Balanced voting

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BALANCED VOTING

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Abstract

This thesis examines a new voting scheme, which we call 'Balanced Voting' and is particularly suitable for making fundamental societal decisions. Such decisions typically involve subgroups that are strongly in favor of, or against, a new fundamental direction, and others that care much less. In a two-stage procedure, Balanced Voting works as follows: Citizens may abstain from voting on a fundamental direction in the first stage. This guarantees them a voting right on the variations of the chosen fundamental direction in the second stage. All losers in the first stage also obtain voting rights in the second stage, while winners do not.

We develop a model with two fundamental directions for which stakes are high for some individuals and with private information about preferences among voters. We demonstrate that if the voting body is sufficiently large, Balanced Voting is superior to simple majority voting, Storable Votes and Minority Voting from the perspective of utilitarian welfare. Moreover, the outcome under Balanced Voting is Pareto-dominant to the outcome under simple majority voting and Minority Voting. We discuss several aspects that need to be considered when Balanced Voting is applied in practice, as well as interesting extensions to our framework. We demonstrate how Balanced Voting can be applied to a real-world policy issue by considering data from two surveys on public opinion about using nuclear power for energy production.

Finally, we analyze Balanced Voting when the preferences of agents are publicly observable. We consider a committee of three agents and compare the equilibrium voting behavior under Balanced Voting with complete and incomplete information. We illustrate that while there exist equilibria with complete information that conform to the equilibrium with incomplete information, there also exist equilibria that do not.

Zusammenfassung

Diese Dissertation untersucht eine neue Abstimmungmethode, die wir 'Balanced Voting' nennen. Sie eignet sich besonders für grundlegende gesellschaftliche Entscheidungen, bei denen typischerwise zwei Arten von Wählern abstimmen. Die einen haben ein starkes Interesse daran, eine bestimmte politische Grundausrichtung einzuschlagen, während die anderen viel weniger stark an der Entscheidung interessiert sind.

Balanced Voting ist ein zweistufiges Verfahren und funktioniert wie folgt. Bei einer ersten Abstimmung, in welcher über die Grundrichtung entschieden wird, können sich Wähler der Stimme enthalten, wenn sie es möchten. Dieser Verzicht garantiert ihr Stimmrecht bei der zweiten Abstimmung, in der entschieden wird, welche Variante der vorher gewählten Grundrichtung implementiert werden soll. Neben denjenigen, die sich in der ersten Runde der Stimme enthalten haben, haben auch alle 'Verlierer' der ersten Abstimmung ein Stimmrecht für die zweite Runde, während die 'Gewinner' der ersten Runde kein Stimmrecht mehr haben.

Wir entwickeln ein Modell mit zwei politischen Grundausrichtungen, für welche das Interesse einiger Personen sehr stark ist. Die Information über die Präferenzen der Wähler ist privat. Wir zeigen, dass Balanced Voting der einfachen Mehrheitsregel, dem Storable-Votes-System und Minority Voting aus der Perspektive der utilitaristischen Wohlfahrt überlegen ist, wenn nur die Zahl der Abstimmenden gross genug ist.

Darüber hinaus ist das Ergebnis unter Balanced Voting Pareto-dominant gegenüber dem Ergebnis unter der einfachen Mehrheitsregel und unter Minority Voting. Wir diskutieren verschiedene Aspekte, die berücksichtigt werden müssen, um Balanced Voting in der Praxis anwenden zu können, und analysieren verschiedene Erweiterungen, die im von uns vorgestellten Rahmen möglich wären. Ausgehend von einer realen politische Frage und

unter Verwendung von Daten aus zwei Umfragen zur Stromerzeugung durch Kernenergie zeigen wir auf, wie Balanced Voting in der Praxis angewendet werden könnte.

Schliesslich analysieren wir Balanced Voting, wenn die Präferenzen der Agenten öffentlich beobachtbar sind. Wir betrachten ein Komitee mit drei Mitgliedern und vergleichen das Gleichgewichts-Abstimmungsverhalten unter Balanced Voting mit vollständiger und unvollständiger Information. Wir zeigen, dass es Gleichgewichte mit vollständiger Information gibt, die mit dem Gleichgewicht mit unvollständiger Information übereinstimmen, allerdings auch Gleichgewichte existieren, welche dies nicht tun.

Contents

1 Introduction

"Wrong does not cease to be wrong because the majority share in it."

— Leo Tolstoy, *A Confession*

Polities are repeatedly confronted with the need to take decisions that are fundamental. Decisions such as exiting nuclear power, reversing the course in public indebtedness, enacting comprehensive labor market reforms or joining the European Union have a large and long-lasting impact on the direction a society takes. When an electorate takes such decisions in a democracy, some subgroups strongly favor or oppose a fundamental new direction, while others may care much less.

Typically, democratic societies use simple majority voting to take such fundamental decisions. This rule has many desirable properties for a democracy. May (1952) shows that in binary decisions, simple majority voting treats all individuals equally, favors both alternatives similarly, produces a unique outcome for all preference profiles and responds positively to individual preferences.

However, simple majority rule is known to be plagued with difficulties. The most pressing of all is the tyranny of the majority, where the power of the majority may lead to the oppression of minority groups. The reality of this problem is made painfully clear by the threat of or the ongoing civil wars around the world. One reason for this drawback of the majority rule is that it does not allow agents to express the *intensity* of their preferences. A majority overrules a minority, even if the minority is much more concerned about the issue than the majority, for instance. This is particularly problematic when making fundamental societal decisions, which have significant and long term impact on the members of a society. Allowing the minority to veto the resulting decision might resolve the problem, but may also be detrimental. The protection of the minority would then turn into an unwelcome tyranny.

To address this problem, we introduce a new scheme, which we call 'Balanced Voting', particularly suited for taking fundamental decisions. The concept of Balanced Voting was developed jointly with Prof. Dr. Hans Gersbach.[1] It aims at striking a balance between taking into account the intensity of preferences and protecting minorities.

The idea underlying Balanced Voting is conveniently explained in a two-stage set up. Suppose a society or a committee of individuals votes on two related binary decisions. The first decision determines the fundamental direction, which is assumed to be irreversible (e.g. using nuclear power in energy production or discontinuing its use). The second decision establishes the variation or the way in which the choice of the fundamental direction may be realized. For example, either investing in new nuclear power plants or improving the safety and efficiency of existing ones are possible nuclear variations. Typical non-nuclear variations are the investment in more solar power or hydro power.

Under Balanced Voting, agents have the option of either voting for a fundamental direction or of abstaining in the first stage. Those who abstain "save " their voting rights for the second stage. The agents who are 'losers' of the first stage obtain, moreover, voting rights in the second stage, while the 'winners' in the first stage are not allowed to vote in the second stage.

Balanced Voting thereby allows individuals who do not feel strongly about the fundamental decision to trade off their voting rights in the first stage for a guaranteed vote in the second stage. Thus, individuals who are only weakly-inclined towards a particular fundamental direction have the opportunity to exert more influence on this second-stage decision. This allows, for instance, strong advocates or opponents of nuclear power to exercise more influence on the first-stage decision, i.e. whether it should be used or not. Those agents that voted for a fundamental direction but belonged to the 'losers' are compensated by receiving the right to vote in the second stage. Hence, if nuclear power is chosen in the first stage, strong opponents of nuclear power will be in a better position to limit the number of nuclear power plants to be built in the future. Similarly, if the decision to discontinue nuclear power is taken in the first stage, strong proponents of nuclear power will have a better chance of selecting an alternative they prefer.

[1]Chapter 3 of this thesis, in particular, is joint research with Prof. Dr. Hans Gersbach.

As already mentioned, Balanced Voting can be applied to any fundamental decision.[2] It may be particularly suitable for collective decisions on whether to increase public debt. Suppose that, in the first stage the decision is whether public debt should be increased or not when current tax laws and government expenditure involve a budget deficit. If the Parliament decides to increase public debt, the second decision could involve determining the projects in which this additional funding is invested. If the Parliament decides not to increase the debt level, the second decision involves the choice about the mix of tax increases or cutting expenditure to meet the debt ceiling.

Several other innovative voting schemes that lead to higher social welfare have been developed in recent years. In the Storable Votes scheme developed by Casella (2005), agents have the option of using their vote in the current decision or storing it for future use. This enables them to concentrate their votes on decisions dealing with issues over which they have intense preferences. Hortala-Vallve (2012) introduced Qualitative Voting, in which each individual is granted a stock of votes that can be freely distributed over a series of binary choices, whereby all votes are cast simultaneously. The Minority Voting scheme introduced by Fahrenberger and Gersbach (2010) and further analyzed in Fahrenberger and Gersbach (2012) focuses on protecting individuals from the accumulation of utility losses over time when they repeatedly wind up in the minority. In a two-stage voting procedure, Minority Voting gives the losers in the first stage exclusive voting rights in the second stage. All these voting schemes link *separate* voting decisions. Jackson and Sonnenschein (2007) show that in general, linking repeated voting decisions leads to welfare gains, and that it is possible to achieve full efficiency as the number of decisions grows large.

Other mechanisms that attempt to capture agents' intensity of preferences are Cumulative Voting and vote-trading. In Cumulative Voting, individuals are endowed with a stock of votes that they can freely distribute over the candidates standing in an election, whereby allocating more than one to a candidate is allowed (see Sawyer and MacRae (1962), Gerber, Morton, and Rietz (1998) and Cox (1990)). Under vote-trading, an agent is permitted to exchange his vote with another in return for monetary compensation (see, for example, Coleman (1966) and Philipson and Snyder (1996)). These works are discussed in further detail in Chapter 2.

[2] One might also apply Balanced Voting in court decisions taken by a group of judges, with the first vote indicating the basic ruling and the second vote the specific application.

Balanced Voting uses the ideas from Storable Votes and Minority Voting, is tailored to fundamental societal decisions and examined for incomplete information. Consequently, Balanced Voting not only allows agents to use their vote on issues about which they feel strongly, but it also provides better protection to organized minorities with strong preferences, in keeping with the argument of Guinier (1994) and Casella, Palfrey, and Riezman (2008) that such minorities deserve some special protection.

Balanced Voting is related to the proportionality principle introduced by Brighouse and Fleurbaey (2010) which requires that the power in collective decision-making processes should be proportional to individual stakes. Brighouse and Fleurbaey (2010) argue that the standard notion of equality in democracy is the reason for drawbacks such as the tyranny of the majority and propose that equality should be replaced with the principle of proportionality which not only alleviates the difficulties of democracy, but also corresponds better to how democracy is practiced in reality. They propose that the proportionality of voting power should be granted via unequal voting weights that correspond to each individual's stakes in the issue. Balanced Voting, on the contrary, preserves equality as all individuals have the same ex-ante voting power, but also adheres to the idea of the principle of proportionality since people having high stakes on one issue are selected endogenously into the group of people that takes the decision. Balanced Voting also avoids the complexities that arise when applying the principle of proportionality in practice and provides better protection to minorities.

Another line of thinking demonstrates that those with strong preferences self-select themselves into making decisions when participation in voting is costly. For example, Börgers (2004), Campbell (1999) and Osborne, Rosenthal, and Turner (2000) illustrate that when voting is costly, only those with intense preferences or low costs of participation cast their votes, and others free-ride on those who vote. Balanced Voting, in contrast, concentrates decision power on individuals with strong preferences, without an exogenous cost component. Under Balanced Voting, the cost of voting is endogenous, as individuals have to trade-off the possibility of influencing the decision in one stage against a better chance of influencing the decision in the other. A vote is never "wasted" by abstention, under Balanced Voting. Those with less intense preferences who would like to abstain in the first stage and "free ride" in the sense of Campbell (1999), are now allowed a higher chance of influencing the decision in the second stage, on which their stakes are comparatively higher.

This dissertation is organized as follows. In Chapter 3 we introduce Balanced Voting in an environment with asymmetric information. We develop a two-stage model with binary decisions where preferences of the voters are private information. We characterize the equilibria under Balanced Voting and show that in equilibrium, individuals with strong preferences participate in the first voting stage, while those who are weakly-inclined abstain, provided that the stakes of strongly-inclined individuals are sufficiently high and those of weakly-inclined individuals are sufficiently low. Moreover, if the society is sufficiently large, Balanced Voting is preferable to several existing voting schemes, with regard to utilitarian welfare and the Pareto criterion.

In Chapter 4 we discuss several aspects that need to be considered when applying Balanced Voting in practice and interesting extensions to the framework that could be pursued in further research. In Chapter 5 we present how Balanced Voting can be applied to a real-world policy issue. We draw on data provided by two recent surveys on public opinion about using nuclear power for energy production and illustrate the predicted voting outcome under Balanced Voting, in comparison to simple majority voting.

In Chapter 6 we analyze Balanced Voting when the preferences of agents are publicly observable. We consider a committee of three agents and examine several cases where individuals have different combinations of preferences. We characterize the subgame perfect Nash equilibria in each case. We then compare the equilibrium voting behavior under Balanced Voting with complete and incomplete information to draw conclusions on the role of symmetric information on the equilibrium voting behavior. We illustrate that while there exist equilibria which conform to the equilibrium with incomplete information, there also exist equilibria that do not.

In Appendix A we summarize the properties of expressions used in the dissertation. In Appendix B we provide the proofs. Appendix C outlines detailed calculations of key expressions and Appendix D contains a list of notation used in the dissertation.

2 Overview: New Voting Rules

Ideas for new voting rules that improve the shortcomings of the majority rule have gained interest in the recent past.[1] The features of Balanced Voting draw on several ideas that are currently under discussion in this literature. This section gives an overview of the existing ideas for new voting rules, and identifies how Balanced Voting contributes to this growing literature. We specifically focus on Storable Votes, Minority Voting, Qualitative Voting, Flexible Majority Rules, Cumulative Voting and vote-trading. In most cases, such voting rules are compared with the veil-of-ignorance approach. Individuals decide about voting rules at a stage when they do not precisely know how they will be affected by future issues for which the voting rules will be applied.[2]

2.1 Storable Votes

Storable Votes introduced by Casella (2005) is a simple and intuitive voting scheme for a committee that meets periodically to make a series of binary decisions. Under Storable Votes, each member receives one vote in each period that he can use in the current decision or store for future use. The ability to shift votes intertemporally allows voters to concentrate more votes on decisions they feel strongly about. This essentially allows agents to increase the probability of winning on issues that matter more, at the expense of winning on issues that matter less. Moreover, each agent has the autonomy to decide how much influence he would exert on which decision, whereby the collective decision

[1]These ideas rest and draw on a rich theory regarding voting, elections, agenda selection, democratization and constitutional design covered, for example, in Austen-Smith and Banks (2000), Austen-Smith and Banks (2005) and Acemoglu and Robinson (2009).

[2]The veil-of-ignorance approach to analyzing majority voting rules is developed in the seminal book by Buchanan and Tullock (1962).

does not determine whether or not he is allowed to vote, as in Minority Voting. Thus, Storable Votes grants agents a richer set of options other than the choice of voting or abstaining and acts as a mechanism of capturing the intensity of voter preferences.

Casella (2005) shows that equilibrium strategies under Storable Votes are intuitive in that the number of votes cast by individuals in equilibrium are monotonically increasing in their intensity of preferences. Moreover, Storable Votes leads to ex-ante welfare gains over simple majority voting even though it does not achieve full efficiency. Casella (2005) rigorously proves welfare gains over simple majority voting for the case with two voters. As the number of voters increases, welfare gains continue to hold as long as one of the following conditions is satisfied: (i) the committee is sufficiently large; (ii) preferences are not too polarized; (iii) the time horizon is sufficiently long. This finding is in line with Jackson and Sonnenschein (2007), where they show that creating a 'link' between repeated voting problems increases welfare, and as the number of decisions grows large, it is possible to achieve the first-best allocation in each decision.

While the idea of Storable Votes is very intuitive, the voting game that arises is complicated, requiring a high level of sophistication in voters to identify the equilibrium strategies. This is because voters essentially need to compare the marginal effect of a vote on the probability of being pivotal today to the probability of being pivotal in a future period, which depends on the voting choices of all individuals. In this light, it is natural to question whether it is practically feasible to reap the potential welfare gains of Storable Votes identified in theory. Casella, Gelman, and Palfrey (2006) address this question in a series of experiments. They study a slightly different version of the original Storable Votes mechanism where each voter is endowed with an initial stock of votes at the beginning of the game, in addition to the regular vote he obtains in each period. Casella, Gelman, and Palfrey (2006) observe that although the experimental subjects often adopted off-equilibrium strategies, they consistently cast more votes when preference intensities were higher. This resulted in the realization of aggregate efficiency levels that matched almost exactly with the predictions of theory, indicating that the complexity of the game may not have much practical relevance.

Casella, Palfrey, and Riezman (2008) study Storable Votes in a setting with a systematic minority, where the preferences of the minority group is consistently in the opposite direction of the majority. Furthermore, they conduct a series of experiments to test the predictions of their model. They adopt the Storable Votes scheme in Casella, Gelman, and Palfrey (2006) where each voter is endowed with an initial stock of votes and study two different models. In the "correlated" model, the majority and minority groups are well-organized and all members of a group agree on the direction as well as the strength of their preferences. In the "basic" model, the groups are not organized and while all members of a groups agree on the direction of their preferences, they do not share the same intensity of preferences. Casella, Palfrey, and Riezman (2008) show in theory and in experiments that the minority fared better under Storable Votes in both models, but gains were higher when the minority coordinated. Specifically, the minority won only on issues over which its strength of preferences was high and at the same time, the majority intensity of preferences was low, increasing the overall efficiency. This result supports the view of Guinier (1994) that well-organized minorities deserve special protection. Similar to Casella, Gelman, and Palfrey (2006), the authors observed equilibrium strategies only rarely in the experiments, but both the minority payoff and aggregate efficiency matched the theoretical predictions.

Balanced Voting attempts to combine the desirable welfare properties of Storable Votes with additional protection for minorities. Under Balanced Voting, an individual does not have to make the decision on how many votes to cast on an issue which on the one hand makes the game simpler, but on the other does not allow agents to express different intensities of preferences. For making a fundamental societal decisions, however, we show that Balanced Voting is more suitable than Storable Votes. Furthermore, Storable Votes could be subjected to agenda manipulation, as the order in which issues are considered affects how an individual shifts votes intertemporaly. Balanced Voting is suitable for making fundamental societal decisions, whereby the fundamental issue considered in the first stage seems to be a natural choice, making it less susceptible to agenda manipulation. However, gathering experimental evidence of the theoretical predictions of Balanced Voting remains an important future research avenue.

2.2 Minority Voting

Fahrenberger and Gersbach (2010) introduce Minority Voting as a voting scheme that provides minorities better protection against the risk of repeated exploitation. When a committee meets twice to decide on two separate issues, Minority Voting grants the losers of the first decision exclusive voting rights in the second. The benefits of Minority Voting are pronounced when the first decision involves long-term impact on the utility of voters. In this case, utility losses accumulate over time if an individual repeatedly belongs to the minority, raising the need for an insurance against such situations.

Fahrenberger and Gersbach (2010) illustrate the idea of Minority Voting in a committee consisting of risk-averse members that decides on adopting two projects with long-term impact in a complete information setting. In equilibrium, some agents vote strategically to ensure that they preserve their voting right for the second period, while not affecting the outcome of the first, thereby resulting in a minimum winning majority. Fahrenberger and Gersbach (2010) show that as long as agents are sufficiently risk averse, Minority Voting is superior to simple majority voting from the perspective of utilitarian welfare.

There is an inherent trade-off in Minority Voting. On the one hand it protects the minority against repeated exploitation while on the other hand, the majority of the first period is not allowed to vote in the second, creating negative externalities. However, if agents are sufficiently risk averse, accumulated loses of repeatedly belonging to the minority becomes undesirable from an ex-ante perspective, such that the gains of the minority outweigh the losses of the majority, leading to overall efficiency gains.

Fahrenberger and Gersbach (2012) analyze Minority Voting when individuals exhibit a preference for harmony. Specifically, they develop a context-dependent model where an individual's preference for harmony results in a utility loss for himself if other individuals belong to the minority repeatedly. However, the direct utility of other agents and envy does not affect one's utility. In a similar two-period setting as in Fahrenberger and Gersbach (2010), the authors show that if agents demonstrate a preference for harmony that is strong enough, the coalition-proof equilibria do not involve a minimum winning majority. This ensures that the power of the minority is increased in the second period, thereby increasing the utility of all members of the committee. The main result of the paper is that Minority Voting is superior to simple majority voting as long as the degree of disharmony aversion is sufficiently high.

While Minority Voting, as its name suggests, focuses mainly on better protection of minorities, Balanced Voting attempts to strike a balance between protecting minorities and capturing agents' intensity of preferences. Balanced Voting is suitable for making fundamental decisions, whereby the fundamental direction a society selects in the first stage clearly exerts long-lasting impact on the utility of all agents in the society. In this sense, Balanced Voting is well-positioned to reap the benefits identified in Minority Voting, which demonstrates higher welfare gains if the impact of projects last longer. Moreover, Fahrenberger and Gersbach (2010) show that agents who favor the first-period project are either worse-off with Minority Voting compared to simple majority voting or it is not clear whether they benefit or lose from Minority Voting. Therefore, it is not clear if there exist conditions under which Minority Voting Pareto-dominates simple majority voting. Balanced Voting, however, Pareto-dominates simple majority voting as long as the strength of preferences of strongly-inclined individuals is sufficiently higher than that of weakly-inclined individuals, and the society consists of both strongly and weakly-inclined individuals.

2.3 Qualitative Voting

Qualitative Voting introduced by Hortala-Vallve (2012) allows agents to express their intensity of preferences and minorities to be decisive on issues they are passionate about, thereby achieving voting outcomes that are Pareto dominant to those under simple majority voting. In contrast to dynamic voting games such as Storable Votes, Minority Voting and Balanced Voting, Qualitative Voting is a static voting game. Under Qualitative Voting, each agent is given a stock of votes which he can distribute over a series of simultaneous, binary decisions as he prefers. This enables individuals to allocate more votes on issues that they care more about, and less on those that matter less. Thus, Qualitative Voting not only acts as a mechanism that captures agents' preferences, but it also allows agents to trade-off their influence on one issue with that of another. This is desirable for resolving conflicts between two parties that have opposite preferences on issues, in terms of both direction and intensity.

Hortala-Vallve (2012) focuses his analysis on a setting with two binary decisions and derive results for when the number of voters is either two or three. He shows that in equilibrium, when there are only two voters, they allocate all of their votes on the issue that matter more to them, and splits the votes evenly between the issues if they are indifferent. This enables them to achieve the only ex-ante optimal outcome by coordinating non-cooperatively. With three voters, the equilibrium voting behavior of indifferent agents remains the same, while the non-indifferent agents allocate all, half or three-quarter of votes on their preferred issue, yielding three possible pure strategy equilibria.

Hortala-Vallve (2012) analyzes the optimality of Qualitative Voting by adopting a mechanism design approach. He proves that in the case of two voters, Qualitative Voting is ex-ante optimal while simple majority voting is not. This result holds in the case of three voters, as long as agents value their preferred issue at least three times as much as their less-preferred issue. This indicates that it is optimal to allow the minority to win on issues about which they feel very strongly and the majority does not feel very strongly about. This result is in line with the finding under Storable Votes in Casella, Palfrey, and Riezman (2008), where efficiency gains over simple majority voting were achieved only when the minorities won on issues that they are passionate about, and the majority is not.

The comparison of Qualitative Voting and Balanced Voting opens up a number of interesting issues. Similar to the idea of Storable Votes, Qualitative Voting in principle allows agents to display various intensities of preferences over issues by tailoring the number of votes they cast on an issue to resemble their valuation of the issue. This feature is not present under Balanced Voting. Moreover, optimality of Balanced Voting remains an open issue and is an important avenue to be explored in future research. Additionally, in the setting under Balanced Voting, all agents are considered to be non-indifferent between the two issues to make the model tractable, while Hortala-Vallve (2012) considers indifferent agents in his framework. In addition, Balanced Voting is not applicable to resolve conflicts between two parties, as the formation of a minority is meaningful only with at least three agents. However, the results under Balanced Voting have been proven for a committee of size N which can be much larger than three. Furthermore, as with Storable Votes, Qualitative Voting could also be susceptible to agenda-manipulation to affect how agents distribute their votes across issues, while the possibility for agenda-manipulation under Balanced Voting remains limited.

2.4 Flexible Majority Rules

Flexible Majority Rules allow the majority needed to approve a proposal depend on the proposal itself. This scheme was first introduced by Erlenmeier and Gersbach (2001), where they analyzed Flexible Majority Rules for the provision of public goods. The idea was later extended to the division of resources in Gersbach (2004) and for decision-making in central banks in Gersbach and Pachl (2009).

Erlenmeier and Gersbach (2001) consider a society that decides on the provision of a public project and the associated financing package, proposed by an agenda setter. The society votes on the proposal using Flexible Majority Rules, where the majority required to approve the proposal depends on the number of taxed individuals. The authors show that it is possible to construct a constitution that implements the first-best solution by combining the following principles: (i) taxation constrained to the majority in favor of the project; (ii) maximal taxation of the agenda setter and costly agenda setting; (iii) a ban on subsidies; (iv) Flexible Majority Rules.

Gersbach (2004) illustrates how Flexible Majority Rules combine the advantages of both simple majority rule and unanimity rule. He adopts Flexible Majority Rules to a community of impatient individuals deciding on how to distribute resources. To approve the proposal of an agenda setter, Flexible Majority Rules require a majority that corresponds to the maximal difference between the shares received by individuals. Erlenmeier and Gersbach (2001) and Gersbach (2004) suggest that there are at least three ways in which Flexible Majority Rules can be defined. First, a Constant Majority Rule requires the majority to be a constant fraction of the voters and not depend on the difference in benefits received. Second, a Threshold Flexible Majority Rule allows simple majority rule to be applied as long as the maximal difference in allocations is below a particular threshold and switch to unanimity rule otherwise. Third, a Linear Flexible Majority Rule requires the majority to be at least 50% and linearly increase with the maximal difference in resource allocations. Gersbach (2004) shows that the combination of Flexible Majority Rules and agenda-planning rules lead to efficient and envy-free allocations.

Gersbach and Pachl (2009) extend Flexible Majority Rules to a central bank committee where a change in the interest-rate needs approval by a majority that monotonically increases with the magnitude of the interest-rate change itself. They demonstrate that when a region is hit by a negative shock, the group of states that are negatively affected can approve an interest-rate cut proportional to the size of their group. Thereby, Flexible Majority Rules indirectly approximate aggregate social loss minimization by leading to an interest-rate reduction that corresponds to the magnitude of the shock itself. The main result of Gersbach and Pachl (2009) is that Flexible Majority Rules is superior to simple majority rule, as they lead to lower social losses.

The setting in which Balanced Voting is introduced is different to the settings that are studied under Flexible Majority Rules. Balanced Voting is a dynamic voting game, while voting under Flexible Majority Rules is static. More crucially, Balanced Voting determines fundamental societal directions which typically do not involve verifiable utilities or allocations, which are necessary to the application of Flexible Majority Rules. Nevertheless, the central idea of Flexible Majority Rules that decisions with larger impact should require higher support is related to the idea of Balanced Voting that individuals with higher stakes in an issue should be allowed larger influence on the decision.

2.5 Cumulative Voting

Cumulative Voting is one of the older voting schemes to substitute the simple majority rule, and has been studied extensively and used in practice at least as early as in 1870 (see Sawyer and MacRae (1962)). In an election with more than two candidates, Cumulative Voting endows voters with a stock of votes that corresponds to the number of candidates that must be selected. Voters are free to distribute their votes on the competing candidates as they prefer, allocating more votes on the candidates they prefer, and less or no votes on those they oppose. In that sense, Cumulative Voting allows individuals to express their strength of preference over candidates by the number of votes they cast on them. Moreover, Cumulative Voting has been recognized for providing better protection of minorities (see Guinier (1994)).

There are several instances where Cumulative Voting has been used in practice. Some examples are, the state of Illinois in the United States until 1982, school boards in England until 1902 and some corporations that still continue to use Cumulative Voting to elect members to their boards of directors (see Sawyer and MacRae (1962) and Gerber, Morton, and Rietz (1998)).

There is theoretical, experimental and empirical literature on Cumulative Voting, confirming its property of better minority protection. Cox (1990) examines equilibrium candidate positions when voters can cumulate their votes and illustrates that Cumulative Voting induces politicians to move away from the median voter position and promote minority representation instead. Gerber, Morton, and Rietz (1998) analyze voting equilibria in multi-member districts and provide supporting experimental evidence that Cumulative Voting allows minority candidates to win significantly more seats. Sawyer and MacRae (1962) conduct a theoretical analysis of Cumulative Voting and present an empirical study on how Cumulative Voting was adopted in the state of Illinois in the United States.

Balanced Voting, similar to Cumulative Voting, provides stronger protection for minorities, albeit in a dynamic setting which contrasts with the static nature of Cumulative Voting. Moreover, Balanced Voting is illustrated for binary decisions, whereas Cumulative Voting is meaningful only if the number of candidates to be selected is larger than or equal to two. For binary decisions, Cumulative Voting is equivalent to simple majority voting and we show that in the framework with binary decisions we consider, Balanced Voting is superior to simple majority voting under certain conditions.

2.6 Vote-Trading

Vote-trading allows agents to exchange votes with other agents in return for monetary compensation. The idea is that the price an individual is willing to pay for a vote should reflect his preference intensity on the issue, and aggregating this information through a market for votes should lead to more efficient outcomes than with simple majority voting.

There are many advocates of vote-trading, especially among the early writers (see Coleman (1966)). However, there is considerable criticism of vote-trading for many reasons. Vote-trading violates the fundamental principle of equity in a democracy, as it allows the wealth of an individual to determine the influence he can exert on a decision. Moreover, Piketty (1994) argues that a vote market can be socially harmful. When preferences of agents are uncertain and correlated to some privately-observed signals, Piketty (1994) shows that a market for votes can lead to the destruction of these signals. Philipson and Snyder (1996) recognize the inefficiencies of a decentralized vote market and develop an organized vote market with central control. They show that any outcome that results in a trade in this centralized market is Pareto superior to the alternative without trade. However, such a market allocates significant power to a central authority that not only mediates the market but also determines the price at which votes trade.

Similar to the idea behind vote-trading, Balanced Voting also allows individuals to indirectly express their intensity of preferences through their decision to participate or abstain in the first stage. However, Balanced Voting does not involve the institutional complexities of organizing a market for votes. More importantly, Balanced Voting does not involve monetary transfers and is therefore more in line with democratic principles. Hence, the acceptance of Balanced Voting by democracies is more likely than any mechanism involving vote-trading.

2.7 Other Voting Rules

There are several other voting mechanisms that have been studied in the voting literature and even applied in practice. In this section we briefly outline a few of them and discuss their differences with Balanced Voting.

Levin and Nalebuff (1995) give an overview of a variety of voting mechanisms available and Myerson and Weber (1993) analyze the voting equilibria under plurality rule, approval voting and Borda rule. We focus on four voting rules that are popularly in use: plurality rule, approval voting, single transferable vote and Borda rule.

Plurality voting is the most common way of ranking candidates in an election (Levin and Nalebuff (1995)). Each voter votes for a single candidate out of many, and the candidate with most votes wins. Under approval voting, voters approve or disapprove of each candidate in one ballot, and the candidate with most approvals wins. Single transferable vote allows voters to rank candidates, and after votes are tallied based on first choice, the last-ranked candidate is eliminated, and his votes are distributed on the remaining candidates based on second choice and so on, until a winner emerges. Borda rule allows voters to rank candidates, and candidates receive points according to the rankings received, leading to the win of the candidate with most points. For example, if there are k candidates, for each ranking at first place, a candidate receives $k - 1$ points, a ranking at second place gives $k - 2$ points and so on.

It is natural to question why the two-stage setup of Balanced Voting cannot be modified to reflect a single-stage decision between the four variations of the two fundamental directions, and apply one of the voting schemes above. The main drawback with schemes such as plurality voting and approval voting is that they do not capture the intensity of voter preferences. The majority is once again allowed to overrule the minority on repeated issues, the proportionality principle is violated as the power of a voter on a decision remains constant and thus, the drawbacks of the majority rule remain unaffected. Mechanisms such as Borda rule or single transferable vote allow voters to rank the alternatives, so that voters are able to convey the strength of their preference for one alternative relative to the other alternatives. These schemes, however, treat individuals both strongly and weakly-inclined towards an alternative the same way, as it is not possible to retrieve information on how strong an individual's preference for an alternative is, in absolute terms. In the framework of making fundamental societal decisions for which Balanced Voting is suitable, we contend that only those who feel strongly about the fundamental direction of the society should participate in its decision, as supported by the argument of the principle of proportionality.

3 Model and Results for Asymmetric Information

3.1 The Model

We consider a society of N ($N \geq 3$) individuals who are indexed by i or i' ($i, i' \in \{1, ..., N\}$). They first take a fundamental decision and then decide on variations of the outcome of the first decision.

3.1.1 The setting

There are two fundamental directions, denoted by A and B. An alternative in the first stage is denoted by $\Omega \in \{A, B\}$. Let each Ω have two variations which we denote by x_Ω^j ($j = 1, 2$). As an example, the setting for the fundamental decision on whether or not a society should continue to use nuclear power for energy production is given in Figure 3.1.

3.1.2 Balanced Voting

Under Balanced Voting, agents have the option of either voting for a fundamental direction or of abstaining in the first stage. Those who abstain receive voting rights for the second stage. The agents who lose in the first stage also obtain voting rights in the

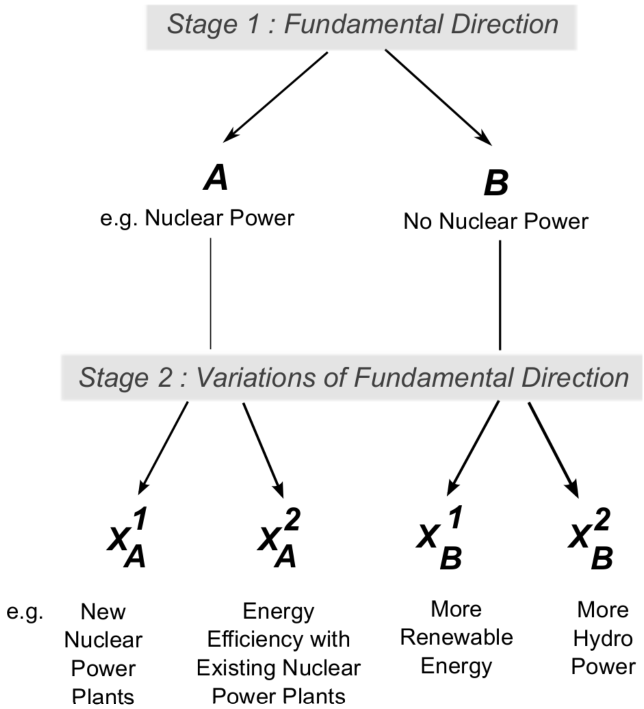

Figure 3.1: The setting.

second stage, while the agents that won in the first stage are not allowed to vote in the second stage.

A comparison of Balanced Voting with simple majority voting, Storable Votes and Minority Voting, with respect to the composition of the voting body in each stage, is given in Figure 3.2.

3.1.3 Utilities

Each individual derives non-negative utility from the final decision x_Ω^j ($\Omega \in \{A, B\}$, $j = 1, 2$). We denote by $U_i(x_\Omega^j|\Omega)$ the utility individual i obtains if Ω is selected in the first stage and x_Ω^j in the second stage. For notational convenience, we write $U_i(x_\Omega^j)$ instead of $U_i(x_\Omega^j|\Omega)$ if Ω has been selected in the first stage.

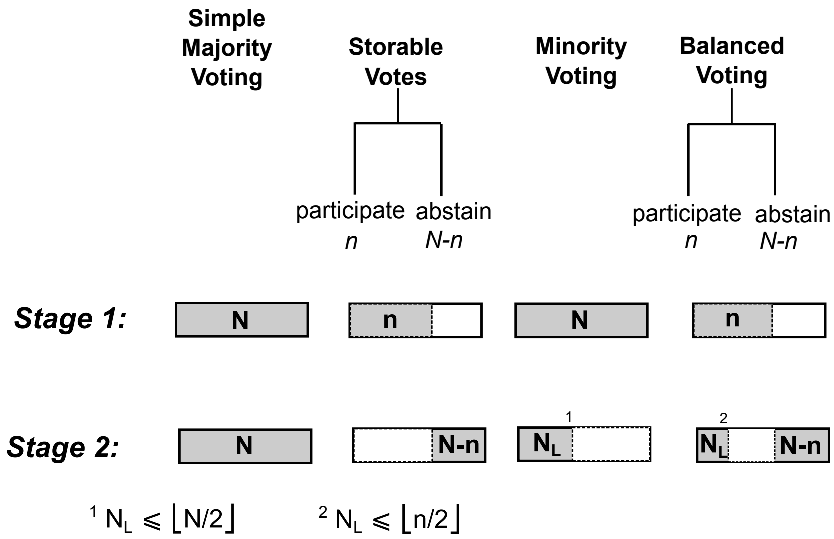

Figure 3.2: The composition of the voting body under different voting schemes.

3.1.4 Preference groups

Every agent has an inclination towards one of the fundamental directions, which may be strong or weak. Furthermore, no agent is indifferent between the variations of a fundamental direction. We denote the group of strongly-inclined individuals by *SI* and the set of weakly-inclined agents by *WI*. N_s and N_w represent the number of individuals belonging to groups *SI* and *WI* respectively. Thus, $N_s + N_w = N$. The utility of agent i is as follows:

$$i \in SI \quad \text{iff} \quad \begin{cases} U_i(x^j_\Omega) = 1 + H & \text{for some } \ \Omega \in \{A, B\} \ \text{ and some } \ j \in \{1, 2\} \\ U_i(x^k_\Omega) = 0 + H & \text{for } \ k \in \{1, 2\}, j \neq k \\ U_i(x^{j'}_{\Omega'}) = 1 & \text{for some } \ j' \in \{1, 2\}, \Omega' \in \{A, B\}, \Omega' \neq \Omega \\ U_i(x^{k'}_{\Omega'}) = 0 & \text{for } \ k' \in \{1, 2\}, k' \neq j' \end{cases}$$

$$i \in WI \quad \text{iff} \quad \begin{cases} U_i(x^j_\Omega) = 1 + \epsilon & \text{for some } \ \Omega \in \{A, B\} \ \text{ and some } \ j \in \{1, 2\} \\ U_i(x^k_\Omega) = 0 + \epsilon & \text{for } \ k \in \{1, 2\}, j \neq k \\ U_i(x^{j'}_{\Omega'}) = 1 & \text{for some } j' \in \{1, 2\}, \Omega' \in \{A, B\}, \Omega' \neq \Omega \\ U_i(x^{k'}_{\Omega'}) = 0 & \text{for } \ k' \in \{1, 2\}, k' \neq j' \end{cases}$$

H and ϵ are positive constants representing the agents' intensity of preferences or stakes.

Our main assumption is

Assumption 1

$$H >> 1 > \epsilon > 0.$$

While *SI* individuals derive higher utility from all variations of their preferred fundamental direction, the utility of *WI* agents is not guaranteed to be higher if their preferred proposal is chosen in the first stage. Depending on which variation is elected in the second stage, there may be instances where *WI* individuals are better off if their less-favored alternative was selected in the first stage.

Suppose, for example, that individual i is strongly-inclined towards A while individual i' is weakly-inclined towards B. Then for individual i, variations have second-order importance, while for individual i', the chosen variation is of first-order importance. Table 3.1 provides an example of a possible realization of their utilities.

	$j = 1$	$j = 2$
$U_i(x_A^j)$	$1 + H$	$0 + H$
$U_i(x_B^j)$	0	1
$U_{i'}(x_A^j)$	1	0
$U_{i'}(x_B^j)$	$1 + \epsilon$	$0 + \epsilon$

Table 3.1: An example of realization of utilities.

At this stage we can provide an initial intuition for the plausible behavior of agents under Balanced Voting. An *SI* agent highly benefits if his fundamental direction is chosen, and therefore, has an incentive to participate in determining the fundamental direction. For a *WI* individual, if ϵ is very small, the additional utility of 1 he derives from his preferred variation may seem more important compared to ϵ. Therefore, *WI* agents may have an incentive to exert more influence on the decision of the second stage.

3.1.5 Informational assumptions

We assume that each individual faces a probability p ($0 < p < 1$) of exhibiting a strong inclination towards one direction, while he has a weak inclination with probability $1-p$. There is a probability of $\frac{1}{2}$ for this inclination to be towards A. Similarly, $\frac{1}{2}$ is the probability that an individual has an additional utility of 1 for one of the variations of a fundamental direction over the other. We assume that these events are stochastically independent. Furthermore, we assume that the realization of the individual utility is stochastically independent across individuals and is privately observed prior to the start of voting. The probability p and values of H and ϵ are common knowledge.

3.1.6 Voting rules

We consider secret voting under the following voting regimes:

1. Simple Majority Voting (SM)
2. Balanced Voting (BV)
3. Storable Votes (ST)
4. Minority Voting (MV)

The specific rules under each voting scheme are as follows:

Under SM, individuals decide whether to vote and how to vote on the fundamental proposals A and B in the first stage, and the proposal receiving the higher number of votes is selected. If both proposals receive the same number of votes, the winner is determined by flipping a fair coin, giving each alternative a probability of $\frac{1}{2}$ to be selected. In the second stage, individuals once again can participate in a simple majority vote on variations of the selected fundamental proposal. The winner is chosen using the same rules as in the first stage.

Under BV, each individual must first decide whether he participates in or abstains from voting in the first stage. The losers and the absentees[1] of the first stage have the right to

[1]We use the term "absentees" in the spirit of Fahrenberger and Gersbach (2010) to denote the individuals who abstain from voting in the first stage.

vote in the second stage. The decision is taken by the simple majority rule in both stages with a similar tie-breaking procedure as in SM. If every member decides to abstain from voting in the first stage, or if there are neither losers nor absentees from the first stage, all individuals are eligible to vote in both stages.

Under ST, agents receive one vote each, which they can either use in the first stage or store for use in the second stage.[2] Following Casella (2005), the proposal receiving a simple majority of votes is selected in both stages, with a tie-breaking procedure similar to the one in SM. If, however, all agents use their votes on the first decision or store them for the second decision, every individual will be allowed to vote in both stages.

Under MV, all citizens can vote in the first stage while only the minority of the first stage has a voting right in the second stage. An individual may abstain from voting in the first stage, in which case he will not be allowed to vote in the second stage. If there are no losers from the first stage, all agents retain their right to vote in the second stage. The decision is taken by the simple majority rule in both stages, with a tie-breaking procedure similar to the one in SM.

3.2 Equilibria

3.2.1 Equilibrium concept and probability of winning

In the following, we look for perfect Bayesian equilibria in pure strategies under each voting rule. We adopt the standard refinement for voting games to exclude implausible voting behavior. In particular, we assume that all agents participating in the second stage eliminate weakly-dominated strategies at this stage. If this procedure yields unique strategies for the second stage, we assume that agents also eliminate weakly-dominated strategies in the first stage, if there are any.

Finally, we look for symmetric equilibria where voters with the same intensity of preference, i.e. *SI* and *WI* individuals, behave similarly with respect to the sincerity of their vote and their possible abstention from the first stage. A perfect Bayesian equilibrium

[2] The interpretation of ST we use is slightly different from the scheme introduced in Casella (2005), where every agent obtains a vote in each stage. The reason is to allow direct comparison with BV.

with these properties will simply be called an 'equilibrium' in the remainder of the dissertation.

A justification of the refinement of symmetric equilibria is in order here. In our model, we assume that an individual is inclined towards A or B with equal probability, and that his utility does not change with the direction he prefers. The same applies for his preference of x_A^1 over x_A^2 and x_B^1 over x_B^2. The only event that may occur with unequal probability and that simultaneously affect his utility is the event that he is inclined either strongly or weakly. Thus, it appears justifiable that we consider symmetric equilibria where agents with the same intensity of preference behave the same way, so that the direction of his preference does not impact his voting behavior. In Section 4.1, we will show that equilibria under BV remain unaffected even if we impose less demanding symmetry requirements.

To derive the equilibria, it is useful to express an individual's probability of winning in an isolated vote on two alternatives. Consider the following isolated voting problem:

- n ($n \leq N$) agents voting,
- two alternatives,
- ex-ante probability of $\frac{1}{2}$ that an individual favors one alternative,
- private information of utilities.

Then, if n individuals cast their votes, the probability of winning for one individual if all other individuals vote sincerely is given by[3]

$$P(n) = \frac{1}{2^n}\binom{n-1}{\lfloor \frac{n-1}{2} \rfloor} + \frac{1}{2}. \tag{3.1}$$

The above expression states that an individual faces a probability greater than $\frac{1}{2}$ of having his preferred option elected by participating in the vote. This probability increases if the number of voting agents decreases. Furthermore, $P(n-1) = P(n)$ when n is an odd number. Moreover, $\lim_{n\to\infty} P(n) = \frac{1}{2}$, as the weight of an individual vote becomes negligible. The plot of $P(n)$ is given in Figure 3.3.

[3]See e.g. Fahrenberger and Gersbach (2010). Note that $\lfloor a \rfloor = \max \{ b \in \mathbb{N} : b \leq a \}$.

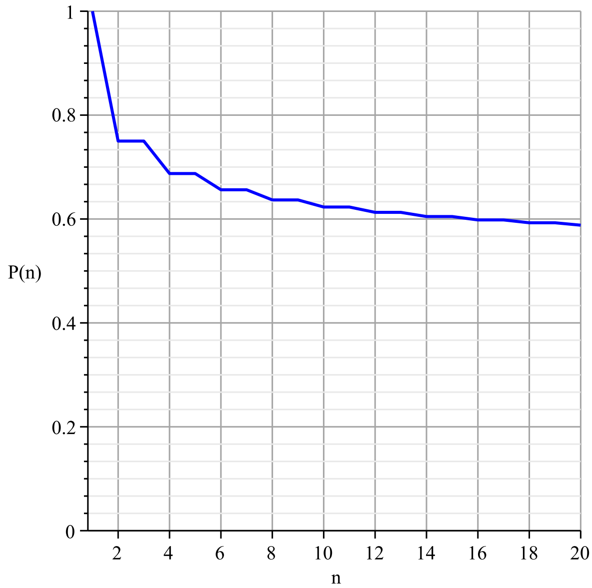

Figure 3.3: The probability of winning $P(n)$ vs. the number of voting agents n.

Before characterizing the equilibria under each voting scheme, we introduce some useful notation. We have already shown that if n agents participate in the first stage, each participating agent wins with probability $P(n)$ and loses with probability $(1 - P(n))$. The number of agents allowed to vote in the second stage, however, varies with the voting scheme. Hence, from an ex-ante point of view, i.e. before voting begins, but after agents have privately observed their preferences, an agent faces different probabilities of winning in the second stage under each voting scheme. It is, thus, important that these probabilities be formally introduced.

Under SM, all agents are allowed to vote in the second stage. In Section 3.2.2, we show that no agent will abstain from voting in either stage under SM. Hence, regardless of whether he wins or loses in the first stage, every individual expects to win in the second stage with probability $P(N)$ if all members of the committee vote in the second stage.

Under BV, the winners will not be allowed to vote in the second stage. Hence, their preferred variation in the second stage will be selected with probability $\frac{1}{2}$. The losers will participate in the second stage, along with the absentees. Suppose $n \in (0, N)$ agents participated in the first stage and $N_L \in [0, \lfloor \frac{n}{2} \rfloor]$ agents lost the vote. Then, $N - n + N_L$ individuals will vote in the second stage. Ex-ante, a voter in the second stage wins with a probability that depends on whether he was allowed to vote as an absentee or loser from the first stage. Using Bayes' Theorem, we obtain these probabilities as follows:

$\Phi(N,n)$:= Ex-ante probability of winning in second stage under BV, for losers of first stage

$$= \begin{cases} \dfrac{\sum_{N_L=1}^{\lfloor \frac{n}{2} \rfloor} \binom{n}{N_L} P(N-n+N_L)}{\sum_{z=1}^{\lfloor \frac{n}{2} \rfloor} \binom{n}{z}} & \text{for } n \text{ odd, } n>1, \\ \dfrac{\sum_{N_L=1}^{\frac{n}{2}-1} \binom{n}{N_L} \frac{1}{2^{(n-1)}} P(N-n+N_L) + \binom{n}{\frac{n}{2}} \frac{1}{2^n} P(N-\frac{n}{2})}{\sum_{z=1}^{\frac{n}{2}-1} \binom{n}{z} \frac{1}{2^{(n-1)}} + \binom{n}{\frac{n}{2}} \frac{1}{2^n}} & \text{for } n \text{ even.} \end{cases} \tag{3.2}$$

For $n = 1$ we define $\Phi(N, 1) = 0$.

