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Outage Analysis of Asynchronous OFDM Non-orthogonal DF Cooperative Networks

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***Abstract*—Outage behavior of non-orthogonal[1] selection decode-and-forward (NSDF) relaying protocol over an *asynchronous* cooperative network is examined when orthogonal frequency division multiplexing (OFDM) is used to combat synchronization error among the transmitting nodes. It is proved that the asynchronous protocol provides diversity gain greater than or equal to the one of the corresponding synchronous counterpart, synchronous NSDF, in the limit of code word length and throughout the range of multiplexing gain.**

I. Introduction

Cooperative diversity was first proposed as a synchronous technique [1], [2] to provide spacial diversity with the help of surrounding terminals; however, because relays are at different locations (i.e., different propagation delays) and they have their own local oscillators with no common timing reference, it is an *asynchronous* technique in nature.

To combat the synchronization error, two major approaches have been proposed: delay tolerant space-time schemes (see [3], [4] and references therein), and OFDM [5]. While it is usually assumed in the former schemes that asynchronous delays are integer factor of the symbol interval, OFDM allows the delays to be any real number. In [6], the effect of the synchronization error on diversity multiplexing gain tradeoff (DMT) [7] of an orthogonal decode-and-forward (DF) cooperative network with two relays is examined when the maximum possible relative delay between the relays is less than a symbol interval. In [8], authors show that by allowing the source and the relays to transmit over proper portions of a cooperative frame, the better diversity gain can be achieved for each multiplexing gain.

In this paper, we analyze the outage behavior of NSDF protocol over a general two-hop relay network when OFDM is used to offset the synchronization error among transmitting nodes. In contrast to [6], we do not restrict the relative delays to be less than a symbol interval. In addition, we let the source and the relays to transmit over non-symmetric portions of a cooperative frame to maximize the diversity gain at each multiplexing gain. It is proved that the asynchronous protocol outperforms the synchronous counterpart in the limit of code word length and throughout the range of the multiplexing gain.

[1]A relaying protocol is called orthogonal if the source and relays transmit in two non-overlapping intervals; otherwise, it is called non-orthogonal.

In the following, the system model and the required background are presented. DMT analysis of the asynchronous OFDM NSDF relaying protocol is detailed afterward. The paper is concluded at the end.

II. Preliminaries

A. Notations, Assumptions, and Definitions

In this work, letters with underline $\underline{x}$, $\underline{X}$ denote vectors, and boldface uppercase letters $\mathbf{X}$ denote matrices. The superscripts $(\cdot)^T$ and $(\cdot)^\dagger$ denote the transpose and conjugate transpose of the corresponding vector or matrix, respectively. $\mathbf{I}_n$ is the identity matrix of dimension n. diag$\{\cdot\}$ indicates a diagonal or a block diagonal matrix of its arguments. The symbol $\otimes$ indicates the Kronecher product. $\doteq$ is used to show the exponential equality. For example, $f(\rho) \doteq \rho^b$ if $\lim_{\rho\to\infty} \frac{\log f(\rho)}{\log \rho} = b$. $(x)^+$ is considered as $\max\{0, x\}$.

We assume half-duplex signal transmission. All channels are assumed to be quasi-static. They are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance $\mathbb{C}\mathcal{N}(0, 1)$. Each node knows channel state information (CSI) of its incoming links. The destination also knows the asynchronous delays of its incoming links.

Define $\{\mathcal{C}(\rho)\}$ as a family of variable rate codes each of them is used at the corresponding signal to noise ratio ρ. This family of codes is said to achieve the multiplexing gain r and the diversity gain $d(r)$ if

$$\lim_{\rho\to\infty} \frac{R(\rho)}{\log \rho} = r, \qquad \lim_{\rho\to\infty} \frac{\log P_e(\rho)}{\log \rho} = -d(r), \tag{1}$$

where $R(\rho)$ is the rate and $P_e(\rho)$ is the average error probability of the code $\mathcal{C}(\rho)$. The outage diversity is obtained by replacing $P_e(\rho)$ with the outage probability $P_{\mathcal{O}}$ in the above formula. It is proved that the outage diversity is a tight upper bound for the diversity gain of a coding scheme [7].

