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ORIGINAL PAPER

Optimum geometric layout of a single cable road

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Abstract Cable-based technologies have been a backbone for harvesting on steep slopes. The layout of a single cable road is challenging because one must identify intermediate support locations and heights that guarantee structural safety and operational efficiency while minimizing set-up and dismantling costs. Our study objectives were to (1) develop an optimization approach for designing the best possible intermediate support layout for a given ground profile, (2) compare optimization procedures between linearized and nonlinear analyses of a cable structure and (3) investigate the effect of simplifying a multi-span representation. Our results demonstrate that the computational effort is 30–60 times greater for an optimization approach based on nonlinear cable mechanical assumptions than when considering linear assumptions. Those nonlinear assumptions also stipulate lower heights for intermediate supports and a larger span length. Finally, compared with the unloaded case, tensile force in the skyline is increased by as much as 80% under load for a single-span skyline configuration. Our approach provides additional value for cable operations because it ensures greater structural safety at a lower cost for installation. Improvements are still needed in developing a stand-alone application that can be easily distributed. Moreover, our rather simple assumptions regarding set-up and dismantling costs must be refined.

Keywords Cable yarding · Cable mechanics · Standing skyline · Intermediate support layout · Graph theory

Introduction

Cable-based technologies have been the backbone of steep-slope harvesting in mountainous regions of the world, such as the Alps in central Europe, the Pacific Northwest of the United States, and Japan. From an operational point of view, the spatially explicit layout of a set of cable roads over a given area is a challenging task. Efforts toward set-up and dismantling must be regarded as part of the fixed cost that is assigned when estimating the total expense of extracting a particular volume of timber. Two factors must be considered in the layout of a single cable road—structural safety and the minimum number of intermediate supports.

Structural analysis of a cable structure is challenging due to the nonlinearity of the problem. The approach associated with European cable road design has been based on linearized analyses along with strong assumptions, for example, constants that represent the tensile forces in a skyline for both loaded and unloaded configurations. The North American approach has focused on "exact" catenary solutions, primarily layouts for single-span skylines.

Our research goal was to develop a method that incorporates "close-to-reality" structural analysis and a minimum number of intermediate supports, resulting in greater structural safety as well as lower set-up and dismantling costs. Our aims were to (1) identify an optimum layout for intermediate supports, (2) compare the optimization

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procedure for two cable mechanics approaches—linearized versus close-to-catenary—and (3) investigate the effect of multi-span simplifications. For experimental purposes, we assumed that both head and tail spar anchors were externally given and that the geometry of the ground profile between those two anchors was available at reasonable accuracy. We first reviewed current methods of structural analysis and those for locating intermediate supports. After developing our representation and optimization model, we evaluated the configuration mass of multiple span skylines for real-world cable road in a specific geographical area.

Background

Mechanical behaviour of cable structures

An exact analysis of a single cable span that utilizes catenary equations is constrained because it is impossible to obtain an explicit solution due to nonlinearity. Simplifications, such as (1) linear distribution of the self-weight of the cable along a span, (2) a constant horizontal component of the tensile forces in the cable and (3) an inelastic cable, result in an equation with six parameters:

- one mid-span deflection (y_m),
- two geometric properties of the cable span (a, horizontal span between anchor points; and c, chord distance between anchor points),
- two load characteristics (Q, moving load; and q_s, self-weight of the skyline) and
- one force component (H, horizontal component of the tensile force in the skyline).

$$y_m = \frac{q_s ca}{8H} + \frac{Qa}{4H} \quad (1)$$

Equation 1, originally used for cable-way design (Findeis 1923), was then introduced by Hauska (1933) for the analysis of forest cable systems. Later known as the Pestal (1961) equation, it is still computed for cable engineering in European forestry operations. Here, we use the LIN acronym to refer to the linear Pestal version.

The North American approaches to skyline engineering developed along a different path. Lysons and Mann (1967) devised a "graphic-tabular handbook" technique or "chain and board" method. This consisted of a board inscribed with the manually drawn ground profile and a small chain that was used as a physical model for the skyline. Another technique, introduced by Suddarth (1970), provided a mathematical solution utilizing mainframe computers. The emergence of desktop computers and plotters at the beginning of the 1970s triggered the development of computer-aided methods, the first of which was presented by Carson et al. (1971). Desktop computer solutions were continuously improved, eventually leading to the "logger PC" program (Sessions 2002).

