An examination of some aspects of factor analysis in damage detection

Conference Paper

Author(s):
Mozaffari, Salma; Bernal, Dionisio P.

Publication date:
2012

Permanent link:
https://doi.org/https://doi.org/10.3929/ethz-b-000487916

Originally published in:
Conference Proceedings of the Society for Experimental Mechanics Series 6, https://doi.org/10.1007/978-1-4614-2419-2_40

An Examination of some Aspects of Factor Analysis in Damage Detection

Salma Mozaffari Kojidi [*], Dionisio P. Bernal[*]

[*]Department of Civil & Environmental Engineering, 400 Snell Engineering Center, Northeastern University, 360 Huntington Avenue, Boston, MA 02115, US.

Abstract

It is well known that dynamic properties can be affected by environmental factors and this complicates the identification of damage. Among procedures explored to mitigate the fluctuations in dynamic features from environmental effects factor analysis is one of the simplest and has the merit that it operates without the need to measure the environmental variables. The method has been examined by several researchers in recent years but experience is limited and questions on some technical details remain open. This paper presents an investigation on three aspects that may affect the performance. The most important is the number of factors used in the analysis. The second relates to whether the diagonal nature on the covariance of random errors is enforced or not, and the third examines the influence of the criterion used to solve the over-determined set of equations that arise in the method. Results are obtained by simulating a system where the stiffness is cubically related to changes in temperature and where the temperature field, in each simulation, is a realization of a random process with a prescribed spatial correlation.

Introduction

The time scale at which environmental conditions fluctuate is typically much larger than the time span used to collect vibration data and, as a consequence, environmental conditions in any data set are typically constant. A feature extracted when the environmental parameters are X, however, does not generally have the same statistical properties as when the environmental parameters are Y. So, one faces the difficulty of deciding if an observed change is due to damage or the result of a change in environmental conditions.

One way to mitigate the mentioned difficulty is to derive a model in which the environmental effects are explicitly included. Although conceptually clear, implementation of this strategy in practice can be difficult. For example, it may be clear that temperature is the key environmental variable but the effect of temperature on the selected feature may be strongly dependent on the spatial distribution of the temperature, pointing to the fact that a complex model is likely to be required. A much simpler alternatively consists in assuming that the effect of the environment on the selected feature can be represented using a model of a prescribed mathematical form but which contains a set of unmeasured variables. The premise being that if the number of variables and the functional relation to the features are accurate, the residual, when damage takes place, will increase notably beyond the values that prevail when the system is in the reference (healthy) state [1-3]. Most often the functional relationship between the features and the unmeasured variables is assumed linear and random fluctuations on the feature are assumed uncorrelated, Gaussian and white. This paper examines how the number of unmeasured variables used to formulate the model affects the performance of the approach, known as linear factor analysis, plus two other aspects that are best discussed within the body of the paper.

Linear Factor Analysis

In linear factor analysis it is assumed that the features, x, are related linearly to a set of unobserved factors ξ, by some transformation Λ that needs to be identified [4-6]. With ε being the random fluctuations in the featured vector one has

$$\mathrm{x} = \mathrm{x}_0 + \Lambda\xi + \varepsilon \tag{1}$$

where x_0 is the feature vector mean. Since its evident that eq.1 can be written as

$$\mathrm{x} = \mathrm{x}_0 + (\Lambda\mathrm{T}^{-1})(\mathrm{T}\xi) + \varepsilon \tag{2}$$

it follows, since T can be selected as needed, that one can assume that the factors are mutually independent, with zero mean and unit variance. Assuming, in addition, that they are normally distributed one has $\xi \sim N(0, I)$. From eq.1the covariance of the features is

$$R = E[xx^T] = E[(\Lambda\xi + \varepsilon)(\Lambda\xi + \varepsilon)^T] = E(\Lambda\xi\xi^T\Lambda^T + \Lambda\xi\varepsilon^T + \varepsilon\xi^T\Lambda^T + \varepsilon\varepsilon^T) \tag{3}$$

where E is the expectation operator. Since ε is assumed white it is uncorrelated with the factors and one gets, recalling that the covariance of the factors can be taken as the identity [5,6]

$$R = \Lambda\Lambda^T + \Psi \tag{4}$$

In practice one gets R from training data and the task is to factor the result into the form of eq.4. Eq.4 is not unique, but the decomposition one seeks is one where the first term on the *rhs* is a basis for the subspace of R associated with the significant singular values and Ψ is diagonally heavy, since we've assumed that the random fluctuation of the features are uncorrelated. From a SVD of R [5,6] one has