B. System Model

We consider a network containing one source node, one destination node, and M relay nodes as shown in Fig. 1. h_i and g_i are fading coefficients represent the links from the i-th transmitting node to the destination and from the source to the i-th relay, respectively. Communication between the source and the destination is carried out in two phases. First, the

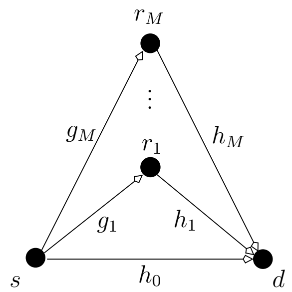

Fig. 1. System structure

source broadcasts its message to relays and the destination in p channel uses. Second, those relays that can fully decode the source message retransmit it to the destination in q channel uses. Assuming ℓ is the length of a cooperative frame, $\ell = p + q$. The cooperation is avoided whenever it is beneficial to do so. In this case, the source transmits to the destination without help of the relays. Each node is supported by an i.i.d. Gaussian random code book which is independent from the other nodes' code books. The source's transmitted signal in the first phase is given by

$$x'_0(t) = \sum_{i=0}^{p-1} x'_0(i)\mathrm{g}_0(t - iT_s), \tag{2}$$

where $\underline{x}'_0 = [x'_0(0), x'_0(1), \ldots, x'_0(p-1)]^T$ is the transmitted code word corresponding to the source message, T_s is the symbol interval, and $\mathrm{g}_0(t)$ is a unit energy shaping waveform with non-zero duration T_s over $t \in [0, T_s]$. The received signals at the destination and the i-th relay are modeled by

$$\begin{aligned} y_d(t) &= h_0 x'_0(t) + z_d(t), \quad 0 \le t \le pT_s, & (3)\\ y_{r_i}(t) &= g_i x'_0(t) + z_{r_i}(t), \quad 0 \le t \le pT_s, & (4) \end{aligned}$$

where $z_d(t)$ and $z_{r_i}(t)$ are additive noises at the destination and the i-th relay modeled by white Gaussian noises with zero mean and variances σ_d^2 and σ_r^2, respectively.

Let $\mathcal{D}$ be a set containing index of the nodes participating in the second phase (not in outage). As the relaying protocol is non-orthogonal, $\mathcal{D}$ contains 0, index of the source. Similarly, the i-th relay, $i \in \mathcal{D}$, uses a unit energy shaping waveform $\mathrm{g}_i(t)$ with nonzero duration T_s to transmit its code words of length q in the second phase. This signal is received at the destination by τ_i second delay with reference to the first received signal. τ_is are finite values less than or equal to τ_{max} which is the maximum amount of asynchronous delay. Without loss of generality, we assume that the source signal is the earliest received signal at the destination, and the delays of the other received signals are measured with reference to this signal, i.e., $\tau_0 = 0$.

Let $x_i(t)$ be the transmitted signal by the i-th node. The received signal at the destination is modeled by

$$y_d(t) = \sum_{i \in \mathcal{D}} h_i x_i(t - \tau_i) + z_d(t), \tag{5}$$

$y_d(t)$ is processed through parallel matched filters corresponding to the transmitting links. The output of the i-th matched filter sampled at $t = (k+1)T_s + \tau_i$, is given by

$$y_{d_i}(k) = \int_{kT_s+\tau_i}^{(k+1)T_s+\tau_i} y_d(t)\mathrm{g}_i^*(t - kT_s - \tau_i)dt. \tag{6}$$

C. Asynchronous OFDM Space-Time Codes

In our work, OFDM is used to combat the synchronization error. Assume that the i-th node participates in the second phase, i.e., $i \in \mathcal{D}$. Its code word of length n, $\underline{x}_i$, is first passed through an inverse discrete Fourier Transform (IDFT) filter, IDFT$\{\underline{x}_i\} = \underline{X}_i$, and then supported by a cyclic prefix (CP) of length $u = \lceil \frac{\tau_{max}}{T_s} \rceil$, where $\lceil x \rceil$ denotes the smallest integer greater than x, to produce $\underline{X}_i^{cp}$ of length $q = n + u$. The received signal at the destination is given by

$$Y_d(t) = \sum_{i \in \mathcal{D}} h_i \sum_{j=0}^{q-1} X_i^{cp}(j)\mathrm{g}_i(t - jT_s - \tau_i) + Z_d(t), \tag{7}$$

where $X_i^{cp}(j)$ is the j-th entry of $\underline{X}_i^{cp}$. For $i \ge j$, $i, j \in \mathcal{D}$, define the relative delay τ_{ij} as

$$\tau_{ij} \triangleq \tau_i - \tau_j. \tag{8}$$

As $i \ge j$, then $\tau_{i,j} \ge 0$. The fractional delay $\tilde{\tau}_{ij}$ is defined as

$$\tilde{\tau}_{ij} \triangleq \tau_{ij} - a_{ij}T_s, \tag{9}$$

where $a_{ij} = \lfloor \frac{\tau_{ij}}{T_S} \rfloor \ge 0$, with $\lfloor x \rfloor$ denoting the largest integer smaller than or equal to x, and $0 \le \tilde{\tau}_{ij} < T_s$.