$\Lambda(N,n)$:= Ex-ante probability of winning in second stage under BV, for absentees of first stage

$$= \begin{cases} \sum_{N_L=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n}{N_L} \frac{1}{2^{(n-1)}} P(N-n+N_L) & \text{for } n \text{ odd,} \\ \sum_{N_L=0}^{\frac{n}{2}-1} \binom{n}{N_L} \frac{1}{2^{(n-1)}} P(N-n+N_L) \\ \qquad + \binom{n}{\frac{n}{2}} \frac{1}{2^n} P(N-\frac{n}{2}) & \text{for } n \text{ even.} \end{cases} \tag{3.3}$$

Detailed derivations of $\Phi(N, n)$ and $\Lambda(N, n)$ are given in Appendix C.[4]

Under ST, losers from the first stage are not allowed to vote in the second stage. Therefore, all participants in the first stage have their preferred variation in the second stage selected with probability $\frac{1}{2}$. All $N - n$ absentees vote in the second stage and win with probability $P(N - n)$.

In Section 3.2.5, we show that no agent with voting rights will abstain from voting in either stage under MV. As in BV, all winners of the first stage win their preferred option in the second stage with probability $\frac{1}{2}$. The losers vote in the second stage and win with

[4]The calculation of $\Phi(N, n)$ requires the Formula (3.4), which is derived below. Further properties of $\Phi(N, n)$ and $\Lambda(N, n)$ are given in Table A.1.

the following probability which is derived in detail in Appendix C:[5]

$$\Theta(N) := \text{Ex-ante probability of winning in second stage under MV, for losers of first stage}$$

$$= \begin{cases} \dfrac{\sum_{N_L=1}^{\lfloor \frac{N}{2} \rfloor} \binom{N}{N_L} P(N_L)}{\sum_{z=1}^{\lfloor \frac{N}{2} \rfloor} \binom{N}{z}} & \text{for } N \text{ odd,} \\[2ex] \dfrac{\sum_{N_L=1}^{\frac{N}{2}-1} \binom{N}{N_L} \frac{1}{2(N-1)} P(N_L) + \binom{N}{\frac{N}{2}} \frac{1}{2N} P(\frac{N}{2})}{\sum_{z=1}^{\frac{N}{2}-1} \binom{N}{z} \frac{1}{2(N-1)} + \binom{N}{\frac{N}{2}} \frac{1}{2N}} & \text{for } N \text{ even.} \end{cases} \tag{3.4}$$

We further recall the following three special cases and the corresponding voting rules.

1. If all agents abstain ($n = 0$) under BV or ST, then all agents can vote in both stages.
2. If all agents participate in the first stage ($n = N$) under ST, then all agents can vote in both stages.
3. Suppose all agents participate in the first stage ($n = N$) under BV and MV. If individuals are unanimously inclined towards A or B, then all agents can vote in both stages. If there are losers in the first stage, they receive exclusive voting rights in the second stage.

We note that in these three special cases, whenever all agents can vote in both stages, no individual has an incentive to abstain from voting, as participation strictly increases his probability of winning. Using the properties of the binomial coefficient and the number of agents expected to vote in each case, we derive the following fact, which holds for all $N \in [3, \infty)$:

Fact 1

- $\Theta(N) > \Lambda(N, n) \geq P(N)$ *when* $0 < n < N$, *where* $\Lambda(N, n) = P(N)$ *if and only if* $n = 1$ *and* N *is odd.*

[5] Further properties of $\Theta(N)$ are given in Table A.1.

- *$\Theta(N) > \Lambda(N,n) \geq \Phi(N,n) \geq P(N)$ when $1 < n < N$, where $\Phi(N,n) = P(N)$ if and only if $n = 2$ and N is odd.*

The conditions in Fact 1 are intuitive and are proven in Appendix B. When the number of agents participating in a vote is less than N, the participating individuals win with a probability greater than or equal to $P(N)$. Thus, min $\{\Theta(N),\ \Lambda(N,n),\ \Phi(N,n)\}$ $\geq P(N)$. When only losers from the first stage vote in the second, the voting body in the second stage is always smaller than or equal to $\lfloor \frac{N}{2} \rfloor$. However, when absentees from the first stage are guaranteed voting rights in the second, the voting body of the second stage can be greater than $\lfloor \frac{N}{2} \rfloor$ with positive probability. Thus, $\Theta(N) >$ max $\{\Lambda(N,n),$ $\Phi(N,n)\}$. Interestingly, although under BV, both losers and absentees of the first stage vote in the second stage, we note that their ex-ante probabilities of winning can differ. That is, $\Lambda(N,n) \geq \Phi(N,n)$, which can be explained as follows: N_L could be zero and the voting body could merely consist of absentees in the second stage. This occurs with positive probability. A loser from the first stage faces a voting body strictly larger than the number of absentees, which makes his probability of winning in the second stage weakly lower than that of the absentees. To illustrate this property with an example, we plot the probabilities for $N = 20$ in Figure 3.4.

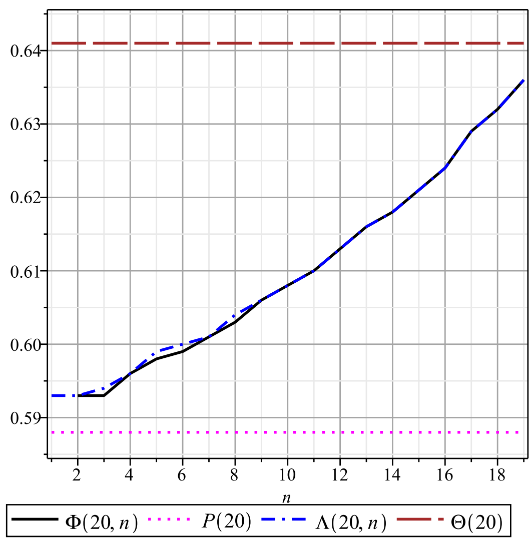

Figure 3.4: $\Theta(N), \Lambda(N,n), \Phi(N,n), P(N)$ vs. n for $N = 20$.

Now, we are ready to examine the equilibria under each voting scheme.

3.2.2 Equilibria under SM

We start by analyzing the voting behavior in the second stage under SM. We first observe that voting weakly-dominates abstention, as there is a chance that the agent is pivotal. Moreover, voting sincerely weakly-dominates voting strategically, i.e. voting for the option that gives one lower utility, as no member can realize any utility gain by voting against his true preference in a binary collective decision when the game ends in the second stage.

Next, we focus on the first stage. Regardless of the variation selected in the second stage, an *SI* agent is strictly better off if his preferred proposal is selected in the first stage. Hence, he has an incentive to vote sincerely and increase the selection chances of his preferred fundamental direction.

Since preference realizations are private information, a *WI* individual i's behavior depends on his expectation of which variation will be chosen in the second stage. Given that all agents will vote sincerely in the second stage, i's preferred variation has an equal probability of being selected, for any fundamental direction chosen in the first stage. Hence, i's expected utility weakly increases if he votes for his preferred fundamental direction in the first stage.

Thus, after sequentially eliminating weakly-dominated strategies, all individuals participate and vote sincerely in both stages under SM. Hence, there exists a unique Bayesian Equilibrium in pure strategies, where all agents vote sincerely in both stages.

3.2.3 Equilibria under BV

Under BV, winners of the first stage are not allowed to vote in the second stage, while losers are compensated for their loss a vote in the second stage. However, with privately-observed preferences, it is ex-ante unclear whether an agent would belong to the losers or not. Thus, individuals face a trade-off in the first stage: abstention lowers the probability of winning in the first stage, while it secures voting rights in the second stage. We characterize the agents' voting behavior in the following proposition:

Proposition 1

I) Suppose that $H > 1$ and $\epsilon \leq \epsilon^{crit}(N)$ for some critical value $\epsilon^{crit}(N) > 0$. Then, there exists a unique equilibrium under BV characterized by

(i) every individual $i \in SI$ participates in the first stage. All individuals $i \in WI$ abstain;

(ii) all votes in both stages are cast sincerely.

II) The critical value on ϵ is given by

$$\epsilon^{crit}(N) := \min\left[\frac{P(N)-\frac{1}{2}}{1-P(N)},\ 4\Lambda(N,1)-\Phi(N,2)-\frac{3}{2}\right].$$

The proof of Proposition 1 is given in Appendix B. Proposition 1 establishes a threshold on the intensity of preferences of *WI* individuals regarding the fundamental direction the society should take. If ϵ is below $\epsilon^{crit}(N)$, an individual $i \in WI$ would always be inclined to abstain from voting in the first stage. As shown in the proof, with our main assumption $H >> 1$, *SI* individuals are always better off voting in the first stage. This is in line with the main motivation behind BV, which is to allow individuals that care more about an issue to exert more influence on a decision, while those who care less choose to abstain to secure the possibility to influence future decisions.

A plot of $\epsilon^{crit}(N)$ is given in Figure 3.5.[6] We note that $\epsilon^{crit}(N)$ is monotonically decreasing in N. As the committee grows larger, the voting body of the second stage also increases in size with positive probability. Therefore, it becomes less attractive to abstain from the first stage for a better chance of winning in the second. The value of ϵ should, thus, be smaller for *WI* individuals to still prefer abstention, which is confirmed by Figure 3.5. For the remainder of the dissertation, we assume that ϵ is such that Proposition 1 holds.

[6] Further properties of $\epsilon^{crit}(N)$ are given in Table A.1.

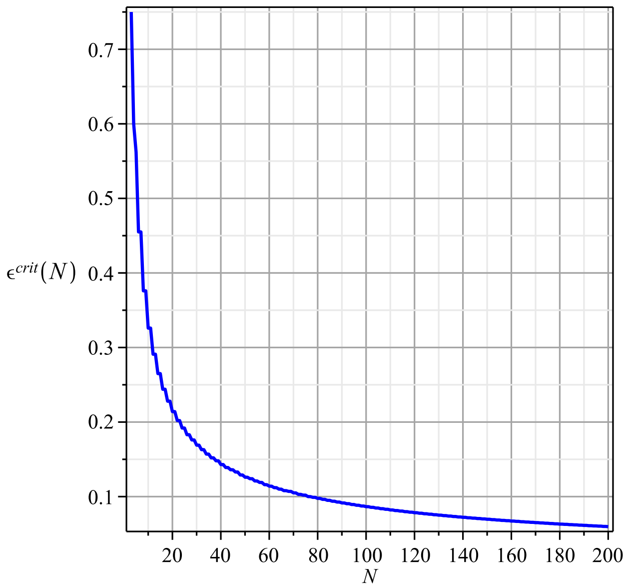

Figure 3.5: $\epsilon^{crit}(N)$ vs. the number of agents N.

3.2.4 Equilibria under ST

Suppose the society adopts ST as the voting scheme, where each agent will receive one vote, which he can either use in the first stage or store for use in the second stage. We characterize the voting behavior of agents under ST in the following lemma:

Lemma 1

There exists a unique equilibrium under ST, in which all individuals $i \in SI$ cast their vote sincerely in the first stage, while all $i \in WI$ store it and vote sincerely in the second stage, for H and ϵ satisfying the following conditions:

- $H \geq H_{st}^{crit}(N) := \frac{1-P(N)}{P(N)-\frac{1}{2}}$,
- $\epsilon \leq \epsilon_{st}^{crit}(N) := \frac{P(N)-\frac{1}{2}}{1-P(N)} \geq \epsilon^{crit}(N)$.

The proof of Lemma 1 follows the proof of Proposition 1 and is given in Appendix B. We plot $H_{st}^{crit}(N)$ and $\epsilon_{st}^{crit}(N)$ in Figure 3.6.[7]

[7]Further properties of $H_{st}^{crit}(N)$ and $\epsilon_{st}^{crit}(N)$ are given in Table A.1.

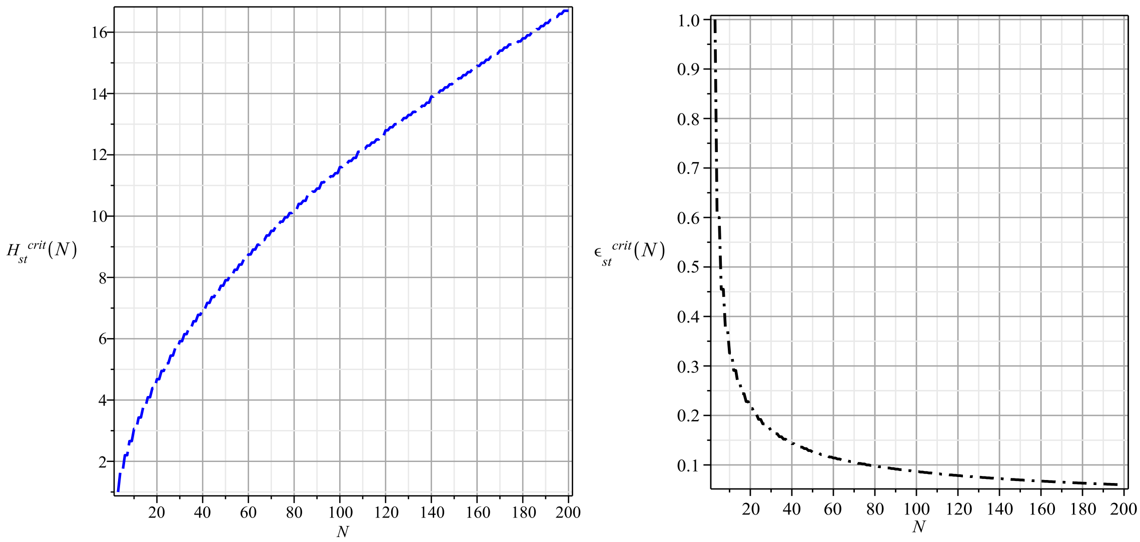

Figure 3.6: $H_{st}^{crit}(N)$ and $\epsilon_{st}^{crit}(N)$ vs. N.

We note that the thresholds on ϵ for the existence of a unique equilibrium under BV and ST are almost the same. Moreover, if $\epsilon \leq \epsilon^{crit}(N)$, $\epsilon \leq \epsilon_{st}^{crit}(N)$ also holds.

Under ST, $H_{st}^{crit}(N)$ imposes a lower bound on H that was not required under BV to ensure that all *SI* individuals are voting in the first stage. Under ST, only absentees of the first stage vote in the second. Under BV, however, losers from the first stage also vote in the second, making the voting body in the second stage larger with positive probability. Thus, abstention under ST is more attractive and H should be high enough for participation in the first stage to be more desirable than abstention.

Moreover, we observe that $H_{st}^{crit}(N)$ is increasing in N. If most of the agents choose to vote in the first stage, the voting body of the first stage grows larger with increasing N, while the voting body of the second becomes comparatively smaller. An individual voting in the first stage should, therefore, be willing to renounce the much higher probability of winning in the second stage, which is possible only if the strength of his preference is sufficiently high.

The additional constraint on H that is required under ST indicates that compared to ST, BV is better-suited for fundamental decision-making, as *SI* individuals are more inclined to vote on the first-stage decision, where their stakes are high.

3.2.5 Equilibria under MV

Now suppose that MV is the voting scheme used to reach the collective decision. Under MV, all agents will vote in the first stage and the resulting losers will receive exclusive voting rights in the second stage. In contrast with the setting with complete information analyzed by Fahrenberger and Gersbach (2010), we focus on a society with incomplete information. Incomplete information makes it impossible to gain by voting strategically. The resulting voting behavior of agents is summarized in the following lemma:

Lemma 2
There exists a unique equilibrium under MV where all individuals vote sincerely in both stages.

The proof of Lemma 2 is given in Appendix B. We note that since abstention from voting in the first stage under MV means being excluded from voting in both stages, no agent has an incentive to abstain and therefore, there are no restrictions on H and ϵ for the existence of the equilibrium under MV.

3.3 An Example

We illustrate BV by the example of a society with three agents ($N = 3$). They vote in a setting of incomplete information, where each agent observes his own utility but not the utility of the other two. We consider the following cases:

Case 1: $N_s = 3$ and $N_w = 0$

First, consider the scenario where all three individuals are SI. Then, as shown in Section 3.2, all individuals will choose to vote in the first stage.

If option Ω ($\Omega \in \{A, B\}$) achieves a majority of two votes, it will be selected as the fundamental direction. The agent who preferred Ω' ($\Omega' \in \{A, B\}, \Omega \neq \Omega'$) will obtain the sole voting right in the second stage, and the variation x_Ω^1 or x_Ω^2 that gives him the highest utility will be selected.

If Ω receives all three votes in the first stage, under the rules of BV, all agents will be eligible to vote in the second stage, which makes BV equivalent to SM.

Case 2: $N_s = 2$ and $N_w = 1$

In this case, two of the three individuals are SI, while the remaining agent is WI. Thus, the SI individuals will choose to vote in the first stage, while the WI individual will abstain.

If the SI agents agree on a fundamental decision Ω, it will be selected unanimously in the first stage. The WI individual will be the sole decision-maker in the second stage, and will select x_Ω^1 or x_Ω^2, according to his preference.

If the SI agents cannot agree on a fundamental decision, A or B will be chosen with a probability of $\frac{1}{2}$ by flipping a fair coin. Suppose option Ω ($\Omega \in \{A, B\}$) was selected this way. Then, the SI agent who preferred Ω', together with the WI individual, will be eligible to vote in the second stage. Variation x_Ω^1 or x_Ω^2 will be then selected, either unanimously or by flipping a fair coin.

Case 3: $N_s = 1$ and $N_w = 2$

Now suppose there is only one SI individual, while the other two are WI. The SI individual will be the only voter of the first stage, as the WI agents will choose to abstain in the first stage. Hence, the desired fundamental direction of the SI agent, Ω, will be the outcome of the first stage. The WI individuals will then select one of the variations x_Ω^1 or x_Ω^2, either unanimously or by flipping a fair coin.

Case 4: $N_s = 0$ and $N_w = 3$

Finally, consider the case where all three individuals are WI. Then, all individuals will choose to abstain in the first stage. Under the rules of BV, all individuals will then be allowed to vote in both stages, making BV equivalent to SM in this particular case.

3.4 Welfare Comparison

We compute social welfare from an ex-ante perspective when the citizens' utilities have not yet been realized, under the four voting schemes considered, i.e. SM, BV, ST and MV. We then compare the welfare under SM, ST and MV to welfare under BV to derive the conditions for which BV is superior to each of the other voting schemes. For this, we denote the aggregate expected utility by

$$W^V := \mathbb{E}\left[\sum_{i=1}^{N} U_i(\cdot)\right], \tag{3.5}$$

where $\mathbb{E}$ denotes the expectation operator before the utilities are realized in the first stage and $V \in \{BV, SM, ST, MV\}$. The welfare comparison is based on the utilitarian criterion, which means that V is strictly superior to V' ($V' \in \{BV, SM, ST, MV\}$, $V \neq V'$) if and only if $W^V > W^{V'}$. In addition to the welfare comparison, the utility differences for the groups of agents identified below are also computed:

(i) *WI* agents when $N_s = 0$,

(ii) *WI* agents when $0 < N_s < N$,

(iii) *SI* agents when $N_s = N$,

(iv) *SI* agents when $0 < N_s < N$.

3.4.1 Welfare under BV

We begin our welfare comparisons by calculating the expected aggregate utility under BV in equilibrium. Recall that all N_s *SI* individuals will vote sincerely in the first stage, while all $N - N_s$ *WI* individuals will abstain. Also recall that the number of losers from the first stage has been denoted by N_L. Hence, a total of $N - N_s + N_L$ agents will participate in the second stage.

For ease of reference, we summarize the ex-ante probabilities of winning for each group of agents in Table 3.2, first in both stages, and the in the second stage only. We note that the probability of winning in the first stage only and the probability of losing in both stages can be easily derived from these expressions.

		BV	**SM**
For WI when $N_s = 0$	Win in both stages Win only in 2^{nd} stage	$P(N)P(N)$ $(1-P(N))P(N)$	$P(N)P(N)$ $(1-P(N))P(N)$
For WI when $0 < N_s < N$	Win in both stages Win only in 2^{nd} stage	$\frac{1}{2}\Lambda(N, N_s)$ $\frac{1}{2}\Lambda(N, N_s)$	$P(N)P(N)$ $(1-P(N))P(N)$
For SI when $N_s = N$	Win in both stages Win only in 2^{nd} stage	$P(N)\left(\left(1-\frac{1}{2^{N-1}}\right)\frac{1}{2} + \left(\frac{1}{2^{N-1}}\right)P(N)\right)$ $(1-P(N))\Theta(N)$	$P(N)P(N)$ $(1-P(N))P(N)$
For SI when $0 < N_s < N$	Win in both stages Win only in 2^{nd} stage	$P(N_s)\frac{1}{2}$ $(1-P(N_s))\Phi(N, N_s)$	$P(N)P(N)$ $(1-P(N))P(N)$

Table 3.2: Ex-ante probabilities of winning: BV vs. SM.

Therefore, W^{BV} is given by

$$
\begin{aligned}
W^{BV} = \quad & \sum_{N_s=1}^{N-1} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} \Bigg[N_s \left\{ P(N_s)\left(\frac{1}{2}+H\right) + \big(1-P(N_s)\big)\Phi(N, N_s) \right\} \\
& \qquad + (N-N_s)\left\{ \Lambda(N, N_s) + \frac{\epsilon}{2} \right\} \Bigg] \\
& + p^N N \left[P(N) \left\{ \left(1 - \frac{1}{2^{N-1}}\right)\left(\frac{1}{2}+H\right) + \left(\frac{1}{2^{N-1}}\right)(P(N)+H) \right\} \right. \\
& \qquad \left. + \big(1-P(N)\big)\Theta(N) \right] \\
& + (1-p)^N N P(N)(\epsilon + 1).
\end{aligned}
$$

The first two lines of the expression on the right-hand side express the welfare when $0 < N_s < N$. The next two lines denote the welfare when $N_s = N$. The remainder of the expression captures the case $N_s = 0$.

3.4.2 Welfare comparison of BV and SM

Welfare expressions

Suppose SM is used as the voting scheme. The social welfare is now as follows:

$$
\begin{aligned}
W^{SM} = & \sum_{N_s=0}^{N} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} \Big[N_s \Big\{ P(N)\Big(P(N)+H\Big) + [1-P(N)]P(N) \Big\} \\
& +(N-N_s) \Big\{ P(N)\Big(P(N)(1+\epsilon) + [1-P(N)]\epsilon\Big) + [1-P(N)]P(N) \Big\} \Big] \\
= & \sum_{N_s=0}^{N} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} \left[N_s P(N)(H+1) + (N-N_s)P(N)(\epsilon+1) \right].
\end{aligned}
$$

BV yields a higher welfare than SM if and only if

$$
\begin{aligned}
& W^{BV} \geq W^{SM} \iff \\
& \sum_{N_s=1}^{N-1} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} N_s \Big[P(N_s)\left(\frac{1}{2}+H\right) + \big(1-P(N_s)\big)\Phi(N,N_s) \\
& \qquad\qquad\qquad\qquad\qquad\qquad - P(N)(H+1) \Big] \\
& + \sum_{N_s=1}^{N-1} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} (N-N_s) \Big[\Lambda(N,N_s) + \frac{\epsilon}{2} - P(N)(\epsilon+1) \Big] \\
& + p^N N \left[P(N) \left\{ \frac{2^{N-1}-1}{2^N} + \frac{P(N)}{2^{N-1}} - 1 \right\} + \big(1-P(N)\big)\Theta(N) \right] \geq 0. \qquad (3.6)
\end{aligned}
$$

The first expression in square brackets is the expected utility difference for an *SI* individual when $0 < N_s < N$, while the second set of square brackets captures the same for a *WI* agent. The third expression in square brackets shows the expected utility difference for an *SI* agent when $N_s = N$. We note that there is no welfare difference between the two voting schemes when $N_s = 0$, as SM and BV coincide in that case.

Utility comparison for agents

We first compare the expected utility under BV with the one under SM for different groups of agents, before proceeding with the comparison of overall welfare. Recall that the probabilities of winning under BV and SM in both stages, as well as the winning probabilities for the second stage only are summarized in Table 3.2.

For *WI* agents if $N_s = 0$

The expected utility is the same under BV and SM in this case. Thus, *WI* agents are indifferent between the two voting schemes if $N_s = 0$.

For *WI* agents if $0 < N_s < N$

In this scenario, *WI* individuals prefer BV to SM if and only if

$$\begin{aligned} P(N)(\epsilon+1) &\leq \Lambda(N, N_s) + \frac{\epsilon}{2} \\ \epsilon &\leq \epsilon^{SM}(N, N_s) := \frac{\Lambda(N, N_s) - P(N)}{P(N) - \frac{1}{2}}. \end{aligned} \tag{3.7}$$

In Figure 3.7, we plot $\epsilon^{SM}(N, N_s)$ against N_s for $N = 20$ and $N = 200$.[8] We also plot the corresponding $\epsilon^{crit}(N)$ value for comparison.

We note that *WI* agents face a trade-off with BV when $0 < N_s < N$, compared to SM. They have a better probability of winning in the second stage, although their probability of winning in the first stage is lower. Their inclination for a fundamental direction ϵ must therefore be adequately low for them to prefer abstention in the first stage in exchange for a guaranteed vote in the second, thereby preferring BV over SM.

As N_s becomes larger for a given value of N under BV, the body of voters in the second stage becomes smaller with positive probability. The benefits of using BV thus become more substantial and $\epsilon \leq \epsilon^{crit}(N)$ is a sufficient condition for *WI* agents to prefer BV over SM. For very small N_s, however, $\epsilon^{SM}(N, N_s)$ is more restrictive than $\epsilon^{crit}(N)$. When

[8]Further properties of $\epsilon^{SM}(N, N_s)$ are given in Table A.1.

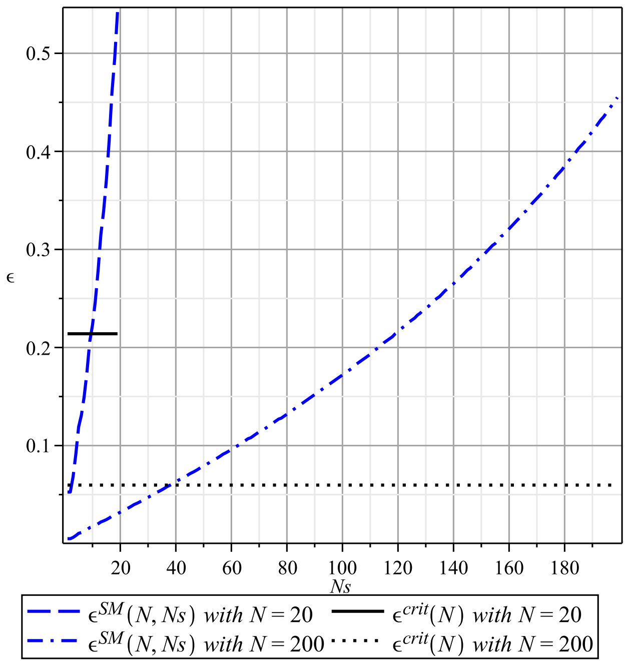

Figure 3.7: $\epsilon^{SM}(N, N_s)$ and $\epsilon^{crit}(N)$ vs. the number of *SI* agents N_s.

most of the society consists of *WI* agents, these agents' probability of winning in the second stage becomes very similar under BV and SM, thus weakening the advantage presented by BV over SM. Hence, for *WI* agents to prefer BV over SM, the inclination towards A or B must be correspondingly small.

We observe that for $N_s = 1$ and N odd, $\Lambda(N, 1) = P(N)$ from Fact 1 and therefore, $\epsilon^{SM}(N, N_s) = 0$. *WI* agents face the same probability of winning in the second stage as with SM, but have a lower probability of winning in the first stage compared to SM. Thus, in this case *WI* agents strictly prefer SM to BV.

For *SI* agents if $N_s = N$

In this case, *SI* agents prefer BV to SM if and only if

$$0 \leq Q(N) := P(N) \left\{ \frac{2^{N-1} - 1}{2^N} + \frac{P(N)}{2^{N-1}} - 1 \right\} + \big(1 - P(N)\big)\Theta(N). \tag{3.8}$$

To examine whether (3.8) holds, we plot $Q(N)$ in Figure 3.8. We note that $Q(N)$ is negative for all N.[9] Therefore, when the whole society is comprised of *SI* agents, these agents strictly prefer SM to BV. We note that the probability of winning in the first stage is the same under SM and BV. In the second stage, losers of the first stage have a higher probability of winning under BV, while this probability is lower for winners. The welfare gains from protecting the minority, however, do not offset the welfare losses resulting from the loss of voting rights for the majority.

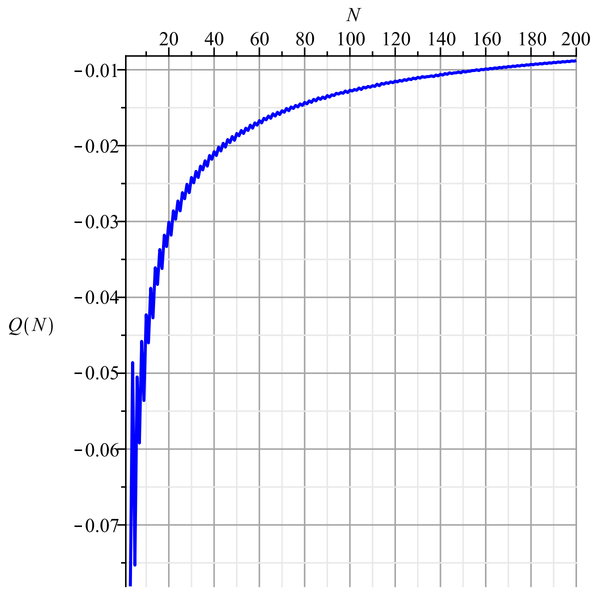

Figure 3.8: $Q(N)$ vs. the number of agents N.

For *SI* agents if $0 < N_s < N$

SI individuals prefer BV to SM if and only if

$$P(N)(H+1) \quad \leq P(N_s)\left(\frac{1}{2}+H\right) + (1-P(N_s))\Phi(N,N_s),$$

[9]This follows from the observation that $P(N)$ is a monotonically-decreasing sequence bounded by $\frac{1}{2}$. We further note that $Q(N)$ converges to 0. This follows from the fact that $P(N)$ and $\Theta(N)$ converge to $\frac{1}{2}$ for $N \to \infty$ while preserving $\Theta(N) > P(N)$ for all N.

which is equivalent to

$$(P(N_s) - P(N))H \ \geq P(N) - \frac{P(N_s)}{2} - (1 - P(N_s))\Phi(N, N_s). \tag{3.9}$$

When $P(N_s) \neq P(N)$, this yields

$$H \geq H^{SM}(N, N_s) := \frac{P(N) - \frac{P(N_s)}{2} - (1 - P(N_s))\Phi(N, N_s)}{P(N_s) - P(N)}. \tag{3.10}$$

The plots of $H^{SM}(N, N_s)$ for $N = 20$ and $N = 200$ are given in Figure 3.9.[10]

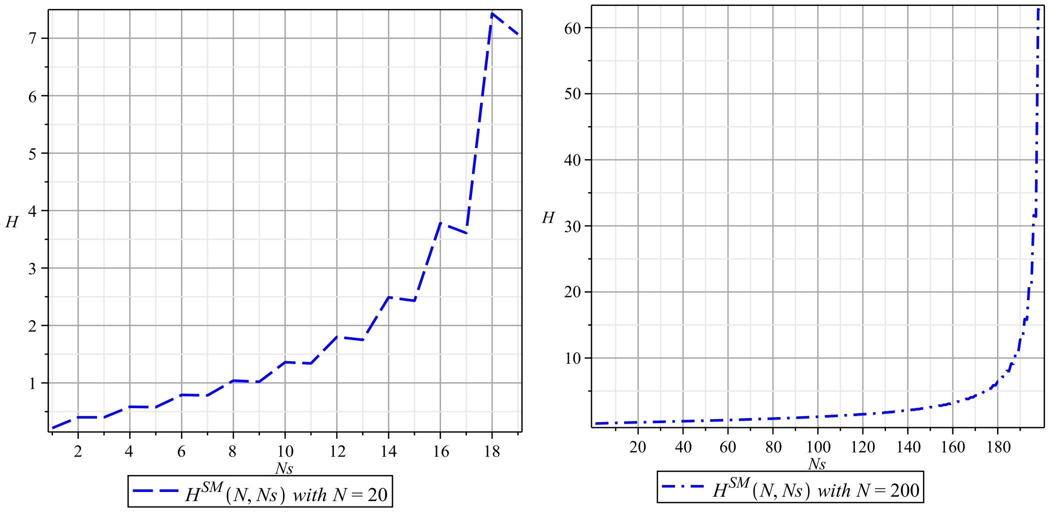

Figure 3.9: $H^{SM}(N, N_s)$ vs. the number of *SI* agents N_s for $N = 20$ and $N = 200$.

We observe that for small N_s, *SI* individuals prefer BV over SM, since $H > 1$. BV restricts the number of voters in the first stage to N_s, thus yielding a higher probability of winning for *SI* agents, which is particularly significant for small groups of *SI* individuals. The majority of these agents, however, face a lower probability of winning in the second stage. This probability becomes more undesirable as N_s increases and the probability of winning in the first stage approaches the one under SM. Therefore, *SI* agents must have a stronger inclination towards a fundamental direction to find BV more favorable despite the risk of losing their voting right for the second stage.

We note that $H^{SM}(N, N_s)$ is undefined for $P(N_s) = P(N)$ as the denominator becomes zero. This occurs when $N_s = N - 1$ and N is odd. In this case, Inequality (3.9) does

[10]Further properties of $H^{SM}(N, N_s)$ are given in Table A.1.

not hold, as the right-hand side is positive for all N.[11] Hence, if $N_s = N-1$ and N is odd, *SI* agents strictly prefer SM over BV. The result is intuitive, because in such circumstances, BV does not offer any additional benefits for *SI* individuals in the first stage, as they face the same probability of winning as under SM. A large fraction of the society, however, loses its voting rights for the second stage, and this utility loss is not compensated by the increase in the probability of winning for the minority.

Aggregate utility comparison

We now compare the aggregate expected utility under BV to the one under SM. Rearranging the Expression (3.6) yields

$$\begin{aligned}
& W^{BV} \geq W^{SM} \iff \\
& \sum_{N_s=1}^{N-1} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} \left[N_s \left\{ P(N_s) - P(N) \right\} H - (N-N_s) \left(P(N) - \frac{1}{2} \right) \epsilon \right] \\
\geq \; & \sum_{N_s=1}^{N-1} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} \Bigg[(N-N_s) \left\{ P(N) - \Lambda(N,N_s) \right\} \\
& \qquad + N_s \left\{ P(N) - \frac{P(N_s)}{2} - (1-P(N_s))\Phi(N,N_s) \right\} \Bigg] \\
& + p^N N \left[P(N) \left\{ 1 - \frac{2^{N-1}-1}{2^N} - \frac{P(N)}{2^{N-1}} \right\} - \big(1-P(N)\big)\Theta(N) \right]
\end{aligned}$$

$$\iff \quad H \geq M(N,p)\,\epsilon + C(N,p), \tag{3.11}$$

where $M(N,p)$ and $C(N,p)$ are given by

$$M(N,p) := \frac{\sum_{N_s=1}^{N-1} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)}(N-N_s)\left(P(N)-\frac{1}{2}\right)}{\sum_{N_s=1}^{N-1} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} N_s (P(N_s)-P(N))} \tag{3.12}$$

and

$$C(N,p) := \frac{\overline{C}(N,p)}{\sum_{N_s=1}^{N-1} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} N_s \left(P(N_s)-P(N)\right)}, \tag{3.13}$$

[11] This conclusion follows from the observation that $Q(N) + P(N) - \frac{P(N_s)}{2} - (1-P(N_s))\Phi(N,N_s) > 0$ while $Q(N) < 0$ for all N.

where

$$
\begin{aligned}
\overline{C}(N,p) := \; & \sum_{N_s=1}^{N-1} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} \Bigg[(N-N_s)\left\{P(N) - \Lambda(N,N_s)\right\} \\
& \qquad + N_s \left\{ P(N) - \frac{P(N_s)}{2} - (1-P(N_s))\Phi(N,N_s) \right\} \Bigg] \\
& + p^N N \left[P(N) \left\{ 1 - \frac{2^{N-1}-1}{2^N} - \frac{P(N)}{2^{N-1}} \right\} - \big(1-P(N)\big)\Theta(N) \right].
\end{aligned}
$$

We have verified numerically that $M(N,p)$ and $C(N,p)$ are finite and positive.[12] To illustrate this with an example, we plot the equality of (3.11) in Figure 3.10 for $N = 20$ and for different values of p. We also denote the corresponding $\epsilon^{crit}(N)$ and $H = 1$ values, and shade the area in which Proposition 1 is satisfied.

The reason why H has to increase when ϵ increases for BV to be welfare-superior to SM is as follows: An increase in ϵ means that *WI* agents may prefer SM over BV when N_s is small. To maintain that welfare under BV is superior to that under SM, H must increase analogously to ensure that more *SI* agents prefer BV over SM when N_s is large.

Moreover, we observe that for small values of p (e.g. $p = 0.3$), welfare is superior under BV to welfare under SM. As p increases, H must increase as well to ensure the superiority of BV over SM. An increase in p indicates an expected increase in the number of *SI* agents in the society. As explained in the preceding analysis of Section 3.4.2, as N_s increases, H has to simultaneously increase for *SI* agents to maintain these agents' preference for BV over SM.

[12] It was verified for the ranges $N \in [3, 1000]$ and $p \in [0.01, 0.99]$ in steps of 0.01.

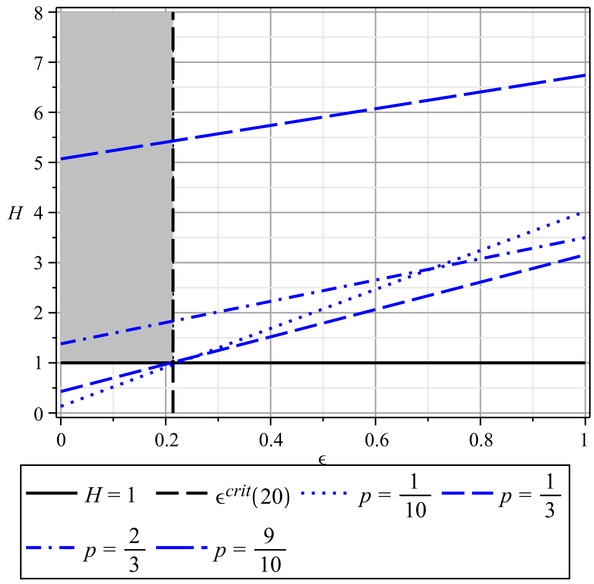

Figure 3.10: H vs. ϵ for $W^{BV} = W^{SM}$ to hold when $N = 20$, according to (3.11).

We summarize the results of welfare comparison between BV and SM as follows:

Proposition 2
The comparison of ex-ante expected utility under SM and BV in equilibrium yields the following insights:

- *The welfare under BV is higher than the welfare under SM if*

$$H \geq M(N, p)\, \epsilon + C(N, p),$$

where $M(N, p)$ and $C(N, p)$ are given by Equations (3.12) and (3.13), respectively.

- *WI individuals are indifferent between BV and SM if $N_s = 0$.*

- *When $0 < N_s < N$, WI agents prefer BV to SM, provided that*

$$\epsilon \leq \min\{\epsilon^{crit}(N),\ \ \epsilon^{SM}(N, N_s)\}.$$

- *When $N_s = N$, SI individuals strictly prefer SM to BV.*

- *When* $0 < N_s < N$, *SI agents prefer BV to SM if*

$$H \geq H^{SM}(N, N_s).$$

- *When* $N_s = N - 1$ *and* N *is odd, SI agents strictly prefer SM to BV.*

We also summarize the conditions under which BV Pareto-dominates SM.

Proposition 3
The equilibrium outcome under BV Pareto-dominates the outcome under SM if and only if the following conditions are satisfied:

(i) $0 < N_s < N$ *when* N *is even and* $0 < N_s < N - 1$ *when* N *is odd,*

(ii) $\epsilon \leq \min\{\epsilon^{crit}(N),\ \epsilon^{SM}(N, N_s)\}$, *and*

(iii) $H \geq H^{SM}(N, N_s)$.

3.4.3 Welfare comparison of BV and ST

Welfare expressions

Suppose the society adopts ST as the voting scheme. For ease of reference, we summarize the probabilities of winning for each group of agents in Table 3.3, first in both stages and then in the second stage only.

Welfare under ST is given by

$$\begin{aligned} W^{ST} &= \sum_{N_s=1}^{N-1} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} \left[N_s \left\{ P(N_s)H + \frac{1}{2} \right\} + (N - N_s) \left\{ P(N - N_s) + \frac{\epsilon}{2} \right\} \right] \\ &\quad + p^N N \left[P(N)\left(H+1\right) \right] + (1-p)^N N \left[P(N)\left(\epsilon + 1\right) \right]. \end{aligned}$$

The first expression in square brackets denotes the expected utility of agents when $0 < N_s < N$. The second and third square brackets describe the cases $N_s = N$ and $N_s = 0$, respectively.

		BV	**ST**
For WI when $N_s = 0$	Win in both stages Win only in 2^{nd} stage	$P(N)P(N)$ $(1-P(N))P(N)$	$P(N)P(N)$ $(1-P(N))P(N)$
For WI when $0 < N_s < N$	Win in both stages Win only in 2^{nd} stage	$\frac{1}{2}\Lambda(N, N_s)$ $\frac{1}{2}\Lambda(N, N_s)$	$\frac{1}{2}P(N-N_s)$ $\frac{1}{2}P(N-N_s)$
For SI when $N_s = N$	Win in both stages Win only in 2^{nd} stage	$P(N)\left(\left(1-\frac{1}{2^{N-1}}\right)\frac{1}{2} + \left(\frac{1}{2^{N-1}}\right)P(N)\right)$ $(1-P(N))\Theta(N)$	$P(N)P(N)$ $(1-P(N))P(N)$
For SI when $0 < N_s < N$	Win in both stages Win only in 2^{nd} stage	$P(N_s)\frac{1}{2}$ $(1-P(N_s))\Phi(N, N_s)$	$P(N_s)\frac{1}{2}$ $(1-P(N_s))\frac{1}{2}$

Table 3.3: Ex-ante probability of winning: BV vs. ST.

BV yields a higher welfare than ST if and only if

$$\begin{aligned}
&W^{BV} \geq W^{ST} \iff \\
&\sum_{N_s=1}^{N-1} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} N_s \Bigg[P(N_s)\left(\frac{1}{2}+H\right) - \left(P(N_s)H + \frac{1}{2}\right) \\
&\qquad\qquad + \big(1-P(N_s)\big)\Phi(N, N_s)\Bigg] \\
&+\sum_{N_s=1}^{N-1} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} (N-N_s)\left[\Lambda(N, N_s) + \frac{\epsilon}{2} - \left(P(N-N_s) + \frac{\epsilon}{2}\right)\right] \\
&+(1-p)^N N\left[P(N)(\epsilon+1) - P(N)(\epsilon+1)\right] \\
&+p^N N\left[P(N)\left\{\frac{2^{N-1}-1}{2^N} + \frac{P(N)}{2^{N-1}} - 1\right\} + \big(1-P(N)\big)\Theta(N)\right] \geq 0. \qquad (3.14)
\end{aligned}$$

The first expression in square brackets captures the welfare difference for SI individuals if $0 < N_s < N$, while the second set of square brackets indicates the welfare difference for WI agents for the same situation. The third expression in square brackets is the welfare difference for WI individuals if $N_s = 0$, which is zero. The remaining expression displays the welfare difference for SI agents if $N_s = N$.

Utility comparison for agents

We first compare the expected utility under BV to the one under ST for different groups of agents.

For *WI* agents if $N_s = 0$

WI individuals are indifferent between ST and BV in this case.

For *WI* agents if $0 < N_s < N$

We note that $\Lambda(N, N_s) < P(N - N_s)$, as $\Lambda(N, N_s)$ gives the ex-ante probability of winning when the committee comprises the *WI* agents and the losers from the first stage, while $P(N - N_s)$ is the probability of winning when the committee consists only of *WI* agents. Therefore, *WI* individuals strictly prefer ST to BV if $0 < N_s < N$. Their probability of winning is higher under ST, as there are no losers from the first stage who take part in voting in the second stage.

For *SI* agents if $N_s = N$

In this case, *SI* agents prefer BV to ST if and only if

$$0 \leq P(N)\left\{\frac{2^{N-1}-1}{2^N} + \frac{P(N)}{2^{N-1}} - 1\right\} + \big(1 - P(N)\big)\Theta(N) = Q(N).$$

From Figure 3.8, we observe that $Q(N)$ is negative for all N. Thus, when $N_s = N$, *SI* agents strictly prefer ST to BV. ST is identical to SM when $N_s = N$, and, as explained in Section 3.4.2, BV is inferior to SM when $N_s = N$. As N becomes larger, $P(N)$ tends to $\frac{1}{2}$, and the gap between losses and gains is reduced, driving $Q(N)$ closer to zero.

For *SI* agents if $0 < N_s < N$

The *SI* agents are strictly better off with BV – as opposed to ST – if $0 < N_s < N$. The reason is that BV provides them with the same benefits as ST, with the additional feature of better protection, should they belong to the minority of the first stage.

Aggregate utility comparison

Next we compare the aggregate expected utility under BV to the one under ST. We rearrange (3.14) to yield

$$\begin{aligned}
&W^{BV} \geq W^{ST} \iff \\
&\sum_{N_s=1}^{N-1} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} \left[N_s \left\{ \big(1-P(N_s)\big)\Phi(N,N_s) + \frac{P(N_s)}{2} - \frac{1}{2} \right\} \right. \\
&\qquad\qquad \left. - (N-N_s)\left\{P(N-N_s) - \Lambda(N,N_s)\right\} \right] \\
&+p^N N \left[P(N) \left\{ \frac{2^{N-1}-1}{2^N} + \frac{P(N)}{2^{N-1}} - 1 \right\} + \big(1-P(N)\big)\Theta(N) \right] \geq 0. \quad (3.15)
\end{aligned}$$

We denote the entire expression on the left-hand side of Inequality (3.15) by $K(N,p)$, i.e.

$$W^{BV} \geq W^{ST} \iff \quad K(N,p) \geq 0.$$

In Figure 3.11, we plot $K(N,p)$ against p for different values of N.

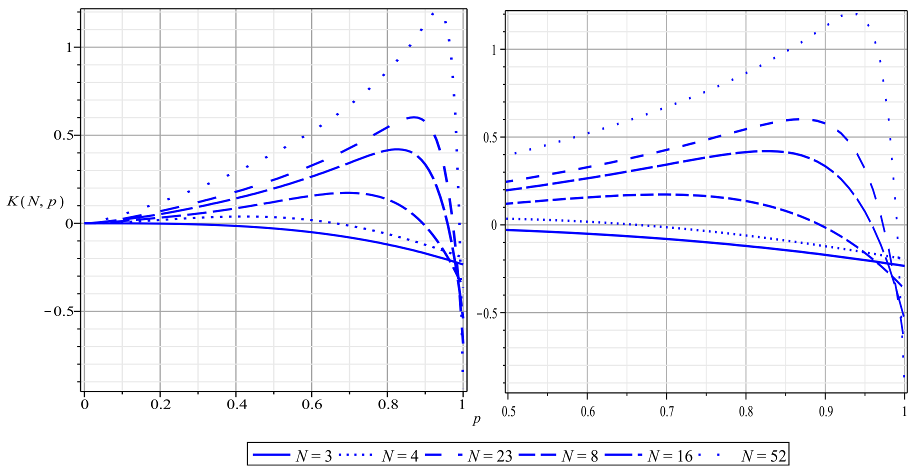

Figure 3.11: $K(N,p)$ vs. p.

We observe that (3.15) holds for sufficiently large N or sufficiently small p. We have verified numerically that for each $N > 3$, there is some p which yields $K(N,p) = 0$.[13]

[13] The analysis was conducted for $N \in [4, 1000]$ and $p \in [0.01, 0.99]$ in steps of 0.001.

For $N = 3$, welfare properties of ST are superior to BV for all values of p. From Figure 3.8, we note that if $N_s = N$, the social losses under BV are rather large if $N = 3$, contributing to the poor welfare performance. We also note that the welfare losses for $N_s = N$ rapidly become smaller as N increases. Hence, for $N > 3$, BV is superior to ST as long as $p \leq 0.6$. As p increases, however, N must increase analogously to keep welfare under BV more desirable than under ST. For $p = 0.9$ and $p = 0.99$, for example, $N > 8$ and $N > 51$ must be satisfied, respectively, for BV to be superior to ST. An increase in p means there is a larger probability that the committee will be comprised of *SI* agents. Hence, the case $N_s = N$ has a higher impact on the overall welfare. Thus, as p increases, N must also increase to ensure $W^{BV} \geq W^{ST}$.

