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Author(s):
Mozaffari, Salma; Bernal, Dionisio P.; Döhler, Michael

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CHANGE DETECTION UNDER ENVIRONMENTAL VARIABILITY

Salma Mozaffari Kojidi[1], Dionisio Bernal[2], Michael Döhler[3]

ABSTRACT

Change detection is the process of announcing, from inspection of the sequence of measured signals, if changes in a system have taken place. Change detection is tantamount to a hypothesis test on the likelihood of the aggregate of the data given a statistical model of reference state. A basic assumption in change detection is that the data points are independent and identically distributed (i.i.d) under the null hypothesis. When the data depends on environmental parameters the i.i.d premise for a sequential stream of points is not justified. Operations on the data can be performed, however, so the premise is reasonably satisfied and the change detection scheme can then be applied. Amomg altenatives that do not require that the environmental parameters be measured the one that has been most widely examined is Principal Component Analysis. In this case the data is projected into a subspace where the variance from the environment is small. The price paid is that the damage related information on the principal directions is then discarded. There are (at least) two other possibilities: one is to decorrelate the data using an autoregressive model and the other is to breake the correlation by scrambling. In the later case the price is that detection has to be carried out with some irreducible delay. This paper examines the relative merit of the alternatives noted.

Keywords: Damage detection, Cumulative Sum control chart, Principal Component Analysis, environmental variability

1. INTRODUCTION

Data-driven, point-in-time, damage detection is a one class classification problem. Namely, a statistical model of the reference state is formulated and the data during monitoring is classified as novel or normal depending on the likelihood that it is from the established model. The fact that the system does not switch back and forth from healthy to damage can be taken advantage of by formulating the problem as one of the change detection, instead of as a point-in-time classification.

[1] Research Assistant, Northeastern University, mozaffarikoj.s@husky.neu.edu
[2] Professor, Northeastern University, bernal@neu.edu
[3] Post-doc., Northeastern University, m.doehler@neu.edu

The difficulty that arises in cvil engineering applications, however, is that the data is affected by the environmental conditions and as a result, the assumption that the points are idependent and identically distributed (i.i.d) is often far from valid. Needless to say, if one can formulate a model for the environmental effects so that the data can be " normalized", then the issues noted are essentially removed. In the opinion of the writers this is the most effective way to treat the problem when feasible [1,2,3].

An alternative to compensate for enviromental fluctuations without formulating a model for environment is projecting the feature vector to a subpace where the fluctuations in the data is small. The price to pay resides in the fact that when damage takes place, all the information due to damage that projects in the principal directions is discarded. The potential for type II error is thus increased and it is difficult to ascertain quantitatively how much because results depend on specifics.

Another alternative to make the i.i.d premise reasonable is by decorrelating the data using an AutoRegressive model that subtracts the trend. A third and final alternative treated here is by scrambling. In this case one can easily ascertain that the damage is identified with an irreducible delay.

In this paper we compare the results of decorrelation and scrambling with preforming PCA. The results shown are for a 10 DOF uniform shear system. To allow dipicting results in 2D we consider the features as the first and the second frequencies.

2. PRINCIPAL COMPONENT ANALYSIS

Let the recorded data be used to obtain a vector $x \in R^{m\times L}$ that depends on system properties. One has, therefore, $\mathrm{X} = \{x_1 \quad x_2 \quad \cdots \quad x_L\}$. Assume it is known that the structure, for *j =1, …, L,* is in the undamaged state. Let $\mu \in R^{m\times 1}$ be the mean of the columns in *X*, define the covariance matrix as

$$E = \frac{1}{L}\sum_{k=1}^{L} x_k x_k^T \tag{1}$$

Perform a SVD of *E* and partition into the subspaces ϑ_1 and ϑ_2, namely

$$E = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} S_1 & \\ & S_2 \end{bmatrix} \begin{bmatrix} U_1^T \\ U_2^T \end{bmatrix} \tag{2}$$

where S_2 is selected such that the largest entry in S_2 is small compared to the smallest entry in S_1. The model of the reference is {μ, S_2 and U_2}. Upon the arrival of new data one computes

$$\overline{x}_{v,k} = x_{v,k} - \mu \tag{3}$$

and the projection in ϑ_2 (sometimes referred to as the reconstruction error) as

$$pr_k = U_2^T \overline{x}_{v,k} \tag{4}$$

the squared Mahalanobis distance [4] (the discriminating metric) is then

$$M(pr_k) = pr_k^T S_2^{-1} pr_k \tag{5}$$

Use of PCA in damage detection under environmental variability appears in [5].

