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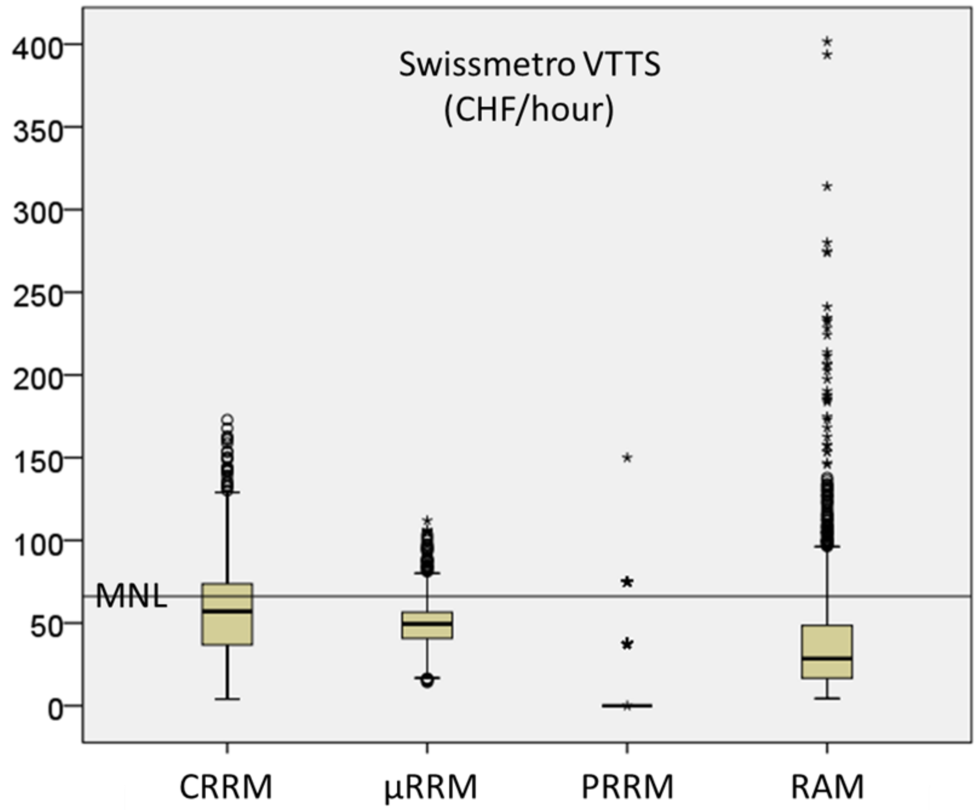

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Abstract

When facing several alternatives, people are often assumed to choose the alternative which maximizes their utilities. This concept is widely known as random utility maximization (RUM). In transportation research, one of the most famous modeling techniques based on this idea, e.g. for modeling mode choice, is the multinomial logit (MNL) approach.

Recently there is a growing interest in an alternative modeling approach, random regret minimization (RRM). In RRM, an individual, when choosing between alternatives, is assumed to minimize anticipated regret as opposed to maximize his/her utility. There are three variants of RRM, the classical CRRM, the µRRM, and the P-RRM. There is also another alternative approach called relative advantage maximization (RAM) turning the idea around and focusing on the gains.

We compare MNL with the four mentioned alternatives. The data used are stated choice data sets collected by the IVT, ETH Zurich which comprise of mode choice, location choice, parking choice, carpooling, car sharing, etc experiments. We compare the performance of those five models by their model fit (Final LL, hit rate, and prediction). We also present a comparison of their VTTS, travel time and cost elasticities.

Keywords

Context-dependent models – Random Regret Minimization – RRM variants – Relative Advantage Maximization

1. Introduction

When facing several alternatives, it is reasonable to assume that people tend to choose an alternative which maximizes their utilities. This concept is widely known as random utility maximization (RUM), when the model allows for perception differences. In transportation research, one of the most famous modeling technique for this is multinomial logit (MNL) (Ben-Akiva and Lerman, 1985; McFadden, 1973). Recently there is a growing interest in implementing an alternative modeling approach called random regret minimization (RRM) (Chorus et al., 2008; Chorus, 2010). In RRM, an individual when choosing between alternatives is assumed to minimize anticipated regret as opposed to maximizing his/her utility. RRM is a context-dependent modeling approach since the decision to choose one alternative depends on the relative performance of the chosen alternative's attributes against other alternatives' attributes. This modelling technique has been implemented for route choice (Chorus, 2012a; Chorus and Bierlaire, 2013; Chorus et al, 2013a), travel information acquisition choice, parking lot, shopping location (Chorus, 2010), automobile fuel choice (Chorus et al, 2013b; Hensher et al. 2013), willingness to pay for advanced transportation services, and salary and travel time trade-off (Hess et al, 2014).

RRM, as a context-dependent modeling alternative to RUM, has several variants, the classical one (Choru, 2010), the GRRM (Chorus, 2014), the μRRM (Van Cranenburgh et al. 2015), and the PRRM (Van Cranenburgh et al. 2015). There have been many attempts that compare the performance of RRMs compare to RUM. Chorus et al. (2014) listed 43 empirical studies comparing RUM and RRM from 2010-2014. Regarding model fit, 15 times RRM outperform RUM and 15 times the other way around. Other 13 empirical studies show neither of these two modeling approaches outperforms each other. Chorus et al. (2014) also listed 7 out of 43 empirical studies that measured hit rate, which is a percentage of observation correctly predicted by the model, and shows that RRM outperforms RUM in three cases. In two cases the RUM hit rate is higher, while for other two cases both models perform equally well.

Leong and Hensher (2015) compare the value of travel time savings (VTTS) from the results of RUM, RRM, Hybrid RRM, and their new context-dependent alternative model, relative advantage maximization (RAM). Leong and Hensher (2015) show that the difference in mean VTTS between RUM-RRM and RUM-Hybrid RRM is small but statistically significant for the seven route choice data sets from Australia and New Zealand. Chorus and Bierlaire (2013) compare RUM and RRM elasticities for the case of route choice and found that travel time elasticities of RRM model are nearly 10% greater compared to RUM. Similarly, for a route choice case, Thiene et al. (2012) showed that for most attributes RRM model elasticities were about 10% greater than the RUM model. For the case of preference of alternative fuel car use, Hensher et al. (2013) compared RRM and RUM elasticities and found a substantial difference in the elasticities with the RRM being higher.

Other than RRM, there is another context-dependent modeling approach that recently has been introduced, RAM (Leong and Hensher, 2015). There have not been many empirical studies comparing the performance of RAM with RUM or RRMs except for route choice models comparison by Leong and Hensher (2015). They found that RAM produces better model fit and obtaining more precise model outputs such as VTTS.

It appears that most empirical studies tested the difference of RUM and other context-dependent modeling approaches in term of model fit. Few exceptions compared them in terms of prediction accuracy, VTTS, and demand elasticities. From most of the cases mentioned above, we cannot say for sure which modeling approach is better. Different data sets and contexts might produce different results and biases.

Therefore, the objective of this paper is to compare RUM, RRMs, and RAM comprehensively in term of model fit, prediction accuracy, VTTS, and demand elasticities. By comparing those different approaches, we might find which model gives the best fit, which modeling approach accurately predicts the choice compared to other approaches. Hopefully, we can contribute to the greater body of RRM and RAM literature through the comparison of Swiss data sets.

In section 2, we discuss the alternative modeling approaches to RUM, their properties, and variants, followed by section 3 where we describe the data sets. In section 4 we present the result of our estimation for different modeling approaches including prediction accuracy. In section 5 we discuss the VTTS followed by section 6 where we discuss the demand elasticities and hit rates. Finally, in Section 7 the conclusions are drawn.

2. Alternatives to RUM

2.1 Random regret minimization

Random regret minimization (RRM) was first introduced by Chorus et al. (2008) as a model of travel choice. According to Chorus at al. (2008) in RRM, individual bases his/her choice between alternatives wishing to avoid a situation where a non-chosen alternative turns out to be more attractive than the chosen one, causing regret. Thus, the individual when choosing between alternatives is assumed to minimize anticipated regret as opposed to maximize his/her utility. Chorus (2010) admitted that this first RRM approach has two limitations. Therefore, he improved the technique to alleviate those limitations with a new RRM-approach. This new RRM approach (Chorus, 2010) is now widely known as Classical RRM (Van Cranenburgh and Prato, 2016).

In the Classical RRM (CRRM) framework, for a person q, the regret associated with an alternative i is obtained given by the following formula (Chorus, 2012a):

$$RR_{iq} = R_{iq} + \varepsilon_{iq} = \alpha_i + \sum_{j \neq i} \sum_k \ln\left(1 + \exp\left[\beta_k \cdot \left(X_{kjq} - X_{kiq}\right)\right]\right) + \varepsilon_{iq} \tag{1}$$

Where, RR_{iq} : random regret for an alternative i for person q

R_{iq} : systematic regret for alternative i for person q

ε_{iq} : unobserved regret for alternative i for person q

α_i : alternative specific constant

β_k : estimable parameter associated with generic attribute X_k

X_{kjq}, X_{kiq} : values associated with generic attribute X_k for, respectively, person q choosing alternative i over competitor alternative j.

Similar to RUM formulation of choice probabilities (McFadden, 1973), for the classical RRM framework the error term in Eq. 1 is assumed to be identically and independently distributed (i.i.d) Extreme Value Type I distribution with a variance of $\pi^2/6$. In the RRM setting, the formulation for the choice probabilities is:

$$P_{iq} = \frac{\exp\left(-R_{iq}\right)}{\sum_{\substack{i \in J \\ j=1}}^{J} \exp\left(-R_{jq}\right)} \tag{2}$$

The next variant of RRM idea proposed by Chorus (2014) is called Generalized-RRM (GRRM). This model generalizes the classical RRM by replacing the 1 inside the logarithm function with a regret-weight parameter γ. Van Cranenburgh et al. (2015) introduced a different version of RRM called µRRM. In this type of RRM, a scale parameter (µ) enters the model as an additional degree of freedom which allows for flexibility of the regret function level attribute. The µRRM generalized the CRRM by allowing to estimate the variance of the error term. The formula for µRRM is as follows (Van Cranenburgh et al. 2015):

$$RR_{iq}^{\mu\mathrm{RRM}} = \alpha_i + R_{iq}^{\mu\mathrm{RRM}} + \varepsilon_{iq} = \alpha_i + \sum_{j \neq i} \sum_k \ln\left(1 + \exp\left[\frac{\beta_k}{\mu} \cdot \left(X_{kjq} - X_{kiq}\right)\right]\right) + \varepsilon_{iq} \tag{3}$$

where $\varepsilon_{iq} \sim i.i.d.EV(0, \mu)$

The formulation for the choice probabilities is as follows:

$$P_{iq}^{\mu RRM} = \frac{\exp\left(-\mu R_{iq}^{\mu RRM}\right)}{\sum_{\substack{i \in J \\ j=1}}^{J} \exp\left(-\mu R_{jq}^{\mu RRM}\right)} \tag{4}$$

The latest version of RRM is also introduced by Van Cranenburgh et al. (2015), P-RRM. The P-RRM is a limiting case of the μRRM model. Classical RRM model and any other RRM variants postulate that both regrets and rejoices are experienced. According to Van Cranenburgh et al. (2015), the P-RRM yields the strongest regret minimization behavior possible within the RRM framework since it postulates no rejoice which is the opposite of regret.

The formula for systematic regret of the P-RRM model (Van Cranenburgh et al., 2015) is as follows:

$$R_{iq}^{P-RRM} = \alpha_i + \sum_k \beta_k X_{kjiq}^{P-RRM} \quad \text{where } X_{kjiq}^{P-RRM} = \begin{cases} \sum_{j \neq i} \max\left(0, X_{kjq} - X_{kiq}\right) \text{ if } \beta_k > 0 \\ \sum_{j \neq i} \min\left(0, X_{kjq} - X_{kiq}\right) \text{ if } \beta_k < 0 \end{cases} \tag{5}$$

The computation of the X-vector (X_{kjiq}^{P-RRM})is linear and can be done prior to the estimation. There is a prerequisite that the signs of the taste parameters are known prior to the estimation. Once the X-vectors are obtained, the estimation of the P-RRM model is similar to the estimation of a linear additive RUM model. The formulation of the choice probabilities is:

$$P_{iq}^{P-RRM} = \frac{\exp\left(-R_{iq}^{P-RRM}\right)}{\sum_{\substack{i \in J \\ j=1}}^{J} \exp\left(-R_{jq}^{P-RRM}\right)} \tag{6}$$

2.2 Relative advantage maximization

Similar to RRM, relative advantage maximization (RAM) also compares the chosen alternative with competing alternatives. However, there is a key difference in the way in which RAM explicitly takes into account the disadvantages and advantages of an alternative. The advantages of alternatives are expressed as a ratio of the sum of advantage and disadvantage.

Leong and Hensher (2015) formulate the disadvantage of the person q choosing alternative i over competitor alternative j for an attribute k, denoted D_{kijq} with this formula.

$$D_{kijq} = \ln\left(1 + \exp\left[\beta_k \cdot \left(X_{kjq} - X_{kiq}\right)\right]\right) \tag{7}$$

Leong and Hensher (2015) assume that disadvantages and advantages are symmetrical, that is if the advantage of the person q choosing alternative i over j with respect to attribute k is the corresponding disadvantage of the person q choosing alternative j over i with respect to the same attribute, then the formula is:

$$A_{kijq} = D_{kjiq} = \ln\left(1 + \exp\left[\beta_k \cdot \left(X_{kiq} - X_{kjq}\right)\right]\right) \tag{8}$$

Now the definition of A_{kijq} is a binary advantage of the person q choosing alternative i over j, and the definition of D_{kijq} is a binary disadvantage of alternative j over i. The formula for both are as follows:

$$A_{ijq} = \sum_k A_{kijq} \quad \text{and} \quad D_{ijq} = \sum_k D_{kijq} \tag{9}$$

The relative advantage of the person q choosing alternative i over j according to Leong and Hensher (2015) is as follow:

$$RA_{ijq} = \frac{A_{ijq}}{A_{ijq} + D_{ijq}} \tag{10}$$

The observed component of utility for the person q choosing an alternative i is written as linear combination of MNL. The formula for systematic utility is as follows:

$$V_{iq}^{RAM} = \alpha_i + \sum_{k'} \beta_{k'} X_{k'iq} + \sum_{\substack{i \in J \\ j \neq i}} RA_{ijq} \tag{11}$$

With $X_{k'iq}$ referring to a context-independent attribute k' for person q choosing an alternative i, the RAM model allows for a combination of context-independent preferences and context-dependent preferences.

In this paper, we compare the standard RUM model (MNL) with the classical RRM (Chorus, 2010), and the µRRM as well as the P-RRM (Van Cranenburgh et al., 2015). We also compare those approaches with the new RAM approach (Leong and Hensher, 2015). Although the RAM approach allows for incorporation of context-independent attributes, for this paper, we only use context dependent generic attributes k.

3. Data Description

Chorus (2010) shows that for binary choice situations, the RRM reduces to the linear-additive RUM. Therefore, in this paper, we select data sets where respondents face at least three alternatives. Table 1 shows the information regarding the data sets used, while the description of the data sets can be found in the next sub-section. The data sets that we use for this study are stated choice data sets, and one RP data set collected in Switzerland. Since RRM is choice set dependent, meaning that choosing an alternative is influenced by the presence of other alternatives in term of their attribute values, therefore for this study we only use a parsimonious model formulation, using two generic attributes i.e.: travel time (TT) and travel cost (TC). We add alternative specific constants for the labeled data sets.

Table 1 Data sets used

Data set	Location	Publication	Sample	Obs.	Choice set composition
Swiss Metro	Residents of Switzerland	Bierlaire et al. (2001)	623	5607	Train, Swissmetro, car
Influence of parking	Residents of Switzerland	Weis et al. (2012)	631	6301	Location A, location B, none of these
Influence of parking	Residents of Switzerland	Weis et al. (2012)	585	5853	Parking A, parking B, none of these
Influence of parking	Residents of Switzerland	Weis et al. (2012)	168	1666	Walk/bike, car, transit
Car-sharing	Residents of Switzerland	Ciari and Axhausen (2012)	735	4350	Carsharing, car, transit
Carpooling	Residents of Switzerland	Ciari and Axhausen (2012)	511	3975	Car, carpooling as driver, carpooling as passenger, transit
RP mode choice	Residents of Switzerland	Schmutz (2015)	33942	33942	Walk, bike, car, transit

3.1 Swissmetro

The Swissmetro was a major innovation proposed for the Swiss transport system. Abay (1999) conducted revealed preference (RP) and stated preference (SP) survey of long-distance road and rail travelers. The details of the data sets can be found in Bierlaire et al. (2001) and Axhausen (2013). For long distance travel, there are three alternatives: Train, Swissmetro (SM), and car. For this paper, we only selected SP data where respondents had all three choice alternatives. Thus SP data where there are only two alternatives (Train and SM) are omitted. In total, 5607 observations from 1192 respondents were used for modeling.

