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Technical Paper

Self-optimizing thermal error compensation models with adaptive inputs using Group-LASSO for ARX-models☆

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ABSTRACT

Thermal errors have a significant impact on the machining accuracy of five-axis machine tools. The thermal adaptive learning control (TALC), which combines adaptable data-based models and on-machine measurements, realizes a precise and robust long-term reduction of thermal errors of machine tools. To further improve the precision and the robustness of data-based models for thermal error compensation, this publication introduces a new method to realize adaptive model inputs. This method combines the Group-LASSO (least absolute shrinkage and selection operator) for autoregressive models with exogenous inputs (ARX) and the particle swarm optimization to realize a simultaneous estimation of the optimal inputs, the model structure, and the model parameters. Additionally, the self-optimization ability of thermal error compensation models, based on the TALC, is increased by introducing error-specific action control limits to define the frequency of model updates. The newly developed methods are applied to compensate the thermal errors of a swiveling and a rotary axis of a five-axis machine tool during a long-term test series of 350 h. Randomly generated speed profiles of the linear and rotary axes as well as the spindle and changing ambient conditions ensure a high variety of thermal load cases within in the analyzed long-term test series. The results show that the prediction accuracy measured as peak-to-peak values and the robustness of the thermal error compensation models are improved by up to 36% and 58% respectively when adaptive instead of static model inputs are used. Furthermore, the compensation results of the new method outperform the previously used sequential input selection method regarding prediction accuracy and repeatability. The average peak-to-peak value of the compensated translational thermal errors is reduced by 23% and the repeatability of the corresponding compensation results is increased by 57%. Consequently, the consideration of the resulting model structure during the selection of the optimal model inputs significantly enhances the performance of the resulting data-based thermal error compensation models.

1. Introduction

Compensation strategies are a sustainable and cost-efficient tool to improve the machining accuracy of manufacturing systems such as five-axis machine tools. Particularly, the compensation of time-invariant geometrical errors in machine tools already have a widespread application in industry. However, the compensation of time-variant thermal errors is still a huge challenge in industry and academia, although thermal errors cause a large fraction of inaccuracies on produced workpieces as discussed by Mayr et al. [1]. Thermal errors of machine tools are strongly influenced by various internal and external heat sources as described by Bryan [2]. Consequently, improving the thermal behavior of machine tools often results in complex optimization problems defined by a trade-off between precision, productivity, and the resulting power consumption. In general, a huge amount of energy is spent for machine cooling and air-conditioning of the workshops to realize precise and robust machining operations. To solve complex optimization problems, artificial intelligence is increasingly applied to realize self-optimizing production systems as shown by Möhring et al. [3] and Chen et al. [4]. Regarding thermal errors, this boosts a transformation from resource-based towards intelligence-based countermeasures, because the energy-intensive tempering measures can be

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replaced by control-based thermal error compensation strategies. The integration of self-learning systems into the thermal error compensation of machine tools empowers the adaption to changing boundary conditions and solves the challenge of long-term robustness of thermal error compensation models. This provides the foundation for precise and energy-efficient machine tools which are able to produce parts with tight tolerances.

So far, predominantly data-based models have been applied to compensate thermal errors of machine tools, as for example presented by Brecher et al. [5], Gebhardt et al. [6], Mayr et al. [7] and Mareš et al. [8]. These models represent the relationship between the considered thermal errors and their main origins, such as environmental influences, drives, bearings, process heat and the heat distribution of relevant fluidic media, which are represented by the model inputs. Consequently, selecting the optimal inputs, which are for example temperatures or data provided by the numerical control of the machine tool, is crucial for the application of data-based compensation models. Recent thermal error compensation approaches also use a combination of thermal camera images and temperature measurements as inputs for data-based compensation models to get more detailed information of the overall temperature distribution [9]. When using temperature measurement signals as model inputs, it is essential to measure at temperature-sensitive locations and to select the optimal measurement signals as inputs for each thermal error model separately. Strategies for autonomous selection of the optimal model inputs are indispensable to ensure a precise and robust data-based models. Furthermore, the thermal behavior of machine tools is significantly influenced by the environmental conditions in their use-phase which are mostly unknown to the machine tool manufacturers. This includes for example, the placement of other machine tools in the workshop or the temperature changes between different seasons. Therefore, an automatic input selection is essential to realize the required adaptability for a widespread industrial application.

In the context of data-based compensation models, statistical approaches are frequently applied to identify the optimal inputs using temperature and thermal error measurements, because physical models to identify the optimal sensor locations, as used by Herzog et al. [10] and Wei et al. [11], are often unavailable. Furthermore, Abdulshahed et al. [12] applied thermal camera images to obtain reasonable positions for the surface mounted temperature sensors. The goal of statistical approaches is to identify the most relevant model inputs out of the available measurement signals before the model parameters are identified. This procedure is commonly divided into two main steps. In the first step, the available input signals are clustered into different groups with similar characteristics by using for example fuzzy clustering [13–15]. In the following screening step, the most significant representative from each cluster is selected as optimal model input by applying mathematical correlation methods such as gray correlation [15,16] or common correlation coefficients [17–19]. As stated by Liu et al. [20], the gray correlation is not capable of representing negative correlations because it only analyzes the shape similarities between the signals. However, inputs with large negative correlations can be a good predictor for thermal errors. Furthermore, the cluster-based methods often exhibit the problem, that the selected inputs are strongly correlated to the number of clusters which are typically defined by engineering experience. To reduce these limitations, Li et al. [21] introduced a systematical temperature information parameter, which includes information about the temperature value, the temperature shape, and the similarity between the temperatures and the thermal errors, to cluster the temperature variables using fuzzy C-means clustering. Considering the thermal error during the clustering allows to define an individual number of clusters for different thermal errors. Subsequently, the correlation coefficient is used to define the optimal model inputs.

To omit the clustering step, Miao et al. [22] applied the principal component regression to avoid multi-collinearity between the selected model inputs. Furthermore, Liu et al. [23] applied the principal component regression and the ridge regression to tackle the multi-collinearity problem after selecting the most relevant model inputs based on correlation coefficients. Tsai et al. [24] developed a method consisting of a ranking and screening step to handle the collinearity between the considered temperature measurements. In the first step, the principal component analysis and the principal component regression were applied to rank the available temperature sensors. Afterwards, singular value decomposition was used to eliminate redundant sensor signals. According to Liu et al. [25], the reduction of multi-collinearity between the potential model inputs, before considering the influence weight in the thermal error model, can result in selecting inputs with a low significance. Therefore, a method using the variance inflation factor and gray correlation was developed to define the optimal model inputs. However, this method suffers from the previously mentioned inability of gray correlation to directly represent negative correlations and requires an additional editing of the sensor signals to avoid an exclusion of negatively correlated input signals. Li et al. [26] developed a method, which is based on the combination of an improved binary grasshopper optimization algorithm and the stepwise regression. This combination also allows the elimination of the clustering step during the selection process of the optimal temperature variables. Tan et al. [27] directly selected the temperature-sensitive points for spindle error compensation by applying the least absolute shrinkage and selection operator (LASSO). However, they applied LASSO only to obtain the optimal set of inputs without using the resulting model parameters. The identified temperatures were applied as inputs for a least squares support vector machine model.

