Multiresolution weighted norm equivalences and applications

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Erratum

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Four equations in statements and proofs of Theorems 3.2 and 3.3 were printed with errors. The text below contains the correct equations.

Theorem 3.2 *The infinite matrix* $M = ((\psi_k^l, \psi_{k'}^{l'})_w)_{(k,l);(k',l')}$ *is bounded in* l_2.

Proof We decompose the matrix M into $M = M_1 + M_2$ where the coefficients in M_2 are $(\psi_k^l, \psi_{k'}^{l'})_w$ iff $0 \in \operatorname{supp} \psi_k^l \cap \operatorname{supp} \psi_{k'}^{l'}$ and M_1 does not contain the interaction of wavelets which are both located at the point zero. By applying Theorem 3.1, Lemma 3.7 and the Schur Lemma to M_1 we have $\| M_1 \|_2 \leq c$. From Lemma

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3.8 we have $\| M_2 \|_1 \leq c$ and $\| M_2 \|_\infty \leq c$ which shows $\| M_2 \|_2 \leq c$. Hence, the assertion is proven. □

We show now the equivalence of the L^2_w norm of a function

$$u = \sum_{l=l_0}^{\infty} \sum_k u_k^l \psi_k^l \in L^2_w\,((0,1))$$

with its discrete l^2_w norm of the coefficients $(u_k^l)_{(k,l)} \in \mathbb{R}$, i.e.

$$|||u_k^l|||_w^2 := \sum_l \sum_k w^2(2^{-l}k)|u_k^l|^2.$$

Theorem 3.3 *Let us assume that Assumptions 3.1 and 3.2 are valid. For any function* $u = \sum_{l=l_0}^{\infty} \sum_k u_k^l \psi_k^l \in L^2_w\,((0,1))$ *holds*

$$\| u \|_w^2 \approx |||u_k^l|||_w^2.$$

Proof From Theorem 3.2 we conclude

$$\| u \|_w^2 = \sum_{l,l'} \sum_{k,k'} u_k^l u_{k'}^{l'} w(2^{-l}k) w(2^{l'}k') \Big(\psi_k^l, \psi_{k'}^{l'}\Big)_w$$
$$\leq \| M \|_2 \sum_l \sum_k \Big(\left|u_k^l\right| w(2^{-l}k) \Big)^2 \leq c|||u_k^l|||_w^2.$$

To prove the lower estimate we consider the dual system

$$\widetilde{v} = \sum_l \sum_k \widetilde{v}_k^l \widetilde{\psi}_k^l = G(\widetilde{v}_k^l)$$

in the dual space $L^2_{w^{-1}}\,((0,1))$. We denote by $\widetilde{M}$ the mass matrix of the dual wavelet basis $\widetilde{\psi}_k^l$ with respect to the $L^2_{w^{-1}}\,((0,1))$ innerproduct. Then, by the same arguments

$$\| \widetilde{v} \|_{w^{-1}}^2 \leq \| \widetilde{M} \|_2 \; |||\widetilde{v}_k^l|||_{w^{-1}}^2.$$

This means $G : l^2_{w^{-1}} \to L^2_{w^{-1}}\,((0,1))$ is bounded. Therefore, the adjoint operator $G^* : L^2_w\,((0,1)) \to l^2_w$ is bounded, too. G^* is explicitly given by

$$G^* u := \Big(\langle u, \widetilde{\psi}_k^l \rangle\Big)_{l,k} = (u_k^l)_{l,k}$$

which proves the lower bound. □

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