Table 2 presents the descriptive analysis of this data set. We present minimum and maximum value, the mean and standard deviation for each attribute used to measure the VTTS in section 5. The minimum travel time for each of the mode is varied within one hour. The maximum travel time, is the longest for car, being 26 hour. The minimum cost for train and SM is zero for those who have an annual season ticket (General abonnement (GA)).

Table 2 Descriptive analysis of the Swissmetro data used

Data Set	Attributes	Observations	Minimum	Maximum	Mean	St. Dev.
Swiss Metro	Train TT (min)	5607	43	1022	172.18	70.54
	Train TC (CHF)	5607	0	576	94.23	62.48
	SM TT (min)	5607	19	796	87.51	48.89
	SM TC (CHF)	5607	0	768	113.95	76.38
	Car TT (min)	5607	32	1560	148.66	79.77
	Car TC (CHF)	5607	8	520	94.94	47.21

3.2 Influence of Parking

Weis et al. (2012) assessed the effect of parking availability on travelers' behavioral responses. They assumed that in addition to the trade-off between travel time and fuel or transit cost, parking search times and cost have a substantial impact on travelers' decision. Therefore, they conducted a stated choice study of parking, location, and mode choice to assess those choices. The detail of the study is explained in Weis et al. (2012; 2013). We use the data sets to run the models on three different choice sets: location choice, parking choice, and mode choice.

For the location choice, there are two alternative locations and one "none of these" option, thus three choices alternatives. Hess et al. (2014) using two different data sets, willingness to pay an advance public transport in Netherlands and tradeoff between salary and travel time in Sweden, showed that with only two alternative choices the model fits result of RUM and RRM is the same. However, with the addition of the opt-out alternative (none of these options), the model fit of RRM is better than RUM. Therefore, in this research, we also estimate RUM and RRMs (as well as RAM) for data sets with an opt-out alternative. In total 6301 observations from 631 respondents were used. For the parking choice, there are three alternative choices: parking A, parking B, and the opt-out alternative. In total 5853 observations from 585 respondents were used.

Finally, we also estimate mode choice model. There are four mode choice alternatives: walk, bicycle, car, transit. For longer distance travel, walk and bicycle might not be available. Moreover, during the experiment, none of the respondents faced all four available alternatives together. Therefore, for this paper, we only take short distance trips where respondents are facing three choices: walk or bike, car, and transit. In total, only 1666 observations from 168 respondents were used.

Table 3 presents the descriptive analysis of these three data sets. For the location of parking, we can see that the second location is slightly more expensive and also takes longer time. For parking alternatives, we can see the minimum cost of zero that is for those who already have a parking pass. Finally, for mode choice, the zero cost for transit is for those who have a GA, and we assume that travel costs for walk and bike are zero. As for the car fuel cost, zero cost is possible for those who already have a parking space and travel a very short distance.

Table 3 Descriptive analysis of the parking data used

Data Sets	Attributes	Observations	Minimum	Maximum	Mean	St. Dev.
Influence of parking (Location)	Location A TT (min)	6301	9	64	29.48	15.68
	Location A TC (CHF)	6301	1.5	14.0	5.10	2.86
	Location B TT (min)	6301	7	64	29.67	15.84
	Location B TC (CHF)	6301	1.5	14.0	5.12	3.00
Influence of parking (Parking)	Parking A TT (min)	5835	8.0	39	22.41	11.80
	Parking A TC (CHF)	5835	0.0	20	5.60	5.24
	Parking B TT (min)	5835	8.0	39	22.05	11.77
	Parking B TC (CHF)	5835	0.0	20	5.52	5.22
Influence of parking (Mode choice)	Walk TT (min)	1666	4.0	170.0	59.25	37.22
	Bike TT (min)	1666	1.0	45.0	15.79	9.89
	Car TT (min)	1666	3.0	50.0	20.25	8.57
	Car TC (CHF)	1666	0.0	22.4	7.14	6.35
	Transit TT (min)	1666	3.0	108.0	23.84	15.07
	Transit TC (CHF)	1666	0.0	7.4	2.08	1.28

3.3 Car-Sharing and Carpooling

Two SP experiments were conducted to estimate the potential of carpooling in Switzerland. In order to gain insight about user perception regarding innovative modes, the SP part was composed of two different experiments, one of them including car sharing as an alternative. The details of the survey are available in Ciari and Axhausen (2012; 2013a; 2013b).

For the experiment, which includes car sharing, there are three alternative modes: car sharing, car, and transit. In total, 4350 observations from 735 respondents. In the other experiment, there are four alternative modes: carpooling as a driver (CPD), carpooling as a passenger (CPP), transit, and car. In total, 3975 observations from 511 respondents were used. Note that car is the only alternative that available across all 3975 observations, however since all observation have three available alternatives, we include 3975 observations in the model.

Table 4 presents the descriptive analysis of these two data sets. For car-sharing data set, the zero cost for transit is for those who have a GA. As for carpooling data set, we have zero travel time and travel cost for all alternatives except the car. That is because, for some observation, the CPP, CPD, and transit are not available. For those who have a GA, we set the travel cost for transit to zero.

Table 4 Descriptive analysis of car sharing and carpooling data used

Data Sets	Attributes	Observation	Minimum	Maximum	Mean	St. Dev.
Car-sharing	Car sharing TT (min)	4350	3.20	308.40	36.85	38.02
	Car sharing TC (CHF)	4350	0.83	439.67	41.45	49.32
	Car TT (min)	4350	3.20	318.00	40.79	37.34
	Car TC (CHF)	4350	0.55	747.40	44.83	63.49
	Transit TT (min)	4350	3.20	418.80	62.00	50.66
	Transit TC (CHF)	4350	0.00	244.80	14.20	20.76
Carpooling	CPP TT (min)	3975	0.0	297.6	31.59	36.08
	CPP TC (CHF)	3975	0.0	43.5	3.19	4.61
	CPD TT (min)	3975	0.0	258.0	18.33	31.66
	CPD TC (CHF)	3975	0.0	37.5	2.38	4.24
	Car TT (min)	3975	4.8	297.6	43.21	38.71
	Car TC (CHF)	3975	0.1	171.0	7.75	11.14
	Transit TT (min)	3975	0.0	372.0	45.83	54.58
	Transit TC (CHF)	3975	0.0	244.8	10.25	18.98

3.4 RP mode choice

Schmutz (2015) used data from the Swiss Microcensus 2010 for his study. The Mobility and Transport Microcensus is a survey conducted every five years that provides detailed information on mobility behavior of the Swiss residents. The official data set includes around 300,000 stages, 210,000 trips and 65,000 tours starting and ending at home. In the work of Schmutz (2015), only travel behavior part of the main survey for travel on one appointed day per individual have been used. Schmutz (2015) presents MNL models for five levels of aggregation: stage, sub-tour, tour, trip, and day plan. In this paper, we only use the trip data set.

The alternatives for mode choice are walk, bike, car, and transit. After some filtering, where we only use observations that have all four alternatives available and reasonable walk and bike travel time for all observations, we obtain 33942 observations. The details of the data set can be found in Schmutz (2015).

Table 5 presents the descriptive analysis of the RP data used. The zero cost for transit is for those who have GA.

Table 5 Descriptive analysis of the RP data used

Data Sets	Attributes	Observations	Minimum	Maximum	Mean	St. Dev.
RP mode choice	Walk TT (min)	33942	5.4	719.9	188.11	147.44
	Bike TT (min)	33942	2.0	240.0	62.71	49.15
	Car TT (min)	33942	4.0	253.0	21.13	11.75
	Car TC (CHF)	33942	1.0	14.0	2.06	1.53
	Transit TT (min)	33942	6.0	830.0	55.92	36.48
	Transit TC (CHF)	33942	0.0	34.0	5.05	4.10

4. Model Estimation

4.1 Estimation Result

For the Swissmetro data set, other than generic attributes, travel time and travel cost, we added alternative specific constants (ASC) for each mode to the utility function/regret function, and we normalize the Swissmetro ASC to zero.

The location choice and parking choice are non-labelled data sets. Therefore, we do not include ASCs. We use a similar method as in Hess et al. (2014) for the opt-out alternative case. In the first and second utility/regret function we multiply time and cost parameters with respective attributes. Then we include the third utility/regret function where there is only one parameter "none" to be estimated. For mode choice, only car and transit are available across 1666 observations. For those who have the walk alternative, there is no bike alternative and vice versa. We added four ASCs, and we normalize transit ASC to zero.

For the case of car sharing, we normalize the transit ASC to zero in our utility/regret function. As for the carpooling case, only car alternative is available across 3975 observations. Therefore, we normalize ASC car to zero in our model. Finally, for RP mode choice we normalize the ASC transit to zero.

All models are estimated using PythonBiogeme (Bierlaire, 2016). The results for MNL, CRRM, µ RRM, PRRM, and RAM for seven data sets are presented in Table 6 below. For brevity, we only present generic attributes and the scale parameter for µRRM.

Table 6 Estimation results

Data Sets	Attribute	MNL est	MNL t-stat	C RRM est	C RRM t-stat	µRRM est	µRRM t-stat	P RRM est	P RRM t-stat	RAM est	RAM t-stat
Swiss Metro (N=5607)	Time	-0.01	-10.9	-0.01	-17.8	-0.01	-9.3	-0.01	-8.4	-0.08	-9.3
	Cost	-0.01	-16.1	-0.01	-16.8	-0.01	-16.6	-0.01	-16.2	-0.08	-10.0
	µ					1.21	4.3				
	F-LL		-4382.490		-4539.672		-4373.356		-4418.252		-4239.245
	AIC		1.5646		1.6207		1.5617		1.5774		1.5136
	BIC		1.5694		1.6254		1.5677		1.5821		1.5183
Parking location choice (N=6301)	Time	-0.06	-31.5	-0.04	-31.4	-0.04	-22.2	-0.03	-28.2	-0.83	-6.0
	Cost	-0.18	-19.8	-0.13	-20.6	-0.14	-18.3	-0.10	-19.4	-1.85	-5.8
	µ					6.22*	1.0				
	F-LL		-5063.745		-4993.869		-4988.037		-5010.554		-5293.729
	AIC		1.6082		1.5861		1.5845		1.5914		1.6812
	BIC		1.6114		1.5893		1.5888		1.5946		1.6844
Parking choice (N=5835)	Time	-0.13	-32.3	-0.09	-30.6	-0.10	-20.0	-0.09	-27.2	-2.25	-2.4
	Cost	-0.16	-18.3	-0.15	-23.1	-0.18	-11.8	-0.14	-21.9	-3.71	-2.9
	µ					3.34**	1.7				
	F-LL		-3160.084		-2933.602		-2930.057		-2925.971		-3964.244
	AIC		1.0842		1.0065		1.0057		1.0039		1.3598
	BIC		1.0876		1.0100		1.0102		1.0074		1.3632
Parking mode choice (N=1666)	Time	-0.05	-14.1	-0.08	-17.9	-0.07	-9.9	-0.07	-10.1	-2.43	-3.5
	Cost	-0.14	-11.2	-0.08	-10.1	0.50	2.3	0.48	5.3	-0.68	-8.2
	µ					1.17	2.3				
	F-LL		-1366.330		-1321.320		-1349.922		-1349.337		-1414.494
	AIC		1.6463		1.5922		1.6278		1.6259		1.7041
	BIC		1.6625		1.6085		1.6473		1.6421		1.7203
Car sharing (N=4350)	Time	-0.02	-16.4	-0.02	-16.2	-0.02	-17.1	-0.02	-17.0	-0.19	-9.0
	Cost	-0.01	-8.5	-0.01	-7.1	-0.01	-7.0	-0.01	-6.9	-0.20	-6.6
	µ					0.12	18.3				
	F-LL		1583.012		1636.519		1680.706		1681.501		1926.19
	AIC		1.8352		1.8229		1.8132		1.8125		1.7563
	BIC		1.8410		1.8287		1.8205		1.8184		1.7621
Car pooling (N=3975)	Time	-0.01	-6.2	-0.01	-5.9	-0.01	-8.4	-0.01	-8.6	-0.10	-7.6
	Cost	-0.05	-5.3	-0.03	-4.3	-0.03	-4.2	-0.03	-4.3	-0.64	-5.6
	µ					0.09	13.9				
	F-LL		-3950.835		-3949.359		-3929.118		-3922.169		-3832.877
	AIC		1.9904		1.9896		1.9799		1.9759		1.9310
	BIC		1.9983		1.9975		1.9894		1.9838		1.9389
RP mode choice (N=33942)	Time	-0.02	-16.2	-0.01	-12.2	-0.01	-12.8	-0.01	-11.5	-0.17	-9.5
	Cost	-0.14	-13.8	-0.06	-16.8	-0.07	-15.0	-0.06	-18.1	-1.50	-7.6
	µ					2.59	9.1				
	F-LL		-15417.741		-15410.937		-15382.139		-15459.477		-14990.85
	AIC		0.9088		0.9084		0.9067		0.9112		0.8836
	BIC		0.9100		0.9096		0.9082		0.9125		0.8849

Note:* = not significant; ** = 10% significant

For all models, the parameter of time and cost are significant with the expected sign (negative). However, we need to be careful in interpreting these parameters. In MNL, a parameter estimate refers to increase or decrease in the utility of an alternative caused by a one-unit or one standard deviation increase in an attribute's value. Therefore in the case of our MNL models, the increase by a standard deviation of travel time and travel cost of an alternative decrease the utility of that alternative.

In the RRM context, a parameter estimate reflects the *potential* increase or decrease in regret associated with comparing a considered alternative with another alternative in term of one unit increase in an attribute's value. In short RRM is context dependent. Whereas in RUM, the attribute of other alternatives is irrelevant, in RRM attribute of others alternatives can influence the increase/decrease the regret of the chosen alternative. For RAM context, a parameter estimate in RAM context reflects the *potential* increase or decrease in relative advantage associated with comparing a considered alternative with another alternative in term of one unit increase in an attribute's value.

We present the model fit comparison in Table 6 that consists of log-likelihood, Akaike information criterion (AIC) and Bayesian information criterion (BIC). Looking at AIC and BIC, we found that RAM outperforms other models in the case of Swissmetro, car sharing, car pooling, and RP mode choice. We also found that µRRM outperforms other models in the case of parking location and parking mode choice, while PRRM outperforms other models in the case of parking choice.

Regarding the comparison of MNL and CRRM in term of model fit, we found that only two times MNL outperforms CRRM in the case of Swissmetro and car pooling. This result underlines the literature result, that none of the models are clearly superior in all cases.

4.2 Prediction Accuracy

Hit rate can also be one of the indicators measuring the goodness of fit of a choice model. Hit rate refers to the fit between actual choice observed from the data and the predicted choice obtained by using the model itself. The higher the hit rate the closer we can say that our model represents reality. In Table 7, we present the prediction accuracy of five modeling approaches across seven data sets. In the first five columns, we present the hit rate of five models. In another column, we present percentage of observations where all models produce the same outcome regardless the observed choice are. This column followed by another column where we show the percentage of observations which all models correctly predict the outcomes.

Table 7 Prediction Accuracy

Data Sets	Hit rate					All models predict the same outcome	All models predict the right outcome
	MNL	CRRM	µRRM	PRRM	RAM		
Swiss Metro (N=5607)	68.50%	68.50%	68.50%	68.50%	69.10%	91.14%	64.38%
Parking location choice (N=6301)	67.80%	68.00%	68.00%	68.00%	67.30%	94.02%	65.40%
Parking choice (N=5835)	81.10%	81.70%	81.90%	81.30%	80.10%	88.47%	78.01%
Parking mode choice (N=1666)	65.49%	61.16%	61.22%	61.34%	62.30%	67.65%	47.84%
Car sharing (N=4350)	59.20%	59.80%	60.00%	60.10%	60.70%	82.76%	52.69%
Car pooling (N=3975)	49.26%	49.08%	49.74%	49.74%	51.30%	81.91%	43.00%
RP mode choice (N=33942)	87.30%	87.30%	87.30%	87.30%	87.40%	99.48%	87.10%

We found two data sets where the hit rate is above 80% in the case of parking choice (SP unlabeled data) and RP mode choice. In other three data sets, Swiss metro, location choice, and parking mode choice, the hit rate is approximately 60%. As for the car pooling data sets the hit rate of all models are below 50% except for the RAM model. RAM model, in general, shows the highest model fit except for the case of the parking data sets. Overall we can say that RUM is outperformed by other approaches in all data sets in term of hit rate.

