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Design and experimental verification of an extended KDamper - based vibration absorber

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Abstract. Recently, researchers have proposed an innovative negative stiffness-based vibration control concept, namely the KDamper absorber. The envisaged mechanism comprises a combination of appropriate stiffness, damping and mass elements, including a negative stiffness element. Previous studies have formulated the mathematical framework of the system, as well as design and optimization algorithms. These take into consideration the application of interest and geometrical and manufacturing limitations, regarding the vibration control components, including the realization of the negative stiffness mechanics. The KDamper has been numerically and analytically implemented as a vibration control concept for seismic protection of bridges, buildings as well as wind turbines and noise mitigation panels, while results indicated its beneficial effect towards vibration attenuation. For the first time, an experimental set-up of the proposed mechanism is designed by adopting an optimization procedure and tested under horizontal harmonic and seismic shaking. Results highlight the vibration control properties of the proposed system and validate previous numerical and analytical studies. The experimental device serves as a proof of concept of the KDamper absorber and showcases its advantages as well as application limitations that should be considered in future research.

1. Introduction

The idea of the KDamper vibration absorber is based on widely used vibration control devices that have been proposed by researchers in the past. As an example, the Tuned Mass Damper (TMD), is one of the well-established and well-documented mechanisms to attain vibration mitigation of structures [1,2]. The idea comprises an oscillating mass, stiffness elements and dampers that are attached to the original, uncontrolled structure and when tuned accordingly, provide significant vibration attenuation properties. Since its introduction and first application, the TMD has been applied in numerous applications and variety of structures including bridges, skyscrapers [3,4], and in the bases of buildings [5–7]. While the Tuned Mass Damper (TMD) is acknowledged as a dependable vibration control technique with extensive testing, it comes with notable drawbacks that cannot be ignored. Firstly, it demands substantial mass compared to the overall structure weight. In addition, environmental factors and uncertainties in

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material behavior can cause gradual detuning of the system, resulting in a loss of its attenuation properties and effectiveness [8].

Inspired by previous advances in vibration absorption devices, Antoniadis et al. [9] introduced the KDamper, an innovative passive vibration control concept. This novel approach combines appropriate stiffness, mass, and damping elements, including a negative stiffness element. Unlike positive stiffness elements that resist movement, negative stiffness elements facilitate motion [10]. Integrating a negative stiffness element enhances the damper's inertial forces, reducing at the same time the need for a large oscillating mass. To this end, a device with extraordinary damping properties is introduced. Moreover, careful allocation of stiffness-mass elements ensures both static and dynamic stability, preserving the initial stiffness of the structure and preventing potential instabilities [11–13]. Antoniadis & Paradeisiotis [9], developed negative stiffness elements using pre-stressed disc (Belleville) springs arranged in specific geometrical configurations. For structural systems, negative stiffness can be achieved using conventional pre-compressed springs arranged appropriately, post-buckled beams, plates, and other pre-stressed mechanical elements [14–16]. Specifically, Sarlis et al. [17,18], utilized a highly compressed machined spring (CS) that generates force in the direction of motion (negative stiffness) and decreases with displacement, ensuring system stability even at large horizontal displacements of the system.

The KDamper vibration control system has been investigated for safeguarding bridges [19,20], wind turbines [21,22], and various structural systems [23–26], successfully reducing the displacement demand at the base level. Notably, Kapasakalis et al. [23], introduced an extended version of the KDamper concept as a vibration absorber for low-rise buildings. In this application, the system was employed in addition to conventional base isolation methods for a typical reinforced concrete structure. The optimization process and subsequent analysis involved a simplified structural model, assuming both linear structural behavior and linear KDamper components.

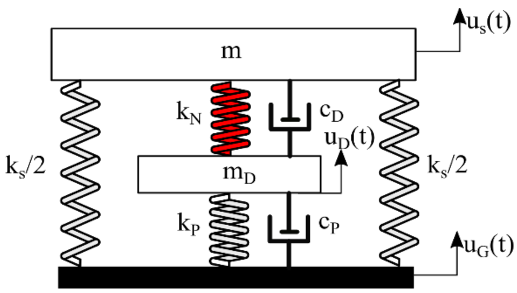


Figure 1. Schematic presentation of the Extended KDamper (EKD) absorber excited at its base.