For further improvement, the performance of their input selection methods, some researchers included additional characteristics of the thermal system into the selection process. Liu et al. [15] and Fu et al. [28] considered for example geometrical information about the temperature sensor locations in their input selection methods. However, this requires more detailed information about the specific sensor position and limits the simple and universal applicability of the method. Another approach, developed by Tan et al. [29] includes a specific cost function, which considers the number of selected model inputs and the achievable thermal error reduction. Furthermore, the input selection process comprises the optimization of the hyperparameters for the applied support vector machine model, so that input selection and modeling are combined.

Another major limitation of the presented methods is the significant influence, that the characteristics of the underlying measurement data have on the selected inputs. Consequently, changing thermal boundary conditions, which are not present in the training data of the data-based models, can result in inaccurate compensation results. Miao et al. [22] described that optimal model inputs can vary over time and significantly depend on the considered thermal load cases. Furthermore, Liu et al. [20] analyzed the identified temperature-sensitive points when using data from different seasons to indicate that the temperature sensitive points vary over time. Consequently, changes of the temperature-sensitive points can result in a decreased prediction accuracy and robustness of data-based compensation models. To counteract this issue, Liu et al. [23] considered data from different seasons and established individual compensation models for different levels of the ambient temperature to obtain precise compensation results. However, this procedure can only be realized when a large amount of data with a high variation of environmental conditions is available. Furthermore, this approach is more time-consuming to implement in comparison to more general compensation models for all of the considered thermal errors. Even a large amount of empirical data cannot ensure that all relevant long-term changes and thermal boundary conditions are represented in the data at the beginning of the life cycle of the considered machine tool.

Therefore, self-optimizing thermal error compensation models are required to realize a precise and robust reduction of the thermal errors even in the case of changing boundary conditions. Zimmermann et al.

[30] developed an adaptive input selection method for data-based thermal error compensation models, which enables an automated and adaptive selection of the optimal model inputs even after the initial model training. The adaptive input selection was applied to the Thermal Adaptive Learning Control (TALC) presented by Blaser et al. [31], which already enables an automatic recalibration of the model parameters when unknown thermal boundary conditions occur. The applied method for the adaptive input selection is an iterative procedure, which includes a statistical signal comparison method based on the Time Series Cluster Kernel (TCK) developed by Mikalsen et al. [32]. However, this iterative procedure decouples the input selection step from the subsequent modeling step as almost all existing methods for optimal input selection of data-based models, so that the interaction between the selected optimal inputs and the model structure cannot be considered. Consequently, this method is also limited regarding the analyzed input combinations. After a certain model input is selected, it cannot be removed to choose another input combination. Furthermore, the signal comparison is not able to represent system dynamics such as a time shift between the measured temperatures and the occurring thermal errors. To overcome these limitations and to improve the prediction accuracy and robustness of the resulting compensation models, this paper introduces a new Group-LASSO based procedure for the adaptive input selection, which allows a simultaneous optimization of the optimal inputs and the resulting model structure. Additionally, the automation level of the TALC is further increased by introducing a new method, which automatically defines the tolerances. The violation of these action control limits by the residual errors triggers a model update.

Section 2 describes the TALC in combination with the adaptive input selection and the autonomously triggered model updates in detail. Afterwards, in Section 3, the new method for the adaptive and error-specific definition of the action control limits is presented. In Section 4 the newly developed method for adaptive input selection of ARX-models based on the Group-LASSO and the particle swarm optimization is introduced. Section 5 describes the experimental setup of the considered five-axis machine tool including the applied measurement cycle to identify the thermal position errors of the A-axis and the thermal position and orientation errors of the C-axis. In Section 6, the results of the new method including a statistical analysis regarding its repeatability are presented. Section 7 summarizes the developments.

2. Thermal adaptive learning control (TALC) with autonomous and adaptive input selection

The TALC, introduced by Blaser et al. [31], realizes robust long-term compensation results for thermal errors of machine tools by combining data-based models and on-machine measurements. To handle the changing sampling rates, Mayr et al. [33] introduced ARX-models in combination with the weighted least squares to model the thermal errors. However, in the initial TALC, the model inputs are predefined before the model calibration starts and only the model structure is individually adapted for each considered thermal error. In the calibration phase, the temperatures which are used as model inputs and the thermal errors are measured with a constant sampling rate. At the end of the calibration phase, the model structure and the model parameters are calculated automatically. The model structure, which is represented by the order of the inputs and outputs, is defined by applying the Akaike Information Criterion (AIC). After the initialization of the models, the compensation phase starts, and periodical on-machine measurements are performed to check whether the residual errors exceed the predefined action control limits (ACL). When an ACL exceedance is detected, a so-called No-Good (NG) mode starts with the same measurement frequency as during the calibration phase to record new training data with the previously unknow thermal load cases. After the (NG) mode, the model parameters are updated and the compensation with the reduced on-machine measurement frequency continues.

To increase the self-learning ability of the TALC, Zimmermann et al.

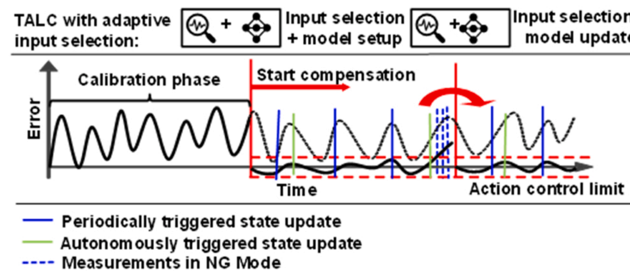

Fig. 1. Concept of the TALC in combination with the adaptive input selection and autonomously triggered on-machine measurements.
Adapted from Zimmermann et al. [34].

[30] developed the adaptive input selection method. This approach enables a fully autonomous selection and recalibration of the used model inputs during the initial model training and the model updates. Furthermore, Zimmermann et al. [34] replaced the periodically performed on-machine measurements by adaptive on-machine measurements which are triggered based on temperature measurements when unknow thermal conditions occur to optimize the trade-off between precision and productivity of the TALC. Fig. 1 summarizes the concept of self-learning thermal error compensation models that combine the TALC with the adaptive input selection and the on-demand triggered on-machine measurements. The self-learning ability of the automated and adaptive input selection represents a universal interface between the machine hardware and the compensation strategy. Consequently, it can be implemented on many different five-axis machine tools, if a measurement cycle for the separation of the axis-specific thermal error is available.

The existing approach to adaptively select inputs for data-based models is based on the TCK which was developed by Mikalsen et al. [32] specifically for time series with missing data. In total, the method for adaptive input selection consists of five steps. In the first step, the temperature and thermal errors are standardized. Afterwards, the standardized temperature measurements are clustered, so that temperature sensors with similar characteristics belong to the same group. This step is necessary to avoid collinearity between the chosen model inputs. In the third step, the standardized temperature and thermal error measurements are compared using the TCK, and the temperature sensor with the highest similarity is identified as first model input. After the selection of one temperature sensor all other temperature sensors from the same cluster are excluded from the further selection process. In the consecutive step, the corresponding compensation model with the selected input is created and the resulting residual, which cannot be predicted by the previous model, is calculated. During the next iterations the residual of the current model is used to identify the next model input which describes the remaining residual most suitably. Consequently, compensation models with a different number of inputs are established until the maximal number of inputs is reached. Then, the optimal compensation model is selected by using for example the AIC or the Bayesian Information Criterion (BIC).

