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Hedging Strategy and Electricity Contract Engineering

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Abstract

This thesis studies risk management in the electricity market in general and the interaction between physical production and electricity contracts in particular. From a risk management point of view, a power portfolio differs substantially from a traditional financial portfolio. Electricity is non-storable, which together with the marginal production cost characteristics creates jumps in the spot price. The return of a power portfolio is hence typically heavy-tailed, and a risk measure, such as CVaR, that captures this heavy-taildness is needed. To be able to compare production and contracts on a unified basis, we identify the set of contracts that corresponds to each power plant. These contracts build up a replicating portfolio of the power plant. This engineering of contracts allows us to risk manage these often complex contracts, through production. Further, a producing electricity company can through a simple absence of arbitrage argument assess these contracts by studying the costs associated with the corresponding power plant. Flexible production units, such as a gas turbine, relate to options whereas inflexible units, such as a nuclear plant, relate to futures.

The electricity market is heavily incomplete, why perfect hedges are not achievable for a number of contracts. Hence we introduce the concept of best hedge. The best hedge is found through an optimization, where risk, measured as CVaR, is minimized subject to a constraint on the expected profit. It turns out that this problem can be solved with linear programming, allowing us to handle problems of substantial size.

When a whole portfolio is considered we try to utilize our risk mandate at the best possible way. This leads us to the well-known problem in finance

of portfolio optimization. However, this problem needs to be tailored for the electricity market because of the special characteristics of power portfolios. An optimal portfolio implies also an optimal dispatch of the production assets. We focus on the challenging hydro storage plant, which because of its flexible nature corresponds to a series of options. These options are however interdependent through the stored water in the reservoir. An exercise of an option, i. e. production, decreases the amount of stored water and may prohibit production at a later point in time. We develop a dynamic dispatch strategy, which takes this interdependence into account. The optimization of a portfolio consisting of a hydro storage plant and electricity contracts hence needs to derive the optimal portfolio of contracts and the optimal dispatch strategy, or with financial terms the optimal exercise conditions for the corresponding options. We solve the problem with linear programming by maximizing the expected profit over a specified time horizon under the constraint that CVaR of the portfolio may not exceed some threshold, typically determined by the risk preferences of the firm.

It turns out that a simultaneous optimization of the dispatch and the contracts is needed, since the dispatch depends on the volume risk in the entered contracts. A main result is the high value related to the operational flexibility of the hydro storage plant. By studying the dual of our linear portfolio optimization problem, we can actually quantify this value. In a performed case study it is shown that this value of flexibility can be substantial. Any valuation that does not take this operational flexibility into account may hence underestimate flexible power plants.

Zusammenfassung

Die vorliegende Schrift hat zum Gegenstand, das Risikomanagement im Allgemeinen und das Zusammenspiel von Elektrizitätsproduktion und Elektrizitätsverträgen im Besonderen, zu studieren. Aus der Sicht des Risiko Managements unterscheiden sich Elektrizitätsportfolios substanziell von traditionellen Finanzportfolios. Preissprünge, die auf die nicht vorhandene Lagerfähigkeit von Elektrizität und die besonderen Grenzkosteneigenschaften bei der Produktion zurückgeführt werden können, implizieren eine langschwänzige Verteilung für den Return. Es scheint sinnvoll, mit einem Risikomass zu arbeiten, dass dieser Langschwänzigkeit Rechnung trägt. Um Produktion und Verträge überhaupt auf einer einheitlichen Basis miteinander vergleichen zu können, werden zunächst für jedes Kraftwerk, die dazugehörigen Verträge bestimmt. Diese Verträge entsprechen einem Replikatsportfolio des Kraftwerks. Dadurch kann das Risiko der oft komplexen Verträge über die Produktion gesteuert werden. Unter Ausschluss von Arbitrage-Möglichkeiten kann eine Elektrizitätsgesellschaft diese Verträge durch das Untersuchen, der für das entsprechende Elektrizitätswerk anfallen Kosten, bewerten. Flexible Produktionseinheiten, wie zum Beispiel Gasturbinen, lassen sich als Optionen interpretieren, unflexible Einheiten, wie Kernkraftwerk, entsprechen Futures.

Da der Elektrizitätsmarkt in grossem Masse unvollkommen ist und somit für die meisten Verträge kein perfekter Hedge existiert, wird der Begriff des bestmöglichen Hedges eingeführt. Der bestmögliche Hedge wird durch Optimieren bestimmt, wobei es das Risiko, gemessen in CVaR, unter Begrenzung des erwarteten Profits, zu minimieren gilt. Dieses Problem kann mittels

Linearer Programmierung gelöst werden, womit auch umfangreiche Probleme betrachtet werden können.

Wenn wir ein Portfolio betrachten, versuchen wir den vorgegebenen Risikospielraum bestmöglich auszunutzen, welches auf das bekannte Problem der Optimierung von Finanzportfolios zurückgeführt werden kann. Ungeachtet dessen, muss aufgrund der speziellen Charakteristika der Elektrizitätsportfolios das Problem den besonderen Eigenschaften des Elektrizitätsmarktes angepasst werden. Ein optimales Portfolio impliziert dabei eine optimale Einplanung der verfügbaren Anlagen. In dieser Arbeit liegt der Schwerpunkt auf der Betrachtung von Speicherkraftwerken, die aufgrund ihrer Flexibilität als eine Folge von Optionen interpretiert werden können. Da die Ausübung einer Option, d.h. die Produktion von Energie, die Menge des gestauten Wassers verringert und unter Umständen eine Produktion zu einem späteren Zeitpunkt unmöglich gemacht wird, gelten die Optionen als voneinander abhängig. Es wird eine dynamische Produktionsstrategie entwickelt, welche dieser Abhängigkeit Rechnung trägt. Das Optimieren eines Portfolios bestehend aus Wasserkraftwerken und Elektrizitätsverträgen führt zu einem optimalen Vertragsportfolio und einer optimalen Produktionsstrategie, oder vom Finanzstandpunkt aus gesehen, zur Bestimmung des optimalen Ausübungskonditionen für die Optionen. Das Problem wird mit Hilfe der Linearen Programmierung gelöst, indem über eine bestimmte Zeitspanne der Profit maximiert wird. Als Nebenbedingung wird dabei gefordert, dass der CVaR des Portfolios eine gewisse Schranke, welche die Risikopräferenzen des Unternehmens widerspiegelt, nicht überschritten werden darf.

Aufgrund der Abhängigkeit zwischen Produktion und Volumenrisiko der abgeschlossenen Verträge, kann gezeigt werden, dass Produktion und Verträge simultan optimiert werden müssen. Die Grundaussage dieser Arbeit ist die, dass das Vorhandensein von operationeller Flexibilität als sehr wertvoll einzustufen ist. Durch das Betrachten des dualen Problems der Portfolio Optimierung kann dieser Wert quantifiziert werden. In einem durchgeführten Fallbeispiel zeigt sich, dass der Wert der Flexibilität bedeutend sein kann. Jede Bewertung welche einer operationellen Flexibilität bei der Stromerzeugung nicht Rechnung trägt, unterschätzt somit den Wert flexibler Kraftwerke.

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Chapter 1

Introduction

1.1. Motivation

The electricity market in Europe is going trough a big transition. From being a regulated market with no or very low uncertainty in future earnings, the market is now becoming liberalized and deregulated. Electricity prices are no longer determined by the regulator, but by the market. From the deregulated markets in, for example, California, Sweden and Norway, one has observed extremely volatile prices. For an electricity producing company this makes future earnings very uncertain and brings along a need for risk management.

Electricity can be traded in a similar manner as stocks at power exchanges in, for example, Germany and Scandinavia. There are however major differences between the traditional financial markets, such as the stock market, and the electricity market, like lacking liquidity, tremendous volatility, non-normal distributions, real option pricing problems and market incompleteness. The electricity market especially distinguishes itself by the transmission constraints and the non-storability of electricity. Because of the specific characteristics of electricity, like price jumps and the fact that producing companies are by nature long in electricity, the risk management ideas developed for the financial markets are not directly applicable to the electricity market.

The uncertainty is however not necessarily negative for electricity producers. As a matter of fact, flexible power plants can take advantage of volatile prices. Traditional approaches to handle uncertainty in the electricity market was focused on meeting a fluctuating load, where prices where assumed to be deterministic. Since this is not the case anymore, a new approach to manage risk in the electricity market is needed.

The liberalization of the electricity markets brings along a lot of risks for the players, but also plenty of possibilities and chances. One of the keys to success in the liberalized market is the ability to manage these new risks.

1.2. Goal and purpose

The goal of this thesis is to investigate risk management in the electricity market in general and the interaction between physical production and financial and physical contracts in particular. The focus of this work is on how an electricity producer can utilize the operational flexibility in power plants as a risk management tool.

1.3. Structure of the thesis

In Chapter 2 the electricity market in general is investigated. Its special characteristics, which are needed as background for the further analysis, are highlighted. In Chapter 3 some general risk management ideas are presented followed by a short investigation of the risk measures currently used in the traditional financial markets. After analyzing their suitability as risk measures in the electricity market, the so-called *Conditional Value at Risk* is presented. The chapter is closed by studying the traditional valuation techniques and their shortcomings in the electricity market. In Chapter 4 the notion of *contract engineering* is introduced and the set of contracts corresponding to each type of power plant is systematically derived. The importance of flexibility in a power plant is highlighted as a key value driver. In Chapter 5 some traditional *hedging strategies* are introduced and once again the limitations of these traditional approaches in the electricity market are pinpointed. Instead the concept of *best* *hedge* is introduced. In Chapter 6 *portfolio optimization* in the electricity market is introduced. A, for the electricity market, new approach, accounting not only for expected profit but also for risk, is presented, where a portfolio of production assets and power contracts is optimized in a profit and Conditional Value at Risk framework. The importance and assessment of operational flexibility in production is investigated and the chapter is concluded by a positioning of contracts and plants, where the attractiveness of different assets is estimated. In Chapter 7 an optimization of a real power portfolio is conducted and the results of this case study are presented. Finally, in Chapter 8 our conclusions are presented.

Chapter 2

The electricity market

2.1. Overview

The electricity industry's basic function is to convert fuel and primary energy into electricity and transport it to customers.[1] Because it is very costly to store electricity, the electricity industry has to match supply with demand in real time. Unlike other businesses, like the gas industry, there is therefore no scope for suppliers to meet demand from stored reserves. The electricity industry can be divided into four main groups corresponding to the four-stage vertical inter-dependent process that is needed to produce and deliver electricity, namely:

1. Generation, i. e. electricity production.
2. Transmission, i. e. transport of electricity at high voltage.
3. Distribution, i. e. transport and delivery of electricity to customers.
4. Supply, i. e. retail.

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[1] The SI-unit of energy is joule [J]. One joule equals a constant power of one watt [W] for one second. In the electricity industry however, energy and power levels are typically very high, why the industry standard for energy is kilo watt hour [kWh], which corresponds to the energy stemming from a one hour constant power of 1000 watt. 1 MWh hence corresponds to 1000 kWh, 1 GWh to 1000 MWh and 1 TWh to 1000 GWh. The industry standard for power is consequently kW, MW and GW.

Generation Generation is the essential activity of the electricity industry. It involves the conversion of chemical, atomic or mechanical energy to electricity. Traditionally, generation is carried out on a large scale, since there are considerable economic advantages associated with using large-scale equipment [Enr01]. However, in 1999 the cheapest new electricity plant used gas fired CCGT[2] technology, which has a low minimum efficient scale [BV99]. The main types of fuel used in electricity generation are coal, nuclear, gas, oil and hydro. Other plants may however use such forms of energy as solar, wind, biomass and waste power.

Within most European electricity markets, there are a large number of power plants and it is thought that the market for generated wholesale bulk electricity is potentially competitive - and hence that active competition should be encouraged. This is because economics of scale in power stations are exhausted at low levels of output relative to the size of the market.

Transmission Transmission is the bulk transport of electricity at high voltage. High voltages are used for transportation in order to minimize losses, since losses are inversely proportional to voltage. The voltage of the electricity produced by generators is therefore stepped up by use of transformers before entering the transmission grid. Transmission lines are very capital intensive, why installing new lines is associated with high capital cost. It is therefore uneconomic to duplicate them, as it leads to under-utilization. Electricity transmission systems exhibit significant economics of scale and are natural monopolies, since marginal cost prices may inadequately generate revenues needed for capital recovery. For this reason transmission is regarded as a monopoly. Electricity is then supplied to distribution networks via step-down transformers, lowering the voltage.

Distribution Distribution is the transportation of power from the step-down transformers to consumers through successively lower voltage circuits. As with transmission, developing distribution networks is very costly and marginal cost prices may also in the distribution insufficiently create revenues needed for capital recovery, so it as well is regarded as a monopoly.

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[2] Combined cycle gas turbine (CCGT) and other plant types are further investigated in Chapter 4.

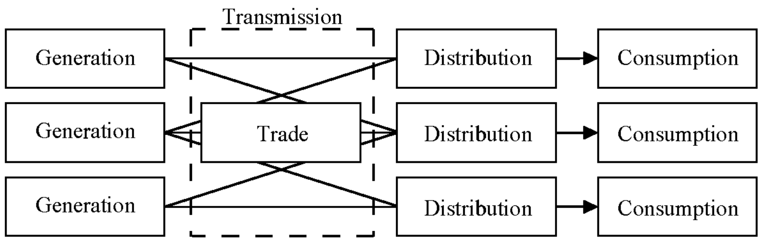

Fig. 2.1: Illustration of trading in wholesale competition.

Supply Retailing or supply is the business of advertising, branding, contract bundling, metering and billing of electricity for end users. Electricity retailing has traditionally been bundled with distribution, but recently liberalization efforts have demonstrated that it is actually separable from distribution and hence competitive. Supply companies can purchase generated electricity and transmission services and compete on the basis of least-cost purchasing, metering and billing costs and quality of customer service.

Trading In the regulated market no market mechanism was essentially established. Traditional utilities needed only to import electricity to their territory in the case of a power shortfall. As a result of deregulation and enforcement of competition at the level of generation and supply, the number of transactions has increased dramatically. To serve this increase in market activity, a number of power exchanges has emerged in the electricity markets. Trading of electricity refers to the business of facilitating exchange of wholesale electricity between generators and suppliers in order to meet contractual obligations. Trading within wholesale competition according to [HS96] is illustrated in Figure 2.1. Electricity contracts are not only traded at the mentioned power exchanges, but can also be traded on a bilateral basis.

The ongoing liberalization of the electricity market and its consequences are discussed in Chapter 2.2. The supply side is investigated in Chapter 2.3, the transmission issues in Chapter 2.4 and the demand characteristics in Chapter 2.5. The special characteristics of electricity is summarized in Chapter 2.6. The, for the electricity market very characteristic, complex contracts are introduced in Chapter 2.7, followed by a discussion of power exchanges in Chapter 2.8. The effects of the peculiarities in the electricity market on the price dynamic is investigated in Chapter 2.9.

2.2. Liberalization of the electricity market

Traditionally, centralized regulation of the electricity supply industry was considered necessary to ensure security of supply and efficient production. Efficiency was achieved through economics of scale. The electricity industry was characterized by a highly vertically integrated market structure with little competition in the potentially competitive segments of the market, with the market defined as a national electricity market. It used to be assumed that electricity generation, transmission, distribution and supply enjoyed significant vertical economics that would be lost if the functions were placed under the control of different companies. Such economics arose from the reduced transaction costs, the improved incentives to invest in specific assets that would not be subsequently held-up, and the advantages of coordinated supply and demand side planning [BV99]. In particular, it was almost universally observed that generation and high voltage transmission were integrated within the same company, as were distribution and retailing. However, many countries have during the last decade restructured and deregulated their electricity industry to introduce competition. In particular in the European Union, many countries have already liberalized their electricity markets.[3] In 1997 the Directive 96/92/EC on the Internal Market in Electricity came into force, providing for a phased up opening of electricity markets to competition. The Directive established common rules for the generation, transmission and distribution of electricity and prescribed a separation of the monopoly elements of the business from the potentially competitive segments so that controllers of the monopoly parts are unable to use their market power to abuse their position in other stages of production. The Directive introduced competition at the wholesale and supply level of the industry and marks the last step to the liberalization of the electricity sector in the European Union.

One can ask oneself what initiated the wave of deregulations in the electricity markets worldwide in the last ten years or so. One reason why this process

[3] For example, competition was introduced in England and Wales 1989, in Norway 1991, in Sweden 1995, in Finland 1997, in Spain 1999 and in Germany in 1999.

started now is the technology improvement. Moving from a vertically integrated utility to a chain of specialized and competing electricity companies is a huge step from a communication point of view. The system operator, being responsible for the stability in the grid, has to gather reliable data from not just as before one vertically integrated monopolist, but from a vast number of competing firms at different levels of the supply chain. This was in the past simply technologically not feasible.

In the light of the possibilities that technology offers today, one has left the idea of one vertically integrated utility. Instead introducing competition was believed to improve cost efficiency, increase diversity of fuel supply and provide additional benefits to the consumer. According to economic theory the transition from a monopoly through oligopoly to a fully competitive market would imply better allocation of resources [GS86]. Microeconomic theory states that the price in a non-regulated monopoly market would decline by introducing competition [GR92]. The electricity market was however regulated, why introduction of competition would not necessarily result in a price decline. In the Nordic markets and elsewhere prices though seem to have gone down as an effect of deregulation.

As mentioned, transmission and distribution systems are regarded as natural monopolies, because of the substantial economics of scales involved in these businesses.[4] Competition can therefore not be established in these businesses. On the other hand, generation and retail are thought to be potentially competitive, why competition could be introduced in these businesses. As a result of the deregulation of the electricity industry, one has to separate the transportation from the thing being transported, or with other words electric energy, as a product has to be separated commercially from transmission as a service. This separation is called *unbundling*.

The liberalization of the electricity market has changed the priorities of the industry and introduced new responsibilities. Companies are now concerned with profitability and maintaining a competitive edge and must consider the interest of all stakeholders. In a competitive electricity market utilities

[4] According to Griffin & Steele [GS86] if the firm's long-run average cost function declines continuously, the firm is considered as a natural monopoly.

cannot automatically pass costs through to all customers, since revenues are determined by market success and not by regulatory formulae. This has the effect of increasing uncertainty and risks born by investors in the electric supply industry with increasing cost of equity and debt finance as a result. Electricity is changing from a primarily technical business, to one in which the product is treated in much the same way as any other commodity, with trading, risk management and customer care as key tools to run a successful business.

The transformation of a utility needed to meet this new environment is a challenging task, where we believe that risk management will be a critical undertaking.

2.2.1. System operator

Electricity transmission and distribution systems require orderly arrangements for the dispatch of power plants to satisfy demand from customers. This requires a system operator, who oversees the process of instructing plants on required availability and physically balancing the system in situations, where actual supply and demand deviate from the planned supply and demand. So although an important part of the system operator's job is to match supply with demand, it is just as important that there is sufficient generating and line capacity available to ensure that supply volume is maintained in times of unforeseen demand spikes. This stability of voltage and frequency[5] is carried out by calling on power plants, which offer rapid response, or large customers who can shed load instantly. A system operator would together with a utility have the possibility to favor own transactions and plants, which is very inconsistent with a liberalized market. The system operator in deregulated markets therefore has to be independent from other electricity companies and be separated from other activities in the supply chain. This is called an *independent system operator (ISO)*.

[5] Voltage and frequency at different nodes in the grid is typically measured to detect mismatch between supply and demand.

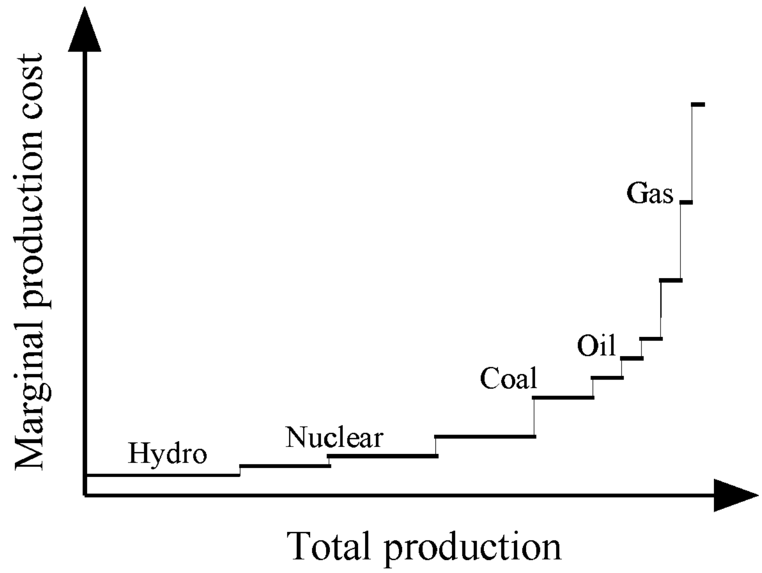

Fig. 2.2: Schematic supply stack.

2.3. Supply stack

The supply stack is the ranking of all generation units of a given utility or of a given set of utilities. The generation units are ordered according to the total amount of production that has to be exceeded until they are brought on-line. This ranking is based on many factors, such as marginal cost of production, which is the most important one, or the response time of the several generation unit types. The utility will typically first dispatch hydro units,[6] if available, followed by nuclear and coal units. These types of plants are generally used to cover the base load, whereas gas-fired plants are used to meet peak-demand. This ordering, the so-called supply stack, can be illustrated in a graph showing the marginal production cost as a function of total production. Figure 2.2 shows an example of such a supply stack. The flat part to the left in the graph represents the base load, which is met by plants with relatively low marginal costs, but often with a low flexibility, implying that the response time may be high or that it has to produce some constant amount of electricity. The middle part represents the intermediate load and is covered by units with intermediate marginal costs, such as coal and oil plants. The steep part to the right represent

[6] In Chapter 4.3 we will however see that this should not necessarily be the case for hydro storage plants.

the peak load. This load has to be met by very flexible plants, since demand is highly stochastic and hence the points in time of peaking demand. This flexibility normally has a negative effect on the marginal cost, why flexible units, such as the gas turbine tend to have high marginal costs, whereas a hydro-storage plant, on the other hand, is an example of a very flexible unit with low marginal costs. Since the available water is limited one can however argue that the shadow price of that water should be included in the marginal cost.

The supply stack is not static in time, since there are many factors that may alter the ordering of the generation units. The price of the fuel used in, for example, oil and gas fired units are highly volatile. The fuel price often makes up the majority of the marginal cost, why changes in these prices may have a substantial impact on the supply stack. Another factor affecting the supply stack is outages of plants. These outages can be due to regular maintenance operations, transmission constraints or unforeseen breakdowns. Such an outage will simply merge the supply stack's part to the left with the part to the right of the unit that becomes unavailable for dispatch.

In [SJR97] base load is defined as the load that is exceeded in 80% of all hours in a year. Generating plants that run nearly all time (>80% of the time) is consequently referred to as base load resources. Intermediate load is the level of demand that occurs between 20% and 80% of the time, and plants that run for this fraction of time are intermediate load resources. Peak load is the level that is exceeded less than 20% of the year and the corresponding plants are called peak load resources. This can be illustrated in a load duration curve, showing the cumulative frequency distribution of load levels, as in Figure 2.3.

2.4. Transmission costs

In contrary to other goods and energy sources, electricity has, because of the limited storage possibilities, to be produced and consumed simultaneously. This rises an important task of managing the production and consumption in such a way that equality between the two holds at every instant of time. This is normally done by the ISO. Unlike railways, roads or see routes, where transports in different directions are conducted sometimes at the same time,

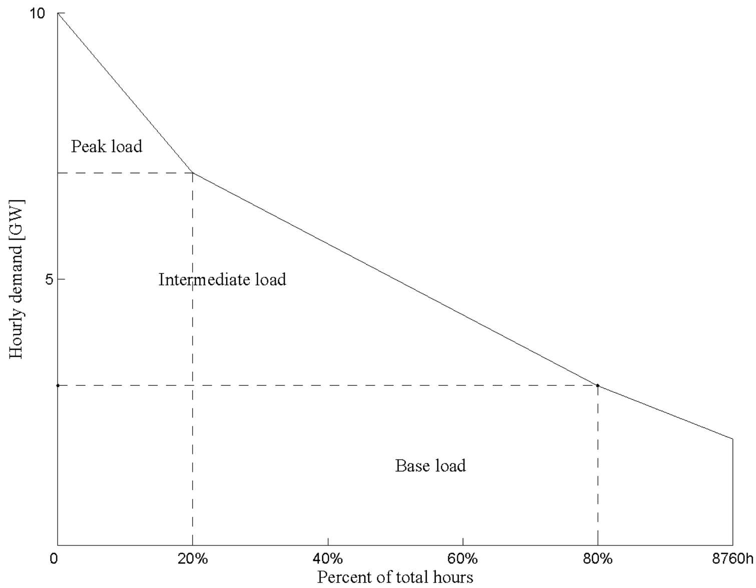

Fig. 2.3: Sample load duration curve.

only the net of all electrical currents is flowing in the electrical cable. As a consequence of the Kirchoff-laws, the electrical current does not necessarily flow through a certain line, but through a number of lines. The effective path of a transaction is therefore in a complex mesh not definable. Since it is not possible to mark the flow of electrons in a line, it is totally misleading to believe that one knows from which plant the electron-flow comes from and by which load it will be used. Still it is important to find a plausible determination of the path of electrical energy in the transmission net. The goal is to find a consequent and fair calculation of the transmission costs in the grid.

The transmission costs can according to [Big00] be divided into three categories: *Infrastructure costs*, *transmission losses* and *ancillary services*.

Infrastructure costs consist of the capital costs of the capital intense grid and operating costs, like personal costs.

Transmission losses are caused by the fact that energy is lost in the whole system, in the lines and in the components. In a transmission grid typically 1-2 percent of the electrical energy is lost due to resistance. In the distribution net even more, roughly 5-6 percent is lost because of the low voltage and hence higher current [Big00].[7]

Ancillary services are necessary to guarantee reliability of the system, such as dealing with congestions, provision for transmission losses and frequency control and provision of reactive power to assure that the system is in balance.

Since it is impossible to correctly determine the actual flow of a transaction, it is also impossible to allocate the grid's total costs to the different transactions in a correct manner. Instead a number of approximate methods to determine the transmission costs have been proposed.

The *postage stamp method* does not consider the contract path; a long transmission is priced equal to a short transmission. A fee per transmitted unit of electrical energy is paid, independent of location of the buyer and seller. The *contract path method* determines a fictitious path between the buyer and seller, the so-called contract path. By allocating each edge in the grid a cost, the transmission cost over the contract path can be calculated. In the *MW Mile method*, the fee is proportional to the geographical distance between the buyer and the seller (mile) and to the power of the transaction (MW). In the *empty grid method* only the current transaction is taken into account and the grid is assumed to be empty except for this transaction. The effect on the concerned lines is compensated on a per MW · km basis. This is the first method that tries to consider the real electric flow. The *marginal participation factors method* calculates the marginal flow contribution of a transaction to each edge in the grid. This divided by the yearly index for the flow in that edge gives the marginal participation of that edge. The sum of marginal participation multiplied by the transmission costs for each edge gives the total transmission price for that transaction. The *benefit factors method* is similar to the previous one, but here the economical utility for a transaction in each edge is the driving force to determine the price. The *nodal prices method* determines the price at every node in the grid as the cost derivative with respect to demand in the respective node. The transmission price between two nodes is then defined as the difference in the nodal prices. The *Staffellauf method* further tries to model the real effect on the grid and traces the electricity from its origin to its

[7] Losses are proportional to current

destination. For more information on transmission pricing see [Big00].

Since the market forces are not working in this monopoly business, transmission pricing must give the ISO correct economic incentives for new investments. Hence correct transmission pricing is crucial. On the other hand, to be market friendly, transmission prices should preferable be known when a transaction is entered. These two objectives are difficult to fulfill at the same time, why a trade-off between being exact and being market-friendly has to be done. The chosen method to determine transmission prices will have a big effect on risk management in the electricity industry. If a rather exact method is used, where all transactions have to be known before the transmission pricing can be determined, only the players closing the last transaction could potentially know their transmission price. This would add one dimension of risk, namely transmission price risk.

In this paper we will assume that we have a transmission pricing system, where the transmission costs are known in advance when entering a contract, such as the Swedish model, similar to the postage stamp method, where prices are geographically differentiated, but constant over time. This is a plausible assumption, since most countries have a transmission pricing mechanism where no transmission uncertainty exists when entering a contract.[8]

2.4.1. Transmission constraints and congestions

The network in an electricity market does not have unlimited capacity. Rather each component, such as a transformer has its limitations on the manageable power level. Some components are instable in the sense that the resistance grows with temperature, which further increases temperature.[9] The system actively has to be monitored and managed to avoid that a too high power level causes an outage in one part of the grid, which would instantaneously put additional pressure on the other still operative parts, potentially causing the

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[8] Denmark, Finland, France, Portugal and Spain have postage stamp method. Italy has postage stamp method with distance correction and Netherlands have postage stamp method with point tariff. Germany has a simple point tariff method and Sweden has a nodal tariff method with geographical difference [Cen00].

[9] Through resistance electrical energy is converted into heat.

ISO's worst nightmare, a so-called black-out.[10] The monitoring and managing of the grid is the major task for the ISO, which at his disposal has some potential tools. One is *geographical price differentiation* to give the players incentives to change their output and load to avoid congestions. Another tool is having own *reservoir generation capacity* at different locations to manage the flow of the electricity. If the ISO has no reservoir capacity at his disposal he can always *buy and sell* electricity at different locations to alter the flow.

The last two tools are expensive for the ISO, either in terms of capital and operating costs for having reservoir capacity at his disposal or by always having to sell and buy electricity at unfavorable rates.[11] The costs for the first tool, on the other hand, are not born by the ISO, but by the market through price alterations. The geographically differentiated electricity prices would be economical incentives to allocate plants in high price areas i. e. with a production deficit and to allocate electricity intensive industries in low price areas i. e. with a production surplus. As long as these areas and the price differences between the areas are stable over time, geographical price differentiation will not affect the market in a negative way, by introducing further uncertainties. But if these areas or the price differences in a stochastic way were changing with time, this model would not be market-friendly in the sense that the price variations between different locations would be unknown when entering a transaction. This would, in a similar way to our arguing about the transmission prices, heavily affect electricity risk management and bring yet one new dimension to the problem. We will in the analysis assume that the congestion risk is borne by the ISO, implying that the only uncertainty on the electricity prices is the spot price. This is the case in, for example, Sweden.

Unlike traditional financial markets, which essentially are global markets, the electricity markets are geographically distinct. There are several electricity markets between which transmitting electricity is either non-economical, because of high transmission costs or impossible, because of transmission constraints. Electricity markets hence tend to be regional and the price of electricity may well differ between geographical locations.

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[10] Black-out denotes loss of load in the whole or part of a grid.

[11] The ISO will have to buy electricity expensive at high demand areas and sell it cheap at high output areas to achieve the wanted result.

2.5. Demand

Compared to primary fuels, such as coal and oil, electricity is clean and safe. No waste is produced at the user's end, since all pollution is borne by the producer, not the end-user. Unlike most other fuels, which require storage and processing, electricity is immediate available and easily controllable at point of use. Precisely for these characteristics, electricity has become a fundamental driver of our economy [Ku95] and electricity is needed by essentially all sectors in the economy, from household to industry. There are generally high costs associated with unserved energy and the value of lost load can for some industries amount to tens of times the typical electricity price [WH97]. Demand is hence inelastic to price changes.

Demand of electricity exhibits seasonal fluctuations, which are essentially driven by the climate. In Europe the demand-peak normally occurs in the winter due to excessive heating. In other geographical regions, like California demand peaks in the summer, since humidity and heat initiate extensive use of air-conditioning. Electricity demand is not even uniform throughout the day. Several electricity end-users are related to the time-of-day, like lighting, cooking and use of computers. The hours of the day during which the highest demand occurs is known as the peak period. The demand fluctuation with a yearly periodicity is exemplified in Figure 2.4, where one can also see the increasing demand over time. Electricity demand has throughout the world been growing steady in the past and seems to be closely related with GNP growth [Ku95], which in the developed world historically has been a few percent per year in real terms. The demand fluctuations with a daily periodicity can be seen in Figure 2.5, where the low demand during weekends, causing a demand variation with a weekly frequency, is also conspicuous. This is a typical pattern for electricity demand, due to the low industrial activity during nights and weekends.

Extraordinary weather conditions can cause sudden and dramatic shocks to the demand, which may increase substantial. The demand is typically falling back to its normal level as soon as the underlying weather phenomenon is over. Demand is normally well correlated with temperature and a rule of thumb in Switzerland is that a temperature decrease of one degree Celsius, increases the

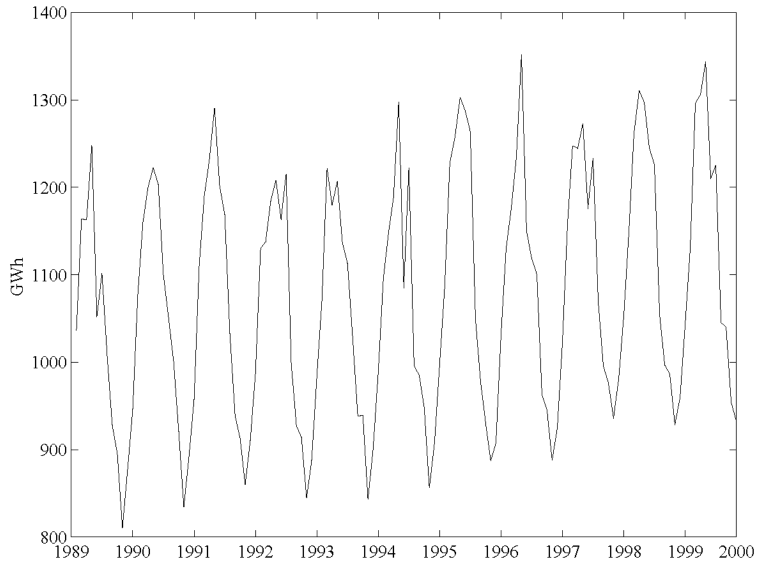

Fig. 2.4: Monthly demand from October 1989 to September 2000 for a typical Swiss utility.

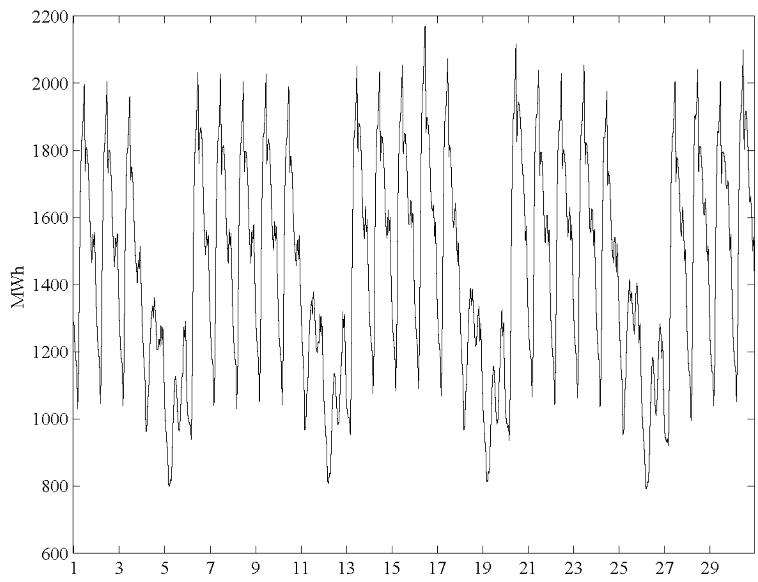

Fig. 2.5: Hourly demand during September 2000 for a typical Swiss utility.

demand of about one percent [RW99].

2.6. Special characteristics of electricity

Electricity is very different from traditional financial products, such as equities. It is also different from other commodities like oil and gas. We will here shortly recapitulate what makes electricity so special.

There are no efficient ways to store electricity, the most efficient way is the use of the limited pump capacity that is installed in some of the hydro storage plants. This efficiency that is regarded very good for electricity storing, is still only around 70%. It is therefore fair to use the conventional wording and say that electricity is *non-storable*, especially as electricity in itself is not storable. The electrical energy actually has to be stored in other forms, such as chemical energy in batteries or potential energy in stored water. A very important effect of this non-storability is the fact that in a power system, *supply has to equal demand* at each instance of time. Further, electricity has to be *consumed exactly at the same time as it is produced.* Following from the non-storability, electricity supplied at different times of the day and in different places are not perfect substitutes, which is different from other commodity markets. Even natural gas, probably the most similar commodity, can be stored, not only in special storage facilities, but also in the pipeline network. Unlike the electricity grid, the gas network allows supply to differ from demand, since the gas pressure in the pipeline network can vary. No such mechanism is possible in the electricity grid, where an increased load, without a corresponding increase in supply would take power from the other units in the grid. This is not only a very challenging task for the engineers building and maintaining such power systems, but also affects the value of electricity in a deregulated market and complicates the modeling of electricity prices to a far more complex level than the modeling of, for example, stock prices. Another special feature is that electricity prices are not expected to follow an up-going trend, i. e. is not expected to generate any return due to risk. This is different from for example the equity market, where an up-ward trend is stipulated by CAPM [Sha64].

Unforeseen outages of generation assets, highly stochastic demand together with its inelasticity to prices, and transmission constraints further makes electricity stand out from the family of commodities and from traditional financial products.

2.6.1. Relationship to other energy markets

Electricity differs from other commodities mainly because of its non-storability. Still electricity is considered to be a commodity and despite its special characteristics, the electricity market is related to other commodities.

The group of commodities used for their high concentration of energy is of special interest for the electricity market. Energy markets like oil, coal and gas are directly influencing the electricity market in two ways. In the long term, these energy commodities are *substitutes* for electricity and if the relative price of, for example, oil declines, some power users will switch their consumption from electricity to oil. In the short term these energy markets are explicitly affecting the electricity price, since oil, coal and gas, to mention the most important ones, are used as *fuel* in thermal power plants. A higher fuel price will bring along a higher marginal cost for the producer and hence a higher electricity price, since microeconomic theory states that in perfect markets prices will equal marginal production costs [GR92]. The uncertainty in energy markets will therefore translate into an uncertainty for the players in the electricity market.

2.7. Electricity contracts

One differentiates between *standardized contracts*, traded at exchanges, where clearing is often offered and *OTC contracts*,[12] which are traded on a bilateral basis.

2.7.1. Traded standardized contracts

There are three types of standardized contracts traded at many of the growing number of power exchanges in Europe and elsewhere: *spots*, *futures* and *op-*

[12] Over the counter (OTC) contracts are not traded on organized exchanges.

tions. Standardized contracts are normally cleared at a power exchange, meaning that the exchange takes the counterparty risk,[13] why these contracts have to be simple to assure a correct risk assessment and have to be well defined to avoid any legal uncertainty.

2.7.1.1. The spot market

The spot market is actually a day-ahead market, but still called a spot market.[14] A pure spot market would not be possible in the electricity market, since the ISO needs advanced notice to verify that the schedule is feasible and lies within the transmission constraints. Further, not all power plants can alter their output within minutes, why also the generation side needs some respite.[15]

The spot is normally an hourly contract, but can be even shorter, like the half-hourly spot contract traded at the Amsterdam Power Exchange. The spot contract has physical delivery and is the underlying of most derivatives. The spot is not traded on a continuous basis, since the pricing mechanism is an auction, conducted once per day.

The spot contract is a contract giving the buyer the obligation to receive one MW of electricity over the period, and the seller the obligation to deliver the same amount of power at a specific geographical location. Depending on how the market is set-up and how the ISO is managing the grid, this specific location can be anywhere in a grid or it can be a single hub in the grid. In any case, the exact location of the supply and the demand has to be communicated to the ISO so that he can manage the power system. In Figure 2.6 the load curve of a spot contract is visualized.

2.7.1.2. The futures market

The electricity futures are normally traded on a continuous basis, which is also the case for most traditional financial markets. The future has the spot price

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[13] Counterparty risk is also called credit risk, which is defined in Chapter 3.3.
[14] The definition of a spot market is a market for immediate or very near delivery
[15] The operational flexibility will be explained in more details in Chapter 4

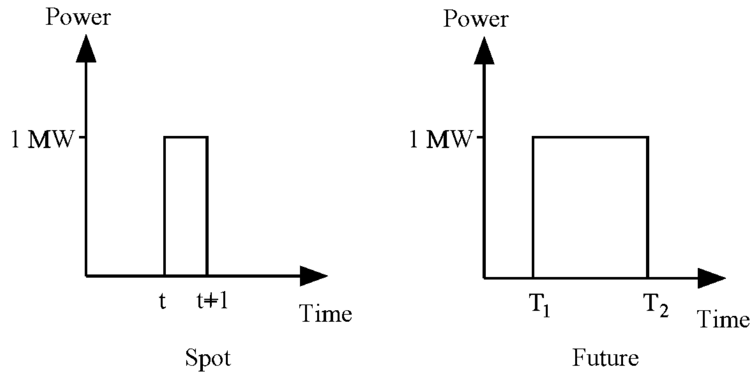


Fig. 2.6: Load curve of spot and futures contract.

as underlying and there are both financially and physically settled contracts, depending on the power exchange. Futures are normally used to assure a fixed price of sold or bought electricity in the future. There are in Europe futures with up to a year of average spot price as underlying and they are traded up to three years in advance.

The physically settled future obligates the buyer to receive a constant power of 1 MW over the period $[T_1, T_2]$ and the seller to deliver the same amount at a specified price, the so-called delivery price K, as illustrated in Figure 2.6. The location is determined by the underlying spot contract's specifications.

To facilitate for financial players to trade in the futures market and hence increase liquidity, many power exchanges have chosen financially settled futures, meaning that no electricity is delivered. Instead, the same profit and loss profile is achieved through a cash payoff given by the difference between the average spot price during the period $\frac{1}{T_2-T_1}\sum_{\tau=T_1}^{T_2} S_\tau$ and the delivery price K as illustrated in Figure 2.7.[16] The players that need to receive or deliver physical energy will have to cover their physical demand or supply in the spot market. The payoff of the financial settled future however assures that the actual price for buying or selling electricity in combination with the future will be exactly the delivery price K.

[16] Since the shortest future typically has a period of a full day, i. e. 24 spot contracts, the payoff is given by the difference between the average spot price and the delivery price and not as in traditional financial markets between the spot price and the delivery price.

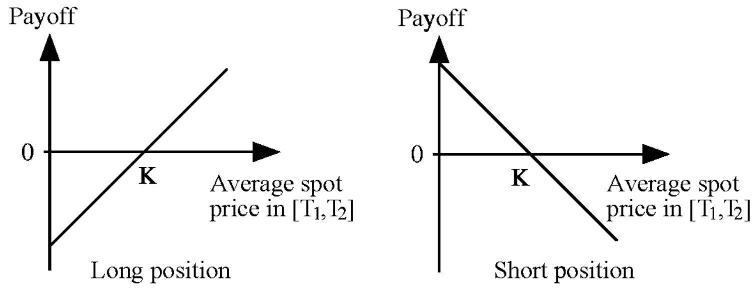


Fig. 2.7: Payoffs from future contracts.

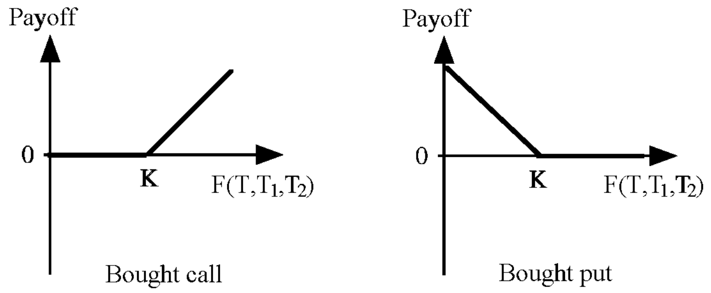


Fig. 2.8: Payoff structure of European options.

