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Distributed Estimator Design for a Formation with Markovian Communication Topology

Maxim V. Subbotin, Roy S. Smith

Abstract— **A solution to the synthesis problem of distributed decentralized estimator for a formation of agents using local measurements and inter-agent communication is proposed. Agents can communicate between each other through digital unidirectional links modeled with two-state Markov chains which results in a stochastic communication topology. The design procedure is based on convex optimization problem formulated with linear matrix inequalities (LMIs).**

I. Introduction

In recent years, there has been a lot of interest in decentralized and distributed estimation problems. An extensive research in that area has been motivated by a growing class of distributed control applications which include formation control, sensor networks, and distributed power systems [3]-[6], [10]-[13]. In many formulations the estimation problem was shown to be closely connected to the control problem [13], [11], with structure and properties of information flow in the system playing a central role. The issues of communication constraints and their influence on the formation stability and achievable performance were addressed by Fax and Murray [3], Smith and Hadaegh [11], and Tatikonda and Mitter [12]. Yan, Kang, and Bitmead [6], [13] considered a coordinated control problem for a formation of vehicles and offered an estimator design procedure for a class of decoupled linear systems with a specific communication architecture.

In this paper we propose a solution to the problem of synthesis of distributed decentralized estimator for a formation of agents described by a discrete linear time-invariant (LTI) system. We develop design procedures for the type of systems considered by Smith and Hadaegh in [11], but use a different and more realistic communication model. We consider formations where agents are coupled by a common formation objective function and each agent carries an entire formation state estimate. This estimation architecture is essential for control problems with formation-wide objective functions, such as keeping precise formations, implementing formation reconfiguration or collision avoidance without higher level control commands. In our work we do not impose any assumptions on the structure of the formation system dynamics and hence consider general linear systems.

The main result of this paper is a procedure for the synthesis of decentralized distributed estimator for a formation of agents communicating though unidirectional links subject to random packet loses. In addition to their own measurements agents of the formation use the information received from other agents to update their formation state estimates. In our work we do not impose any assumptions on the structure of information flow between agents and consider general communication topologies. Each agent of the formation is viewed as a node of a directed graph describing communication topology. Each unidirectional communication link transferring data between two agents is considered to be a discrete channel with possibility that the transmitted information can be completely lost. To capture the stochastic nature of a channel we model it as a two-state Markov chain, which results in a Markovian communication topology. To describe the formation with the Markovian communication topology and develop the design tools, we use results of Costa and Fragoso [1]. As a performance measure in the design problem we use a norm of an estimation error correlation matrix. Utilizing the recent results of Oliveira et al. [7], [8], we propose a synthesis procedure for the estimator design, formulated with linear matrix inequalities.

In Section 2 we describe class of systems we consider through out the paper and introduce the notation and variables we use for the synthesis of distributed estimator. In Section 3 we develop the synthesis procedure. Section 4 presents a design example with experimental results.

Throughout the paper we use the following notation. The symbol $\otimes$ is used to denote the Kronecker product. The identity matrix with dimension $n \times n$ is defined as I_n and a column vector with the dimension n and all elements equal to 1 is defined as 1_n. A block diagonal matrix B with submatrices $B_i,\ i = 1, ..., n$ on the diagonal is denoted by $B = \text{diag}(B_1, ..., B_n)$ or $B = \text{diag}_i(B_i)$.

II. Problem Formulation

We consider discrete LTI systems described by

$$x(k+1) = Ax(k) + B_u u(k) + B_v v(k), \quad (1)$$

where $x(k) \in I\!R^{n_x}$ is the system state, $u(k) \in I\!R^{m_u}$ is the actuation input, and $v(k) \in I\!R^{m_v}$ is a zero-mean, Gaussian process noise with covariance Q_v. The state dynamics (1) represent the collective formation dynamics of N vehicles; the agents of the formation. The control input is composed of individual control inputs of each agent, $u(k) = \sum_{i=1}^{N} u_i(k)$, and i^{th} agent's control signal, which corresponds to the control of local actuators, is defined by $u_i(k) = \Pi_i u(k) \in I\!R^{m_u}$, where Π_i is the projection matrix and $\sum_{i=1}^{N} \Pi_i = I$. Each agent is able to measure the signal,

$$y_i(k) = C_i x(k) + n_i(k), \quad (2)$$

where $y_i(k) \in I\!R^{k_{y_i}}$ is the system output, agent's local measurements, available to agent i, and $n_i(k) \in I\!R^{k_{y_i}}$ is a zero-mean, Gaussian measurement noise with variance Q_{n_i}.