3. Adaptive ACL

In the previously described concept of self-optimizing thermal error compensation models, the ACL is defined by the operator and is constant over time. However, defining a suitable ACL is crucial for the accuracy and productivity of the considered machine tool because it is substantial for triggering the model updates. On the one hand, an undersized ACL results in frequently conducted model updates without improving the prediction accuracy and thus in a reduced productivity. On the other hand, setting the ACL limit too high results in a reduced prediction accuracy of the TALC. Therefore, the self-optimization ability of the TALC is increased by including error-specific adaptive ACLs, which do not

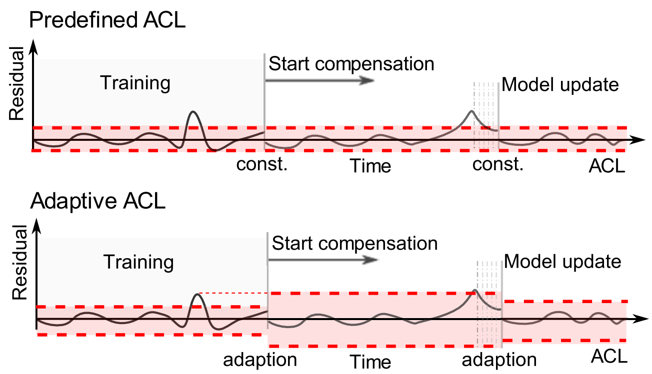

Fig. 2. Comparison between the predefined and the error-specific adaptive ACL, which is based on the maximum residual error during model training.

require a detailed knowledge about the thermal behavior of the machine tool. The comparison between the previous ACL and newly developed adaptive ACL is given in Fig. 2.

The individual and adaptive ACL is defined by the maximum residual of each thermal error in the training phase of the data-based model. After each model update the ACL is recalibrated considering the residual of the new model.

4. Adaptive input selection for ARX-models based on Group-LASSO

The developed method realizes adaptive inputs for ARX-models using Group-LASSO and particle swarm optimization to achieve a simultaneous estimation of the optimal inputs and the corresponding model parameters. Consequently, the interaction between the optimal inputs and the selected model structure is considered in the selection process, which is not the case in the previously developed method. The main steps of the developed method are summarized in Fig. 3.

4.1. Standardization

Standardization is a commonly applied method to enable the comparison of data with different scales and units. Using the z-score standardization, the transformed time series exhibits a mean of zero and a standard deviation of one. The mathematical description of this transformation is given as:

$$z_i = \frac{x_i - \mu}{\sigma} \tag{1}$$

The parameters μ and σ represent the mean and the standard deviation of the considered time series. The parameter x_i is the corresponding measurement value at a time step i before and z_i after the transformation. Before the input selection is conducted, the temperature and the thermal errors are standardized by using the z-score.

4.2. ARX-models

In general, thermal errors can be described by a linear time-invariant dynamic system, which is represented by a weighted sum

$$y[k]+a_1\cdot y[k-1]+\ldots+a_{n_a}\cdot y[k-n_a]=b_0\cdot u[k]+b_1\cdot u[k-1]+\ldots+b_{n_b}\cdot u[k-n_b] \tag{2}$$

consisting of the current and past outputs y, describing the thermal error, the current and past inputs u representing for example a temperature sensor, the corresponding model parameters a_i and b_j and the time step variable k. The order n_a describes the number of past system outputs that influence the current time step and n_b represents the number of time steps of the corresponding input that are considered to calculate the current system output. Such a model structure is indicated as an autoregressive model with exogenous inputs (ARX), which belongs to the group of auto regressive moving average models. The prediction of an ARX-model is calculated by

$$\hat{y}[k,\theta] = \underline{\varphi}\ \underline{\theta} \tag{3}$$

where the vector $\underline{\varphi}$ contains the shifted input and output data of the system and the vector $\underline{\theta}$ summarizes the corresponding model parameters. The parameters of ARX-models can be calculated by using linear least square

$$\underline{\theta} = \left(\underline{\underline{\Phi}}^T\underline{\underline{\Phi}}\right)^{-1}\underline{\underline{\Phi}}^T\underline{y} \tag{4}$$

where the matrix $\underline{\underline{\Phi}}$ contains the vectors $\underline{\varphi}$ for all considered time steps. When applying ARX-models within the TALC, according to Mayr et al. [33] the weighted least square methods has to be used to realize a correct calculation of the model parameter within the model updates. Then, during model updates only measurement values and no predictions may be used to calculate the model parameters. Compared to the least square, the weighted least squares

$$\underline{\theta} = \left(\underline{\underline{\Phi}}^T\underline{\underline{W}}\,\underline{\underline{\Phi}}\right)^{-1}\underline{\underline{\Phi}}^T\underline{\underline{W}}\,\underline{y}. \tag{5}$$

includes an additional weight matrix $\underline{\underline{W}}$ which enables the handling of the irregular sampling rates by weighting measurements with a one and predictions with a zero. In the TALC, multiple input single output (MISO) models are used to model the considered thermal errors.

4.3. Group-LASSO for ARX-models

In regressing analysis, LASSO is a widespread method to combine variable selection and regularization. Due to the variable selection, irrelevant inputs are eliminated to increase the prediction accuracy and

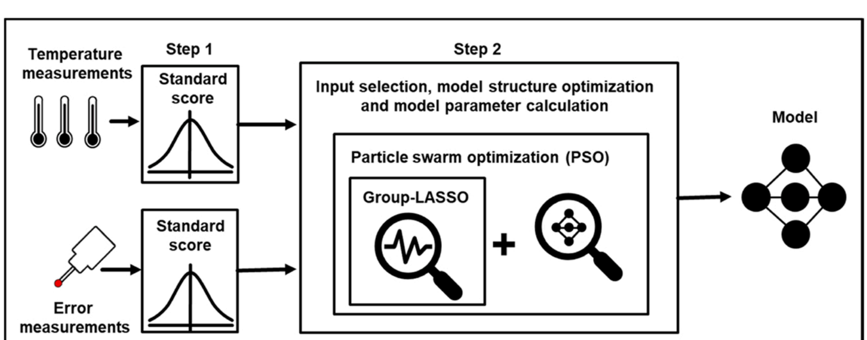

Fig. 3. Concept of the optimal input selection method for ARX-models based on Group-LASSO and particle swarm optimization.

robustness of the data-based models. The LASSO introduced by Tibshirani [35] represents an extension of the common regression problem by adding an L1-regularization as it is shown in Eq. (6).

$$\underline{\beta} = \text{argmin}_{\underline{\beta}} \left(\left\| \underline{y} - \underline{\underline{X}}\,\underline{\beta} \right\|^2 + \lambda \left\| \underline{\beta} \right\|_1 \right) \tag{6}$$

The L1-regularization eliminates insignificant elements from the model parameter vector $\underline{\beta}$. The parameter $\lambda \geq 0$ controls the significance of the regularization term in the optimization problem. Consequently, it defines the number of selected inputs and the parameters of the obtained regression model, because the elimination of a model parameter removes the corresponding model input. Considering ARX-models, all parameters that describe the influence of one input on the output have to be minimized to eliminate the corresponding input from the model structure. Therefore, Klingspor et al. [36] apply the Group-LASSO, which eliminates groups of variables, to define the optimal inputs for ARX-models. The resulting mathematical optimization problem presents itself as:

$$\widehat{\underline{\theta}}_{GL}\left(\lambda, n_a, \underline{n}_b\right) = argmin_{\underline{\theta}} \left(\left\| \underline{y} - \underline{\underline{\Phi}}\,\underline{\theta} \right\|_2^2 + \lambda \sum_{k=1}^{N} \sqrt{\underline{n}_b(k)} \left\| \underline{\theta}_b(k) \right\|_2 \right) \tag{7}$$