2.7.1.3. The options market

There are two types of traded options in Europe, namely *European options* with futures as underlying and *Asian options* with spot contracts as underlying. Both option types are typically traded on a continuous basis. There are two types of options, *calls* and *puts*. The buyer of a European call option is entitled, but not obligated to buy the underlying future at the strike price K at the time of expiration of the option T. The buyer of a European put option, on the other hand, is entitled, but not obligated to sell the underlying future at the strike price K. If we by $F(t, T_1, T_2)$ denote the market price at time t of the underlying future with delivery in $[T_1, T_2]$, where $T \leq T_1 < T_2$ then the payoff at time T of a call option is given by $\left(F(T, T_1, T_2)-K\right)^+$ and of a put option by $\left(K-F(T, T_1, T_2)\right)^+$, as shown in Figure 2.8, where $t^+ = max(0, t)$. Since the European options are typically cash settled no delivery of the underlying future takes place and the payoff is paid in cash. The Asian options are cash settled and the buyer of an Asian call option has a payoff at expiration T given by the difference between the average spot price in the period $[T_1, T]$, where $T_1 < T$

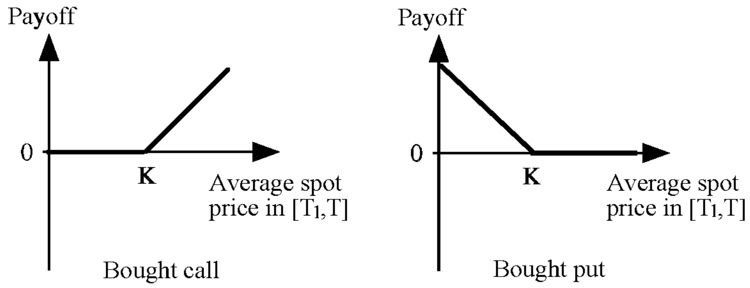


Fig. 2.9: Payoff structure of Asian options.

and the strike price K if it is positive, otherwise a zero payoff is achieved, i. e. $\left(\frac{1}{T_1-T}\sum_{\tau=T_1}^{T} S_\tau - K\right)^+$. The buyer of an Asian put option, on the other hand has a payoff given by $\left(K - \frac{1}{T_1-T}\sum_{\tau=T_1}^{T} S_\tau\right)^+$, as illustrated in Figure 2.9.

2.7.2. OTC contracts

In contrary to the rather straightforward nature of the standardized contracts, the OTC contracts can be very complicated. There are a vast number of different contracts from plain forwards via swing options to interruptible contracts traded on a bilateral basis. As is generally the case with OTC products, the liquidity is thin and the liquidity risk can hence be substantial.[17]

2.7.2.1. Fixed price fixed quantity contracts

These so called bulk contracts are used to cover the bulk of an electricity users demand. The seller of such a contract is obligated to deliver a constant power at a predefined geographical location at a fixed price over a given time period. In financial terms this is a forward with physical delivery and the payoff is identical to the one of a future contract.[18]

[17] Liquidity risk and other risk affecting a player in the electricity market will be investigated in Chapter 3.3.

[18] However, since futures are marked-to-market during their life time, whereas forwards are not, the exact time of the payoff may differ.

2.7.2.2. Indexed contract

For many industries the electricity cost makes up a substantial part of the total costs. An uncertain electricity price thus makes the total costs uncertain. Further, for many industries the price of sold goods makes up for most of the uncertainty on the revenue side (together with the amount of sold goods). One way to hedge against these two risks would be to buy electricity on a fixed price basis and sell the produced goods on a fixed price basis. This could theoretically be arranged for by using future contracts. The problem is that the amount of electricity and sold goods is not exactly known, why a perfect hedge against the two price risks is not achievable. This volume uncertainty can however be avoided by linking the electricity price to the output price. The electricity market offers such so-called indexed products for some industries. The electricity price that the player pays is determined by an index, based on, for example, aluminum prices. An aluminum producer could through such an indexed contract hedge the margin, given by the ratio between costs and revenues to assure a fixed margin. This is of course a simplification of the reality, since also other costs may vary, such as personal costs and costs of other input resources. Still the electricity-output spread will be hedged. An American utility, Bonneville Power Administration already in 1985 introduced aluminum-linked products.

2.7.2.3. Cross-market contracts

Whereas an electricity consumer may be interested in an indexed contract to offset some of his risks, a producer may be interested in so-called cross-market contracts. The fuel costs for a thermal producer makes up the absolute majority of the variable costs and a substantial part of the total costs. Such a producer may therefore want to hedge away the fuel price uncertainty. The amount of fuel to hedge is however unknown, since it depends on the future dispatch,[19] why a normal future or forward hedge will not do the job. Instead, there are products linking fuel price with electricity price to offset this spread risk. These cross-market contracts can be a fuel-electricity swap for example, or options on this swap. A widely used cross-market contract is the spark spread option.[20] A

[19] As will be shown in Chapter 4 the future optimal dispatch will for most plants depend on unknown stochastic factors, such as electricity prices and demand.

[20] The spark spread will be discussed more in Chapter 4.

buyer of such contract has the option to switch one unit of gas for one unit of electricity at a specified strike price.

2.7.2.4. Floating contract

A contract with a fixed quantity, but floating price is a long-term contract, where the buyer however pays a short-term price in each period. This price is typically based on the spot price. The floating price contract can thus be seen as an indexed contract, where the index is the spot price or any other short-term reference price. A floating contract has the same cost structure as constantly buying electricity on the spot market. This can be compared with a fixed income contract with a floating interest rate, like the FRN.[21]

2.7.2.5. Caps and floors

A capped contract is a floating product, but with a maximum level on these floating prices. A capped contract can basically be divided into the underlying floating contract and a series of call options on the underlying contract with a strike price equal to the capped level. A capped contract is therefore always more expensive than the pure floating one. To achieve a lower price the buyer can agree also on a floor level, in practice meaning that he sells back a series of put options to the seller on the underlying contract. The effect that caps and floors has on a floating product is illustrated in Figure 2.10.

2.7.2.6. Contracts for difference

When a future or option is bought in order to, for example, hedge away some undesirable risks, the underlying spot price has to be defined. Because of transmission costs and congestions, the spot price at different locations may differ, as discussed in Chapter 2.4.1. In Nord Pool, for example, there are a number

[21] A floating rate note (FRN) is a dept security with a long life, where the yield is periodically reset relative to a reference index rate to reflect changes in the short- or intermediate-term interest rates.

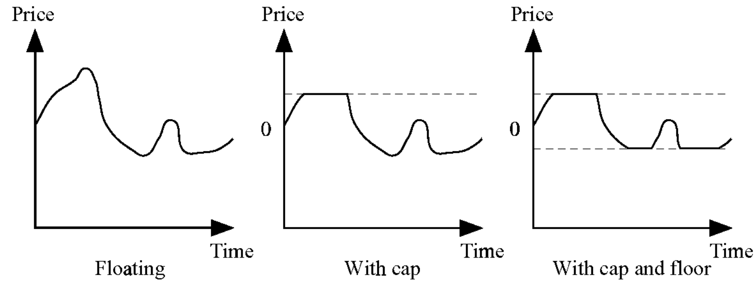


Fig. 2.10: Effect of caps and floors.

of geographically distinct spot prices, but all derivatives are based on a fictitious so-called system price.[22] The spot prices may however differ from the system price, meaning that if a player hedges the uncertain spot price at one location through, for example, a future based on the system price (since this is the only available underlying) he will not be perfectly hedged. The player will be fully exposed to the difference between the spot price and the system price, and will carry a so-called *basis risk*.[23] This is an example of why the *contract for difference* was brought to the market. This contract allows players to hedge against locational price risk and these types of contracts are called locational spread contracts. The buyer of such contract is obligated to change electricity at a predefined location **A** for electricity at another predefined location **B** at a given price K. These contract are however often cash settled and the payoff is naturally given by $(S^A - S^B) - K$, where S^A denotes the price at location **A** and S^B the price at **B**.

2.7.2.7. Swing option

The OTC contracts described so far are basically used to, except for delivering electrical energy, transfer the *price risk*[24] from one player to another. A user of electricity may however not know how much electricity he will need and hence hedge. This uncertainty known as *volume risk* consequently needs to be managed as well. Hence many OTC contracts have a flexible underlying volume of electricity. The buyer of such a contract has the right to *swing* his load, that is

[22] The system price is the price that would apply if there were no transmission congestions. The Nord Pool system will be discussed further in Chapter 2.8.1.
[23] Basis risk and other risks are defined in Chapter 3.3
[24] See Chapter 3.3 for more information on the different risks

why these contracts are generally called *swing options*. A broad definition of a swing option with a life time from 0 to T is that both the instantaneous load D_t and the total energy $\int_0^T D_t dt$ are constrained as

$$\begin{aligned} A_1 \leq D_t \leq A_2, \quad \forall t \in [0, T] \\ B_1 \leq \int_0^T D_t dt \leq B_2. \end{aligned} \tag{2.1}$$

The contracts that the retail customers enter with their electricity suppliers are essentially swing options, where the maximum load is determined by the fuse, say 30 Ampere. Below this threshold the customer is allowed to swing his demand. The volume risk of a retailer, who has to deliver the demanded power, with a large number of customers, will be diversified down a bit. Still, a substantial uncertainty in the demanded volume will remain. A retailer without own flexible production capacity will therefore need to transfer this risk to other players. This is typically done through swing options in different forms.

A typical swing option is the *load factor contract*. It has a fixed amount of energy and is often a one year contract with 5000 hours utilization time. Since there are $24 \cdot 365 = 8760$ hours in a year, the contract is said to have a *load factor*[25] of $5000/8760 = 57\%$, a measure of the flexibility inherited in the contract. Assume that the load factor contract has a 10 GWh limitation, then the maximum power of the contract, the maximum drawdown, is 10GWh/5000h= 2 MW.

2.7.2.8. Interruptible contract

Some contracts have a clause of interruptibility, meaning that, normally the seller has the right to curtail the supply at predefined number of occasions, in exchange of course for lower prices. Hence an interruptible contract has built-in options. The intuitive connection between the interruptible contract and options is verified in [GV92], where the equivalence between interruptible contracts and forward contracts bundled with a call option is described. These types of contracts was introduced as part of demand-side management programs, where

[25] The load factor is given by the ratio of the average load to the peak load.

utilities gave the demand side incentives to be more flexible in their electricity consumption, as an alternative to build capital intensive new capacity.[26] The price discount will depend on how often these curtailments may occur and how far in advance the notification has to occur. An early notification is less harmful to a consumer than a late notification, which will be reflected in the price of such a contract.

2.7.2.9. Weather derivatives

The swing option and the interruptible contract are adapted to hedge volume risk. None of these contracts however look at the real source of the demand and supply uncertainty. One major factor influencing both demand and supply is weather. The temperature affects demand and precipitation affects the supply. There are now a vast number of weather derivatives on an OTC basis, like the ones based on temperature, such as heating degree-days (HDD) and cooling degree-days (CDD).[27] These temperature-based derivatives can be used to hedge volume risk, since demand is normally well correlated with temperature as described in Chapter 2.5. Futures based on HDD and CDD are traded at the Chicago Mercantile Exchange and in Europe, London International Financial Futures Exchange (LIFFE) is in a process to develop such products. For more information on weather derivatives see [ADS00].

2.8. Power exchanges and pricing mechanisms

Several countries have deregulated their power markets and power exchanges have emerged in the light of the deregulation wave that has swept over us during the last years. Power exchanges show structural similarities to the well known financial trading floors. Each power exchange has its own pricing mechanism, its own contracts and its own settlement principals. One can however see similarities between different exchanges and we have chosen to exemplify these

[26] For more information on DSM see for example [SJR97].

[27] Heating degree days is normally defined as HDD=$(18-\bar{T})^+$ and cooling degree days as CDD=$(\bar{T}-18)^+$, where $\bar{T}$ is the average temperature in degrees Celsius in a certain day at a certain location. This special definition comes from the fact that heating is typically required for temperatures below 18 degrees Celsius, whereas air-conditioning tends to be used for temperatures above 18.

Price Hour	0 NOK	Possibly 12 additional price buckets	5000 NOK
00-01	11 MWh		-80 MWh
01-02	9 MWh		0 MWh
02-03	10 MWh		-90 MWh
...			

Tab. 2.1: Exemplification of bid to be filled in by all participants in the spot market.

by studying the generally regarded most developed power exchange in Europe, Nord Pool.

2.8.1. Nord Pool

The Norwegian deregulation came into force in 1991 when grid owners were compelled to open up their grids to competitors. In 1993 Nord Pool started its business as a power exchange for the Norwegian market. In 1996 Sweden was integrated into the exchange. One year later, in 1997, Nord Pool started to offer clearing also of some bilateral contracts as a service to eliminate the counterparty risk. In 1998 Finland was integrated and in 1999 the western part of Denmark also joined the exchange.

Nord Pool offers two types of standardized contracts, physically settled and financially settled contracts. The physically settled contracts are limited to the spot market. Every day is divided into 24 hourly contracts. Before noon, the previous day, all participants send in their bids for each hour as exemplified in Table 2.1.[28] Each participant specifies how much he is willing to buy or sell and at which price for each hour. This bid is binding and the procedure is called an auction, in contrary to continuous trading. This is done in order to gather all the liquidity to one point in time. From the bids Nord Pool achieves a supply and demand curve for each participant and hour, which is thereafter aggregated to a total supply and demand curve, from which the system price S_p, is calculated as the equilibrium for each of the 24 hours, as illustrated in Figure 2.11.

[28] Norwegian Kronas (NOK) is the currency used by Nord Pool, 1 NOK $\approx$ 0.19 USD.

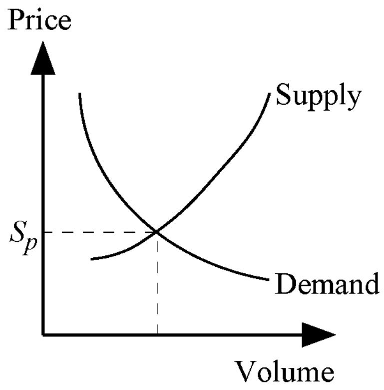


Fig. 2.11: Determination of the system price.

The system price is a theoretical price in the sense that it assumes that no congestions will occur and is the same in the whole Nordic area. To avoid congestions the Nordic market is divided into zones. In each zone the demand- and supply-curve is, if needed, shifted through a zone specific spot price in such a way that congestions between the zones are avoided. The price in the surplus areas will be lowered to decrease the supply of electricity, whereas the price in the deficit areas will be increased to increase the supply in order to regulate the power flow down below the capacity limit. Norway may be divided into many areas, while Sweden always is one area, as is the case for Finland and Denmark. The system price and the zone specific spot prices are disclosured at 15.00.

The financial settled contracts are futures up to three years and options up to one year with a delivery period of maximum one year. The market is according to Nord Pool not yet ready for contracts expiring further out in the forward curve.[29] For futures contracts, the value of each participant's contract portfolio is calculated daily, reflecting changes in the market price of the contracts. The daily changes in value are settled financially between the buyer and the seller. Through this process, a portfolio manager can quickly identify and realize losses as well as profits, which keeps the credit risk at a low level.

Previously both European options with futures as underlying and Asian options with the average spot price as underlying were traded at Nord Pool. However,

[29] A forward curve is a plot of future prices against their maturity.

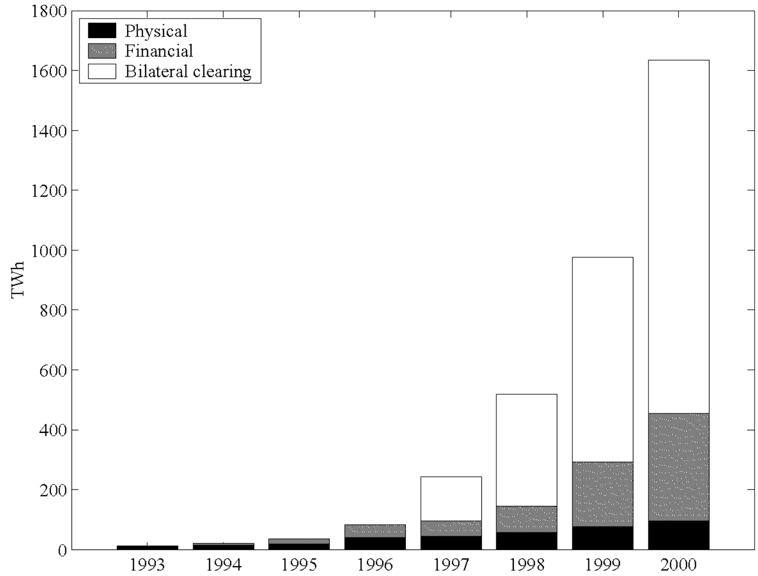


Fig. 2.12: Turnover at Nord Pool from 1993 to 2000 divided into physical, financial and bilateral clearing market.

liquidity of the Asian option was not sufficient, why now only European options are traded. The liquidity is pretty thin also in these European option contracts, which stands in contrary to the futures contracts, where liquidity is good. Beside the standardized contracts, where the price is transparent, Nord Pool is also clearing bilateral contracts with simple constructions. Since Nord Pool is counterpart to every player, they have to calculate margins, to be disposed at Nord Pool, in order to eliminate, or at least to lower the credit risk. The bilateral contracts that they clear are, up to today, only contracts very similar to the standardized contracts traded at Nord Pool.

The traded volume has increased substantially since Nord Pool's launch in 1993. The turnover of the financial markets now dominates the physical market and the clearing turnover of bilateral contracts has soared since 1997, as seen in Figure 2.12. The volume of cleared bilateral contracts is now much larger than the volume of traded contracts. The reason for this is, according to Nord Pool, that some players are not willing to disclosure the price information and therefore chooses OTC contracts. This in combination with some player's beliefs that they can achieve better deals on a bilateral basis explains why the cleared OTC contracts show higher volumes.

To regulate unpredictable imbalances between demand and supply and to deal with grid capacity constraints within each zone, a second physical market has evolved, called the *balance market*. Active participants in the balancing market must be consumers and producers who can respond quickly to unanticipated power imbalances by quickly adjusting their power load or production. The balance market can be seen as a spot market traded on a very short notice, used by the ISO to fulfill the goal of a stable grid.

2.9. Price dynamic

According to basic economic ideas the price of a good is determined by the intersection of demand and supply, which explicit is the pricing mechanism that is chosen by, for example, Nord Pool. The price will therefore be directly influenced by the supply and demand of electricity. The non-storability of electricity produces some oddness in the spot price behavior, which creates challenges in modeling the spot price dynamic. Here we will present the most important features of the spot price.

In finance uncertain price processes are typically modeled in stochastic differential equations (SDE), which are similar to partial differential equation, but with the important extension that parts of the differential equations contains random variables. The mother of all price models is probably the one used in the Black & Scholes model [BS73]. In their stock model the spot price S_t follows a so-called geometric Brownian motion

$$dS_t = aS_t dt + \sigma S_t dW_t, \tag{2.2}$$

where W_t is a Brownian motion. The process is called a geometric Brownian motion because when the SDE is solved with ITO calculus[30] one sees that the Brownian motion appears in the exponent

$$S_t = S_0 e^{(a-\frac{\sigma^2}{2})t+\sigma W_t}.$$

[30] For information on ITO calculus see [Øks95].

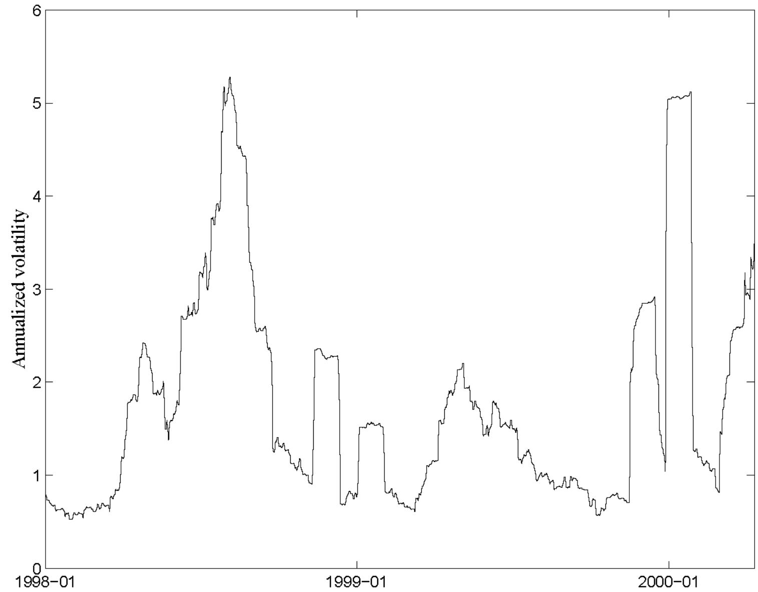


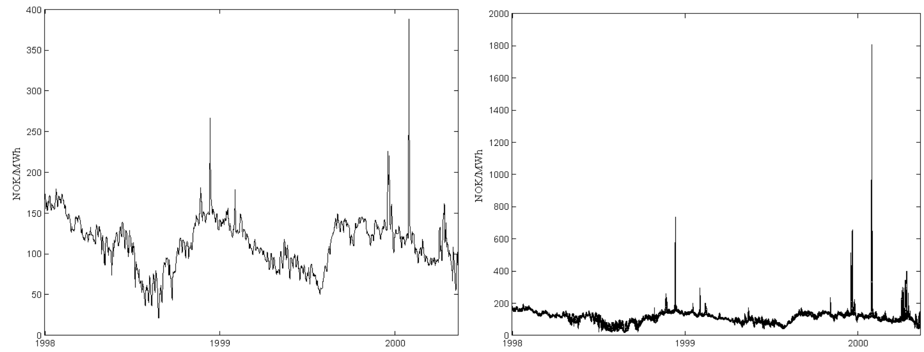
Fig. 2.13: Rolling volatility calculated from a monthly window of the daily spot price at Nord Pool from 1997-12-30 to 2000-05-06.

The spot increments dS_t in (2.2) has a deterministic part, given by $aS_t dt$, where a is the drift in the spot price. It also has a stochastic component, given by $\sigma S dW_t$, where the constant σ denotes volatility. Figure 2.13 shows that the volatility of electricity prices is not constant, why a time dependent volatility σ_t may be preferred when modeling electricity prices.

Electricity prices are however fundamentally different from stock prices why the simple geometric Brownian motion is not sufficient to model electricity prices.

2.9.1. Mean reversion

Energy spot prices in general are mean reverting [GS90] and electricity prices in particular seem to move around some sort of equilibrium level, a so-called mean level. Electricity prices are hence regarded to be *mean reverting*. This feature is normally modeled by having a drift term that is negative if the spot price is higher than the mean reversion level μ and positive if the spot price is lower than the mean reversion level and the simple geometric Brownian motion



(a) Daily average (b) Hourly

Fig. 2.14: Spot prices at Nord Pool from 1997-12-30 to 2000-05-06.

can be extended as

$$dS_t = \alpha\big(\mu_t - S_t\big)dt + \sigma S_t dW_t. \tag{2.3}$$

The speed of mean reversion α determines how fast the price will revert to its mean level. In the SDE (2.3) the level of mean reversion is made time dependent to reflect the fact that electricity prices tend to revert to different levels over the year.

2.9.2. Seasonal fluctuations

As already mentioned, demand follows seasonal fluctuations, mainly influenced by climate and in Europe the demand peaks in the winter. Also the supply side shows seasonal variations in output. Hydro units, for example, are heavily dependent on precipitation and snow melting, which vary with seasons. These seasonal fluctuations in demand and supply are directly translated into seasonal fluctuating electricity prices. The left part of Figure 2.14, showing the daily average spot prices at Nord Pool, clearly shows this seasonal component, with high prices in the winter and low prices in the summer.

These seasonal fluctuations is typically modeled by letting the level of mean reversion follow a deterministic sinus function

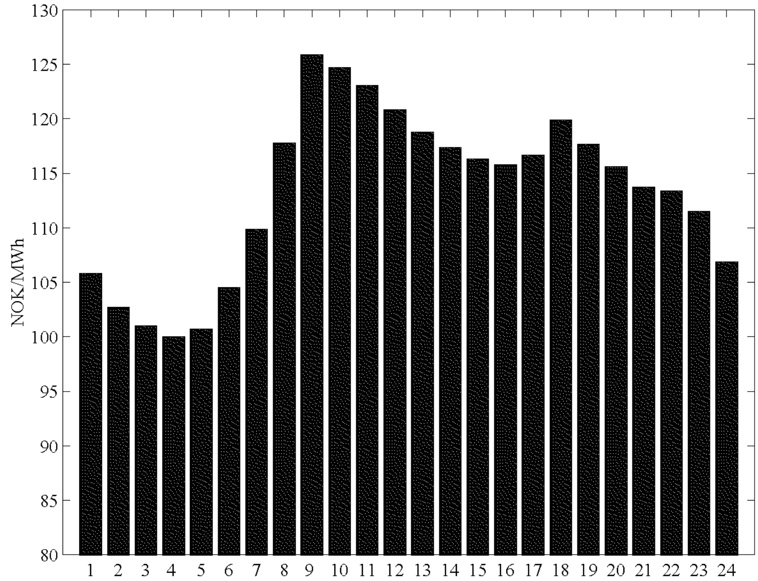


Fig. 2.15: Average spot prices per hour at Nord Pool from 1997-12-30 to 2000-05-06.

$$\mu_t = \bar{\mu} + \mu_y^{amp} \sin(\omega_y t + \phi_y),$$

where $\bar{\mu}$ denotes the level around which prices fluctuates over the seasons and μ_y^{amp} denotes the amplitude of the yearly fluctuations. The ω_y is chosen such that a yearly frequency is achieved. If t is measured in years ω_y should consequently be given by 2π. The phase ϕ_y is to be chosen such that the maximum of μ_t occurs when prices are peaking, typically in January.

2.9.3. Intra-day variations

The electricity price is not even uniform throughout the day. The intra-day variations of demand caused by different level of working activities will also translate into intra-day variations in the electricity prices. The peak and off-peak demand naturally correspond to high respective low prices. Figure 2.15 showing the average spot prices per hour exemplifies this phenomenon. The spot price seems to have two peaks during one day, with an obvious peak before noon and a less distinctive peak in the late after noon. This is a typical pattern throughout Europe. Even though these intra-day variations do not follow a simple sinus function, but a more complicated function, these variations are for computational tractability usually modeled with a simple sinus function, which is added to the function with a yearly periodicity

$$\mu_t = \bar{\mu} + \mu_y^{amp} \sin(\omega_y t + \phi_y) + \mu_d^{amp} \sin(\omega_d t + \phi_d), \tag{2.4}$$

where μ_d^{amp} denotes the amplitude of the daily variations. The ω_d is chosen such that a daily frequency is achieved. If t is measured in years ω should consequently be given by $365 \cdot 2\pi$. The phase ϕ_d is to be chosen such that the maximum of μ_t occurs when prices are peaking, typically around noon.

In addition to the yearly and daily periodicity also a weekly periodicity could be built into the level of mean reversion to capture the fact that demand and hence prices tend to be lower on weekends compared to weekdays.

2.9.4. Jumps

Electricity prices exhibit infrequent, but large jumps. The spot price can increase with several hundred percent in one hour, which is illustrated in the right part of Figure 2.14. This is an effect of the non-storable nature of electricity, which cannot be substituted for electricity available shortly after or before, since it has to be consumed at the same time as it is produced. Therefore, electricity sold at different times of the day actually should be viewed as different commodities. Jumps in the electricity price are an effect of dramatic load fluctuations, caused by extreme weather conditions often in combination with generation outages or transmission failures, possibly cutting of some plants. These spikes are normally quite short-lived, and as soon as the weather phenomena or outage is over, prices fall back to a normal level.

The spot price S, given as the intersection between demand D, and supply, is not very sensitive to demand shifts when the demand is low, since the supply stack typically is flat in the low-demand region. When demand is high however only small increments in demand can have huge effects on the price, since demand then intersects with the steep part of the supply stack. If demand increases so much that the supply side simply is not there to meet this

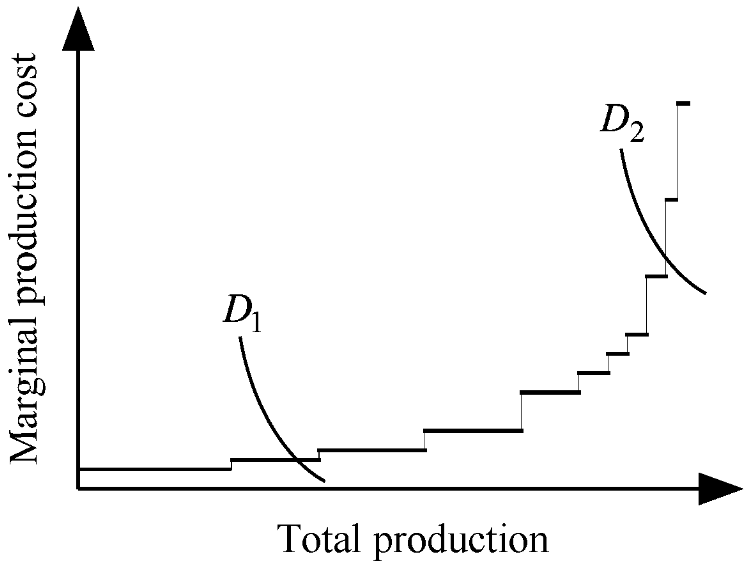


Fig. 2.16: Schematic supply stack with 2 potential demand curves.

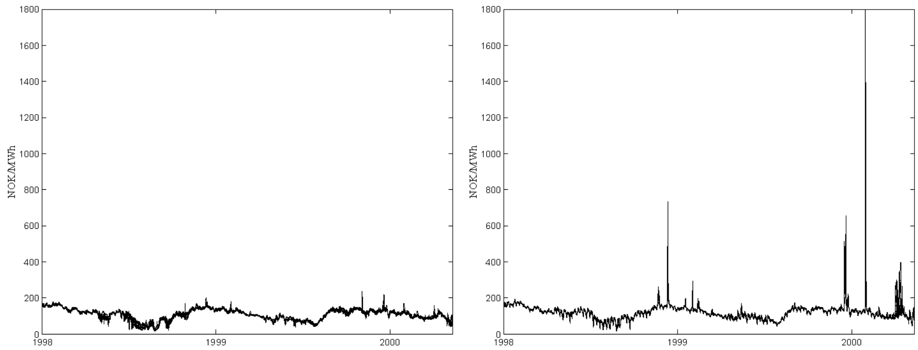
extra demand, i. e. when the supply stack is vertical, prices can be driven exceptionally high with no real connection to the marginal cost. Because of the typical shape of the supply stack which is non-decreasing and, except for the distortions when a new technology is brought online, convex in demand, we can conclude that the price derivative with respect to demand is increasing in demand

$$\frac{\partial S(D_1)}{\partial D} \leq \frac{\partial S(D_2)}{\partial D} \qquad \forall D_1 \leq D_2,$$

which is illustrated in Figure 2.16.

The lumpiness of the supply stack with jumps in the marginal costs as more expensive technology is dispatched, creates a need for a spot price modeling, which is not continuous, since the discontinuities in marginal cost are directly translated into spikes in the price. The SDEs presented so far does not capture this feature, since they all result in continuous price processes.

In the literature this phenomena is usually accounted by adding a jump component to a process driven only by a Brownian motion, such as (2.3). Johnson and Barz [JB98] propose the following model



(a) Off-peak hours (20.00-8.00) (b) Peak hours (8.00-20.00)

Fig. 2.17: Spot prices in peak hours and off-peak hours at Nord Pool from 1997-12-30 to 2000-05-06.

$$dS_t = \alpha\big(\mu_t - S_t\big)dt + \sigma S_t dW_t + \nu dq_t, \tag{2.5}$$

where the last part represents the unpredictable price jumps. Without specifying its process, q_t accounts for the frequency of the jumps, and ν is a random variable describing the severity of each jump.

Eydeland and Geman [EG00] propose a similar model, where the jump part is given by $US_t dN_t$. N_t is a Poisson process and the severity U is normally distributed. To account for the fact that jumps tend to be more severe during high price periods, such as during winter times, the random variable describing the severity of each jump could have a distribution dependent of time. In (2.5) that variable would then be given by ν_t. Figure 2.17, showing the spot price of each hour divided into off-peak and peak hours, clearly illustrates that jumps solely occur during peak hours, and Figure 2.14 shows that jumps only occur in peak months. Hence an intra-day and seasonal component should preferably be incorporated in the jump process.

Modeling the jump feature with a Poisson process has become the industry standard in the electricity market and is proposed also by [CS00] and [Kam97]. Modeling jumps in prices with a Poisson process was actually introduced already in 1976 by Merton [Mer76], although for the stock market. Empirical studies shows that the electricity price rapidly reverts to its normal level after a jump [CS00]. In the models mentioned above the price is forced back by the mean reversion after a jump, which according to [Bau99] is not fast enough, why he proposes a process, where a positive jump is always followed by a negative jump to capture the rapid decline of electricity prices after a jump.

Candidate models for electricity prices face two conflicting performance objectives. The models need to be detailed enough to capture the complex behaviour of electricity markets. At the same time these models need to be simple enough to be used for derivative pricing, but also for other tasks, such as simulation of prices and serve as input to stochastic optimization models. To make the price process more attractive from the second point of view many researchers have proposed models with no jump component, but with a stochastic volatility to achieve a more heavy tailed distribution, such as the model by [EG00]

$$\begin{aligned} dS_t &= m_1(t, S_t)dt + \sigma S_t dW_t^1, \\ d\sigma_t^2 &= m_2(t, \sigma_t^2)dt + y(t, \sigma_t^2)dW_t^2. \end{aligned} \tag{2.6}$$

The two Brownian motions W_t^1 and W_t^2 may be correlated and the terms $m_1(t, S_t)$ and $m_2(t, \sigma^2)$ may account for some mean reversion either in the spot price or in the volatility. For more information on electricity price models, see [Bar99].

2.10. Summary

The liberalization of electricity markets have created additional risks and new challenges for players in the market. The special nature of electricity in general, such as transmission issues, highly fluctuating demand and supply, and its non-storability in particular distinguishes the electricity market from other markets. The non-storability in combination with the volatile demand and the lumpiness and convexity of the supply stack creates volatile prices, and complex price models capturing mean reversion, seasonality and jumps are needed. In the case study presented in Chapter 7 we actually model the spot price with all these features. Despite its complexity, the spot price model in the case study serves as input to a stochastic optimization.

To deal with the uncertainty on electricity prices, a number of power exchanges have evolved. In combination with these exchanges a large number of very complex OTC contracts, typically with built-in options also on the underlying volume, are available. This complexity of the electricity market makes risk management a truly challenging task. Risk management will be investigated in the following chapter.

Chapter 3

Risk management

3.1. Overview

Risk Management is the theory about how to handle risks. The need for risk management comes with potentially risky and complex products, such as derivatives. The first standardized options were traded in 1973 when the Chicago Board Options Exchange began its operations. The same year the paper on valuation of options by Black and Scholes [BS73] was published, which helped to increase the number of derivative deals.

There are to date no operational valuation model of electricity derivatives based on a sound academic ground, such as the Black & Scholes model [BS73] for equity derivatives. Nevertheless, the deregulation and the, with it coming, uncertain prices has developed a need for risk transferring products like options and futures. The difference is that while the traditional finance industry had tens of years to develop their methods to handle risk, the electricity industry is facing at least the same risks, but has had only a few years to learn how to manage them.

In Chapter 3.2 a motivation and introduction to risk management is given. The risks in the electricity market are introduced in Chapter 3.3. Some traditional risk management models are investigated in Chapter 3.4. Their shortcomings and the need for a new risk measure are discussed in Chapter 3.5. Multi period risk management is briefly introduced in Chapter 3.6. Valuation models in general and electricity valuation models in particular are discussed in Chapter 3.7.

3.2. Motivation of and introduction to risk management

It is an established fact that in the financial markets there are no excess returns without risk, why a zero risk tolerance would result in zero excess returns [GL98]. To deliver on shareholders demand on return, risk must be taken, implying that some losses cannot be prevented. The goal of risk management is to monitor those risks and to contain those losses within pre-specified tolerances.

The numerous examples of firms that have suffered because of lacking risk management skills, like Barings, Metallgesellschaft and Orange to mention a few, show how costly it can be when risk management is not taken seriously.

A firm that does not have its risks under control will have a greater probability to default. If the firm would default it would cause problems for all parties that have connections with the firm, the so-called stakeholders. The bondholders could lose a substantial part of their lent money, the firm's counterparties could potentially lose built-up profits and the customers could suffer a lot of problems if their electricity supplier defaulted. In all cases the stakeholders will demand something for this uncertainty, a so-called risk premium. The bondholder will demand a higher interest rate, the counterparties will demand bigger spreads and better collateral of securities, and the customers better prices. Banks and other financial institutions are enforced by regulation to carry a certain amount of capital, depending on the risk level, to decrease the probability to default. Well functioning risk management procedures within the firm can avoid or decrease this costly risk premium[1] and hence improve the profitability.

Players in the deregulated electricity market are facing substantial risks. In such markets in Europe and in the US extremely volatile prices have been observed. This in combination with the complex contracts and the relatively

[1] Either in terms of the stake holder's demand for a risk premium or in terms of the low yielding regulatory capital.

immature markets has put electricity firms under pressure. Because of the uncertainty that arose along with the deregulation a number of utilities have already been downgraded by rating institutes like Standard & Poors. To mention a few European examples, the French incumbent, EdF and the Spanish incumbent, Endesa, were both downgraded in 1999. UNA in the Netherlands and TXU in the UK were also downgraded that year, whereas United Utilities was downgraded in 2000.[2]

In the third week of June 1998, the spot price in the Midwest soared to more than 7000 US$/MWh, whereas it usually trades at about 30 to 40 US$/MWh. Some market players were not able to meet their obligations and went out of business, which created difficulties for all players that were doing business with them. A number of electricity firms lost a lot of money during this price spike and in the US a number of utilities have defaulted and went bankrupt since the deregulation process started. A few utilities in California have been badly injured by the deregulation process, where a disastrous retail cap was enforced in combination with unlimited wholesale prices. California's largest utility, Pacific Gas & Electricity went bankrupt last year because of its inability to hedge against this odd one-sided price cap.[3] The bankruptcy of Pacific Gas & Electricity has raised concerns about the creditworthiness of other utilities in California, like Southern California Edison. Their counterparties have therefore increased their demand on collateral to decrease their credit risk.

The need for and importance of risk management in the electricity industry is thus indeed substantial.

3.3. The risks in the electricity market

The risks that players in the electricity market are facing can in our view be divided into two groups. The first part consists of the important risks that appear also in traditional financial markets. The second group is electricity- or at least commodity-specific risks in the sense that they do not play an important role in traditional financial markets.

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[2] Source: Reuters, Dow Jones.

[3] RISK Magazine, Vol 15/No 5 May 2001.

3.3.1. Traditional financial risks

Price risk - the risk to a firm's financial condition arising from adverse movements in the level or volatility of market prices.

Credit risk - the risk that a counterpart will fail to perform on an obligation.

Settlement risk - the risk that a firm will not receive funds or instruments from its counterparties at the expected time. Probably relative negligible in the electricity industry, since business are done between counterparts within the same or adjacent time zones and the settled money will therefore not be stuck in the bank system for long.

Liquidity risk - the risk that a firm may not be able to, or cannot easily, unwind or offset a particular position at or near the previous market price, due to inadequate market depth.

Operational risk - the risk that deficiencies in information systems or internal controls will result in unexpected loss. This risk is associated with human error, system failure and inadequate procedures and controls. This risk is relatively big in the electricity market because of the low knowledge of risk management in combination with the complex contracts.

3.3.2. Electricity specific risks

Volume risk - OTC contracts are often complex in the sense that options are built in. Some of these options refer to the underlying amount of energy. The seller of such a contract hence does not know how much electricity that he has to deliver. This uncertainty, known as volume risk is converted to a price risk as soon as the volume is known.

Basis risk - the risk that the price relationship between two or more traded commodities or price references will change. The basis is the difference between the commodity underlying the derivative used for hedging and the spot commodity that the company has to sell or buy. The basis risk can be further divided into three sub-groups.

Locational basis risk - the risk of a change in the price relationship between energy at a contracts delivery point and the actual point of energy consumption.

Cross commodity price risk - the risk that the price relationship of electricity to other commodities, such as natural gas or aluminum will change.

Cash/futures basis risk - the risk of the price of the futures contract deviating from the physical market price of electricity for the same time period. The lack of convergence between cash and futures contracts can be caused by an illiquid market.

Physical risk - the risk that electricity is not physically delivered or received as agreed due to capacity constraints or production and delivery problems. The economic effect to a chemical industry, for example, of an outage can be huge. An interrupted chemical process may have to be totally restarted with severe consequences on the production as a result. The value of lost load can exceed 75 $/kWh [WH97] to be compared with a typical electricity price of a few cents.

Regulatory risk - changes in value due to unexpected regulatory intervention. Examples of such interventions are the two-year price cap enforced 1994 in UK and the changed price cap from 250 $/MWh to 700 $/MWh in North Carolina in 1999.

Political risk - changes in value due to unexpected political decisions. The decision in 1999 to close down one nuclear reactor in Sweden is an example of such a political decision that affected the whole market. The long planning horizons of the electricity industry, like 30 to 40 years, mean that industry life cycles are much longer that the length of a government in office. Political uncertainty relates to the uncertain implications of changes in the government or policy legislation.

The five first risks; price-, credit-, settlement-, liquidity- and operational-risk are familiar from the financial markets. However, because of the highly volatile electricity prices, the price risk and hence also the credit risk can be argued to be even more severe in the electricity market. One could also argue that the basis risks are nothing but price risk for the whole portfolio. The physical-, regulatory-, and political-risk are, as we see it, far less important in the traditional financial market than in the electricity market today. Volume risk does not exist in the traditional financial markets, but as we know from Chapter 2.5 electricity end-users have a very variable consumption, why they and typically also retailers are exposed to volume risk. Many electricity contracts are still OTC and often tailored for each player. The liquidity of such contracts is naturally extremely low. The liquidity problem in the electricity markets is accentuated by the fact that there is not one global market, but several regional markets as an effect of the finite transmission capacity and the transmission losses. The liquidity consequently tends to be low relative to traditional markets also for standardized contracts. The immature nature of the electricity market as a deregulated market, where liquidity has not yet turned to the exchanges, but where the majority of the transactions still are made on a bilateral basis, makes it problematic to close a position fast. The liquidity risk in the electricity market is hence substantial. The immaturity in combination with complex contracts naturally also raises concern about the operational risk. The electricity market faces at least the same risks as the traditional financial market, whereas as mentioned the electricity market have had only a short time to learn how to handle these risks.

One could of course add a number of new risks to the list, which is more on a strategic level rather than on an operational level, such as technical risk. If a new technology makes it possible to transmit electricity at a much lower cost than today it would change the playground and markets would converge towards one more global market. One result would be that inefficient producers in high price markets would be out-performed by more efficient producers in low price markets. We do however not see this as an issue for the risk manager, but for the CEO and the board

Only price risk and volume risk are explicitly addressed in this work, even though, for example, credit risk and liquidity risk naturally also are of great concern to a player in the electricity market.

3.4. Traditional risk management models

Since the traditional financial markets have been exposed to financial risks for a long time, the most sophisticated risk management models were developed for these markets. The differences between the traditional financial markets and the electricity markets are as already mentioned many, but studying these risk management models can however give some insight into the challenges that the electricity market is facing.

3.4.1. Risk measures

Whereas deterministic problems are characterized by ’real’ numbers, such as costs or returns, stochastic problems are characterized by random variables. Naturally it is this stochastic variability that adds the risk component to the problem. We know that risk should be managed, but a natural question is how this risk should be measured. The simile taken from Gumerlock & Litterman [GL98] exemplifies the problem of how to measure risk:

> Leaving aside risk for a moment, consider the measurement of human size. Everyone knows qualitatively what *large* and *small* mean, but life gets more difficult when we want to express size in a single number. Either height or weight can be useful, depending on the problem being addressed. Each metric is appropriate for a given problem, and neither serves all purposes. Indeed, if one asks for a definite answer to the question of which metric is the best measure of size, the answer is that neither height, nor weight, nor a linear combination of them is the best measure of size. The best measure of size is the one most appropriate to the purpose for which it is intended.

As with human size, the same problem arises when measuring risk. Actually risk is too complex to characterize with one number. On the other hand, management calls for a simple measure of risk, which should be easy to interpret. One single number has therefore traditionally characterized risk. A typical risk measure developed for this purpose is *Value at Risk*. Another type of risk measure is *variance*, which however does not possess a natural interpretation from a management point of view. On the other hand, it has been derived from the utility maximizing agent theory as the risk measure that would concern such an agent, of course under certain assumptions.

We will here present the traditional risk measure, variance, the quantile based Value at Risk and the blunt stress test to give an idea of the variety of risk measures that are used in the industry. For more information on risk measures in general, see for example [Pfl99, Pfl01, ADEH97, BLS00].

Generally a risk measure is a measure of how much one could lose or how uncertain a profit or loss is within a given time-period, the so-called time horizon, in a subset of all possible outcomes.

