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# A dynamic macroscopic parking pricing model

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**Abstract**

Demand-responsive pricing has been gaining attention in recent research, but its interdependency on searching-for-parking traffic is still unknown and a model is needed. In this paper, a dynamic macroscopic parking pricing model with its influence on cruising vehicles is developed. The methodology builds on a previous dynamic macroscopic urban traffic and parking study, whose model is extended by parking pricing to better replicate reality.

This study describes a demand-responsive pricing scheme which takes parking search phenomena into consideration. The parking fee also changes in response to the demand, compared to previous studies that only focus on the current occupancy rate of parking spaces. In addition, the model investigates how parking pricing can affect the decision of searchers between paying the current parking cost or keep on searching to obtain a lower cost. In other words, it describes the trade-off of a user between a longer cruising time and a lower parking price. To do so, multiple user classes are considered with respect to their origin and valuation of time. Several cost variables are taken into consideration such as the predicted parking cost at future time and the penalty cost for the past cruising distance in the network.

Compared to most literature, this macroscopic pricing model has rather low data requirements, mostly related to average values and probability distributions across time at the network level. Hence, this macroscopic pricing approach saves on data collection efforts and reduces the computation costs significantly. Moreover, the model can be easily solved with a simple numerical solver without the use of complex simulation software.

The model also provides a preliminary idea for city councils regarding the impacts of parking fee policies on the searching-for-parking traffic (cruising), the congestion in the network (traffic performance), the total driven distance (environmental conditions) and the revenue created by parking fees for the city. Based on the model, the influence of demand-responsive parking pricing on traffic can be better understood. It is then hoped that the knowledge and methodology obtained here can be transferred and used in urban areas.


*Keywords:* Dynamic macroscopic parking pricing model; Demand-responsive parking pricing; Parking-related traffic state; Cruising-for-parking; System dynamics of urban traffic.

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## 1. Introduction

In nearly all major cities, parking pricing policies can lead to significant changes on the performance of a transportation network. Short-term pricing strategies, for example, can have an influence on the performance of both the urban parking and traffic system, e.g., parking pricing can affect the parking availability, the congestion and traffic performance, or the traffic composition in the network. In this research, we provide a dynamic macroscopic parking pricing model which analyzes the interdependency between demand-responsive pricing and searching-for-parking traffic. It includes several cost variables (e.g., predicted parking cost at future time, penalty cost for the past cruising distance in the network) to provide a more realistic parking pricing model with its influence on cruising vehicles.

In general, the definition of a parking fee is normally based on empirical or modelling approaches. Empirical approaches usually collect data by using parking meters for on-street parking spaces, e.g., SF*park* (2009), Xerox® (implemented in Los Angeles's LA ExpressPark™), or through questionnaires, e.g., Auchincloss et al. (2015); Bianco (2000). Moreover, several companies invest heavily in their "smart parking" technologies (e.g., Deteq, Fybr, Streetline, Libelium, etc.). Based on dynamic information, it is possible to predict real-time demand (Caicedo et al. (2012)). In our macroscopic model, we have the advantage that we do not require any specific parking or parking pricing data. Without any physical devices nor data collection efforts, we provide general results regarding the influences of parking pricing on a dynamic traffic network under more realistic generalized conditions.

For modeling parking pricing, Arnott et al. (1991) were among the first to study parking in network equilibrium models. By using a parking fee policy to control the congestion of the city, low-income workers would try to avoid paying high parking fees and try to park further away from their destination in the center of the city. This model, however, does not represent traffic performance, i.e., the traffic performance parameters (e.g., travel speed value) are assumed as fixed for all conditions in the models. Mackowski et al. (2015) developed a dynamic Stackelberg leader-follower game theory approach to model variable parking prices in real-time for effective parking access and space utilization. This model provides a long-term demand management strategy capturing user competition and considering market equilibrium while our model provides an aggregated parking pricing methodology focusing on short-term effects. Ayala et al. (2012) work on a pricing model that sets the parking fees such that the total driving distance is minimized in the system. A static parking demand is assumed, i.e., the model cannot replicate a dynamic real-world environment. In recent research, Van Nieuwkoop (2014) develops a dynamic link-based model formulation as a mixed complementarity problem (MCP) that combines a traffic assignment model in the tradition of Wardrop (1952) and a parking search model into one single model. By having a distinction between curbside and garage parking spaces and a differentiation of user classes with respect to their origin and value of time (VOT), the objectives of the model are to analyze the efficiency and distributional effects of different parking fee policies and to impose a demand-responsive pricing scheme for parking. This agent-based MCP model has interesting results for the impact of parking fee policies on cruising and on congestion, whereas our research model considers these dynamic effects macroscopically with much less simulation data requirements. Qian and Rajagopala (2013) model a system optimal parking flow minimization problem that follows a real-time pricing approach for a parking lot based on its occupancy rate. This paper assumes a user equilibrium travel behavior and focuses on the pricing of off-street parking facilities that may not be transferred directly to an on-street parking pricing model. Arnott and Rowse (2009) develop an on-street parking policy with identical agents for a medium-sized city. Their outcome is that garage parking fees are overpriced compared to the underpricing of on-street parking fees. Other researches (Anderson and de Palma (2004)) analyze the parking pricing economics more formally and show that the social optimum can be achieved if the parking garages are owned privately. In this study, we only consider the aggregated number of vehicles that are looking for on-street parking in the network.

In this research, we focus on a macroscopic demand-responsive pricing scheme which takes parking search phenomena into consideration. The parking fee also changes in response to the demand, compared to previous studies that only concentrate on the current occupancy rate of parking spaces. In addition, the model investigates how parking pricing can affect the decision of searchers between paying the current parking cost or keep on searching to obtain a lower cost, i.e., it describes the trade-off of a user between a longer cruising time and a lower parking price. This dynamic model can later be used to provide a preliminary idea for city councils regarding short-term effects of parking fee policies on traffic systems. Although different parking pricing schemes have been analyzed, proposed and implemented; to the author's knowledge, no study has provided yet a parking pricing scheme built upon the macroscopic view of the interaction between parking pricing, parking demand, parking availability, and traffic conditions.

This paper builds on a previous study from Cao and Menendez (2015a), in which a parking-state-based matrix was developed to model the interaction between urban parking and traffic macroscopically over time. The general model provides an approximation of the proportion of cars searching for parking, as well as an approximation of the time cars spent searching for parking, or traveling through the system. These approximations are now computed under the consideration of the influence of our demand-responsive parking pricing fee over time with its corresponding probability of deciding to park.

In general, in the parking pricing model, when a searcher finds an available parking space, he makes the decision to stay or to keep on searching based on several cost factors:

- the drivers' VOT depending on where they enter the network;
- the parking fee of the current parking space;
- the expected cost of keep on searching (which include the predicted future parking fee at the next parking spot, the costs associated with traveling to the next parking possibility, and the penalty for past cruising time).

All travelers come to a decision based on the comparison of parking costs, then the traffic and parking conditions can be found over time. Based on that, we analyze the efficiency of the proposed parking pricing scheme with its influence on the urban traffic and parking system. These short-term effects include:

- the searching-for-parking traffic (cruising);
- the congestion in the network (traffic performance);
- the total driven distance (environmental conditions); and
- the revenue created by parking fees for the city.

The paper is organized as follows. Section 2 presents the overall methodology of the macroscopic parking pricing model. Section 3 reviews the probability of finding parking over time based on different drivers' VOT. It is integrated into the matrix that describes the system dynamics of urban traffic based on its parking-related states. Section 4 introduces the analytical concept/framework of the dynamic macroscopic demand-responsive parking pricing. It determines the probability of deciding to park based on the cost of staying and the cost of keep on searching. Section 5 shows a numerical example to explore the use of the concept and the proposed methodology. Section 6 summarizes the findings of this paper.