3. CHANGE DETECTION USING CUMULATIVE SUM CHART

The Cumulative Sum (CUSUM) chart [6] is designed to perform a hypothesis test regarding the shift in any parameter of a distribution that is assumed to prevail when the null hypothesis is true. An essential assumption is the sample points are independent. In this section we present general expressions for the exponential distribution family and particular results for the Gaussian and the Gamma density functions.

Let y be the novelty metric and θ be the parameter that is allowed to vary in the distribution (parameters vary one at a time) one has, for the exponential family

$$f\left(y|\theta\right) = e^{\{a(y)b(\theta)+c(y)+d(\theta)\}} \tag{6}$$

Assuming independence, the probability of the sequence of values $Y=y_i$ and $i=1,\ldots, n$, follows from Equation 6 as

$$f\left(Y|\theta\right) = e^{\left\{b(\theta)\sum_{j=1}^{n} a(y_j)+\sum_{j=1}^{n} c(y)+nd(\theta)\right\}} \tag{7}$$

Consider two postulated parameter θ_0 and θ_1, we want to determine which of the two is more likely given the data. The answer is given by Equation 7 evaluated for $\theta = \theta_1$ divided by the same expression evaluated for $\theta = \theta_0$. If the ratio is larger than1, then θ_1 is more likely and the opposite is true if the value is smaller than 1. The ratio is

$$\Upsilon = \frac{f\left(Y|\theta_1\right)}{f\left(Y|\theta_0\right)} = e^{\left\{(b(\theta_1)-b(\theta_0)\sum_{j=1}^{n} a(y_j)+n(d(\theta_1)-d(\theta_0))\right\}} \tag{8}$$

or, designating $\Im = \ln(\Upsilon)$

$$\Im_n = (b(\theta_1)-b(\theta_0))\sum_{j=1}^{n} a(y_j) + n(d(\theta_1)-d(\theta_0)) \tag{9}$$

From inspection of Equation 9 it follows that

$$\Im_n = \Im_{n-1} + (b(\theta_1)-b(\theta_0))a(y_n) + (d(\theta_1)-d(\theta_0)) \tag{10}$$

or

$$\Im_n = \Im_{n-1} + (b(\theta_1)-b(\theta_0))\left\{a(y_n)+k\right\} \tag{11}$$

where one has

$$k = \frac{d(\theta_1)-d(\theta_0)}{b(\theta_1)-b(\theta_0)} \tag{12}$$

Negative values of $\Im_n$ indicate that $\theta = \theta_0$ is most likely, a trend in the upward direction indicates that the alternative hypothesis is the more likely one. While one can focus on the slope of $\Im_n$, it is more convenient to inspect the attainment of a critical upper movement once the slope turns positive. This is easily done by reinitializing $\Im_n$ whenever it goes negative and setting some limiting positive value, h^+ as a level which, when crossed, damage is announced. Our experience is that taking $\theta_1 = 1.5\theta_0$ (or 1/1.5 if damage decreases the parameter) and selecting h^+ as a hard threshold using data for the training phase is adequate. Expressions for a(y), b(θ) and d(θ) for specific parameters and distributions are presented in Table1.

Step by step procedure

- Compute L values of the metric in the reference state
- Fit a distribution to the metric (if neither Gaussian nor Gamma works, then results for some others can be found in [7]).
- Determine if θ increases or decreases due to damage and select θ_1 accordingly
- Start with $\Im_0 = 0$, compute k, and evaluate $\Im_n$ using Equation 11 for the healthy state. Recall that if $\Im$ goes negative at any step it is replaced by zero before executing the next step.
- From inspection of the results, select h^+.
- Process the data of the test phase exactly as before and announce damage when the threshold h^+ is crossed.

Table 1 Parameters for implementation of the CUSUM

Distribution	θ	a(y)	b(θ)	d(θ)
Gaussian N(μ,σ)	μ	y	$\frac{\mu}{\sigma^2}$	$-\frac{\mu^2}{2\sigma^2}$
	σ	$(y-\mu)^2$	$-\frac{1}{2\sigma^2}$	$-\ln(\sigma)$
Gamma Γ(α,β)	α	$\ln(y)$	α-1	$\ln(\Gamma(\alpha)) - \alpha\ln(\beta)$
	β	y	$-\frac{1}{\beta}$	$-\alpha\ln(\beta)$

4. CUSUM WITH CORRELATED DATA

4.1. Decorrelation

A simple approach often used to allow the assumption of i.i.d is to formulate a model for the trend of the metric, subtract out the trend, and perform the detection on the residual. One of the simplest models for the trend is the first order AutoRegressive model, AR(1), and this is one alternative we use in the numerical example.

