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Train trajectory optimization in the presence of external factors: The example of wind

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The train trajectory optimization problem consists in determining the speed profile of a train between two stations that minimizes energy consumption while respecting the scheduled arrival time and operational constraints such as speed limits. The problem is well-known in the literature but has so far been studied without accounting for external factors as weather conditions or train load that in reality vary in each journey. These factors have an impact on the train resistance, which in turn can affect energy consumption. In this paper, we focus on wind uncertainty and propose a novel train resistance equation that accounts for the impact of wind intensity and direction. For different wind conditions, we determine optimal trajectories as dynamic programs defined on a space-speed network that embeds the physical train motion relations updated with the actual wind information. Numerical experiments show that our "wind-aware" train trajectories are more energy-efficient than traditional solutions computed independent of wind information.

1. Introduction

Improving energy efficiency is a major challenge in modern railway transportation due to energy consumption being one of the largest operating costs (Railenergy 2016). This topic has in fact drawn considerable attention in the railway operations literature (see Yang et al. 2016 and De Martinis and Corman 2018 for recent surveys). In particular, optimizing the speed profiles (or trajectories) of individual trains while satisfying all operational constraints can lead to potential energy savings of 5–20% (Hansen and Pachl 2014) and is an attractive option for railway companies to reduce energy consumption as it does not require infrastructural changes or investments.

This problem, henceforth the *train trajectory optimization problem* (TTOP), has been studied in the scientific literature using a multitude of approaches including: (ii) direct methods, that formulate and solve the problem as a mathematical program (usually mixed-integer or non-linear; Wang et al. 2013, Wang and Goverde 2016), (i) indirect methods, where the TTOP is formulated as a boundary value problem using differential equations (Howlett 2000, Howlett and Pudney 2012), and (iii) dynamic programming (DP; Ko et al. 2004, Haahr et al. 2017, Zhou et al. 2017). Underlying the latter approach is a network constructed with discrete space, time, and speed quantities; for instance, a space-speed network (Haahr et al. 2017) or a space-speed-time network (Zhou et al. 2017).

The extant literature has studied the TTOP in a static environment in which speed profiles are computed independently from actual journey-dependent conditions. In reality, factors such as wind, rain, and train load, vary across journeys on the same track can affect energy consumption and optimal driving strategy. Our intuition is that if we were able to account for these factors, then the optimal trajectory could potentially be more energy-efficient because it adapts to more information neglected in previous research. Thus, our research questions is: Can we exploit knowledge of journey-dependent factors, such as weather conditions, to compute more energy-efficient speed profiles?

To address this question, we focus on the example of wind and propose a new train resistance equation that accounts for wind intensity and direction by decomposing the impact of wind on the train into traversal and longitudinal components. Although train motion is governed by well-established physical equations (see §2, or Wang and Rakha 2018 for a recent study on train dynamics), the wind effect on train resistance has not been studied to our knowledge.

Given the current wind state, our goal is hence determining the optimal train trajectory for this state (we call it a *wind-aware* trajectory) based on updated resistances. To compute this trajectory, we propose a method that combines a simple line search algorithm with the known DP approach in a space-speed network with arc costs adjusted for the actual wind state. We perform a numerical study showing that our wind-aware trajectories deviate from classical trajectories computed independent of wind and can be more energy-efficient. Although the resulting energy saving is rather limited (usually below 0.5%), we prove with this research that there is a practical benefit in including external factors such as weather in the computation of speed profiles. Our next step is to use this model, but extend the uncontrollable externals factors to a pervasive description of uncertainty factors in railway networks (like people onboard, friction coefficients, energy conversion efficiency, etc.). This would allow to classify their relative effect and opportunities.

To summarize, this paper contributes to the TTOP and railway operations literature: (i) by formulating a novel train resistance equation that incorporates wind direction and intensity, and (ii) by numerically showing that accounting for the actual wind state (or in general, journey-dependent conditions, uncontrollable conditions that can be known with limited knowledge at some specific time only) when optimizing speed profiles has a practical impact in terms of energy consumption.

The rest of the paper is structured as follows. In §2, we present the new train resistance equation that embeds wind. In §3, we describe our optimization approach to find wind-aware trajectories. In §4, we introduce the setup of our numerical experiments and discuss the results. We conclude in §5.