3.4.1.1. Time horizon

The dispersion of the profit and loss distribution of a portfolio normally grows with time. The uncertainty on the value of the portfolio next week is typically larger than the uncertainty on the value tomorrow. A risk measure will therefore depend on the time horizon studied. The one-day risk will differ from the one-week risk. For a portfolio value process following a geometric Brownian motion (2.2), for example, the dispersion measured as variance will grow linear in time. For other value processes and indeed for other risk measures the risk will have a different time dependence.

Normally the time horizon, over which the risk is calculated, is given by the time it takes to liquidate a position. This is less than a day for most currency positions, a few days for interest and equity positions and longer for many credit positions. Certainly there are foreign exchange positions that are very illiquid and equity positions that can be closed intra-day, but generally the time horizon for these tree most traditional financial markets are of this magnitude. In the traditional financial markets the time horizon used in risk management is normally chosen to be between one and ten days.

In the electricity market with its liquidity issues the liquidation time will depend on, for example, the size of a position, not only on the type of position. This is certainly the case also in the traditional markets, but not to the same extent. It is therefore questionable to use one horizon for all positions within a portfolio. These issues are addressed by Nagornii & Dozeman [ND00], where they propose to add a liquidity risk component to the price risk, and state some interesting properties on the liquidity at Nord Pool, such as that the liquidity differs with time to expiration.

The liquidation time can be substantial for some OTC contracts and very long for power plants. We will later show that plants actually can be seen as contracts[4] and will have to be incorporated in the portfolio to determine the risk. The time to liquidate, for example, a nuclear plant, i. e. to sell the whole plant is difficult to estimate, but it will certainly be months or even years. The price risk in a plant could be substantially lowered through entering similar but inverse contracts in the market. Still, a significant open position would remain. A utility having a portfolio of plants, OTC contracts and standardized contract would face a difficult problem of choosing a common time horizon. In the traditional financial markets it is not seldom that different time horizon are chosen for FX, FI and equity positions. Though in the case of a utility such a differentiation would heavily penalize the plants with their tremendous liquidation times. It is questionable if this would give a reasonable risk assessment of the different positions. Either one would have to assign a very long time horizon to the illiquid positions, such as the plants or one would have to relax the connection between liquidation time and time horizon. We have in this thesis chosen to relax this connection and will work with a common time horizon over all positions. We are aware that the liquidity risk component will not be captured by this approach, but it is out of the scope of this work to further exploit liquidity risk, and we note that it is an area for further research.

3.4.1.2. Variance

Modern forms of risk quantification find their origins in the article by Markowitz [Mar52], where he published his work on the mean-variance portfolio problem. The portfolio variance is minimized subject to a constraint on the expected return

$$\begin{aligned} \min_{x} \quad & x^T V x \\ \text{s.t.} \quad & x^T R = R_p, \end{aligned} \tag{3.1}$$

where $V \in \mathbb{R}^{n \times n}$ is the covariance matrix $R \in \mathbb{R}^n$ is a vector of mean returns and $x \in \mathbb{R}^n$ is the weight in each position. One can note that V per definition as a covariance matrix is symmetric and positive definite. Hence $x^T Vx$ is strict convex. The linear constraint $x^T R = R_p$ defines a convex set and the problem

[4] See Chapter 4.

therefore has a unique solution and only the first order conditions need to be obtained. The famous diversification principle was here quantified by the non-perfect correlation between individual assets. The idea of not putting all your eggs in the same basket was not new, but Markowitz was the first to formalize and apply it to financial instruments.

It can be shown that no matter what assumptions are made about the distribution of asset returns, if an investor has a quadratic utility function, utility is increasing in the mean return and decreasing in the variance and that only these two first moments matter. Therefore quadratic utility maximizing investors will only choose portfolios according to the mean-variance portfolio problem [Arr71]. The quadratic utility however has the unrealistic property of satiation in the sense that the utility will decrease with wealth beyond a certain point. Further, the absolute risk aversion will increase with wealth meaning that the demand for a risky asset will decrease with increased wealth [HL88].

Chamberlain [Cha83] shows that the most general class of distributions that allow investors to rank portfolios based only on the first two moments is the family of elliptical distributions. In the electricity market returns are not elliptically distributed, because of the spikes observed in the prices, as indicated earlier, why the mean-variance portfolio is very questionable. Actually in any market a portfolio with options will have asymmetric distributed returns. Further variance, which is a symmetric measure of risk, where potential upside is penalized just as much as potential downside is not a preferable property for a risk measure.

3.4.1.3. Value at Risk

Triggered by the mentioned difficulties with variance, a number of so-called downside-risk measures have been proposed in the literature [Roy52, Mar59, Bav78]. Value at Risk (VaR) is the downside risk measure that is most widely used today. The reason for this is the simple interpretation of VaR and because it is the risk measure that the *Bank for International Settlements*[5] has enforced.

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[5] An organization of central banks.

Let $l(x, Y)$ denote the loss function of a decision variable $x \in \mathbb{R}^n$, which we can see as a portfolio, and a random vector $Y \in \mathbb{R}^m$ representing the future values of stochastic variables of interest, such as spot price or demand. The density of Y is denoted by f_Y.

We assume that $l(x, \cdot)$ is measurable and let $F(x, \cdot)$ for each $x \in X$ denote the distribution function for the loss $l(x, Y)$, $F(x, \alpha) = P(Y|l(x, Y) \leq \alpha)$, where X is a subset of $\mathbb{R}^n$ and can be interpreted as the set of available portfolios.

At a given confidence level $\beta \in (0, 1)$, which in risk management typically could be 0.99, the β-percentile of the loss distribution is called VaR.

Definition 3.1
VaR with confidence level β of the loss associated with a decision x is the value

$$\alpha_\beta = \inf\{\alpha|F(x, \alpha) \geq \beta\}.$$

In Figure 3.1 the 95% percentile of the profit and loss distribution $VaR_{95\%}$, together with $CVaR_{95\%}$, which will be defined soon, are illustrated.

VaR attempts to provide a meaningful answer to the important question

> How much are we likely to lose?

but actually only states that we are β% certain that we will not lose more than VaR_β in the given time horizon.

There are in the industry three traditional methods to calculate VaR; the *variance/covariance method*, *historical simulation* and *Monte Carlo simulation*. Further the by Embrechts et al. [EKM97] proposed *extreme value theory* approach has increased much attention from the industry in the last years.

Variance/covariance method In this approach the risk is calculated analytically based on the statistical properties of the risk factors. The assumption here is that the returns of the positions are multi normal distributed, why only the first two moments are of interest. This analytical method is therefore called

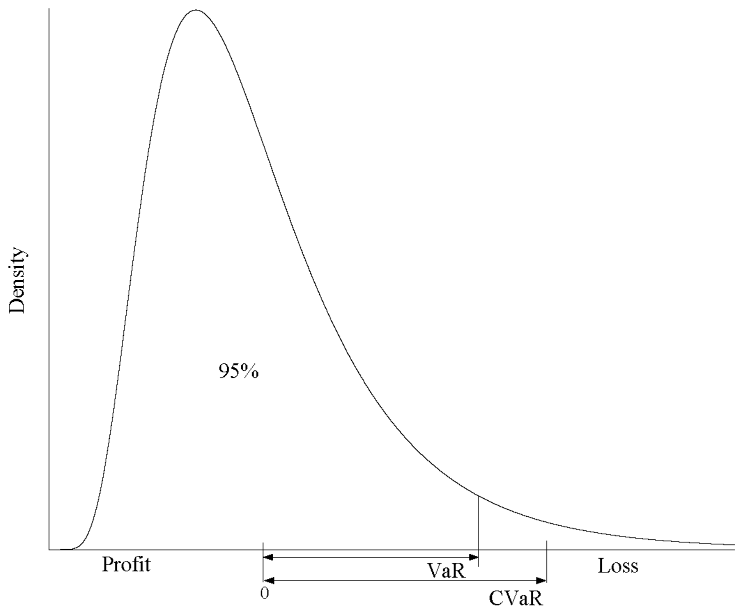

Fig. 3.1: VaR and CVaR with a 95% confidence interval for a log normally distributed loss function.

the variance/covariance method. The methodology Risk Metrics, developed by JP Morgan, is probably the best known.

The tractability of this analytical approach is its simplicity and its efficiency regarding computing time. Even for fairly complex portfolios VaR can be calculated in a spreadsheet. The problem is the assumption about normally distributed returns. This is fairly true for traditional linear products such as equities and bonds, but definitely not true for non-linear products such as options. In the electricity market it is however highly questionable if returns are normally distributed, even for the linear products, such as the electricity spot, why the analytical approach is not well suited for the power market.

Historical simulation The historical simulation approach makes no assumption about the distribution of the returns. Historical returns of the portfolio are taken as the distribution also for the future returns. The simulated value changes of the portfolio are then assembled into a return distribution and the quantile can easily be estimated. The historical simulation has the advantage of not being subjective when it comes to determining distribution. The disadvantage is that it requires very accurate historical prices of all positions in the portfolio, which in many cases do not exist. In the electricity market the majority of contracts are still OTC with no price history. The historical simulation is therefore not a good approach in the power market.

Monte Carlo simulation In the Monte Carlo approach a large number of scenarios are generated based on the assumptions made about the stochastic processes driving each position. The portfolio is then assessed in each scenario and the quantile, VaR is calculated. The approach is very flexible, but hence subject to subjectivity. It has the disadvantage of using large computer power. Because of the complex contracts and the complicated stochastic price process in the electricity market the flexibility of the Monte Carlo approach is however needed, why this approach seems to be best suited for electricity risk management purposes.

Extreme value theory In the three approaches mentioned above one always has the problem of estimating VaR outside the data sample for high quantiles. Extreme value theory can then be used to extrapolate outside the observed sample, to estimate the size of *a yet unseen disaster* [EKM97].

3.4.1.4. Stress test

The stress test is normally conducted as a compliment to the VaR calculation, but it can also be used as a stand-alone risk measure. First a potential movement within the time horizon in the underlying risk factors is defined. That could, for example, be a 200 basis point shift in the yield curve or a 30% shift in all equity positions or a foreign exchange movements of 15%. Then the loss that would occur given these shifts is calculated, which serves as an estimate of the risk in the portfolio. No dependencies are taken into account, why stress test is conservative and very blunt. The advantage is its simplicity, but it is strongly recommended that a more sophisticated measure should be used instead of, or in addition to, stress test.

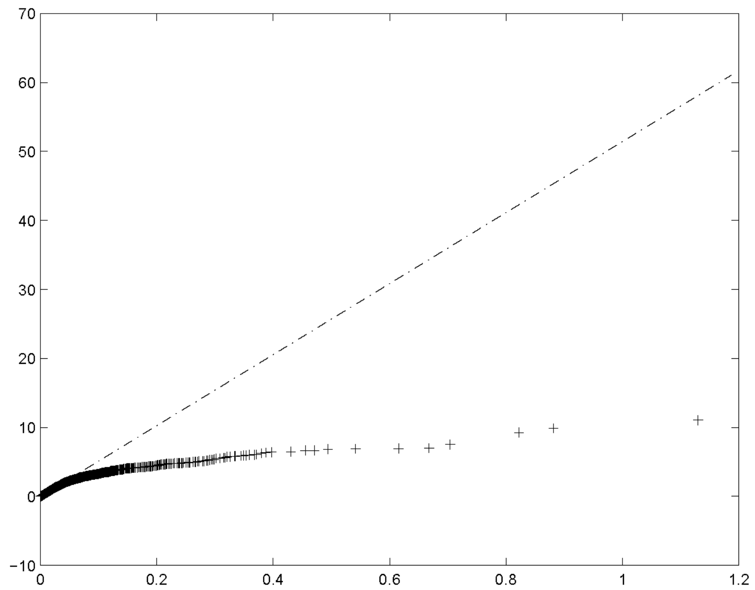

Fig. 3.2: QQ-plot of log returns of the Nord Pool hourly spot prices from 1997-12-30 to 2000-05-06, against the Exp(1) distribution, showing the typical curvature of a more heavy tailed distribution.

3.5. The need for a new risk measure

VaR has received much criticism for its inability to differentiate between a possible loss that is slightly bigger than VaR and a possible loss that is far greater than VaR. The reason for this is that for VaR the probability of losses is of concern, not the magnitude. This is especially crucial in the electricity market, where the jumps in the spot prices will cause the return distribution of a power portfolio to be non-normal and heavy tailed. The *heavy taildness* stemming from these jumps can be seen in the QQ-plot in Figure 3.2, where returns from the spot market is plotted against the medium-tailed exponential distribution. The probability of a large loss is consequently non-negligible and a risk measure that truly penalizes large losses is needed.

Further VaR is not a coherent risk measure in the sense of Artzner et. al. [ADEH97, ADEH99], as it lacks subadditivity, implying that VaR of a combination of two portfolios can be higher than the sum of the risks of the individual portfolios. VaR hence may violate the fundamental financial idea of diversification.

From an optimization point of view Mausser and Rosen [MR99] and McKay and Keefer [MK96] showed that VaR can be ill-behaved as a function of the portfolio positions and that VaR can have multiple local minimums, why a global optimum can be difficult to determine. Rockafellar & Uryasev [RU01] state that VaR is being unstable and difficult to work with numerically when losses are not normally distributed.

Basak & Shapiro [BS98] have shown that the market is destabilized when utility-maximizing investors also must manage a VaR constraint. They show that VaR investors often optimally choose a larger exposure to risky assets than non-VaR investors, and consequently incur larger losses, when losses occur. Because of the shortcoming of VaR of not being able to penalize a potential very large loss more than a large loss, they propose a risk management model based on the expectation of the loss rather than the probability of the loss.

As a result of the critics against variance for its symmetric features and against VaR for its inability to differentiate between large and very large losses, there is a need for a non-symmetric risk measure that penalizes large losses. One such risk measure with very promising features is *Conditional Value at Risk* (CVaR), which is closely related to Value at Risk.

In addition to the assumption made in Chapter 3.4.1.3 that $l(x, \cdot)$ is measurable, we assume that $l(x, Y)$ is integrable for each x, i. e. that $E[|l(x, Y)|] < \infty, \quad \forall x \in X$.

Definition 3.2
CVaR, with confidence level β, of the loss associated with a decision x is the value

$$\phi_\beta(x) = E[l(x, Y)|l(x, Y) \geq \alpha_\beta] = \frac{1}{1-\beta} \int_{l(x,y)\geq\alpha_\beta} l(x, y) f_Y(y) dy.$$

CVaR is as seen an expectation of the loss, conditioned that the loss is greater than or equal to VaR.[6] Unlike VaR, this risk measure has the desired property

[6] Which motivates the name, Conditional Value at Risk.

of focusing on the size of the loss and not just the frequency of losses. CVaR will therefore penalize large losses, which is a desired risk measure characteristic. CVaR is together with VaR illustrated in Figure 3.1.

Variance has a well-documented motivation as a risk measure, since it comes out as the natural risk measure from an expected utility maximization. Consider an investment choice based on such maximization with an investor specific utility function $u(\cdot)$. A portfolio with a random return X is then preferred to one with random return Z if $E[u(X)] \geq E[u(Z)]$. Variance however, as mentioned, comes out as risk measure only if the portfolio returns are elliptically distributed or if the utility is quadratic, where the appropriateness of a quadratic utility function is highly questionable. For elliptical distributions a CVaR minimizer will choose the same portfolio as a variance minimizer [RU00], while for asymmetric distributions a portfolio based on a true downside risk measure will be chosen. CVaR hence generalizes variance as a risk measure.

A further motivation for CVaR as utility derived risk measure can be given. Let U_2 be the commonly considered class of utility functions that are increasing (more is preferred to less) and concave (risk aversion). Levy [Lev92] has shown the following theorem connecting utility maximization and CVaR.

Theorem 3.3
$E[u(X)] \geq E[u(Z)], \quad \forall u \in U_2$ *if and only if* $E[X|X \leq \alpha_\beta(X)] \geq E[Z|Z \leq \alpha_\beta(Z)], \quad \forall \beta \in (0, 1)$.

A minimization of the conditional value at risk may not be achieved simultaneously for all $\beta \in (0, 1)$ with a single portfolio.[7] But Theorem 3.3 states that a portfolio chosen to minimize CVaR for a fixed β and a given mean is non-dominated, i. e. there exists no other portfolio with the same mean which would be preferred to the chosen portfolio by all investors with utilities $u \in U_2$. Bertsimas et al. [BLS00] state that one thus is naturally led to minimize CVaR for some β. As a matter of fact the CAPM[8] and the two-fund separation derived from the mean variance problem can actually be performed also for

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[7] Observe that $\beta E[X|X < \alpha_\beta(X)]$ is equivalent to $E[X] - (1 - \beta) \underbrace{E[X|X \geq \alpha_\beta(X)]}_{\phi_\beta(X)}$.

[8] Capital asset pricing model, see [Sha64].

the mean CVaR problem [BLS00].

Pflug [Pfl99] has shown that CVaR under certain assumptions is a coherent risk measure, which implies that it encourages diversification. Furthermore, CVaR has because of its convexity in the portfolio positions very tractable properties in terms of optimization [RU01]. A global minimum is relatively easy to find, as only the first order conditions has to be fulfilled. For more information on coherent risk measures in general and CVaR in particular, see [Del00].

3.5.1. CVaR as an appropriate risk measure in the electricity market

We believe that CVaR is an appropriate risk measure in the electricity market. The skewness in the distribution of profit and loss caused by the price spikes and the options built into basically any power contract makes an asymmetric risk measure that can really penalize extreme events, such as CVaR interesting. The tractable computational features of CVaR do indeed strengthen its position. Further the connection between a utility maximizing agent and CVaR is strong enough to motivate this risk measure over others also from a utility theory point of view. Hence we will use CVaR as risk measure in this work.

3.5.2. Calculation of CVaR

There are several ways to calculate CVaR. It can actually be calculated analytically for certain distributions. As with VaR, CVaR can be calculated using the variance/covariance method, where the assumption of normally distributed returns is made. For that case it can be easily shown that CVaR and VaR are simply given by a scaling of the standard deviation [RU01]. CVaR as risk measure is however motivated by heavy tailed distributions, as opposed to the normal distribution, and CVaR can actually be calculated analytically for heavy-tailed distributions with extreme value theory [EKM97]. This approach, called *peaks over threshold* (POT), is however not straight-forward and certain assumptions have to be made to derive the analytical CVaR. Most important is probably that this method currently only handles one-dimensional problems, implying that one has to have a historical data set, to estimate the parameters of the extreme value distribution, of the whole portfolio. This data is seldom available, especially not in the immature electricity market with its OTC contracts. One is thus left with no choices, but to work with a non-analytical estimation of CVaR. One can either work with historical simulation or Monte Carlo simulation to estimate CVaR. In the former case one, once again, faces the problem of lacking historical data and the only feasible approach is probably to simulate the value of the often very complex portfolio with Monte Carlo simulation, based on the chosen process for the underlying driver, e. g. a mixed diffusion and jump process.

Let $l(x, y_1), \ldots, l(x, y_J)$ be a sample of J losses of the portfolio x, ordered with increasing severity $l(x, y_1) \geq \ldots \geq l(x, y_J)$ and let $K = \lfloor (1-\beta)J \rfloor$. An intuitive estimator of CVaR with a confidence level of β proposed by [BLS00] is then given by

$$\hat{\phi}_\beta(x) = \frac{1}{K} \sum_{j=1}^{K} l(x, y_j).$$

3.5.3. Optimizing with CVaR

Measuring risk is a passive activity. Simply knowing the amount of risk does not provide much guidance on how to manage risk. Rather risk management is a dynamic process and it requires tools to optimize the utilization of risk. In this chapter we will describe such a tool, namely how a portfolio can be optimized using the risk measure CVaR. The approach was developed independently by Rockafellar & Uryasev [RU00] and Bertsimas et al. [BLS00].

With the motivation of CVaR as an appropriate risk measure in the electricity market it is natural to introduce the notion of an optimal portfolio x solving

$$\begin{aligned} \min_{x \in \mathbb{R}^n} \quad & \phi_\beta(x) \\ \text{s.t.} \quad & E[-l(x, Y)] \geq R, \end{aligned} \tag{3.2}$$

where the risk, measured as CVaR with a confidence level of β is minimized, subject to constraint, R on the expected profit. Or differently formulated

$$\max_{x \in \mathbb{R}^n} \quad E[-l(x, Y)] \qquad (3.3)$$
$$\text{s.t.} \quad \phi_\beta(x) \leq C,$$

where the expected profit is maximized, subject to a constraint C on the risk, measured as CVaR with a confidence level of β.

The function $\phi_\beta(x)$ is unfortunately in general not easy to handle, especially not when involved in an optimization problem. This problem can however be avoided by introducing the similar function

$$F_\beta(x, \alpha) = \alpha + \frac{1}{1-\beta} E\Big[\big(l(x, Y) - \alpha\big)^+\Big],$$

and by sharpening the assumptions needed to define VaR and CVaR, and also require that the distribution of Y does not depend on x and that $l(x, Y)$ is continuous in x. Under these assumptions [RU00] have shown the following two theorems

Theorem 3.4
$F_\beta(x, \alpha)$ is convex and continuously differentiable with respect to α, and $\phi_\beta(x)$ is given as the minimization of $F_\beta(x, \alpha)$ with respect to α

$$\phi_\beta(x) = \min_{\alpha \in \mathbb{R}} F_\beta(x, \alpha).$$

Furthermore the α that minimizes $F_\beta(x, \alpha)$ equals $\alpha_\beta(x)$

$$\alpha_\beta(x) = \textit{left endpoint of} \operatorname*{argmin}_{\alpha \in \mathbb{R}} F_\beta(x, \alpha).$$

The function $F_\beta(x, \alpha)$ cannot only be used to measure CVaR, it can also be used to manage a portfolio in a CVaR perspective, as seen in Theorem 3.5.

Theorem 3.5
Minimizing $\phi_\beta(x)$ with respect to $x \in X$ is equivalent to minimizing $F_\beta(x, \alpha)$ over all $(x, \alpha) \in X \times \mathbb{R}$ in the sense that

$$\min_{x \in X} \phi_\beta(x) = \min_{(x,\alpha) \in X \times \mathbb{R}} F_\beta(x, \alpha)$$

where moreover

$$(x^\star, \alpha^\star) \in \underset{(x,\alpha)\in X\times\mathbb{R}}{argmin}\ F_\beta(x,\alpha) \iff$$
$$x^\star \in \underset{x\in X}{argmin}\ \phi_\beta(x),\ \alpha^\star \in \underset{\alpha\in\mathbb{R}}{argmin}\ F_\beta(x^\star,\alpha)$$

A proof of Theorem 3.4 and 3.5 is given in [RU00]. If $l(x, Y)$ is convex with respect to x, then (3.2) and (3.3) can with help of $F_\beta(x,\alpha)$ be solved with convex programming, since both the objective function and the constraint are then convex.

The integral in $F_\beta(x,\alpha)$ can be approximated by sampling from the probability distribution of Y.[9] If we sample a collection of vectors $y_1, \ldots, y_J$ then $F_\beta(x,\alpha)$ can be approximated by

$$\tilde{F}_\beta(x,\alpha) = \alpha + \frac{1}{(1-\beta)J}\sum_{j=1}^{J}\big(l(x,y_j)-\alpha\big)^+.$$

If the loss function $l(x, Y)$ is linear in x, the optimization problem involving CVaR can be solved with linear programming. This is a very nice feature of CVaR, since linear programming can handle very large problems efficiently. The terms $\big(l(x,y_j)-\alpha\big)^+$ in $\tilde{F}_\beta(x,\alpha)$ are not linear, but piecewise linear. This can however be resolved by replacing these terms by the auxiliary variables z_j, and imposing the constraints $z_j \geq l(x,y_j)-\alpha$, $z_j \geq 0$, $j = 1, \ldots, J$. The optimization problem (3.2) can then be reduced to the linear program

$$\begin{array}{lll} \min\limits_{x\in\mathbb{R}^n, z\in\mathbb{R}^J, \alpha\in\mathbb{R}} & \alpha + \frac{1}{(1-\beta)J}\sum_{j=1}^{J} z_j & (3.4) \\ \text{s.t.} & z_j \geq l(x,y_j)-\alpha,\ z_j \geq 0,\ j = 1,\ldots,J & \\ & -\frac{1}{J}\sum_{j=1}^{J} l(x,y_j) \geq R, & \end{array}$$

and (3.3) similarly to

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[9] There is a whole theory on how to sample and generate random numbers. We have in this work used the Monte Carlo method, but there are a vast number of variance-reduction techniques, such as Latin hypercube sampling. For more on sampling see, for example, [Nie92].

$$\begin{array}{lll} \max\limits_{x\in\mathbb{R}^n, z\in\mathbb{R}^J, \alpha\in\mathbb{R}} & -\frac{1}{J}\sum_{j=1}^{J} l(x, y_j) & \\ \text{s.t.} & \alpha + \frac{1}{(1-\beta)J}\sum_{j=1}^{J} z_j \leq C & \\ & z_j \geq l(x, y_j) - \alpha,\ z_j \geq 0,\ j = 1, \ldots, J. & \end{array} \tag{3.5}$$

If X is a polyhedral set, i. e. if the constraints are built up from a set of linear inequalities, then the general constraint $x \in X$ can be added to the above problems without losing the linear feature. The two problems (3.4) and (3.5) deliver the same solution if C is chosen to correspond to R and they will both be used in this work. The former for determining hedging strategies and the latter for portfolio optimization.

3.6. Multi period risk management

The risk management models used in the industry basically all measure the risk today over a specified horizon. This horizon is, as mentioned, typically determined by the time it takes to liquidate the positions. In some industries, such as the electricity and the freight industry, this liquidation time can be substantial. Especially for long time horizons the question arises if one, by measuring the potential losses only at the end of the horizon, may underestimate the risk prior to the time horizon.

Let current time be t, the time horizon be T and the risk over the whole period be denoted by $r(t, T)$. More formally the question can be restated if the intermediate risks $r(t, \tau)$ for any $\tau \in [t, T]$ may exceed the end horizon risk $r(t, T)$.

One approach to cope with the non-static nature of risk and avoid to miss any information about the riskiness of a portfolio by only measuring the risk over the whole period $[t, T]$ would be to define the overall risk as

$$R(t, T) = \max_{\tau\in[t,T]} r(t, \tau).$$

It should be noted that for portfolios with normally distributed returns, (i. e. where the underlying price process follows a geometric Brownian motion)

$R(t, T) = r(t, T)$ for most risk measure, such as variance, VaR and CVaR, since variance grows linear in time.[10] For other underlying price processes, which are not stationary, as is the case in the electricity market with its seasonality and mean reversion, the statement made above about the overall risk $R(t, T)$ is not obvious anymore. In any case the measurement of the intermediate risks $r(t, \tau)$ can give additional information on how much capital that is needed in each period as insurance against adverse market movements.[11] The intermediate risks together with knowledge about certain cash flows affecting the capital base can give insight in how a firm best can utilize the risk during the horizon.

The risk models used today in practice are static in the sense that they by nature are one-periodic. The portfolio is assumed to be constant over the whole period. This type of model is not forward looking, since it does not care about how the risk evolves over time. One approach to investigate the dynamics of the risk, would be to study the process $r(\tau, T)$ for $\tau \in [t, T]$. It should be noted that the risk itself will be a random variable for $t < \tau$, which complicates the picture. Although a complex task, measuring the risk in a multi-periodic set up could add important aspects to risk management for financial portfolios in general and for electricity portfolios with their long liquidation times in particular. We will not go further into multi period risk management, but note that it is an area for future research. For more information on multi-period risk, see Artzner et al. [ADEH01].

3.7. Valuation models

There are two groups of valuation models to assess financial contracts. One group is the valuation models that use absence of arbitrage to value contracts in terms of other assets. The other group contains the valuation models that derive the value from an economic equilibrium.

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[10] Observe that VaR and CVaR for normally distributed returns are given by a scaling of the standard deviation.

[11] Corresponds to the regulatory capital in the banking sector.

3.7.1. Absence of Arbitrage

The *Arbitrage Pricing Theory* (APT) model was developed by Ross [Ros76]. The idea is to assume that the return on any security is perfectly correlated to a number of common factors. By assuring absence of arbitrage the security can then be priced in terms of these common factors.

In 1973 Black and Scholes [BS73] presented their option pricing theory, which revolutioned the derivatives pricing. They showed that the payoff an option, under certain assumptions, could be perfectly replicated with a continuously adjusted holding in the underlying asset and the risk free bond. Under the assumption of perfect and complete markets, given that the underlying price process is a geometric Brownian motion, they derived an analytical price formula of European call and put options. Since the risk of writing an option can be completely eliminated, the price formula is independent of the risk preferences of market participants.

More generally, under the assumption that the market is arbitrage free and that the interest rate r is constant, the time t price of any contingent claim is given by the discounted expected time T payoff X under the equivalent martingale measure Q (equivalent to P)[12] given the filtration at time t, $\mathcal{F}_t$, see for example [Nai95]

$$\Psi_t = e^{-r(T-t)} E^Q[X|\mathcal{F}_t]. \tag{3.6}$$

The market is arbitrage free if there exist no arbitrage strategies, where such a strategy in [Bjö94] is defined as

Definition 3.6
The portfolio x is an arbitrage strategy if its value process $\Psi_t(x)$ satisfies

$$\Psi_0(x) < 0, \quad P(\Psi_t(x) \geq 0) = 1$$

Definition 3.7
A market is complete if all contingent claims can be replicated

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[12] For terminology see [HP81].

If the market is complete then this probability measure Q is unique and a unique pricing formula that is independent on the utility functions of the different players, such as the B&S formula, can be derived. The probability measure Q is then often referred to as the risk neutral probability measure, since all contingent claims can be perfectly replicated and hence the risk can be eliminated.

In an incomplete market however the probability measure Q is not unique and a wide range of arbitrage free prices can be derived, depending on which measure that is chosen, as shown in for example [EJ97]. This non-uniqueness can be interpreted as if the utility function of the different players comes into play. A pricing model that does not involve the special risk aversion of the different players will therefore probably fail to give a precise assessment.

3.7.2. Equilibrium

An approach that does not make the assumption of complete markets is the family of equilibrium pricing models.

The *Capital Asset Pricing Model* (CAPM) was developed by Sharpe [Sha64]. Under the assumptions of normal distributed log-returns he shows that the equilibrium expected return of a security can be written as a function of the expected market return, the interest rate and a dependence measure between the market and the asset (β). Merton [Mer73] presented the *Intertemporal Capital Asset Pricing Model* (ICAPM), where he showed that if the investment opportunity set (interest rates, volatilities etc) is not constant, then the equilibrium model by Sharpe does not hold. He showed that for each stochastic factor in the opportunity set one additional term is added to the function describing the equilibrium expected return of a security. The *Consumption-based Capital Asset Pricing Model* (CCAPM) was developed by Breeden [Bre79]. The equilibrium expected return of a security is here given as a function of the expected market return, the interest rate and a dependence measure between the consumption and the security return.

Mainly because of the special characteristics of electricity, such as the non-storability, the electricity market differs substantial from the traditional financial markets. Still there are some resemblances to, in particular, the fixed income market. Like the fixed income contracts, the electricity contracts always have a lifetime greater than zero. Electrical energy is the time integral of the power, why no electrical work can be carried out unless power is delivered over a time interval. The shortest standardized power contract is the spot contract and the shortest fixed income contract is essentially the over-night interest rate. The pricing of fixed income products is not unique, since the price per unit of risk has to be exogenously determined [Vas77]. Because of the jumps, the volume uncertainty in many contracts and the transmission constraints, all electricity derivatives cannot be replicated, why the electricity market is an incomplete market [EG00]. The power pricing is consequently also not necessarily unique. Both the fixed income and the power market build up their expectations in forward curves, even though it is normally presented as a yield curve in the bond market. Some of the pricing models that are used by the power players hence have their roots as fixed income model. One such equilibrium model is the HJM model [HJM92], which extended has been proposed for the electricity market [End99]. As for all mentioned equilibrium models the assumption of a price process driven solely by a Brownian motion, however does not fit well with the jumps in electricity prices.

3.7.3. Valuation of contingent power claims

Since the electricity market is incomplete, the models based on absence of arbitrage will fail to give a unique price. Even though the equilibrium models are frequently used in the financial industry, they do not, because of their assumptions on simple underlying price processes, give us much operational guidance on how to price the complex electricity derivatives. Still the players in the electricity market need to price their contracts and a wide range of pricing models are used in the industry.

3.7.3.1. Commodity arbitrage approaches

As mentioned in Chapter 2.6, electricity is very different from traditional financial products. The electricity market therefore started to use pricing models developed for the commodity markets in general and for energy markets, such as the oil market in particular, despite the fact that electricity also differs substantially from other commodities.

Black - 76 In the electricity market the concept of being able to replicate options by continuously trading the underlying asset is unrealistic, since spot electricity is non-storable. Instead it was argued that the option could be replicated by trading futures on the spot. Futures naturally does not invoke any storage problem, because they are simply financial papers. Hence the analytical pricing formula by Black [Bla76] for options on forwards is frequently used in the electricity market [Won01]. The spot price is assumed to follow a simple GBM with a drift under the martingale measure Q equal to the risk free interest rate minus the so-called convenience yield[13]

$$dS_t = (r - \delta)S_t dt + \sigma S_t dW_t. \tag{3.7}$$

Schwarz The constant volatility assumption of the Black model is however not consistent with empirical observations that longer dated forwards are less volatile than short dated forwards. Schwartz [Sch90] introduced a more realistic model, where mean reversion is introduced in the spot price process

$$dS_t = \alpha(\mu - \lambda - lnS_t)S_t dt + \sigma S_t dW_t,$$

where λ is the market price of risk. The volatility of the forwards prices are then given by $\sigma_F(t, T) = \sigma e^{-\alpha(T-t)}$, which better describe the empirical results of commodity forward prices.

Gibson and Schwartz The Schwartz model (and the Black-76 model) is a single factor models, since only one factor is stochastic, namely the spot price. One of the deficiencies of that model is the simple volatility structure and its convergence to zero with increasing maturity. This can however be resolved

[13] Brennan [Bre91] defines the convenience yield as

> ...the flow of services which accrues to the owner of a physical inventory but not to the owner of a contract for future delivery

by introducing a second stochastic factor into the model. In addition to the GBM (3.7) model of the spot price, Gibson and Schwartz [GS90] introduces a stochastic convenience yield that is assumed to follow a mean reverting process

$$d\delta_t = \alpha_\delta(\bar{\delta} - \delta)dt + \sigma_\delta dW_t^\delta.$$

Despite the two stochastic factors, the model permits a high level of analytical tractability and analytical formulas for plain options can, as in the Schwarz model, be derived. However since convenience yield is not a traded variable, the prices of forwards or options from the model will depend on investor risk preferences. Philipovic [Pil98] proposes a two factor mean reverting model, where spot prices revert to a long term equilibrium, which in itself is stochastic.

There are several shortcomings of using these commodity models for pricing electricity derivatives. The major problem is the relationship between the spot market and forward prices used, where it is assumed that the futures prices satisfies a version of the cost of carry relationship[14]

$$F(t, T) = Se^{(r-\delta)(T-t)}. \tag{3.8}$$

This does naturally not hold for the electricity market, since electricity is non-storable, which precludes the cash-and-carry strategy.[15] Further the special characteristics of electricity mentioned in Chapter 2.9, like seasonality and severe jumps, are not modeled. The absence of jumps will result in a too thin-tailed distribution of the prices, which will underestimate the value of in particular out-of-the-money options.

3.7.3.2. Electricity fuel arbitrage approaches

Although electricity cannot be stored, the fuels used to generate it can be stored, which is used in the fuel arbitrage models to derive an electricity forward curve. In Chapter 2.6.1 we mentioned that the electricity market is tightly coupled

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[14] This relationship applies for the Black-76 model, whereas it becomes slightly more complicated for the other models.

[15] The cash-and-carry strategy involves buying the underlying (cash) and storing it (carry) to replicate a future.

with the energy markets in the sense that marginal production costs and hence electricity prices depend on the fuel prices. The marginal production cost and hence electricity price S^e in a thermal plant is essentially given by the price of the input fuel S^f multiplied with the efficiency of the plant, expressed as the heat rate H[16]

$$S^e = H \cdot S^f, \tag{3.9}$$

where it is assumed that the fuel costs are the only cost contributing to the marginal cost, that electricity is priced at its marginal production cost and that thermal plants are dominating the market and hence determining the price. A basic electricity forward curve can via (3.9) be derived from the fuel forward curve, which in itself can be derived from a cash-and-carry strategy, such as in (3.8). By assuming a constant value of the heat rate the shape of the electricity forward curve should resemble the forward curve of the input fuel. For more information on electricity fuel arbitrage, see [Leo97].

The assumptions made are however fairly strict and from Chapter 2.3 it is known that the fuel used will depend on the demand in the sense that for a low demand plants using cheap fuels, such as nuclear plants will be dispatched, whereas for a high demand expensive fuels, like gas will have to be used. The electricity forward curve should hence depend on demand and the supply stack and their evolution over time.

3.7.3.3. Pure electricity arbitrage approaches

The commodity based pricing approach fails to capture the specific characteristics of the electricity prices, such as jumps. The fuel arbitrage pricing approach, so far, does not capture the fact that the efficiency and fuel costs depend on the demand, why the jumps stemming from dispatching a new plant technology cannot be incorporated. To take these more realistic, but hence more complicated price processes, such as (2.5) into account, a different approach is needed. The price of a derivative can be derived by using the arbitrage free relationship (3.6). Because of the complex spot price process, typically no analytical price

[16] Heat rate is given by the number of units of fuel needed to produce one unit of electricity. In Chapter 4 the conversion process of fuel into electricity will be investigated further.

formula can be derived. Instead the price can be calculated by numerical methods and often Monte Carlo simulation is needed. The spot price is then sampled under the probability measure Q, the derivative payoff in each scenario j $X_{T,j}$ calculated and discounted, whereby the derivative's price Ψ_t is given by the expectation [CS00]

$$\Psi_t = e^{-r(T-t)} \frac{1}{J} \sum_{j=1}^{J} X_{T,j}, \tag{3.10}$$

assuming that J scenarios where sampled. The drawback of Monte Carlo simulation is the computational time and a trader typically needs immediate price information.

However the arbitrage free price in (3.6) and consequently (3.10) is because of the incompleteness of the electricity market not unique. The only constraint that can be put on the probability measure Q is that the arbitrage pricing formula should resemble the current market prices. This however leaves a large set of possible measures Q, why a wide range of derivatives prices can be obtained. By definition, in an incomplete market replication of certain contracts cannot be done, yet one could ask how can one replicate a payoff that is equal to or greater than a given contract's payoff in the cheapest possible way. Such a trading strategy is called a minimum cost strategy for a given payoff. The cost of such a strategy does according to [Nai95] give an upper bound on the price of the contingent claim. A lower bound can be derived similarly by taking the position of the buyer. For more information on pricing in incomplete markets, see for example [KQ95] or [DS94].

A weakness of using a pricing model based on absence of arbitrage in the electricity market is that it is based on the idea of building a replicating portfolio. This is however, because of the non-storability, not possible if spot contracts are considered as underlying. And we would like to stress that (3.6) and hence (3.10) only make sense if replication is possible.

3.8. Summary

The complex contracts, the limited liquidity, the immaturity of the market and the highly volatile prices makes the electricity market risky. Managing risks is of great importance in general and in the risky electricity market in particular, in order to decrease costs and to avoid losses and bankruptcy. Because of the special characteristics of electricity and the differences to traditional financial markets, a new risk measure is needed. We believe that CVaR is an appropriate risk measure in the electricity market and the utilization of CVaR can be optimized with linear programming. Pricing of electricity contingent claims is a challenging task and today there exists no operational pricing models that rely on a sound theoretical base, where the special characteristics of electricity is taken into account. Instead, the industry uses more or less motivated models to price their books. It is beyond the scope of this work to go further into pricing, but in the following chapter we will show how electricity derivatives can be replicated by physical assets and assessed internally.

Chapter 4

Contract engineering

4.1. Overview

Some power plants allow the owner to quickly change the output at low costs. Other plants however, does not facilitate this flexibility. As is the case with contracts, some give the owner flexibility, like an option. Other contracts lock in the owner and forces him to fulfill certain obligations, such as a future. This analogy can actually help us to view any production plant as a series of financial instruments [Tha00]. *Contract engineering* is a structured way to engineer contracts that can be hedged and assessed in terms of production.

Different plant types and their corresponding contracts are investigated in Chapter 4.2-4.6. Production outages and their related contracts are discussed in Chapter 4.7. Transmission assets are treated similarly in Chapter 4.8. The closely related theory on *real options* is introduced in Chapter 4.9. The value of different plant types is discussed in Chapter 4.10. The engineering of new contracts that through own production are risk manageable is introduced in Chapter 4.11.

4.2. Gas turbine

Unlike most other thermal plants, the gas turbine does not use steam to propel a turbine. Instead, the combustion of natural gas produces expanding gases, which are forced through a turbine, thereby generating electricity.[1] A gas turbine is typically a small scale plant with a capacity of a few MW up to some hundred MW. Some 14% of the worlds demand on electricity is produced with gas [HL00].

The gas turbine is, because of the direct link between combustion and propelling of the turbine, the most flexible plant and can be started up and closed down within minutes. The owner of such a plant therefore has an option in each period to produce or not. The marginal cost to produce electricity from a gas turbine is essentially the fuel cost. Efficiency differs between gas turbines, where efficiency is defined as the amount of gas needed to produce a certain amount of electrical energy. The industry notation for this efficiency is the so-called *heat rate*, which correspond to the number of *Btus*[2] required to produce one kilowatt of electricity. Thus, the lower the heat rate, the more efficient the facility. A typical heat rate for a gas turbine is 10.000 Btu/kWh [Kam97]. The marginal cost C_m for a gas turbine can therefore be expressed as the gas price S^g times the heat rate H

$$C_m = S^g H.$$

This marginal cost is high compared to other plants, why a gas turbine is used solely for peak load. The total cost to produce is naturally higher, since it also includes fixed costs, like capital costs and depreciations, but since the owner wants to spread these fixed costs over as many produced units as possible the optimal dispatch would be to produce as soon as the electricity price S^e is higher than or equal to the marginal cost $S^g H$. This is consistent with microeconomic theory stating that only marginal results are important for determining decisions [MCWG95], and if we denote the maximum capacity of the gas turbine with p_{max} [MW] then the optimal dispatch strategy x [MW] is given by

$$x(S^g, S^e, H) = \begin{cases} p_{max} & \text{if } S^e \geq S^g H \\ 0 & \text{otherwise} \end{cases}.$$

[1] Combustion of other types of fuels, such as light oil distillates or crude oil do occur, but natural gas is the preferred fuel.

[2] One British thermal unit (Btus) is the amount of energy required to raise the temperature of one pound of pure water by one degree Fahrenheit. This is equal to 3.411 Wh or 252 calories.

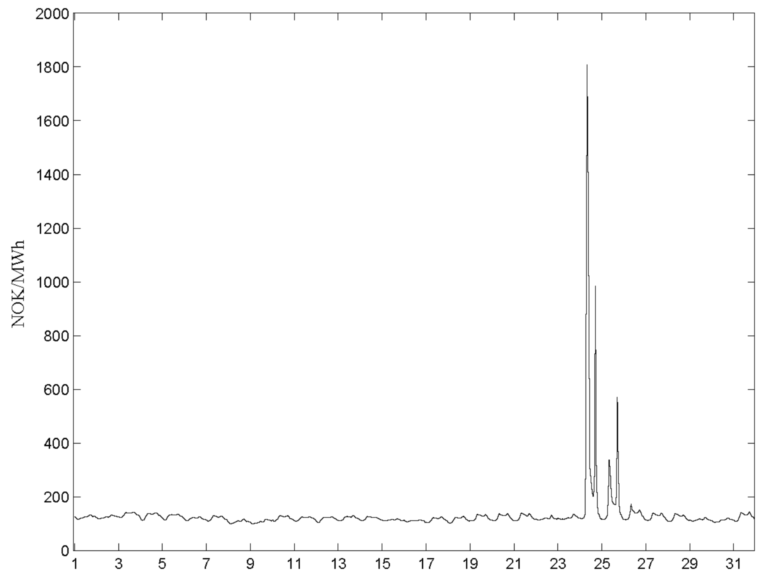

Fig. 4.1: Hourly prices at Nord Pool in January 2000.

In each period the payoff Π would then be $S^e - S^g H$ multiplied with the capacity of the plant, given that we produce, i. e. given that $S^e \geq S^g H$ and zero if we do not produce

$$\Pi(S^g, S^e, H) = p_{max}(S^e - S^g H)^+.$$

This payoff reminds us of a simple call option with the electricity spot as underlying and the marginal cost as strike price.[3] If we for simplicity assume that the gas price is constant, then the marginal cost would also be constant[4] and a gas turbine would in contract terms equal a series of call options on the electricity spot with the marginal cost as strike price.