It is interesting to see the distribution of the prediction rate where all models predict the same outcome. For the most of our data sets, the prediction rate is above 80%, more specifically for three data sets, the prediction rate is above 90%. The highest prediction rate can be found in the case of RP mode choice which is almost 100%. The lowest prediction rate is in the case of parking mode choice; this might be due to the difference in the choice set, some have no walk alternative while the rests have no bike alternative.

The prediction rates for all models predict the right outcome can be seen in the last column. All the percentage is slightly below the hit rate of all the five modeling approach for each respective data set. The substantial difference between the percentage of all models predict correctly, and the hit rate can be found in the case of parking mode choice. This might be due to some observations facing zero alternative for a particular mode (walk or bike). The same reason might be applied to the car pooling data set wherein the case of car pooling not all of the observations facing all four alternatives.

4.3 Probability Plot

In this section present the probability of each alternative predicted by the five models. At the y-axis is the probability range from 0 to 1. At the x-axis is the observations. There are six lines in the figure each represent each modeling approach, and also one line represent the observed choice, with 0 means the alternative is not chosen, and 1 means the alternative is chosen. To plot this graph, we grouped together the observed choice, and the predicted choice from five modeling approaches for an alternative i. Then we sorted them based on the observed choice followed by MNL predicted choice as the base. That means the observation-n in x-axis for the alternative i is not necessarily the same as observation-n in x-axis for an alternative j.

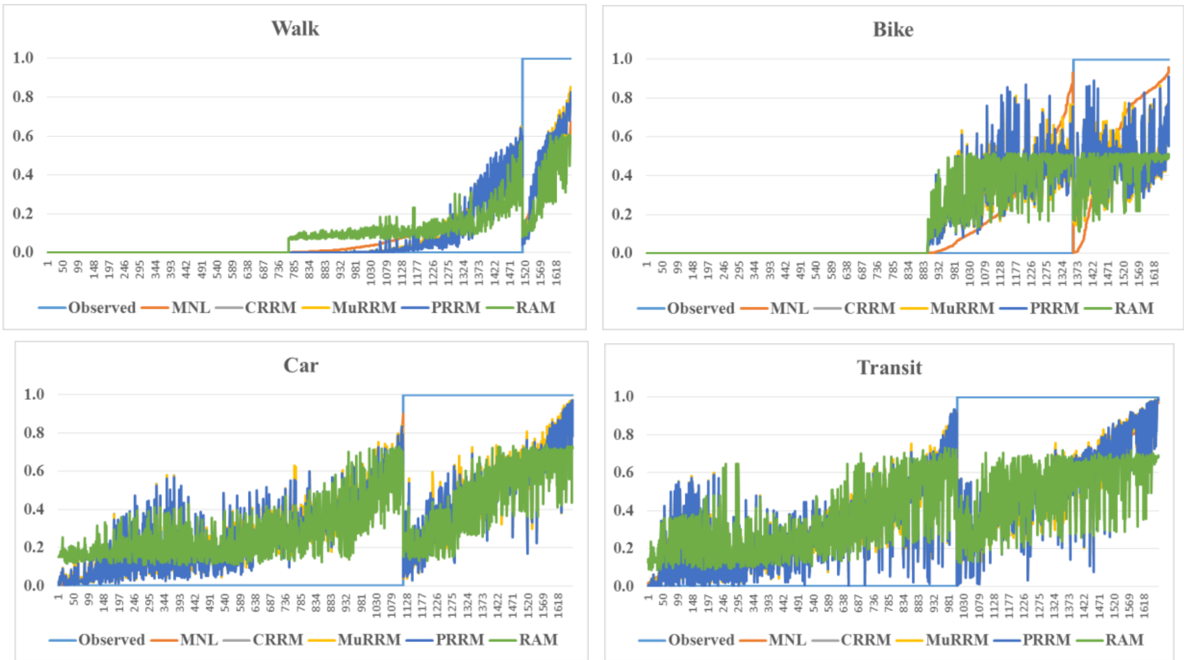
In Figure 1 we can see the probability plot for Swissmetro data set. For the train alternative, we can observe for those who did not choose a train, the probabilities of choosing train are very low which as expected. However, the probabilities of choosing train for those who chose train are very low which means that none of those people will be predicted to choose the train. Interestingly the probability for CRRM is higher than the other modeling approach.

Figure 1 Swissmetro probability plot

For the Swissmetro alternative, we can see that for those who chose Swissmetro, the probabilities plot is as expected even though there are some observations which have low probabilities. However, for those who did not choose Swissmetro, there are a number of observations which have high probabilities. This might be the reason why the hit rates for Swissmetro are only around 68%. For car alternative, the probabilities for those who did not choose a car is as expected, however, for those who choose a car, the probabilities for more than half of them are quite low.

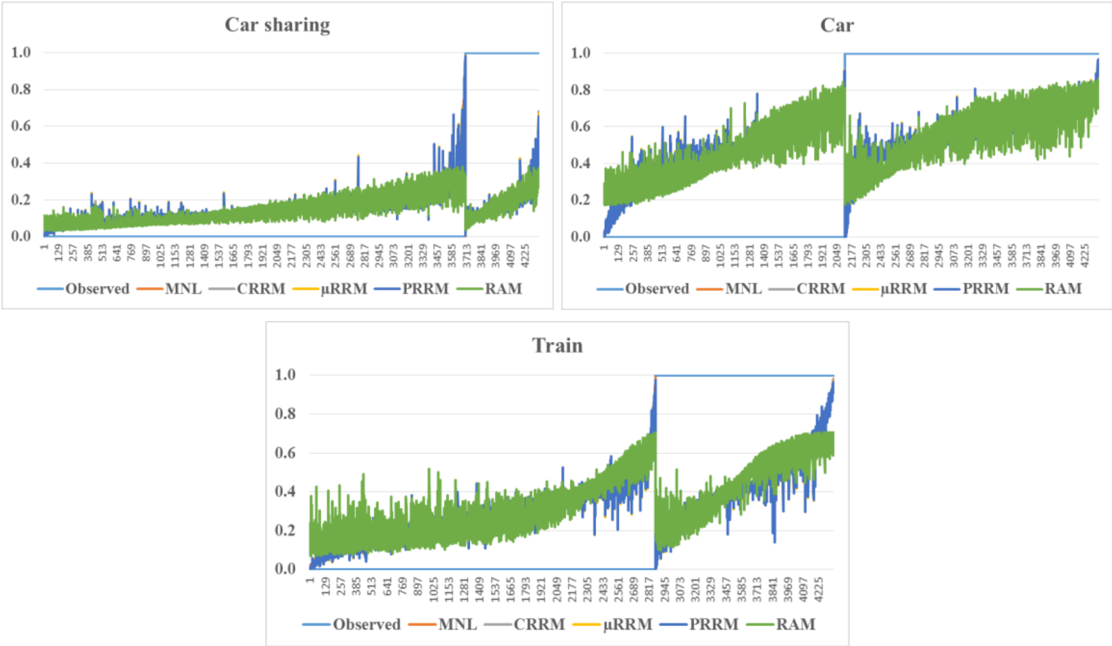
Since parking location and parking choice both are unlabeled data set, the probability plot of each alternative might not tell much information; we decide not to show the probability plot. In Figure 2, we show the probability plot for Parking mode choice. Those who are facing walk alternative are not facing bike alternative and vice versa. Therefore we can see zero probability for all observations on the left side of walk and bike alternative. Unlike in Figure 1 where the probabilities of µRRM and PRRM are mostly similar to RAM, here the probabilities of µRRM and PRRM are varied. For all of four alternatives, the probabilities plot are as expected, even though there are some cases where we find high probabilities on the non-chosen side for car and transit.

Figure 2 Parking mode choice



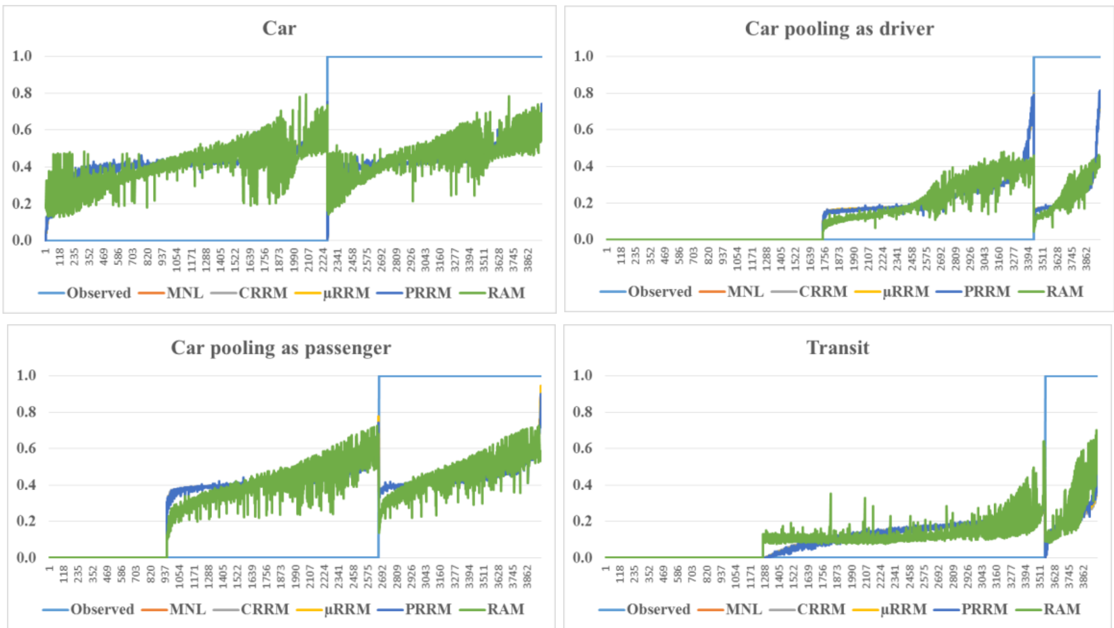
In Figure 3 we present the probability plot for car sharing data set. The probabilities plot for car alternative is as expected, however for the train alternative especially those who chose the train, the probabilities are quite low. Interestingly if we look at the car-sharing alternative, we can see that the probabilities for PRRM for some observations, in the non-chosen and chosen case, are quite high. It is also interesting that the probabilities for those who chose car sharing are very low.

Figure 3 Car sharing



We plot the probabilities for car pooling in Figure 4. Car is the only alternative which was faced by all observations, which explains the zero probabilities in the left side of the other three alternatives.

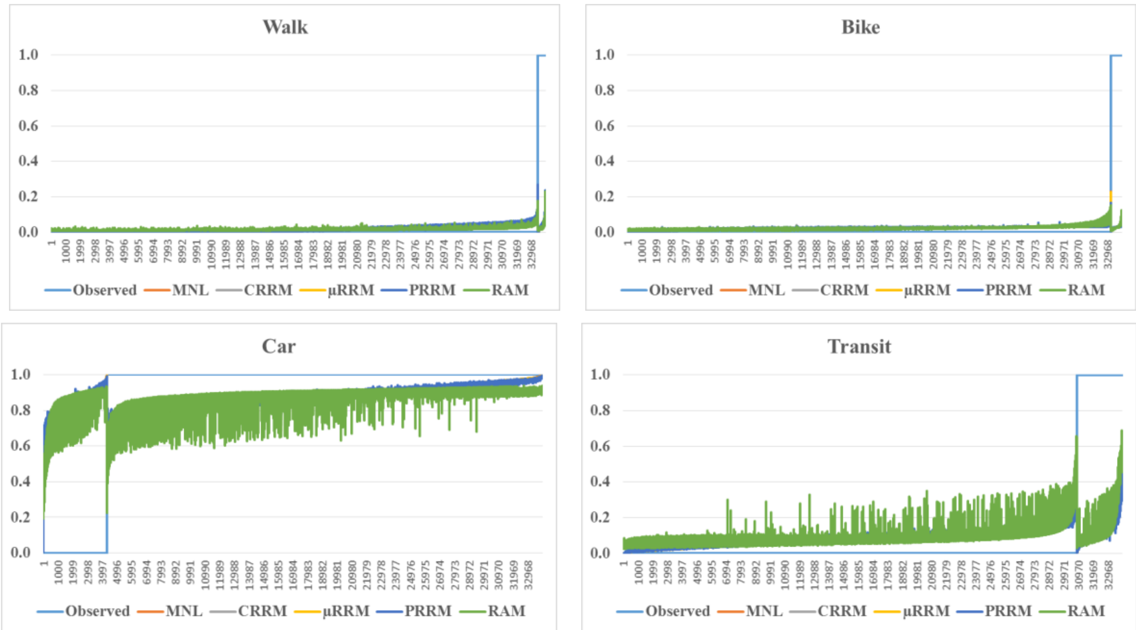
Figure 4 Car pooling



In our car pooling plot, we can see that for all alternatives except car, the probabilities are not that high for the chosen alternative side.

Finally, we present the probability of our RP mode choice data set in Figure 5. It is interesting to see that there is no high probability for walk and bike. We can also see that for transit alternative especially the chosen side, the probability to choose transit are not high. We can see high probabilities to choose a car which most of them above 0.5.

Figure 5 RP mode choice



4.4 Non-trading Behavior

Non-traders refer to respondents in stated preference survey who always make the same choice decision regardless of the available alternative's attributes. In our SP data sets, we have found some percentage of non-traders which is shown in Table 8. In this section, we show how many percents of non-traders can be predicted from our five modeling approaches.

In the case of Swissmetro, the non-traders of train alternative is only 2.09% which only 13 people. The five modeling approaches can not predict those for this alternative. This is understandable since in Figure 1 we can see that the probabilities for choosing train are very low. For Swissmetro alternative, we can see that there are 21.67% Swissmetro non-traders which is about 135 people. From this number, MNL and µRRM can correctly predict the non-traders by 83.70%, which is about 113 people. For car alternative, PRRM can give a higher prediction rate.

We do not present the non-labelled data set as it does not matter which alternative is chosen. Overall looking at the table, we might say that there is no modeling approach better than the others for all contexts. There is a case where MNL outperforms other, but that is also happening for RRMs and RAM.

Table 8 Non-trading prediction

Data Sets	Alter-native	Observed non-traders	Percentage non-traders predicted from observed non-traders				
			MNL	CRRM	μRRM	P RRM	RAM
Swiss Metro (sample=623)	Train	2.09%	0%	0%	0%	0%	0%
	Swissmetro	21.67%	83.70%	76.30%	83.70%	82.96%	77.78%
	Car	6.10%	28.95%	28.95%	28.95%	34.21%	39.47%
Parking mode choice (sample=168)	Walk	5.95%	10.00%	70.00%	70.00%	10.00%	20.00%
	Bike	7.14%	100%	8.33%	8.33%	0%	83.33%
	Car	5.36%	22.22%	11.11%	11.11%	11.11%	11.11%
	Transit	9.52%	0%	6.25%	6.25%	6.25%	18.75%
Car sharing (sample=735)	Car sharing	2.31%	0%	0%	0%	0%	0%
	Car	25.44%	81.28%	77.01%	71.66%	70.59%	58.29%
	Transit	21.90%	21.74%	22.98%	24.84%	24.84%	36.65%
Car pooling (sample=511)	Car	8.22%	52.38%	52.38%	52.38%	54.76%	52.38%
	CP driver	1.37%	14.29%	14.29%	14.29%	14.29%	0%
	CP passenger	5.87%	3.33%	3.33%	3.33%	3.33%	3.33%
	Transit	2.94%	0%	0%	0%	0%	0%

5. Value of Travel Time Savings

The value of travel time savings (VTTS) is an important concept for travel demand analysis. It measures how much money (e.g.CHF) a person is willing to pay for a unit reduction in travel time (e.g. an hour). The VTTS for the MNL model can be obtained from Eq.12 below

$$VTTS_{iq}^{MNL} = 60 \times \frac{\partial V_{iq} / \partial TT_{iq}}{\partial V_{iq} / \partial TC_{iq}} = 60 \times \frac{\beta_{TT}}{\beta_{TC}} \tag{12}$$

Where V_{iq} represents systematic utility for an alternative i for person q , TT_{iq} represents travel time associated with a person q choosing an alternative i , and TC_{iq} represent travel cost associated with a person q choosing an alternative i . The parameters of travel time and travel cost are represented by β_{TT} and β_{TC} respectively. Since RUM is not context dependent, the VTTS for an alternative is not influenced by other alternatives as in the case of RRM. The methods to measure VTTSs for context-dependent choice models are described below.