2. Wind effect on train motion

The motion of railway vehicles is governed by the well-known equations (Hansen and Pachl 2014):

$$\frac{dv(s)}{ds} = \frac{f(s) - R_{train}(v) - R_{line}(s)}{\rho \cdot m \cdot v(s)}; \tag{1a}$$

$$\frac{dt(s)}{ds} = \frac{1}{v(s)}, \tag{1b}$$

where s is the traversed distance, $v(s)$ the train speed, $t(s)$ the time, $f(s)$ represents the traction force when positive and braking force when negative, and $R_{train}(v)$ and $R_{line}(s)$ the train and line resistance, respectively. The parameter m is the train mass and ρ the rotating mass factor. In addition to (1), the train movement is constrained by upper bounds on the engine power $f(s) \cdot v(s) \leqslant p^{\text{MAX}}$, traction force $f(s) \leqslant f^{\text{MAX}}$, and speed $v(s) \leqslant v^{\text{MAX}}(s)$, where $v^{\text{MAX}}(s)$ is the speed limit at location s. Moreover, the acceleration is bounded by $a^{\text{MIN}} \leqslant dv(s)/dt(s) \leqslant a^{\text{MAX}}$ to ensure riding comfort to passengers, and time windows at passage points could also be enforced (Wang and Goverde 2016).

The train resistance $R_{train}(v)$, in particular, is composed by rolling, bearing, and air resistances and is generally expressed as a quadratic function of speed (Hansen and Pachl 2014):

$$R_{train}(v) = \alpha + \beta \cdot v + \gamma \cdot v^2, \tag{2}$$

where $\alpha > 0$, $\beta > 0$, and $\gamma > 0$ are constant scalars. In reality, the train resistance is not only a function of speed but it is also affected by external factors like wind, rain, and train load (number of passengers) that change from journey to journey. Therefore, we propose to incorporate these factors in (2) to obtain a more precise, journey-dependent description of the actual resistances.

In the rest of the paper we focus on the effect of wind on resistance. It is reasonable to assume that wind affects the quadratic term (i.e., the air resistance) of (2) in a manner that depends on its speed (w) and the angle between train movement and wind (θ). We introduce this relation by:

$$R_{train}(v) = \alpha + \beta \cdot v + \gamma \cdot \max\{v - v_e(w, \theta),\, 0\}^2, \tag{3}$$

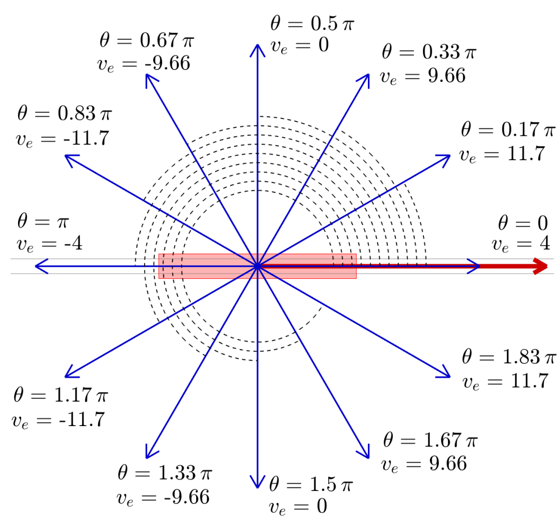
where $v_e(w, \theta)$ is dubbed the *wind effect* and intuitively represents how much the relative train speed in the quadratic term increases or decreases as a result of wind. We describe the wind effect by:

$$v_e(w, \theta) := w \cdot \cos\theta \left(\xi_1 \cdot L \cdot h \cdot |\sin\theta| + \xi_2 \cdot l \cdot h \cdot |\cos\theta|\right), \tag{4}$$

where L, l, and h are the train length, width, and height, while ξ_1 and ξ_2 are scaling parameters.

To build some intuition, we can think of (4) as the sum of traversal and longitudinal wind components. The traversal component describes the force that wind exerts on the lateral side of the train, and is hence proportional to the side area $L \cdot h$ and the impact angle via $|\sin\theta|$. Similarly, the longitudinal component is the force that wind exerts on the front/back of the train and is proportional to the front area $l \cdot h$ (approximated by a rectangle) and the impact angle via $|\cos\theta|$. The resulting sum is multiplied by the wind speed w, and the outer cosine function that adjusts the sign of v_e and ensures the wind effect is null when wind is perpendicular to the track ($\theta = \pm\pi/2$). To illustrate, in Figure 1 we display the wind effect corresponding to the parameters $w = 20\,\mathrm{km/h}$, $L = 160\,\mathrm{m}$, $l = 3.2\,\mathrm{m}$, $h = 4.4\,\mathrm{m}$, $\xi_1 = 0.0014$, $\xi_2 = 0.014$, and different θ values. Here, v_e is larger when there is a traversal component that hits the side of the train (e.g., $\theta = 0.33\,\pi$), compared to a fully parallel wind ($\theta = 0$) that only hits the back of the train (a much smaller area).