Example 4.1 *To exemplify, assume that we had to buy a constant flow of 1 MW of electricity on the spot market at Nord Pool in January 2000. It turned out that prices spiked in the end of that month and during the 24th and 25th of January the spot price soared from its typical level of around 130 NOK/MWh up to 1800 NOK/MWh, as seen in Figure 4.1. This would not have been a pleasant*

[3] Compare with the payoff of a simple European call option in Chapter 2.7.

[4] Efficiency of a plant normally decreases with time, but the time frame is then typically years, why one for short horizons can assume that the heat rate is constant.

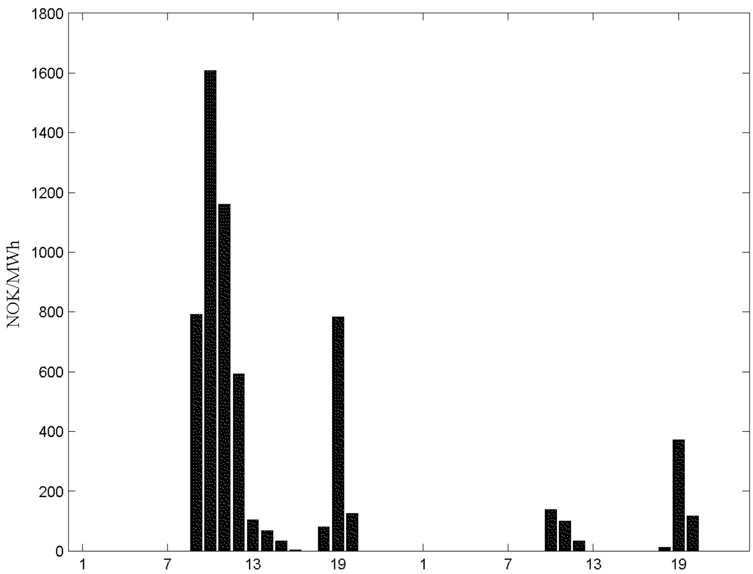

Fig. 4.2: Hourly payoff of the gas turbine, corresponding to the spot prices exceeding the marginal cost of 200NOK/MWh during the 24 and 25 of January 2000.

situation for a spot buyer, since the electricity costs would increase substantially due to these spikes. Assume further that we had a 1 MW gas turbine with a constant marginal cost of 200 NOK/MWh at our disposal. With the reasoning above we would then choose to produce instead of buying electricity, as soon as prices exceed 200 NOK/MWh. This month we would exercise 17 *of our* $24 \cdot 31 = 744$ *hourly options, namely during the hours with prices higher than 200 NOK/MWh, which is illustrated in Figure 4.2 showing the hourly payoffs of the gas turbine. Our actual cost for achieving electricity under that month would equal the spot price for all hours with a spot price lower than 200 and equal 200 for all hours with a spot price exceeding 200, i. e.* $\min(S^e, 200)$. *This corresponds to a so-called capped spot contract and in Figure 4.3 one clearly sees the capping effect of the gas turbine.*[5]

The problem with this reasoning is that the gas price is not constant, but rather very volatile. The volatile gas prices are exemplified in Figure 4.4. The marginal cost hence is stochastic, why also the strike price will be stochastic. The gas price can be fixed through, for example, gas futures. The problem

[5] See Chapter 2.7 for a description of a capped contract.

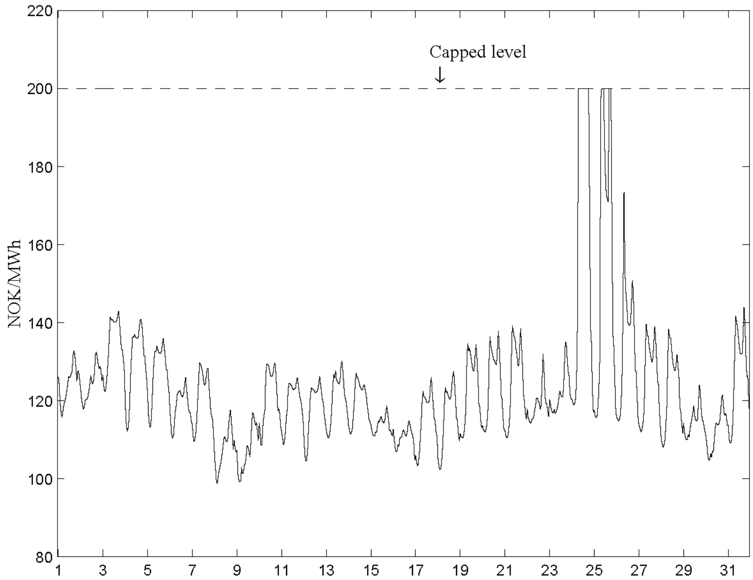

Fig. 4.3: Capped spot price as the effect of gas turbine at Nord Pool in January 2000.

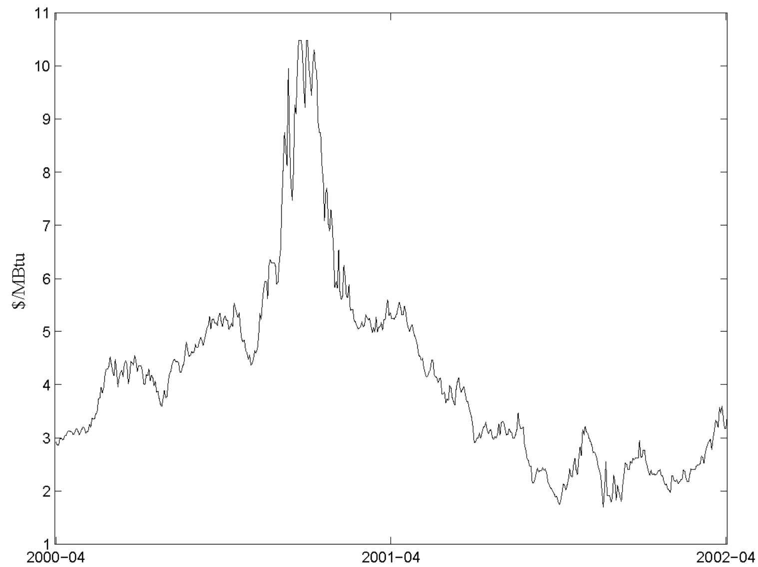

Fig. 4.4: Gas spot price at Henry Hub, Louisiana, April 2000 to April 2002.

however is that the quantity to buy in the futures market is unknown. It will depend on how often the electricity price exceeds the, by the gas price implied, marginal cost, and since the electricity price is stochastic also the demanded gas volume will be stochastic.

A stochastic strike price is not a desired property when identifying a plant's corresponding contracts, rather one wants to allocate all the stochastics in the underlying process. This can actually be done, since a gas turbine essentially is a process where natural gas is 'refined' to electricity, i. e. gas is changed for electricity. Hence, in each period we have the option to change gas at a value of $p_{max}S^gH$ for electricity at the value of $p_{max}S^e$ at zero costs. Each such option is called a spark-spread[6] option, why a gas turbine equals a series of such spark-spread options. With the same reasoning as above this can also be seen as a cap on the spark spread, defined by $S^e - HS^g$, where the capped level equals zero.

A plant with increasing popularity is the combined cycle gas turbine (CCGT), which uses the high temperature of the gas turbine exhaust to boil water. The steam is then used to propel a steam turbine. The efficiency is increased compared to a pure gas turbine, but the flexibility suffers from the same disadvantages as the coal plant and the oil plant that are discussed in Chapter 4.4. Like the gas turbine, the CCGT is associated with relatively short construction times, which is an important advantage over other plant types. The CCGT plant is in general larger than the gas turbine and typically has a capacity of 50-500 MW.

4.3. Hydro storage plant

A hydro storage plant uses the potential energy of water stored in a reservoir, typically located on high altitudes in the mountains. By utilizing the difference in altitude between the reservoir and the turbines, this potential energy is converted into mechanical energy by letting the water, under high velocity, propel the turbines. In the generators this mechanical energy is converted into electrical energy. Some of the hydro storage plants also have pumps,

[6] Spark spread denotes the difference of the price of electricity and the price of the gas needed to produce it.

facilitating the opposite process, where water from a lower located lake is pumped up to the reservoir. Hydro plants, given by hydro storage plants and run river plants,[7] cover some 19% of the world's demand on electricity [HL00].

The hydro storage plant is flexible, within certain limits, depending on the possibility to change the level of stored water. In some countries, e. g. Sweden, these possibilities are restricted for environmental reasons. The marginal cost to produce electricity from a hydro storage plant is very low, since production is not subject to any fuel costs. The marginal cost is low enough to motivate selling base load power. On the other hand, since the stored water is limited an owner of such a plant would try to sell this power during peak hours to maximize the profit. The more water that is available, the more the storage plant will be used for base power and the more limited the water is the more the plant will be focused on delivering peak load.

The flexibility is a result of the possibility to change output essentially immediately at no cost. The owner of such a plant consequently has the option in each period to produce or not. And if pumps are installed the owner also has the option to pump up water, i. e. to convert electrical energy back into potential energy. It is thus natural to identify options as the corresponding contracts. These options are however far more complex than those corresponding to the gas turbine and because of the inherent complexity of the hydro storage plant we need to model it more formally to understand its corresponding contracts.

The stored water is kept in a so-called storage dam and new water is floating into the dam from precipitation and melting snow. This inflow is random and normally very seasonal dependent. In countries with cold winters like Switzerland or Sweden the inflow is limited during the winter months, since precipitation gets stuck as snow and ice in the mountains, whereas the inflow is abundant in the spring months because of melting snow.

The total amount of energy stored in the dam at time t is denoted by V_t [MWh], the inflow rate of water to the dam at time t by i_t [MW], the spill over rate at time t, caused by heavy inflow in combination with a full dam, by l_t [MW] and the turbined power at time t by p_t [MW]. Notice that this variable can be

[7] Run river plants are discussed later in Chapter 4.6.

negative if pumping is possible.

The rate of change in the stored water level at time t can be written in the following difference equation

$$dV_t = \big(i_t - l_t - p_t\big)dt$$

and the amount of stored energy is consequently given by

$$V_t = V_0 + \int_{s=0}^{t} i_s ds - \int_{s=0}^{t} l_s ds - \int_{s=0}^{t} p_s ds, \qquad (4.1)$$

where V_0 is the initial amount of stored energy. Note that the stored energy in the dam V_t is expressed in the corresponding electrical energy that can be generated. All losses in, for example, the turbines and pipelines are already taken into account. This means that stored energy is not equal, but less than the *potential energy* of the water given by $h \cdot m \cdot g$, where h is the difference in height [m], m is the mass of the stored water [kg] and g is a constant of around 9.81 [m/s^2]. Further, for a given amount of water in a given hydro plant, the stored energy will be altered when, for example, old generators are changed for new more efficient generators.

Electricity is however not traded on a continuous time axes, but on a discrete axes, where the shortest period to trade electricity normally is an hour.[8] We will therefore work in a discrete set-up and the discrete version of the energy equation (4.1) is given by

$$V_k = V_0 + \sum_{i=1}^{k} I_i - \sum_{i=1}^{k} L_i - \sum_{i=1}^{k} x_i, \qquad (4.2)$$

where I_k is the inflow of water in period k [MWh], L_k is the spill over loss in period k [MWh], x_k is the turbined or pumped energy in period k [MWh] and V_k is the stored energy at the end of period k [MWh], as illustrated in Figure 4.5. The reason for changing p for x, for turbined or pumped energy, is because that will later on be our decision variable in the so-called static

[8] See Chapter 2.7.

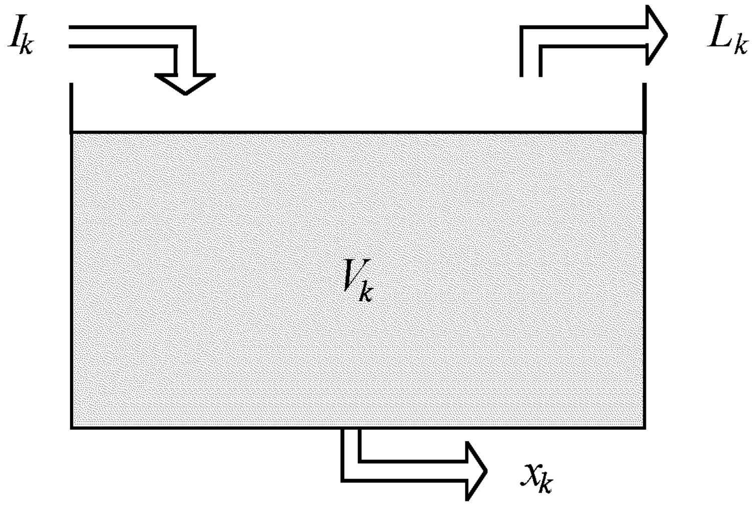

Fig. 4.5: Illustration of a hydro storage plant in discrete time.

dispatch strategy[9] and since the optimization later will boil down to a LP, the standard notation, namely x for the decision variables is introduced already here.

The boundaries on x_k is determined by the technical characteristics of the hydro storage plant, such as height of water column, dimension of water pipes, pumps, turbines etc. The maximum capacity of the turbines and pumps is denoted p_{max} and p_{min} respectively as expressed in (4.3), where we state that the dispatch must lie within the technical limitations in each period

$$-p_{min} \leq x_k \leq p_{max}, \quad \forall k. \tag{4.3}$$

In reality p_{max} and p_{min} are actually functions of the water level and hence V. The water column and hence the velocity of the water stream hitting the turbine blades is dependent on the water level. This dependency is however negligible for a typical hydro storage plants in, for example, Switzerland, where the water column down to the generators and the pumps normally is a few hundred meters, whereas the dam water level can differ with some tens of meters.

[9] See Chapter 6.4.5.1.

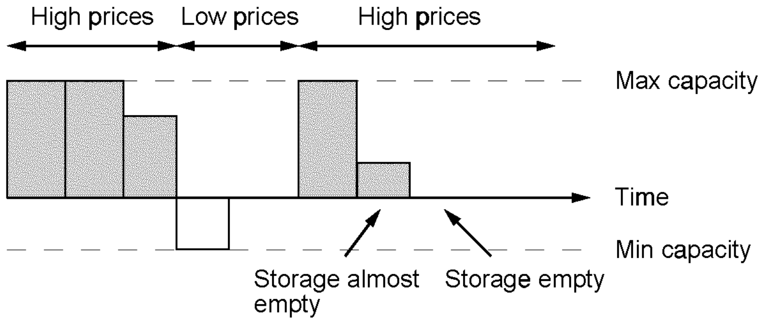

Fig. 4.6: Illustration of the dispatch of a storage hydro plant.

The physical constraint that we cannot go 'short' in water is expressed as the probabilistic constraint

$$P(V_k \geq 0) = 1, \quad \forall k, \tag{4.4}$$

since the stored water is a random variable.[10] The owner of a hydro storage plant has, in each period, the option to produce at a marginal cost, which is very low. Since the electricity prices will basically always be higher than the marginal cost, one would like to produce in every period. The resource, the stored water, is however scarce, why the dispatch decision will be far more complex. By exercising an option the water level decreases. And since the water inflow is stochastic, the probability that an option in the future can be exercised

$$P(V_{k'} \geq 0 | x_k > 0), \quad k' > k$$

decreases. On the other hand, by pumping up water, this probability increases as the water level rises. The decision made today about producing or pumping and how much, will thus affect the future possibilities to dispatch the plant. Figure 4.6 tries to visualize this interdependence between the options, where an extensive production in the beginning leads to an empty storage later on, which preclude the option to produce. A hydro storage plant hence equals a series of interdependent options. This corresponds to a swing option, where the maximum capacity of the turbines and the pumps respectively corresponds to the upper and lower limit on the load in the swing option, and the stored water

[10] Observe that V is stochastic, since the inflow I, is stochastic.

plus the stochastic inflow in the dam plus the pumped water is corresponding to the swing option's maximum energy. When pumping capacity is taken into consideration, the swing option can be reverted, but at a higher 'price', since the pump efficiency is lower than 100% in transforming electrical energy into potential energy.

V has a natural lower limit of zero as expressed in (4.4), but one could here introduce a positive minimal water level to account for environmental constraints. The stored water also has an upper limit V_{max} determined by the size of the dam, expressed by the following probabilistic constraint

$$P(V_k \leq V_{max}) = 1, \quad \forall k. \tag{4.5}$$

The similarities between a hydro storage plant and a swing option becomes more apparent when comparing the limitations on power and energy in the hydro storage plant (4.3), (4.4) and (4.5) with the definition of a swing option (2.1).

The efficiency of a pump χ is defined as the ratio between pumped energy in terms of water and used electrical energy. A typical efficiency for a hydro pump is 70%. The effect on the electricity balance will therefore not be the same if water is turbined or pumped. In the first case the effect on the electricity balance will be exactly x, whereas in the latter case x/χ. Because of this asymmetry we divide x into two part as

$$x_k = x_k^+ - x_k^-, \quad x_k^+ \geq 0, \quad x_k^- \geq 0, \quad \forall k,$$

where x_k^+ and x_k^- denote production and pumping respectively in period k. The dispatch will however be written only as x, unless the differentiation between the two parts is explicitly needed.

The uncertainty on the available water for a utility with several hydro storage plants can be reduced if the inflow into the different dams is not perfectly correlated. This diversification effect is typical for a portfolio consisting of hydro storage plants located both in the northern and southern part of Switzerland, because of the relatively low dependence between the weather in the two parts.

4.4. Coal plant and oil plant

The coal plant and the oil plant are two thermal plant types, where the fuel, coal or oil is burned to boil water. The steam from the boiled water is then used to propel a turbine, driving a generator to produce electricity. Coal plants are typically large scale units with a capacity between 250 and 1000 MW and coal plays a major role as a source of energy and provides some 39% of the world's electricity. Oil plants are in general smaller and some 11% of the world's electricity is produced with oil [HL00].

The coal plant and the oil plant are flexible to some extent. The output can be varied[11] with a stable operation within the range of 25-100% of the maximum capacity [CIAB96]. Thermal losses however occur in the boiler when the plant is shut down or when output is decreased. Further, there are costs and lead times associated wit start-up, since a large amount of water has to be heated to a boiling point to get steam to propel the turbines.

The marginal cost is, as in the gas turbine case, mainly attributed to fuel costs. Since coal and oil is cheaper than gas, the variable costs are lower than these for a gas turbine, but higher than, for example, these for a nuclear plant. The coal plant and the oil plant are because of their intermediate marginal costs not typical base load or peak load units, but rather something in between, why they are normally referred to as intermediate load facilities.

The price of the fuel used, coal or crude oil, is as the gas price stochastic. The volatility is however lower. One may be tempted to take the same approach as for the gas turbine, arguing that the plant in each period corresponds to an option to exchange the fuel used, coal or oil, for electricity at a given price, determined by the efficiency of the plant. The lower flexibility of the coal plant and the oil plant, caused by the slower response time by start-up and the thermal losses by shut-down, however complicates the picture.

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[11] Changes of load (ramping) are typically 3-5% per minute, but can be up to 8% [CIAB96].

4.4.1. Start-up times

Starting with the start-up time, the problem arises when the decision about producing or not has to be taken about a period, for which no spot price yet exists. The bidding process is terminated and the spot prices for the next day are communicated at 15.00 at Nord Pool. Hence a utility has first at 15.00 knowledge about the next day's spot prices. As long as the start-up time is shorter than the time between the disclosure of the next day's prices, and the first production hour the following day (24.00), then it does not affect the flexibility of the plant. But if the start-up time is longer than these roughly 9 hours (24-15), some of the optionality is lost.[12] The value of an option comes from the fact that the holder can wait with his exercise decision, until he has the information about the underlying price, in this case the knowledge about the spot price and hence the payoff.[13] If the start-up time is longer than these 9 hours we would have to decide whether to start up the plant or not, i. e. exercise the option for some hours next day, without knowing the corresponding spot price. The longer prior to the exercise day that one has to make the decision, the less precise the price forecasts will be. Hence, the longer this gap is, the more of the option value will be lost and the more the corresponding contract turns into a future. In the limit, when this gap goes to infinity all option value is lost and the plant will correspond to a series of plain futures.

To exemplify, assume that a plant has a start-up time of 14 hours and that the prices for the coming day are disclosed at 15.00. The owner of the plant has to decide latest at 10.00 (24-14) whether to produce, or not, in the first hour next day without knowing the spot price for that hour. Latest at 11.00, whether he will produce or not in the second hour next day, also without knowing the spot price. The dispatch decision can be taken with a known spot price first for the sixth hour the next day. For a start-up time greater than 33 hours the dispatch decision for all periods the coming day will have to be taken without a known spot price. Entering hourly futures could eliminate this uncertainty, since the producer would have access to a market price far earlier than when the spot

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[12] A typical coal or oil plant requires between 4 and 8 hours for a cold start-up, but it can be longer [CIAB96].

[13] Of course the information about the fuel price is also of great importance. In the case of coal and oil the fuel is however typically bought in advance and stored at cite, why this uncertainty is hedged through storage.

price is available. The problem is that such short futures are not offered in Europe, the shortest futures now available are the daily futures.

4.4.2. Shut-down costs

The shut-down costs, in form of thermal losses, also causes deviations from the option feature. One can handle the shut-down cost as a fixed cost that has to be spread out on the periods in one connected production phase, adding up the marginal cost. The longer the production phase is, the lower the additional marginal cost.

Let us for simplicity assume that the start-up time is short enough to base the production decision on known spot prices. Then, as long as the spot price S, in one period is higher than the sum of the marginal cost C_m, and the shut-down cost C_{sd}, i. e. a spot price higher than the one-periodic total marginal cost, the decision is trivial, namely to produce. And if the spot price is lower than the marginal cost the decision is also trivial, namely not to produce. The problem arises when the spot price lies within the open interval $(C_m, C_m + C_{sd})$. Then the question arises whether to produce or not, i. e. to exercise the option or not. If the production would last for only one period, the total marginal cost C_{mTot} would equal $C_m + \frac{C_{sd}}{1}$, which obviously exceeds spot prices in that interval. It is therefore not economically defendable to produce only in one period. A production phase with at least two periods is clearly needed. The total marginal cost given that the production phase lasts for n periods is $C_{mTot} = C_m + \frac{C_{sd}}{n}$.

In the existence of shut-down costs, the producer may have to take a decision about the dispatching based on spot prices in periods where they are unknown. This will, for example, be the case if the production phase extends over the next day, where the spot prices not yet are determined. As with start-up times, shut-down costs will destroy some of the option value in the plants, because of the inability to make the exercise decision with perfect information.

Since the shut-down cost forces us to extend the production phase, our decision is not anymore whether to produce in a specific period or not, but rather whether to produce in one *and* its following say $n - 1$ periods. This corresponds in this contract engineering framework to a series of fewer but longer options. Per day, for example, the plant does not anymore correspond to 24 hourly options, but maybe to 2 twelve-hourly options.

The option payoff is asymmetric in the sense that the upside is almost unlimited while the down-side is limited. Fewer but longer options will therefore always be less (or equal) valuable, which is illustrated in the example below.

Example 4.2 *The spot price in period 1 is 50 CHF/MWh and in the second period 30 CHF/MWh. The marginal cost to produce is 40 CHF/MWh, which is assumed to be independent of the production phase length. The capacity of the plant is 1 MW.*

Case 1. The plant corresponds to two one-periodic options *In the first period the spot price is higher than the marginal cost so we decide to produce and achieve a positive payoff of 50-40=10 CHF. In the second period we decide not to produce, since the spot price is lower than the marginal cost with a natural payoff of zero. The total payoff is 10 CHF.*

Case 2. The plant corresponds to one two-periodic options *The 2-hour price, i. e. the average spot price over the two periods is 40 CHF/MWh, which is not exceeding our marginal cost. We decide not to produce with a total payoff of zero.*

In the first case we could benefit from the high price in the first period without suffering from the low price in the second period. In the second case we could not make this selection. We had to choose to be exposed to the upside and downside of both periods. The alternative is not to produce.

The example also illustrates the importance of using a fine discretization, corresponding to the length of the spot contract when modelling flexible plants. Otherwise the operational flexibility will be underestimated. A cruder discretization would naturally correspond to Case 2, where some of the option value is lost.

The decision whether to continue to produce or not is somewhat different. If the spot price exceeds the marginal cost, we should keep on producing, since the total marginal costs decreases, while we have a positive additional cash flow in that period. Also when the spot price equals the marginal cost we should continue, even though it does not contribute to a positive cash flow, since that keeps our option open for the coming periods. Even in the case where $S^e < C_m$ it is not clear that one should quit producing. A trade-off between having to take an additional shut-down cost and inquiring a negative cash flow of $S^e - C_m$ has to be done.

To summarize, there are two complications with the coal plant and the oil plant, compared to the gas turbine. The first one is the start-up time, which may force us take the dispatch decision without knowing the prices. This destroys some of the plant's option value and shifts the plant in our contract engineering framework towards a future contract. The second complication is shut-down costs, which forces us to produce, not in one-periodic buckets, but in multi-periodic buckets. This feature also destroys some of the option value and may also force us to make a dispatch decision based on uncertain spot price forecasts. The exercise conditions, when to produce or not, are therefore not explicitly known. The decision to continue to produce or not will also differ from the decision to start a production phase or not.

By assuming that the start-up time is short enough and that the shut-down costs are negligible, the coal plant and the oil plant equal a series of hourly call options with the marginal cost as strike. If the fuel price is not hedged through, for example, storage then the plants equal a series of call options on the fuel electricity spread, as is the case for the gas turbine. For oil plants these spread contracts were developed already in the 1980s by Morgan Stanley [Tha00].

4.5. Nuclear plant

In the nuclear plant uranium fissions, producing heat in a continuous process called a chain reaction. As in fossil-fuel thermal plants, the heat is used to produce steam, which spins a turbine to drive a generator, producing electricity. Nuclear plants are typically large scale production units with a typical size of 1000 MW. Over 16% of the world's electricity is generated from nuclear plants [HL00].

The nuclear plant has very low marginal costs, which typically amounts to a third of the marginal costs in a coal plant and between a fourth and a fifth of those for a gas combined-cycle plant [OEC98].[14] The flexibility is unfortunately very low, caused by the high costs associated with varying the output. There have actually been occasions in, for example, France when EdF has been willing to sell nuclear produced electricity at negative prices due to these high costs. Nuclear plants are because of the low marginal costs and low flexibility used for base load. Not only are nuclear plants subject to the start-up times and shut-down losses that coal and oil plants suffer from, they are also subject to a much more complex heating procedure. Whereas a coal plant or an oil plant fairly easy can change output by simply regulating the amount of fuel and air, the fission reaction is more sensitive to quick ramp ups and ramp downs.

The low operational flexibility, forcing the plant to run over long periods of time, implies that the owner has to take a decision whether to produce or not in the future where the spot prices are uncertain. However, a series of long future contracts would perfectly replicate this stable amount of produced electricity, which consequently are the nuclear plant's corresponding contracts. But even if the flexibility would be high and the output could be changed instantly, the option value would be small. Assuming a high operational flexibility, the owner would in each period have the possibility to decide whether to produce or not and the plant would hence equal a series of hourly options. Since the marginal cost and hence the corresponding option's strike price is so low, the decision would however essentially solely be to produce. The options would normally be deep in-the-money,[15] since the spot price by far would exceed the low marginal cost. A possibility to not produce would consequently have a low value. Generally, the more an option is in-the-money, the more the asymmetric payoff is lost and therefore more of the option features are lost,[16] which is il-

[14] Marginal cost is here defined as the total fuel cost, including waste management.

[15] A call option is out-of-the-money if the underlying price S is lower than strike price K and vice versa for a put option. An option is at-the-money if $S = K$. A call option is in-the-money if $S > K$ and vice versa for a put option.

[16] One important option feature is the possibility to achieve high returns with only mod-

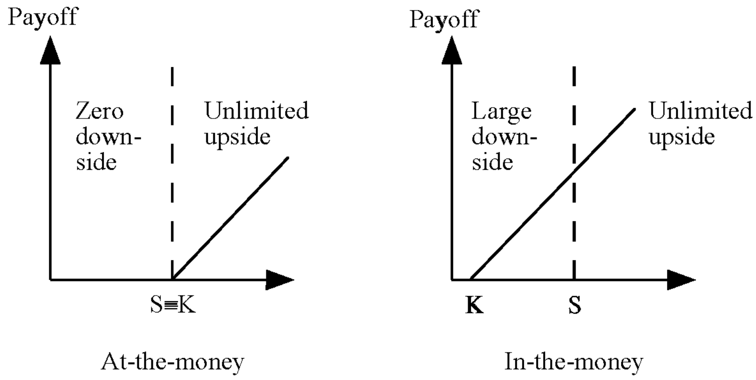

Fig. 4.7: Difference between at- and in-the-money options.

lustrated in Figure 4.7. A deep in-the-money option is however of course more valuable than an at-the money-option and the reason why nuclear plants have not outperformed other plant types are, except for safety concerns, their very high capital costs. These are typically more than double as high compared to those of a peak plant, such as a gas turbine [OEC98]. In financial terms this would correspond to the well known fact that the more in-the-money an option is, the more expensive it should be.

4.6. Run river plant, wind plant and solar plant

The procedure how electricity is generated differ substantially between the run river plant, the wind plant and the solar plant. The run river plant uses the potential energy of the water in a river, the wind plant uses the kinetic energy of the wind and the solar plant take advantage of the sun's radiation to produce electricity. However, the characteristics of these plants from a contract engineering point of view are very similar. The advantage is the marginal costs, which for the three plant types essentially equal zero, since no fuel is needed. The disadvantage, on the other hand, is the flexibility, or rather the absence of flexibility.

On the contrary to the hydro storage plant, the run river plant has no storage

erate changes in the underlying price. This possibility naturally decreases the more an option is in-the-money, and deep in-the-money options essentially behaves like the underlying asset.

possibilities, which in combination with the low marginal costs forces the run river plant to produce as soon as water is available, and the dispatch is given by

$$x_k = \min(I_k, p_{max}), \quad \forall k.$$

Since the inflow of water and hence the potential energy of the water is stochastic, the output will be highly uncertain. Wind velocity and sun radiation are two other very uncertain quantities, why the output from the wind plant and the solar plant will also be random.

In contrast to, for example, the gas turbine, where the owner can determine the output, or the nuclear plant, where the output hardly can be changed, but where the output is at least known, the run river plant, the wind plant and the solar plant have a negative flexibility in the sense that the output is not only non-controllable, but also stochastic. Volume risk is therefore indeed present, which makes these three plant types the most inflexible production units. This of course should be reflected in the value of such a plant, especially for a risk averse investor. This negative flexibility corresponds to having sold swing options, since the owner of the plant is not in the position to determine the output. Someone else makes the 'decision', namely the weather. Hence the run river, wind and solar plant are not only inflexible, they exhibit a negative flexibility. There is however a difference between a swing option and these plant types. Whereas the owner of a swing option will typically withdraw power during peak hours, when prices are high and it hurts the seller the most, the owner of an inflexible plant will not be subject to these rational decisions. The corresponding contract would hence be a swing option bought by an irrational player, where the maximum power A_2 in (2.1) is given by the maximum output of the plant, the minimum power A_1 by zero. The maximum and minimum energy B_2 and B_1 are however unknown and 'determined' by nature.

4.7. All production plants

The possibility of a power plant to benefit from the optionality is dependent on its flexibility and availability. So far we have only discussed the first factor, the flexibility, but just as important is the availability of the plant. Every plant is subject to planned outages due to, for example, yearly maintenance, but also unplanned outages caused by, for example, technical failure.

4.7.1. Planned outages

Planned outages like maintenance mean that the electricity production for a certain time will be reduced, and in many cases equal zero. During these periods the corresponding contract parameters have to be changed to mirror the new reduced output. A total outage of a plant naturally corresponds to a zero position in the corresponding contract during the time of the outage. From a risk management point of view a planned outage is not very severe, since the outage can be compensated by positions in the market, even though the flexible nature of, for example, a hydro storage plant will be difficult to mirror. To exemplify, a gas turbine with a planned maintenance in August, reducing the possible output to zero, corresponds to a series of call option for each hour, except for those in August. Planned outages hence reduces the option value of flexible plants.

4.7.2. Unplanned outages

The unplanned or forced outages, like unforeseen plant breakdowns, are features of every plant and have to be modeled to achieve the correct correspondence between production and contracts. Planned outages are costly, since the corresponding contract falls away during the outage. The unplanned outages are however much more severe, especially in a risk framework, since not only the corresponding contract falls away, but also an uncertainty regarding when this outage will take place comes into play. This enforces a volume risk in any plant and the uncertain output can be viewed as the curtailable part of an interruptible load contract. The buyer of such a contract has the right to interrupt the load in a predefined manner, such as 5 days in a year. Unplanned outages actually correspond to sold options and will therefore in a risk framework be very costly.

In Table 4.1 the typical unplanned outage time as a percentage of the total time according to [BA96] is presented for a number of plants, where one can see that unplanned outages do occur with a non-negligible probability. Further,

Unit type	Size [MW]	FOR(%)
Fossil steam	12	2
Combustion turbine	20	10
Hydro plant	50	1
Fossil steam	76	2
Fossil steam	100	4
Fossil steam	155	4
Fossil steam	197	5
Fossil steam	350	8
Nuclear steam	400	12

Tab. 4.1: Typical force outages rates (FOR).

this probability seems to increase with the plant size and hydro plants seem to have a high availability. The cost of unplanned outages are twofold. Firstly, the potential profit is reduced, since the plant will not be able to run for a certain time. Secondly, the potential loss and hence risk increases, since the time of the outage is unknown.

4.8. Transmission lines

Transmission assets are not power plants. However, transmission lines have interesting corresponding contracts similar to the contracts corresponding to power plants. Transmission costs and constraints have the effect that price of electricity in different locations can differ substantially. A transmission line between location **A** and **B** makes it possible to transmit electricity from one of the locations to the other. A transmission asset therefore enables the owner to change electricity in one location for electricity in another. Some of the electrical energy will however be lost due to resistance in the cable. Since the resistance is proportional to voltage, which is constant over time, it is fair to assume that the losses $0 \leq K^l \leq 1$, are proportional to the energy transmitted. One unit of electricity in location **A** can therefore be exchanged for $1 - K^l$ units of electricity in **B** and vice versa.

Let the price in location **A** and **B** be given by S^A and S^B respectively. In each period the owner has the option to transmit electricity from **A** to **B** whose payoff is given by $(S^B(1-K^l)-S^A)^+$. But the owner also has the option to transmit electricity from **B** to **A**, whose payoff is given by $\big(S^A(1-K^l)-S^B\big)^+$. Each of these options is called a locational spread call option. These types of contracts are already traded in the Nordic market under the name *contracts for difference*,[17] though not as options but as futures.

By owning such a transmission line we have the option in each period to exchange electricity in **A** for **B** and vice versa at the cost of $1-K^l$. We can hence conclude that a transmission line equals a series of locational spread options.

4.9. Real option theory

The traditional approach to value real assets, such as a power plant, is to use the *discounted cash flow* (DCF) method. The expected future cash flows $E[CF_t]$ are discounted at a risk-adjusted rate r_a and integrated over the life time of the asset (t, T) to achieve the value of the asset

$$DCF = \int_t^T e^{-r_a(\tau-t)}E[CF_\tau]d\tau.$$

The risk adjusted rate could typically be the equilibrium return derived from the CAPM, introduced in Chapter 3.7.2. In many assets the owner has the possibility to control these cash flows, since he has the option to pursue different actions like postponing an investment or in our case to choose whether to produce or not. The DCF method cannot take these options into account [CA01] and hence assesses an incorrect value to the real asset. In fact, risk is seen as something purely negative, since the discount rate increases with risk,[18] whereas we know that flexible plants like the gas turbine benefit from risky, i. e. volatile prices. It is hence clear that an approach is needed where this flexibility, these built-in options are taken into account.

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[17] These contracts were introduced in Chapter 2.7.

[18] Risk in the CAPM framework, for example, is given by the β, determined by the volatility of the asset's return and the correlation to the market return.

The science concerned with valuing real assets by taking these options into account is called *real option* theory. The tools to value managerial options in real asset are closely related to the tools used to value financial options.[19] The similarities arise because the ability to control or manage a cash flow stream represents an option. Another important similarity is according to [BT00] that equivalent martingale pricing techniques, as in equation (3.6), are appropriate to both real and financial options. The major difference, on the other hand, is that while financial options are almost always options on traded assets, the rights to controllable cash flows typically cannot be reduced to claims on traded assets. As [BT00] note, the determination of the risk neutral probability measure therefore is more complicated than is the case for financial options.

In the case of power plants in a liberalized market these controllable cash flows can however theoretically be reduced to claims on traded assets, namely futures and options on electricity as shown in this chapter. Because of the incompleteness of the electricity market one however does not get much guidance on how to choose the risk neutral probability measure, as discussed in Chapter 3.7.1.

Deng et al. [DJS01] gives an example on how a gas turbine can be assessed by valuing the corresponding spark spread options. By assuming that the start-up and shut-down times are short and that the facility's operation and maintenance costs are constant, they state that the value of the plant Ψ_{gas} is given by $\Psi_{gas} = \int_0^T C(t)dt$, where $C(t)$ is the value of a spark spread option with expiration at t and with the corresponding characteristics of the plant, namely p_{max} and H, and T is the remaining life of the plant.

In the same manner they show how a transmission asset can be assessed by valuing the corresponding locational spread options. Let $C_{A,B}(t)$ be the value of a locational spread option between location **A** and **B** with expiration at time t. Let further $C_{B,A}(t)$ be the value of an locational spread option between location **B** and **A** with expiration at time t, both with the characteristics of the specific transmission asset, namely its maximum power capacity and transmission losses. The value of a transmission line connecting **A** and **B** with a remaining lifetime of T is then given by $\Psi_{tr} = \int_0^T C_{A,B}(t) + C_{B,A}(t)dt$.

[19] See Chapter 3.7 for an introduction to the valuation of options.

As seen in the example from Deng et al. [DJS01] our notion of contract engineering is closely related to the real option theory. But whereas the real option theory normally tries to value a real asset by valuing the corresponding managerial options we have so far only derived the corresponding financial contracts of the different plants, not valued these contracts. It should once again be mentioned that it is very challenging to value such options in the electricity market and the attempts so far rely on heavy assumptions, such as in the case of Deng et al. [DJS01], where they assume that hourly futures are traded out on the forward curve until the remaining life of the asset T, typically decades to avoid the problem of the non-storability of electricity, and that the underlying price process does not exhibit jumps. Instead, we will in Chapter 4.11 shortly describe how a utility can internally price contracts through their corresponding production plant.

4.10. Value of different production plants

The value of a power plant basically comes from the difference between the electricity price and the operation costs. The price that the electricity on average is effectively sold at depends on the flexibility of the plant. As shown in Chapter 4.2 the gas turbine, for example, will sell electricity at a higher average price than, for example, a nuclear plant. On the other hand, the gas turbine will not generate a continuous cash flow to cover deprecations and capital costs, but only run occasionally when electricity prices exceed marginal cost of production.

Most technologies exhibit an inverse relationship between their capital costs and generation costs. Base load plants typically have high capital costs and low generation costs as compared to peak load plants, which have relatively low capital costs and high generation costs [OEC98]. This is illustrated in Figure 4.8, showing the proportionally much higher fixed costs for a base load plant (nuclear plant) compared to an intermediate load plant (coal plant) and a peak load plant (gas turbine) and vice versa for the marginal costs.

The plants that will survive in a deregulated market must either have low

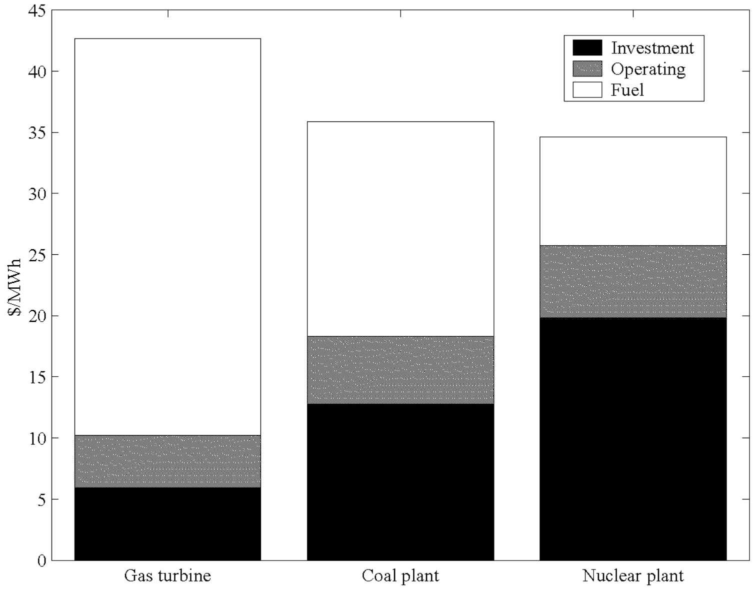

Fig. 4.8: Total generation costs divided into fixed (investment cost) and variable costs (fuel and to some extent operating costs) according to [OEC98].

marginal costs, like the nuclear plant or high flexibility like the gas turbine or some features of the two, like the coal plant. To illustrate the relative position of the different plants, we have ranked them according to flexibility in Figure 4.9. Flexibility is here arbitrarily and subjectively defined as the possibility to alter output in combination with its associated costs (e. g. start-up and shut-down costs and times, and ramping). Another ranking, which is closely related to the flexibility is presented in the same figure, namely the excess value over the DCF derived value, stemming from the operational flexibility. This value is highest for the plants on the top, corresponding to bought options, is low or zero for the plants, corresponding to futures with low or no option value and is negative for the plants, corresponding to sold options, having a negative option value.[20] Finally the plants are ranked according to the prevailing cost type, where the already mentioned inverse relationship between fixed costs and marginal costs is illustrated.

[20] Case studies by McKinsey & Co [LLMR$^+$00], show as expected, that the excess value over the DCF value increases with flexibility.

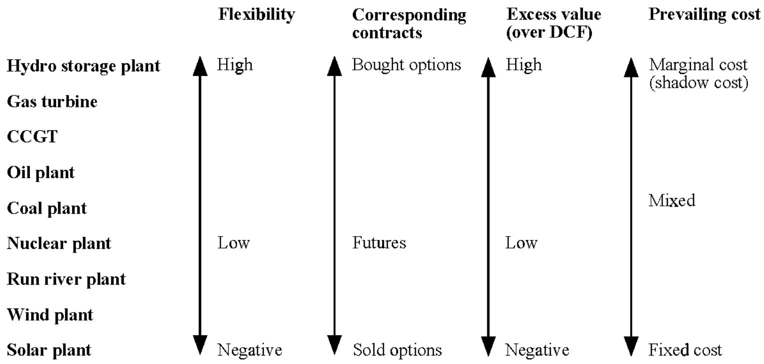

Fig. 4.9: Ranking of plant types according to flexibility, corresponding contracts, excess value and prevailing cost type.

4.11. Engineering of contracts

A good is homogenous if the objects are considered equal by the consumers. They have no reasons for preferring one supplier for another. This is the case for electricity; no one can separate whether the electricity they are consuming is coming from Eon or EdF, if security of supply is taken for granted.

As known from traditional economics, essentially the only competitive weapon that a company can use in a commodity market is the price. The margin on commodities therefore tends to be low [Sam97]. From a sellers point of view there is hence a driving force to de-commodities their products and differentiate from their competitors. From Chapter 2.7 we know that electricity contracts even in fairly standardized forms can be complex. Such de-commoditiesed products will have to be differentiated enough to motivate a higher price and will consequently be quite complex and typically not be hedgeable in the standardized market.

A golden rule from the financial markets is to never enter a contract that one cannot price or risk manage. That indeed also applies to the power markets, where it is even more important, since the tools to price and manage risk are less exploited and since the contracts generally are much more complicated. However, in this chapter we have shown how very complex series of contracts can be replicated by power plants. Hence with the back-up of own production capacity, a large number of contracts can be engineered and also risk managed, like the capped spot contract that is hedged by the gas turbine and the swing option, whose volume risk can be hedged by a hydro storage plant.

Producing utilities should take advantage of this, for them unique, possibility to construct new, not only differentiated, but also risk manageable contracts to increase their margins at an acceptable risk level. In Chapter 6.6.9 we will show how a utility in a structured way can utilize the valuable flexibility in its own production plant park by entering the right types of contracts.

