



# Earthquake-Induced Damage Estimation in Structural Systems using Parametric Physics-Based Reduced-Order Models

## Other Conference Item

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# Earthquake induced damage estimation in structural systems using parametric physics-based Reduced-Order Models (ROMs)

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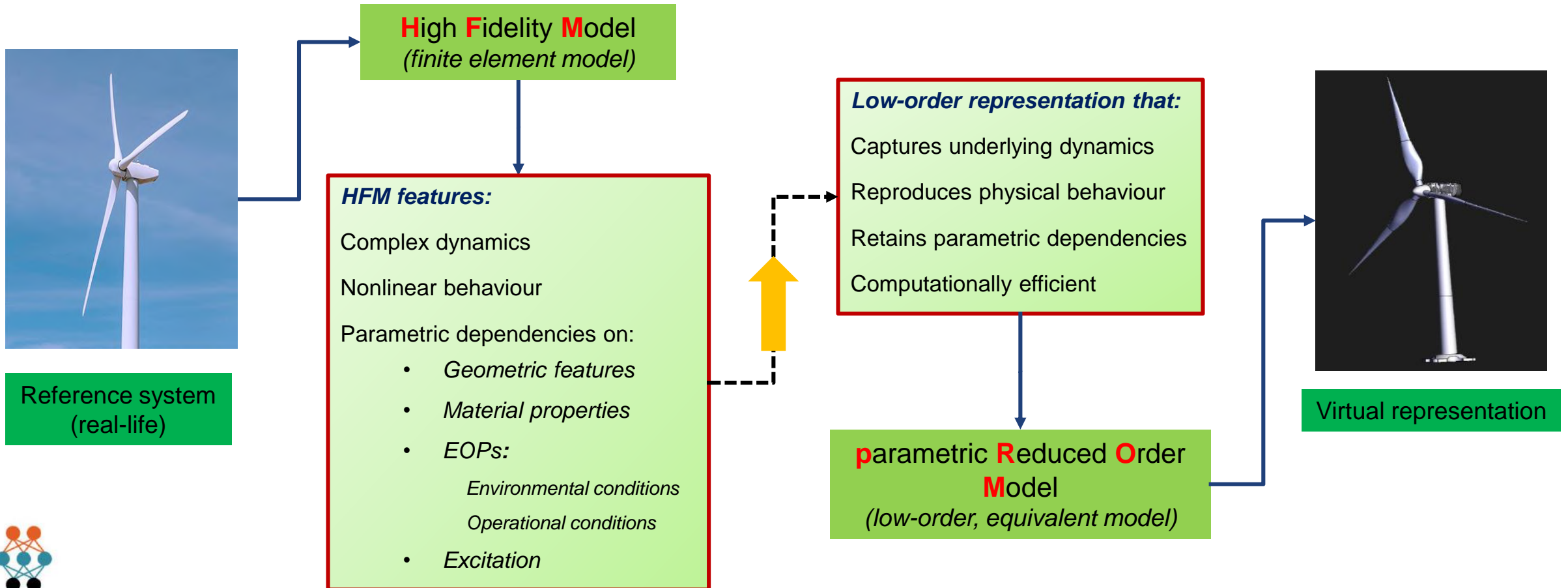
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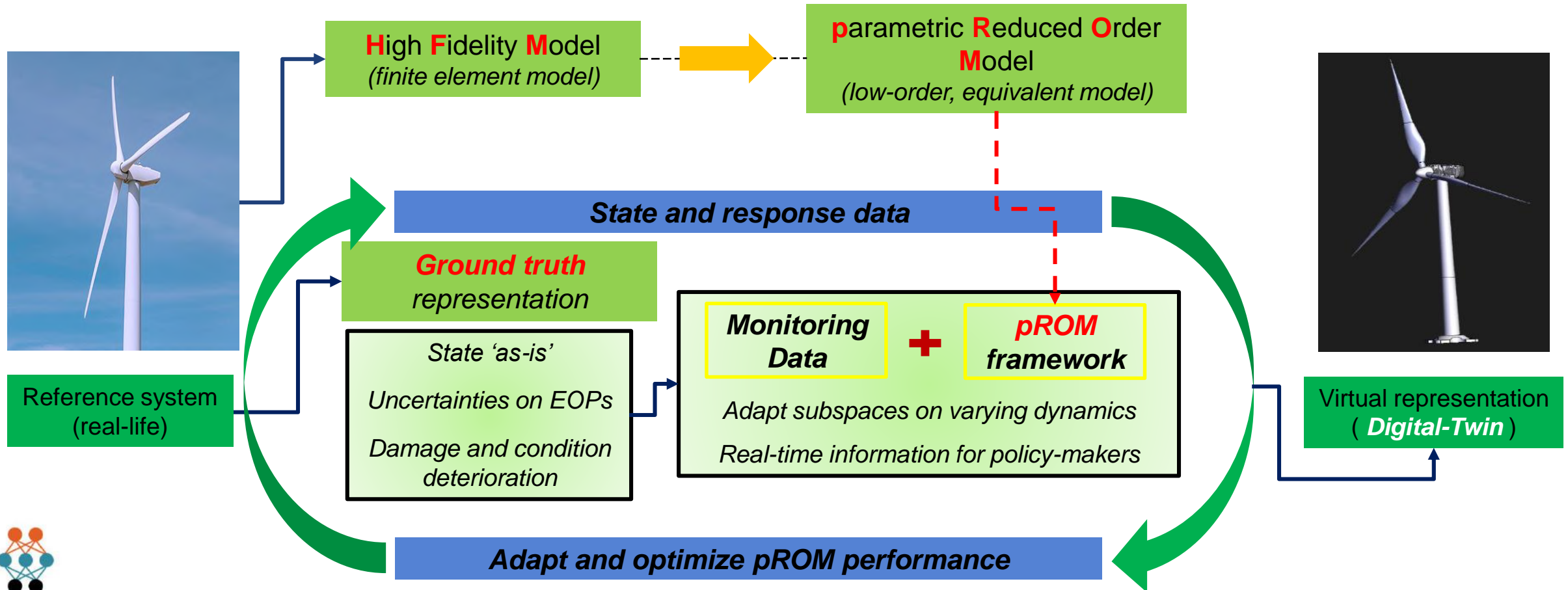
# Problem Statement