The vector $\underline{\theta}_b$ includes the model parameter of one input and consequently its length is defined by the corresponding order $n_b(k)$ of the considered model input. The number of available inputs is represented by the variable N. The relevant inputs and parameters of the ARX-model can be calculated using Eq. (7) for a given parameter set consisting of λ and the model orders n_a and $\underline{n}_b$. The input selection based on the resulting ARX-model structure also enables a consideration of the system dynamics which are exemplarily represented by a time-lag between the measured temperatures and the resulting thermal errors. However, to realize an adaptive input selection for model updates, the previously mentioned weight matrix has to be integrated into the optimization problem

$$\widehat{\underline{\theta}}_{GL}\left(\lambda, n_a, \underline{n}_b\right) = \text{argmin}_{\underline{\theta}} \left(\left\| \underline{\underline{W}}\,\underline{y} - \underline{\underline{W}}\,\underline{\underline{\Phi}}\,\underline{\theta} \right\|_2^2 + \lambda \sum_{k=1}^{N} \sqrt{\underline{n}_b(k)} \left\| \underline{\theta}_b(k) \right\|_2 \right). \tag{8}$$

In contrary to the common weighted least squares method, which can be solved by a closed-form solution as shown in Eq. (5), the presented optimization problem does not have a closed-form solution. Therefore, a numerical solver is applied to solve the optimization. Boyd et al. [37] describe in detail that the Alternating Direction Method of Multipliers (ADMM) solves the Group-LASSO problem efficiently. Consequently, this solver is used in the adaptive input selection to compute the Group-LASSO optimization. However, the orders of the ARX-model, which significantly influence the prediction accuracy and the robustness of the model cannot be optimized within the Group-LASSO problem. To solve this issue, another optimization algorithm is included into the overall method.

4.4. Order computation based on particle swarm optimization

An additional optimization algorithm is applied to simultaneously optimize the inputs and the orders of ARX-models. In this optimization procedure the Group-LASSO problem is solved for several different combinations of the parameters λ, n_a and $\underline{n}_b$. The complexity of the obtained optimization problem significantly increases with the number of available inputs and the desired model orders. This high-dimensional optimization problem is then solved using the particle swarm optimization presented by Kennedy and Eberhart [38]. The cost function for the particle swarm optimization is an adapted BIC [39],

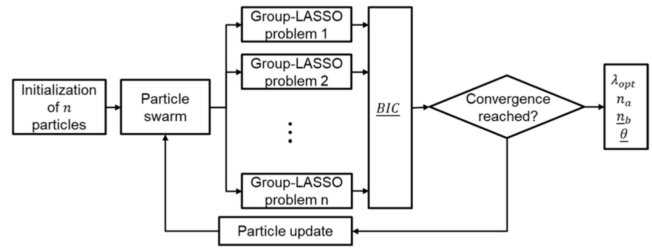

Fig. 4. Procedure of the particle swarm optimization to simultaneously define the optimal inputs, orders, and parameters of ARX-models.

$$BIC = \ln(n)\cdot\left(n_a + \sum_{k=1}^{N} n_b(k) + N^2\right) + n\cdot\ln\left(\sum_{i=1}^{n} E_{comp}^2(t_i)\right)\cdot\frac{\max\left(\underline{E}_{comp}\right)}{S_T\cdot T} \tag{9}$$

which additionally considers the maximal residual error, the desired tolerance, and the number of used model inputs. The additional quadratic term is introduced into the elaborated cost function because the used number of selected model inputs N should be minimized, and an additional input should only be added when the prediction accuracy significantly increases. The second additional term is the quotient between the maximum residual of the model and the desired tolerance T of the model. This term is only considered if the maximum residual is larger than the desired tolerances, because then the corresponding model has to be additionally punished. With this additional term the optimization problem can be pushed in the direction to meet the required tolerance. The parameter S_T represents an additional safety factor, that can be set for example to 0.75. The procedure to solve the described optimization problem is illustrated in Fig. 4.

5. Experimental setup

The developed Group-LASSO method for adaptive inputs of ARX-models is exemplarily applied to compensate the thermal position errors of a swelling and the thermal position and orientation errors of rotary axis of a five-axis machine tool.

5.1. Five-axis machine tool

The kinematic chain of the analyzed machine tool can be described according to ISO 10791-1:2015 [40] as following:

V [w – C′ – A′ – X′ – b – Y – Z – S – t].

The temperature sensors are placed by engineering knowledge close to the relevant thermal heat sources and on the structural components which are not considered in the control loop as for example the swiveling table unit. In total, 20 temperature sensors measure the structural temperatures of the machine tool. Another four temperature sensors evaluate the ambient temperatures around the machine tool and two sensors are placed in the working space. Additionally, three sensors measure the temperature of the inlet and outlet of the machine cooling and the temperature of the metalworking fluid. Furthermore, an eddy current sensor which is mounted in the spindle measures the elongation at the spindle nose. The configuration of the temperature sensors, which can be potentially used as model inputs, is presented in Fig. 5.

5.2. Measurement cycle

The thermal position errors of the A-axis and the thermal position and orientation errors of the C-axis are measured by applying a slightly adapted version of the on-machine measurement cycle presented by Zimmermann et al. [41]. This measurement cycle is based on a touch trigger probe and a precision sphere which is mounted on the machine

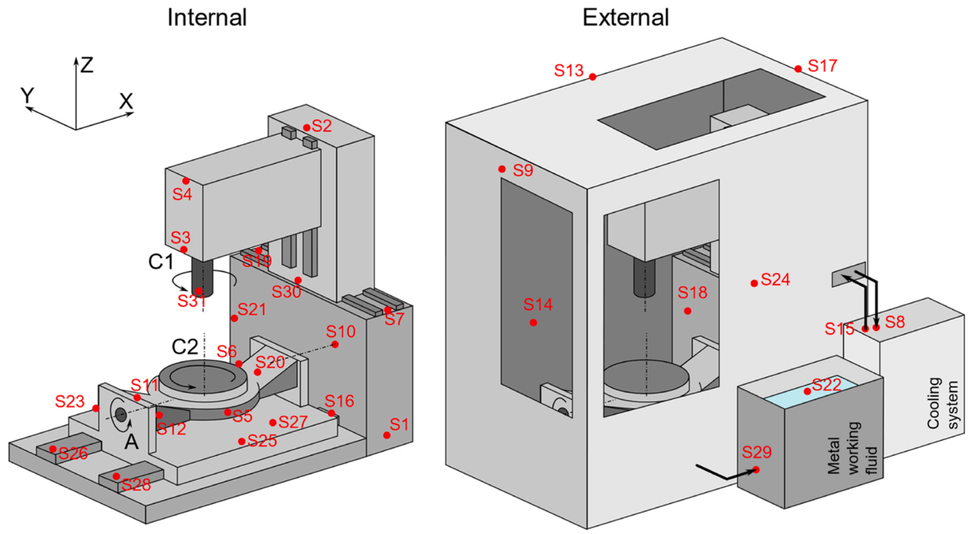

Fig. 5. Temperature sensor location on the analyzed five-axis machine tool.