As a matter of fact, this dual consideration of power plants makes it possible to internally assess complex contracts, such as a swing option, despite the fact that it might be impossible to price that contract with traditional pricing models. The costs associated with the corresponding production plant would then by a simple absence of arbitrage argument determine the internal price of such a contract. This price is in no way a market price, but rather a company specific internal price, dependent on accounting standards, allocating costs to the plant. However, this internal price may give some additional guidance when entering complex contracts.

4.12. Summary

Power plants can be viewed as a set of electricity contracts. As a matter of fact, all investigated plant types corresponds to a set of futures and options. In a natural way one identifies flexibility as options in the corresponding contracts. Flexibility in production is identified as a value lever in the deregulated market, allowing the owner to make use of the volatile prices. Outages, on the other hand, are identified as value destroyers, putting the owner in a position of an option seller. Transmission assets, in a similar way to production assets, have a set of corresponding contracts, enabling the owner to change electricity in one location for electricity in another. Electricity producing companies should take advantage of this possibility to use contract engineering to replicate a set of contracts with physical assets and hence being able to sell complex and hopefully high margin products that however are risk manageable.

To be able to perform power risk management on an aggregated portfolio level, the identification of the power plant's corresponding contracts enables us to get a better understanding of how different production plants contribute to the companies total risk. Further, it allows us to compare and relate production plants with financial and physical contracts on a unified basis.

Chapter 5

Hedging strategies

5.1. Overview

Hedging is a very common term in the financial world. A formal definition of *hedge* is however difficult, since it has as a slightly different meaning to the various players in the industry. To some, *hedge* means eliminate the risk in a position or in a portfolio. To others, it means limit the risk. Our definition of *hedge*, which is similar to the one in, for example, [Gas92] lies somewhere in between.

Definition 5.1
A hedge is an action, which reduces risk, usually at the expense of potential reward.

A crucial question is whether hedging should be done at all by a utility. Modigliani & Miller [MM58] in their analysis of capital structure imply that it is not necessary to hedge on a corporate level, since the investors rather could do it on their own. Their results, however are conditioned on a market with no transaction costs, no taxes and no information symmetries. If these assumptions are violated, which is the case in practice, one could argue that corporations should hedge, if they can do it at a lower cost than their investors. And as pointed out by Fleten [Fle00], since there are fixed fees, information costs etc. associated with participating in the electricity derivatives markets, it is probably cheaper for the utility than for the investor to hedge.[1] Several papers have

[1] There are thus economics of scale to hedging [Mia96].

been published motivating corporate hedging in imperfect markets, like the Smith & Stulz [SS85] paper about convex taxation and the paper by Brealey & Myers [BM00] about bankruptcy and financial distress costs. Ross [Ros96] addresses that hedging permits greater leverage and thus a more efficient tax shield. Since the electricity market is indeed imperfect, caused by, for example, transaction costs to mention one imperfection, we can conclude that the literature supports corporate hedging in the electricity market

In Chapter 5.2 we give an introduction to some traditional hedging strategies. These approaches are then in Chapter 5.3 evaluated for the electricity market and because of their shortcomings we introduce our approach *best hedge* in Chapter 5.4.

5.2. Traditional hedging

Replicating hedge The most simple way to hedge a position is to enter an identical, but opposite position to off-set all the risk. The currency risk, for example, of a known future positive cash flow in an foreign currency can be off-set by selling the same amount of that currency on a future basis. One tries to *replicate* the risky position that is to be hedged and takes a short position in that replication. For linear positions, whose price is linear in the underlying price, futures are generally the simplest hedging instrument.[2] If the goal is to minimize the risk with a future that does not behave equivalent to the position that is to be hedged, it might not be optimal from a hedging point of view to enter a future with the same underlying amount as the position to be hedged. Under certain assumptions one can actually find the optimal future position that minimizes the risk.

Optimal hedge ratio Assume that a company holds a long spot position that it wants to hedge with a future. Let ΔS define the change in spot price S, during the period of time equal to the life of the hedge. ΔF defines the change in futures price F, during the same period. The standard deviation of ΔS and ΔF are given by σ_S and σ_F respectively. The correlation between ΔS and ΔF is given by ρ and the hedge ratio, defined as the position in the future divided

[2] Basically all instruments except derivatives with assymetric payoffs, like options are linear.

by the position in the spot, is given by h.

The change in value of the hedged position will then be given by

$$\Delta S - h\Delta F. \tag{5.1}$$

The variance σ^2, of the change in value of the hedged position is

$$\sigma^2 = \sigma_S^2 + h^2\sigma_F^2 - 2h\rho\sigma_S\sigma_F \tag{5.2}$$

and the derivative with respect to the hedge ration is

$$\frac{\partial \sigma^2}{\partial h} = 2h\sigma_F^2 - 2\rho\sigma_S\sigma_F. \tag{5.3}$$

Since $\partial^2\sigma^2/\partial h^2 = 2\sigma_p^2$ is positive, the first order condition is sufficient to find the h that minimizes the variance, namely

$$\frac{\partial \sigma^2}{\partial h} = 0 \quad \Rightarrow \quad h = \rho\frac{\sigma_S}{\sigma_F}. \tag{5.4}$$

It is certain that the hedge ratio h, will minimize the variance, but it is debatable if it is *optimal*, since we implicitly state that variance is the risk measure of concern. If we assume that the spot price follows a geometric Brownian motion and that the good is storable, then the cash-and-carry strategy implies that also the future price will follow the same price process. The returns of both the spot and the future will therefore be normally distributed, why variance or standard deviation will be the natural risk measure, and a variance minimization is appropriate.[3]

Delta hedge For non-linear positions, where the price is non-linear in the underlying's price, the two simple approaches described above have to be expanded. A static hedging strategy, like the optimal hedge ratio, cannot be used anymore, since the dependency between a non-linear position and the underlying position will differ with, for example, time and price of underlying. The

[3] As argued in Chapter 3.4.1.

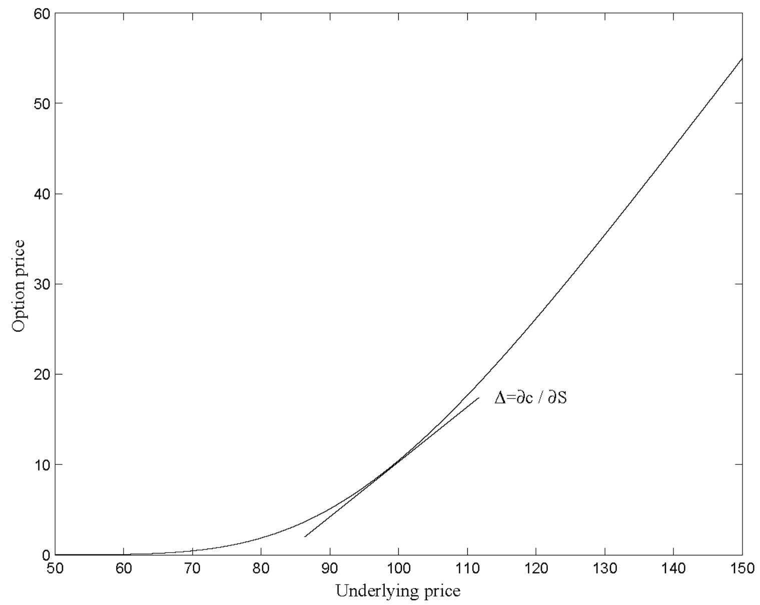

Fig. 5.1: Price of a European call option, according to the Black & Scholes model, as a function of the underlying price. Strike equals 100, time to maturity is one year, the volatility 20% and the risk free interest rate 5%.

price of a European call option in the Black & Scholes economy will have a typical convex shape when plotted against the underlying price, as seen in Figure 5.1, where also the delta, defined as the price derivative with respect to the underlying price S is shown. For a call option with price c the delta is given by

$$\Delta = \frac{\partial c}{\partial S}. \tag{5.5}$$

By hedging so that the delta of the hedged position equals zero, for example, through being short Δ of the underlying, the position is said to be locally risk free. For infitesmal changes in the underlying price dS, the changes in value of the hedged position $d\Phi$, will be given by

$$d\Phi = \Delta_p \cdot dS \tag{5.6}$$

and since the delta of the portfolio Δ_p equals zero ($\Delta_p = \Delta - \Delta \cdot 1$) the value of the hedged position will for infitesmal changes in the underlying price be constant. There are however two complications. First, the delta of a non-linear position is changing with underlying price, volatility, time to maturity and interest rate, why the hedge will have to be revised as soon as any of these parameters change. Since, for example, the underlying price typically will change at least every other minute, the delta hedge has to be *dynamically rebalanced*. The second complications comes from the fact that the delta hedged position is only *locally immune* to changes in the underlying price and will for any movement actually be exposed to some risk.

The value of the hedged position can be Taylor expanded, like any function $\Phi \in \mathcal{C}^{\infty}$, with respect to the underlying price S

$$d\Phi = \frac{\partial \Phi}{\partial S} dS + \frac{1}{2}\frac{\partial^2 \Phi}{\partial S^2}(dS)^2 + \ldots + \frac{1}{n!}\frac{\partial^n \Phi}{\partial S^n}(dS)^n + \ldots \tag{5.7}$$

The error caused by approximating the value Φ, with only its first component in the Taylor expansion grows with the changes in the underlying dS. One way to improve the approximation is to add the second component in the expansion.

Delta/gamma hedge By requiring that not only the delta, but also the second derivative with respect to the underlying price, the gamma

$$\Gamma = \frac{\partial^2 \Phi}{\partial S^2} \tag{5.8}$$

shall equal zero, one conducts a so-called delta/gamma hedge. The approximation is of course better than in the delta case, but one still has the problem of dynamic rebalancing, which because of transaction costs can be extremely costly if done on a continuous basis.

Duration hedge The more sophisticated hedging schemes using the sensitivities Δ and Γ, are typically mainly used for simple underlying contracts without a time component, like stocks. For interest contracts, which always have such a time component, adding a dimension of complexity, a simpler approach is often called for. One such method is the *duration hedge*. The duration of a bond is a measure of how long, on average, the holder of the bond has to wait before receiving cash payments. A zero coupon bond that matures in n years also has a duration of n years, since all payments are made then. A coupon bearing bond however, maturing in n years has a duration of less than n years, since some cash flows occur prior to maturity.

Let current time be 0 and suppose there is a bond providing the holder with n yearly payments c_i at time t_i $(i = 1, \ldots, n)$. The price of the bond B, and the continuously compounded yield y, are related by

$$B = \sum_{i=1}^{n} c_i e^{-yt_i}. \tag{5.9}$$

The duration D, of the bond is then given by

$$D = \sum_{i=1}^{n} t_i \frac{c_i e^{-yt_i}}{B}. \tag{5.10}$$

From (5.9) we can derive the bond price's sensitivity to changes in the yield y

$$\frac{\partial B}{\partial y} = -\sum_{i=1}^{n} c_i t_i e^{-yt_i}, \tag{5.11}$$

which together with (5.10) can be written as

$$\frac{\partial B}{\partial y} = -BD. \tag{5.12}$$

The relative change in the bond price $\Delta B/B$ will for small parallel shifts in the yield curve Δy, therefore be given by

$$\frac{\Delta B/B}{\Delta y} = -D\Delta y. \tag{5.13}$$

From (5.13) one can see that the relative bond price changes are proportional to the duration, why duration is often used as a measure of risk in the fixed income market. By hedging a position such that the duration equals zero, the hedged position will be locally immune to parallel shifts in the yield curve. A mayor drawback of the duration approach is however just the assumption that the yieldcurve will move in parallel shifts - not a very realistic assumption.

5.3. Relevance for electricity hedging

An important question is how one could leverage these traditional hedging approaches to the electricity market. The idea of closing down a positions and hence its risk by entering a *replicating* position is certainly a sound approach also in the electricity market. However, because of the complexity and the OTC characteristics of most electricity contracts, the replicating hedge approach would only be able to cover a small subset of all contracts.

As already argued in Chapter 3.5, variance is not an appropriate risk measure in the electricity market, for example, because of the heavy tailed returns, why an *optimal hedge ratio* as described above would need to revised to fit the electricity market.

Since the electricity market is incomplete, it may be difficult to obtain unique prices of electricity derivatives[4] and consequently sensitivities may not be obtained. The idea of *delta* and *delta/gamma* hedging will therefore be difficult to pursue.

Even given that unique prices would exist for electricity derivatives, the non-storability prevents us from taking the delta or delta/gamma hedge approach. The idea is to hold a certain amount (Δ) of the underlying in order to make the portfolio of the derivative and the underlying locally immune, but one cannot, because of its non-storability hold on to electricity. For options on a storable underlying, such as a future, the approach would however be theoretically feasible.

A duration-like method is appropriate also in the electricity market only in the sense that a long position should be hedged with a short position of the same

[4] See Chapter 3.7.

maturity, and vice versa. A duration-like weighted life time of an electricity contract could probably be introduced, but since the duration method already in the fixed income market has mayor drawbacks, it would not be a suitable candidate for an electricity hedging approach.

The complicated price process of electricity, the complex contracts involving not only price risk, but also volume risk and the additional dimension of the production side calls for a more sophisticated and general hedging approach than the presented traditional approaches. We will in the next section introduce a hedging approach that we believe is suited for the electricity market, which we call *best hedge*.

5.4. Best Hedge

The non-storability of electricity and hence the impossibility to pursue cash-and-carry strategies in combination with the fact that futures and other standardized derivatives only are available at a limited number of nodes in the grid makes it difficult to find *perfect hedges* for electricity positions. With perfect hedge we mean a hedge that totally eliminates the price risk in a position. We know from Definition 3.7 that the electricity market is incomplete and hence that only a subset of all contingent claims are replicatable. It is therefore in the electricity market in general not possible to find such a perfect hedge .

Our approach is instead to find the best possible hedge. If we return to Definition 5.1, where hedge is defined as *an action that reduces risk, usually at the expense of potential reward*, it seems natural to state that the *best hedge* is a hedge that minimizes the risk, under some constraint on the expense of potential reward. This sounds similar to the optimal hedge ratio approach, but as mentioned, variance is not a good measure of risk in the electricity market, and the optimal hedge ratio does not consider the costs associated with the hedge. In Chapter 3.5 we argued that CVaR is an appropriate risk measure in this market, hence CVaR will instead of variance be used as measure of risk. A hedging approach is needed that can handle all types of complex electricity contracts. We believe that the *best hedge* is one such approach, which is obtained by finding the hedge that minimizes the risk, in terms of CVaR under the constraint that the expected profit is greater or equal to some threshold R, to assure that the *expense of potential reward* is not too high.

This problem can luckily be reduced to the following linear program

$$\begin{array}{ll}\min\limits_{x\in\mathbb{R}^n, z\in\mathbb{R}^J, \alpha\in\mathbb{R}} & \alpha + \frac{1}{(1-\beta)J}\sum_{j=1}^{J} z_j \\ \text{s.t.} & z_j \geq l(x, y_j) - \alpha,\ z_j \geq 0,\ j = 1, \ldots, J \\ & -\frac{1}{J}\sum_{j=1}^{J} l(x, y_j) \geq R, \end{array} \tag{5.14}$$

as shown in Chapter 3.5.3.

The contract or contracts to be hedged x^h are held fixed, whereas the other contracts $x \setminus x^h$ are our decision variables to be chosen, such that the best hedge is found.

Observe that (5.14) is a generalization of a pure risk minimizing, since the expected profit constraint can be made non-binding by assigning R the value $-\infty$. For some simple contracts with the expected profit constraint relaxed, the best hedge will actually coincide with the perfect hedge given that the contracts are replicatable. One can note that in a complete market the best hedge with non-binding expected profit constraint will for *all* contracts coincide with the perfect hedge.

Excess production capacity can in many cases be a much more appropriate hedging instrument than financial instruments, such as options and futures. The dual consideration of production capacity, presented in Chapter 4, is important to understand how production can be used to replicate other positions in order to hedge away undesired risks. Example 4.1, for instance, shows how a gas turbine can hedge a capped spot contract. The flexibility in some production plants, such as the hydro storage plant, even makes them suitable to hedge not only price risk, but also volume risk, which currently is not possible in the standardized market. To exemplify such a procedure, where a contract is to be hedged through the approach of *best hedge* with the help of contracts and production capacity, we assume that a load factor contract with an expected load of 11 MW is to be hedged over the coming week. At our disposal we have a gas turbine, a hydro storage plant and the traded contracts; spots, futures and options. Both the gas turbine and the hydro storage plant corresponds to

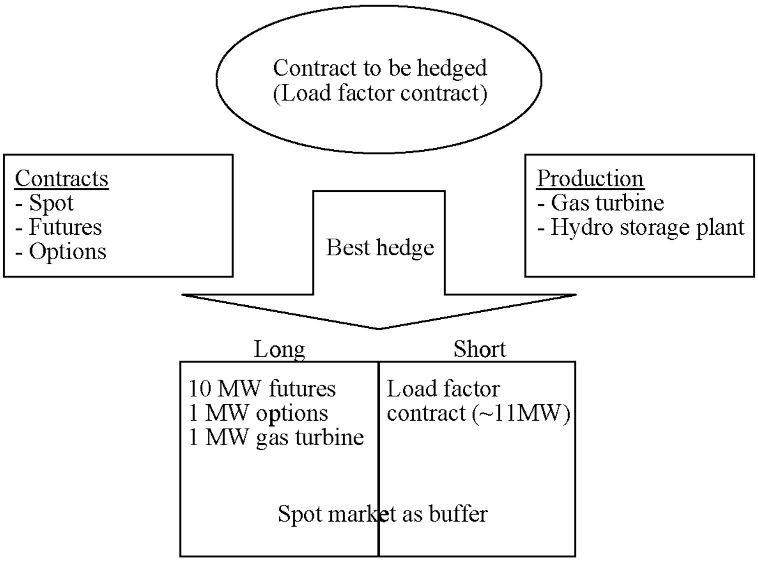

Fig. 5.2: Illustration of best hedge procedure.

options and we need an internal price to be assigned to both, otherwise we would optimally choose to allocate the whole capacity of the gas turbine and the hydro plant in our best hedge.[5] This would naturally put the next contract to be hedged in a bad situation, having to rely only on traded contracts. With our notation the contract to be hedged x^h is the load factor contract with its stochastic demand, whereas the contracts used in the best hedge $x \setminus x^h$ are given by spots, futures, options, the gas turbine and the hydro storage plant.

Figure 5.2 illustrates such a procedure, where the best hedge is achieved through going long 10 MW in the futures market, buying call options on 1 MW, allocating 1 MW of the gas turbine and using the spot market as a buffer for the stochastic demand. In this case the hydro storage plant was too expensive. The illustration is only schematic and is not a result from a real optimization. In Chapter 7 we will however perform a similar optimization of a real power portfolio.

The players in a financial market are normally categorized as *hedgers*, *speculators* and *arbitrageurs* [Hul97]. Hedgers are interested in reducing the risk

[5] Observe that if the plants are not assigned an internal price we could buy 'options' at zero cost.

that they already face, whereas speculators wish to take a risky position in the market in order to possibly make a profit. Arbitrageurs take advantage of arbitrage opportunities to make riskless returns, but are of no importance in this context. The difference between hedgers and speculators can however be rather fine. A player that finds a best hedge, but with a very high constraint on the expected profit, is he a hedger or a speculator? He knows that a high constraint on the profit will result in a high risk, why one could view him as a speculator. On the other hand, he is minimizing the risk and could also be viewed as a hedger. Our notion of *best hedge*, where a constraint on the expected profit is introduced therefore generalizes the notion of hedgers to comprise also speculators. In the limit when the best hedge is found under as high expected profit constraint as possible resulting in at least one feasible portfolio, it naturally corresponds to a profit maximization without any concerns on the risk level. A speculator could hence also be characterized by a player that maximizes the expected profit with a soft constraint on the risk level.

The *best hedge* is actually a small scale portfolio optimization, where typically only part of the whole portfolio is considered, namely the contract or contracts that are to be hedged. In Chapter 6 we will introduce the power portfolio optimization, where a similar optimization is used and where the whole portfolio is considered. We will therefore not go into details on, for example, how the production side is actually modeled here, but leave that for the next chapter.

Chapter 6

Power portfolio optimization

6.1. Overview

Measuring risk is a passive activity. Simply knowing the amount of risk does not provide much guidance on how to manage risk. Rather, as noted already in Chapter 3.5.3, risk management requires tools to optimize the utilization of risk. In particular for a utility in the electricity industry with its built-in operational flexibility an optimization of the portfolio has to be conducted to determine the most efficient use of this flexibility.

Along with the deregulated market and the high uncertainty in the return of a power portfolio, comes the need to change the attitude from only maximizing expected profit or minimizing costs to also incorporate risk as a factor. Some of the traditional portfolio approaches in the electricity market are presented in Chapter 6.2. The peculiarities of power portfolios are described in Chapter 6.3. Modeling issues of the operational flexibility in power plants, with special focus on hydro storage plants, are investigated in Chapter 6.4. Mathematical results are presented for some special power portfolio optimization cases in Chapter 6.5. Our proposed optimization approach using CVaR is then described in Chapter 6.6.

6.2. Traditional power portfolio optimizations

Traditionally, optimization approaches in the electricity market have been focused on the reliability of the whole power system and was developed for central planning purposes, such as *least cost planning* [Sto89] and *integrated resource planning* [SJR97, Gar00]. Prices were regarded to be deterministic and the goal was typically cost-efficient and reliable supply of electricity

From an optimization point of view the power portfolio optimization started with capacity planning, which was formulated as a least cost investment problem. Solved with linear programming the total cost was minimized subject to fuel availability, demand and capacity to mention the most important constraints. The objective function typically summed up capital cost and generation cost, like fuel cost, over the whole planning period. The constraints as a rule included forecasted demand and plant availability. This deterministic approach was pioneered by EdF in 1954 that developed a schematic linear programming model with only 4 constraints and 5 variables. Described by Massé and Gibrat [MG57] this was the first application of LP to electricity planning. To account for the start-up times and shut-down costs of thermal units, like coal plants, Schaeffer and Cherene [SC89] introduced integer variables and solved their mixed-integer linear program by finding the optimal dispatch of an existing array of generating plants. The generation cost was minimized subject to meeting short-term demand. These so-called unit commitment problems have been extended to allow for short-term transactions, i. e. entering of contracts. Takriti et al. [TKW98] solve this problem using Lagrangian relaxation and Bender's decomposition. The goal is still to minimize the generation costs while meeting the electric load. An optimal production schedule for a longer period, typically a year, for hydro systems with reservoir capacities with the possibility to enter contracts has also been studied. For example, Bart et al. [BBC+98] examines a utility with both nuclear and hydro plants having access to a spot market. The objective is to minimize the generation costs and the problem is solved with linear programming. For an extensive survey of optimization approaches in regulated power markets, see [Ku95].

Following the deregulation and the corresponding uncertainty on future earnings, producers will however have to change their focus from reliable and cost-efficient supply of electricity to more profit oriented objectives, including financial risk.

Papers on portfolio optimization in the electricity market, where also risk is considered, are rare. Fleten et al. [FWZ99] penalize risk through a piecewise linear target shortfall cost function in their study of a single hydro producer. The objective is to maximize the expected profit in which this shortfall cost has been assigned. The producer has access to a spot market, to a forwards market and to an options market. The five periodic problem is solved with stochastic dual dynamic programming. Güssow [Güs01] takes a similar approach and optimizes a portfolio consisting of hydro and thermal units. To take the high operational flexibility of, for example, a hydro storage plant into account, one would as illustrated in Example 4.2 have to work with a very fine discretization. Since the size of the stochastic dual dynamic programming models used in [FWZ99] and [Güs01] grow heavily with the number of periods, they had to use a rough discretization of weeks. Herzog [Her02] uses a stochastic control theory approach in continuous time to optimize the dispatch of a single hydro storage plant. To be able to efficiently solve this problem he uses a, for the electricity market, questionable variance-like risk measure. Further, jumps in the spot price were not modeled, a short-coming also in [FWZ99] and [Güs01].

In this chapter we will provide a portfolio optimization model for a utility with hydro storage plants, having access to a spot, a futures and an OTC market, where the operational flexibility is taken into account, an appropriate risk measure is used and a realistic modeling of the stochastic factors, including jumps is possible.

6.3. Power portfolio optimization in general

In the traditional financial markets the standard portfolio optimization approach is the mean-variance portfolio problem by Markowitz [Mar52], where the portfolio variance is minimized subject to a constraint on the expected return. There are however fundamental differences between, for example, an equity portfolio and a power portfolio. The differences, described below, calls for a new optimization approach.

Production assets A power portfolio consists not only of financial and physical contracts, hereby called *contract portfolio*, but also of production assets, hereby called *production portfolio*. Numerous of the power plants exhibit operational flexibility, as discussed in Chapter 4, where we concluded that each plant type corresponds to a specific set of contracts. By viewing production facilities as contracts, it is natural to state that an optimal power portfolio implies not only an optimal contract portfolio, but also an optimal production portfolio. The production portfolio can be optimized on two levels. The first level is to find the optimal use of the operation flexibility, i. e. to find the optimal dispatch strategy for the existing plants. The second level is to find the the best portfolio of power plants, i. e. to allow for acquisitions and sell-offs of these assets.

Complex contracts The contracts in the power market can be very complex in comparison with traditional financial contracts. A major difference is that many OTC power contracts have an uncertain underlying quantity of electricity as described in Chapter 2.7. In addition to the price risk that players in the traditional financial markets are facing, the electricity players often also face a volume risk stemming from swing options or from outages of plants. Such volume uncertainty cannot be handled by the simple Markowitz approach (3.1), but naturally needs to be dealt with by a power portfolio optimization.

Simultaneous optimization Holthausen [Hol79] stated a separation theorem that production scheduling can be done independently from hedging, under the assumption of no production uncertainty and no basis risk. In the electricity market there is generally a presence of basis risk, due to the finite number of nodes in the grid at which derivatives are traded, and definitely a production uncertainty, why the separation theorem does not hold. Anderson & Danthine [AD80] showed that production must be determined jointly with hedging in the case of basis risk. Viaene & Zilcha [VZ98] have showed that even in the absence of basis risk the separation theorem breaks down in the case of correlation between input costs and output price. This correlation exists in the electricity market through, for example, the correlation between fuel prices or water inflow and electricity prices. Thus we can conclude that a *simultaneous optimization* of the production and the contract portfolio, which could typically be used to hedge future profits from the production portfolio, is needed. It is obvious that the overall portfolio risk and expected profit depends on the dispatch strategy. The dispatch will consequently influence the possibility to take on risky positions in, for example, the contract portfolio. The dispatch strategy of some plants will, maybe less obvious, depend on the contract portfolio. The price risk would automatically be reasonably hedged by a strategy like the gas turbine strategy, namely to produce when spot prices exceed marginal costs. The volume risk, on the other hand, would not be explicitly hedged by such a strategy, why the hydro dispatch strategy would also be influenced by the volume risk in the portfolio, stemming from swing options, interruptible contracts and stochastic outages of production units.

Non-normality Whereas one often assumes that returns are normally distributed in traditional financial markets, the return distribution of a power portfolio will typically be non-normal and heavy tailed, for example, caused by jumps in the spot price as described in Chapter 2.9. Naturally a risk measure is needed that captures the non-normality of the return distribution.

Probabilistic problems The last, but not the least difference to a traditional financial portfolio is the fact that when modeling a production portfolio consisting of hydro storage plants the available resource, water, is a stochastic quantity. This calls for a probabilistic approach. For example, the probability that the water level falls below zero must not exceed zero.

These aspects has to be captured by a power portfolio optimization approach and we believe, as expressed already in Chapter 3.5, that CVaR is an appropriate risk measure in the electricity industry, capturing the heavy-taildness of electricity portfolios by penalizing large losses. Hence a portfolio optimization based on CVaR seems suitable. In Chapter 3 two similar such portfolio optimization approaches were introduced. In the first one, CVaR was minimized under an expected profit constraint and in the second, the expected profit was maximized under a CVaR constraint. Whereas the first approach was used for finding the *best hedge*, we believe that the latter one is best suited for optimizing a whole portfolio. The maximum risk level that may not be exceeded is typically determined by the board. This risk level depends on the company's risk appetite, the company's wanted credit rating and hence interest rate costs and is typically fixed for longer periods of time. The goal to constantly be able to utilize the given risk level naturally motivates the second approach, given by

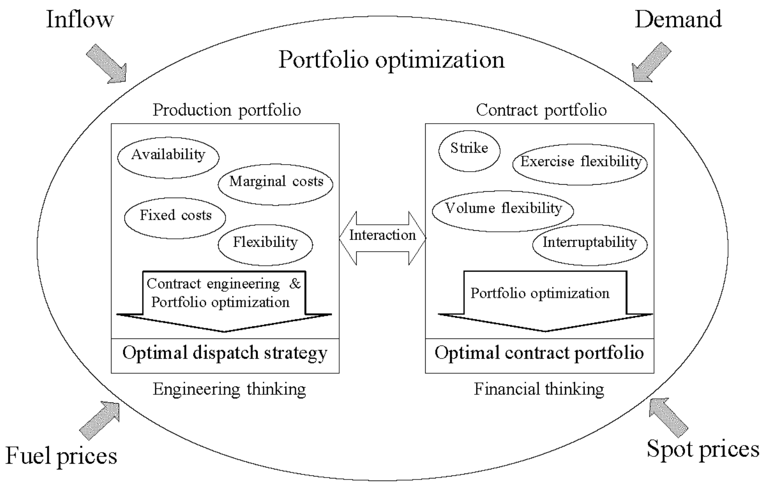

Fig. 6.1: Schematic figure of portfolio optimization.

$$\begin{aligned} \max_{x \in \mathbb{R}^n} \quad & E[-l(x,Y)] \\ \text{s.t.} \quad & \phi_\beta(x) \leq C. \end{aligned} \tag{6.1}$$

We will in the coming chapters show that (6.1) actually can capture also the other aspects, in terms of handling the production side and simultaneously optimize the contract and production portfolio, and in terms of managing the complex electricity contracts. Until just a few years ago linear programming was thought to be unable to cope with uncertainty in power optimization. The quote from Ku [Ku95] illustrates this:

> ...it [LP] is unable to deal with uncertainty without relying on numerous assumptions, approximations and post-LP analysis

We will however later in this chapter describe how linear programming in an efficient way can be used to capture the risk aspect in power portfolio optimization.

In Figure 6.1 a power portfolio optimization is illustrated, where the interaction between the production portfolio and the contract portfolio is stressed. Some important characteristics of the two portfolios are highlighted; availability, marginal costs, fixed costs and flexibility for the production portfolio and strike price, exercise flexibility, volume flexibility and interruptability for the contract portfolio. The marginal cost of a plant, as already mentioned, corresponds to the strike price of an option, and the availability of a plant corresponds to the interruptability of an OTC contract. Before the production portfolio is optimized we identify the contracts corresponding to each plant, which we call *contract engineering*. By doing this we can more easily compare the production and the contract portfolio and derive the optimal dispatch strategy for the different plant types. The power portfolio optimization problem is very tractable from an academic point of view, since the engineering skills and ways of thinking are needed to understand the production portfolio, whereas financial thinking and skills are needed for the understanding of the contract portfolio. The return of a power portfolio is affected by four major sources of uncertainty; spot price, demand in swing options creating volume risk, inflow into water dams adding production output uncertainty and fuel prices for thermal plants causing production cost uncertainty. The importance of the fuel price effect on the profit and loss should not be underestimated. For example, PowerGen [plc92] attributes almost 70% of their operating costs to fuel. Further fuel prices are volatile, in 1974 oil prices quadrupled and doubled in 1979. Derivatives of oil, like natural gas, followed a similar pattern.

In general we will assume that the time, over which we study profit and risk corresponds to K periods, where the length of each period is given by the length of the spot contract, typically one hour.

6.4. Modeling of plants and their operational flexibility

For some plant types it is trivial to find the dispatch strategy making full use of the operational flexibility, for other plants it is however a complex task. Before we investigate this strategy for the most challenging plant, the hydro storage plant in depth, we as an introduction shortly discuss the strategy corresponding to the other plant types.

6.4.1. Gas turbine

The optimal dispatch strategy for a gas turbine is trivial and actually independent on the owner's risk preferences. As already stated in Chapter 4.2 the optimal strategy is given by

$$x(S^g, S^e, H) = \begin{cases} p_{max} & \text{if } S^e \geq S^g H, \\ 0 & \text{otherwise} \end{cases}.$$

6.4.2. Nuclear plant

Because of the low operational flexibility of a nuclear plant, it corresponds to long futures and hence no decision problem is involved. The optimal dispatch strategy is simply to produce a constant and high amount, which is equivalent to the strategy used in regulated markets, where prices are essentially deterministic.

6.4.3. Coal plant and oil plants

The optimal dispatch strategy of a coal plant and oil plant would in the case of no start-up and shut-down times and costs resemble the gas turbine strategy

$$x(S^f, S^e, H) = \begin{cases} p_{max} & \text{if } S^e \geq S^f H, \\ 0 & \text{otherwise} \end{cases},$$

where S^f denotes the spot price of the fuel used, i. e. coal or oil. However, as motivated in Chapter 4.4 the decision problem involved in optimally running such thermal plants is complex, due to the costs and times associated with start-up and shut-down of the plant. The efficiency of this plant type differs with its operating status. A cold plant, for example, has to be handled differently from a warm plant. Hence power portfolio optimization problems involving semi-flexible thermal plants typically have to have integer variables associated with the operating status.

6.4.4. Run river plant, wind plant and solar plant

As discussed in Chapter 4.6 there are no decisions involved in running these plant types. Their negative flexibility puts the decision, not in our hands, but in the hands of the weather. However, when modeling these plants it is important that the highly stochastic output is taken into account, since it negatively affects the risk level.

6.4.5. Hydro storage plant

Hydro management is an interesting and complex problem because water is a storable commodity, whereas electricity is not. The hydro management thus involves a continuous process of deciding whether to release water now or to store it and release it later on. The natural inflow does not have an explicit cost, but using the water for power production represents an opportunity cost to the owner, since there is only a limited supply available. Consequently, the operations of a hydro storage plant becomes a temporal resource allocation problem under uncertainty.

6.4.5.1. Static dispatch strategy

From Chapter 4.3 we know that a hydro storage plant equals a series of interdependent options. However, we will here, as an introduction derive a static dispatch strategy that does not take this optionality into account. Today, the dispatch in each period over the planning horizon is determined. The choice to produce or pump will not react to new information about, for example, inflow or spot price, but be totally static. Since the decision about exercising the option in each period is taken already today, we actually model the hydro storage plant as a series of hourly futures, not options.

We recall the physical and the technical constraints of a hydro storage plant that any dispatch strategy $(x_k)_{k=1,\ldots,K}$ has to fulfill

$$P(0 \leq V_k \leq V_{max}) = 1, \quad k = 1, \ldots, K \tag{6.2}$$

and

$$-p_{min} \leq x_k \leq p_{max}, \quad k = 1, \ldots, K, \tag{6.3}$$

where the amount of stored water in period k is given by

$$V_k = V_0 + \sum_{i=1}^{k} I_i - \sum_{i=1}^{k} L_i - \sum_{i=1}^{k} x_i, \quad k = 1, \ldots, K. \tag{6.4}$$

We can however not guarantee that a static dispatch strategy, which is only allowed to be a function of time will fulfill the probabilistic constraint (6.2). It is actually very likely that it will be violated for any such strategy if the inflow is random. We note this immediate drawback of the static dispatch strategy and relax (6.2) to

$$0 \leq \bar{V}_k \leq V_{max}, \quad k = 1, \ldots, K, \tag{6.5}$$

where $\bar{V}_k$ denotes the average amount of water, which is naturally given by

$$\bar{V}_k = V_0 + \sum_{i=1}^{k} \bar{I}_i - \sum_{i=1}^{k} \bar{L}_i - \sum_{i=1}^{k} x_i, \quad k = 1, \ldots, K. \tag{6.6}$$

Observe that $E[x_k] = x_k$, since x is not a random variable. As a matter of fact the decision variables associated with dispatching the hydro storage plant is in this static approach given by $x^p = (x_1, \ldots, x_K)$. Further, note that the average spill over $\bar{L}_k$ will be non-zero, only when the average inflow $\bar{I}_k$ is greater than the maximum possible production p_{max} in combination with a full dam, when applied in a expected profit maximization. If we are not modeling the plant over its whole life time, which can be over 50 years, we need to avoid a myopic strategy, that could leave an empty dam for the periods following after the horizon. Hence we impose a lower limit V_{end} on the average amount of water left for subsequent periods $\bar{V}_K$

$$\bar{V}_K \geq V_{end}. \tag{6.7}$$

Note, as seen in (6.6) our modeling of the plant (6.3), (6.5) and (6.7) is now actually deterministic.

Recall from Chapter 4.3 that because of the non-perfect pump efficiency $\chi < 1$ we have to divide the dispatch in each period into one production part and one pumping part

$$x_k = x_k^+ - x_k^-, \quad x_k^+ \geq 0, \quad x_k^- \geq 0, \quad k = 1, \ldots, K.$$

Typically, it is not possible to simultaneously produce and pump, since the same pipelines are used for production and pumping. Hence we would need to impose that either production or pumping equals zero in each period

$$x_k^+ x_k^- = 0, \quad k = 1, \ldots, K. \tag{6.8}$$

However, when the objective is to maximize the expected profit, this constraint will for a non-perfect pump be redundant and hence not needed. A violation of (6.8) would imply that pumping was conducted simultaneously with production. Let Δ_k denote $min(x_k^+, x_k^-)$, where x_k^+ and x_k^- is a feasible dispatch in period k. One could change the dispatch in that period to $\tilde{x}_k^+$ and $\tilde{x}_k^-$ respectively given by

$$\tilde{x}_k^+ = x_k^+ - \Delta_k$$

$$\tilde{x}_k^- = x_k^- - \Delta_k,$$

where feasibility is assured by definition of Δ_k. This change in dispatch would not affect the stored water level V_k. The effect on the electricity balance, on the other hand, would change as

$$\left(x_k^+ - \Delta_k - \frac{x_k^- - \Delta_k}{\chi}\right) - \left(x_k^+ - \frac{x_k^-}{\chi}\right) = -\Delta_k + \frac{\Delta_k}{\chi} \tag{6.9}$$

By assumption that (6.8) is violated $\Delta_k > 0$ and the efficiency of the non-perfect pump is $0 < \chi < 1$, why (6.9) is strictly positive. The profit in period k would therefore increase by

$$\left(-\Delta_k + \frac{\Delta_k}{\chi}\right)S_k > 0 \quad \forall S_k > 0.$$

A similar argumentation can be conducted for the improbable, but possible case of negative prices. Hence we have shown by contradiction that (6.8) will hold in optimum, why we will not explicitly require that pumping is not conducted simultaneously with production.

Even though the static dispatch strategy is naive in the sense that it does not respond to new information and fails to capture the option features of the hydro storage plant, it is illustrative to understand the value of a hydro plant. Further, it serves as an introduction to the dynamic dispatch strategy, which will be introduced in the next section. By modeling the inflow as deterministic, for example as the average inflow, there is a way to theoretically lock in the future earnings, i. e. to fully hedge the stochastic cash flows stemming from the hydro storage plant. The strategy is to sell the known, but time-varying water in the dam in the electricity futures market. In order to be able to replicate the hourly future positions $x_1^f, \ldots, x_K^f$ with the dispatch, it is obvious that $x_k = x_k^f$ has to solve (6.2), (6.5) and (6.7). Given that the inflow actually would be deterministic, such static dispatch strategy would be the one chosen by an extremely risk averse utility with no other exposures, since the price risk would be perfectly hedged away. Such a utility, however still naturally wants to maximize these deterministic cash flows. How this is done is shown later in Chapter 6.6.3.

6.4.5.2. Dynamic dispatch strategy

In the static dispatch approach the production schedule over the whole period is determined today and is fixed. We know that the hydro storage plant in contract terms is a series of interdependent options and not a series of interdependent futures. The value of an option, contrary to a future, comes from the fact that one does not have to make the decision in advance whether to exercise it or not, as a result a true asymmetric payoff is achieved. Our static modeling of the hydro plant cannot capture the value of this flexibility. The model does not react to, for example, a high price in a certain period. Instead if a high price was not probable the schedule could have been to actually pump in that period, which naturally would be counterproductive.

To capture the value of the flexibility and to be able to fulfill the tighter, but realistic constraints that had to be relaxed for the static dispatch strategy

$$P(0 \leq V_k \leq V_{max}) = 1, \quad k = 1, \ldots, K \tag{6.10}$$

and

$$-p_{min} \leq x_k \leq p_{max}, \quad k = 1, \ldots, K, \tag{6.11}$$

a dynamic approach is needed. The decision to exercise the option, i. e. how much to produce or to pump at time t, should be a function of the realization of the stochastic factors up to t.

The simplest example of such an exercise decision is probably a European call option on a single stock. The optimal strategy at the exercise date is simply to exercise the option if and only if the stock price is higher than the option's strike price. A slightly more difficult exercise condition can be found in the American put option. An American option is an option where you have the right to exercise the option during the maturity and not only on the exercise day. For an American call option it can be shown in the absence of dividends, that the optimal exercise time is always to wait until the option expires and the exercise conditions are the same as for the European call option [Øks95]. For the American put option it becomes more difficult. The optimal exercise time could be at expiration date, but also before expiration. The exercise conditions are not analytical and can only be solved numerically or quasi-analytically. The decision to exercise the put option is however a function of spot price, strike price, dividend, volatility and time to expiration $x = x(S, K, \delta, \sigma, \tau)$.

Unfortunately the hydro storage plant is far more complicated than an American put option, because of the interdependence between the options and since the decision is not binary, i. e. to exercise or not, but how much to exercise. Since problems arise already by finding an analytical exercise condition for an American put option, the task of deriving the optimal exercise conditions for the swing options, corresponding to a hydro storage plant, i. e. to derive the optimal dispatch strategy, will be non-trivial.

The most obvious factor that intuitively should influence the exercise decision in a hydro storage plant is the spot price. A high spot price should trigger the exercise of an option and hence production, whereas a low spot price should imply no production or even to pump. An almost full dam also indicates that one should produce, since this would decrease the probability of the value destroyer, water spill. An empty dam, on the other hand, should signalize a cautious dispatch and maybe initiate pumping. Further, if the utility has volume risk in its portfolio, caused for example by swing option contracts, then intuitively high demand from such sold swing options should trigger the exercise of an option, as a hedge against the volume risk. With the same reasoning a low demand should not trigger production, but maybe allow for pumping.

Since the optimal exercise conditions are unknown, we take the following heuristic approach. Let the dispatch for a given hydro storage plant in period k be given by

$$x_k^+ = \sum_{i=1}^{r} \gamma_i^+ g_i^+(S_k, D_k, I_k^a, t), \quad k = 1, \ldots, K, \tag{6.12}$$

$$x_k^- = \sum_{i=1}^{\tau} \gamma_i^- g_i^-(S_k, D_k, I_k^a, t), \quad k = 1, \ldots, K, \tag{6.13}$$

where g_i^+ and g_i^- are functions $\mathbb{R}^4 \to \mathbb{R}$, and γ_i^+ and γ_i^- are weighting factors corresponding to the functions g_i^+ and g_i^- respectively. $I_k^a = \sum_{i=1}^{k} I_i$ is the aggregated inflow until period k and D_k is the total demand in the portfolio's swing options in period k.

As one can see, the dispatch is not explicitly a function of the water level V. Implicitly the dispatch is however a function of V, since the factors that determine V are in the functions g_i^+ and g_i^-. The reason for not having the dispatch as an explicit function of the water level is that itself is a function of the dispatch in previous periods as seen in (6.4). Unless the functions g_i^+ and g_i^- would be linear in V, the constraints on the water level would indeed be non linear. Since we have natural boundaries on the dispatch, given by the technical constraints (6.3), it is not feasible to let g_i^+ and g_i^- be linear functions of V, except of course for the trivial case, where the dispatch function is constant in V. The only possibility to explicitly invoke the water level as an exercise conditions would therefore be to introduce non-linearity in the model. This is something that we want to avoid to be able to take advantage of the superior computational features of linear programming. The dispatch decision will hence not directly be a function of the water level. That is why the aggregated inflow is modeled as a variable in the exercise functions and not the instantaneous inflow. It is natural to have the instantaneous spot price and demand as variables, since these two parameters directly influence the profit and loss function and hence the risk. The inflow, on the other hand, does not directly influence the profit and as seen in (6.4) the aggregated inflow describes the current water level better than the instantaneous inflow.