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We assume that a stabilizing state feedback, $u(k) = -Kx(k)$, which satisfies a formation wide objective function, is given and specifies the desired closed-loop dynamics of the formation through the matrix $A_{clp} = A - B_u K$. Since we focus on the estimation part of the problem, we do not consider a particular method for choice of K. The formation control law, $u(k)$, is calculated and implemented by each agent individually using available measurements and information transmitted from other agents, hence resulting in a decentralized and distributed architecture. Each i^{th} agent's control system consists of a combination of a full order formation state estimator which provides $\hat{x}_i(k) \in \mathbb{R}^{n_x}$ and state feedback for the calculation of $u_i(k)$. As a result, the i^{th} agent's contribution to the control input is given by,

$$u_i(k) \quad = \quad -\Pi_i K \hat{x}_i(k). \tag{3}$$

To be able to approach the distributed estimator design problem, we first derive the equations describing the complete closed-loop formation dynamics. For simplicity, we start our derivations for the case where each agent's estimator uses only agent's local measurements to update its formation state estimate, and then introduce communication in the estimation structure. If there is no communication between agents, the state estimator of the i^{th} agent, used to provide formation state estimate, $\hat{x}_i(k)$, is given by,

$$\hat{x}_i(k+1) = (A - B_u K)\hat{x}_i(k) + L_i(y_i(k) - C_i \hat{x}_i(k)), \tag{4}$$

We can define an estimation error for each agent,

$$e_i(k) \quad = \quad x(k) - \hat{x}_i(k).$$

Then the closed-loop plant dynamics are given by,

$$x(k+1) = A_{clp} x(k) + B_u \sum_{i=1}^{N} \Pi_i K e_i(k) + B_v v(k). \tag{5}$$

And the i^{th} estimation error dynamics are,

$$e_i(k+1) = x(k+1) - \hat{x}_i(k+1) = (A_{clp} - L_i C_i) e_i(k) + $$
$$B_u \sum_{i=1}^{N} \Pi_i K e_i(k) + B_v v(k) - L_i n_i(k). \tag{6}$$

If we collect estimation errors in one vector, $e(k) = [e_1(k)' \ e_2(k)' \ \dots \ e_N(k)']' \in \mathbb{R}^{N n_x}$, we can write the collected estimation error dynamics which together with (5) describe completely dynamics of the closed-loop system,

$$e(k+1) = (A_B - L_f C_f) e(k) + \Gamma v(k) - L_f n(k), \tag{7}$$

where $A_B = I_N \otimes A_{clp} + M_N$, $M_N = 1_N \otimes M_1$, $M_1 = [B_u \Pi_1 K \ \dots \ B_u \Pi_N K]$, $L_f = \operatorname{diag}(L_1, \dots, L_N)$, $C_f = \operatorname{diag}(C_1, \dots, C_N)$, $\Gamma = 1_N \otimes B_v$, and $n(k) = [n_1' \ \dots \ n_N']' \in \mathbb{R}^{\sum_{i=1}^{N} k_{y_i}}$ with $Q_n = \operatorname{diag}(Q_{n_1}, \dots, Q_{n_N})$.

It is clear that the block-diagonal matrix $L_f C_f$ is limited in its ability to stabilize the collected estimation error dynamics given by $A_B - L_f C_f$. To improve that situation, we add communication into the distributed estimation structure. By communication we mean a transfer of information about the formation state estimates, $\hat{x}_i(x)$, between agents' estimators. As will be evident from our derivations, communication introduces new terms into the collected estimation error dynamics matrix and modifies all blocks of A_B, not only those which are on the diagonal. We assume that the agents of the formation can transmit information between each other through unidirectional links.

In this paper we consider communication links which can be modeled as channels able to carry vectors of real numbers without any corruption, but the information transmitted though a channel can be completely lost. This model is often used in the literature [5] to describe digital channels which are used to transfer information in packets, and the failure to transmit a packet though a channel corresponds to a packet dropout due to, for example, transmission delays or clock synchronization errors.

With this argument in mind, we consider the following model for a single unidirectional communication link used to transmit information from agent j to agent i,

$$t_{ij}(k) = \mu_{ij}(k) H_{ij} \hat{x}_j(k), \tag{8}$$

where $t_{ij}(k) \in \mathbb{R}^{k_{ij}}$ is the signal received by the estimator of the i^{th} agent from the estimator of the j^{th} agent. $H_{ij} \in \mathbb{R}^{k_{ij} \times n_x}$ is the transmitter gain matrix, and $\mu_{ij}(k)$ is a binary parameter describing success of transmission, so that $\mu_{ij}(k) = 1$ if the transmission is successful and $\mu_{ij}(k) = 0$ if not. The binary parameter $\mu_{ij}(k)$ modeling success or failure of the transmission can be defined to be either stochastic or deterministic variable. The most common and widely accepted method for specifying $\mu_{ij}(k)$ is modeling it as a stochastic variable described by either a Bernoulli process or a finite-state Markov chain (see [5] and the references therein). In this paper we consider a model where $\mu_{ij}(k)$ can be described by a two-state Markov chain represented graphically in Figure 1. State 0 corresponds to a failure in a link and state 1 corresponds to a successful transmission. Transition probabilities p_{ij}^0 and p_{ij}^1 describe probabilities of staying in state 0 and state 1 correspondingly.