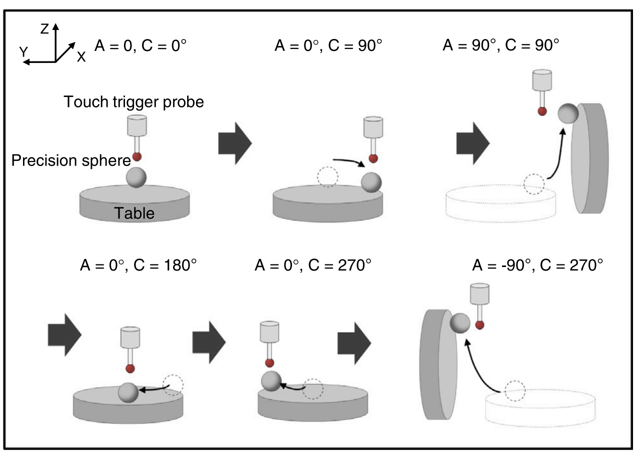

Fig. 6. Measurement procedure of the extended discrete R-Test to identify the thermal position errors of the A-axis and the thermal position and orientation errors of the C-axis.
(Adapted from [41]).

Table 1
Position errors of the A-axis and the position and orientation errors of C-axis according to [42] with the addition of the table related errors according to [43].

Error	Description
E_{Y0A}	Error of the position of the A-axis in Y-direction
E_{Z0A}	Error of the position of the A-axis in Z-direction
E_{X0C}	Error of the position of the C-axis in X-direction
E_{Y0C}	Error of the position of the C-axis in X-direction
E_{Z0T}	Axial error of the machine table in Z-direction
E_{R0T}	Radial error of the machine table
E_{A0C}	Error of the orientation of the C-axis in A-direction
E_{B0C}	Error of the orientation of the C-axis in B-direction
E_{C0C}	Zero position error of the C-axis

table. In total, six measurement positions with different inclination angles of the swiveling and the rotary axis are included in the measurement cycle. The sequence of the reached measurement positions is presented in Fig. 6. This on-machine measurement cycle takes approximately 120 s and identifies nine axis-specific thermal errors which are summarized in Table 1. Using the measurement values of the considered positions in X-, Y- and Z-direction, the corresponding thermal errors can be calculated

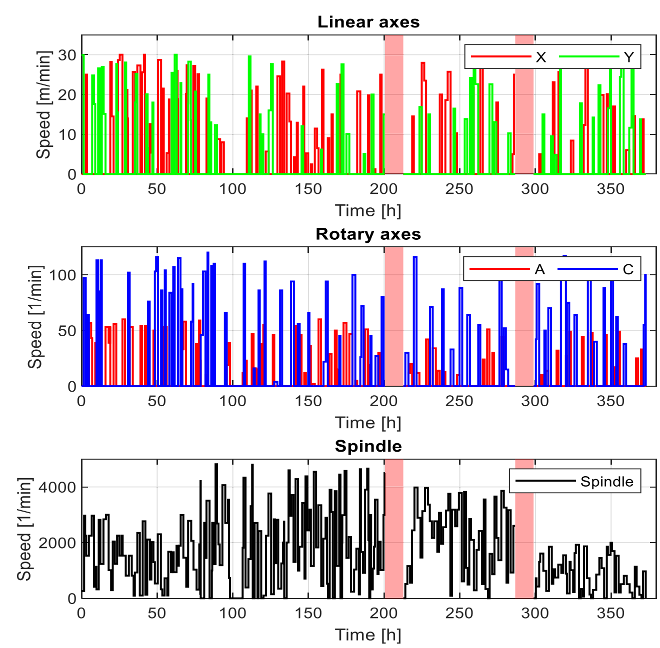

Fig. 7. Speed profile of the linear and rotary axes as well as the spindle of the conducted long-term test series. The red area is a not-true-to-scale representation of the interruption between the measurements. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

by applying the equations given by Gebhardt. [43] and Zimmermann et al. [41].

5.3. Experiments

The thermal load cases of the long-term test series consisting of three experiments, which are realized by randomly generated speed profiles. The speed profiles include sequentially changing single axis movements of the linear and rotary axes in combination with varying spindle speeds. The considered linear axes, namely the X- and the Y-axis, oscillate with the defined speed over their complete traveling distance. The A-axis oscillates between the swelling angles 90° and −90° and the C-axis and the spindle rotate with the defined rotational speeds. The applied speed profiles of the considered axes are presented in Fig. 7.

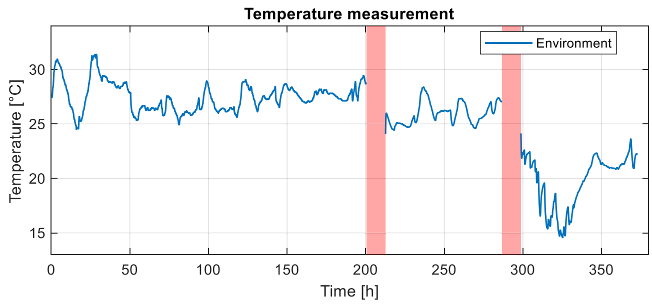

Fig. 8. Environmental temperature during the conducted long-term test series. The red area is a not-true-to-scale representation of the interruption between the measurements. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

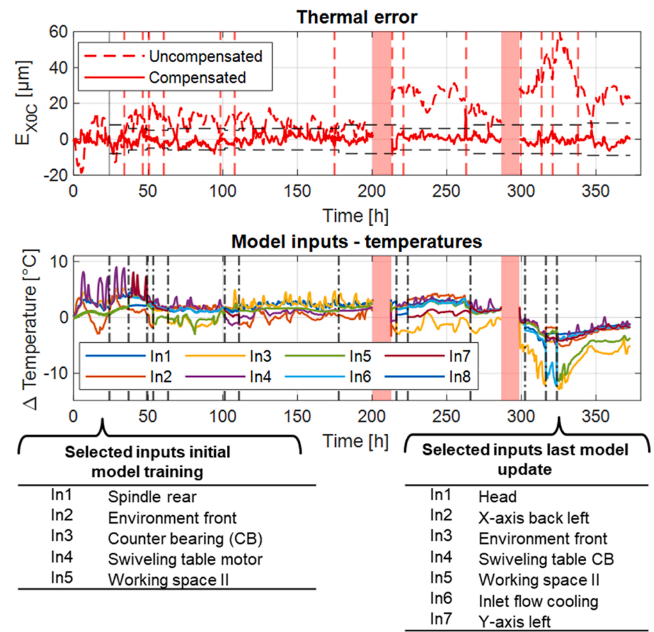

Fig. 9. Uncompensated and compensated thermal error E_{X0C} for the thermal load cases shown in Figs. 7 and 8. The listed sensors are the selected model inputs defined by the Group-LASSO method during the initial model setup and the last model update. The horizontal black dashed lines represent the ACL. The black vertical line shows the model setup and the red vertical dashed line the exceedance of the ACL of any considered thermal error. The black dashed dotted lines indicate the reselection of the model inputs after a NG mode. The red area is a not-true-to-scale representation of the interruption between the measurements. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 2
Predefined parameters of the TALC (CP: calibration phase, NG: MT out of precision).