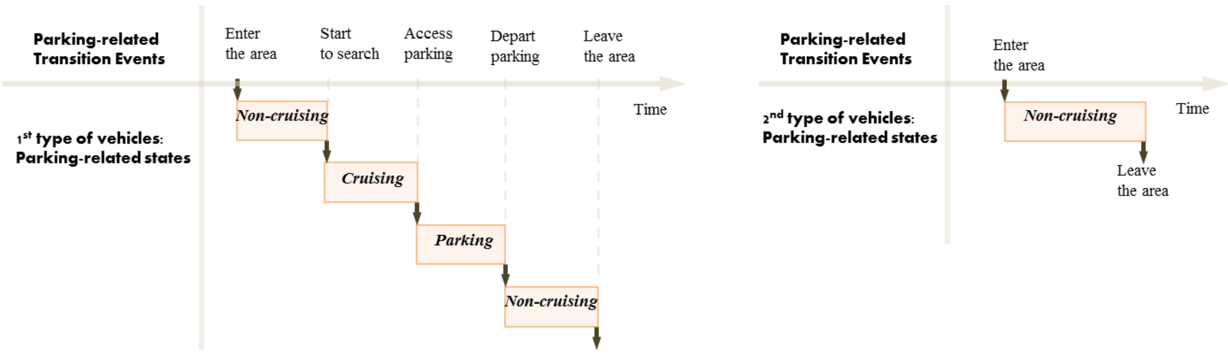
## 2. Framework

In this section, the methodology of the dynamic macroscopic parking pricing model is developed. It enhances the dynamic urban parking and traffic system based on a matrix from Cao and Menendez (2015a).

### *2.1. Overview of the parking-state-based matrix*

The entire model is described in detail in Cao and Menendez (2015a), but for the readers' convenience, the core parts are summarized here. The study considers a small urban area as a simple spatially symmetric network where traffic is homogenously distributed. Inputs to this model are the demand, the size of the network, the amount of parking supply, and the distribution of parking durations. The parking-state-based matrix, as an output of this model, estimates the proportion of cars cruising-for-parking and the cruising time, as well as the traffic conditions and parking usage over time.

Only identical on-street parking places are included in this analysis. The parking searchers have an indifferent attitude towards the parking spaces and choose the first available spot they find. The total time domain is split into small time slices (e.g., 1 minute), and the traffic/parking conditions are assumed steady within each time slice but they can change over time. Two types of traffic demands are considered in this network and they are generated simultaneously in each time slice. The first group of vehicles searches for parking, i.e., they look for on-street parking places in the network. This portion of the traffic demand experiences five transition events in the area of interest as seen in Fig. 1(a). During one single time slice a vehicle may experience at most one of the transition events. The three parking-related traffic states and the five transition events are shown in details in Table 1. The second group of vehicles does not search for parking, i.e., the vehicles can be considered as through traffic and/or heading to a given parking garage. This portion of vehicles in Fig. 1(b) only experiences the transition events "enter the area" and "leave the area" in Table 1, and the decision to leave the area or not depends on their driven distance in the network. Each traffic state and transition event in Table 1 is explicitly modelled in Cao and Menendez (2015a).




(a) First group of vehicles. (b) Second group of vehicles.

Fig. 1. The transition events of urban traffic in-between different parking-related states (Source: Cao and Menendez (2015b)).

It is assumed that all trips are exclusively made by car in this network (i.e., the mode choice has been previously made). At the beginning of each time slice, the parking searchers are homogenously distributed within the overall driving traffic, while the available identical parking spaces are on average uniformly distributed on the network. These assumptions aim at replicating typical conditions in a downtown area, where traffic and parking spaces are more or less homogenously distributed.

Table 1. Relevant key variables for matrix per time slice.

| Notation | Definition |
|---|---|
| $N_{ns}^{i}$ | Number of vehicles in the state "non-searching" at the beginning of time slice $i$ (Non-searching). |
| $N_{s}^{i}$ | Number of vehicles in the state "searching" at the beginning of time slice $i$ (Searching). |
| $N_{p}^{i}$ | Number of vehicles in the state "parking" at the beginning of time slice $i$ (Parking). |
| $n^{i}{}_{/ns}$ | Number of vehicles that enter the area and transition to "non-searching" during time slice $i$ (Enter the area). |
| $n_{ns/s}^{i}$ | Number of vehicles that transition from "non-searching" to "searching" during time slice $i$ (Start to search). |
| $n_{s/p}^{i}$ | Number of vehicles that transition from "searching" to "parking" during time slice $i$ (Access parking). |

Based on some initial conditions, the output of the model is the parking-state related matrix. It represents the number of vehicles experiencing each transition event as well as the resulting parking and traffic conditions (e.g., parking occupancy and average travel speed). These conditions affect the transition events in the next time slice and update iteratively the matrix. The iterations of the model continue over time until a defined criteria is reached (e.g., all the cars leave the area). This model is used as an underlying model and is extended to include a dynamic macroscopic parking pricing methodology.

*2.2. Macroscopic parking pricing model*

The purpose of this research is to develop a dynamic macroscopic parking pricing model with its interdependency on searching-for-parking traffic. It is included into a traffic system with a parking search model over time to replicate reality.

In Cao and Menendez (2015a), the transition event "Access parking" describes the vehicles' transition from the "searching" to the "parking" state. In this paper, this transition event is enhanced with the parking pricing process in Fig. 2.

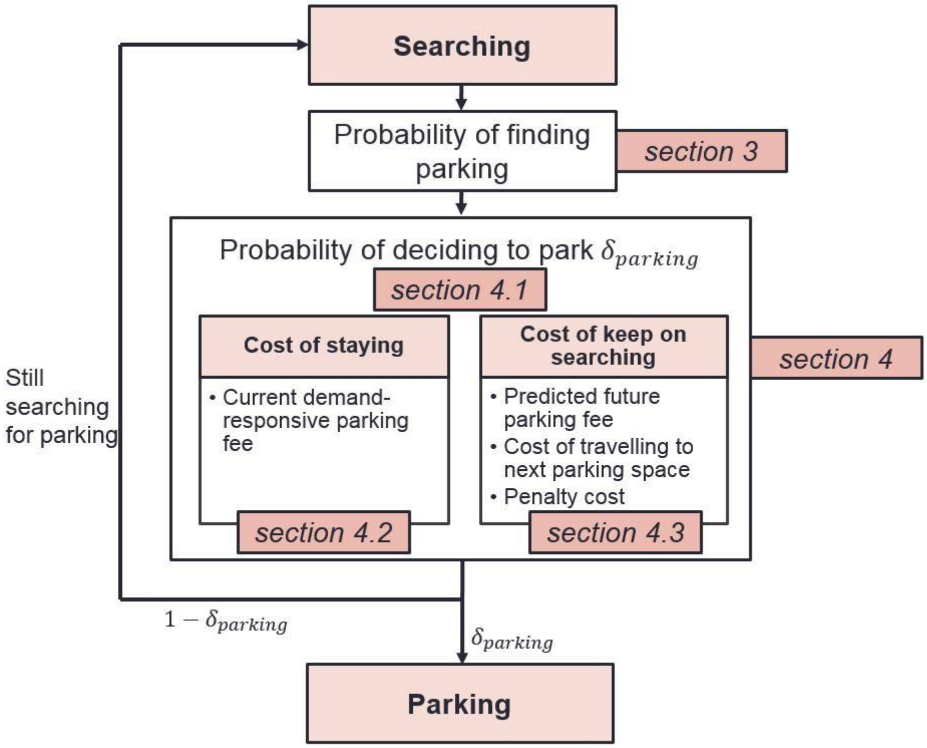



Fig. 2. Modelling parking pricing enhancements between the "searching" and "parking" state.

Fig. 2 shows an overview of our dynamic demand-responsive parking pricing model within the structure of this paper, that includes the probability of finding parking (section 3) and the probability of deciding to park (section 4). This last section explains the main model to determine the probability of deciding to park (section 4.1), that refers to the computation of the cost of staying (section 4.2), and the cost of keep on searching (section 4.3).

When a searcher finds an available parking space, he/she makes the decision to park or to keep on searching for the next parking possibility. This decision needs to be modelled mathematically and it is dependent on various influence factors. Below is an overview of those influence factors and the parking pricing process shown in Fig. 2.