Robust digital virtualization of nonlinear dynamical systems



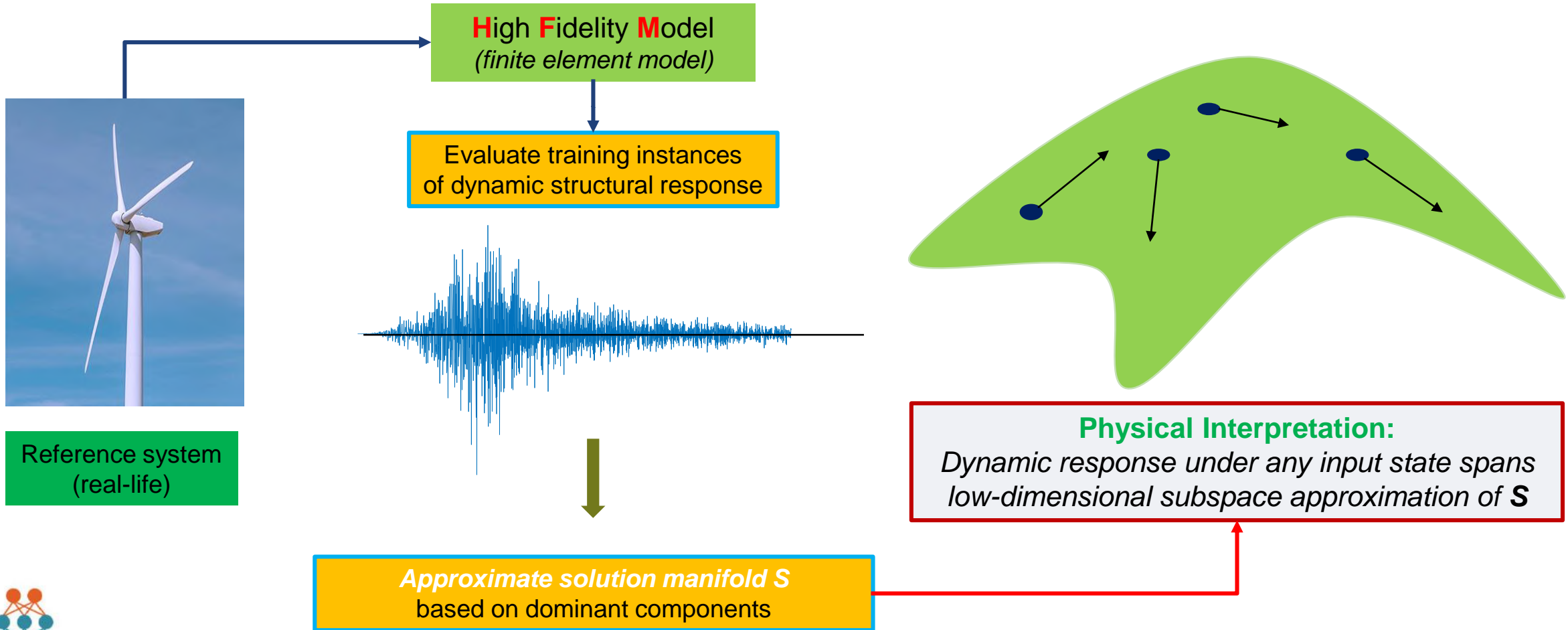
# Problem Statement

Condition deterioration or damage during operation



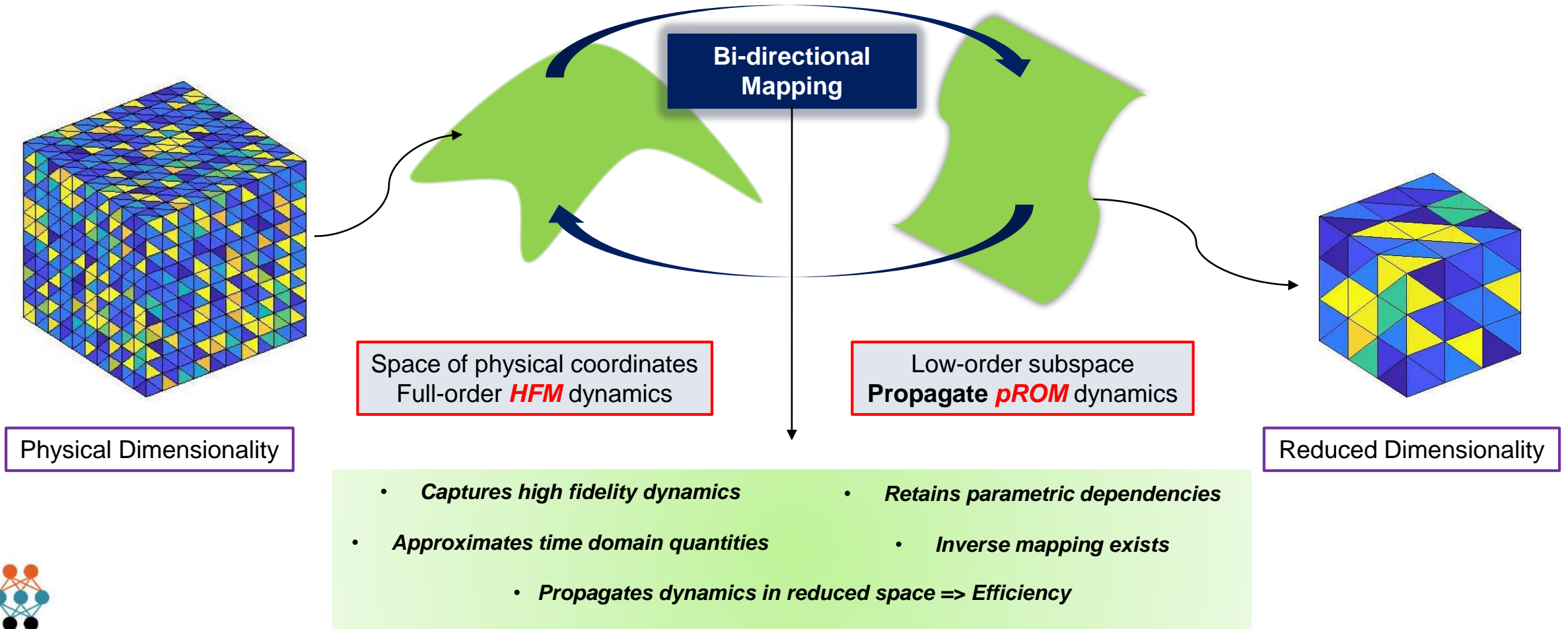
# Approach conceptualization

Parametric ROM (pROM) as forward simulator



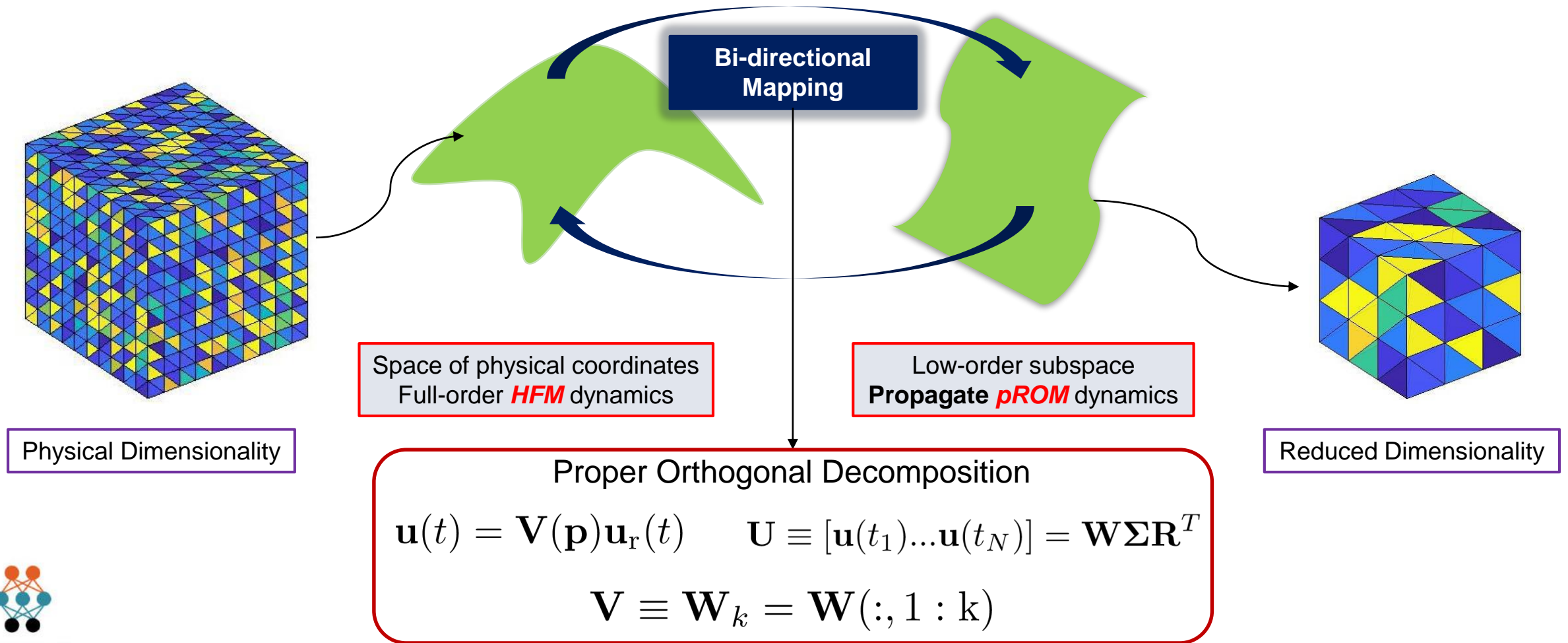
# Approach conceptualization

Projection-based pROM as forward simulator



# Approach conceptualization

Projection-based pROM as forward simulator



# Approach conceptualization

## Adaptive pROM for robust Structural Health Monitoring

### ( Initial ) parametric ROM framework

- *Projection-based approach relying on POD subspaces*
- *Propagates dynamics forward in time in reduced coordinates*
- *Utilizes local ROMs through clustering to retain dependencies throughout domain of operation*

### Earthquake induced damage / System deterioration

The pROM is no longer able to perform estimation tasks accurately  
Subspaces on training set do not sufficiently capture occurring phenomena  
*=> Performance bottleneck*

### Adaptivity in a pROM context

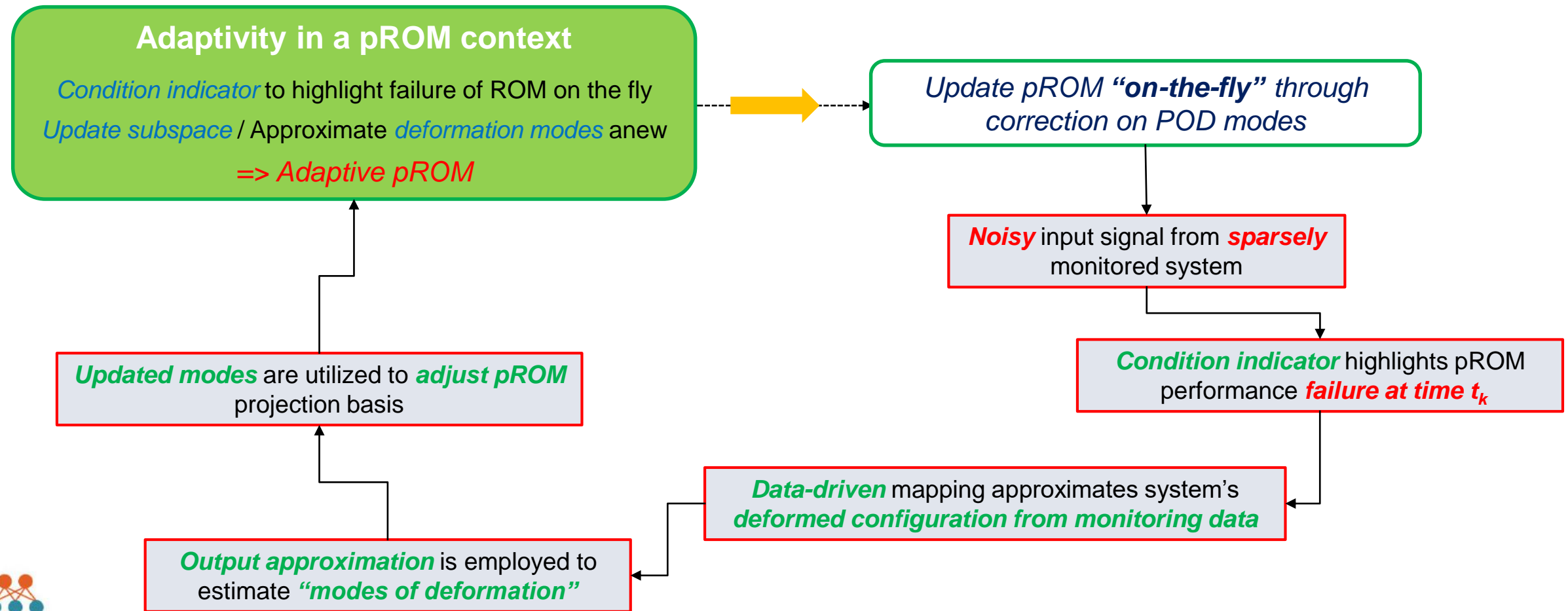
*Condition indicator* to highlight failure of ROM on the fly  
*Update subspace* / Approximate *deformation modes* anew  
*=> Adaptive pROM*