In this research work, for the first time, an experimental set-up of the proposed Extended KDamper vibration control mechanism is designed and constructed by adopting an optimization procedure and tested under horizontal harmonic and seismic shaking. Results highlight the vibration control properties of the proposed system and validate previous numerical and analytical studies. The experimental device serves as a proof of concept of the KDamper absorber and showcases its advantages as well as application limitations that should be considered in future research.

2. The Extended-KDamper Concept

The proposed vibration control system, an extension of the KDamper referred to as the EKD system (shown in Figure 1), follows a similar framework to the KDamper. Like its predecessor, the EKD system incorporates masses, negative stiffness (NS), positive stiffness elements, and artificial dampers. However, the main modification lies in the system's arrangement: the positive stiffness element (k_P) links the damper mass (m_D) to the system's base, while the negative stiffness element (k_N), is positioned between the damper mass (m_D) and the oscillator's mass (m). Additionally, an extra damper is introduced and located in parallel with the NS element, resulting in the presence of two dampers, denoted as c_D and c_P.

The following equations of motion for the EKD are derived for base excitation u_G :

$$
\begin{aligned}
m\ddot{u}_S + c_{NS}(\dot{u}_S - \dot{u}_D) + k_{NS}(u_S - u_S) + k_S u_S &= -m\ddot{u}_G \\
m_D\ddot{u}_D - c_N(\dot{u}_S - \dot{u}_D) - k_{NS}(u_S - u_D) + k_{PS}u_D + c_{PS}\dot{u}_D &= -m_D\ddot{u}_G
\end{aligned}
\tag{1}
$$

Assuming a harmonic excitation and steady-state response, the equations of motion (1) of the EKD system can be written as:

$$
\begin{aligned}
-\omega^2 m\tilde{U}_S + j\omega c_{NS}(\tilde{U}_S - \tilde{U}_D) + k_{NS}(\tilde{U}_S - \tilde{U}_D) + k_S\tilde{U}_S &= -m\ddot{U}_G \\
-\omega^2 m_D\tilde{U}_D - j\omega c_{NS}(\tilde{U}_S - \tilde{U}_D) - k_{NS}(\tilde{U}_S - \tilde{U}_D) + k_{PS}\tilde{U}_D + j\omega c_{pS}U_D &= -m_D\ddot{U}_G
\end{aligned}
\tag{2}
$$

Based on the above, the EKD system Transfer Functions result:

$$
\begin{bmatrix} \tilde{H}_{US} \\ \tilde{H}_{UD} \end{bmatrix} = \begin{bmatrix} \tilde{U}_S / \ddot{U}_G \\ \tilde{U}_D / \ddot{U}_G \end{bmatrix} = -\tilde{H}^{-1} \begin{bmatrix} m \\ m_D \end{bmatrix}
\tag{3}
$$

$$
\tilde{H}_{AS} = \ddot{U}_S / \ddot{U}_G = 1 - \omega^2 \tilde{H}_{US}
\tag{4}
$$

$$
\tilde{H}_{AD} = \ddot{U}_D / \ddot{U}_G = 1 - \omega^2 \tilde{H}_{UD}
\tag{5}
$$

$$
\tilde{H} = \begin{bmatrix} -\omega^2 m + j\omega c_{NS} + k_{NS} + k_S & -j\omega c_{NS} - k_{NS} \\ -j\omega c_{NS} - k_{NS} & -\omega^2 m_D + j\omega(c_{NS} + c_{PS}) + k_{PS} + k_{NS} \end{bmatrix}
\tag{6}
$$

The major goal of the extended KDamper concept is to ensure that displacements and velocities remain within acceptable limits and display a realistic and efficient design. The following non-dimensional parameters that concern the EKD are introduced:

$$
\begin{gathered}
\mu = m_D / m \\
k_D = k_{NS} + k_{PS} \\
\omega_D = \sqrt{k_D / m_D} = \sqrt{(k_{NS} + k_{PS}) / m_D} \\
\zeta_{NS} = c_{NS} / 2\omega_D m_D = c_{NS} / 2\sqrt{(k_{NS} + k_{PS}) m_D} \\
\zeta_{PS} = c_{PS} / 2\omega_D m_D = c_{PS} / 2\sqrt{(k_{NS} + k_{PS}) m_D} \\
m_{tot} = m + m_D \\
f_0 = \sqrt{k / m_{tot}} / 2\pi = \sqrt{(k_S + k_{PS}k_{NS} / k_{PS} + k_{NS}) / (m + m_D)} / 2\pi
\end{gathered}
\tag{7}
$$

where μ is the mass ratio of the additional mass of the EKD, ζ_{NS} and ζ_{PS} are the damping ratios of the artificial dampers c_{NS} and c_{PS}, respectively, and f_0 is the EKD nominal frequency.