- **Probability of finding parking:** In section 3, we adapt the transition events "Enter the area" and "Start to search" to include vehicles from different VOT origins. The probability of finding parking is determined in the transition event "Access parking" based on the number of vehicles $n^i_{s/p}$ transitioning from "searching" state to "parking" state in the matrix as shown in Cao and Menendez (2015a).
- **Probability of deciding to park $\boldsymbol{\delta_{parking}}$:** In section 4.1, the main model to compute the probability of deciding to park $\delta_{parking}$ is determined. This probability models the decision process to park or to keep on searching

dynamically. Incorporating the cost of staying and the cost of keep on searching, the probability $\delta_{parking}$ refers to the vehicles that decide for parking at the current parking space.

- **Cost of staying:** The cost of staying in section 4.2 shows the influence factors corresponding to the decision that the vehicles would like to stay at the current parking space and pay for parking. The dynamic demand-responsive parking fee over time is computed macroscopically dependent on the number of vehicles $N_s^i$ in the "searching" state and the number of available parking spaces at the beginning of time slice $i$.
- **Cost of keep on searching:** We consider in section 4.3 the cost of keep on searching, that are part of the model to determine the probability of deciding to park $\delta_{parking}$. It includes
  - the predicted parking fee at the next parking spot in a future time slice (section 4.3.1),
  - the costs associated with traveling to the next parking possibility (section 4.3.2), and
  - the penalty term for the past cruising distance in the network (section 4.3.3).

Depending on the probability of deciding to park $\delta_{parking}$, the number of searching vehicles $\delta_{parking} \cdot N_s^i$ will access parking while all other searching vehicles $\left(1 - \delta_{parking}\right) \cdot N_s^i$ will keep on searching for a next parking possibility.

The model can be used to provide a preliminary idea for city councils regarding the impacts of parking fee policies on the searching-for-parking traffic (cruising), the congestion in the network (traffic performance), the total driven distance (environmental conditions) and the revenue created by parking fees for the city.

## 3. Probability of finding parking

This section shows an overview of the assumptions, inputs and outputs of the parking pricing model. In addition, we determine the probability of finding parking over time based on the drivers' VOT.

### 3.1. Basic information for analytical model

Basic model assumptions, inputs, and expected outputs are briefly described below.

*Assumptions:*

Basic assumptions from Cao and Menendez (2015a) for the matrix are kept here. They include, knowledge on the traffic demand over a period of time (e.g., a day), a homogeneous urban traffic network, knowledge on the distribution of parking durations, and knowledge on the length and traffic properties of the network.

The searching time and distance depend on the current traffic conditions (i.e., $N_s^i$, $A^i$, etc.) and on drivers' luck finding an available parking spot (based on their own location, that of the available parking spots and the competitors). To specify now each vehicles' driving time and driving distance, one normally needs to record the location of all cars and parking spots throughout the different time slices in the system. In this model, we can avoid that by using our macroscopic approach. We only consider the average number of vehicles that access parking during a time slice, and the total/average searching distance driven during this time slice. The location of parking searchers and available parking spots are reset at the beginning of each time slice $i$, such that the macroscopic model is not influenced by the randomness of vehicles' location at that specific time slice $i$. Due to the fact that the locations of the parking spaces and searching vehicles are not tracked, the following two assumptions are used in this model. First, we assume the stochastically independent distribution of parking availability on the network, i.e., at the beginning of each time slice $i$ the locations of available parking spots are assumed as random. The second assumption guarantees that the traffic demand is homogeneously generated and the locations of all parking searchers are uniformly distributed on the network at the beginning of each time slice $i$. The second assumption of having a homogeneously generated demand is required for the macroscopic model that provides an average number of vehicles that access parking and an average amount of parking spots being taken. This, however, limits the model. In reality, searchers can focus on one street to find parking while parking spots are easily available in other areas of the network. In this case, the model would likely overestimate the amount of parking spots being taken. However, the model presents an idea of the influence of parking to traffic performance under general conditions, because we are interested on whether there is on average at least one car that is able to take an available parking spot.

In addition to these assumptions, we need to add some parking pricing specific assumptions to the macroscopic model. We assume that the VOT is different for individual vehicles depending on their origin. This VOT cost affects the parking decision of the cruising vehicles. The proportion of new arrivals that correspond to traffic that is not searching for parking is assumed to be independent of vehicles' VOT. The same assumption is valid for the distance that must be driven by a vehicle before it starts to search for parking.

Vehicles that use parking garages do not typically search for parking and they treat the parking garage as their target destination. Thus, it is not realistic to model them as searching traffic. We define a portion of travel demand as through traffic which represents trips that do not search for parking. These vehicles either want to park at off-street, dedicated/private parking facilities, or they simply drive through the network. In this way, it is not necessary to model parking garages explicitly, but the vehicles using them are still taken into account.

*Inputs:*

Corresponding to the assumptions described above, Table 2 shows all the model's independent variables.

The first set corresponds to the travel demand and supply. These variables can be estimated based on some historical data, e.g., traffic data on main roads to enter the network; parking data from one day's data collection, etc.

The second set corresponds to the traffic network. These variables can be estimated based on real measurements, the macroscopic fundamental diagram, and/or simulation results.

The third set corresponds to the initial conditions of the parking-related states. These variables can be measured, assumed or simulated.

The fourth set corresponds to parking pricing specific input parameters. These variables can be estimated based on historical parking pricing data.

Table 2. Independent variables (inputs to the model).

| Notation | Definition |
|---|---|
| $K$ | Total number of origins for demand input of the network. Each origin has a different VOT. |
| $VOT_k$ | VOT for origin $k \in K$. |
| $n^i_{k,/ns}$ | New arrivals to the network for origin $k \in K$ during time slice $i$ (i.e., travel demand per VOT origin) |
| $\beta^i$ | Proportion of new arrivals during time slice $i$ that correspond to traffic that is not searching for parking. |
| $A$ | Total number of existing on-street parking spots (for public use) in the area. |
| $l_{ns/s}$ | Distance that must be driven by a vehicle before it starts to search for parking. |
| $L$ | Size (length) of the network. |
| $t$ | Length of a time slice. |
| $v$ | Free flow speed, i.e., maximum speed in the network. |
| $N^0_{ns}$ | Initial condition for the non-searching state. |
| $N^0_s$ | Initial condition for the searching state. |
| $N^0_p$ | Initial condition for the parking state. |
| $p_0(x_r)$ | Initial parking pricing for parking place $x_r$. |
| $\Delta_{max}$ | Maximum increase/decrease of pricing per time slice. |
| $\eta_{pred}$ | Fixed number of time interval slices to include for approximation of predicted parking pricing. |
| $p_{dist}$ | Price per kilometer driven on the network (i.e., external costs as petrol, wear and tear of vehicles). |

*Outputs:*

The model provides, amongst others, the interactions between the dynamic demand-responsive parking pricing system, the urban parking system, and the urban traffic system. The demand-responsive pricing output over time and its interdependency on cruising vehicles can be studied. The short-term effects of parking pricing on traffic conditions can be investigated, i.e., the distance driven and the time spent for both, vehicles searching and vehicles not searching. Besides these environmental, cruising-for-parking and traffic performance effects, we also analyze the revenue created by parking fees for the city.

Table 3 shows a list of new variables we define and use for our methodology. The first set is used to quantify the number of vehicles that experience each transition event in a time slice. The second set corresponds to modelling of the dynamic demand-responsive pricing. The third set corresponds to the cost variables that are used to compute the probability of deciding to park. The fourth set is used to compute these costs variables.