Figure 1: Illustration of the wind effect. The red arrow indicates the train moving direction.



3. Trajectory optimization with wind

In §3.1, we construct a space-speed network embedding the new resistance equation. In §3.2, we present a speed optimization method that combines dynamic programming with line search.

3.1 Space-speed network

We define our space-speed network as a directed acyclic graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ as follows. We start by considering a discrete set of locations $s \in \mathcal{S} = \{0, \dots, S\}$, where $s = 0$ corresponds to the departure station and $s = S$ to the arrival station, and a set of speed points $v \in \mathcal{V}(s) = \{0, \ldots, v^{\text{MAX}}(s)\}$. We use a uniform discretization of location and speed with intervals Δs and Δv, respectively. A node $n \in \mathcal{N}$ in the network is represented by the space-speed pair $n = (s, v) \in \mathcal{S} \times \mathcal{V}(s)$. We denote by $s(n)$ and $v(n)$, respectively, the location and speed associated with node n.

For each pair of "consecutive" nodes $n_1, n_2 \in \mathcal{N}$, i.e. with $s(n_2) = s(n_1) + \Delta s$, we insert the arc $a = (n_1, n_2)$ from n_1 to n_2 if the latter node is reachable from the previous node when taking into account the physical relations of train motion including maximum acceleration, braking, traction force, and engine power (see §2). Therefore, an arc $a \in \mathcal{A}$ represents a feasible train movement (hence a control) between two consecutive space-speed points n_1 and n_2. We associate with each arc $a \in \mathcal{A}$ the time t_a needed to travel the arc, and the energy consumption e_a. While the time t_a is assumed not to be affected by the train resistance (since space and speed points are fixed), the force, and consequently the energy e_a, needed for the train to change its speed between two target values in the interval Δs depends on the train resistance. Thus, given a wind state (w, θ), for each $a \in \mathcal{A}$ we compute the energy consumption $e_a = e_a(w, \theta)$ using the resistance equations (3)–(4).

3.2 Dynamic programming

It is well-known that finding the minimum time train trajectory in graph $\mathcal{G}$ is equivalent to solving a shortest path problem from $(s, v) = (0, 0)$ to $(s, v) = (S, 0)$ with arc costs equal to the travel time $c_a = t_a$, for $a \in \mathcal{A}$. Since $\mathcal{G}$ is directed acyclic, the shortest path can be found efficiently in $\mathcal{O}(|\mathcal{A}|)$ using DP (Ahuja et al. 1993). The same is true if the objective is to minimize energy consumption regardless of time, by assigning arc costs $c_a(w, \theta) = e_a(w, \theta)$. However, in TTOP we must deal with both energy consumption and travel time jointly by minimizing energy with a constraint on time. One strategy is to formulate a resource-constrained shortest path problem (Hassin 1992) with energy and time being respectively the unconstrained and constrained resource. This problem is weakly NP-hard and solving it usually requires excessive computational memory for large networks. To overcome this challenge, we opted for a second strategy where we solve standard shortest path problems iteratively with arc costs defined by $c_a(\eta, w, \theta) = t_a + \eta \cdot e_a(w, \theta)$, for $a \in \mathcal{A}$, i.e. linear combinations of time and energy. We optimize the trade-off parameter η using a line search scheme, ensuring that the scheduled arrival time T^{S} is respected. The procedure is outlined in Algorithm 1.

Algorithm 1: Line search DP for train trajectory optimization

Inputs: Graph $\mathcal{G}$; Wind (w, θ); $\eta^{\text{MAX}} > 0$ (high value); Maximum iterations I; Time tolerance ϵ.

Initialization: $T(w, \theta) = +\infty$, $E(w, \theta) = +\infty$, $\eta^{\text{MIN}} = 0$.

For iteration $i = 1$ to I **do**:

1. Set $\eta := (\eta^{\text{MAX}} + \eta^{\text{MIN}})/2$ and $c_a(\eta, w, \theta) := t_a + \eta\, e_a(w, \theta)$, $\forall a \in \mathcal{A}$, obtaining $\mathcal{G} = \mathcal{G}(\eta, w, \theta)$;
2. Solve DP on $\mathcal{G}(\eta, w, \theta)$, resulting in trajectory X_η, travel time T_η, and energy E_η;
3. **If** $|T_\eta - T^{\text{S}}| < |T(w, \theta) - T^{\text{S}}|$, update solution $X(w, \theta) = X_\eta$, $T(w, \theta) = T_\eta$, $E(w, \theta) = E_\eta$;
4. **If** $T_\eta < T^{\text{S}}$, redefine $\eta^{\text{MIN}} = \eta$, **else**, redefine $\eta^{\text{MAX}} = \eta$;
5. **If** $|T(w, \theta) - T^{\text{S}}| < \epsilon$, **break**.

