

# Reflection seismic 1 script

## **Educational Material**

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# Outline

- Correlation
- Auto-correlation
- Filter
- Convolution
- Deconvolution
- Wiener filter

## Numerical evaluation of Cross-correlation

$$\phi_{xy}(\tau) = \sum_{i=1}^{n-\tau} x_{i+\tau} y_i$$

$x_i$ : (i=0 ... n)

$y_i$ : (i= 0 ... n)

$\phi_{xy}(\tau)$  : (-m <  $\tau$  < +m)

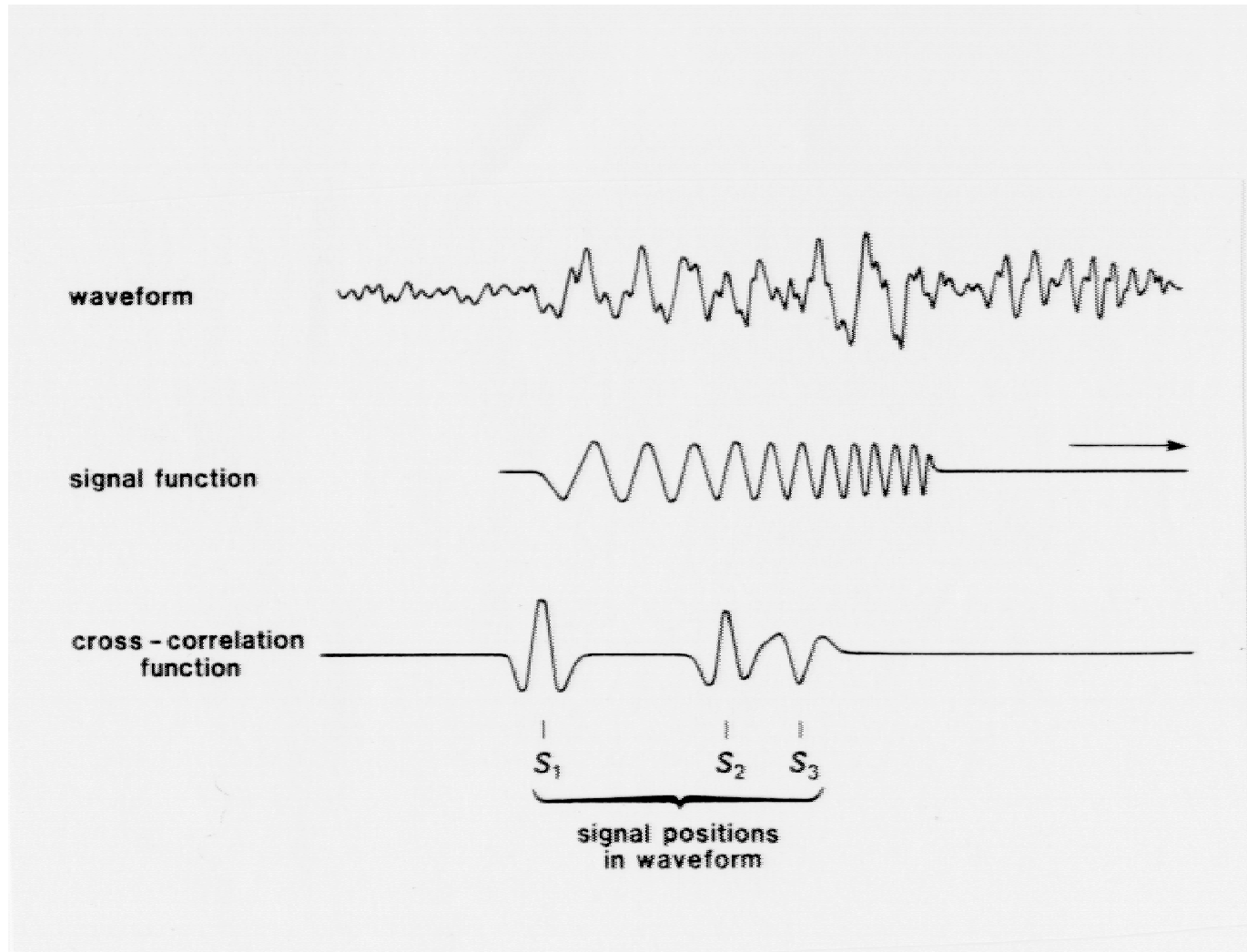
m = max. displacement

In Fourier domain:

**Cross-correlation = Multiplication of Amplitude spectrum  
and Subtraction of Phase spectrum**



# Cross-correlation function



# Autocorrelation

**Cross-correlation of a Function with itself**

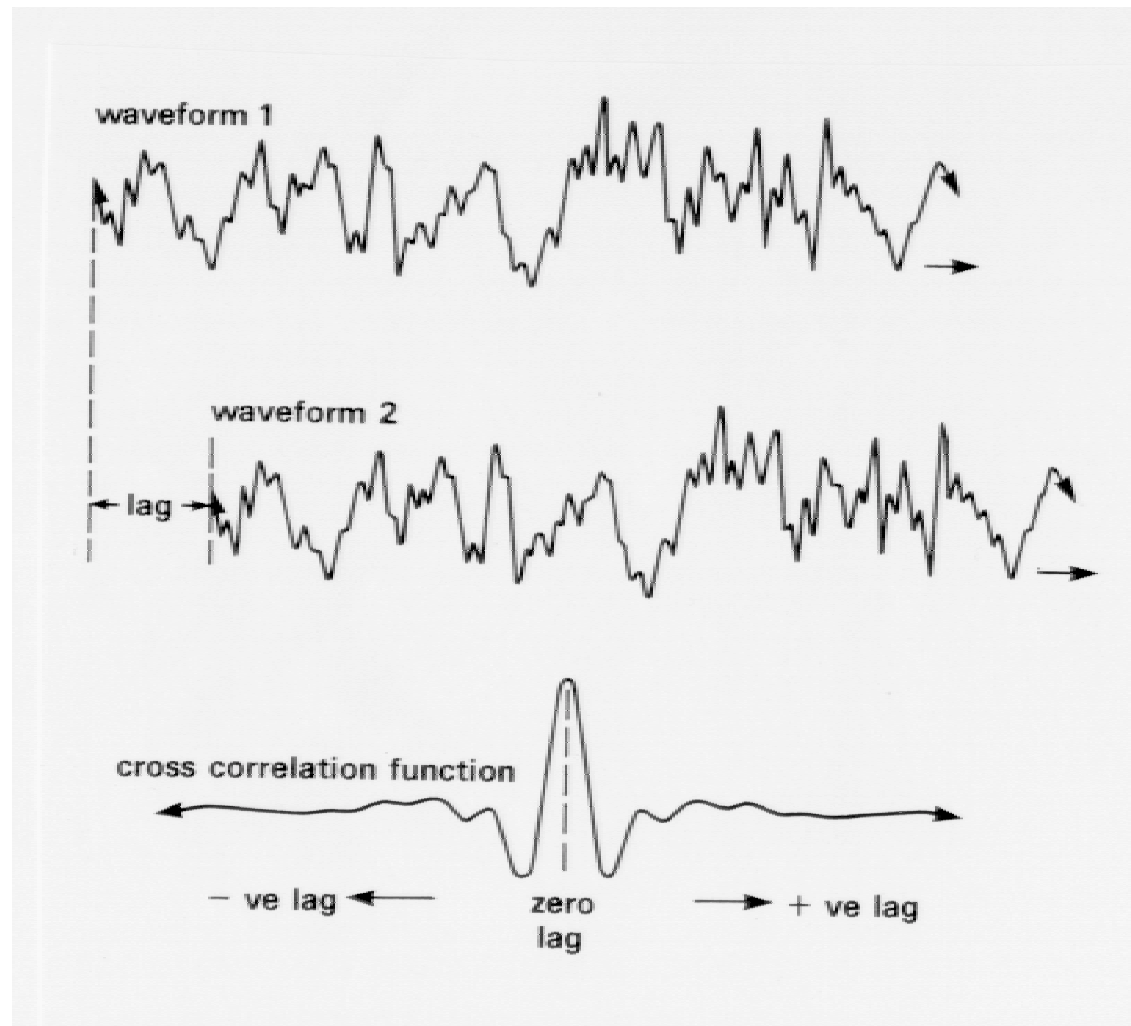
$$\phi_{xx}(\tau) = \sum_{i=1}^{n-\tau} X_{i+\tau} X_i$$

$$x_i = (i=0 \dots n)$$

$$\phi_{xx}(\tau) = (-m < \tau < +m)$$

**m = max. displacement**

# Auto-correlation of two identical waveforms



# Normalization of correlation

**Auto-correlation**

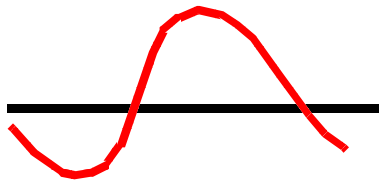
$$\phi_{xx}(\tau)_{norm} = \frac{\phi_{xx}(\tau)}{\phi_{xx}(0)}$$

