of emission control when abatement costs are stochastic is not only inconvenient for the regulator but can result in a huge loss of welfare.

3.4. General Market Designs with Zero Windfall Profits

One of the main arguments in favor of the relative schemes studied in the previous section is the fact that they reduce windfall profits. However, this reduction comes with slightly higher reduction costs than in the case of the standard scheme. Hence, it is of great theoretical interest to understand how and why one can design schemes that give exactly zero windfall profits at exactly the same reduction costs as the standard cap-and-trade scheme. In order to do so, we need to identify the generalized schemes which are in a one-to-one correspondence with the production policies of the standard scheme. The latter are given by a subclass of generalized schemes for which $Z_t^{i,j,k}$ and $Y_t^{i,j,k}$ depend only on k, and we often use the notation $Y_t^{i,j,k} = Y_t^k$ and $Z_t^{i,j,k} = Z_t^k$.

Moreover, the terminal wealth of firm $i \in I$ under such a scheme reads:

$$H^{A,S,i}(\theta^i, \xi^i) := -\sum_{t=0}^{T-1} V_t^i + \sum_{t=0}^{T-1} \sum_{(j,k)\in M_i} (S_t^k - C_t^{i,j,k} - Z_t^k)\xi_t^{i,j,k} \tag{3.13}$$
$$+ \sum_{t=0}^{T-1} \theta_t^i (A_{t+1} - A_t) - \theta_T^i A_T$$
$$- \pi\left(\Delta^i + \Pi^i(\xi^i) - \sum_{t=0}^{T-1}\left(X_t^i + \sum_{(j,k)\in M_i} Y_t^k \xi_t^{i,j,k}\right) - \theta_T^i\right)^+ .$$

The results of this section will demonstrate the versatility and the flexibility of the generalized framework introduced in this chapter. However, because of the level of complexity of their implementations, and despite the high degree of control they provide the regulator with, it is unlikely that the schemes identified here will be used by policy makers or regulators.

At this stage we recall the notation of equilibria: (A^*, S^*) is used for any equilibrium of the standard scheme, while $(\overline{A}, \overline{S})$ is the unique equilibrium of the standard schemes as defined in Theorem 1. Thereby $\overline{S}$ denotes the lowest possible prices of goods. Moreover, $\hat{S}$ is used for the target price.

3.4.1. Equivalence between Equilibria

The main thrust of this subsection is to identify one-to-one correspondences between equilibria of generalized schemes with allocations given by (3.1) and equilibria of standard schemes with allocations of the form

$$\Lambda_t^i = X_t^i + \Xi_t^i \quad \text{for all} \quad i \in I, t = 0, \ldots, T-1, \tag{3.14}$$

where the stochastic process Ξ in $\mathcal{L}^1_{T-1}(\mathbb{R}^{|I|})$ is chosen so that:

$$\sum_{i\in I} \Xi_t^i = \sum_{k\in K} Y_t^k D_t^k \quad \text{for all} \quad t = 0, 1, \cdots, T-1. \tag{3.15}$$

The following proposition is the main theoretical result of this section.

Proposition 5 *Let the processes $Y^{i,j,k}$ and $Z^{i,j,k}$ depend only upon the good $k \in K$ and not on the firm $i \in I$ or the technology $j \in J^{i,k}$. Then it holds under Assumptions 1 and 3 that: If (A^*, S^*) is an equilibrium with production strategy ξ^* for a standard cap-and-trade scheme with an allocation given by (3.14) then the couple of price processes (A^*, S'^*) where S'^* is given by*

$$S'^{*k}_t = S^{*k}_t + Z^k_t - Y^k_t A^*_t \quad \text{for all } k \in K, t = 0, \ldots, T-1 \tag{3.16}$$

form an equilibrium of the generalized cap-and-trade scheme with the same production strategies ξ^*. The converse statement also holds.

In order to prove Proposition 5, we shall need the following lemma.

Lemma 4 *Let A be a bounded martingale, $\xi^i \in \mathcal{U}^i, \theta^i \in \mathcal{V}^i(A, \Lambda^i), \theta'^i \in \mathcal{V}^i(A, \hat{\Lambda}^i(\xi^i))$, and S, S' be two integrable price processes, such that:*

$$\theta'^i_t = \theta^i_t \quad \text{for all } t \leq T-1 \tag{3.17}$$

$$\theta'^i_T = \theta^i_T + \sum_{t=0}^{T-1} \left(\Xi^i_t - \sum_{(j,k) \in M_i} Y^k_t \xi^{i,j,k}_t \right) \tag{3.18}$$

$$S'^k_t = S^k_t + Z^k_t - Y^k_t A_t \quad \text{for all } k \in K, t = 0, \ldots, T-1.$$

If moreover, (3.14) and Assumption 3 are satisfied, then it holds that:

$$\mathbb{E}[L^{A,S,i}(\theta^i, \xi^i)] - \mathbb{E}\left[\sum_{t=0}^{T-1} \Xi^i_t A_t\right] = \mathbb{E}[H^{A,S',i}(\theta'^i, \xi^i)] + \mathbb{E}\left[\sum_{t=0}^{T-1} V^i_t\right]. \tag{3.19}$$

Proof. The right hand side of (3.19) is equal to

$$\mathbb{E}\left[\sum_{t=0}^{T-1} \sum_{(j,k) \in M_i} (S'^k_t - C^{i,j,k}_t - Z^k_t)\xi^{i,j,k}_t - \theta'^i_T A_T \right.$$
$$\left. -\pi\left(\Delta^i + \Pi^i(\xi^i) - \sum_{t=0}^{T-1} \left(X^i_t + \sum_{(j,k) \in M_i} Y^k_t \xi^{i,j,k}_t\right) - \theta'^i_T\right)^+\right]$$

and using the martingale property of A, it can be expressed as

$$\mathbb{E}\left[\sum_{t=0}^{T-1}\sum_{(j,k)\in M_i}(S_t^k - C_t^{i,j,k} - Y_t^k A_t)\xi_t^{i,j,k} \right.$$
$$-\theta_T^i A_T - \sum_{t=0}^{T-1}\left(\Xi_t^i - \sum_{(j,k)\in M_i}\xi_t^{i,j,k}Y_t^k\right)\mathbb{E}(A_T|\mathcal{F}_t)$$
$$\left. -\pi\left(\Delta^i + \Pi^i(\xi^i) - \sum_{t=0}^{T-1}(X_t^i + \Xi_t^i) - \theta_T^i\right)^+\right]$$
$$= \mathbb{E}\left[\sum_{t=0}^{T-1}\sum_{(j,k)\in M_i}(S_t^k - C_t^{i,j,k})\xi_t^{i,j,k} - \theta_T^i A_T\right.$$
$$\left. -\pi\left(\Delta^i + \Pi^i(\xi^i) - \sum_{t=0}^{T-1}\Lambda_t^i - \theta_T^i\right)^+ - \sum_{t=0}^{T-1}\Xi_t^i A_t\right]$$

which is the left hand side of (3.19). □

We can now turn to the proof of Proposition 5.

Proof. (i) Let (A^*, S^*) be equilibrium price processes of a standard scheme with strategies (θ^*, ξ^*). Then Proposition 1 implies that A^* is a bounded martingale. Moreover, let θ'^* be the adjusted optimal strategy as in Lemma 4. It is straightforward to prove that $(\theta'^{*i}, \xi^{*i}) \in \mathcal{V}^i(A^*, \hat{\Lambda}^i(\xi^{*i})) \times \mathcal{U}^i$ hence the assertion follows by checking that conditions (i) to (iii) of Definition 1 are fulfilled by the pair of price processes (A^*, S'^*) and strategies (θ'^*, ξ^*). Since θ^* satisfies the market clearing condition (2.23), so does θ'^* due to (3.15) . Condition (ii) is obvious. Moreover, given $(\theta'^i, \xi^i) \in \mathcal{V}^i(A^*, \hat{\Lambda}^i(\xi^i)) \times \mathcal{U}^i$, we define strategies θ^i such that (3.17) and (3.18) hold, which guaranties that $\theta^i \in \mathcal{V}^i(A^*, \Lambda^i)$ is satisfied as well. Hence the result of Lemma 4 yields

$$\begin{aligned}\mathbb{E}[H^{A^*,S'^*,i}(\theta'^i, \xi^i)] &= \mathbb{E}[L^{A^*,S^*,i}(\theta^i, \xi^i)] - \mathbb{E}\left[\sum_{t=0}^{T-1}(V_t^i + \Xi_t^i A_t^*)\right] \\ &\leq \mathbb{E}[L^{A^*,S^*,i}(\theta^{*i}, \xi^{*i})] - \mathbb{E}\left[\sum_{t=0}^{T-1}(V_t^i + \Xi_t^i A_t^*)\right] \qquad (3.20) \\ &= \mathbb{E}[H^{A^*,S'^*,i}(\theta'^{*i}, \xi^{*i})]\end{aligned}$$

where we used the optimality of the equilibrium strategies (ξ^*, θ^*) of the standard scheme in (3.20). This holds for all $(\theta'^i, \xi^i) \in \mathcal{V}^i(A^*, \hat{\Lambda}^i(\xi^i)) \times \mathcal{U}^i$, which proves condition (iii). The converse can be proven in exactly the same way. □

Note that the above proof also shows that, not only do allocation prices coincide, but also equilibrium production strategies: Thus the switching costs of the generalized cap-and-trade schemes are the same as for the standard cap-and-trade schemes.

The above discussion suggests that windfall profits could be reduced with a relative allocation rule constant over time. This motivates the following analysis.

3.4.2. Reduction of Windfall Profits with Generalized Schemes

In this subsection, we discuss the design of financial incentives. We propose to adjust the financial positions of each firm $i \in I$ by

$$-\sum_{t=0}^{T-1}(V_t^i + \sum_{(j,k)\in M_i} \xi_t^{i,j,k} Z_t^k) \tag{3.21}$$

depending on its production strategy ξ^i. Obviously, the results of Proposition 5 still hold in this case. In equilibrium, allowance prices, production strategies, and penalty are identical to those of a standard scheme. However, the price of goods increases by the amount Z_t^k at $t = 0, \ldots, T-1$. Hence, as depicted in Figure 3.9 the scheme induces a money transfer

$$\sum_{t=0}^{T-1}(V_t^i + \sum_{(j,k)\in M_i} \xi_t^{i,j,k} Z_t^k)$$

from producers to the regulator. In the meantime, the quantity

$$\sum_{t=0}^{T-1} \sum_{(j,k)\in M_i} \xi_t^{i,j,k} Z_t^k$$

is entirely passed on to the consumer, so that the price increase $(Z_t^k)_{t=0}^{T-1} \geq 0$ results in a money transfer from consumers to the regulator.

Remark 5 *This discussion is a good illustration of the difficulties in reducing windfall profits. As proposed in Ecologic-Institute (2005) it seems natural to think that one could do so by keeping track of the number of MWh traded, the marginal technology used, and the price of carbon with the intention to charge the windfall profits to the producers and reimburse consumers later. In this case, given a production strategy ξ^i and a price process S, each agent $i \in I$ would be charged the amount*

$$\sum_{t=0}^{T-1} \sum_{(j,k)\in M_i} (S_t^k - \hat{S}_t^k)\xi_t^{i,j,k}. \tag{3.22}$$

This would correspond to a standard scheme where (3.22) would be withdrawn from (2.17), or alternatively S^k would be replaced by $\hat{S}^k$. But since $\bar{S}^k$ is the lowest price for good $k \in K$, the demand cannot be satisfied if $\hat{S}^k$ drops below $\bar{S}^k$. Due to (2.73) this happens whenever $e^{i,j,k} > 0$ for some marginal production plant (i,j). Hence, it is very unlikely that an equilibrium exists in such a scheme. On the other hand, if we change the amount to be charged to

$$\sum_{t=0}^{T-1} \sum_{(j,k)\in M_i} (\overline{S}_t^k - \hat{S}_t^k)\xi_t^{i,j,k}, \tag{3.23}$$

it follows from the above discussion that the entire amount is passed on to the end-consumer and windfall profits are not reduced.

Proposition 5 happens to be a very versatile tool when it comes to designing new schemes with required properties. As corollaries to this proposition, two appropriate adjustments with zero windfall profits are given in Subsections 3.4.3 and 3.4.4.

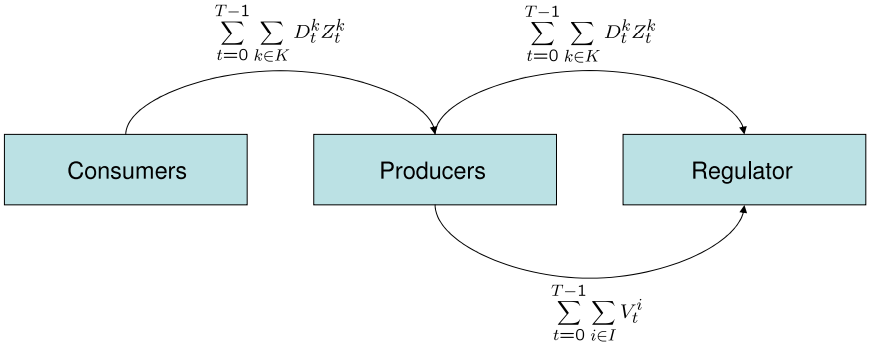

Fig. 3.9: $V > 0$ gives a money transfer from producers to the regulator, while $Z > 0$ gives a money transfer from consumer to the regulator. By choosing V and Z in an appropriate way it is possible to avoid a money transfer to/from the regulator.

3.4.3. Zero Windfall Profit Scheme with Tax and Subsidy

In this section we consider the generalized allocation scheme given by

$$X_t^i = \Lambda_t^i$$
$$Y_t^{i,j,k} = 0$$

for all $i \in I$, $k \in K$, $j \in J^{i,k}$ and $t = 0, \ldots, T-1$. We show here how the tax/subsidy system comprised into the generalized scheme can theoretically lead to zero windfall profits at equilibrium. This result is a direct corollary of Proposition 5.

Corollary 1 *Consider a generalized cap-and-trade scheme such that for all $i \in I$ and $k \in K$*

$$\begin{aligned} V_t^i &= \sum_{(j,k)\in M_i} \overline{\xi}_t^{i,j,k}(\overline{S}_t^k - \hat{S}_t^k) \text{ for all } t = 0, \ldots, T-1 \\ Z_t^k &= 0 \text{ for all } k \in K, t = 0, \ldots, T-1. \end{aligned} \tag{3.24}$$

Under Assumptions 1 and 3 each equilibrium (A^*, S^*) of the standard cap-and-trade scheme is also an equilibrium of this generalized scheme. If furthermore Assumptions 2 and 4 are satisfied then the equilibrium with lowest product prices is given by $(\overline{A}, \overline{S})$ from Theorem 1. For this equilibrium the windfall profits for the aggregated producing sector are zero.

Remark 6 *The processes $\overline{S}$, $\hat{S}$ and $\overline{\xi}$ appearing in (3.24) are defined in Theorem 1 and formula (2.73). They are independent of the actually realized equilibrium (A^*, S^*) and its corresponding strategies (θ^*, ξ^*). Consequently, their computation involves solving the global optimal control problem.*

In order to maintain the consumer costs at a reasonable level, the amount $\sum_{i\in I}\sum_{t=0}^{T-1} V_t^i$ has to be redistributed from the regulator to the end consumers in an appropriate way. As shown in Figure 3.9 this is not needed if the financial incentives fulfill

$$\sum_{t=0}^{T-1}\sum_{i\in I} V_t^i = -\sum_{t=0}^{T-1}\sum_{k\in K} D_t^k Z_t^k, \tag{3.25}$$

and in this case, there is no money transfer from producers to regulators. This is the object of the following corollary.

Corollary 2 *Consider a generalized cap-and-trade scheme such that for all $i \in I$ and $k \in K$*

$$\begin{aligned} V_t^i &= \sum_{(j,k)\in M_i} \overline{\xi}_t^{i,j,k}(\overline{S}_t^k - \hat{S}_t^k) \qquad \text{for all } t = 0, \dots, T-1 \\ Z_t^k &= -(\overline{S}_t^k - \hat{S}_t^k) \qquad \text{for all } t = 0, \dots, T-1. \end{aligned}$$

Under Assumptions 1 and 3 each equilibrium (A^*, S^*) of the standard cap-and-trade scheme corresponds to an equilibrium of the generalized cap-and-trade scheme given by $(A^*, S^* - (\overline{S} - \hat{S}))$. If moreover Assumptions 2 and 4 are satisfied then the equilibrium with lowest product prices is given by $(\overline{A}, \hat{S})$. In this equilibrium, the aggregated producing sector has zero windfall profits.

If a firm i follows the production strategy ξ^i, then the net money transfer to the regulator it is responsible of is given by

$$\sum_{(j,k)\in M_i} (\overline{\xi}_t^{i,j,k} - \xi_t^{i,j,k})(\overline{S}_t^k - \hat{S}_t^k). \tag{3.26}$$

Notice that this is zero if the firm follows the strategy $\overline{\xi}$. If it decides to produce more or less than in equilibrium, it is rewarded or respectively

penalized by (3.26). This increases the incentives to produce, and hence results in lower product prices. As shown by (3.25), the only money transfer in this scheme is the transfer of payments from penalized firms to rewarded firms.

3.4.4. Zero Windfall Profit Scheme with Dynamic Allocation

Here we set $V \equiv Z \equiv 0$, and show how dynamic stochastic allocation of allowances can also lead to null windfall profits for the producers.

Corollary 3 *Consider a cap-and-trade scheme with dynamic allowance allocation where*

$$X_t^i = -\sum_{(j,k)\in M_i} \overline{\xi}_t^{i,j,k} \frac{\overline{S}_t^k - \hat{S}_t^k}{\overline{A}_t} + \Lambda_t^i$$

$$Y_t^{i,j,k} = Y_t^k = \frac{\overline{S}_t^k - \hat{S}_t^k}{\overline{A}_t}$$

at each time point $t = 0, \dots, T-1$ and for the convention $0/0 = 0$. Under Assumptions 1-4 each equilibrium (A^*, S^*) of the standard cap-and-trade scheme corresponds to an equilibrium of the generalized cap-and-trade scheme given by $(A^*, S^* - (\overline{S} - \hat{S}))$. Moreover the equilibrium with lowest prices for goods is given by $(\overline{A}, \hat{S})$ and the windfall profits of each firm are zero for this equilibrium.

Proof. Notice that both X_t^i and Y_t^k are well defined despite the presence in the denominator of $\overline{A}_t$. Indeed by definition, $\overline{S}_t^k = \hat{S}_t^k$ whenever $\overline{A}_t = 0$. The equilibrium result is a direct consequence of Proposition 5. Hence it remains only to prove that $(X_t^i)_{t=0}^{T-1} \in \mathcal{L}_{T-1}^1(\mathbb{R})$ and $(Y_t^k)_{t=0}^{T-1} \in \mathcal{L}_{T-1}^\infty(\mathbb{R})$ for all $i \in I$. Let us first prove that $\frac{\overline{S}_t^k - \hat{S}_t^k}{\overline{A}_t}$ is bounded from above. Since

$$\begin{aligned}
\overline{S}_t^k &= \max_{i\in I, j\in J^{i,k}} (C_t^{i,j,k} + e^{i,j,k}\overline{A}_t)\mathbf{1}_{\{\overline{\xi}_t^{i,j,k}>0\}} \\
&\leq \max_{i\in I, j\in J^{i,k}} C_t^{i,j,k} 1_{\{\overline{\xi}_t^{i,j,k}>0\}} + \max_{i\in I, j\in J^{i,k}} e^{i,j,k}\overline{A}_t 1_{\{\overline{\xi}_t^{i,j,k}>0\}} \\
&\leq \hat{S}_t^k + \overline{A}_t \max_{i\in I, j\in J^{i,k}} e^{i,j,k}
\end{aligned}$$

it follows that

$$\frac{\overline{S}_t^k - \hat{S}_t^k}{\overline{A}_t} \leq \max_{i \in I, j \in J^{i,k}} e^{i,j,k} \text{ for all } k \in K, t = 0, \ldots, T-1 \tag{3.27}$$

which gives the upper bound. To prove the lower bound, we notice that for all $k \in K$ and $t = 0, \ldots, T-1$ it holds that $\overline{S}_t^k - \hat{S}_t^k \geq 0$ and $\overline{A}_t \geq 0$. Thus

$$\frac{\overline{S}_t^k - \hat{S}_t^k}{\overline{A}_t} \geq 0 \text{ for all } k \in K, t = 0, \ldots, T-1 \tag{3.28}$$

which concludes the proof. □

This scheme ensures the existence of an equilibrium with zero windfall profits and exactly the same reduction costs as for the standard cap-and-trade scheme, without any extra money transfer from consumer/producer to regulator, as e.g. would be the case for auctioning. A further advantage compared to auctioning is that it allows to distribute the amount $\sum_{i \in I} X_t^i$ differently to control the incentives to build cleaner plants.

The drawback of this scheme is that it requires a random allocation (following a prespecified rule) of allowances dynamically through time. Moreover, the daily allocation can be negative at times for some producers. But this should not be a problem as seen in the green certificate schemes.

Notice further that it suffices to give a relative allocation only to those plants which are frequently on the margin. Therefore, in the electricity markets, the relative allocation does not need to be given to nuclear plants.

The theoretical results of this section suppose market rules that may be difficult to enforce in a real market. However, they show how the different levers brought by the generalized schemes can be used to keep the prices of goods at a low level.

Chapter 4

Auction and Hybrid Allocation Schemes

Chapter Overview

In this chapter we prove social optimality of standard cap-and-trade schemes and propose a hybrid cap-and-trade scheme, where the allocation procedure is a mixture of relative allocation and auctioning. This scheme reduces prices of goods by relative allocation while, in contrast to relative schemes, social optimality is preserved (in the case of inelastic demand).