These approaches are valid for only single-span skyline configurations. Although that type of design is predominant among North American operations, the European practice has a long tradition of multiple span configurations, such that we must consider additional boundary conditions for skyline length. Whereas the total length is held constant for a specific configuration, that of a single span varies according to the location of the load. If a load is moving from one span to the next, the skyline is feeding over the support, shortening the skyline in the first span and lengthening it in the second span. To our knowledge, this effect has not yet been included in analyses of forest cable systems. Zweifel (1960) introduced a "close-to-catenary" approach for multiple span configurations of cable ways. There, one assumes that (1) anchoring is fixed at the head and tail spars, (2) the cable has elastic properties, and (3) the skyline is freely fed over supports as the load moves from one span to the next. Zweifel approximated catenary equations through a Taylor series and developed an algorithm for manually solving the system of equations. This algorithm delivered a design value for the horizontal component of the tensile force of a loaded cable, which allowed one to calculate mid-span deflections for all spans. Although this approach (herein referred to as "close-to-catenary" or CTC) has been widely taken in the cable industry, it is only occasionally used for the analysis of forest cable systems.

Location of intermediate supports

For multi-span skyline configurations, an additional design issue must be addressed, that is, the location of intermediate supports over a given ground profile. This problem has historically been solved by intuition or trial and error. Pestal (1961) described some rules of thumb that are followed to this day. First, one must start with a single span between the head spar and tail spar and then draw the shape of the unloaded skyline over the ground profile. Second, the distance between the ground profile and the shape of the skyline must be minimal, or even negative, when examining those ground profile points. Third, intermediate support locations should eventually be placed into the profile, and each cable span should be evaluated for minimum ground clearance.

The automatic search for alternative procedures to locate intermediate supports began with research by Sessions (1992), who instituted the design that placed intermediate supports at all protruding profile points. Sessions then used a heuristic algorithm that eliminated the second of three consecutive intermediate supports if ground clearance was greater than the minimum required (Chung and Sessions 2003). This process continued until the number of supports was smaller than the user-defined maximum. Although the solutions that resulted from this approach were likely to be good, they did not prove to be optimum.

Leitner et al. (1994) presented a solution for identifying both the best location of intermediate supports over a ground profile and their optimum height. If *f* locations were possible for intermediate supports and each had *g* possible heights, there were *f* times *g* possible support points. When the head and tail spars were introduced, all possible spans and, therewith, all potential solutions could be illustrated by a directed graph (Fig. 1) in which the nodes indicated possible support points and the arcs, possible spans. The weight of the arcs was a quadratic function of the end-support height of the span. However, a subset of all possible spans was infeasible because a minimum ground clearance was not achieved.

For this current research, we opted for the problem representation of Leitner et al. (1994), which includes a directed graph to identify the optimum support configuration using a shortest path algorithm. Here, it was adequate to adopt the LIN assumptions of Findeis (1923) to describe the mechanical behaviour of the cable system when defining our problem.

Model development

The purpose of our study was to develop an approach that minimizes the number and height of intermediate supports required for a cable road. In doing so, we considered both the minimum ground clearance for the carriage and the capacity to keep tensile forces within acceptable limits. We made the following assumptions: a standing skyline configuration, nonlinear behaviour of the cable structure under load, a multi-span configuration and frictionless movement of the skyline over supports. Our solution comprised four components. First, we presented the problem as a directed mathematical graph. Second, we devised a scheme to solve the problem with cable mechanics. Third, we developed a procedure to construct that mathematical graph, while also considering mechanical feasibility. Finally, we created optimization procedures to operate on that mathematical graph.

Representation of the solution space

A multi-span skyline structure has a head spar and a tail spar, with n_f intermediate support locations in between, each with n_g possible support heights. This solution space can be presented as a directed graph with support locations as nodes and spans as edges. The related mathematical structure is an adjacency matrix. Such a representational approach was first described by Leitner et al. (1994).