**Cross-correlation**

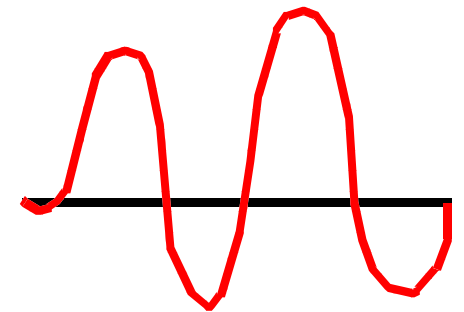
$$\phi_{xy}(\tau)_{norm} = \frac{\phi_{xy}(\tau)}{\sqrt{\phi_{xx}(0)\phi_{yy}(0)}}$$



# General Filter

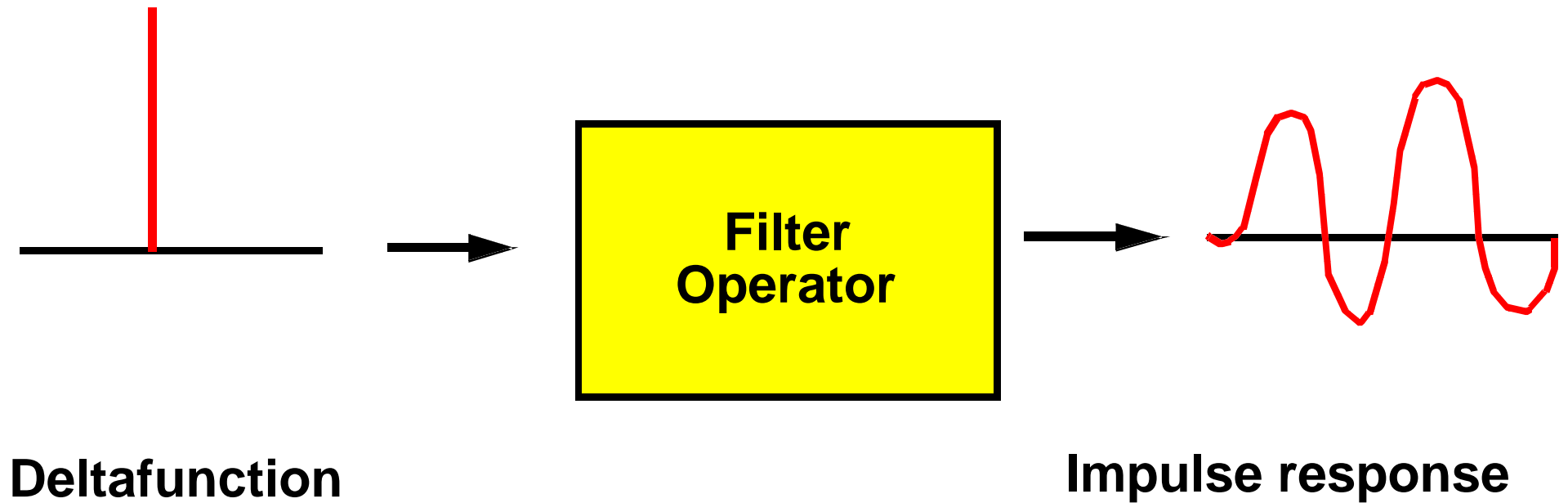


**Input function**

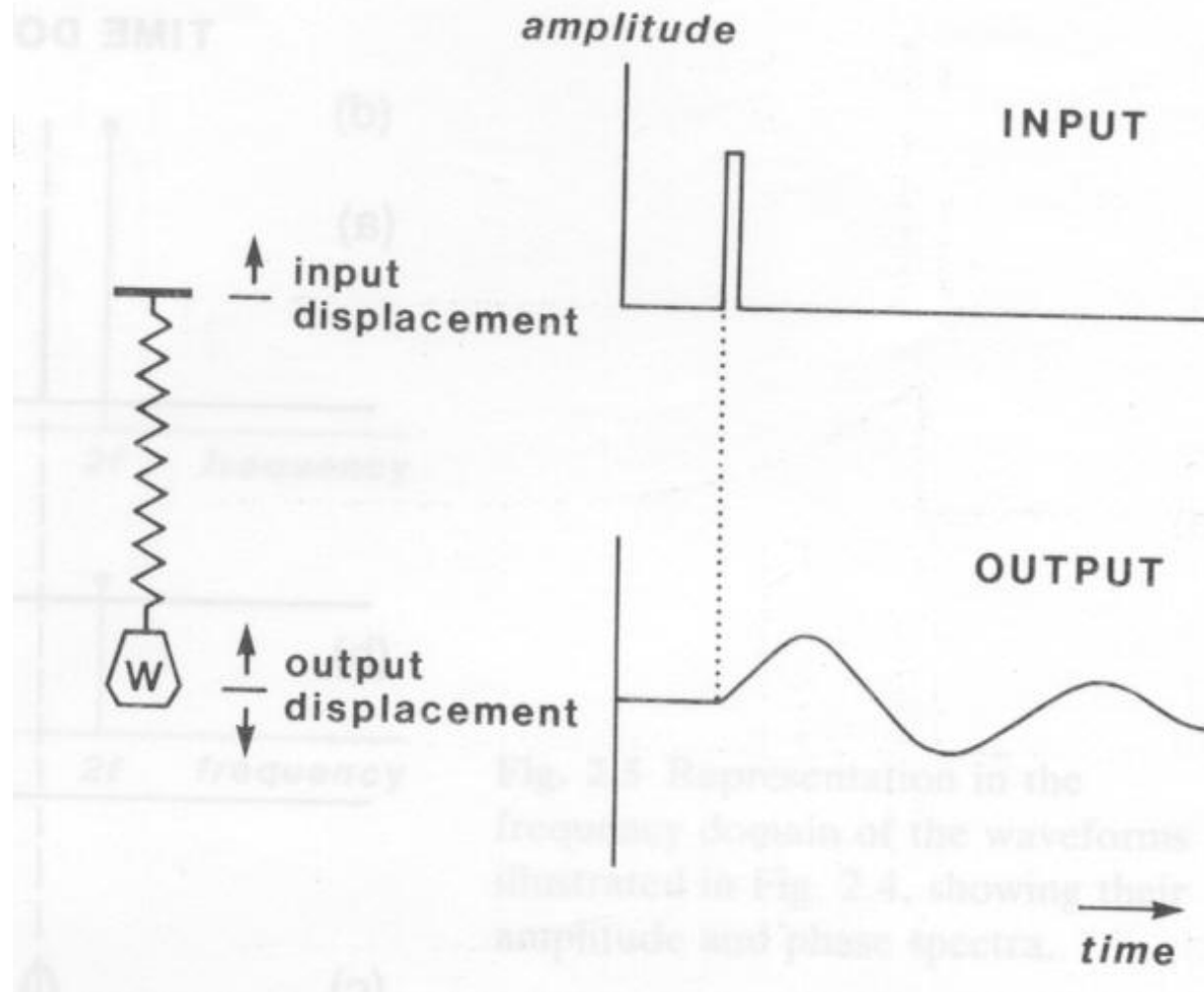


**Output function**

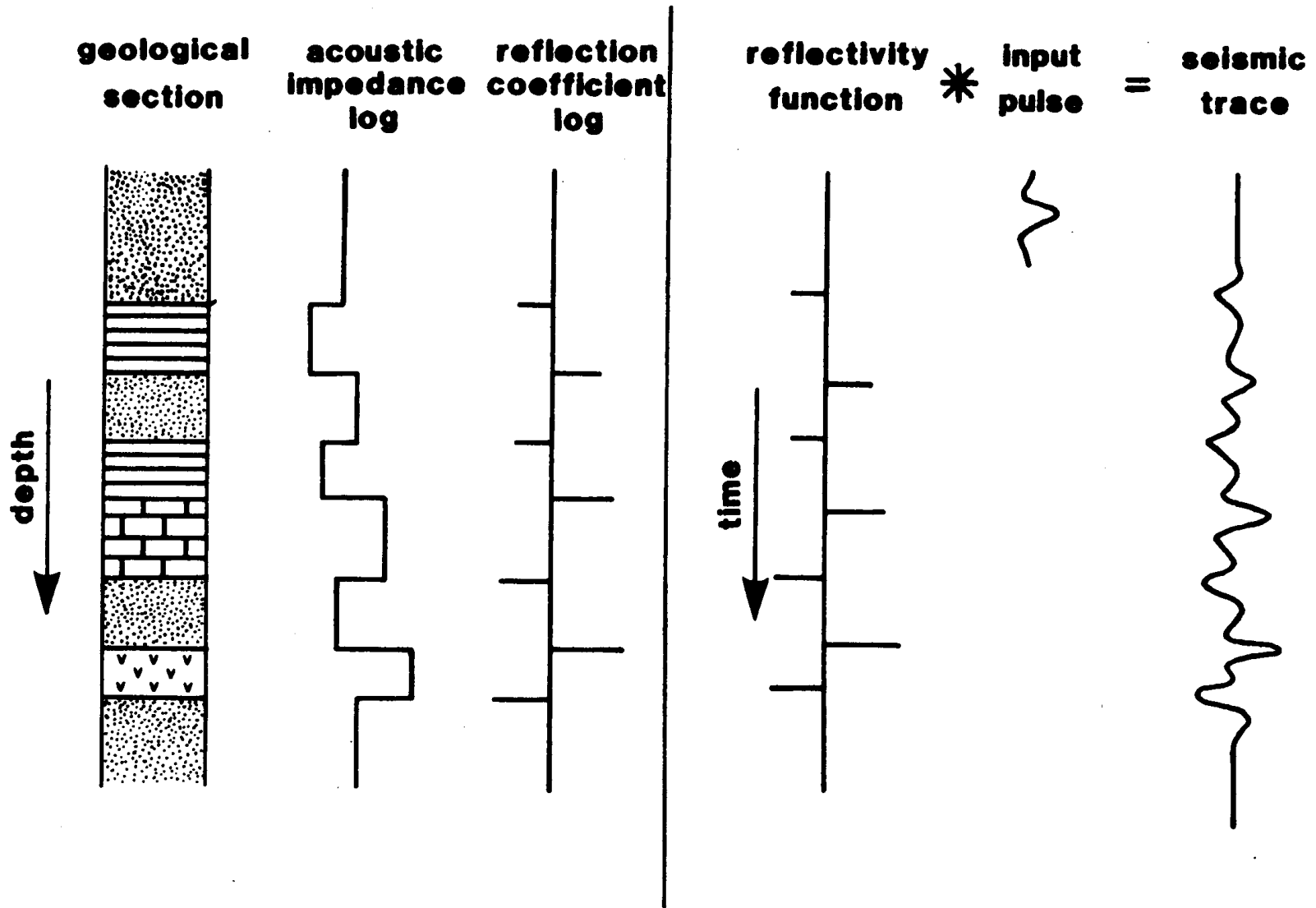
# General Filter



# Example of input response



# From geology to seismogram



# Convolution

## Convolution

$y(t) = g(t) * f(t)$

**outputfunction**

Seismic trace

**Inputfunction**

Source wavelet

**Filterfunction**

Reflectivity function

# Numerical implementation of convolution

$$y_k = \sum_{i=0}^m g_i f_{i-k}$$

$$k = 0 \dots m+n$$

$$g_i = (i=0 \dots m)$$