Social optimality for cap-and-trade schemes is a well known result for cap-and-trade schemes in the original layout of Montgomery (1972) with a stringent emission target. In Section 4.2 this result is carried over to a stochastic setting for emission trading schemes allowing excess emissions modulo a penalty π, in the spirit of EU ETS. Thereby we consider emission targets formulated in terms of expected excess emission above a given cap. Given any reasonable choice of such a target one can find a penalty level such that in equilibrium the standard cap-and-trade scheme has minimal costs of production under the constraint that the emission target is fulfilled and that the demand for goods has to be met.

Motivated by the discussion of the shortcomings of standard cap-and-trade schemes in Chapter 2, we considered in Chapter 3 proportional allocation schemes, and showed that they did lead to lower equilibrium prices, reducing windfall profits at their origin. They can be calibrated to reduce prices of goods to a level that covers reduction costs without incorporating the costs for allowances. At the same time they can be set up with a partial free initial allocation to still allow the regulator to set incentives.

However, the drawback of the relative scheme is the fact that the actual amount of allowances injected in the market is not known in advance since it depends upon the demand for goods. Moreover, the relative scheme is not socially optimal, even though as we saw in Section 3.3 the reduction costs are nearly the same as for the standard scheme.

Therefore we propose in Section 4.1 a hybrid cap-and-trade scheme, which incorporates the best properties of the standard scheme with those of the relative scheme. Namely it is socially optimal, while at the same time it has a fixed cap and provides a tight control of windfall profits. In Subsection 4.1.4 we define a notion of market equilibrium

suited to this scheme.

From a mathematical point of view, the main highlight of this chapter is the proof of the equivalence of equilibria in Section 4.3. This result identifies a one-to-one correspondence between equilibria of standard cap-and-trade schemes and hybrid cap-and-trade schemes. In particular, this result shows why and how the prices of goods are reduced by the hybrid scheme.

In Section 4.4 we show how to calibrate the different parameters of the scheme and compare the qualitative properties of the hybrid scheme with those of standard schemes both with and without auction.

4.1. Equilibrium for Hybrid Cap and Trade Schemes

In this section we propose a hybrid allocation rule being a mixture of relative allocation and auctioning, which reduces prices of goods by relative allocation while the amount of allowances in the market is fixed.

4.1.1. Hybrid Allocation Mechanism

For the hybrid cap-and-trade scheme, part of the initial allocation is put into a pot from which the relative allocation is withdrawn during a time period. If the relative allocation exceeds the amount of allowances in the pot, each agent is given a fixed amount of allowances (which is computed using only the production strategies of the other agents). The rest of the allowances are auctioned. If the relative allocation does not exceed the amount of allowances in the pot, the allowances that remain in the pot are auctioned.

For each unit of produced goods agents obtain a certain amount of allowances for free in the case that the relative allocation does not exceed the size of the pot. Since allowances, which are given away for free, relieve the agents from buying these later in the auction, this allocation mechanism reduces the marginal production costs. The same holds for the representative planer. Note that it is crucial that the rest of the allowances are auctioned. If the remaining allowances were to be

given away for free, there would be no benefit of receiving allowances relative to production.

For practical applications we propose a simplified allocation rule which gives similar results but for which there is no theoretical correspondence to the equilibrium of a standard cap-and-trade scheme. In a numerical study of the Japanese electricity market we show that for reasonable parameters the probability of the relative allocation to exceed the pot is negligible. Hence, in practice the equilibrium will not change much if we simplify the allocation rule in such a scenario. A realistic allocation rule would be to stop the relative allocation at the point in time where it exceeds the pot.

4.1.2. Mathematical Formulation

In a standard scheme, each installation is provided with a free allocation. In the relative scheme introduced and studied in Chapter 3 and in Carmona et al. (2009b), each installation is not only provided with a free initial allocation of a specific number of allowances at time $t = 0$, but throughout the compliance period, it is also provided with free allowances on the basis of its production.

We now describe the hybrid allocation scheme we propose in the present study. As explained in the introduction, this scheme is characterized by a cap $\tilde{\Lambda}$, an initial allocation $\tilde{\Lambda}_0^i$ given at time $t = 0$ to each participating installation $i \in I$, a proportional distribution of free allowances over a period $[0, \tau]$ with $\tau \in \{0, \dots, T-1\}$ and at time τ, an auction of the remaining allowances. The details of this allocation procedure are as follows. Once the free initial allocation of the $\tilde{\Lambda}_0^i$ allowances to firm i are taken care of, the remaining

$$\Upsilon := \tilde{\Lambda} - \sum_{i \in I} \tilde{\Lambda}_0^i \tag{4.1}$$

allowances are set for proportional distribution and auction. We use the suggestive terminology of a *pot* from which the allowances will be allocated relatively to production levels and possibly auctioned off.

Given coefficients of proportionality y^k chosen by the regulator, for each

admissible strategy $\xi^i \in \mathcal{U}^i$ of agent $i \in I$, we denote by

$$Y^i(\xi^i) := \sum_{k\in K} \sum_{j\in J^{i,k}} \sum_{t=0}^{\tau-1} y^k \xi_t^{i,j,k} \tag{4.2}$$

the amount of allowances potentially earned by firm i in the name of the relative allocation up to time $\tau - 1$, and we denote by

$$Y(D) := \sum_{k\in K} \sum_{t=0}^{\tau-1} y^k D_t^k \tag{4.3}$$

the total allocation that the proportional scheme would require before time τ for a production schedule satisfying the demand.

In order to omit that the allocation in the hybrid scheme exceeds the cap we limit the relative allocation of every agent $i \in I$ by the amount

$$\Gamma(\xi^{-i}) := \left(\Upsilon - \sum_{i'\in I\setminus\{i\}} Y^{i'}(\xi^{i'})\right)^+, \tag{4.4}$$

where $\xi^{-i} := (\xi^{i'})_{i'\in I\setminus\{i\}}$ denotes the strategies of the other agents. So given the production strategies $\xi^{-i} = (\xi^{i'})_{i'\in I\setminus\{i\}}$ of the other firms and its own admissible production strategy $\xi^i \in \mathcal{U}^i$, the relative allocation of firm $i \in I$ is always given by $\Gamma(\xi^{-i}) \wedge Y^i(\xi^i)$. Thus if $\sum_{i\in I} Y^i(\xi^i) < \Upsilon$, then each agent $i \in I$ is given $Y^i(\xi^i)$ allowances for free. However, if $\sum_{i\in I} Y^i(\xi^i) \geq \Upsilon$ then agent $i \in I$ is given the amount $\Gamma(\xi^{-i})$.

In any case, after this free distribution there are

$$\Upsilon(\xi) = \begin{cases} \Upsilon - \sum_{i\in I} Y^i(\xi^i) & \text{if } \sum_{i\in I} Y^i(\xi^i) < \Upsilon \\ \Upsilon - \sum_{i\in I} \Gamma(\xi^{-i}) & \text{if } \sum_{i\in I} Y^i(\xi^i) \geq \Upsilon \end{cases}$$

allowances remaining *in the pot* and they are auctioned in an Open Ascending-Bid Auction (English Auction) at time τ, and we denote by the $\mathcal{F}_\tau$-measurable random variable P the price of the auctioned allowances and by φ^i the $\mathcal{F}_\tau$-measurable auction strategy, i.e. the number of allowances bought by firm i at the auction. For simplicity we assume that the allowances are paid and delivered at time T, even though they are auctioned at time τ.

Remark 7 *This allocation mechanism prevents the hybrid allocation to exceed the size of the pot since in the case that* $\sum_{i\in I} Y^i(\xi^i) \geq \Upsilon$ *we have*

$$\sum_{i\in I} \Gamma(\xi^{-i}) \leq \Upsilon. \tag{4.5}$$

This is seen as $\sum_{i\in I} \Gamma(\xi^{-i})$ *is monotonically decreasing in the* $Y^i(\xi^i)$ *while* (4.5) *is fulfilled with equality if* $\sum_{i\in I} Y^i(\xi^i) = \Upsilon$.

Since the rest of the allowances in the pot are auctioned, the total amount of allowances in the market corresponds to the cap $\tilde{\Lambda}$, as in the standard scheme. Notice that for $y^k = 0$ we recover the standard scheme where the totality Υ of the pot is auctioned. If on the other hand $\Upsilon = 0$ then we are in the realm of the standard cap-and-trade scheme without auction and without relative allocation.

4.1.3. Profits

If we denote by φ^i the number of allowances purchased by firm i at the time τ of the auction and by θ^i_T its final position due to trading, the penalty payment due by firm i at time T for using the strategy ξ^i while the other firms are using strategies ξ^{-i} is:

$$\pi\Big(\Delta^i + \Pi^i(\xi^i) - \tilde{\Lambda}^i_0 - \varphi^i - \theta^i_T - \Gamma(\xi^{-i}) \wedge Y^i(\xi^i)\Big)^+. \tag{4.6}$$

Combining (2.13), (2.14) and (4.6) together with (2.12), we obtain the following expression for the terminal wealth (profits and losses at time T) of firm i

$$\begin{aligned} H^{A,S,P,\xi^{-i},i}(\theta^i,\xi^i,\varphi^i) := &\sum_{t=0}^{T-1} \sum_{(j,k)\in M_i} (S^k_t - C^{i,j,k}_t)\xi^{i,j,k}_t \\ &+ \sum_{t=0}^{T-1} \theta^i_t(A_{t+1} - A_t) - \theta^i_T A_T - \varphi^i P \\ &- \pi\Big(\Delta^i + \Pi^i(\xi^i) - \tilde{\Lambda}^i_0 - \varphi^i - \theta^i_T - \Gamma(\xi^{-i}) \wedge Y^i(\xi^i)\Big)^+. \end{aligned}$$

4.1.4. Market Equilibrium

We now define the notion of market equilibrium appropriate for the regulation given by the cap-and-trade scheme built on the hybrid allocation scheme described in the previous section. We will use the following spaces of admissible strategies

$$\tilde{\mathcal{Q}}^{\xi^{-i},i}(A,P) := \Big\{ (\theta^i,\xi^i,\varphi^i) \,\big|\, \xi^i \in \mathcal{U}^i \,,\ \varphi^i \geq 0 \text{ a.s. and } \mathcal{F}_\tau\text{-measurable}, \\ \varphi^i P \text{ and } R_T^A(\theta^i) \text{ are integrable}, \\ \theta_T^i \geq -\Big(\tilde{\Lambda}_0^i + \Gamma(\xi^{-i}) \wedge Y^i(\xi^i) + \varphi^i \Big) \text{ a.s.} \Big\}.$$

and

$$\tilde{\mathcal{Q}}(A,P) := \Big\{ (\theta^i,\xi^i,\varphi^i)_{i\in I} \,\big|\, (\theta^i,\xi^i,\varphi^i) \in \tilde{\mathcal{Q}}^{\xi^{-i},i}(A,P) \text{ for all } i \in I \Big\}.$$

The following technical assumption will help us refine the statements of some of the results leading to the equilibria.

Assumption 5 *We assume that the potentially earned amount of allowances $Y(D)$ matches the size of the pot only on sets of zero probability, i.e.* $\mathbb{P}[Y(D) = \Upsilon] = 0$.

In our case study of the Japanese electricity market we model the demand based on an Ornstein-Uhlenbeck process, for which this assumption is fulfilled. Following the intuition that given prices $A = \{A_t\}_{t=0}^T$, $S = \{(S_t^k)_{k\in K}\}_{t=0}^{T-1}$ and P, and the production strategies ξ^{-i} of the other firms, each firm aims at increasing its own wealth by maximizing the function

$$(\theta^i,\xi^i,\varphi^i) \hookrightarrow \mathbb{E}[H^{A,S,P,\xi^{-i},i}(\theta^i,\xi^i,\varphi^i)], \tag{4.7}$$

over its admissible investment and production strategies, we are led to define equilibrium in the following way:

Definition 2 *The triple* $(A^*, S^*, P^*) \in \mathcal{L}^1_T(\mathbb{R}) \times \mathcal{L}^1_{T-1}(\mathbb{R}^{|K|}) \times L^1(\mathcal{F}_\tau)$ *of integrable processes and random variable are an equilibrium of the market if there exists* $(\theta^*, \xi^*, \varphi^*) \in \tilde{\mathcal{Q}}(A^*, P^*)$ *such that:*
(i) All financial positions are in zero net supply, i.e.

$$\sum_{i \in I} \theta^{*i}_t = 0, \qquad t = 0, \dots, T \tag{4.8}$$

(ii) All allowances that are auctioned are bought, i.e.

$$\sum_{i \in I} \varphi^{*i} = \Upsilon(\xi^*) \tag{4.9}$$

(iii) Supply meets demand for each good

$$\sum_{i \in I} \sum_{j \in J^{i,k}} \xi^{*i,j,k}_t = D^k_t, \qquad k \in K, \;\; t = 0, \dots, T-1 \tag{4.10}$$

(iv) Each firm $i \in I$ *is satisfied by its own strategy given the strategies* $\xi^{*-i} = (\xi^{*i'})_{i' \in I \setminus \{i\}}$ *of the other firms, namely:*

$$\mathbb{E}[H^{A^*, S^*, P^*, \xi^{*-i}, i}(\theta^{*i}, \xi^{*i}, \varphi^{*i})] \geq \mathbb{E}[H^{A^*, S^*, P^*, \xi^{*-i}, i}(\theta^i, \xi^i, \varphi^i)] \tag{4.11}$$

for all $(\theta^i, \xi^i, \varphi^i) \in \tilde{\mathcal{Q}}^{\xi^{*-i}, i}(A^*, P^*)$.

From this definition it is obvious that $\varphi^{*i}(P^* - A^*_\tau) = 0$ almost surely for all $i \in I$. If it would hold that $\mathbb{P}[\{P^* < A^*_\tau\}] > 0$ then the auction and trading strategies $\varphi^{*i} + \mathbf{1}_{\{P^* < A^*_\tau\}}$ and $\theta^{*i}_T - \mathbf{1}_{\{P^* < A^*_\tau\}}$ would outperform the original strategies φ^{*i} and θ^{*i}. However, this does not lead to an equilibrium as on $\{P^* < A^*_\tau\}$ it holds that

$$\sum_{i \in I} \varphi^{*i} + \mathbf{1}_{\{P^* < A^*_\tau\}} > \sum_{i \in I} \varphi^{*i} = \Upsilon(\xi^*)$$

contradicting condition (ii) of an equilibrium. On the other hand on $\{P^* > A^*_\tau\}$ it holds that $\varphi^{*i} = 0$ since it is cheaper buying allowances in the market than in the auction. Hence we conclude that $\varphi^{*i}(P^* - A^*_\tau) = 0$.

Since in equilibrium all allowances in the auction are bought the total amount of allowances under a standard scheme and a hybrid scheme

with the same cap $\Lambda = \tilde{\Lambda}$ is the same. Hence we expect the hybrid scheme to give the same allowance price processes as the standard scheme. But due to the relative allocation we anticipate that the prices of goods will be lower.

Later we will relate the equilibria of hybrid schemes to the equilibria of standard schemes, where the allocation is only given at time $t = 0$, i.e. $\Lambda_t^i = 0$ for all $i \in I$ and $t = 1, \ldots, T-1$. Therefore, it will be convenient to denote the feasible production strategies in the standard scheme by

$$\mathcal{Q}^i(A) := \left\{ (\theta^i, \xi^i) \,\middle|\, \xi^i \in \mathcal{U}^i \,, R_T^A(\theta^i) \text{ are integrable} \,, \theta_T^i \geq -\Lambda_0^i \text{ a.s.} \right\}$$
$$\mathcal{Q}(A) := \prod_{i \in I} \mathcal{Q}^i(A).$$

4.2. Emission Targets and Social Optimality

Montgomery proved in a deterministic setting that emission trading schemes are socially optimal in the sense that a given emission target is reached at lowest possible cost. See Montgomery (1972). Because he is working in a deterministic setting, the emission target is a hard constraint, i.e. emissions in equilibrium have to stay below the cap. However, emissions are stochastic in real life, and a stringent emission target can rapidly become prohibitive. Hence emission trading schemes, as e.g. EU ETS, allow for excess emissions modulo a penalty π, which at the same time serves as a safety valve for the allowance price.

In a stochastic setting, we need a different notion of emission target.

4.2.1. Definition of the Emission Target

A natural choice is to control the emission distribution by a risk measure, as was done in Chapters 2 and 3 using a quantile measure. Like Value At Risk, this measure does a poor job at controlling the tail of the distribution since it controls only the frequencies of exceedances and not their actual sizes. In complete analogy with the expected shortfall,

we propose here to control the emissions by setting an upper bound η on the expected excess emission

$$E^{\lambda}(\xi) = \mathbb{E}\left[\left(\Delta + \sum_{(i,j,k)\in M}\sum_{t=0}^{T-1} e^{i,j,k}\xi_t^{i,j,k} - \lambda\right)^+\right] \tag{4.12}$$

above some threshold λ under a production strategy $\xi \in \mathcal{U}$. Due to demand inelasticity, there exists for every λ a lower bound on excess emissions given by

$$\tilde{a}(\lambda) = \inf_{\xi\in\mathcal{U}} E^{\lambda}(\xi). \tag{4.13}$$

This is seen as the lower bound is attained for a specific production schedule $\tilde{\xi}$ guaranteed by Proposition 3.

4.2.2. Social Optimality of Standard Cap-and-Trade Schemes

In the sequel we say that a cap-and-trade scheme is socially optimal if given any choice of such a reduction target, one can find control parameters (e.g. cap and penalty level) such that the scheme in equilibrium reaches the emission target at minimal costs of production given that the demands for goods have to be met. More precisely for the emission regulations discussed in this thesis we define:

Definition 3 *An emission regulation is said to be socially optimal if for every choice of the threshold $\lambda > 0$ and upper bound $\eta > \tilde{a}(\lambda)$ there exist regulatory parameters such that (at least) one corresponding equilibrium production schedule ξ^* is a solution of*

$$\begin{aligned} \inf \quad & \mathbb{E}[C(\xi)] && (4.14)\\ \text{s.t.} \quad & E^{\lambda}(\xi) \leq \eta && (4.15)\\ & \xi \in \mathcal{U}. \end{aligned}$$

The meaning of equilibrium production schedule depends on the specific design of the emission regulation and is given by Definitions 1 and 2 respectively.

We now show that standard cap-and-trade schemes are socially optimal in this sense. To be more specific, we show that for every choice of

threshold $\lambda > 0$ and for every upper bound $\eta > \tilde{a}(\lambda)$ ever so slightly greater than the minimum emission level possible, the corresponding cap is given by $\Lambda = \lambda$ and the penalty π is found as the Lagrange multiplier of the constraint (4.15). I.e. we compute it as the solution of

$$\sup \left\{ \inf_{\xi \in \mathcal{U}} L^{\eta,\lambda}(\xi, \pi) \middle| \pi \geq 0 \right\} \tag{4.16}$$

where

$$L^{\eta,\lambda}(\xi, \pi) := \mathbb{E}[G^{\pi,\lambda}(\xi)] - \pi\eta = \mathbb{E}[C(\xi)] + \pi(E^{\lambda}(\xi) - \eta) \tag{4.17}$$

denotes the Lagrangian of problem (4.14) while

$$G^{\pi,\lambda}(\xi) = C(\xi) + \pi(\Delta - \lambda + \Pi(\xi))^{+} \tag{4.18}$$

denotes the societal costs from production and penalty payments. This is the same as $G(\xi)$ with $\Lambda = \lambda$ highlighting its dependence on λ and π. Corollary 4 concludes that under appropriate assumptions the standard cap-and-trade scheme is socially optimal. For this proof we need following Proposition.

Proposition 6 *Under Assumption 3 it holds for every $\lambda \geq 0$ and $\eta > \tilde{a}(\lambda)$ that:*

(i) There exists both a solution $\overline{\xi} \in \mathcal{U}$ to the optimal control problem

$$\inf \left\{ \sup_{\pi \geq 0} L^{\eta,\lambda}(\xi, \pi) \middle| \xi \in \mathcal{U} \right\} \tag{4.19}$$

and a solution $\overline{\pi} \in [0, \infty)$ of problem (4.16).

(ii) The solution $(\overline{\pi}, \overline{\xi})$ forms a saddle point of the Lagrangian, i.e.

$$L^{\eta,\lambda}(\xi, \overline{\pi}) \geq L^{\eta,\lambda}(\overline{\xi}, \overline{\pi}) \geq L^{\eta,\lambda}(\overline{\xi}, \pi) \quad \text{for all } \xi \in \mathcal{U},\ \pi \geq 0.$$

(iii) For a standard cap-and-trade scheme with penalty $\overline{\pi}$ and cap $\Lambda = \lambda$ the strategy $\overline{\xi}$ is a solution of the global optimal control problem (2.43) *and* (4.14).

Proof. (i) As proven in Lemma 1, the function $\xi \hookrightarrow L^{\eta,\lambda}(\xi, \pi) = \mathbb{E}[G^{\pi,\lambda}(\xi)] - \pi\eta$ is weak* lower semicontinuous. Since the pointwise

supremum of a family of lower semicontinuous functions is also lower semicontinuous, the function $\xi \hookrightarrow \sup_{\pi\geq 0} L^{\eta,\lambda}(\xi,\pi)$ is lower semicontinuous in the weak* topology. Moreover, since $\mathcal{U}$ is weak* compact we conclude that its infimum is attained at a point $\bar{\xi} \in \mathcal{U}$.