We define the threshold on N that yields $K(N,p) \geq 0$ for a given value of p as $N_{st}(p)$, i.e.

$$K(N,p) \gtreqless 0 \quad \text{if} \quad N \gtreqless N_{st}(p). \tag{3.16}$$

For $p \in [0.6, 0.99]$ in steps of 0.01, we calculate $N_{st}(p)$, which determines the threshold on N that yields $K(N,p) \geq 0$. In Figure 3.12, we plot $N_{st}(p)$ against p.

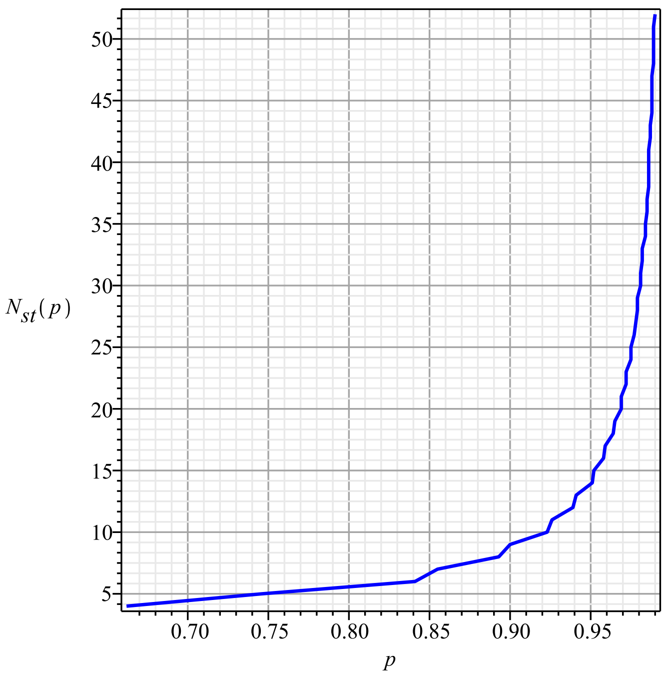

Figure 3.12: $N_{st}(p)$ vs. p.

We summarize the results of welfare comparison of BV and ST in the following proposition:

Proposition 4
The comparison of expected utility under ST and BV in equilibrium yields the following insights:

- *The welfare under BV is higher than the welfare under ST for sufficiently large N, i.e. if*
$$N \geq N_{st}(p),$$
where $N_{st}(p)$ is defined in (3.16).
- *WI individuals are indifferent between BV and ST if $N_s = 0$.*
- *When $0 < N_s < N$, WI individuals strictly prefer ST to BV.*
- *When $N_s = N$, SI agents strictly prefer ST to BV.*
- *When $0 < N_s < N$, SI individuals strictly prefer BV to ST.*

We note that the welfare comparison between BV and ST imposes a restriction on the number of agents that depends on p for BV to be superior to ST. The intuition is as follows: A large p indicates a high probability that the entire society consists of *SI* agents. Consider the case where the entire society does consist of *SI* agents indeed. Then, ST allows all individuals to vote in both stages, while under BV, the majority is not allowed to vote in the second stage. When N is low, this effectively means that the majority loses voting rights on a decision for which it would have had a high chance of being pivotal. This leads to large welfare losses under BV. As N increases, however, the chances of being pivotal in the second stage decrease, and the utility losses under BV diminish. Thus, for sufficiently large N, BV is superior to ST from a utilitarian welfare perspective.

3.4.4 Welfare comparison of BV and MV

Welfare expressions

Now assume that MV is the voting scheme used for a collective decision. For ease of reference, we summarize the probabilities of winning for each group of agents in Table 3.4, first in both stages and then in the second stage only.

		BV	**MV**
For *WI* when $N_s = 0$	Win in both stages	$P(N)P(N)$	$P(N)\left(\left(1-\frac{1}{2^{N-1}}\right)\frac{1}{2} + \left(\frac{1}{2^{N-1}}\right)P(N)\right)$
	Win only in 2^{nd} stage	$(1-P(N))P(N)$	$(1-P(N))\Theta(N)$
For *WI* when $0 < N_s < N$	Win in both stages	$\frac{1}{2}\Lambda(N, N_s)$	$P(N)\left(\left(1-\frac{1}{2^{N-1}}\right)\frac{1}{2} + \left(\frac{1}{2^{N-1}}\right)P(N)\right)$
	Win only in 2^{nd} stage	$\frac{1}{2}\Lambda(N, N_s)$	$(1-P(N))\Theta(N)$
For *SI* when $N_s = N$	Win in both stages	$P(N)\left(\left(1-\frac{1}{2^{N-1}}\right)\frac{1}{2} + \left(\frac{1}{2^{N-1}}\right)P(N)\right)$	$P(N)\left(\left(1-\frac{1}{2^{N-1}}\right)\frac{1}{2} + \left(\frac{1}{2^{N-1}}\right)P(N)\right)$
	Win only in 2^{nd} stage	$(1-P(N))\Theta(N)$	$(1-P(N))\Theta(N)$
For *SI* when $0 < N_s < N$	Win in both stages	$P(N_s)\frac{1}{2}$	$P(N)\left(\left(1-\frac{1}{2^{N-1}}\right)\frac{1}{2} + \left(\frac{1}{2^{N-1}}\right)P(N)\right)$
	Win only in 2^{nd} stage	$(1-P(N_s))\Phi(N, N_s)$	$(1-P(N))\Theta(N)$

Table 3.4: Ex-ante probability of winning: BV vs. MV.

Welfare under MV is given by

$$
\begin{aligned}
W^{MV} = & \sum_{N_s=0}^{N} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} \left[N_s \left\{ P(N) \left[\left(1 - \frac{1}{2^{N-1}}\right)\left(\frac{1}{2} + H\right) \right.\right.\right. \\
& \left.\left. + \left(\frac{1}{2^{N-1}}\right)(P(N) + H) \right] + [1 - P(N)]\Theta(N) \right\} \\
& + (N - N_s) \left\{ P(N) \left[\left(1 - \frac{1}{2^{N-1}}\right)\left(\frac{1}{2} + \epsilon\right) + \left(\frac{1}{2^{N-1}}\right)(P(N) + \epsilon) \right] \right. \\
& \left.\left. + [1 - P(N)]\Theta(N) \right\} \right] \\
= & \sum_{N_s=0}^{N} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} \left[N_s \left\{ P(N) \left(\frac{2^{N-1} - 1}{2^N} + \frac{P(N)}{2^{N-1}} + H \right) \right.\right. \\
& \left. + (1 - P(N))\,\Theta(N) \right\} \\
& \left. + (N - N_s) \left\{ P(N) \left(\frac{2^{N-1} - 1}{2^N} + \frac{P(N)}{2^{N-1}} + \epsilon \right) + (1 - P(N))\,\Theta(N) \right\} \right].
\end{aligned}
$$

Welfare under BV is higher than welfare under MV if and only if

$$
\begin{aligned}
& W^{BV} \geq W^{MV} \iff \\
& \sum_{N_s=1}^{N-1} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} N_s \left[P(N_s)\left(\frac{1}{2} + H\right) + (1 - P(N_s))\Phi(N, N_s) \right. \\
& \qquad \left. - P(N)\left(\frac{2^{N-1} - 1}{2^N} + \frac{P(N)}{2^{N-1}} + H \right) - (1 - P(N))\Theta(N) \right] \\
& + \sum_{N_s=1}^{N-1} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} (N - N_s) \left[\Lambda(N, N_s) + \frac{\epsilon}{2} - (1 - P(N))\Theta(N) \right. \\
& \qquad \left. - P(N)\left(\frac{2^{N-1} - 1}{2^N} + \frac{P(N)}{2^{N-1}} + \epsilon \right) \right] \\
& -(1-p)^N N \left[P(N)\left(\frac{2^{N-1} - 1}{2^N} + \frac{P(N)}{2^{N-1}} - 1 \right) + (1 - P(N))\Theta(N) \right] \geq 0. \qquad (3.17)
\end{aligned}
$$

The first expression in square brackets captures the welfare difference for *SI* individuals if $0 < N_s < N$, while the second pair of square brackets comprises the welfare difference for *WI* agents in the same case. The third expression in square brackets shows the welfare difference for *WI* individuals if $N_s = 0$.

Utility comparison for agents

We first compare the expected utility under BV with the one under MV for different groups of agents.

For ***WI*** agents if $N_s = 0$

WI individuals prefer BV to MV in this case if and only if

$$P(N) \geq \ P(N)\left(\frac{2^{N-1}-1}{2^N}+\frac{P(N)}{2^{N-1}}\right)+(1-P(N))\Theta(N),$$

or equivalently,

$$0 \geq \ P(N)\left(\frac{P(N)}{2^{N-1}}-\frac{1}{2}-\frac{1}{2^N}\right)+(1-P(N))\Theta(N)=Q(N). \tag{3.18}$$

Recall that Figure 3.8 gives the plot of $Q(N)$. Hence, Inequality (3.18) holds for all N. When the entire society consists of *WI* agents, everyone votes in both stages under BV, while under MV, only losers of the first stage can vote in the second stage. For the majority, the loss of voting rights on the decision they would have preferred to exert more influence on results in a welfare loss under MV, thus making BV strictly more desirable.

For ***WI*** agents if $0 < N_s < N$

In this case, BV is preferred to MV by *WI* individuals if and only if

$$\Lambda(N,N_s)+\frac{\epsilon}{2}-P(N)\left(\frac{2^{N-1}-1}{2^N}+\frac{P(N)}{2^{N-1}}+\epsilon\right)-(1-P(N))\Theta(N)\geq 0 \tag{3.19}$$

$$\epsilon \leq \ \underbrace{\frac{1}{P(N)-\frac{1}{2}}\left[\Lambda(N,N_s)-P(N)\left(\frac{2^{N-1}-1}{2^N}+\frac{P(N)}{2^{N-1}}\right)-(1-P(N))\Theta(N)\right]}_{=:\,\epsilon^{MV}(N,N_s)}. \tag{3.20}$$

We note that for a given N, $\Lambda(N, N_s)$ is monotonically increasing in N_s. Therefore, so is the right-hand side of Inequality (3.20). Since we have assumed $\epsilon \le \epsilon^{crit}(N)$, we examine if Inequality (3.20) holds for the least possible value of N_s, which is $N_s = 1$, when $\epsilon = \epsilon^{crit}(N)$. In Figure 3.13, we plot $\epsilon^{MV}(N,1) - \epsilon^{crit}(N)$ for $N \in [3, 200]$ and observe that it is positive if N is not too small, i.e. if $N > 7$.[14]

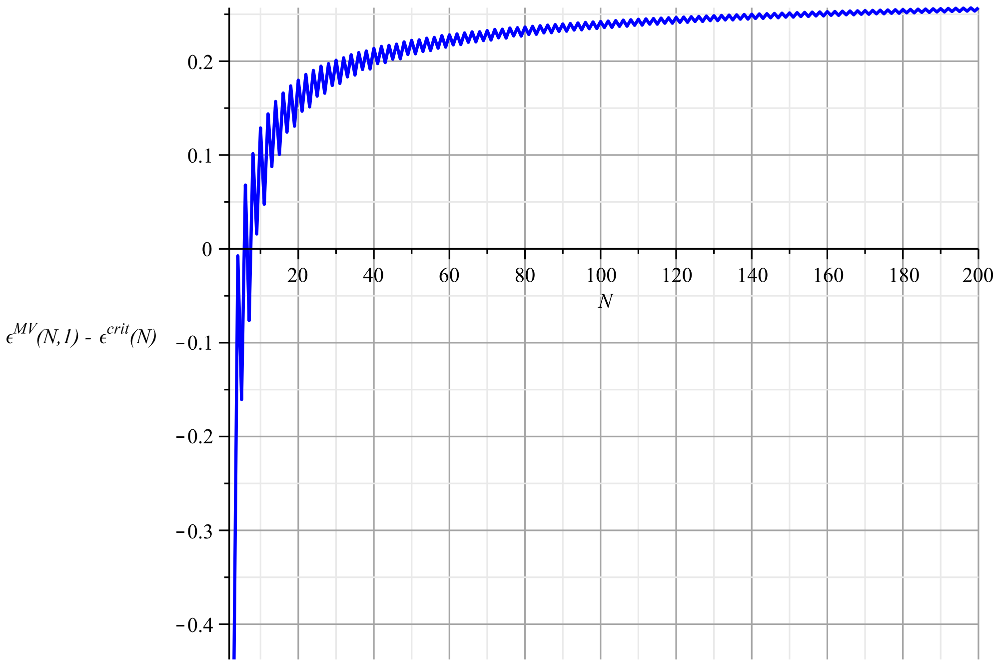

Figure 3.13: $\epsilon^{MV}(N,1) - \epsilon^{crit}(N)$ vs. N.

Thus, our assumption $\epsilon \le \epsilon^{crit}(N)$ is sufficient for *WI* agents to prefer BV over MV when $N > 7$. Under MV, *WI* individuals vote with the rest of the society in the first stage, but face the possibility of losing their right to vote in the second stage when the voting body is much smaller. This leads to a welfare loss that is not offset by the comparatively higher probability of winning in the first stage, which is small if the society is large.

When $N \le 7$, the probability of winning in the first stage, $P(N)$, is much larger than $\frac{1}{2}$, which renders participation in the first stage more attractive. Therefore, ϵ must satisfy the more stringent criteria $\epsilon \le \epsilon^{MV}(N,1) < \epsilon^{crit}(N)$ for *WI* individuals to find BV more attractive than MV.

[14]The generalization of the result to all N follows from the observation that for all N_s, the left-hand side of Inequality (3.19) is positive and tends to zero as $N \to \infty$.

For $\boldsymbol{SI}$ agents if $N_s = N$

SI individuals are indifferent between BV and MV if $N_s = N$, as BV and MV coincide in this case.

For $\boldsymbol{SI}$ agents if $0 < N_s < N$

BV is preferred to MV by SI individuals in this case if and only if

$$P(N_s)\left(\frac{1}{2} + H\right) + (1 - P(N_s))\Phi(N, N_s) - P(N)\left(\frac{2^{N-1} - 1}{2^N} + \frac{P(N)}{2^{N-1}} + H\right) \\ - (1 - P(N))\Theta(N) \geq 0$$

$$\Rightarrow \quad H\big(P(N_s) - P(N)\big) \geq P(N)\left(\frac{2^{N-1} - 1}{2^N} + \frac{P(N)}{2^{N-1}}\right) + (1 - P(N))\Theta(N) \\ - (1 - P(N_s))\Phi(N, N_s) - \frac{P(N_s)}{2}. \qquad (3.21)$$

When $P(N_s) \neq P(N)$ this means,

$$H \geq \frac{1}{P(N_s) - P(N)}\left[P(N)\left(\frac{2^{N-1} - 1}{2^N} + \frac{P(N)}{2^{N-1}}\right) + (1 - P(N))\Theta(N) \right. \\ \left. - (1 - P(N_s))\Phi(N, N_s) - \frac{P(N_s)}{2}\right] =: H^{MV}(N, N_s). (3.22)$$

Since the relationship between $H^{MV}(N, N_s)$ and N_s is not straightforward, we numerically calculate the maximum value of $H^{MV}(N, N_s)$ for each $N \in [3, 200]$ and plot it in Figure 3.14.

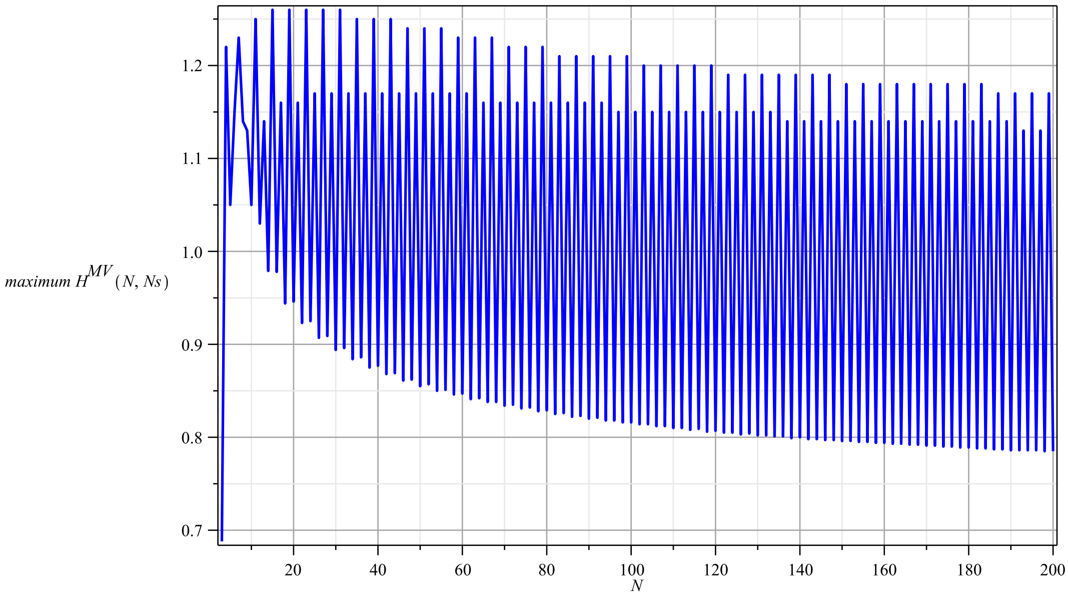

Figure 3.14: $\max_{N_s} H^{MV}(N, N_s)$ vs. the number of agents N.

We note that (3.22) holds if $H > H^{MV}(N, N_s) \leq 1.26$.[15] Since we have assumed $H >> 1$, it is therefore possible to conclude that (3.22) holds for all N. Under MV, the entire society participates in the first stage, while only *SI* agents do so under BV. This gives them a better chance of winning in the decision that matters more to them. MV, however, offers a higher probability of winning in the second stage. Yet, the benefits of BV still outweigh those of MV. Therefore, *SI* agents strictly prefer BV over MV when $0 < N_s < N$.

We note that $H^{MV}(N, N_s)$ is undefined for $P(N_s) = P(N)$, that is, when $N_s = N - 1$ and N is odd. In this case, Inequality (3.21) does not hold, as the right-hand side is strictly positive for all N and tends to zero as $N \to \infty$. Hence, if $N_s = N - 1$ and N is odd, *SI* agents strictly prefer MV over BV. Although there is one agent less who votes in the first stage under BV, *SI* individuals face the same probability of winning as the one under MV, since $P(N - 1) = P(N)$. In the second stage, however, the *WI* agent participates in the voting under BV, which decreases the *SI* minority's chances of winning.

[15]The generalization of the result to all N follows from the observation that the right-hand side of Inequality (3.22) tends to zero as $N \to \infty$, since $\Phi(N, N_s) \approx \Theta(N)$ for large N and N_s. A large value of N_s is considered, since the trend of $H^{MV}(N, N_s)$ is increasing with N_s.

Aggregate utility comparison

Next, we compare the aggregate expected utility under BV to the one under MV. We rearrange (3.17) to obtain

$$
\begin{aligned}
& W^{BV} \geq W^{MV} \iff \\
& \sum_{N_s=1}^{N-1} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} \left[N_s H \big(P(N_s) - P(N) \big) - (N - N_s) \left(P(N) - \frac{1}{2} \right) \epsilon \right] \\
\geq \quad & \sum_{N_s=1}^{N-1} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} N_s \Big[(1 - P(N))\Theta(N) - \big(1 - P(N_s)\big) \Phi(N, N_s) \\
& \qquad + P(N) \left(\frac{2^{N-1}-1}{2^N} + \frac{P(N)}{2^{N-1}} \right) - \frac{P(N_s)}{2} \Big] \\
& + \sum_{N_s=1}^{N-1} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} (N - N_s) \Big[P(N) \left(\frac{2^{N-1}-1}{2^N} + \frac{P(N)}{2^{N-1}} \right) \\
& \qquad + (1 - P(N))\Theta(N) - \Lambda(N, N_s) \Big] \\
& + (1-p)^N N \left[(1 - P(N))\Theta(N) - P(N) \left(1 - \frac{2^{N-1}-1}{2^N} - \frac{P(N)}{2^{N-1}} \right) \right] \\
\iff \quad & H \geq M(N,p)\, \epsilon + D(N,p), \qquad (3.23)
\end{aligned}
$$

where $M(N,p)$ is given in (3.12) and $D(N,p)$ is defined by

$$
D(N,p) := \frac{\overline{D}(N,p)}{\sum_{N_s=1}^{N-1} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} N_s \left(P(N_s) - P(N) \right)}, \qquad (3.24)
$$

where

$$
\begin{aligned}
\overline{D}(N,p) := \sum_{N_s=1}^{N-1} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} \Bigg[(N-N_s) \Bigg\{ P(N)\left(\frac{2^{N-1}-1}{2^N} + \frac{P(N)}{2^{N-1}}\right) \\
+ (1-P(N))\Theta(N) - \Lambda(N,N_s) \Bigg\} \\
+ N_s \Bigg\{ P(N)\left(\frac{2^{N-1}-1}{2^N} + \frac{P(N)}{2^{N-1}}\right) + (1-P(N))\Theta(N) \\
- \frac{P(N_s)}{2} - (1-P(N_s))\Phi(N,N_s) \Bigg\} \Bigg] \\
+(1-p)^N N \left[\big(1-P(N)\big)\Theta(N) - P(N)\left(1 - \frac{2^{N-1}-1}{2^N} - \frac{P(N)}{2^{N-1}}\right)\right].
\end{aligned}
$$

We note that $D(N,p) < C(N,p)$[16] and therefore $D(N,p)$ is finite.[17] To illustrate, we plot the Equality (3.23) in Figure 3.15 for $N = 20$ and for different values of p. We also plot the values of $H = 1$ and $\epsilon^{crit}(20)$, and mark the area in which H and ϵ must lie to satisfy Proposition 1. We have verified that for all $N \notin \{3,5,7\}$, welfare under BV is always superior to the welfare under MV.[18] In Section 3.4.4, we showed that in almost all situations, individuals either strictly or weakly prefer BV over MV, given $H >> 1$ and $\epsilon \leq \epsilon^{crit}(N)$ are satisfied. This results in the welfare under BV being strictly better than under MV.

For $N \in \{3,5,7\}$, welfare under BV is superior to welfare under MV only if H and ϵ satisfy Expression (3.23). In Section 3.4.4, we illustrated that $\epsilon \leq \epsilon^{crit}(N)$ is not sufficient for *WI* agents to prefer BV over MV for $N \leq 7$. Furthermore, we showed that when N is odd and $N_s = N-1$, *SI* agents strictly prefer MV to BV. The combination of these two effects yields the result that welfare under BV is not always superior to welfare under MV for $N \in \{3,5,7\}$. Welfare properties of BV are better only if ϵ is sufficiently low, or analogously, if H is sufficiently high, such that Expression (3.23) is satisfied.

[16]This observation follows from the fact that the denominator is the same for both $C(N,p)$ and $D(N,p)$, and $\overline{D}(N,p)$ is finite and is strictly smaller than $\overline{C}(N,p)$, since $Q(N)$ is negative but finite for all N.

[17]This assertion holds for all $N \in [3,1000]$ and $p \in [0.01,0.99]$, since $C(N,p)$ was verified to be finite in these ranges.

[18]This was verified numerically for the ranges $N \in [3,1000]$ and $p \in [0.01,0.99]$ in steps of 0.01.

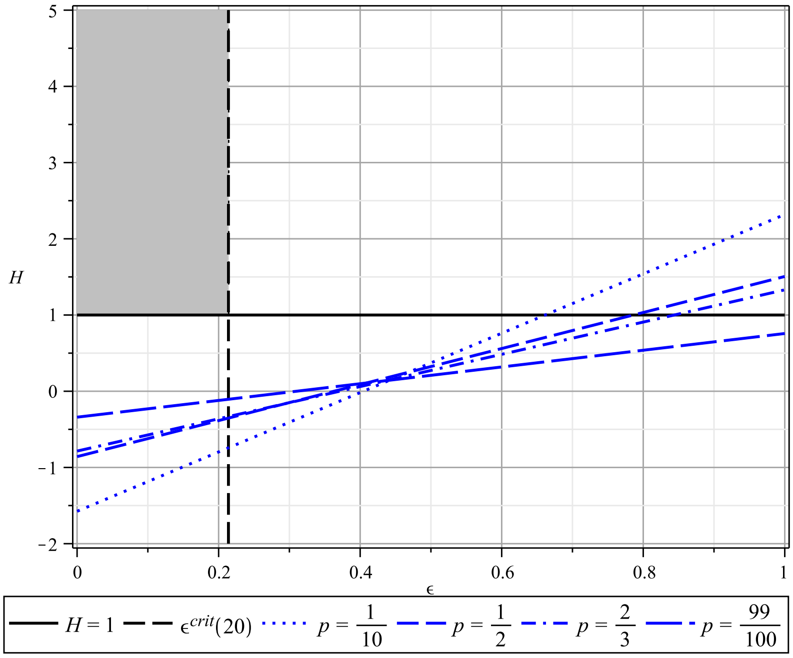

Figure 3.15: H vs. ϵ to satisfy $W^{BV} \geq W^{MV}$ for $N = 20$.

We summarize the outcomes of the welfare comparison between BV and MV in the following proposition:

Proposition 5

Suppose that $H >> 1$ and $\epsilon \leq \epsilon^{crit}(N)$. Then the comparison of ex-ante expected utility under MV and BV in equilibrium yields the following results.

- *The welfare under BV is strictly higher than the welfare under MV for all $N \notin \{3, 5, 7\}$.*
- *For $N \in \{3, 5, 7\}$, welfare under BV is superior to welfare under MV if*

$$H \geq M(N, p)\,\epsilon + D(N, p),$$

  where $M(N, p)$ and $D(N, p)$ are given by Equations (3.12) and (3.24), respectively.
- *WI individuals strictly prefer BV to MV if $N_s = 0$.*

- *When $0 < N_s < N$, WI agents strictly prefer BV to MV, provided that*

$$\epsilon \leq \min\{\epsilon^{crit}(N),\ \ \epsilon^{MV}(N, N_s)\}.$$

- *When $N_s = N$, SI individuals are indifferent between BV and MV.*
- *When $0 < N_s < N$, SI agents strictly prefer BV to MV.*
- *When $N_s = N - 1$ and N is odd, SI agents strictly prefer MV to BV.*

We also summarize the conditions under which BV Pareto-dominates MV.

Proposition 6
The equilibrium outcome under BV Pareto-dominates the outcome under MV if and only if the following conditions are satisfied:

(i) $N_s < N$ *when* N *is even and* $N_s < N - 1$ *when* N *is odd, and*

(ii) $\epsilon \leq \min\{\epsilon^{crit}(N),\ \ \epsilon^{MV}(N, N_s)\}$.

3.5 Main Result

Now we summarize the findings of Section 3.4 to present our main result:

Theorem 1
For the ranges $N \in [3, 1000]$ and $p \in [0, 1]$, BV is superior to SM, ST and MV with respect to utilitarian welfare if the following conditions are satisfied:

- $\epsilon \leq \epsilon^{crit}(N)$,
- $H \geq \max\{H_{st}^{crit}(N), M(N, p)\,\epsilon + C(N, p)\}$, *and*
- $N \geq N_{st}(p)$,

where $M(N, p)$, $C(N, p)$, and $N_{st}(p)$ are given in Equations (3.12), (3.13), *and* (3.16), *respectively, and $H_{st}^{crit}(N)$ and $\epsilon^{crit}(N)$ are defined in Lemma 1 and Proposition 1, respectively.*

Theorem 1 shows that when the stakes, expressed by the value of H, are sufficiently high in comparison with ϵ, BV is superior to other common voting rules which could be used in such circumstances. Essentially, BV splits the society into those agents who have large stakes and those agents with small stakes. Individuals with high stakes decide on the fundamental societal issue. Still, to allow as many citizens as possible to participate in the decision on variations of the chosen fundamental direction, and thus to increase the aggregate welfare that can be achieved in the second stage, BV requires losers from the first stage to have voting rights in the second stage.

A note on the robustness of the model is in order here. We have assumed a two-stage voting procedure and assumed a simple structure with individual preferences that are privately-observed. All these assumptions are essential for our results. In Chapter 4, we discuss how these assumptions can be relaxed to reveal interesting extensions to our framework.

3.6 Conclusion

We have introduced Balanced Voting as a new voting scheme that aims at striking a balance between expressing the intensity of preferences and protecting minorities. It is particularly suitable for fundamental societal decisions, which inherently means intense preferences or high stakes for some groups of citizens. We have shown that capturing this preference intensity and providing additional protection to a minority with high stakes through BV lead to welfare gains.

4 Extensions and Discussion

We have limited our analysis to a very simple framework. A host of interesting extensions emerge as soon as we deviate from our basic framework. Moreover, a variety of issues arise where the practical implementation of BV is considered. Specifically, applications of BV require further procedural rules. In this chapter, we discuss some possible extensions to our framework and practical issues that need to be considered for the application of BV.

We begin by focusing on two particular extensions. First, in Section 4.1 we analyze the implications on the established equilibrium if weaker symmetry assumptions are imposed. Second, in Section 4.2 we investigate equilibria that may arise when BV is used to making decisions that are non-fundamental in nature. In Section 4.3 we detail further extensions that may be pursued in future research. Sections 4.4 and 4.5 provide a discussion on the procedural rules and practical issues that need to be considered before BV can be applied in practice.

4.1 Weaker Symmetry Assumptions

Here we analyze the equilibrium strategies of agents if we relax the definition of symmetric equilibria used in Chapter 3. We show that the uniqueness of Proposition 1 holds even if we require weaker symmetry assumptions where the grouping of agents with symmetric behavior is not restricted to their intensity of preference, but to the realization of their utilities.

Recall from the proof of Proposition 1, Step B4, that the established equilibrium is unique if we have $Pr[j_\Omega] = \frac{1}{2}$, where $Pr[j_\Omega]$ is the probability that individual j votes for Ω where $j \neq i$ and j participates in voting in the first stage. Now we show that all conceivable symmetric equilibria yield $Pr[j_\Omega] = \frac{1}{2}$, in which case the voting behavior in Proposition 1 remains unique.

Step 1: Preference groups

We consider all conceivable symmetric equilibria where the voters with the same preferences behave in the same way. We begin by categorizing the preference groups in our model. Since there are 4 dimensions defining an individual's preference, we have $2^4 = 16$ different groups with the same utility realizations which we denote by $G1$ to $G16$ as follows.

G1: *SI* towards A, $x_A^1 \succ x_A^2$, $x_B^1 \succ x_B^2$

G2: *SI* towards A, $x_A^1 \succ x_A^2$, $x_B^2 \succ x_B^1$

G3: *SI* towards A, $x_A^2 \succ x_A^1$, $x_B^1 \succ x_B^2$

G4: *SI* towards A, $x_A^2 \succ x_A^1$, $x_B^2 \succ x_B^1$

G5: *SI* towards B, $x_A^1 \succ x_A^2$, $x_B^1 \succ x_B^2$

G6: *SI* towards B, $x_A^1 \succ x_A^2$, $x_B^2 \succ x_B^1$

G7: *SI* towards B, $x_A^2 \succ x_A^1$, $x_B^1 \succ x_B^2$

G8: *SI* towards B, $x_A^2 \succ x_A^1$, $x_B^2 \succ x_B^1$

G9: *WI* towards A, $x_A^1 \succ x_A^2$, $x_B^1 \succ x_B^2$

G10: *WI* towards A, $x_A^1 \succ x_A^2$, $x_B^2 \succ x_B^1$

G11: *WI* towards A, $x_A^2 \succ x_A^1$, $x_B^1 \succ x_B^2$

G12: *WI* towards A, $x_A^2 \succ x_A^1$, $x_B^2 \succ x_B^1$

G13: *WI* towards B, $x_A^1 \succ x_A^2$, $x_B^1 \succ x_B^2$

G14: *WI* towards B, $x_A^1 \succ x_A^2$, $x_B^2 \succ x_B^1$

G15: *WI* towards B, $x_A^2 \succ x_A^1$, $x_B^1 \succ x_B^2$

G16: *WI* towards B, $x_A^2 \succ x_A^1$, $x_B^2 \succ x_B^1$

We note that an individual may belong to a group from $G1$ to $G8$ with probability $\frac{p}{8}$, while he may belong to a group from $G9$ to $G16$ with probability $\frac{1-p}{8}$. Recall from the proof of Proposition 1 that the decision of individuals to abstain from the first stage or not can be disregarded for the question whether or not an individual votes sincerely in the first stage. Therefore, any equilibrium under BV can be categorized according to the decision of groups $G1$ to $G16$ to vote sincerely or insincerely. Therefore, there are a total of $\sum_{w=0}^{16}\binom{16}{w} = 2^{16}$ possible equilibria, where w denotes the number of groups voting sincerely, while the remaining $16-w$ groups vote insincerely in equilibrium.

Step 2: Probability of winning without sincere voting

We next derive the expression for $P(n)$ that would be valid for all conceivable symmetric equilibria, i.e. *without* the assumption that other voters will vote sincerely.

Suppose individual i (weakly or strongly) prefers direction Ω, $\Omega \in \{A, B\}$ and he has decided to vote in the first stage. We denote by $P_\Omega(n,p)$ the probability of winning for i if he votes for Ω and if a total of n agents participate in the vote. $P_\Omega(n,p)$ is constructed with the assumption that agents other than i vote for Ω with probability $Pr[j_\Omega]$ which may not be equal to $\frac{1}{2}$. We note that $Pr[j_A] = 1 - Pr[j_B]$ and *vice versa*. We further note that the value of $Pr[j_\Omega]$ may depend on p, as the voting behavior of agents is symmetric across a preference group, and the probability with which an individual belongs to a certain preference group depends on p.

We obtain,

$$P_\Omega(n,p) = \begin{cases} \sum_{m=\frac{n-1}{2}}^{n-1}(Pr[j_\Omega])^m(1-Pr[j_\Omega])^{n-m-1}\binom{n-1}{m}, & \text{if } n \text{ is odd} \\ \sum_{m=\frac{n}{2}}^{n-1}(Pr[j_\Omega])^m(1-Pr[j_\Omega])^{n-m-1}\binom{n-1}{m} & \\ \qquad + \frac{1}{2}(Pr[j_\Omega])^{\frac{n}{2}-1}(1-Pr[j_\Omega])^{\frac{n}{2}}\binom{n-1}{\frac{n}{2}-1}, & \text{if } n \text{ is even.} \end{cases} \tag{4.1}$$

(4.2)

We observe that $P_\Omega(n,p)$ is monotonically decreasing in n if $Pr[j_\Omega] < \frac{1}{2}$ but has an increasing trend if $Pr[j_\Omega] > \frac{1}{2}$.[1] To illustrate, in Figure 4.1 we plot $P_\Omega(n,p)$ against n for $Pr[j_A] = \frac{1}{4}$ and $Pr[j_B] = \frac{3}{4}$.

[1]Further properties of $P_\Omega(n,p)$ are given in Table A.2.

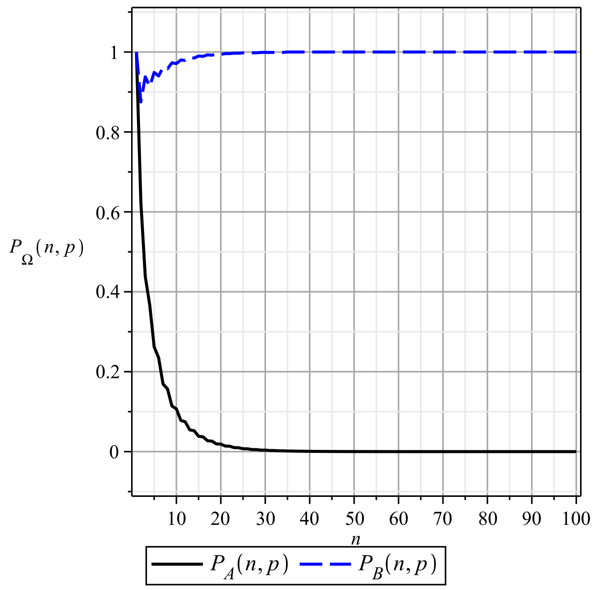

Figure 4.1: $P_\Omega(n,p)$ vs. n for $Pr[j_A] = \frac{1}{4}$ and $Pr[j_B] = \frac{3}{4}$.

Step 3: Decision to vote sincerely or insincerely

The probability of winning or losing in each stage, introduced in Step A2 of proof of Proposition 1, should now modified as follows. Let the indices WW, WL, LW and LL represent the same outcomes as before, and the index V_Ω stand for voting for Ω in the first stage.

$$\Gamma^{V_\Omega}_{WW}(N,x_i,p) = \begin{cases} P_\Omega(x_i+1,p)\frac{1}{2} & \text{if } 0 \le x_i < N-1 \\ P_\Omega(N,p)\left(\frac{P(N)}{2^{N-1}} + \left(1-\frac{1}{2^{N-1}}\right)\frac{1}{2}\right) & \text{if } x_i = N-1 \end{cases}$$

$$\Gamma^{V_\Omega}_{WL}(N,x_i,p) = \begin{cases} P_\Omega(x_i+1,p)\frac{1}{2} & \text{if } 0 \le x_i < N-1 \\ P_\Omega(N,p)\left(\frac{1-P(N)}{2^{N-1}} + \left(1-\frac{1}{2^{N-1}}\right)\frac{1}{2}\right) & \text{if } x_i = N-1 \end{cases}$$

$$\Gamma^{V_\Omega}_{LW}(N,x_i,p) = \begin{cases} (1-P_\Omega(x_i+1,p))\Phi(N,x_i+1) & \text{if } 0 < x_i < N-1 \\ (1-P_\Omega(N,p))\Theta(N) & \text{if } x_i = N-1 \end{cases}$$

$$\Gamma^{V_\Omega}_{LL}(N,x_i,p) = \begin{cases} (1-P_\Omega(x_i+1,p))(1-\Phi(N,x_i+1)) & \text{if } 0 < x_i < N-1 \\ (1-P_\Omega(N,p))(1-\Theta(N)) & \text{if } x_i = N-1, \end{cases}$$

For ease of representation, let us use the notation $\Gamma_F^{V_\Omega} = \Gamma_F^{V_\Omega}(N, x_i, p)$ for all $F \in \{WW, WL, LW, LL\}$. Without loss of generality, let us assume that i prefers A over B. The expected utility of i if he votes for A is,

$$\Gamma_{WW}^{V_A}(1+M) + \Gamma_{WL}^{V_A}(M) + \Gamma_{LW}^{V_A}$$

where $M = H$ is i if *SI* and $M = \epsilon$ if i is *WI*. The expected utility of i if he votes for B is,

$$\Gamma_{WW}^{V_B} + \Gamma_{LW}^{V_B}(1+M) + \Gamma_{LL}^{V_B}(M)$$

Following the same logic as in Step A5 of the proof of Proposition 1, since we do not know the probability distribution of x_i at this stage, we obtain a sufficient condition on M for sincere voting to be the best response for i. Sincere voting is a best response for i if for each $x_i \in [0, N-1]$ we have,

$$\Gamma_{WW}^{V_A}(1+M) + \Gamma_{WL}^{V_A}(M) + \Gamma_{LW}^{V_A} \geq \Gamma_{WW}^{V_B} + \Gamma_{LW}^{V_B}(1+M) + \Gamma_{LL}^{V_B}(M) \tag{4.3}$$

Assuming $\Gamma_{WW}^{V_A} + \Gamma_{WL}^{V_A} - \Gamma_{LW}^{V_B} - \Gamma_{LL}^{V_B} > 0$, this expression can be written as

$$M \geq \frac{\Gamma_{LW}^{V_B} + \Gamma_{WW}^{V_B} - \Gamma_{WW}^{V_A} - \Gamma_{LW}^{V_A}}{\Gamma_{WW}^{V_A} + \Gamma_{WL}^{V_A} - \Gamma_{LW}^{V_B} - \Gamma_{LL}^{V_B}}.$$

This condition simplifies to the following inequalities.

$$M \geq \begin{cases} 0 & \text{for} \quad x_i = 0 \\ M_1^{V_A}(N, x_i) := \frac{\left(P_B(x_i+1,p) - P_A(x_i+1,p)\right)\cdot\left(\frac{1}{2} - \Phi(N, x_i+1)\right)}{P_A(x_i+1,p)+P_B(x_i+1,p)-1} & \text{for} \quad 0 < x_i < N-1 \\ M_2^{V_A}(N) := \frac{\left(P_B(N,p) - P_A(N,p)\right)\cdot\left(\frac{P(N)}{2^{N-1}} + (1 - \frac{1}{2^{N-1}})\frac{1}{2} - \Theta(N)\right)}{P_A(N,p)+P_B(N,p)-1} & \text{for} \quad x_i = N-1. \end{cases}$$

We note that if $P_A(x_i+1,p) + P_B(x_i+1,p) \geq 1$ and $P_B(x_i+1,p) \geq P_A(x_i+1,p)$ for all $0 < x_i \leq N-1$, then we have $M_1^{V_A}(N, x_i) \leq 0$ and $M_2^{V_A}(N) \leq 0$ and therefore sincere voting is always a best response for i. Using the properties of the binomial coefficient, we observe the following lemma.

Lemma 3
For all $0 < x_i \le N-1$,

(1) $P_A(x_i+1,p) + P_B(x_i+1,p) \ge 1$

(2) $P_B(x_i+1,p) \ge P_A(x_i+1,p)$ *if and only if* $Pr[j_A] \le \frac{1}{2}$

(3) $P_B(x_i+1,p) < P_A(x_i+1,p)$ *if and only if* $Pr[j_A] > \frac{1}{2}$

Proof of Lemma 3:
We proceed in two steps.

1) $P_A(x_i+1,p) + P_B(x_i+1,p) \ge 1$

Suppose first that x_i+1 is odd. Then,

$$P_A(x_i+1,p) = \sum_{m=\frac{x_i}{2}}^{x_i} (Pr[j_A])^m (1-Pr[j_A])^{x_i-m} \binom{x_i}{m} \tag{4.4}$$
$$P_B(x_i+1,p) = \sum_{m=\frac{x_i}{2}}^{x_i} (1-Pr[j_A])^m (Pr[j_A])^{x_i-m} \binom{x_i}{m} \tag{4.5}$$

Using the Binomial Theorem we obtain

$$\begin{aligned}(Pr[j_A]+1-Pr[j_A])^{x_i} &= \sum_{m=0}^{x_i} (Pr[j_A])^{x_i-m}(1-Pr[j_A])^m \binom{x_i}{m} \\ &= \sum_{m=0}^{x_i} (1-Pr[j_A])^{x_i-m}(Pr[j_A])^m \binom{x_i}{m} = 1. \end{aligned}\tag{4.6}$$

As x_i is an even number and the expressions for both $P_A(x_i+1,p)$ and $P_B(x_i+1,p)$ start with the term $m = \frac{x_i}{2}$,

$$\begin{aligned}P_A(x_i+1,p) + P_B(x_i+1,p) &= \sum_{m=0}^{x_i} (Pr[j_A])^{x_i-m}(1-Pr[j_A])^m \binom{x_i}{m} \\ &\quad + (Pr[j_A])^{\frac{x_i}{2}}(1-Pr[j_A])^{\frac{x_i}{2}} \binom{x_i}{\frac{x_i}{2}} \\ &= 1 + (Pr[j_A])^{\frac{x_i}{2}}(1-Pr[j_A])^{\frac{x_i}{2}} \binom{x_i}{\frac{x_i}{2}} > 1 \end{aligned}\tag{4.7}$$

Now suppose that $x_i + 1$ is even. Then,

$$P_A(x_i+1,p) = \sum_{m=\frac{x_i+1}{2}}^{x_i} (Pr[j_A])^m (1-Pr[j_A])^{x_i-m} \binom{x_i}{m} + \frac{1}{2}(Pr[j_A])^{\frac{x_i-1}{2}} (1-Pr[j_A])^{\frac{x_i+1}{2}} \binom{x_i}{\frac{x_i-1}{2}} \tag{4.8}$$

$$P_B(x_i+1,p) = \sum_{m=\frac{x_i+1}{2}}^{x_i} (1-Pr[j_A])^m (Pr[j_A])^{x_i-m} \binom{x_i}{m} + \frac{1}{2}(1-Pr[j_A])^{\frac{x_i-1}{2}} (Pr[j_A])^{\frac{x_i+1}{2}} \binom{x_i}{\frac{x_i-1}{2}} \tag{4.9}$$

As x_i is an odd number, from (4.6) we have

$$\begin{aligned} P_A(x_i+1,p) + P_B(x_i+1,p) &= \sum_{m=0}^{x_i} (Pr[j_A])^{x_i-m} (1-Pr[j_A])^m \binom{x_i}{m} \\ &\quad + \frac{1}{2}(Pr[j_A])^{\frac{x_i-1}{2}} (1-Pr[j_A])^{\frac{x_i+1}{2}} \binom{x_i}{\frac{x_i-1}{2}} \\ &\quad + \frac{1}{2}(1-Pr[j_A])^{\frac{x_i-1}{2}} (Pr[j_A])^{\frac{x_i+1}{2}} \binom{x_i}{\frac{x_i-1}{2}} \\ &= 1 + \frac{1}{2}(Pr[j_A])^{\frac{x_i-1}{2}} (1-Pr[j_A])^{\frac{x_i+1}{2}} \binom{x_i}{\frac{x_i-1}{2}} \\ &\quad + \frac{1}{2}(1-Pr[j_A])^{\frac{x_i-1}{2}} (Pr[j_A])^{\frac{x_i+1}{2}} \binom{x_i}{\frac{x_i-1}{2}} \\ &> 1. \end{aligned} \tag{4.10}$$

Therefore, for all $x_i + 1$ we obtain $P_A(x_i + 1, p) + P_B(x_i + 1, p) \geq 1$, where the equality occurs when $x_i + 1$ is large enough such that the terms on the right-hand side of (4.7) and (4.10) in excess of 1 become negligibly small.

2) $P_B(x_i + 1, p) \geq P_A(x_i + 1, p)$ if and only if $Pr[j_A] \leq \frac{1}{2}$ and *vice versa*

First suppose $x_i + 1$ is odd. Then, $P_A(x_i + 1, p)$ and $P_B(x_i + 1, p)$ are given by

(4.4) and (4.5), respectively. We obtain

$$P_B(x_i+1,p) - P_A(x_i+1,p) = \sum_{m=\frac{x_i}{2}+1}^{x_i} \Big[(1-Pr[j_A])^m (Pr[j_A])^{x_i-m} - (Pr[j_A])^m (1-Pr[j_A])^{x_i-m}\Big] \binom{x_i}{m} \quad (4.11)$$

We observe that (4.11) is greater than zero if and only if $1 - Pr[j_A] > \frac{1}{2}$, i.e. $Pr[j_A] < \frac{1}{2}$ and *vice versa*. Similarly, (4.11) is equal to zero if and only if $Pr[j_A] = \frac{1}{2}$.

Next suppose $x_i + 1$ is even. Then, $P_A(x_i+1,p)$ and $P_B(x_i+1,p)$ are given by (4.8) and (4.9), respectively. We obtain

$$\begin{aligned}
&P_B(x_i+1,p) - P_A(x_i+1,p) = \\
&\sum_{m=\frac{x_i+1}{2}}^{x_i} \Big[(1-Pr[j_A])^m (Pr[j_A])^{x_i-m} - (Pr[j_A])^m (1-Pr[j_A])^{x_i-m}\Big] \binom{x_i}{m} \\
&+\frac{1}{2}\Big[(1-Pr[j_A])^{\frac{x_i-1}{2}} (Pr[j_A])^{\frac{x_i+1}{2}} - (Pr[j_A])^{\frac{x_i-1}{2}} (1-Pr[j_A])^{\frac{x_i+1}{2}}\Big] \binom{x_i}{\frac{x_i-1}{2}} \\
&= \sum_{m=\frac{x_i+3}{2}}^{x_i} (1-Pr[j_A])^m (Pr[j_A])^{x_i-m} \binom{x_i}{m} + \frac{1}{2}(1-Pr[j_A])^{\frac{x_i+1}{2}} (Pr[j_A])^{\frac{x_i-1}{2}} \binom{x_i}{\frac{x_i-1}{2}} \\
&- \sum_{m=\frac{x_i+3}{2}}^{x_i} (Pr[j_A])^m (1-Pr[j_A])^{x_i-m} \binom{x_i}{m} - \frac{1}{2}(Pr[j_A])^{\frac{x_i+1}{2}} (1-Pr[j_A])^{\frac{x_i-1}{2}} \binom{x_i}{\frac{x_i-1}{2}}
\end{aligned} \quad (4.12)$$

We observe that (4.12) is greater than zero if and only if $1 - Pr[j_A] > \frac{1}{2}$, i.e. $Pr[j_A] < \frac{1}{2}$ and *vice versa*. Similarly, (4.12) is equal to zero if and only if $Pr[j_A] = \frac{1}{2}$.

Therefore, for $x_i + 1$ odd or even, we have $P_B(x_i+1,p) \gtreqless P_A(x_i+1,p)$ if and only if $Pr[j_A] \lesseqgtr \frac{1}{2}$. This concludes the proof of Lemma 3.