Table 3. Intermediate model variables.

| Notation | Definition |
|---|---|
| $A^i$ | Number of available parking spots at the beginning of time slice $i$. |
| $\Delta\frac{N_s^i}{A^i}$ | Ratio of number of searching vehicles $N_s^i$ to available parking spaces $A^i$ between time slices $i-1$ and $i$. |
| $v^i$ | Average travel speed in time slice $i$. |
| $d^i$ | Maximum driven distance of a vehicle in time slice $i$. |
| $x_r$ | Location of parking spots for all $r \in \{1,2,\dots,A^i\}$. |
| $x_c$ | Initial positions searching vehicles for all $c \in \{1,2,\dots,N_s^i\}$. |
| $\Delta p^i(x_r)$ | Maximum increase/decrease of demand-responsive pricing per time slice. |
| $p^i(x_r)$ | Fixed number of last time interval slices to include for approximation of predicted parking pricing. |
| $C_{tot}^i$ | Total costs to keep on searching starting from parking space $x_r$ at time slice $i$. |
| $\Delta C_{pay}^i$ | Maximum increase/decrease of demand-responsive predicted future pricing per time slice. |
| $C_{pay}^i$ | Predicted future parking pricing $C_{pay}^i \approx p^{i+1}(x_j)$ at next parking spot $x_j$ for next future time slice $i+1$ determined in time slice $i$. |
| $C_{dist}^i$ | Costs for driving the distance from parking space $x_r$ to next parking location $x_j$ (i.e., external costs as petrol, wear and tear of vehicles). |
| $C_{pen}^i$ | Costs for penalty term (i.e., driving costs in past old iterations). |
| $E_{\mathrm{VOT}}^i$ | Expectation value for all VOT costs considering all origins $k \in K$ and all time slices $\{1,\dots,i\}$. |
| $\tau_{x_r x_j}^i$ | Travel time from parking space $x_r$ to next parking location $x_j$. |
| $d_{x_r x_j}^i$ | Travel distance from parking space $x_r$ to next parking location $x_j$. |
| $\delta_{parking}$ | Dynamic probability of deciding to park depending on demand-responsive parking pricing. |
| $\gamma_r^i$ | Boolean variable to model the decision to stay at current parking space $x_r$ or to keep on driving to next parking location $x_j$. |
| $\delta(d^i)$ | Probability of finding parking dependent on $d^i$. |
| $occ^i$ | Parking occupancy in time slice $i$. |
| $\eta_{ACT}^i$ | Average cruising time expressed in time slices for time slice $i$. |
| $\eta_{\infty}^i$ | Average cruising time for searchers that will not find parking, i.e., the drivers search for an infinite distance. |

### *3.2. Probability of finding parking over time based on VOT*

Based on the assumption that the urban network is abstracted as one ring road with cars driving in a single direction, we introduce the modification of the transition events “Enter the area”, “Start to search” and “Access parking” as presented in Cao and Menendez (2015a). This assumption simplifies the model without affecting the model results, because the homogeneously distributed traffic demand and thus the demand-responsive parking pricing can be seen independently of the vehicles traveling in a single direction or two.

The transition events are adapted to include vehicles from different VOT origins in the network and to access parking following the computation of the dynamic probability of deciding to park $\delta_{parking}$ in section 4.

The number of vehicles ${n^i}_{/ns}$ entering the area and transitioning to “non-searching“ state (transition event “Enter the area”) during time slice $i$ is an input to the model. A percentage $\beta^i$ of the number of vehicles ${n^i}_{/ns}$ does not search for parking and will directly leave the area after driving a distance $l_{ns/s}$, whereas the remaining percentage of $1-\beta^i$ will go through all transition events. For our parking pricing model we consider different VOTs and corresponding VOT costs $VOT_k$ for all origins $k \in K$, where $K$ is the total number of different VOT origins in the network. This leads to the fact that the total number of vehicles $n^i_{/ns}$ is split into $k$ different numbers of input vehicles $n^i_{k,/ns}$. The percentage $\beta^i$ of the number of vehicles $n^i_{k,/ns}$ is assumed to be $k$-independent for $k \in K$.

Based on these different inputs $n^i_{k,/ns}$ for all origins $k \in K$ the next transition event "Start to search" is determined for all origins $k \in K$. We assume that the vehicles start to search after driving a distance $l_{ns/s}$ since they enter the area. $l_{ns/s}$ can assumed to be fixed or to be determined by a given probability function. For simplicity, we assume it is fixed and not dependent on the individual origin $k \in K$. Eq. (1) shows the number of vehicles $n^i_{k,ns/s}$ starting to search for parking during time slice $i$.

$$n^i_{k,ns/s} = \sum_{i'=1}^{i-1} \underbrace{\left(1-\beta^{i'}\right)\cdot n^{i'}_{k,/ns}}_{\text{term 1}} \cdot \underbrace{\gamma^{i'}_{ns/s}}_{\text{term 2}} \tag{1}$$

with

$$\gamma^{i'}_{ns/s} = \begin{cases} 1, & \text{if } l_{ns/s} \le \sum_{j=i'}^{j=i-1} d^j \text{ and } \sum_{j=i'}^{j=i-1} d^j \le l_{ns/s} + d^{i-1} \\ 0, & \text{if otherwise} \end{cases}$$

The number of vehicles $n^i_{k,ns/s}$ in Eq. (1) consists of vehicles from origin $k \in K$ that have entered the network area in any slice between 1 and $i-1$. Here we use $i' \in [1, i-1]$ to denote such a time slice. Term 1 in Eq. (1) shows all the vehicles that are not searching yet for parking spaces, whereas term 2 indicates as a binary variable whether these vehicles will start to search for parking in time slice $i$ or keep on driving without searching for parking. The vehicles start to search in case they have driven the required distance $l_{ns/s} \le \sum_{j=i'}^{j=i-1} d^j$ and in case they have not started to search in a former time slice (condition: $\sum_{j=i'}^{j=i-1} d^j \le l_{ns/s} + d^{i-1}$). The remaining vehicles that are not part of through traffic and/or vehicles heading to a given parking garage, keep on driving in the "non-searching" state until these conditions in Eq. (1) are fulfilled.

We consider the probability of finding parking determined in the transition event "Access parking" in Cao and Menendez (2015a). This number of vehicles that transition from "searching" to "parking" state are adapted based on the probability of deciding to park (section 4.1), the cost of staying (section 4.2), and the cost of keep on searching (section 4.3).

## 4. Probability of deciding to park

In this section, the modelling parts and the analytical formulations for the dynamic macroscopic demand-responsive parking pricing methodology are shown. We propose the dynamic algorithms for the probability of deciding to park, including the corresponding cost of staying and the cost of keep on searching. The goal is to model the decision whether to park or keep on searching for a next parking possibility.

### *4.1. Main model*

Recall, that $p^i(x_r)$ is the actual parking fee at parking spot $x_r$. For a parking spot in reach of a car $x_r \in [x_c, x_c + d^i]$ for all $r \in \{1, \dots, A^i\}$ we check the following condition

$$p^i(x_r) \le C^i_{tot}, \tag{2}$$

i.e., we check whether the parking price at the current parking space $x_r$ is smaller than the total costs $C^i_{tot}$ of keep on searching. If condition (2) is fulfilled, the driver decides to park at parking location $x_r$ in time slice $i$. Otherwise the driver will keep on searching for a next parking spot and as soon as he/she arrives at this next parking possibility, condition (2) will be checked again.

Now the probability of deciding to park $\delta_{parking}$ is defined as the probability such that condition (2) is fulfilled.

$$\mathrm{P}(0 \leq p^i(x_r) \leq C_{tot}^i) = \delta_{parking} = \frac{\sum_{r=1}^{A^i} \gamma_r^i}{A^i} \tag{3}$$

where:

$$\gamma_r^i = \begin{cases} 1, & if \quad 0 \leq p^i(x_r) \leq C_{tot}^i \\ 0, & if \quad otherwise \end{cases}$$

Eq. (3) returns the dynamic probability of deciding to park $\delta_{parking}$, whereas we consider the computation of its elements in the following sections. We start with the computation of $p^i(x_r)$ in section 4.2 and keep on with $C_{tot}^i$ in section 4.3.

To illustrate condition (2), we use a simple example of three uniformly distributed parking places on a ring road with vehicles driving in a single direction as seen in Fig. 3.

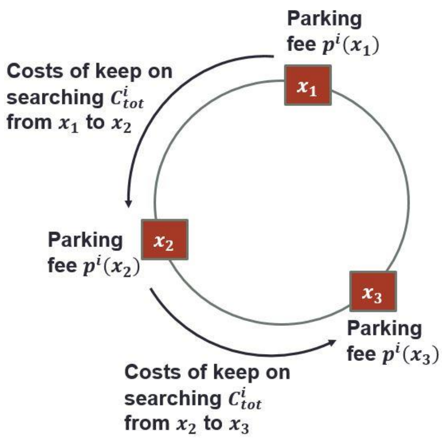



Fig. 3. Simple example of three uniformly distributed parking places to illustrate condition (2).