Parameter	Value
Calibration phase	24 h
Measurement interval	5 min
Measurement interval (post CP)	Adaptive
Measurement interval (NG)	5 min
Action control limit (ACL)	Adaptive
NG mode duration	16 measurements
Measurement cycle duration	120 s
Max. n_a	5
Max. n_b	5

Furthermore, the thermal load cases are significantly influenced by the changing ambient temperature, because the machine tool is located in a shop floor without an exact air-conditioning system. The conducted test series has a duration of approximately 340 h and consists of three different experiments that are conducted during different seasons to cover a high variety of ambient temperatures as shown in Fig. 8. The first experiment lasts approximately 196 h, and the two following experiments have a duration of 72 h. During these experiments the conducted thermal load cases have a duration of five minutes and after each load case the thermal errors and the temperatures are measured. The compensation method is implemented on a desktop PC which sends the predicted thermal errors given in Table 1 to the NC control. The parameters which are used for the TALC are summarized in Table 2.

6. Results

To evaluate the performance of the developed Group-LASSO method for the adaptive input selection of ARX-models, it is applied to the presented long-term test series. The first subsection discusses the time series of the compensation results obtained from applying the TALC in combination with the newly developed Group-LASSO method. Afterwards, the new method for adaptive input selection is statistically analyzed regarding its robustness and repeatability. Furthermore, its performance is statistically compared to the previously used TCK-method.

6.1. Compensation results for the adaptive input selection based on the Group-LASSO method for ARX-models

The compensation result for the thermal error E_{X0C}, using the TALC in combination with the adaptive input selection based on the Group-LASSO method and the adaptive ACL, is shown in Fig. 9. This figure includes the uncompensated und compensated thermal error as well as the adaptively selected model inputs. The lists of the selected model inputs are exemplarily presented for the initial model training and the last model update. The results show that the considered thermal error can be significantly reduced by applying the proposed compensation strategy. Furthermore, the adaptive ACL limit increases over time from 5 to 9 μm. This already indicates an increase in complexity of the underlying thermal behavior. This is also highlighted by the significant change of the used model inputs between the initial model training and the last model update. Only two model inputs from the initial model training, namely environment front and working space II, are still used after the last model update.

Considering for example the compensation results of the thermal error E_{Z0A} presented in Fig. 10, the model input set changes less between the initial model training and the last model update. In this case only one input is added, and another model input has changed after considering all available data from the model updates. Although the change of the used inputs is minimal, the thermal error is robustly reduced over the whole long-term test series.

The changes of the selected optimal model inputs for the thermal errors E_{X0C} and E_{Z0A} are illustrated in Fig. 11. This comparison demonstrates, that the changes of the optimal inputs depend on the modeled thermal error. For example, the identified optimal inputs for the compensation models of E_{X0C} visibly change more frequently than the optimal inputs for the thermal error E_{Z0A} due to the model updates. The compensation results of all considered thermal errors are summarized in Table 3.

6.2. Comparison between adaptive and static model inputs

To further evaluate the performance of the newly developed method for adaptive model inputs, the compensation results of the TALC with static and adaptive model inputs are compared. In the case of static inputs, the optimal inputs are only selected during the setup of the data-based model and during the compensation phase only the model

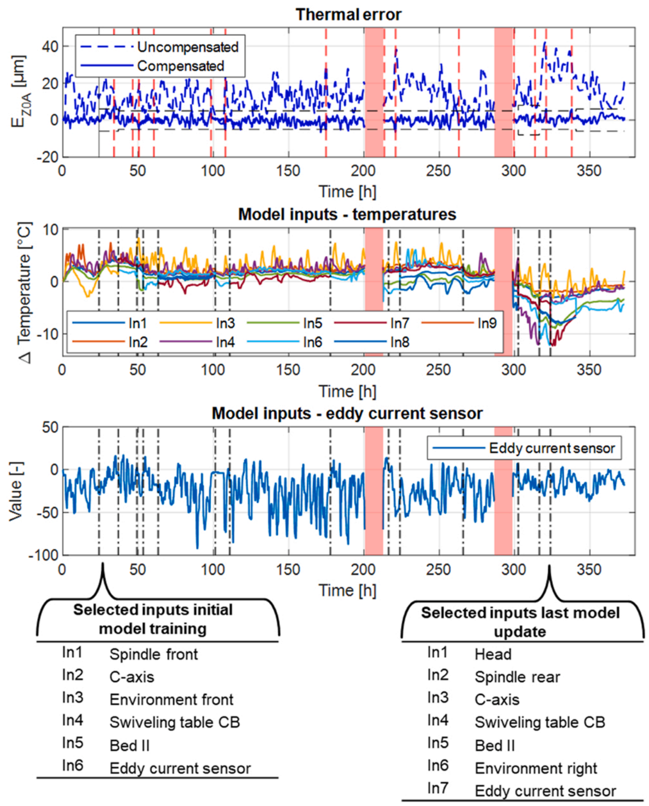

Fig. 10. Uncompensated and compensated thermal error E_{Z0A} for the thermal load cases shown in Figs. 7 and 8. The listed sensors are the selected model inputs defined by the Group-LASSO method during the initial model setup and the last model update. The horizontal black dashed lines represent the ACL. The black vertical line shows the model setup and the red vertical dashed line the exceedance of the ACL of any considered thermal error. The black dashed dotted lines indicate the reselection of the model inputs after a NG mode. The red area is a not-true-to-scale representation of the interruption between the measurements. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

parameter and the model structure are updated. The TALC with adaptive model inputs, enables an automated recalibration of the inputs as well as the orders and the parameters of the ARX-models. This comparison is visualized in Fig. 12.

Fig. 13 shows the residuals of the thermal errors E_{Z0A} and E_{X0C} for the TALC with static and adaptive model inputs. The results indicate that particularly the compensation result of the thermal error E_{X0C} is improved by applying the adaptive input selection. This affects especially the compensation results of the second and the third experiment of the long-term test series. In contrast, the compensation results of E_{Z0A} do not significantly differ for the static and the adaptive model inputs. Consequently, the modeling of complex thermal errors benefits from the adaptive input selection, because for those it is unfeasible to consider all relevant thermal load cases in the training data. Furthermore, the results indicate that the number of model updates is slightly increased from 9 to 12 for the TALC with adaptive inputs. This increase is caused by the effect of adaptive ACL. For example, considering the thermal error E_{X0C}, the adaptive ACL is 15 µm for the static inputs and 9 µm for adaptive inputs at the end of the long-term test series. In contrast, a static ACL would result in an increased number of ACL exceedances, because the limited modeling accuracy of models with static inputs is not able to fulfill the defined ACL as shown in the previous publication [30]. Consequently, the adaptive ACL automatically decreases the achievable accuracy, if the ability to adapt to changing thermal boundary conditions is limited instead of conducting unnecessary model updates which cannot improve the prediction accuracy of the data-based models.