3. Design & Experimental Setup

This section briefly presents the design of the Extended KDamper (EKD) experimental prototype, which serves as a proof of concept of the original analytical and numerical frameworks. This simple experimental setup, depicted in Figure 2 (a), consists of steel plates, aluminium parts, roller bearings and a prismatic coil spring. The configuration is designed for an oscillating mass (*m*) of approximately

16 kg and for an internal (m_D) mass of approximately 0.82 kg (5% of the mass m). The negative stiffness spring and mass m_D configurations are easily removed when needed, in order to apply the testing protocol to the equivalent SDof oscillator. This serves as a comparison between the NS enhanced and uncontrolled structure, and provides insight of the beneficial vibration control effects of the KDamper to an existing structure/base.

To ensure the effectiveness of the EKD system, it necessitates high values of negative stiffness (NS). The practical option for realizing these NS elements in such applications involves incorporating elastic forces. Various mechanical designs, including conventional positive stiffness prestressed elastic mechanical elements like post-buckled beams, plates, shells, and pre-compressed springs arranged in specific geometric configurations, can generate controlled NS with the desired magnitude. In this study, the proposed mechanism draws inspiration from the NS design introduced by Kapasakalis et al. [23]. A schematic representation of this configuration is depicted in Figure 2. Herein, the negative stiffness element is realized by a prestressed prismatic spring and a rigid link (steel) connecting the internal mass (m_D) to the oscillating mass (m) of the EKD through hinged connections (ball bearings). This configuration leads to a non-linear force (negative stiffness - as the force is in the direction of movement) that decreases as horizontal displacement increases. In this way, the overall stability of the system is reassured.

The following expressions can be derived for the potential energy U, the nonlinear force N, and the equivalent nonlinear stiffness k_N of this mechanism:

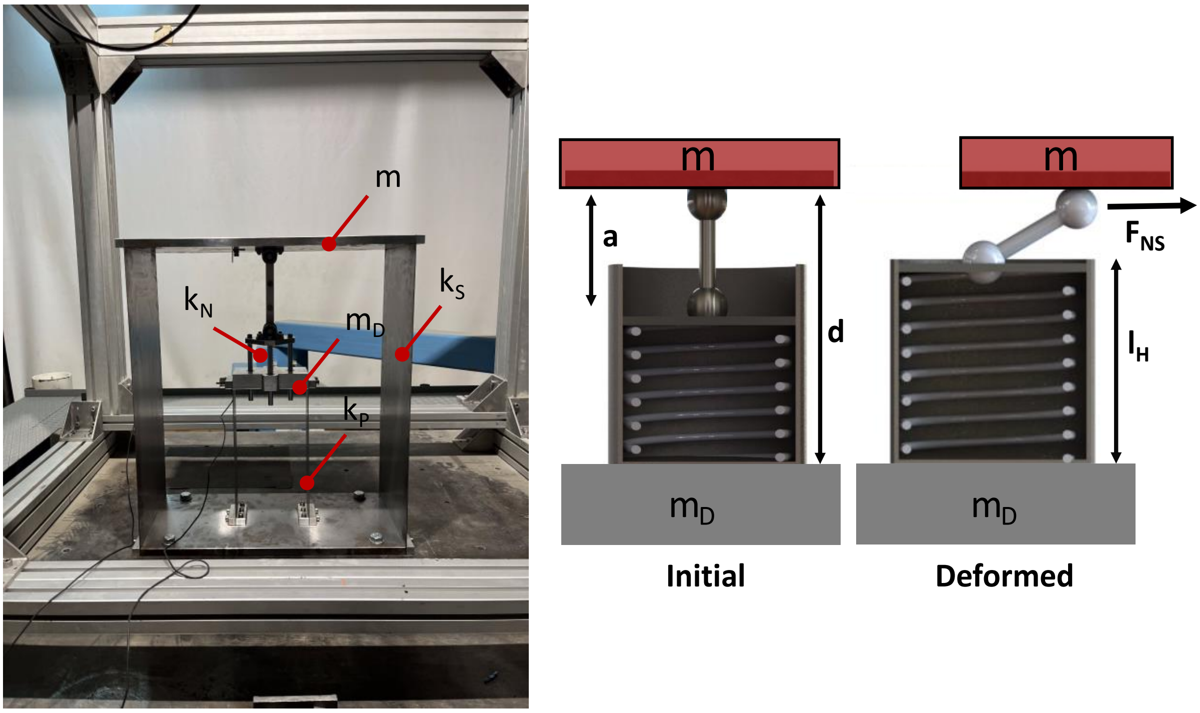
$$U(u_{NS}) = \frac{1}{2} k_c \left(l_H - l_{Hi} \right)^2 \tag{8}$$