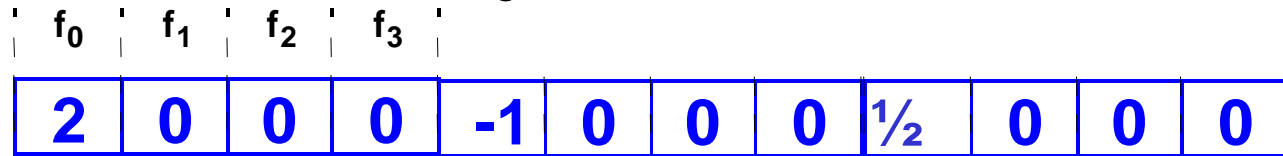
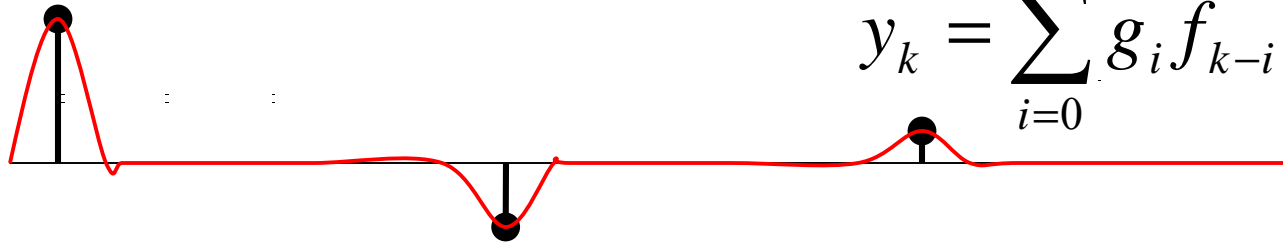
$$f_j = (j= 0 \dots n)$$

**In Fourier domain:**

**Convolution = Multiplication (of Amplitudes  
and Addition of Phasespectrum)**

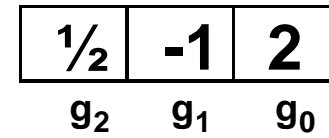
# Example of Convolution

$$y_k = \sum_{i=0}^m g_i f_{k-i}$$

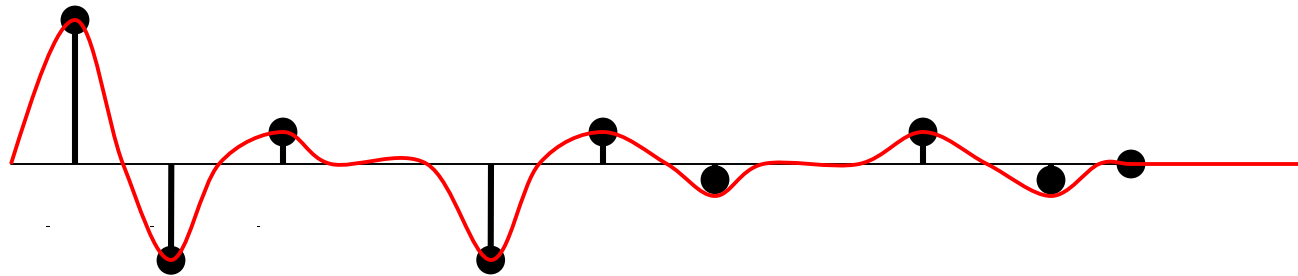


$f_k$

$g_k$



$y_k$



# Convolution model of the Earth

$W_t$

(equivalent Wavelet)

$$g_t = k\delta_t * s_t * n_t * p_t * e_t + \text{Noise}$$

Impulse  
of source

Near-surface  
zone of source

Reflectivity  
of the Earth

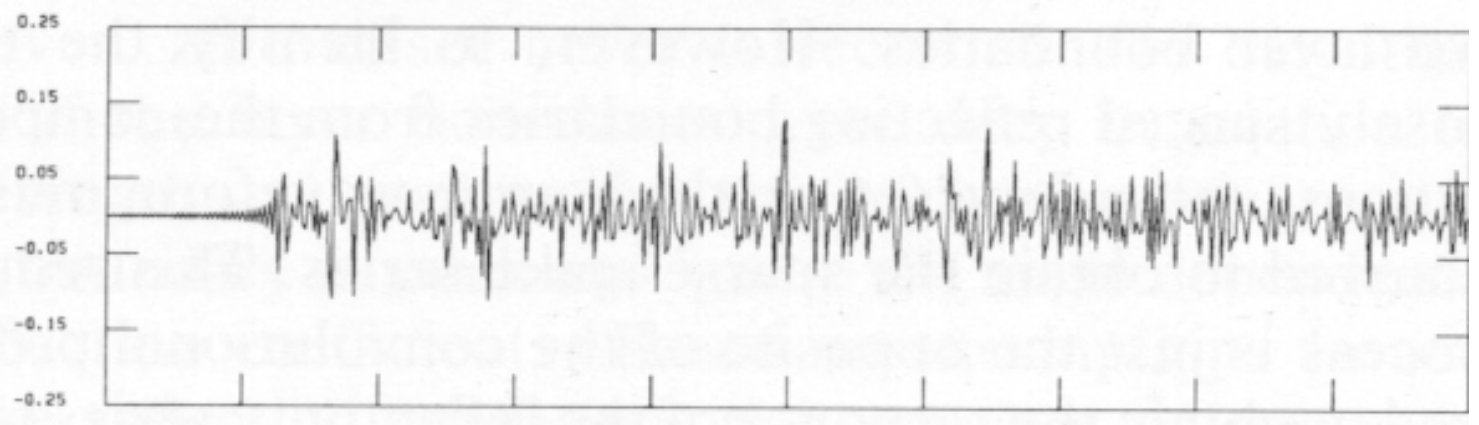
Source effect

Additional modifying  
Effects (absorption, wave conversion)