To formalize our ideas on how the dispatch should react to changes in spot price, demand and aggregated inflow, we impose the conditions that g_i^+ should be increasing and g_i^- decreasing in spot price, demand and aggregated inflow respectively, which is consistent with basic economic ideas, such as selling at high prices and buying at low prices

$$\begin{aligned} &g_i^+(S+\Delta_S, D+\Delta_D, I^a+\Delta_I, t) \geq g_i^+(S, D, I^a, t), \\ &\quad \forall(\Delta_S, \Delta_D, \Delta_I) \geq 0,\ i=1,\ldots,r, \end{aligned} \tag{6.14}$$

$$\begin{aligned} &g_i^-(S+\Delta_S, D+\Delta_D, I^a+\Delta_I, t) \leq g_i^-(S, D, I^a, t), \\ &\quad \forall(\Delta_S, \Delta_D, \Delta_I) \geq 0,\ i=1,\ldots,\tau. \end{aligned} \tag{6.15}$$

In order to fulfill the technical constraints and to allow a full range in the dispatch we further impose that

$$\max_{S,D,I^a,t}(g_i^+) = p_{max},\ \min_{S,D,I^a,t}(g_i^+) = 0, \quad i=1,\ldots,r \tag{6.16}$$

$$\max_{S,D,I^a,t}(g_i^-) = p_{min},\ \min_{S,D,I^a,t}(g_i^-) = 0, \quad i=1,\ldots,\tau, \tag{6.17}$$

and that the weighting factors sum up to 1 together with non-negativity

$$\sum_{1=1}^{r} \gamma_i^+ = 1, \quad \gamma_i^+ \geq 0, \quad i=1,\ldots,r, \tag{6.18}$$

$$\sum_{1=1}^{r} \gamma_i^- = 1, \quad \gamma_i^- \geq 0, \quad i = 1, \ldots, \tau. \tag{6.19}$$

Through (6.16)-(6.19) the constraints $0 \leq x_k^+ \leq p_{max}$ and $0 \leq x_k^- \leq p_{min}, \quad k = 1, \ldots, K$ will be fulfilled and the dispatch will be able to take the extreme values hence allowing the dispatch to cover its full technical range.

The decision to produce or pump is solely determined by the spot price, demand, aggregated inflow and time. Even though it seems reasonable that the dispatch should be a function of those variables, we do not know the shape of these functions. The key in this dynamic approach is therefore to let the weighting factors $\gamma = (\gamma_1^+, \ldots, \gamma_r^+, \gamma_1^-, \ldots, \gamma_\tau^-)$ be the decision variables, instead of letting the dispatch x_k in itself be a decision variable, as is the case in the static dispatch strategy. The optimization problem will thus give us the best convex combination of the, exogenously given, exercise functions g_i^+ and g_i^-.

One could let g_i^+ and g_i^- be functions of also other variables, such as volatility or jump frequency. However, these variables are in this work assumed to be deterministic and for a given volatility and jump frequency, the state variable time is actually sufficient to take these possibly time varying variables into account. If we change the volatility or jump frequency, the weighting factors, building up the optimal exercise condition, may however change.

6.5. Analysis of optimal dispatch strategy

A crucial issue is to derive the correct type of exercise function. We state that the step function is a natural choice. A step function is a function that can only take two values. It is hence easy to interpret, it is limited and can fulfill the conditions (6.14)-(6.17). But more important, we can show that under certain assumptions the step function is indeed the correct one.

Consider the hydro dispatch problem, where the expected profit is maximized under the sustainability constraint that the expected end-water level must exceed or equal a threshold

$$\begin{aligned} \max_{\gamma} \quad & E[-l(\gamma, Y)] \\ s.t. \quad & \bar{V}_K \geq V_{end}. \end{aligned} \tag{6.20}$$

We hence assume that the water constraint (6.10) is non-binding. Since risk is of no concern, the hydro dispatch is independent of the contract portfolio and naturally only depends on the spot price and time. To simplify we assume that the distribution function of the spot price f_S is constant over time, why the dispatch only depends on the spot price.

6.5.1. Hydro production strategy

To start with, we consider a hydro storage plant with no pumping capacity. The exercise functions are given as step functions

$$g_i^+ = \begin{cases} p_{max} & \text{if } S \geq S_i^+ \\ 0 & \text{otherwise} \end{cases}, \quad i = 1, \ldots, r,$$

where $0 = S_1^+ < S_2^+ < \ldots < S_r^+$ is a given ordered set of spot prices. The dispatch problem (6.20) can then be formulated as

$$\begin{aligned} \max_{\gamma^+ \in \mathbb{R}^r} \quad & K \cdot E\left[S \sum_{i=1}^{r} \gamma_i^+ g_i^+\right] \\ s.t. \quad & K \cdot E\left[\sum_{i=1}^{r} \gamma_i^+ g_i^+\right] \leq W \\ & \sum_{i=1}^{r} \gamma_i^+ = 1, \ \gamma_i^+ \geq 0, \quad i = 1, \ldots, r, \end{aligned} \tag{6.21}$$

where $W = V_0 - V_{end} + \sum_{i=1}^{K} \bar{I}_i$ denotes the average available water for production.

To facilitate reading denote

$$a_i^+ = \int\limits_{S \geq S_i^+} S f_S dS, \quad i = 1, \ldots, r,$$

$$b_i^+ = \int\limits_{S \geq S_i^+} f_S dS, \quad i = 1, \ldots, r.$$

Evidently

$$b_r^+ \leq b_{r-1}^+ \leq \ldots \leq b_1^+$$

and

$$a_r^+ \leq a_{r-1}^+ \leq \ldots \leq a_1^+$$

and the inequalities will be strict if $f_S > 0$ on $[S_1^+, S_r^+]$, which we will assume to simplify the analysis.

The optimal dispatch problem (6.21) can now be written as

$$\begin{aligned} \max_{\gamma^+ \in \mathbb{R}^r} \quad & K p_{max} \sum_{i=1}^{r} \gamma_i^+ a_i^+ \\ s.t. \quad & K p_{max} \sum_{i=1}^{r} \gamma_i^+ b_i^+ \leq W \\ & \sum_{i=1}^{r} \gamma_i^+ = 1, \ \gamma_i^+ \geq 0, \quad i = 1, \ldots, r. \end{aligned} \tag{6.22}$$

Note that by defining $W^\star = \frac{W}{K p_{max}}$, where $K p_{max} > 0$, (6.22) reduces to

$$
\begin{aligned}
\max_{\gamma^+ \in \mathbb{R}^r} \quad & \sum_{i=1}^{r} \gamma_i^+ a_i^+ \\
s.t. \quad & \sum_{i=1}^{r} \gamma_i^+ b_i^+ \le W^\star \\
& \sum_{i=1}^{r} \gamma_i^+ = 1, \ \gamma_i^+ \ge 0, \quad i = 1, \dots, r.
\end{aligned}
\tag{6.23}
$$

To avoid a trivial solution we assume that $r \ge 2$, i. e. that the owner of the plant has a choice of at least two thresholds, above which he should produce.

Theorem 6.1
For $b \in [b_i^+, b_{i-1}^+]$, let λ be such that $b = \lambda b_i^+ + (1-\lambda) b_{i-1}^+$, where $i = 2, \dots, r$ and define $a(b) = \lambda a_i^+ + (1-\lambda) a_{i-1}^+$, where $\lambda = \frac{b_{i-1}^+ - b}{b_{i-1}^+ - b_i^+}$.

Then $a(b)$ is a concave, monotone increasing function in b as illustrated in Figure 6.2.

This theorem, proven in Appendix A on page 193, will now facilitate our investigation of the optimal dispatch strategy.

Corollary 6.2
The solution of (6.23) is given by

(i) In the trivial case, where $W^\star \ge b_1^+$, $\gamma_1^+ = 1$

(ii) If $W^\star \le b_r^+$ then the problem is infeasible

(iii) Else $W^\star \in [b_i^+, b_{i-1}^+]$ for some i and $\gamma_i^+ = \frac{b_{i-1}^+ - W^\star}{b_{i-1}^+ - b_i^+}$, $\gamma_{i-1}^+ = 1 - \gamma_i^+$.

A proof is given in Appendix A on page 195.

Corollary 6.3
Evidently the optimal policy when maximizing the expected profit given a constraint on the averaged produced water is to choose $S^\star$ such that

$$\int_{S \ge S^\star} f_S dS = W^\star.$$

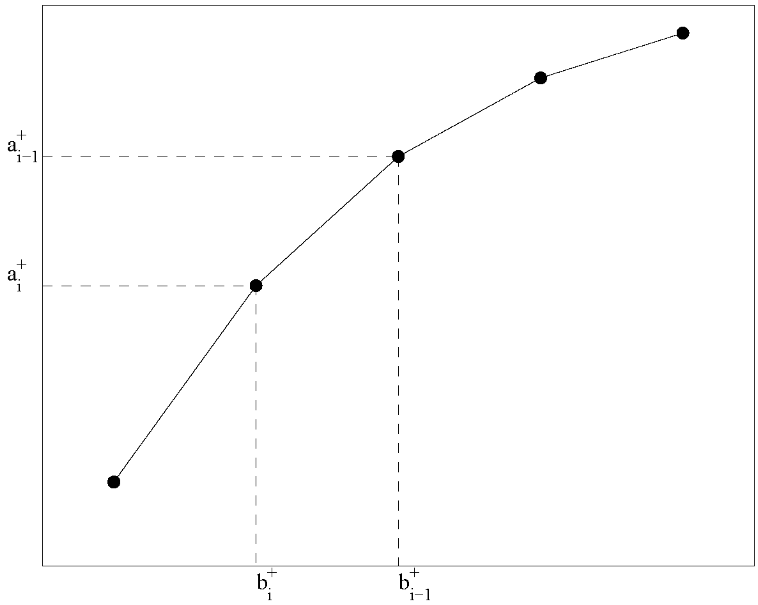

Fig. 6.2: a being concave and monotone increasing in b.

A proof is given in Appendix A on page 195. Hence we have shown that step functions in the case of no pumping are the correct exercise functions for a hydro storage plant when the risk constraint and the probabilistic water constraints are inactive.

We now introduce the so-called *marginal value of water*, which we will need later on in Chapter 6.6.8 when investigating the value of a hydro storage plant's operational flexibility.

Corollary 6.4
The marginal value of water $\frac{z(W^\star+\Delta)-z(W^\star)}{\Delta}$

(i) Is in the trivial case, where $W^\star > b_1^+$ *zero*

(ii) Is in the general case, where $W^\star \in [b_i^+, b_{i-1}^+)$ *for some* i*, given by* $\frac{a_{i-1}^+-a_i^+}{b_{i-1}^+-b_i^+}$*, and is constant in the interval* $\Delta \in [b_i^+ - W^\star, b_{i-1}^+ - W^\star]$.

The corollary is proven in Appendix A on page 195.

Also the average price of produced electricity will be of importance when investigating the flexibility.

Proposition 6.5
The marginal value of water is never exceeding the average price of produced electricity $\frac{z(W^\star)}{W^\star} \geq \frac{z(W^\star+\Delta)-z(W^\star)}{\Delta}$.

The proposition is proven in Appendix A on page 196.

6.5.2. Hydro production and pump strategy

Now we introduce also pumping and the corresponding exercise functions

$$g_j^- = \begin{cases} p_{min} & \text{if } S < S_j^- \\ 0 & \text{otherwise} \end{cases}, \quad j = 1, \ldots, \tau,$$

where $0 = S_1^- < S_2^- < \ldots < S_\tau^-$ is a ordered set of spot prices. Similarly to the pure production problem we define

$$a_j^- = \int_{S<S_j^-} S f_S dS, \quad j = 1, \ldots, \tau,$$

$$b_j^- = \int_{S<S_j^-} f_S dS, \quad j = 1, \ldots, \tau.$$

For a pump efficiency of χ the production-pumping problem is then given by

$$\max_{\gamma^+\in\mathbb{R}^r,\ \gamma^-\in\mathbb{R}^\tau} \quad Kp_{max}\sum_{i=1}^{r}\gamma_i^+a_i^+ - \frac{Kp_{min}}{\chi}\sum_{j=1}^{\tau}\gamma_j^-a_j^- \tag{6.24}$$
$$s.t. \quad Kp_{max}\sum_{i=1}^{r}\gamma_i^+b_i^+ - Kp_{min}\sum_{j=1}^{\tau}\gamma_j^-b_j^- \le W$$
$$\sum_{i=1}^{r}\gamma_i^+ = 1,\ \gamma_i^+ \ge 0, \quad i = 1,\ldots,r$$
$$\sum_{j=1}^{\tau}\gamma_j^- = 1,\ \gamma_j^- \ge 0, \quad j = 1,\ldots,\tau.$$

Define $\rho = \frac{p_{min}}{p_{max}} \ge 0$ and again $W^\star = \frac{W}{Kp_{max}}$. Then (6.24) becomes equivalent to

$$\max_{\gamma^+\in\mathbb{R}^r,\ \gamma^-\in\mathbb{R}^\tau} \quad \sum_{i=1}^{r}\gamma_i^+a_i^+ - \frac{\rho}{\chi}\sum_{j=1}^{\tau}\gamma_j^-a_j^- \tag{6.25}$$
$$s.t. \quad \sum_{i=1}^{r}\gamma_i^+b_i^+ - \rho\sum_{j=1}^{\tau}\gamma_j^-b_j^- \le W^\star$$
$$\sum_{i=1}^{r}\gamma_i^+ = 1,\ \gamma_i^+ \ge 0, \quad i = 1,\ldots,r$$
$$\sum_{j=1}^{\tau}\gamma_j^- = 1,\ \gamma_j^- \ge 0, \quad j = 1,\ldots,\tau.$$

Now by definition

$$a_j^- = \int_{S<S_j^-} Sf_S dS = \int_S Sf_S dS - \int_{S\ge S_j^-} Sf_S dS = E[S] - a_j^+, \quad j = 1,\ldots,\tau$$

and similarly

$$b_j^- = (1 - b_j^+), \quad j = 1, \ldots, \tau,$$

where j denotes an index from the set of pumping thresholds $\{S_j^-\}_{j=1}^{\tau}$ as opposed to an index from the set of production thresholds $\{S_i^+\}_{i=1}^{r}$.

Using these expressions in (6.25) results in

$$\begin{aligned} \max_{\gamma^+ \in \mathbb{R}^r,\ \gamma^- \in \mathbb{R}^\tau} \quad & \sum_{i=1}^{r} \gamma_i^+ a_i^+ + \frac{\rho}{\chi}\Big[\sum_{j=1}^{\tau} \gamma_j^- a_j^+ - E[S]\Big] \qquad (6.26) \\ s.t. \quad & \sum_{i=1}^{r} \gamma_i^+ b_i^+ + \rho \sum_{j=1}^{\tau} \gamma_j^- b_j^+ \leq W^\star + \rho \\ & \sum_{i=1}^{r} \gamma_i^+ = 1,\ \gamma_i^+ \geq 0, \quad i = 1, \ldots, r \\ & \sum_{j=1}^{\tau} \gamma_j^- = 1,\ \gamma_j^- \geq 0, \quad j = 1, \ldots, \tau. \end{aligned}$$

Again $\frac{\rho}{\chi} a$ is concave and monotone increasing in b as shown by Theorem 6.1 and hence we can conclude

Corollary 6.6
In the solution of (6.26)

(i) *If $W^\star \geq b_1^+$ then $\gamma_1^+ = 1$ and $\gamma_1^- = 1$ is optimal*

(ii) *If $W^\star + \rho < b_r^+ + \rho b_\tau^+$ then the problem is infeasible*

(iii) *Else at most two adjacent production weights γ_{i-1}^+ and γ_i^+ and at most two adjacent pumping weights γ_{j-1}^- and γ_j^- will be non-zero.*

A proof is given in Appendix A on page 196. To actually derive the solution we however need to study the dual problem. Consider the dual to the production-pumping problem (6.26)

$$\begin{aligned}\min_{u\in\mathbb{R},\ v\in\mathbb{R},\ w\in\mathbb{R}} \quad & (W^\star+\rho)u+v+w \\ s.t. \quad & b_i^+u+v\geq a_i^+, \quad i=1,\ldots,r \\ & \rho b_j^+u+w\geq \frac{\rho}{\chi}a_j^+, \quad j=1,\ldots,\tau \\ & u\geq 0.\end{aligned} \tag{6.27}$$

Define

$$v(u)=\max_i(a_i^+-b_i^+u)$$

and

$$w(u)=\rho\max_j(\frac{a_j^+}{\chi}-b_j^+u).$$

Evidently $v(u),\ u\geq 0$ forms the lower envelope of the feasible region $(u,v)\subset\mathbb{R}^2$, as illustrated in Figure 6.3. Similarly $w(u),\ u\geq 0$ forms the lower envelope of the feasible region $(u,w)\subset\mathbb{R}^2$.

Observe that $v(u)$ can be specified by the points

$$\begin{aligned}&[\hat{u}_1=0,a_1^+]\\ &[\hat{u}_i,a_i^+-b_i^+\hat{u}_i],\ i=2,\ldots,r\\ &[\hat{u}_{r+1}=\frac{a_r^+}{b_r^+},0],\end{aligned} \tag{6.28}$$

where $\hat{u}_i$ is the solution to $a_{i-1}^+-b_{i-1}^+u=a_i^+-b_i^+u$, given by

$$\hat{u}_i=\frac{a_{i-1}^+-a_i^+}{b_{i-1}^+-b_i^+},\ i=2,\ldots,r.$$

Following from Theorem 6.1 it is easy to see that $v(u)$ is specified by these points by observing that the point $\hat{u}_1<\hat{u}_2<\ldots<\hat{u}_{r+1}$, the intersection

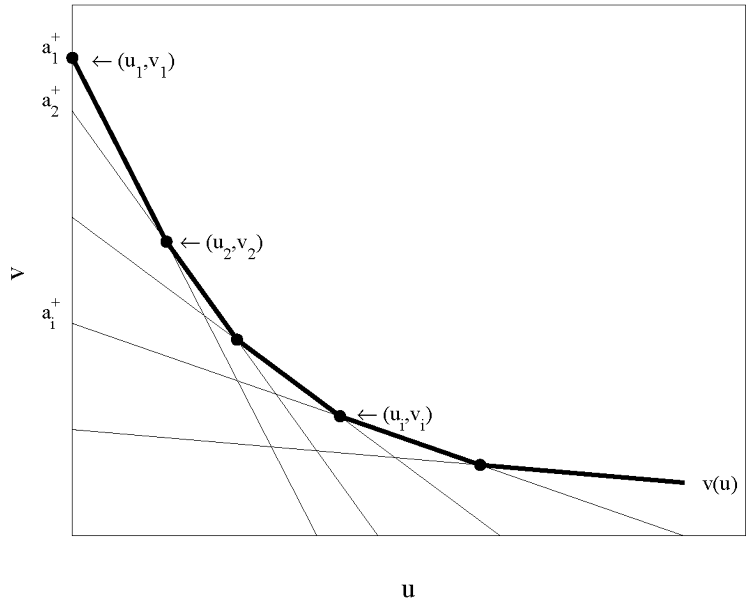

Fig. 6.3: Lower envelope of the dual variable v.

$a_1^+ > a_2^+ > \ldots > a_r^+ \geq 0$ and the slope $b_1^+ > b_2^+ > \ldots > b_r^+ \geq 0$.

The lower envelope $w(u)$ has similar properties and can be specified by

$$
\begin{aligned}
&[\bar{u}_1 = 0, \frac{\rho}{\chi} a_1^+] \\
&[\bar{u}_j, \frac{\rho}{\chi} a_j^+ - \rho b_j^+ \bar{u}_j], \; j = 2, \ldots, \tau \\
&[\bar{u}_{\tau+1} = \frac{a_\tau^+}{b_\tau^+ \chi}, 0],
\end{aligned}
\tag{6.29}
$$

where

$$
\bar{u}_j = \frac{1}{\chi} \cdot \frac{a_{j-1}^+ - a_j^+}{b_{j-1}^+ - b_j^+}, \; j = 2, \ldots, \tau.
$$

Hence the dual (6.27) can be reduced to

$$
\min_{u \geq 0} z(u) = (W^\star + \rho)u + v(u) + w(u) \tag{6.30}
$$

and solved by checking the functional value at $[\hat{u}_i],\ i = 1, \ldots, r+1$ and at $[\bar{u}_j],\ j = 1, \ldots, \tau + 1$.

At $\hat{u}_i$ either $\exists j$ such that $\hat{u}_i \in [\bar{u}_j, \bar{u}_{j+1}]$ and $z(\hat{u}_i)$ is given by

$$z(\hat{u}_i) = (W^\star + \rho)\hat{u}_i + v(\hat{u}_i) + \big(\lambda w(\bar{u}_j) + (1-\lambda) w(\bar{u}_{j+1})\big),$$

where $\lambda = \frac{\bar{u}_{j+1} - \hat{u}_i}{\bar{u}_{j+1} - \bar{u}_j}$, or else $\hat{u}_i > \bar{u}_{\tau+1} \Longrightarrow w(\hat{u}_i) = 0$ and $z(\hat{u}_i)$ is consequently given by

$$z(\hat{u}_i) = (W^\star + \rho)\hat{u}_i + v(\hat{u}_i).$$

Similarly, at $\bar{u}_j$ either $\exists i$ such that $\bar{u}_j \in [\hat{u}_i, \hat{u}_{i+1}]$ and $z(\bar{u}_j)$ is given by

$$z(\bar{u}_j) = (W^\star + \rho)\bar{u}_j + \big(\lambda v(\bar{u}_i) + (1-\lambda) v(\bar{u}_{i+1})\big) + w(\bar{u}_j),$$

where $\lambda = \frac{\hat{u}_{i+1} - \bar{u}_j}{\hat{u}_{i+1} - \hat{u}_i}$, or else $\bar{u}_j > \hat{u}_{r+1} \Longrightarrow v(\bar{u}_j) = 0$ and $z(\bar{u}_j)$ is hence given by

$$z(\bar{u}_j) = (W^\star + \rho)\bar{u}_j + w(\bar{u}_j).$$

The optimum is consequently found by choosing the minimum among

$$\big\{z(\hat{u}_i)_{i=1,\ldots,r+1},\ z(\bar{u}_j)_{j=1,\ldots,\tau+1}\big\}.$$

With the help of the dual we can now sharpen the results of Corollary 6.6.

Proposition 6.7
In the solution of (6.26) at the most three weights are non-zero and given by

(i) *In the trivial case, where there is abundant water* $\gamma_1^+ = \gamma_1^- = 1$.

(ii) *In the general case where water is scarce, either one production weight equals one, i. e.* $\gamma_i^+ = 1$ *for an* $i \in \{1, \ldots, r\}$, *and two adjacent pumping weights* γ_{j-1}^- *and* γ_j^- *for a* $j \in \{2, \ldots, \tau\}$ *are non-zero and given by*

$$\gamma_j^- = \frac{W^\star + \rho - b_i^+ - \rho b_{j-1}^+}{\rho(b_j^+ - b_{j-1}^+)}, \quad \gamma_{j-1}^- = 1 - \gamma_j^-.$$

Or else one pumping weight equals one, i. e. $\gamma_j^- = 1$ *for an* $j \in \{1, \dots, \tau\}$ *and two adjacent production weights* γ_{i-1}^+ *and* γ_i^+ *for an* $i \in \{2, \dots, r\}$ *are non-zero and given by*

$$\gamma_i^+ = \frac{W^\star + \rho - \rho b_j^+ - b_{i+1}^+}{b_i^+ - b_{i+1}^+}, \quad \gamma_{i-1}^+ = 1 - \gamma_i^+.$$

The proposition is proven in Appendix A on page 197.

To further exploit the production and pumping problem (6.26), we study the special case, where $r = \tau$ and $S_1 = S_1^+ = S_1^-, \dots, S_r = S_r^+ = S_\tau^-$.

Corollary 6.8
The pumping decision is related to the production decision by the marginal value of water and

(i) Assume production at price S_i*, then the marginal value of water is* $\hat{u}_i$*. Pumping occurs below price* S_j $(j < i)$*, where the marginal value of water* $\hat{u}_j \le \chi \hat{u}_i \le \hat{u}_{j+1}$*.*

(ii) Assume pumping at price S_j *with marginal value of water* $\hat{u}_j$*. Then production occurs at price* S_i $(j < i)$ *where* $\chi \hat{u}_i \le \hat{u}_j \le \chi \hat{u}_{i+1}$*.*

The corollary is proven in Appendix A on page 198.

6.5.3. Conclusion and choice of exercise functions

We have in this section, for the special case, where the probabilistic water constraints and the risk constraint are inactive, derived a number of results and showed that step functions indeed are the correct exercise functions. In the general case, where risk and intermediate water level is of concern we can rely upon the approximation theorem, proved in, for example, [Rud76], that any finite and real valued function can be well approximated by a linear combination of step functions. This implies that *any* reasonable (i.e finite and real valued) dispatch can be well approximated by a linear combination of step functions. We have therefore chosen to work directly with such step functions also in the general case in the following manner

$$g_i^+ = \begin{cases} p_{max} & \text{if } S \geq S_i^+(t) \text{ and } D \geq D_i^+(t) \text{ and } I^a \geq I_i^{a+}(t) \\ 0 & \text{otherwise} \end{cases}, \tag{6.31}$$

$$g_i^- = \begin{cases} 0 & \text{if } S \geq S_i^-(t) \text{ or } D \geq D_i^-(t) \text{ or } I^a \geq I_i^{a-}(t) \\ p_{min} & \text{otherwise} \end{cases}. \tag{6.32}$$

The thresholds $S_i^+(t)$, $S_i^-(t)$, $D_i^+(t)$, $D_i^-(t)$, $I_i^{a+}(t)$ and $I_i^{a-}(t)$ are, as seen, functions of time. Assume a long time horizon of say one year. We know that electricity prices, demand and inflow show seasonal pattern and in Europe prices and demand are generally higher in the winter compared to the summer. The limited amount of water that can be stored implies that we should take these seasonal effects into account and, for example, allow production at lower price thresholds in the summer than in the winter. Price and demand thresholds that are functions of time are therefore preferable and the importance of having the aggregated inflow thresholds as a function of time follows also from its strong seasonality.

One possible and intuitive approach would be to choose the thresholds to be quantiles of the respective random variable's distribution. The spot price thresholds in the production functions g_i^+ could naturally be fairly high quantiles of the spot price, whereas the corresponding thresholds for the pumping functions g_i^- could be rather low quantiles. To exemplify, assume that we have four exercise functions for production and pumping respectively. The four spot price thresholds for production could then be assigned the 30%, 50%, 70% and 90% quantile respectively, whereas the pumping spot price thresholds could be given by the 10%, 30%, 50% and 70% quantile respectively. The time component is then in a natural way incorporated, since the quantiles vary over time as illustrated in Figure 6.4.

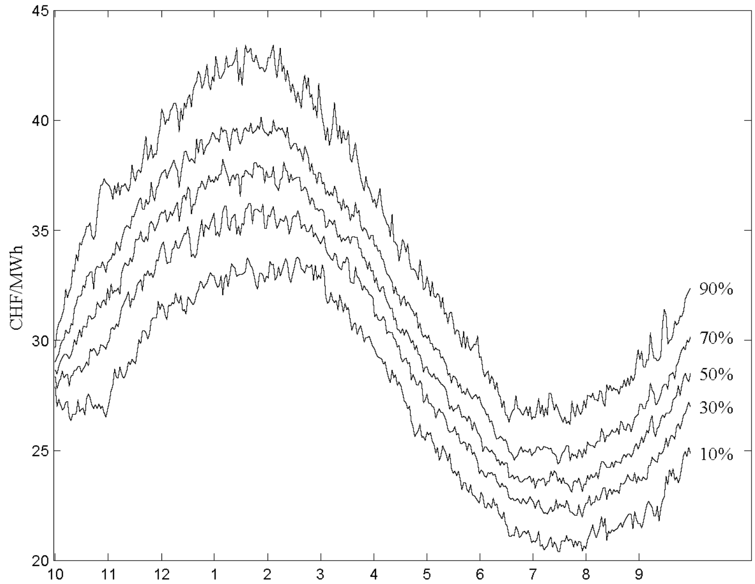

Fig. 6.4: Quantiles of typical spot price over one hydrological year starting in the beginning of October.

6.6. Power portfolio optimization with CVaR

We have now exploited the modeling of the production portfolio, where two approaches were presented for modeling a hydro storage plant. In this chapter we will add the contract portfolio, jointly optimize the two portfolios and analyze the result of the two different optimal portfolios stemming from the static dispatch strategy and the dynamic dispatch strategy respectively. We will only incorporate hydro storage plants in the production portfolio, but other plant types, except for the coal and oil plant, could because of their simple dispatch strategies easily be included. To make the model operational we now introduce J scenarios sampled from the probability distribution of the stochastic factors $Y = (S, D, I)$ according to their density f_Y. Each scenario $y_j,\ j = 1, \ldots, J$ is a joint path of S, D and I, between which dependences may exist, over the periods $1, \ldots, K$.

6.6.1. Contract portfolio

The contract portfolio in this work potentially consists of three different asset classes. The utility has access to a futures and a spot market and is assumed to have sold swing options and is hence carrying a short position, causing a price and volume risk. Yet more asset classes, such as plain options, could easily be introduced. This would however not contribute to analysis, but introduce more notation.

We assume that there exist $m-2K$ different future contracts in the market. The position in each of them are denoted $x_{2K+1}, \ldots, x_m$ [MWh/period] respectively.[1] The utility may go long or short in any of these traded future contracts and $x_{2K+1}, \ldots, x_m$ are the decision variables corresponding to the first asset class. Since different futures have distinct periods of underlying spot contracts, we let X_k denote the set of futures, which have the spot contract in period k as underlying. A future with only the spot contract in period 1 or a future with only the spot contracts in period 3 and 4 as underlying would hence not belong to X_2, whereas the first future belongs to X_1 and the latter to X_3 and X_4.

Further, we assume that there are $n - m$ different available swing options in the OTC market. The corresponding positions in each swing option are denoted $x_{m+1}, \ldots, x_n$ [number of contracts],[2] which are the decision variables corresponding to the second asset class. Since these contracts are OTC, they may lack liquidity, why a trading constraint on these positions may be imposed. In the case where the positions in the swing options are allowed to be altered, we need to know the demand stochastics of each swing option, $D_{k,j,m+1}, \ldots, D_{k,j,n}$ [MWh], in each period k and each scenario j. When, on the other hand, these positions cannot be adjusted due to, for example, lack of liquidity, then only the aggregated stochastic demand $D_{k,j} = \sum_{i=m+1}^{n} D_{k,j,i}$ [MWh] is of interest.

Because of the non-storability of electricity, the utility has to be in balance in

1 Observe that the first $2K$ subscripts are denoting production and pumping respectively in period $1, \ldots, K$.

2 Note that the load in the swing options are unknown, hence the unit [number of contracts].

the sense that the electricity that is bought or produced has to equal the electricity that is sold or used in each instance of time. Therefore the spot position in each period k and scenario j $x^{spot}_{k,j}$ is uniquely determined by the dispatch, the future positions and the swing option positions with their corresponding demand

$$x^{spot}_{k,j} = -\left[x^{+}_{k,j} - \frac{x^{-}_{k,j}}{\chi} + \sum_{i=2K+1}^{m} x_i \mathbf{1}_{x_i \in X_k} + \sum_{i=m+1}^{n} x_i D_{k,j,i}\right]. \qquad (6.33)$$

The spot price in period k and scenario j is denoted $S_{k,j}$ [CHF/MWh].

The decision variables in the contract portfolio, i. e. the universe of available contracts,[3] can hence be written as $x^c = (x_{2K+1}, \ldots, x_n) \in \mathbb{R}^{n-2K}$. These decision variables can be restricted by the general constraint $x^c \in X^c$, where for example the problematics to adjust an OTC position may be enforced.

6.6.2. Loss function

The loss function can be divided into four parts, where the first part is attributed to the marginal cost to run the hydro storage plant. Let c^+ [CHF/MWh] denote the marginal production costs and c^- [CHF/MWh] the marginal pumping costs, exclusive the cost of electricity, then the total marginal costs in period k and scenario j are given by $c^+ x^+_{k,j} + c^- x^-_{k,j}$. The second part is the profit and loss derived from the futures positions. Let c_i, $i = 2K+1, \ldots, m$ [CHF/MWh] denote the price of respective future at the beginning of the planning period, then the futures loss in period k and scenario j is given by $\sum_{i=2K+1}^{m}(c_i - S_{k,j})x_i \mathbf{1}_{x_i \in X_k}$. The third part is the profit and loss stemming from the swing options. Let c_i, $i = m+1, \ldots, n$ [CHF/MWh] denote the price of respective swing option at the beginning of the planning period, then the swing option loss in period k and scenario j is given by $\sum_{i=m+1}^{n}(c_i - S_{k,j})x_i D_{k,j,i}$. The fourth part is the profit and loss derived from selling and buying electricity as an effect of the hydro storage plant operations. This loss is in period k and scenario j given by $-S_{k,j}(x^+_{k,j} - x^-_{k,j}/\chi)$. If we gather all components depending on the

[3] Except for the spot position that is uniquely determined by the other positions.

spot price in $x_{k,j}^{spot}$ as in (6.33) we get the following loss function in period k and scenario j

$$l(x, y_j)_k = c^+ x_{k,j}^+ + c^- x_{k,j}^- + \sum_{i=2K+1}^{m} c_i x_i \mathbf{1}_{x_i \in X_k} + \sum_{i=m+1}^{n} c_i x_i D_{k,j,i} + S_{k,j} x_{k,j}^{spot},$$

where x denotes the decision variables, which in the static case is given by (x^p, x^c), and in the dynamic case by (γ, x^c). If we discount these periodic losses and sum them up over the whole horizon, we get the overall loss function in scenario j

$$\begin{aligned} l(x, y_j) &= \sum_{k=1}^{K} e^{-kr} \Big(c^+ x_{k,j}^+ + c^- x_{k,j}^- + \sum_{i=2K+1}^{m} c_i x_i \mathbf{1}_{x_i \in X_k} + \\ &+ \sum_{i=m+1}^{n} c_i x_i D_{k,j,i} + S_{k,j} x_{k,j}^{spot} \Big), \end{aligned}$$

where r is a one-periodic continuously compounded discount rate. The importance of the spot position x^{spot} should not be underestimated. The profit from the production is created here and also much of the risk arises from this synthetic position. Actually all volume risk is modeled here.

Note that the positions are not closed at the end of the horizon. We assume that the maturity of the futures and swing options do not exceed the horizon. This assumption can easily be relaxed but, would introduce yet more notations, since the contracts would need to be priced at the end of the horizon. Observe further the great flexibility that our approach offers by working with scenarios, and that essentially any contract can be modeled.

6.6.3. Static portfolio optimization

In this static optimization problem the goal is to find, except for the optimal contract portfolio x^c, the optimal production portfolio x^p, given by

$$x^p = (x_1^+, \dots, x_K^+, x_1^-, \dots, x_K^-)$$

where $x_1^+, \dots, x_K^+$ and $x_1^-, \dots, x_K^-$ denote production and pumping respectively in each period. This dispatch is in the contract engineering framework equivalent to K futures that summed up must not exceed the amount of energy determined by the constraint on the average water level $\bar{V}$. Our obvious decision variables are consequently given by

$$x = (x^p, x^c) = (x_1^+, \dots, x_K^+, x_1^-, \dots, x_K^-, x_{2K+1}, \dots, x_n).$$

The hydro storage plant however has yet another component of decision variables, namely the average spill over losses, $\bar{L}_k$. The spill over loss can be seen as a dummy variable being assigned the value $(\bar{V}_k - V_{max})^+$, but since $\bar{V}_k$ is a function of our decisions x^p, also the average spill over losses $\bar{L}_k$ have to be modeled as decision variables.

Our objective is as in (6.1) to maximize the expected profit given that the risk, measured as CVaR, is kept below an acceptable level C. In Chapter 3.5.3 we stated that this problem, under certain assumptions, can be reduced to the linear program

$$\begin{array}{ll} \max\limits_{x\in\mathbb{R}^n, z\in\mathbb{R}^J, \alpha\in\mathbb{R}} & -\frac{1}{J}\sum_{j=1}^{J} l(x, y_j) \\ \text{s.t.} & \alpha + \frac{1}{(1-\beta)J}\sum_{j=1}^{J} z_j \leq C \\ & z_j \geq l(x, y_j) - \alpha, \; z_j \geq 0, \; j = 1, \dots, J \\ & x \in X. \end{array} \tag{6.34}$$

To recap, the five assumptions are: 1) The probability distribution of Y does not depend on x, 2) $l(x, Y)$ is continuous in x, 3) $l(x, \cdot)$ is measurable, 4) $l(x, Y)$ is integrable $\forall x \in X$ and 5) $l(x, Y)$ is linear in (x) and that X is a polyhedral set. The first assumption implies that we have to assume that there exists no *transaction costs*. The transaction cost in terms of the bid-ask spreads, is probably negligible compared to the effect that a dominant utility could have on a thin spot market. Especially in the electricity market with auctioning, where the price explicitly is a function of the supply and demand, a big player may move the market when his portfolio is adjusted. We here have to assume that the market is liquid enough to swallow any trade of our utility without affecting the price. This assumption about no transaction costs is used elsewhere in finance and is one of the prerequisites for a *frictionless* and *perfect market*, see for example [Mer90]. We also assume that the player is a price-taker and not a price-maker, i. e. that the player has no market power. The literature basically supports this assumption for the Scandinavian electricity market. Amundsen et al. [ABA98] find that the Cournot equilibrium prices are close to perfectly competitive prices and Fleten [Fle00] conclude that pure hydro producers have no or little market power and that the simulated prices in an oligopoly case is less than 7% higher than under perfect competition. Johnsen et al. [JVW99] though find evidence for the use of market power in the southern part of Norway. In other electricity markets the literature gives a diversified picture on market power. Borenstein & Bushnell [BB99] find that the potential for market power in California is particularly large in the low hydro-generation seasons. Bushnell [Bus98] state that hydro producers may profitable schedule hydro to off-peak from peak periods. For more information on market power in the electricity industry, see for example [Mou01, OWP01, Sch01] The second, third and fourth assumptions are of rather technical form and will be fulfilled for any reasonable stochastic modeling and for any reasonable portfolio. And unless we invoke non-linearities in the general constraint $x^c \in X^c$ the fifth assumption will also be fulfilled, since only linear constraints limit the production portfolio x^p. It is obvious that the above assumptions are fulfilled also for the auxiliary decision variables $\bar{L}_k$.

A simultaneous optimization of the static hydro dispatch and the contract portfolio in a CVaR and profit framework as in (6.34) will thus be given by the solution of the following linear program

$$\max_{x\in\mathbb{R}^n, z\in\mathbb{R}^J, \alpha\in\mathbb{R}, \bar{L}\in\mathbb{R}^K} -\frac{1}{J}\sum_{j=1}^{J} l(x, y_j) \tag{6.35}$$

subject to

$$\alpha + \frac{1}{(1-\beta)J}\sum_{j=1}^{J} z_j \leq C \tag{6.36}$$

$$z_j \geq l(x, y_j) - \alpha, \quad j = 1, \ldots, J \tag{6.37}$$

$$0 \leq \bar{V}_k = V_0 + \sum_{i=1}^{k} \bar{I}_i - \sum_{i=1}^{k} \bar{L}_i - \sum_{i=1}^{k} \left(x_i^+ - x_i^-\right), \quad k = 1, \ldots, K \tag{6.38}$$

$$\bar{V}_k = V_0 + \sum_{i=1}^{k} \bar{I}_i - \sum_{i=1}^{k} \bar{L}_i - \sum_{i=1}^{k} \left(x_i^+ - x_i^-\right) \leq V_{max}, \quad k = 1, \ldots, K \tag{6.39}$$

$$\bar{L}_k \geq 0, \quad k = 1, \ldots K \tag{6.40}$$

$$\bar{V}_K = V_0 + \sum_{i=1}^{K} \bar{I}_i - \sum_{i=1}^{K} \bar{L}_i - \sum_{i=1}^{K} \left(x_i^+ - x_i^-\right) \geq V_{end} \tag{6.41}$$

$$0 \leq x_k^+ \leq p_{max}, \quad k = 1, \ldots, K \tag{6.42}$$

$$0 \leq x_k^- \leq p_{min}, \quad k = 1, \ldots, K \tag{6.43}$$

$$x^c \in X^c. \tag{6.44}$$

The first two constraints (6.36) and (6.37) governs that the risk is kept below an acceptable level. The third restriction (6.38) prevents us from producing when, on average, no water is available. One could here introduce a positive minimal water level to account for e. g. environmental restrictions. Constraint (6.39) assures that we, on average, do not pump water above the maximal accepted level V_{max} and together with (6.40) that excessive water inflow is lost through spillover. Through (6.41) we assure sustainability. By assuming positive spot prices the optimization would otherwise try to empty the dam. This myopic solution would of course heavily penalize the cash flows stemming from the hydro storage plant after the horizon, which we try to avoid. Constraint (6.42) and (6.43) put technical limits on the dispatch and assures that that the optimal static dispatch lies within the feasible power range. Finally, the general constraint (6.44) can enforce any linear contract portfolio constraint, such as limit the possibility to adjust some OTC positions.

6.6.3.1. End-water level

The minimal end-water level V_{end} can be determined from, for example, a long-term optimization, where the boundary conditions on the water level are exogenous. One approach that seems reasonable is to conduct a yearly optimization corresponding to a hydrological year. In Switzerland, for example, the year starts in the beginning of October and the reservoirs should then in principal be full because of the dry and cold coming winter. The very low inflow during winter months, in conjunction with the high prices is indicating that stored water has a large value in the winter. The value of water, converted to electricity has a lower value in the summer and fall, as a result of the lower prices, compared to the winter, why a basically full dam at the beginning of the hydrological year seems reasonable. Observe that we say basically full. A hundred percent full dam would intuitively never be optimal when the inflow is stochastic. The payoff of additional inflow would be perfectly asymmetric, but not in our favor. More inflow than we can produce would be lost through spill over with no value creation. The situation with a full dam can hence be viewed as having sold an option from a payoff view. Some safety margin on the water level should consequently be obtained, to be able to benefit from large inflow. The size of the security margin depends on the dispersion of the inflow, on the maximum capacity of the turbines and on the price difference between summer and winter.

This rule of thumb of having a basically full dam at the beginning and hence end of a hydrological year has been used for decades, however in a regulated environment.

6.6.4. Size of static problem

The standard form for a linear program is given by

$$\begin{array}{ll} \max & c^T x \\ \text{s.t.} & Ax \leq b. \end{array} \tag{6.45}$$

Eq.	x^p	x^c	$\bar{L}$	z	α	#
(6.36)				$\vdots\vdots\vdots$	$\vdots\vdots\vdots$	1
(6.37)	$\vdots\vdots\vdots$	$\vdots\vdots\vdots$		$\ddots$	$\vdots\vdots\vdots$	J
(6.38)	$\vdots\ddots$		$\vdots\ddots$			K
(6.39)	$\vdots\ddots$		$\vdots\ddots$			K
(6.41)	$\vdots\vdots\vdots$		$\vdots\vdots\vdots$			1
(6.42)	$\ddots$					K
(6.43)	$\ddots$					K
#	$2K$	$n-2K$	K	J	1	

Tab. 6.1: Structure of the A matrix. $\vdots\vdots\vdots$ denotes a full sub-matrix, $\vdots\ddots$ denotes a triangular sub-matrix and $\ddots$ denotes a diagonal sub-matrix.

To get an idea of the size of the problem (6.35) - (6.43) we can study its corresponding A matrix in (6.45). Important factors determining the size of the linear program is the size of A, i. e. the number of variables and the number of constraints, and the density of A, i. e. the percentage non-zero elements.

The size of A is given by $A \in \mathbb{R}^{(4K+J+2)\times(n+K+J+1)}$.[4] For large problems, i. e. many periods and many scenarios, the number of variables will essentially be given by $3K + J$ and the number of constraints by $4K + J$. If we however study the structure of A as in Table 6.1 we see that most elements will be zero and that the density of A is given by

$$\frac{(n+3)J + 3K^2 + 5K + 1}{(4K + J + 2)(n + K + J + 1)}.$$

6.6.5. Analysis of static solution

The results from the optimization are interesting in many ways. It will deliver the optimal contract portfolio in the sense of tradable and OTC contracts, $x^\star_{2K+1}, \ldots, x^\star_n$. Further the optimal static dispatch will be given by $x^\star_1, \ldots, x^\star_K$.

[4] The general constraint (6.44) is not yet known, but could otherwise also be analysed.

Since the hydro storage plant is modeled as a series of hourly futures, the optimal static dispatch can be seen as the optimal way to sell the available water in the futures market. Interestingly, the optimal dispatch will not only sell the stored water in peak hours in the best possible way, it will also 'arbitrage' between futures in different periods.

