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Other Conference Item

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Publication date:
2021-01

Permanent link:
https://doi.org/10.3929/ethz-b-000454493

Funding acknowledgement:
795917 - Simulation-Driven and On-line Condition Monitoring with Applications to Aerospace (EC)

14th World Congress on Computational Mechanics (WCCM)
ECCOMAS Congress 2020
19–24 July 2020, Paris, France

A DEFLATED CONJUGATE GRADIENT SOLVER FOR EXTENDED FINITE ELEMENT MODELS

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Key Words: Conjugate Gradients, Deflation, XFEM, Crack

In finite element analysis for solid mechanics, the solution of the linear system of equations represents the performance bottleneck in terms of both computation time and memory storage. For this task, direct solvers are a common choice due to their simplicity and robustness, however the very large amount of memory required to factorize the stiffness matrix precludes the solution of medium to large-sized problems. For this reason, substantial research effort in the field of iterative solvers has resulted in the adoption of the Conjugate Gradient (CG) method. In this procedure, explicit factorization and thus large quantities of additional memory are not necessary, due to the use of a gradient-descent optimization process. Among many available improvements of the CG, the Deflated Conjugate Gradient (DCG) solver [2] includes information from approximate eigenvectors (known as the deflation space) to achieve a faster convergence.

In this contribution, we present an application of the DCG for fracture mechanics with the eXtended Finite Element Method (XFEM) [1]. Among other features, such as holes and inclusions, cracks can be effectively modeled with this method, due to the addition of degrees of freedom and functions that enable the representation of discontinuous displacements and singular stresses within the solution space. In addition to the perspective of accelerated numerical analysis, the motivation to employ an iterative solver lies in the fact that no explicit assembly of the entire stiffness matrix is required, which has positive implications in crack propagation and large scale analyses.

With this approach, the main challenge lies in the construction of the deflation space in the presence of additional displacement patterns provided by the enrichment functions. Different methods to achieve this feature are studied and tested. With respect to the CG, the decrease in the number of necessary iterations provided by the optimal method should be comparable to the one delivered in a setup without enrichment. Finally, one can observe that, especially for large simulations, such acceleration justifies the higher implementation effort required.

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