# Approach conceptualization

## Adaptivity through data assimilation



# Approach conceptualization

## Adaptive pROM framework based on data assimilation

### Offline / Training Strategy:

- ✓ Derive initial **pROM as forward simulator** :

Examples:

- **Initial linear state** and **nonlinearities** during operation **to represent damage**
- **Initial nonlinear state** and **deterioration effects** during operation

- ✓ Assemble **Damage Indicator** :

- **Deterministic nature** based on response comparison metrics
- Relies on **limited nodal measurements**
- Includes input noise / exploit **noise statistics to define activation threshold**

- ✓ **Gaussian Process Regression (GPR)** trained on **residual response**:

- GPR **trained on pool of snapshots**, without compromising efficiency

Examples:

- GPR trained on certain parametric states representing damage

### Online / During Operation:

- **Monitor residual response** between pROM and monitoring data
- **If indicator signals** "ROM Performance Deteriorates":
  - ✓ Employ GPR estimation to **reconstruct full residual state**
  - ✓ **Enrichment mode** = **pROM approximation + GPR residual**
  - ✓ **Enrich pROM** by using corrected modes in Basis



# Implementation details

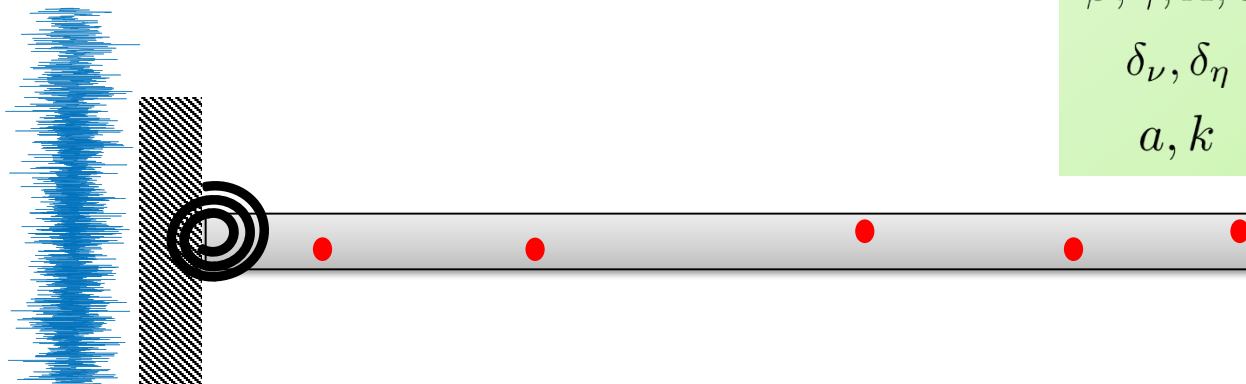
## Configurations and scenarios

### Cantilever Beam Case Study

- Stochastic *ground motion excitation*
- Parametrized Boundary => *Nonlinear rotational spring*
- *Limited number* of nodes monitored

#### Damage Scenario:

- ✓ Derive *ROM based on "design"* case study
- ✓ Induce **damage** by *activating parametric boundary*
- ✓ Use *indicator to detect* failure
- ✓ Employ *GPR-based scheme* to assemble deformed modes
- ✓ *Refine* POD-Basis



### Hysteretic spring model

- *Total restoring force:*

$$\mathbf{R} = \mathbf{R}_{linear} + \mathbf{R}_{hysteretic} = \alpha k \mathbf{u} + (1 - \alpha) k \mathbf{z}$$

- *Bouc-Wen equation with degradation/deterioration effects:*

$$\dot{\mathbf{z}} = \frac{A \dot{\mathbf{u}} - \nu(t) (\beta |\dot{\mathbf{u}}| \mathbf{z} |\mathbf{z}|^{w-1} - \gamma \dot{\mathbf{u}} |\mathbf{z}|^w)}{\eta(t)}$$

$$\nu(t) = 1.0 + \delta_\nu \epsilon(t), \quad \eta(t) = 1.0 + \delta_\eta \epsilon(t), \quad \epsilon(t) = \int_0^t \mathbf{z} \dot{\mathbf{u}} \delta t$$

#### Characteristics of the Bouc-Wen links:

$\beta, \gamma, A, w$  : Control smoothness and shape of hysteresis

$\delta_\nu, \delta_\eta$  : *Degradation/Deterioration* effects

$\alpha, k$  : *Linear/Hysteretic* contribution weighting

# Implementation details

## Configurations and scenarios

### Cantilever Beam Case Study

- Stochastic *ground motion excitation*
- Parametrized Boundary => *Nonlinear rotational spring*
- *Limited number* of nodes monitored

#### Damage Scenario:

- ✓ Derive *ROM based on "design"* case study
- ✓ Induce **damage** by *activating parametric boundary*
- ✓ Use *indicator to detect* failure
- ✓ Employ *GPR-based scheme* to assemble deformed modes
- ✓ *Refine* POD-Basis

#### Scenario B:

- *Initial* "design" case study is *nonlinear*
- **Damage** is represented through *degradation / deterioration* effects during *operation*

### Hysteretic Bouc-Wen spring model

- *Total restoring force:*

$$\mathbf{R} = \mathbf{R}_{linear} + \mathbf{R}_{hysteretic} = \alpha k \mathbf{u} + (1 - \alpha) k \mathbf{z}$$

- *Bouc-Wen equation with degradation/deterioration effects:*

$$\dot{\mathbf{z}} = \frac{A \dot{\mathbf{u}} - \nu(t) (\beta |\dot{\mathbf{u}}| |\mathbf{z}|^{w-1} - \gamma \dot{\mathbf{u}} |\mathbf{z}|^w)}{\eta(t)}$$

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#### Characteristics of the Bouc-Wen links:

$\beta, \gamma, A, w$  : Control smoothness and shape of hysteresis

$\delta_\nu, \delta_\eta$  : *Degradation/Deterioration* effects

$\alpha, k$  : *Linear/Hysteretic* contribution weighting

#### Scenario A:

- *Initial* "design" case study is *linear*
- **Nonlinear spring** is activated during *operation*



# Implementation details

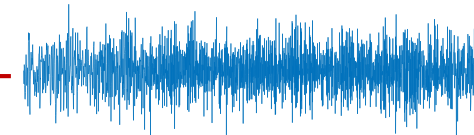
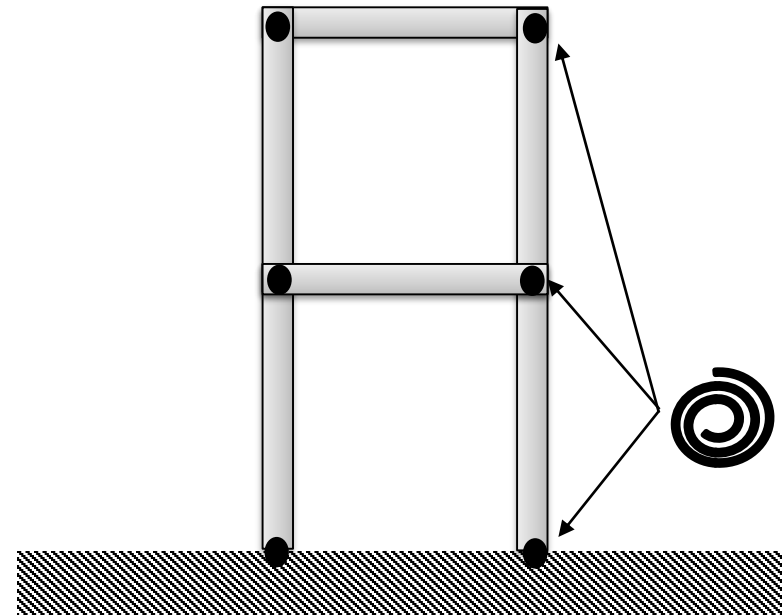
## Configurations and scenarios

### Plane Frame Case Study

- Stochastic *parametrized* ground motion *excitation ( Amplitude )*
- *Nonlinear* parametric rotational *spring on all nodal connections*
- *Limited number* of nodes monitored

#### Damage Scenario:

- ✓ Derive *ROM based on “design”* case study
- ✓ Induce *damage* by *activating parametric springs*
- ✓ Use *indicator to detect* failure
- ✓ Employ *GPR-based scheme* to assemble deformed modes
- ✓ *Refine* POD-Basis