$$g_t = w_t * e_t + \text{Noise}$$

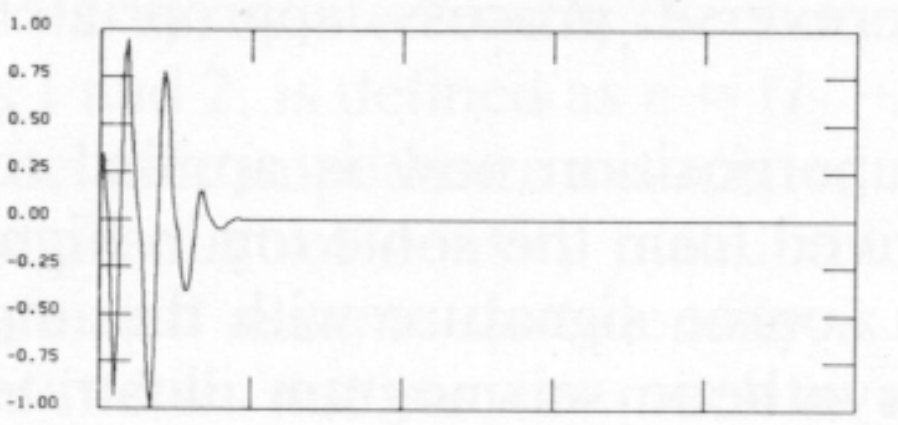


**Impulse  
Response**



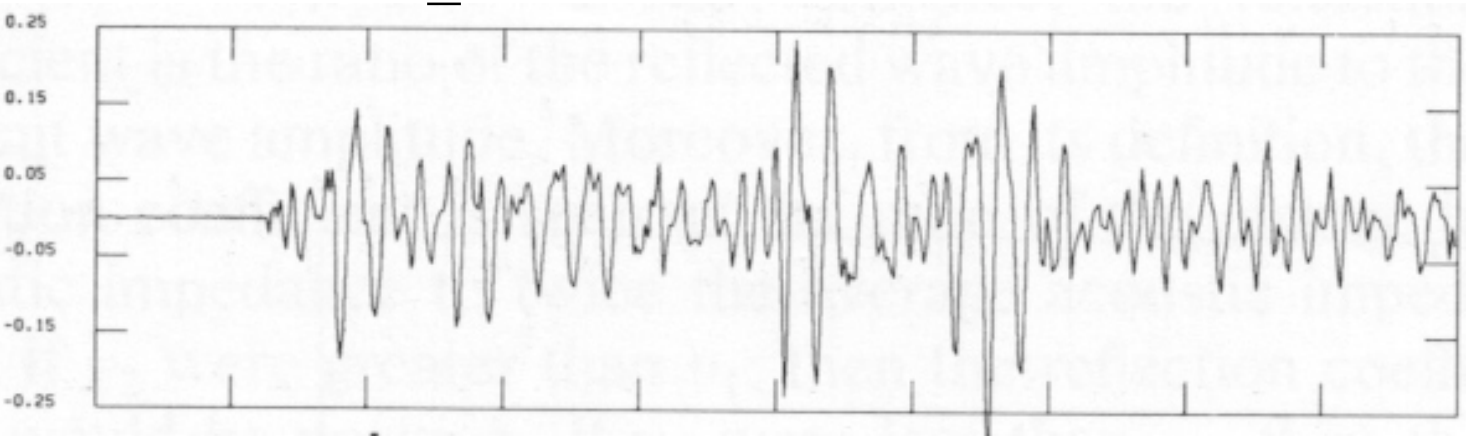
\*

**Source  
Wavelet**

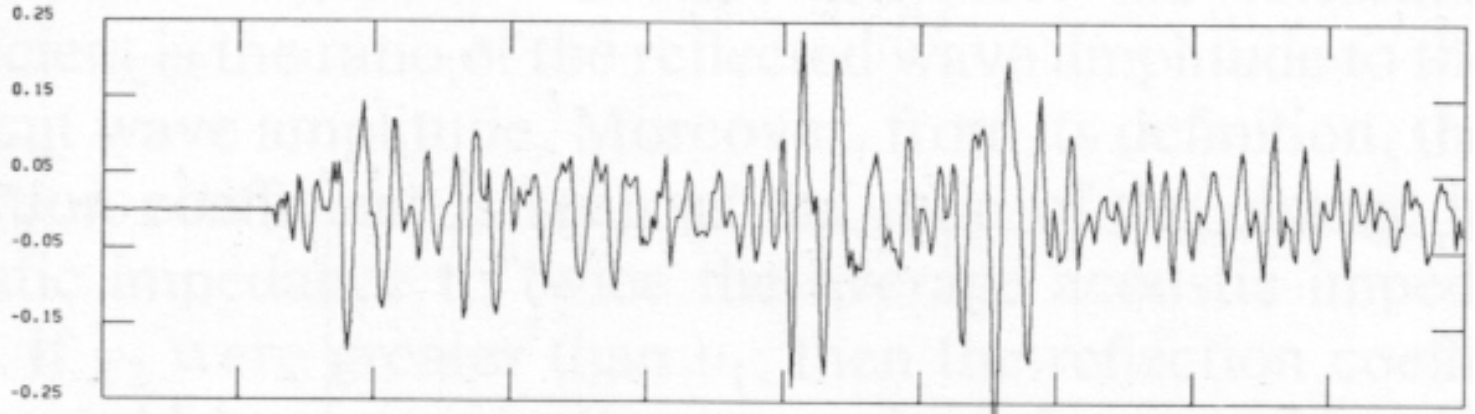


=

**Noise-Free  
Seismogram**

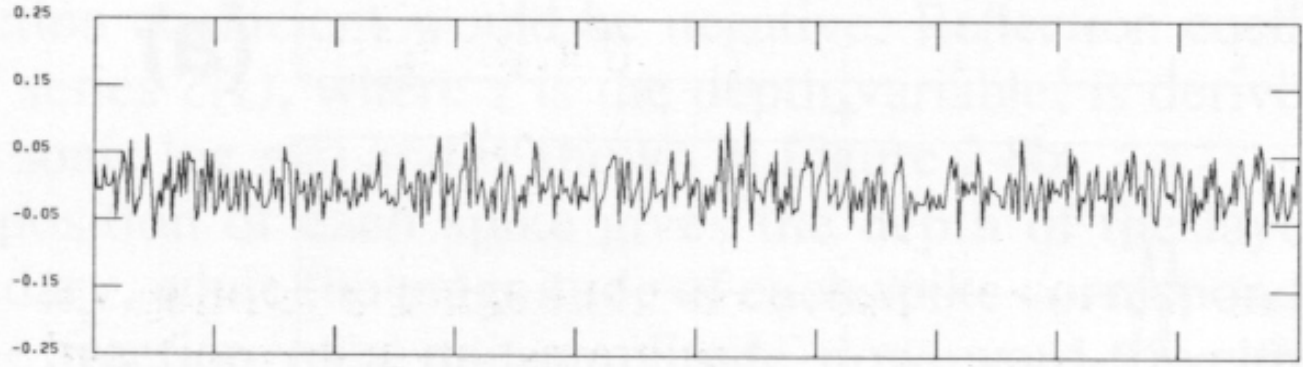


Noise-Free  
Seismogram



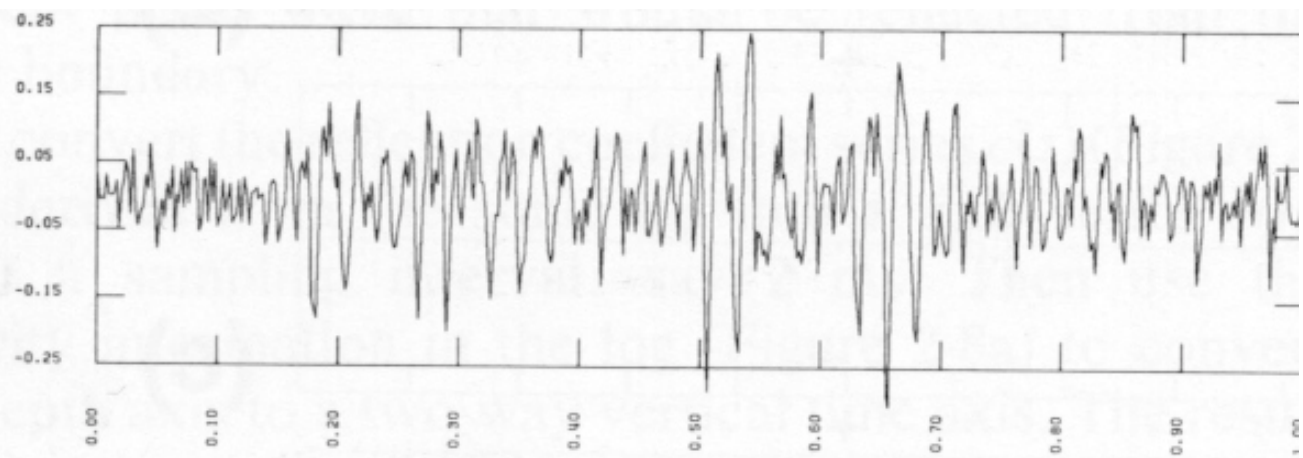
+

Noise



=

Seismogram



## **Aim of Deconvolution**

**Theoretical:**

**Reconstruction of the Reflectivity function**

**Practical:**

**Shorting of the Signal**

**Suppression of Noise**

**Suppression of Multiples**