The vehicles arrive at location $x_1$ of Fig. 3, where the first decision is modelled. The drivers can now decide to pay the parking fee $p^i(x_1)$ or to keep on driving until they find the next parking place $x_2$. Please note that it is not possible to skip parking places. Having the underlying assumption of a ring road with vehicles driving in a single direction the mathematical condition is executed at every parking space on their way. In case the total costs of keep on searching $C_{tot}^i$ from parking location $x_1$ to parking location $x_2$ are less than the initial parking fee $p^i(x_1)$ at location $x_1$, the drivers will keep on driving to location $x_2$ where the condition is executed again. Otherwise the drivers decide to park at location $x_1$ and thus they enter the “parking“ state of the matrix.

## *4.2. Cost of staying*

The cost of staying, i.e., the demand-responsive parking fee over time are now presented. In the transition event “Access parking”, we compute the demand-responsive parking pricing macroscopically. For this algorithm $A^i$ and $N_s^i$ at the beginning of each time slice are found based on the parking-state-based matrix over time. Assume the uniformly distributed parking spots are located at $x_r$ for all $r \in \{1,2,\ldots,A^i\}$, and the searching vehicles' initial positions are $x_c$, for all $c \in \{1,2,\ldots,N_s^i\}$. Then, we define our dynamic demand-responsive pricing model for all parking spots in reach of a car, i.e., for all $x_r \in [x_c, x_c + d^i]$ for all $r \in \{1,2,\dots,A^i\}$, where $d^i$ is defined as the maximum driven distance of a vehicle in time slice $i$. Recall, $p_0(x_r)$ is the initial reference price, an input parameter to our parking pricing model (see Table 2).

The ratio between the number of searchers $N_s^i$ and the number of available parking spots $A^i$ changes from one time slice to the next. This change is formulated as written in Eq. (4).

$$\Delta \frac{N_s^i}{A^i} = \frac{N_s^i}{A^i} - \frac{N_s^{i-1}}{A^{i-1}} \tag{4}$$

Depending on $\Delta \frac{N_s^i}{A^i}$, the parking pricing then increases or decreases. The parking pricing increases, if $\Delta \frac{N_s^i}{A^i} > 0$; it decreases if $\Delta \frac{N_s^i}{A^i} < 0$.

The change of parking fee is formulated as Eq. (5). It describes the pricing difference for parking spot $x_r$ between consecutive time slices, because we model parking pricing here with dependency on demand-responsiveness and its initial start price.

$$\Delta p^i(x_r) = \underbrace{p_0(x_r)}_{\text{term 1}} \cdot \underbrace{\left(\left|\Delta \frac{N_s^i}{A^i}\right|\right)^{\frac{1}{y}}}_{\text{term 2}} \tag{5}$$

In Eq. (5), term 1 is the initial parking price $p_0(x_r)$ and term 2 represents the demand-responsive impact to $\Delta p^i(x_r)$. Within term 2, $y$ is the influence factor of the demand-responsivity, it changes the level of influence of $\Delta \frac{N_s^i}{A^i}$ to the delta pricing value $\Delta p^i(x_r)$. For further studies in this paper, we assume a square root dependency and set $y = 2$.

After computing $\Delta p^i(x_r)$, the updated parking fee can be found with Eq. (6).

$$p^i(x_r) = \begin{cases} p^{i-1}(x_r) + min\{\Delta p^i(x_r), \Delta_{max}\}, & if\ \Delta \frac{N_s^i}{A^i} > 0 \\ p^{i-1}(x_r), & if\ \Delta \frac{N_s^i}{A^i} = 0 \\ p^{i-1}(x_r) - min\{\Delta p^i(x_r), \Delta_{max}\}, & if\ \Delta \frac{N_s^i}{A^i} < 0 \end{cases} \tag{6}$$

$\Delta_{max}$ is used to reduce the oscillations in the pricing model (i.e., avoid drastic price fluctuations). For $\Delta \frac{N_s^i}{A^i} \gtrless 0$, we only consider $\Delta p^i(x_r)$ to increase or decrease the parking fee $p^i(x_r)$, if it is smaller than the maximum pricing change input parameter $\Delta_{max}$. Otherwise this pricing change is performed by $\Delta_{max}$. For $\Delta \frac{N_s^i}{A^i} = 0$, no pricing change is made. In the next section, we have a more detailed investigation of how to compute the cost of keep on searching for another parking possibility.

*4.3. Cost of keep on searching*

In this section, we focus on the total costs $C_{tot}^i$ of keep on searching for a next parking space $x_j$ in condition (2). Due to the simplification of a one ring road, the next parking spot is defined as a modulo operation $x_j = x_{(r+1)mod(A^i)}$. For every time slice, the searchers endeavor to minimize the total costs $C_{tot}^i$ as defined in Eq. (7),

$$C_{tot}^i = C_{pay}^i + C_{dist}^i + C_{pen}^i, \tag{7}$$

including

- the cost $C_{pay}^{i} \approx p^{i+1}(x_j)$ for the predicted parking fee at the next parking spot $x_j$ for the next future time slice $i+1$, predicted at the beginning of time slice $i$ (subsection 4.3.1),
- the cost $C_{dist}^{i}$ of traveling from parking space $x_r$ to next parking location $x_j$ (subsection 4.3.2),
- the penalty cost $C_{pen}^{i}$, i.e., the driving cost associated with past iterations (subsection 4.3.3).

In the following subsections, we derive these cost terms and show the details for their computation.

*4.3.1. Predicted future parking fee, $C_{pay}^{i}$*

The significant term $C_{pay}^{i}$ is the parking fee for the next parking location $x_j$ within reach of the car in the next future time slice $i+1$, considered as the predicted future parking pricing. We predict this future price by using existing values from past iterations, i.e., we need to have historical information available about the traffic and parking system. The approximation for this prediction differs in the amount of time slices to include to compute the predicted parking fee in time slice $i+1$. Based on the existing demand-responsive delta pricing in Eq. (5), we focus first on the approximation of the demand-responsive term 2 in Eq. (5) for the predicted time slice $i+1$. This term is estimated by the total average of increases/decreases over the last fixed $\eta_{pred}$ time slices, whereas $1 \le \eta_{pred} \le i-1$ is an input parameter to the model. Thus, we consider all ratios $\varDelta \frac{N_s^k}{A^k} \neq 0$ for $k \in \{i-\eta_{pred}+1, \ldots, i\}$ to get the aggregated predictive estimate in Eq. (8).

$$\left(\left|\varDelta \frac{N_s^{i+1}}{A^{i+1}}\right|\right)^{\frac{1}{y}} \approx \underbrace{\left(\left|\varDelta \frac{N_s^{i}}{A^{i}}\right|\right)^{\frac{1}{y}}}_{\text{term 1}} \cdot \underbrace{\left|\sum_{k=i-\eta_{pred}+1}^{i} \frac{\varDelta \frac{N_s^k}{A^k}}{\varDelta \frac{N_s^{k-1}}{A^{k-1}}}\right|}_{\text{term 2}}, \tag{8}$$

where $\varDelta \frac{N_s^{k-1}}{A^{k-1}} \neq 0$. Term 1 in Eq. (8) refers to the current demand-responsive term in Eq. (5). For the input parameters $\eta_{pred} = 1$, the approximation term 2 in Eq. (8) only considers the increase/decrease of the ratio $\varDelta \frac{N_s^k}{A^k}$ from the time slice $i-1$ to the current slice $i$, while for $\eta_{pred} = i-1$ this estimate covers the total average of increases/decreases of the ratio $\varDelta \frac{N_s^k}{A^k}$ for all time slices $k \in \{2, \ldots, i\}$. Due to the input parameter $\eta_{pred}$ in Eq. (8), the ratios $\varDelta \frac{N_s^k}{A^k}$ for initial time slices $k$ have no impact on the predictive pricing for long simulations runs. By using the estimate in Eq. (8), we get the dynamic predicted demand-responsive delta pricing in Eq. (9) analog to Eq. (5).