5.1 Method to measure context-dependent choice VTTS

5.1.1 CRRM VTTS

To measure the VTTS for CRRM we need to derive the systematic regret of the person q choosing the alternative i with respect to attribute X_{kiq}. The derivation is shown in Eq. 13 below, with more details in Appendix 1.

$$\begin{aligned}\frac{\partial R_{iq}}{\partial X_{kiq}} &= \sum_{\substack{i\in J\\ j\neq i}} \frac{-\beta_k \cdot \exp\left[\beta_k \cdot \left(X_{kjq}-X_{kiq}\right)\right]}{1+\exp\left[\beta_k \cdot \left(X_{kjq}-X_{kiq}\right)\right]} = \sum_{\substack{i\in J\\ j\neq i}} \frac{-\beta_k}{\dfrac{1}{\exp\left[\beta_k \cdot \left(X_{kjq}-X_{kiq}\right)\right]}+1} \\ &= \sum_{\substack{i\in J\\ j\neq i}} \frac{-\beta_k}{\exp\left[-\beta_k \cdot \left(X_{kjq}-X_{kiq}\right)\right]+1}\end{aligned} \tag{13}$$

Eq. 13 enters the VTTS formula as shown in Eq. 14 below, which also presented in Chorus (2012b).

$$VTTS_{iq}^{CRRM} = 60\times\frac{\partial R_{iq}/\partial TT_{iq}}{\partial R_{iq}/\partial TC_{iq}} = 60\times\frac{\sum_{j\neq i} -\beta_{TT}\Big/\left(\exp\left[-\beta_{TT}\cdot\left(TT_{jq}-TT_{iq}\right)\right]+1\right)}{\sum_{j\neq i} -\beta_{TC}\Big/\left(\exp\left[-\beta_{TC}\cdot\left(TC_{jq}-TC_{iq}\right)\right]+1\right)} \tag{14}$$

Eq. 14 implies that VTTS measures will generally change when choice set changes in terms of alternatives. Changes in attributes of competing for an alternative as well as changes in attributes of the chosen alternative will influence the VTTS.

5.1.2 µRRM VT

The derivative of the systematic regret of the µRRM model is shown in Appendix 2. The formula for deriving the µRRM VTTS is shown in Eq. 15 below.

$$VTTS_{iq}^{\mu RRM} = 60\times\frac{\partial R_{iq}^{\mu RRM}/\partial TT_{iq}}{\partial R_{iq}^{\mu RRM}/\partial TC_{iq}} = 60\times\frac{\sum_{j\neq i} -\dfrac{\beta_{TT}}{\mu}\Bigg/\left(\exp\left(-\dfrac{\beta_{TT}}{\mu}\left[TT_{jq}-TT_{iq}\right]\right)+1\right)}{\sum_{j\neq i} -\dfrac{\beta_{TC}}{\mu}\Bigg/\left(\exp\left(-\dfrac{\beta_{TC}}{\mu}\left[TC_{jq}-TC_{iq}\right]\right)+1\right)} \tag{15}$$

5.1.3 PRRM VTTS

Van Cranenburgh and Prato (2016) derive the derivation of the systematic regret for PRRM model with respect to attribute X_{kiq} as shown in Eq.16 below:

$$\frac{\partial R_{iq}}{\partial x_{kiq}} = \begin{cases} -\beta_k \sum\limits_{j \neq i} 1 & \text{if } \beta_k < 0 \text{ and } x_{kjq} < x_{kiq} \text{ or } \beta_k > 0 \text{ and } x_{kjq} > x_{kiq} \\ 0 & \text{if } \beta_k < 0 \text{ and } x_{kjq} > x_{kiq} \text{ or } \beta_k > 0 \text{ and } x_{kjq} < x_{kiq} \end{cases} \tag{16}$$

Since we only have two generic attributes and we already know in advanced that our parameter estimates are both negatives, then we use the upper left part of Eq.16. Thus part of Eq.16 enter Eq.17 for deriving the PRRM VTTS

$$VTTS_{iq}^{PRRM} = 60 \times \frac{\partial R_{iq} / \partial TT_{iq}}{\partial R_{iq} / \partial TC_{iq}} = 60 \times \frac{-\beta_{TT} \sum\limits_{\substack{j \neq i \\ TT_{jq} < TT_{iq}}} 1}{-\beta_{TC} \sum\limits_{\substack{j \neq i \\ TC_{jq} < TC_{iq}}} 1} \tag{17}$$

Let us recall the properties of PRRM as shown in Eq.5, X_{kjiq}^{PRRM} is obtained by the summation of $\min(0, X_{kiq} - X_{kjq})$ in the case of a negative parameter. Therefore in the condition where the chosen alternative is outperformed by the competing for an alternative, the derivative of systematic regret with respect to travel time or travel cost will become zero. If that is the case, we will have an infinite VTTS for the respected person and respected alternative.

5.1.4 RAM VTTS

Leong and Hensher (2015) have already derived an equation for measure RAM VTTS, as shown in Eq. 18:

$$VTTS_{iq}^{RAM} = 60 \times \frac{\partial V_{iq}^{RAM} / \partial TT_{iq}}{\partial V_{iq}^{RAM} / \partial TC_{iq}} = 60 \times \frac{\sum\limits_{j \neq i} \dfrac{D_{ijq} \dfrac{\partial A_{ijq}}{\partial TT_{iq}} - A_{ijq} \dfrac{\partial D_{ijq}}{\partial TT_{iq}}}{\left[A_{ijq} + D_{ijq}\right]^2}}{\sum\limits_{j \neq i} \dfrac{D_{ijq} \dfrac{\partial A_{ijq}}{\partial TC_{iq}} - A_{ijq} \dfrac{\partial D_{ijq}}{\partial TC_{iq}}}{\left[A_{ijq} + D_{ijq}\right]^2}} \tag{18}$$

The derivation of advantage and disadvantage of the person q choosing alternative i over j is in Eq. 19 below:

$$\frac{\partial A_{ijq}}{\partial X_{kiq}} = \frac{\beta_k}{\exp\left[\beta_k \cdot \left(X_{kjq} - X_{kiq}\right)\right] + 1} \text{ and } \frac{\partial D_{ijq}}{\partial X_{kiq}} = \frac{-\beta_k}{\exp\left[-\beta_k \cdot \left(X_{kjq} - X_{kiq}\right)\right] + 1} \tag{19}$$

5.2 VTTS result and discussion

In this subsection, we present the result of VTTS, mean value and standard deviation for each alternative for the five models in Table 9.

Table 9 Value of travel time savings (CHF/hour)

Data Sets	Alter-native	MNL		C RRM		μRRM		P RRM		RAM	
		Mean	St.d	Mean	St.d	Mean	St.d	Mean	St.d	Mean	St.d
Swiss Metro (N=5607)	Train			150.91	28.7	84.66	11.9	70.59	69.1	38.92	23.5
	SM	66.10	0	56.51	24.7	48.62	11.6	51.90	18.7	35.02	25.9
	Car			134.18	114.9	78.63	34.8	30.45	37.6	112.94	164.0
Parking location choice (N=6301)		19.60	0	19.29	4.6	18.50	1.1	21.77	10.4	32.40	92.9
Parking choice (N=5835)		46.63	0	42.10	8.6	36.61	4.1	38.83	11.5	$7*10^{10}$	$9*10^{11}$
Parking mode choice (N=1666)	Walk			101.31	24.6	99.56	29.4	116.19	19.2	16.85	33.7
	Bike	20.29	0	38.83	10.0	55.01	16.1	87.26	30.6	86.42	180.5
	Car			38.58	17.0	39.18	15.8	43.19	21.2	147.31	369.3
	Transit			54.54	23.5	53.60	21.2	64.17	33.1	33.55	62.8
Car sharing (N=4350)	CS			91.88	18.0	59.60	37.9	95.44	46.5	84.97	118.8
	Car	104.94	0	95.22	17.2	68.20	70.8	86.85	52.6	95.09	185.4
	Transit			159.26	105.2	$9*10^{10}$	$5*10^{12}$	16.81	53.8	117.54	239.3
Car pooling (N=3975)	Car			14.14	1.6	19.24	12.5	19.46	8.7	15.06	31.3
	CPD	9.83	0	14.27	2.0	55.17	677.5	33.67	11.2	10.56	11.2
	CPP			14.88	2.6	$5*10^{5}$	$2*10^{7}$	29.65	10.7	12.06	16.2
	Transit			14.21	1.9	23.90	16.6	21.48	7.1	41.01	75.3
RP mode choice (N=33942)	Walk			19.42	4.5	15.34	2.9	33.13	4.3	1.71	1.0
	Bike	9.25	0	11.29	1.4	11.08	0.3	16.30	6.3	4.84	1.5
	Car			7.38	2.0	9.29	1.4	0.18	1.0	3.36	1.5
	Transit			9.05	2.1	10.03	1.4	6.01	2.2	8.16	7.3

For unlabelled SPs it does not make sense to present values for the two alternatives as the order in the choice experiment (left or right alternative) does not matter and it is quite random. Therefore we only present the mean VTTS from two alternatives.

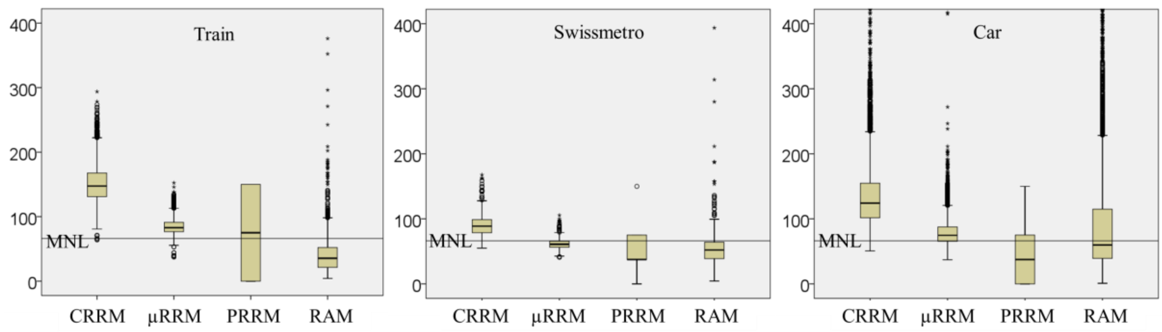
The VTTSs results are in the expected range except for some strange results in the case of parking choice data for RAM model and also μRRM transit alternative for car sharing data set.

To do a better depiction of the VTTSs distribution, we plot the VTTS by choice situation for each alternative with a box plot. At the x-axis, we present the four context-dependent models.

At the y-axis, we present the VTTS in CHF per hour. The reference line attached to the y-axis represents MNL VTTS.

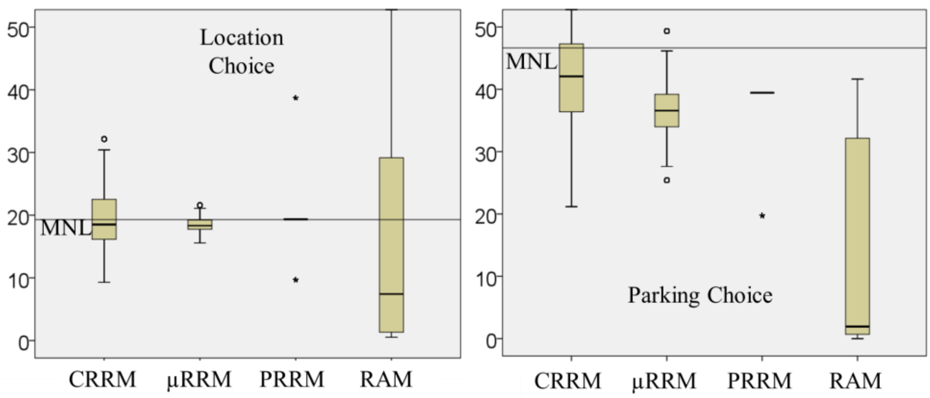
For the Swissmetro data set, the depiction of VTTS can be seen in Figure 6. We can see for train case, the CRRM and μRRM VTTS are mostly above MNL VTTS. For the Swissmetro case, the VTTS of other modeling approaches is below the MNL. Finally, for car VTTS, we can see that for the CRRM, and μRRM VTTS are mostly above MNL. The VTTS for PRRM is below MNL VTTS. In Car alternative, we can see many and quite substantial outliers for the car alternative.

Figure 6 Swissmetro VTTS (CHF/hour)



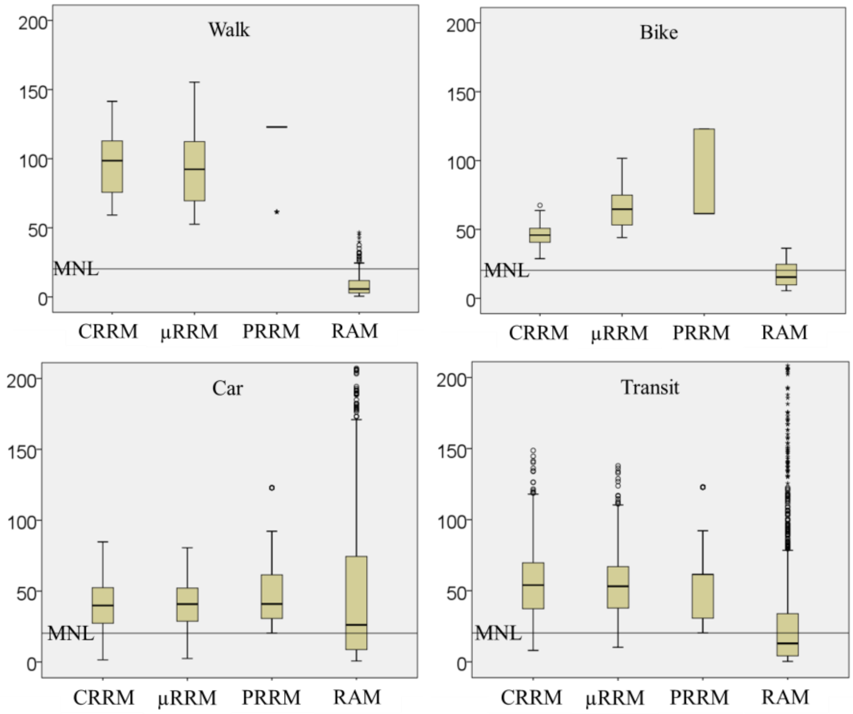
For the unlabeled data sets with an opt-out alternative, the depiction of VTTSs for location choice and parking choice can be seen in Figure 7. For these unlabeled cases, the RAM VTTS distribution is very high.

Figure 7 Location and parking choice VTTS (CHF/hour)



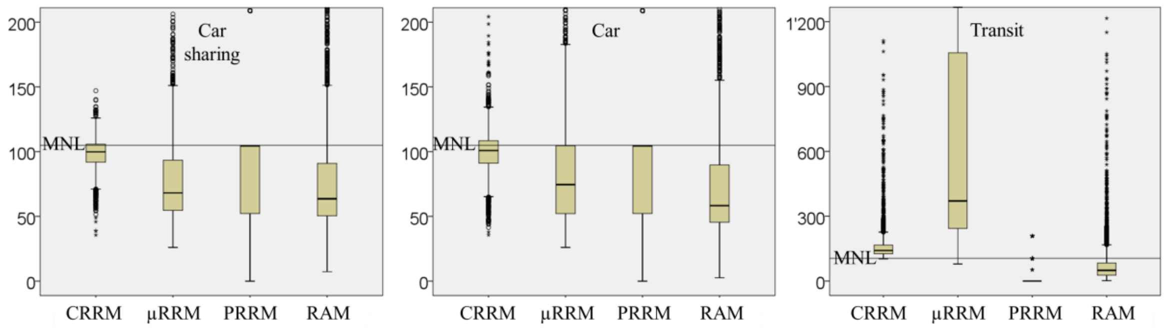
In Figure 8 we present the plot of parking mode choice VTTS. For all alternatives, the VTTS for PRRM and RAM are below MNL.