Let us now prove that the supremum of $0 \leq \pi \hookrightarrow \inf_{\xi\in\mathcal{U}} L^{\eta,\lambda}(\xi,\pi)$ is attained at some $\bar{\pi} < \infty$. Therefore, recall that $\tilde{\xi}$ denotes the cleanest production schedule from (4.13). Moreover, assume momentarily that $\xi^* \in \mathcal{U}$ is a minimizer of $\xi \hookrightarrow G^{\pi,\lambda}(\xi)$ and $E^\lambda(\xi^*) > \eta$ and $\pi = \mathbb{E}[C(\tilde{\xi})]/(\eta - \tilde{a}(\lambda))$, then

$$\begin{aligned}\mathbb{E}[G^{\pi,\lambda}(\xi^*)] \geq \pi E^\lambda(\xi^*) > \pi\eta &= \mathbb{E}[C(\tilde{\xi})] + \pi\tilde{a}(\lambda) \\ &= \mathbb{E}[C(\tilde{\xi})] + \pi\mathbb{E}[\Delta + \Pi(\tilde{\xi}) - \lambda] = \mathbb{E}[G^{\pi,\lambda}(\tilde{\xi})],\end{aligned}$$

which contradicts the optimality of ξ^*, and proves that for any optimal ξ^* it holds that

$$E^\lambda(\xi^*) \leq \eta \quad \text{if} \quad \eta = \frac{\mathbb{E}[C(\tilde{\xi})]}{\pi} + \tilde{a}(\lambda).$$

Hence for all $\pi \geq 0$ it holds that

$$\begin{aligned}\inf_{\xi\in\mathcal{U}} L^{\eta,\lambda}(\xi,\pi) &\leq \sup_{\xi\in\mathcal{U}} \mathbb{E}[C(\xi)] + \pi\big(\tilde{a}(\lambda) + \frac{C(\tilde{\xi})}{\pi} - \eta\big) \\ &= \sup_{\xi\in\mathcal{U}} \mathbb{E}[C(\xi)] + \pi(\tilde{a}(\lambda) - \eta) + C(\tilde{\xi}).\end{aligned}$$

Analog to above argumentation the supremum is attained and it follows that

$$\lim_{\pi\to\infty} \inf_{\xi\in\mathcal{U}} L^{\eta,\lambda}(\xi,\pi) = -\infty$$

whereas Proposition 3 implies that

$$\inf_{\xi\in\mathcal{U}} L^{\eta,\lambda}(\xi,\pi) > -\infty$$

for all $0 \leq \pi < \infty$, which proves that there exists a constant $b(\eta,\lambda) \in \mathbb{R}$ such that

$$\sup_{\pi\geq 0} \inf_{\xi\in\mathcal{U}} L^{\eta,\lambda}(\xi,\pi) = \sup_{0\leq\pi\leq b(\eta,\lambda)} \inf_{\xi\in\mathcal{U}} L^{\eta,\lambda}(\xi,\pi).$$

The function $\pi \hookrightarrow L^{\eta,\lambda}(\xi,\pi)$ is affine for each $\xi \in \mathcal{U}$, hence it's infimum over $\xi \in \mathcal{U}$ is an upper semicontinuous function for which the supremum over the compact set $[0, b(\eta,\lambda)]$ is attained at some $\bar{\pi} \in [0,\infty)$.

(ii) With these choices of $\overline{\xi}$ and $\overline{\pi}$ it holds that

$$\sup_{\pi\geq 0}\inf_{\xi\in\mathcal{U}} L^{\eta,\lambda}(\xi,\pi) = \inf_{\xi\in\mathcal{U}} L^{\eta,\lambda}(\xi,\overline{\pi}) \leq L^{\eta,\lambda}(\overline{\xi},\overline{\pi}) \\ \leq \sup_{\pi\geq 0} L^{\eta,\lambda}(\overline{\xi},\pi) = \inf_{\xi\in\mathcal{U}}\sup_{\pi\geq 0} L^{\eta,\lambda}(\xi,\pi).$$

Since moreover $\mathcal{U}$ is convex and weak* compact and $\xi \hookrightarrow L^{\eta,\lambda}(\xi,\pi)$ is convex and weak* lower semicontinuous for all $\pi \geq 0$ while $\pi \hookrightarrow L^{\eta,\lambda}(\xi,\pi)$ is concave for all $\xi \in \mathcal{U}$ it holds due to Theorem 2.10.2 in Zalinescu (2002) that

$$\sup_{\pi\geq 0}\inf_{\xi\in\mathcal{U}} L^{\eta,\lambda}(\xi,\pi) = \inf_{\xi\in\mathcal{U}}\sup_{\pi\geq 0} L^{\eta,\lambda}(\xi,\pi).$$

Hence we conclude that

$$\inf_{\xi\in\mathcal{U}} L^{\eta,\lambda}(\xi,\overline{\pi}) = L^{\eta,\lambda}(\overline{\xi},\overline{\pi}) = \sup_{\pi\geq 0} L^{\eta,\lambda}(\overline{\xi},\pi)$$

and in particular $(\overline{\xi},\overline{\pi})$ is a saddle point of the Lagrangian $L^{\eta,\lambda}$ since

$$L^{\eta,\lambda}(\xi,\overline{\pi}) \geq L^{\eta,\lambda}(\overline{\xi},\overline{\pi}) \geq L^{\eta,\lambda}(\overline{\xi},\pi) \quad \text{for all } \xi\in\mathcal{U},\ \pi\geq 0.$$

(iii) The saddle point property implies that for all $\xi \in \mathcal{U}$ it holds that

$$\mathbb{E}[G^{\overline{\pi},\lambda}(\xi)] - \overline{\pi}\eta \geq \mathbb{E}[G^{\overline{\pi},\lambda}(\overline{\xi})] - \overline{\pi}\eta$$

or equivalently

$$\mathbb{E}[G^{\overline{\pi},\lambda}(\xi)] \geq \mathbb{E}[G^{\overline{\pi},\lambda}(\overline{\xi})]$$

proving that $\overline{\xi}$ is a solution of the global optimal control problem (2.43) with cap $\Lambda = \lambda$ and hence an equilibrium strategy.

Now let us prove that $\overline{\xi}$ is also a solution to (4.14). Clearly for each $\xi \in \mathcal{U}$ we have:

$$\sup_{\pi\geq 0} L^{\eta,\lambda}(\xi,\pi) = \begin{cases} \mathbb{E}[C(\xi)] & \text{if } E^\lambda(\xi)\leq\eta \\ \infty & \text{if } E^\lambda(\xi)>\eta \end{cases}.$$

so that problem (4.14) rewrites

$$\inf_{\xi\in\mathcal{U},\, E^\lambda(\xi)\leq\eta} \mathbb{E}[C(\xi)] = \inf\Big\{\sup_{\pi\geq 0} L^{\eta,\lambda}(\xi,\pi)\Big|\xi\in\mathcal{U}\Big\}.$$

□

From this Proposition it is straight forward to conclude that standard cap-and-trade schemes are socially optimal:

Corollary 4 *Under Assumptions 1-4 the standard cap-and-trade scheme is socially optimal.*

Proof. For every $\lambda > 0$ and $\eta > \tilde{a}(\lambda)$ Proposition 6 shows the existence of a penalty $\overline{\pi}$ and cap $\Lambda = \lambda$ such that there exists a strategy $\overline{\xi}$ which is a solution of the global optimal control problem (2.43) and (4.14). If Assumptions 1-4 are fulfilled Theorem 1 implies that $\overline{\xi}$ is also an equilibrium strategy, which concludes the proof. □

4.3. Equivalence of Standard and Hybrid Schemes

In this section we show the equivalence of standard cap-and-trade schemes with hybrid cap-and-trade schemes, proving that every equilibrium production strategy in the standard cap-and-trade scheme is also an equilibrium production strategy for the hybrid cap-and-trade scheme and vice versa if penalty and cap are the same. Therefore, we assume that the allocation in the standard scheme is only given at time $t = 0$, i.e. $\Lambda_t^i = 0$ for all $t = 1, \ldots, T-1$, and that the caps in both schemes are identical, i.e. $\tilde{\Lambda} = \Upsilon + \sum_{i \in I} \tilde{\Lambda}_0^i = \sum_{i \in I} \Lambda_0^i = \Lambda$. The main result of this chapter is:

Theorem 2 *Under Assumptions 1, 3 and 5 the following holds. If (A^*, S^*) is an equilibrium with production strategies ξ^* for the standard scheme then the prices (A^*, S'^*, P^*) where*

$$S_t'^{*k} = S_t^{*k} - y^k \mathbb{E}[A_\tau^* \mathbf{1}_{\{Y(D) < \Upsilon\}} | \mathcal{F}_t] \quad k \in K, t = 0, \ldots, \tau - 1 \tag{4.20}$$

$$S_t'^{*k} = S_t^{*k} \quad k \in K, t = \tau, \ldots, T-1 \tag{4.21}$$

$$P^* = A_\tau^* \tag{4.22}$$

define an equilibrium for the cap-and-trade scheme with the hybrid allocation having the same production strategies ξ^*. The converse statement is true as well.

Theorem 2 implies that the results of Theorem 1 and Corollary 4 can be transferred to the hybrid scheme:

Corollary 5 *Under Assumptions 1-5 it holds that:*

(i) There exists a solution $\overline{\xi} \in \mathcal{U}$ *of the global optimal control problem (2.43) and the triple* $(\overline{A}, \overline{S}', \overline{P})$ *defined by*

$$\overline{A}_t = \pi \mathbb{P}_t[\Gamma + \Pi(\overline{\xi}) \geq 0]$$

for all $t = 0, \ldots, T$, $\overline{P} = \overline{A}_\tau$ *and*

$$\overline{S}_t'^k = \max_{i \in I,\, j \in J^{i,k}} (C_t^{i,j,k} + e^{i,j,k}\overline{A}_t) 1_{\{\overline{\xi}_t^{i,j,k} > 0\}} - y^k \mathbb{E}[\overline{A}_\tau \mathbf{1}_{\{Y(D) < \Upsilon\}} | \mathcal{F}_t] \mathbf{1}_{\{t < \tau\}}$$

for all $t = 0, \ldots, T-1$, $k \in K$ *form a market equilibrium of the hybrid cap-and-trade scheme. Moreover, the equilibrium allowance price process is almost surely unique, while the process* $\overline{S}'$ *is the smallest equilibrium price in the sense of Theorem 1.*

(ii) The hybrid cap-and-trade scheme is socially optimal.

Proof. (i) This follows directly from Theorem 1 and Theorem 2.

(ii) Corollary 4 implies that for every threshold $\lambda > 0$ and upper bound $\eta > \tilde{a}(\lambda)$ there exists an equilibrium of the standard scheme with corresponding equilibrium strategy $\overline{\xi}$ which is a solution of (4.14). Due to Theorem 2 there exists an equilibrium of the hybrid scheme with the same corresponding equilibrium strategy $\overline{\xi}$ proving the assertion. □

Notice that the conclusions of Theorem 2 are stronger than assertion (i) of Corollary 5. Namely Corollary 5 implicitly relates the lowest possible product prices $\overline{S}$ of the standard scheme with the lowest possible product prices $\overline{S}'$ of the hybrid scheme. On the other hand Theorem 2 relates *all* equilibrium prices S^* of the standard scheme to equilibrium prices S'^* of the hybrid scheme and vice versa. In the case study presented in Section 4.4, we will see that the hybrid scheme in contrast to a pure auction scheme can reduce windfall profits to zero in average, while at the same time it preserves the social optimality of the standard scheme. But let us first prove Theorem 2.

4.3.1. Proof of Theorem 2

In this section we prove Theorem 2. To this end we show that the existence of equilibrium strategies (θ^*, ξ^*) for the standard scheme with

prices (A^*, S^*) guarantee existence of equilibrium strategies $(\theta'^*, \xi^*, \varphi^*)$ for the hybrid scheme with prices (A^*, S'^*, P^*) and vice versa. In fact we can prove this without knowing the specific form of the equilibrium strategies.

The rationale is as follows: If there exist strategies (θ^*, ξ^*) that are feasible and fulfill conditions (i)-(iii) of Definition 1 under (A^*, S^*) then the strategies $(\theta'^*, \xi^*, \varphi^*)$ related to (θ^*, ξ^*) by

$$\begin{aligned}
\varphi^{*i} &= \Upsilon(\xi^*)/|I| \\
\theta'^{*i}_T &= \theta^{*i}_T + \Lambda^i_0 - \tilde{\Lambda}^i_0 - \Gamma(\xi^{*-i}) \wedge Y^i(\xi^{*i}) - \varphi^{*i} \\
\theta'^*_t &= \theta^*_t \text{ for all } t = 0, \ldots, T-1
\end{aligned}$$

are feasible and fulfill conditions (i)-(iv) of Definition 2 under (A^*, S'^*, P^*) and vice versa.

Feasibility of above strategies is based on the following Lemma:

Lemma 5 *Let A an integrable allowance price process and consider an agent $i \in I$ with auction strategy φ^i such that $\varphi^i A_\tau$ is integrable. Moreover, fix $\hat{\xi} \in \mathcal{U}$ as well as $\xi^i \in \mathcal{U}^i$ and assume that the trading strategies θ^i and θ'^i are related by*

$$\theta^i_t = \theta'^i_t \text{ for all } t = 0, \ldots, T-1 \tag{4.23}$$

and

$$\Lambda^i_0 + \theta^i_T = \tilde{\Lambda}^i_0 + \varphi^i + \theta'^i_T + \Gamma(\hat{\xi}^{-i}) \wedge Y^i(\xi^i) \tag{4.24}$$

almost surely. Then, $(\theta^i, \xi^i) \in \mathcal{Q}^i(A)$ if and only if $(\theta'^i, \xi^i, \varphi^i) \in \tilde{\mathcal{Q}}^{\hat{\xi}^{-i}, i}(A, A_\tau)$.

Proof. Since θ^i and θ'^i differ only at time $t = T$, the equivalence between the integrability of $R^A_T(\theta^i)$ and $R^A_T(\theta'^i)$ relies on the equivalence between the integrability of $\theta^i_T A_T$ and $\theta'^i_T A_T$. But this is clear from (4.24) because $\varphi^i A_\tau$ is integrable and $\Gamma(\hat{\xi}^{-i}) \wedge Y^i(\xi^i)$ is bounded. Considering now the inequality constraint, if we assume that

$$\theta^i_T \geq -\Lambda^i_0,$$

then both the left and right hand side of (4.24) are positive, which implies that

$$\theta'^i_T \geq -\left(\tilde{\Lambda}^i_0 + \Gamma(\hat{\xi}^{-i}) \wedge Y^i(\xi^i) + \varphi^i\right).$$

The converse statement is proven analogously. □

The following Lemma shows that if (θ^*, ξ^*) fulfill conditions (i) and (ii) of Definition 1 then it follows that $(\theta^*, \xi^*, \varphi^*)$ fulfill conditions (i), (ii) and (iii) of Definition 2 and vice versa.

Lemma 6 *Let (A, S) be integrable allowance and product price processes. Moreover, suppose that for all agents $i \in I$ the strategies $(\theta^i, \xi^i)_{i\in I} \in \mathcal{Q}(A)$ and $(\theta'^i, \xi^i, \varphi^i)_{i\in I} \in \tilde{\mathcal{Q}}(A, A_\tau)$ satisfy conditions (4.23), (4.24) and*

$$\sum_{i\in I} \varphi^i = \Upsilon(\xi) \tag{4.25}$$

almost surely. Then it holds that $(\theta^i, \xi^i)_{i\in I}$ fulfill conditions (i) and (ii) of Definition 1 if and only if $(\theta'^i, \xi^i, \varphi^i)_{i\in I}$ fulfill conditions (i) and (iii) of Definition 2.

Proof. Condition (4.25) implies that

$$\sum_{i\in I} \varphi^i = \Upsilon - \sum_{i\in I} (\Gamma(\xi^{-i}) \wedge Y^i(\xi^i)).$$

Summing the left and right hand side of (4.24) this yields

$$\begin{aligned}\sum_{i\in I} \Lambda_0^i + \sum_{i\in I} \theta_T^i = &\sum_{i\in I} \tilde{\Lambda}_0^i + \sum_{i\in I} \Gamma(\xi^{-i}) \wedge Y^i(\xi^i) \\ &+ \Upsilon - \sum_{i\in I} \Gamma(\xi^{-i}) \wedge Y^i(\xi^i) + \sum_{i\in I} \theta_T'^i,\end{aligned}$$

while (4.1) implies that

$$\sum_{i\in I} \theta_T^i = \sum_{i\in I} \theta_T'^i.$$

Hence if $(\theta^i, \xi^i)_{i\in I}$ satisfy condition (i) of Definition 1 then $(\theta'^i, \xi^i, \varphi^i)_{i\in I}$ fulfill conditions (i) of Definition 2 and vice versa. If $(\theta^i, \xi^i)_{i\in I}$ satisfy condition (ii) of Definition 1 then $(\theta'^i, \xi^i, \varphi^i)_{i\in I}$ fulfill conditions (iii) of Definition 2 and vice versa. □

Until now we have shown that all equilibrium conditions but the individual optimality conditions (condition (iii) of Definition 1 and condition

(iv) of Definition 2) are fulfilled. It remains to prove following assertion: If (ξ^*, θ^*) fulfill

$$\mathbb{E}[L^{A^*,S^*,i}(\theta^{*i}, \xi^{*i})] - \mathbb{E}[L^{A^*,S^*,i}(\theta^i, \xi^i)] \geq 0 \tag{4.26}$$

for all $(\theta^i, \xi^i) \in \mathcal{Q}^i(A^*)$ then $(\theta'^*, \xi^*, \varphi^*)$ fulfill

$$\mathbb{E}[H^{A^*,S'^*,P^*,\xi^{*-i},i}(\theta'^{*i}, \xi^{*i}, \varphi^{*i})] - \mathbb{E}[H^{A^*,S'^*,P^*,\xi^{*-i},i}(\theta^i, \xi^i, \varphi^i)] \geq 0 \tag{4.27}$$

for all $(\theta^i, \xi^i, \varphi^i) \in \tilde{\mathcal{Q}}^i(A^*, A^*)$ and vice versa.

A simple concavity argument gives us lower bounds for which it is easier to prove non-negativity than for (4.26) and (4.27). A family of lower bounds is given in the following Lemma:

Lemma 7 *Let (A, S) be integrable price processes and P an integrable auction price. Moreover, fix an agent $i \in I$ and suppose that the strategies $\xi^{*i}, \xi^i \in \mathcal{U}^i$ and θ^{*i}, θ^i, φ^{*i}, φ^i are such that $\varphi^{*i}P$, $\varphi^i P$, $R_T^A(\theta^{*i})$ and $R_T^A(\theta^i)$ are integrable and consider the following linear combinations*

$$\begin{aligned} \xi^i(\mu) &:= \xi^{*i} + \mu(\xi^i - \xi^{*i}) \\ \theta^i(\mu) &:= \theta^{*i} + \mu(\theta^i - \theta^{*i}) \\ \varphi^i(\mu) &:= \varphi^{*i} + \mu(\varphi^i - \varphi^{*i}) \end{aligned}$$

for $0 < \mu \leq 1$. If in addition Assumption 3 is satisfied, then it holds that

$$\begin{aligned} &\mathbb{E}\Big[L^{A,S,i}(\theta^{*i}, \xi^{*i})\Big] - \mathbb{E}\Big[L^{A,S,i}(\theta^i, \xi^i)\Big] \\ &\qquad \geq \frac{1}{\mu}\mathbb{E}\Big[L^{A,S,i}(\theta^{*i}, \xi^{*i}) - L^{A,S,i}(\theta^i(\mu), \xi^i(\mu))\Big] \end{aligned}$$

in case of the standard scheme as well as

$$\begin{aligned} &\mathbb{E}\Big[H^{A,S,P,\xi^{*-i},i}(\theta^{*i}, \xi^{*i}, \varphi^{*i})\Big] - \mathbb{E}\Big[H^{A,S,P,\xi^{*-i},i}(\theta^i, \xi^i, \varphi^i)\Big] \\ &\quad \geq \frac{1}{\mu}\mathbb{E}\Big[H^{A,S,P,\xi^{*-i},i}(\theta^{*i}, \xi^{*i}, \varphi^{*i}) - H^{A,S,P,\xi^{*-i},i}(\theta^i(\mu), \xi^i(\mu), \varphi^i(\mu))\Big] \end{aligned}$$

in case of the hybrid scheme.

Proof. From the concavity of L we conclude for all $0 < \mu \leq 1$ that

$$\mathbb{E}[L^{A,S,i}(\theta^i(\mu), \xi^i(\mu))] \geq (1-\mu)\mathbb{E}[L^{A,S,i}(\theta^{*i}, \xi^{*i})] + \mu\mathbb{E}[L^{A,S,i}(\theta^i, \xi^i)]$$

which translates to

$$\begin{aligned}\mathbb{E}[L^{A,S,i}(\theta^i(\mu),\xi^i(\mu))] - \mathbb{E}[L^{A,S,i}(\theta^{*i},\xi^{*i})] \\ \geq \mu\big(\mathbb{E}[L^{A,S,i}(\theta^i,\xi^i)] - \mathbb{E}[L^{A,S,i}(\theta^{*i},\xi^{*i})]\big)\end{aligned}$$

and proves the assertion. A similar argument holds for the hybrid scheme. □

For all agents $i \in I$ following Lemma helps us to express the lower bounds from Lemma 7 for the standard/hybrid scheme in terms of the objective function of the hybrid/standard scheme and a function $\mathcal{U}^i \ni \xi^i \mapsto K^{A,\xi^{-i},i}(\xi^i)$ defined as

$$K^{A,\xi^{-i},i}(\xi^i) = -Y^i(\xi^i)A_\tau \mathbf{1}_{\{Y(D)<\Upsilon\}} + \Big(Y^i(\xi^i) \wedge \Gamma(\xi^{-i})\Big)A_\tau$$

for all processes A and ξ^{-i}. For small perturbations of equilibrium strategies this gives a simple relation between the objective functions in the standard and the hybrid schemes since Lemma 9 shows that the terms originating from K vanish as μ converges to zero.