Cable mechanics

When assuming a standing skyline configuration, the skyline is fixed to anchors at both the head and tail spars. This means that the unstretched skyline length remains constant for any load configuration. The response by such a cable structure is four-fold: (1) it changes the shape of the skyline along the single spans, (2) it feeds skyline length from adjacent unloaded spans into the loaded span, (3) it increases the tensile forces in the skyline, and (4) it elastically stretches its total length. By contrast, the widely used Pestal approach considers only changes in shape and neglects those three other factors. Therefore, to ensure that the improved mechanical model is close to reality, thereby encompassing all four responses, our new approach

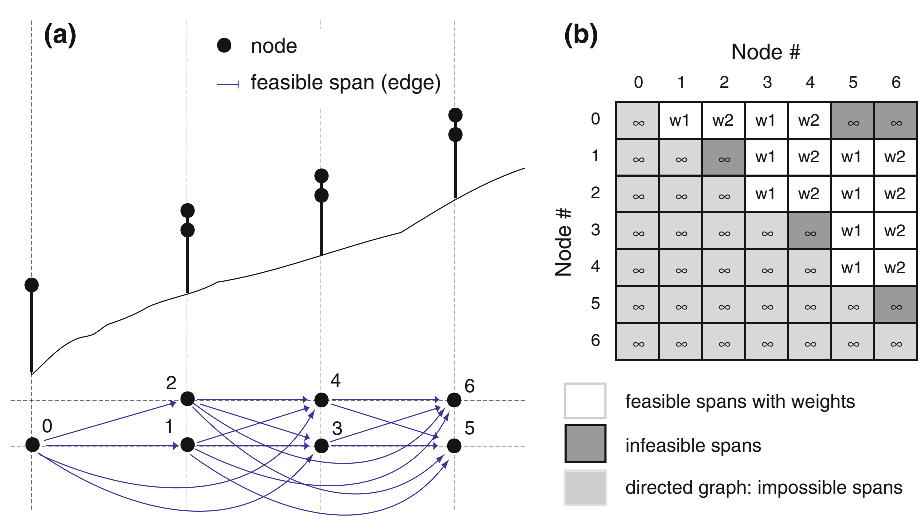

Fig. 1 Representation of the intermediate support solution space, based on graph theory: **a** graph network (directed graph) and **b** adjacency matrix

included features used for cable-way design. As stated earlier, this is known as the CTC approach because Zweifel (1960) approximated the catenary equations with a Taylor series. This numerical procedure iteratively identifies the increase in a skyline's tensile force for a load moving over a span as follows:

- start with a basic tensile force (H^0: horizontal component of the tensile force for the unloaded skyline) and calculate the unstretched, unloaded skyline length;
- put load Q at the mid-span position of the largest span;
- increase that basic tensile force of the cable by one unit ($+\Delta H$);
- calculate the unstretched length for the loaded span with this enhanced tensile force; and
- continue to increase the basic tensile force until the unstretched length of the loaded skyline equals the unstretched length of the unloaded skyline.

This procedure can be used to calculate two critical values for the horizontal component of the basic tensile force—the maximum allowed, $H^0_{\max}$, which guarantees that the design strength is not exceeded; and the minimum, $H^0_{\min}$, which ensures the lowest ground clearance.