$$N(u_{NS}) = \frac{\partial U}{\partial u_{NS}} = -k_c \left(1 + \frac{l_{Hi} - d}{\sqrt{a^2 - u_{NS}^2}} \right) u_{NS} \tag{9}$$

$$K_N(u_{NS}) = \frac{\partial N}{\partial u_{NS}} = -k_c \left(1 + \frac{\left(l_{Hi} - d \right) / a}{\sqrt[3]{\left(1 - u_{NS}^2 / a^2 \right)}} \right) \tag{10}$$

where a is the undeformed height of the NS device, d the length of the lever arm, l_{HI} the length of the undeformed conventional spring, and k_c the spring stiffness. The negative stiffness configuration is schematically presented in Figure 2 (b) and the parameters of the NS mechanism are presented in detail in Table 1.

Following the formulation of the mathematical framework, the goal is to identify effective parameters as design variables for the proposed concept, aiming to attain the most efficient vibration attenuation strategy. Through a constrained optimization process, these design variables, along with other device parameters, are carefully selected. The objective is to obtain an optimal combination of EKD properties that minimize the relative displacement (drift) of the oscillating mass, ensuring at the same time that the maximum accelerations do not surpass a predefined percentage of the peak ground acceleration (acceleration filter). To facilitate this optimization, a new metaheuristic algorithm, namely the harmony search algorithm (HS), is employed [27]. More details of the optimization process are provided in previous research works of the authors [23,26].



(a) (b)

Figure 2. (a) View of the EKD prototype. (b) Conceptual design of the negative stiffness spring.

Table 1. Properties of the Negative Stiffness (NS) mechanism

k_c [kN/m]	a [m]	l_{HI} [m]	d [m]	F_{NS}
4100	0.146	0.1	0.203	73.8

The oscillating mass (m) is constructed by a steel plate with dimensions 0.25x0.15x0.01, the internal mass of the damper (m_D) by an aluminium plate with dimensions 0.1x0.5x0.015 and the positive stiffness elements k_P and k_S by steel sheets with dimensions 0.27x0.07x0.001 and 0.585x0.25x0.0015, respectively. The optimized parameters of the EKD device are presented in Table 2.

Table 2. Optimized parameters of the EKD device.

m [kg]	m_D [kg]	k_S [kN/m]	k_P [kN/m]	k_N $(u_{NS}=0)$ [kN/m]
16.3	0.815	1770	1440	-500

A series of single frequency-harmonic input tests were carried out, at the shaking table of the Soil Mechanics Laboratory, National Technical University of Athens, which has one degree of freedom and is able to provide motion in the horizontal direction. In addition, the setup is tested under a selection of real strong earthquake excitations. For the experimental measurements, accelerometers were installed at the base of the absorber, the oscillating mass (m) and the internal mass (m_D) while small amplitude excitations were applied to ensure the linear behaviour of the absorber. Two different configurations are tested; (a) the EKD and (b) a SDoF mechanism with the same mass and spring stiffness properties. No extra damping elements are added in neither configuration while the structural damping ratio of the EKD was calculated from a free vibration test to approximately $\xi = 8\%$ and the damping ratio of the SDoF $\xi = 2\%$.

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4. Results

Figure 3 illustrates the experimentally and theoretically estimated frequency response of the EKD absorber and the SDoF oscillator. Results validate the theoretical expected behaviour of the KDamper, while the analytically derived equations of motion are confirmed. The comparison of the corresponding system's magnification factor when the seismic mass is applied only on the k_S stiffness springs (black line) indicates the effectiveness of the device. Namely, the fundamental resonance peak is reduced by approximately 60% and by extension, this leads to an isolation frequency of ≈ 1.4 Hz which is in line with the intended goal of the device.