Lemma 8 *Let A, S, S' be integrable price processes such that*

$$S_t'^k = S_t^k - y^k \mathbb{E}\big[A_\tau \mathbf{1}_{\{Y(D)<\Upsilon\}} \big| \mathcal{F}_t\big] \mathbf{1}_{\{t \leq \tau - 1\}} \tag{4.28}$$

holds almost surely for all $k \in K$ and $t = 0, \ldots, T-1$. Moreover, let P be an integrable auction price and suppose that for all firms $i \in I$ the strategies $(\theta^i, \xi^i)_{i\in I} \in \mathcal{Q}(A)$ and $(\theta'^i, \xi^i, \varphi^i)_{i\in I} \in \tilde{\mathcal{Q}}(A,P)$ satisfy the conditions (4.23), (4.24) as well as $\varphi^i(P - A_\tau) = 0$ almost surely. Then it holds under Assumption 3 that

$$\mathbb{E}\Big[H^{A,S',P,\xi^{-i},i}(\theta'^i,\xi^i,\varphi^i)\Big] = \mathbb{E}\Big[L^{A,S,i}(\theta^i,\xi^i) - (\Lambda_0^i - \tilde{\Lambda}_0^i)A_T + K^{A,\xi^{-i},i}(\xi^i)\Big]$$

is satisfied for all the firms $i \in I$.

Proof. Due to (4.23) and (4.24) it holds that

$$
\begin{aligned}
&\mathbb{E}\Big[H^{A,S',P,\xi^{-i},i}(\theta'^i,\xi^i,\varphi^i)\Big] \\
&= \mathbb{E}\Big[\sum_{t=0}^{T-1}\sum_{(j,k)\in M_i}(S_t'^k - C_t^{i,j,k})\xi_t^{i,j,k} + \sum_{t=0}^{T-1}\theta_t'^i(A_{t+1}-A_t) - \theta_T'^i A_T - \varphi^i P \\
&\quad - \pi\Big(\Delta^i + \Pi^i(\xi^i) - \tilde{\Lambda}_0^i - \Gamma(\xi^{-i}) \wedge Y^i(\xi^i) - \varphi^i - \theta_T'^i\Big)^+\Big] \\
&= \mathbb{E}\Big[\sum_{t=0}^{T-1}\sum_{(j,k)\in M_i}(S_t^k - C_t^{i,j,k} - 1_{\{t\le\tau-1\}}y^k\mathbb{E}\left[A_\tau \mathbf{1}_{\{Y(D)<\Upsilon\}}|\mathcal{F}_t\right])\xi_t^{i,j,k} \\
&\quad + \sum_{t=0}^{T-1}\theta_t^i(A_{t+1}-A_t) - \theta_T^i A_T - (\Lambda_0^i - \tilde{\Lambda}_0^i)A_T - \varphi^i(P-A_\tau) \\
&\quad + \big(\Gamma(\xi^{-i}) \wedge Y^i(\xi^i)\big)A_\tau - \pi\left(\Delta^i + \Pi^i(\xi^i) - \Lambda_0^i - \theta_T^i\right)^+\Big].
\end{aligned}
$$

With (4.2) this reads

$$
\begin{aligned}
&\mathbb{E}\Big[\sum_{t=0}^{T-1}\sum_{(j,k)\in M_i}(S_t^k - C_t^{i,j,k})\xi_t^{i,j,k} + \sum_{t=0}^{T-1}\theta_t^i(A_{t+1}-A_t) - \theta_T^i A_T - (\Lambda_0^i - \tilde{\Lambda}_0^i)A_T \\
&\quad - \varphi^i(P-A_\tau) - Y^i(\xi^i)A_\tau\mathbf{1}_{\{Y(D)<\Upsilon\}} + \big(\Gamma(\xi^{-i}) \wedge Y^i(\xi^i)\big)A_\tau \\
&\quad - \pi\left(\Delta^i + \Pi^i(\xi^i) - \Lambda_0^i - \theta_T^i\right)^+\Big] \\
&= \mathbb{E}\Big[L^{A,S,i}(\theta^i,\xi^i) - (\Lambda_0^i - \tilde{\Lambda}_0^i)A_T + K^{A,\xi^{-i},i}(\xi^i)\Big]
\end{aligned}
$$

which concludes the proof. □

Using the results from Lemma 8 to rewrite the lower bounds from Lemma 7 the terms originating from K vanish as μ converges to zero. This is shown in the following Lemma.

Lemma 9 *Let $\xi^* \in \mathcal{U}$ and $\xi^i \in \mathcal{U}^i$ for some $i \in I$. Moreover, assume that Assumption 5 is fulfilled and define $\xi^i(\mu) := \xi^{*i} + \mu(\xi^i - \xi^{*i})$ for all $0 < \mu \leq 1$. Then taking the limit $\mu \searrow 0$ along a countable set $(0,1] \cap \mathbb{Q}$ it holds that*

$$\lim_{\mu \to 0} \frac{1}{\mu} \mathbb{E}\big[K^{A,\xi^{*-i},i}(\xi^{*i}) - K^{A,\xi^{*-i},i}(\xi^i(\mu))\big] = 0$$

for all integrable allowance price processes A.

Proof. On $\{Y(D) < \Upsilon\}$ it holds that $Y^i(\xi^{*i}) < \Gamma(\xi^{*-i})$ and hence

$$\lim_{\mu \to 0} \frac{1}{\mu} \Big(Y^i(\xi^{*i}) \wedge \Gamma(\xi^{*-i}) - Y^i(\xi^i(\mu)) \wedge \Gamma(\xi^{*-i}) \Big) = Y^i(\xi^{*i}) - Y^i(\xi^i).$$

On the other hand it holds on $\{Y(D) > \Upsilon\}$ that $Y^i(\xi^{*i}) > \Gamma(\xi^{*-i})$ and therefore

$$\lim_{\mu \to 0} \frac{1}{\mu} \Big(Y^i(\xi^{*i}) \wedge \Gamma(\xi^{*-i}) - Y^i(\xi^i(\mu)) \wedge \Gamma(\xi^{*-i}) \Big) = 0.$$

Since moreover Assumption 5 implies $\mathbb{P}[\{Y(D) = \Upsilon\}] = 0$ we conclude that

$$\begin{aligned} \lim_{\mu \to 0} \frac{1}{\mu} \Big(Y^i(\xi^{*i}) \wedge \Gamma(\xi^{*-i}) - Y^i(\xi^i(\mu)) \wedge \Gamma(\xi^{*-i}) \Big) & \\ = \big(Y^i(\xi^{*i}) - Y^i(\xi^i) \big) \mathbf{1}_{\{Y(D) < \Upsilon\}} & \\ = \frac{1}{\mu} \Big(Y^i(\xi^{*i}) - Y^i(\xi^i(\mu)) \Big) \mathbf{1}_{\{Y(D) < \Upsilon\}} & \end{aligned}$$

holds almost surely. Hence it follows that

$$\lim_{\mu \to 0} \frac{1}{\mu} \Big(K^{A,\xi^{*-i},i}(\xi^{*i}) - K^{A,\xi^{*-i},i}(\xi^i(\mu)) \Big) = 0$$

almost surely. Moreover, it holds by the triangle inequality for all $0 <$

$\mu \leq 1$ that

$$\begin{aligned}
&\frac{1}{\mu}\Big|K^{A,\xi^{*-i},i}(\xi^{*i}) - K^{A,\xi^{*-i},i}(\xi^{i}(\mu))\Big| \\
&\leq \frac{1}{\mu}\Big|\Big(Y^i(\xi^{*i}) - Y^i(\xi^i(\mu))\Big)A_\tau \mathbf{1}_{\{Y(D)<\Upsilon\}}\Big| \\
&\quad + \frac{1}{\mu}\Big|\Big(Y^i(\xi^{*i}) \wedge \Gamma(\xi^{*-i}) - Y^i(\xi^i(\mu)) \wedge \Gamma(\xi^{*-i})\Big)A_\tau\Big| \\
&\leq \frac{2}{\mu}\Big|\Big(Y^i(\xi^{*i}) - Y^i(\xi^i(\mu))\Big)A_\tau\Big| \\
&= \frac{2}{\mu}\Big|\sum_{k\in K}\sum_{j\in J^{i,k}}\sum_{t=0}^{\tau-1} y^k \mu(\xi_t^{*i,j,k} - \xi_t^{i,j,k})A_\tau\Big| \\
&\leq 2\Big|\sum_{k\in K}\sum_{j\in J^{i,k}}\sum_{t=0}^{\tau-1} y^k \kappa^{i,j,k} A_\tau\Big|
\end{aligned}$$

while A_τ is integrable by assumption. Hence the assertion follows from dominated convergence. □

We can now turn to the proof of Theorem 2.

Proof. Assume first that (A^*, S^*) is an equilibrium of the standard scheme with strategies $(\theta^*, \xi^*) \in \mathcal{Q}(A^*)$.

Based on these strategies we define optimal strategies $(\theta'^*, \xi^*, \varphi^*)$ for the hybrid scheme, where for all $i \in I$ the strategies φ^{*i} and θ'^{*i} are given by

$$\varphi^{*i} = \Upsilon(\xi^*)/|I| \tag{4.29}$$

$$\theta'^{*i}_T = \theta^{*i}_T + \Lambda^i_0 - \tilde{\Lambda}^i_0 - \Gamma(\xi^{*-i}) \wedge Y^i(\xi^{*i}) - \varphi^{*i} \tag{4.30}$$

and $\theta'^*_t = \theta^*_t$ for all $t = 0, \ldots, T-1$. Since $\varphi^{*i}A_\tau$ is integrable and φ^{*i} is almost surely nonnegative for all $i \in I$ we deduce from Lemma 5 that $(\theta'^{*i}, \xi^{*i}, \varphi^{*i}) \in \tilde{\mathcal{Q}}^{\xi^{*-i},i}(A^*, A^*_\tau)$. Further $(\varphi^{*i})_{i\in I}$ so defined fulfills condition (ii) of Definition 2. This together with Lemma 6 implies that $(\theta'^{*i}, \xi^{*i}, \varphi^{*i})_{i\in I}$ fulfill conditions (i) and (iii) of Definition 2. To show that (A^*, S'^*, P^*) is an equilibrium of the auction scheme it remains to prove that condition (iv) of Definition 2 is satisfied for any $(\theta'^i, \xi^i, \varphi^i) \in$

$\tilde{\mathcal{Q}}^{\xi^{*-i},i}(A^*, A^*_\tau)$. To this end we define linear combinations

$$\begin{aligned}\xi^i(\mu) &:= \xi^{*i} + \mu(\xi^i - \xi^{*i})\\ \theta'^i(\mu) &:= \theta^{*i} + \mu(\theta'^i - \theta^{*i})\\ \varphi^i(\mu) &:= \varphi^{*i} + \mu(\varphi^i - \varphi^{*i}).\end{aligned}$$

Based on these strategies we define also a trading strategy $\theta^i(\mu)$ for the standard scheme given by

$$\theta^i_T(\mu) := -\Lambda^i_0 + \tilde{\Lambda}^i_0 + \Gamma(\xi^{*-i}) \wedge Y^i(\xi^i(\mu)) + \varphi^i(\mu) + \theta'^i_T(\mu)$$

and $\theta^i_t(\mu) := \theta'^i_t(\mu)$ for all $t = 0, \ldots, T-1$.

Note that since θ^{*i}_T and θ'^i_T are feasible they satisfy

$$\begin{aligned}\theta'^i_T(\mu) &= (1-\mu)\theta^{*i}_T + \mu\theta'^i_T\\ &\geq -\tilde{\Lambda}^i_0 - (1-\mu)(\Gamma(\xi^{*-i}) \wedge Y^i(\xi^{*i}) + \varphi^{*i})\\ &\qquad - \mu(\Gamma(\xi^{*-i}) \wedge Y^i(\xi^i) + \varphi^i).\end{aligned}$$

Since moreover $\xi^i \hookrightarrow \Gamma(\xi^{*-i}) \wedge Y^i(\xi^i)$ is concave this implies for all $0 \leq \mu \leq 1$ that

$$\theta'^i_T(\mu) \geq -\tilde{\Lambda}^i_0 - \Gamma(\xi^{*-i}) \wedge Y^i(\xi^i(\mu)) - \varphi^i(\mu)$$

meaning that $(\theta'^i(\mu), \xi^i(\mu), \varphi^i(\mu)) \in \tilde{\mathcal{Q}}^{\xi^{*-i},i}(A^*, A^*_\tau)$. This together with Lemma 5 implies that $(\theta^i(\mu), \xi^i(\mu)) \in \mathcal{Q}^i(A^*)$. Knowing this we are ready to prove condition (iv) of Definition 2. From Lemmas 7 and 8 we derive that

$$\begin{aligned}&\mathbb{E}\Big[H^{A^*,S'^*,P^*,\xi^{*-i},i}(\theta'^{*i}, \xi^{*i}, \varphi^{*i})\Big] - \mathbb{E}\Big[H^{A^*,S'^*,P^*,\xi^{*-i},i}(\theta'^i, \xi^i, \varphi^i)\Big]\\ &\geq \frac{1}{\mu}\mathbb{E}\Big[H^{A^*,S'^*,P^*,\xi^{*-i},i}(\theta'^{*i}, \xi^{*i}, \varphi^{*i})\\ &\qquad\qquad - H^{A^*,S'^*,P^*,\xi^{*-i},i}(\theta'^i(\mu), \xi^i(\mu), \varphi^i(\mu))\Big]\\ &= \frac{1}{\mu}\mathbb{E}\Big[L^{A^*,S^*,i}(\theta^{*i}, \xi^{*i}) + K^{A^*,\xi^{*-i},i}(\xi^{*i})\\ &\qquad\qquad - L^{A^*,S^*,i}(\theta^i(\mu), \xi^i(\mu)) - K^{A^*,\xi^{*-i},i}(\xi^i(\mu))\Big]\\ &\geq \frac{1}{\mu}\mathbb{E}\Big[K^{A^*,\xi^{*-i},i}(\xi^{*i}) - K^{A^*,\xi^{*-i},i}(\xi^i(\mu))\Big] \qquad (4.31)\end{aligned}$$

for all $0 < \mu \leq 1$, where the last inequality holds due to the optimality of (θ^{*i}, ξ^{*i}) for the standard scheme. Hence we conclude from Lemma 9 that

$$\begin{aligned}
&\mathbb{E}\Big[H^{A^*,S'^*,P^*,\xi^{*-i},i}(\theta'^{*i}, \xi^{*i}, \varphi^{*i})\Big] - \mathbb{E}\Big[H^{A^*,S'^*,P^*,\xi^{*-i},i}(\theta'^{i}, \xi^{i}, \varphi^{i})\Big] \\
&\geq \lim_{\mu \to 0} \frac{1}{\mu} \mathbb{E}\Big[K^{A^*,\xi^{*-i},i}(\xi^{*i}) - K^{A^*,\xi^{*-i},i}(\xi^i(\mu))\Big] = 0 \qquad (4.32)
\end{aligned}$$

where we take the limit $\mu \searrow 0$ along a countable set $(0,1] \cap \mathbb{Q}$. This argumentation holds for any $(\theta'^i, \xi^i, \varphi^i) \in \tilde{\mathcal{Q}}^{\xi^{*-i},i}(A^*, A^*_\tau)$ proving condition (iv) of Definition 2.

To prove the converse statement assume that (A^*, S'^*, P^*) is an equilibrium of the hybrid scheme with corresponding strategies $(\theta'^{*i}, \xi^{*i}, \varphi^{*i}) \in \tilde{\mathcal{Q}}^{\xi^{*-i},i}(A^*, P^*)$. For condition (iii) of Definition 2 to be fulfilled it is necessary that $\varphi^{*i}(P^* - A^*_\tau) = 0$ almost surely for all agents $i \in I$.

Based on the optimal strategies in the hybrid scheme we define optimal strategies (θ^*, ξ^*) for the standard scheme where for all $i \in I$ the trading strategy θ^{*i} is given by

$$\theta^{*i}_T := \theta'^{*i}_T - \Lambda^i_0 + \tilde{\Lambda}^i_0 + \Gamma(\xi^{*-i}) \wedge Y^i(\xi^{*i}) + \varphi^{*i}$$

and $\theta^{*i}_t := \theta'^{*i}_t$ for all $t = 0, \ldots, T-1$. Since $\varphi^{*i}(P^* - A^*_\tau) = 0$ and $(\theta'^{*i}, \xi^{*i}, \varphi^{*i}) \in \tilde{\mathcal{Q}}^{\xi^{*-i},i}(A^*, P^*)$ it holds that $\varphi^{*i} A^*_\tau$ is integrable. Hence, Lemma 5 implies that $(\theta^*, \xi^*) \in \mathcal{Q}(A^*)$. Moreover, Lemma 6 implies that $(\theta^{*i}, \xi^{*i})_{i \in I}$ fulfill conditions (i) and (ii) of Definition 1.

It remains to prove that condition (iii) of the equilibrium definition for standard schemes is fulfilled. Therefore, we compare the strategy (θ^*, ξ^*) with all other strategies $(\theta^i, \xi^i) \in \mathcal{Q}^i(A^*)$ and $i \in I$ and define linear combinations by

$$\begin{aligned}
\xi^i(\mu) &:= \xi^{*i} + \mu(\xi^i - \xi^{*i}) \\
\theta^i(\mu) &:= \theta^{*i} + \mu(\theta^i - \theta^{*i}) \\
\theta'^i_T(\mu) &:= \Lambda^i_0 - \tilde{\Lambda}^i_0 - \Gamma(\xi^{*-i}) \wedge Y^i(\xi^i(\mu)) - \varphi^{*i} + \theta^i_T(\mu). \\
\theta'^i_t(\mu) &:= \theta^i_t(\mu) \quad \text{for all } t = 0, \ldots, T-1.
\end{aligned}$$

Again it holds that $(\theta^i(\mu), \xi^i(\mu)) \in \mathcal{Q}^i(A^*)$ and hence $(\theta'^i(\mu), \xi^i(\mu), \varphi^{*i}) \in \tilde{\mathcal{Q}}^{\xi^{*-i},i}(A^*, A^*_\tau)$ by Lemma 5. Moreover, it holds for any $(\theta^i, \xi^i) \in \mathcal{Q}^i(A^*)$

and $0 < \mu$ that

$$\begin{aligned}
&\mathbb{E}[L^{A^*,S^*,i}(\theta^{*i},\xi^{*i})] - \mathbb{E}[L^{A^*,S^*,i}(\theta^i,\xi^i)] \\
&\geq \frac{1}{\mu}\mathbb{E}[L^{A^*,S^*,i}(\theta^{*i},\xi^{*i}) - L^{A^*,S^*,i}(\theta^i(\mu),\xi^i(\mu))] \\
&= \frac{1}{\mu}\mathbb{E}[(H^{A^*,S'^*,P^*,\xi^{*-i},i}(\theta'^{*i},\xi^{*i},\varphi^{*i}) - K^{A^*,\xi^{*-i},i}(\xi^{*i})) \\
&\qquad - (H^{A^*,S'^*,P^*,\xi^{*-i},i}(\theta'^{i}(\mu),\xi^i(\mu),\varphi^{*i}) - K^{A^*,\xi^{*-i},i}(\xi^i(\mu)))] \\
&\geq \frac{1}{\mu}\mathbb{E}[K^{A^*,\xi^{*-i},i}(\xi^i(\mu)) - K^{A^*,\xi^{*-i},i}(\xi^{*i}))]
\end{aligned}$$

where the first and second equality are consequences of Lemma 7 and 8 respectively. The last inequality holds due to the optimality of $(\theta^{*i},\xi^{*i},\varphi^{*i})$ for the hybrid scheme. Hence taking the limit $\mu \searrow 0$ along a countable set $(0,1] \cap \mathbb{Q}$ we derive from Lemma 9 that

$$\begin{aligned}
&\mathbb{E}[L^{A^*,S^*,i}(\theta^{*i},\xi^{*i})] - \mathbb{E}[L^{A^*,S^*,i}(\theta^i,\xi^i)] \\
&\geq \lim_{\mu\to 0}\frac{1}{\mu}\mathbb{E}[K^{A^*,\xi^{*-i},i}(\xi^i(\mu)) - K^{A^*,\xi^{*-i},i}(\xi^{*i}))] = 0
\end{aligned}$$

holds for any $(\theta^i,\xi^i) \in \mathcal{Q}^i(A^*)$. This proves condition (iii) of Definition 1 and concludes the proof. □

4.4. Comparison of Standard, Auction and Hybrid Schemes

We illustrate our arguments comparing the three alternative reduction schemes discussed above in the case study of the Japanese electricity market with data expected for 2012.

4.4.1. Calibration of Parameters

The numerical results reported below are based on the assumption that the goal of the regulator is to reach a reduction target formulated in terms of expected excess emissions with $\lambda = 300$Mt and $\eta = 48$kt at lowest overall cost. This is reached for $\Lambda = \tilde{\Lambda} = 300$Mt and $\pi = 100$\$.

To be able to compare these results with the results from previous chapters we have chosen η such that the socially optimal parameters are the same parameters that we used in Chapters 2 and 3. I.e. these parameters give also an equilibrium with emissions that are less than or equal to the cap of 300Mt with probability of 95%.

Controls available to the regulator (depending upon the market design)

i) For the standard scheme, this is the size of the penalty and the total number of allocated allowances.
ii) For the standard scheme with auction, this is the size of the penalty, the total number of allocated allowances and the amount that is auctioned.
iii) For the hybrid cap-and-trade scheme introduced in this chapter, this is the size of the penalty, the total number of allowances allocated up front, the size of the pot and the factor of proportional allocation.

We choose the pot size and the proportional allocation factor according to subsection 4.4.3.