Figure 8 Parking mode choice VTTS (CHF/hour)



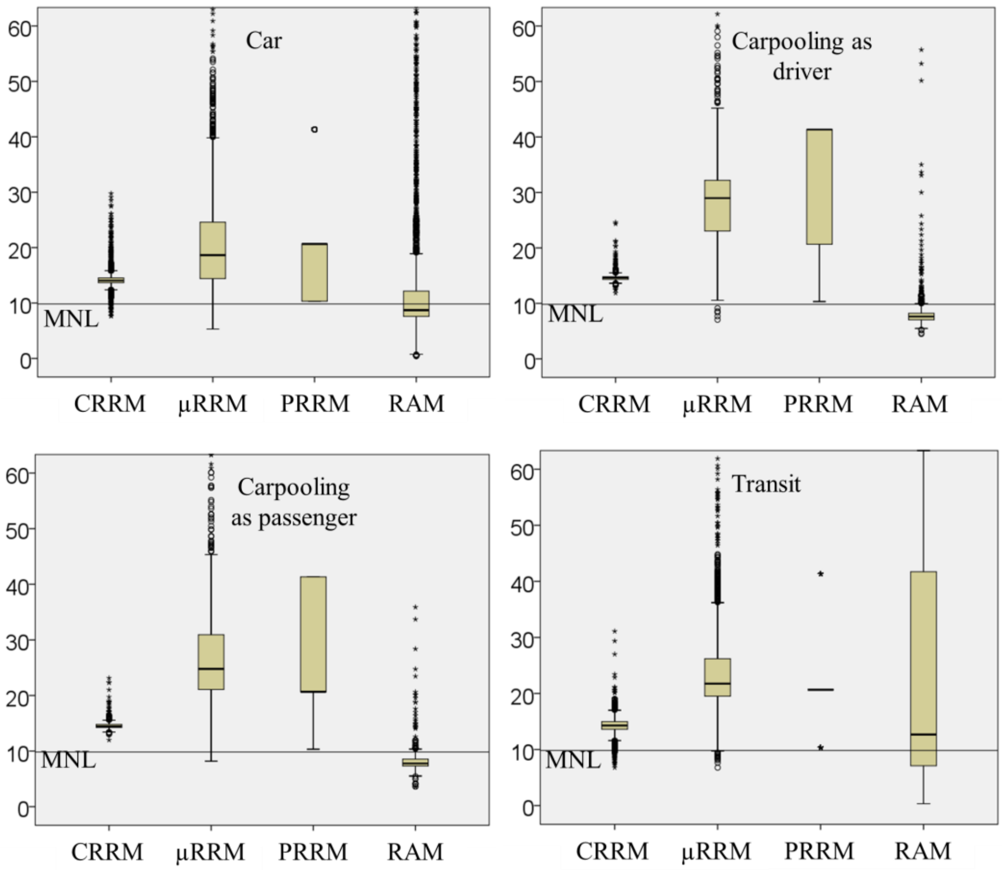
In Figure 9 we present the plot of car sharing VTTS. There are many outliers for RAM for three alternatives. For car sharing and car alternative, the VTTS are below MNL for all modeling approaches. Only in the case of transit, we can see that the µRRM VTTS is substantially higher than MNL.

Figure 9 Car sharing VTTS (CHF/hour)



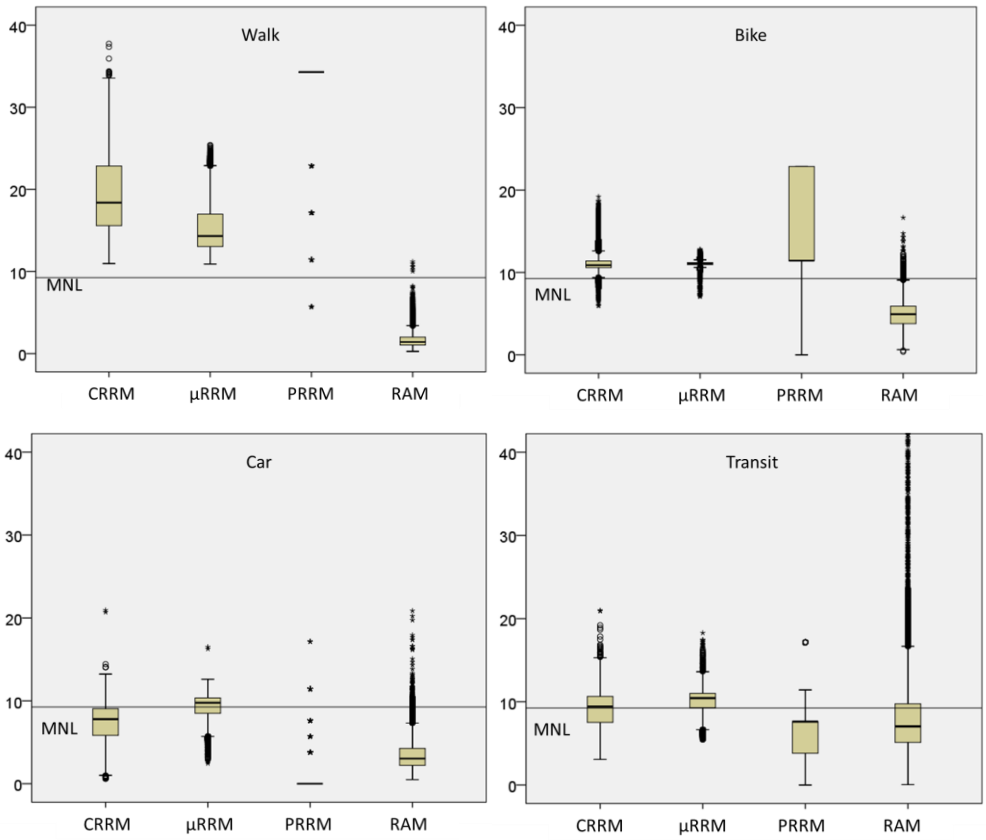
The VTTS for car pooling data set can be seen in Figure 10. The distribution for CRRM for Car and car pooling as passenger alternatives are very low. For the same alternatives, the MNL VTTS is below all the other modeling approaches VTTS. For car pooling as driver and transit alternatives, we can see a high distribution of VTTS in four modeling approaches.

Figure 10 Car pooling VTTS (CHF/hour)



Finally, in Figure 11, we present the VTTS for RP mode choice. For Walk and Bike alternatives, only RAM VTTS is below MNL. For car alternative, we can see that all modeling approaches VTTS are below MNL. For transit alternative, we can see that µRRM VTTS is above MNL while other approaches' VTTS are below MNL.

Figure 11 RP mode choice VTTS (CHF/hour)



6. Travel Time and Cost Elasticities

Direct elasticities measure is another way to compare the behavior implication of RUM and RRM. The direct elasticities derive from the RUM and RRM model shows the relationship between a percentage change in the magnitude of the attribute and the percentage change in the probability of choosing an alternative based on respected attribute. To measure the direct elasticities of RUM model, we can use the formula from Ben-Akiva and Lerman (1985) shown in Eq. 20.

$$E_{iqX_{kiq}} = \frac{\partial P_{iq}}{\partial X_{kiq}} \cdot \frac{X_{kiq}}{P_{iq}} = (1 - P_{iq}) \cdot \beta_k \cdot X_{kiq} \tag{20}$$

In the following subsection, we show how to measure direct elasticities for three variant RRM models.

6.1 RRM Elasticities

Hensher et al. (2013) derive for the first time an equation to measure RRM (CRRM) elasticities as shown in Eq. 21. This equation according to Van Cranenburgh and Prato (2016) can also be used to measure PRRM and also μRRM.

$$E_{iqX_{kiq}} = \left(-\frac{\partial R_{iq}}{\partial X_{kiq}} + \sum_{\substack{i \in J \\ j \neq i \\ j=1}}^{J} P_{jq} \cdot \frac{\partial R_{jq}}{\partial X_{kiq}} \right) \cdot X_{kiq} \tag{21}$$

6.1.1 CRRM elasticities

The formula to measure the direct elasticities for CRRM (as discussed in Hensher et al. 2013) is as follow.

$$E_{iqX_{kiq}}^{CRRM} = \left(-\frac{\partial R_{iq}^{CRRM}}{\partial X_{kiq}} + \sum_{\substack{i \in J \\ j \neq i \\ j=1}}^{J} P_{jq} \cdot \frac{\partial R_{jq}^{CRRM}}{\partial X_{kiq}} \right) \cdot X_{kiq} =$$
$$\left(\left(-\sum_{\substack{i \in J \\ j \neq i \\ j=1}}^{J} \frac{-\beta_k}{\exp\left[-\beta_k \cdot \left(X_{kjq} - X_{kiq}\right)\right] + 1} \right) + \left[\begin{array}{l} \sum_{\substack{i \in J \\ j \neq i \\ j=1}}^{J} P_{iq} \frac{-\beta_k}{\exp\left[-\beta_k \cdot \left(X_{kjq} - X_{kiq}\right)\right] + 1} + \\ \left(\sum_{\substack{i \in J \\ j \neq i \\ j=1}}^{J} P_{jq} \frac{\beta_k}{\exp\left[\beta_k \cdot \left(X_{kjq} - X_{kiq}\right)\right] + 1} \right) \end{array} \right] \right) \cdot X_{kiq} \tag{22}$$

6.1.2 μRRM elasticities

From Appendix 1, we find that the derivative of μRRM with respect to attribute X_k as shown

$$\frac{\partial R_{iq}^{\mu RRM}}{\partial X_{kiq}} = \sum_{\substack{i \in J \\ j \neq i \\ j=1}}^{J} \frac{-\frac{\beta_k}{\mu}}{\exp\left(-\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right] \right) + 1} \tag{23}$$

In order to measure μRRM direct elasticities, we derive the formula from Eq. 21 and Appendix 2 to get Eq. 24 below.

$$E_{iqX_{kiq}}^{\mu RRM} = \left(-\frac{\partial R_{iq}^{\mu RRM}}{\partial X_{kiq}} + \sum_{\substack{i\in J\\ j\neq i\\ j=1}}^{J} P_{jq} \cdot \frac{\partial R_{jq}}{\partial X_{kiq}} \right) \cdot X_{kiq}$$

$$= \left(\left(-\sum_{\substack{i\in J\\ j\neq i\\ j=1}}^{J} \frac{-\frac{\beta_k}{\mu}}{\exp\left(-\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right) + 1} \right) + \left[\begin{array}{l} \left(\sum_{\substack{i\in J\\ j\neq i\\ j=1}}^{J} P_{iq} \frac{-\frac{\beta_k}{\mu}}{\exp\left(-\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right) + 1} \right) + \\ \left(\sum_{\substack{i\in J\\ j\neq i\\ j=1}}^{J} P_{jq} \frac{\frac{\beta_k}{\mu}}{\exp\left(\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right) + 1} \right) \end{array} \right] \right) \cdot X_{kiq} \qquad (24)$$

6.1.3 PRRM elasticities

We can use the same formula as Eq. 16 to derive PRRM direct elasticities. Van cranenburgh and Pato (2016) already derived the formula as shown in Eq. 25 below.

$$E_{iqX_{kiq}}^{PRRM} = \left(-\frac{\partial R_{iq}}{\partial X_{kiq}} + \sum_{\substack{i\in J\\ j\neq i\\ j=1}}^{J} P_{jq} \cdot \frac{\partial R_{jq}}{\partial X_{kiq}} \right) \cdot X_{kiq}$$

$$\text{where } \frac{\partial R_{iq}}{\partial X_{kiq}} = \begin{cases} -\beta_k \sum_{j\neq i} 1 \text{ if } X_{kjq} < X_{kiq} \\ 0 \text{ if } X_{kjq} > X_{kiq} \end{cases} \text{ and } \frac{\partial R_{jq}}{\partial X_{kiq}} = \begin{cases} \beta_k \text{ if } X_{kjq} > X_{kiq} \\ 0 \text{ if } X_{kjq} < X_{kiq} \end{cases} \qquad (25)$$

6.1.4 RAM elasticities

We derive the formula to measure RAM elasticities in Appendix 3, as follows:

$$E_{iqX_{kiq}}^{RAM} = \frac{\partial P_{iq}}{\partial X_{kiq}} \cdot \frac{X_{kiq}}{P_{iq}} = \frac{\partial \ln P_{iq}}{\partial X_{kiq}} \cdot X_{kiq} = \left(\frac{\partial RA_{ijq}}{\partial X_{kiq}} - \sum_{\substack{i\in J\\ j\neq i\\ j=1}}^{J} P_{jq} \frac{\partial RA_{jiq}}{\partial X_{kiq}} \right) \cdot X_{kiq} \qquad (26)$$

The derivation for $\frac{\partial RA_{ijq}}{\partial X_{kiq}}$ can be seen in Leong and Hensher (2015) to measure RAM VTTS as shown in Eq. 18. While the derivation of $\frac{\partial RA_{jiq}}{\partial X_{kiq}}$ is as follows:

$$\frac{\partial RA_{jiq}}{\partial X_{kiq}} = \frac{D_{jiq} \cdot \frac{\partial A_{jiq}}{\partial X_{kiq}} - A_{jiq} \cdot \frac{\partial D_{jiq}}{\partial X_{kiq}}}{\left[A_{jiq} + D_{jiq}\right]^2} \tag{27}$$

The derivation of advantage and disadvantage of choosing alternative *j* over alternative *i* is similar to Eq. 19 as shown below:

$$\frac{\partial A_{jiq}}{\partial X_{kiq}} = \frac{-\beta_k}{\exp\left[-\beta_k \cdot \left(X_{kjq} - X_{kiq}\right)\right] + 1} \text{ and } \frac{\partial D_{jiq}}{\partial X_{kiq}} = \frac{\beta_k}{\exp\left[\beta_k \cdot \left(X_{kjq} - X_{kiq}\right)\right] + 1} \tag{28}$$

Substituting Eq. 28 to Eq. 26, the formula to derive direct elasticities of RAM is as follows:

$$\begin{aligned} E^{RAM}_{iqX_{kiq}} &= \left(\frac{\partial RA_{ijq}}{\partial X_{kiq}} - \sum_{\substack{i \in J \\ j \neq i \\ j=1}}^{J} P_{jq} \frac{\partial RA_{jiq}}{\partial X_{kiq}} \right) \cdot X_{kiq} \\ &= \left(\begin{array}{l} \left(\sum_{\substack{i \in J \\ j \neq i \\ j=1}}^{J} \frac{D_{ijq} \cdot \frac{\partial A_{ijq}}{\partial X_{kiq}} - A_{ijq} \cdot \frac{\partial D_{ijq}}{\partial X_{kiq}}}{\left[A_{ijq} + D_{ijq}\right]^2} \right) + \left(-\sum_{\substack{i \in J \\ j \neq i \\ j=1}}^{J} P_{iq} \frac{D_{ijq} \cdot \frac{\partial A_{ijq}}{\partial X_{kiq}} - A_{ijq} \cdot \frac{\partial D_{ijq}}{\partial X_{kiq}}}{\left[A_{ijq} + D_{ijq}\right]^2} \right) \\ + \left(-\sum_{\substack{i \in J \\ j \neq i \\ j=1}}^{J} P_{jq} \frac{D_{jiq} \cdot \frac{\partial A_{jiq}}{\partial X_{kiq}} - A_{jiq} \cdot \frac{\partial D_{jiq}}{\partial X_{kiq}}}{\left[A_{jiq} + D_{jiq}\right]^2} \right) \end{array} \right) \cdot X_{kiq} \end{aligned} \tag{29}$$

6.2 Travel Time Elasticities

We present the measurement of travel time elasticities across five models and seven data sets in Table 10. In the first column, we present the alternatives followed by the MNL measurement and RRMs in the next column. As we all know that RRM is the alternative model to the well-established RUM, therefore in the next column we present the difference between RUM and four other context-dependent models. All figures presented here is the mean elasticities of the sample.

The sign of all the time elasticities measurement is as expected which means that a percentage increase in travel time will have an effect on an average of some percentage reduction in the probability of choosing a respected alternative.

From the Swissmetro data set, we can see that travel time for Swissmetro alternative is nearly inelastic across all models. That means 10% increase in travel time for Swissmetro on average will not give substantial impact in the reduction of Swissmetro probability. But that is not the case for train alternative, for MNL model, the 10% increase of travel time for a train on average will give a result of 20% reduction of train choice probability. In the context of RRM, the 10% increase of train travel time, takes into account the level of travel time associated with car travel time and Swissmetro travel time. The 81.9% difference with RRM being higher than MNL might suggest that the idea that the wrong choice may have been taken amplifies the response away from normal RUM based elasticity.