$$R = U\Sigma U^T \tag{5}$$

where we've accounted for the fact that R is symmetric. Partitioning Σ into Σ_1, which contains the first m singular values and Σ_2 [5,7] one has

$$\mathrm{R} = [\,\mathrm{U}_1 \quad \mathrm{U}_2]\begin{bmatrix}\Sigma_1 & 0\\ 0 & \Sigma_2\end{bmatrix}\begin{bmatrix}\mathrm{U}_1\\ \mathrm{U}_2\end{bmatrix} = \mathrm{U}_1\Sigma_1\mathrm{U^T}_1 + \mathrm{U}_2\Sigma_2\mathrm{U^T}_2 = \Lambda\Lambda^\mathrm{T} + \Psi \tag{6}$$

Therefore, it is evident that Λ and Ψ can be taken as

$$\Lambda = U_1\sqrt{\Sigma_1} \tag{7}$$

and

$$\Psi = U_2 \Sigma_2 U_2^T \tag{8}$$

An issue with the solution in eqs.7 and 8 is that, contrary to the assumption that the errors in the feature vector entries are uncorrelated, Ψ does not prove diagonal. An iterative procedure to modify the solution in eqs.7 and 8 that forces Ψ to be diagonally heavy exists and examination of its merit is one of the issues that are looked at in the numerical part of the paper.

Number of Factors

In real cases inevitable noise makes R full rank but one looks to take the number of factors as the effective rank R. This selection is obvious when there is a clear gap in the ordered singular value sequence but there are many instances where there is no such clear gap and judgment has to be used. It is of interest to determine how the decision on this regard affects the performance of factor analysis as a damage detection technique[5,6,8,9]. We make some observations following presentation of the numerical results.

Diagonalizing Ψ

As mentioned previously, the covariance matrix of the random errors is assumed to be diagonal, yet the matrix Ψ that is obtained from eq.8 is not generally diagonal. An iterative algorithm that makes Ψ diagonally heavy proceeds as follows [7]

$$R - \Psi^{(i-1)} = U_1^{(i)} \Sigma_1^{(i)} U_1^{(i)^T} + U_2^{(i)} \Sigma_2^{(i)} U_2^{(i)^T} = \Lambda^{(i)} \Lambda^{(i)^T} + U_2^{(i)} \Sigma_2^{(i)} U_2^{(i)^T} \tag{9}$$

Where Ψ is updated in each step as; $\Psi^{(i)} = diag[R - \Lambda^{(i)} \Lambda^{(i)^T}]$ (10)

Solving the Over-Determined Set of Equations

Once Λ is determined the factors ξ can be estimated by solving

$$\Lambda\xi = (\mathrm{x} - \mathrm{x}_0) - \varepsilon \tag{11}$$

Since Λ is a tall matrix eq.11 is an over-determined set and one can obtain a solution in a number of different ways [3,6,10]. One way is to use a weighted least square solution since the covariance of the noise is known. In the literature this alternative is known as Bartlett's method, namely

$$\tilde{\xi} = (\Lambda^T \Psi^{-1} \Lambda)^{-1} \Lambda^T \Psi^{-1} (x - x_0) \tag{12}$$

Another alternative derived from Bayesian principles is known as Thomson's solution and is given by

$$\tilde{\xi} = (I + \Lambda^T \Psi^{-1} \Lambda)^{-1} \Lambda^T \Psi^{-1} (x - x_0) \tag{13}$$

And an expression based on linear regression is:

$$\tilde{\xi} = \Lambda^T (\Psi + \Lambda\Lambda^T)^{-1} (x - x_0) = \Lambda^T R^{-1} (x - x_0) \tag{14}$$

Once the factors for a given data set are computed the residual is obtained from

$$\varepsilon = x - \Lambda\xi \tag{15}$$

To detect damage one defines some metric based on ε and sets up some criterion to decide if the computed metric comes from data of the reference condition or not.

Numerical investigation

We considered a system with 8 equal masses and springs as shown in Fig.1.