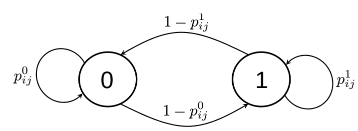

Fig. 1. Two-state Markov chain.

Assume that there are N_l unidirectional links in the topology describing communication between agents' estimators in the formation. The fact that each of N_l agents received or did not receive its corresponding information at step k can be described by a vector $\Theta(k) = [\Theta_1(k) \ \Theta_2(k) \ \dots \ \Theta_{N_l}(k)] \in \mathbb{R}^{N_l}$, where each element of the vector $\Theta_l(k)$, $l = 1, \dots, N_l$ is equal to the state of the Markov chain $\mu_{ij}(k)$ for one of the links in the topology. If the transitions between states of the Markov chains for individual links are independent, then $\Theta(k)$ is itself an element of a finite-state Markov chain with M states, $M = 2^{N_l}$, since each element of $\Theta(k)$ vector can take one of two values, 0 or 1, independently of other elements of the vector. To define the M-state Markov chain modeling communication, we introduce a state $\theta(k)$ which takes values in $\{1, \dots, M\}$ and corresponds to one of M vectors $\Theta(k)$. We also define $\pi_s(k) = P\{\theta(k) = s\}$, $s = 1, \dots, M$, a probability of being in state s at time k, and $\mathcal{P} \in \mathbb{R}^{M \times M}$, a transition probability matrix. The elements of matrix $\mathcal{P}$, p_{st}, $s = 1, ..., M$, $t = 1, ..., M$ can be calculated using the transition probabilities of Markov chains for individual links, p^0_{ij}, p^1_{ij}, by simply taking the products of M probabilities describing transition from state $\theta(k)$ to $\theta(k+1)$. For later derivations we define a row vector of the probability distribution for the states of the chain, $\pi(k) = [\pi_1(k)\ \pi_2(k)\ ...\ \pi_M(k)] \in I\!R^M$, and its evolution is described by,

$$\pi(k+1) = \pi(k)\mathcal{P}. \tag{9}$$

Using the proposed communication model we derive new equations for the collected estimation error dynamics. We consider the estimators where each i^{th} agent of the formation receiving information from other agents' estimators updates its formation state estimate according to the following model,

$$\hat{x}_i(k+1) = (A - B_u K)\hat{x}_i(k) + L_i(y_i(k) - C_i\hat{x}_i(k)) + \sum_j \mu_{ij}(k) F_{ij}(t_{ij} - H_{ij}\hat{x}_i(k)), \tag{10}$$

where F_{ij} is the receiver gain matrix which corresponds to the transmitter gain matrix H_{ij} and the sum is taken over all received signals. This structure for applying the information communicated from other estimators preserves the separation between the collected estimation error dynamics and the closed-loop plant dynamics. The use of H_{ij} allows generality in the choice of information sent between agents, and we consider both F_{ij} and H_{ij} as design variables in the estimator synthesis problem. After the time interval allocated for the transmission, each i^{th} agent of the formation is able to say if it received or did not receive the information from other agents. This fact is reflected in binary variable $\mu_{ij}(k)$ present in the update equation, so if agent i did not receive information from agent j, then $\mu_{ij}(k) = 0$ and $\mu_{ij}(k) = 1$ otherwise.

Now we would like to specify communication topology describing the information flow between the agents, for that purpose we take an approach similar to the one in [3]. We consider each agent of the formation to be a node of a graph. To be able to specify the topology of the graph we use matrices $\mathcal{L}_j \in I\!R^{N\times N}$, $j = 1, ..., N$. We call the matrix $\mathcal{L}_j$ a Laplacian, though our definition does not coincide with the standard one used in graph theory. Each Laplacian, $\mathcal{L}_j$, specifies communication between the j^{th} agent and all other agents of the formation and is defined as follows: elements l_{st} of the Laplacian, L_j, satisfy, $l_{sj} = -1$, $l_{ss} = 1$ if there is a communication link from agent j to agent s and $l_{st} = 0$ otherwise. To specify the communication topology of the whole formation, we introduce the collected Laplacian, $\mathcal{L}_f = [(\mathcal{L}_1 \otimes I_{n_x})'\ (\mathcal{L}_2 \otimes I_{n_x})'\ ...\ (\mathcal{L}_N \otimes I_{n_x})']' \in I\!R^{N^2 n_x \times N n_x}$.

It is clear that for the considered communication model the Laplacians $\mathcal{L}_j$ and the collected Laplacian $\mathcal{L}_f$ are stochastic variables described by the states of the Markov chain, $\theta(k)$. To emphasize this fact we further use the notation $\mathcal{L}_{j\{\theta(k)\}}$ and $\mathcal{L}_{f\{\theta(k)\}}$. So for any given choice of the Markov state $\theta(k) = i$, $i = 1, ..., M$, the corresponding collected Laplacian $\mathcal{L}_{f\{\theta(k)=i\}}$ carries the information about which links in the specified communication topology failed and which were successful.