Table 10 Travel time elasticities

Alternatives	MNL	CRRM	µRRM	PRRM	RAM	Percent difference MNL-CRRM	Percent difference MNL-µRRM	Percent difference MNL-PRRM	Percent difference MNL-RAM
Train	-2.03	-3.69	-1.88	-4.03	-1.34	-81.9%	7.4%	-98.6%	34.0%
Swissmetro	-0.46	-0.74	-0.37	-0.16	-0.40	-59.2%	20.3%	65.6%	14.2%
Car	-1.37	-2.55	-1.21	-2.28	-1.00	-86.3%	11.7%	-66.9%	26.6%
Location choice	-1.13	-1.23	-0.19	-1.60	-0.55	-8.8%	83.0%	-41.2%	51.6%
Parking choice	-1.15	-1.26	-0.20	-1.60	-0.52	-9.2%	82.9%	-38.9%	54.8%
Walk	-1.88	-2.54	-0.74	-3.43	-0.93	-35.3%	60.7%	-83.0%	50.3%
Bike	-1.96	-2.77	-0.80	-3.88	-0.94	-40.9%	59.2%	-97.4%	51.9%
Car	-2.35	-9.77	-5.54	-10.58	-0.72	-316.8%	-136.2%	-351.1%	69.5%
Transit	-0.63	-1.10	-0.93	-2.74	-0.81	-74.9%	-46.9%	-335.3%	-29.1%
Car Sharing	-0.66	-1.65	-1.41	-1.92	-0.57	-150.2%	-114.1%	-191.0%	13.8%
Car	-0.72	-1.69	-1.45	-3.69	-0.65	-136.3%	-102.0%	-415.1%	8.6%
Transit	-0.75	-0.80	-6.35	-0.93	-1.13	-6.8%	-744.0%	-23.7%	-49.8%
Car	-0.52	-0.57	-4.51	-0.55	-0.63	-9.6%	-770.1%	-5.5%	-20.6%
Car Pooling Driver	-0.98	-1.24	-11.91	-2.04	-0.86	-26.7%	-1119.1%	-108.7%	12.4%
Car Pooling Passenger	-0.19	-0.25	-3.61	-0.51	-0.29	-26.8%	-1768.3%	-163.4%	-47.8%
Transit	-0.11	-0.25	-4.15	-0.88	-0.52	-132.0%	-3695.4%	-703.4%	-374.1%
Walk (RP)	-0.13	-0.22	-3.25	-0.43	-0.43	-69.4%	-2420.6%	-234.5%	-230.5%
Bike (RP)	-0.30	-0.86	-15.31	-1.56	-0.28	-181.8%	-4940.9%	-412.7%	6.8%
Car (RP)	-4.15	-6.16	-2.25	-7.98	-0.68	-48.6%	45.8%	-92.4%	83.6%
Transit (RP)	-1.37	-1.29	-0.55	-1.76	-1.07	6.0%	59.6%	-28.5%	22.2%

In general, across all data sets and alternatives, we can see a substantial difference between MNL and CRRM with CRRM being higher. From the behavioral perspective, this might suggest that the potential regret one might have been experienced had the wrong choices being made amplifies the behavioral response.

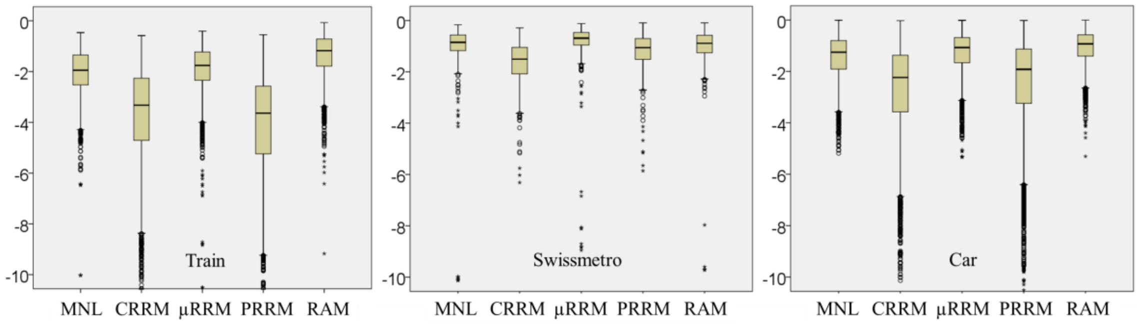
In the next column, we can see that the time elasticities difference between MNL and µRRM are substantially higher with MNL being highest overall. Van Cranenburgh et al. (2015) note that when the scale parameter is arbitrarily large, then the attribute level regret function are almost linear, and hardly yield difference between the regret generated by a loss and the rejoice generated by equivalent gain. In Table 6 we have seen that our scale parameter is mostly higher than 1. In this situation, the µRRM might yield the same choice probabilities as the linear-additive RUM model.

The next column shows the difference between MNL and PRRM. PRRM is a special case of µRRM where it postulates no rejoices, just pure regret. Therefore, it makes sense that across all data sets and alternatives we can see a higher difference compare to the difference between RUM and CRRM. Since it is pure regret, we might say that the potential regret one might have been experienced had the wrong choices being made without any rejoices, substantially amplifies the behavioral responses.

Finally, we can see the column for the RAM model; it is interesting that across all alternatives and data sets, the RAM models are inelastic. RAM is opposite of RRM where instead of measuring regret, it measures the relative advantage or the ratio of rejoice over total regret plus rejoice. Therefore, it makes sense that we see a difference in elasticity between MNL and RAM with MNL being higher. We might say that for the case of Swissmetro alternative, the 117.8% difference might suggest that the right choice might have been taken amplifies the behavioral responses.

We present the plot of each data set time elasticities for each alternative in Figure 12-17. For Swissmetro data sets, Figure 12, the distribution for all alternatives particularly Swissmetro is not too high. We can see some outliers for all modeling approaches.

Figure 12 Swissmetro time elasticities



For location and parking choice, Figure 13, we can see high distribution in MNL, CRRM, and PRRM. For parking mode choice, Figure 14, we can see low distribution in all modeling approaches except for CRRM in walk alternative. For car sharing data set, Figure 15, we can see high distribution in PRRM modeling approach. On the contrary for car pooling data set, Figure 16, the PRRM modeling approach time elasticities distribution is the lowest compared to other modeling approaches. For RP mode choice, Figure 17, we can see that only car alternative time elasticities distribution are mostly similar for all models. For transit, the time elasticities distribution is quite high, higher than bike distribution. We only found one strange case for MNL elasticity which is very high.

Figure 13 Location and parking choice time elasticities

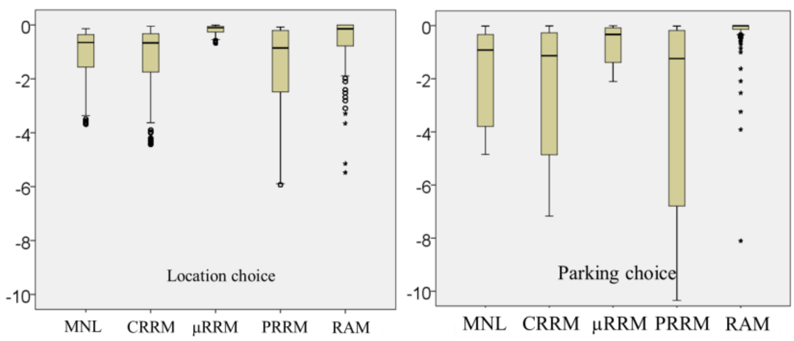


Figure 14 Parking mode choice time elasticities

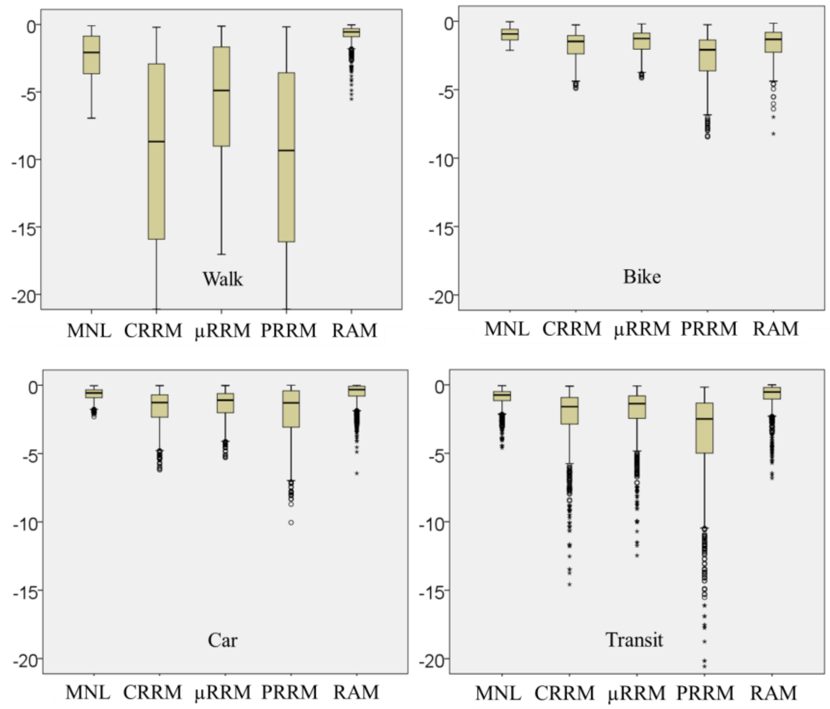


Figure 15 Car sharing time elasticities

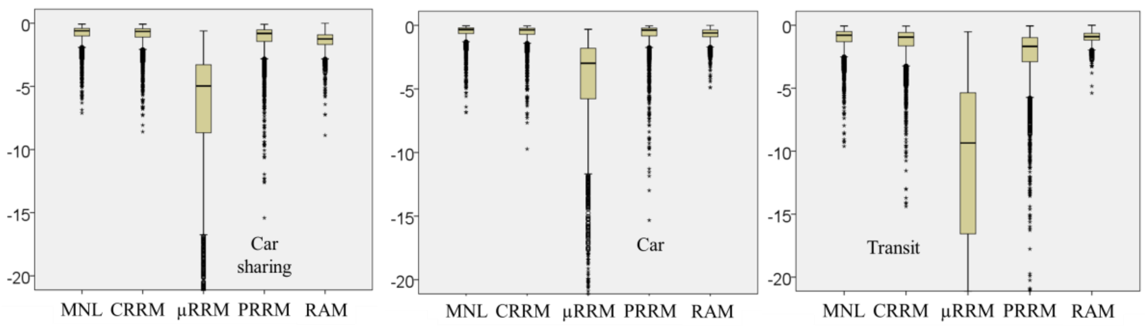


Figure 16 Carpooling time elasticities

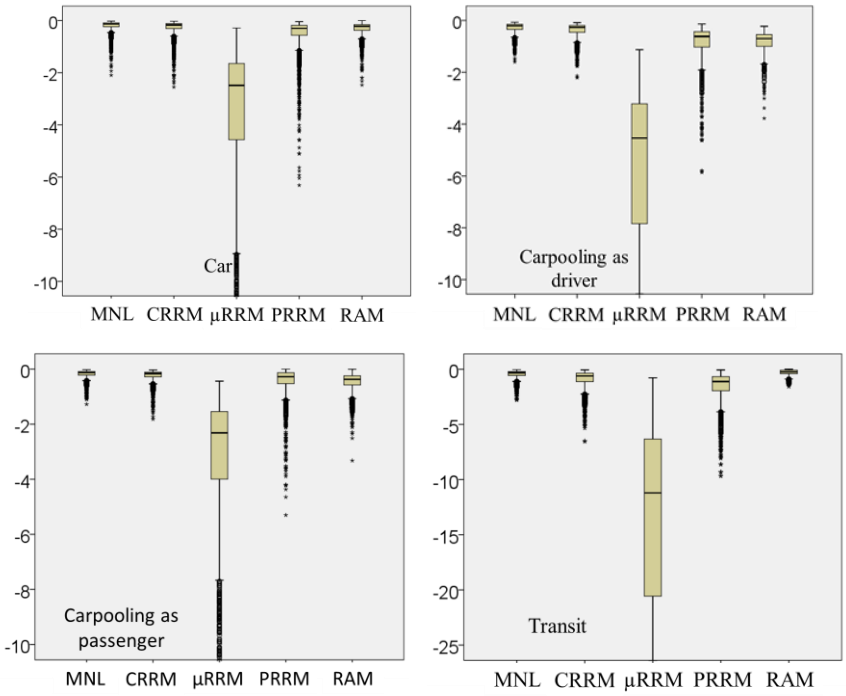
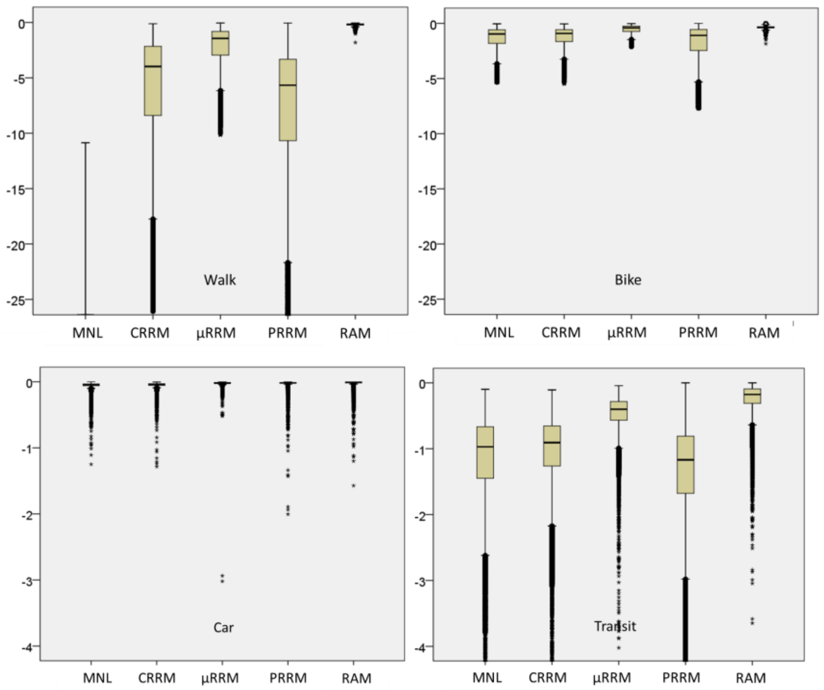


Figure 17 RP mode choice time elasticities



6.3 Cost Elasticities

We present the average of cost elasticities across four models and six data sets in Table 11. Overall the travel attributes are less elastic compared to traveling time attributes. Only in CRRM and PRRM, we can see the travel cost are elastic. We can also see a similar pattern with the travel time elasticity where CRRMs overall higher than MNLs and PRRMs are overall higher than CRRMs. RAMs are overall lower than the MNL.

Table 11 Cost elasticities

Alternatives	MNL	CRRM	µRRM	PRRM	RAM	Percent difference MNL-CRRM	Percent difference MNL-µRRM	Percent difference MNL-PRRM	Percent difference MNL-RAM
Train	-1.01	-1.03	-0.85	-0.64	-1.29	-2.0%	15.6%	36.3%	-27.8%
Swissmetro	-0.59	-0.69	-0.52	-1.06	-0.69	-18.4%	10.7%	-81.0%	-17.9%
Car	-0.79	-0.91	-0.69	-0.89	-0.74	-15.8%	12.8%	-13.5%	6.4%
Location choice	-0.57	-0.63	-0.10	-0.77	-0.28	-12.2%	82.1%	-35.9%	51.2%
Parking choice	-0.57	-0.65	-0.10	-0.79	-0.25	-12.6%	82.1%	-37.1%	56.2%
Car	-0.66	-1.09	-0.34	-1.47	-0.66	-63.6%	48.4%	-121.0%	0.3%
Transit	-0.68	-1.14	-0.36	-1.56	-0.71	-68.2%	47.1%	-130.8%	-5.5%
Car Sharing	-0.78	-0.84	-0.72	-1.09	-0.22	-8.3%	7.7%	-40.1%	71.5%
Car	-0.22	-0.18	-0.16	-0.22	-0.18	19.4%	30.0%	2.6%	18.9%
Transit	-0.47	-0.52	-4.87	-0.79	-0.92	-10.8%	-945.2%	-69.0%	-97.9%
Car	-0.36	-0.42	-3.95	-0.67	-0.49	-16.9%	-1001.0%	-87.2%	-35.6%
Car Pooling Driver	-0.12	-0.13	-0.91	-0.03	-0.27	-2.2%	-644.2%	78.4%	-122.6%
Car Pooling Passenger	-0.23	-0.20	-2.23	-0.32	-0.23	10.7%	-874.4%	-37.8%	-2.4%
Transit	-0.08	-0.12	-1.15	-0.17	-0.32	-52.9%	-1392.1%	-119.8%	-316.7%
Car (RP)	-0.08	-0.09	-0.86	-0.06	-0.25	-13.7%	-1007.7%	26.5%	-223.8%
Transit (RP)	-0.43	-0.86	-10.22	-1.06	-0.25	-100.5%	-2292.1%	-147.8%	40.6%

We present the plot of each data set cost elasticities for each alternative in Figure 18-23. For Swissmetro data set, Figure 18, the cost elasticities distribution is quite low for all models and all alternatives. For location and parking cost elasticities, Figure 19, the distribution of PRRM and RAM is the lowest, which is also the similar case for transit alternative for parking mode choice in Figure 20. For car sharing, Figure 21, the distribution for cost elasticities are quite low except for PRRM which is opposite case for car pooling in Figure 22. For RP data set,

Figure 23, the cost elasticities for car alternative are quite low for all alternatives. However, the cost elasticities for transit are quite high except for RAM case.