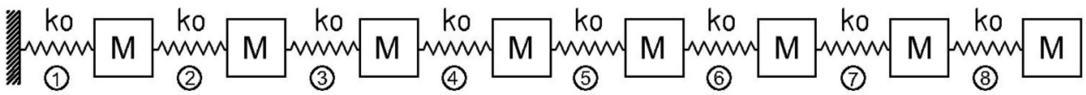


Fig.1 mass-spring system

We assume that the stiffness of the springs is related to the temperature change T by

$$k = k_0(1 + \frac{0.5}{20^3}T^3) \tag{16}$$

where T is in degrees and k_0 is the stiffness at the reference temperature, which is taken as 100 KN/m. The feature vector is the set of identified frequencies. Training data is obtained by performing 1000 simulations where the temperature change on each spring is selected from a spatially correlated Gaussian field. Having established a model, an additional 1000 simulations are carried out to establish the limit of normal behavior. Finally, damage is introduced by reducing the stiffness of various springs (one at a time) and an additional 1000 simulations are carried out (for each damage scenario) to determine what fraction of the cases fall outside the range of normal behavior. This result is known as the power of the test.

Observations

<u>Number of Factors</u>

Figs.2 and 3 shows the Power of the test vs the severity of the damage for two different springs. In this case the solution of the equations is carried out with weighted LS (Bartlett's method) and the covariance of the error is made diagonal heavy using the iterative procedure previously mentioned. As can be seen, the best performance for significant damage is realized when two factors are used but for low damage the results for two and three factors are essentially the same. The singular values of the covariance matrix of the data, normalized to the larges value are (1, 0.052, 0.019, 0.012, 0.008, 0.0049, 0.0027, 0.0008). Examination of these results would lead one to postulate either one or two factors with three being a less likely candidate.

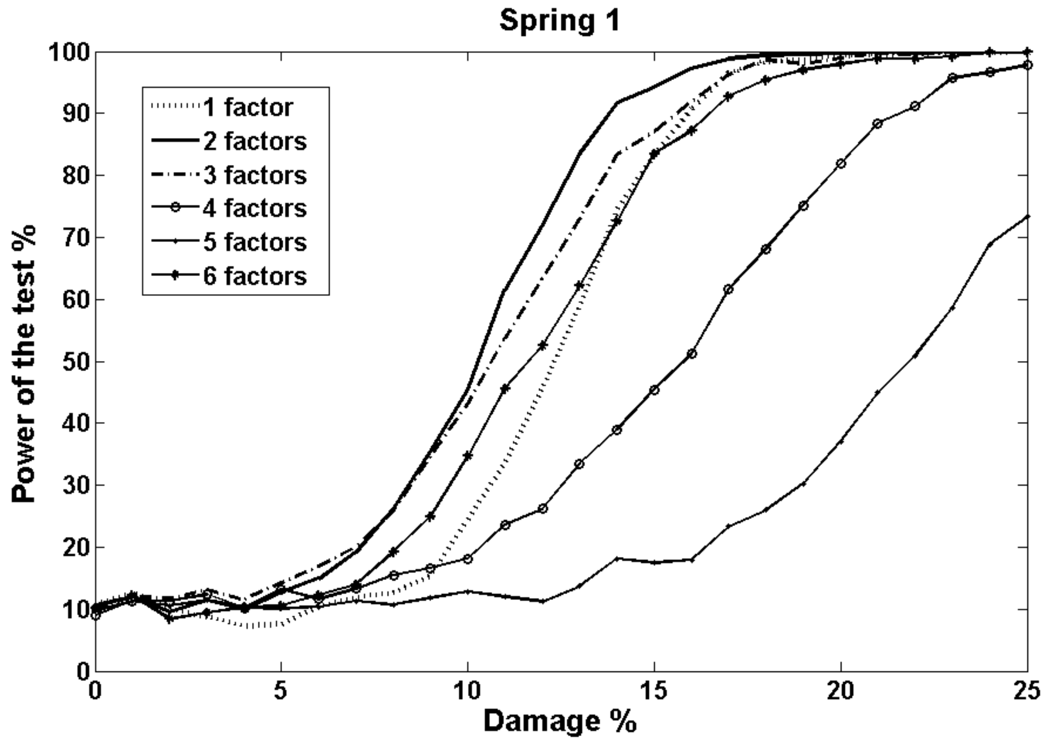


Fig.2 The effect of using different number of factors in the power of the test-spring 1