$\blacksquare$

Following the properties of Lemma 3, we note that if $Pr[j_A] \leq \frac{1}{2}$ holds, (4.3) is satisfied for all $x_i \in [0, N-1]$ and sincere voting is the best response for i. However, if $Pr[j_A] > \frac{1}{2}$, sincere voting is not always a best response for i for finite values of M.

Step 4: Possible symmetric equilibria with insincere voting

We now check if there exist symmetric equilibria where any group of agents will vote insincerely and therefore, $P_\Omega(n, p)$ may hold with $Pr[j_\Omega] \neq \frac{1}{2}$.

We start by analyzing if there exist any equilibrium where agent i, who is (strongly or weakly) inclined towards A, has an incentive to vote insincerely. Following the properties of Lemma 3, i may vote insincerely only if $Pr[j_A] > \frac{1}{2}$. We now analyze if there exist any equilibrium where $Pr[j_A] > \frac{1}{2}$ holds.

We note that,

$$Pr[j_A] \quad = Pr[j_A|j_{PA}]Pr[j_{PA}] + Pr[j_A|j_{PB}]Pr[j_{PB}].$$

Given that $Pr[j_{PA}] = Pr[j_{PB}] = \frac{1}{2}$, a subset of individuals inclined towards B must vote insincerely for $Pr[j_A] > \frac{1}{2}$ to hold. However, $Pr[j_A] > \frac{1}{2}$ implies $Pr[j_B] = 1 - Pr[j_A] < \frac{1}{2}$. Conducting the same analysis for an individual inclined towards B yields that sincere voting is the best response for all agents inclined towards B, if $Pr[j_B] < \frac{1}{2}$. Therefore, there cannot exist an equilibrium where i inclined towards A will vote insincerely. Analogously, no agent inclined towards B will vote insincerely either.

Therefore, in any conceivable symmetric equilibrium all individuals vote sincerely in both stages.

Step 5: Decision to abstain

Given that all agents vote sincerely in both stages, and by construction of H and ϵ, all *SI* agents will participate in the vote in the first stage, while the *WI* agents will abstain under BV. Therefore, we conclude that the equilibrium in Proposition 1 is unique even with weaker symmetry assumptions.

4.2 Non-fundamental Decisions

An inherent assumption in our framework is that there are large utility differences in the electorate regarding fundamental decisions. It is useful to examine the suitability of BV for making decisions that may not be fundamental in nature. In our model, this relates to situations where $H < 1$ or $\epsilon > \epsilon^{crit}(N)$ or both. We observe that in this case, the constructed equilibrium under BV given in Proposition 1 may not be unique. Furthermore, there is a possibility for the occurrence of mixed-strategy equilibria. In this section we provide a simple example to demonstrate both these properties.

Suppose a small society with $N = 3$ agents. For simplicity, we assume $p = 1$ and therefore, the entire society consists of individuals strongly-inclined towards a fundamental direction. We denote by q, $0 \leq q \leq 1$ the probability with which they participate in the first stage. We note that voting sincerely is the best response of all individuals in both stages. Sincere voting in second stage is obvious as the game reaches the end in the second stage. Sincere voting in the first stage follows from the observation that *SI* agents are strictly better off if their preferred fundamental direction is selected in the first stage and can be verified easily by comparing the expected utility from voting sincerely and voting insincerely. We now analyze the equilibrium behavior with respect to agents' decision to participate in the first stage.

Suppose individual i is considering voting in the first stage. Then, he would face the following probabilities of winning:

- Probability of winning in both stages:

$$q^2\left[\frac{1}{2^2}P(3)P(3) + \left(1 - \frac{1}{2^2}\right)P(3)\frac{1}{2}\right] + 2q(1-q)P(2)\frac{1}{2} + (1-q)^2 P(1)\frac{1}{2}$$

- Probability of winning in the first stage but losing in the second:

$$q^2 P(3)\left[\frac{1}{2^2}(1 - P(3)) + \left(1 - \frac{1}{2^2}\right)\frac{1}{2}\right] + 2q(1-q)P(2)\frac{1}{2} + (1-q)^2 P(1)\frac{1}{2}$$

- Probability of losing in the first stage but winning in the second:

$$q^2(1 - P(3))\Theta(3) + 2q(1-q)\Phi(3,2)(1 - P(2))$$

- Probability of losing in both stages:

$$q^2(1-P(3))(1-\Theta(3))+2q(1-q)(1-P(2))(1-\Phi(3,2))$$

The expected utility from participation in voting in the first stage for i is given by,

$$\begin{aligned}U_i^{Par} &= \Big[q^2\Big(\frac{P(3)P(3)}{4}+\frac{3}{4}.\frac{1}{2}P(3)\Big)+q(1-q)P(2)+\frac{(1-q)^2}{2}\Big](1+H)\\ &+\Big[q^2\Big(\frac{P(3)(1-P(3))}{4}+\frac{3}{4}.\frac{1}{2}P(3)\Big)+q(1-q)P(2)+\frac{(1-q)^2}{2}\Big]H\\ &+q^2(1-P(3))\Theta(3)+2q(1-q)(1-P(2))\Phi(3,2).\end{aligned}$$

Using $\Theta(3)=P(1)=1$ and $\Phi(3,2)=P(2)=P(3)=\frac{3}{4}$,

$$\begin{aligned}U_i^{Par} &= \Big[q^2\Big(\frac{9}{64}+\frac{9}{32}\Big)+q(1-q)\frac{3}{4}+\frac{(1-q)^2}{2}\Big](1+H)\\ &+\Big[q^2\Big(\frac{3}{64}+\frac{9}{32}\Big)+q(1-q)\frac{3}{4}+\frac{(1-q)^2}{2}\Big]H+q^2\frac{1}{4}+2q(1-q)\frac{3}{16}\\ &=\frac{43}{64}q^2+q(1-q)\frac{9}{8}+\frac{(1-q)^2}{2}+\Big[\frac{3}{4}q^2+q(1-q)\frac{3}{2}+(1-q)^2\Big]H.\end{aligned}$$

If i abstains from voting in the first stage, he would have the following probabilities of winning.

- Probability of winning in both stages:

$$q^2\frac{1}{2}\Lambda(3,2)+2q(1-q)P(2)\frac{1}{2}+(1-q)^2P(3)P(3)$$

- Probability of winning in the first stage but losing in the second:

$$q^2\frac{1}{2}(1-\Lambda(3,2))+2q(1-q)(1-P(2))\frac{1}{2}+(1-q)^2P(3)(1-P(3))$$

- Probability of losing in the first stage but winning in the second:

$$q^2\frac{1}{2}\Lambda(3,2)+2q(1-q)P(2)\frac{1}{2}+(1-q)^2(1-P(3))P(3)$$

- Probability of losing in both stages:

$$q^2\frac{1}{2}(1-\Lambda(3,2)) + 2q(1-q)(1-P(2))\frac{1}{2} + (1-q)^2(1-P(3))(1-P(3))$$

The expected utility from abstention for i is given by,

$$\begin{aligned}
U_i^{Abs} &= \Big[q^2\frac{1}{2}\Lambda(3,2) + q(1-q)P(2) + (1-q)^2P(3)P(3)\Big](1+H) \\
&+\Big[q^2\frac{1}{2}(1-\Lambda(3,2)) + q(1-q)(1-P(2)) + (1-q)^2P(3)(1-P(3))\Big]H \\
&+q^2\frac{1}{2}\Lambda(3,2) + q(1-q)P(2) + (1-q)^2(1-P(3))P(3).
\end{aligned}$$

Using $P(2) = P(3) = \frac{3}{4}$ and $\Lambda(3,2) = \frac{1}{2} + \frac{1}{2}P(2) = \frac{7}{8}$,

$$\begin{aligned}
U_i^{Abs} &= \Big[\frac{7}{16}q^2 + q(1-q)\frac{3}{4} + (1-q)^2\frac{9}{16}\Big](1+H) \\
&+\Big[\frac{1}{16}q^2 + q(1-q)\frac{1}{4} + (1-q)^2\frac{3}{16}\Big]H + q^2\frac{7}{16} + \frac{3}{4}q(1-q) + (1-q)^2\frac{3}{16} \\
&= \frac{7}{8}q^2 + q(1-q)\frac{3}{2} + (1-q)^2\frac{3}{4} + \Big[\frac{q^2}{2} + q(1-q) + (1-q)^2\frac{3}{4}\Big]H.
\end{aligned}$$

We look for symmetric equilibria where all SI agents behave the same and participate in the first stage with probability q. For i to play a mixed strategy between participation and abstention given that all other SI agents participate with probability q, he must be indifferent between the expected utilities from the two strategies. That is,

$$U_i^{Par} = U_i^{Abs}.$$

This implies,

$$\begin{aligned}
\Big[\tfrac{q^2}{4} + \tfrac{q(1-q)}{2} + \tfrac{(1-q)^2}{4}\Big]H &= \frac{13}{64}q^2 + \frac{3}{8}q(1-q) + \frac{(1-q)^2}{4} \\
\Rightarrow \qquad H &= \frac{5}{16}q^2 - \frac{q}{2} + 1 =: H_{ms}(q) \qquad (4.13)
\end{aligned}$$

We observe that $H_{ms}(0) = 1$ and $H_{ms}(1) = 0.8125$. For the remaining values of q, we plot $H_{ms}(q)$ against q in Figure 4.2.

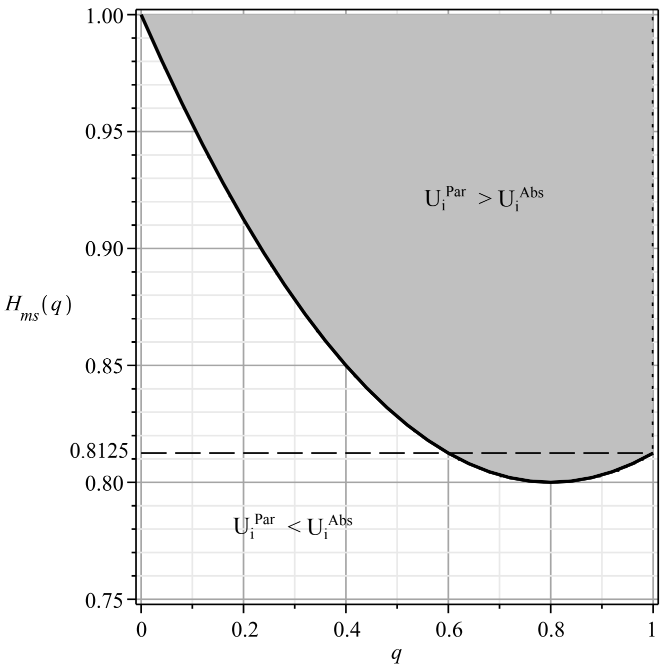

Figure 4.2: $H_{ms}(q)$ vs. q.

Since we are looking for symmetric equilibria, the equilibria for a given value of H can be determined by drawing a horizontal line across the H value in Figure 4.2 and examining which regions it crosses. We note that the unique equilibrium when $U_i^{Par} > U_i^{Abs}$ is the pure strategy equilibrium where all agents participate ($q = 1$) and the unique equilibrium when $U_i^{Par} < U_i^{Abs}$ is the pure strategy equilibrium where all agents abstain ($q = 0$). If $H = H_{ms}(q)$ for some q, then there exists a mixed strategy equilibrium where agents participate with probability $q(H)$. The equilibria for each H value in this example can be summarized as follows.

(i) If $H \geq 1$:

- One pure strategy equilibrium: all agents participate ($q = 1$)

(ii) If $1 > H > 0.8125$:

- Two pure strategy equilibria: all agents participate ($q = 1$) or all agents abstain ($q = 0$)
- One mixed strategy equilibrium: all agents participate with probability $q(H)$ ($0 < q(H) < 1$)

(iii) If $0.8125 \geq H > 0.8$:

- Two pure strategy equilibria: all agents participate ($q = 1$) or all agents abstain ($q = 0$)
- Two mixed strategy equilibrium: all agents participate with probability $q_1(H)$ or $q_2(H)$ ($0 < q_1(H), q_2(H) < 1$)

(iv) If $H = 0.8$:

- One pure strategy equilibrium: all agents abstain ($q = 0$)
- One mixed strategy equilibrium: all agents participate with probability $q(H)$ ($0 < q(H) < 1$)

(v) If $H < 0.8$:

- One pure strategy equilibrium: all agents abstain ($q = 0$)

This example, therefore, illustrates that for non-fundamental decisions with $H < 1$, multiple equilibria and mixed-strategy equilibria are possible under BV.

4.3 Further Extensions

In this section, we discuss several other interesting extensions to our framework.

Publicly-observable agent preferences
An interesting extension is the case where the preferences of agents are common knowledge. If individuals can observe each other's preferences, voters may coordinate to reach a minimal majority in the first stage. This would lead to a large group of individuals (close to 50% of voters in the first stage) being eligible to vote in the second stage. This would further enhance the welfare properties under BV.

Deliberation before voting
An intriguing new dimension to our framework would be to introduce pre-vote deliberation, i.e. agents engaging in communication before voting. The contributors of Elster

(1998) argue that there are several advantages of deliberation compared with other methods of collective decision-making. Since fundamental societal decisions have significant long-term impact on the members of society, an opportunity to share information and views and engage in a relevant public debate may be desirable. How deliberation – including the possibility to misinterpret one's preferences or the presence of persuasive agents – affects the performance of BV is an important avenue of further research.

Alternative models

It is possible to consider alternative models with different informational assumptions. There could be a fraction of the society that has a fixed type while the remaining agents' preferences would be random. One could also analyze situations where a society has a higher tendency to lean towards one of the fundamental directions, by considering unequal probabilities for an individual to prefer A over B and *vice versa*. The same can be extended to an agent's preference over variations of the fundamental directions. While these constellations require new and elaborate analyses, it is likely that BV continues to be performing well as long as the stakes in the first decision are high.

Extension to several stages

BV can also be applied to sequential voting processes involving several stages. For example, suppose that the society decides on a fundamental direction in the first stage and determines its implementation through a sequence of smaller decisions to follow. BV can be applied to this scenario in two ways. First, if the number of stages is even, BV can be applied on segments of two consecutive stages, as described in this dissertation. Second, agents who abstain from voting in the first stage and the losers of the first stage, can be guaranteed participation in all stages to follow.

Other equilibrium refinements

We have restricted our analysis to symmetric equilibria where individuals with the same preference intensity display the same voting behavior. This excludes the possible symmetric equilibria that may arise if we allow agents who prefer the same fundamental direction or the same variation of a fundamental direction to behave in different ways. Whether such equilibria exist under ST and MV is an open question. It is clear that no other equilibria would arise under SM if we impose less restrictive symmetry assumptions. As shown in Section 4.1, no further equilibria arise under BV if we impose less demanding symmetry requirements.

4.4 Procedural Rules

To apply BV, several additional procedural rules must be established.

First, a *pre-decision* is necessary to assess whether an issue is decisive for a fundamental direction. In many cases, this appears to be intuitive. Examples are exiting nuclear power, legalizing abortion, joining a currency union or reducing public debt significantly. Still, there is a need for a procedure that defines an issue as fundamental. There are several possibilities to clarify this matter. It could be a question a small and selected committee decides upon, e.g. a special committee in Parliament. This question could also be linked to its constitutional rank.

Second, one might be concerned that the group of citizens allowed to vote on the variants of a fundamental decision may be too small, which would reduce the efficiency of such votes. Such cases can be avoided by requiring a minimal size for the electorate of the second stage. This can be achieved by randomly-selecting additional voters from the group of individuals who belonged to the majority in the first stage.

Third, there are consequences if the composition of the society changes from one stage to another. In situations where the final decision depends on the votes of two separate voting bodies (the United States Congress for example), BV would have to be modified and applied to each body separately. In other situations, one committee may be a subset of the other. In the democracies that comply with the Westminster System, the Cabinet of Ministers is appointed out of the current Members of Parliament, and the Ministers decide on the implementation details of the legislature that is passed by the Parliament. In this scenario, it is possible to apply BV to those agents who are allowed to vote in both stages. It is important however, that procedural rules are put into place to avoid very small decision-making bodies, as discussed above.

Fourth, BV excludes some individuals from the vote in the first stage or the second. Losing their right to vote may seem inequitable for some individuals. Thus, it is important that the adoption of BV be supported by a large consensus preceding its implementation. Requiring a unanimous vote in favor of BV would give individuals the freedom to veto the adoption of a voting scheme in which they might lose their voting right in one of the stages.

4.5 Practical Implementation

Once the necessary procedural rules are established, the practical implementation of BV requires close examination.

First, the voting process associated with BV can be organized in two different ways. BV may be implemented practically in two voting rounds or in a single voting round without changing the results we have established. In two-round voting, on the one hand, the votes of the two stages should be cast sequentially, i.e., individuals vote on the fundamental direction first and once a fundamental direction is selected, they vote on its variations. In one-round voting, on the other hand, individuals could cast all their choices simultaneously, where their vote on the variations would be valid conditional on the outcome of the first stage and the voters having a right to vote in the second stage or not. As an example, an individual may cast his vote as *"First Stage: vote for A. Second Stage: x_A^1 if A is selected, x_B^2 if B is selected (and I have a right to vote for the second decision)"*.

Second, we have assumed that voting is secret. Despite secret ballots, one might wonder whether anonymity can be preserved, as some agents lose their voting rights after the first stage. One could infer that such agents were in the majority in the first stage. With paper ballots or electronic ballots, anonymity can be preserved, as other citizens cannot observe whether a particular citizen votes in the first stage or obtains a ballot in the second stage. Thus preferences remain private information.

Third, BV can also be applied to open ballots. As long as voting takes place simultaneously in both stages, our results apply. Individual voting strategies will remain the same in the second stage regardless of the secrecy of the vote. Deviating from sincere voting will not reap any benefits for any individual as the game ends with the second stage. Equilibrium voting strategies will remain best responses if voting occurs simultaneously and openly in the first stage. But our results may need to be modified if the votes are publicly observable and are not strictly simultaneous. For example, an *SI* individual would have an incentive to abstain from the first stage if he observes that his preferred fundamental direction has received sufficient votes to be selected.

5 Application: Decision to Continue or Discontinue Nuclear Power

5.1 Introduction

We now examine how BV could be applied to a real-world policy issue. As an illustration, we focus on the current issue of using nuclear power for energy production. This is a typical example of a fundamental societal decision for which the application of BV will be suitable.

Nuclear technology plays an important role in meeting the energy demands of the world today. As of April 2014, there are over 430 commercial nuclear power reactors operating in 31 countries. The total nuclear energy production amounts to over 11% of the world's electricity. Furthermore, nuclear power is attractive as a continuous, reliable base-load power without carbon dioxide emissions (World Nuclear Association (2014a)).

Despite nuclear power's attractiveness as an energy source, the risks associated with it have triggered an ongoing debate on the suitability of nuclear technology for energy production in many societies. In fact, the repercussions of a nuclear accident can be substantial and extremely harmful. "Nuclear power is a relatively tiny industry with huge economic, technical, safety, environmental, and political problems."(Black (2011)).

In many countries, the year 2011 marked a fundamental shift in public perception with regard to the use of nuclear power as the Fukushima accident reminded the world of the risks associated with nuclear reactors. In March 2011, an earthquake and a subsequent tsunami caused a major nuclear accident in the Fukushima-Daiichi power plant in Japan, releasing radioactive material into the environment (World Health Organization (2012)). This tragedy re-ignited the nuclear debate all around the world, with the opposition to nuclear power gaining strength in many countries and Germany and Switzerland enacting measures to phase-out their nuclear stations in the coming years (The Economist (2013)).

Over the years, several surveys have tracked public opinion on nuclear power programs. We focus on two of these surveys to analyze how BV would perform in practice. The first survey under consideration was conducted in the European Union before the Fukushima accident and the second survey was conducted in Canada, after it. We use these surveys to construct two numerical examples to which BV can be applied and yield outcomes that are different from the repeated application of SM. Specifically, the first example demonstrates how BV can lead to the selection of a fundamental direction that is different from the one that would be chosen under SM. The second example illustrates how even if the chosen fundamental direction is the same under SM and BV, the minority of the first stage is granted more power in the second stage to select a variation they prefer.

5.2 Example 1: Policy Decision in the European Union

We consider a survey on European public opinion on nuclear energy, conducted for the European Commission in 2009 (European Commission (2010)). The survey was conducted in the 27 member states of the European Union, with the main objective of understanding public perception on nuclear issues and safety. Nuclear energy amounted to 31% of the electricity generated in the European Union in 2009. But the use of nuclear power remained controversial, with incidents such as the Chernobyl disaster dominating concerns of nuclear safety in the region (European Commission (2010)).

To construct an example to which BV can be applied, suppose that among those surveyed, the countries operating or planning to operate nuclear power plants make a two-stage decision. In the first stage, each society makes the fundamental decision whether they should continue to use nuclear power (A) or not (B). In the second stage, a decision is taken on two variations of the selected fundamental decision. If the first voting yields A, the nuclear variations are whether to build new nuclear power plants (x_A^1) or maintain the existing ones (x_A^2). If the first voting yields B, the non-nuclear variations are whether to invest in solar (x_B^1) or in wind (x_B^2) power.

We consider two exhibits from the survey. They are reproduced in Figure 5.1 and Figure 5.2. Table 5.1 lists the countries operating or planning to operate nuclear power plants according to World Nuclear Association (2014b), and their official abbreviations used in the figures. For the purpose of constructing an example to which BV can be applied, we now make specific assumptions that would allow us to apply this data to our setup.

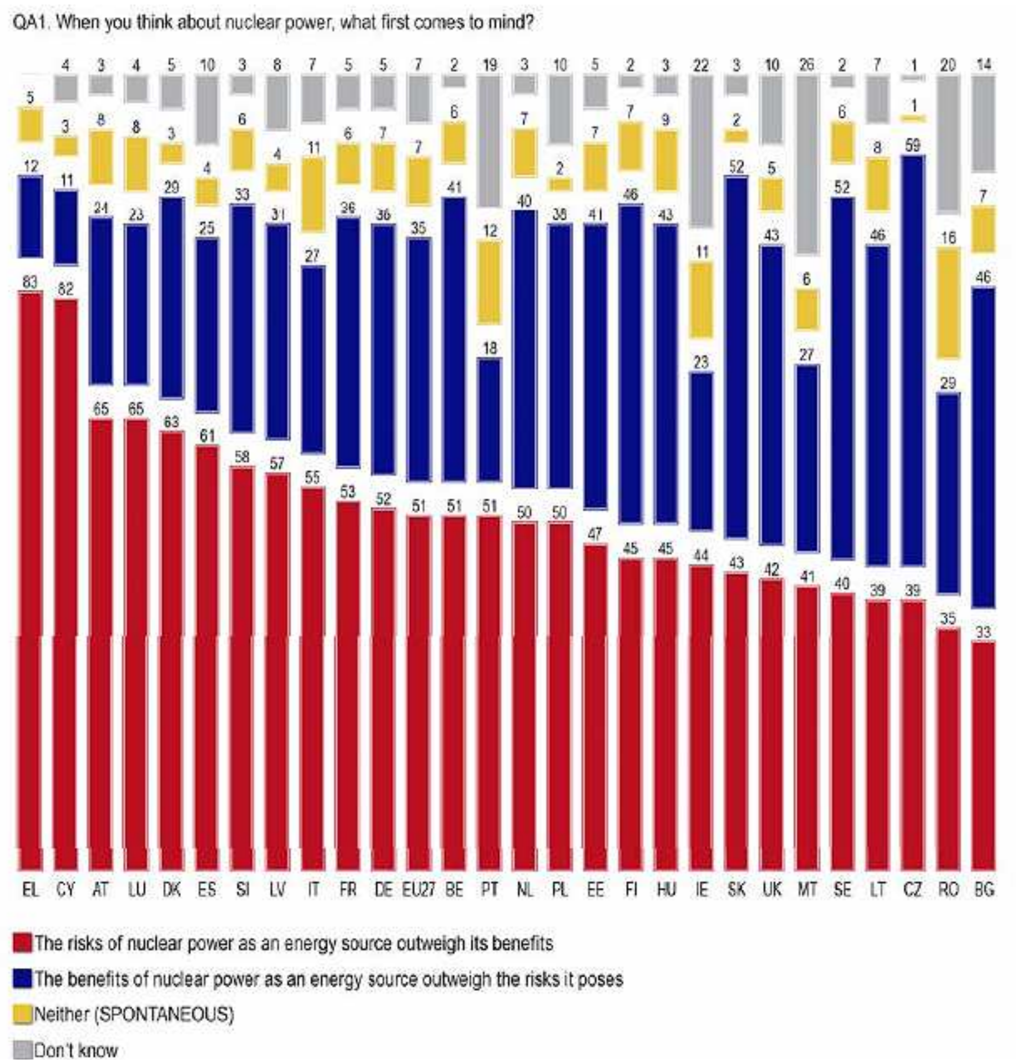

Figure 5.1: Preferences over nuclear power. Source: European Commission (2010).

Figure 5.1 indicates the portion of each society that believes that the benefits of nuclear power outweigh its risks and *vice versa*. We assume that those who believe that the benefits of nuclear energy outweigh its risks are inclined towards A, and those who believe the risks outweigh the benefits are inclined towards B.[1]

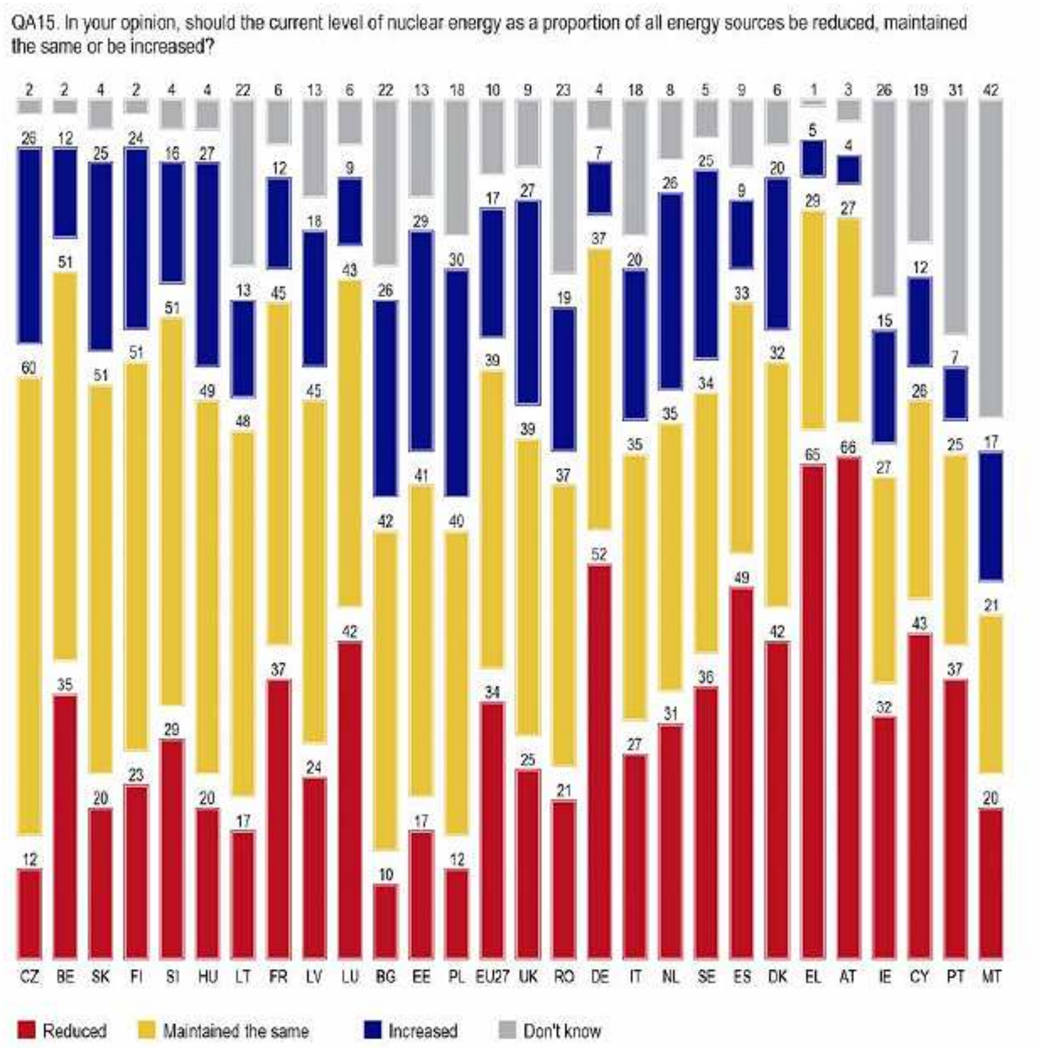

Figure 5.2: Preferences over nuclear variants. Source: European Commission (2010).

Figure 5.2 depicts the response of the participants to the question whether they would like to reduce, maintain or increase the number of nuclear power plants in operation. We

[1]We cannot directly observe whether agents are inclined towards A or B from the survey data. For our purpose of constructing an example that demonstrates how BV can be applied, we make assumptions about agent preferences.

are unable to directly verify whether an agent is *SI* or *WI* from this data. However, for the purpose of constructing an example, we make assumptions that allow us to quantify the agents that are *SI* or *WI* towards A or B. We assume that those who would like to increase the number of nuclear power plants are strongly-inclined towards A, while those who want to reduce the number of nuclear power plants are strongly-inclined towards B. Those that opted for maintaining the same number of nuclear power plants are *WI* either towards A or B depending on how many agents are inclined towards A or B in each country, according to our assumptions on the data from Figure 5.1. As our nuclear variations do not include the option of *reducing* the number of nuclear power plants, we make the reasonable assumption that those who are strongly-inclined towards B would prefer maintaining the current number of power plants (x_A^2) to increasing it (x_A^1).

We note three limitations in our assumptions. First, we assume that all those who are strongly-inclined towards B prefer x_A^2 to x_A^1. Second, we do not allow agents that are strongly-inclined towards A to prefer x_A^2 over x_A^1. Third, we do not allow WI agents to prefer x_A^1 to x_A^2, as we assume that all those who favor x_A^1 are strongly-inclined towards A. The first assumption is intuitive and can be justified, as no rational agent who feels strongly about a full nuclear phase-out in the future would support investment in new nuclear power plants. The second is also somewhat intuitive, as a strong advocate of nuclear technology would be more likely to support building more nuclear power plants. The last is not so intuitive and can be considered as a drawback of our assumptions. However, given the data available in the survey and the setup we are considering, we proceed with these assumptions for illustration purposes.

Based on our assumptions, in Table 5.1, we summarize the size of preference groups in each country that is currently operating or plans to operate nuclear power plants, according to World Nuclear Association (2014b). We note that the respondents who answered "don't know" or "neither" in Figure 5.1 and Figure 5.2 were not categorized into any preference group. Moreover, after determining the size of each preference group according to our assumptions, a small portion of the group that favored maintaining the same level of nuclear energy in some countries was not allocated to any preference group. In a few other countries, the number of agents who were assumed to be inclined towards A or B, marginally exceeded the number of respondents who chose to increase, maintain or reduce the number of nuclear power plants. We acknowledge these errors as further drawbacks of our assumptions.

We apply the equilibrium behavior under BV and SM with incomplete information to each society, and indicate the predicted outcome in each case. We note that due to the lack of data, we are unable to make suitable assumptions to quantify agent preferences over the variations of B, and are therefore unable to predict the outcome in the second stage if B is the winner of the first stage.

We observe that in this example, both BV and SM yield the same fundamental direction in most cases, except in four instances. In Hungary and Poland, the group of individuals strongly-inclined towards A outnumbers those that are strongly-inclined towards B. Thus, under BV, A is chosen in the first stage. But under SM, the strong preferences of the SI agents are diluted in the large pool of WI agents, and B is selected as the winner in the first stage. In Lithuania and Sweden, the reverse can be observed. In Sweden, for example, there is a large group strongly opposing nuclear power. However, their concerns are not addressed under SM, as there is an equally large group of individuals weakly favoring nuclear power, thereby electing A as the winner. Under BV, however, the preferences of the SI agents are given priority and B is chosen in the first stage.

In the second stage, the voting body is strictly smaller under BV than under SM in all cases. Therefore, the voting individuals would, in principle, realize their preferred variation with higher probability under BV. This effect, however, cannot be observed in this example, due to the limitations of the assumptions and the unavailability of all relevant data. The survey we consider next in Section 5.3, allows us to construct an example that demonstrates this effect.

Country	**SI :** A $x_A^1 \succ x_A^2$	**WI :** A $x_A^2 \succ x_A^1$	**WI :** B $x_A^2 \succ x_A^1$	**SI :** B $x_A^2 \succ x_A^1$	**BV**	**SM**
Belgium (BE)	12%	29%	16%	35%	B	B
Bulgaria (BG)	26%	20%	23%	10%	A; x_A^2	A; x_A^2
Czech Republic (CZ)	26%	33%	27%	12%	A; x_A^2	A; x_A^2
Finland (FI)	24%	22%	22%	23%	A; x_A^2	A; x_A^2
France (FR)	12%	24%	16%	37%	B	B
Germany (DE)	7%	29%	–	52%	B	B
Hungary (HU)	27%	16%	25%	20%	A; x_A^2	B
Italy (IT)	20%	7%	28%	27%	B	B
Lithuania (LT)	13%	33%	22%	17%	B	A; x_A^2
Netherlands (NL)	26%	14%	19%	31%	B	B
Poland (PL)	30%	8%	38%	12%	A; x_A^2	B
Romania (RO)	19%	10%	14%	21%	B	B
Slovakia (SK)	25%	27%	23%	20%	A; x_A^2	A; x_A^2
Slovenia (SI)	16%	17%	29%	29%	B	B
Spain (ES)	9%	16%	12%	49%	B	B
Sweden (SE)	25%	27%	4%	36%	B	A; x_A^2
United Kingdom (UK)	27%	16%	17%	25%	A; x_A^2	A; x_A^2

Table 5.1: Preference groups as a percentage of total population and the predicted voting outcome under BV and SM. Based on specific assumptions on the data from European Commission (2010).

5.3 Example 2: Policy Decision in Canada

We next consider a survey conducted in Canada in 2012 (Canadian Nuclear Association (2012)), with the main motivation of understanding Canadian opinions towards nuclear power generation and the impact of the Fukushima tragedy on their attitudes. The survey was conducted as a national proportionate survey to represent the attitudes of each region of Canada as well as the nation as a whole (Canadian Nuclear Association (2012)). In June 2012, when the survey was published, 15.3% of the Canadian electricity was generated by 17 nuclear reactors in operation, with several more nuclear reactors planned or proposed to be built in the future (World Nuclear Association (2014b)).

To construct an example to which BV can be applied,, now suppose that the residents of Canada participate in the two-stage decision whether or not to exit nuclear power generation as detailed in Section 5.2. We consider two exhibits from the survey which are reproduced in Figure 5.3 and Figure 5.4.

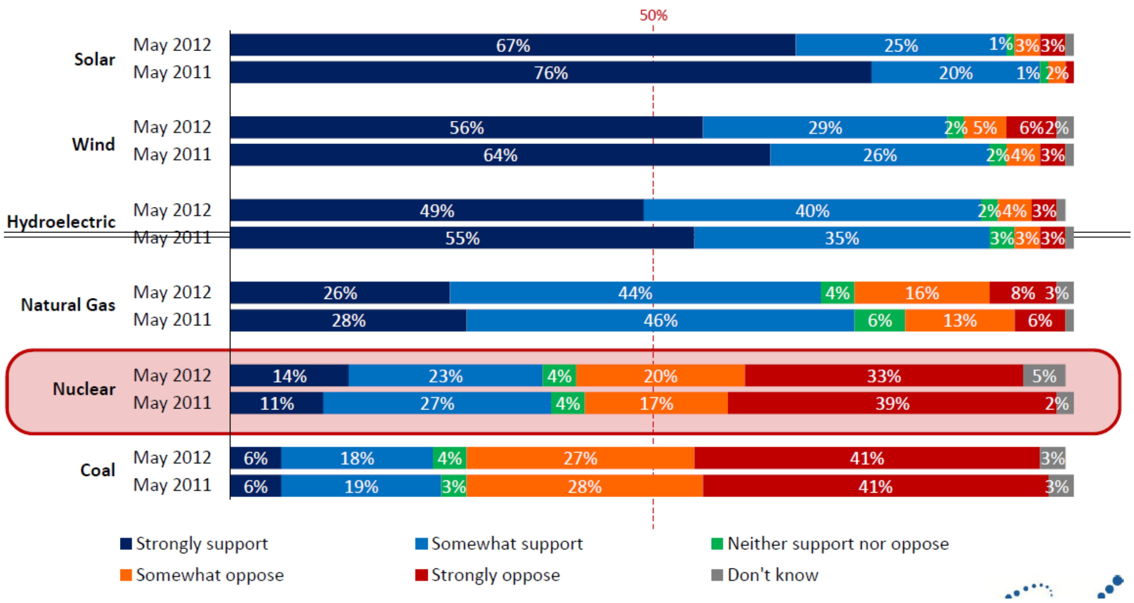

Figure 5.3: Preferences over nuclear power and non-nuclear variations. Source: Canadian Nuclear Association (2012).

Figure 5.3 indicates which portion of the respondents supports or opposes nuclear power, and whether these respondents' preferences are strong or weak.[2] To construct an example that illustrates the application of BV, we assume that those who "strongly support" nuclear power are strongly-inclined towards A, that those who "somewhat support" nuclear power are weakly-inclined towards A, that those who "somewhat oppose" nuclear power are weakly-inclined towards B, and that those who "strongly oppose" nuclear power are strongly-inclined towards B.

Furthermore, we use the data in Figure 5.3 to formulate assumptions about agent preferences over the non-nuclear variations of solar power (x_B^1) and wind power (x_B^2). Figure 5.3 indicates that 92% of the respondents declared support for solar power and 85% supported the use of wind power. However, we cannot infer agent preferences on solar power *versus* wind power from this data, which would be the decision the society is faced with if B is chosen in the first stage in our setup. Thus, for our purpose, we make the specific assumption that if the society is to decide between x_B^1 and x_B^2, the support for one alternative over the other can be derived as follows.

$$x_B^1 : x_B^2 = 92 : 85 = 52\% : 48\%.$$

We note that this derivation of preferences is based on the assumption that the set of individuals who declared support for solar power is disjoint from the set of individuals who supported wind power. This is clearly *not* what the survey data indicates and is thus a strong assumption. However, for illustration purposes of our example, we proceed with this derivation of preferences that captures the marginally higher support for solar power compared to wind power in the survey data and assumes that this support extends to the situation where the decision would be to choose solar *or* wind power.

Moreover, our formulation of preferences over the non-nuclear variations assumes that every agent has a preference towards solar power or wind power. That is, we disregard the small percentage of people who declared neither support nor opposition for any of the two technologies, by answering "don't know" or "neither support nor oppose" for both solar power and wind power.

[2] We cannot directly infer from this data whether agents are strongly or weakly inclined towards A or B in the sense of our setup. Yet, for our purpose of constructing an example that demonstrates how BV can be applied, we make assumptions about agent preferences.

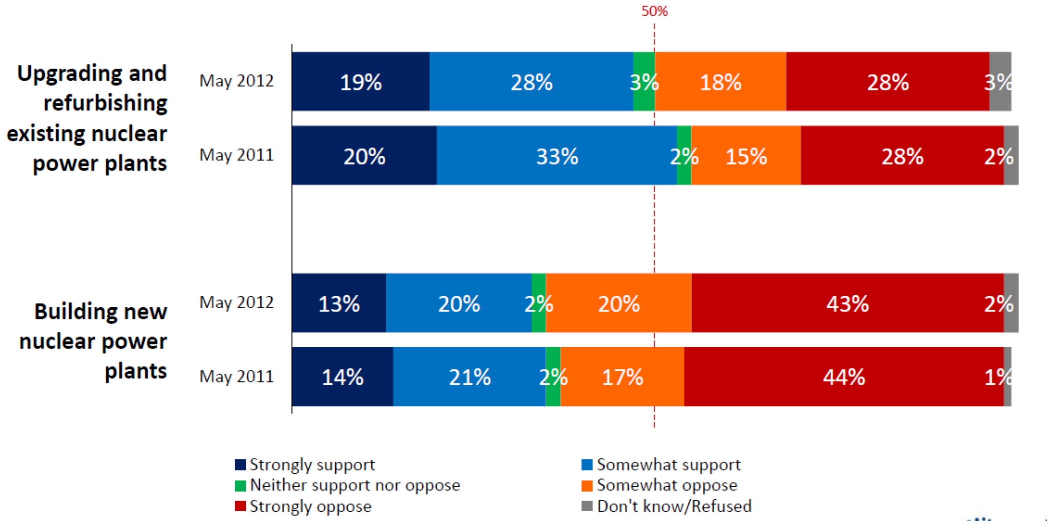

Figure 5.4: Preferences over nuclear variations.
Source: Canadian Nuclear Association (2012).

Figure 5.4 summarizes the responses in favor of upgrading existing nuclear power plants (x_A^2) and building new nuclear power plants (x_A^1). 47% of the respondents expressed their support for option x_A^2, while 33% supported option x_A^1. In our example, however, the society is confronted with the decision of x_A^1 *versus* x_A^2. Analogously to our assumptions with the non-nuclear variations, we assume that the portion of agents preferring one alternative over the other can be derived as follows:

$$x_A^1 : x_A^2 = 33 : 47 = 41\% : 59\%.$$

	SI agents	**WI agents**
$A \succ B$	14%	23%
$B \succ A$	33%	20%

Table 5.2: Preference groups by fundamental direction as a percentage of the total population. Based on specific assumptions on the data from Canadian Nuclear Association (2012).

In Table 5.2, we summarize the portion of the society that is strongly-inclined or weakly-inclined towards A and B according to our assumptions on the data in Figure 5.3. We observe the following feature in our example, which is due to the nature of our assumptions: While all agents in the society exhibit a preference for one variation over the other for both A and B, only 90% of the society are classified as *SI* or *WI* towards A or B. In our equilibrium analysis in Chapter 3, we assumed that all agents who voted were either *SI* or *WI* towards A or B. We continue to make the same assumption in this example. However, given that 10% of the society that were not classified as *SI* or *WI* in this example are not indifferent between the variations in the second stage, it is unrealistic to assume they would refrain from voting in the second stage. This is considered to be a further drawback of our assumptions, and we try to minimize its effect on our comparison of the outcomes under SM and BV by assuming these agents refrain from voting under both voting schemes.

We now apply the equilibrium voting behavior under BV and SM with incomplete information. Under SM, all agents with an inclination towards A or B vote in the first stage, and A receives 37% of the votes, while B receives 53% of the votes, making B the fundamental direction of the society. Under BV, only *SI* agents vote in the first stage. Then, A receives 14% of the votes, while B receives 33% of the votes, again electing B as the winner of the first stage. The Canadian society exhibits substantial opposition to the use of nuclear power, with over 50% of its agents declaring their disapproval for this technology. Therefore, regardless of whether SM or BV is used, the fundamental direction of the society would be to abandon the use of nuclear power.

We note that BV highlights the preferences of those agents that are very passionate about either using or discontinuing nuclear power. The percentage of agents that is weakly-inclined towards A or B is almost the same (23% and 20% respectively). However, those agents that strongly oppose nuclear technology are more than twice as numerous as those who strongly support it (33% and 14% respectively). This is depicted in the large winning margin of B over A under BV, illustrating the particular feature of BV that it allows those who care more to exert more influence on a decision. Although the voting outcome is the same under SM and BV in this example, there can be several other examples – as shown in Section 5.2 – where the fundamental direction of the society could change if only those who have high stakes on an issue participate in its decision-making.

Next, we focus on the second-stage decision. We have assumed that the societal preferences over solar power and wind power are given by $x_B^1 : x_B^2 = 52\% : 48\%$. To illustrate the differences between SM and BV, let us suppose that agents inclined towards different fundamental directions exhibit different preferences over x_B^1 and x_B^2. Specifically, let us assume that 45% of agents inclined towards A prefer x_B^1 over x_B^2, while 57% of agents inclined towards B prefer x_B^1 over x_B^2. According to our previous assumption that every agent has a preference towards x_B^1 or x_B^2, this means that 55% of the agents inclined towards A and 43% of the agents inclined towards B prefer x_B^2 to x_B^1. We assume that the remaining 10% of the population who do not display any inclination towards A or B are equally divided between x_B^1 or x_B^2. These assumptions satisfy our initial estimate of societal preferences for $x_B^1 : x_B^2 = 52\% : 48\%$ since

$$\begin{aligned} x_B^1 \succ x_B^2 \Rightarrow &\quad 45\% \times 37\% + 57\% \times 53\% + 50\% \times 10\% \simeq 52\% \\ x_B^2 \succ x_B^1 \Rightarrow &\quad 55\% \times 37\% + 43\% \times 53\% + 50\% \times 10\% \simeq 48\%. \end{aligned}$$

Now let us focus on the voting outcome of the second stage. In Table 5.3, we show the size of each preference group according to its preferences for the fundamental direction and the non-nuclear variations derived from our assumptions.

		SI agents	**WI agents**
$A \succ B$	$x_B^1 \succ x_B^2$ (45%) $x_B^2 \succ x_B^1$ (55%)	6.3% 7.7%	10.35% 12.65%
$B \succ A$	$x_B^1 \succ x_B^2$ (57%) $x_B^2 \succ x_B^1$ (43%)	18.81% 14.19%	11.4% 8.6%

Table 5.3: Preference groups by fundamental direction and non-nuclear variations as a percentage of the total population. Based on specific assumptions on the data from Canadian Nuclear Association (2012).

We now apply the equilibrium voting behavior in the second stage under BV and SM, and summarize the votes received by each variation in Table 5.4.

	BV	SM
Votes for x_B^1	28.05%	46.86%
Votes for x_B^2	28.95%	43.14%

Table 5.4: Vote totals under BV and SM. Based on specific assumptions on the data from Canadian Nuclear Association (2012).

We note that the voting outcome of the second stage depends on the voting scheme used. Under SM, x_B^1 is selected, while x_B^2 is the winner under BV. The majority of agents inclined towards B prefer x_B^1 to x_B^2 (57%). Since the majority of the society is inclined towards B, its influence dominates the voting outcome under SM. This is a typical illustration of the tyranny of the majority. The majority overruled the minority in the first stage by selecting B over A, and now, in the second stage, it *again* selects its preferred variation over the one preferred by the minority.

Under BV, the possibility for the minority to suffer from repeated losses is reduced. The agents that are strongly-inclined towards B, who won in the first stage, are not given a ballot in the second stage. Thus, the agents inclined towards A are able to exert more influence on the final decision in the second stage. As a result, x_B^2, which is the preferred variation of the majority of the agents that are inclined towards A, emerges as the winner. This illustrates the power of BV in challenging the tyranny of the majority and giving better protection to minorities.

5.4 Conclusion

We considered two real-world surveys on public opinion about using nuclear power for energy production, and illustrated with examples how BV could be applied in practice. One survey was conducted in the member states of the European Union before the Fukushima disaster and the other was conducted after, in Canada. We constructed two examples using the survey data, to which BV can be applied, by making specific assumptions about the agents' preferences and about the strength of these preferences.

Given these preferences, we assumed that the surveyed societies participate in a two-stage decision where in the first stage, they decide whether or not to continue using nuclear power for energy production, and in the second, they choose between variations of the selected fundamental decision. We applied the equilibrium voting behavior with incomplete information under BV and SM to each society and examined the predicted outcome. We observed that while the voting outcome is sometimes similar under BV and SM, there are instances where it differs.

The impact of BV was two-fold: First, BV allowed agents who care more about the fundamental issue to exert more influence on the decision, thereby sometimes allowing the agents to select a different fundamental direction which would not be possible under SM. Second, even if the fundamental direction selected in the first stage were the same under SM and BV, in the second stage, BV granted more power to the minorities to select a variation that is more desirable to them, thereby successfully weakening the tyranny of the majority.

6 Model and Results for Symmetric Information

6.1 Introduction

We analyze BV when the preferences of agents are publicly observable. We consider a committee of $N = 3$ agents with the same informational assumptions as in the case with incomplete information, except that individual utilities from all voting outcomes are publicly observable prior to voting. We continue to use the notation introduced in Chapter 3.

We examine several cases where the three individuals have different preference combinations. We characterize the subgame perfect Nash equilibria in each case. We then compare the equilibrium voting behavior under BV with complete and incomplete information to draw conclusions about the role of symmetric information on the equilibrium voting behavior.

When individual preferences are publicly observable, the strategic implications of BV become much more complex compared to when preferences are privately observable. Thus, we focus mainly on these strategic implications, the resulting equilibria and their properties. We will not perform any welfare comparisons between different voting schemes when preferences are publicly observable.

The rest of the chapter is organized as follows. In Section 6.2 we summarize the features of the equilibrium under BV when preferences are observable only privately. In Section 6.3, we explain the equilibrium refinement we will use to rule out implausible equilibria. In Sections 6.4, 6.5, 6.6 and 6.7, we present examples of the voting game with complete information when the committee comprises 3, 2, 1 and no *SI* agents, respectively. We derive the subgame perfect equilibria in each case. In Section 6.8, we compare the features of equilibria under incomplete information and complete information to draw conclusions summarized in Section 6.9.