4.4.2. Costs

In this section we compare three different allocation mechanisms in the case study of the Japanese electricity market with data expected for 2012. The allocation mechanisms that we consider are first a free upfront allocation in the spirit of the first phase of the EU ETS, an upfront auctioning of the total initial allocation (as proposed for the electricity sector in the EU ETS after 2012) and a hybrid cap-and-trade scheme as proposed in this chapter. Notice that all three schemes possess the same production policies, reduction costs and emission processes. Hence we are particularly interested in how consumer costs and windfall profits relate to reduction costs and penalty payments under the different allocation mechanisms. To make this section self contained we summarize in the next two subsections the main findings of Section 3.3 for the costs of standard schemes both with and without auction.

4.4.2.1. Free upfront allocation

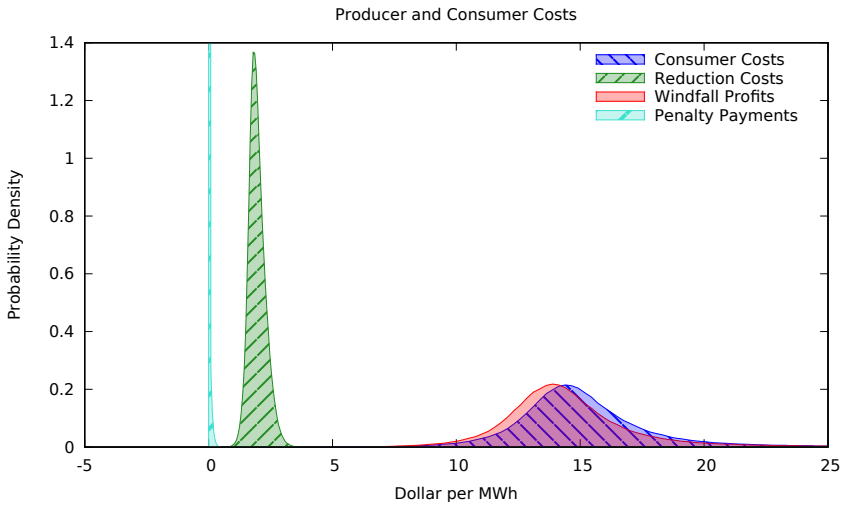

Fig. 4.1: Histograms of consumer costs, reduction costs, windfall profits and penalty payments of a standard cap-and-trade scheme calibrated to reach the emissions target with 95% probability.

Obviously, costs of production are higher in the presence of a cap-and-trade scheme. This is because, due to the emission constraints, producers switch to cleaner and more expensive technologies to avoid paying the penalty. However, from Figure 4.1 we conclude that for a 20% reduction target, average abatement costs are only 1.96$ per MWh of produced electricity! Though as observed in the EU ETS consumers costs (in average 15.13$ per MWh) exceed the overall reduction costs by far ($\times 8$ in the present case). This is the reason for the huge windfall profits, which were observed in the first phase of the EU ETS and have been the core of the main criticism of cap-and-trade schemes by consumer advocates.

4.4.2.2. Auction

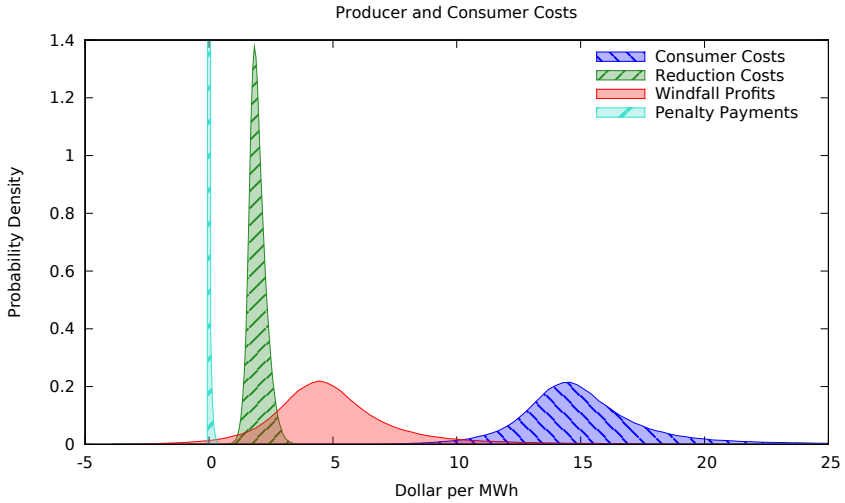

Fig. 4.2: Histograms of consumer costs, reduction costs, windfall profits and penalty payments of a standard cap-and-trade scheme where all allowances are auctioned at the beginning of the trading period.

The most frequent approach to reduce windfall profits is to replace free allocation of allowances through an auction. This is tempting because the income from the auction is an additional income for the state. This income can be used to return revenues to consumers, can be invested in other emission reduction projects or even used as a subsidy for the poorest income classes (as planned for the USA). However, what is commonly over looked is that consumer costs of the scheme can exceed the auction revenue by far.

As shown in Figure 4.2 for our Japanese case study, auctioning neither reduces consumer costs (in average 15.13$ per MWh) nor reduces windfall profits to a satisfactory level. These are on average still 5.1$ per MWh. Because the auction revenue corresponds to 9.5$ per MWh, consumer costs exceed by far the revenue from the auction. This amount can only cover about half of the consumer costs. Hence, the commonly believed argument that auction revenues can be used to cover costs

of end-consumers is wrong and there is still significant wealth transfer from consumer to producer.

Another drawback of auctioning that we already discussed is that a 100% auction leaves no free allocation crippling the regulatory mechanism to control incentives by benchmarking of the initial allocation.

4.4.2.3. Hybrid Cap-and-Trade Scheme

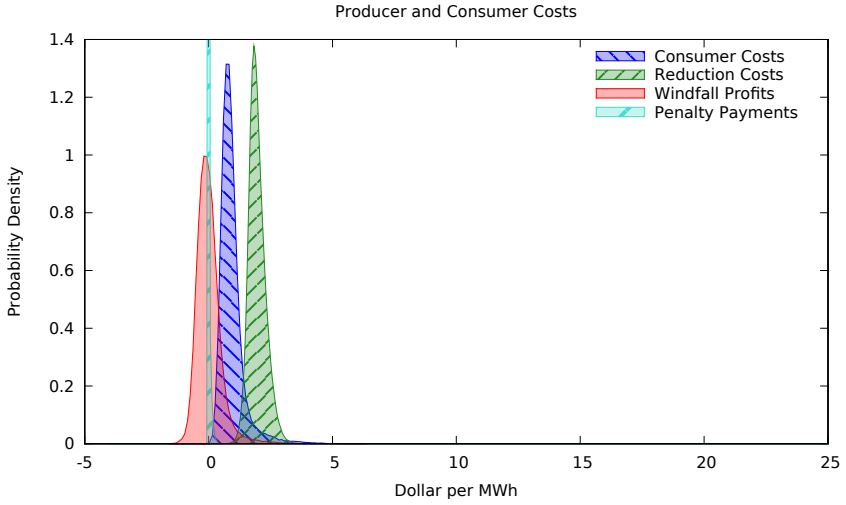

Fig. 4.3: Histograms of consumer cost, reduction cost, windfall profits and penalty payments in the case of a hybrid cap-and-trade scheme.

Because equilibrium electricity spot price is given by

$$\overline{S}_t'^k = \max_{i\in I,\, j\in J^{i,k}} (C_t^{i,j,k} + e^{i,j,k}\overline{A}_t) 1_{\{\bar{\xi}_t^{i,j,k}>0\}} - \mathbf{1}_{\{t\leq\tau-1\}} y^k \mathbb{E}[\overline{A}_\tau \mathbf{1}_{\{Y(D)<\Upsilon\}} | \mathcal{F}_t]$$

for all $t = 0, \ldots, T-1$ and $k \in K$. The hybrid scheme reduces windfall profits at its origin, the electricity price. Hence, the reduction of windfall profits is related to the product units consumed within 0 and $\tau - 1$ rather than the number of allowances in the market, as in the case of an auctioning of allowances. This results in a much more efficient reduction of windfall profits, where even a significant amount of allowances is

left over for free allocation and can be used to set incentives to develop a sustainable production portfolio.

In this case study we used relative allocation factor $y = 0.44$ and pot size $\Upsilon = 2.4 \times 10^8$, which leaves 20% of the allowances for a free upfront allocation. As in Section 3.1 the relative allocation is only given for electricity that was not produced by nuclear power, as nuclear power is never the marginal technology.

Figure 4.3 shows that in contrast to the pure auction scheme, the reduction in windfall profits comes with a reduction of consumer costs, which are 0.93$ per MWh in average, which implies that reduction costs (in average 1.96$ per MWh) are split nearly equally between consumers and producers. Windfall profits are on average also reduced to zero.

Moreover, notice that consumer costs are much less volatile than under an upfront allocation or an auction.

4.4.3. Calibration of Parameters of Pot Size and Relative Allocation Factor

As equilibrium production strategies of the hybrid cap-and-trade scheme and the standard scheme with upfront allocation are the same, we refer to subsection 4.4.1 for the calibration of penalty π and cap $\Lambda = \tilde{\Lambda}$. Here we focus on the pot size Υ and the relative allocation factors $(y^k)_{k \in K}$. For the Japan case study we consider two goods, electricity in the eastern and in the western part of the market, for which we use the same relative allocation factor, which we denote by y^e. In Figure 4.4 we plot the expected windfall profits for different parameters Υ and y^e.

Imagine for a moment that the pot size Υ is fixed. As long as the probability that the relative allocation exceeds the pot size is small, increasing the relative allocation factor results in a reduction of the electricity price and hence of the windfall profits. However, increasing the relative allocation factor increases the probability that the relative allocation will exceed the pot size. If the relative allocation is in the same range as the pot size, this effect becomes dominant and electricity price begins to increase until it reaches the price of the standard scheme.

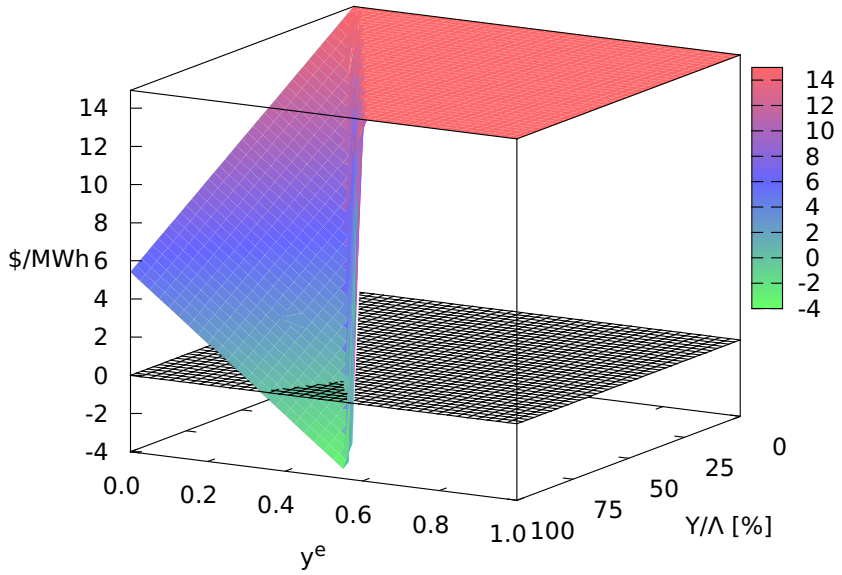

Fig. 4.4: Windfall profits for different parameters Υ and y^e. For $y^e = 0$ we recover a standard scheme where the total amount Υ is auctioned. The parameters $y^e = 0$ and $\Upsilon = 0$ correspond to a standard scheme with free upfront allocation. The black surface denotes the zero windfall profit level.

Hence for fixed pot size there is a y^e with minimum windfall profits. For pot size Υ above $2.4 \times 10^8 t_{CO_2}$ this minimum is even negative. Obviously there are different parameter y^e and Υ such that windfall profits are zero. These are found at the intersection of the windfall profit surface with the black surface giving the zero windfall profits level.

In Figure 4.5 we display the corresponding level sets. As argued above, the free initial allocation plays an important role when it comes to set incentives for clean technologies. Consequently, the parameters of the hybrid cap-and-trade scheme should be chosen such that the initial allocation is as large as possible, i.e. the size of the pot is as small as possible.

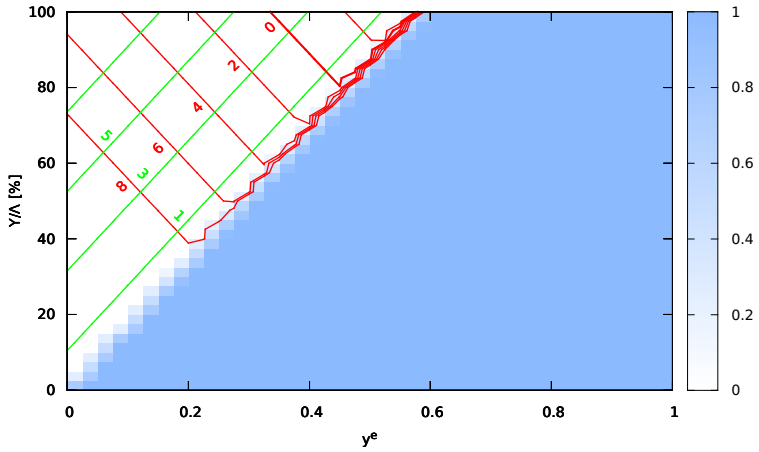

Fig. 4.5: The red lines show the level sets of windfall profits at a distance of 2$ per MWh. The green lines show the level sets of the auction revenue at the same distance. The blue background color indicates the levels of the probability for the pot to become empty at time τ.

For our Japanese case study we recover these parameters at $y^e = 0.44$ and $\Upsilon = 2.4 \times 10^8$ corresponding to the point on the level set of zero average windfall profits in Figure 4.5 where Υ is minimal. Also, we could choose the parameters to reach a certain level of windfall profits and a certain expected revenue from the auction (as indicated by the green level sets).

Figure 4.5 shows that there are several choices of parameters where the probability that the relative allocation exceeds the pot size is negligible. For these parameters we could define a different (and simpler) allowance distribution procedure when the pot becomes empty. The equilibrium equivalence with a standard cap-and-trade scheme would not hold any longer however, it is likely that production strategies would not change much. For example, a possible allocation would be

$$\sum_{t=0}^{\tau-1} \sum_{(j,k)\in J^{i,k}} y^k \xi_t^{i,j,k} \mathbf{1}_{\{\sum_{s=0}^{t} \sum_{(i,j,k)} y^k \xi_s^{i,j,k} < \Upsilon\}}.$$

Chapter 5

Numerical Procedures and Error Analysis

Chapter Overview

The global optimal control problem (2.43) cannot be solved exactly. The reason is the computational complexity coming from the continuous state space of the stochastic processes and the control. Hence we solve a discrete version of the optimal control problem by dynamic programming as shown in Section 5.1 and use this solution to construct a feasible (but not optimal) strategy $\xi^\dagger \in \mathcal{U}$ for the continuous state space.

As the control policy $\xi^\dagger$ is only an approximate solution it is crucial to be able to control the resulting errors. Therefore we show in Section 5.2 how to control the error based on duality theory by constructing upper and lower bounds for the distance of the optimal value $\mathbb{E}[G(\bar{\xi})]$ to the value $\mathbb{E}[G(\xi^\dagger)]$ of the approximate solution. These error bounds do not allow the error for a given discretization to be predicted without solving the corresponding optimal control problem. However, for any discretization we can construct a solution $\xi^\dagger$ and then compute the error. Consequently we can narrow the discretization until the required accuracy is reached.

As for most applications we use $\xi^\dagger$ to compute the allowance prices following Theorem 1 we are particularly interested in the error of this approximated allowance price. Hence in Section 5.3 we construct for every discretization upper and lower bounds for the allowance price at time zero.

Moreover, in Section 5.4 we show how the error bounds behave when the discretization is refined. Note that for this chapter we assume that Assumption 3 is fulfilled, this guarantees together with Proposition 3 the existence of an optimal solution to the global optimal control problem (2.43).

5.1. Numerical Implementation

In contrast to our Japanese case study, where we had two goods electricity in the Eastern and in the Western part of the market, we choose to describe the numerical procedure for one good only. Moreover, we

assume that electricity is produced only from two different technologies, gas and coal. Stochastic costs of production are equal to $C_t^{i,j,k} = H^j P_t^j$, where $j \in \{g, c\}$, H^j is the heat rate of technology j and P_t^j is the corresponding fuel price. D_t stands for the electricity demand from which nuclear capacity has already been subtracted. We set the emission rates to 0.42 tonne carbon dioxide per MWh for gas technology (typical CCGT-like plant) and 0.85 tonne carbon dioxide per MWh for coal technology respectively. These average emission rates have been chosen to give a faithful representation of Japan's park of power plants.

The global optimal control problem reads:

$$\inf_{\xi \in \mathcal{U}} \mathbb{E}\left[\sum_{t=0}^{T-1}(C_t^g \xi_t^g + C_t^c \xi_t^c) + \pi\left(\Gamma + \sum_{t=0}^{T-1}(e^g \xi_t^g + e^c \xi_t^c)\right)^+\right] \tag{5.1}$$

where Γ is defined in (2.41) as the negative net amount of allowances. In the particular case of two technologies, we define the maximum emission at each time point $t = 0, \ldots, T-1$ by

$$\breve{\mathcal{E}}_t = e^c \min(D_t, \kappa^c) + e^g (D_t - \kappa^c)^+$$

and proceed to the change of variable $(\xi_t^g, \xi_t^c) \longmapsto \mathcal{E}_t$, where

$$\mathcal{E}_t = \breve{\mathcal{E}}_t - (e^c \xi_t^c + e^g \xi_t^g)$$

is the emission reduction with respect to the maximum emission. Using the constraint that the demand has to be met, we obtain an equivalent formulation in terms of an emission abatement problem:

$$\inf_{0 \le \mathcal{E}_t \le \breve{\kappa}_t} \mathbb{E}\left[\sum_{t=0}^{T-1}(D_t(e^c F_t + C_t^c) - F_t(\breve{\mathcal{E}}_t - \mathcal{E}_t)) + \pi\left(\breve{\Gamma} - \sum_{t=0}^{T-1}\mathcal{E}_t\right)^+\right] \tag{5.2}$$

where:

$$\begin{aligned}
\breve{\kappa}_t &:= \breve{\mathcal{E}}_t - (e^g \min(D_t, \kappa^g) + e^c (D_t - \kappa^g)^+) \\
\breve{\Gamma} &:= \Gamma + \sum_{t=0}^{T-1} \breve{\mathcal{E}}_t \\
F_t &:= \frac{C_t^g - C_t^c}{e^c - e^g}.
\end{aligned}$$

are respectively the greatest possible emission reduction at $t = 0, \ldots, T-1$, maximum net emissions up to time T and the abatement cost per tonne of carbon dioxide. The stochastic process F is also called fuel spread and represents the marginal switching cost necessary to decrease emissions by 1 unit. Notice that the fuel spread can also become negative, in which case the emission abatement is carried out even in a Business as Usual Scenario. The stochastic control problem (5.2) can up to a constant be expressed in terms of reduction costs and reads

$$\inf_{0 \leq \mathcal{E}_t \leq \breve{\kappa}_t} \mathbb{E}\left[\sum_{t=0}^{T-1} F_t \mathcal{E}_t + \pi\left(\breve{\Gamma} - \sum_{t=0}^{T-1} \mathcal{E}_t\right)^+\right]. \tag{5.3}$$

Notice that for constant $\breve{\kappa}_t$ this is the same problem as stated in Carmona et al. (2009a).

5.1.1. Dynamic Programming and Simulation

To construct an approximate control policy we start by solving a discrete version of the optimal control problem by backward induction on a trinomial forest, which in the case of our Japan case study has 1.76 billion variables (for an exact description of the tree building procedure we refer to Fehr (2005)). Figure 5.1 graphically illustrates this technique.

Each tree corresponds to a certain emission level, whereas branches of each tree describe the dynamics of the exogenous processes. At each node of each tree, we see a splitting into three vertical and three horizontal directions, giving nine branches in all. The vertical direction describes the movement of the fuel switch price, whereas the horizontal branches model the demand dynamics. As we are only interested in an approximate solution we neglect the random variable Δ in the discretization. At the end T of the compliance period, paths finish either at positive or non-positive excess emissions giving either π or 0 allowance price.

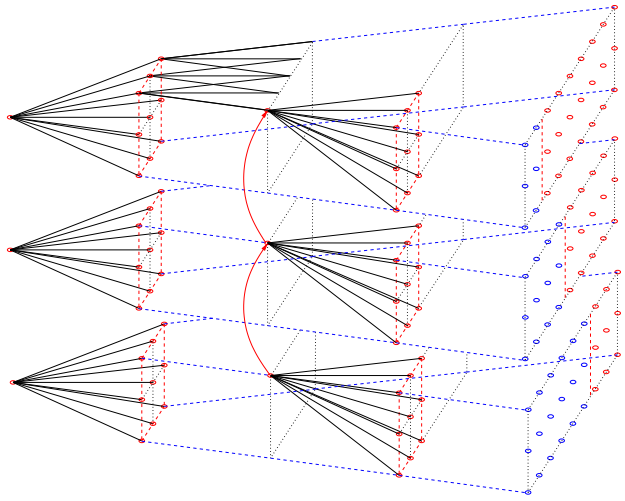

Fig. 5.1: Backward induction optimal control schematic, as applied to the trinomial tree discretization to each component of the processes $(F(t), D(t))_{t=0}^{T-1}$.

The optimally controlled fuel switch process is calculated by backward induction: Starting at time $t = T - 1$ at each node, the expected allowance price is computed based on the allowance price of subsequent nodes at time $t + 1$. Only if the expected allowance price exceeds the fuel switch price a fuel switch is performed. Depending on the resulting strategy a corresponding emission level is realized indicated by the move to higher (as emissions increase over time) trees in the forest diagram. This procedure gives us the optimal abatement (or production) policy and in particular allowance prices at discrete nodes.