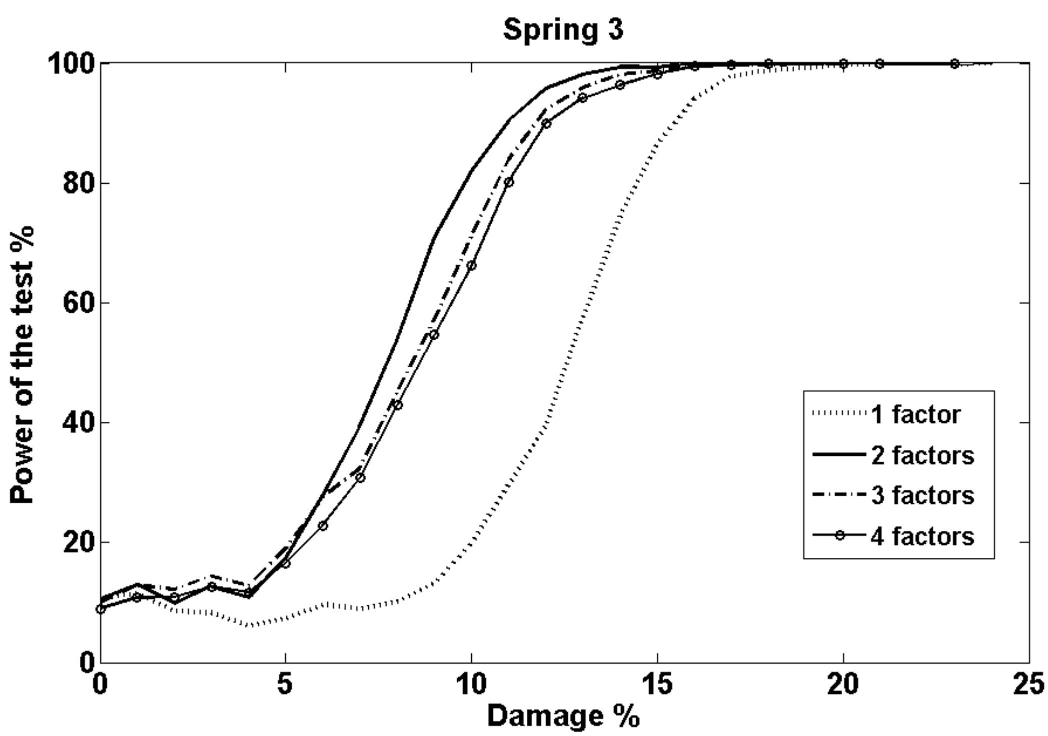
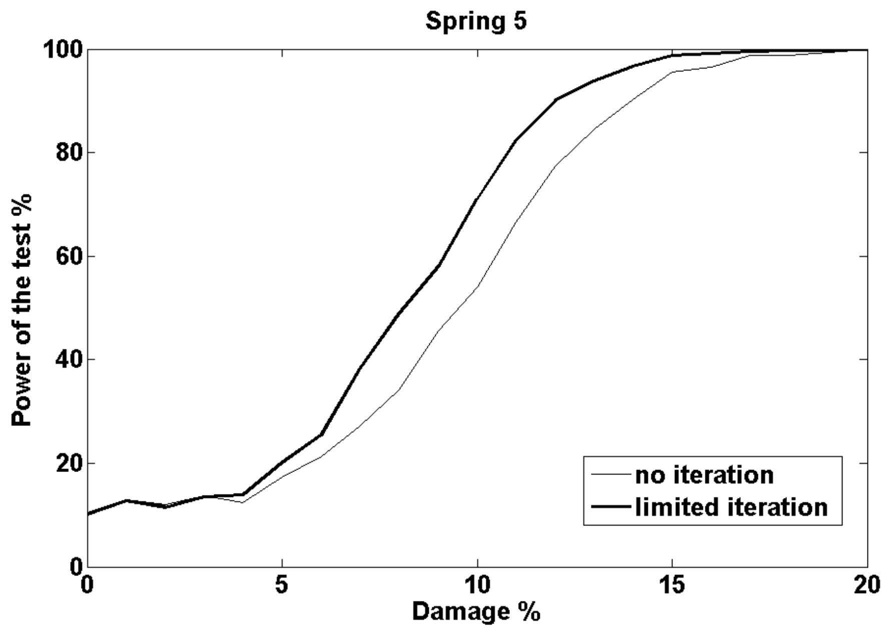