6.2 Equilibrium with Incomplete Information

Recall from Chapter 3 that there exists a unique equilibrium under BV with incomplete information, given that $H > 1$ and $\epsilon \leq \epsilon^{crit}(N)$. The main characteristics of this equilibrium are summarized below.

(i) All *SI* agents vote in the first stage if $H > 1$.

(ii) All *WI* agents abstain from voting in the first stage if $\epsilon \leq \epsilon^{crit}(N)$.

(iii) All votes are cast sincerely in the first stage.

(iv) All votes are cast sincerely in the second stage.

In the following, we derive the equilibria under BV when preferences are publicly observable, and examine whether these features are present in equilibria with complete information.

6.3 Equilibrium Refinement

As in the analysis in the setting with incomplete information, we adopt the standard refinement for voting games to exclude implausible voting behavior. We assume that all agents participating in the second stage eliminate weakly-dominated strategies at this stage. If this yields unique strategies for the second stage, we assume that agents also eliminate weakly-dominated strategies in the first stage, if there are any.

We note that regardless of the fundamental direction selected in the first stage, agents are strictly better off if their preferred variation is chosen in the second stage. Hence, voting insincerely in the second stage is always weakly-dominated by voting sincerely for agent i when $i = 1, 2, 3$. This follows from the observation that individual i is pivotal for some combination of strategies by the other two agents. Even if i is not pivotal in equilibrium, if an individual has a very small probability of playing an unintended strategy through a tremble or 'slip of a hand', i becomes pivotal with a small positive probability. In that case, voting sincerely in the second stage is strictly better than voting insincerely.

As introduced in Chapter 3, we use the following notation: Fundamental directions are denoted by $\Omega, \Omega' \in \{A, B\}$ with $\Omega \neq \Omega'$. Variations of Ω are denoted by x_Ω^j and x_Ω^k while variations of Ω' are denoted by $x_{\Omega'}^{j'}$ and $x_{\Omega'}^{k'}$ where $j, k = 1, 2$ with $j \neq k$, $j', k' = 1, 2$ with $j' \neq k'$. Sincere voting is abbreviated by SV and insincere voting by ISV. Specifically, sincere voting in the second stage means voting for the variation that gives a higher utility. Sincere voting in the first stage means voting for the fundamental direction one is inclined towards.

6.4 Equilibria when $N_s = 3$ and $N_w = 0$

Suppose that the committee consists of all *SI* individuals. In Sections 6.4.1, 6.4.2 and 6.4.3 we examine three cases which represent different realizations of individual preferences.

6.4.1 All agents prefer Ω and x_Ω^j

We start with the second stage. After eliminating weakly-dominated strategies, agents that choose or are allowed to vote in the second stage will vote sincerely, as there is no incentive for any individual to deviate.

Next, we focus on the first stage. All strongly-inclined individuals are strictly better off if their preferred fundamental direction Ω is selected in the first stage. If Ω is chosen in the first stage, x_Ω^j will be chosen in the second stage. Therefore, none of the agents has an incentive to vote insincerely in the first stage and to increase the probability of selecting Ω'. Following an argument similar to the second stage in Section 6.3, voting sincerely weakly-dominates voting insincerely in the first stage. After eliminating weakly-dominated strategies with respect to voting sincerity, any agent opting to vote in the first stage would vote sincerely.

Given sincere voting in both stages, there will be no losers from the first stage, and x^j_Ω will be selected as the final decision. Any combination of individuals may choose to vote in the first stage without changing the voting outcome. Hence, there is a total of $\binom{3}{0} + \binom{3}{1} + \binom{3}{2} + \binom{3}{3} = 8$ possible equilibria that may arise. These equilibria are detailed in Table 6.1.

	Absentees	**First Stage**	**Second Stage**	**Outcome**
8 possible equilibria	0/1/2/3 absentees	3/2/1 agents (SV)	3/1/2 agents (SV)	x^j_Ω

Table 6.1: Nash equilibria if all agents prefer Ω and x^j_Ω.

We observe that in all possible equilibria, votes are cast sincerely in both stages, as in the equilibrium voting behavior with incomplete information. However, some *SI* agents may choose to abstain from voting in the first stage in some equilibria. This behavior arises from the fact that *SI* agents are indifferent between participation and abstention in the first stage, rather than having an incentive to waive their voting right in the first stage to obtain a guaranteed voting right for the second stage. We can eliminate such uninteresting equilibria with an additional assumption that if an individual is indifferent between participation and abstention in the first stage, he will choose to participate. Under such an assumption, there exists a unique equilibrium with complete information where all *SI* agents will vote in the first stage. This equilibrium is then compatible with the features of the equilibrium with incomplete information.

6.4.2 All agents are inclined towards Ω. Two agents prefer x^j_Ω, while the other prefers x^k_Ω.

Without loss of generality, we assume that $i = 1, 2$ prefer x^j_Ω and $i = 3$ prefers x^k_Ω. As in the previous case, after eliminating weakly-dominated strategies in the second stage, all agents who choose or are allowed to vote in the second stage will vote sincerely.

In the first stage, the voting behavior is not as straightforward as in the previous case. The committee still consists of only SI agents inclined towards the same fundamental direction. They are all better off if Ω is selected in the first stage, which guarantees a utility of $H >> 1$. However, given that Ω is selected in the first stage, the committee is divided on the decision about its variation. Hence, an agent faces a higher probability of gaining an additional utility of 1 by voting in the second stage. Therefore, given that there are still enough votes in favor of Ω, an individual would have an incentive to abstain from voting or to vote insincerely in the first stage to obtain a guaranteed voting right in the second.

Given that votes in the second stage are cast sincerely, we represent the game in normal form in Table 6.2.

	$i=2$ \ $i=3$	**SV**	**Abstain**	**ISV**
$i=1$ SV	SV	$H+1,\quad H+1,\quad H$	$H,\quad H,\quad H+1$	$H,\quad H,\quad H+1$
	Abstain	$H+1,\quad H+1,\quad H$	$H+\frac{1}{2},\quad H+\frac{1}{2},\quad H+\frac{1}{2}$	$\frac{H}{2}+\overset{(h)}{\frac{1}{2}}$ or $\overset{(i)}{\frac{3}{4}}$, $\frac{H}{2}+\overset{(h)}{\frac{1}{2}}$ or $\overset{(i)}{\frac{3}{4}}$, $\frac{H}{2}+\overset{(j)}{\frac{1}{4}}$ or $\overset{(h)}{\frac{1}{2}}$ or $\overset{(d)}{\frac{3}{4}}$
	ISV	$H+1,\quad H+1,\quad H$	$\frac{H}{2}+\overset{(a)}{\frac{1}{2}}$ or $\overset{(b)}{\frac{3}{4}}$, $\frac{H}{2}+\overset{(c)}{\frac{1}{4}}$ or $\overset{(a)}{\frac{1}{2}}$ or $\overset{(d)}{\frac{3}{4}}$, $\frac{H}{2}+\overset{(a)}{\frac{1}{2}}$ or $\overset{(b)}{\frac{3}{4}}$	1, $\overset{(h)}{0}$ or $\overset{(i)}{1}$, $\overset{(a)}{0}$ or $\overset{(b)}{1}$
$i=1$ Abstain	SV	$H+1,\quad H+1,\quad H$	$H+\frac{1}{2},\quad H+\frac{1}{2},\quad H+\frac{1}{2}$	$\frac{H}{2}+\overset{(h)}{\frac{1}{2}}$ or $\overset{(i)}{\frac{3}{4}}$, $\frac{H}{2}+\overset{(h)}{\frac{1}{2}}$ or $\overset{(i)}{\frac{3}{4}}$, $\frac{H}{2}+\overset{(j)}{\frac{1}{4}}$ or $\overset{(h)}{\frac{1}{2}}$ or $\overset{(d)}{\frac{3}{4}}$
	Abstain	$H+1,\quad H+1,\quad H$	$H+1,\quad H+1,\quad H$	$\overset{(h)}{\frac{1}{2}}$ or $\overset{(i)}{1}$, $\overset{(h)}{\frac{1}{2}}$ or $\overset{(i)}{1}$, $\overset{(j)}{0}$ or $\overset{(h)}{\frac{1}{2}}$ or $\overset{(d)}{1}$
	ISV	$\frac{H}{2}+\overset{(a)}{\frac{3}{4}}$ or $\overset{(b)}{1}$, $\frac{H}{2}+\overset{(c)}{\frac{1}{2}}$ or $\overset{(a)}{\frac{3}{4}}$ or $\overset{(d)}{1}$, $\frac{H}{2}+\overset{(a)}{\frac{1}{4}}$ or $\overset{(b)}{\frac{1}{2}}$	$\overset{(a)}{\frac{1}{2}}$ or $\overset{(b)}{1}$, $\overset{(c)}{0}$ or $\overset{(a)}{\frac{1}{2}}$ or $\overset{(d)}{1}$, $\overset{(a)}{\frac{1}{2}}$ or $\overset{(b)}{1}$	1, $\overset{(h)}{0}$ or $\overset{(i)}{1}$, $\overset{(a)}{0}$ or $\overset{(b)}{1}$
$i=1$ ISV	SV	$H+1,\quad H+1,\quad H$	$\frac{H}{2}+\overset{(e)}{\frac{1}{4}}$ or $\overset{(f)}{\frac{1}{2}}$ or $\overset{(d)}{\frac{4}{3}}$, $\frac{H}{2}+\overset{(f)}{\frac{1}{2}}$ or $\overset{(g)}{\frac{3}{4}}$, $\frac{H}{2}+\overset{(f)}{\frac{1}{2}}$ or $\overset{(g)}{\frac{3}{4}}$,	$\overset{(h)}{0}$ or $\overset{(i)}{1}$, 1, $\overset{(f)}{0}$ or $\overset{(g)}{1}$
	Abstain	$\frac{H}{2}+\overset{(e)}{\frac{1}{2}}$ or $\overset{(f)}{\frac{3}{4}}$ or $\overset{(d)}{1}$, $\frac{H}{2}+\overset{(f)}{\frac{3}{4}}$ or $\overset{(g)}{1}$, $\frac{H}{2}+\overset{(f)}{\frac{1}{4}}$ or $\overset{(g)}{\frac{1}{2}}$	$\overset{(e)}{0}$ or $\overset{(f)}{\frac{1}{2}}$ or $\overset{(d)}{1}$, $\overset{(f)}{\frac{1}{2}}$ or $\overset{(g)}{1}$, $\overset{(f)}{\frac{1}{2}}$ or $\overset{(g)}{1}$	$\overset{(h)}{0}$ or $\overset{(i)}{1}$, 1, $\overset{(f)}{0}$ or $\overset{(g)}{1}$
	ISV	$\overset{(a)}{0}$ or $\overset{(b)}{1}$, $\overset{(f)}{0}$ or $\overset{(g)}{1}$, 1	$\overset{(a)}{0}$ or $\overset{(b)}{1}$, $\overset{(f)}{0}$ or $\overset{(g)}{1}$, 1	$\overset{(e)}{0}$ or $\overset{(b)/(i)}{1}$, $\overset{(c)}{0}$ or $\overset{(g)/(i)}{1}$, $\overset{(j)}{0}$ or $\overset{(b)/(g)}{1}$

Table 6.2: The game in 6.4.2 in normal form where Conditions (a) – (n) are given in Table 6.3.

The expected payoffs of individuals in the first stage sometimes depend on individual preferences over the variations of Ω'. We denote by $R_i(\Omega')$ the preference relation of agent i, $i = 1, 2, 3$, over $x^1_{\Omega'}$ and $x^2_{\Omega'}$. That is, $x^1_{\Omega'}\ R_i(\Omega')\ x^2_{\Omega'}$, where $R_i(\Omega') \in \{\succ_i, \prec_i\}$. The conditions on which the expected payoffs depend are given in Table 6.3.

Condition	**Meaning of Condition**
(a)	$R_1(\Omega') \neq R_3(\Omega')$
(b)	$R_1(\Omega') = R_3(\Omega')$
(c)	$R_1(\Omega') = R_3(\Omega') \neq R_2(\Omega')$
(d)	$R_1(\Omega') = R_2(\Omega') = R_3(\Omega')$[1]
(e)	$R_2(\Omega') = R_3(\Omega') \neq R_1(\Omega')$
(f)	$R_2(\Omega') \neq R_3(\Omega')$
(g)	$R_2(\Omega') = R_3(\Omega')$
(h)	$R_1(\Omega') \neq R_2(\Omega')$
(i)	$R_1(\Omega') = R_2(\Omega')$
(j)	$R_1(\Omega') = R_2(\Omega') \neq R_3(\Omega')$
(k)	$\epsilon \geq 0.5$
(l)	$\epsilon < 0.5$
(m)	$H \geq 1.5$
(n)	$H < 1.5$

Table 6.3: Conditions determining the expected payoffs in the first stage.

We solve for all possible Nash equilibria in pure strategies. In Table 6.4 we denote individual best responses using underlined boldface lettering and shade the constituting Nash equilibria.

[1]For ease of presentation, suppose that in this case, all agents prefer $x^{j'}_{\Omega'}$ over $x^{k'}_{\Omega'}$, where $j', k' \in \{1, 2\}$, $j' \neq k'$.

$i=1$	$i=2$ \ $i=3$	SV	Abstain	ISV
$i=1$ SV	SV	$\underline{\boldsymbol{H+1}}$, $\underline{\boldsymbol{H+1}}$, H	H, H, $\underline{\boldsymbol{H+1}}$	$\underset{\boldsymbol{(h)/(i)\&(m)}}{\underline{\boldsymbol{H}}}$, $\underset{\boldsymbol{(h)/(i)\&(m)}}{\underline{\boldsymbol{H}}}$, $\underline{\boldsymbol{H+1}}$
	Abstain	$\underline{\boldsymbol{H+1}}$, $\underline{\boldsymbol{H+1}}$, H	$H+\frac{1}{2}$, $\underline{\boldsymbol{H+\frac{1}{2}}}$, $\underline{\boldsymbol{H+\frac{1}{2}}}$	$\underline{\boldsymbol{\frac{H}{2}+\overset{(h)}{\frac{1}{2}}\text{ or }\overset{(i)}{\frac{3}{4}}}}$, $\underline{\boldsymbol{\frac{H}{2}+}}\overset{\boldsymbol{(h)}}{\boldsymbol{\frac{1}{2}}}$ or $\underset{\boldsymbol{(n)}}{\overset{\boldsymbol{(i)}}{\boldsymbol{\frac{3}{4}}}}$, $\frac{H}{2}+\overset{(j)}{\frac{1}{4}}$ or $\overset{(h)}{\frac{1}{2}}$ or $\overset{(d)}{\frac{3}{4}}$
	ISV	$\underline{\boldsymbol{H+1}}$, $\underline{\boldsymbol{H+1}}$, $\underset{\boldsymbol{(a)/(b)\&(m)}}{\underline{\boldsymbol{H}}}$	$\underline{\boldsymbol{\frac{H}{2}+\overset{(a)}{\frac{1}{2}}\text{ or }\overset{(b)}{\frac{3}{4}}}}$, $\frac{H}{2}+\overset{(c)}{\frac{1}{4}}$ or $\overset{(a)}{\frac{1}{2}}$ or $\overset{(d)}{\frac{3}{4}}$, $\underline{\boldsymbol{\frac{H}{2}+}}\overset{(a)}{\frac{1}{2}}$ or $\underset{\boldsymbol{(n)}}{\overset{\boldsymbol{(b)}}{\boldsymbol{\frac{3}{4}}}}$	$\underline{\boldsymbol{1}}$, $\overset{(h)}{0}$ or $\overset{(i)}{1}$, $\overset{(a)}{0}$ or $\overset{(b)}{1}$
$i=1$ Abstain	SV	$\underline{\boldsymbol{H+1}}$, $\underline{\boldsymbol{H+1}}$, H	$\underline{\boldsymbol{H+\frac{1}{2}}}$, $H+\frac{1}{2}$, $\underline{\boldsymbol{H+\frac{1}{2}}}$	$\underline{\boldsymbol{\frac{H}{2}+}}\overset{\boldsymbol{(h)}}{\boldsymbol{\frac{1}{2}}}$ or $\underset{\boldsymbol{(n)}}{\overset{\boldsymbol{(i)}}{\boldsymbol{\frac{3}{4}}}}$, $\underline{\boldsymbol{\frac{H}{2}+\overset{(h)}{\frac{1}{2}}\text{ or }\overset{(i)}{\frac{3}{4}}}}$, $\frac{H}{2}+\overset{(j)}{\frac{1}{4}}$ or $\overset{(h)}{\frac{1}{2}}$ or $\overset{(d)}{\frac{3}{4}}$
	Abstain	$\underline{\boldsymbol{H+1}}$, $\underline{\boldsymbol{H+1}}$, $\underline{\boldsymbol{H}}$	$\underline{\boldsymbol{H+1}}$, $\underline{\boldsymbol{H+1}}$, $\underline{\boldsymbol{H}}$	$\overset{(h)}{\frac{1}{2}}$ or $\overset{(i)}{1}$, $\overset{(h)}{\frac{1}{2}}$ or $\overset{(i)}{1}$, $\overset{(j)}{0}$ or $\overset{(h)}{\frac{1}{2}}$ or $\overset{(d)}{1}$
	ISV	$\frac{H}{2}+\overset{(a)}{\frac{3}{4}}$ or $\overset{(b)}{1}$, $\frac{H}{2}+\overset{(c)}{\frac{1}{2}}$ or $\overset{(a)}{\frac{3}{4}}$ or $\overset{(d)}{1}$, $\underline{\boldsymbol{\frac{H}{2}+\overset{(a)}{\frac{1}{4}}\text{ or }\overset{(b)}{\frac{1}{2}}}}$	$\overset{(a)}{\frac{1}{2}}$ or $\overset{(b)}{1}$, $\overset{(c)}{0}$ or $\overset{(a)}{\frac{1}{2}}$ or $\overset{(d)}{1}$, $\overset{(a)}{\frac{1}{2}}$ or $\overset{(b)}{1}$	$\underline{\boldsymbol{1}}$, $\overset{(h)}{0}$ or $\overset{(i)}{1}$, $\overset{(a)}{0}$ or $\overset{(b)}{1}$
$i=1$ ISV	SV	$\underline{\boldsymbol{H+1}}$, $\underline{\boldsymbol{H+1}}$, $\underset{\boldsymbol{(f)/(g)\&(m)}}{\underline{\boldsymbol{H}}}$	$\frac{H}{2}+\overset{(e)}{\frac{1}{4}}$ or $\overset{(f)}{\frac{1}{2}}$ or $\overset{(d)}{\frac{4}{3}}$, $\underline{\boldsymbol{\frac{H}{2}+\overset{(f)}{\frac{1}{2}}\text{ or }\overset{(g)}{\frac{3}{4}}}}$, $\underline{\boldsymbol{\frac{H}{2}+}}\overset{\boldsymbol{(f)}}{\frac{1}{2}}$ or $\underset{\boldsymbol{(n)}}{\overset{\boldsymbol{(g)}}{\boldsymbol{\frac{3}{4}}}}$	$\overset{(h)}{0}$ or $\overset{(i)}{1}$, $\underline{\boldsymbol{1}}$, $\overset{(f)}{0}$ or $\overset{(g)}{1}$
	Abstain	$\frac{H}{2}+\overset{(e)}{\frac{1}{2}}$ or $\overset{(f)}{\frac{3}{4}}$ or $\overset{(d)}{1}$, $\frac{H}{2}+\overset{(f)}{\frac{3}{4}}$ or $\overset{(g)}{1}$, $\underline{\boldsymbol{\frac{H}{2}+\overset{(f)}{\frac{1}{4}}\text{ or }\overset{(g)}{\frac{1}{2}}}}$	$\overset{(e)}{0}$ or $\overset{(f)}{\frac{1}{2}}$ or $\overset{(d)}{1}$, $\overset{(f)}{\frac{1}{2}}$ or $\overset{(g)}{1}$, $\overset{(f)}{\frac{1}{2}}$ or $\overset{(g)}{1}$	$\overset{(h)}{0}$ or $\overset{(i)}{1}$, $\underline{\boldsymbol{1}}$, $\overset{(f)}{0}$ or $\overset{(g)}{1}$
	ISV	$\overset{(a)}{0}$ or $\overset{(b)}{1}$, $\overset{(f)}{0}$ or $\overset{(g)}{1}$, $\underline{\boldsymbol{1}}$	$\overset{(a)}{0}$ or $\overset{(b)}{1}$, $\overset{(f)}{0}$ or $\overset{(g)}{1}$, $\underline{\boldsymbol{1}}$	$\overset{(e)}{0}$ or $\overset{\boldsymbol{(b)/(i)}}{\underline{\boldsymbol{1}}}$, $\overset{(c)}{0}$ or $\overset{\boldsymbol{(g)/(i)}}{\underline{\boldsymbol{1}}}$, $\overset{(j)}{0}$ or $\overset{\boldsymbol{(b)/(g)}}{\underline{\boldsymbol{1}}}$

Table 6.4: The game in 6.4.2 and Nash equilibria before eliminating weakly-dominated strategies in the first stage. Conditions $(a)-(n)$ are given in Table 6.3.

We observe that there are six possible Nash equilibria in pure strategies. We summarize them in Table 6.5.

	Conditions for Existence	**Absentees**	**First Stage**	**Second Stage**	**Outcome**
(1)		$i = 1, 2$	$i = 3$ (SV)	$i = 1, 2$ (SV)	x^j_Ω
(2)		all	all (SV)	all (SV)	x^j_Ω
(3)	If (a) OR $[(b)$ & $(m)]$	none	all $i = 1, 3$ (SV) $i = 2$ (ISV)	$i = 2$ (SV)	x^j_Ω
(4)	If (h) OR $[(i)$ & $(m)]$	none	all $i = 1, 2$ (SV) $i = 3$ (ISV)	$i = 3$ (SV)	x^j_Ω
(5)	If (f) OR $[(g)$ & $(m)]$	none	all $i = 2, 3$ (SV) $i = 1$ (ISV)	$i = 1$ (SV)	x^j_Ω
(6)	If (d)	none	all (ISV)	all (SV)	$x^{j'}_{\Omega'}$

Table 6.5: Nash equilibria of the game in 6.4.2 before eliminating weakly-dominated strategies in the first stage.

Next, we focus on eliminating weakly-dominated strategies in the first stage. We observe that abstention weakly-dominates insincere voting for all agents. Moreover, sincere voting and abstention do not weakly-dominate one another for any agent. After eliminating equilibria with insincere voting, we obtain our refined set of equilibria, which we summarize in Table 6.6. We note that these equilibria are path-independent.

The voting behavior under both equilibria is intuitive. Individuals $i = 1$ and $i = 2$ prefer to abstain, because together, they can ensure the selection of x^j_Ω in the second stage. Since $i = 3$ is strongly-inclined towards Ω, he has no incentive to vote for Ω' in the first stage, as he is strictly better off with any variation of Ω than with a variation of Ω'. Therefore, $i = 1, 2$ are certain of the selection of Ω in the first stage and are thus confident that by abstaining in the first stage, they can select x^j_Ω as the final outcome.

	Absentees	First Stage	Second Stage	Outcome
(1)	$i = 1, 2$	$i = 3$ (SV)	$i = 1, 2$ (SV)	x_Ω^j
(2)	all	all (SV)	all (SV)	x_Ω^j

Table 6.6: Refined Nash equilibria of the game in 6.4.2.

Note that neither of them has an incentive to vote because this would yield the possibility that $i = 3$ may abstain and consequently, that x_Ω^k is selected with positive probability. Given that both $i = 1, 2$ abstain, $i = 3$ is indifferent between participation or abstention in the first stage, yielding two possible equilibria.

We note that in this case, votes are cast sincerely in both stages, as in the equilibrium with incomplete information. However, two *SI* agents have an incentive to abstain from voting in the first stage for a guaranteed voting right in the second, which is inconsistent with the voting behavior of *SI* agents with incomplete information.

6.4.3 Two agents are inclined towards Ω and one towards Ω'. All agents prefer x_Ω^j over x_Ω^k.

Without loss of generality, we assume that $i = 1, 2$ are inclined towards Ω and $i = 3$ is inclined towards Ω'. As in the previous cases, all agents that choose or are allowed to vote in the second stage will vote sincerely, after eliminating weakly-dominated strategies in the second stage.

Given that votes in the second stage are cast sincerely, we represent the game in normal form in Table 6.8. We solve for all possible Nash equilibria in pure strategies. As before, we denote individual best responses using underlined boldface lettering and shade the constituting Nash equilibria. The Nash equilibria of the game before eliminating weakly-dominated strategies in the first stage are summarized in Table 6.7.

	Conditions for Existence	**Absentees**	**First Stage**	**Second Stage**	**Outcome**
(1)		none	all (SV)	$i = 3$ (SV)	x_Ω^j
(2)		$i = 3$	$i = 1, 2$ (SV)	$i = 3$ (SV)	x_Ω^j
(3)		none	all $i = 1, 2$ (SV) $i = 3$ (ISV)	all (SV)	x_Ω^j
(4)	If (d)	none	all $i = 1, 2$ (ISV) $i = 3$ (SV)	all (SV)	$x_{\Omega'}^{j'}$

Table 6.7: Nash equilibria of the game in 6.4.3 before eliminating weakly-dominated strategies in the first stage.

	$i=2$ \ $i=3$	SV	Abstain	ISV
$i=1$ SV	SV	$\underline{\boldsymbol{H+1}},\ \underline{\boldsymbol{H+1}},\ \underline{\boldsymbol{1}},$	$\underline{\boldsymbol{H+1}},\ \underline{\boldsymbol{H+1}},\ \underline{\boldsymbol{1}}$	$\underline{\boldsymbol{H+1}},\ \underline{\boldsymbol{H+1}},\ \underline{\boldsymbol{1}}$
	Abstain	$\underline{\boldsymbol{\frac{H}{2}+\overset{(h)}{\frac{3}{4}}\text{ or }\overset{(i)}{1}}},\ \frac{H}{2}+\overset{(h)}{\frac{3}{4}}\text{ or }\overset{(i)}{1},\ \underline{\boldsymbol{\frac{H}{2}+\overset{(j)}{\frac{1}{2}}\text{ or }\overset{(h)}{\frac{3}{4}}\text{ or }\overset{(d)}{1}}},$	$\underline{\boldsymbol{H+1}},\ \underline{\boldsymbol{H+1}},\ 1,$	$\underline{\boldsymbol{H+1}},\ \underline{\boldsymbol{H+1}},\ 1$
	ISV	$\underline{\boldsymbol{1}},\ \overset{(h)}{0}\text{ or }\overset{(i)}{1},\ \underline{\boldsymbol{H+}}\underset{\boldsymbol{(m)}}{\overset{\boldsymbol{(a)}}{\underline{\boldsymbol{0}}}}\text{ or }\underline{\overset{\boldsymbol{(b)}}{\boldsymbol{1}}}$	$\underline{\boldsymbol{\frac{H}{2}+\overset{(a)}{\frac{3}{4}}\text{ or }\overset{(b)}{1}}},\ \frac{H}{2}+\overset{(c)}{\frac{1}{2}}\text{ or }\overset{(a)}{\frac{3}{4}}\text{ or }\overset{(d)}{1},\ \underline{\boldsymbol{\frac{H}{2}+}}\underset{\boldsymbol{(n)}}{\overset{\boldsymbol{(a)}}{\underline{\boldsymbol{\frac{3}{4}}}}}\text{ or }\overset{\boldsymbol{(b)}}{\boldsymbol{1}}$	$\underline{\boldsymbol{H+1}},\ \underline{\boldsymbol{H+1}},\ 1$
$i=1$ Abstain	SV	$\frac{H}{2}+\overset{(h)}{\frac{3}{4}}\text{ or }\overset{(i)}{1},\ \underline{\boldsymbol{\frac{H}{2}+\overset{(h)}{\frac{3}{4}}\text{ or }\overset{(i)}{1}}},\ \underline{\boldsymbol{\frac{H}{2}+\overset{(j)}{\frac{1}{2}}\text{ or }\overset{(h)}{\frac{3}{4}}\text{ or }\overset{(d)}{1}}}$	$\underline{\boldsymbol{H+1}},\ \underline{\boldsymbol{H+1}},\ 1$	$\underline{\boldsymbol{H+1}},\ \underline{\boldsymbol{H+1}},\ 1$
	Abstain	$\overset{(h)}{\frac{1}{2}}\text{ or }\overset{(i)}{1},\ \overset{(h)}{\frac{1}{2}}\text{ or }\overset{(i)}{1},\ \underline{\boldsymbol{H+\overset{(j)}{0}\text{ or }\overset{(h)}{\frac{1}{2}}\text{ or }\overset{(d)}{1}}}$	$\underline{\boldsymbol{H+1}},\ \underline{\boldsymbol{H+1}},\ 1$	$\underline{\boldsymbol{H+1}},\ \underline{\boldsymbol{H+1}},\ 1$
	ISV	$\underline{\boldsymbol{1}},\ \overset{(h)}{0}\text{ or }\overset{(i)}{1},\ \underline{\boldsymbol{H+}}\underset{\boldsymbol{(m)}}{\overset{\boldsymbol{(a)}}{\underline{\boldsymbol{0}}}}\text{ or }\overset{\boldsymbol{(b)}}{\boldsymbol{1}},$	$\overset{(a)}{\frac{1}{2}}\text{ or }\overset{(b)}{1},\ \overset{(c)}{0}\text{ or }\overset{(a)}{\frac{1}{2}}\text{ or }\overset{(d)}{1},\ \underline{\boldsymbol{H+}}\underset{\boldsymbol{(m)}}{\overset{\boldsymbol{(a)}}{\underline{\boldsymbol{\frac{1}{2}}}}}\text{ or }\overset{\boldsymbol{(b)}}{\boldsymbol{1}}$	$\frac{H}{2}+\overset{(a)}{\frac{3}{4}}\text{ or }\overset{(b)}{1},\ \frac{H}{2}+\overset{(c)}{\frac{1}{2}}\text{ or }\overset{(a)}{\frac{3}{4}}\text{ or }\overset{(d)}{1},\ \underline{\boldsymbol{\frac{H}{2}+}}\underset{\boldsymbol{(n)}}{\overset{\boldsymbol{(a)}}{\underline{\boldsymbol{\frac{3}{4}}}}}\text{ or }\overset{\boldsymbol{(b)}}{\boldsymbol{1}}$
$i=1$ ISV	SV	$\overset{(h)}{0}\text{ or }\overset{(i)}{1},\ \underline{\boldsymbol{1}},\ \underline{\boldsymbol{H+}}\underset{\boldsymbol{(m)}}{\overset{\boldsymbol{(f)}}{\underline{\boldsymbol{0}}}}\text{ or }\overset{\boldsymbol{(g)}}{\underline{\boldsymbol{1}}},$	$\frac{H}{2}+\overset{(e)}{\frac{1}{2}}\text{ or }\overset{(f)}{\frac{3}{4}}\text{ or }\overset{(d)}{1},\ \underline{\boldsymbol{\frac{H}{2}+\overset{(f)}{\frac{3}{4}}\text{ or }\overset{(g)}{1}}},\ \underline{\boldsymbol{\frac{H}{2}+}}\underset{\boldsymbol{(n)}}{\overset{\boldsymbol{(f)}}{\underline{\boldsymbol{\frac{3}{4}}}}}\text{ or }\overset{\boldsymbol{(g)}}{\boldsymbol{1}}$	$\underline{\boldsymbol{H+1}},\ \underline{\boldsymbol{H+1}},\ 1$
	Abstain	$\overset{(h)}{0}\text{ or }\overset{(i)}{1},\ \underline{\boldsymbol{1}},\ \underline{\boldsymbol{H+}}\overset{\boldsymbol{(f)}}{\boldsymbol{0}}\text{ or }\overset{\boldsymbol{(g)}}{\underline{\boldsymbol{1}}},$	$\overset{(e)}{0}\text{ or }\overset{(f)}{\frac{1}{2}}\text{ or }\overset{(d)}{1},\ \overset{(f)}{\frac{1}{2}}\text{ or }\overset{(g)}{1},\ \underline{\boldsymbol{H+\overset{(f)}{\frac{1}{2}}\text{ or }\overset{(g)}{1}}},$	$\frac{H}{2}+\overset{(e)}{\frac{1}{2}}\text{ or }\overset{(f)}{\frac{3}{4}}\text{ or }\overset{(d)}{1},\ \frac{H}{2}+\overset{(f)}{\frac{3}{4}}\text{ or }\overset{(g)}{1},\ \frac{H}{2}+\overset{(f)}{\frac{3}{4}}\text{ or }\overset{(g)}{1}$
	ISV	$\overset{(e)}{0}\text{ or }\overset{\boldsymbol{(b)/(i)}}{\underline{\boldsymbol{1}}},\ \overset{(c)}{0}\text{ or }\overset{\boldsymbol{(g)/(i)}}{\underline{\boldsymbol{1}}},\ \underline{\boldsymbol{H+}}\ \overset{(j)}{0}\text{ or }\overset{\boldsymbol{(b)/(g)}}{\underline{\boldsymbol{1}}}$	$\overset{(a)}{0}\text{ or }\overset{(b)}{1},\ \overset{(f)}{0}\text{ or }\overset{(g)}{1},\ \underline{\boldsymbol{H+1}}$	$\overset{(a)}{0}\text{ or }\overset{(b)}{1},\ \overset{(f)}{0}\text{ or }\overset{(g)}{1},\ \underline{\boldsymbol{H+1}}$

Table 6.8: The game in 6.4.3 and Nash equilibria before eliminating weakly-dominated strategies in the first stage. Conditions $(a)-(n)$ are given in Table 6.3.

The agents are divided as to the fundamental direction that is best for society. Since all agents are strongly-inclined, $i = 1, 2$ have an incentive to vote sincerely in the first stage, to ensure that Ω will be selected in the first stage. We observe that sincere voting weakly-dominates abstention and insincere voting for $i = 1, 2$. For $i = 3$, abstention weakly-dominates insincere voting. Furthermore, sincere voting weakly-dominates abstention for $i = 3$ if Condition (d) is satisfied.

We note that the iterated elimiation of weakly-dominated strategies is path-dependent in this case. If we iteratively eliminate weakly-dominated strategies, so that strategies for $i = 3$ are eliminated last, our refined set of equilibria will consist of Equilibria (1), (2) and (3) in Table 6.7. However, if the iterative elimination process starts with strategies of $i = 3$, our refined set of equilibria consists of two Nash equilibria given in Table 6.9.

	Conditions for Existence	**Absentees**	**First Stage**	**Second Stage**	**Outcome**
(1)		none	all (SV)	$i = 3$ (SV)	x^j_Ω
(2)	If not (d)	$i = 3$	$i = 1, 2$ (SV)	$i = 3$ (SV)	x^j_Ω

Table 6.9: Refined Nash equilibria of the game in 6.4.3, when the iterative elimination of weakly-dominated strategies starts with $i = 3$.

In either case, the voting behavior of $i = 1, 2$ is intuitive, as both would want to participate in the first stage and vote sincerely to ensure that Ω is chosen. Given this behavior, $i = 3$ would be indifferent between participation, abstention and insincere voting, as his behavior will not change the final outcome.

The remaining cases to be analyzed when the whole committee comprises strongly-inclined agents are

- Two agents are inclined towards Ω and prefer x^j_Ω over x^k_Ω. The other agent is inclined towards Ω' and prefers x^k_Ω over x^j_Ω.
- Two agents are inclined towards Ω and one prefers x^j_Ω while the other prefers x^k_Ω. The third agent is inclined towards Ω' and prefers x^j_Ω over x^k_Ω.

6.5 Equilibria when $N_s = 2$ and $N_w = 1$

Now suppose that the committee consists of two *SI* and one *WI* individuals. Without loss of generality, suppose that $i = 1, 2$ are *SI* and $i = 3$ is *WI*. In Sections 6.5.1, 6.5.2 and 6.5.3 we examine three cases which represent different realizations of individual preferences.

6.5.1 All agents prefer Ω and x_{Ω}^{j}

We note that after eliminating weakly-dominated strategies, all agents that choose or are allowed to vote in the second stage will vote sincerely. Similarly, any agent opting to vote in the first stage would vote sincerely. Both *SI* and *WI* individuals are inclined towards Ω and they obtain their highest payoff from x_{Ω}^{j}. Hence, no one has an incentive to follow a strategy that would yield any other outcome. Given that the final outcome is the one preferred by all agents, any combination of agents may abstain in the first stage. Similar to the Case 6.4.1, there are eight possible Nash equilibria, detailed in Table 6.1.

We note that all agents vote sincerely in both stages, as in the equilibrium with incomplete information. However, the participation of *SI* agents in the first stage and the abstention of *WI* agents may contrast with the equilibrium voting behavior with incomplete information. As in Section 6.4.1, this behavior arises from the indifference between participation and abstention rather than being a calculated trade-off of voting rights between the two stages. If we impose the additional assumption that if an individual is indifferent between participation and abstention in the first stage, he will choose to participate, then, there exists a unique equilibrium where all agents will vote in the first stage. This equilibrium contrasts with the equilibrium with incomplete information insofar that the *WI* agent will participate in the first stage.

6.5.2 All agents are inclined towards Ω. The SI agents prefer x_{Ω}^{j}, while the WI agent prefers x_{Ω}^{k}.

As in the previous cases, elimination of weakly-dominated strategies in the second stage yields sincere voting by all agents who choose or are allowed to vote in the second stage.

Given that votes in the second stage are cast sincerely, we represent the game in normal form in Table 6.10. We solve for all possible Nash equilibria in pure strategies. We note that although all agents are inclined towards Ω, the weakly-inclined agent may not always be better off if Ω is selected in the first stage. Depending on which agents will vote in the second stage and on how agents agree on the variations of Ω', the *WI* agent may be better off if Ω' is selected in the first stage.

	$i=2$ \ $i=3$	SV	Abstain	ISV
$i=1$ SV	SV	$\underline{\boldsymbol{H+1}}$, $\underline{\boldsymbol{H+1}}$, ϵ	H, H, $\underline{\boldsymbol{\epsilon+1}}$	$\underset{(h)/(i)\&(m)}{\underline{\boldsymbol{H}}}$, $\underset{(h)/(i)\&(m)}{\underline{\boldsymbol{H}}}$, $\underline{\boldsymbol{\epsilon+1}}$
	Abstain	$\underline{\boldsymbol{H+1}}$, $\underline{\boldsymbol{H+1}}$, ϵ	$H+\frac{1}{2}$, $\underline{\boldsymbol{H+\frac{1}{2}}}$, $\underset{(j)/(h)/(d)\&(k)}{\underline{\boldsymbol{\epsilon+\frac{1}{2}}}}$	$\underline{\boldsymbol{\frac{H}{2}+\overset{(h)}{\frac{1}{2}}\text{ or }\overset{(i)}{\frac{3}{4}}}}$, $\underline{\boldsymbol{\frac{H}{2}+}}\overset{(h)}{\frac{1}{2}}\text{ or }\underset{(n)}{\overset{(i)}{\boldsymbol{\frac{3}{4}}}}$, $\underline{\boldsymbol{\frac{\epsilon}{2}+}}\overset{(j)}{\frac{1}{4}}\text{ or }\overset{(h)}{\frac{1}{2}}\text{ or }\underset{(l)}{\overset{(d)}{\underline{\boldsymbol{\frac{3}{4}}}}}$
	ISV	$\underline{\boldsymbol{H+1}}$, $\underline{\boldsymbol{H+1}}$, ϵ	$\underline{\boldsymbol{\frac{H}{2}+\overset{(a)}{\frac{1}{2}}\text{ or }\overset{(b)}{\frac{3}{4}}}}$, $\frac{H}{2}+\overset{(c)}{\frac{1}{4}}\text{ or }\overset{(a)}{\frac{1}{2}}\text{ or }\overset{(d)}{\frac{3}{4}}$, $\underline{\boldsymbol{\frac{\epsilon}{2}+\overset{(a)}{\frac{1}{2}}}}\text{ or }\underset{(k)}{\overset{(b)}{\underline{\boldsymbol{\frac{3}{4}}}}}$	$\underline{\mathbf{1}}$, $\overset{(h)}{0}\text{ or }\overset{(i)}{1}$, $\overset{(a)}{0}\text{ or }\underset{(l)}{\overset{(b)}{\underline{\mathbf{1}}}}$
$i=1$ Abstain	SV	$\underline{\boldsymbol{H+1}}$, $\underline{\boldsymbol{H+1}}$, ϵ	$\underline{\boldsymbol{H+\frac{1}{2}}}$, $H+\frac{1}{2}$, $\underset{(j)/(h)/(d)\&(k)}{\underline{\boldsymbol{\epsilon+\frac{1}{2}}}}$	$\underline{\boldsymbol{\frac{H}{2}+}}\overset{(h)}{\frac{1}{2}}\text{ or }\underset{(n)}{\overset{(i)}{\underline{\boldsymbol{\frac{3}{4}}}}}$, $\underline{\boldsymbol{\frac{H}{2}+}\overset{(h)}{\frac{1}{2}}\text{ or }\overset{(i)}{\boldsymbol{\frac{3}{4}}}}$, $\underline{\boldsymbol{\frac{\epsilon}{2}+}}\overset{(j)}{\frac{1}{4}}\text{ or }\overset{(h)}{\frac{1}{2}}\text{ or }\underset{(l)}{\overset{(d)}{\underline{\boldsymbol{\frac{3}{4}}}}}$
	Abstain	$\underline{\boldsymbol{H+1}}$, $\underline{\boldsymbol{H+1}}$, $\underset{(j)/(h)\&(k)}{\underline{\boldsymbol{\epsilon}}}$	$\underline{\boldsymbol{H+1}}$, $\underline{\boldsymbol{H+1}}$, $\underset{(j)/(h)\&(k)}{\underline{\boldsymbol{\epsilon}}}$	$\overset{(h)}{\frac{1}{2}}\text{ or }\overset{(i)}{1}$, $\overset{(h)}{\frac{1}{2}}\text{ or }\overset{(i)}{1}$, $\overset{(j)}{0}\text{ or }\underset{(l)}{\overset{(h)}{\underline{\boldsymbol{\frac{1}{2}}}}}\text{ or }\overset{(d)}{\underline{\mathbf{1}}}$
	ISV	$\frac{H}{2}+\overset{(a)}{\frac{3}{4}}\text{ or }\overset{(b)}{1}$, $\frac{H}{2}+\overset{(c)}{\frac{1}{2}}\text{ or }\overset{(a)}{\frac{3}{4}}\text{ or }\overset{(d)}{1}$, $\underline{\boldsymbol{\frac{\epsilon}{2}+}}\underset{(k)}{\overset{(a)}{\underline{\boldsymbol{\frac{1}{4}}}}}\text{ or }\overset{(b)}{\boldsymbol{\frac{1}{2}}}$	$\overset{(a)}{\frac{1}{2}}\text{ or }\overset{(b)}{1}$, $\overset{(c)}{0}\text{ or }\overset{(a)}{\frac{1}{2}}\text{ or }\overset{(d)}{1}$, $\underset{(l)}{\overset{(a)}{\underline{\boldsymbol{\frac{1}{2}}}}}\text{ or }\overset{(b)}{\underline{\mathbf{1}}}$	$\underline{\mathbf{1}}$, $\overset{(h)}{0}\text{ or }\overset{(i)}{1}$, $\overset{(a)}{0}\text{ or }\overset{(b)}{\underline{\mathbf{1}}}$
$i=1$ ISV	SV	$\underline{\boldsymbol{H+1}}$, $\underline{\boldsymbol{H+1}}$, ϵ	$\frac{H}{2}+\overset{(e)}{\frac{1}{4}}\text{ or }\overset{(f)}{\frac{1}{2}}\text{ or }\overset{(d)}{\frac{4}{3}}$, $\underline{\boldsymbol{\frac{H}{2}+\overset{(f)}{\frac{1}{2}}\text{ or }\overset{(g)}{\frac{3}{4}}}}$, $\underline{\boldsymbol{\frac{\epsilon}{2}+\overset{(f)}{\frac{1}{2}}}}\text{ or }\underset{(k)}{\overset{(g)}{\underline{\boldsymbol{\frac{3}{4}}}}}$	$\overset{(h)}{0}\text{ or }\overset{(i)}{1}$, $\underline{\mathbf{1}}$, $\overset{(f)}{0}\text{ or }\underset{(l)}{\overset{(g)}{\underline{\mathbf{1}}}}$
	Abstain	$\frac{H}{2}+\overset{(e)}{\frac{1}{2}}\text{ or }\overset{(f)}{\frac{3}{4}}\text{ or }\overset{(d)}{1}$, $\frac{H}{2}+\overset{(f)}{\frac{3}{4}}\text{ or }\overset{(g)}{1}$, $\underline{\boldsymbol{\frac{\epsilon}{2}+}}\underset{(k)}{\overset{(f)}{\underline{\boldsymbol{\frac{1}{4}}}}}\text{ or }\overset{(g)}{\boldsymbol{\frac{1}{2}}}$	$\overset{(e)}{0}\text{ or }\overset{(f)}{\frac{1}{2}}\text{ or }\overset{(d)}{1}$, $\overset{(f)}{\frac{1}{2}}\text{ or }\overset{(g)}{1}$, $\underset{(l)}{\overset{(f)}{\underline{\boldsymbol{\frac{1}{2}}}}}\text{ or }\overset{(g)}{\mathbf{1}}$	$\overset{(h)}{0}\text{ or }\overset{(i)}{1}$, $\underline{\mathbf{1}}$, $\overset{(f)}{0}\text{ or }\overset{(g)}{\underline{\mathbf{1}}}$
	ISV	$\overset{(a)}{0}\text{ or }\overset{(b)}{1}$, $\overset{(f)}{0}\text{ or }\overset{(g)}{1}$, $\underline{\mathbf{1}}$	$\overset{(a)}{0}\text{ or }\overset{(b)}{1}$, $\overset{(f)}{0}\text{ or }\overset{(g)}{1}$, $\underline{\mathbf{1}}$	$\overset{(e)}{0}\text{ or }\overset{(b)/(i)}{\underline{\mathbf{1}}}$, $\overset{(c)}{0}\text{ or }\overset{(g)/(i)}{\underline{\mathbf{1}}}$, $\overset{(j)}{0}\text{ or }\overset{(b)/(g)}{\underline{\mathbf{1}}}$

Table 6.10: The game in 6.5.2 and Nash equilibria before eliminating weakly-dominated strategies in the first stage. Conditions $(a)-(n)$ are given in Table 6.3.

The Nash equilibria of the game before eliminating weakly-dominated strategies in the first stage are summarized in Table 6.11.

	Conditions for Existence	**Absentees**	**First Stage**	**Second Stage**	**Outcome**
(1)	If (j) OR $[(h)$ & $(k)]$	$i = 1, 2$	$i = 3$ (SV)	$i = 1, 2$ (SV)	x^j_Ω
(2)	If (j) OR $[(h)$ & $(k)]$	all	all (SV)	all (SV)	x^j_Ω
(3)	If (d) & (n) & (l)	$i = 2$	$i = 1$ (SV) $i = 3$ (ISV)	$i = 1$ or $3, 2$ (SV)	x^j_Ω (prob. $\frac{1}{4}$) x^k_Ω (prob. $\frac{1}{4}$) $x^{j'}_{\Omega'}$ (prob. $\frac{1}{2}$)
(4)	If (d) & (n) & (l)	$i = 1$	$i = 2$ (SV) $i = 3$ (ISV)	$i = 2$ or $3, 1$ (SV)	x^j_Ω (prob. $\frac{1}{4}$) x^k_Ω (prob. $\frac{1}{4}$) $x^{j'}_{\Omega'}$ (prob. $\frac{1}{2}$)
(5)	If (h) OR $[(i)$ & $(m)]$	none	all $i = 1, 2$ (SV) $i = 3$ (ISV)	$i = 3$ (SV)	x^k_Ω
(6)	If (d)	none	all (ISV)	all (SV)	$x^{j'}_{\Omega'}$

Table 6.11: Nash equilibria of the game in 6.5.2 before eliminating weakly-dominated strategies in the first stage.

Next we focus on eliminating weakly-dominated strategies in the first stage. We observe that abstention weakly-dominates insincere voting for all *SI* agents $i = 1, 2$. For the *WI* agent, abstention weakly-dominates insincere voting if Condition (k) is satisfied and Condition (d) is *not* satisfied. Moreover, abstention weakly-dominates sincere voting for the *WI* agent if Condition (l) is satisfied.

It is obvious that Equilibrium (6) is implausible after eliminating weakly-dominated strategies. However, refining the remaining equilibria is not as straightforward, due to the conditions underpinning the (weak) dominance of strategies for $i = 3$. We thus, first establish the precise conditions under which abstention does not weakly-dominate the other strategies for $i = 3$. We note that

- abstention does not weakly-dominate insincere voting for $i = 3$ if and only if $[(d)]$ or $\Big[(l)$ and $\Big((b)$ or (g) or $(h)\Big)\Big]$;
- abstention does not weakly-dominate sincere voting for $i = 3$ if and only if (k) and $[(a)$ or $(f)]$.