$$\varDelta C_{pay}^{i} = p_0(x_r) \cdot \left(\left|\varDelta \frac{N_s^{i}}{A^{i}}\right|\right)^{\frac{1}{y}} \cdot \left|\sum_{k=i-\eta_{pred}+1}^{i} \frac{\varDelta \frac{N_s^k}{A^k}}{\varDelta \frac{N_s^{k-1}}{A^{k-1}}}\right|, \tag{9}$$

where $\varDelta \frac{N_s^{k-1}}{A^{k-1}} \neq 0$. By using Eq. (9) we compute in Eq. (10) the actual predictive parking pricing $C_{pay}^{i}$ for parking spot $x_j$ in time slice $i+1$.

$$C_{pay}^{i} = \begin{cases} C_{pay}^{i-1} + min\{\Delta C_{pay}^{i}, \Delta_{max}\}, & if\ \Delta \dfrac{N_s^i}{A^i} > 0 \\ C_{pay}^{i-1}, & if\ \Delta \dfrac{N_s^i}{A^i} = 0 \\ C_{pay}^{i-1} - min\{\Delta C_{pay}^{i}, \Delta_{max}\}, & if\ \Delta \dfrac{N_s^i}{A^i} < 0 \end{cases} \tag{10}$$

Depending on the positive or negative sign of ratio $\Delta \frac{N_s^i}{A^i}$, Eq. (10) determines an increase or decrease of the dynamic predictive demand-responsive parking fee for future time slice $i+1$.

*4.3.2. Costs of traveling to next parking space, $C_{dist}^{i}$*

$C_{dist}^{i}$ represents the cost of traveling from one parking space to the next parking possibility. This cost is associated with the driving distance from location $x_r$ to the next parking space $x_j$ and with the VOT of the drivers from origin $k \in K$.

Recall that $L$ is the length of the network and $v^i$ the average travel speed in time slice $i$. We define the travel distance $d_{x_r x_j}^{i}$ from parking space $x_r$ to $x_j$ in Eq. (11).

$$d_{x_r x_j}^{i} = \frac{L}{A^i} \tag{11}$$

With the aid of $d_{x_r x_j}^{i}$ we get the travel time $\tau_{x_r x_j}^{i}$ from parking location $x_r$ to $x_j$ in Eq. (12).

$$\tau_{x_r x_j}^{i} = \frac{d_{x_r x_j}^{i}}{v^i} \tag{12}$$

We introduce the input variable $p_{dist}$, the price per distance unit. Then, $d_{x_r x_j}^{i}$ and $\tau_{x_r x_j}^{i}$ are both used to model $C_{dist}^{i}$ in Eq. (13),

$$C_{dist}^{i} = \underbrace{p_{dist} \cdot d_{x_r x_j}^{i}}_{\text{term 1}} + \underbrace{E_{VOT}^{i} \cdot \tau_{x_r x_j}^{i}}_{\text{term 2}}, \tag{13}$$

where term 1 is associated with the actual driving distance (i.e., external costs as petrol, wear and tear of vehicles) and term 2 refers to the drivers' VOT. The variable $E_{VOT}^{i}$ in term 2 shows the expectation value for all VOT costs for all origins in time slice $i$. Recall $VOT_k$ are the input parameters showing the VOT costs for all origins $k \in K$. We assume the number of vehicles $n_{k,\,ns/s}^{i}$ starting to search for parking from origin $k$ in time slice $i$ being an indicator for $E_{VOT}^{i}$ for all possible VOTs. $E_{VOT}^{i}$ is dependent on both, the searching vehicles coming originally from origin $k$ compared to the total number of currently searching vehicles, and the VOT costs $VOT_k$ for $k \in K$. By averaging over all times $i$ and all $k \in K$, we get Eq. (14) with $n_{ns/s}^{t} = \sum_{k=1}^{K} n_{k,\,ns/s}^{t}$.

$$E_{VOT}^{i} = \frac{1}{i} \sum_{k=1}^{K} \sum_{t=1}^{i} \frac{n_{k,\,ns/s}^{t}}{n_{ns/s}^{t}} \cdot VOT_k \tag{14}$$

With Eq. (14) the travel time $\tau_{x_r x_j}^{i}$ from the actual to the next possible parking location is transferred to cost related to VOT.

### 4.3.3. Penalty cost, $C_{pen}^{i}$

Now we need to consider as well the driving cost $C_{pen}^{i}$ in past iterations and take into consideration of how long the driver already travelled in total. We model this by using the average cruising time on the network that is the same for every car. This penalty cost term is defined as

$$C_{pen}^{i} = p_{dist} \cdot \sum_{k=i-\eta_{ACT}^{i}+1}^{i} d^{k}, \tag{15}$$

where $d^k$ is the maximum driven distance of a vehicle in time slice $k$ and $\eta_{ACT}^{i}$ shows the average number of time slices where the vehicles are in "searching" state. In other words, $\eta_{ACT}^{i}$ in Eq. (15) denotes the average cruising time divided by the length of a time slice $t$. We focus now on the estimation of $\eta_{ACT}^{i}$.

In Cao and Menendez (2015b) the probability of finding parking $\delta(d^i)$ is determined. This estimation describes the probability of finding parking assuming that the conditions remain the same over a period of time. In other words, the equation looks at a single time slice, where the period is comparatively short (e.g., smaller than 10 minutes).

Denote $occ^i$ as the parking occupancy, hence the number of currently available parking spaces is $(1 - occ^i) \cdot A^i$. Notice that when $occ^i = 1$, there is currently no available parking. Searchers have to drive while waiting for parking spaces to become available and thus, $\delta(d^i) = 0$. The cruising distance $d^i$ of vehicles depends on the future departure time of parked vehicles, i.e., before any parking space becomes available, the searchers are cruising for nothing. The relationship between $\delta(d^i)$ and $d^i$ is explained in Cao and Menendez (2015b). By integrating $\delta(d^i)$ with respect to $d^i$ we get the average cruising distance for a low occupancy rate $occ^i \in \left[0, 1 - \frac{N_s^i}{A^i}\right]$ and for a high occupancy rate $occ^i \in \left[1 - \frac{N_s^i}{A^i}, 1\right]$. Here it is important to differ between $(1 - occ^i) \cdot A^i$ searchers that will find parking, and $N_s^i - (1 - occ^i) \cdot A^i$ searchers that will not find parking. The drivers that will not find parking search for an infinite distance and still never find parking. Thus, their average cruising distance is infinite and their corresponding average cruising time is defined as $\eta_{\infty}^{i}$. With the aid of the average cruising distance and $d^i = v^i \cdot t \cdot \eta_{ACT}^{i}$, we estimate $\eta_{ACT}^{i}$ in Eq. (16).

$$\eta_{ACT}^{i} = \begin{cases} \dfrac{L - \int_0^L \delta(d^i)\boldsymbol{d}d^i}{t \cdot v^i} & ,for\ all\ searchers \quad ,if\ occ^i \in \left[0, 1 - \dfrac{N_s^i}{A^i}\right] \\ \begin{cases} \dfrac{L \cdot \left(\dfrac{A^i \cdot (1 - occ^i)}{N_s^i}\right)^2 - \int_0^{\frac{A^i \cdot L \cdot (1-occ^i)}{N_s^i}} \delta(d^i)\boldsymbol{d}d^i}{t \cdot v^i} & ,for\ (1 - occ^i) \cdot A^i\ searchers \\ \eta_{\infty}^{i} & ,for\ N_s^i - (1 - occ^i) \cdot A^i\ searchers \end{cases} & ,if\ occ^i \in \left[1 - \dfrac{N_s^i}{A^i}, 1\right] \end{cases} \tag{16}$$

We determine the integrals in Eq. (16) by using analytical integration and a Taylor series approximation to its third polynomial term.