To be able to write compactly the equation for the estimation error dynamics, we introduce the collected receiver gain matrix, $F_f = [F_1\ \ F_2\ ...\ F_N] \in I\!R^{Nn_x \times \sum_{i=1}^N \sum_{j=1}^N k_{ij}}$, where $F_j = \mathrm{diag}(F_{1j}, ..., F_{Nj})$ contains the receiver gains of agents receiving signals from the j^{th} agent, $F_{ij} \in I\!R^{n_x \times k_{ij}}$, and the collected transmitter gain matrix, $H_f = \mathrm{diag}(H_1, ..., H_N) \in I\!R^{\sum_{i=1}^N \sum_{j=1}^N k_{ij} \times N^2 n_x}$, where $H_j = \mathrm{diag}(H_{1j}, ..., H_{Nj})$ contains the transmitter gains of the j^{th} agent, and $H_{ij} \in I\!R^{k_{ij}\times n_x}$. We set $k_{ij} = 1$ if there is no communication link from agent j to agent i, and ensure that $F_{ij} = [0\ \ 0\ ...\ 0]' \in I\!R^{n_x\times 1}$ and $H_{ij} = [0\ \ 0\ ...\ 0] \in I\!R^{1\times n_x}$. Note, that the introduced matrices allow specifying all possible communication interconnections in the formation with different transmitter-receiver pairs, H_{ij}, F_{ij}, assigned for each individual link.

Using the introduced notation the collected estimation error dynamics can be written compactly as,

$$e(k+1) = (A_B - L_f C_f - F_f H_f \mathcal{L}_{f\{\theta(k)\}})e(k) + \Gamma v(k) - L_f n(k) = \bar{A}_{\{\theta(k)\}} e(k) + \bar{B}\bar{u}(k), \tag{11}$$

where $\bar{A}_{\{\theta(k)\}} = A_B - L_f C_f - F_f H_f \mathcal{L}_{f\{\theta(k)\}}$, $\bar{B} = [\Gamma\ \ -L_f]$, and $\bar{u}(k) = [v(k)'\ n(k)']'$.

The collected estimation error dynamics (11) together with (5) describe completely closed-loop dynamics of the formation with N agents and estimators exchanging the information between each other through links modeled with the 2-state Markov chains. As we can see from (11) and (5) the estimation error dynamics are decoupled from the closed-loop plant dynamics, while the latter are driven by the estimation error. The closed-loop plant dynamics are specified by a choice of the state feedback gain K. At the same time the estimation error dynamics are determined by the choice of the design variables L_f, F_f, H_f, and the Laplacian $\mathcal{L}_{f\{\theta(k)\}}$.

III. Estimator Design

The estimation error dynamics (11) is a description of a discrete-time Markovian jump linear system. To be able to approach our design problem we use the results of Costa and Fragoso [1], which allow us to formulate the design problem as a set of LMIs and in the case of their feasibility, guarantee mean square stability (MSS) of system (11).

We start with defining new vector variables $z_j(k) := E\{e(k)\ 1_{\theta(k)=j}\} \in I\!R^{Nn_x}$, where $1_{\theta(k)=j}$ is the Dirac measure, $j = 1, ..., M$. Hence $z_j(k)$ is the estimation error expectation depending on the state j of the Markov chain at time step k. We also define the collected vector, $z(k) := [z_1(k)'\ z_2(k)'\ ...\ z_M(k)']' \in I\!R^{Nn_xM}$, and matrix, $Z_j(k) := E\{z_j(k)z_j(k)'\} \in I\!R^{Nn_x\times Nn_x}$, $j = 1, ..., M$. As shown in [1],

$$Z(k) := E\{z(k)z(k)'\} = \mathrm{diag}(Z_1(k), ..., Z_M(k)), \tag{12}$$

$$\text{and,} \qquad Z(k+1) = \zeta(k), \tag{13}$$

$\zeta(k) := \mathrm{diag}_j\Big(\sum_{i=1}^M p_{ij}\bar{A}_i Z_i(k)\bar{A}_i' + \bar{B}Q\bar{B}' \sum_{i=1}^M \pi_i(k)p_{ij}\Big)$, $Q = \mathrm{diag}(Q_v, Q_n)$ and we used the fact that the input matrix $\bar{B}$ is independent of states of the Markov chain. Equation (13) is the update equation for the augmented estimation error correlation matrix, $Z(k)$. The estimation error correlation matrix is then, $P(k) := E\{e(k)e(k)'\} =$

$E\{\sum_{j=1}^{M} z_j(k) \sum_{j=1}^{M} z_j(k)'\} = [I \ \dots \ I]Z(k)[I \ \dots \ I]' = \sum_{j=1}^{M} Z_j(k)$. In this paper, we would like to design the distributed estimator with constant gains and for that purpose consider a time-invariant formulation which corresponds to a steady-state solution, $Z := Z(k) = Z(k+1)$, or a long-run average solution of (13).