4.2. Scrambling

Another possibility that eliminates the correlation in the data is to scramble it randomly. In this case the reference condition can be treated without difficulty, but the price to pay is that detection cannot be carried out in "near real time" but has to be performed in delayed mode. Namely, it is necessary in this case to wait for a sufficient amount of time during testing so that scrambling of the data is meaningful. As one gathers, in this approach damage may or may not be detected in the interval that it occurs because it may occur late in the segment. In these cases it is detected in the next cycle. The longest time from damage to detection, therefore is somewhat longer than the scrambling segment duration. If the damage is present for a substantial time during the segment, it is detected in the segment and the time from damage to detection is a fraction of the scrambling segment duration.

5. NUMERICAL EXAMINATIONS

Consider a uniform 10 DOF mass-spring system with initial stiffness k_0. The spring stiffness is assumed to be a function of temperature as:

$$k = k_0(1 - 0.22(\frac{T - T_0}{T_0})^2 sgn(T - T_0)) \tag{13}$$

where T_0 = 60°F. Figure 1a shows how temperature varies during a year, and Figure 1b shows how the sampling is performed during different temperature ranges.

Data points are vectors of first and second frequencies, and a reasonable variation due to noise compare to temperature variation is added to the data. The data pattern is depicted in Figure 2a. Figure 3b shows the subspace angle between the change in the feature vector and U_2 due to 15% stiffness reduction in each spring. The severity of damage is found to have a negligible on this plot.

Since the subspace angle in Figure 2b is minimum for spring #4 and maximum for spring #6, damage in the former is easy to detect but it is difficult in the later when PCA is used.

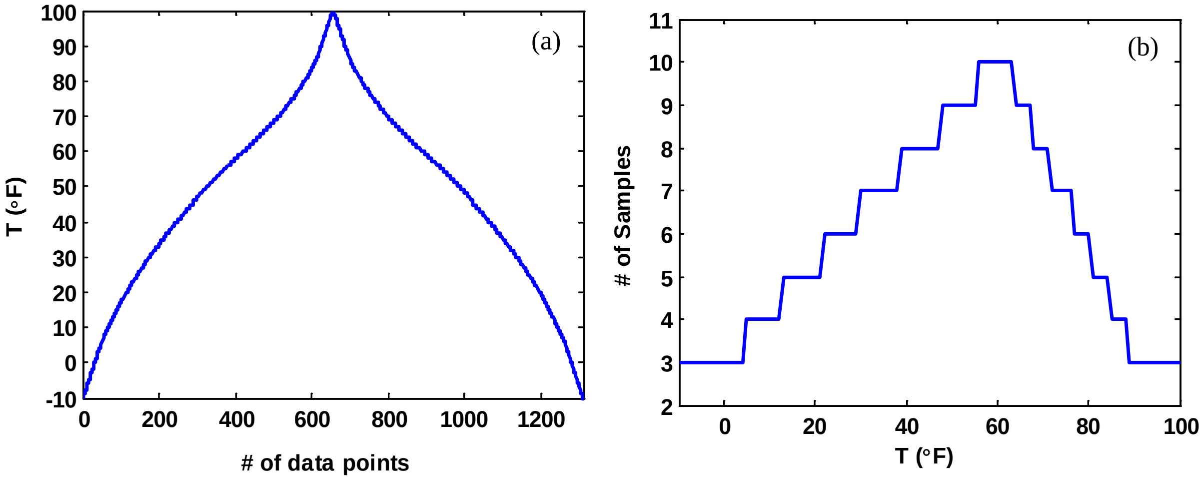

Figure 1 (a) yearly temperature variation, (b) sampling rate vs. temperature range