Applying these conditions together with the conditions required for the existence of equilibria, we obtain our refined set of plausible equilibria, which we summarize in Table 6.12. We note that these equilibria are path-independent.

	Conditions for Existence	**Absentees**	**First Stage**	**Second Stage**	**Outcome**
(1)	If (k) & $[(j)$ or (e) or $(c)]$	$i = 1, 2$	$i = 3$ (SV)	$i = 1, 2$ (SV)	x^j_Ω
(2)	If (j) OR $[(h)$ & $(k)]$	all	all (SV)	all (SV)	x^j_Ω
(3)	If (d) & (n) & (l)	$i = 2$	$i = 1$ (SV) $i = 3$ (ISV)	$i = 1$ or $3, 2$ (SV)	x^j_Ω (prob. $\frac{1}{4}$) x^k_Ω (prob. $\frac{1}{4}$) $x^{j'}_{\Omega'}$ (prob. $\frac{1}{2}$)
(4)	If (d) & (n) & (l)	$i = 1$	$i = 2$ (SV) $i = 3$ (ISV)	$i = 2$ or $3, 1$ (SV)	x^j_Ω (prob. $\frac{1}{4}$) x^k_Ω (prob. $\frac{1}{4}$) $x^{j'}_{\Omega'}$ (prob. $\frac{1}{2}$)
(5)	If $[(d)$ & $(m)]$ OR $[(h)$ & $(l)]$	none	all $i = 1, 2$ (SV) $i = 3$ (ISV)	$i = 3$ (SV)	x^k_Ω

Table 6.12: Refined Nash equilibria of the game in 6.5.2.

Let us now explain the intuition behind each equilibrium.

Equilibrium (1): When $\epsilon > 0.5$ (Condition (k)) and the *WI* agent, $i = 3$, is confident that his preferred variation of Ω' would not be selected with certainty in the second stage, he will vote sincerely, as his expected payoff from the variations of Ω is higher than for Ω'. Given this behavior, the *SI* agents $i = 1, 2$ would abstain to ensure that their preferred variation is selected in the second stage. For this strategy of *SI* agents, voting sincerely is still a best response for $i = 3$.

Equilibrium (2): Regardless of the value of ϵ, if $i = 3$ anticipates that his less preferred variation of Ω' will be selected in the second stage if he votes for Ω' in the first stage (Condition (j)), then, he has no incentive to vote for Ω'. Anticipating this, $i = 1, 2$ abstain to ensure that x^j_Ω is selected in the second stage. Given this behavior, abstention is a best response for $i = 3$. In equilibrium, all agents would choose to abstain in the first stage, and as no voters have chosen to participate in the first stage, all agents will vote sincerely in both stages.

Equilibria (3) and (4): If $\epsilon < 0.5$, (Condition (l)), the *WI* agent is inclined to vote for Ω' if he sees that all agents agree on the variations of Ω' (Condition (d)). Additionally, if the value of H is not very large, i.e. if $H < 1.5$ (Condition (n)), the *SI* agents are less inclined to waive voting rights in the second stage to ensure that Ω is selected as the fundamental direction. Thus, given that $i = 3$ will vote insincerely in the first stage, one *SI* agent would abstain to ensure that x^j_Ω has a positive probability of being selected if Ω is selected in the first stage. The strategy of $i = 3$ remains a best response for this behavior of *SI* agents.

Equilibrium (5): If the value of H is large, i.e. if $H > 1.5$ (Condition (m)), both *SI* agents participate in voting in the first stage to ensure that Ω will be selected. The *WI* agent then votes insincerely to ensure that his preferred variation x^k_Ω will be selected in the second stage. The same equilibrium is possible when $\epsilon < 0.5$, (Condition (l)). $i = 3$ is then inclined to vote insincerely, as his expected payoff is higher when voting for Ω' as opposed to Ω. Given this strategy of $i = 3$, the *SI* agents $i = 1, 2$ vote sincerely in the first stage to ensure that the selection of Ω as the fundamental direction. For this behavior of *SI* agents, insincere voting remains a best response for $i = 3$.

We observe that in this case, the similarities between the equilibrium voting behavior with complete and incomplete information are limited to sincere voting in the second stage. In Equilibria (1) - (4), *SI* agents choose to abstain from the first stage, and in Equilibrium (5), the *WI* agent chooses to participate in the first stage. Moreover, in Equilibria (3) - (5), the *WI* agent engages in insincere voting in the first stage but *SI* agents do not vote insincerely in equilibrium.

6.5.3 All agents are inclined towards Ω. One SI agents prefers x^j_Ω, while the other prefers x^k_Ω.

Without loss of generality, we assume that $i = 1, 3$ prefer x^j_Ω and $i = 2$ prefers x^k_Ω. As in the previous case, after eliminating weakly-dominated strategies in the second stage, all agents that choose or are allowed to vote in the second stage will vote sincerely.

As in the previous case, we note that the weakly-inclined agent may not always be better off if Ω is selected in the first stage. Depending on which agents will vote in the second stage, and how agents agree on the variations of Ω', the *WI* agent may better off if Ω' is selected in the first stage. The *SI* agents are strictly better off if Ω is chosen in the first stage. However, as they disagree about the variations of Ω, they have an incentive to try and participate in voting in the second stage, given sufficient votes are cast for Ω in the first stage.

We represent the game in normal form in Table 6.13, when votes in the second stage are cast sincerely. We solve for all possible Nash equilibria in pure strategies.

$i=1$	$i=2$ \ $i=3$	**SV**	**Abstain**	**ISV**
$i=1$ SV	SV	$\underline{\boldsymbol{H+1}}$, H, $\underline{\boldsymbol{\epsilon+1}}$	$\underline{\boldsymbol{H+1}}$, H, $\underline{\boldsymbol{\epsilon+1}}$	$\underline{\boldsymbol{H+1}}$, $\underset{\boldsymbol{(h)/(i)\&(m)}}{\underline{\boldsymbol{H}}}$, $\underline{\boldsymbol{\epsilon+1}}$
	Abstain	H, $\underline{\boldsymbol{H+1}}$, ϵ	$H+\frac{1}{2}$, $\underline{\boldsymbol{H+\frac{1}{2}}}$, $\underset{\boldsymbol{(j)/(h)/(d)\&(k)}}{\underline{\boldsymbol{\epsilon+\frac{1}{2}}}}$	$\underline{\boldsymbol{\frac{H}{2}+\overset{(h)}{\frac{1}{2}}\text{ or }\overset{(i)}{\frac{3}{4}}}}$, $\underline{\boldsymbol{\frac{H}{2}+}}\overset{(h)}{\frac{1}{2}}\text{ or }\underset{\boldsymbol{(n)}}{\underline{\overset{\boldsymbol{(i)}}{\boldsymbol{\frac{3}{4}}}}}$, $\underline{\boldsymbol{\frac{\epsilon}{2}+}}\overset{(j)}{\frac{1}{4}}\text{ or }\overset{(h)}{\frac{1}{2}}\text{ or }\underset{\boldsymbol{(l)}}{\underline{\overset{\boldsymbol{(d)}}{\boldsymbol{\frac{3}{4}}}}}$
	ISV	$\underset{\boldsymbol{(a)/(b)\&(m)}}{\underline{\boldsymbol{H}}}$, $\underline{\boldsymbol{H+1}}$, ϵ	$\underline{\boldsymbol{\frac{H}{2}+\overset{(a)}{\frac{1}{2}}\text{ or }\overset{(b)}{\frac{3}{4}}}}$, $\frac{H}{2}+\overset{(c)}{\frac{1}{4}}\text{ or }\overset{(a)}{\frac{1}{2}}\text{ or }\overset{(d)}{\frac{3}{4}}$, $\underline{\boldsymbol{\frac{\epsilon}{2}+\overset{(a)}{\frac{1}{2}}}}\text{ or }\underset{\boldsymbol{(k)}}{\underline{\overset{\boldsymbol{(b)}}{\boldsymbol{\frac{3}{4}}}}}$	$\underline{\boldsymbol{1}}$, $\overset{(h)}{0}\text{ or }\overset{(i)}{1}$, $\overset{(a)}{0}\text{ or }\underset{\boldsymbol{(l)}}{\underline{\overset{\boldsymbol{(b)}}{\boldsymbol{1}}}}$
$i=1$ Abstain	SV	$\underline{\boldsymbol{H+1}}$, H, $\underline{\boldsymbol{\epsilon+1}}$	$\underline{\boldsymbol{H+1}}$, $\underline{\boldsymbol{H}}$, $\underline{\boldsymbol{\epsilon+1}}$	$\frac{H}{2}+\overset{(h)}{\frac{3}{4}}\text{ or }\overset{(i)}{1}$, $\underline{\boldsymbol{\frac{H}{2}+\overset{(h)}{\frac{1}{4}}\text{ or }\overset{(i)}{\frac{1}{2}}}}$, $\frac{\epsilon}{2}+\overset{(j)}{\frac{1}{2}}\text{ or }\overset{(h)}{\frac{3}{4}}\text{ or }\overset{(d)}{1}$
	Abstain	$\underline{\boldsymbol{H+\frac{1}{2}}}$, $\underline{\boldsymbol{H+\frac{1}{2}}}$, $\epsilon+\frac{1}{2}$	$\underline{\boldsymbol{H+1}}$, $\underline{\boldsymbol{H}}$, $\underline{\boldsymbol{\epsilon+1}}$	$\overset{(h)}{\frac{1}{2}}\text{ or }\overset{(i)}{1}$, $\overset{(h)}{\frac{1}{2}}\text{ or }\overset{(i)}{1}$, $\overset{(j)}{0}\text{ or }\overset{(h)}{\frac{1}{2}}\text{ or }\overset{(d)}{1}$
	ISV	$\underline{\boldsymbol{\frac{H}{2}+}}\overset{\boldsymbol{(a)}}{\boldsymbol{\frac{1}{2}}}\text{ or }\underset{\boldsymbol{(n)}}{\underline{\overset{\boldsymbol{(b)}}{\boldsymbol{\frac{3}{4}}}}}$, $\frac{H}{2}+\overset{(c)}{\frac{1}{4}}\text{ or }\overset{(a)}{\frac{1}{2}}\text{ or }\overset{(d)}{\frac{3}{4}}$, $\underline{\boldsymbol{\frac{\epsilon}{2}+\overset{(a)}{\frac{1}{2}}}}\text{ or }\underset{\boldsymbol{(k)}}{\underline{\overset{\boldsymbol{(b)}}{\boldsymbol{\frac{3}{4}}}}}$	$\overset{(a)}{\frac{1}{2}}\text{ or }\overset{(b)}{1}$, $\overset{(c)}{0}\text{ or }\overset{(a)}{\frac{1}{2}}\text{ or }\overset{(d)}{1}$, $\overset{(a)}{\frac{1}{2}}\text{ or }\underset{\boldsymbol{(l)}}{\underline{\overset{\boldsymbol{(b)}}{\boldsymbol{1}}}}$	$\underline{\boldsymbol{1}}$, $\overset{(h)}{0}\text{ or }\overset{(i)}{1}$, $\overset{(a)}{0}\text{ or }\underset{\boldsymbol{(l)}}{\underline{\overset{\boldsymbol{(b)}}{\boldsymbol{1}}}}$
$i=1$ ISV	SV	$\underline{\boldsymbol{H+1}}$, $\underset{\boldsymbol{(f)/(g)\&(m)}}{\underline{\boldsymbol{H}}}$, $\underline{\boldsymbol{\epsilon+1}}$	$\frac{H}{2}+\overset{(e)}{\frac{1}{2}}\text{ or }\overset{(f)}{\frac{3}{4}}\text{ or }\overset{(d)}{1}$, $\underline{\boldsymbol{\frac{H}{2}+\overset{(f)}{\frac{1}{4}}\text{ or }\overset{(g)}{\frac{1}{2}}}}$, $\frac{\epsilon}{2}+\overset{(f)}{\frac{3}{4}}\text{ or }\overset{(g)}{1}$	$\overset{(h)}{0}\text{ or }\overset{(i)}{1}$, $\underline{\boldsymbol{1}}$, $\overset{(f)}{0}\text{ or }\overset{(g)}{1}$
	Abstain	$\frac{H}{2}+\overset{(e)}{\frac{1}{4}}\text{ or }\overset{(f)}{\frac{1}{2}}\text{ or }\overset{(d)}{\frac{3}{4}}$, $\underline{\boldsymbol{\frac{H}{2}+}}\overset{\boldsymbol{(f)}}{\boldsymbol{\frac{1}{2}}}\text{ or }\underset{\boldsymbol{(n)}}{\underline{\overset{\boldsymbol{(g)}}{\boldsymbol{\frac{3}{4}}}}}$, $\underline{\boldsymbol{\frac{\epsilon}{2}+\overset{(f)}{\frac{1}{2}}}}\text{ or }\underset{\boldsymbol{(k)}}{\underline{\overset{\boldsymbol{(g)}}{\boldsymbol{\frac{3}{4}}}}}$	$\overset{(e)}{0}\text{ or }\overset{(f)}{\frac{1}{2}}\text{ or }\overset{(d)}{1}$, $\overset{(f)}{\frac{1}{2}}\text{ or }\overset{(g)}{1}$, $\overset{(f)}{\frac{1}{2}}\text{ or }\underset{\boldsymbol{(l)}}{\underline{\overset{\boldsymbol{(g)}}{\boldsymbol{1}}}}$	$\overset{(h)}{0}\text{ or }\overset{(i)}{1}$, $\underline{\boldsymbol{1}}$, $\overset{(f)}{0}\text{ or }\underset{\boldsymbol{(l)}}{\underline{\overset{\boldsymbol{(g)}}{\boldsymbol{1}}}}$
	ISV	$\overset{(a)}{0}\text{ or }\overset{(b)}{1}$, $\overset{(f)}{0}\text{ or }\overset{(g)}{1}$, $\underline{\boldsymbol{1}}$	$\overset{(h)}{0}\text{ or }\overset{(i)}{1}$, 1, $\overset{(f)}{0}\text{ or }\underline{\overset{\boldsymbol{(g)}}{\boldsymbol{1}}}$	$\overset{(e)}{0}\text{ or }\underline{\overset{\boldsymbol{(b)/(i)}}{\boldsymbol{1}}}$, $\overset{(c)}{0}\text{ or }\underline{\overset{\boldsymbol{(g)/(i)}}{\boldsymbol{1}}}$, $\overset{(j)}{0}\text{ or }\underline{\overset{\boldsymbol{(b)/(g)}}{\boldsymbol{1}}}$

Table 6.13: The game in 6.5.3 and Nash equilibria before eliminating weakly-dominated strategies in the first stage. Conditions (a) – (n) are given in Table 6.3.

In Table 6.14, we denote the Nash equilibria of the game before eliminating weakly-dominated strategies in the first stage.

	Conditions for Existence	**Absentees**	**First Stage**	**Second Stage**	**Outcome**
(1)		$i = 1, 3$	$i = 2$ (SV)	$i = 1, 3$ (SV)	x^j_Ω
(2)		all	all (SV)	all (SV)	x^j_Ω
(3)	If (d) & (n) & (l)	$i = 2$	$i = 1$ (SV) $i = 3$ (ISV)	$i = 1$ or $3, 2$ (SV)	x^j_Ω (prob. $\frac{1}{4}$) x^k_Ω (prob. $\frac{1}{4}$) $x^{j'}_{\Omega'}$ (prob. $\frac{1}{2}$)
(4)	If (f) OR $[(g)$ & $(m)]$	none	all $i = 2, 3$ (SV) $i = 1$ (ISV)	$i = 1$ (SV)	x^j_Ω
(5)	If (h) OR $[(i)$ & $(m)]$	none	all $i = 1, 2$ (SV) $i = 3$ (ISV)	$i = 3$ (SV)	x^j_Ω
(6)	If (d)	none	all (ISV)	all (SV)	$x^{j'}_{\Omega'}$

Table 6.14: Nash equilibria of the game in 6.5.3 before eliminating weakly-dominated strategies in the first stage.

Next, we focus on eliminating weakly-dominated strategies in the first stage. We observe that abstention weakly-dominates insincere voting for all *SI* agents $i = 1, 2$. For the *WI* agent, abstention weakly-dominates insincere voting if Condition (k) is satisfied.

As in the previous case, Equilibrium (6) is implausible after eliminating weakly-dominated strategies. But refining the remaining equilibria is not very straightforward, due to the conditions underpinning the (weak) dominance of strategies for $i = 3$. We thus first establish the conditions under which abstention does not weakly-dominate insincere voting for $i = 3$. We note that

- abstention does not weakly-dominate insincere voting for $i = 3$ if and only if $[(l)]$ and $\Big[(b) \text{ or } (d)\Big]$ are satisfied.

Applying these conditions together with the conditions required for existence, we refine the conditions of existence for Equilibrium (5). In Table 6.15 we present our refined set of plausible equilibria. We note that these equilibria are path-independent.

	Conditions for Existence	**Absentees**	**First Stage**	**Second Stage**	**Outcome**
(1)		$i = 1, 3$	$i = 2$ (SV)	$i = 1, 3$ (SV)	x^j_Ω
(2)		all	all (SV)	all (SV)	x^j_Ω
(3)	If (d) & (n) & (l)	$i = 2$	$i = 1$ (SV) $i = 3$ (ISV)	$i = 1$ or $3, 2$ (SV)	x^j_Ω (prob. $\frac{1}{4}$) x^k_Ω (prob. $\frac{1}{4}$) $x^{j'}_{\Omega'}$ (prob. $\frac{1}{2}$)
(4)	If (f) OR $[(g)$ & $(m)]$	none	all $i = 2, 3$ (SV) $i = 1$ (ISV)	$i = 1$ (SV)	x^j_Ω
(5)	If $[(l)$ & $(c)]$ OR $[(l)$ & (d) & $(m)]$	none	all $i = 1, 2$ (SV) $i = 3$ (ISV)	$i = 3$ (SV)	x^j_Ω

Table 6.15: Refined Nash equilibria of the game in 6.5.3.

In the following, we explain the intuition behind each equilibrium.

Equilibria (1) and (2): In equilibrium, since $i = 2$ is *SI*, he will vote for Ω if he participates in the first stage. Anticipating this, $i = 1, 3$ will abstain from voting in the first stage to ensure that their preferred variation is selected in the second stage. Given this behavior, $i = 2$ is indifferent between participation and abstention, yielding two equilibria.

Equilibrium (3): Given that all agents agree on the variations of Ω' (Condition (d)) and $\epsilon < 0.5$ (Condition (l)), $i = 3$ is inclined to vote for Ω' in the first stage. Knowing this, $i = 1$ votes for Ω in the first stage. If the value is not very large, i.e. $H < 1.5$ (Condition (n)), $i = 2$ abstains from voting in the first stage to increase the selection chances of his preferred variation x^k_Ω. Given the strategies of the other two agents, no individual has an incentive to deviate from his strategy.

Equilibrium (4): Given the behavior in Equilibrium (4), we observe that no agent has an incentive to deviate. Agents $i = 1, 3$ obtain their highest-possible utility from x^j_Ω, and therefore, have no incentive to deviate from their strategy. If agents $i = 2$ and $i = 3$ do not agree on the variations of Ω' (Condition (f)), or if $H \geq 1.5$ (Condition (m)), agent $i = 2$ has no incentive to deviate from his strategy either.

Equilibrium (5): If $\epsilon < 0.5$ (Condition (l)), and particularly if all agents agree on the variations of Ω' (Condition (d)), agent $i = 3$ is inclined to vote for Ω' in the first stage. Given this voting behavior, the *SI* agents are inclined to vote for Ω, especially if $H \geq 1.5$ (Condition (m)). Given the strategies of other individuals, no agent has an incentive to deviate from his strategy.

As in the previous case, the similarities between the equilibrium voting behavior with complete and incomplete information end with sincere voting in the second stage. In Equilibria (1) - (3) *SI* agents choose to abstain from the first stage, and in Equilibria (3) - (5), the *WI* agent chooses to participate in the first stage. Moreover, in Equilibria (3) and (5), the *WI* agent engages in insincere voting in the first stage, while in Equilibrium (4), an *SI* agent votes insincerely in the first stage.

The cases remaining to be analyzed are

- *SI* agents $i = 1, 2$ prefer Ω and the *WI* agent $i = 3$ prefers Ω'. All agents prefer x^j_Ω.
- *SI* agents $i = 1, 2$ prefer Ω and x^j_Ω. The *WI* agent $i = 3$ prefers Ω' and x^k_Ω.
- *SI* agents $i = 1, 2$ prefer Ω and the *WI* agent $i = 3$ prefers Ω'. Moreover, $i = 1, 3$ prefer x^j_Ω and $i = 2$ prefers x^k_Ω.
- $i = 1$ prefers Ω and $i = 2, 3$ prefer Ω'. All agents prefer x^j_Ω.
- $i = 1$ prefers Ω and x^j_Ω while $i = 2, 3$ prefer Ω' and x^k_Ω.

- $i = 1$ prefers Ω and $i = 2, 3$ prefer Ω'. Moreover, $i = 1, 2$ prefer x^j_Ω and $i = 3$ prefers x^k_Ω.
- $i = 1$ prefers Ω and $i = 2, 3$ prefer Ω'. Moreover, $i = 1, 3$ prefer x^j_Ω and $i = 2$ prefers x^k_Ω.

6.6 Equilibria when $N_s = 1$ and $N_w = 2$

Now suppose that the committee consists of one *SI* and two *WI* individuals. Without loss of generality, suppose that $i = 1$ is *SI* and $i = 2, 3$ are *WI*. In Sections 6.6.1, 6.6.2 and 6.6.3, we examine three cases which represent different realizations of individual preferences.

6.6.1 All agents prefer Ω and x^j_Ω

We note that after eliminating weakly-dominated strategies, all agents that choose or are allowed to vote in the second stage will vote sincerely. Similarly, any agent opting to vote in the first stage would vote sincerely. Both *SI* and *WI* individuals are inclined towards Ω, and they obtain their highest payoff from x^j_Ω. Hence, no one has an incentive to follow a strategy that would yield any other outcome. Given that the final outcome is the one preferred by all agents, any combination of agents may abstain from the first stage. As in the Case 6.4.1, there are eight possible Nash equilibria detailed in Table 6.1.

6.6.2 All agents are inclined towards Ω. The SI agent prefers x^j_Ω, while the WI agents prefer x^k_Ω.

Similar to the previous cases, elimination of weakly-dominated strategies in the second stage yields sincere voting by all agents that choose or are allowed to vote in the second stage.

Given that votes in the second stage are cast sincerely, we represent the game in normal form in Table 6.16. We solve for all possible Nash equilibria in pure strategies. The *SI* individual is always better off if Ω is selected in the first stage. We note, however, that the majority of the committee comprises of *WI* agents with identical preferences on the variations of their preferred direction Ω. Hence, it would only be in their interest to support the selection of Ω in the first stage if they can ensure that their preferred variation x_{Ω}^{k} is selected in the second stage.

$i=1$	$i=2$ \ $i=3$	SV	Abstain	ISV
$i=1$ SV	SV	H, $\underline{\epsilon+1}$, $\underline{\epsilon+1}$	H, $\underline{\epsilon+1}$, $\underline{\epsilon+1}$	$\underset{(h)/(i)\&(m)}{\underline{H}}$, $\underline{\epsilon+1}$, $\underline{\epsilon+1}$
	Abstain	H, $\underline{\epsilon+1}$, $\underline{\epsilon+1}$	$\underline{H}$, $\underline{\epsilon+1}$, $\underline{\epsilon+1}$	$\underline{\frac{H}{2}+\overset{(h)}{\frac{1}{4}}\text{ or }\overset{(i)}{\frac{1}{2}}}$, $\frac{\epsilon}{2}+\overset{(h)}{\frac{3}{4}}\text{ or }\overset{(i)}{1}$, $\frac{\epsilon}{2}+\overset{(j)}{\frac{1}{2}}\text{ or }\overset{(h)}{\frac{3}{4}}\text{ or }\overset{(d)}{1}$
	ISV	$\underset{(a)/(b)\&(m)}{\underline{H}}$, $\underline{\epsilon+1}$, $\underline{\epsilon+1}$	$\underline{\frac{H}{2}+\overset{(a)}{\frac{1}{4}}\text{ or }\overset{(b)}{\frac{1}{2}}}$, $\frac{\epsilon}{2}+\overset{(c)}{\frac{1}{2}}\text{ or }\overset{(a)}{\frac{3}{4}}\text{ or }\overset{(d)}{1}$, $\frac{\epsilon}{2}+\overset{(a)}{\frac{3}{4}}\text{ or }\overset{(b)}{1}$	$\underline{1}$, $\overset{(h)}{0}\text{ or }\overset{(i)}{1}$, $\overset{(a)}{0}\text{ or }\overset{(b)}{1}$
$i=1$ Abstain	SV	$\underline{H+1}$, ϵ, ϵ	$\underline{H+\frac{1}{2}}$, $\epsilon+\frac{1}{2}$, $\underset{(j)/(h)/(d)\&(k)}{\underline{\epsilon+\frac{1}{2}}}$	$\underline{\frac{H}{2}}+\overset{(h)}{\frac{1}{2}}\text{ or }\underset{(n)}{\overset{(i)}{\frac{3}{4}}}$, $\underline{\frac{\epsilon}{2}+\overset{(h)}{\frac{1}{2}}}\text{ or }\overset{(i)}{\frac{3}{4}}$, $\underline{\frac{\epsilon}{2}}+\overset{(j)}{\frac{1}{4}}\text{ or }\overset{(h)}{\frac{1}{2}}\text{ or }\underset{(n)}{\overset{(d)}{\frac{3}{4}}}$
	Abstain	$\underline{H+\frac{1}{2}}$, $\underset{(c)/(a)/(d)\&(k)}{\underline{\epsilon+\frac{1}{2}}}$, $\epsilon+\frac{1}{2}$	$\underline{H}$, $\underline{\epsilon+1}$, $\underline{\epsilon+1}$	$\overset{(h)}{\frac{1}{2}}\text{ or }\overset{(i)}{1}$, $\overset{(h)}{\frac{1}{2}}\text{ or }\overset{(i)}{\underline{1}}$, $\overset{(j)}{0}\text{ or }\overset{(h)}{\frac{1}{2}}\text{ or }\overset{(d)}{1}$
	ISV	$\underline{\frac{H}{2}}+\overset{(a)}{\frac{1}{2}}\text{ or }\underset{(n)}{\overset{(b)}{\frac{3}{4}}}$, $\underline{\frac{\epsilon}{2}}+\overset{(c)}{\frac{1}{4}}\text{ or }\overset{(a)}{\frac{1}{2}}\text{ or }\underset{(l)}{\overset{(d)}{\frac{3}{4}}}$, $\underline{\frac{\epsilon}{2}}+\overset{(a)}{\frac{1}{2}}\text{ or }\underset{(k)}{\overset{(b)}{\frac{3}{4}}}$	$\overset{(a)}{\frac{1}{2}}\text{ or }\overset{(b)}{1}$, $\overset{(c)}{0}\text{ or }\overset{(a)}{\frac{1}{2}}\text{ or }\overset{(d)}{1}$, $\overset{(a)}{\frac{1}{2}}\text{ or }\underset{(l)}{\overset{(b)}{\underline{1}}}$	$\underline{1}$, $\overset{(h)}{0}\text{ or }\overset{(i)}{\underline{1}}$, $\overset{(a)}{0}\text{ or }\underset{(l)}{\overset{(b)}{\underline{1}}}$
$i=1$ ISV	SV	$\underline{H+1}$, ϵ, ϵ	$\frac{H}{2}+\overset{(e)}{\frac{1}{4}}\text{ or }\overset{(f)}{\frac{1}{2}}\text{ or }\overset{(d)}{\frac{4}{3}}$, $\underline{\frac{\epsilon}{2}+\overset{(f)}{\frac{1}{2}}}\text{ or }\underset{(k)}{\overset{(g)}{\frac{3}{4}}}$, $\underline{\frac{\epsilon}{2}+\overset{(f)}{\frac{1}{2}}}\text{ or }\underset{(k)}{\overset{(g)}{\frac{3}{4}}}$	$\overset{(h)}{0}\text{ or }\overset{(i)}{1}$, $\underline{1}$, $\overset{(f)}{0}\text{ or }\underset{(l)}{\overset{(g)}{\underline{1}}}$
	Abstain	$\frac{H}{2}+\overset{(e)}{\frac{1}{4}}\text{ or }\overset{(f)}{\frac{1}{2}}\text{ or }\overset{(d)}{\frac{3}{4}}$, $\underline{\frac{\epsilon}{2}+\overset{(f)}{\frac{1}{2}}}\text{ or }\underset{(k)}{\overset{(g)}{\frac{3}{4}}}$, $\underline{\frac{\epsilon}{2}+\overset{(f)}{\frac{1}{2}}}\text{ or }\underset{(k)}{\overset{(g)}{\frac{3}{4}}}$	$\overset{(e)}{0}\text{ or }\overset{(f)}{\frac{1}{2}}\text{ or }\overset{(d)}{1}$, $\overset{(f)}{\frac{1}{2}}\text{ or }\underset{(l)}{\overset{(g)}{\underline{1}}}$, $\overset{(f)}{\frac{1}{2}}\text{ or }\underset{(l)}{\overset{(g)}{\underline{1}}}$	$\overset{(h)}{0}\text{ or }\overset{(i)}{1}$, $\underline{1}$, $\overset{(f)}{0}\text{ or }\underset{(l)}{\overset{(g)}{\underline{1}}}$
	ISV	$\overset{(a)}{0}\text{ or }\overset{(b)}{1}$, $\overset{(f)}{0}\text{ or }\underset{(l)}{\overset{(g)}{\underline{1}}}$, $\underline{1}$	$\overset{(a)}{0}\text{ or }\overset{(b)}{1}$, $\overset{(f)}{0}\text{ or }\underset{(l)}{\overset{(g)}{\underline{1}}}$, $\underline{1}$	$\overset{(e)}{0}\text{ or }\overset{(b)/(i)}{\underline{1}}$, $\overset{(c)}{0}\text{ or }\overset{(g)/(i)}{\underline{1}}$, $\overset{(j)}{0}\text{ or }\overset{(b)/(g)}{\underline{1}}$

Table 6.16: The game in 6.6.2 and Nash equilibria before eliminating weakly-dominated strategies in the first stage. Conditions (a) – (n) are given in Table 6.3.

The Nash equilibria of the game before eliminating weakly-dominated strategies in the first stage, are summarized in Table 6.17.

	Conditions for Existence	**Absentees**	**First Stage**	**Second Stage**	**Outcome**
(1)		$i = 2, 3$	$i = 1$ (SV)	$i = 2, 3$ (SV)	x^k_Ω
(2)		all	all (SV)	all (SV)	x^k_Ω
(3)	If (d) & (l)	$i = 1$	$i = 2, 3$ (ISV)	$i = 1$ (SV)	$x^{j'}_{\Omega'}$
(4)	If (h) OR $[(i)$ & $(m)]$	none	all $i = 1, 2$ (SV) $i = 3$ (ISV)	$i = 3$ (SV)	x^k_Ω
(5)	If (a) OR $[(b)$ & $(m)]$	none	all $i = 1, 3$ (SV) $i = 2$ (ISV)	$i = 2$ (SV)	x^k_Ω
(6)	If (d)	none	all (ISV)	all (SV)	$x^{j'}_{\Omega'}$

Table 6.17: Nash equilibria of the game in 6.6.2 before eliminating weakly-dominated strategies in the first stage.

Next, we focus on eliminating weakly-dominated strategies in the first stage. We observe that abstention weakly-dominates insincere voting for *SI* agent $i = 1$. For *WI* agents $i = 2, 3$, abstention weakly-dominates insincere voting if Condition (k) is satisfied.

As in the previous case, Equilibrium (6) is implausible after eliminating weakly-dominated strategies. We establish the conditions under which abstention does not weakly-dominate insincere voting for *WI* agents. We note that

- abstention does not weakly-dominate insincere voting for *WI* agents $i = 2, 3$ if and only if $[(l)]$ and $\Big[(d)$ or $(g)\Big]$ are satisfied.

Applying these conditions together with the conditions required for existence, we refine the conditions of existence for Equilibria (4) and (5). In Table 6.18, we present our refined set of plausible equilibria, which are path-independent.

	Conditions for Existence	**Absentees**	**First Stage**	**Second Stage**	**Outcome**
(1)		$i = 2, 3$	$i = 1$ (SV)	$i = 2, 3$ (SV)	x^k_Ω
(2)		all	all (SV)	all (SV)	x^k_Ω
(3)	If (d) & (l)	$i = 1$	$i = 2, 3$ (ISV)	$i = 1$ (SV)	$x^{j'}_{\Omega'}$
(4)	If $[(l)$ & $(e)]$ OR $[(l)$ & (d) & $(m)]$	none	all $i = 1, 2$ (SV) $i = 3$ (ISV)	$i = 3$ (SV)	x^k_Ω
(5)	If $[(l)$ & $(e)]$ OR $[(l)$ & (d) & $(m)]$	none	all $i = 1, 3$ (SV) $i = 2$ (ISV)	$i = 2$ (SV)	x^k_Ω

Table 6.18: Refined Nash equilibria of the game in 6.6.2.

In the following, we explain the intuition behind each equilibrium.

Equilibria (1) and (2): Since $i = 1$ is *SI*, he is strictly better off if Ω is chosen in the first stage. Thus, he will vote for Ω if he participates in the first stage. Anticipating this, $i = 2, 3$ will abstain from voting in the first stage to ensure that their preferred variation is selected in the second stage. Given this behavior, $i = 1$ is indifferent between participation and abstention, giving rise to two equilibria.

Equilibrium (3): If all agents prefer the same variation of Ω' (Condition (d)) and if $\epsilon < 0.5$ (Condition (l)) the *WI* agents $i = 2, 3$ will prefer Ω' being selected in the first stage. They will thus vote insincerely in the first stage, while $i = 1$ abstains. Given these strategies, no agent has an incentive to deviate from his own strategy.

Equilibria (4) and (5): If $H \geq 1.5$ (Condition (m)), the *SI* agent $i = 1$ will vote sincerely in the first stage, as he wants to ensure that the selection of Ω. Thus, $i = 2$ or $i = 3$ will vote insincerely to obtain voting rights for the second stage and realize their preferred direction x^k_Ω, yielding two equilibria. Given the behavior of the *WI* agents,

$i = 1$ has no incentive to deviate from his strategy. Deviation would select Ω' in the first stage with positive probability, which would be a worse outcome for $i = 1$ when H is large or when he is certain that his preferred variation of Ω' will not be selected in the second stage (Condition (e)).

As in the previous case, the similarities between the equilibrium voting behavior with complete and with incomplete information end with sincere voting in the second stage. In Equilibria (2) and (3),the *SI* agent chooses to abstain from the first stage and in Equilibria (3) - (5), the *WI* agents participate in the first stage. Moreover, in Equilibria (3) and (5), one or both *WI* agents engage in insincere voting in the first stage, but in equilibrium, the *SI* agent never votes insincerely in the first stage.

6.6.3 All agents are inclined towards Ω. One WI agents prefers x_Ω^j, while the other prefers x_Ω^k.

Without loss of generality, we assume that $i = 1, 3$ prefer x_Ω^j and $i = 2$ prefers x_Ω^k. After eliminating weakly-dominated strategies in the second stage, all agents that choose or are allowed to vote in the second stage will vote sincerely.

As in the previous case, the *SI* individual is always better off if Ω is selected in the first stage. We note that now, the *WI* agents are divided about the variation of Ω. We represent the game in normal form in Table 6.19, when votes in the second stage are cast sincerely. We solve for all possible Nash equilibria in pure strategies.

$i=1$	$i=2$ \ $i=3$	**SV**	**Abstain**	**ISV**
$i=1$ SV	SV	$\underline{\boldsymbol{H+1}}$, ϵ, $\underline{\boldsymbol{\epsilon+1}}$	$\underline{\boldsymbol{H+1}}$, ϵ, $\underline{\boldsymbol{\epsilon+1}}$	$\underline{\boldsymbol{H+1}}$, ϵ, $\underline{\boldsymbol{\epsilon+1}}$
	Abstain	H, $\underline{\boldsymbol{\epsilon+1}}$, ϵ	$H+\frac{1}{2}$, $\underset{(c)/(a)/(d)\&(k)}{\underline{\boldsymbol{\epsilon+\frac{1}{2}}}}$, $\underset{(j)/(h)/(d)\&(k)}{\underline{\boldsymbol{\epsilon+\frac{1}{2}}}}$	$\underline{\frac{H}{2}+\overset{(h)}{\frac{1}{2}}\text{ or }\overset{(i)}{\frac{3}{4}}}$, $\underline{\frac{\epsilon}{2}+\overset{(h)}{\frac{1}{2}}}\text{ or }\underset{(k)}{\overset{(i)}{\frac{3}{4}}}$, $\underline{\frac{\epsilon}{2}+\overset{(j)}{\frac{1}{4}}}\text{ or }\overset{(h)}{\frac{1}{2}}\text{ or }\underset{(l)}{\overset{(d)}{\frac{3}{4}}}$
	ISV	$\underset{(a)/(b)\&(m)}{\underline{\boldsymbol{H}}}$, $\underline{\boldsymbol{\epsilon+1}}$, ϵ	$\underline{\frac{H}{2}+\overset{(a)}{\frac{1}{2}}\text{ or }\overset{(b)}{\frac{3}{4}}}$, $\underline{\frac{\epsilon}{2}+}\overset{(c)}{\frac{1}{4}}\text{ or }\overset{(a)}{\frac{1}{2}}\text{ or }\underset{(l)}{\overset{(d)}{\frac{3}{4}}}$, $\underline{\frac{\epsilon}{2}+\overset{(a)}{\frac{1}{2}}}\text{ or }\underset{(k)}{\overset{(b)}{\frac{3}{4}}}$	$\boxed{\underline{\boldsymbol{1}}}$, $\overset{(h)}{0}\text{ or }\boxed{\underset{(l)}{\overset{(i)}{\underline{\boldsymbol{1}}}}}$, $\overset{(a)}{0}\text{ or }\boxed{\underset{(l)}{\overset{(b)}{\underline{\boldsymbol{1}}}}}$
$i=1$ Abstain	SV	$\underline{\boldsymbol{H+1}}$, ϵ, $\underline{\boldsymbol{\epsilon+1}}$	$\underline{\boldsymbol{H+1}}$, $\underset{(c)/(a)\&(k)}{\underline{\boldsymbol{\epsilon}}}$, $\underline{\boldsymbol{\epsilon+1}}$	$\frac{H}{2}+\overset{(h)}{\frac{3}{4}}\text{ or }\overset{(i)}{1}$, $\underline{\frac{\epsilon}{2}+}\underset{(k)}{\overset{(h)}{\frac{1}{4}}}\text{ or }\overset{(i)}{\frac{1}{2}}$, $\frac{\epsilon}{2}+\overset{(j)}{\frac{1}{2}}\text{ or }\overset{(h)}{\frac{3}{4}}\text{ or }\overset{(d)}{1}$
	Abstain	$\underline{\boldsymbol{H+\frac{1}{2}}}$, $\underset{(c)/(a)/(d)\&(k)}{\underline{\boldsymbol{\epsilon+\frac{1}{2}}}}$, $\epsilon+\frac{1}{2}$	$\underline{\boldsymbol{H+1}}$, $\underset{(c)/(a)\&(k)}{\underline{\boldsymbol{\epsilon}}}$, $\underline{\boldsymbol{\epsilon+1}}$	$\overset{(h)}{\frac{1}{2}}\text{ or }\overset{(i)}{1}$, $\underset{(l)}{\overset{(h)}{\underline{\frac{1}{2}}}}\text{ or }\overset{(i)}{\underline{\boldsymbol{1}}}$, $\overset{(j)}{0}\text{ or }\overset{(h)}{\frac{1}{2}}\text{ or }\overset{(d)}{1}$
	ISV	$\underline{\frac{H}{2}+}\overset{(a)}{\frac{1}{2}}\text{ or }\underset{(n)}{\overset{(b)}{\frac{3}{4}}}$, $\underline{\frac{\epsilon}{2}+}\overset{(c)}{\frac{1}{4}}\text{ or }\overset{(a)}{\frac{1}{2}}\text{ or }\underset{(l)}{\overset{(d)}{\frac{3}{4}}}$, $\underline{\frac{\epsilon}{2}+\overset{(a)}{\frac{1}{2}}}\text{ or }\underset{(k)}{\overset{(b)}{\frac{3}{4}}}$	$\overset{(a)}{\frac{1}{2}}\text{ or }\overset{(b)}{1}$, $\overset{(c)}{0}\text{ or }\underset{(l)}{\overset{(a)}{\underline{\frac{1}{2}}}}\text{ or }\overset{(d)}{\underline{\boldsymbol{1}}}$, $\overset{(a)}{\frac{1}{2}}\text{ or }\underset{(l)}{\overset{(b)}{\underline{\boldsymbol{1}}}}$	$\boxed{\underline{\boldsymbol{1}}}$, $\overset{(h)}{0}\text{ or }\boxed{\overset{(i)}{\underline{\boldsymbol{1}}}}$, $\overset{(a)}{0}\text{ or }\boxed{\underset{(l)}{\overset{(b)}{\underline{\boldsymbol{1}}}}}$
$i=1$ ISV	SV	$\underline{\boldsymbol{H+1}}$, ϵ, $\underline{\boldsymbol{\epsilon+1}}$	$\frac{H}{2}+\overset{(e)}{\frac{1}{2}}\text{ or }\overset{(f)}{\frac{3}{4}}\text{ or }\overset{(d)}{1}$, $\frac{\epsilon}{2}+\overset{(f)}{\frac{1}{4}}\text{ or }\overset{(g)}{\frac{1}{2}}$, $\frac{\epsilon}{2}+\overset{(f)}{\frac{3}{4}}\text{ or }\overset{(g)}{1}$	$\overset{(h)}{0}\text{ or }\overset{(i)}{1}$, $\underline{\boldsymbol{1}}$, $\overset{(f)}{0}\text{ or }\overset{(g)}{1}$
	Abstain	$\frac{H}{2}+\overset{(e)}{\frac{1}{4}}\text{ or }\overset{(f)}{\frac{1}{2}}\text{ or }\overset{(d)}{\frac{3}{4}}$, $\underline{\frac{\epsilon}{2}+\overset{(f)}{\frac{1}{2}}}\text{ or }\underset{(k)}{\overset{(g)}{\frac{3}{4}}}$, $\underline{\frac{\epsilon}{2}+\overset{(f)}{\frac{1}{2}}}\text{ or }\underset{(k)}{\overset{(g)}{\frac{3}{4}}}$	$\overset{(e)}{0}\text{ or }\overset{(f)}{\frac{1}{2}}\text{ or }\overset{(d)}{1}$, $\overset{(f)}{\frac{1}{2}}\text{ or }\overset{(g)}{\underline{\boldsymbol{1}}}$, $\overset{(f)}{\frac{1}{2}}\text{ or }\underset{(l)}{\overset{(g)}{\underline{\boldsymbol{1}}}}$	$\overset{(h)}{0}\text{ or }\overset{(i)}{1}$, $\underline{\boldsymbol{1}}$, $\overset{(f)}{0}\text{ or }\underset{(l)}{\overset{(g)}{\underline{\boldsymbol{1}}}}$
	ISV	$\overset{(a)}{0}\text{ or }\overset{(b)}{1}$, $\overset{(f)}{0}\text{ or }\underset{(l)}{\overset{(g)}{\underline{\boldsymbol{1}}}}$, $\underline{\boldsymbol{1}}$	$\overset{(h)}{0}\text{ or }\overset{(i)}{1}$, $\underline{\boldsymbol{1}}$, $\overset{(f)}{0}\text{ or }\overset{(g)}{\underline{\boldsymbol{1}}}$	$\overset{(e)}{0}\text{ or }\boxed{\overset{(b)/(i)}{\underline{\boldsymbol{1}}}}$, $\overset{(c)}{0}\text{ or }\boxed{\overset{(g)/(i)}{\underline{\boldsymbol{1}}}}$, $\overset{(j)}{0}\text{ or }\boxed{\overset{(b)/(g)}{\underline{\boldsymbol{1}}}}$

Table 6.19: The game in 6.6.3 and Nash equilibria before eliminating weakly-dominated strategies in the first stage. Conditions (a) – (n) are given in Table 6.3.

The Nash equilibria of the game before eliminating weakly-dominated strategies in the first stage, are summarized in Table 6.20.

	Conditions for Existence	**Absentees**	**First Stage**	**Second Stage**	**Outcome**
(1)	If (c) OR $[(a)$ & $(k)]$	$i = 1, 3$	$i = 2$ (SV)	$i = 1, 3$ (SV)	x^j_Ω
(2)	If (c) OR $[(a)$ & $(k)]$	all	all (SV)	all (SV)	x^j_Ω
(3)	If (d) & (l)	none	all $i = 1$ (SV) $i = 2, 3$ (ISV)	$i = 1$ (SV)	$x^{j'}_{\Omega'}$
(4)	If (d) & (l)	$i = 1$	$i = 2, 3$ (ISV)	$i = 1$ (SV)	$x^{j'}_{\Omega'}$
(5)	If (d)	none	all (ISV)	all (SV)	$x^{j'}_{\Omega'}$

Table 6.20: Nash equilibria of the game in 6.6.3 before eliminating weakly-dominated strategies in the first stage.

Next, we focus on eliminating weakly-dominated strategies in the first stage. We observe that abstention weakly-dominates insincere voting for the *SI* agent $i = 1$. For the *WI* agent $i = 3$, abstention weakly-dominates insincere voting if Condition (k) is satisfied. For $i = 2$, abstention weakly-dominates insincere voting if Condition (k) is satisfied and Condition (d) is *not* satisfied. Moreover, abstention weakly-dominates sincere voting for the *WI* agent $i = 2$ if Condition (l) or (d) is satisfied.

We next establish the conditions under which abstention does not weakly-dominate the other strategies for the *WI* agents. We note that

- abstention does not weakly-dominate insincere voting for $i = 3$ if and only if $[(l)]$ and $\Big[(b) \text{ or } (d)\Big]$,
- abstention does not weakly-dominate insincere voting for $i = 2$ if and only if $[(d)]$ or $\Big[(l) \text{ and } \Big((i) \text{ or } (a) \text{ or } (g)\Big)\Big]$,

- abstention does not weakly-dominate sincere voting for $i = 2$ if and only if $[(k)]$ and $\Big[(h) \text{ or } (f)\Big]$.

Applying these conditions together with the conditions required for the existence of equilibria, we note that Equilibrium (5) can be eliminated and that the conditions for existence of Equilibrium (1) are modified. We denote our refined set of plausible equilibria, which are path-independent, in Table 6.21.

	Conditions for Existence	**Absentees**	**First Stage**	**Second Stage**	**Outcome**
(1)	If (k) & $[(c)$ or (e) or $(j)]$	$i = 1, 3$	$i = 2$ (SV)	$i = 1, 3$ (SV)	x^j_Ω
(2)	If (c) OR $[(a)$ & $(k)]$	all	all (SV)	all (SV)	x^j_Ω
(3)	If (d) & (l)	none	all $i = 1$ (SV) $i = 2, 3$ (ISV)	$i = 1$ (SV)	$x^{j'}_{\Omega'}$
(4)	If (d) & (l)	$i = 1$	$i = 2, 3$ (ISV)	$i = 1$ (SV)	$x^{j'}_{\Omega'}$

Table 6.21: Refined Nash equilibria of the game in 6.6.3.

In the following, we explain the intuition behind each equilibrium.

Equilibria (1) and (2): If $\epsilon \geq 0.5$ (Condition (k)) and $i = 1, 3$'s preferences over the variations of Ω' are such that $i = 2$ will not realize his preferred variation of Ω' in the second stage, then $i = 2$ is inclined to vote for Ω in the first stage. Thus, $i = 1, 3$ will abstain from the first stage to realize their preferred variation x^j_Ω in the second stage. Given this voting behavior, $i = 2$ becomes indifferent between participation and abstention in the first stage, yielding two equilibria. Note that $i = 1, 3$ have no incentive to deviate, since they receive their highest-possible utility from x^j_Ω. Similarly, $i = 2$ is worse off by voting insincerely in the second stage, and has no incentive to deviate from his strategy.

Equilibria (3) and (4): If all agents agree on the variations of Ω' (Condition (d)) and if ϵ is very small (Condition (l)), then the *WI* agents $i = 2, 3$ will vote for Ω' in the first stage. Then, $i = 1$ becomes indifferent between participation and abstention, which yields two equilibria. Note that since the *WI* agents do not agree on the variations of Ω, none of them has an incentive to vote for Ω, given that ϵ is small.