The possibility to pump is actually a way to exchange electricity in one period for electricity in a later period, i. e. to virtually 'store' electricity. Since the static dispatch strategy is solely a function of time, the value of the pumping ability will be determined by the differences in futures prices between periods. Or if hourly futures do not exists, the value of the pumping will be determined by the daily and seasonal variations in the expected spot price. A risk free portfolio can then however not be achieved. But by assuming that hourly futures do exists, the optimal dispatch will try to 'arbitrage' between futures in different periods, through the hydro storage plant's virtual storage. The price difference needed to conduct pumping of course depends on the efficiency of the pump χ.

Let Φ denote the value of the hydro storage plant with pumps in terms of the expected profit given the optimal static strategy

$$\Phi = -\frac{1}{J}\sum_{j=1}^{J} l(x^\star, y_j),$$

which thus is given by the solution to (6.35)-(6.44). Let $\tilde{\Phi}$ similarly denote the value in terms of the expected profit given the optimal static strategy $\tilde{x}^\star$, where no pumping is allowed. The upper boundary on the pumping in constraint (6.43) p_{min} is hence set to zero. The value of the pump capacity can then be written as $\Phi - \tilde{\Phi}$.

To exemplify how this virtual storage is used, let the future prices in period 1 to 7 be as in Figure 6.5, where the price in period 0 for a future with delivery in period k is denoted by $F(0, k)$. Now assume that the following holds

$$\frac{F(0,1)}{F(0,3)} < \chi < \frac{F(0,5)}{F(0,7)}. \tag{6.46}$$

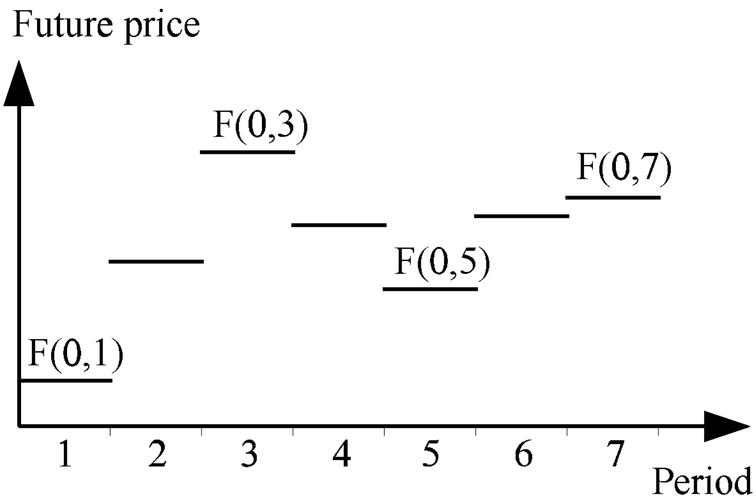

Fig. 6.5: Illustration of 'arbitrage' between futures with pumping capacity.

The relative difference between the future price in the first off-peak period $F(0,1)$ and in the first peak period $F(0.3)$ is according to (6.46) big enough to offset the non-perfect efficiency of the pump χ. If the optimal dispatch in period 3, where no pumping is allowed $\tilde{x}^\star < p_{max}$, and if there is enough capacity in the dam to swallow the pumped up water, then pumping will be conducted in period 1. This water will then be stored and used for production in period 3, adding the following deterministic profit

$$\Delta x_3^+ F(0,3) - \frac{\Delta x_1^-}{\chi} F(0,1), \quad \Delta x_3^+ = \Delta x_1^-. \tag{6.47}$$

Since the transaction does not involve any risk and according to the assumption (6.46), the value in (6.47) will be strictly positive, this does fulfill the definitions of arbitrage (see Definition 3.6). The optimal choice of Δx_3^+ and hence Δx_1^- will be given by $max(p_{max} - \tilde{x}^\star, p_{min})$, since one naturally wants to maximize this risk free profit. From (6.46) we note that between the second off-peak and second peak period, no such 'arbitrage' strategy exists, since the relative price difference between $F(0,5)$ and $F(0,7)$ simply is too low compared to the efficiency of the pump.

We can now draw two conclusions. Firstly, if hourly futures do exist, the value of pumping $\Phi - \tilde{\Phi}$ will be given by the optimal combination of such arbitrage deals. Observe that the optimal portfolio, where no pumping is allowed, will utilize as much risk as possible. Hence the risk constraint (6.36) will, if the general constraint (6.44) allows it, be binding. Further, since we assume that hourly futures are traded, we are not adding a new instrument to our optimization by allowing pumping, except for the possibility to virtually store electricity. Consequently the optimal production portfolio, where pumping is allowed $x^\star$ will equal the optimal production portfolio, without pumping $\tilde{x}^\star$ plus the optimal combination of these arbitrage deals. Secondly, if the market would be dominated by hydro storage plants with a pump efficiency of χ, then the future prices in different periods would be related by a absence of arbitrage argument as

$$\frac{e^{r(k-j)}}{\chi} F(i,j) \geq F(i,k), \quad \forall i \text{ and } \forall k > j.$$

One should however keep in mind that a hydro storage plant is associated with substantial capital costs, why this arbitrage argument is only applicable for existing plants. Further, the pump and turbine capacity is in most market small compared to the total capacity and in reality the inflow is stochastic.

Even without the pumping capacity the dam however facilitates storage possibilities, which certainly has some value. Let us in the same manner as in the case of pumping define the value of a hydro plant with no dam. i.e with no storage capacity with $\hat{\Phi}$ as the expected profit given by the optimization program (6.35)-(6.44) with $V_0 = V_{max} = V_{end} = 0$. What we actually model is now a *run river plant* and the dispatch will consequently be given by

$$x_k^+ = \min(I_k, p_{max}), \; x_k^- = 0, \quad k = 1, \ldots, K.$$

Production has to equal inflow in each period and if the technical constraints on the turbines prohibits this, then the production will be given by that maximum possible capacity and the excess inflow will be lost through spill over. Similarly to the characterization of the value of pumping we can write the value of storage as $\tilde{\Phi} - \hat{\Phi}$. The value of a hydro storage plant with pump capacity can hence in the static case be decomposed as in Figure 6.6,

In Chapter 5 we mentioned that the hydro storage plant can be suitable for hedging volume risk. This is however not possible with the static strategy, since

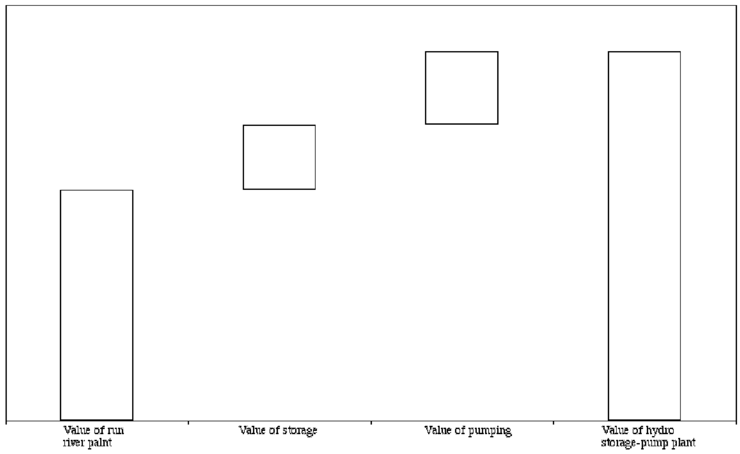

Fig. 6.6: Schematic value decomposition of a static hydro storage-pump plant.

the dispatch is not allowed to react to new information on, for example, swing option demand. In the following chapter we will however further investigate the dynamic dispatch strategy that does capture this hedging aspect and more realistically models the hydro storage plant.

6.6.6. Dynamic portfolio optimization

In the dynamic dispatch strategy we again impose the tighter, but more realistic constraints on the water stored in the dam, and stochastically model the hydro storage plant.

The only uncertainty directly influencing the plant is the inflow, which in period k and scenario j is given by $I_{k,j}$. The stored energy and spill over loss will also be stochastic, but influenceable by the owner and are given by $V_{k,j}$ and $L_{k,j}$ respectively. The probabilistic constraint on the water level, (6.2) can with help of (6.4) be written as

$$0 \leq V_{k,j} = V_0 + \sum_{i=1}^{k} I_{i,j} - \sum_{i=1}^{k} L_{i,j} - \sum_{i=1}^{k} x_{i,j}, \tag{6.48}$$
$$k = 1, \ldots, K, \quad j = 1, \ldots, J,$$
$$V_{k,j} = V_0 + \sum_{i=1}^{k} I_{i,j} - \sum_{i=1}^{k} L_{i,j} - \sum_{i=1}^{k} x_{i,j} \leq V_{max}, \tag{6.49}$$
$$k = 1, \ldots, K, \quad j = 1, \ldots, J,$$
$$L_{k,j} \geq 0, \quad k = 1, \ldots, K, \quad j = 1, \ldots, J.$$

Further as in Chapter 6.5 we impose a constraint on the average end-water level to avoid a myopic solution of an empty water dam at the end of the horizon

$$\bar{V}_K = J^{-1} \sum_{j=1}^{J} \left(V_0 + \sum_{i=1}^{K} I_{i,j} - \sum_{i=1}^{K} L_{i,j} - \sum_{i=1}^{K} x_{i,j} \right) \geq V_{end}. \tag{6.50}$$

$x^+_{1,j}, \ldots, x^+_{K,j}$ and $x^-_{1,j}, \ldots, x^-_{K,j}$ denote generating and pumping respectively in each period in scenario j [MWh], which as before are given by

$$x^+_{k,j} = \sum_{i=1}^{r} \gamma_i^+ g_i^+(S_{k,j}, D_{k,j}, I^a_{k,j}), \quad k = 1, \ldots, K, \ j = 1, \ldots, J \tag{6.51}$$

$$x^-_{k,j} = \sum_{i=1}^{\tau} \gamma_i^- g_i^-(S_{k,j}, D_{k,j}, I^a_{k,j}), \quad k = 1, \ldots, K, \ j = 1, \ldots, J. \tag{6.52}$$

The exercise functions g_i^+ and g_i^- are given by the step functions (6.31) and (6.32). The weighting factors, i. e. the decision variables building up the dispatch from the individual exercise functions are denoted by $\gamma = \left(\gamma_1^+, \ldots, \gamma_r^+, \gamma_1^-, \ldots, \gamma_\tau^-\right)$. As in the static case, the hydro storage plant has yet another component of decision variables, namely the spill over losses, $L_{k,j}$.

Similarly to the static case, we would like to take advantage of the superior computational aspects of linear programming and want to reduce our optimization problem (6.1) to (6.34). The assumptions needed in the static case also have to be fulfilled for γ. The constraints on γ (6.18) and (6.19) are obviously linear. Also the loss function is linear in γ, since it can be written in the following form $l(x, \gamma, Y) = \sum_{i=2K+1}^{n} x_i f_i(Y) + \sum_{i=1}^{r} \gamma_i^+ f_{n+i}(Y) + \sum_{i=1}^{\tau} \gamma_i^- f_{n+r+i}(Y)$, where the functions $f_i(Y)$ only depends on the stochastic factors Y. The assumptions are obviously fulfilled and a simultaneous optimization of the dynamic hydro dispatch and the contract portfolio in a CVaR and profit framework as in (6.34) will thus be given by the solution of the following linear program

$$\max_{x^c \in \mathbb{R}^{n-2K}, \gamma \in \mathbb{R}^{r+\tau}, z \in \mathbb{R}^J, \alpha \in \mathbb{R}, L \in \mathbb{R}^{K \times J}} \quad -J^{-1} \sum_{j=1}^{J} l(x^c, \gamma, y_j) \tag{6.53}$$

subject to

$$\alpha + \frac{1}{(1-\beta)J} \sum_{j=1}^{J} z_j \leq C \tag{6.54}$$

$$z_j \geq l(x^c, \gamma, y_j) - \alpha, \quad z_j \geq 0, \quad j = 1, \ldots, J \tag{6.55}$$

$$0 \leq V_{k,j} = V_0 + \sum_{i=1}^{k} I_{i,j} - \sum_{i=1}^{k} L_{i,j} - \sum_{i=1}^{k} \left(x_{i,j}^+ - x_{i,j}^-\right), \tag{6.56}$$
$$k = 1, \ldots K, \; j = 1, \ldots, J$$

$$V_{k,j} = V_0 + \sum_{i=1}^{k} I_{i,j} - \sum_{i=1}^{k} L_{i,j} - \sum_{i=1}^{k} \left(x_{i,j}^+ - x_{i,j}^-\right) \leq V_{max}, \tag{6.57}$$
$$k = 1, \ldots K, \; j = 1, \ldots, J$$

$$L_{k,j} \geq 0, \quad k = 1, \ldots K, \; j = 1, \ldots, J \tag{6.58}$$

$$\bar{V}_K = V_0 + \frac{1}{J}\left(\sum_{i=1}^{K} I_{i,j} - \sum_{i=1}^{K} L_{i,j} - \sum_{i=1}^{K} \left(x_{i,j}^+ - x_{i,j}^-\right)\right) \geq V_{end} \tag{6.59}$$

$$\sum_{i=1}^{r} \gamma_i^+ = 1, \; \gamma_i^+ \geq 0, \quad i = 1, \ldots, r \tag{6.60}$$

$$\sum_{i=1}^{\tau} \gamma_i^- = 1, \; \gamma_i^- \geq 0, \quad i = 1, \ldots, \tau \tag{6.61}$$

$$x^c \in X^c. \tag{6.62}$$

As in the static case, (6.54) and (6.55) governs that the risk is kept below an acceptable level, (6.56) prevents us from producing when no water is available, (6.57) assures that we do not pump up water above the maximal accepted level and together with (6.58) that excessive water inflow is lost through spill over. Sustainability is again governed by (6.59). Constraint (6.60) and (6.61) put technical limits on the dispatch and again the general constraint (6.62) can enforce any linear contract portfolio constraint.

To penalize the scenarios where the end-water level is below the threshold V_{end}

and vice versa, a linear penalty function could be introduced to the loss function $\kappa(V_{end} - V_{K,j})$, where κ is a positive constant, representing the value of water. Another possibility to avoid that some scenarios leave a low amount of water for coming periods is to sharpen the constraint on the end-water level and state that $V_{K,j} \geq V_{end}$, $j = 1, \ldots, J$ where every trajectory of the water level, $V_{K,j}$ has to exceed the threshold V_{end}. The threshold V_{end} can again be determined by a one-year optimization as described for the static case.

6.6.7. Size of dynamic problem

Compared to the static model, the dynamic model will typically have fewer variables describing the dispatch. We here only need one weighting factor per exercise function, which will give us $r + \tau$ variables, compared to twice the number of periods in the static case. Normally the number of periods will exceed the number of exercise functions, why one could expect this optimization to be faster. However, whereas the constraints on the water level in the static case were scenario independent, in the dynamic case they are not, why the size of the dynamic problem is typically substantially larger. To get an idea on the size of the problem we again study the A matrix. The size of A is given by $A \in \mathbb{R}^{(2KJ+J+4)\times(r+\tau+n-2K+J+KJ+1)}$. For large problems, i. e. many periods and many scenarios, the number of variables will essentially be given by KJ and the number of constraints by $2KJ$ and we can immediately conclude that the size of the dynamic problem will then be substantial. The density of the matrix, defined as the percentage non-zero elements is given by

$$\frac{(K^2 + 2(r + \tau)K + 4r + n + 3)J + 4r + 1}{(2KJ + J + 4)(r + \tau + n - 2K + J + KJ + 1)},$$

and will for large problems be determined by the ratio

$$\frac{K^2 J}{2K^2 J^2} = \frac{1}{2J}.$$

Even though the size of the matrix grows fast for large problems, the density will obviously be fairly low.

Eq.	γ	x^c	L	z	α	#
(6.54)				⁝⁝⁝	⁝⁝⁝	1
(6.55)	⁝⁝⁝	⁝⁝⁝		⋱	⁝⁝⁝	J
(6.56)	⁝⁝⁝		⋱ ⋱			KJ
(6.57)	⁝⁝⁝		⋱ ⋱			KJ
(6.59)	⁝⁝⁝		⁝⁝⁝			1
(6.60)	⋮⋱					1
(6.61)	⋮⋱					1
#	$r+\tau$	$n-2K$	KJ	J	1	

Tab. 6.2: Structure of the A matrix. ⁝⁝⁝ denotes a full, ⋮⋱ denotes a triangular, ⋱ denotes a diagonal and ⋱ ⋱ denotes a diagonal band submatrix.

As seen in Table 6.2 the constraint (6.56) and (6.57) and the spill over variables regulating the water level build up the majority of the matrix. One could however relax some of these constraints. For example, in a short horizon optimization in the winter, when the dams are basically full, the constraint that the water level must be non-negative could be excluded from the problem. Further, in the summer when the water level is low, the upper constraint on the water level can be relaxed and the spill over variables could be eliminated. This would dramatically reduce the size of the problem. The feasibility of the solution, although very probable, must then be checked ex-post. To keep the size of the problem within reasonable boundaries, the proposed relaxation of the water level constraints and the corresponding spill-over variables should however be done whenever possible. Another possibility to decrease the size of the problem is to aggregate a number of periods into one. This would however only serve as an approximate solution. The hydro storage plant's options would then have an average spot price as underlying and not the actual spot price. This should hence preferably be done during times when the stochastic factors are fairly stable, for example, by dividing each day into only one peak period and one off-peak period. The well known fact that the value of a portfolio of options is greater than the value of an option on a portfolio, however implies that an aggregation of periods would underestimate the value of flexible production units, as shown in Example 4.2. One should therefore proceed with care when aggregating periods.

6.6.8. Analysis of dynamic solution

As a solution from the optimization (6.53)-(6.62) we will obtain the optimal contract portfolio $x^{\star}_{2K+1}, \ldots, x^{\star}_{n}$. We will also get the optimal weighting factors $\gamma_1^{+\star}, \ldots, \gamma_r^{+\star}, \gamma_1^{-\star}, \ldots, \gamma_\tau^{-\star}$ building up the optimal dispatch strategy, i. e. the optimal exercise conditions. In any instant within the time horizon we can measure the state variables S, D, I^a and t and with these variables as input to the exercise functions (6.51) and (6.52) get the optimal dispatch.

In this dynamic dispatch strategy we are allowed to react to new information and, for example, meet unexpected high demand by producing more or meet a price peak in a typical off-peak period. This flexibility offers an excellent way to manage these type of uncertain events, which is yet another wording for the hydro storage plant's superb hedging capacity.

An interesting question is what is this flexibility worth. One way to quantify this value would be to compare the average price of produced electricity $\bar{S}^h$ given by

$$\bar{S}^h = \frac{1}{J}\sum_{j=1}^{J}\left[\frac{\sum_{k=1}^{K} S_{k,j}\left(\sum_{i=1}^{r}\gamma_i^{+} g^{+}_{i,k,j} - \frac{1}{\chi}\sum_{i=1}^{\tau}\gamma_i^{-} g^{-}_{i,k,j}\right)}{\sum_{k=1}^{K}\left(\sum_{i=1}^{r}\gamma_i^{+} g^{+}_{i,k,j} - \sum_{i=1}^{\tau}\gamma_i^{-} g^{-}_{i,k,j}\right)}\right], \tag{6.63}$$

with the average spot price given by

$$\bar{S} = \frac{1}{JK}\sum_{j=1}^{J}\sum_{k=1}^{K} S_{k,j}. \tag{6.64}$$

The difference between the average price of produced electricity, $\bar{S}^h$ and the average spot price $\bar{S}$ actually gives us the total value of having a flexible dispatch if the risk constraint (6.54) is inactive. In this case the problem is equivalent to a pure expected profit maximization and hedging loses its validity, since risk is not of interest. The only value of the operational flexibility then comes from the ability to take advantage of the volatile prices and produce at high prices and to take advantage of the price structure by pumping at low cost in off-peak periods, store the water and produce electricity in peak periods. An inflexible plant such as the nuclear plant, producing over long periods, would produce at the average price of $\bar{S}$. We hence believe that $\bar{S}^h - \bar{S}$ is an appropriate assessment of operational flexibility in a pure profit maximization. As soon as risk is of concern the actual value of this flexibility is however larger than $\bar{S}^h - \bar{S}$. The reason for this is that the flexible hydro storage plant will allow us to take more risky and hence more profitable positions in the contract portfolio, since additional risk can be off-set by the hedging capabilities of the stored water. This hedging value of the stored water can be motivated by studying the *dual problem*.

A linear program, such as (6.53)-(6.62) written on its standard primal form

$$\begin{aligned} z = \quad & \max \quad c^T x \\ & \text{s.t.} \quad Ax \leq b, \end{aligned} \tag{6.65}$$

where $x \in \mathbb{R}^n$, $A \in \mathbb{R}^{n \times m}$, $c \in \mathbb{R}^n$ and $b \in \mathbb{R}^m$, has an interesting corresponding dual problem, introduced already in Chapter 6.5, given by

$$\begin{aligned} \min \quad & b^T y \\ \text{s.t.} \quad & A^T y = c \\ & y \geq 0. \end{aligned} \tag{6.66}$$

The dual variables $y \in \mathbb{R}^m$ are connected to the primal problem in the sense that the ith dual variable y_i corresponds to the ith constraint $\left(Ax \leq b\right)_i$. Given that the primal problem (6.65) has a non-degenerate solution, the ith dual variable, being the solution of (6.66) $y_i^\star$ will equal the marginal value of the 'resource' b_i. For more information on linear programming in general, see [Chv83]. The interesting resource in this case is the stored water. The available water can be manipulated by adjusting the right hand side of the sixth constraint (6.59) stating that $\bar{V}_K \geq V_{end}$. Given that $V_{max} \geq V_{end}$ one unit decrease in V_{end} would allow us to utilize one more unit of water during the time horizon studied. Hence, the optimal dual variable $y_6^\star$ corresponding to the sixth constraint and consequently the 'resource' V_{end} will give us the value of one unit of additional water $\frac{z(V_{end}-\Delta)-z(V_{end})}{\Delta}$, i. e. the marginal value of water. One could argue that for an almost empty dam the value of having the possibility to leave one unit less of water to the periods after the horizon will differ from having one additional unit of water at disposal already at period one. When the constraints (6.56) and (6.57), stating that for all k and j, $0 \leq V_{k,j} \leq V_{max}$, are not binding then these values will however coincide. In any case the dual variable $y_6^\star$ will give the marginal value of water made available in the end of the horizon.

The dimension of the dual variable $y_6^\star$ is expected profit divided by amount of water [CHF/MWh] and is hence comparable with the average price of produced electricity $\bar{S}^h$. A marginal value of water that exceeds the value that this water, on average, has on the market, given the current dispatch strategy $\bar{S}^h$ is a result of the hydro storage plant's ability to manage risk. This ability allows us, for example, to take on more aggressive positions in the contract portfolio, which motivates a high marginal value of water. We hence quantify the hedging value of water as the difference between the marginal value of water and the average price of produced electricity

Definition 6.9
The hedging value of water is given by

$$\frac{z(V_{end}-\Delta)-z(V_{end})}{\Delta} - \bar{S}^h,$$

which in the non-degenerate case will equal

$$y_6^\star - \bar{S}^h.$$

From Proposition 6.5 we know that the hedging value will be negative if the risk constraint and the water constraints (6.56) and (6.57) are non-binding. This difference will however typically be close to zero.[5]

In the risk averse case the value of the operational flexibility thus can be seen as the sum of the ability to produce at high prices and arbitrage between peak and

[5] This is verified in the case study in Chapter 7.

off-peak periods, and the hedging value of water. We hence define the value of the operational flexibility as the difference between the marginal value of water and the average spot price

Definition 6.10
The total value of the operational flexibility is given by

$$\left(\frac{z(V_{end}-\Delta)-z(V_{end})}{\Delta} - \bar{S}^h\right) + \left(\bar{S}^h - \bar{S}\right) = \frac{z(V_{end}-\Delta)-z(V_{end})}{\Delta} - \bar{S}.$$

In the case studies performed in Chapter 7, we will see that especially the hedging value stemming from the optionality in the hydro storage plant can be substantial.

In order to analyze the marginal value of water, we introduce the so-called optimal value function $z(b)$.

Proposition 6.11
The optimal value function $z(b)$ is piecewise linear and concave.

A proof is given in Appendix A on page 198.

Corollary 6.12
The marginal value of water, $\frac{z(V_{end}-\Delta)-z(V_{end})}{\Delta}$ is piecewise constant and increasing in V_{end}.

Again a proof in given in Appendix A on page 199.

Observe that the value of the water stored in the dam can be expressed as the integral of the marginal value of water over the available water, ranging from 0 to the actual available amount of water. Hence following from Corollary 6.12, stating that the marginal value is decreasing in the available amount of water, $\frac{z(V_{end}-\Delta)-z(V_{end})}{\Delta}$ is actually a lower bound for the value of the water in the dam.

6.6.9. Positioning of contracts and plants

The dual problem does however not only give us insight into the hedging value of water. Instead of studying the value of additional water, we can study the value of additional risk. The optimal dual variable y_1^* corresponding to the risk constraint (6.54) tells us how much the objective function (6.53) would increase by increasing the risk constraint C by one unit. Hence $y_1^\star$ will, in the non-degenerate case, give us the marginal value of risk $\frac{z(C+\Delta)-z(C)}{\Delta}$. With our optimal portfolio we know that we would increase the expected profit by exactly $y_1^\star$ if we were allowed to utilize one more unit of risk. It is clear that the marginal value of risk given by the dual problem will depend on the maximum risk level C, which in itself depends on the risk appetite of the utility. Further, the marginal value of risk will depend on the portfolio constraints in the sense that a player that owns hydro storage plants, for example, will have a different dual solution than a player who does not have any production capacity. The marginal value of risk is hence not unique within the market, but rather specific for each player.

The marginal value of risk tells us how we in the optimum can utilize one additional unit of risk with the contracts and production assets made available in the optimization. Assume that we are offered to enter a new type of contract with a ratio between marginal contribution of expected profit, and marginal contribution of risk $\frac{\Delta_{profit}}{\Delta_{risk}}$ that exceeds our marginal value of risk of the original portfolio

$$\frac{\Delta_{profit}}{\Delta_{risk}} > y_1^\star. \tag{6.67}$$

Should we then buy the contract? Well, standard financial theory does not give us much operational guidance in this matter, since the incompleteness of the electricity market implies that no unique risk neutral valuation can be found, as was described in Chapter 3.7.1. The marginal value of risk however implies that we at least should consider to buy the contract, since we could decrease our risk utilization with one unit at the expected cost of $y_1^\star$ by adjusting the positions in the original portfolio. Then we could enter the new contract with one unit of marginal risk, summing up the total risk to the same level as before, but with an expected profit exceeding the original one with

$$\frac{\Delta_{profit}}{\Delta_{risk}} - y_1^\star > 0.$$

For a new contract priced such that $\frac{\Delta_{profit}}{\Delta_{risk}} < y_1^\star$ on the other hand, we would

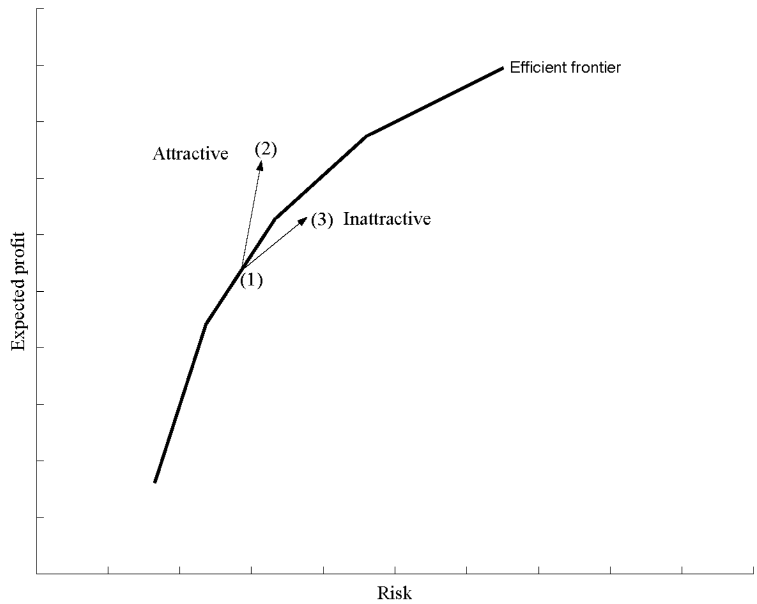

Fig. 6.7: Positioning of contracts and plants.

with the same arguing as before go short in that contract reducing the risk by one unit, but also penalizing the expected profit with $\frac{\Delta_{profit}}{\Delta_{risk}}$. The reduced risk utilization would however allow us to further leverage on the contracts made available in the original optimization and increase the expected profit with $y_1^\star - \frac{\Delta_{profit}}{\Delta_{risk}} > 0$.

The efficient frontier[6] can be very helpful in positioning contracts and plants according to the marginal value of risk, since the slope of the efficient frontier gives the marginal price of risk.

Corollary 6.13
The efficient frontier $z(C)$ is piecewise linear and concave.

A proof is given in Appendix A on page 199.

Assume that our optimal portfolio conditioned on the investment opportunity set is given by (1) in Figure 6.7. A contract or plant added to the investment

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[6] The efficient frontier is the plot $\big(z(C), C\big)$. The term *efficient frontier* is normally used for a whole market and not as in this case for a specific player.

opportunity set would then be attractive if the portfolio, consisting of the original portfolio (1) including the new contract or plant is located above the original efficient frontier (2). A new portfolio located below the efficient frontier (3), on the other hand, would be unattractive.

One is with this arguing tempted to define the price of a contract or a plant such that $\frac{\Delta_{profit}}{\Delta_{risk}} = y_1^\star$, since this is the price making the utility indifferent between buying or selling the contract or plant. We do not go that far, since that price would then depend on the current portfolio and hence the current investment opportunity set. The order in which contracts are added to the investment opportunity set would consequently matter. We however believe that a utility can get additional information on the attractiveness of contracts and plants by investigating the marginal value of risk. And by doing that the utility can get a guidance on which contracts or plants that it should consider to acquire or sell.

Operationally the attractiveness of a new contract or plant could be determined by simply calculating the new portfolio's (consisting of the original portfolio and the new contract or plant) expected profit and risk, and compare it with the efficient frontier of the original portfolio. An example of such a positioning is presented in the case study in the next chapter.

6.7. Summary

The proposed power portfolio optimization model using CVaR manages to incorporate the production assets and allows us, through the dynamic dispatch strategy, to take advantage of the hydro storage plant's operational flexibility. We have also shown how a simultaneous optimization of the contract and the production portfolio can be carried out. As already stated in Chapter 3.5, we believe that CVaR is an appropriate risk measure in heavy-tailed portfolios, such as power portfolios, and we have shown how the non-normal and heavy-tailed feature of power portfolios can be captured by our optimization approach using CVaR. The probabilistic constraints on the water level could also be handled by working with scenarios, which also allows us to incorporate and model the, for the electricity market typical, complex contracts.

We hence believe that the proposed optimization approach using CVaR, where a dynamic dispatch strategy is used for the hydro storage plant, do capture the peculiarities of a power portfolio and that it sheds additional light on the complex challenge of optimizing a power portfolio.

Chapter 7

Case study

In this chapter a portfolio of a typical Swiss utility is studied, where the production park consists of a few nuclear plants and a number of hydro storage plants, whereof some are also equipped with pumps. On the short side, the utility has sold a number of swing option contracts. The utility has the possibility to trade in the spot and futures market. As known from Chapter 4, the dispatch of a nuclear plant is trivial and the corresponding contracts is a series of futures. Hence we will not model the nuclear plants and the focus will rather be on the hydro storage plants. These plants are however only modeled on an aggregated level by a super hydro storage plant, where the characteristics of each hydro storage plant, such as inflow, dam size and turbine capacity is summed up. We will hence get no guidance on which specific hydro storage plant to dispatch, but rather how we on a portfolio level should dispatch. We could model each hydro storage plant individually, however at the costs of an increased problem size.

A time horizon of two weeks is chosen, which corresponds to 336 hourly periods. We are standing at the beginning of a hydrological year, with the first period being the first hour of a Monday in the beginning of October.

The short positions in the swing option contracts are due to illiquidity assumed to be long term in the sense that they cannot be changed within the short horizon.

Spot price process					
α	2000	[-]	$\bar{\mu}$	39	[CHF/MWh]
σ	0.45	[-]	$\bar{\mu}_d^{amp}$	1.5	[CHF/MWh]
λ	2.2	[-]	$\bar{\mu}_y^{amp}$	5.6	[CHF/MWh]
ν	90	[CHF/MWh]			
Demand process					
α	2000	[-]	$\bar{\mu}$	1650	[MWh]
σ	0.15	[-]	$\bar{\mu}_d^{amp}$	150	[MWh]
λ	2.3	[-]	$\bar{\mu}_y^{amp}$	418	[MWh]
ν	3000	[MWh]			

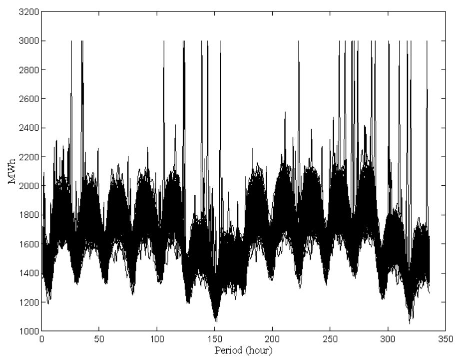
Tab. 7.1: Parameters describing the spot price process and the demand process.

The stochastic factors are modelled in 250 scenarios and since the positions in the swing options cannot be changed within the two week horizon, only the aggregated demand $D_k = \sum_{i=m+1}^{n} D_{k,i}$ together with the spot price S_k and inflow I_k are of interest and will be modelled. The driving process of the spot price as for the swing option demand is a mixed mean reverting diffusion and jump process as in (2.5), where the seasonality is modeled as in (2.4). A jump only lasts for one period and a spot price jump occurs with a frequency λ of 2.2, whereas a demand jump occurs with a frequency of 2.3. The other relevant parameters describing the spot price process and demand process are given in Table 7.1.[1] The spot price is slightly correlated with the swing option demand in the sense that the random variable determining if a spot price jump occurs or not has a correlation of 0.10 to the random variable determining if a demand jump occurs or not. The inflow is modelled by the simple diffusion process

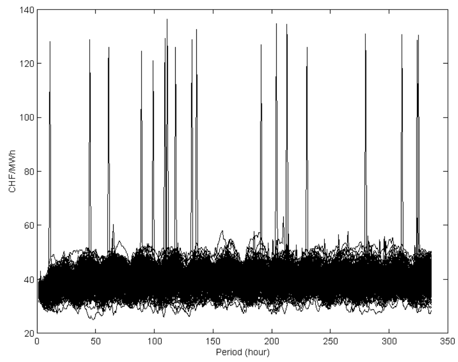
$$dI_t = \alpha dt + \sigma dW_t,$$

where α is given by -1080 [MWh] and σ by 90 [MWh]. The parameters are estimated on the basis of historical data.

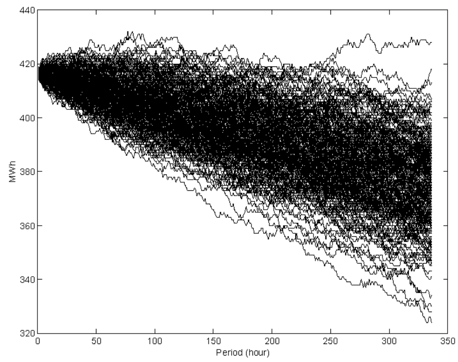
[1] To account for the weekend effect on demand, an additional variable is added to the demand mean reversion level in (2.4). This variable has the value -150 [MW] for all hours on Saturdays, -200 [MW] for all hours on Sundays and 0 for all hours on weekdays. Observe further that time is measured in years.



(a) Demand



(b) Spot price



(c) Inflow

Fig. 7.1: Scenarios of the three stochastic processes.

The 250 scenarios of the respective stochastic factors are illustrated in Figure 7.1, where one clearly sees the jumps in spot price and demand. The negative trend in inflow, as an effect of approaching the winter season, and an upper limit on the demand in the swing option of 3000 MW is also seen. Further, the intra-day variation is obvious for demand, whereas it is difficult to see for the spot price because of the higher volatility hiding these intra-day variations.

In Table 7.2 the average, minimum and maximum of the simulated stochastic factors are presented and one can see that for demand and spot price, the average is far closer to the minimum than to the maximum. This skewness is caused by the infrequent, but large jumps.

Stochastic factor	Statistical property	Value	Unit
Demand	Average	1666	[MW]
	Minimum	1051	[MW]
	Maximum	3000	[MW]
Spot price	Average	39.66	[CHF/MWh]
	Minimum	25.08	[CHF/MWh]
	Maximum	136.5	[CHF/MWh]
Inflow	Average	395.9	[MW]
	Minimum	324.0	[MW]
	Maximum	432.0	[MW]

Tab. 7.2: Average, minimum and maximum of the simulated hourly demand, spot price and inflow.

The swing options are priced at 40 CHF/MWh, which is only marginally higher than the average spot price during these two weeks. One could though argue that the swing option contracts with its built-in flexibility for the buyer on the volume side, should yield a yet higher price compared to the expected spot price. The average peak price[2] over the two weeks is slightly below 41 CHF/MWh and the average off-peak price is around 38.5 CHF/MWh. The peak price is hence, on average, only 6% more expensive than the off-peak price. One could therefore expect that pumping, with an efficiency of 70% will not be of great importance.

For simplicity we introduce only one future having the spot contract in all 336 periods as underlying and only allow to buy or sell the future at the beginning of the planning period. The future price is given by 39 CHF/MWh. To avoid a too large spot position a constraint on the future gross position is set at 10000 MW. This will however not limit the utility's possibility to hedge the spot position.

[2] Peak periods are here defined as the periods between eight in the morning and eight in the evening, whereas off-peak as the periods between nine in the evening and seven in the morning.

The exercise functions, are in this case study only functions of spot price and demand, not inflow. The reason for this is simply to be able to visualize the exercise conditions in a 3D plot[3] and to simplify the interpretation of the results. These exercise functions are as before given by step functions

$$g_{i,l}^{+} = \begin{cases} p_{max} & \text{if } S \geq S_i^{+} \text{ and } D \geq D_l^{+}, \\ 0 & \text{otherwise} \end{cases}, \tag{7.1}$$

$$g_{i,l}^{-} = \begin{cases} 0 & \text{if } S \geq S_i^{-} \text{ or } D \geq D_l^{-}, \\ p_{min} & \text{otherwise} \end{cases}. \tag{7.2}$$

Due to the short horizon of two weeks no time component was taken into account in the thresholds. These thresholds are presented in Table 7.3 and the five spot thresholds and six demand thresholds build up 30 different exercise functions for production and pumping respectively as in (7.1) and (7.2). One could introduce even more exercise functions, which of course would result in an equal or better expected profit. It would however not change the typical results that we want to analyze.

The end water level constraint V_{end} is obtained by running a one-year optimization starting in the beginning of October, corresponding to a hydrological year. The one-year optimization was a static one where the schedule was determined at the beginning of the period for the whole year as in (6.35)-(6.44). The reason for using a static dispatch is to keep the problem on a computationally manageable size. As argued already in Chapter 6.6.3.1 the low inflow in combination with the high prices in the winter implies that a basically full dam at the beginning and hence end of the hydrological year, which in Switzerland starts in October, is a fair assumption. The margin in this one-year optimization was chosen to be 50 GWh. This together with a maximum water level of 1500 GWh gives us a starting and end level of 1450 GWh in the one year optimization. The only output that interests us from this optimization is the optimal water level over time. The result is presented in Figure 7.2, where one can see the typical shape of the optimal water level with a low level in the summer and a high level in the winter. One can see that major production is conducted in

[3] By introducing inflow in the exercise functions the expected profit increased only slightly with up to 1%, depending on the risk constraint C.

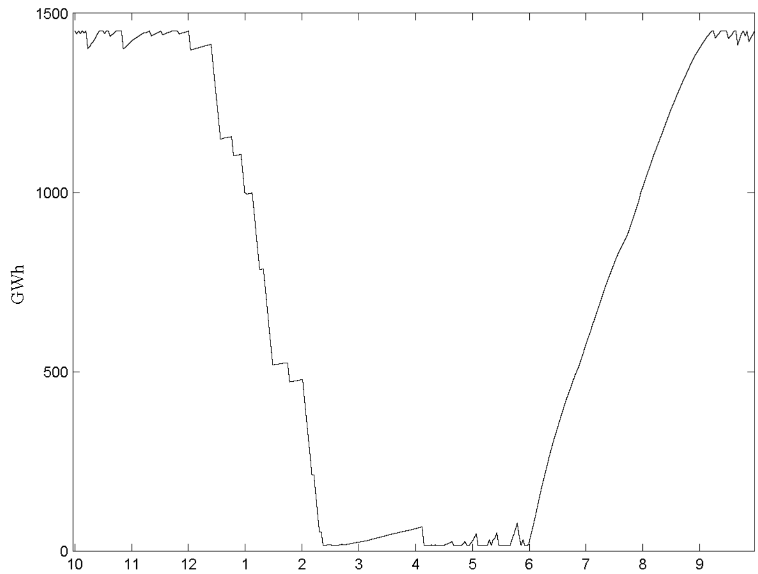


Fig. 7.2: Optimal water level over one hydrological year starting in beginning of October.

the end of December, which is the start of a sharp decline of the water level that continuous until the middle of February. An increase in the water level is starting in the middle of June and ends in the beginning of September. The starting water level is 1450 GWh and the optimal end water level after two weeks also happens to be 1450 GWh, which will be our constraint on the end water level in our actual two-weeks optimization.

A much-debated issue is the confidence level used in VaR and CVaR. For VaR the quantile is typically 90%, 95% or 99%. We have chosen to have a fairly low confidence level of 90% simply because a higher level would leave us with a small data sample to calculate the expectation in the tail.

The parameters specific for this optimization are, as partially already presented, given in Table 7.4. And the general constraint $x^c \in X^c$ consists of the limit on the single available future, $-10000 \leq x \leq 10000$ [MW].

The decision variables are the spill over losses $L_{k,j}$, the future position x and the weighting factors γ_i^+ and γ_i^-, and the optimal contract portfolio and opti-

Production				Pumping			
Spot price [CHF/MWh]		Demand [MW]		Spot price [CHF/MWh]		Demand [MW]	
S_1^+	25	D_1^+	0	S_1^-	25	D_1^-	0
S_2^+	37	D_2^+	1500	S_2^-	30	D_2^-	1200
S_3^+	43	D_3^+	1550	S_3^-	33	D_3^-	1300
S_4^+	49	D_4^+	1600	S_4^-	35	D_4^-	1400
S_5^+	53	D_5^+	1650	S_5^-	37	D_5^-	1500
		D_6^+	1700			D_6^-	1600

Tab. 7.3: Exercise thresholds.

$J = 250$	$V_0 = 1450000$ [MWh]
$K = 336$	$V_{end} = 1450000$ [MWh]
$\beta = 0.90$	$V_{max} = 1500000$ [MWh]
$r = \tau = 30$	$p_{max} = 2261$ [MW]
$\chi = 0.70$	$p_{min} = -285$ [MW]

Tab. 7.4: Parameters used in case study.

mal production portfolio is given by the following optimization problem

$$\max_{x\in\mathbb{R},\gamma\in\mathbb{R}^{60},z\in\mathbb{R}^{250},\alpha\in\mathbb{R},L\in\mathbb{R}^{336\times 250}} -\frac{1}{250}\sum_{j=1}^{250} l(x,\gamma,y_j) \tag{7.3}$$

subject to

$$\alpha + \frac{1}{(1-0.90)250}\sum_{j=1}^{250} z_j \le C \tag{7.4}$$

$$z_j \ge l(x,\gamma,y_j) - \alpha, \quad j = 1,\dots,250 \tag{7.5}$$

$$0 \le V_{k,j}, \quad k = 1,\dots,336,\ j = 1,\dots,250 \tag{7.6}$$

$$V_{k,j} \le 1500000, \quad k = 1,\dots 336,\ j = 1,\dots,250 \tag{7.7}$$

$$L_{k,j} \ge 0, \quad k = 1,\dots 336,\ j = 1,\dots,250 \tag{7.8}$$

$$1450000 \le \frac{1}{250}\sum_{j=1}^{250} V_{336,j} \tag{7.9}$$

$$\sum_{i=1}^{30} \gamma_i^+ = 1,\ \gamma_i^+ \ge 0, \quad i = 1,\dots,30 \tag{7.10}$$

$$\sum_{i=1}^{30} \gamma_i^- = 1,\ \gamma_i^- \ge 0, \quad i = 1,\dots,30 \tag{7.11}$$

$$-10000 \le x \le 10000, \tag{7.12}$$

where the water level in period k and scenario j is given by

$$\begin{aligned} V_{k,j} &= 1450000 + \sum_{i=1}^{k} I_{i,j} - \sum_{i=1}^{k} L_{i,j} - \sum_{i=1}^{k}\Bigg(\sum_{l=1}^{30}\gamma_l^+ g_l^+(S_{i,j}D_{i,j}) - \\ &\quad -\sum_{l=1}^{30}\gamma_l^- g_l^-(S_{i,j}D_{i,j})\Bigg) \end{aligned} \tag{7.13}$$

and the loss function, where no discounting is performed due to the short horizon, is in scenario j given by

$$
\begin{aligned}
l(x, \gamma, y_j) \;=\; & 39 \cdot 336x - 40 \sum_{i=1}^{336} D_{i,j} + \sum_{i=1}^{336} S_{i,j}\Bigg(- \sum_{l=1}^{30} \gamma_l^{+} g_l^{+}(S_{i,j}, D_{i,j}) + \\
& + \frac{1}{0.70} \sum_{l=1}^{30} \gamma_l^{+} g_l^{-}(S_{i,j}, D_{i,j}) - x + D_{i,j} \Bigg). \qquad (7.14)
\end{aligned}
$$

The marginal costs for production and pumping are, as seen, not present. They are typically very low and are negligible in this analysis.