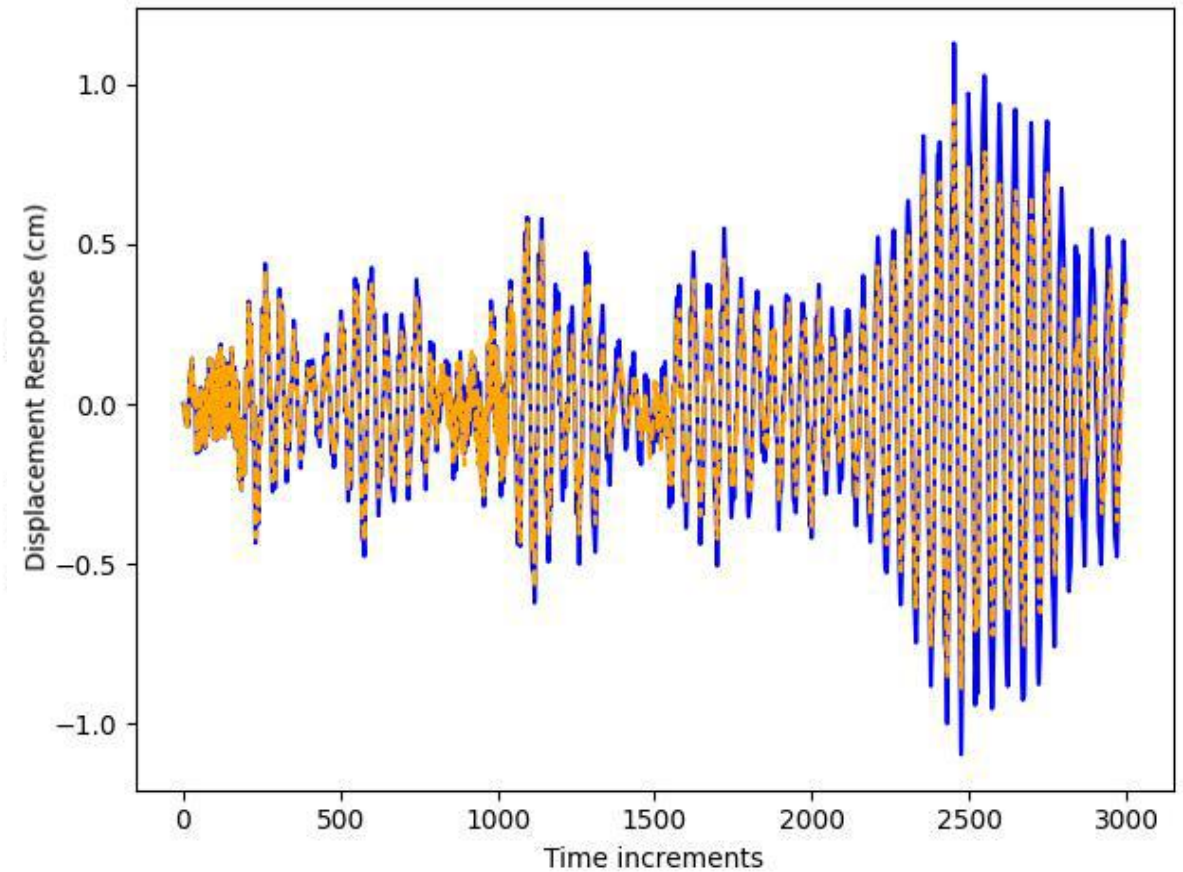
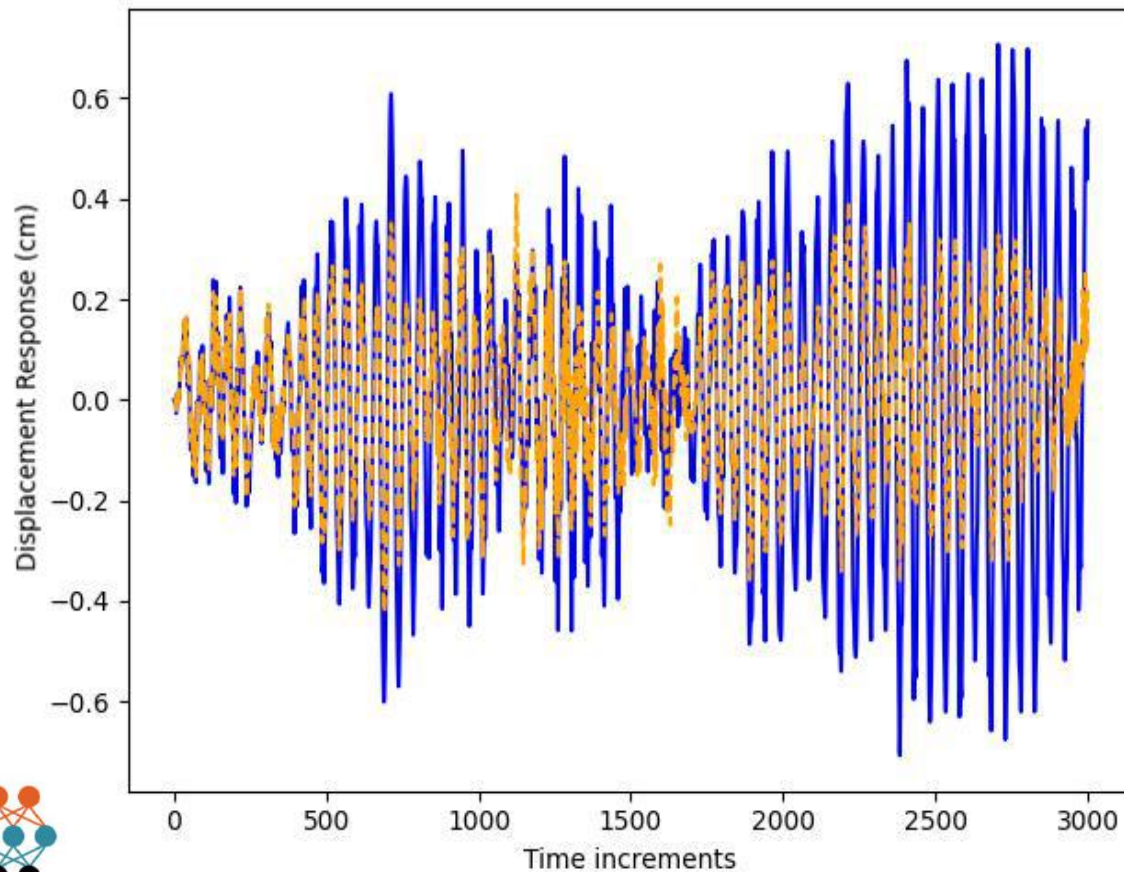
#### Scenario C:

- *Initial* “design” case study is *linear*
- *Nonlinear spring* is activated during *operation*
- Evaluation earthquake not included in training set