Figure 18 Swissmetro Cost Elasticities

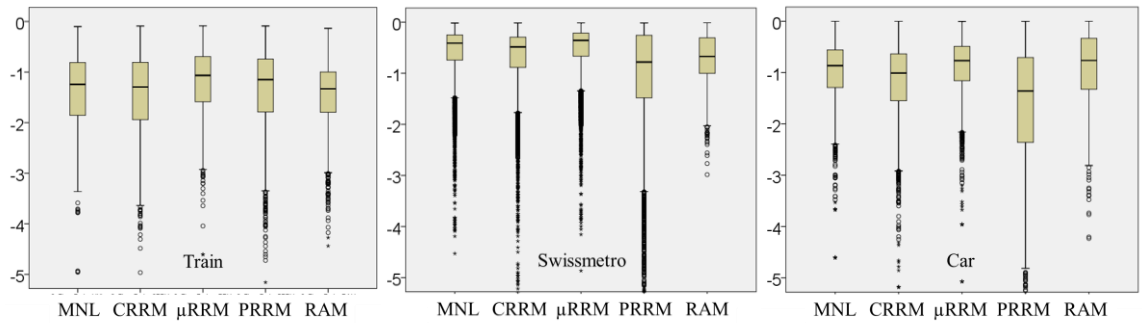


Figure 19 Location and parking cost elasticities

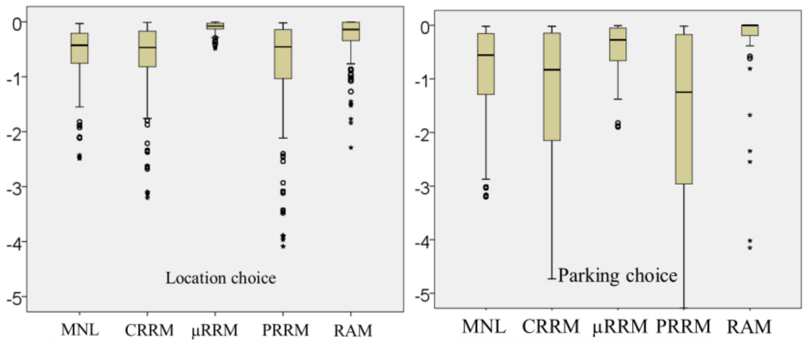


Figure 20 Parking mode choice cost elasticities

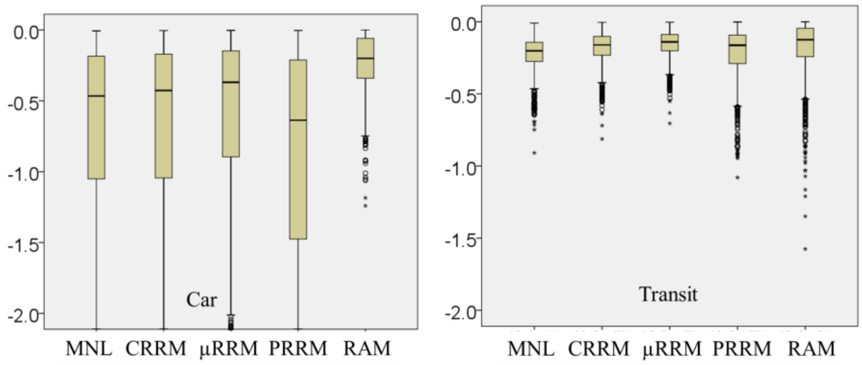


Figure 21 Car sharing choice cost elasticities

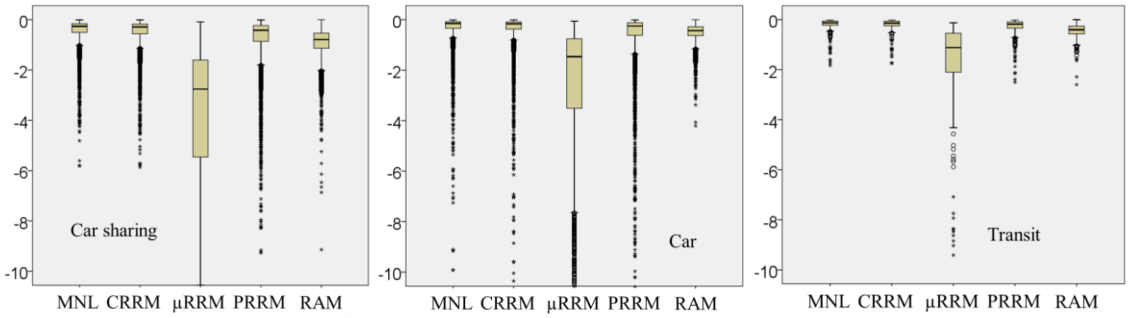


Figure 22 Carpooling choice cost elasticities

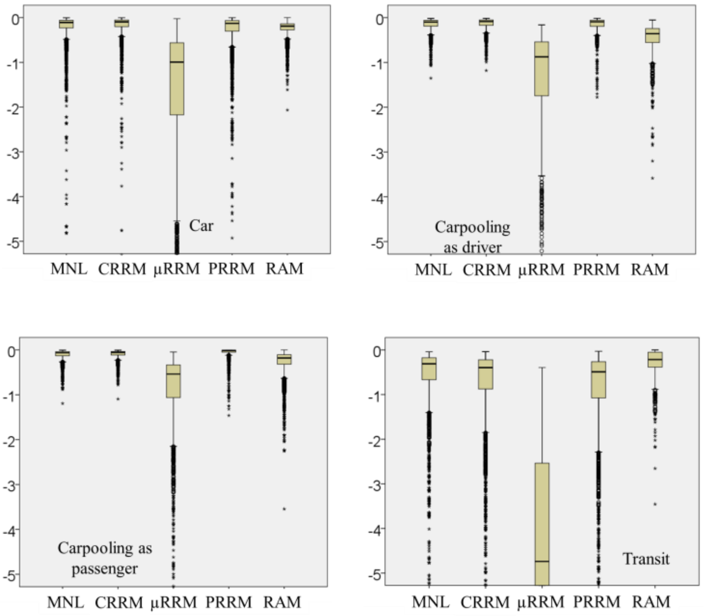
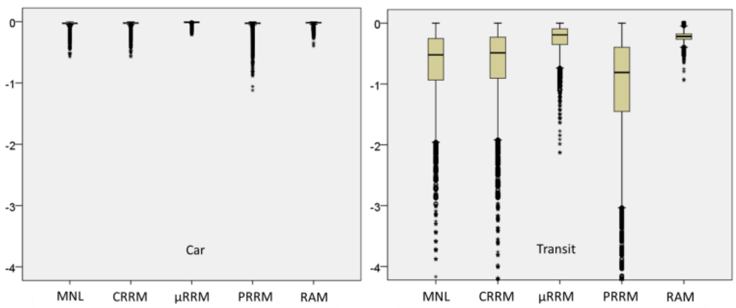


Figure 23 RP mode choice cost elasticities



7. Conclusion

There have been many empirical studies which compare the performance of RUM with context-dependent alternative modeling approaches such as RRM. In many cases, several empirical studies reported the better goodness of fit of RRM compares to RUM. However, there are also similar numbers of other empirical studies which reported that RUM model fit is better than RRM. Apparently, there has not been a consistent result of which modeling approaches is better than another. While most previous empirical studies reported model fit, few of them presented the prediction accuracy, VTTS and also demand elasticity to compare those modeling approaches.

Our goal is to comprehensively compare five different modeling approaches, RUM, CRRM, µRRM, PRRM, and RAM using Swiss data sets. We presented model fit, prediction accuracy, prediction plot, non-traders, VTTS and demand elasticity across five modeling approaches and seven data sets. With only two generic attributes, time and cost, we found in our model that the parameters of those attributes are significant with expected sign.

Our comparison of MNL and CRRM underlines the literature results that none of the two approaches, RUM and RRM are confirmed to be superior in all cases. In term of prediction rate, our results in many cases show that RUM and RRM hit rate is almost similar. Surprisingly the hit rate of the new RAM model is slightly higher than others especially for labelled data and RP data. We found interesting result that more than 80% all models predict the same outcome. This indicates that whatever model we use, we might ended up obtaining the same outcome. If we want to use the model which has higher model fit, then we might be able to use these comparisons, but in term of prediction, it might be a different case.

For the VTTS, we find that for MNL the result is the same for all alternatives. But for the case of other four models we can obtain different VTTS. We found some under-estimated value in the case of RP mode choice, where the VTTS is too low. We also some cases that show VTTS very high for example $9*10^{10}$ CHF/hour. These strange results require further examination. In term of time and cost elasticities, the sign is as expected. But we found that in many cases the different between MNL and other models are substantially high. For regret case, this might be due to the potential regret that will be faced by the person choosing that alternative. Similar explanation might be applied to RAM.

There are some limitations of this study. First, we only use two generic attributes for all our models. While it might give a better comparison, but we can not capture other significant factors that influence the decision. Second, we have tried to be as comprehensive as possible in which we include both labeled and unlabeled data sets in our presentation. However, the unlabeled data sets that we have are the one where we have an opt-out alternative. We do not have an unlabeled data set where there are three alternatives with three generic attributes. For future study, it would be better to add more RP data so that we can better compare and draw more conclusion. Since the modeling approaches that we presented here are a context-dependent model, different choice sets and different context might produce different results. Therefore more empirical results are necessary.

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9. Appendix

9.1 Appendix 1 Derivation of CRRM

In this appendix, we present comprehensively how to derive the systematic regret of CRRM in order to measure VTTS and elasticities. The derivation of RRM elasticities has previously shown in Hensher et al. (2013) as well as in Van Cranenburgh and Prato (2016). The derivation of systematic regret for an alternative i for person q with respect to attribute X_{kiq} is shown below:

$$\frac{\partial R_{iq}^{CRRM}}{\partial x_{iq}} = \frac{\partial}{\partial x_{kiq}} \sum_{\substack{i\in J\\ j\neq i}} \ln\left(1+\exp\left(\beta_k\left[x_{kjq}-x_{kiq}\right]\right)\right) = \sum_{\substack{i\in J\\ j\neq i}} \frac{\partial}{\partial x_{kiq}} \ln\left(1+\exp\left(\beta_k\left[x_{kjq}-x_{kiq}\right]\right)\right)$$

$$= \sum_{\substack{i\in J\\ j\neq i}} \frac{1}{1+\exp\left(\beta_k\left[x_{kjq}-x_{kiq}\right]\right)} \cdot \frac{\partial}{\partial x_{kiq}} \left(1+\exp\left(\beta_k\left[x_{kjq}-x_{kiq}\right]\right)\right)$$

$$= \sum_{\substack{i\in J\\ j\neq i}} \frac{\frac{\partial}{\partial x_{kiq}}(1) + \frac{\partial}{\partial x_{kiq}}\left(\exp\left(\beta_k\left[x_{kjq}-x_{kiq}\right]\right)\right)}{1+\exp\left(\beta_k\left[x_{kjq}-x_{kiq}\right]\right)} = \sum_{\substack{i\in J\\ j\neq i}} \frac{0+\exp\left(\beta_k\left[x_{kjq}-x_{kiq}\right]\right)\cdot \frac{\partial}{\partial x_{kiq}}\left(\beta_k\left[x_{kjq}-x_{kiq}\right]\right)}{1+\exp\left(\beta_k\left[x_{kjq}-x_{kiq}\right]\right)}$$

$$= \sum_{\substack{i\in J\\ j\neq i}} \frac{\exp\left(\beta_k\left[x_{kjq}-x_{kiq}\right]\right)\cdot \beta_k\left(\frac{\partial}{\partial x_{kiq}}\left(x_{kjq}\right) - \frac{\partial}{\partial x_{kiq}}\left(x_{kiq}\right)\right)}{1+\exp\left(\beta_k\left[x_{kjq}-x_{kiq}\right]\right)} = \sum_{\substack{i\in J\\ j\neq i}} \frac{\exp\left(\beta_k\left[x_{kjq}-x_{kiq}\right]\right)\cdot \beta_k(0-1)}{1+\exp\left(\beta_k\left[x_{kjq}-x_{kiq}\right]\right)}$$

$$= \sum_{\substack{i\in J\\ j\neq i}} \frac{-\beta_k \cdot \exp\left(\beta_k\left[x_{kjq}-x_{kiq}\right]\right)}{1+\exp\left(\beta_k\left[x_{kjq}-x_{kiq}\right]\right)} = \sum_{\substack{i\in J\\ j\neq i}} \frac{\left(-\beta_k \cdot \exp\left(\beta_k\left[x_{kjq}-x_{kiq}\right]\right)\right)/\exp\left(\beta_k\left[x_{kjq}-x_{kiq}\right]\right)}{\left(1+\exp\left(\beta_k\left[x_{kjq}-x_{kiq}\right]\right)\right)/\exp\left(\beta_k\left[x_{kjq}-x_{kiq}\right]\right)}$$

$$= \sum_{\substack{i\in J\\ j\neq i}} \frac{-\beta_k}{\frac{1}{\exp\left(\beta_k\left[x_{kjq}-x_{kiq}\right]\right)}+1} = \sum_{\substack{i\in J\\ j\neq i}} \frac{-\beta_k}{\exp\left(-\beta_k\left[x_{kjq}-x_{kiq}\right]\right)+1}$$

(Appendix 1.1)

The formula to measure direct elasticities for RRM is as follows:

$$E_{iqX_{kiq}} = \frac{\partial P_{iq}}{\partial X_{kiq}} \cdot \frac{X_{kiq}}{P_{iq}} = P_{iq} \cdot \frac{\partial \ln P_{iq}}{\partial X_{kiq}} \cdot \frac{X_{kiq}}{P_{iq}} = \frac{\partial \ln P_{iq}}{\partial X_{kiq}} \cdot X_{kiq}$$

$$= \left(\frac{\partial \ln \left(\frac{\exp(-R_{iq})}{\sum_j \exp(-R_{jq})} \right)}{\partial X_{kiq}} \right) \cdot X_{kiq} = \left(-\frac{\partial R_{iq}}{\partial X_{kiq}} - \frac{\partial \ln \left(\sum_j \exp(-R_{jq}) \right)}{\partial X_{kiq}} \right) \cdot X_{kiq}$$

$$= \left(-\frac{\partial R_{iq}}{\partial X_{kiq}} - \frac{\partial \ln \left(\sum_j \exp(-R_{jq}) \right)}{\partial X_{kiq}} \right) \cdot X_{kiq} = \left(-\frac{\partial R_{iq}}{\partial X_{kiq}} - \frac{1}{\sum_j \exp(-R_{jq})} \cdot \frac{\partial \left(\sum_j \exp(-R_{jq}) \right)}{\partial X_{kiq}} \right) \cdot X_{kiq}$$

$$= \left(-\frac{\partial R_{iq}}{\partial X_{kiq}} - \frac{1}{\sum_j \exp(-R_{jq})} \cdot \sum_J \frac{\partial \left(\exp(-R_{jq}) \right)}{\partial X_{kiq}} \right) \cdot X_{kiq}$$

$$= \left(-\frac{\partial R_{iq}}{\partial X_{kiq}} + \frac{1}{\sum_j \exp(-R_{jq})} \cdot \sum_J \exp(-R_{jq}) \frac{\partial R_{jq}}{\partial X_{kiq}} \right) \cdot X_{kiq} = \left(-\frac{\partial R_{iq}}{\partial X_{kiq}} + \sum_J P_{jq} \frac{\partial R_{jq}}{\partial X_{kiq}} \right) \cdot X_{kiq}$$

(Appendix 1.2)