In equation (13) the probability distribution, $\pi(k) \in I\!R^M$, of the states of the Markov chain is a dynamic variable with an evolution described by (9). To be able to consider the time-invariant case we make several observations about the properties of the Markov chain. First, observe that according to our definition of the transition probability matrix, $\mathcal{P}$, the Markov chain describing communication topology can exhibit both aperiodic and periodic behavior. For the aperiodic case there exists a steady-state solution of (9), $\pi = \lim_{k\to\infty} \pi(k)$, which generally depends on the initial value of distribution $\pi(0)$. We can find the steady-state distribution, π, and consider the steady-state solution of (13) with $Z = Z(k) = Z(k+1)$.

In the periodic case when $\lim_{k\to\infty} \pi_i(k)$ does not exist, we can consider a Cesaro limit, the long-run average solution of (13), given by,

$$\lim_{k\to\infty} \frac{1}{k} \sum_{l=0}^{k-1} \left[Z(l+1) = \zeta(l) \right]. \tag{14}$$

From standard results of the Markov chain theory we know that for a periodic case there exists a limit $\pi := \lim_{k\to\infty}(\pi(0)+\pi(1)+\dots+\pi(k-1))/k$ and hence the limit for the last term of the right-hand side of (14) is defined. For the left-hand side of (14), $\lim_{k\to\infty} \frac{1}{k}\sum_{l=0}^{k-1} Z(l+1) = \lim_{k\to\infty} \frac{1}{k}\left(\sum_{l=0}^{k-1}[Z(l)] - Z(0) + Z(k)\right)$, and if $Z(0)$ and $Z(k)$ are bounded, then $\lim_{k\to\infty} \frac{1}{k}\sum_{l=0}^{k-1} Z_i(l+1) = \lim_{k\to\infty} \frac{1}{k}\sum_{l=0}^{k-1} Z_i(l)$, $i = 1, \dots, M$. We assume that this limit exits and $Z_i := \lim_{k\to\infty} \frac{1}{k}\sum_{l=0}^{k-1} Z_i(l)$.

With these assumptions for any of two types of the Markov chain the steady state or the long-run average solution of (13) should satisfy,

$$Z = \operatorname*{diag}_{j} \left(\sum_{i=1}^{M} p_{ij} \bar{A}_i Z_i \bar{A}_i' + \bar{B} Q \bar{B}' \sum_{i=1}^{M} \pi_i p_{ij} \right).$$

Both sides of the above equality have block-diagonal structure and for $j = 1, \dots, M$,

$$Z_j = \sum_{i=1}^{M} p_{ij} \bar{A}_i Z_i \bar{A}_i' + \bar{B} Q \bar{B}' \sum_{i=1}^{M} \pi_i p_{ij}. \tag{15}$$

At this point we can consider the problem of designing distributed parallel estimator which stabilizes the collected estimation error dynamics (11) and minimizes a steady-state or a long-run average estimation error correlation matrix, $P := \sum_{j=1}^{M} Z_j$. In this design problem we can use the estimator gain matrix, L_f, the transmitter gain matrix, H_f, and the receiver gain matrix, F_f, as our design variables. We state this optimization problem as follows:

$$\min_{L_f, F_f, H_f, X_j,\ j=1,\dots,M} \|\bar{P}\|,$$

subject to $\bar{P} = \bar{P}' = \sum_{j=1}^{M} X_j > 0$, and for all $j = 1, \dots, M$,

$$X_j - \sum_{i=1}^{M} p_{ij} \bar{A}_i X_i \bar{A}_i' - \bar{B} Q \bar{B}' \sum_{i=1}^{M} \pi_i p_{ij} > 0, \tag{16}$$

where L_f, F_f, and H_f satisfy previously defined structural constraints.

Feasibility of the matrix inequalities (16) is equivalent to the MSS of the Markovian jump linear system described by (11) due to the result of Costa and Fragoso [1]. The MSS of the system (11) implies the existence of $e \in I\!R^{Nn_x}$ and $P \in I\!R^{Nn_x \times Nn_x}$ independent of $e(0)$ such that, $\|E\{e(k)\} - e\| \to 0$ and $\|E\{e(k)e(k)'\} - P\| \to 0$ as $k \to \infty$.

To be able to use inequalities (16) in the design procedure, we state the following lemma.