Construction of the mathematical graph

Our solution to the problem of laying out an optimum design of intermediate supports started with gathering information about terrain conditions between head and tail spars, as described in a longitudinal section. Afterward, we stated the technical specifications of the yarding system, such as type and self-weight of the skyline (q_S), its coefficients of elasticity (E) and cross-sectional area (A), and the load weight (Q). The set of possible intermediate support locations F was then defined. Here, x and y represented the horizontal and vertical coordinates of the profile. We selected the x-coordinate of the base of the headspar and added multiples of δl to this to obtain the x-coordinates of possible intermediate support locations. The y-coordinate of a possible base was the y-coordinate of the terrain line corresponding to the x-coordinate of the possible base. Certain support locations were neglected that would never be selected, for example, those for concave terrain points. Those points were defined with the following logic. For each possible support location i, the height coordinate was y_i. Heights of the neighbouring points (both with distance δl) were y_{i-1} and y_{i+1}. If $(y_{i-1} + y_{i+1})/2 > y_i$, then location i was defined as concave and excluded as a potential location. To reduce the number of potential combinations, we defined minimum and maximum horizontal lengths of a span as $l_{\min}$ and $l_{\max}$. To fit the horizontal length of the profile l_p, the last element of F, f_n, was placed at a lower distance than δl from f_{n-1}, if l_p was not a multiple value of δl. The set (G) of possible intermediate support heights (difference in elevation from the base of the support to the top) was described by three parameters—minimum height ($h_{\min}$), maximum height ($h_{\max}$) and the height interval (δh) between two consecutive height options at a specific support. This set included all values $g_x = h_{\min} + x^* \delta h$ where $g_x \leq h_{\max}$ and x was an integer. If the last element of G, $g_n = h_{\min} + n^* \delta h < h_{\max}$, then $h_{\max}$ was set to g_n. Assuming that $h_{\min} = 8$ m, $h_{\max} = 14$ m and $\delta h = 1$ m, there are 7 height options (8, 9 …14 m). For $\delta h = 2$ m, we have 4 height options (8, 10, 12, 14 m). When $\delta h = 4$ m, we have height options of 8 and 12 m, where $h_{\max} = 14$ m is no longer possible. So, there are parameter values of δh, for which $h_{\max}$ is excluded as a height option. By following this procedure for location and height identification, we could determine all the nodes for the graph $\{f,g\}$, where $f \in F$ and $g \in G$.

The next step was to analyse all potential paths between the head spar and the tail spar for structural safety and serviceability (i.e. minimum ground clearance, minimum gradient of the load path for gravity-affect carriages, and maximum allowable tensile stress; Fig. 2). Although quite time-consuming, this had to be done for all possible consecutive span sequences. To minimize calculation efforts, we found a "three-span representation" to be useful because it simplified potential, consecutive sequences as follows: head spar—intermediate support node i (f_B,g_B) at the beginning of the observed span—intermediate support node j (f_E,g_E) at the end of the observed span—tail spar. Two critical values were then calculated for the horizontal component of the tensile force—$H^0_{\min}$ and $H^0_{\max}$. The former was the basic tensile force required to guarantee minimum ground clearance; the latter, the basic force that resulted in maximum allowable tensile stress. The

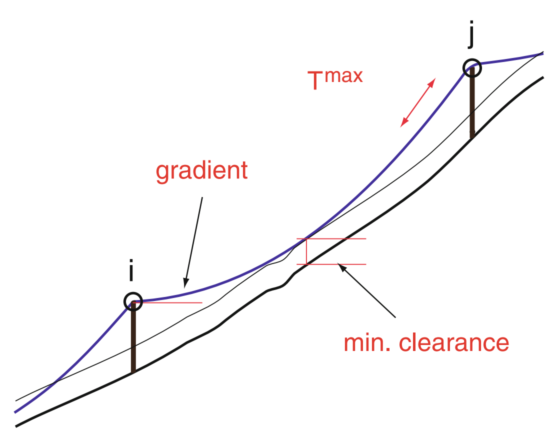

Fig. 2 Feasibility analysis of a single cable span ij, defined through nodes i and j. The range of basic tensile forces $\left(H^0_{\min} \text{ to } H^0_{\max}\right)$ was evaluated for which the span fulfilled the constraints of minimum clearance, gradient (optional) and maximum cable breaking strength ($T^{\max}$)

minimum ground clearance was checked by default over 1-m horizontal intervals. If $H^0_{\min}$ was greater than $H^0_{\max}$, then span i $(f_B, g_B) - j$ (f_E, g_E) was deemed non-feasible and its weight was set to infinity. However, if $H^0_{\min}$ proved smaller than $H^0_{\max}$, then span i $(f_B, g_B) - j$ (f_E, g_E) was feasible. Its weight was then set to a value representing the cost for rigging and taking-down the intermediate support j (f_E, g_E). After performing this "feasibility analysis", we obtained a range of H^0 values for each span to become feasible. To consider all possible configurations, we varied the basic tensile force H^0 between H^0_{absmin} (0 kN) and H^0_{absmax} (horizontal component of the design strength) in increments of 1 kN. When feasibility was checked for each span, the result was an adjacency matrix for each H^0.