As mentioned by Van Cranenburgh and Prato, the formula in Appendix 1.2 can be used to measure elasticities for all RRMs. The formula to measure systematic regret for an alternative j for person q with respect to attribute X_{kiq} is shown below:

$$\frac{\partial R_{jq}^{CRRM}}{\partial x_{kiq}} = \frac{\partial}{\partial x_{kiq}} \sum_{\substack{i \in J \\ j \neq i}} \ln\left(1 + \exp\left(\beta_k \left[x_{kiq} - x_{kjq}\right]\right)\right) = \sum_{\substack{i \in J \\ j \neq i}} \frac{\partial}{\partial x_{kiq}} \ln\left(1 + \exp\left(\beta_k \left[x_{kiq} - x_{kjq}\right]\right)\right)$$

$$= \sum_{\substack{i \in J \\ j \neq i}} \frac{1}{1 + \exp\left(\beta_k \left[x_{kiq} - x_{kjq}\right]\right)} \cdot \frac{\partial}{\partial x_{kiq}} \left(1 + \exp\left(\beta_k \left[x_{kiq} - x_{kjq}\right]\right)\right)$$

$$= \sum_{\substack{i \in J \\ j \neq i}} \frac{\frac{\partial}{\partial x_{kiq}}(1) + \frac{\partial}{\partial x_{kiq}} \left(\exp\left(\beta_k \left[x_{kiq} - x_{kjq}\right]\right)\right)}{1 + \exp\left(\beta_k \left[x_{kiq} - x_{kjq}\right]\right)} = \sum_{\substack{i \in J \\ j \neq i}} \frac{0 + \exp\left(\beta_k \left[x_{kjq} - x_{kiq}\right]\right) \cdot \frac{\partial}{\partial x_{kiq}} \left(\beta_k \left[x_{kiq} - x_{kjq}\right]\right)}{1 + \exp\left(\beta_k \left[x_{kiq} - x_{kjq}\right]\right)}$$

$$= \sum_{\substack{i \in J \\ j \neq i}} \frac{\exp\left(\beta_k \left[x_{kiq} - x_{kjq}\right]\right) \cdot \beta_k \left(\frac{\partial}{\partial x_{kiq}} (x_{kiq}) - \frac{\partial}{\partial x_{kiq}} (x_{kjq}) \right)}{1 + \exp\left(\beta_k \left[x_{kiq} - x_{kjq}\right]\right)} = \sum_{\substack{i \in J \\ j \neq i}} \frac{\exp\left(\beta_k \left[x_{kiq} - x_{kjq}\right]\right) \cdot \beta_k (1 - 0)}{1 + \exp\left(\beta_k \left[x_{kiq} - x_{kjq}\right]\right)}$$

$$= \sum_{\substack{i \in J \\ j \neq i}} \frac{\beta_k \cdot \exp\left(\beta_k \left[x_{kiq} - x_{kjq}\right]\right)}{1 + \exp\left(\beta_k \left[x_{kiq} - x_{kjq}\right]\right)} = \sum_{\substack{i \in J \\ j \neq i}} \frac{\left(\beta_k \cdot \exp\left(\beta_k \left[x_{kiq} - x_{kjq}\right]\right)\right) / \exp\left(\beta_k \left[x_{kiq} - x_{kjq}\right]\right)}{\left(1 + \exp\left(\beta_k \left[x_{kiq} - x_{kjq}\right]\right)\right) / \exp\left(\beta_k \left[x_{kiq} - x_{kjq}\right]\right)}$$

$$= \sum_{\substack{i \in J \\ j \neq i}} \frac{\beta_k}{\frac{1}{\exp\left(\beta_k \left[x_{kiq} - x_{kjq}\right]\right)} + 1} = \sum_{\substack{i \in J \\ j \neq i}} \frac{\beta_k}{\exp\left(\beta_k \left[x_{kjq} - x_{kiq}\right]\right) + 1}$$

(Appendix 1.3)

9.2 Appendix 2 Derivation of µRRM

The derivation of systematic regret for an alternative i for person q with respect to attribute X_{kiq} is shown below:

$$\frac{\partial R_{iq}^{\mu RRM}}{\partial x_{kiq}} = \frac{\partial}{\partial x_{kiq}} \sum_{\substack{i \in J \\ j \neq i}} \ln\left(1 + \exp\left(\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right)\right) = \sum_{\substack{i \in J \\ j \neq i}} \frac{\partial}{\partial x_{kiq}} \ln\left(1 + \exp\left(\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right)\right)$$

$$= \sum_{\substack{i \in J \\ j \neq i}} \frac{1}{1 + \exp\left(\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right)} \cdot \frac{\partial}{\partial x_{kiq}} \left(1 + \exp\left(\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right)\right)$$

$$= \sum_{\substack{i \in J \\ j \neq i}} \frac{\frac{\partial}{\partial x_{kiq}}[1] + \frac{\partial}{\partial x_{kiq}} \exp\left(\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right)}{1 + \exp\left(\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right)} = \sum_{\substack{i \in J \\ j \neq i}} \frac{0 + \exp\left(\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right) \cdot \frac{\partial}{\partial x_{kiq}} \left(\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right)}{1 + \exp\left(\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right)}$$

$$= \sum_{\substack{i \in J \\ j \neq i}} \frac{\exp\left(\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right) \cdot \frac{\beta_k}{\mu}\left(\frac{\partial}{\partial x_{kiq}}\left[x_{kjq}\right] - \frac{\partial}{\partial x_{kiq}}\left[x_{kiq}\right]\right)}{1 + \exp\left(\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right)} = \sum_{\substack{i \in J \\ j \neq i}} \frac{\exp\left(\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right) \cdot \frac{\beta_k}{\mu}(0 - 1)}{1 + \exp\left(\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right)}$$

$$= \sum_{\substack{i \in J \\ j \neq i}} \frac{-\frac{\beta_k}{\mu} . \left(\exp\left(\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right)\right)}{1 + \exp\left(\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right)} = \sum_{\substack{i \in J \\ j \neq i}} \frac{\left(-\frac{\beta_k}{\mu} . \left(\exp\left(\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right)\right)\right) \Big/ \left(\exp\left(\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right)\right)}{\left(1 + \exp\left(\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right)\right) \Big/ \left(\exp\left(\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right)\right)}$$

$$= \sum_{\substack{i \in J \\ j \neq i}} \frac{-\frac{\beta_k}{\mu}}{\frac{1}{\exp\left(\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right)} + 1} = \sum_{\substack{i \in J \\ j \neq i}} \frac{-\frac{\beta_k}{\mu}}{\exp\left(-\frac{\beta_k}{\mu}\left[x_{kjq} - x_{kiq}\right]\right) + 1}$$

(Appendix 2.1)

The formula in Appendix 1.2 can be used to measure µRRM elasticities. The derivation of systematic regret for an alternative j for person q with respect to attribute X_{kiq} is shown below:

$$\frac{\partial R_{jq}^{\mu RRM}}{\partial x_{kiq}} = \frac{\partial}{\partial x_{kiq}} \sum_{\substack{i\in J\\ j\neq i}} \ln\left(1+\exp\left(\frac{\beta_k}{\mu}\left[x_{kiq}-x_{kjq}\right]\right)\right) = \sum_{\substack{i\in J\\ j\neq i}} \frac{\partial}{\partial x_{kiq}} \ln\left(1+\exp\left(\frac{\beta_k}{\mu}\left[x_{kiq}-x_{kjq}\right]\right)\right)$$

$$= \sum_{\substack{i\in J\\ j\neq i}} \frac{1}{1+\exp\left(\frac{\beta_k}{\mu}\left[x_{kiq}-x_{kjq}\right]\right)} \cdot \frac{\partial}{\partial x_{kiq}} \left(1+\exp\left(\frac{\beta_k}{\mu}\left[x_{kiq}-x_{kjq}\right]\right)\right)$$

$$= \sum_{\substack{i\in J\\ j\neq i}} \frac{\frac{\partial}{\partial x_{kiq}}[1] + \frac{\partial}{\partial x_{kiq}} \exp\left(\frac{\beta_k}{\mu}\left[x_{kiq}-x_{kjq}\right]\right)}{1+\exp\left(\frac{\beta_k}{\mu}\left[x_{kiq}-x_{kjq}\right]\right)} = \sum_{\substack{i\in J\\ j\neq i}} \frac{0+\exp\left(\frac{\beta_k}{\mu}\left[x_{kiq}-x_{kjq}\right]\right) \cdot \frac{\partial}{\partial x_{kiq}}\left(\frac{\beta_k}{\mu}\left[x_{kiq}-x_{kjq}\right]\right)}{1+\exp\left(\frac{\beta_k}{\mu}\left[x_{kiq}-x_{kjq}\right]\right)}$$

$$= \sum_{\substack{i\in J\\ j\neq i}} \frac{\exp\left(\frac{\beta_k}{\mu}\left[x_{kiq}-x_{kjq}\right]\right) \cdot \frac{\beta_k}{\mu}\left(\frac{\partial}{\partial x_{kiq}}\left[x_{kiq}\right] - \frac{\partial}{\partial x_{kiq}}\left[x_{kjq}\right]\right)}{1+\exp\left(\frac{\beta_k}{\mu}\left[x_{kiq}-x_{kjq}\right]\right)} = \sum_{\substack{i\in J\\ j\neq i}} \frac{\exp\left(\frac{\beta_k}{\mu}\left[x_{kiq}-x_{kjq}\right]\right) \cdot \frac{\beta_k}{\mu}(1-0)}{1+\exp\left(\frac{\beta_k}{\mu}\left[x_{kiq}-x_{kjq}\right]\right)}$$

$$= \sum_{\substack{i\in J\\ j\neq i}} \frac{\frac{\beta_k}{\mu} . \left(\exp\left(\frac{\beta_k}{\mu}\left[x_{kiq}-x_{kjq}\right]\right)\right)}{1+\exp\left(\frac{\beta_k}{\mu}\left[x_{kiq}-x_{kjq}\right]\right)} = \sum_{\substack{i\in J\\ j\neq i}} \frac{\left(\frac{\beta_k}{\mu} . \left(\exp\left(\frac{\beta_k}{\mu}\left[xx_{kiq}-x_{kjq}\right]\right)\right)\right) \Big/ \left(\exp\left(\frac{\beta_k}{\mu}\left[x_{kiq}-x_{kjq}\right]\right)\right)}{\left(1+\exp\left(\frac{\beta_k}{\mu}\left[x_{kiq}-x_{kjq}\right]\right)\right) \Big/ \left(\exp\left(\frac{\beta_k}{\mu}\left[x_{kiq}-x_{kjq}\right]\right)\right)}$$

$$= \sum_{\substack{i\in J\\ j\neq i}} \frac{\frac{\beta_k}{\mu}}{\frac{1}{\exp\left(\frac{\beta_k}{\mu}\left[x_{kiq}-x_{kjq}\right]\right)}+1} = \sum_{\substack{i\in J\\ j\neq i}} \frac{\frac{\beta_k}{\mu}}{\exp\left(\frac{\beta_k}{\mu}\left[x_{kjq}-x_{kiq}\right]\right)+1}$$

(Appendix 2.2)

Van Cranenburgh and Prato (2016) have shown a formula to measure PRRM elasticities therefore in this paper we do not present the measurement for PRRM elasticities.

9.3 Appendix 2 Derivation of RAM

Leong and Hensher (2015) have shown the derivatives of the systematic utility of RAM in order to measure VTTS.

The derivation of systematic utility for an alternative i for person q with respect to attribute X_{kiq} is shown below:

$$\frac{\partial V_{iq}^{RAM}}{\partial x_{kiq}} = \sum_{\substack{i\in J\\ j\neq i}} \frac{\partial\left(\frac{A_{ijq}}{A_{ijq}+D_{ijq}}\right)}{\partial X_{kiq}} = \sum_{\substack{i\in J\\ j\neq i}} \frac{\frac{\partial A_{ijq}}{\partial X_{kiq}}\cdot\left(A_{ijq}+D_{ijq}\right) - A_{ijq}\cdot\frac{\partial\left(A_{ijq}+D_{ijq}\right)}{\partial X_{kiq}}}{\left(A_{ijq}+D_{ijq}\right)^2}$$

$$= \sum_{\substack{i\in J\\ j\neq i}} \frac{A_{ijq}\cdot\frac{\partial A_{ijq}}{\partial X_{kiq}} + D_{ijq}\cdot\frac{\partial A_{ijq}}{\partial X_{kiq}} - A_{ijq}\cdot\frac{\partial A_{ijq}}{\partial X_{kiq}} - A_{ijq}\cdot\frac{\partial D_{ijq}}{\partial X_{kiq}}}{\left(A_{ijq}+D_{ijq}\right)^2}$$

$$= \sum_{\substack{i\in J\\ j\neq i}} \frac{D_{ijq}\cdot\frac{\partial A_{ijq}}{\partial X_{kiq}} - A_{ijq}\cdot\frac{\partial D_{ijq}}{\partial X_{kiq}}}{\left(A_{ijq}+D_{ijq}\right)^2} \tag{Appendix 3.1}$$

where $\frac{\partial A_{ijq}}{\partial X_{kiq}} = \frac{\partial R_{jq}^{CRRM}}{\partial x_{kiq}} = \frac{\beta_k}{\exp\left(\beta_k\left[x_{kjq}-x_{kiq}\right]\right)+1}$

and $\frac{\partial D_{ijq}}{\partial X_{kiq}} = \frac{\partial R_{iq}^{CRRM}}{\partial x_{kiq}} = \frac{-\beta_k}{\exp\left(-\beta_k\left[x_{kjq}-x_{kiq}\right]\right)+1}$

The formula to measure direct elasticities for RAM is as follows:

$$E_{iqX_{kiq}}^{RAM} = \frac{\partial P_{iq}}{\partial X_{kiq}}\cdot\frac{X_{kiq}}{P_{iq}} = P_{iq}\cdot\frac{\partial \ln P_{iq}}{\partial X_{kiq}}\cdot\frac{X_{kiq}}{P_{iq}} = \frac{\partial \ln P_{iq}}{\partial X_{kiq}}\cdot X_{kiq}$$

$$= \left(\frac{\partial \ln\left(\frac{\exp\left(V_{iq}\right)}{\sum_j \exp\left(V_{jq}\right)}\right)}{\partial X_{kiq}}\right)\cdot X_{kiq} = \left(\frac{\partial V_{iq}}{\partial X_{kiq}} - \frac{\partial \ln\left(\sum_j \exp\left(V_{jq}\right)\right)}{\partial X_{kiq}}\right)\cdot X_{kiq}$$

$$= \left(\frac{\partial V_{iq}}{\partial X_{kiq}} - \frac{\partial \ln\left(\sum_j \exp\left(V_{jq}\right)\right)}{\partial X_{kiq}}\right)\cdot X_{kiq} = \left(\frac{\partial V_{iq}}{\partial X_{kiq}} - \frac{1}{\sum_j \exp\left(V_{jq}\right)}\cdot\frac{\partial\left(\sum_j \exp\left(V_{jq}\right)\right)}{\partial X_{kiq}}\right)\cdot X_{kiq}$$

$$= \left(\frac{\partial V_{iq}}{\partial X_{kiq}} - \frac{1}{\sum_j \exp\left(V_{jq}\right)}\cdot\sum_J \frac{\partial\left(\exp\left(V_{jq}\right)\right)}{\partial X_{kiq}}\right)\cdot X_{kiq}$$

$$= \left(\frac{\partial V_{iq}}{\partial X_{kiq}} - \frac{1}{\sum_j \exp\left(V_{jq}\right)}\cdot\sum_J \exp\left(V_{jq}\right)\frac{\partial V_{jq}}{\partial X_{kiq}}\right)\cdot X_{kiq}$$

$$= \left(\frac{\partial V_{iq}}{\partial X_{kiq}} - \sum_J P_{jq}\frac{\partial V_{jq}}{\partial X_{kiq}}\right)\cdot X_{kiq} \tag{Appendix 3.2}$$

The derivation of systematic utility for an alternative j for person q with respect to attribute X_{kiq} is shown below:

$$\frac{\partial V_{jq}^{RAM}}{\partial x_{kiq}} = \sum_{\substack{i\in J\\ j\neq i}} \frac{\partial\left(\frac{A_{jiq}}{A_{jiq}+D_{jiq}}\right)}{\partial X_{kiq}} = \sum_{\substack{i\in J\\ j\neq i}} \frac{\frac{\partial A_{jiq}}{\partial X_{kiq}}\cdot\left(A_{jiq}+D_{jiq}\right) - A_{jiq}\cdot\frac{\partial\left(A_{jiq}+D_{jiq}\right)}{\partial X_{kiq}}}{\left(A_{jiq}+D_{jiq}\right)^2}$$

$$= \sum_{\substack{i\in J\\ j\neq i}} \frac{A_{jiq}\cdot\frac{\partial A_{jiq}}{\partial X_{kiq}} + D_{jiq}\cdot\frac{\partial A_{jiq}}{\partial X_{kiq}} - A_{jiq}\cdot\frac{\partial A_{jiq}}{\partial X_{kiq}} - A_{jiq}\cdot\frac{\partial D_{jiq}}{\partial X_{kiq}}}{\left(A_{jiq}+D_{jiq}\right)^2}$$

$$= \sum_{\substack{i\in J\\ j\neq i}} \frac{D_{jiq}\cdot\frac{\partial A_{jiq}}{\partial X_{kiq}} - A_{jiq}\cdot\frac{\partial D_{jiq}}{\partial X_{kiq}}}{\left(A_{jiq}+D_{jiq}\right)^2} \qquad \text{(Appendix 3.3)}$$

where $\frac{\partial A_{jiq}}{\partial X_{kiq}} = \frac{\partial R_{iq}^{CRRM}}{\partial x_{kiq}} = \frac{-\beta_k}{\exp\left(-\beta_k\left[x_{kjq}-x_{kiq}\right]\right)+1}$

and $\frac{\partial D_{jiq}}{\partial X_{kiq}} = \frac{\partial R_{jq}^{CRRM}}{\partial x_{kiq}} = \frac{\beta_k}{\exp\left(\beta_k\left[x_{kjq}-x_{kiq}\right]\right)+1}$

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