$$\int_0^L \delta(d^i)\boldsymbol{d}d^i = L\left[1 - \frac{1}{(1 - occ^i)A^i + 1}\left(1 - \left(1 - \frac{1}{N_s^i}\right)^{(1-occ^i)A^i+1}\right) + \frac{A^i(1 - occ^i)}{N_s^i} e^{\frac{(occ^i-1)A^i}{N_s^i}} \left[\frac{1}{A^i(1 - occ^i)}(1 - N_s^i) + \frac{13}{36} - \frac{1}{4(N_s^i)^2} - \frac{1}{9(N_s^i)^3} + \frac{A^i(1 - occ^i)}{N_s^i}\left(\frac{1}{24(N_s^i)^3} - \frac{1}{24}\right)\right]\right]$$

$$\int_0^{\frac{(1-occ^i)\cdot A^i\cdot L}{N_s^i}} \delta(d^i)\boldsymbol{d}d^i = L\left[\frac{A^i(1-occ^i)}{N_s^i} - \frac{1}{(1-occ^i)A^i+1}\left(1-\left(1-\frac{1}{N_s^i}\right)^{(1-occ^i)A^i+1}\right)\right.$$
$$+\frac{A^i(1-occ^i)}{N_s^i}e^{\frac{(occ^i-1)A^i}{N_s^i}}\left[\frac{1}{A^i(1-occ^i)} - 1 - \frac{1}{4(N_s^i)^2} - \frac{1}{9(N_s^i)^3} + \frac{A^i(1-occ^i)}{24(N_s^i)^4}\right.$$
$$\left.\left.+\frac{(A^i)^2(1-occ^i)^2}{4(N_s^i)^2} + \frac{(A^i)^3(1-occ^i)^3}{9(N_s^i)^3} - \frac{(A^i)^4(1-occ^i)^4}{24(N_s^i)^4}\right]\right]$$

For implementation purposes, $\eta_\infty^i$ is approximated in Eq. (17) by dividing the total cruising time until now (expressed in time slices) by the total number of vehicles that cruised and already found parking and the number of vehicles that are still cruising in the current time slice $i$. Note that $\eta_\infty^i$ also includes the cruised time for vehicles that already found parking compared to the other cases described in Eq. (16).

$$\eta_\infty^i = \frac{\sum_{m=1}^{i} N_s^m}{\sum_{m=1}^{i}\sum_{k=1}^{K} n_{k,\,ns/s}^m} \tag{17}$$

Eq. (16) shows an approximation for the average cruising time $\eta_{ACT}^i$ expressed in time slices to determine $C_{pen}^i$ in Eq. (15).

## 5. Applications

In this section, a numerical example is provided to illustrate the influences of dynamic demand-responsive parking pricing on the traffic system. We present the results obtained from multiple simulation runs and discuss the findings regarding parking pricing, and the corresponding revenue with its impact on the average/total searching time/distance in the network.

### *5.1. Numerical example*

The study is conducted with the aid of a simple numerical solver such as Matlab, and employs an abstraction of the network that considers a one ring road with cars driving in a single direction. We have three different origins to the network that are associated to different VOTs ($VOT_1 = 14$ CHF/h; $VOT_2 = 18$ CHF/h; $VOT_3 = 16$ CHF/h). The total travel demand contains $200$ trips, whereas $40$ trips start from origin 1, $60$ trips from origin 2 and $100$ trips from origin 3. The entry time of the vehicles to the area from these origins obeys a gamma distribution, where the average arrival time is $20$ min after the observation period starts (more precisely, the shape parameter is 4 and the scale parameter is 5). Each time slice lasts for 1 min, i.e., $t = 1$ min. For simplicity and due to space constraints, we assume there is no through traffic (i.e., $\beta^i = 0, \forall i$). The initial parking fee is set to $p_0(x_r) = 2.50$ CHF for all parking spots and the price per distance driven is $p_{dist} = 0.3$ CHF/km. The maximum pricing change per time slice is set to $\Delta_{max} = 0.5$ CHF/min and the predictive future parking fee is based on the past $\eta_{pred} = 10$ time slices. Other inputs include: $L = 1$ km; $A = 23$ parking spaces, and $v = 30$ km/h. All further input parameters to the general macroscopic model can be found in the numerical example in Cao and Menendez (2015a).

In the following section, we analyze the outputs with focus on the revenue that can be collected with the aid of demand-responsive parking fees with its influence on the average/total searching time in the area.

### *5.2. Impacts of parking pricing*

The focus of this paper is to add parking pricing to the macroscopic traffic and parking model in Cao and Menendez (2015a) to provide a realistic parking pricing model with its influence on searching-for-parking traffic.

Pricing might increase the cruising time of vehicles, but in the long-term drivers might avoid high parking fees and quit their trips. This could change the demand, but this potential long-term effect is out-of-scope of this paper. Instead we concentrate on short-term effects as the financial benefits of parking fees that can lead to increasing revenue for the city.

For this numerical example, we get the parking pricing output over time in Fig. 4. This plot is obtained by grouping the pricing fees over 4 consecutive time slices, i.e., the parking price is updated every 4 minutes. In addition, the parking fee is rounded to the next 0.5 CHF value to simplify the pricing structure.

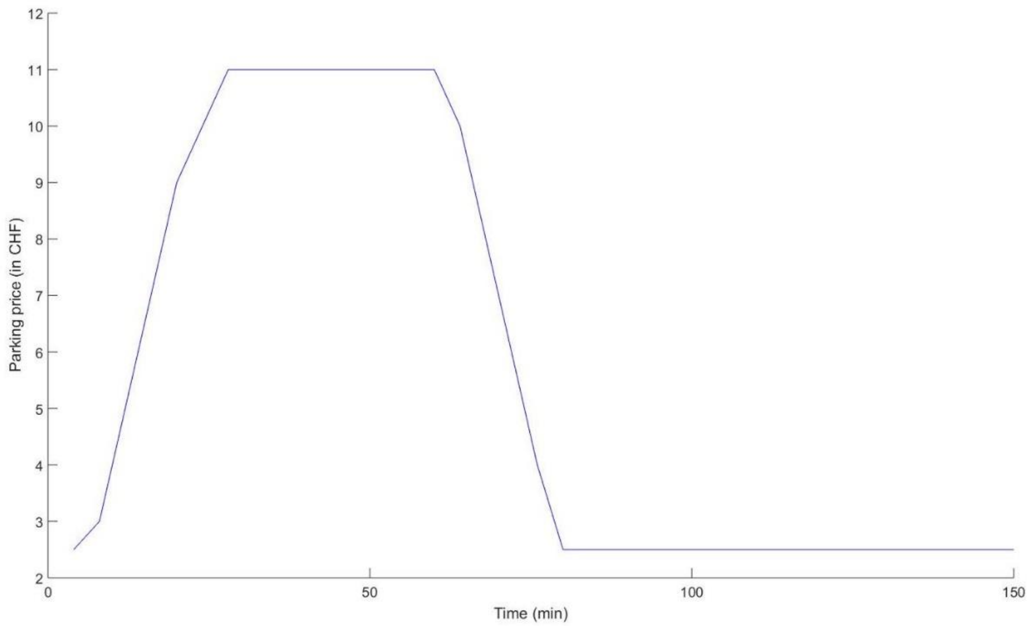



Fig. 4. Demand-responsive parking pricing fee over time.

We realize that with the increasing cruising-for-parking traffic the parking fee increases from its initial value to 11 CHF, where it stays as long as all parking spaces are occupied. In case a driver departs from the location, the parking spot is taken immediately by another cruising vehicle. After 60 minutes the ratio of the number of searching vehicles to the available parking spaces decreases. Thus, the parking fee decreases back to its initial value of 2.50 CHF. Cumulating this parking fee for all vehicles that decide to park results in the cumulative revenue over time in Fig. 5. A total revenue of 1414 CHF is reached after 100 minutes.