# Deconvolution

## Reverse of Convolution

$$\boxed{x_t = w_t * e_t} \quad \longrightarrow \quad \boxed{e_t = x_t * w_t^{-1}}$$

**=> Inverse Filtering**

**Problem:**

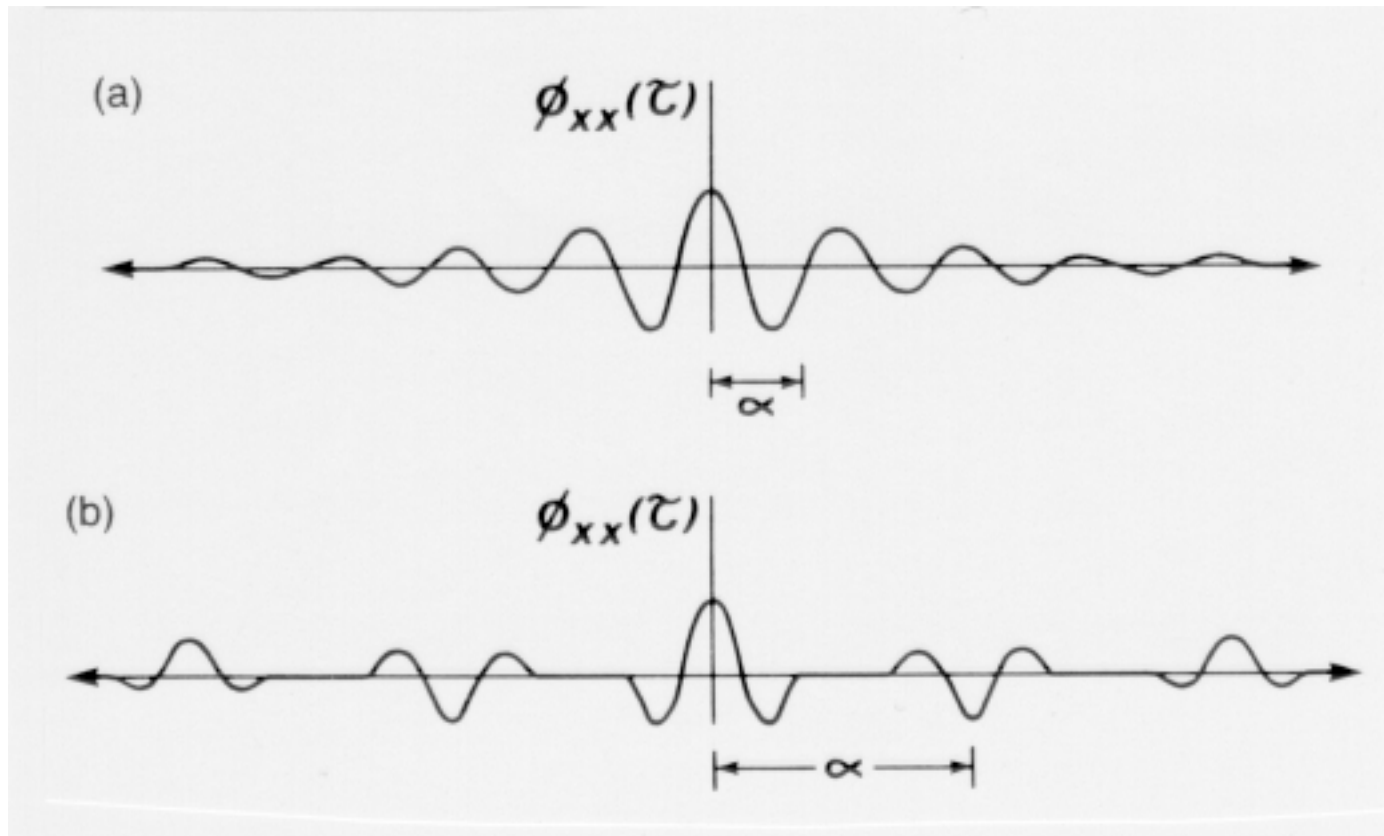
**w(t) is in general not known,**

**i.e.  $w^{-1}(t)$  Can not be determined directly**

# Auto-correlation: multiples

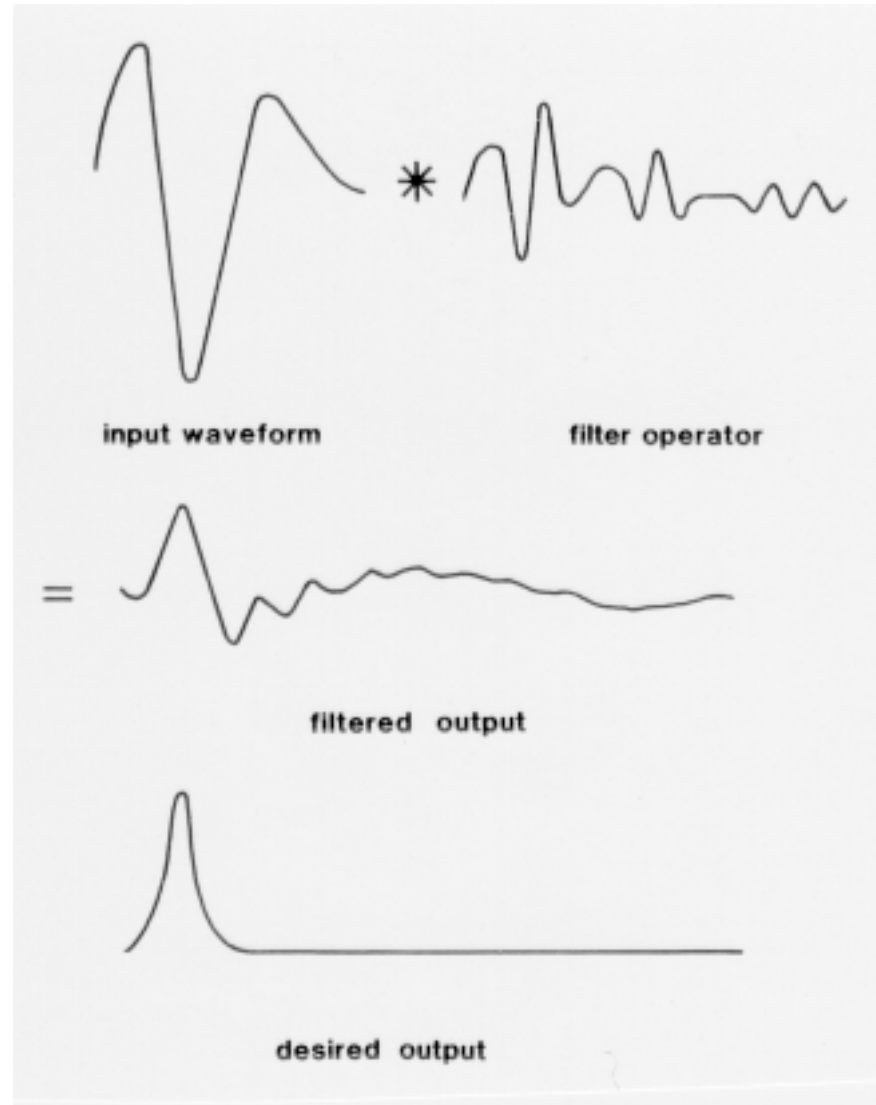


## Autocorrelation functions contain reverberations



- a) A gradually decaying function indicative of short-period reverberation
- b) A function with separate side lobes indicative of long-period reverberations: multiples