III. Asynchronous OFDM NSDF Protocol

A. Signal Model

Let E_m, the event of any m relays participates in the second phase, occurs. E_0 corresponds to the case that only the source node transmits in the second phase. $\mathcal{D} = \{0, 1, 2, \ldots, m\}$ is the index set pointing out to participating nodes in the second phase. Without loss of generality, we assume that $0 = \tau_0 \le \tau_1 \le \tau_2 \le \ldots \le \tau_m$. The sampled signal at the output of the i-th matched filter $(i = 0, 1, \ldots, m)$ is modeled by [9]

$$\begin{aligned} Y_{d,i}(k) &= h_i X_i^{cp}(k) + Z_{d,i}(k) + \\ &\sum_{j=0}^{i-1} h_j [X_j^{cp}(k + a_{ij} + 1)B_{ij}^* + X_j^{cp}(k + a_{ij})C_{ij}^*] + \\ &\sum_{j=i+1}^{m} h_j [X_j^{cp}(k - a_{ji} - 1)B_{ji} + X_j^{cp}(k - a_{ji})C_{ji}], \end{aligned} \tag{10}$$

where $Y_{d,i}(k)$ is the k-th entry of the output of the i-th matched filter, and for $i \ge j$, $B_{ij} = \int_0^{T_s} \mathrm{g}_i(t+T_s-\tilde{\tau}_{ij})\mathrm{g}_j^*(t)dt$, $C_{ij} = \int_0^{T_s} \mathrm{g}_i(t - \tilde{\tau}_{ij})\mathrm{g}_j^*(t)dt$. Define

$$\alpha_{ij}(k) \triangleq [C_{ij} + B_{ij}e^{-j\frac{2\pi}{n}k}]e^{j\frac{2\pi}{n}k\tilde{a}_{ij}}, \tag{11}$$

where $\tilde{a}_{ij} = 0$ when $\tilde{\tau}_{i0} \ge \tilde{\tau}_{j0}$, and $\tilde{a}_{ij} = 1$ when $\tilde{\tau}_{i0} < \tilde{\tau}_{j0}$. It can be checked that, for $j > i, \alpha_{ij}(k) = \alpha_{ji}^*(k)$, $k = 0, 1, \ldots, n-1$ and $i, j = 0, 1, \ldots, m$. Let

$$\mathbf{D}_{ij} = \mathrm{diag}\{\alpha_{ij}(0), \alpha_{ij}(1), \ldots, \alpha_{ij}(n-1)\}, \tag{12}$$

$$\mathbf{E}_i = \mathrm{diag}\{1, e^{-j\frac{2\pi}{n}i}, \ldots, e^{-j\frac{2\pi}{n}(n-1)i}\}. \tag{13}$$

At the output of each matched filter CP is discarded. The result is then passed through a Discrete Fourier Transform (DFT) filter. The outputs can be written in a matrix form as

$$\underline{y} = \mathbf{H}\underline{x} + \underline{z}, \tag{14}$$

where

$$\begin{aligned}
\underline{x} &= \left[\underline{x}_0^T \ \underline{x}_1^T \ \dots \ \underline{x}_m^T\right]^T \\
\underline{y} &= \left[\underline{y}_{d,0}^T \ \left(\mathbf{E}_1^\dagger \underline{y}_{d,1}\right)^T \ \dots \ \left(\mathbf{E}_1^\dagger \underline{y}_{d,m}\right)^T\right]^T, \\
\underline{z} &= \left[\underline{z}_{d,0}^T \ \left(\mathbf{E}_1^\dagger \underline{z}_{d,1}\right)^T \ \dots \ \left(\mathbf{E}_1^\dagger \underline{z}_{d,m}\right)^T\right]^T, \\
\mathbf{H} &= \Xi(\mathbf{I}_n \otimes \hat{\mathbf{H}})\mathbf{U}.
\end{aligned} \tag{15}$$