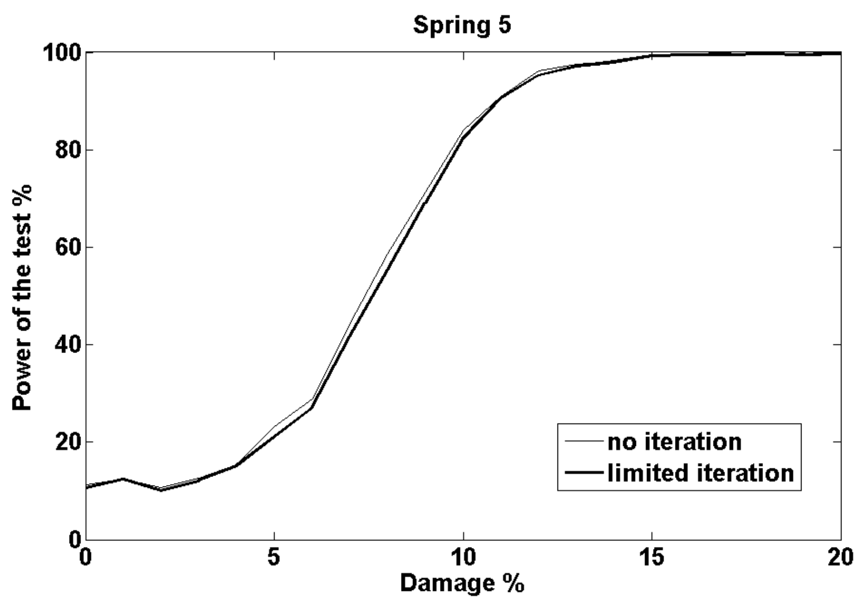
Fig.3 The effect of using different number of factors in the power of the test-spring 3

Making Ψ diagonally Heavy

The effect of doing iteration to make Ψ diagonal is examined next. Fig.4 (a) compares the power of the test when three factors are used and the diagonalization of Ψ is implemented or not. Part (b) of the figure shows results in the case where the number of factors used is two. In both cases Bartlett's method is used to obtain the solution. As can be seen, diagonalization helps notably when three factors are used but has a marginal influence in the case of two factors. Based on these limited results it appears that diagonalization of Ψ is generally useful and that it can make a notable difference in some cases.



(a)



(b)

Fig.4 Effect of implementing iterations to on power of the test-spring5 using three factors (a) and two factors (b)

Solving the Over-determined set of Equations (eqs.12-14)

Results for two and three factors were obtained using the solutions in eqs.12-14 and the power of the test in all cases proved so similar that a graphical display essentially shows the same curve. In this example, at least, this aspect is of negligible importance.

Conclusions

A brief examination of factor analysis in the context of a simple system is done. It appears that the effect of the number of factors on the power of the test is larger in the region where this measure of performance is significant, which is the practical range. Observations regarding whether it is preferable to over-specify or under-specify the number of factors cannot be made based on the limited results obtained here since the results are not conclusive in this regard. In addition, forcing the covariance of errors to be diagonally heavy was found to improve the power of the test, in some cases only marginally but in others quite substantially, the operation, therefore, is recommended. Finally, the specific approach used to solve the system of equations that gives the factors was found to have a very small effect in performance.

References

[1] Sohn H, Effects of environmental and operational variability on structural health monitoring, Phi. Trans. A of the Royal Society on structural health monitoring, 365(1851), pp.539-560.

[2] A.-M. Yan, G. Kerschen, P. De Boe, J.-C. Golinval, Structural damage diagnosis under varying environmental conditions—Part I: A linear analysis, Mechanical Systems and Signal Processing 19 (2005) 847–864

[3] Kullaa, J. Is temperature measurement essential in structural health monitoring. 4th Int. Workshop on Structural Health Monitoring, Stanford University, CA, pp. 717–724, 2003.

[4] A. Deraemaeker, A. Preumont, J. Kullaa, Modeling and removal of environmental effects for vibration based SHM using spatial filtering and factor analysis, in: Proceedings of the IMAC XXIV, S.E.M., St. Louis, USA, 2006.

[5] A. Deraemaeker, E.Reynders, G.De Roeck, J.Kullaa, Vibration-based structural health monitoring using output-only measurements under changing environment, ULB, Active Structures Laboratory, Brussels, Belgium. K.U.Leuven, Department of Civil Engineering, Heverlee, Belgium. Helsinki Polytechnic Stadia, Department of Mechanical and Production Engineering, City of Helsinki, Finland, 2007.

[6] R.A. Johnson and D.W. Wichern. Applied multivariate statistical analysis. Prentice-Hall, Englewood Cliffs, New Jersey, 2002.

[7] A. Deraemaeker, K. Worden (Eds). New trends in vibration based structural health monitoring, CISM international center for mechanical sciences vol. 520, Springer, Wien New York, 2011

[8] R.L. Gorsuch. Factor analysis. Lawrence Erlbaum, Hillsdale, New Jersey, 1983.

[9]T. Hill, P. Lewicki. Statistics methods and applications a comprehensive reference for science, industry, and data mining. StatSoft, Tulsa, Oklahoma, 2006.

[10] Kullaa, Structural health monitoring of a crane in variable configurations, in: Proceedings of ISMA 2004, Leuven, Belgium, 2004, pp. 457–469.