# Principle of Wiener filtering



## Principle of Wiener-Filters

**Input-Function** \* **Filter** = **Output-Function**  
(known) (wanted) (known)

$$\begin{bmatrix} g_0 \\ g_1 \\ \dots \end{bmatrix} * \begin{bmatrix} f_0 \\ \vdots \\ f_1 \\ \dots \end{bmatrix} \rightarrow \begin{bmatrix} g_0 f_0 = y_0 \\ g_1 f_0 + g_0 f_1 = y_1 \\ \dots \end{bmatrix}$$

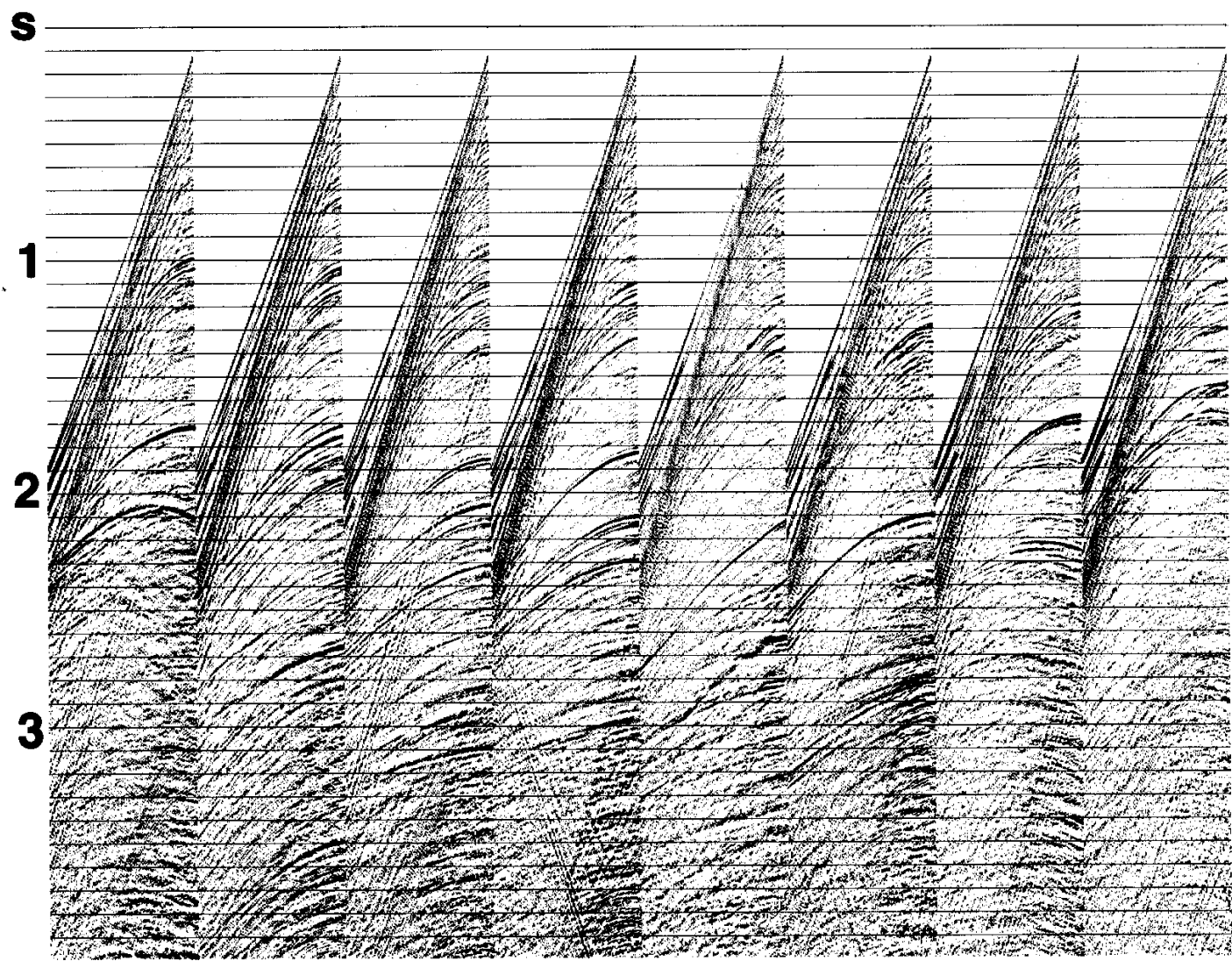
**=> Solve system of equations**



# Wiener filter

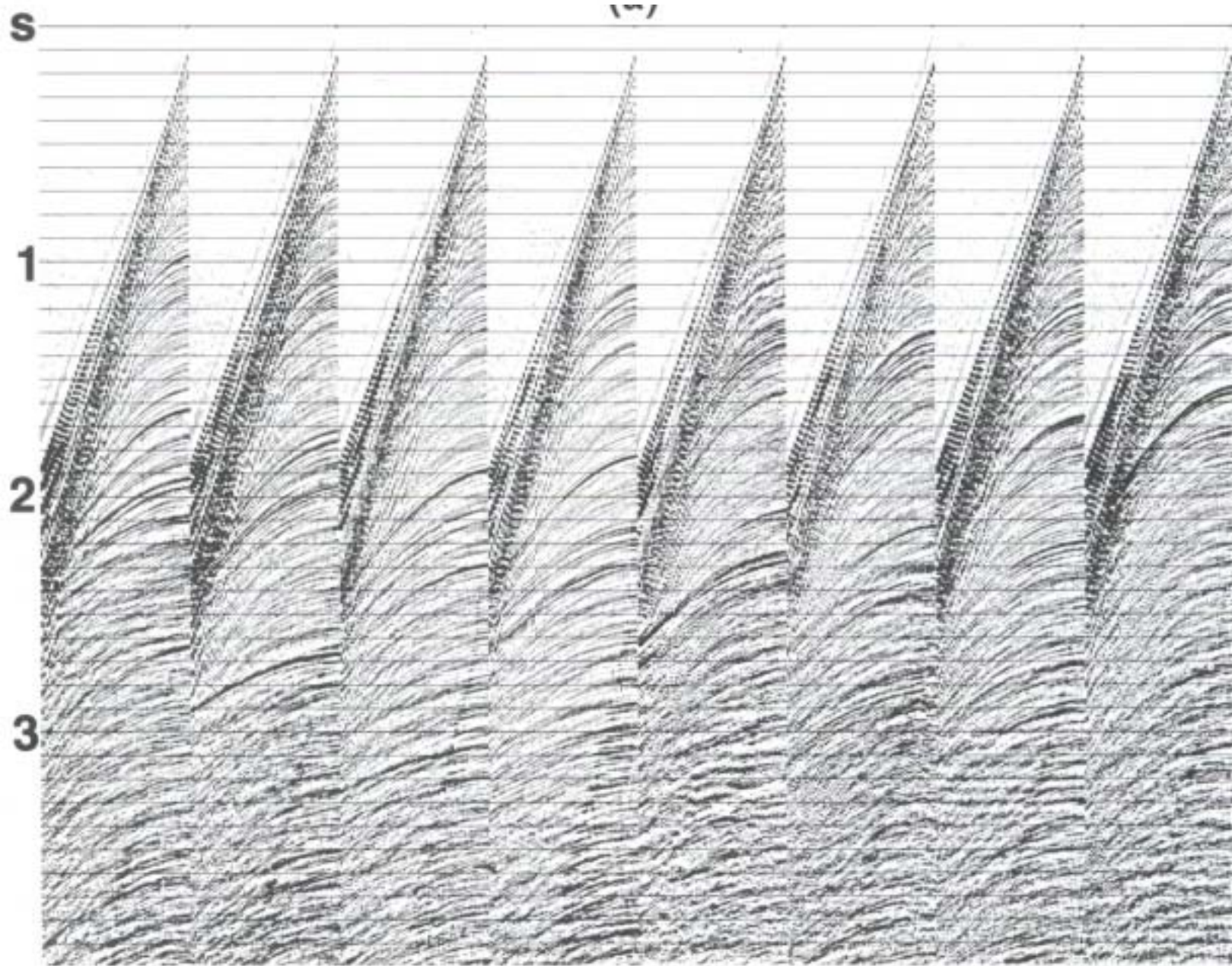
- Spiking deconvolution:  
desired output is a spike
- Predictive deconvolution:  
attempts to remove the effect of multiples