Finding the optimum solution

Optimization aims at minimizing the installation costs for a cable system. Because real-cost functions were not available in this example, we sought a solution that contained a minimum number of intermediate supports (1st priority) and a minimum square sum of the heights (2nd priority). This led to the following objective function (Eq. 2):

$$\mathrm{MinV} = \sum_{f \in F} \sum_{g \in G} \left((g + 100)^2 x_{fg} \right) \tag{2}$$

where MinV optimized objective value, G set of heights for intermediate support nodes, F set of possible intermediate support locations, $x_{fg} = 1$, if the span that ends in the node at location f with support height g is selected for the solution; =0, otherwise.

The term "+100" was introduced to find, as a first priority, a solution with the fewest intermediate supports and, as the second priority, a solution with a minimum sum of support heights. The quadratic term was used when assuming that the cost of rigging an intermediate support would increase disproportionately to its height.

Identifying the optimum solution required two main steps. First, we calculated the shortest path for the entire set of adjacency matrices. Second, we looked for the entire set of shortest paths and selected the path with the minimum value. The graph was topologically sorted and could be solved by Bellmann's (1958) shortest path algorithm. The corresponding basic tensile force of the optimum solution was named $T^{0,\mathrm{opt}}$, while the best horizontal component was labelled $H^{0,\mathrm{opt}}$.

Graph parameters

Changing the parameters that defined the graph (δl, δh) was always a trade-off between accuracy of the results and calculation time. The latter increased with the number of arcs in the graph. If all of our terrain points were assumed to be convex, we determined the number of arcs (N_A) per Eq. 3, as derived by Näsberg (1985).

$$N_A \approx \left(N_l - z_{\max} + \left(\frac{\Delta z - 1}{2} \right) \right) * N_h^2 * \Delta z \tag{3}$$

where

$$N_h = \frac{h_{\max} - h_{\min}}{\delta h} + 1;$$

$$N_l = \mathrm{ceil}\left(\frac{l_P}{\delta l} \right) + 1; \text{ ceil: round toward infinity}$$

$$\Delta z = z_{\max} - z_{\min} + 1;$$

$$z_{\max} = \frac{l_{\max}}{\delta l};$$

$$z_{\min} = \frac{l_{\min}}{\delta l}$$

Here, default values for graph parameters were assumed to be the following: $\delta l = 10$ m, $l_{\min} = 30$ m, $l_{\max} = 400$ m, $h_{\min} = 8$ m, $h_{\max} = 14$ m and $\delta h = 1$ m. Term l_P was the horizontal length of the profile.

The process of modifying the parameters that defined a longitudinal section ($l_{\min}$, $l_{\max}$, and δl) demonstrated that term δl had the greatest influence on the number of spans. In the range of $\delta l = 1$–10 m, that number of spans varied by a factor of 100; for range $\delta l = 1$–30 m, by a factor of 1,000 (Fig. 3c). By comparison, the influence of $l_{\min}$ and $l_{\max}$ was negligible, especially if one considered that the values of these parameters also depended on technical constraints. Therefore, δl was the focus here.

For experimental purposes, we ran an optimization procedure with LIN assumptions along a randomly selected profile. The length profile was generated from a DEM (digital elevation model), with a 2-m by 2-m horizontal resolution, as well as from a 10-m by 10-m DEM that was generated by the 2-m version. Because that profile did not run in the orientation of the coordinate system, but rather in a diagonal orientation, the resolution of the DEM did not fit with the resolution of the length profile. For example, if we assumed the DEM had a resolution of 10 m and we set $\delta l = 1$, then the first 10 potential support locations would not all have the same elevation coordinate and, indeed, the grade breaks would have been more frequent.