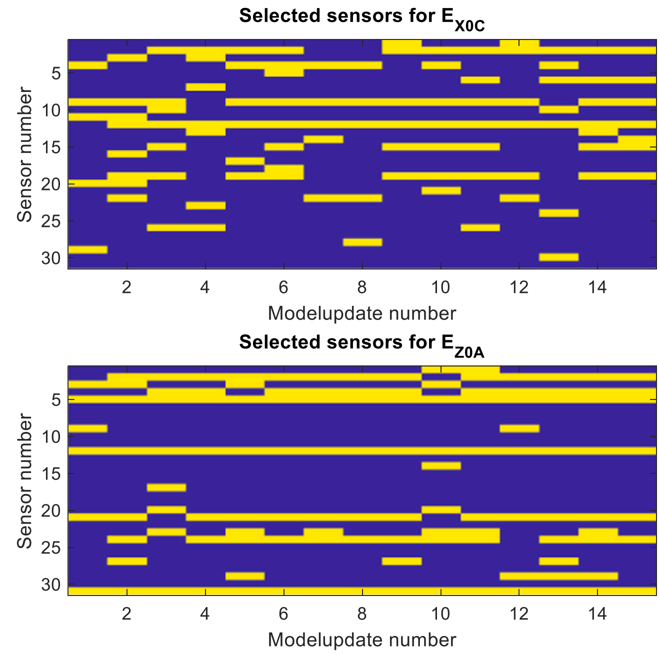

Fig. 11. Chronological changes of the selected model inputs represented by the yellow areas for the thermal errors E_{X0C} and E_{Z0A}. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 3
Uncompensated and compensated thermal errors and the corresponding reduction using ARX-models with adaptive inputs based on the Group-LASSO method.

Error	Peak to peak value			RMSE		
	Uncomp [µm] / [µm/m]	Comp [µm] / [µm/m]	R [%]	Uncomp [µm] / [µm/m]	Comp [µm] / [µm/m]	R [%]
E_{Y0A}	8.8	10.7	-22.3	1.3	1.5	-11.1
E_{Z0A}	44.1	15.0	65.9	15.9	2.0	87.4
E_{X0C}	72.7	20.4	71.9	20.3	2.5	87.6
E_{Y0C}	8.8	9.5	-8.3	1.4	1.6	-15.3
E_{Z0T}	22.5	7.0	68.8	4.9	1.0	79.3
E_{R0T}	12.4	4.0	67.8	2.5	0.6	76.6
E_{A0C}	23.8	13.5	43.3	4.2	2.2	48.2
E_{B0C}	62.1	36.8	40.8	11.7	5.1	56.6
E_{C0C}	22.1	13.9	37.3	5.5	2.0	63.0

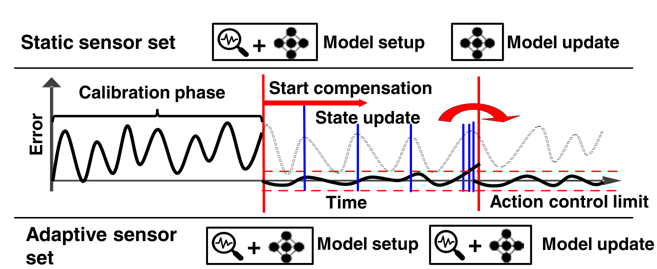

Fig. 12. Comparison between the TALC with static and adaptive model inputs.

6.3. Statistical analysis

This section aims at identifying the robustness and the repeatability of the compensation results obtained from the TALC in combination with the newly developed method for optimal input selection. The results of this analysis also include the influence of the applied numerical

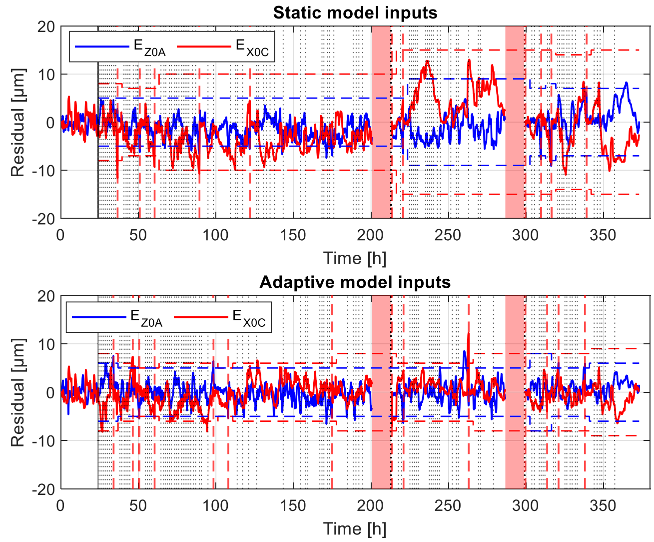

Fig. 13. Residuals of the thermal errors E_{X0C} and E_{Z0A} using the TALC with static and adaptive model inputs based on the Group-LASSO method. The horizontal blue and red dashed lines represent the adaptively estimated ACL of E_{Z0A} and E_{X0C}. The solid vertical line shows the time of the model setup and the red dashed vertical lines the exceedance of the ACL. The dotted lines represent the autonomously triggered ST updates. The red area is a not-true-to-scale representation of the interruption between the measurements. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

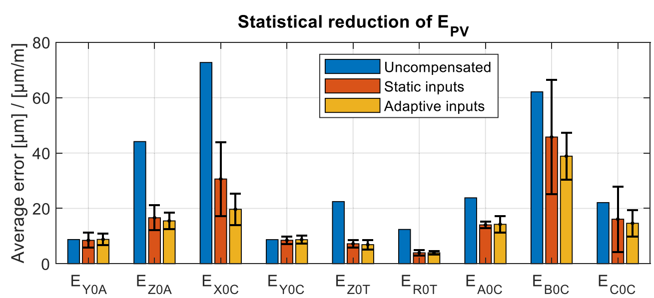

Fig. 14. E_{PV} of the uncompensated thermal errors and the average E_{PV} of the compensated thermal errors for static and adaptive model inputs including the standard deviation ($k = 2$) computed from 25 simulations.

solvers on the compensation results. To realize a statistical evaluation, the compensation strategy is repeatedly applied to the presented long-term test series. These simulations are conducted for static as well as adaptive model inputs. Due to calculation time limitations for each set up 25 simulations are conducted.

The peak-to-peak error (E_{PV}) and the root mean squared error (E_{RMS}) are used as evaluation criteria for the simulation results. Fig. 14 compares the peak-to-peak value of the uncompensated thermal errors and the averaged peak-to-peak values of the compensated thermal errors computed from the repeated simulations using the TALC with static and adaptive model inputs. The error bars in the figure denote the standard deviations ($k = 2$) of the compensation results between the conducted simulations. The corresponding results for the root mean squared errors are presented in Fig. 15.

The results show, that completely self-optimizing thermal error compensation models based on the TALC and the developed Group-LASSO method for ARX-models reliably reduce the thermal errors of machine tools. The average reduction for the evaluation criteria E_{PV} and E_{RMS} of the compensation results for static and adaptive model inputs are summarized in Table 4. The application of adaptive model inputs significantly reduces several thermal errors. For instance, the

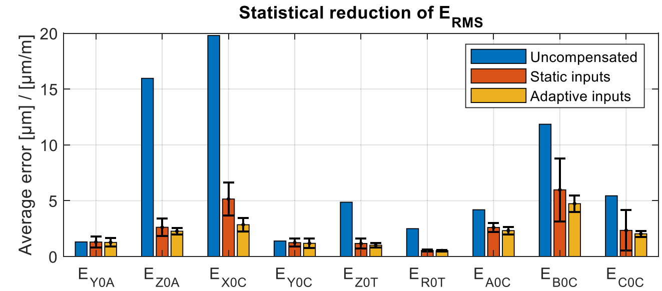

Fig. 15. E_{RMS} of the uncompensated thermal errors and the average E_{RMS} of the compensated thermal errors for static and adaptive model inputs including the standard deviation ($k = 2$) computed from 25 simulations.

Table 4
Reduction of the considered thermal errors by applying the TALC with static and adaptive model inputs based on E_{PV} and E_{RMS}.