$\mathbf{U} = \text{diag}\{\mathbf{I}_n, \mathbf{E}_{a_{10}}, \dots, \mathbf{E}_{a_{m0}}\}$, $\hat{\mathbf{H}} = \text{diag}\{h_0, h_1, \dots, h_m\}$, and

$$\Xi = \begin{bmatrix} \mathbf{I}_n & \mathbf{D}_{10} & \mathbf{D}_{20} & \dots & \mathbf{D}_{m0} \\ \mathbf{D}_{10}^\dagger & \mathbf{I}_n & \mathbf{D}_{21} & \dots & \mathbf{D}_{m1} \\ \vdots & \vdots & \vdots & & \vdots \\ \mathbf{D}_{m0}^\dagger & \mathbf{D}_{m1}^\dagger & \mathbf{D}_{m2}^\dagger & \dots & \mathbf{I}_n \end{bmatrix}. \tag{16}$$

Equation (14) represents a simple multiple-input multiple-output (MIMO) channel model with correlated noise vector $\underline{z}$ for the underlying system. The covariance matrix of $\underline{z}$ is calculated as [9]

$$\mathbf{\Phi} = n\sigma_d^2 \ \Xi. \tag{17}$$

Clearly, $\mathbf{\Phi}^{-1}$ exists if and only if Ξ^{-1} exists.

Proposition 1: Ξ is semi-positive definite. i.e., $\det \Xi \geq 0$. The equality holds if and only if $\exists \ \underline{c} \in \mathbb{C}^{1\times m}, \exists \ k \in \{0, \dots, n-1\}$ such that [9].

$$\left(\sum_{i=0}^{1} \underline{\text{g}}(t + iTs)e^{-j\frac{2\pi}{n}ki}\right)\underline{c}^\dagger = 0, \quad \forall t \in [0, T_s], \tag{18}$$

where $\underline{\text{g}}(t) \triangleq [\text{g}_0(t), \text{g}_1(t-\tilde{\tau}_{10}), \text{g}_2(t-\tilde{\tau}_{20}), \dots, \text{g}_m(t-\tilde{\tau}_{m0})]$, and $\mathbb{C}$ is the field of complex numbers.

B. DMT Analysis

The outage probability $P_\mathcal{O}$ is calculated as follows.

$$P_\mathcal{O} \ = \ \sum_{m=0}^{M} Pr(I_{E_m} < R \mid \ E_m)Pr(E_m),$$

where I_{E_m} is the mutual information between the source and the destination when E_m occurs.

Lemma 1: $Pr(E_m)$ is given by [9]

$$Pr(E_m) \doteq \begin{cases} \rho^{-(1-\frac{\ell r}{p})(M-m)}, & 0 \leq r \leq \frac{p}{\ell}, \\ 0, & \frac{p}{\ell} < r \leq 1, \ 1 \leq m \leq M \\ 1, & \frac{p}{\ell} < r \leq 1, \ m = 0. \end{cases} \tag{19}$$

When E_m occurs, the mutual information between the source and the destination is given by [9]

$$\begin{aligned} I_{E_m} \ &= \ \frac{p}{\ell}\log(1 + \rho|h_0|^2) + \\ &\frac{1}{\ell}\log\det\left(\mathbf{I}_{(m+1)n} + n\mathcal{E}\mathbf{H}\mathbf{H}^\dagger\mathbf{\Phi}^{-1}\right), \end{aligned} \tag{20}$$

where the first and the second terms on the right hand side are the resulted mutual information between the transmitting nodes and the destination, respectively, in the first and the second phases. Define $\mathcal{A} \triangleq \mathbf{I}_{(m+1)n} + n\mathcal{E}\mathbf{H}\mathbf{H}^\dagger\mathbf{\Phi}^{-1}$. By substituting (15) and (17) into (20) and considering the fact that $\mathbf{U}$ is a Hermitian matrix, we have

$$\det\mathcal{A} = \det\left(\mathbf{I}_{(m+1)n} + \rho\Xi(\mathbf{I}_n \otimes \hat{\mathbf{H}}\hat{\mathbf{H}}^\dagger)\right). \tag{21}$$