Common shot gathers just after deconvolution  
After trace balancing  
Corrected for spiking deconvolution

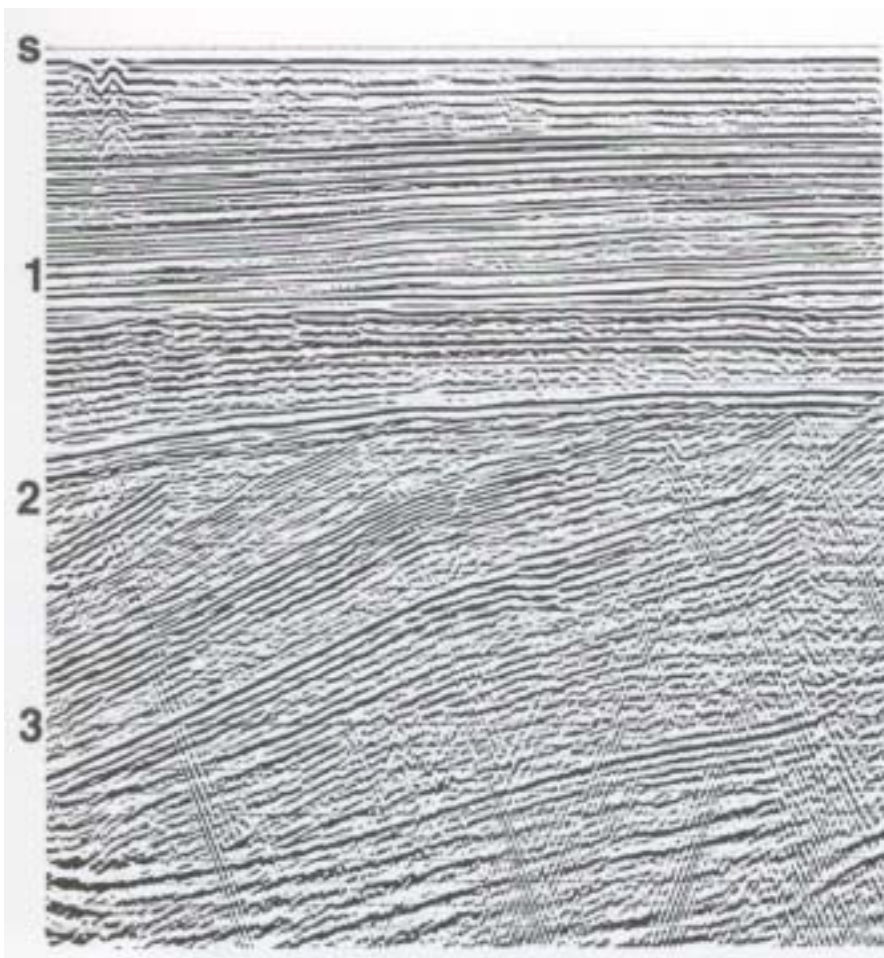


Yilmaz, 1987

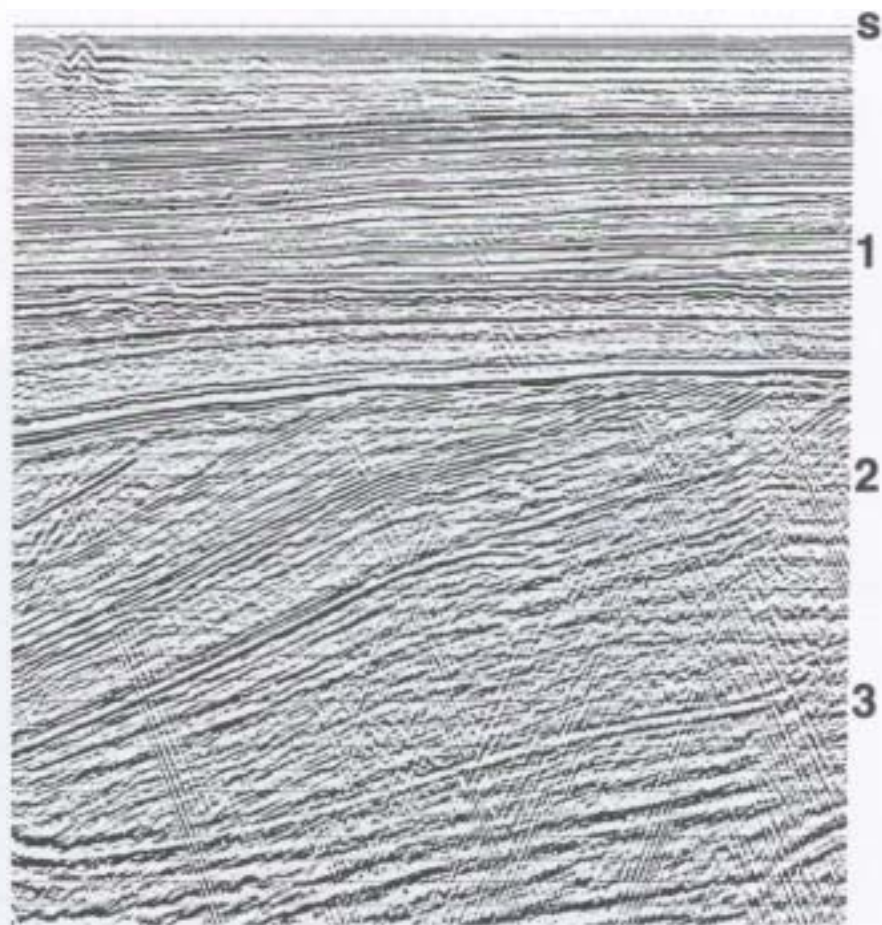
Underlined gather note the prominent reflections