# Implementation details

## Configurations and scenarios

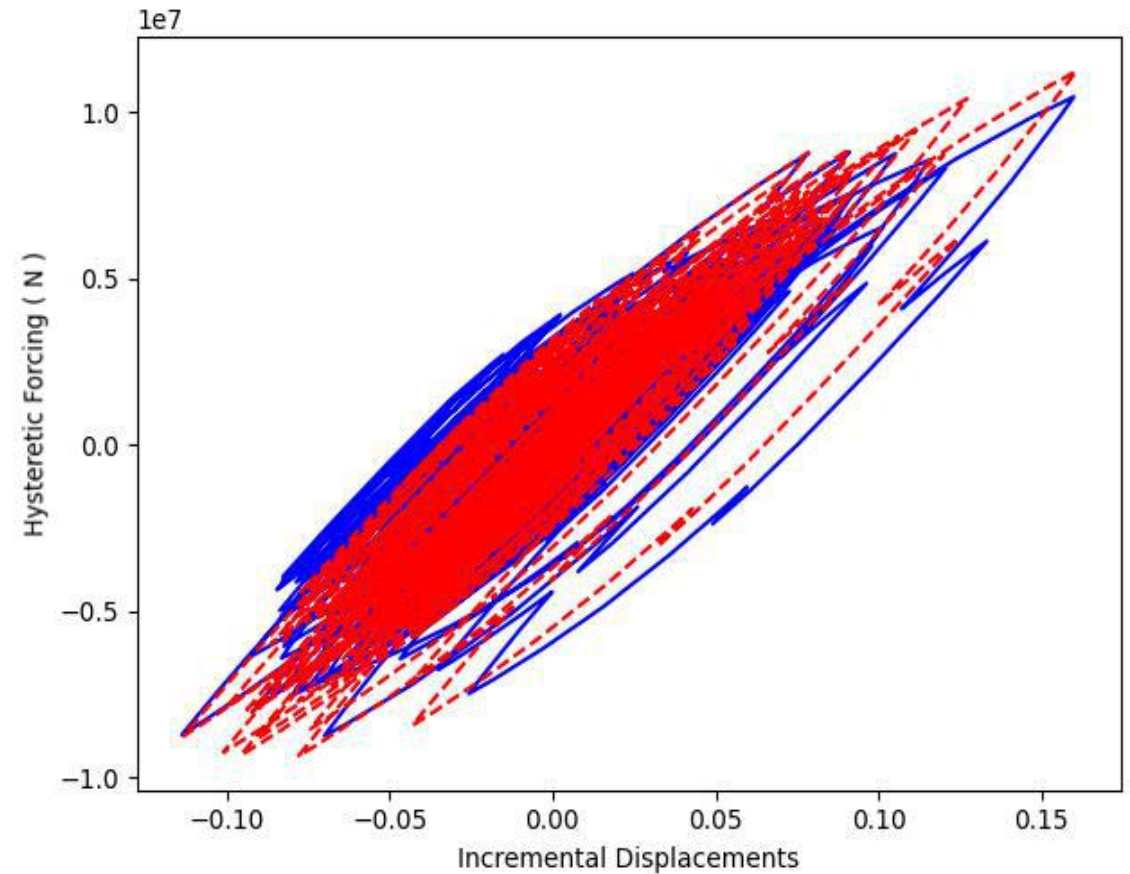
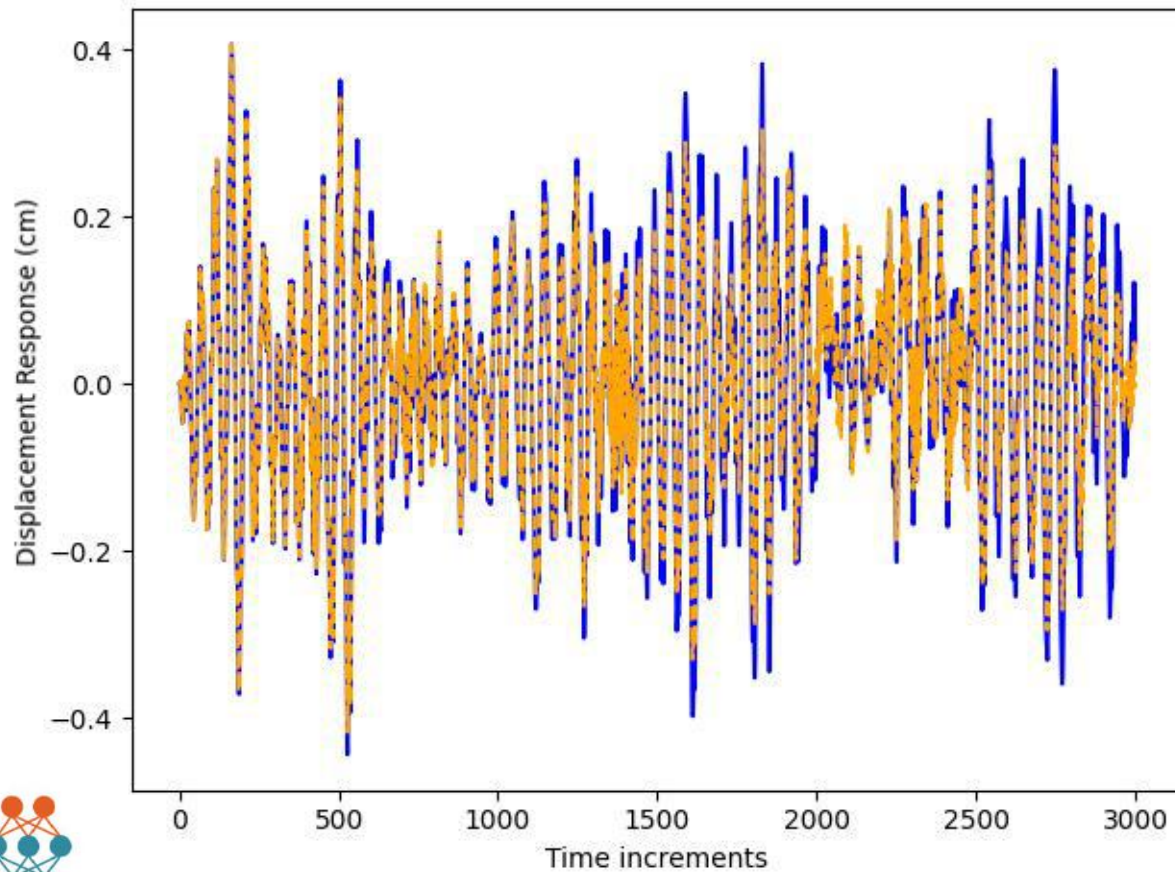
*Linear vs Nonlinear* response examples for *different Bouc-Wen activation parameters*



# Implementation details

## Configurations and scenarios

Response examples with *Bouc-Wen degradation phenomena* during operation



# Implementation details

## Damage indicator and GPR-scheme

### Damage Indicator

- **Deterministic nature** based on response comparison metrics  
=> **Mahalanobis distance (MD) measure**
- Relies on **limited nodal measurements** ( 5-10% nodal output measured )
- Includes input **noise ( 3% )** / **exploit noise statistics** to define activation threshold  
=> **Alert threshold from Chi-Square** distribution ( 0.01% significance level)

### Gaussian Process Regression (GPR)

- Trained based on **residual responses** between monitoring data and pROM
- GPR **trained on pool of snapshots**, without compromising online efficiency
- **Input:** Response information from monitoring channels  
**Output:** Additive correction on full coordinate space
- Leverage **local** and **physical degree-of-freedom correlations**
- **Software:** **gpytorch** implementation with MultitaskGPMoel and RBFKernel()

Measurement Data  $\mathbf{d}_k \in \mathbb{R}^{n_d}$

Vector of random values  $\mathbf{r}_k \in \mathbb{R}^{n_d}$

St. Dev. of measurement signals  $\sigma_\delta \in \mathbb{R}^{n_d \times n_d}$

Noise level  $\delta$

**Noisy measurement data**

$$\tilde{\mathbf{d}}_k = \mathbf{d}_k + \delta \sigma_d \mathbf{r}_k$$

### Damage Indicator

Input  $\in \mathbb{R}^{N_{channels} * 2 \times 1}$

Response on monitoring channels ( displacements & rotations)

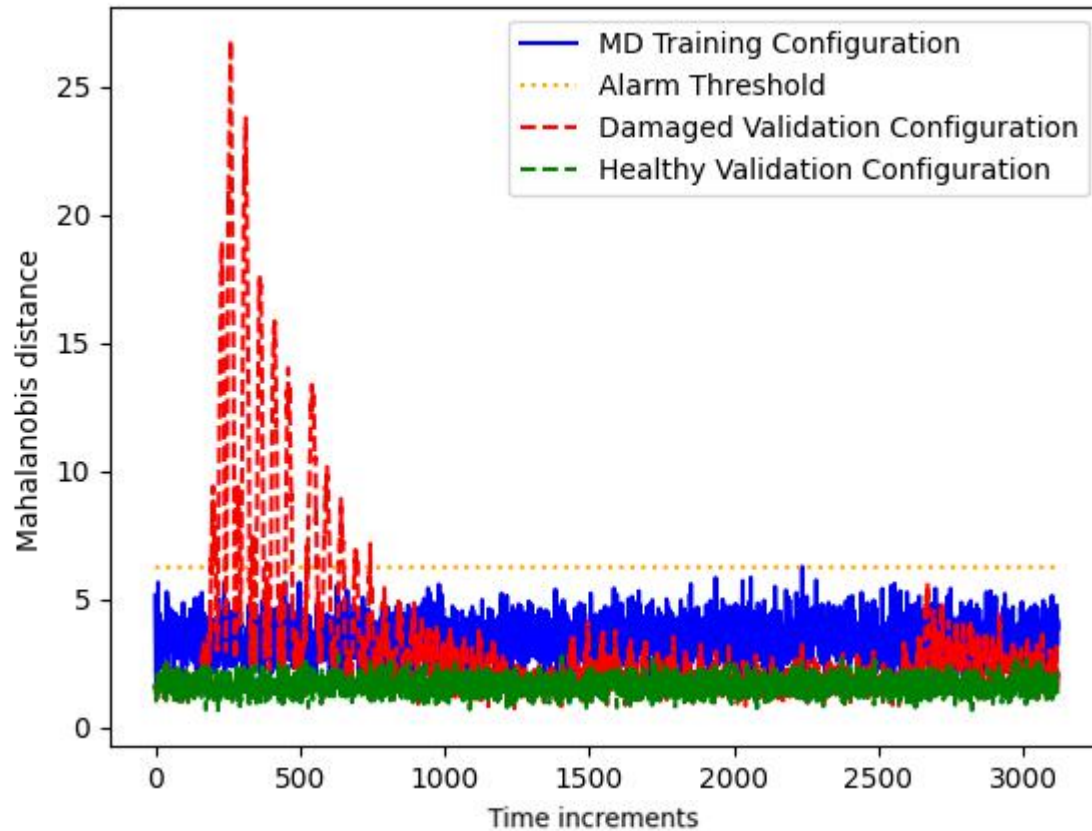
**Output => Performance failure alert signal**