By interpolating these allowance prices linearly between nodes and discrete emission levels at each time point t we obtain an allowance price function $\hat{\mathcal{A}}_t : \mathbb{R}^3 \to \mathbb{R}$. An approximate solution $\mathcal{E}^\dagger$ of the stochastic control problem 5.3 is constructed by a recursion started at $t = 0$ and $E_{-1} = 0$ given by

$$\mathcal{E}_t^\dagger = \max_{0 \le q \le \breve{\kappa}_t, q \in \mathbb{Q}} (\breve{\kappa}_t - q) \mathbf{1}_{\{\hat{\mathcal{A}}_t(F_t, D_t, E_{t-1} + \breve{\mathcal{E}}_t - (\breve{\kappa}_t - q)) \ge F_t\}} \tag{5.4}$$

and $E_t = E_{t-1} + \breve{\mathcal{E}}_t - \mathcal{E}_t^\dagger$ where E_t denotes the emissions of the fuel switch strategy up to time t. This means that if $\hat{\mathcal{A}}_t(F_t, D_t, E_{t-1} + \breve{\mathcal{E}}_t - \breve{\kappa}_t) \ge F_t$

then the optimal fuel switch intensity is $\breve{\kappa}_t$ whereas if $\hat{\mathcal{A}}_t(F_t, D_t, E_{t-1} + \breve{\mathcal{E}}_t) < F_t$ then the optimal fuel switch intensity is 0. However, if none of the above hold then the algorithm searches for a fuel switch level $\breve{\kappa}_t - q$, for which the allowance price $\hat{\mathcal{A}}_t(F_t, D_t, E_{t-1} + \breve{\mathcal{E}}_t - (\breve{\kappa}_t - q))$ comes close to the fuel switch price F_t. This mimics the optimal solution, which only runs at intermediate fuel switches in the case that allowance price equals fuel switch price.

It follows by induction started at $t = 0$ that E_t and $\mathcal{E}_t^\dagger$ are $\mathcal{F}_t$-measurable for all $t = 0, \ldots, T-1$. To perform the inductive step we assume that E_{t-1} is $\mathcal{F}_t$-measurable. Since by construction $\hat{A}_t : \mathbb{R}^3 \to \mathbb{R}$ is continuous and F_t, D_t, $\breve{\kappa}_t$ as well as $\breve{\mathcal{E}}_t$ are $\mathcal{F}_t$-measurable it follows that $(\breve{\kappa}_t - q)\mathbf{1}_{\{\hat{\mathcal{A}}_t(F_t, D_t, E_{t-1} + \breve{\mathcal{E}}_t - (\breve{\kappa}_t - q)) \geq F_t\}}$ is $\mathcal{F}_t$-measurable for each q. Hence we conclude that $\mathcal{E}_t^\dagger$ and hence also E_t are $\mathcal{F}_t$-measurable as the maximum in (5.4) is taken over a countable family of $\mathcal{F}_t$-measurable random variables. Since moreover by construction it holds that $0 \leq \mathcal{E}_t^\dagger \leq \breve{\kappa}_t$ we conclude that $\mathcal{E}^\dagger$ is a feasible strategy. For each path $(F_t, D_t)_{t=0}^{T-1}(\omega)$ above recursion defines a unique abatement path $\mathcal{E}^\dagger(\omega)$ and the Monte Carlo simulation of abatement policies reduces to the generation of Monte Carlo samples of $(F_t, D_t)_{t=0}^{T-1}$. Notice however that even though the simulation can be done pathwise the optimization problem can not be solved pathwise. The policies must be adapted.

Moreover, based on the simulation of production strategies we obtain allowance prices by $A_t^\dagger = (\mathbb{E}[\pi 1_{\{\breve{\Gamma} + \sum_{t=0}^{T-1} \mathcal{E}_t^\dagger\}} | \mathcal{F}_t])_{t=0}^{T-1}$ (Notice that these include also Δ). In practice, the expected value is computed as the average of Monte Carlo samples of abatement strategies. This is the method that we consider later in this chapter. However, in Chapters 2, 3 and 4 we use $\hat{\mathcal{A}}_t(F_t, D_t, E_{t-1} + \breve{\mathcal{E}}_t - \mathcal{E}_t^\dagger)$ as allowance price, which is computationally faster, but does not necessarily give a martingale. Simulation of electricity prices is straightforward and follows from Theorem 1.

5.2. Error Bounds for the Global Optimal Control Problem

In the preceding section we constructed an approximate abatement strategy $\mathcal{E}^\dagger$. Based on this strategy we can construct an approximate production strategy $\xi^\dagger$ for the global optimal control problem (5.1). In this section we are concerned with the error of the strategy $\xi^\dagger$ with respect to the optimal control problems (2.43) and (5.1). As any feasible solution gives us an upper bound of the optimal value when plugged into the primal problem this chapter is merely concerned with the construction of the lower bound, namely the dual problem. As a consequence particular focus is given to the proof of weak and strong duality.

5.2.1. Primal and Dual Problems

Under Assumption 3 the global optimal control problem (2.43) is well defined and we rephrase it as

$$
\begin{aligned}
\inf \quad & \mathbb{E}\Big[\sum_{t=0}^{T-1}\sum_{(i,j,k)\in M} C_t^{i,j,k}\xi_t^{i,j,k} + \pi\zeta\Big] \\
\text{s.t.} \quad & \xi_t^{i,j,k} \le \kappa^{i,j,k} \text{ a.s. for all } (i,j,k)\in M, t=0,\dots,T-1 \\
& \sum_{(i,j,k)\in M}\sum_{t=0}^{T-1} e^{i,j,k}\xi_t^{i,j,k} - \zeta \le -\Gamma \text{ a.s.} \\
& \sum_{i\in I}\sum_{j\in J^{i,k}} \xi_t^{i,j,k} = D_t^k \text{ a.s. for all } k\in K, t=0,\dots,T-1 \\
& \zeta \ge 0\,,\ \xi_t^{i,j,k} \ge 0 \text{ a.s. for all } (i,j,k)\in M, t=0,\dots,T-1 \\
& \zeta \in L^1(\mathcal{F}_T), \xi \in \mathcal{L}^\infty_{T-1}(\mathbb{R}^{|M|}),
\end{aligned}
$$

which we call the (primal) central planer problem (P). Moreover, we denote by (P*) its optimal value.

As in Carmona and Fehr (2009b) we could derive the dual problem using LP theory for general vector spaces. Since we have a concrete primal problem we will rather state the dual problem and prove that

it fulfills weak as well as strong duality. Following this approach requires only straightforward arguments omitting the lengthy formalism required to apply standard results. For general results on linear optimization in infinite dimensional spaces, we refer the interested reader to Kretschmer (1961), Anderson and Nash (1987), Hernandez-Lerma and Lasserre (1996) as well as Ponstein (2004). In the sequel we call

$$\begin{aligned}
&\sup \mathbb{E}\big[- \sum_{(i,j,k)\in M} \sum_{t=0}^{T-1} \kappa^{i,j,k} v_t^{i,j,k} + \Gamma A_T + \sum_{k\in K} \sum_{t=0}^{T-1} D_t^k S_t^k \big] \\
&\text{s.t. } S_t^k - e^{i,j,k}\mathbb{E}[A_T|\mathcal{F}_t] - v_t^{i,j,k} \leq C_t^{i,j,k} \text{ a.s. } (i,j,k) \in M, t = 0,\dots,T-1 \\
&\quad A_T \leq \pi \text{ a.s.} \\
&\quad A_T \geq 0, v_t^{i,j,k} \geq 0 \text{ a.s. for all } (i,j,k) \in M \\
&\quad |A_T| \in L^\infty(\mathcal{F}_T), S \in \mathcal{L}^1_{T-1}(\mathbb{R}^{|K|}), v \in \mathcal{L}^1_{T-1}(\mathbb{R}^{|M|}).
\end{aligned}$$

the dual program (D) and refer to its optimal value by (D*). Here the notation emphasizes that the S-component of the optimal dual solution will turn out to be the equilibrium price processes of goods and the corresponding A-component will turn out to be the equilibrium allowance price at the last time point T. Without loss of generality we can assume that $\kappa^{i,j,k} > 0$ almost surely. Then given the dual variables (A, S) the optimal v is attained at

$$v_t^{*i,j,k}(A,S) = (S_t^k - e^{i,j,k}\mathbb{E}[A_T|\mathcal{F}_t] - C_t^{i,j,k})^+$$

for all $(i,j,k) \in M$ and $t = 0,\dots,T-1$. Hence the dual problem reduces to

$$\begin{aligned}
\sup \ \ &\mathbb{E}\Big[- \sum_{(i,j,k)\in M} \sum_{t=0}^{T-1} \kappa^{i,j,k} (S_t^k - e^{i,j,k}\mathbb{E}[A_T|\mathcal{F}_t] - C_t^{i,j,k})^+ + \Gamma A_T \\
&\quad + \sum_{k\in K} \sum_{t=0}^{T-1} D_t^k S_t^k \Big] \\
\text{s.t. } \ &A_T \leq \pi \text{ a.s.} \\
&A_T \geq 0 \text{ a.s.} \\
&|A_T| \in L^\infty(\mathcal{F}_T), S \in \mathcal{L}^1_{T-1}(\mathbb{R}^{|K|}).
\end{aligned}$$

5.2.2. Duality Results

In this subsection we prove that weak and strong duality hold between the primal and the dual problem. The weak duality gives a lower bound on the optimal solution.

Proposition 7 Weak Duality *For all feasible solutions* (ξ, ζ) *and* (v, A, S) *of the primal and dual problem it holds that*

$$\mathbb{E}\Big[\sum_{t=0}^{T-1}\sum_{(i,j,k)\in M} C_t^{i,j,k}\xi_t^{i,j,k} + \pi\zeta\Big] \geq \mathbb{E}\Big[-\sum_{t=0}^{T-1}\sum_{(i,j,k)\in M}\kappa^{i,j,k}v_t^{i,j,k} + \Gamma A_T + \sum_{t=0}^{T-1}\sum_{k\in K} D_t^k S_t^k\Big].$$

Proof. This follows from

$$\begin{aligned}
&\mathbb{E}\Big[\sum_{t=0}^{T-1}\sum_{(i,j,k)\in M} C_t^{i,j,k}\xi_t^{i,j,k} + \pi\zeta\Big] \\
&\geq \mathbb{E}\Big[\sum_{t=0}^{T-1}\sum_{(i,j,k)\in M}(S_t^k - e^{i,j,k}\mathbb{E}[A_T|\mathcal{F}_t] - v_t^{i,j,k})\xi_t^{i,j,k} + A_T\zeta\Big] \\
&= \mathbb{E}\Big[\sum_{t=0}^{T-1}\sum_{k\in K} S_t^k D_t^k - A_T\Pi(\xi) - \sum_{t=0}^{T-1}\sum_{(i,j,k)\in M} v_t^{i,j,k}\xi_t^{i,j,k} + A_T\zeta\Big] \\
&\geq \mathbb{E}\Big[\sum_{t=0}^{T-1}\sum_{k\in K} S_t^k D_t^k - A_T\Pi(\xi) - \sum_{t=0}^{T-1}\sum_{(i,j,k)\in M} v_t^{i,j,k}\kappa^{i,j,k} + A_T(\Gamma + \Pi(\xi))\Big] \\
&= \mathbb{E}\Big[\sum_{t=0}^{T-1}\sum_{k\in K} D_t^k S_t^k - \sum_{t=0}^{T-1}\sum_{(i,j,k)\in M}\kappa^{i,j,k}v_t^{i,j,k} + \Gamma A_T\Big]
\end{aligned}$$

which holds due to primal and dual feasibility of (ξ, ζ) and (v, A, S). □

Given any dual feasible solution weak duality gives a lower bound for the primal problem. In the next Proposition we prove that strong duality also holds. This guaranties that the bound is stringent, i.e. the lower

bound is attained for an optimal solution. Notice that in infinite dimensional spaces strong duality must not hold for LPs. See e.g. Kretschmer (1961). Besides strong duality we also show that the optimal dual solution is attained at $(\overline{S}_t)_{t=0}^{T-1}$ and $\overline{A}_T$ as defined in Theorem 1.

Proposition 8 Strong Duality *If Assumption 4 is satisfied then strong duality holds between (P) and (D). In particular if* $\overline{\xi}$ *and* $(\overline{A}, \overline{S})$ *are defined as in Theorem 1, then it holds that* $(\overline{A}, \overline{S})$ *is dual feasible and*

$$\begin{aligned} \mathrm{P}^* &= \mathbb{E}\Big[\sum_{t=0}^{T-1} \sum_{(i,j,k)\in M} C_t^{i,j,k} \overline{\xi}_t^{i,j,k} + \pi(\Gamma + \Pi(\overline{\xi}))^+\Big] \\ &= \mathbb{E}\Big[- \sum_{(i,j,k)\in M} \sum_{t=0}^{T-1} \kappa^{i,j,k}(\overline{S}_t^k - e^{i,j,k}\mathbb{E}[\overline{A}_T|\mathcal{F}_t] - C_t^{i,j,k})^+ \\ &\qquad + \Gamma\overline{A}_T + \sum_{k\in K}\sum_{t=0}^{T-1} D_t^k \overline{S}_t^k\Big] = \mathrm{D}^* \end{aligned}$$

i.e. $(\overline{S}_t)_{t=0}^{T-1}$, $\overline{A}_T$ *and* $\overline{v}_t^{i,j,k} = (\overline{S}_t^k - e^{i,j,k}\mathbb{E}[\overline{A}_T|\mathcal{F}_t] - C_t^{i,j,k})^+$ *for all* $(i,j,k) \in M$ *and* $t = 0, \ldots, T-1$ *are dual optimal solutions.*

Proof. The Definition of $\overline{S}$ as well as Assumptions 3 and 4 together with Lemma 3 part (ii) imply the inclusions (2.61) and (2.62), i.e.

$$\begin{aligned} \{\overline{S}_t^k - C_t^{i,j,k} - e^{i,j,k}\overline{A}_t > 0\} &\subseteq \{\overline{\xi}_t^{i,j,k} = \kappa^{i,j,k}\}, \\ \{\overline{S}_t^k - C_t^{i,j,k} - e^{i,j,k}\overline{A}_t < 0\} &\subseteq \{\overline{\xi}_t^{i,j,k} = 0\} \end{aligned}$$

(for a proof see Theorem 1). From this we derive that

$$\kappa^{i,j,k}(\overline{S}_t^k - e^{i,j,k}\mathbb{E}[\overline{A}_T|\mathcal{F}_t] - C_t^{i,j,k})^+ = \overline{\xi}_t^{i,j,k}(\overline{S}_t^k - e^{i,j,k}\mathbb{E}[\overline{A}_T|\mathcal{F}_t] - C_t^{i,j,k}) \tag{5.5}$$

which together with the optimality of D* implies

$$\begin{aligned} D^* \geq \mathbb{E}\Big[&- \sum_{(i,j,k)\in M} \sum_{t=0}^{T-1} \kappa^{i,j,k}(\overline{S}_t^k - e^{i,j,k}\mathbb{E}[\overline{A}_T|\mathcal{F}_t] - C_t^{i,j,k})^+ \\ &+ \Gamma\overline{A}_T + \sum_{k\in K}\sum_{t=0}^{T-1} D_t^k \overline{S}_t^k \Big] \\ = \mathbb{E}\Big[&- \sum_{(i,j,k)\in M} \sum_{t=0}^{T-1} \overline{\xi}_t^{i,j,k}(\overline{S}_t^k - e^{i,j,k}\overline{A}_t - C_t^{i,j,k}) + \Gamma\overline{A}_T + \sum_{k\in K}\sum_{t=0}^{T-1} D_t^k\overline{S}_t^k \Big]. \end{aligned} \tag{5.6}$$

Due to primal feasibility of $\overline{\xi}$ it holds moreover that

$$\sum_{i\in I}\sum_{j\in J^{i,k}} \overline{\xi}_t^{i,j,k} = D_t^k$$

for all $t = 0, \ldots, T-1$, which together with (5.6) yields

$$\begin{aligned} D^* &\geq \mathbb{E}\big[- \sum_{(i,j,k)\in M} \sum_{t=0}^{T-1} \overline{\xi}_t^{i,j,k}(-e^{i,j,k}\overline{A}_t - C_t^{i,j,k}) + \Gamma\overline{A}_T\big] \\ &= \mathbb{E}\big[+ \sum_{(i,j,k)\in M} \sum_{t=0}^{T-1} \overline{\xi}_t^{i,j,k} C_t^{i,j,k} + (\Gamma + \Pi(\overline{\xi}))\overline{A}_T\big] \\ &= \mathbb{E}\big[+ \sum_{(i,j,k)\in M} \sum_{t=0}^{T-1} \overline{\xi}_t^{i,j,k} C_t^{i,j,k} + \pi(\Gamma + \Pi(\overline{\xi}))^+\big] \\ &\geq P^*. \end{aligned} \tag{5.7}$$

Here we used equation (2.55)

$$\overline{A}_T = \pi \mathbf{1}_{\{\Gamma+\Pi(\overline{\xi})\geq 0\}}$$

in the second last step. Due to weak duality all inequalities hold with equality proving the assertion. □

Notice that the conditional expectation in the dual problem is crucial for the equality in equation (5.5) and hence also for strong duality to hold.

Corollary 6 *The dual of the stochastic optimal control problem (5.3) reads*

$$\begin{aligned} \sup \quad & \mathbb{E}\Big[\sum_{t=0}^{T-1} -\breve{\kappa}_t(\mathbb{E}[A_T|\mathcal{F}_t] - F_t)^+ + \breve{\Gamma} A_T\Big] \\ \text{s.t.} \quad & A_T \leq \pi \ \ a.s. \\ & A_T \geq 0 \ \ a.s. \\ & |A_T| \in L^\infty(\mathcal{F}_T). \end{aligned} \tag{5.8}$$

Moreover, weak and strong duality hold between (5.3) and (5.8).

Proof. This result is a straightforward adaption of the proofs of Propositions 7 and 8. □

5.3. Error Bounds for the Allowance Price

As the error bound for the allowance price process is closely related to the abatement mechanism we will assume the setting of Section 5.1 and in particular equation (5.3) where the global optimal control problem is expressed in terms of fuel switch policies and reads

$$\inf_{0\leq\mathcal{E}_t\leq\breve{\kappa}_t} \mathbb{E}\left[\sum_{t=0}^{T-1} F_t\mathcal{E}_t + \pi\left(\breve{\Gamma} - \sum_{t=0}^{T-1}\mathcal{E}_t\right)^+\right].$$

Moreover, we assume that Assumptions 1-4 are fulfilled. Due to Theorem 1 each optimal fuel switch policy $\overline{\mathcal{E}} = (\overline{\mathcal{E}}_t)_{t=0}^{T-1}$ gives the equilibrium allowance process $(\overline{A}_t)_{t=0}^{T}$ as

$$\overline{A}_t = \pi\mathbb{E}\Big[\mathbf{1}_{\{\breve{\Gamma}-\sum_{t=0}^{T-1}\overline{\mathcal{E}}_t>0\}}\Big|\mathcal{F}_t\Big] \quad \text{for all } t = 0,\dots,T.$$

Moreover, let in the sequel $\mathfrak{T} = \{0,\dots,T-1\}$ denote the set of all time steps where production takes place.

Proposition 9 *Let $\mathcal{E}^\dagger = (\mathcal{E}_t^\dagger)_{t=0}^{T-1}$ be a feasible production policy and $(A_t^\dagger)_{t=0}^T$ an allowance price process given by*

$$A_t^\dagger = \pi\mathbb{E}\Big[\mathbf{1}_{\{\check{\Gamma}-\sum_{t=0}^{T-1}\mathcal{E}_t^\dagger>0\}}\Big|\mathcal{F}_t\Big] \quad \text{for all } t=0,\dots,T.$$

Then the equilibrium allowance price $\overline{A}_0$ at time $t=0$ is bounded by

$$A_0^\dagger - \mathbb{E}\Big[\sup_{t\in\mathfrak{T}}(F_t-A_t^\dagger)^-\mathbf{1}_{\{\mathcal{E}_t^\dagger<\check{\kappa}_t\}}\Big] \le \overline{A}_0 \le A_0^\dagger + \mathbb{E}\Big[\sup_{t\in\mathfrak{T}}(F_t-A_t^\dagger)^+\mathbf{1}_{\{\mathcal{E}_t^\dagger>0\}}\Big]$$

almost surely.