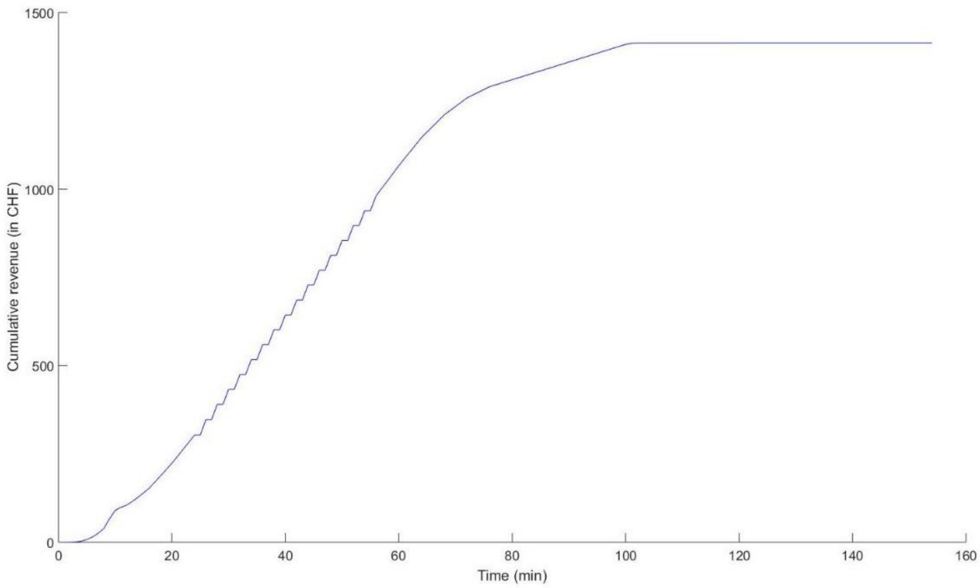



Fig. 5. Cumulative revenue (in CHF) resulting from parking pricing over time.

Besides this financial benefit, parking pricing leads to a short-term increase of the average/total searching time and thus, to more congestion in the network. Compared to the scenario without parking pricing the total searching time increases by 150 minutes to 5740 minutes in the pricing scenario (Table 4). The VOT expectation value for all origins is determined as $E_{VOT}^{i} = 0.27$ CHF/min. Hence, the total searching time in costs converted through VOT is 1549.8 CHF for the parking pricing scenario. This is an increase by 40.5 CHF for all vehicles in the whole time period. We can also investigate an increase of the total non-searching time by 69 minutes that is equivalent to a rise of approx. 18.7 CHF.

Table 4. Average/Total time vehicles spent in (non-)searching states (with/without parking pricing).

| State | Average time per vehicle (min/veh) | Total time (min) | Total costs (converted through VOT) |
|---|---|---|---|
| Searching state (without parking pricing) | 27.9 | 5590 | 1509.3 |
| Searching state (with parking pricing) | 28.7 | 5740 | 1549.8 |
| Non-searching state (without parking pricing) | 6.7 | 1339 | 361.5 |
| Non-searching state (with parking pricing) | 7 | 1408 | 380.2 |

Considering the average/total driven distance per vehicle in Table 5, we realize that the total driven distance decreases in the scenario with parking pricing for both, vehicles searching and vehicles non-searching. For the searching state we get a total decrease of 4.7 km and for the non-searching state a total decrease of 1.3 km. Hence, the average driven distance also decreases minimally per vehicle for both states. The increased searching and non-searching time for all vehicles leads to deteriorated traffic conditions, hence lower network speeds and a decreasing driven distance. This, in turn, slightly improves environmental conditions for the pricing scenario.

Table 5. Average/Total driven distance (with/without parking pricing).

| State | Average driven distance (km/veh) | Total driven distance (km) |
|---|---|---|
| Searching state (without parking pricing) | 6.123 | 1224.6 |
| Searching state (with parking pricing) | 6.099 | 1219.9 |
| Non-searching state (without parking pricing) | 1.111 | 222.2 |
| Non-searching state (with parking pricing) | 1.104 | 220.9 |

Adding of parking pricing not only provides a realistic model, it also leads to financial revenues that significantly exceed the decrease regarding the total searching time in the network. By comparing the total increase in time by 59.2 CHF (i.e., 219 minutes) with the total revenue of 1414 CHF, the model leads to significant improvements for city councils or private agencies in the area.

But now the usage of the model is not limited to these specific results and it allows you to optimize the parking pricing scheme for different object criteria (e.g., maximize revenues, minimize travel distance, and minimize cruising traffic) in the network.

## 6. Conclusions

In this study, we develop a dynamic macroscopic parking pricing model and analyze the interdependency between demand-responsive pricing and searching-for-parking traffic. A focus is set on the influence of parking pricing on cruising vehicles. The model is integrated to an existing urban traffic and parking methodology to replicate reality. Based on the study from Cao and Menendez (2015a), we use the parking-state based matrix of aggregated vehicles between different parking-related states within the urban area as an underlying model.

The main contributions from this paper are summarized in the following.

- In our demand-responsive pricing scheme the parking fee also changes in response to the number of searching vehicles, compared to previous studies that only focus on the current occupancy rate of parking spaces. This makes our methodology to an actual demand-responsive model that takes parking search phenomena into consideration.
- The model investigates how parking pricing can affect cruising vehicles. The decision of searchers between paying the current parking cost or keep on searching for another parking possibility to obtain a lower cost is modelled. In other words, the model describes the trade-off of a user between a longer cruising time and a lower parking price. Thus, multiple user classes are considered with respect to their origin and VOT. The probability of deciding to park results from its influencing factors, the costs of staying at the current parking place and the costs of keep on searching for a next parking possibility. These cost variables include the predicted parking cost at future time at the next possible parking location, the costs of traveling from the actual parking spot to a next parking place associated with driving distance and VOT, and the penalty cost associated with the cruising time in past iterations.
- The model also provides a preliminary idea for city councils regarding the impacts of parking fee policies on the searching-for-parking traffic (cruising), the congestion in the network (traffic performance), the total driven distance (environmental conditions) and the revenue created by parking fees for the city. By incorporating parking pricing to the macroscopic traffic and parking model, pricing might increase the cruising time of the vehicles in the network. But in long-term drivers might avoid high parking fees and quit their journeys. This could change the demand, but this potential long-term effect is out-of-scope of this paper. Instead we concentrate on short-term effects as the financial benefits of parking fees that can lead to increasing revenue for the city.

The whole framework provides an easy to implement generalized methodology to macroscopically model dynamic parking pricing. The methodology is based on very limited data inputs, including travel demand, VOT, number of existing on-street parking places, the traffic network, and initial parking pricing specifications. All data requirements correspond to aggregated values at the network level over time, such that no pricing data for individual parking spots is needed. Compared to existing models, this macroscopic pricing approach saves on data collection efforts and reduces the computation costs significantly. Moreover, the model can be easily solved with a simple numerical solver without the use of complex simulation software.

Overall, the usage/application of the proposed model is far beyond what we have illustrated in the numerical example. The model can provide the relation between through traffic or vehicles that are not searching for parking, parking pricing and the traffic conditions. This is not included in the paper as the through traffic was assumed to be zero for simplification purposes. In addition, for simplicity we have assumed in the numerical example that all individual parking spaces have the same initial parking price for on-street parking. The influence of individual initial parking prices on the traffic system can be analyzed in future studies. Furthermore, the pricing of off-street parking facilities can be modeled explicitly, such as the pricing of not uniformly distributed parking spaces over the network. In reality, vehicles can focus on parking possibilities in a central street or area of the network. Future research can also include the parking pricing for a non-homogenous environment. Besides that, we can include a traffic demand split with a fixed (low subsidized) parking fee in all garages and/or on-street parking spaces. This can be motivated by, e.g., the subsidy by a company or the city for its residents. The remaining portion of the demand is treated demand-responsively, reflecting the external costs for parking.

In summary, the proposed model, despite its simplicity, can be used to efficiently evaluate a dynamic demand responsive pricing scheme macroscopically. With the aid of scarce aggregated data, this model can be used to investigate, how parking pricing can affect searching-for-parking traffic and traffic performance (e.g., average time searching for parking, and average distance driven); and how different traffic conditions (e.g., number of vehicles cruising for parking, and available parking spaces in the network) can affect the demand-responsive parking pricing and the probability of deciding to park at the current parking space over time. Based on the model, the influence of demand-responsive parking pricing on traffic can be better understood. It is then hoped that the knowledge and methodology obtained here can be transferred and used in urban areas.

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