Lemma 1: *If there exists a matrix $G \in I\!R^{Nn_x \times Nn_x}$ and matrices $Y_j = Y_j' > 0$, $j = 1, \dots, M$, $Y = \mathrm{diag}(Y_1, \dots, Y_M)$ such that for all $j = 1, \dots, M$,*

$$\begin{bmatrix} Y & \hat{A}_j' G' & 0 \\ G\hat{A}_j & G + G' - Y_j & \sqrt{\bar{\Sigma}_j} G\bar{B} \\ 0 & \bar{B}' G' \sqrt{\bar{\Sigma}_j} & Q^{-1} \end{bmatrix} > 0, \tag{17}$$

with $\bar{\Sigma}_j = \sum_{i=1}^{M} \pi_i p_{ij}$ and $\hat{A}_j = \left[\sqrt{p_{1j}}\bar{A}_1 \ \dots \ \sqrt{p_{Mj}}\bar{A}_M\right]$, then there exist $X_j = X_j' > 0$, $j = 1, \dots, M$ such that (16) holds.

The line of argument in the proof of this lemma is similar to the proof of Theorem 1 in [7] due to Oliveira et al. and we omit it here due to space limitations. To use LMIs (17) for the design of the gains L_f, F_f, and H_f, we define a block-diagonal structured variable $G := \mathrm{diag}(G_1, \dots, G_N)$ with $G_i \in I\!R^{n_x \times n_x}$, $i = 1, \dots, N$. We also define $R := GL_f = \mathrm{diag}(R_1, \dots, R_N) \in I\!R^{Nn_x \times \sum_{i=1}^{N} k_{y_i}}$ with $R_i = G_i L_i$, and $E := GF_f H_f = [E_1 \ \ E_2 \ \dots \ E_N] \in I\!R^{Nn_x \times N^2 n_x}$, where $E_j = \mathrm{diag}(E_{1j}, \dots, E_{Nj})$ and $E_{ij} = G_i F_{ij} H_{ij}$. Note that each $G\bar{A}_i = GA_B - RC_f - E\mathcal{L}_{f\{i\}}$ and as a consequence all $G\hat{A}_j$ are linear in new variables G, R, and E. The product $G\bar{B} = [G\Gamma \ \ -R]$ is also linear in G and R, and hence all inequalities in (17) are linear in matrix variables Y_j, $j = 1, \dots, M$, G, R, and E. Now we can redefine the design problem as a convex optimization problem with LMIs:

$$\max_{G,R,E,Y_j,\ j=1,\dots,M} \gamma, \tag{18}$$

subject to $0 < \gamma I \le \sum_{j=1}^{M} Y_j$, and M LMIs from (17) with variables G, R, and E which satisfy structural constraints. When the feasible solution which minimizes the upper bound on the estimation error correlation matrix P is found, we can calculate L_f and $F_f H_f$ from $L_f = G^{-1}R$ and $F_f H_f = G^{-1}E$ since G is nonsingular.

Note that the solution to the proposed optimization problem has the transmitter, H_f, and the receiver, F_f, matrix gains as a product, $F_f H_f$, which does not allow us to use this result directly for construction of the distributed parallel estimator. To be able to use the proposed algorithm we have to extract matrices H_f and F_f from the product.

Another issue which should be addressed, is the rank constraint on each individual block, $F_{ij}H_{ij}$, of the product $F_f H_f$ and consequently the variable E. Since the dimension of a transmitter gain matrix, H_{ij}, is $k_{ij} \times n_x$ and $F_{ij}H_{ij} \in I\!R^{n_x \times n_x}$, the ranks of all N_l, where N_l is the number of links in the topology, nonzero products $F_{ij}H_{ij}$ should be less or equal than the corresponding k_{ij}. To tackle this issue we can use results of Fazel et al. [2] or Orsi et al. [9] and impose additional LMI constraints on N_l nonzero blocks of the matrix variable E, $E_{ij} = G_i F_{ij} H_{ij}$.

Once all the nonzero blocks E_{ij} in the solution to the optimization problem satisfy the rank constraints, we can find F_{ij} and H_{ij} by taking a singular value decomposition (SVD) of each product $F_{ij}H_{ij}$,

$$F_{ij}H_{ij} = [U_{ij}^1 \;\; U_{ij}^2] \begin{bmatrix} D_{ij} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{ij}^1 \\ V_{ij}^2 \end{bmatrix}, \qquad (19)$$

where $D_{ij} \in I\!R^{k_{ij} \times k_{ij}}$ is a diagonal matrix with possibly some zeros on the diagonal. Then a possible choice of transmitter and receiver gains is $H_{ij} = V_{ij}^1$ and $F_{ij} = U_{ij}^1 D_{ij}$. In any practical application some limitations are usually imposed on a transmitter power. We can guarantee that each H_{ij} satisfies the desired limitation by simply scaling the matrices in the SVD product.

IV. Experimental results

Now we illustrate the described design procedure on an experimental example. The experimental configuration is a formation with three agents, where each agent is a motor cart able to move along its track. This experimental setup was implemented with three Quanser motor cart modules and one computer station with an acquisition board. All three controllers, each consisting of the full formation state estimator and state feedback, were implemented in Simulink on one diagram. The communication links between estimators were also implemented by links on the same diagram, along with the measurement and process noises. Failures in the links were implemented with binary signals generated by a Markov chain model from Matlab.