Proof. Assume that the optimal fuel switch policy $\overline{\mathcal{E}}$ is known and define rescheduling policies

$$\zeta_t^+ = (\overline{\mathcal{E}}_t - \mathcal{E}_t^\dagger)^+ \qquad \zeta_t^- = (\overline{\mathcal{E}}_t - \mathcal{E}_t^\dagger)^-$$

for all $t=0,\dots,T-1$. Based on $\mathcal{E}^\dagger$ they increase or decrease respectively fuel switch intensity giving the optimal fuel switch policy $\overline{\mathcal{E}}$. Notice that

$$\begin{aligned}\{\zeta_t^+>0\} &\subseteq \{\overline{\mathcal{E}}_t>0\}\cap\{\mathcal{E}_t^\dagger<\check{\kappa}_t\}\\ &\subseteq \{F_t\le\overline{A}_t\}\cap\{\mathcal{E}_t^\dagger<\check{\kappa}_t\} \quad \text{for all } t\in\mathfrak{T} \qquad (5.9)\end{aligned}$$

and

$$\begin{aligned}\{\zeta_t^-<0\} &\subseteq \{\overline{\mathcal{E}}_t<\kappa_t\}\cap\{\mathcal{E}_t^\dagger>0\}\\ &\subseteq \{F_t\ge\overline{A}_t\}\cap\{\mathcal{E}_t^\dagger>0\} \quad \text{for all } t\in\mathfrak{T}. \qquad (5.10)\end{aligned}$$

Further let $(A_t^+)_{t=0}^T$ and $(A_t^-)_{t=0}^T$ given by

$$A_t^+ = \pi\mathbb{E}\Big[\mathbf{1}_{\{\check{\Gamma}-\sum_{t=0}^{T-1}(\mathcal{E}_t^\dagger+\zeta_t^+)>0\}}\Big|\mathcal{F}_t\Big] \quad \text{for all } t=0,\dots,T$$

and

$$A_t^- = \pi\mathbb{E}\Big[\mathbf{1}_{\{\check{\Gamma}-\sum_{t=0}^{T-1}(\mathcal{E}_t^\dagger-\zeta_t^-)>0\}}\Big|\mathcal{F}_t\Big] \quad \text{for all } t=0,\dots,T$$

denote marginal expected penalty associated with rescheduling policies that solely turn on or off capacity. Consequently we obtain

$$A^- \ge A^\dagger \qquad A^+ \le A^\dagger. \qquad (5.11)$$

Moreover, define stopping times

$$\tau^+ = \inf\{t\in\{0,\dots,T\}|\zeta_t^+>0 \text{ or } t=T\}$$

and

$$\tau^- = \inf\{t \in \{0, \ldots, T\} | \zeta_t^- > 0 \text{ or } t = T\},$$

giving the first time point where switch intensity is increased or reduced respectively for the first time, or T if no switch occurs. For events where no switch capacity is turned on or off respectively it follows that

$$\{\tau^+ = T\} \subseteq \{\overline{A}_T = A_T^-\} \qquad \{\tau^- = T\} \subseteq \{\overline{A}_T = A_T^+\}. \tag{5.12}$$

First let us prove

$$\overline{A}_0 \geq A_0^\dagger - \mathbb{E}\left[\sup_{t\in\mathfrak{T}}(F_t - A_t^\dagger)^- \mathbf{1}_{\{\mathcal{E}_t^\dagger < \breve{\kappa}_t\}}\right].$$

It follows that

$$\begin{aligned}
\overline{A}_0 &= \mathbb{E}[\overline{A}_{\tau^+}] = \mathbb{E}\Big[\overline{A}_{\tau^+}\mathbf{1}_{\{\tau^+<T\}} + \overline{A}_T\mathbf{1}_{\{\tau^+=T\}}\Big] \\
&\geq \mathbb{E}\Big[F_{\tau^+}\mathbf{1}_{\{\tau^+<T\}} + A_T^-\mathbf{1}_{\{\tau^+=T\}}\Big] && (5.13)\\
&\geq \mathbb{E}\Big[F_{\tau^+}\mathbf{1}_{\{\tau^+<T\}} + A_T^\dagger\mathbf{1}_{\{\tau^+=T\}}\Big] && (5.14)\\
&= A_0^\dagger + \mathbb{E}\Big[(F_{\tau^+} - A_{\tau^+}^\dagger)\mathbf{1}_{\{\tau^+<T\}}\Big] \\
&= A_0^\dagger + \mathbb{E}\Big[(F_{\tau^+} - A_{\tau^+}^\dagger)\mathbf{1}_{\{\tau^+<T\}}\mathbf{1}_{\{\mathcal{E}_{\tau^+}^\dagger < \breve{\kappa}_t\}}\Big] \\
&\geq A_0^\dagger + \mathbb{E}\left[\inf_{t\in\mathfrak{T}}(F_t - A_t^\dagger)\mathbf{1}_{\{\tau^+<T\}}\mathbf{1}_{\{\mathcal{E}_t^\dagger < \breve{\kappa}_t\}}\right] && (5.15)\\
&\geq A_0^\dagger - \mathbb{E}\left[\sup_{t\in\mathfrak{T}}(F_t - A_t^\dagger)^-\mathbf{1}_{\{\mathcal{E}_t^\dagger < \breve{\kappa}_t\}}\right].
\end{aligned}$$

Here (5.13) holds due to (5.9) and (5.12), while (5.14) holds due to (5.11). (5.15) follows from (5.9). Next let us prove

$$\overline{A}_0 \leq A_0^\dagger + \mathbb{E}\left[\sup_{t\in\mathfrak{T}}(F_t - A_t^\dagger)^+\mathbf{1}_{\{\mathcal{E}_t^\dagger > 0\}}\right].$$

This holds since

$$\begin{aligned}
\overline{A}_0 &= \mathbb{E}[\overline{A}_{\tau^-}] \leq \mathbb{E}\Big[\overline{A}_{\tau^-}\mathbf{1}_{\{\tau^-<T\}} + \overline{A}_T\mathbf{1}_{\{\tau^-=T\}}\Big] \\
&\leq \mathbb{E}\Big[\mathcal{E}_{\tau^-}1_{\{\tau^-<T\}} + A_T^+\mathbf{1}_{\{\tau^-=T\}}\Big] \qquad (5.16)\\
&\leq \mathbb{E}\Big[F_{\tau^-}1_{\{\tau^-<T\}} + A_T^\dagger\mathbf{1}_{\{\tau^-=T\}}\Big] \qquad (5.17)\\
&= A_0^\dagger + \mathbb{E}\Big[(F_{\tau^-} - A^\dagger_{\tau^-})\mathbf{1}_{\{\tau^-<T\}}\Big] \\
&\leq A_0^\dagger + \mathbb{E}\left[\sup_{t\in\mathfrak{T}}(F_t - A_t^\dagger)\mathbf{1}_{\{\tau^-<T\}}\mathbf{1}_{\{\mathcal{E}_t^\dagger>0\}}\right] \qquad (5.18)\\
&\leq A_0^\dagger + \mathbb{E}\left[\sup_{t\in\mathfrak{T}}(F_t - A_t^\dagger)^+\mathbf{1}_{\{\mathcal{E}_t^\dagger>0\}}\right].
\end{aligned}$$

Here (5.16) holds due to (5.10) and (5.12), while (5.17) holds due to (5.11). (5.18) follows from (5.10). □

Remark Again the error bound is stringent as $\overline{\mathcal{E}} = \mathcal{E}^\dagger$ implies that

$$A_0^\dagger \leq \overline{A}_0 \leq A_0^\dagger$$

almost surely. Notice moreover that these bounds do not carry over to $t > 0$ as the bounds of Proposition 9 are based on the fact that the state of the control problem is known at the time $t = 0$.

5.4. Error Bounds versus Discretization

In Subsection 5.1.1 we described how to construct an approximate control policy by backward induction on a trinomial forest. In this section we quantify the impact of the forest discretization on the error bounds from Sections 5.2 and 5.3 at the example of the Japan case study.

The discretization of the emission level is straightforward. As for all feasible strategies it holds that

$$\sum_{s=0}^{t}(\xi_s^c e^c + \xi_s^g e^g) \in \left[0, \sum_{s=0}^{T-1}(\kappa^c e^c + \kappa^g e^g)\right] \quad \text{a.s. for all } t = 0, \ldots, T-1$$

we discretize this interval with $m \in \mathbb{N}$ equally spaced points at a distance of $\frac{1}{m}\sum_{t=0}^{T-1}(\kappa^c e^c + \kappa^g e^g)$.

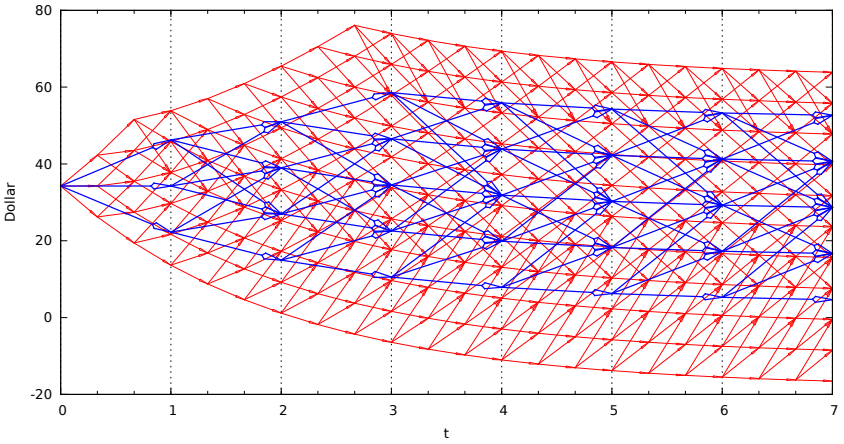

Fig. 5.2: This Figure depicts the trinomial trees for the Japanese fuel switch price process (for details on this process we refer the reader to the Appendix A.2) for different time samplings. In order to obtain a visible branching we discretize one year by $T = 10$ time points indicated by the dashed vertical lines. Discretizing the price process on these time points (i.e. $\Delta t = 1$) gives the coarse blue discretization. If in addition the process is sampled at the 2 intermediate time points (i.e. $\Delta t = 1/3$) which we indicate by the minor tics on the x-axis we obtain the narrow red discretization. Notice that fuel switches take only place at $t = 0, 1, \ldots, T-1$, i.e. at the dashed vertical lines.

On the other hand the discretization of the trinomial trees which represent the demand and fuel switch price processes can not be changed as easily, it is rather determined by the time discretization Δt at which the processes are sampled, see e.g. Fehr (2005) and Brigo and Mercurio (2005). E.g. for an Ornstein-Uhlenbeck process as we used for our Japan case study the distance between nodes at each time point is given by

$$\sqrt{3}\frac{\sigma^2}{2\gamma}(1 - e^{-2\gamma\Delta t}) \tag{5.19}$$

where σ denotes the (local) volatility and γ the rate of mean reversion of the process. Since the time sampling of the stochastic processes can be finer than $t = 0, \ldots, T-1$ we introduce $n \in \mathbb{N}$ intermediate time

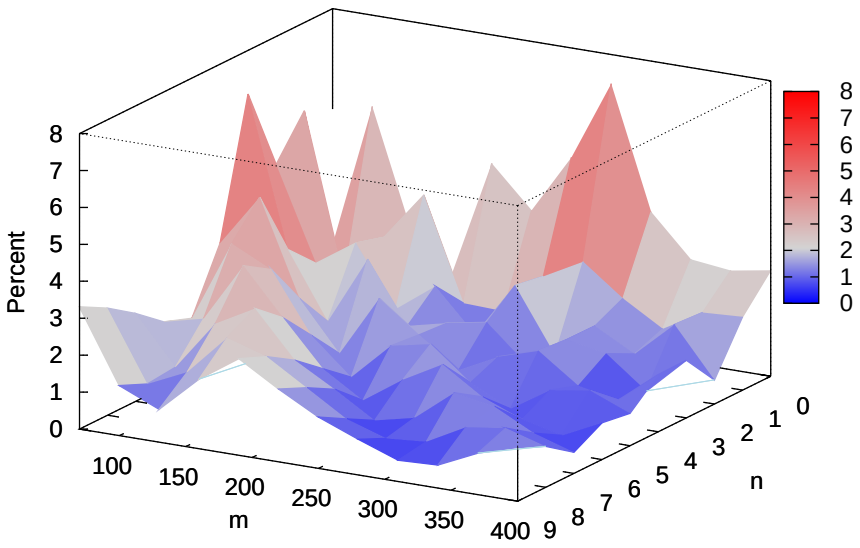

Fig. 5.3: This Figure shows the Duality Gap for different discretizations. The duality gap is small for large enough m and n.

steps between all time points t and $t+1$, and sample the processes at a period of $\Delta t = 1/n$. Due to equation (5.19) this refines the mesh at the time points $t = 0, \ldots, T-1$ where we can compute a finer (and often better) control policy.

Figure 5.2 shows how the price discretization changes with the number of intermediate time points. Introducing two intermediate time steps the price space (for $t \geq 3$) is sampled with 11 nodes instead of the original 5 nodes. Thereby the discretization not only becomes finer but also broader. This broadening of the mesh improves the exercise policy as long as it stays in the interval $[0, \pi]$. Outside this interval the exercise policy is independent of the exact fuel switch price. A similar argument holds for the demand process. Notice that in contrast to Brigo and Mercurio (2005) we do not get a symmetric tree as we used the slightly different tree building procedure described in Fehr (2005). Moreover the growth of the trees stops at $t = 3$ because of the mean reversion of the Ornstein-Uhlenbeck process. This is a nice feature as it contains

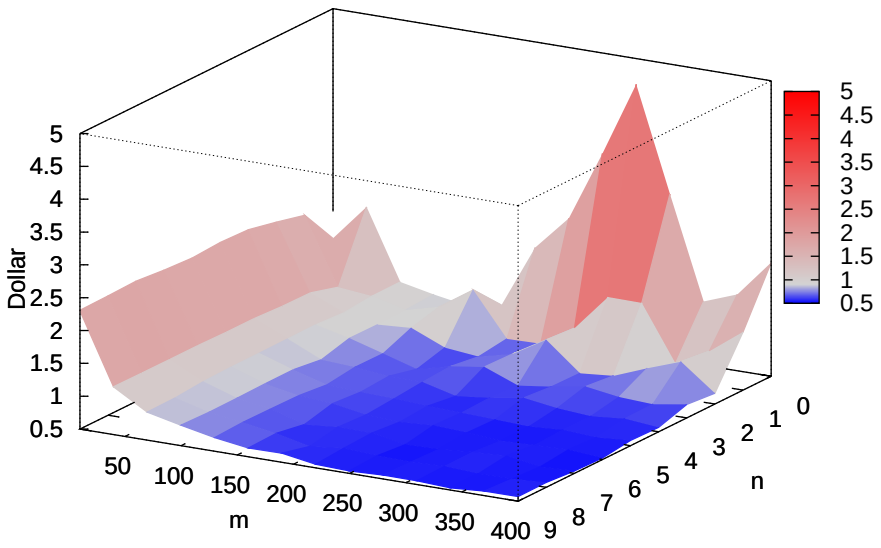

Fig. 5.4: Depicted is the gap between the upper and the lower allowance price bound depending on the discretization parameters. Again the gap is small for large enough m and n.

the growth of the computational complexity in T.

To quantify the behavior of the error bounds from Sections 5.2 and 5.3 when the discretization is refined we consider only the last month of the Japanese trading scheme, merge the eastern and the western interconnection into one market, and assume that fuel switches can only take place at $T = 10$ time steps equally spread over the month. We start with a coarse discretization and gradually refine the discretization by adding intermediate time points and more discretization steps for the emission space. Hence by purpose we do not choose $T = 30$, which would be natural for one month, as to start with, this would give too narrow of a discretization.

Figures 5.3 and 5.4 show the duality gap and the gap between the upper and lower allowance price bounds for different discretization parameters m and n. As expected both the duality gap and the gap between upper and lower bound for the allowance price are small if both m and

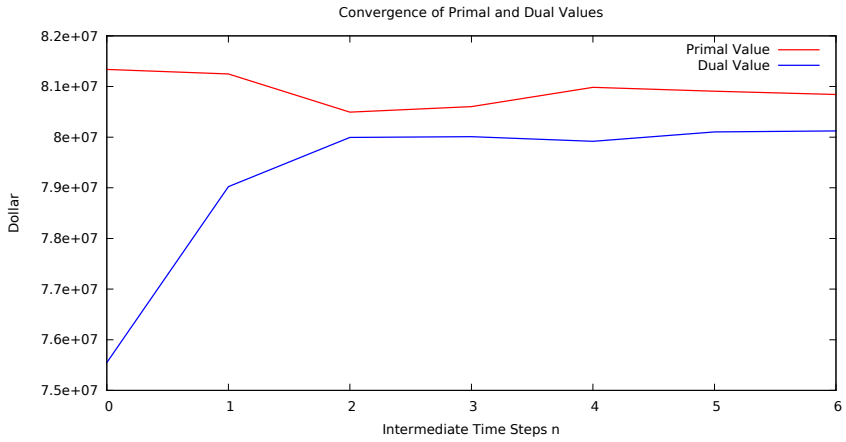

Fig. 5.5: For fixed $m = 280$ this plot depicts how the values of the primal and the dual problem approach each other. Already for two intermediate time steps we have a relative error of 0.625%. The zigzag movement in the convergence of the primal and dual value is characteristic for stochastic optimal control problems.

n are chosen large enough. While the error bound for the allowance price decreases quite fast and nearly monotonically the duality gap decreases with a zigzag movement, being characteristic for optimal control problems (see e.g. Fehr (2005)). The reason for this is best seen for American options. Focusing on one time step, one understands that the distance of the theoretical exercise boundary and the discrete nodes assigned to the exercise boundary alternates between increasing and decreasing, while refining the mesh. Hence, refining the mesh sometimes lead to a better solution and sometimes to a worse solution.

While Figures 5.3 and 5.4 depict the differences between upper and lower bounds, Figures 5.5 and 5.6 illustrate the converging behavior of the upper and the lower bound separately. For illustrative reasons we choose to display only the behavior with respect to the number of intermediate time steps n at fixed $m = 280$.

As already seen in Figures 5.3 and 5.4 the error bounds seem to converge while refining the discretization. Already for two or three intermediate time steps we reach a reasonable accuracy with a relative error of 0.625%

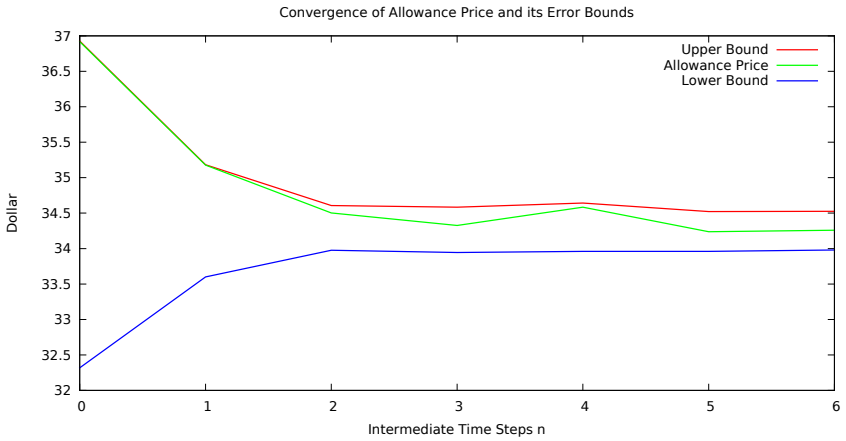

Fig. 5.6: Here we depict the simulated allowance price at time zero and its upper and lower error bounds. For 6 intermediate time steps we have an error of $\pm 0.25\$$.

for the value of the primal and the dual problem and an error of $+0.1\$/-0.5\$$ for the allowance price at time zero. This justifies the choice of parameters $n = 0$, $m = 10220$ and $T = 365$ which we used for the Japan case study, since these give the same displacement between nodes as above monthly example with $n = 2$, $m = 280$ and $T = 10$.

Chapter 6

Conclusions

Chapter Overview

In the following, we provide a general summary of the relevant parts of this thesis and summarize the main findings. Finally an outlook will refer to possible continuation of this work.

6.1. Review

This thesis introduces a new mathematical framework for competitive equilibrium, in which emission trading schemes can be analyzed. This framework is general enough to accommodate tax based abatement policies, existing cap-and-trade schemes such as those implemented by the European Union, as well as new market designs.

The main thrust of the thesis is to provide policy makers and regulators with the tools necessary to design and implement cap-and-trade schemes capable of reaching reasonable pollution targets at low social costs while controlling windfall profits and incentives for cleaner plants.

On the one hand, we develop a rigorous mathematical theory for competitive economic models in which stochastic demand and production costs are given exogenously. We prove existence and uniqueness, of an equilibrium in which price processes for goods and pollution appear endogenously.

On the other hand, we provide analytic and computational tools to analyze and compare the various emissions trading schemes. Regulators and policy makers need to understand the structure and the role of these new markets vis-a-vis pollution control, and we view these tools as crucial in the design and the implementation of sound environmental economic policies.

The computational tools developed for the analysis provide, for each market design, Monte Carlo scenario generators for equilibrium prices of goods and emission allowances. In each scenario we can compute:

- emission levels
- reduction costs

- consumer costs
- producers windfall profits
- error bounds

Last but not least we use our computational tools to provide a thorough comparison of a sample of alternative schemes at the example of the Japanese electricity market: the standard scheme inspired by the first EU implementation phase, an emission tax scheme, and a relative scheme in which the distribution of allowances is driven by instantaneous (as opposed to historical) production levels.

Deciding which public policy tools to use in order to curb Green House Gas (GHG) emissions is paramount. This thesis approaches the problem from three different points of view: social, the consumer's and the producer's. The main findings are:

Standard cap-and-trade schemes are socially optimal in a stochastic setting if they are given the right emission targets and penalties. However, the numerical case study shows that the consumers' burden far exceeds the overall reduction costs (8 times the reduction cost for this case study), giving rise for huge windfall profits for electricity companies.

Despite the fact that they are easy to explain and implement, **taxes** (see for example Nordhaus (2007) for a discussion in the context of pollution abatement) are the least efficient of the schemes considered in this study. Because of the uncertainties in the demand for goods and the cost of production they are less efficient and more costly than cap-and-trade schemes. In particular, we demonstrate in this study that when it comes to reaching emission targets they do not perform as well as properly designed cap-and-trade schemes. Moreover, taxes are unpopular and in most countries, synonymous of *political suicide* for policy makers. Therefore, it seems that taxation will be unlikely to be considered as an alternative to cap-and-trade mechanisms.

Auctioning is very popular among the supporters of cap-and-trade schemes puzzled by the magnitude of the windfall profits of the first phase of EU ETS. However, we show that auctions cannot lower windfall profits to a reasonable level. They merely help the re-distribution of

these costs. Indeed the revenues of the auctions (expected to be in the range of $9,5$ \$ per MWh) remain significantly smaller than the consumer costs, covering only approximately two thirds of the latter. Hence the commonly believed argument that auction revenues can be used to cover consumer costs needs to be substantiated, as there is significant wealth transfer from consumer to producer.