Ξ can be decomposed as $\Xi = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^\dagger$, where $\mathbf{V}$ is a unitary matrix and $\mathbf{\Lambda}$ is a diagonal matrix containing eigenvalues of Ξ on its main diagonal. By assuming proper design of the shaping waveforms, all eigenvalues of Ξ are finite positive values bounded from zero. Hence, their ρ exponents at high SNR regime is zero. By replacing all the eigenvalues by the smallest one, say ξ, the mutual information between the source and the destination is lower bounded. Since the ρ exponent of ξ is zero, this bound is tight. We have,

$$\begin{aligned} \det\mathcal{A} \ &\doteq \ \det\left(\mathbf{I}_{(m+1)n} + \rho\xi(\mathbf{I}_n \otimes \hat{\mathbf{H}}\hat{\mathbf{H}}^\dagger)\right) \\ &\doteq \ \prod_{i=0}^{m}(1 + \rho|h_i|^2)^n. \end{aligned}$$

Define $\gamma_i \triangleq -\frac{\log|h_i|^2}{\log\rho}$. For large values of ρ, $(1 + \rho|h_i|^2) \simeq \rho^{(1-\gamma_i)^+}$. After some mathematical manipulation, we obtain

$$I_{E_m} = \left[\frac{p+n}{\ell}(1-\gamma_0)^+ + \frac{n}{\ell}\sum_{i=1}^{m}(1-\gamma_i)^+\right]\log\rho. \tag{22}$$

As can be seen, the resulted mutual information among the transmitting nodes and the destination behaves similar to the one of a parallel channel with (m+1) independent links. $P_{\mathcal{O}|E_m}$ is obtained as follows [9].

$$P_{\mathcal{O}|E_m} \ = \ P\left(I_{E_m} < R\right) \ \doteq \ \rho^{-d_{E_m}(r)}$$

where

$$d_{E_m}(r) = \inf_{\frac{p+n}{\ell}(1-\gamma_0)^+ + \frac{n}{\ell}\sum_{i=1}^{m}(1-\gamma_i)^+ < r} \ \sum_{i=0}^{m}\gamma_i. \tag{23}$$

By solving the above optimization problem, we have [9]

Lemma 2:

$$d_{E_m}(r) = \begin{cases} 1 + m - \frac{\ell}{n}r, & 0 \leq r \leq \frac{mn}{\ell}, \\ 1 + \frac{mn}{p+n} - \frac{\ell}{p+n}r, & \frac{mn}{\ell} < r \leq \frac{p+n}{\ell}. \end{cases}$$

Define $\kappa \triangleq \frac{p}{n}$. When $m \geq \kappa + 1$, then $\frac{mn}{\ell} \geq \frac{p+n}{\ell}$. Hence,

$$d_{E_m}(r) = 1 + m - \frac{\ell}{n}r, \quad 0 \leq r \leq \frac{p+n}{\ell}.$$

For the single relay network, theorem 1 concludes the results.

Theorem 1: DMT of the asynchronous OFDM NSDF protocol over the single relay cooperative network for a fix $\kappa \geq 1$ is as follows [9].

If $\quad 1 \leq \kappa \leq \hat{\kappa}$

$$d(r) = \begin{cases} (1 - \frac{\ell}{p}r) + (1 - \frac{\ell}{p+n}r), & 0 \leq r \leq \eta_1 \\ 1 - r, & \eta_1 \leq r \leq 1, \end{cases}$$

else if $\quad \kappa \geq \hat{\kappa}$

$$d(r) = \begin{cases} 2(1-\frac{\ell}{2n}r), & 0 \le r \le \eta_2 \\ 1+\frac{n}{p+n}-\frac{\ell}{p+n}r, & \eta_2 \le r \le \eta_3 \\ (1-\frac{\ell}{p}r)+(1-\frac{\ell}{p+n}r), & \eta_3 \le r \le \eta_1 \\ 1-r, & \eta_1 \le r \le 1, \end{cases}$$

where $\hat{\kappa} = \frac{1+\sqrt{5}}{2}$, $\eta_1 = \frac{(p+n)p}{(2p+n)\ell-(p+n)p}$, $\eta_2 = \frac{n}{\ell}$, and $\eta_3 = \frac{p^2}{(p+n)\ell}$. For the case that κ varies to maximize the diversity gain, for large length code words we have

$$d(r) = \begin{cases} [1-(1+\frac{1}{\hat{\kappa}})r]+(1-r), & 0 \le r \le \frac{1}{\hat{\kappa}+1} \\ (1-\sqrt{r})+(1-r), & \frac{1}{\hat{\kappa}+1} \le r \le 1. \end{cases}$$

The optimum κ corresponding to each r is given by

$$\kappa = \begin{cases} \hat{\kappa}, & 0 \le r \le \frac{1}{\hat{\kappa}+1} \\ \frac{\sqrt{r}}{1-\sqrt{r}}, & \frac{1}{\hat{\kappa}+1} \le r \le 1. \end{cases}$$

Fig. 2 depicts the DMT curves of the asynchronous OFDM NSDF and the corresponding synchronous protocol over a single relay network when κ varies to maximize the diversity gain at each multiplexing gain r. As can be seen, the DMT performance of the asynchronous protocol performs is the same as that of the synchronous one in low multiplexing gains and is better than that in high multiplexing gains.