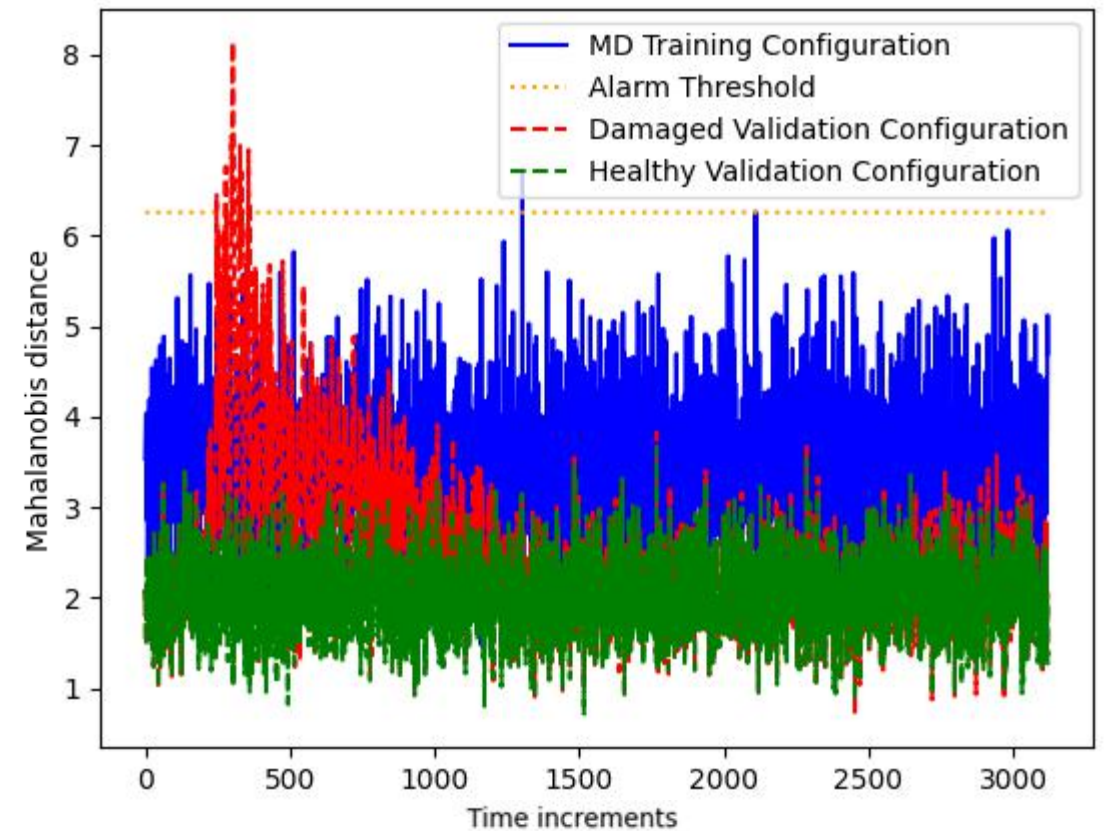


# Implementation details

## Damage indicator and GPR-scheme



*Linear vs Nonlinear*  
response example (**Scenario A**)



*Bouc-Wen degradation phenomena*  
during operation (**Scenario B**)



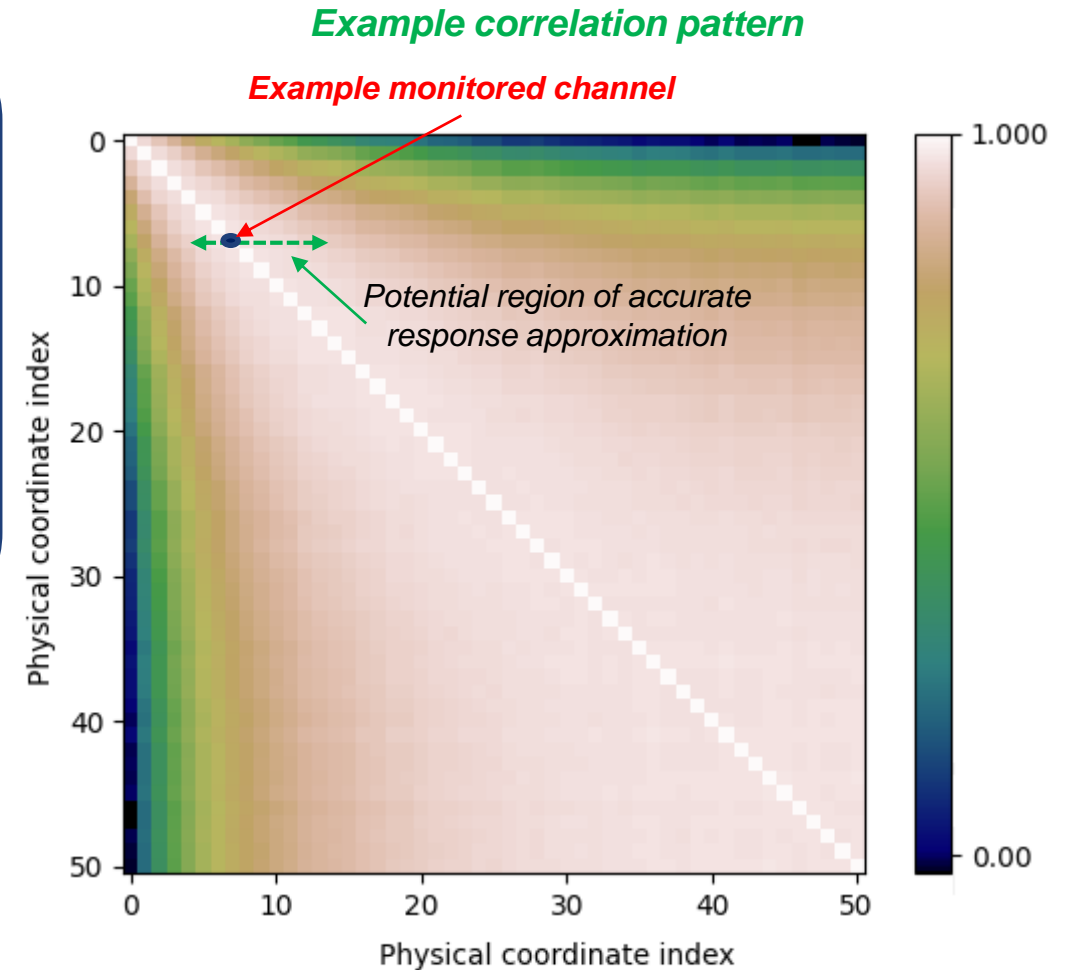
# Implementation details

## Damage indicator and GPR-scheme

### Gaussian Process Regression (GPR)

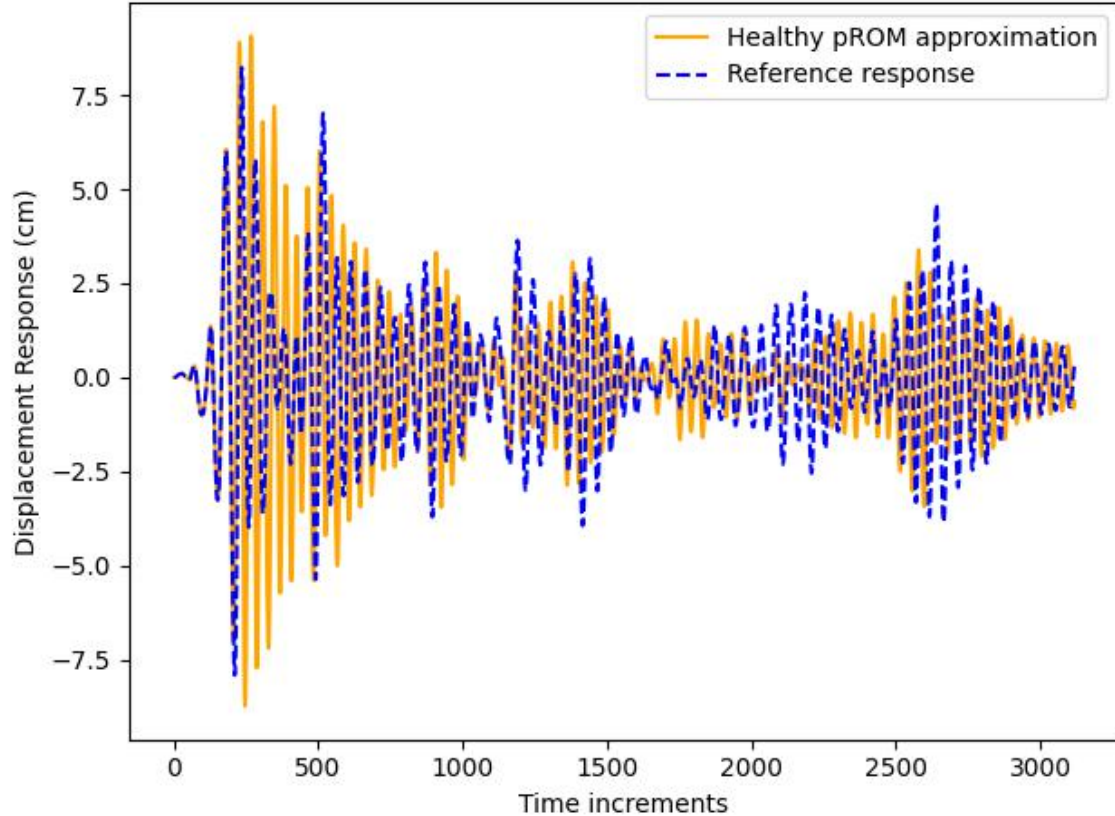
- Trained based on *residual* between monitoring data and pROM
- **Input**: Response information in monitoring channels (displacements & rotations)  
 $\text{Input} = \text{True response} - \text{pROM estimation} \text{ (monitored coordinates)}$
- **Output**: Response approximation through additive correction on full coordinate space  
 $\text{Output} = \text{True response} - \text{pROM estimation} \text{ (all coordinates)}$   
 $\Rightarrow$  pROM **Basis Enrichment** mode =  $\text{pROM approximation} + \text{GPR residual}$
- GPR trained on pool of snapshots, without compromising online efficiency
- Leverage *local* and *physical degree-of-freedom correlations*

- ✓ Assemble indirect *correlation* matrices between *response in each physical coordinate* / degree-of-freedom
- ✓ Leverage correlations to *define output window* for each monitored input channel
- ✓ **Overlapping** to ensure quality of approximation