Outputs: Optimized train trajectory $X(w, \theta)$, time $T(w, \theta)$, and energy consumption $E(w, \theta)$.

To evaluate our wind-aware trajectories, we generate a set of K wind scenarios $\{(w_k, \theta_k), k \in \mathcal{K}\}$ and for each $k \in \mathcal{K}$ solve Algorithm 1 using $(w, \theta) = (w_k, \theta_k)$. This procedure results in the energy consumption distribution $\{E(w_k, \theta_k),\ k \in \mathcal{K}\}$. To evaluate the traditional "no-wind" trajectory under the same wind conditions, we instead solve Algorithm 1 for $(w, \theta) = (0, 0)$, and evaluate the outcome solution $X(0, 0)$ using the true consumption $e_a(w_k, \theta_k)$, $a \in \mathcal{A}$, for all scenarios $k \in \mathcal{K}$, giving a second energy consumption distribution. We compare these two energy distributions in the next section.

4. Numerical study

In this section, we first describe the instance in §4.1 and then present and discuss the results in §4.2.

4.1 Instance

In Table 1, we report the vehicle and instance parameters used in our experiments. The track is 20 km long and contains six different speed limits (which are displayed in Figures 2–3). We do not use gradients in this example but they could easily be handled by our method. The resulting graph $\mathcal{G}$ has approximately 40 000 nodes and 1.9 million feasible arcs. When running Algorithm 1, we use $\eta^{\text{MAX}} = 10$, $I = 100$, and $\epsilon = 0.5$, that is, our trajectories are within 0.5 seconds from the target T^{S}.

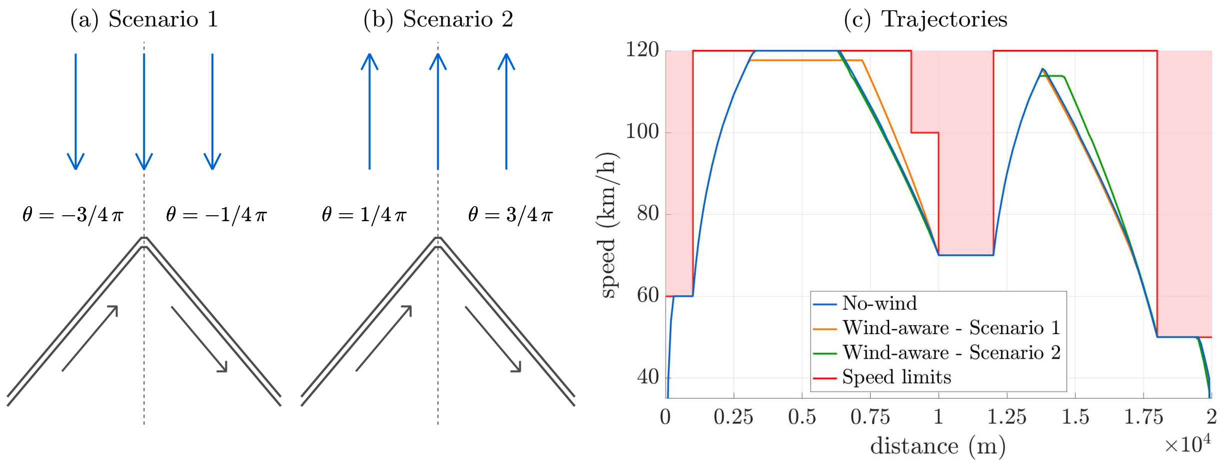
Table 1: Vehicle and instance parameters.

Name	Value	Unit	Name	Value	Unit	Name	Value	Unit	Name	Value	Unit
m	220	t	α	5.8	kN	a^{MAX}	0.6	m/s^2	Δs	100	m
ρ	1.06	-	β	0.02	kN h/km	a^{MIN}	-0.8%	m/s^2	$\lvert\mathcal{V}\rvert$	251	-
L	160	m	γ	0.002	kN(h/km)^2	p^{MAX}	1918	kW	$\lvert\mathcal{S}\rvert$	201	-
l	3.2	m	ξ_1	0.002	-	f^{MAX}	170	kN	S	20	km
h	4.4	m	ξ_2	0.021	-	Δv	0.48	km/h	T^{S}	900	s

4.2 Results

We start with an illustrative example. Assume that at 10 km (i.e. half-way) the track turns by 90° and a strong wind of 50 km/h can blow according to the scenarios of Figures 2(a) and 2(b). In scenario 1, the wind is initially opposed to the train motion (resistance increases) and afterwards in favor (resistance decreases). Intuitively, one could exploit this information by e.g. keeping a lower speed during the first part of the journey but start coasting later, or doing the opposite in the second part.