Dynamics of each motor cart can be described by the discrete state-space model,

$$\bar{x}_i(k+1) = A_i \bar{x}_i(k) + B_i u_i(k).$$

The full formation dynamics are then described by,

$$\bar{x}(k+1) \;\; = \;\; \bar{A}\bar{x}(k) + \bar{B}_u u(k) + \bar{B}_v v(k), \qquad (20)$$

where $\bar{x}(k) = [\bar{x}_1(k)' \; \bar{x}_2(k)' \; \bar{x}_3(k)']' \in I\!R^6$, $u(k) = [u_1(k)' \; u_2(k)' \; u_3(k)']' \in I\!R^3$, $\bar{A} = \mathrm{diag}(A_1, \; A_2, \; A_3)$, $\bar{B}_u = \mathrm{diag}(B_1, \; B_2, \; B_3)$, and we augmented the original system dynamics with the zero-mean Gaussian process noise $v(k) \in I\!R^3$ with covariance $Q_v = 10^{-6} I$ entering the system through $\bar{B}_v = \mathrm{diag}(\bar{b}, \; \bar{b}, \; \bar{b})$, $\bar{b} = [0 \; 1]'$.

We define the formation consisting of three agents by specifying relative distances between agents in the formation. The control objective is to guarantee that the agents converge to and keep relative distances specified with a vector $d = [d_{12} \; d_{23} \; d_{13}]$, where d_{ij} is the distance between agent i and agent j, $d_{12} = 0.2 \; m$, $d_{23} = 0.2 \; m$, $d_{13} = 0.4 \; m$. We assume that agent 1 is able to measure its distance to agent 2, agent 2 measures the distance to agent 3, and agent 3 its distance to agent 1. With these measurements three output matrices for the agents are: $\bar{C}_1 = [-1 \; 0 \; 1 \; 0 \; 0 \; 0]$, $\bar{C}_2 = [0 \; 0 \; 1 \; 0 \; -1 \; 0]$, $\bar{C}_3 = [1 \; 0 \; 0 \; 0 \; -1 \; 0]$. System (20) with the output matrix $\bar{C} = [\bar{C}_1' \; \bar{C}_2' \; \bar{C}_3']'$ is not observable, due to the fact that the absolute position of the formation on the tracks is not present at the measured output. Hence we can reduce the dimension of the system (20) and drop unobservable part without influencing performance of the formation. We apply similarity transformation $x(k) = T\bar{x}(k)$ and truncate unobservable states of the system to arrive at,

$$\begin{aligned} x(k+1) \;\; &= \;\; Ax(k) + B_u u(k) + B_v v(k), \qquad (21) \\ y_i(k) \;\; &= \;\; C_i x(k) + n_i(k), \; i = 1, 2, 3, \end{aligned}$$

where $x(k) \in I\!R^4$, and $n_i(k) \in I\!R$ are the zero-mean Gaussian measurement noises with covariances $Q_{n_i} = 10^{-6}$.

First, we design a stabilizing state feedback, $u(k) = -Kx(k)$ using a standard LQR design method. The individual control inputs for each cart are defined by,

$$u_i(k) = -\Pi_i K \hat{x}_i(k).$$

Here $\hat{x}_i(k)$ is the estimate of state $x(k)$ at i^{th} agent's estimator and Π_i is the corresponding projection matrix, $\Pi_1 = \mathrm{diag}(1, \; 0, \; 0)$, $\Pi_2 = \mathrm{diag}(0, \; 1, \; 0)$, $\Pi_3 = \mathrm{diag}(0, \; 0, \; 1)$.

For our example we allow agent 1 to communicate its estimates to agent 2 with $k_{21} = 4$, and agent 2 to communicate its estimates to agents 1 and 3 with $k_{12} = 4$ and $k_{32} = 4$. The described communication topology has $N_l = 3$ links and we model each link with the two-state Markov chain represented on Figure 1, with the same transition probabilities for all links, $p_{21}^0 = p_{12}^0 = p_{32}^0 = p^0 = 0.1$ and $p_{21}^1 = p_{12}^1 = p_{32}^1 = p^1 = 0.95$. Then according to our definition the result of communication at step k can be described by the state of the Markov chain with $M = 2^3 = 8$ states:

$\theta(k) = 1 : \; \Theta(k) = [0 \; 0 \; 0]$, $\theta(k) = 2 : \; \Theta(k) = [0 \; 1 \; 0]$,
$\theta(k) = 3 : \; \Theta(k) = [1 \; 0 \; 0]$, $\theta(k) = 4 : \; \Theta(k) = [1 \; 1 \; 0]$,
$\theta(k) = 5 : \; \Theta(k) = [0 \; 0 \; 1]$, $\theta(k) = 6 : \; \Theta(k) = [0 \; 1 \; 1]$,
$\theta(k) = 7 : \; \Theta(k) = [1 \; 0 \; 1]$, $\theta(k) = 8 : \; \Theta(k) = [1 \; 1 \; 1]$.