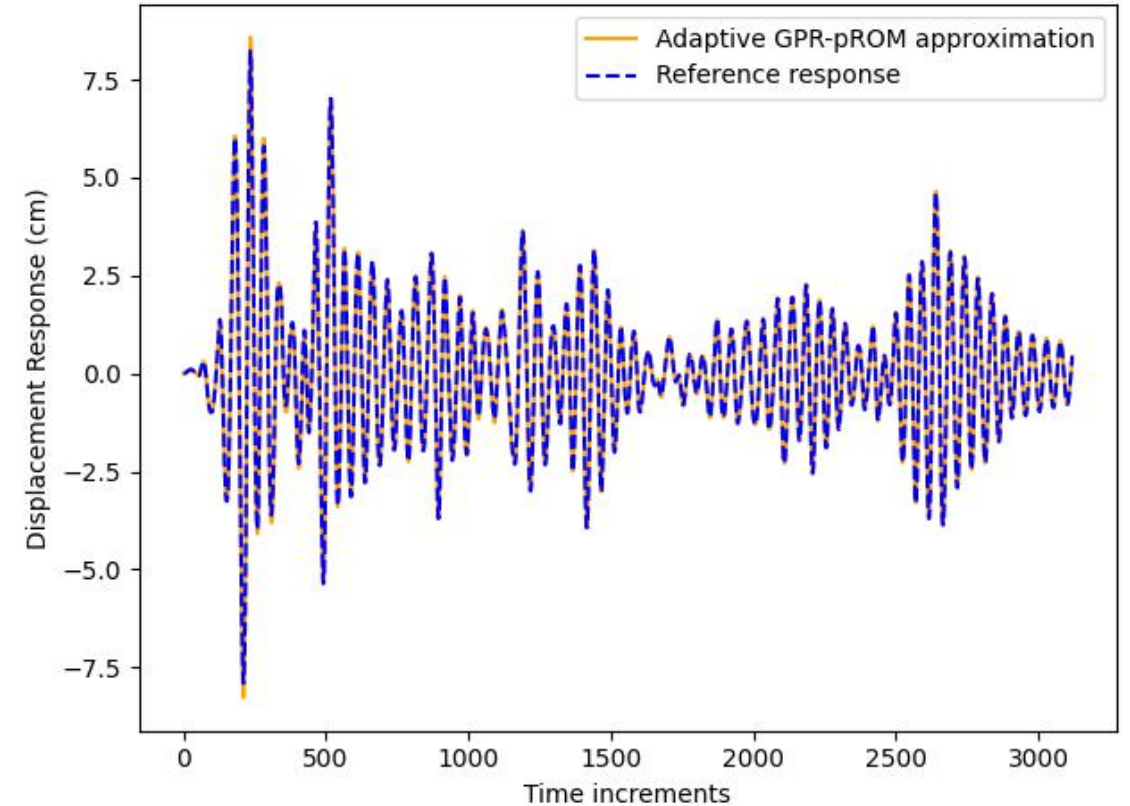


# Case studies

## Accuracy performance of the framework



*Healthy pROM* uses  
initial linear Basis (*Scenario A*)

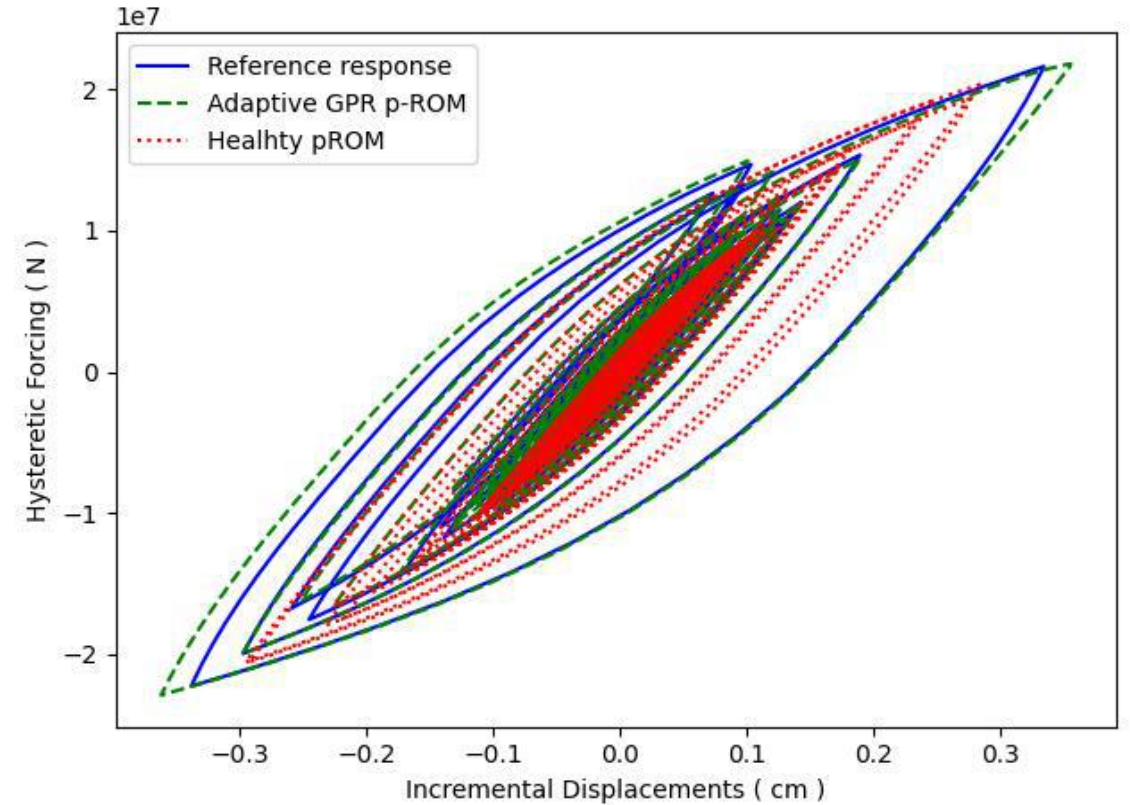
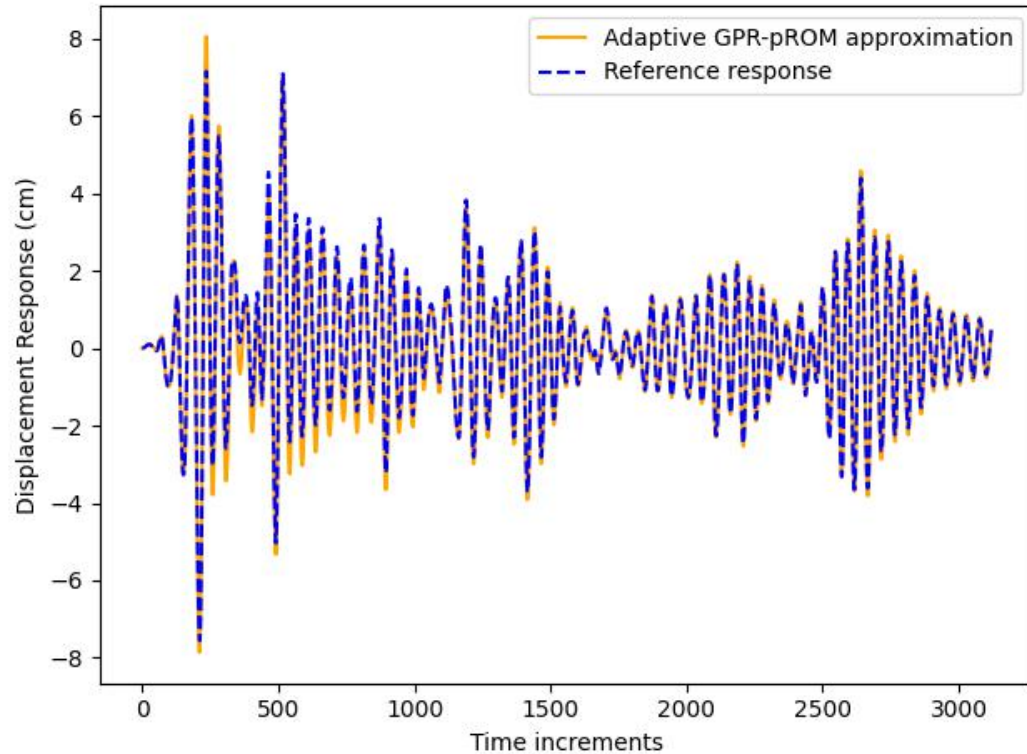


*GPR-pROM* adapts  
projection Basis (*Scenario A*)