In the **relative allocation scheme** allowances are distributed proportionally to the production of goods at the source of emissions. We show that such a proportional allocation scheme when calibrated correctly can reach emission targets (even though the cap is not fixed) and offers a good control of windfall profits at low (however not minimal) reduction costs.

This thesis proposes and advocates **hybrid cap-and-trade schemes**, which incorporate the best properties of the standard scheme with those of the relative scheme. Namely, it is socially optimal while at the same time it has a fixed cap and provides a tight control of windfall profits. The reason for the good control of windfall profits is because those are reduced at their origin, by reducing the factor by which emissions are priced into products at the source of emissions. This is in contrast to a wide range of economic literature claiming that carbon should be priced into product prices, since this gives an efficient demand response. In this thesis we argue that efficiency should not be the only objective when designing cap-and-trade schemes. This is highlighted using the example of electricity markets where the demand response is relatively small, giving only a small improvement in efficiency, that comes at disproportional huge consumer costs (8 times the reduction cost for this case study). Policy makers should weigh consumer costs and windfall profits against efficiency and control these quantities with a hybrid (or relative) scheme. In real life, different industry sectors can be covered by the same emission trading scheme, where sectors for which demand response is desirable are covered by a standard allocation and sectors with a poor demand response could be covered by a hybrid allocation mechanism.

The following table summarizes the results of our comparative analysis of standard cap-and-trade schemes (whether or not they include auctioning of allowances) with tax schemes, relative schemes and hybrid

schemes, which we identified in this thesis.

Comparison of Schemes				
	Red. Target	Windfall	Red. Cost	Cons. Cost
Standard	+	−	+	−
Std&Auct	+	−	+	−
Relative	∘	+	∘	+
Tax	−	+	−	−
Hybrid	+	+	+	+

(The circle indicates that the result is good but not optimal.)

The theoretical and quantitative analysis of cap-and-trade schemes for the purpose of emissions reduction are still in their infancy. However, we hope that this contribution will convince regulators and policy makers of the importance of the insights, which could be gained from using the tools developed for the purpose of this study.

6.2. Outlook

This thesis was mostly concerned with allocation mechanisms for cap-and-trade schemes. For the envisaged post 2012 regulations other important topics are the linking between compliance periods as well as the linking between different emission trading schemes and CDM/JI.

The article Carmona and Fehr (2009b) gives the theoretical basis for equilibria in such a setting, including also long term abatement mechanisms. It extends the model from Chapter 2 to several emission markets interconnected by CDM/JI and takes into account linking of compliance periods such as banking of allowances from one compliance period to the next one as well as the penalty mechanism of EU ETS. Such a setting would allow the impact of banking, penalty regulations and limits on the amount of CERs/ERUs (that can be used for compliance within a scheme) on emissions, leakage, reduction costs, end consumer costs and sustainable development of production portfolios to be investigated.

Some of these topics can be treated with parallelized versions of the numerical methods developed in this thesis. However for the full-fledged

problem, the course of dimensionality requires other/new numerical procedures. In particular the Least Squares Monte Carlo Algorithm suggests itself for these high dimensional problems, see e.g. Tsitsiklis and Van Roy (2001) or Longstaff and Schwartz (2001). For this procedure, namely the error bounds developed in Chapter 5 are important when choosing the basis functions for the regression.

From a market point of view other interesting questions are how CDM/JI projects should be exercised optimally and how the risks associated with these projects should be managed.

In order to complete the competitive equilibrium theory for cap-and-trade schemes the equilibrium model developed in Chapter 2 needs to allow nonlinear expected utility functions: It should be shown that even if the market is incomplete equilibrium exists and again its solution reduces to the same optimal stochastic control problem as for linear expected utility functions, but in contrast to Chapter 2 under a risk neutral measure. Here the main issue is to prove the existence of the equilibrium.

A different topic concerns the explicit stochastic modeling of allowance price dynamics. The models used in this thesis as well as in Carmona and Fehr (2009b) and Carmona et al. (2009a) do not allow for an explicit formulation of allowance price processes. Instead the allowance price process can only be simulated by an involved numerical procedure. However, first results indicate that considering simpler reduction mechanisms than discussed in this thesis, one obtains a PDE for allowance price dynamics that can be solved explicitly. The goal of such research would be to extend this setting to EU ETS, i.e. to its banking, penalty regulations and its linking to CDM/JI.

Appendix A

Japan Case Study

Chapter Overview

This final section contains the technical details of the implementations used to produce the numerical results presented in the paper. We chose to run the numerical experiments with data coming from the Japanese electricity market, because it forms an independent grid. We can thus address the question of the impact of introducing an emission reduction policy in this country without taking into account emission leakage. The Japanese electricity market is divided into an eastern and a western part, with only 1 Giga Watt interconnection capacity. Hence, for the purpose of this case study, we can safely assume that we are dealing with two separated electricity markets sharing a joint emission cap. Since the electricity demand correlation between east and west is 0.92 we use one stochastic demand driver $(D_t)_{t=0}^{T-1}$. A historical calibration shows that the demand process in the east and the west are approximately recovered as $0.42 \times (D)_{t=0}^{T-1}$ and $0.58 \times (D)_{t=0}^{T-1}$ respectively. In Sections A.1 and A.2 electricity demand and fuel switch price process are specified using continuous-time pendants

$$(D(t))_{t\in[0,T]}, \quad (F(t))_{t\in[0,T]} \tag{A.1}$$

for both processes. Note that we write the time parameter in parenthesis instead of using a subscript, to indicate continuous-time processes. Moreover, the horizon for continuous time is $[0, T]$, where we suppose that the time unit equals to one year. By sampling (A.1) at discrete times, we obtain discrete-time versions of the processes on a daily time step resolution, which is used for numerical computations. Section A.3 shows how the parameters for $(D(t))_{t\in[0,T]}$ and $(F(t))_{t\in[0,T]}$ are estimated by a standard linear regression method.

A.1. Electricity Demand Process

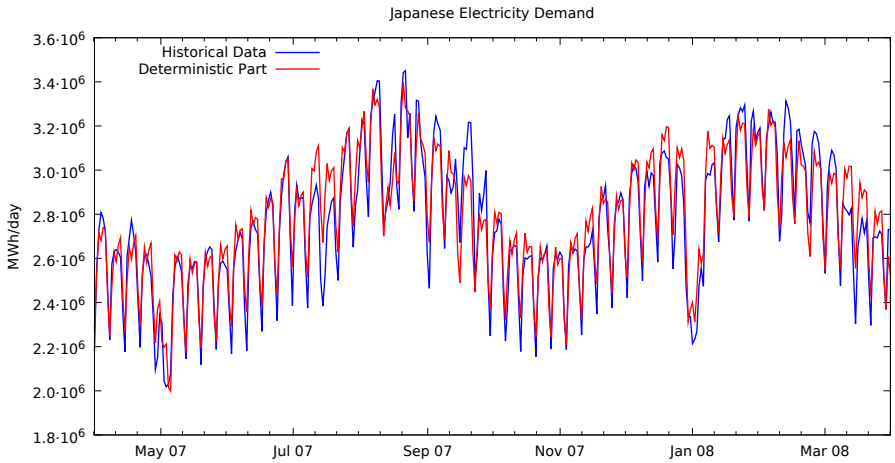

Fig. A.1: Depicted are the historical daily electricity demand for the Japanese electricity market from 01/04/2007 to 31/03/2008 and the corresponding deterministic part $(P(t))_{t=[0,T]}$.

The continuous–time demand process is modeled by

$$D(t) = \min\{(P_D(t) + X_D(t))^+, \kappa^n + \kappa^c + \kappa^g\} \qquad t \in [0,T]$$

where the deterministic part

$$P_D(t) = a_D + b_D t + \sum_{j=0}^{6} c_j \cos(2\pi\varphi_j t + l_j) \qquad t \in [0,T] \tag{A.2}$$

accounts for a linear demand increase superimposed by seasonal and weekly demand fluctuations. The stochastic part $(X(t))_{t\in[0,T]}$ is modeled by an Ornstein-Uhlenbeck process whose evolution follows the stochastic differential equation

$$dX_D(t) = \gamma_D(\alpha_D - X_D(t))dt + \sigma_D dW(t) \tag{A.3}$$

driven by Brownian motion $(W(t))_{t\in[0,T]}$ with parameters $\gamma_D, \alpha_D, \sigma_D \in \mathbb{R}$. The parameter estimation is based on historical load data for the time period April 2005 - April 2008 available on the website of the Electric Power System Council of Japan, depicted in the Figure A.1. The

parameters of the process were identified in two steps. First the deterministic harmonics (A.2) in the demand process are identified with parameters obtained from peaks in the Fourier transform. Secondly after removing the deterministic part $(P_D(t))_{t\in[0,T]}$ (red line in this figure) the residual component $(X_D(t))_{t\in[0,T]}$ is estimated by the standard linear regression described in Subsection A.3. The resulting parameters for the stochastic part are

Stochastic Part $(X_D(t))_{t\in[0,T]}$		
γ_D	α_D	σ_D
177	0	2.242×10^6

those for the affine part of $(P_D(t))_{t\in[0,T]}$ read

Affine Part of $(P_D(t))_{t\in[0,T]}$	
a_D	b_D
2760749	84611

and the 7 main oscillations of $(P_D(t))_{t\in[0,T]}$ are given by

Periodic Part of $(P_D(t))_{t\in[0,T]}$							
	$i=0$	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$
c_i	62584	258240	48653	46785	179396	110187	45208
φ_i	1	2	3	4	52.14	104.29	208.57
l_i	2.80	2.04	−0.35	2.31	2.38	1.72	−0.76

Notice from $(\varphi_i)_{i=0}^6$ that long term periodicities where computed with a yearly periodic Fourier transform while short term periodicities were computed with a weekly periodic Fourier transform.

A.2. Fuel Switch Price Process

The continuous–time fuel switch price process is modeled by

$$F(t) = a_F + b_F t + X_F(t) \qquad t \in [0,T]$$

where the stochastic part $(X_F(t))_{t\in[0,T]}$ is again modeled by an Ornstein-Uhlenbeck process whose evolution follows the stochastic differential equation

$$dX_F(t) = \gamma_F(\alpha_F - X_F(t))dt + \sigma_F dW(t) \tag{A.4}$$

driven by Brownian motion $(W(t))_{t\in[0,T]}$ with parameters $\gamma_F, \alpha_F, \sigma_F \in \mathbb{R}$.

For estimation of the parameters of the fuel spread we used Japanese LNG import prices (Bloomberg) and Japanese coal prices (Argus Media Group) from January 2006 to July 2008. Taking into account expected long time gas and coal price we fixed $a = 60\$$ neglecting the recent fuel switch price increase. As for the electricity demand process the parameters of the stochastic component $(X_F(t))_{t\in[0,T]}$ were calibrated using the procedure described in Subsection A.3. The parameters of the Fuel Switch Price Process are:

Fuel Switch Price Process $(F(t))_{t\in[0,T]}$				
a_F	b_F	γ_F	α_F	σ_F
33.68	4.81	5.18	0	28.21

A.3. Linear Regression

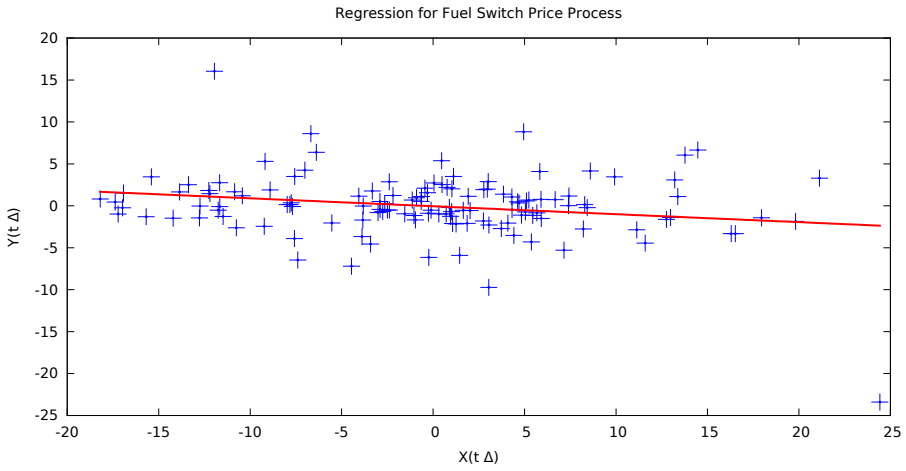

Fig. A.2: Scatter plot of $(X(t\Delta), Y(t\Delta))$ calculated by (A.5) based on historical fuel switch prices for the Japanese electricity market. The straight lines depicts the estimated linear regression.

The parameters $\gamma_i, \alpha_i, \sigma_i$ for $i \in \{D, F\}$ of the Ornstein-Uhlenbeck processes (A.3) and (A.4) are estimated by a standard linear regression method applied as follows: From the formulas for conditional mean and variance

$$\begin{aligned} \mathrm{E}(X(t)|\mathcal{F}_s) &= X(s)e^{-\gamma(t-s)} + \alpha(1 - e^{-\gamma(t-s)}) \quad s \leq t \\ \mathrm{Var}(X(t)|\mathcal{F}_s) &= \frac{\sigma^2}{2\gamma}(1 - e^{-2\gamma(t-s)}) \quad s \leq t \end{aligned}$$

we obtain the regression

$$Y(t\Delta) := X((t+1)\Delta) - X(t\Delta) = \beta_0 + \beta_1 X(t\Delta) + \beta_2 \epsilon_t \qquad t = 1, \ldots, n-1 \tag{A.5}$$

where $(\epsilon_t)_{t=1}^{n-1}$ are independent, standard Gaussian random variables and $\beta_0, \beta_1, \beta_2$ are connected to α, γ, σ by

$$\begin{aligned} \alpha &= -\frac{\beta_0}{\beta_1} \\ \gamma &= -\frac{1}{\Delta}\ln(1 + \beta_1) \\ \sigma &= \sqrt{\frac{2\gamma\beta_2}{1 - e^{-2\gamma\Delta}}}. \end{aligned}$$

Notation and Glossary

The following table of notation lists the most important quantities of this thesis.

$(\Omega, \mathcal{F}, \mathbb{P})$	Probability space
I	Set of all the firms in the economy
K	Set of goods produced in the economy
$J^{i,k}$	Set of technologies that firm $i \in I$ has to produce good $k \in K$
M_i	The index set $\{(j,k) : k \in K, j \in J^{i,k}\}$ for all $i \in I$
M	The index set $\{(i,j,k) : i \in I, k \in K, j \in J^{i,k}\}$
T	Horizon of the model, end of the compliance period
τ	Time point at which the pot is auctioned in the hybrid scheme
$\xi^{i,j,k}$	Production strategy of firm $i \in I$ for good $k \in K$ with technology $j \in J^{i,k}$
$\kappa^{i,j,k}$	Maximum production capacity of firm $i \in I$ for good $k \in K$ with technology $j \in J^{i,k}$
D_t^k	Demand for good $k \in K$ at time $t = 0, \ldots, T-1$
π	Financial penalty for over-emission

Λ	Cap in the standard scheme
Λ_t^i	Allocation of allowances at time $t = 0, \ldots, T-1$ for firm $i \in I$ in the standard scheme
Λ^i	Total endowment of allowances up to time $T-1$ of firm $i \in I$ in the standard scheme
$\hat{\Lambda}_t^i(\xi^i)$	Allocation of allowances at time $t = 0, \ldots, T-1$ for firm $i \in I$ in the generalized scheme
$\hat{\Lambda}^i(\xi^i)$	Total endowment of allowances up to time $T-1$ of firm $i \in I$ in the generalized scheme
$\tilde{\Lambda}$	Cap in the hybrid scheme
$\tilde{\Lambda}_0^i$	Initial allocation for firm $i \in I$ in the hybrid scheme
A_t	Allowance price at time $t = 0, \ldots, T$
θ_t^i	Number of allowance forward contracts held by firm $i \in I$ at time $t = 0, \ldots, T$
$R_T^A(\theta)$	Net cash position at time T resulting from trading
S_t^k	Discounted price of good $k \in K$ at time $t = 0, \ldots, T-1$
$\hat{S}_t^k$	Discounted target price of good $k \in K$ at time $t = 0, \ldots, T-1$, i.e. marginal production cost without allowance price factored into it
$B_t(T)$	Price at time $t = 0, \ldots, T$ of zero coupon bond maturing at time T
$C^{i,j,k}$	Discounted production costs of firm $i \in I$ for good $k \in K$ with technology $j \in J^{i,k}$
$C(\xi)$	Total production costs of the entire economy under strategy $\xi \in \mathcal{U}$
$\Pi^i(\xi^i)$	Cumulated emissions by firm $i \in I$ at time T following strategy $\xi^i \in \mathcal{U}^i$
Δ^i	Uncontrolled emissions of firm $i \in I$
Δ	Total uncontrolled emissions
Γ^i	Negative net amount of allowances of firm $i \in I$ at time T

Γ	Negative net amount of allowances of the entire economy
$\Gamma(\xi^{-i})$	Upper bound on the relative allocation of firm $i \in I$ in the hybrid scheme
$L^{A,S,i}(\theta^i, \xi^i)$	P&L of firm $i \in I$ in the standard scheme
$H^{A,S,i}(\theta^i, \xi^i)$	P&L of firm $i \in I$ in the generalized scheme
$H^{A,S,P,\xi^{-i},i}$	P&L of firm $i \in I$ in the hybrid scheme
$G(\xi)$	Societal costs from production and penalty payments
$G^{\pi,\Lambda}(\xi)$	Societal costs from production and penalty payments, where we highlight the penalty π and the cap Λ
$L^{\eta,\lambda}(\xi, \pi)$	Lagrange function defined in (4.17)
$\mathcal{U}^i$	Admissible production strategies of firm $i \in I$
$\mathcal{U}$	Admissible production strategies of the entire market such that the demand is satisfied
$\mathcal{V}^i(A, \Lambda^i)$	Admissible trading strategies of firm $i \in I$
$\mathcal{V}(A, \Lambda^i)$	Admissible trading strategies of all firms in the economy
(A^*, S^*)	Any equilibrium of standard or generalized schemes
(A^*, S^*, P^*)	Any equilibrium of the hybrid scheme
(ξ^*, θ^*)	Any equilibrium strategies of the standard scheme
$(\overline{\xi}, \overline{\theta})$	Equilibrium strategies based on the solution of the global optimal control problem 2.43 as in Theorem 1
$\overline{A}$	Allowance price process based on the solution $\overline{\xi}$ of the global optimal control problem 2.43 as in Theorem 1
$\overline{S}$	Prices of goods based on the solution $\overline{\xi}$ of the global optimal control problem 2.43 as in Theorem 1
$\hat{\Gamma}, \hat{e}, \hat{C}$	Adjusted parameters for the generalized scheme
X_t^i	Free allocation in the generalized scheme
$Y_t^{i,j,k}$	Free relative allocation in the generalized scheme depending on firm $i \in I$, production good $k \in K$ and technology $j \in J^{i,k}$ at time $t = 0, \ldots, T-1$

V_t^i	Predetermined tax for agent $i \in I$ at time $t = 0, \ldots, T-1$
$Z_t^{i,j,k}$	Relative tax in the generalized scheme depending on firm $i \in I$, production good $k \in K$ and technology $j \in J^{i,k}$ at time $t = 0, \ldots, T-1$
y^k	Relative allocation factor for production of good $k \in K$ in the relative scheme and in the hybrid scheme
x^i	Initial allocation for firm $i \in I$ in the relative scheme
z	Emission tax
Υ	Size of the pot in the hybrid scheme
$\Upsilon(\xi)$	Allowances remaining in the pot under strategy $\xi \in \mathcal{U}$
$Y^i(\xi^i)$	Amount of allowances potentially earned by firm $i \in I$ under the strategy $\xi^i \in \mathcal{U}^i$ in a relative allocation up to time τ
$Y(D)$	Amount of allowances potentially earned by all producers in a relative allocation up to time τ in equilibrium
φ^i	Amount of allowances bought in the auction at time τ of firm $i \in I$ in the hybrid scheme
$\tilde{\mathcal{Q}}^{\xi^{-i},i}(A,P)$	Admissible strategies of agent $i \in I$ in the hybrid scheme
$\tilde{\mathcal{Q}}(A,P)$	Admissible strategies for the entire economy in the hybrid scheme
$\mathcal{Q}^i(A)$	Admissible strategies of agent $i \in I$ in the standard scheme
$\mathcal{Q}(A)$	Admissible strategies for the entire economy in the standard scheme
λ	Threshold for the expected excess emissions
$E^\lambda(\xi)$	Expected excess emissions above the threshold λ
$\tilde{a}(\lambda)$	Lower bound on the expected excess emissions under the threshold λ
η	Target for the expected excess emissions
P	Price in the auction at time τ
P^*	Equilibrium price in the auction at time τ
$\overline{P}$	Auction price based on $\overline{A}$ as in Corollary 5

$\breve{\mathcal{E}}_t$	Maximum emissions at time $t = 0, \ldots, T-1$ in the Japan case study
$\mathcal{E}_t$	Fuel switch strategy at time $t = 0, \ldots, T-1$ in the Japan case study
$\breve{\kappa}_t$	Maximum fuel switch intensity at time $t = 0, \ldots, T-1$ in the Japan case study
$\breve{\Gamma}$	Maximum emission up to time T in the Japan case study
F_t	Fuel switch price at time $t = 0, \ldots, T-1$ in the Japan case study
$\mathcal{E}_t^\dagger$	Approximated fuel switch strategy at time $t = 0, \ldots, T-1$ based on the solution of the discrete optimal control problem
$\xi^\dagger$	Approximated production strategy based on the approximated fuel switch strategy $\mathcal{E}^\dagger$
$A^\dagger$	Approximated allowance price based on the approximated fuel switch strategy $\mathcal{E}^\dagger$
m	Number of discretization points for the emission level in the discretized optimal control problem
n	Number of intermediate time steps in the discretized optimal control problem

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