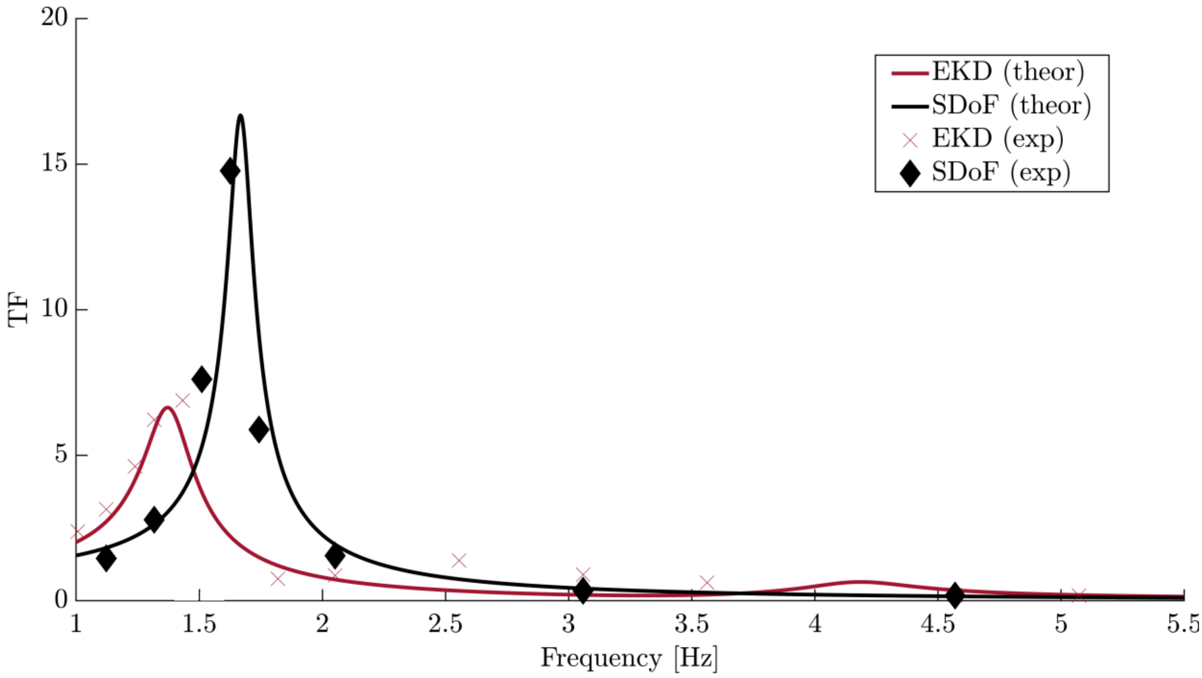

Figure 3. Theoretical and experimental dynamic response of the EKD device vs the SDoF oscillator.

Apart from the initial harmonic excitations, the EKD device was subjected to dynamic time-history analyses, for various real earthquake records. **Figure 4** illustrates the comparison between the theoretical and experimental dynamic response results of the EKD device for the Kocaeli (1999) earthquake; the upper story acceleration time-histories are depicted in (a), while the first-story drift time-histories in (b). It is clearly shown that the analytically calculated acceleration and first story drifts describe accurately the experimental response of the device, validating once more the EKD concept.

Figure 5 portrays a comparison between the SDoF oscillator's and EKD's dynamic response, again for the Kocaeli (1999) earthquake, in terms of (a) accelerations at mass m, and (b) relative displacements (drifts) between the ground and the oscillating mass (m). Specifically, compared to the initial SDoF oscillator, the EKD reduces the maximum acceleration values by approximately 62%, and the maximum drift values by approximately 19%. By all means, a wider range of seismic excitations should be implemented to assess the capabilities of the prototype. However, these initial results are promising, clearly revealing the compelling capabilities of the KDamping concept.

