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Rigorous bounds for vertex corrections On the conjecture named "Migdal's Theorem"

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Summary

The basis of the modern physical description of metals is formed by electron-phonon theory, which describes the interaction of the electrons with oscillations of the ion lattice structure, or sound waves. In electron-phonon theory the standard method of computing physical quantities by means of a perturbative expansion in the coupling constant breaks down because the coupling constant is typically not small.

Having to take all higher-order terms into account makes the calculations very difficult. Therefore it is common to make an approximation first proposed by A.B. Migdal in 1958. It consists of leaving out the higher-order contributions to the interaction vertex. This greatly simplifies solving the main equations of theory. Migdal justified this approximation by claiming that, at zero temperature, the higher order contributions vanish linearly in the sound velocity, c. For ordinary metals, this velocity is small relative to the other relevant parameters. This claim has become known as "Migdal's Theorem" although, to our knowledge, no rigorous proof has ever been published. Migdal gave a sketchy argument for the lowest order correction, the "one-loop correction", and then claimed that higher orders would work the same. Other authors have followed him in doing so and have extended the claim to non-zero temperature.

In this thesis the most simple form of electron-phonon theory, the Jellium model, is considered as a statistical quantum field theory at finite temperature in the presence of an ultra-violet cut-off. A rigorous bound is found for the one-loop correction that is indeed $O(c)$ except for a correction term which vanishes along with the temperature. This done using very explicit calculations using a Feynman-trick and repeated integration by parts.

Proper formulation of the theory so that the zero-temperature limit exists requires renormalization of the theory. Here, a so called Fermi-surface renormalization is done, where counter terms are added to the band relation. Using renormalization group techniques and a scale decomposition argument these counter terms are define precisely and it is shown that the zero temperature limit of the renormalized theory is well defined by establishing temperature independent bounds for the values of graphs occurring in the theory.

Finally it is shown that for the renormalized theory, for all $0 < \epsilon < 1$ the vertex corrections of order r are bounded by

$$M_r(\epsilon)\left\{c^{1-\epsilon} + \left(\frac{(\log\beta + 1)^2}{\beta}\right)^{1-\epsilon}\right\}$$

for some ϵ-dependent constant $M_r(\epsilon)$, with β the inverse temperature. This is done by combining the Feynman-trick and the integration by parts with the scale decomposition.