As in the previous case, the similarities between the equilibrium voting behavior with complete and with incomplete information are limited to sincere voting in the second stage. In Equilibria (1), (2) and (4), the *SI* agent chooses to abstain in the first stage, while in Equilibria (1), (3) and (4), one or both *WI* agents participate in the first stage. Moreover, in Equilibria (3) and (4), the *WI* agents engage in insincere voting in the first stage. In equilibrium, however, the *SI* agent never votes insincerely in the first stage.

The cases remaining to be analyzed are

- $i = 2, 3$ prefer Ω and $i = 1$ prefers Ω'. All agents prefer x^j_Ω,
- $i = 2, 3$ prefer Ω and x^j_Ω while $i = 1$ prefers Ω' and x^k_Ω,
- $i = 2, 3$ prefer Ω and $i = 1$ prefers Ω'. Moreover, $i = 1, 2$ prefer x^j_Ω and $i = 3$ prefers x^k_Ω,
- $i = 1, 2$ prefer Ω and $i = 3$ prefers Ω'. All agents prefer x^j_Ω,
- $i = 1, 2$ prefer Ω and $i = 3$ prefers Ω'. Moreover, $i = 1$ prefers x^j_Ω and $i = 2, 3$ prefer x^k_Ω,
- $i = 1, 2$ prefer Ω and x^j_Ω while $i = 3$ prefers Ω' and x^k_Ω,
- $i = 1, 2$ prefers Ω and $i = 3$ prefer Ω'. Moreover, $i = 1, 3$ prefer x^j_Ω and $i = 2$ prefers x^k_Ω.

6.7 Equilibria when $N_s = 0$ and $N_w = 3$

Suppose that the committee consists of all *WI* individuals. In Sections 6.7.1, 6.7.2 and 6.7.3, we examine three cases which represent different realizations of individual preferences.

6.7.1 All agents prefer Ω and x_{Ω}^{j}

We note that after eliminating weakly-dominated strategies, all agents that choose or are allowed to vote in the second stage will vote sincerely. Similarly, any agent choosing to vote in the first stage would vote sincerely. All *WI* individuals are inclined towards Ω and they obtain their highest payoff from x_{Ω}^{j}. Hence, no one has an incentive to follow a strategy that would yield any other outcome. Given the final outcome is the one preferred by all agents, any combination of agents may abstain from the first stage. Similar to the Case 6.4.1, there are eight possible Nash equilibria, detailed in Table 6.1.

6.7.2 All agents are inclined towards Ω. Two agents prefer x_{Ω}^{j}, while the other prefers x_{Ω}^{k}.

Without loss of generality, we assume that $i = 1, 2$ prefer x_{Ω}^{j} and $i = 3$ prefers x_{Ω}^{k}. As in the previous case, after eliminating weakly-dominated strategies in the second stage, all agents that choose or are allowed to vote in the second stage will vote sincerely.

The committee now only consists of *WI* agents. Hence, no agent is strictly better off if Ω or Ω' is chosen in the first stage. The decision in the first stage depends on individual preferences over the variations of Ω' and the value of ϵ. Given that votes in the second stage are cast sincerely, we represent the game in normal form in Table 6.22.

$i=1$	$i=2$ \ $i=3$	SV	Abstain	ISV
$i=1$ SV	SV	$\underline{\epsilon+1}$, $\underline{\epsilon+1}$, ϵ	ϵ, ϵ, $\underline{\epsilon+1}$	ϵ, ϵ, $\underline{\epsilon+1}$
	Abstain	$\underline{\epsilon+1}$, $\underline{\epsilon+1}$, ϵ	$\epsilon+\frac{1}{2}$, $\underset{(c)/(a)/(d)\&(k)}{\underline{\epsilon+\frac{1}{2}}}$, $\underset{(j)/(h)/(d)\&(k)}{\underline{\epsilon+\frac{1}{2}}}$	$\underline{\frac{\epsilon}{2}+\overset{(h)}{\frac{1}{2}}}\text{ or }\underset{(k)}{\underline{\overset{(i)}{\frac{3}{4}}}}$, $\underline{\frac{\epsilon}{2}+\overset{(h)}{\frac{1}{2}}}\text{ or }\underset{(k)}{\underline{\overset{(i)}{\frac{3}{4}}}}$, $\underline{\frac{\epsilon}{2}+}\overset{(j)}{\frac{1}{4}}\text{ or }\overset{(h)}{\frac{1}{2}}\text{ or }\underset{(l)}{\underline{\overset{(d)}{\frac{3}{4}}}}$
	ISV	$\underline{\epsilon+1}$, $\underline{\epsilon+1}$, ϵ	$\underline{\frac{\epsilon}{2}+\overset{(a)}{\frac{1}{2}}}\text{ or }\underset{(k)}{\underline{\overset{(b)}{\frac{3}{4}}}}$, $\underline{\frac{\epsilon}{2}+}\overset{(c)}{\frac{1}{4}}\text{ or }\overset{(a)}{\frac{1}{2}}\text{ or }\underset{(l)}{\underline{\overset{(d)}{\frac{3}{4}}}}$, $\underline{\frac{\epsilon}{2}+\overset{(a)}{\frac{1}{2}}}\text{ or }\underset{(k)}{\underline{\overset{(b)}{\frac{3}{4}}}}$	$\underline{1}$, $\overset{(h)}{0}\text{ or }\underset{(l)}{\underline{\overset{(i)}{1}}}$, $\overset{(a)}{0}\text{ or }\underset{(l)}{\underline{\overset{(b)}{1}}}$
$i=1$ Abstain	SV	$\underline{\epsilon+1}$, $\underline{\epsilon+1}$, ϵ	$\underset{(e)/(f)/(d)\&(k)}{\underline{\epsilon+\frac{1}{2}}}$, $\epsilon+\frac{1}{2}$, $\underset{(j)/(h)/(d)\&(k)}{\underline{\epsilon+\frac{1}{2}}}$	$\underline{\frac{\epsilon}{2}+\overset{(h)}{\frac{1}{2}}}\text{ or }\underset{(k)}{\underline{\overset{(i)}{\frac{3}{4}}}}$, $\underline{\frac{\epsilon}{2}+\overset{(h)}{\frac{1}{2}}}\text{ or }\underset{(k)}{\underline{\overset{(i)}{\frac{3}{4}}}}$, $\underline{\frac{\epsilon}{2}+}\overset{(j)}{\frac{1}{4}}\text{ or }\overset{(h)}{\frac{1}{2}}\text{ or }\underset{(l)}{\underline{\overset{(d)}{\frac{3}{4}}}}$
	Abstain	$\underline{\epsilon+1}$, $\underline{\epsilon+1}$, $\underset{(j)/(h)\&(k)}{\underline{\epsilon}}$	$\underline{\epsilon+1}$, $\underline{\epsilon+1}$, $\underset{(j)/(h)\&(k)}{\underline{\epsilon}}$	$\overset{(h)}{\frac{1}{2}}\text{ or }\underset{(l)}{\underline{\overset{(i)}{1}}}$, $\overset{(h)}{\frac{1}{2}}\text{ or }\underset{(l)}{\underline{\overset{(i)}{1}}}$, $\overset{(j)}{0}\text{ or }\underset{(l)}{\underline{\overset{(h)}{\frac{1}{2}}}}\text{ or }\underline{\overset{(d)}{1}}$
	ISV	$\frac{\epsilon}{2}+\overset{(a)}{\frac{3}{4}}\text{ or }\overset{(b)}{1}$, $\frac{\epsilon}{2}+\overset{(c)}{\frac{1}{2}}\text{ or }\overset{(a)}{\frac{3}{4}}\text{ or }\overset{(d)}{1}$, $\frac{\epsilon}{2}+\overset{(a)}{\frac{1}{4}}\text{ or }\overset{(b)}{\frac{1}{2}}$	$\overset{(a)}{\frac{1}{2}}\text{ or }\underset{(l)}{\underline{\overset{(b)}{1}}}$, $\overset{(c)}{0}\text{ or }\overset{(a)}{\frac{1}{2}}\text{ or }\overset{(d)}{1}$, $\underline{\overset{(a)}{\frac{1}{2}}\text{ or }\overset{(b)}{1}}$	$\underline{1}$, $\overset{(h)}{0}\text{ or }\underset{(l)}{\underline{\overset{(i)}{1}}}$, $\overset{(a)}{0}\text{ or }\underline{\overset{(b)}{1}}$
$i=1$ ISV	SV	$\underline{\epsilon+1}$, $\underline{\epsilon+1}$, ϵ	$\underline{\frac{\epsilon}{2}+}\overset{(e)}{\frac{1}{4}}\text{ or }\overset{(f)}{\frac{1}{2}}\text{ or }\underset{(l)}{\underline{\overset{(d)}{\frac{4}{3}}}}$, $\underline{\frac{\epsilon}{2}+\overset{(f)}{\frac{1}{2}}}\text{ or }\underset{(k)}{\underline{\overset{(g)}{\frac{3}{4}}}}$, $\underline{\frac{\epsilon}{2}+\overset{(f)}{\frac{1}{2}}}\text{ or }\underset{(k)}{\underline{\overset{(g)}{\frac{3}{4}}}}$	$\overset{(h)}{0}\text{ or }\underset{(l)}{\underline{\overset{(i)}{1}}}$, $\underline{1}$, $\overset{(f)}{0}\text{ or }\underset{(l)}{\underline{\overset{(g)}{1}}}$
	Abstain	$\frac{\epsilon}{2}+\overset{(e)}{\frac{1}{2}}\text{ or }\overset{(f)}{\frac{3}{4}}\text{ or }\overset{(d)}{1}$, $\frac{\epsilon}{2}+\overset{(f)}{\frac{3}{4}}\text{ or }\overset{(g)}{1}$, $\frac{\epsilon}{2}+\overset{(f)}{\frac{1}{4}}\text{ or }\overset{(g)}{\frac{1}{2}}$	$\overset{(e)}{0}\text{ or }\overset{(f)}{\frac{1}{2}}\text{ or }\overset{(d)}{1}$, $\overset{(f)}{\frac{1}{2}}\text{ or }\underset{(l)}{\underline{\overset{(g)}{1}}}$, $\underline{\overset{(f)}{\frac{1}{2}}\text{ or }\overset{(g)}{1}}$	$\overset{(h)}{0}\text{ or }\underset{(l)}{\underline{\overset{(i)}{1}}}$, $\underline{1}$, $\overset{(f)}{0}\text{ or }\underline{\overset{(g)}{1}}$
	ISV	$\overset{(a)}{0}\text{ or }\overset{(b)}{1}$, $\overset{(f)}{0}\text{ or }\overset{(g)}{1}$, $\underline{1}$	$\overset{(a)}{0}\text{ or }\underset{(l)}{\underline{\overset{(b)}{1}}}$, $\overset{(f)}{0}\text{ or }\underset{(l)}{\underline{\overset{(g)}{1}}}$, $\underline{1}$	$\overset{(e)}{0}\text{ or }\underline{\overset{(b)/(i)}{1}}$, $\overset{(c)}{0}\text{ or }\underline{\overset{(g)/(i)}{1}}$, $\overset{(j)}{0}\text{ or }\underline{\overset{(b)/(g)}{1}}$

Table 6.22: The game in 6.7.2 and Nash equilibria before eliminating weakly-dominated strategies in the first stage. Conditions $(a)-(n)$ are given in Table 6.3.

The Nash equilibria of the game before eliminating weakly-dominated strategies in the first stage are summarized in Table 6.23.

	Conditions for Existence	**Absentees**	**First Stage**	**Second Stage**	**Outcome**
(1)	If (j) OR $[(h)$ & $(k)]$	all	all (SV)	all (SV)	x^{j}_{Ω}
(2)	If (d) & (l)	none	all $i=1$ (SV) $i=2,3$ (ISV)	$i=1$ (SV)	$x^{j'}_{\Omega'}$
(3)	If (d) & (l)	$i=1,2$	$i=3$ (ISV)	$i=1,2$ (SV)	$x^{j'}_{\Omega'}$
(4)	If (d) & (l)	$i=1$	$i=2,3$ (ISV)	$i=1$ (SV)	$x^{j'}_{\Omega'}$
(5)	If (d) & (l)	$i=3$	$i=1,2$ (ISV)	$i=3$ (SV)	$x^{j'}_{\Omega'}$
(6)	If (d) & (l)	none	all $i=2$ (SV) $i=1,3$ (ISV)	$i=2$ (SV)	$x^{j'}_{\Omega'}$
(7)	If (d) & (l)	$i=2$	$i=1,3$ (ISV)	$i=2$ (SV)	$x^{j'}_{\Omega'}$
(8)	If (d)	none	all (ISV)	all (SV)	$x^{j'}_{\Omega'}$
(9)	If (j) OR $[(h)$ & $(k)]$	$i=1,2$	$i=3$ (SV)	$i=1,2$ (SV)	x^{j}_{Ω}

Table 6.23: Nash equilibria of the game in 6.7.2 before eliminating weakly-dominated strategies in the first stage.

Next, we focus on eliminating weakly-dominated strategies in the first stage. We note that abstention weakly-dominates sincere voting for $i = 3$. Moreover, abstention weakly-dominates insincere voting for $i = 1$ and $i = 2$ if Condition (k) is satisfied. For $i = 3$, abstention weakly-dominates insincere voting if Condition (k) is satisfied and Condition (d) is *not* satisfied. We establish the precise conditions under which abstention does not weakly-dominate the other strategies for each agent. We note that

- abstention does not weakly-dominate insincere voting for $i = 1$ and $i = 2$ if and only if $[(l)]$ and $\Big[(d) \text{ or } (i)\Big]$,
- abstention does not weakly-dominate insincere voting for $i = 3$ if and only if $[(d)]$ or $\Big[(l) \text{ and } \Big((b) \text{ or } (h) \text{ or } (g)\Big)\Big]$.

Applying these conditions together with the conditions required for the existence of equilibria, we note that Equilibrium (9) can be eliminated. We denote our refined set of plausible equilibria, which are path-independent, in Table 6.24.

	Conditions for Existence	**Absentees**	**First Stage**	**Second Stage**	**Outcome**
(1)	If (j) OR $[(h)$ & $(k)]$	all	all (SV)	all (SV)	x^{j}_{Ω}
(2)	If (d) & (l)	none	all $i = 1$ (SV) $i = 2, 3$ (ISV)	$i = 1$ (SV)	$x^{j'}_{\Omega'}$
(3)	If (d) & (l)	$i = 1, 2$	$i = 3$ (ISV)	$i = 1, 2$ (SV)	$x^{j'}_{\Omega'}$
(4)	If (d) & (l)	$i = 1$	$i = 2, 3$ (ISV)	$i = 1$ (SV)	$x^{j'}_{\Omega'}$
(5)	If (d) & (l)	$i = 3$	$i = 1, 2$ (ISV)	$i = 3$ (SV)	$x^{j'}_{\Omega'}$
(6)	If (d) & (l)	none	all $i = 2$ (SV) $i = 1, 3$ (ISV)	$i = 2$ (SV)	$x^{j'}_{\Omega'}$
(7)	If (d) & (l)	$i = 2$	$i = 1, 3$ (ISV)	$i = 2$ (SV)	$x^{j'}_{\Omega'}$
(8)	If (d) & (l)	none	all (ISV)	all (SV)	$x^{j'}_{\Omega'}$

Table 6.24: Refined Nash equilibria of the game in 6.7.2.

In the following, we explain the intuition behind each equilibrium.

Equilibrium (1): If $\epsilon \geq 0.5$ (Condition (k)) or if agents $i = 1, 2$ prefer the variation of Ω' that is less preferred by $i = 3$ (Condition (j)), agent $i = 3$ has no incentive to vote for Ω'. As voting for Ω costs him his voting right in the second stage, $i = 3$ prefers to abstain. Given this voting behavior, $i = 1, 2$ also abstain, as by voting for Ω, they would lose their right to vote in the second stage, and $i = 3$ will select x_Ω^k. Given the strategies of the other players, no individual has an incentive to deviate from his own strategy.

Equilibria (2) to (8): When all agents have the same preferences on the variations of Ω' (Condition (d)), and $\epsilon < 0.5$ (Condition (l)), agent $i = 3$ has no incentive to vote for Ω in the first period, knowing that his preferred variation of Ω will never be selected with certainty in the second period. If $i = 3$ votes for Ω', then $i = 1, 2$ have no incentive to vote for Ω either, as the expected utility from Ω is less than from Ω', since x_Ω^j will never be selected with certainty. This yields several equilibria where $i = 3$ votes for Ω' and the agents $i = 1, 2$ play different strategies without altering the final outcome. In Equilibrium (5), $i = 3$ abstains, given that $i = 1, 2$ vote for Ω' in the first stage.

In this case, equilibria with complete and incomplete information consist of sincere voting in the second stage. However, unlike the equilibrium with incomplete information, *WI* agents choose to participate in the first stage in Equilibria (2) to (8), and in most cases, they also vote insincerely.

6.7.3 Two agents are inclined towards Ω and the other towards Ω'. All agents prefer x_Ω^j over x_Ω^k.

Without loss of generality, we assume that $i = 1, 2$ are inclined towards Ω and that $i = 3$ is inclined towards Ω'. As in the previous cases, all agents that choose or are allowed to vote in the second stage will vote sincerely, after eliminating weakly-dominated strategies in the second stage.

Given that votes in the second stage are cast sincerely, we represent the game in normal form in Table 6.26. We solve for all possible Nash equilibria in pure strategies. As only the two individuals $i = 1, 2$ are now inclined towards Ω, but all agents prefer the same variation x_Ω^j over x_Ω^k, $i = 1, 2$ have an incentive to ensure that Ω will be selected in the first stage.

The Nash equilibria of the game before eliminating weakly-dominated strategies in the first stage are summarized in Table 6.25.

	Conditions for Existence	**Absentees**	**First Stage**	**Second Stage**	**Outcome**
(1)		$i=3$	$i=1,2$ (SV)	$i=3$ (SV)	x^j_Ω
(2)		none	all (SV)	$i=3$ (SV)	x^j_Ω
(3)		none	$i=1,2$ (SV) $i=3$ (ISV)	all (SV)	x^j_Ω
(4)	If (j) OR $[(h)$ & $(l)]$	$i=2,3$	$i=1$ (SV)	$i=2,3$ (SV)	x^j_Ω
(5)	If (j) OR $[(h)$ & $(l)]$	$i=2$	$i=1$ (SV) $i=3$ (ISV)	$i=2$ (SV)	x^j_Ω
(6)	If (a) & (l)	none	$i=1$ (SV) $i=2,3$ (ISV)	$i=2$ (SV)	x^j_Ω
(7)	If (c) OR $[(h)$ & $(l)]$	$i=1,3$	$i=2$ (SV)	$i=1,3$ (SV)	x^j_Ω
(8)	If (c) OR $[(h)$ & $(l)]$	$i=1$	$i=2$ (SV) $i=3$ (ISV)	$i=1$ (SV)	x^j_Ω
(9)	If (j) OR (h)	all	all (SV)	all (SV)	x^j_Ω
(10)	If (j) OR (h)	$i=1,2$	$i=3$ (ISV)	$i=1,2$ (SV)	x^j_Ω
(11)	If (f) & (l)	none	$i=2$ (SV) $i=1,3$ (ISV)	$i=1$ (SV)	x^j_Ω
(12)	If (d)	none	$i=3$ (SV) $i=1,2$ (ISV)	all (SV)	$x^{j'}_{\Omega'}$

Table 6.25: Nash equilibria of the game in 6.7.3 before eliminating weakly-dominated strategies in the first stage.

	$i=2$ \ $i=3$	SV	Abstain	ISV
$i=1$ SV	SV	$\underline{\epsilon+1}$, $\underline{\epsilon+1}$, $\underline{1}$,	$\underline{\epsilon+1}$, $\underline{\epsilon+1}$, $\underline{1}$	$\underline{\epsilon+1}$, $\underline{\epsilon+1}$, $\underline{1}$
	Abstain	$\underline{\frac{\epsilon}{2}+\overset{(h)}{\frac{3}{4}}\text{ or }\overset{(i)}{1}}$, $\frac{\epsilon}{2}+\overset{(h)}{\frac{3}{4}}\text{ or }\overset{(i)}{1}$, $\underline{\frac{\epsilon}{2}}+\overset{(j)}{\frac{1}{2}}\text{ or }\underset{(k)}{\overset{(h)}{\frac{3}{4}}}\text{ or }\overset{(d)}{\underline{1}}$	$\underline{\epsilon+1}$, $\underline{\epsilon+1}$, $\underset{(j)/(h)\&(l)}{\underline{1}}$	$\underline{\epsilon+1}$, $\underline{\epsilon+1}$, $\underset{(j)/(h)\&(l)}{\underline{1}}$
	ISV	$\underline{1}$, $\overset{(h)}{0}\text{ or }\overset{(i)}{1}$, $\underline{\epsilon}+\overset{(a)}{0}\text{ or }\overset{(b)}{\underline{1}}$	$\underline{\frac{\epsilon}{2}+\overset{(a)}{\frac{3}{4}}\text{ or }\overset{(b)}{1}}$, $\frac{\epsilon}{2}+\overset{(c)}{\frac{1}{2}}\text{ or }\overset{(a)}{\frac{3}{4}}\text{ or }\overset{(d)}{1}$, $\underline{\frac{\epsilon}{2}}+\underset{(k)}{\overset{(a)}{\frac{3}{4}}}\text{ or }\overset{(b)}{1}$	$\underline{\epsilon+1}$, $\underline{\epsilon+1}$, $\underset{(a)\&(l)}{\underline{1}}$
$i=1$ Abstain	SV	$\frac{\epsilon}{2}+\overset{(h)}{\frac{3}{4}}\text{ or }\overset{(i)}{1}$, $\underline{\frac{\epsilon}{2}+\overset{(h)}{\frac{3}{4}}\text{ or }\overset{(i)}{1}}$, $\underline{\frac{\epsilon}{2}}+\overset{(c)}{\frac{1}{2}}\text{ or }\underset{(k)}{\overset{(h)}{\frac{3}{4}}}\text{ or }\overset{(d)}{\underline{1}}$	$\underline{\epsilon+1}$, $\underline{\epsilon+1}$, $\underset{(c)/(h)\&(l)}{\underline{1}}$	$\underline{\epsilon+1}$, $\underline{\epsilon+1}$, $\underset{(c)/(h)\&(l)}{\underline{1}}$
	Abstain	$\overset{(h)}{\frac{1}{2}}\text{ or }\overset{(i)}{1}$, $\overset{(h)}{\frac{1}{2}}\text{ or }\overset{(i)}{1}$, $\underline{\epsilon}+\overset{(j)}{0}\text{ or }\overset{(h)}{\frac{1}{2}}\text{ or }\overset{(d)}{\underline{1}}$	$\underline{\epsilon+1}$, $\underline{\epsilon+1}$, $\underset{(j)/(h)}{\underline{1}}$	$\underline{\epsilon+1}$, $\underline{\epsilon+1}$, $\underset{(j)/(h)}{\underline{1}}$
	ISV	$\underline{1}$, $\overset{(h)}{0}\text{ or }\overset{(i)}{1}$, $\underline{\epsilon}+\overset{(a)}{0}\text{ or }\overset{(b)}{\underline{1}}$,	$\overset{(a)}{\frac{1}{2}}\text{ or }\overset{(b)}{1}$, $\overset{(c)}{0}\text{ or }\overset{(a)}{\frac{1}{2}}\text{ or }\overset{(d)}{1}$, $\underline{\epsilon}+\underset{(k)}{\overset{(a)}{\frac{1}{2}}}\text{ or }\overset{(b)}{\underline{1}}$	$\frac{\epsilon}{2}+\overset{(a)}{\frac{3}{4}}\text{ or }\overset{(b)}{1}$, $\frac{\epsilon}{2}+\overset{(c)}{\frac{1}{2}}\text{ or }\overset{(a)}{\frac{3}{4}}\text{ or }\overset{(d)}{1}$, $\underline{\frac{\epsilon}{2}}+\underset{(l)}{\overset{(a)}{\frac{3}{4}}}\text{ or }\overset{(b)}{1}$
$i=1$ ISV	SV	$\overset{(h)}{0}\text{ or }\overset{(i)}{1}$, $\underline{1}$, $\underline{\epsilon}+\overset{(f)}{0}\text{ or }\overset{(g)}{\underline{1}}$,	$\frac{\epsilon}{2}+\overset{(e)}{\frac{1}{2}}\text{ or }\overset{(f)}{\frac{3}{4}}\text{ or }\overset{(d)}{1}$, $\underline{\frac{\epsilon}{2}+\overset{(f)}{\frac{3}{4}}\text{ or }\overset{(g)}{1}}$, $\underline{\frac{\epsilon}{2}}+\underset{(k)}{\overset{(f)}{\frac{3}{4}}}\text{ or }\overset{(g)}{1}$	$\underline{\epsilon+1}$, $\underline{\epsilon+1}$, $\underset{(f)\&(l)}{\underline{1}}$
	Abstain	$\overset{(h)}{0}\text{ or }\overset{(i)}{1}$, $\underline{1}$, $\underline{\epsilon}+\overset{(f)}{0}\text{ or }\overset{(g)}{\underline{1}}$,	$\overset{(e)}{0}\text{ or }\overset{(f)}{\frac{1}{2}}\text{ or }\overset{(d)}{1}$, $\overset{(f)}{\frac{1}{2}}\text{ or }\overset{(g)}{1}$, $\underline{\epsilon}+\underset{(k)}{\overset{(f)}{\frac{1}{2}}}\text{ or }\overset{(g)}{\underline{1}}$,	$\frac{\epsilon}{2}+\overset{(e)}{\frac{1}{2}}\text{ or }\overset{(f)}{\frac{3}{4}}\text{ or }\overset{(d)}{1}$, $\frac{\epsilon}{2}+\overset{(f)}{\frac{3}{4}}\text{ or }\overset{(g)}{1}$, $\underline{\frac{\epsilon}{2}}+\underset{(l)}{\overset{(f)}{\frac{3}{4}}}\text{ or }\overset{(g)}{1}$
	ISV	$\overset{(e)}{0}\text{ or }\overset{(b)/(i)}{\underline{1}}$, $\overset{(c)}{0}\text{ or }\overset{(g)/(i)}{\underline{1}}$, $\underline{\epsilon}+\overset{(j)}{0}\text{ or }\overset{(b)/(g)}{\underline{1}}$	$\overset{(a)}{0}\text{ or }\overset{(b)}{1}$, $\overset{(f)}{0}\text{ or }\overset{(g)}{1}$, $\underline{\epsilon+1}$	$\overset{(a)}{0}\text{ or }\overset{(b)}{1}$, $\overset{(f)}{0}\text{ or }\overset{(g)}{1}$, $\underline{\epsilon+1}$

Table 6.26: The game in 6.7.3 and Nash equilibria before eliminating weakly-dominated strategies in the first stage. Conditions (a) – (n) are given in Table 6.3.

Next, we focus on eliminating weakly-dominated strategies in the first stage. We note that for the two *WI* agents who are inclined towards Ω, $i = 1, 2$, sincere voting weakly-dominates both abstention and insincere voting. For $i = 3$, abstention weakly-dominates insincere voting if Condition (k) is satisfied. Moreover, for $i = 3$, abstention also weakly-dominates sincere voting if Conditions (j) and (l) are satisfied. We note that

- abstention does not weakly-dominate insincere voting for $i = 3$ if and only if $[(l)]$ and $\Big[(a) \text{ or } (f)\Big]$,
- abstention does not weakly-dominate sincere voting for $i = 3$ if and only if $[(d)]$ or $[(b)]$ or $[(g)]$ or $\Big[(h) \text{ and } (k)\Big]$.

Applying these conditions together with the conditions required for the existence of equilibria, we note that most of the equilibria can be eliminated. We denote our refined set of plausible equilibria in Table 6.27.

	Conditions for Existence	**Absentees**	**First Stage**	**Second Stage**	**Outcome**
(1)		$i = 3$	$i = 1, 2$ (SV)	$i = 3$ (SV)	x^j_Ω
(2)	If (d) OR (b) OR (g) OR $[(h)$ & $(k)]$	none	all (SV)	$i = 3$ (SV)	x^j_Ω
(3)	If (l) & $[(a)$ or $(f)]$	none	$i = 1, 2$ (SV) $i = 3$ (ISV)	all (SV)	x^j_Ω

Table 6.27: Refined Nash equilibria of the game in 6.7.3.

As all agents agree on the variations of Ω, agents $i = 1, 2$ have an incentive to vote for Ω, as this ensures that x^j_Ω will be selected in the second stage, which gives them the highest-possible utility. Given that $i = 1, 2$ vote for Ω in the first stage, $i = 3$ is indifferent between abstention, sincere or insincere voting, yielding three equilibria.

As in the previous case, while agents always vote sincerely in the second stage in equilibrium, *WI* agents do not always abstain, and in Equilibrium (3), one *WI* individual also votes insincerely.

The remaining cases to be analyzed when the whole committee comprises of weakly-inclined agents are

- Two agents are inclined towards Ω and prefer x_{Ω}^{j} over x_{Ω}^{k}. The other agent is inclined towards Ω' and prefers x_{Ω}^{k} over x_{Ω}^{j},
- Two agents are inclined towards Ω and one prefers x_{Ω}^{j}, while the other prefers x_{Ω}^{k}. The third agent is inclined towards Ω' and prefers x_{Ω}^{j} over x_{Ω}^{k}.

6.8 Comparison of Features of Equilibria

In this section, we provide a detailed comparison between the features of equilibria with incomplete and with complete information. We analyze the similarities and differences between equilibria in the two information environments according to the features listed in Section 6.2.

6.8.1 All SI agents vote in the first stage if $H > 1$

Similarities:

There exist equilibria with complete information where *SI* agents vote in the first stage, irrespective of the value of H. When the whole society comprises *SI* agents, for example, the value of H does not play any role in an agent's decision to participate in the first stage. *SI* agents are always better off if their preferred fundamental direction is selected in the first stage, and especially if there is a risk that their preferred direction may not be selected, *SI* agents will participate in voting in the first stage. In the equilibria in Case 6.4.3, for instance, agents $i = 1, 2$ vote in the first stage to select their preferred direction Ω, as $i = 3$ is strongly-inclined towards Ω' and would thus vote for Ω'.

In some equilibria, however, *SI* agents participate in the first stage because they are indifferent between participation and abstention. In Case 6.4.2, as agents $i = 1, 2$ always abstain in equilibrium, $i = 3$ participates in voting in the first stage in Equilibrium (1), and abstains in Equilibrium (2), as he is indifferent between participation and abstention.

In other equilibria, SI agents vote in the first stage if H is at least 1.5. In Equilibrium (5) of Case 6.5.2, for instance, the SI agents $i = 1, 2$ vote in the first stage, waiving their voting right for the second stage and thus waiving the possibility of realizing their preferred variation of Ω if $H \geq 1.5$ (Condition (m)). Given the agents' preferences about Ω', deviation is non-profitable only if H is sufficiently high.

Differences:

There exist equilibria with complete information where SI agents abstain from voting in the first stage, regardless of the value of H. If an SI agent is certain that there will be enough votes in favor of his preferred fundamental direction, he will choose to abstain to enhance his chances to realize his preferred variation, and thereby, an additional utility of 1. The two equilibria in Case 6.4.2 are typical examples. There, SI agents $i = 1, 2$, who prefer x_{Ω}^{j}, abstain in equilibria, knowing that $i = 3$, who is also strongly-inclined towards Ω, would never vote for Ω' in the first stage.

Sometimes, SI agents abstain out of indifference. In Equilibrium (2) of Case 6.4.2, $i = 3$ abstains because he is indifferent between participation and abstention. As $i = 1, 2$ always abstain in equilibrium, the strategy of $i = 3$ does not affect the final outcome.

There also exist equilibria where SI agents abstain if H is not large enough, that is if $H < 1.5$. In Equilibria (3) and (4) of Case 6.5.2, one of the SI agents abstains from the first stage, although this means that his less preferred fundamental direction will be selected with probability $\frac{1}{2}$. When H is not large enough, the additional utility of 1 that they reap from their preferred variation becomes significant, and therefore, it becomes desirable to abstain, to increase the selection chances of their preferred variation.

6.8.2 All WI agents abstain from the first stage if $\epsilon \leq \epsilon^{crit}(N)$

Similarities:

There exists equilibria with complete information where WI agents abstain from voting in the first stage, irrespective of the value of ϵ. In Equilibrium (1) of Case 6.5.3, for instance, WI agent $i = 3$ abstains from voting in the first stage, so that he is able to realize his preferred variation in the second stage. Since the SI agent $i = 2$ votes for his preferred fundamental direction, by abstention, $i = 3$ realizes his highest-possible utility in this case. The behavior of $i = 3$ does not depend on the value of ϵ.

Moreover, there are cases where *WI* agents abstain, irrespective of the value of ϵ, even when it is clear that their preferred variation will not be selected in the second stage. In Equilibrium (1) of Case 6.7.2, given that $i = 1, 2$, who prefer x_{Ω}^{j}, abstain, $i = 3$, who prefers x_{Ω}^{k}, also abstains, although it's clear that the final outcome will be x_{Ω}^{j}. This is only the case, however, if Condition (j) is satisfied, which means that if $i = 3$ votes for Ω', $i = 1, 2$ will choose the variation of Ω' that gives $i = 3$ a utility of 0.

In other equilibria, *WI* agents abstain if ϵ is less than 0.5, which may be less than $\epsilon^{crit}(N)$ in some cases. A low ϵ means that *WI* agents prefer the fundamental direction towards which they are *not* inclined if their preferred variation of this direction is certain to be selected in the second stage. Given that there are enough votes in favor of this fundamental direction, *WI* agents may abstain from voting in the first stage. In Equilibria (3), (4), (5) and (7) of Case 6.7.2, for example, *WI* agents abstain from voting when $\epsilon < 0.5$ (Condition (l)). However, this is more because they are indifferent between participation and abstention, than for a better chance of selecting their preferred variation, because in these equilibria, all agents agree on the variations of Ω'.

Differences:

There exist equilibria with complete information where *WI* agents vote in the first stage regardless of the value of ϵ. In Equilibrium (1) of Case 6.7.3, for instance, since $i = 3$ is *WI* towards Ω', agents $i = 1, 2$, who are *WI* towards Ω, vote in the first stage to ensure that Ω is chosen as the fundamental decision. As all agents prefer the same variation of Ω, $i = 1, 2$ achieves the highest-possible payoff of $1 + \epsilon$ from the final decision.

There also exist equilibria where *WI* agents vote if ϵ is large enough, that is if $\epsilon \geq 0.5$. In Equilibrium (1) of Case 6.5.2, for example, if ϵ is sufficiently large, the *WI* agent $i = 3$ votes for his preferred fundamental direction Ω, while the *SI* agents $i = 1, 2$ abstain. Agent preferences over the variations of Ω' are such that agent $i = 3$'s preferred variation of Ω' will not be elected with certainty in the second stage if Ω' is selected in the first stage. Thus, $i = 3$ votes in the first stage to select Ω as the fundamental decision, which gives him a higher expected utility than Ω'.

WI agents also vote in the first stage if ϵ is small enough, that is if $\epsilon < 0.5$. When ϵ is small, *WI* agents' expected utility from their preferred fundamental direction Ω may be less than from Ω'. Thus, depending on the realization of agent preferences over the variations of Ω', *WI* agents may choose to vote in the first stage to ensure that Ω' will be selected as the fundamental decision. Equilibrium (3) of Case 6.6.2 is a suitable example. Since all agents agree on the variations of Ω' (Condition (d)), the *WI* agents $i = 2, 3$ vote for Ω' in the first stage, knowing that the *SI* agent will select $x_{\Omega'}^{j'}$ in the second stage, which gives them a utility of 1.

6.8.3 All votes in the first stage are cast sincerely

Similarities:

There exists equilibria with complete information in which all votes in the first stage are cast sincerely. These votes were cast by *SI* agents or *WI* agents or by both kinds of agents.

Sincere voting by *SI* agents is intuitive, as they reap strictly higher utility from the variations of their preferred fundamental direction than the other agents. In the equilibria in Cases 6.4.2 and 6.4.3, all *SI* agents vote sincerely, for example.

WI agents vote sincerely in the first stage for numerous reasons. In Equilibrium (1) of Case 6.5.2, the *WI* agent $i = 3$ votes for his preferred direction Ω because individual preferences over Ω' are such that if he votes for Ω', the two abstaining agents $i = 1, 2$ will never elect his preferred variation with probability 1. In Equilibrium (4) of Case 6.5.3, $i = 3$ votes sincerely to ensure that the selection of Ω in the first stage, so that agent $i = 1$, who prefers the same variation of Ω, can save his voting right for the second stage to ensure that their preferred variation is selected. In some equilibria, *WI* agents vote sincerely to guarantee his voting right for the second stage. In Equilibrium (2) of Case 6.7.2, for example, agent $i = 1$ votes sincerely, while the other agents vote insincerely to ensure that the fundamental direction towards which they are *not* inclined, is selected. Sincere voting guarantees that $i = 1$ will belong to the losers of the first stage and will therefore obtain a ballot for the second stage. In Equilibrium (2) of Case 6.7.3, $i = 3$ votes sincerely out of indifference. In all equilibria of Case 6.7.3, agents $i = 1, 2$ vote for Ω in the first stage, knowing that $i = 3$ will vote for their preferred variation in the second stage. This makes $i = 3$ indifferent between abstention, sincere voting and insincere voting, yielding three equilibria.

Differences:

There exist equilibria with complete information where all or some of the votes are cast insincerely in the first stage. In some equilibria, only *WI* agents vote insincerely, while there are other equilibria where *SI* agents also engage in insincere voting.

Equilibria (4), (5), (7) and (8) of Case 6.7.2 are examples for equilibria where all votes in the first stage are cast insincerely. In these equilibria, the entire society consists of *WI* agents, ϵ is rather small, and all agents prefer the same variation of Ω', but not of Ω. Thus, all agents are better off by selecting Ω' in the first stage. Hence, all votes in the first stage are cast insincerely.

Equilibrium (4) of Case 6.5.3 is an example for an equilibrium where only a part of the votes in the first stage is cast insincerely, and also an example of a situation where an *SI* agent votes insincerely in the first stage. Given that agents $i = 2, 3$ vote for Ω, $i = 1$ votes insincerely, so that he will lose in the first stage and therefore obtain a ballot for the second stage.

Similar to the reasons behind sincere voting, there are several reasons for a *WI* to vote insincerely. In some equilibria, it is to increase the winning chances of his less preferred fundamental direction, because the final outcome would give him higher expected utility than his preferred fundamental direction. Equilibria (3) and (4) of Case 6.5.2 and Equilibria (2) – (8) of Case 6.7.2 are typical examples. In some equilibria, *WI* agents vote insincerely to purposely lose their vote in the first stage to ensure that they have a ballot for the second stage. Equilibrium (5) of Case 6.5.2 is one such example. Sometimes, *WI* agents vote insincerely in the first stage out of indifference. In Equilibrium (3) of Case 6.7.3, agent $i = 3$ votes insincerely out of indifference, as the other two agents vote for Ω in all equilibria, and $i = 3$'s voting behavior will not change the final outcome.

6.8.4 All votes in the second stage are cast sincerely

Similarities:

In all equilibria with complete information, all votes are cast sincerely in the second stage. Both *SI* and *WI* agents vote sincerely if they chose to or were allowed to vote in the second stage. No agent could reap any additional benefit by voting insincerely, as the game ends in the second stage.

Differences:

There does not exist an equilibrium with complete information where all or some of the votes are cast insincerely in the second stage. As explained before, no agent has an incentive to engage in insincere voting.

6.9 Conclusion

We examined BV in a simple society of three agents, first voting on a fundamental direction and then on its variants. Voting is carried out in a setting of complete information. We analyze equilibria under several different possible realizations of the agents' utilities. We provide a comparison between the equilibrium voting behavior with complete information and the unique equilibrium with incomplete information. We find that while there exist equilibria which conform to the equilibrium with incomplete information, there also exist equilibria that do not. The only behavior that remains unchanged across different information environments is sincere voting in the second stage. We have developed the intuition for each equilibrium and identified reasons for changes in equilibrium voting strategies, when agent preferences become publicly observable.

Appendices

Appendix A: Summary of Properties of Key Expressions

In this appendix, we summarize the properties of the key expressions used in the main text. Specifically, we analyze each expression at the limit, and state the results obtained. When analytical verification is not possible, we provide the ranges for which an expression has been numerically verified to fulfill the properties considered in the main text.

Expression	**Formula**	**Analytical Results**	**Numerical Verifications**	**Illustration**
$P(N)$	Equation (3.1)	$\lim_{n\to\infty} P(n) = \frac{1}{2}$		vs N for $N \in [1, 20]$ Figure 3.3
$\Theta(N), \Lambda(N, n), \Phi(N, n)$	$\Theta(N) = (3.4)$ $\Lambda(N, n) = (3.3)$ $\Phi(N, n) = (3.2)$	$\lim_{N\to\infty} \Theta(N) = \frac{1}{2}$ $\forall n, \lim_{N\to\infty} \Lambda(N, n) = \frac{1}{2}$ $\forall n, \lim_{N\to\infty} \Phi(N, n) = \frac{1}{2}$		vs n for $N = 20$ Figure 3.4
$\epsilon^{crit}(N)$	Proposition 1	$\lim_{N\to\infty} \epsilon^{crit}(N) = 0$		vs N for $N \in [3, 200]$ Figure 3.5
$H^{crit}_{st}(N)$, $\epsilon^{crit}_{st}(N)$	Lemma 1	$\lim_{N\to\infty} \epsilon^{crit}(N) = 0$ $\lim_{N\to\infty} H^{crit}(N) = \infty$		vs N for $N \in [3, 200]$ Figure 3.6
$\epsilon^{SM}(N, N_s)$	Equation (3.7)	$\forall N_s$ when N even and $\forall N_s \neq 1$ when N odd, $\lim_{N\to\infty} \epsilon^{SM}(N, N_s) = \infty$		vs N_s for $N = 20$ and $N = 200$ Figure 3.7
$Q(N)$	Equation (3.8)	$\lim_{N\to\infty} Q(N) = 0$		vs N for $N \in [3, 200]$ Figure 3.8
$H^{SM}(N, N_s)$	Equation (3.10)	$\forall N_s$ when N even and $\forall N_s \neq N - 1$ when N odd, $\lim_{N\to\infty} H^{SM}(N, N_s) = \infty$		vs N_s for $N = 20$ and $N = 200$ Figure 3.9
$H = M(N, p)\, \epsilon + C(N, p)$	Equation (3.11)		$N \in [3, 1000]$ $p \in [0.01, 0.99]$, in steps of 0.01	H vs ϵ for $N = 20$, $p = \frac{1}{10}, \frac{1}{3}, \frac{2}{3}, \frac{9}{10}$ Figure 3.10
$K(N, p)$	Equation (3.15)		$N \in [4, 1000]$ $p \in [0.01, 0.99]$, in steps of 0.001	vs $p \in [0.01, 0.99]$ for $N = 3$, $N = 4$, $N = 8$, $N = 16$, $N = 23$, $N = 52$ Figure 3.11

Table A.1: Summary of Properties of Key Expressions.

Expression	**Formula**	**Analytical Results**	**Numerical Verifications**	**Illustration**
$N_{st}(p)$	Equation (3.16)		$p \in [0.6, 0.99]$ in steps of 0.01	vs $p \in [0.6, 0.99]$ Figure 3.12
$\epsilon^{MV}(N,1) - \epsilon^{crit}(N)$	Equation (3.20)	$\forall N_s$, $\lim_{N\to\infty} \epsilon^{MV}(N, N_s) = \infty$		vs N for $N \in [3, 200]$ Figure 3.13
$\max_{N_s} H^{MV}(N, N_s)$	Equation (3.22)	for large N_s, $\lim_{N\to\infty} H^{MV}(N, N_s) = 0$		vs N for $N \in [3, 200]$ Figure 3.14
$H = M(N,p)\,\epsilon + D(N,p)$	Equation (3.23)		$N \in [3, 1000]$ $p \in [0.01, 0.99]$, in steps of 0.01	H vs ϵ for $N = 20$, $p = \frac{1}{10}, \frac{1}{2}, \frac{2}{3}, \frac{99}{100}$ Figure 3.15
$P_\Omega(n,p)$	Equation (4.1)	for $Pr(j_\Omega) < \frac{1}{2}$, $\lim_{n\to\infty} P_\Omega(n,p) = 0$ for $Pr(j_\Omega) > \frac{1}{2}$, $\lim_{n\to\infty} P_\Omega(n,p) = 1$ for $Pr(j_\Omega) = \frac{1}{2}$, $\lim_{n\to\infty} P_\Omega(n,p) = \frac{1}{2}$		vs n for $Pr[j_A] = \frac{1}{4}$ and $Pr[j_B] = \frac{3}{4}$ Figure 4.1
$M_1(N, x_i)$	Equation (B.8)		$N \in [3, 1000]$ $0 < x_i < N - 1$	vs x_i for $N = 200$ Figure B.1
$M_0(N)$, $M_1(N, N-2)$, $M_1(N, 1)$, $M_2(N)$	Equation (B.8)		$N \in [3, 1000]$	vs N for $N \in [3, 200]$ Figure B.2
$H^{crit}(N)$ and $\epsilon^{crit}(N)$	Equation (B.9) Equation (B.10)		$N \in [3, 1000]$ for $H^{crit}(N)$	vs N for $N \in [3, 200]$ Figure B.3
$M_0(N)$, $M_3(N, N-2)$, $M_3(N, 1)$, $M_4(N)$	Equation (B.11)	$\lim_{N\to\infty} M_0(N), M_3(N,1) = 0$ $\lim_{N\to\infty} M_3(N, N-2), M_4(N) = \infty$		vs N for $N \in [3, 200]$ Figure B.4

Table A.2: Summary of Properties of Key Expressions.

Appendix B: Proofs

Proof of Fact 1:

Step 1: $\Phi(N,n) \geq P(N)$ **when** $1 < n < N$ **and** $\Lambda(N,n) \geq P(N)$ **when** $0 < n < N$

$\Phi(N,n)$ is a weighted average of the terms $P(N-n+N_L)$ for $N_L \in \left[1, \left\lfloor \frac{n}{2} \right\rfloor \right]$. Similarly, $P(N)$ can be written using the same weights as a weighted average of $P(N)$. For each N_L, $P(N-n+N_L) \geq P(N)$, where the equality occurs if and only if $n = 2$ and N is odd. Therefore, $\Phi(N,n) \geq P(N)$.

Following the same logic, we obtain, $\Lambda(N,n) \geq P(N)$ when $0 < n < N$, where the equality occurs if and only if $n = 1$ and N is odd.

Step 2: $\Lambda(N,n) \geq \Phi(N,n)$ **when** $1 < n < N$

$\Phi(N,n)$ is a weighted average of the terms $P(N-n+N_L)$ for $N_L \in \left[1, \left\lfloor \frac{n}{2} \right\rfloor \right]$, while $\Lambda(N,n)$ is a weighted average of the terms $P(N-n+N_L)$ for $N_L \in \left[0, \left\lfloor \frac{n}{2} \right\rfloor \right]$. In the expression of $\Phi(N,n)$, each $P(N-n+N_L)$ for $N_L \in \left[1, \left\lfloor \frac{n}{2} \right\rfloor \right]$ is assigned a weight of $\frac{\binom{n}{N_L}}{2^{n-1}-1}$, while under $\Lambda(N,n)$, it is assigned a weight of $\frac{\binom{n}{N_L}}{2^{n-1}} < \frac{\binom{n}{N_L}}{2^{n-1}-1}$. The differences in the weights are multiplied by $P(N-n)$ under $\Lambda(N,n)$, where $P(N-n) > P(N-n+N_L)$ for all $N_L \in \left[1, \left\lfloor \frac{n}{2} \right\rfloor \right]$. Therefore, $\Lambda(N,n) \geq \Phi(N,n)$, where the equality holds for large n where the difference in weights become negligible, i.e. $\frac{1}{2^{n-1}} \approx \frac{1}{2^{n-1}-1}$.

Step 3: $\Theta(N) > \Lambda(N,n)$ **when** $0 < n < N$

$\Theta(N)$ is a weighted average of the terms $P(N_L)$ for $N_L \in \left[1, \lfloor \frac{N}{2} \rfloor\right]$, while $\Lambda(N,n)$ is a weighted average of the terms $P(N-n+N_L)$ for $N_L \in \left[0, \lfloor \frac{n}{2} \rfloor\right]$. For $n < \lfloor \frac{N}{2} \rfloor$, it is obvious that $\Theta(N) > \Lambda(N,n)$ holds, as all $P(N_L)$ terms in $\Theta(N)$ are greater than all $P(N-n+N_L)$ in $\Lambda(N,n)$.

It is, however, not straightforward whether this relationship holds for larger n. We observe that for some n, the terms $P(N-n+N_L)$ for $N_L \in \left[0, \lfloor \frac{n}{2} \rfloor\right]$ in $\Lambda(N,n)$ are equal to or greater than those terms in $\Lambda(N,n)$ for smaller n. We thus verify whether $\Theta(N) > \Lambda(N,n)$ when n is as large as possible. If $\Theta(N) > \Lambda(N,n)$ holds for n as large as possible, we can conclude that it holds for all $n \in (0,N)$. There are four cases to consider.