Error	Reduction E_{PV} [%]		Reduction E_{RMS} [%]	
	Static	Adaptive	Static	Adaptive
E_{Y0A}	3	-1	1	3
E_{Z0A}	62	65	84	86
E_{X0C}	58	73	74	86
E_{Y0C}	3	0	11	15
E_{Z0T}	68	69	76	79
E_{R0T}	68	68	78	79
E_{A0C}	41	40	38	45
E_{B0C}	26	37	50	60
E_{C0C}	27	34	57	63

compensation results of the thermal errors E_{X0C} and E_{B0C} show an additional reduction for the adaptive model inputs. The evaluation criteria E_{PV} of E_{X0C} is reduced by 36% from 30.6 µm to 19.7 µm when adaptive instead of static model inputs are used. The evaluation criteria E_{RMS} is even reduced from 5.2 µm to 2.9 µm which corresponds a reduction of 44%. Consequently, the compensation results of thermal errors, which are caused by a high variety of thermal influences, benefit most from the increased adaptability of the data-based models. This results in an additional reduction of the average E_{PV} for the translational and the rotational errors of 7% compared to the compensation results with static inputs. Furthermore, the average E_{RMS} is reduced by 6%. Additionally, the compensated E_{PV} of models with static inputs exhibit higher standard deviations between the conducted simulations, because the initial input selection has a much larger impact on the overall compensation results. Furthermore, adaptive model inputs reduce the average standard deviation of the translational errors by 40% and the average standard deviation of the rotational errors by 52%. Furthermore, the average standard deviation of the E_{RMS} for the translational and rotational thermal errors is reduced by 52% respectively 73%. This shows that different input combinations, which only have a slightly different cost function value after the calibration phase can later have a significant impact on the compensation results although the model structure and model parameters are updated.

Due to the adaptability of the ACL, the number of conducted model updates for the TALC with static and adaptive inputs are similar. The average number of model updates computed from 25 simulations is 12.7 with a standard deviation of 4.8 ($k = 2$) for the static model inputs and 13.0 with a standard deviation of 3.5 for the adaptive model inputs. This indicates that data-based models with adaptive inputs enable more precise compensation results without further decreasing the productivity of the machine tool. Therefore, fully self-optimizing thermal error compensation models are required for precise and long-term robust compensation results when only a limited amount of empirical data is available for model training.

To close the statistical analysis, the effectiveness of the developed

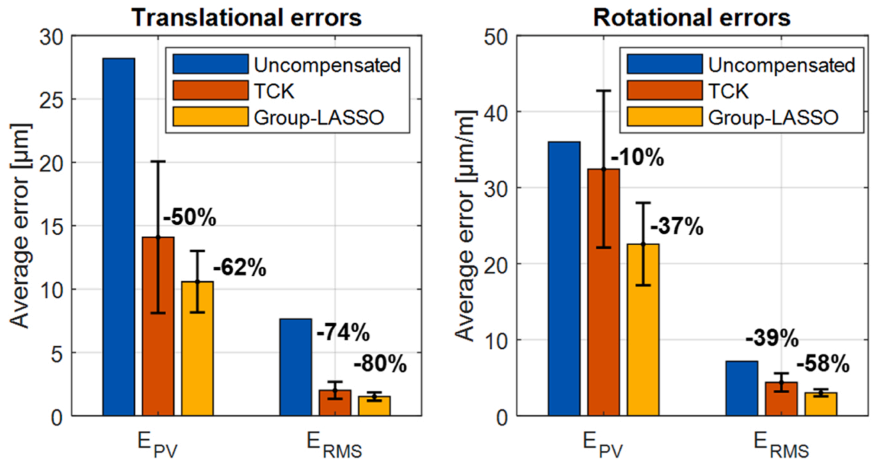

Fig. 16. Statistical compensation results of the adaptive input selection based on the TCK- and the Group-LASSO method.

Group-LASSO method and the previously used TCK method are compared by using the TALC with adaptive model inputs. Fig. 16 indicates that the new method outperforms the TCK-method, because the average E_{PV} reduction is 12% higher for the translation errors and 27% higher for the rotational errors. The average E_{RMS} reduction of the uncompensated translational and rotational errors is improved by 6% and 19%. Furthermore, the standard deviation of the evaluation criteria E_{PV} for the translational errors is reduced by 57% when applying the Group-LASSO method instead of the TCK-method. For an individual error, the largest reduction of the standard deviation of E_{PV} is even 86% in the case of the thermal error E_{R0T}. Consequently, the parallelization of the input selection, the model structure optimization and the model parameter calculation have a positive impact on the compensation results, because the modeled system dynamic of the resulting ARX-model is also considered during the selection process. This results in more precise and robust long-term compensation results for the Group-LASSO method compared to the TCK-method.

7. Conclusion

The introduced method autonomously and adaptively selects the optimal inputs for ARX-models using the Group-LASSO in combination with particle swarm optimization. In contrast to the previously used TCK-method with separate selection and modeling step, the new method allows to consider the modeled system dynamics of the ARX-models during the input selection. The system dynamics are exemplarily represented by the time lag between the measured temperatures and the resulting thermal errors. The Group-LASSO for ARX-models is extended by a weight matrix to obey the irregular sampling rates of the TALC. The particle swarm optimization is applied to realize a simultaneous optimization of the optimal inputs and the model structure. This optimization is based on an adapted BIC which additionally considers the number of chosen model inputs and the desired precision level. Furthermore, self-optimization ability of the TALC is further extended by introducing an adaptive and error specific ACL defined by the maximal residual of each thermal error during the model training.

The combination of the TALC and the newly developed method for optimal inputs selection achieves precise and robust compensation results for static and adaptive model inputs when it is applied to a long-term test series containing a high variety of thermal load cases. This shows that the TALC is able to reduce the thermal errors of machine tools even if they result from complex load cases such as multiple sequentially moving axes. Additionally, the results verify that adaptive model inputs outperform static model inputs regarding the prediction accuracy and the statistical repeatability. The conducted statistical analysis indicates that the average peak-to-peak value of the residuals of the translational thermal errors are reduced from 12.6 µm to 10.6 µm, which corresponds to a reduction of 16% when applying adaptive instead of static model inputs. The repeatability of these compensation results is improved by 40%, which correspond to a reduction of the average standard deviation ($k = 2$) from 4.1 µm to 2.4 µm. The thermal error E_{X0C} exhibits an average reduction of the compensated peak-to-peak value by 36% and a reduction of the standard deviation between the measurements by 58% when adaptive model inputs are used. Consequently, especially the compensation results of thermal errors which are difficult to model benefits from the adaptive input selection. Additionally, the new method outperforms the previously used sequential method for adaptive input selection, which is based on the TCK signal comparison. For instance, the peak-to-peak value of the average translational thermal error is reduced from 13.8 µm to 10.6 µm when the Group-LASSO method is used instead of the TCK-method. This results in an additional reduction of 23%. The standard deviation between the simulation is even reduced by 57% compared to the TCK-method. Consequently, the new approach for adaptive input selection significantly improves two crucial characteristics that are relevant to receive a widespread application of thermal error compensation.

In summary, the combination of the TALC, the adaptive input selection, the autonomously triggered state updates, and the adaptive ACLs further improves the self-optimization abilities of thermal error compensation models. Consequently, precise and robust long-term compensation results are achieved without requiring a detailed knowledge of the thermal behavior of the machine tool or manual adjustments.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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