7.1. Efficient frontier

The tightest risk constraint for which there exists a feasible solution to (7.3)-(7.12) is $C_{min} = -5.657$ million CHF. This means that the polyeder of feasible solutions is empty for risk constraints lower than C_{min}, which hence is the minimum achievable risk.

A number of optimizations were performed for different constraints on the risk, varying from C_{min} to 2 million CHF, where the latter corresponds to a pure profit maximization without any regards to the riskiness of the portfolio. The maximum expected profit plotted against the risk constraint gives us the utility specific *efficient frontier*, which is shown in Figure 7.3. Any portfolio under the efficient frontier is inefficient, since there exists a portfolio with a higher expected profit at the same risk. This corresponds to the efficient frontier in the Markowitz portfolio optimization approach, where however risk is measured as variance [Mar52]. The efficient frontier is shown to be piecewise linear and concave, which is consistent with Corollary 6.13. The expected profit as a function of the maximum risk is strictly increasing for risk constraints tighter than 1325000 CHF. For looser risk constraints the expected profit is constant, as seen in Figure 7.3. The reason for this is that the risk constraint is inactive for $C \geq 1325000$ CHF. The future position is then already at its maximum, $x = 10000$ MW, as can be seen in Figure 7.4, and the production cannot be changed towards a more profit oriented dispatch. This would in the mean-variance case correspond to a full investment in the single stock with the highest expected profit and with short positions prohibited. The marginal value

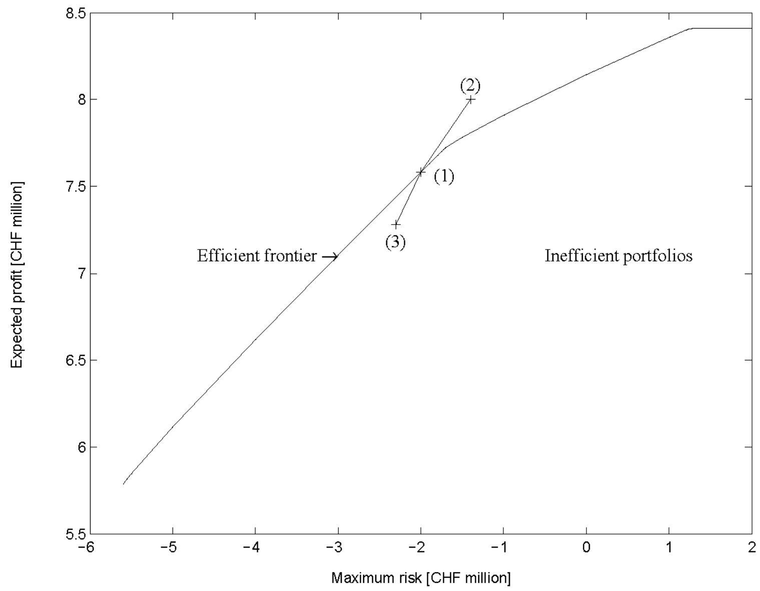


Fig. 7.3: Expected profit versus risk constraint.

of risk, given by the slope of the efficient frontier consequently equals zero for risk constraints looser than 1325000 CHF. Since the risk constraint is inactive, an additional unit of risk would not penalize the objective function, as it would not change the optimal solution. A low marginal value of risk means that an additional unit of risk would not penalize the expected profit that much. The utility should then consider if it has a too loose risk constraint, or if it simply under-utilize the risk that it may take. In the latter case it may be wise to let the utility trade high risk and high margin products to make a better utilization of the risk mandate. An interesting feature of the marginal value of risk and hence of the efficient frontier is the possibility to determine the attractiveness of new such products. The idea is to investigate how much risk and profit a contract or a plant would contribute to the portfolio and compare this with the marginal price of risk in the original optimal portfolio.

Since the marginal price of risk is indeed not constant, it is better to work directly with the efficient frontier. Assume that the utility has an efficient portfolio with a risk of -2 million CHF, marked with (1) in Figure 7.3. The utility is offered to buy a weather derivative to indirectly hedge for demand spikes at the cost of 0.3 million CHF. Because of the internal hedging capabilities, in terms of the hydro storage plant, the risk in the portfolio consisting of the old portfolio and the weather derivative, only decreases to -2.3 million CHF. As seen in Figure 7.3, the weather derivative is not attractive to the utility, since the new portfolio, marked (3) is suboptimal even with respect to the smaller investment opportunity set building up the efficient frontier of the original portfolio. Instead the utility wants to take more advantage of the operational flexibility and is considering to sell a load factor contract, giving the buyer a large flexibility both on the load and energy side. The price is consequently high and the product is priced at 43 CHF/MWh. By adding this load factor contract to the original portfolio, the expected profit would increase to 8 million CHF and the risk would increase to -1.4 million CHF, marked by (2). Despite the large increase in risk, the contract would be attractive to the utility, since it could not utilize the additional risk with such an increase in expected profit given the original investment opportunity set. Together with performing contract engineering to identify the corresponding contracts of a plant, the positioning of contracts and plants can in this sense be very helpful in pinpointing the contracts or plants that make optimal use of the current assets and their flexibility characteristics.

7.2. Spot position

The average spot price over the two weeks is 39.66 CHF/MWh, whereas the two-week future is traded at 39 CHF/MWh just before the beginning of the first period.[4] The future is hence priced almost 1.7% lower than the average spot price. When we impose no, or a very loose, risk constraint the optimal portfolio goes long in the futures market and sells the corresponding electricity in the spot market at a somewhat higher expected price. When we tighten the risk constraint, the slightly higher expected spot price does not make up for the highly uncertain and hence risky profit any longer, which inevitably is the effect of a large spot position. The future position is therefore increasing in the risk constraint C until it hits its upper boundary of 10000 MW, as seen in Figure 7.4.

Absence of volume risk,[5] would in the case of a risk minimization imply

[4] This implies that the producers require a risk premium for selling electricity on the risky spot market instead of on the futures market.

[5] This would in our case correspond to no uncertainty in production volume and no un-

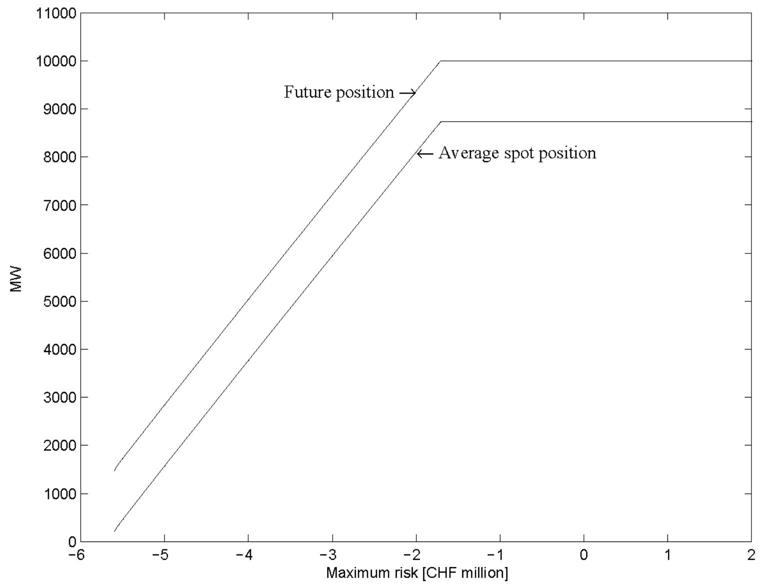


Fig. 7.4: Future position and average spot position versus risk constraint.

that the future position perfectly hedges away the spot position. However, as we see in Figure 7.4, the average spot position, $\frac{1}{JK}\sum_{j=1}^{J}\sum_{k=1}^{K}x_{k,j}^{spot}$ is in this case always positive, also for the risk constraint C_{min}, corresponding to a pure risk minimization. The reason for this 'over hedge' with the future, giving us an, on average, long position in electricity, i. e. long position in the spot market is the skewness in the spot price distributions in combination with the volume risk. The risk associated with being exposed to the heavy tailed part of the skewed spot distribution is naturally higher than being exposed to the thinner tail. The uncertainty on the volume side, which is correlated to the spot price, is directly influencing the spot position as shown in (6.33). This implies that an, on average, short position in electricity, i. e. short spot position, will be penalized more than an, on average, long position.

7.3. Hedging value of water

The average price that produced electricity is sold at depends on the risk constraint, because the optimal dispatch strategy differs with the risk constraint.

certainty in swing option demand.

The average price of produced electricity lies between 39.8 CHF/MWh for a tight risk constraint and 45.2 CHF/MWh for a loose risk constraint. The dispatch strategy is obviously more focused on selling at a high price when the risk is not an issue, whereas when the risk is tightly constrained the strategy tries, not only to fulfill the implicit goal of selling the electricity at a high price, but also to keep the risk below an acceptable level. The optimal strategy responds in two ways to cope with a tight risk constraint, through both the contract portfolio and the production portfolio. Firstly, the spot position is decreased through a reduced future position, resulting in a diminished gross position of the portfolio and secondly, the dispatch is more focused on decreasing the risk.

The flexibility of the hydro storage plant is, as described in Chapter 6.6.8, partially demonstrated by the difference between the average price of produced electricity and the average spot price. The excess price of produced electricity over the average spot price $\bar{S}^h - \bar{S}$ decreases with a tighter risk constraint, since the dispatch strategy is forced to handle the risk, which consequently limit the production flexibility. Still, the produced electricity is on average always sold at a higher price than the average spot price, as seen in Figure 7.5.

The excellent hedging performance, i. e. risk reducing capabilities of the hydro storage plant as a result of its great flexibility is however not visible in Figure 7.5. If we study the marginal value of water in the dam, i. e. the dual price of the constraint on the end water level, this outstanding risk management potential becomes apparent. We have plotted this marginal value of water together with the average price of produced electricity as a function of the risk constraint in Figure 7.6. Obviously, for tight risk constraints an extra unit of water is assessed far higher than the value that water on average has on the market, given the chosen dispatch strategy. The difference can be rather substantial. For an optimal portfolio with a maximum risk of -2 million CHF, the marginal value of water is 48% higher than the average price of produced electricity. For tighter risk constraint the difference becomes even greater. On the other hand, when the risk constraint is relaxed the difference almost disappears and for the pure maximization problem the hedging value of water is slightly negative, which is consistent with Proposition 6.5. Logically the marginal value of water should decrease with a looser risk constraint. As the risk constraint becomes non-binding the problem is transformed into a pure

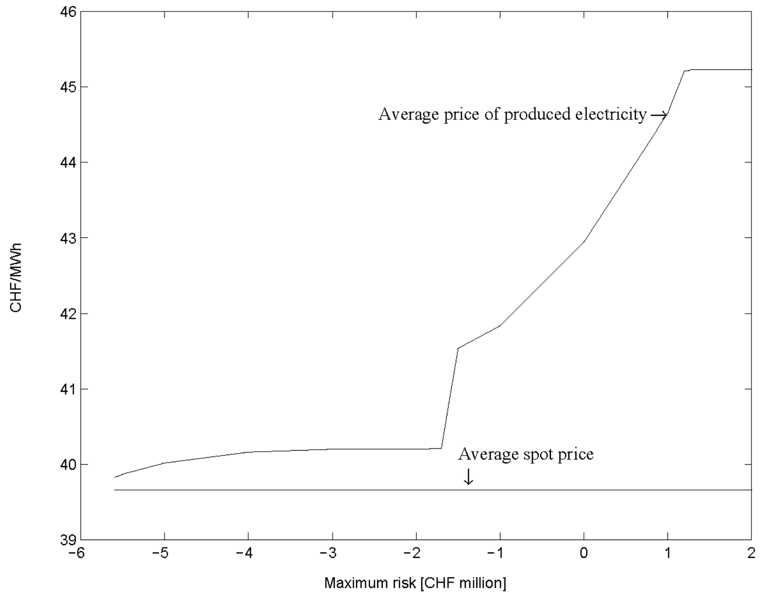


Fig. 7.5: Average price of produced electricity and average spot price versus risk constraint.

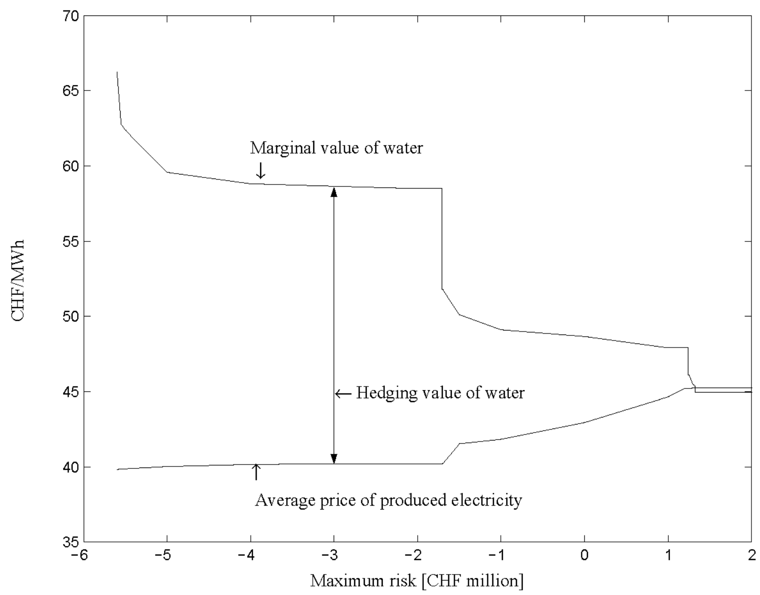


Fig. 7.6: Marginal value of water and average price of produced electricity versus risk constraint.

expected profit maximization, without any risk concerns. The marginal value of water is then simply given by the average price of produced electricity at a dispatch strategy slightly worse than the current one. When the risk constraint is active the superb hedging value of the stored water comes in to play. An additional unit of water namely allows us to take riskier positions, in for example the spot market, without increasing the risk in the overall portfolio. Apparently the value of being able to take a riskier position exceeds the immediate market value of that water. This is a very important observation, since we cannot just verify the value of the hydro storage plant stemming from its optionality, but despite the incomplete market also actually quantify it.

If we were allowed to trade in even more risky instruments, such as interruptible contracts, with not only a price risk, but also a volume risk component, the risk reducing capabilities of the hydro storage plant could be even more obvious. These contracts should according to basic financial theory be priced higher because of their riskiness. This risk could though be manageable for the hydro storage plant, why the value of additional water could differ even more from the average price of produced electricity. The hedging value of the hydro storage plant will hence not only depend on its own characteristics, but also on the contract portfolio. The, for the utility, interesting contracts could be identified by finding the contracts that correspond to the plant, since these contracts are natural replications of the plant itself. For fine tuning of the ideal contracts, positioning of products according to their attractiveness, as described in Chapter 7.1, would be suitable.

7.4. Value decomposition

The value of water in the hydro dam can be divided into three components building up the total value. The first component is the static value that would be achieved by producing a constant amount without an intelligent strategy and equals the average spot price $\bar{S}$. The second component is the value stemming from the dynamic strategy resulting in a price of produced electricity that differs from the average spot price $\bar{S}^h - \bar{S}$. The last component is the hedging value of the water, allowing us to take riskier positions in the contract portfolio $y_6^\star - \bar{S}^h$, as motivated in Chapter 6.6.8.

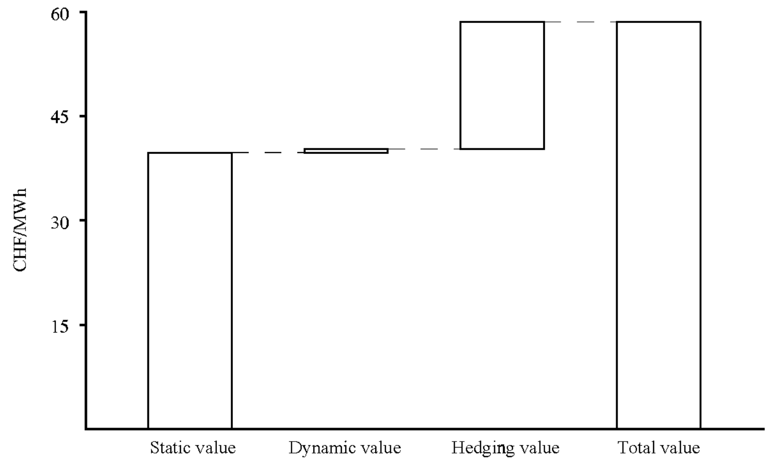


Fig. 7.7: Value decomposition of water for a risk constraint of -2 million CHF.

The two latter components, having their origins in the operational flexibility of the hydro storage plant, differ strongly with the risk constraint that is imposed. For a fairly tight constraint the hedging capability evidently has a large value, whereas the value stemming from having a dynamic dispatch strategy, that allows us to produce at high prices, is small. The contrary is true for a loose risk constraint, which is illustrated in Figure 7.7 and 7.8.

7.4.1. Value of pumping

Let us, as in Chapter 6.6.5, define the value of pumping as the expected profit given by an optimization, where pumping is allowed minus the expected profit when no pumping is allowed. The small differences between peak and off-peak prices erode much of this potential value and this additional option is almost never used, as shown in Figure 7.9, 7.10 and 7.11. The value of pumping is hence small. The highest value occurs for a pure profit maximization ($C \geq 1325000$), but the value is however less than 700 CHF, which corresponds to a difference of less than 0.01 percent. For tighter risk constraint the value of pumping is even more negligible.

The conclusion that pumping in general has no value is however indeed wrong,

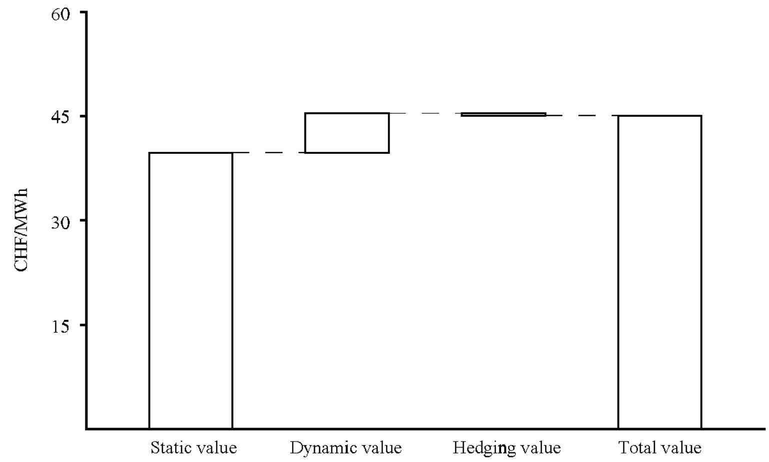


Fig. 7.8: Value decomposition of water for a risk constraint of 2 million CHF.

since the low value coming from our case study is a result of the fairly low intra-day variations. The value in general of course also heavily depends on the efficiency of the pumps, here 70%.

7.4.2. Value of additional turbine capacity

From Chapter 4.3 we know that the hydro plant equals a series of interdependent options. An increase in the turbine capacity would in financial terms imply that we could exercise a greater amount of underlying electricity per period. This is of course valuable, but the load factor measured as the ratio between the average available water per period and the maximum possible production per period, $\frac{V^{end}-V_0+J^{-1}\sum_{j=1}^{J} I^a_{K,j}}{Kp_{max}}$ is already fairly low and amounts to less than 18%. The limitation is therefore not the turbine capacity, but rather the amount of available water, i. e. the coupling of the options is the limitation not the underlying amount in each of the option. The majority of the production is actually conducted at a dispatch below the maximum capacity because of the water limitations, why intuitively the value of increased turbine capacity should be limited.

One could imagine that the value of increasing the turbine capacity would be highest for a loose risk constraint when the dispatch strategy is focused on producing at high prices, since peak prices could then be utilized even better. By

Risk constraint (CHF million)	-5.6	-4	-2	-1.5	0	2
Increased expected profit (%)	0.006	0.05	0.04	0.05	0.06	0.07

Tab. 7.5: Effect of pumping for different risk levels.

increasing the maximum turbine capacity with 10% and optimizing the same portfolio, but with this new technical constraint we see that the value of this increased turbine capacity is limited. As assumed the value is highest for a pure profit maximization and amounts to an additional expected profit of 0.07%, as seen in Table 7.5.

7.5. Dispatch strategy

The abstract weighting factors, $\gamma_1^+, \ldots, \gamma_{30}^+$ and $\gamma_1^-, \ldots, \gamma_{30}^-$ building up the dispatch from the corresponding exercise functions $g_1^+, \ldots, g_{30}^+$ and $g_1^-, \ldots, g_{30}^-$, are our decision variables for the production portfolio. They however do not in them selves visualize the optimal dispatch very well. We have therefore plotted the dispatch as a function of the spot price and the demand, which are the only factors influencing our dispatch decision, for some risk constraints. By starting with the optimal dispatch for a very loose risk constraint of 2 million CHF, one can in Figure 7.9 see that production is solely a function of the spot price, which is consistent with our hypothesis that for loose risk constraints the strategy is focused only on a high profit, implying that a large amount of the water is dispatched during high price periods. Further it is consistent with Corollary 6.6, stating that at the most two weighting factors for producing and pumping respectively will be non-zero. Some pumping is conducted for low prices and low demand. This is however a fairly small amount of, on average, slightly more than 200 MWh over the two weeks. This can be compared with the average production of more than 130000 MWh. The pumping hence only amounts to less than 0.2 percent of the production. For tighter risk constraint the pumped amount is even less and often non-existing. The limited appetite for pumping is, as mentioned a result of the small difference of, on average, 6% between peak and off-peak prices, which should be compared with the pump efficiency of 70%. Hence it does not, on average, payoff to pump during off-peak to 'store' cheap electricity for production during a peak period.

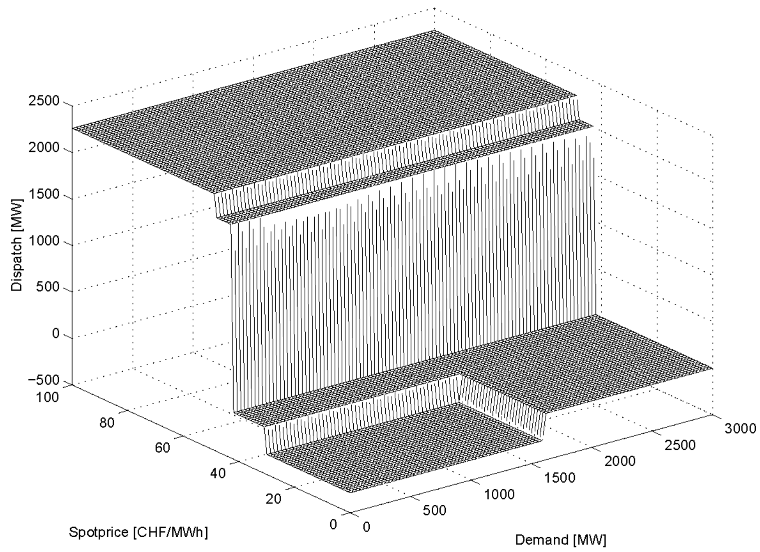


Fig. 7.9: Optimal dispatch for a risk constraint of 2 million CHF.

When the risk constraint is strengthened from 2 million to 0 and -5.6 million CHF respectively the strategy becomes more concerned in dealing with the volume risk, as seen in Figure 7.10 and 7.11. The dispatch is now responding not only to the spot price, but also to the swing option demand, by producing more when the demand increases. Consequently the dispatch strategy tries to avoid a large spot position in the case of a peak in demand by producing. Obviously, the high immediate payoff that production would cause in the case of high prices is lower than the value of leaving that water in the dam to meet a future potential demand spike. No pumping is conducted.

7.6. Summary

The optimal portfolio copes with risk in two ways. When the risk constraint gets tighter the contract portfolio reacts by decreasing the spot position. At the same time the production portfolio manages the risk constraint by decreasing the volume risk through a dispatch strategy that matches demand spikes. The fact that dispatch so heavily depends on the swing option demand is a verification that a simultaneous optimization of the contract portfolio and the production portfolio has to be conducted.

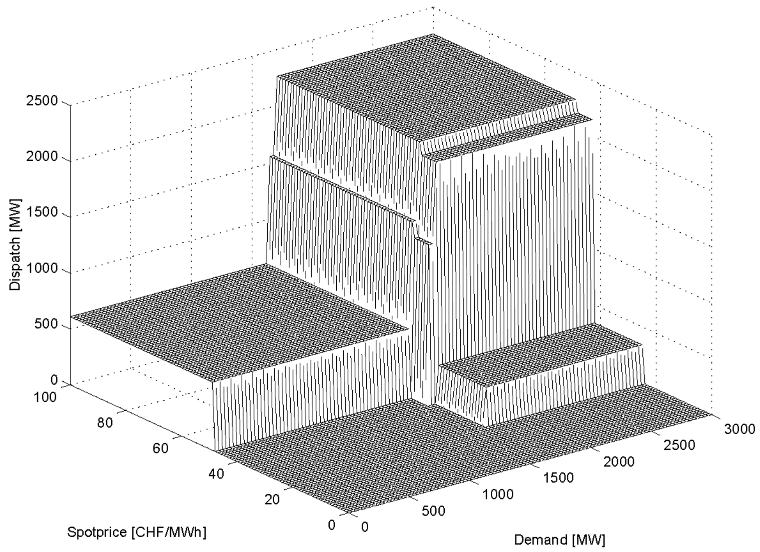


Fig. 7.10: Optimal dispatch for a risk constraint of 0 million CHF.

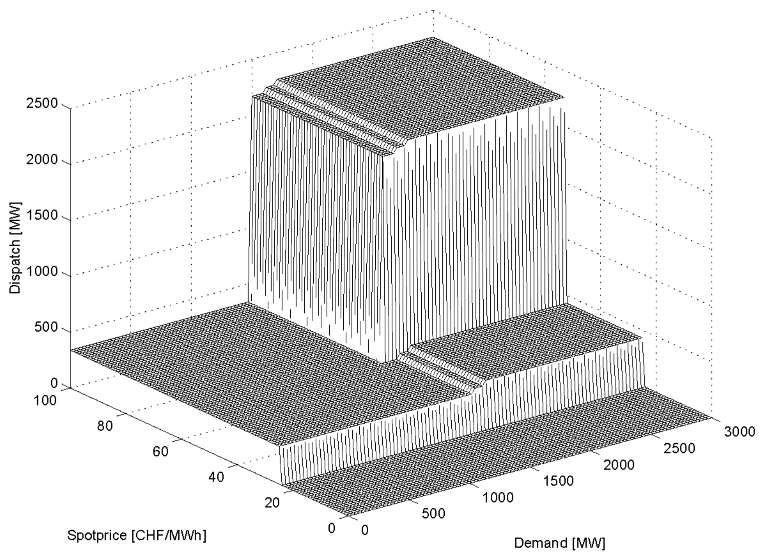


Fig. 7.11: Optimal dispatch for a risk constraint of -5.6 million CHF.

We have quantified the value of the stored water and it turns out that this hedging value can be substantial. The hedging value however depends on the risk constraint and on the contract portfolio. To utilize the hedging potential in the hydro storage plants one needs to identify high margin contracts that from a risk perspective can be managed.

Chapter 8

Conclusions

The special characteristics of electricity as a commodity and the complexity of the electricity market have huge impact on how to manage risk. The incompleteness of the electricity market and the non-storability of electricity makes valuation of electricity contracts a truly challenging task. When appraising risk, a risk measure that penalizes large losses is needed, because of the heavy-tailed nature of power portfolios. A risk measure that we believe is well suited for the electricity market is CVaR. The incompleteness of the market implies that one cannot perfectly hedge all contracts. We therefore introduce the concept of a *best hedge*, which is found by minimizing risk subject to a constraint on the expected profit. A major difference from traditional financial markets is the interaction of the production portfolio and the contract portfolio. We present the idea of *contract engineering*, where the corresponding contracts of a plant type are identified. This allows us to compare the two different asset types and helps us to identify financial risk factors in production. Despite the difficulties to value electricity contracts with traditional concepts, we can with a simple absence of arbitrage argument internally assess the often complex contracts corresponding to a certain power plant by studying the internal costs associated with the power plant.

When optimizing a power portfolio one has to take both the production portfolio and the contract portfolio into account and a simultaneous optimization is needed. We have presented a risk management like portfolio optimization suited for the electricity market, which will not only give us the optimal contract portfolio, but also the optimal production portfolio in terms of the optimal dispatch strategy. We could prove under certain assumptions that this strategy should be of the type step function. The fact that the dispatch in our case study turns out to be a function of demand in the contract portfolio is in itself a verification that the production portfolio depends on the contract portfolio and that a simultaneous optimization of the two portfolios has to be performed to achieve an optimal solution. A key objective of this thesis is to investigate the operational flexibility and its value of power plants. The operational flexibility of hydro storage plants is modeled through a dynamic dispatch strategy, which allows us to, not only decompose the value of this flexibility, but also to quantify it. One should note that a plain discounted cash flow analysis would underestimate the true value of a hydro plant. This is the case also when cash flows are calculated from the average price of produced electricity, which exceeds the average spot price, since the value of the stored water has yet another component, namely its hedging value. In the case study performed, this hedging value, allowing us to take risky positions in the contract portfolio, showed to be substantial. Because of the skewness in the spot price distribution and the volume uncertainty in a power portfolio, a player will optimally 'over-hedge' this volume uncertainty by assuring that he, on average, is a seller in the spot market.

Even though we believe that our proposed risk management ideas has shed some light on the complex problem of handling risk in the electricity market, a number of questions remains to be solved. To mention some, the issue of liquidity risk, which is indeed large in the immature electricity market is a subject for future research. Further, the optimal dispatch of semi-flexible thermal plants, such as coal and oil plants remains to be analyzed.

Appendix A

Proofs

Theorem 6.1
For $b \in [b_i^+, b_{i-1}^+]$, let λ be such that $b = \lambda b_i^+ + (1-\lambda)b_{i-1}^+$, where $i = 2, \ldots, r$ and define $a(b) = \lambda a_i^+ + (1-\lambda)a_{i-1}^+$, where $\lambda = \frac{b_{i-1}^+ - b}{b_{i-1}^+ - b_i^+}$.

Then $a(b)$ is a concave, monotone increasing function in b.

Proof For the proof we observe that the points $[b_i^+, a_i^+] \in \mathbb{R}_+^2$ are in convex positions, i. e. let

$$b_q^+ = \lambda b_i^+ + (1-\lambda)b_j^+, \quad \forall \lambda \in [0, 1]$$

then

$$a_q^+ \geq \lambda a_i^+ + (1-\lambda)a_j^+, \tag{A.1}$$

where we assume w.l.o.g. that $b_i^+ < b_q^+ < b_j^+$ and hence by definition $a_i^+ < a_q^+ < a_j^+$. Again by definition $S_i^+ > S_q^+ > S_j^+$.

To show (A.1) note that

$$a_q^+ = \lambda a_q^+ + (1-\lambda)a_q^+,$$

and hence it is equivalent to prove that

$$\lambda(a_i^+ - a_q^+) + (1-\lambda)(a_j^+ - a_q^+) = -\lambda(a_q^+ - a_i^+) + (1-\lambda)(a_j^+ - a_q^+) \leq 0. \quad \text{(A.2)}$$

Applying the two inequalities

$$(a_q^+ - a_i^+) = \int_{S_q^+}^{S_i^+} S f_S dS \geq S_q^+ \int_{S_q^+}^{S_i^+} f_S dS$$

and

$$(a_j^+ - a_q^+) = \int_{S_j^+}^{S_q^+} S f_S dS \leq S_q^+ \int_{S_j^+}^{S_q^+} f_S dS,$$

(A.2) follows from

$$-\lambda S_q^+ \int_{S_q^+}^{S_i^+} f_S dS + (1-\lambda) S_q^+ \int_{S_j^+}^{S_q^+} f_S dS \leq 0 \iff$$

$$S_q^+ \Big[\underbrace{-\lambda(b_q^+ - b_i^+) + (1-\lambda)(b_j^+ - b_q^+)}_{-\lambda b_q^+ + \lambda b_i^+ + (1-\lambda) b_j^+ - (1-\lambda) b_q^+ = 0} \Big] \leq 0$$

by definition of b_q^+. ■

Corollary 6.2
The solution of (6.23) is given by

(i) In the trivial case, where $W^\star \geq b_1^+$, $\gamma_1^+ = 1$

(ii) If $W^\star \leq b_r^+$ then the problem is infeasible

(iii) Else $W^\star \in [b_i^+, b_{i-1}^+]$ for some i and $\gamma_i^+ = \frac{b_{i-1}^+ - W^\star}{b_{i-1}^+ - b_i^+}$, $\gamma_{i-1}^+ = 1 - \gamma_i^+$.

Proof The results follow immediately from Theorem 6.1. Case (i) corresponds to abundant water and we optimally produce at as low prices as possible, since a is monotone increasing in b. In case (ii) there is not enough water, even if we chose the most conservative strategy to produce only at $S \geq S_r^+$. In the general case (iii) the concavity of a in b implies that a convex combination of two adjacent points is optimal, whereas a being monotone increasing in b gives the actual weights. ■

Corollary 6.3
Evidently the optimal policy when maximizing the expected profit given a constraint on the averaged produced water is to choose $S^\star$ such that

$$\int_{S \geq S^\star} f_S dS = W^\star.$$

Proof Let the distance between the thresholds $S_i^+ - S_{i-1}^+$ go to zero and consider (6.23) in continuous time, then the results follow immediately from Theorem 6.1. ■

Corollary 6.4
The marginal value of water $\frac{z(W^\star+\Delta)-z(W^\star)}{\Delta}$

(i) Is in the trivial case, where $W^\star > b_1^+$ zero

(ii) Is in the general case, where $W^\star \in [b_i^+, b_{i-1}^+)$ for some i, given by $\frac{a_{i-1}^+ - a_i^+}{b_{i-1}^+ - b_i^+}$, and is constant in the interval $\Delta \in [b_i^+ - W^\star, b_{i-1}^+ - W^\star]$.

Proof In the special case (i) an additional unit of water will not have any value, since we cannot even utilize the currently available water. The general case is easily proven by observing that the marginal value of water is given by the slope of $a(b)$. ■

Proposition 6.5
The marginal value of water is never exceeding the average price of produced electricity $\frac{z(W^\star)}{W^\star} \geq \frac{z(W^\star+\Delta)-z(W^\star)}{\Delta}$.

Proof The trivial case where we have abundance of water is obvious, and in the general case the average price of produced electricity is given by

$$\frac{(1-\gamma_i^+)a_{i-1}^+ + \gamma_i^+ a_i^+}{(1-\gamma_i^+)b_{i-1}^+ + \gamma_i^+ b_i^+}.$$

Hence we are comparing the slope of the line connecting the points

$$\left(0,0\right) \text{ and } \left((1-\gamma_i^+)b_{i-1}^+ + \gamma_i^+ b_i^+, (1-\gamma_i^+)a_{i-1}^+ + \gamma_i^+ a_i^+\right)$$

with the line between

$$\left(b_i^+, a_i^+\right) \text{ and } \left(b_{i-1}^+, a_{i-1}^+\right).$$

Observe that a_r and b_r will go to zero when the threshold S_r^+ goes to infinity, hence following from the concavity of a in b, it is obvious that the latter slope cannot exceed the former slope. ■

Corollary 6.6
In the solution of (6.26)

(i) If $W^\star \geq b_1^+$ then $\gamma_1^+ = 1$ and $\gamma_1^- = 1$ is optimal

(ii) If $W^\star + \rho < b_r^+ + \rho b_\tau^+$ then the problem is infeasible

(iii) Else at most two adjacent production weights γ_{i-1}^+ and γ_i^+ and at most two adjacent pumping weights γ_{j-1}^- and γ_j^- will be non-zero.

Proof The results again follows from Theorem 6.1. In case (i) we optimally produce and pump at as low prices as possible, which again follows from a being monotone increasing in b. Case (ii) is obvious and case (iii) again follows directly from the concavity. ■

Proposition 6.7
In the solution of (6.26) at the most three weights are non-zero and given by

(i) In the trivial case, where there is abundant water $\gamma_1^+ = \gamma_1^- = 1$.

(ii) In the general case where water is scarce, either one production weight equals one, i. e. $\gamma_i^+ = 1$ for an $i \in \{1, \dots, r\}$, and two adjacent pumping weights γ_{j-1}^- and γ_j^- for a $j \in \{2, \dots, \tau\}$ are non-zero and given by

$$\gamma_j^- = \frac{W^\star + \rho - b_i^+ - \rho b_{j-1}^+}{\rho(b_j^+ - b_{j-1}^+)}, \quad \gamma_{j-1}^- = 1 - \gamma_j^-.$$

Or else one pumping weight equals one, i. e. $\gamma_j^- = 1$ for an $j \in \{1, \dots, \tau\}$ and two adjacent production weights γ_{i-1}^+ and γ_i^+ for an $i \in \{2, \dots, r\}$ are non-zero and given by

$$\gamma_i^+ = \frac{W^\star + \rho - \rho b_j^+ - b_{i+1}^+}{b_i^+ - b_{i+1}^+}, \quad \gamma_{i-1}^+ = 1 - \gamma_i^+.$$

Proof Assume $\bar{u}_j$ minimizes the dual $\Longrightarrow \exists$ unique $i \in \{1, \dots, r\}$ such that $\bar{u}_j \in [\hat{u}_i, \hat{u}_{i+1}]$. Then by applying the complementarity condition

$$\gamma_k^+(b_k^+\bar{u}_j + v - a_k^+) = 0, \ \forall k = 1, \dots, r$$

and by observing that $b_k^+\bar{u}_j + v - a_k^+ \neq 0, \ \forall k \neq i$, it directly follows that $\gamma_i^+ = 1$. Further by again applying the complementarity condition

$$\gamma_q^-(\rho b_q^+\bar{u}_j + w - \frac{\rho}{\chi}a_q^+) = 0, \ \forall q = 1, \dots, \tau$$

and observing that $\rho b_q^+\bar{u}_j + w - \frac{\rho}{\chi}a_q^+ \neq 0, \ \forall q \neq \{j-1, j\}$, it follows that $\gamma_q^- = 0, \ \forall q \neq \{j-1, j\}$ and hence that $\gamma_{j-1}^- + \gamma_j^- = 1$. Observe that the special case, where $\bar{u}_1 = \hat{u}_1 = 0$ minimizes the dual, corresponds to abundant water and $\gamma_1^+ = \gamma_1^- = 1$ follows from the complementarity conditions. Else observe that, since the water constraint is binding

$$b_i^+ + \rho(\gamma_j^- b_j^+ + (1 - \gamma_j^-)b_{j-1}^+) = W^\star + \rho$$

we can directly solve for $(\gamma_{j-1}^-, \gamma_j^-)$.

Else $\hat{u}_i$ minimizes the dual $\Longrightarrow \exists$ unique $j \in \{1, \dots, \tau\}$ such that $\hat{u}_i \in [\bar{u}_j, \bar{u}_{j+1}]$. A similar procedure now applies and by the complementarity conditions it follows that $\gamma_j^- = 1$ and the production weights can again be solved for by using the active water constraint. ■

Corollary 6.8
The pumping decision is related to the production decision by the marginal value of water and

(i) Assume production at price S_i, then the marginal value of water is $\hat{u}_i$. Pumping occurs below price S_j $(j < i)$, where the marginal value of water $\hat{u}_j \leq \chi \hat{u}_i \leq \hat{u}_{j+1}$.

(ii) Assume pumping at price S_j with marginal value of water $\hat{u}_j$. Then production occurs at price S_i $(j < i)$ where $\chi \hat{u}_i \leq \hat{u}_j \leq \chi \hat{u}_{i+1}$.

Proof Observe that $\bar{u}_j = \frac{1}{\chi} \hat{u}_j$ and that $\exists (i, j)$ such that $\hat{u}_i \in [\bar{u}_j, \bar{u}_{j+1}]$ and consequently $\hat{u}_i \in \frac{1}{\chi}[\hat{u}_j, \hat{u}_{j+1}] \iff \chi \hat{u}_i \in [\hat{u}_j, \hat{u}_{j+1}]$. By definition $\hat{u}_i = \frac{a^+_{i-1} - a^+_i}{b^+_{i-1} - b^+_i}$ is the marginal value of water in the interval $[b^+_i, b^+_{i-1}]$ and from the assumption about (i, j), $\chi \hat{u}_i \geq \frac{a^+_{j-1} - a^+_j}{b^+_{j-1} - b^+_j}$, which is the marginal value of water in the interval $[b^+_j, b^+_{j-1}]$. ■

Proposition 6.11
The optimal value function $z(b)$ is piecewise linear and concave.

Proof The optimal value function can through the strong duality be written as

$$z(b) = \max\{c^T x | Ax \leq b\} = min\{b^T y | A^T y = c, \ y \geq 0\}. \tag{A.3}$$

Trough projection (A.3) can written as the minimum of all dual basis solutions

$$z(b) = \min\{b^T y^k | k = 1, \ldots, q\}, \ \forall b \text{ such that } \{x | Ax \leq b\} \neq \emptyset, \tag{A.4}$$

which proves that $z(b)$ is piecewise linear.

To prove concavity we need to show that

$$z(b_3) \geq \lambda z(b_1) + (1 - \lambda) z(b_2),$$

where $b_3 = \lambda b_1 + (1-\lambda)b_2$ and $\lambda \in [0, 1]$. Let $x_1^\star$ and $x_2^\star$ be optimal and hence feasible solutions to $\max\{c^T x | Ax \leq b_1\}$ and $\max\{c^T x | Ax \leq b_2\}$ respectively. A feasible solution to $\max\{c^T x | Ax \leq b_3\}$ is given by $x_3 = \lambda x_1^\star + (1 - \lambda) x_2^\star$ since

$$Ax_3 = \lambda Ax_1^\star + (1 - \lambda) Ax_2^\star \leq \lambda b_1 + (1 - \lambda) b_2 = b_3.$$

By definition we have that

$$\lambda z(b_1) + (1 - \lambda) z(b_2) = \lambda c^T x_1^\star + (1 - \lambda) c^T x_2^\star = c^T x_3$$

and since $c^T x_3$ is not necessarily optimal, $z(b_3) \geq c^T x_3$. ■

Corollary 6.12
The marginal value of water, $\frac{z(V_{end}-\Delta)-z(V_{end})}{\Delta}$ is piecewise constant and increasing in V_{end}.

Proof The marginal value $\frac{z(b+\Delta)-z(b)}{\Delta}$ is given by the gradient of the optimal value function $z(b)$, which according to Proposition 6.11 is piecewise linear and concave. Hence $\frac{z(b+\Delta)-z(b)}{\Delta}$ will be piecewise constant and decreasing in b and the marginal value of water $\frac{z(V_{end}-\Delta)-z(V_{end})}{\Delta}$ will, because of the opposite inequality sign ($\geq$ instead of $\leq$) consequently be piecewise constant and increasing in V_{end}. ■

Corollary 6.13
The efficient frontier $z(C)$ is piecewise linear and concave.

Proof Observe that $z(C)$ is the one-dimensional version of the optimal value function $z(b)$. The results then immediately follows from Proposition 6.11. ■

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