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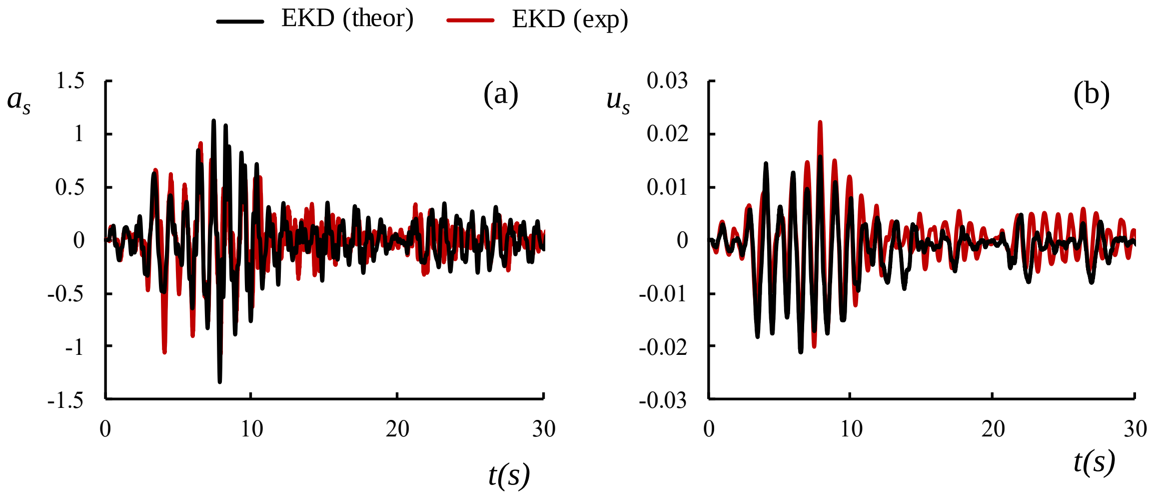

Figure 4. Comparison between the theoretical and experimental dynamic response results of the EKD device for the Kocaeli (1999) earthquake: (a) acceleration at mass *m*, (b) relative displacement ($u_s - u_G$) time histories.

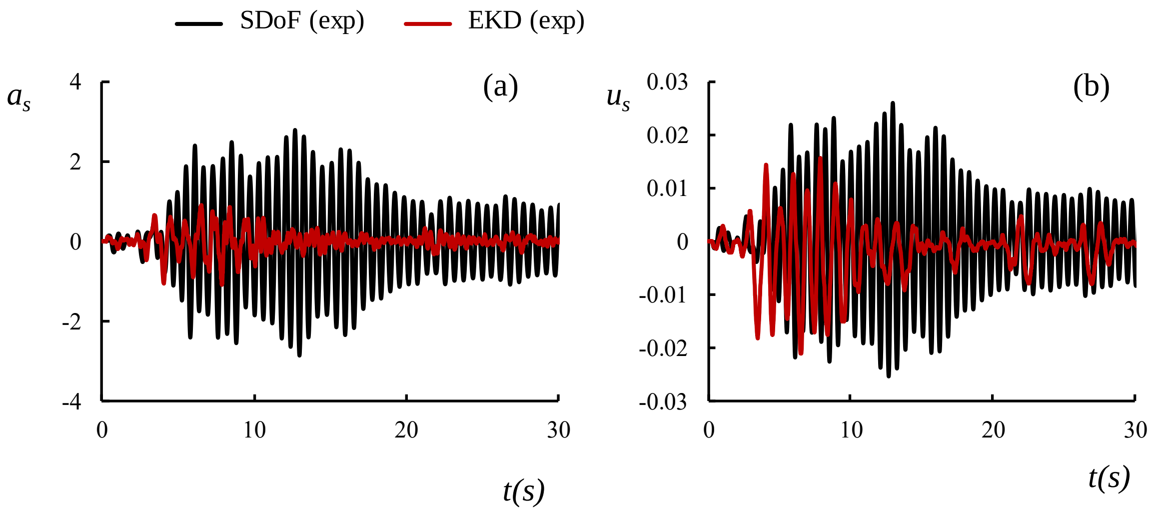

Figure 5. Experimental dynamic response of the EKD device vs the SDoF oscillator for the Kocaeli (1999) earthquake: (a) acceleration at mass *m*, (b) relative displacement ($u_s - u_G$) time histories.

5. Summary & Conclusions

This paper presents an experimental setup of a negative stiffness-based vibration absorber, namely the Extended – KDamper (EKD). The mathematical framework of the concept is initially derived and subsequently the configuration is designed and optimized using a constrained optimization algorithm. The prototype is tested under dynamic base excitation (shaking table tests) using both single-frequency harmonic excitations to derive its frequency response and real earthquake motions. A comparison between an equivalent SDof oscillator is subsequently undertaken both in terms of maximum measured accelerations and displacements.

Experimental results validate the analytical framework, showcasing the beneficial effect of the NS enhanced mechanism both in terms of induced damping (reduction of maximum dynamic responses) and alteration of the natural frequency of the system. Time-history results of real seismic motions also indicate a significant reduction of the acceleration and displacement values of the KDamper system, compared to the equivalent SDof. As a conclusion, results serve as a proof-of-concept and indicate the feasibility of this novel dynamic absorber, hence rendering the concept a compelling vibration control mechanism and seismic retrofitting technology.

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