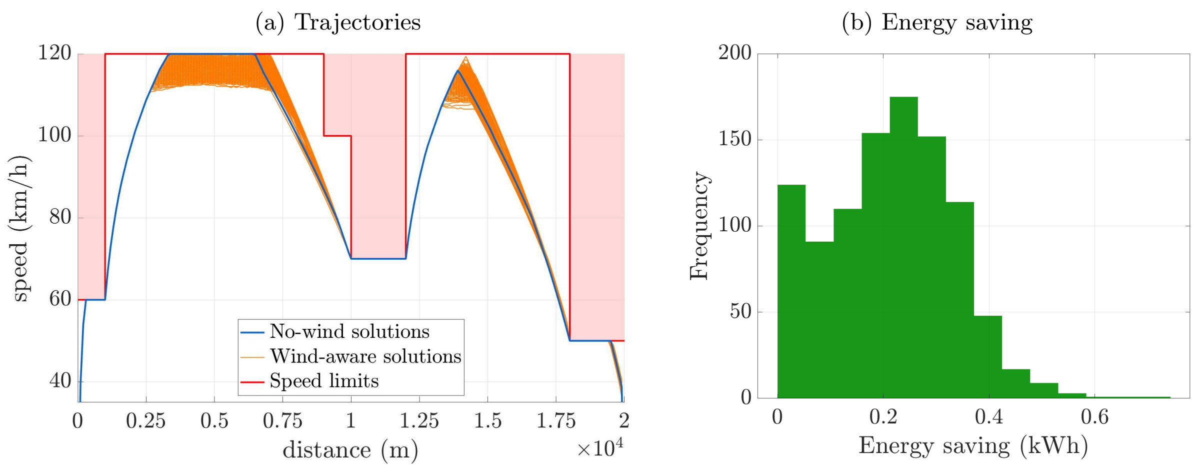
Figure 2: Wind-aware vs. no-wind train trajectories computed under two wind scenarios.



Similar considerations apply to scenario 2. Our results confirm this intuition and the two wind-aware trajectories deviate from the original one, allowing for an average saving of about 0.4 kWh (0.3%).

We now assume the track is a straight line, and evaluate the no-wind and wind-aware trajectories using $K = 1000$ scenarios (w_k, θ_k). We model the wind direction with a uniform distribution $\theta \sim U[0, 2\pi]$ and the wind speed with a Weibull distribution, which is the standard choice in the wind energy literature (Hennessey Jr. 1977). We chose shape and scale parameter of the Weibull equal to 2 and 20, respectively (to have w in km/h). We display the results in Figure 3. As we can see from Figure 3(a), the new trajectories differ from the original trajectory and, as expected, the deviations mainly occur for high speeds for which the air resistance is higher, and consequently the potential impact of wind. Figure 3(b) shows the distribution of savings achieved by the new trajectories, with a mean of 0.23 kWh per journey (0.15%). Although wind-aware trajectories are more energy-efficient, the average saving is overall rather small because most energy is clearly spent on accelerating the train to a high speed which has to be done regardless of wind.

Figure 3: Wind-aware vs. no-wind trajectories and energy consumption under 1000 wind scenarios.



The algorithm was implemented in C++ and run on a i7-8650U processor with 16 GB RAM. Solving a single DP took on average 0.03 s (plus 0.1 s to update the resistances), and executing the full line search, i.e. Algorithm 1, took 3.2 s on average for a given wind scenario.

5. Conclusion

In this paper, we have developed a train speed optimization model accounting for external factors that can vary across journeys on the same track (e.g., weather conditions). We focused on the effect of wind and proposed a new train resistance equation that embeds wind direction and intensity. Using this equation, we combined a line search algorithm with dynamic programming on a space-speed network to optimize speed in different wind scenarios. We performed a numerical study revealing that our wind-aware trajectories outperform trajectories computed independent of wind. Although the resulting energy saving is limited, this paper is the first showing the benefit of designing train speed profiles incorporating external factors that depend on the specific journey. We expect the saving to be larger when including more factors in the model, e.g., rain and train load.

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