Calculating DMT in a general network with any number of relays, say M, is straightforward. However, because too many regions for r and κ should be considered, it is cumbersome. Alternatively, this procedure is easier if we assume that DMT of a simpler network containing $(M-1)$ relays is known. Let $d^M(r)$ be the DMT of an M relay cooperative network when the cooperation is not avoided throughout the range of the multiplexing gain. We have,

Theorem 2: DMT of the asynchronous OFDM NSDF relaying protocol over a general two-hop cooperative network with M relays for a fix $\kappa \ge 1$ is as follows [9].

If $\kappa \le \dfrac{M+\sqrt{M^2+4M}}{2}$

$$d^M(r) = \begin{cases} (1-\frac{\ell}{p}r)+d^{M-1}(r), & 0 \le r \le \frac{p}{\ell} \\ 1-\frac{\ell}{p+n}r, & \frac{p}{\ell} \le r \le \frac{p+n}{\ell}, \end{cases}$$

else if $\kappa > \dfrac{M+\sqrt{M^2+4M}}{2}$

$$d^M(r) = \begin{cases} (1-\frac{\ell}{p}r)+d^{M-1}(r), & 0 \le r \le \eta_1 \\ 1+M-\frac{\ell}{n}r, & \eta_1 \le r \le \eta_2 \\ 1+\frac{Mn}{p+n}-\frac{\ell}{p+n}r, & \eta_2 \le r \le \eta_3 \\ M(1-\frac{\ell}{p}r)+1-\frac{\ell}{p+n}r, & \eta_3 \le r \le \eta_4 \\ 1-\frac{\ell}{p+n}r, & \eta_4 \le r \le \eta_5, \end{cases}$$

where $\eta_1 = \frac{(M-1)p^2n}{\ell(p^2-np-n^2)}$, $\eta_2 = \frac{Mn}{\ell}$, $\eta_3 = \frac{p^2}{\ell(p+n)}$, $\eta_4 = \frac{p}{\ell}$, and $\eta_5 = \frac{p+n}{\ell}$. The resulted DMT is compared to $(1-r)$ to determine wether or not avoiding the cooperation. When κ is allowed to vary to maximize the diversity gain at each multiplexing gain r, for large length code words we have

$$d(r) = \begin{cases} M[1-(1+\frac{1}{\hat{\kappa}})r]+(1-r), & 0 \le r \le \frac{1}{1+\hat{\kappa}} \\ M(1-\sqrt{r})+(1-r), & \frac{1}{1+\hat{\kappa}} \le r \le 1. \end{cases}$$

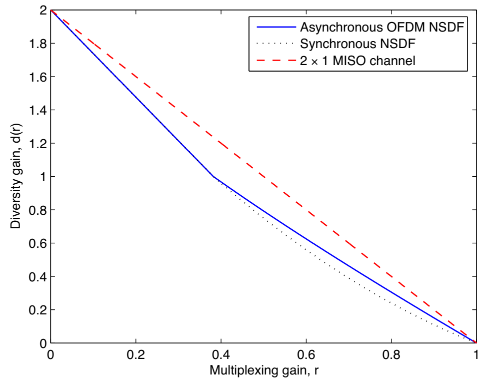

Fig. 2. DMT of the asynchronous OFDM NSDF and the synchronous NSDF protocols over a single relay network with optimum values of κ.

where $\hat{\kappa} = \frac{1+\sqrt{5}}{2}$. The corresponding optimum κ is the same as that of the single relay network.

For $M \ge 2$ the resulted DMT is always better than that of the corresponding synchronous protocol. DMT of the asynchronous orthogonal selection DF (OSDF) relaying protocol is calculated in a similar manner [9].

IV. Conclusion

DMT of the asynchronous OFDM NSDF protocol over a general one-hop cooperative network was examined. It was shown that asynchronous delays among transmitting nodes not only decrease the diversity gain, but also increase it particularly at high multiplexing gains for large length code words.

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