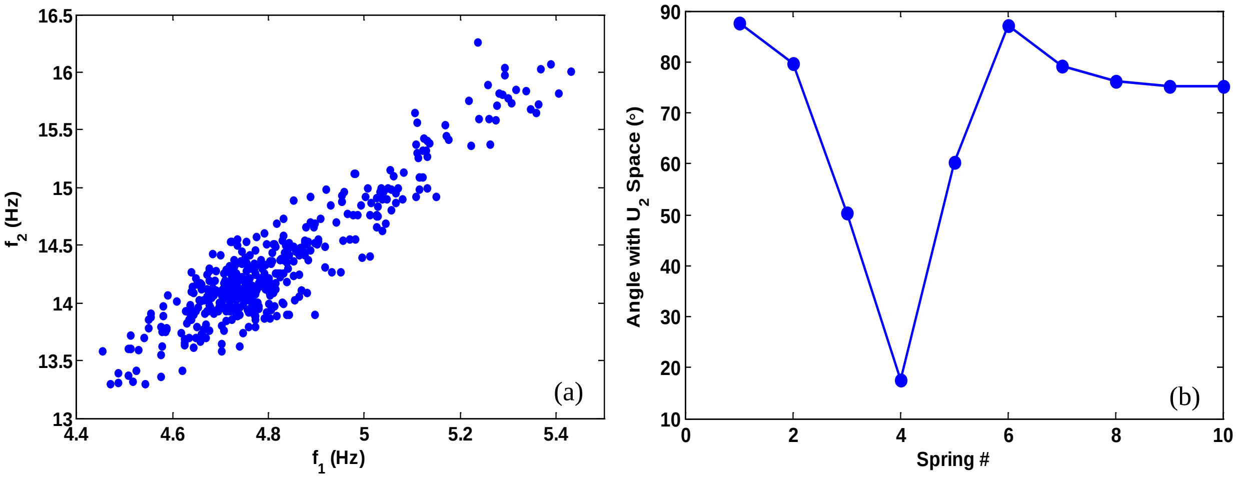

Figure 2 (a) First frequency vs. second frequency, (b) subspace angle of the change due to 15% damage with U_2 for all springs

5.1. Results

Data for 6 month is used to formulate the model, the threshold is selected in the following 6 months of validation, and in the last 6 months, testing is done where damage is introduced to the system 2.5 months after monitoring starts. The novelty metric in all cases except for the decorrelation is the squared Mahalanobis distance, where the CUSUM chart tests the shift in the α parameter of the Gamma distribution (the results for the shift in β is similar). In decorrelation case, the metric is the residual of squared Mahalanobis distance after performing AR(1). In this case, the CUSUM chart examines the shift in the mean of Gaussian distribution.

Figure 3 shows the result for projected data while spring #6 is damaged by 30%, and shows that damage cannot be recognized. In Figure 4, spring #4 is damaged by 15%, and damage is readily identified. The two plots confirm the previous discussion on the angle between the direction of the change due to damage and the U_2 space.