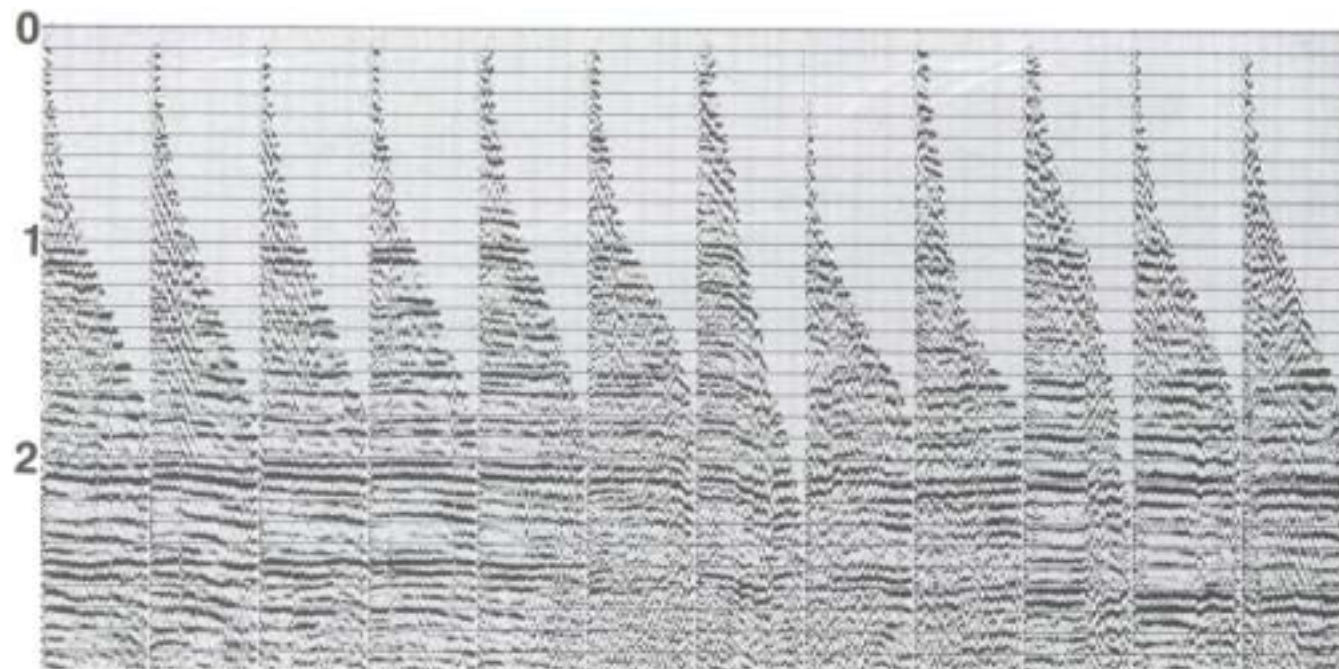
Undeconvolved gathers



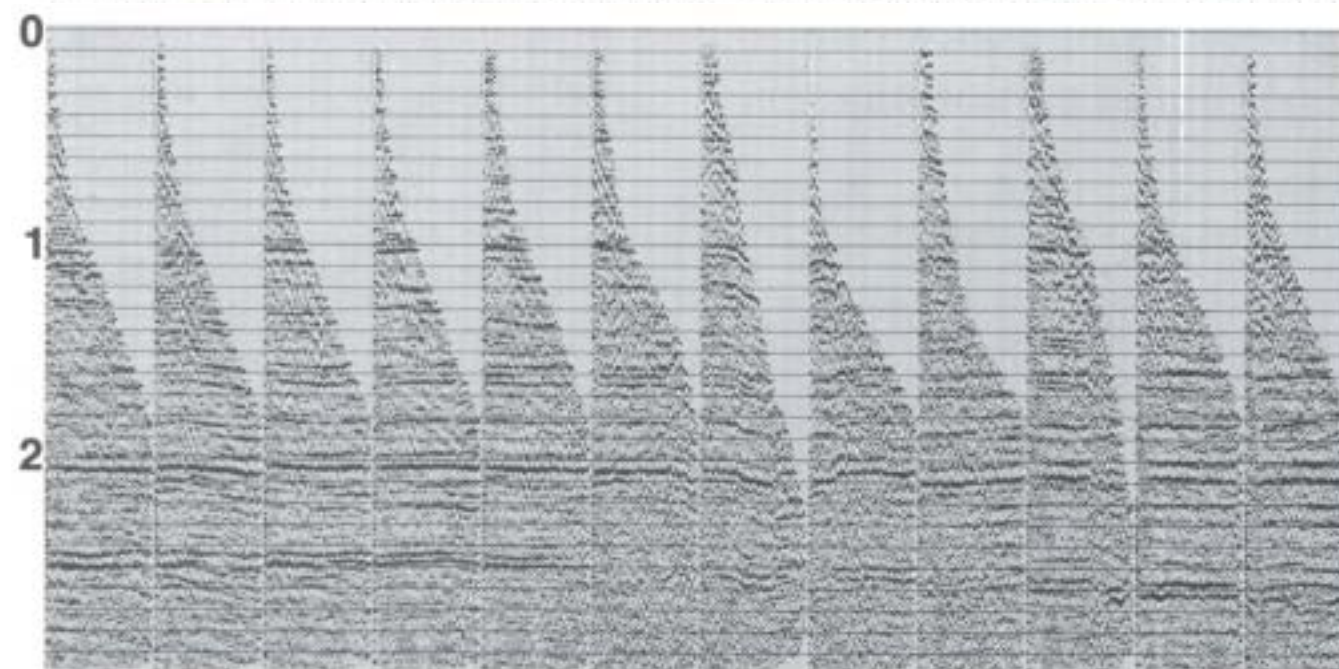
Deconvolved gathers



Before

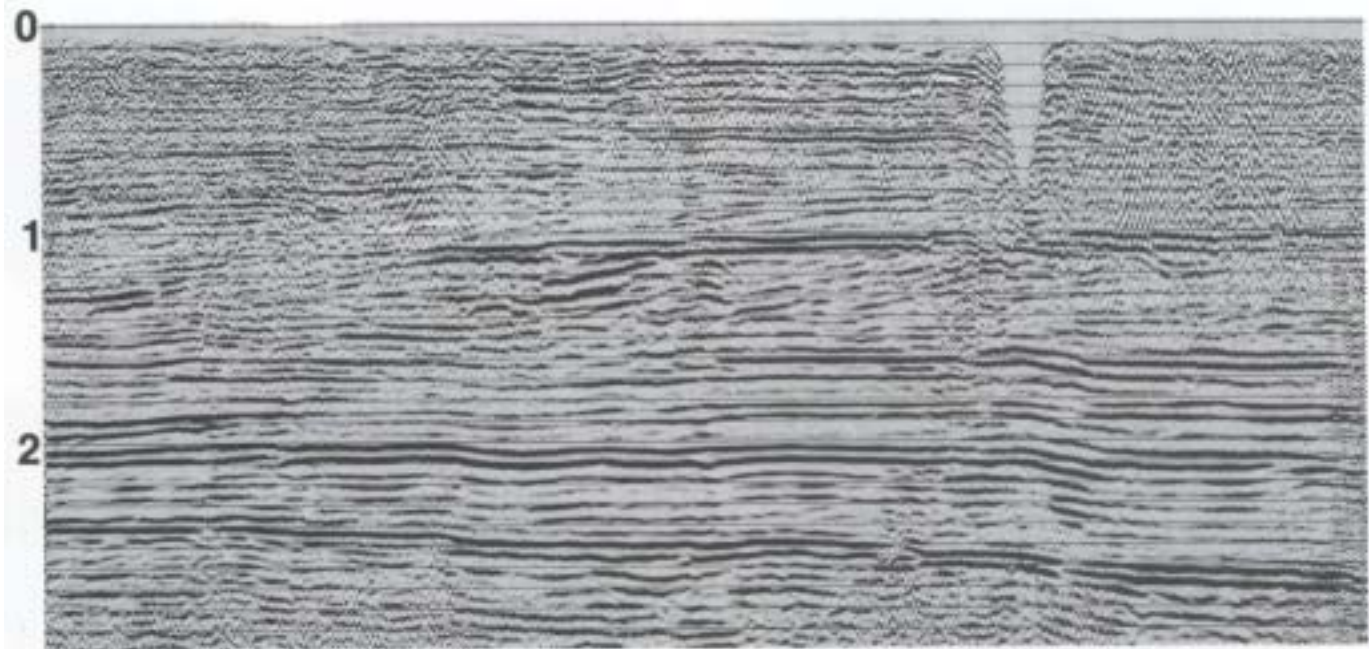


After

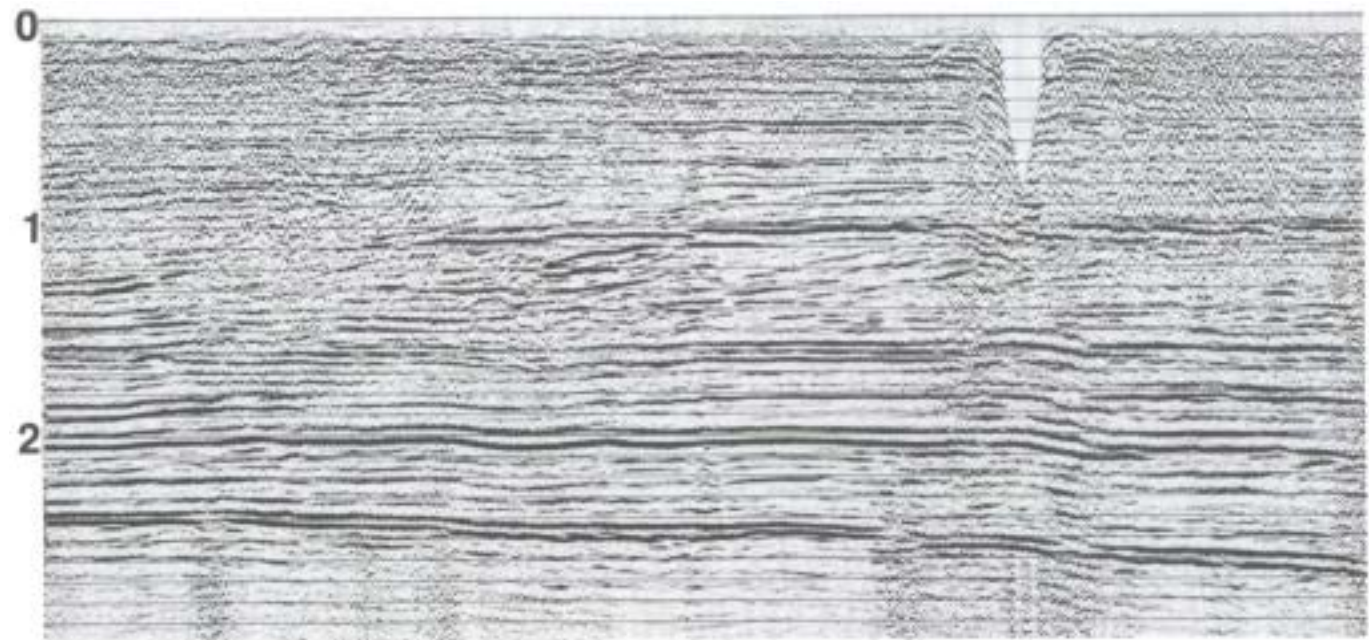


Deconvolution

Before



After



Deconvolution