1) N **and** $n = N-2$ **are odd**

   In this case, $\Theta(N)$ is a weighted average of the terms $P(1)$, $P(2)$,...$P(\lfloor \frac{N}{2} \rfloor)$ and $\Lambda(N,n)$ is a weighted average of $P(2)$, $P(3)$,...$P(\lfloor \frac{N}{2} \rfloor + 1)$. The weightings given to the common terms are lower under $\Lambda(N,n)$. Furthermore, the weighted probabilities under $\Theta(N)$ consists of the highest-possible value $P(1) = 1$, which is not a component of $\Lambda(N,n)$. The difference in weightings under $\Lambda(N,n)$ is multiplied by $P(\lfloor \frac{N}{2} \rfloor + 1)$, which is less than all $P(N_L)$ terms in $\Theta(N)$. Therefore, $\Theta(N) > \Lambda(N,n)$.

   Illustration: Assume $N = 7$ and $n = 5$. Then,

$$\begin{aligned} \Theta(7) = &\quad \frac{7}{63} \cdot P(1) + \frac{21}{63} \cdot P(2) + \frac{35}{63} \cdot P(3) = 0.7778 \\ \Lambda(7,5) = &\quad \frac{1}{16} \cdot P(2) + \frac{5}{16} \cdot P(3) + \frac{10}{16} \cdot P(4) = 0.7113. \end{aligned}$$

2) N **is odd and** $n = N-1$ **is even**

   In this case, $\Theta(N)$ is a weighted average of the terms $P(1)$, $P(2)$,...$P(\lfloor \frac{N}{2} \rfloor)$ and $\Lambda(N,n)$ is a weighted average of $P(1)$, $P(3)$,...$P(\lfloor \frac{N}{2} \rfloor + 1)$. As in the previous case, the weightings given to the common terms are lower under $\Lambda(N,n)$. The differences in the weights are multiplied by $P(\lfloor \frac{N}{2} \rfloor + 1)$ under $\Lambda(N, n)$, which is less than all probabilities making up $\Theta(N)$. Therefore, $\Theta(N) > \Lambda(N, n)$.

Illustration: Assume $N = 7$ and $n = 6$. Then,

$$\begin{aligned}
\Theta(7) = & \ \frac{7}{63} \cdot P(1) + \frac{21}{63} \cdot P(2) + \frac{35}{63} \cdot P(3) = 0.7778 \\
\Lambda(7, 6) = & \ \frac{1}{32} \cdot P(1) + \frac{6}{32} \cdot P(2) + \frac{15}{32} \cdot P(3) + \frac{10}{32} \cdot P(4) = 0.7384.
\end{aligned}$$

3) **N is even and $n = N - 1$ is odd**

In this case, both $\Theta(N)$ and $\Lambda(N, n)$ are a weighted average of the terms $P(1)$, $P(2)$,...$P(\frac{N}{2})$. The weightings given to the terms $P(1)$, $P(2)$,...$P(\frac{N}{2} - 1)$ are lower under $\Lambda(N, n)$. The difference in weightings is given to $P(\frac{N}{2})$ under $\Lambda(N, n)$. But $P(\frac{N}{2})$ is the smallest of all the probabilities that make up the two expressions. Therefore, $\Theta(N) > \Lambda(N, n)$.

Illustration: Assume $N = 6$ and $n = 5$. Then,

$$\begin{aligned}
\Theta(6) = & \ \frac{6}{31} \cdot P(1) + \frac{15}{31} \cdot P(2) + \frac{10}{31} \cdot P(3) = 0.7984 \\
\Lambda(6, 5) = & \ \frac{1}{16} \cdot P(1) + \frac{5}{16} \cdot P(2) + \frac{10}{16} \cdot P(3) = 0.7656.
\end{aligned}$$

4) **N and $= N - 2$ are even**

In this case, $\Theta(N)$ is a weighted average of the terms $P(1)$, $P(2)$,...$P(\frac{N}{2})$ and $\Lambda(N, n)$ is a weighted average of $P(2)$, $P(3)$,...$P(\frac{N}{2} + 1)$. The weightings given to the terms $P(2)$, $P(3)$,...$P(\frac{N}{2} - 1)$ under $\Lambda(N, n)$ are lower than those under $\Theta(N)$, and the weighting for $P(\frac{N}{2})$ is higher. $P(\frac{N}{2})$, however, is the smallest probability in $\Theta(N)$. Moreover, the weighted probabilities under $\Theta(N)$ consists of the highest possible value $P(1) = 1$, which is not a component under $\Lambda(N, n)$. Furthermore, a large weight is given to $P(\lfloor \frac{N}{2} \rfloor + 1)$ under $\Lambda(N, n)$, which is lower than all the probabilities in $\Theta(N)$. Therefore, $\Theta(N) > \Lambda(N, n)$.

Illustration: Assume $N = 6$ and $n = 4$. Then,

$$\begin{aligned}\Theta(6) = &\quad \frac{6}{31} \cdot P(1) + \frac{15}{31} \cdot P(2) + \frac{10}{31} \cdot P(3) = 0.7984 \\ \Lambda(6,4) = &\quad \frac{1}{8} \cdot P(2) + \frac{4}{8} \cdot P(3) + \frac{3}{8} \cdot P(4) = 0.7268.\end{aligned}$$

In all four cases, the effects that yield $\Theta(N) > \Lambda(N, n)$ are amplified for smaller n values. Therefore, for all n and N we have $\Theta(N) > \Lambda(N, n)$. This concludes the proof.

■

Proof of Proposition 1:

A) **Existence**

We start with the proof of *(ii)* and show that all agents vote sincerely in both stages if they have – and use – the right to vote. We then analyze an individual's decision to abstain from the first stage, given this voting behavior.

Step A1: Sincere voting in the second stage

As voting in the second stage is governed by the simple majority rule, every member with the right to vote supports his preferred variation upon elimination of weakly-dominated strategies. Voting insincerely would lower the probability that his preferred variation is selected.

Step A2: Sincere voting in the first stage

We assume that individual i votes in the first stage. We compare i's expected utility when voting sincerely and insincerely, and show that voting sincerely is his best response.

Regardless of how i votes, his utility depends on the voting behavior of other agents. We assume that all individuals voting in the first stage, except i, vote sincerely. It is, however, unclear how many agents might choose to abstain. Let

x_i denote the number of agents, excluding i, who are voting in the first stage, that is, $0 \leq x_i \leq N-1$. Since the probability distribution of x_i is unknown at this stage, as no equilibrium has been identified yet, it is not possible to compute i's expected utility explicitly. Therefore, we derive i's expected utility conditional on x_i, and for all possible values of x_i.

We start by formulating the probability of winning or losing in each stage, expressed as functions of x_i. Let the indices WW, WL, LW and LL represent the following outcomes for individual i, respectively: winning in both stages, winning in the first stage but losing in the second, losing in the first stage but winning in the second, and losing in both stages. Then, i's expected probabilities of winning are as follows:

$$\Gamma_{WW}^{Vo}(N,x_i) = \begin{cases} P(x_i+1)\frac{1}{2} & \text{if } 0 \leq x_i < N-1 \\ P(N)\left(\frac{P(N)}{2^{N-1}} + \left(1-\frac{1}{2^{N-1}}\right)\frac{1}{2}\right) & \text{if } x_i = N-1 \end{cases}$$

$$\Gamma_{WL}^{Vo}(N,x_i) = \begin{cases} P(x_i+1)\frac{1}{2} & \text{if } 0 \leq x_i < N-1 \\ P(N)\left(\frac{1-P(N)}{2^{N-1}} + \left(1-\frac{1}{2^{N-1}}\right)\frac{1}{2}\right) & \text{if } x_i = N-1 \end{cases}$$

$$\Gamma_{LW}^{Vo}(N,x_i) = \begin{cases} (1-P(x_i+1))\Phi(N,x_i+1) & \text{if } 0 < x_i < N-1 \\ (1-P(N))\Theta(N) & \text{if } x_i = N-1 \end{cases}$$

$$\Gamma_{LL}^{Vo}(N,x_i) = \begin{cases} (1-P(x_i+1))(1-\Phi(N,x_i+1)) & \text{if } 0 < x_i < N-1 \\ (1-P(N))(1-\Theta(N)) & \text{if } x_i = N-1, \end{cases}$$

where the expressions of $\Theta(N)$ and $\Phi(N,x_i+1)$ are given by (3.4) and (3.2), respectively, and where the index Vo stands for voting in the first stage. We note that $\Gamma_{LW}^{Vo}(N,x_i)$ and $\Gamma_{LL}^{Vo}(N,x_i)$ are not defined for $x_i = 0$, since losing in the first stage entails more than one agent participating in the vote.

Now we examine i's decision to vote either sincerely or insincerely in the first stage. By voting sincerely, i derives an expected utility of

$$\Gamma_{WW}^{Vo}(N,x_i)(1+M) + \Gamma_{WL}^{Vo}(N,x_i)(0+M) + \Gamma_{LW}^{Vo}(N,x_i), \tag{B.1}$$

where $0 \leq x_i \leq N-1$ and M represents an agent's strength of preference, with $M = H$ for SI individuals and $M = \epsilon$ for WI agents.

When voting insincerely, the expected utility of i amounts to

$$\Gamma^{Vo}_{WW}(N, x_i) + \Gamma^{Vo}_{LW}(N, x_i)(1 + M) + \Gamma^{Vo}_{LL}(N, x_i)(0 + M). \tag{B.2}$$

Voting sincerely is a best response if for all $0 \leq x_i \leq N-1$,

$$M\big(\Gamma^{Vo}_{WW}(N, x_i) + \Gamma^{Vo}_{WL}(N, x_i) - \Gamma^{Vo}_{LW}(N, x_i) - \Gamma^{Vo}_{LL}(N, x_i)\big) > 0. \tag{B.3}$$

Since $P(x_i+1) > \frac{1}{2} > 1-P(x_i+1)$, for all $x_i \in [0, N-1]$, every individual is better off voting sincerely in the first stage, irrespective of whether they are strongly – or weakly – inclined, given that other agents will also vote sincerely.

In summary, voting sincerely is a best response in the first stage.

Step A3: Decision to abstain from first stage

Next, we evaluate i's decision to abstain in the first stage. We assume again that all agents participating in the first stage, except i, vote sincerely. In Step A2, we have shown that should i decide to participate in the first stage, he will also vote sincerely. Now we calculate i's expected utility by abstaining from voting in the first stage.

Suppose i has decided to abstain from voting in the first stage. Then, his expected probabilities of winning are as follows, where x_i and the indices WW, WL, LW and LL represent the same as in Step A2, the index Ab stands for abstention from voting in the first stage, and $\Lambda(N, x_i)$ is given in (3.3):

$$\Gamma^{Ab}_{WW}(N,x_i) = \begin{cases} P(N)P(N) & \text{if } x_i = 0 \\ \frac{1}{2}\Lambda(N,x_i) & \text{if } 0 < x_i \leq N-1 \end{cases}$$

$$\Gamma^{Ab}_{WL}(N,x_i) = \begin{cases} P(N)\big(1-P(N)\big) & \text{if } x_i = 0 \\ \frac{1}{2}\big(1-\Lambda(N,x_i)\big) & \text{if } 0 < x_i \leq N-1 \end{cases}$$

$$\Gamma^{Ab}_{LW}(N,x_i) = \begin{cases} \big(1-P(N)\big)P(N) & \text{if } x_i = 0 \\ \frac{1}{2}\Lambda(N,x_i) & \text{if } 0 < x_i \leq N-1 \end{cases}$$

$$\Gamma^{Ab}_{LL}(N,x_i) = \begin{cases} \big(1-P(N)\big)\big(1-P(N)\big) & \text{if } x_i = 0 \\ \frac{1}{2}\big(1-\Lambda(N,x_i)\big) & \text{if } 0 < x_i \leq N-1. \end{cases}$$

For any $x_i \in [0, N-1]$, the expected utility of i by abstaining from the first stage and voting sincerely in the second stage is

$$\Gamma^{Ab}_{WW}(N,x_i)(1+M) + \Gamma^{Ab}_{WL}(N,x_i)(0+M) + \Gamma^{Ab}_{LW}(N,x_i). \tag{B.4}$$

If i derives a higher expected utility from abstaining, as compared to voting sincerely in the first stage, he will be inclined to abstain.

Step A4: Critical Conditions

To determine which, of participation or abstention in the first stage, gives i a higher expected utility, we need to compare his expected utility from each decision, weighted over the possible values of x_i. We obtain this by weighing the expected utility calculated in (B.1) and (B.4) with the probability distribution of x_i, and taking the sum over all possible values of x_i. That is, i will participate in voting in the first stage if and only if

$$\begin{aligned} &\sum_{x_i=0}^{N-1} Pr(x_i)\Big[\Gamma^{Vo}_{WW}(N,x_i)(1+M) + \Gamma^{Vo}_{WL}(N,x_i)M + \Gamma^{Vo}_{LW}(N,x_i)\Big] \\ &\geq \sum_{x_i=0}^{N-1} Pr(x_i)\Big[\Gamma^{Ab}_{WW}(N,x_i)(1+M) + \Gamma^{Ab}_{WL}(N,x_i)M + \Gamma^{Ab}_{LW}(N,x_i)\Big], \end{aligned} \tag{B.5}$$

where $Pr(x_i)$ denotes the probability that the number of agents voting in the first stage, except i, is x_i. For a given N, this condition imposes an lower bound on M, such that i is better off participating in the first stage if and only if

$$M \geq \frac{\sum_{x_i=0}^{N-1} Pr(x_i)\left[\Gamma^{Ab}_{LW}(N,x_i)-\Gamma^{Vo}_{LW}(N,x_i)+\Gamma^{Ab}_{WW}(N,x_i)-\Gamma^{Vo}_{WW}(N,x_i)\right]}{\sum_{x_i=0}^{N-1} Pr(x_i)\left[\Gamma^{Vo}_{WW}(N,x_i)-\Gamma^{Ab}_{WW}(N,x_i)+\Gamma^{Vo}_{WL}(N,x_i)-\Gamma^{Ab}_{WL}(N,x_i)\right]}. \tag{B.6}$$

Therefore, SI individuals with sufficiently large H prefer to participate in the first stage and WI individuals with sufficiently small ϵ refrain from voting in the first stage.

Step A5: Sufficient Conditions

Since we do not know the probability distribution of x_i at this stage, it is not yet possible to explicitly calculate the critical value of M such that participation is more attractive than abstention. Therefore, we derive a sufficient condition on M such that if satisfied, it ensures that the critical condition is also satisfied.

We obtain the sufficient condition by calculating the value of M such that Expression (B.5) is satisfied for each term in the summation from $x_i = 0$ to $N-1$. That is, for each $x_i \in [0, N-1]$, we have

$$\begin{aligned} 0 \leq \quad & \Big(\Gamma^{Vo}_{WW}(N,x_i) - \Gamma^{Ab}_{WW}(N,x_i)\Big)(1+M) + \Gamma^{Vo}_{LW}(N,x_i) \\ & - \Gamma^{Ab}_{LW}(N,x_i) + \Big(\Gamma^{Vo}_{WL}(N,x_i) - \Gamma^{Ab}_{WL}(N,x_i)\Big)M \\ \Rightarrow M \geq \quad & \frac{\Gamma^{Ab}_{LW}(N,x_i) - \Gamma^{Vo}_{LW}(N,x_i) + \Gamma^{Ab}_{WW}(N,x_i) - \Gamma^{Vo}_{WW}(N,x_i)}{\Gamma^{Vo}_{WW}(N,x_i) - \Gamma^{Ab}_{WW}(N,x_i) + \Gamma^{Vo}_{WL}(N,x_i) - \Gamma^{Ab}_{WL}(N,x_i)}. \end{aligned} \tag{B.7}$$

This condition simplifies to the following inequalities:

$$M \geq \begin{cases} M_0(N) := \frac{P(N)-\frac{1}{2}}{1-P(N)} & \text{for} \quad x_i = 0, \\ M_1(N,x_i) := \frac{\Lambda(N,x_i)-\frac{P(x_i+1)}{2}-(1-P(x_i+1))\Phi(N,x_i+1)}{P(x_i+1)-\frac{1}{2}} & \text{for} \quad 0 < x_i < N-1, \\ M_2(N) := \frac{\Lambda(N,N-1)-P(N)\left[\frac{P(N)}{2N-1}+(1-\frac{1}{2N-1})\frac{1}{2}\right]-(1-P(N))\Theta(N)}{P(N)-\frac{1}{2}} & \text{for} \quad x_i = N-1. \end{cases} \tag{B.8}$$

We observe that for any given value of N, $M_1(N, x_i)$ is monotonically increasing in x_i.[1] As an illustration, in Figure B.1 we plot $M_1(N, x_i)$ against x_i for $N = 200$.

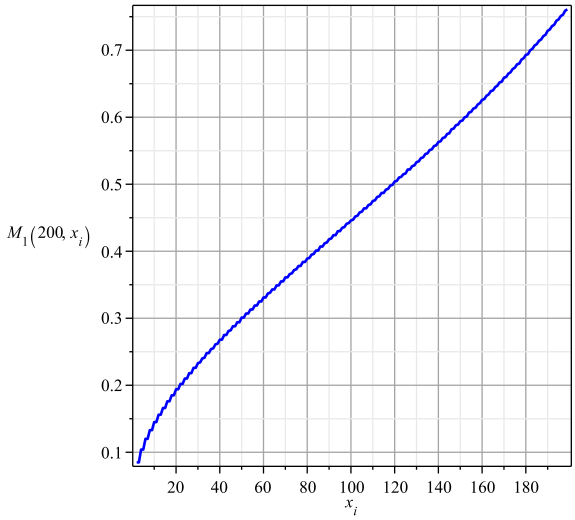

Figure B.1: $M_1(N, x_i)$ vs. x_i for $N = 200$.

As $M_1(N, x_i)$ is monotonically increasing in x_i, it reaches its maximum for $x_i = N - 2$ and minimum for $x_i = 1$. In Figure B.2, we plot $M_0(N)$, $M_1(N, N-2)$, $M_1(N, 1)$ and $M_2(N)$.

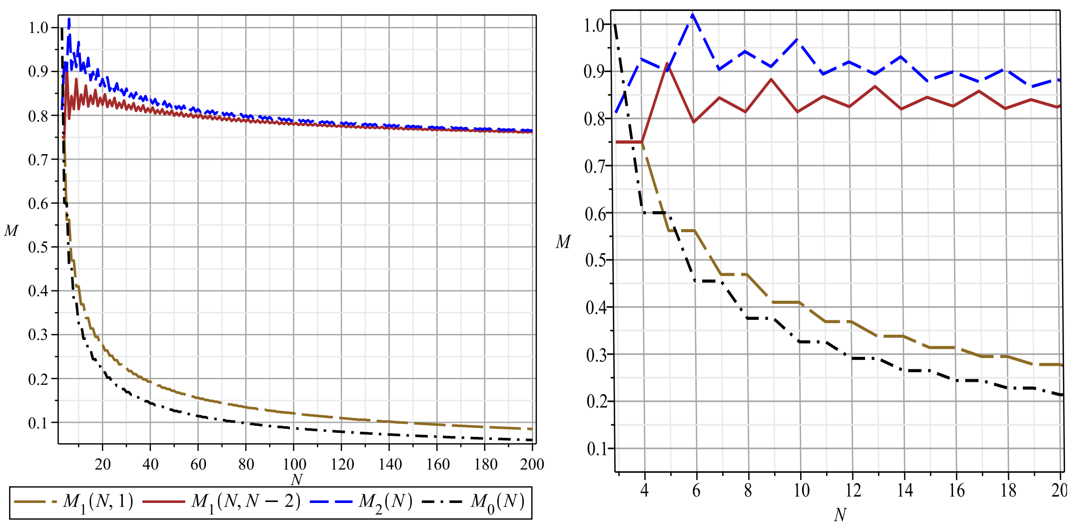

Figure B.2: $M_0(N)$, $M_1(N, N-2)$, $M_1(N, 1)$ and $M_2(N)$ vs. N.

[1] It was verified numerically for $N \in [3, 1000]$ and $0 < x_i < N - 1$.

We note that if x_i is very low (e.g. $x_i = 0$ or 1), the threshold on M monotonically decreases with N. This result is intuitive because for low values of x_i, an increase in N is directly proportional to the increase in the number or agents voting in the second stage. Therefore, as N grows large, it becomes less attractive to abstain from the first stage, and participation becomes attractive even for low M.

If x_i is very large (e.g. $x_i = N-2$ or $N-1$), the threshold on M fluctuates for small N, but decreases asymptotically to 0.7 as $N \to \infty$.[2] It is, however, strictly higher than if x_i is very small for the same N. This is because when x_i is very large, the voting body will always be smaller in the second stage than when x_i is very small – even very small voting bodies being possible with positive probability in the second stage. Therefore, there is a higher incentive to abstain in the first stage and to vote in the second stage, and M has to be sufficiently high for agents to still prefer participation over abstention.

The downward trend of the curve when x_i is high can be explained as follows. As the size of the committee becomes substantially large, the voting body in the second stage also grows larger with positive probability. The desirability of abstaining from the first stage, therefore, shrinks with N, and agents find participation more attractive, even for low M. Note, however, that the threshold on M is still higher than when x_i is very low for the same N. A low value of x_i represents a large voting body in the second stage with certainty, whereas a high x_i indicates that a large voting body in the second stage can occur with high probability.

We set

$$H \geq H^{crit}(N) := \max\left[M_0(N), M_1(N, N-2), M_2(N)\right] \tag{B.9}$$

$$\begin{aligned} \epsilon \leq \epsilon^{crit}(N) &:= \min\left[M_0(N), M_1(N,1), M_2(N)\right] \\ &= \min\left[M_0(N), M_1(N,1)\right] \\ &= \min\left[\frac{P(N)-\frac{1}{2}}{1-P(N)},\ 4\Lambda(N,1) - \Phi(N,2) - \frac{3}{2}\right], \end{aligned} \tag{B.10}$$

[2]Numerically verified for the range $N \in [3, 1000]$.

so that all SI agents participate in voting in the first stage, while all WI agents abstain. In Figure B.3, we plot $H^{crit}(N)$ and $\epsilon^{crit}(N)$.

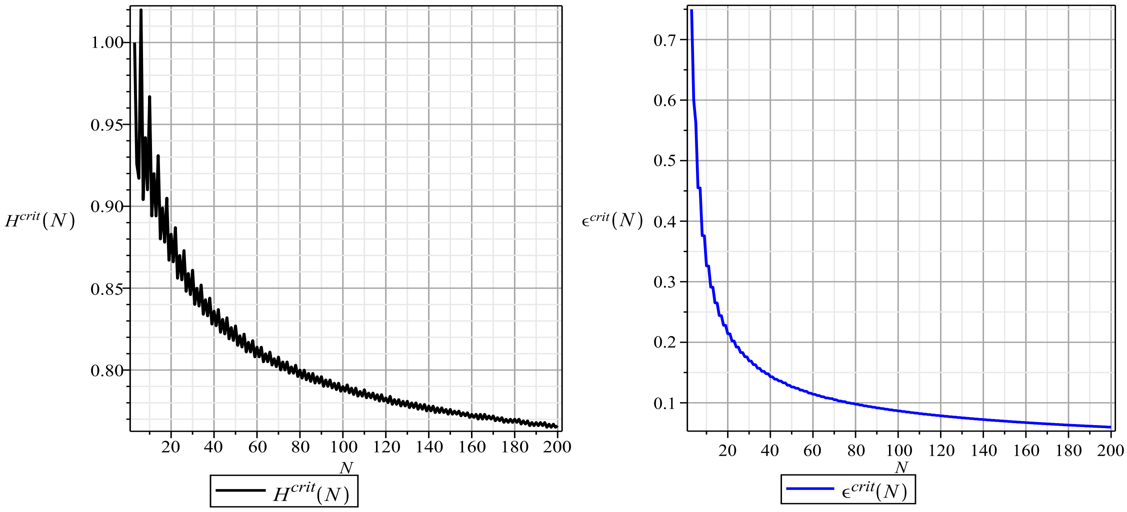

Figure B.3: $H^{crit}(N)$ and $\epsilon^{crit}(N)$ vs. the number of agents N.

We observe that $H^{crit}(N) \leq 1$ for all $N \neq 6$,[3] and $H^{crit}(6) \approx 1$. Under our main assumption that $H >> 1$, all SI individuals will participate in voting in the first stage. Therefore, (B.10) and $H > 1$ are sufficient conditions for all SI agents to participate in voting in the first stage, and for all WI agents to abstain.

B) **Uniqueness**

We analyze the uniqueness of the constructed equilibrium by first considering sincere voting in the second stage, then sincere voting in the first stage, and finally, the decision to abstain from the first stage.

Step B1: Sincere voting in second stage

After eliminating weakly-dominated strategies, it is clear that agents voting in the second stage have no incentive to deviate from sincere voting in any conceivable equilibrium.

[3]It was verified numerically for $N \in [3, 1000]$.

Step B2: Sincere voting in first stage and the role of $P(n)$

Now let us consider the first stage. We start by analyzing individual i's decision to vote sincerely or insincerely, given that he has chosen to participate in voting in the first stage. Recall that $P(n)$ is the probability of winning in the first stage for i, if $n-1$ agents, excluding i, vote in the first stage and if they all vote sincerely. That is, $P(n)$ is based on the critical assumption that all $n-1$ agents vote for a fundamental direction with an ex-ante probability of $\frac{1}{2}$.

Further recall from Step A2 that given $P(n)$, voting sincerely is the best response of i regardless of the number of agents voting in the first stage (x_i). Therefore, i's decision to vote sincerely or insincerely in the first stage is not affected by the other agents' decision to abstain from the first stage or not. Rather, it depends only on $P(n)$, which, in turn, depends on whether other voters vote sincerely or not. Therefore, we will recalculate $P(n)$ in Steps B3 and B4 *without* the assumption that other voters will vote sincerely, and then analyze if voting sincerely is still a dominant strategy for i.

Step B3: Possible symmetric equilibria

We next consider all conceivable symmetric equilibria where the voters with the same preference intensity behave the same way. From Step B2, we know that the decision of individuals to abstain from the first stage or not can be disregarded for the question whether or not an individual votes sincerely in the first stage. Thus, any equilibrium under BV must belong to one of the following categories:

1. *SI* agents vote sincerely and *WI* agents vote insincerely,
2. *SI* agents vote insincerely and *WI* agents vote sincerely,
3. All agents vote sincerely,
4. All agents vote insincerely.

Suppose individual j, $j \neq i$, has also chosen to vote in the first stage. We introduce the following notation where Ω, $\Omega' \in \{A, B\}$, $\Omega \neq \Omega'$, VS (VI) denotes voting sincerely (insincerely) and $k \in \{1, 2, 3, 4\}$ denotes the equilibrium category above:

- $j_\Omega :=$ the event that j will vote for Ω,

- $j_{P\Omega}$:= the event that j prefers the direction Ω,
- j^k_{VS} := the event that j will vote sincerely in equilibrium k,
- j^k_{VI} := the event that j will vote insincerely in equilibrium k,
- $Pr[x]$:= the probability of event x.

For each equilibrium category $k \in \{1, 2, 3, 4\}$, we have $Pr[j^k_{VI}] = 1 - Pr[j^k_{VS}]$ and

$$\begin{aligned} Pr[j^1_{VS}] &= p, \\ Pr[j^2_{VS}] &= 1 - p, \\ Pr[j^3_{VS}] &= 1, \\ Pr[j^4_{VS}] &= 0. \end{aligned}$$

Step B4: Probability of voting for a fundamental direction

For each equilibrium category $k \in \{1, 2, 3, 4\}$, we now have

$$\begin{aligned} Pr[j_\Omega] &= Pr[j_\Omega | j_{P\Omega}]Pr[j_{P\Omega}] + Pr[j_\Omega | j_{P\Omega'}]Pr[j_{P\Omega'}] \\ &= Pr[j^k_{VS} | j_{P\Omega}]Pr[j_{P\Omega}] + Pr[j^k_{VI} | j_{P\Omega'}]Pr[j_{P\Omega'}]. \end{aligned}$$

Due to our definition of symmetric equilibria, the event that j votes sincerely is related to whether he is strongly – or weakly – inclined, and therefore, stochastically independent from the event that his preference is towards Ω or Ω'. Hence, we now have

$$\begin{aligned} Pr[j_\Omega] &= Pr[j^k_{VS}]Pr[j_{P\Omega}] + Pr[j^k_{VI}]Pr[j_{P\Omega'}] \\ &= Pr[j^k_{VS}]\frac{1}{2} + (1 - Pr[j^k_{VS}])\frac{1}{2} \\ &= \frac{1}{2}. \end{aligned}$$

Thus, in all possible symmetric equilibria under consideration from the perspective of individual i, individual j will vote for a fundamental direction with probability $\frac{1}{2}$. Therefore, even without the assumption of sincere voting by other agents, the construction of $P(n)$ remains unchanged. Hence, the analysis in Step A2 is still valid, and allows to conclude that in all possible equilibria, voting sincerely is a dominant strategy for individual i if he participates in voting in the first stage, no matter how other agents vote.

Step B5: Decision to abstain

Given that all agents vote sincerely in both stages, and by construction of H and ϵ, it is clear that the only constellation in which no agent has an incentive to deviate with respect to his choice whether or not to participate in the first stage is the behavior described in Proposition 1. Therefore, the constructed equilibrium is unique.

$\blacksquare$

Proof of Lemma 1:

The proof of Lemma 1 follows the exact steps of the proof of Proposition 1. The only difference is in the utility calculations which are presented below.

Step 1: Sincere voting in both stages

It is clear that agents who save their vote for the second stage vote sincerely. Suppose agent i decides to use his vote in the first stage and all other agents voting in the first stage vote sincerely. Then, i's expected probabilities of winning, derived in the proof of

Proposition 1, are modified as follows:

$$\Gamma^{Vo}_{WW}(N,x_i) = \begin{cases} P(x_i+1)\frac{1}{2} & \text{if } 0 \le x_i < N-1 \\ P(N)P(N) & \text{if } x_i = N-1 \end{cases}$$
$$\Gamma^{Vo}_{WL}(N,x_i) = \begin{cases} P(x_i+1)\frac{1}{2} & \text{if } 0 \le x_i < N-1 \\ P(N)\big(1-P(N)\big) & \text{if } x_i = N-1 \end{cases}$$
$$\Gamma^{Vo}_{LW}(N,x_i) = \begin{cases} \big(1-P(x_i+1)\big)\frac{1}{2} & \text{if } 0 \le x_i < N-1 \\ \big(1-P(N)\big)P(N) & \text{if } x_i = N-1 \end{cases}$$
$$\Gamma^{Vo}_{LL}(N,x_i) = \begin{cases} \big(1-P(x_i+1)\big)\frac{1}{2} & \text{if } 0 \le x_i < N-1 \\ \big(1-P(N)\big)\big(1-P(N)\big) & \text{if } x_i = N-1. \end{cases}$$

As in the proof of Proposition 1, voting sincerely is a best response if and only if Condition (B.3) is satisfied. After applying the modified probabilities of winning, it is clear that Condition (B.3) is satisfied as $P(x_i+1) > 1-P(x_i+1)$ for all $x_i \in [0, N-1]$. Therefore, if an individual decides to use his vote in the first stage, voting sincerely is a dominant strategy, no matter whether he is strongly or weakly-inclined.

Step 2: Decision to store the vote

Now suppose that i has decided to store his vote for the second stage. Then, his probabilities of winning are modified as follows:

$$\Gamma^{Ab}_{WW}(N,x_i) = \begin{cases} P(N)P(N) & \text{if } x_i = 0 \\ \frac{1}{2}P(N-x_i) & \text{if } 0 < x_i \le N-1 \end{cases}$$
$$\Gamma^{Ab}_{WL}(N,x_i) = \begin{cases} P(N)\big(1-P(N)\big) & \text{if } x_i = 0 \\ \frac{1}{2}\big(1-P(N-x_i)\big) & \text{if } 0 < x_i \le N-1 \end{cases}$$
$$\Gamma^{Ab}_{LW}(N,x_i) = \begin{cases} \big(1-P(N)\big)P(N) & \text{if } x_i = 0 \\ \frac{1}{2}P(N-x_i) & \text{if } 0 < x_i \le N-1 \end{cases}$$
$$\Gamma^{Ab}_{LL}(N,x_i) = \begin{cases} \big(1-P(N)\big)\big(1-P(N)\big) & \text{if } x_i = 0 \\ \frac{1}{2}\big(1-P(N-x_i)\big) & \text{if } 0 < x_i \le N-1. \end{cases}$$

As in the proof of Proposition 1, voting in the first stage is a best response if and only if critical Condition (B.6) is satisfied. Since the probability distribution of x_i is unknown at this stage, we once again derive a sufficient condition on M, as in the proof of Proposition 1. Applying the modified probabilities of winning to Expression (B.5), we obtain that an individual i will vote in the first stage if

$$M \geq \begin{cases} M_0(N) := \frac{P(N)-\frac{1}{2}}{1-P(N)} & \text{for} \quad x_i = 0, \\ M_3(N, x_i) := \frac{P(N-x_i)-\frac{1}{2}}{P(x_i+1)-\frac{1}{2}} & \text{for} \quad 0 < x_i < N-1, \\ M_4(N) := \frac{1-P(N)}{P(N)-\frac{1}{2}} & \text{for} \quad x_i = N-1. \end{cases} \tag{B.11}$$

Therefore, *SI* individuals with sufficiently large H use their vote in the first stage, while *WI* individuals with sufficiently small ϵ store their vote for the second stage.

Step 3: Existence of equilibrium

We note that $M_3(N, x_i)$ is monotonically increasing in x_i. Therefore, it reaches its maximum value for $x_i = N-2$ and minimum for $x_i = 1$. In Figure B.4, we plot $M_0(N)$, $M_3(N, N-2)$, $M_3(N, 1)$ and $M_4(N)$ against N.[4]

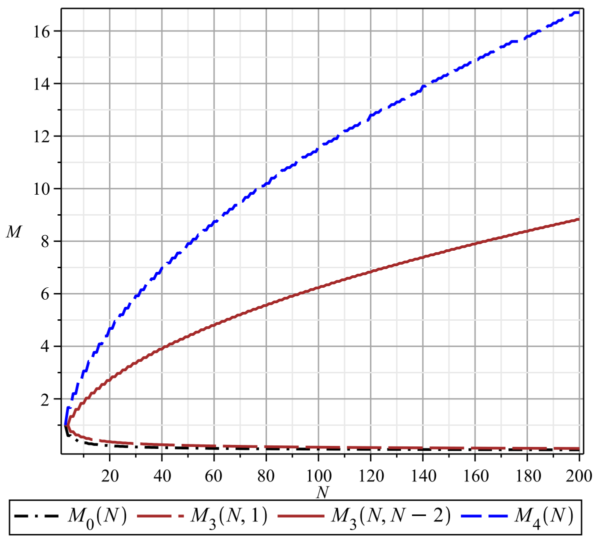

Figure B.4: $M_0(N)$, $M_3(N, N-2)$, $M_3(N, 1)$ and $M_4(N)$ vs. N.

We note that if x_i is very low (e.g. $x_i = 0$ or 1), the threshold on M monotonically

[4]Further properties of these expressions are given in Table A.2.

decreases with N for the same reason as with BV. The curve for $x_i = 0$ is, in fact, identical to the one under BV.

For x_i very large, however, the threshold on M is higher than under BV. A very large x_i under ST means a very small voting body in the second stage, as only those who abstain will vote in the second stage. Therefore, it is more attractive for individuals to abstain under ST, and M has to be higher than under BV for participation to be more attractive than abstention.

Moreover, for very large x_i (e.g. $x_i = N-1$ or $N-2$), the threshold on M is increasing with N. Under ST, as N increases, the number of agents voting in the first stage increases proportionately. The probability of winning in the first stage thus grows smaller, making the very high probability of winning in the second stage more attractive. Therefore, M has to increase analogously to keep participation more attractive than abstention.

We set

$$\begin{aligned} H &\geq H_{st}^{crit}(N) := \max\Big[M_0(N), M_3(N, N-2), M_4(N)\Big] = \frac{1-P(N)}{P(N)-\frac{1}{2}} \\ \epsilon &\leq \epsilon_{st}^{crit}(N) := \min\Big[M_0(N), M_3(N,1), M_4(N)\Big] = \frac{P(N)-\frac{1}{2}}{1-P(N)} \geq \epsilon^{crit}(N), \end{aligned}$$

so that all SI agents will participate in voting in the first stage. We refer to Figure 3.6 for the plot of $H_{st}^{crit}(N)$ and $\epsilon_{st}^{crit}(N)$ against N.

Step 4: Uniqueness

Following the same logic as in the proof of Proposition 1, the constructed equilibrium is unique.

■

Proof of Lemma 2:

We proceed in two steps.

Step 1: Sincere voting in second stage

As in the other voting schemes, there are no gains from voting strategically in the second stage. Hence, all individuals with voting rights cast their votes sincerely in the second stage, as they have a positive probability to be pivotal.

Step 2: Sincere voting in first stage

We note that no agent has an incentive to abstain from voting under MV. Abstention under MV means being excluded from voting in both stages, while participation in a vote strictly increases an individual's chances of winning. Furthermore, following the logic outlined in Proposition 1, in equilibrium, an agent i assumes that other individuals vote sincerely in the first stage because in all symmetric equilibria under consideration, an agent will vote for a fundamental direction with probability $\frac{1}{2}$. As a consequence, given that all N agents will vote in the first stage, i estimates that by voting sincerely, he will derive an ex-ante expected utility of

$$\begin{aligned} & P(N)\left[\frac{1}{2}(1+M)+\frac{1}{2}(0+M)\right]+\Big(1-P(N)\Big)\Theta(N) \\ = \; & P(N)\left[\frac{1}{2}+M\right]+\Big(1-P(N)\Big)\Theta(N), \end{aligned}$$

where $M \in \{H, \epsilon\}$ and $\Theta(N)$ is the ex-ante probability of winning in the second stage for losers of the first stage under MV, given by (3.4).

If i votes strategically, his expected utility is

$$\begin{aligned} & P(N)\frac{1}{2}+\Big(1-P(N)\Big)\left[\Theta(N)(1+M)+(1-\Theta(N))\,(0+M)\right] \\ = \; & \frac{1}{2}P(N)+\Big(1-P(N)\Big)\left[\Theta(N)+M\right]. \end{aligned}$$

Since $P(N) > \frac{1}{2} > 1 - P(N)$, voting sincerely is a dominant strategy for both SI and WI individuals.

■

Appendix C: Detailed Calculations

Derivation of $\Theta(N)$

$$\Theta(N) := \text{ Ex-ante probability of winning in the second stage under MV for losers of the first stage.}$$

We begin by defining the following probabilities.

$$\begin{aligned}
Prob(N_L) &:= \text{Probability that the first stage results in } N_L \text{ losers,}\\
Prob(Y) &:= \text{Probability that the first stage results in at least one loser,}\\
Prob(N_L|Y) &:= \text{Probability that the first stage results in } N_L \text{ losers, given that } N_L \geq 1.
\end{aligned}$$

We now obtain

Lemma 4

$$\Theta(N) = \sum_{N_L=1}^{\left\lfloor \frac{N}{2} \right\rfloor} P(N_L) \cdot Prob(N_L|Y).$$

We analyze two cases:

- Case 1: N is odd
  Now, $1 \leq N_L \leq \lfloor \frac{N}{2} \rfloor$. Losers can either prefer A or B.

$$\begin{aligned} Prob(N_L) = & \binom{N}{N_L} \frac{2}{2^N} = \binom{N}{N_L} \frac{1}{2^{N-1}}. \\ Prob(Y) = & \sum_{N_L=1}^{\lfloor \frac{N}{2} \rfloor} \binom{N}{N_L} \frac{1}{2^{N-1}}. \end{aligned}$$

- Case 2: N is even
  Now, $1 \leq N_L \leq \frac{N}{2}$. If $N_L = \frac{N}{2}$, the winner is chosen by tossing a fair coin.

$$Prob(N_L) = \begin{cases} \binom{N}{N_L} \frac{1}{2^{N-1}} & \text{for } 1 \leq N_L < \frac{N}{2}, \\ \\ \binom{N}{N_L} \frac{1}{2^N} & \text{for } N_L = \frac{N}{2}. \end{cases}$$

$$Prob(Y) = \sum_{N_L=1}^{\frac{N}{2}-1} \binom{N}{N_L} \frac{1}{2^{N-1}} + \binom{N}{\frac{N}{2}} \frac{1}{2^N}.$$

Using Bayes' Rule, we obtain

$$Prob(N_L|Y) = \frac{Prob(Y|N_L)Prob(N_L)}{Prob(Y)}.$$

We note that $Prob(Y|N_L) = 1$, given that $N_L \geq 1$. Substituting the values for $Prob(N_L)$ and $Prob(Y)$ obtained above in the expression for $\Theta(N)$ in Lemma 4 yields the expression of $\Theta(N)$ in the main text.

Derivation of $\Phi(N, n)$

We next derive the expression for $\Phi(N, n)$ for which we need $\Theta(N)$ calculated above.

$\Phi(N, n) :=$ Ex-ante probability of winning in the second stage under BV for losers of the first stage.

The derivation of $\Phi(N, n)$ follows the same steps as the derivation of $\Theta(N)$, with the following changes:

- Substituting n for N (since only n individuals vote in the first stage under BV),
- Substituting $P(N - n + N_L)$ for $P(N_L)$ (since $N - n$ agents who abstain from the first stage are also given the right to vote in the second stage under BV).

Derivation of $\Lambda(N, n)$

Next we derive the expression for $\Lambda(N, n)$.

$$\Lambda(N, n) := \text{ Ex-ante probability of winning in the second stage under BV for absentees of the first stage.}$$

$\Lambda(N, n)$ deals with the case $N_L = 0$. Hence, its derivation is straightforward.

$$\begin{aligned}
\Lambda(N, n) = & \sum_{N_L=0}^{\lfloor \frac{n}{2} \rfloor} P(N - n + N_L) \cdot \left(\text{probability of } N_L \text{ losers in the first stage}\right) \\
= & \sum_{N_L=0}^{\lfloor \frac{n}{2} \rfloor} P(N - n + N_L) \cdot Prob(N_L).
\end{aligned}$$

$Prob(N_L)$ is given by

- Case 1: n is odd

$$Prob(N_L) = \binom{n}{N_L} \frac{2}{2^n} = \binom{n}{N_L} \frac{1}{2^{n-1}}.$$

- Case 2: n is even

$$Prob(N_L) = \begin{cases} \binom{n}{N_L}\frac{1}{2^{n-1}} & \text{for } 0 \le N_L < \frac{n}{2}, \\ \\ \binom{n}{N_L}\frac{1}{2^n} & \text{for } N_L = \frac{n}{2}. \end{cases}$$

Substituting these values yields the expression for $\Lambda(N, n)$ in the main text.

Derivation of W^{BV}

$$\begin{aligned} W^{BV} = \; & \text{Aggregate expected utility for SI individuals when } 0 < N_s < N \\ & + \text{ Aggregate expected utility for WI individuals when } 0 < N_s < N \\ & + \text{ Aggregate expected utility for SI individuals when } N_s = N \\ & + \text{ Aggregate expected utility for WI individuals when } N_s = 0 \\ = \; & \sum_{N_s=1}^{N-1} \binom{N}{N_s} p^{N_s}(1-p)^{(N-N_s)} \Bigg[N_s \Bigg\{ P(N_s) \left(\frac{1}{2}(1+H) + \frac{1}{2}(0+H) \right) \\ & \qquad + \big(1 - P(N_s)\big)\Phi(N, N_s) \Bigg\} \\ & + (N - N_s) \left\{ \frac{1}{2} \Bigg(\Lambda(N, N_s)[1+\epsilon] + (1 - \Lambda(N, N_s))[0+\epsilon] \Bigg) + \frac{1}{2}\Lambda(N, N_s) \right\} \Bigg] \\ & + p^N N \left[P(N) \left\{ \left(1 - \frac{1}{2^{N-1}}\right) \left(\frac{1}{2}(1+H) + \frac{1}{2}(0+H) \right) \right. \right. \\ & \qquad \left. \left. + \left(\frac{1}{2^{N-1}} \right) \Bigg(P(N)(1+H) + (1 - P(N))H \Bigg) \right\} + \big(1 - P(N)\big)\Theta(N) \right] \\ & + (1-p)^N N \left[P(N) \Bigg(P(N)(1+\epsilon) + (1 - P(N))(\epsilon) \Bigg) + (1 - P(N))P(N) \right]. \end{aligned}$$

Simplifying this expression yields

$$\begin{aligned} W^{BV} = \; & \sum_{N_s=1}^{N-1} \binom{N}{N_s} p^{N_s} (1-p)^{(N-N_s)} \Bigg[N_s \left\{ P(N_s) \left(\frac{1}{2} + H \right) + \big(1 - P(N_s)\big) \Phi(N, N_s) \right\} \\ & \qquad\qquad\qquad\qquad + (N - N_s) \left\{ \Lambda(N, N_s) + \frac{\epsilon}{2} \right\} \Bigg] \\ & + p^N N \left[P(N) \left\{ \left(1 - \frac{1}{2^{N-1}} \right) \left(\frac{1}{2} + H \right) + \left(\frac{1}{2^{N-1}} \right) (P(N) + H) \right\} \right. \\ & \qquad \left. + \big(1 - P(N)\big) \Theta(N) \right] + (1-p)^N N P(N) (\epsilon + 1). \end{aligned}$$

Appendix D: List of Notation

Symbol	***Meaning***
N	Number of voters
A, B	Fundamental directions
i, i'	Denotes an individual in the society
x_Ω^j	Variations of fundamental directions, $\Omega \in \{A, B\}$, $j = 1, 2$
j, k, j', k'	Used in the notation for variations: x_Ω^b , where $b = j, k, j', k' = 1, 2$, $\Omega \in \{A, B\}$
Ω, Ω'	Used in the notation for variations: x_c^j , where $j = 1, 2$, $c = \Omega, \Omega'$
$U_i(x_\Omega^j \vert \Omega)$	Utility of individual i if Ω and x_Ω^j are selected with $\Omega \in \{A, B\}$, $j = 1, 2$
$U_i(x_\Omega^j)$	Same as $U_i(x_\Omega^j \vert \Omega)$
SI	The group of strongly-inclined individuals
WI	The group of weakly-inclined individuals
N_s	The number of strongly-inclined individuals
N_w	The number of weakly-inclined individuals
H	A constant denoting the intensity of preference of strongly-inclined individuals
ϵ	A constant denoting the intensity of preference of weakly-inclined individuals
p	The probability with which an individual may have a strong inclination towards A or B
SM	Simple majority voting
BV	Balanced Voting
ST	Storable Votes
MV	Minority Voting
$P(n)$	Probability of winning for an individual if n agents cast their votes,

	in an isolated voting problem characterized in Section 3.2.1
$H^{crit}(N)$	The critical threshold on H $(H \geq H^{crit}(N))$ for the existence of a unique equilibrium under BV
$\epsilon^{crit}(N)$	The critical threshold on ϵ $(\epsilon \leq \epsilon^{crit}(N))$ for the existence of a unique equilibrium under BV
$H^{crit}_{st}(N)$	The critical threshold on H $(H \geq H^{crit}_{st}(N))$ for the existence of a unique equilibrium under ST
W^V	Aggregate expected utility before utilities are realized, $V \in \{SM, BV, ST, MV\}$
$\mathbb{E}$	Expectation operator before voting begins
n	The number of agents voting in the first stage
N_L	The number of losers in the first stage
$\Theta(N)$	Ex-ante probability of winning in the second stage for losers of first stage, under MV
$\Phi(N, n)$	Ex-ante probability of winning in the second stage for losers of first stage, under BV
$\Lambda(N, n)$	Ex-ante probability of winning in the second stage for absentees of first stage, under BV
$\epsilon^{SM}(N, N_s)$	The threshold on ϵ for *WI* agents to prefer BV over SM when $0 < N_s < N$
$Q(N)$	Expression given in (3.8)
$H^{SM}(N, N_s)$	The threshold on H for *SI* agents to prefer BV over SM when $0 < N_s < N$
$M(N, p)$	Expression given in (3.12)
$C(N, p)$	Expression given in (3.13)
$K(N, p)$	Expression given in (3.15)
$N_{st}(p)$	Expression given in (3.16), denoting the threshold on N for $W^{BV} \geq W^{ST}$
$\epsilon^{MV}(N, N_s)$	The threshold on ϵ for *WI* agents to prefer BV over MV when $0 < N_s < N$
$H^{MV}(N, N_s)$	The threshold on H for *SI* agents to prefer BV over MV when $0 < N_s < N$
$D(N, p)$	Expression given in (3.24)
M	Intensity of an individual's preferences, $M \in \{H, \epsilon\}$

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