# Case studies

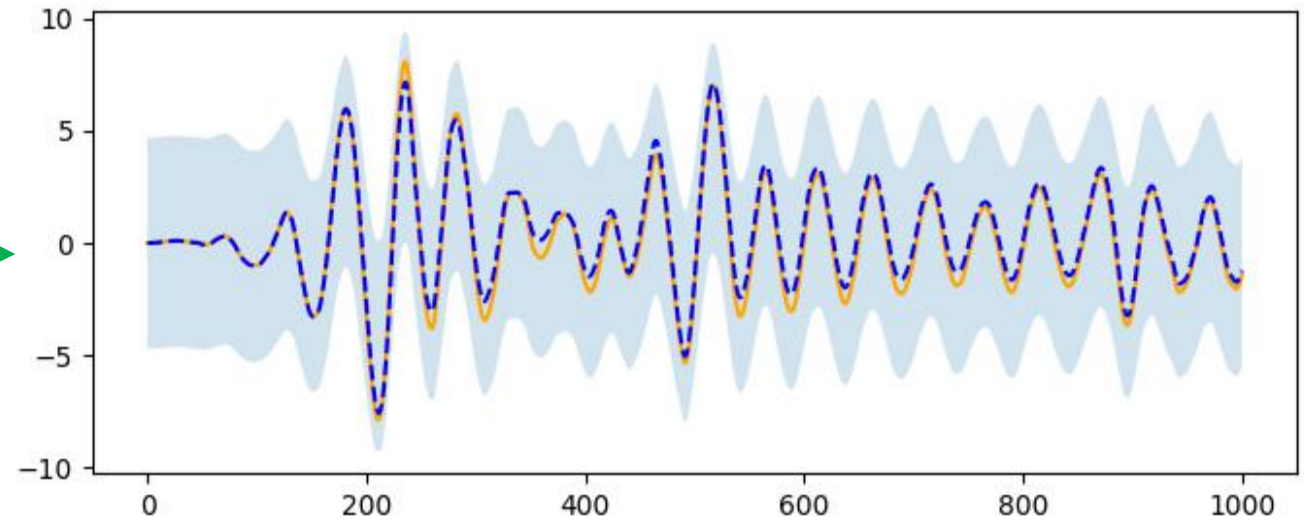
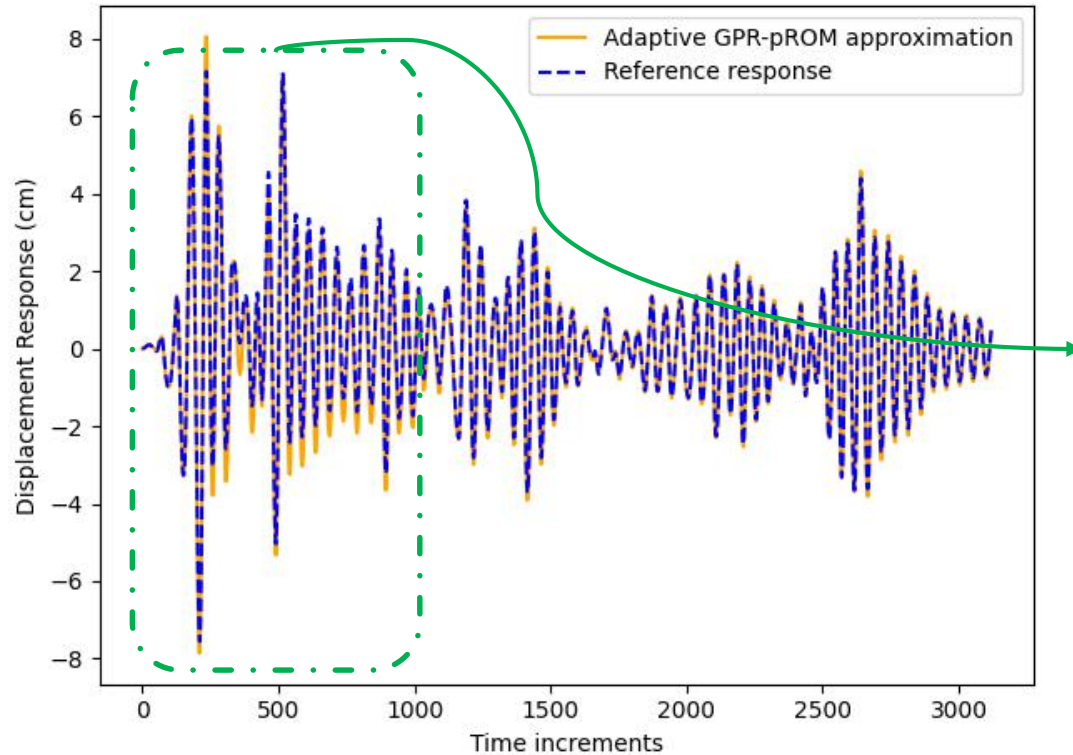
## Accuracy performance of the framework



**GPR-pROM** adapts  
projection Basis (**Scenario C**)

# Case studies

## Accuracy performance of the framework



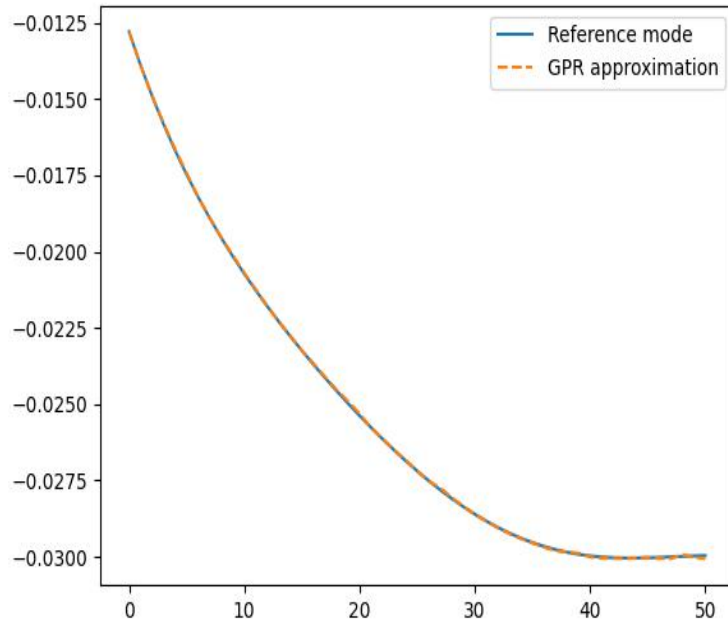
**GPR-pROM** adapts  
projection Basis (**Scenario C**)



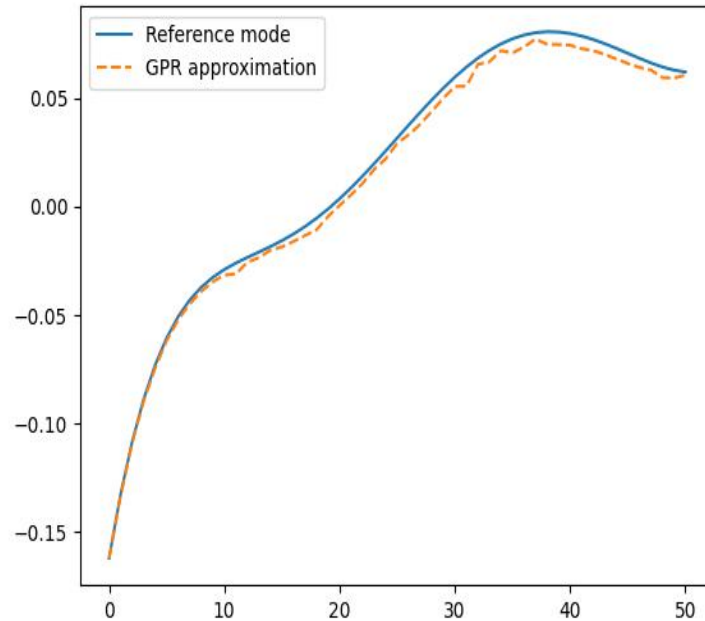
# Case studies

## Accuracy performance of the framework

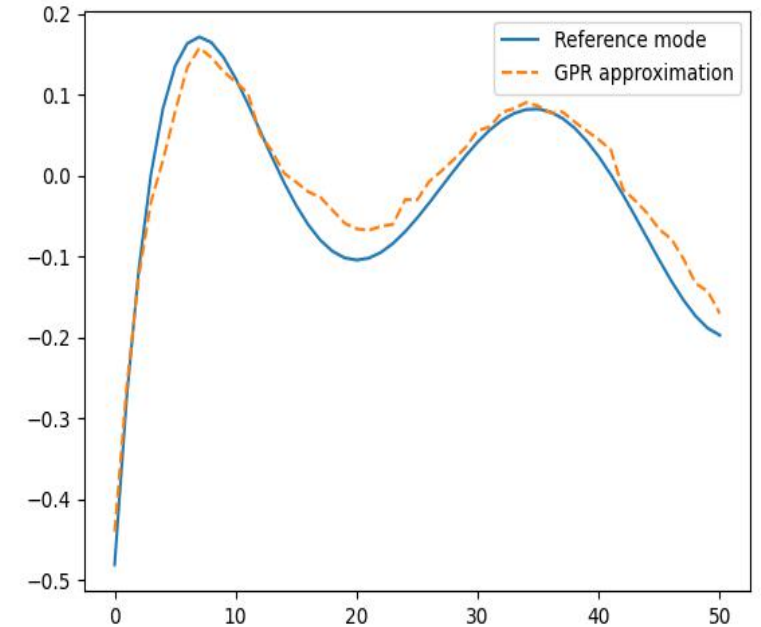
### Reduced-order of pROM : 4 modes



GPR approximation  
on **first mode** (Scenario C)



GPR approximation  
on **fourth mode** (Scenario C)



GPR approximation  
on **sixth mode** (Scenario C)

# Concluding remarks

## Limitations and outlook

### *The proposed adaptive GPR-pROM framework*

- ✓ *Extends performance range* of traditional projection-based pROMs
- ✓ *Captures underlying dynamics* and dependencies *during damage or condition deterioration scenarios*
- ✓ Achieves **on the fly correction** of the pROM **based on sparse measurements**
- ✓ Provides *confidence bounds* for response estimation
- ✓ May be adapted as an **approximative, online low-cost surrogate** for *Structural Health Monitoring* applications

- **Hyper-Reduction implications** for additional efficiency need further investigation
- GPR approximation scheme **fails to capture higher order modes**
- GPR approximation **performance is strongly dependent on noise** level
- GPR input-output channels discretization needs to be automated and optimized

#### *Next short-term steps:*

- ❖ Generalize implementation – adjust overall scope:  
Train pROM on earthquake database => Estimate damage in real-case scenarios
- ❖ Couple with filtering scheme to demonstrate potential on parameter/state/input estimation



## Question session