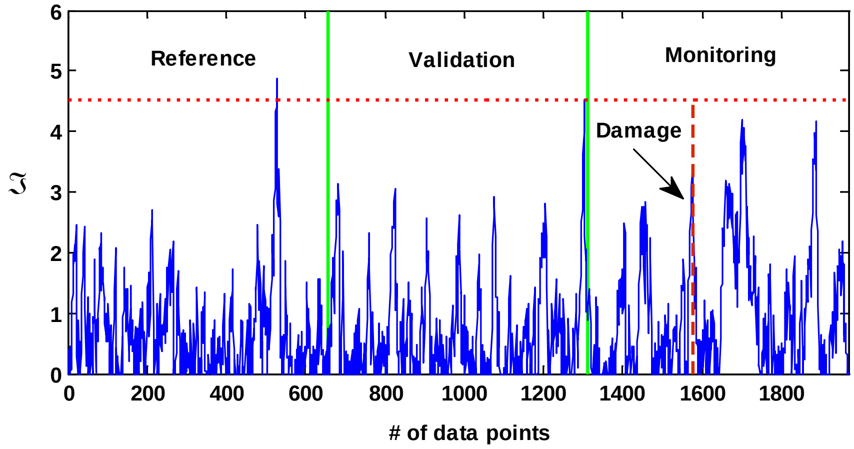

Figure 3 Projection performance with 30% damage on spring #6

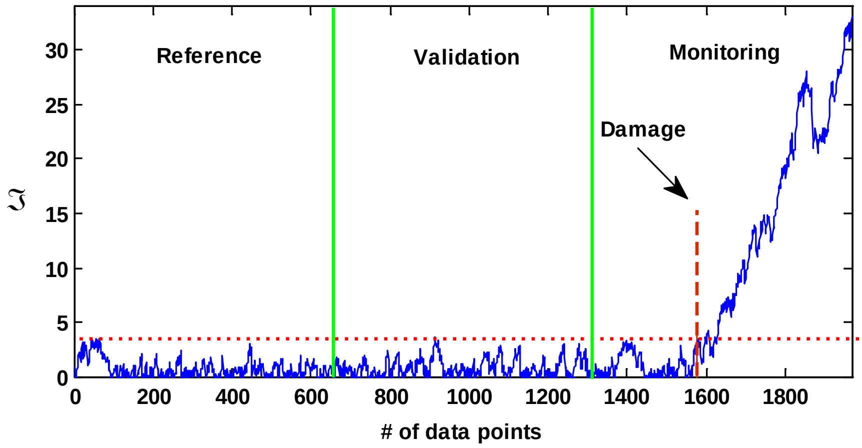

Figure 4 Projection performance with 15% damage on spring #4

The results when decorrelation is used are poor and are shown in Figure 5a. Figure 5b depicts the results when the data is scrambled. In this case the damage is clearly detected. The results for spring #4 with 15% damage are not shown for brevity but are similar to those in Figure 5.

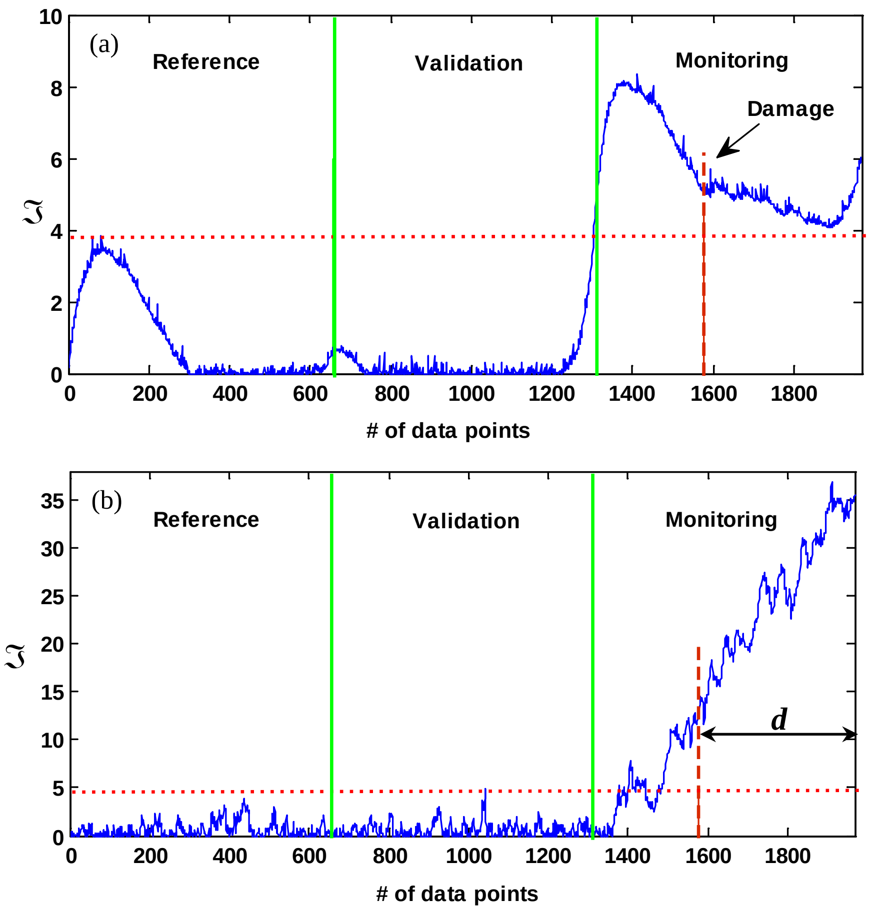

Figure 5 Performance with 30% damage on spring #6 (a) decorrelating (b) scrambling (d = time from damage to the announcement of damage at the end of the segment)

6. CONCLUSIONS

This paper examines change detection in data that depicts linear preferential diretions in input space. The issue is one of ensuring that the assumption of i.i.d is reasonably satisfied while retaining maximum sensitivity to damage. PCA is ideal for ensuring i.i.d in the reference state but it also leads to significant loss of sensitivity since the full projection of damage information in the principal directions is discarded. Decorrelation was not studied in detail, but a pilot study using an AR(1) model to eliminate the trend did not display good performance. Scrambling is not (in principle) as good as PCA in the reference state because the frequency at which the system is sampled (at different temperatures) plays some role, but the approach was found to perform well in the numerical study. The price paid in the scrambling scheme is the fact that one must necessarily wait before damage is announced; the waiting time being a random variable with statistics that depend on the length of the scrambling segment.

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