With $\pi(k) \in I\!R^8$ and the elements of $\mathcal{P}$ calculated from individual probabilities of each link, for example $p_{24} = (1 - p^0)p^1 p^0$, the Markov chain modeled with (9) is aperiodic. With $\pi(0) = [0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 0 \; 1]$ we found the steady-state solution of (9) to be $\pi = [0.0001 \; 0.0026 \; 0.0026 \; 0.0472 \; 0.0026 \; 0.0472 \; 0.0472 \; 0.8503]$ and used this π in the design calculations. The binary signals implementing success or failure of the links for the Markov chain are shown on Figure 5 for the first second. The decentralized distributed estimator-controller structure for the described example is illustrated in Figure 2.

Using our algorithm we designed the distributed estimator gains L_i, $i = 1, 2, 3$, transmitter gains H_{21}, H_{12}, and H_{32}, and receiver gains F_{21}, F_{12}, and F_{32}. To find these gains we implemented the proposed synthesis procedure using *yalmip* [14] in Matlab.

The experimental results for the proposed system architecture and calculated gains are shown in Figures 3 and 4. Figure 3 shows the positions of carts along the tracks versus time. At $t = 0 \; sec.$ the system was initialized with carts located at the same point on their respective tracks. Once the controller was on, they started moving into formation and by the time $t = 4 \; sec.$ the formation was in order. From time $t = 5.8 \; sec.$ for about $2 \; sec.$ we applied a force to cart 2, causing the formation to drift in the direction of applied force. At time $t = 11 \; sec.$ the formation was arranged again and at time $t = 12 \; sec.$ we applied a force to cart 3 and maintained it for about 2 sec. As a result the formation drifted in the opposite direction and by the time $t = 19$ sec. it was in the nominal formation again.

Figure 4 shows the estimates of relative distance errors recorded from all three estimators along with the measurements recorded during the experiment. As can be seen from the plot, all estimators converge very fast and produce accurate estimates of the relative distance errors. After a small transient all curves on the plot lie on top of each other with some small variation.

V. Conclusion

We proposed the solution to the problem of suboptimal synthesis of distributed decentralized estimator for a formation of agents using local measurements and inter-agent communication. Agents of the formation were allowed to transmit information between each other through a communication network subject to link failures modeled with Markov chains. The experimental results demonstrate that the described method is applicable to practical systems.

VI. Acknowledgments

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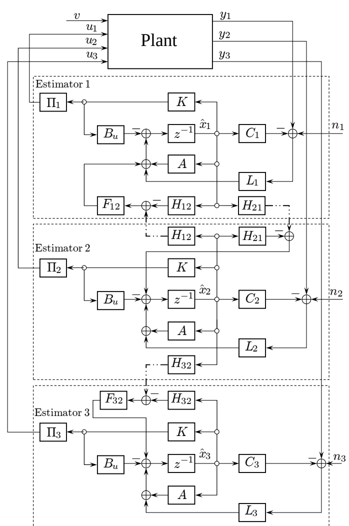

Fig. 2. System structure.

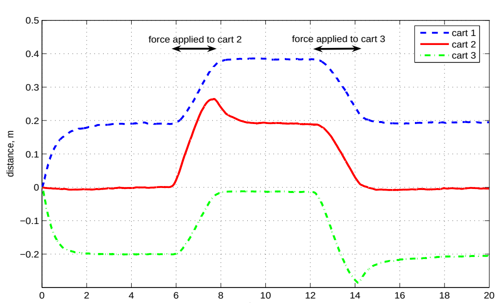

Fig. 3. Positions of carts on the tracks.

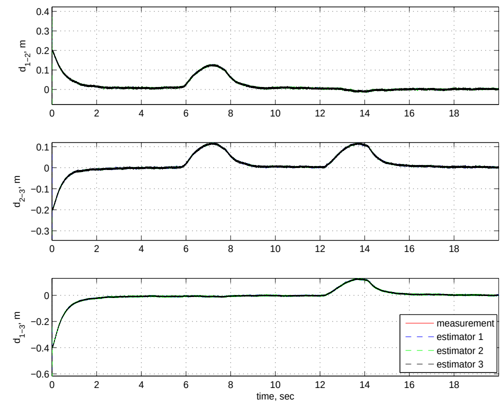

Fig. 4. Measurements of relative distance errors for three carts along with the estimates.

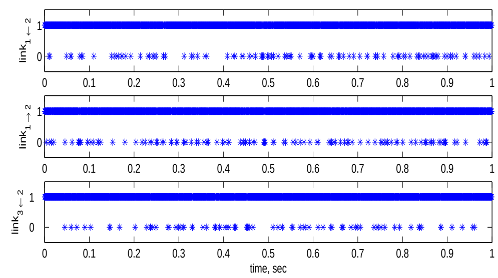

Fig. 5. Random binary signals implementing success, 1, or failure, 0, of the communication links.

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