Fluctuations for the 10-m DEM in δl indicated that, for a range of $\delta l = 1$–15 m, the objective value varied only marginally, whereas for $\delta l \geq 15$ m that value increased (Fig. 3d). This meant that a better objective value could be achieved by reducing δl. To illustrate the influence of the resolution of the DEM, we also calculated the MinV depending on δl on a 2-m DEM (Fig. 3e). In this case, we observed only a marginal variation for $\delta l < 10$. For that,

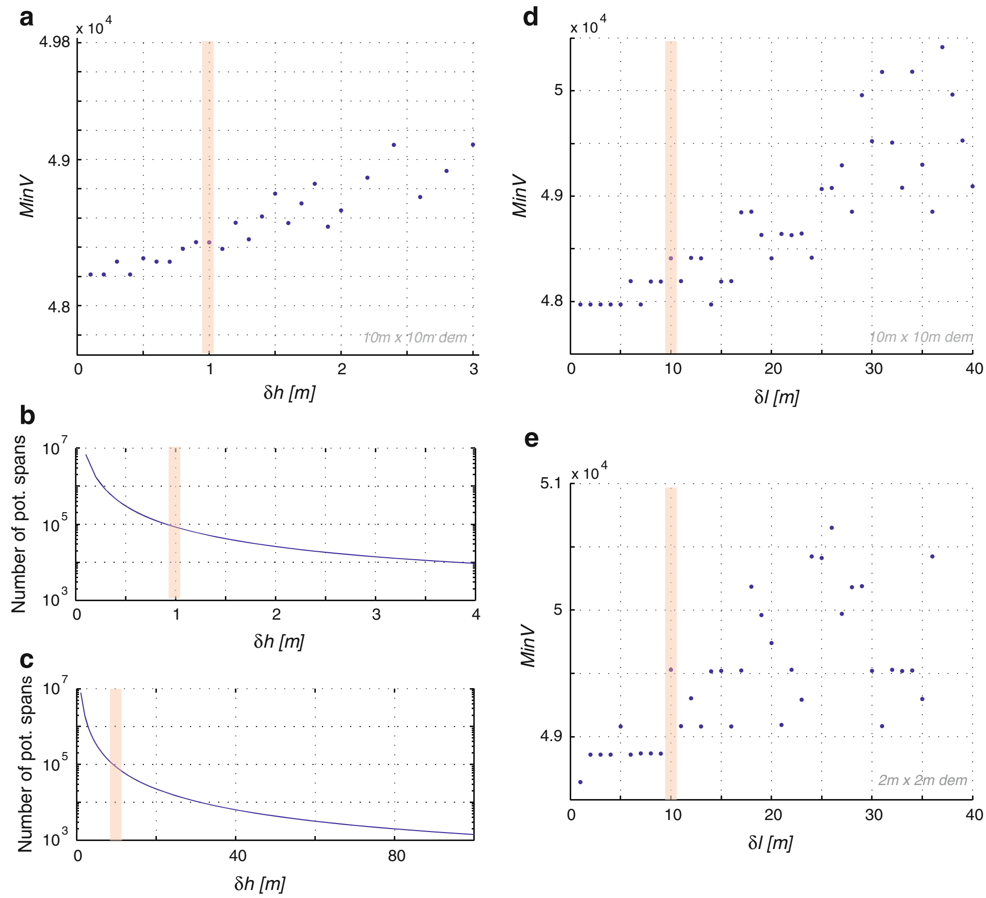

Fig. 3 Effect of support distance interval (δl) and support height interval (δh) on the objective value (**a**, **d**, **e**) and the number of potential spans (**b**, **c**). Recommended values were $\delta l \approx 10$ m and $\delta h \approx 1$ m (highlighted in *red*). (Color figure online)

we would have recommended choosing $\delta l \leq 10$ m to arrive at suitable results for practical applications. The corresponding support heights for Fig. 3d were for the $\delta l = 10$-m resolutions 13, 9, 11, 12, and 8 m, whereas for $\delta l = 1$ m, those heights were 13, 8, 12, 10, or 8 m.

If we wanted to achieve the absolutely minimum objective value, we applied the following consideration when selecting δl. Assuming that the length profile ran in the orientation of the coordinate system (not diagonally), we could then expect similar MinV if the resolution of the terrain model divided by δl was an integer. This was because, over short intervals, the critical locations for the intermediate supports fell on the data points (i.e. where peaks and grade breaks occurred). For example, if δl was 1 m and the horizontal resolution was 10 m, then the possible critical point at 10 m from the headspar could serve as a potential intermediate support. This was also true for $\delta l = 2$, 5 and 10 m, which provided the same MinV. In our case, we predicted a diagonal cable line that would cross 505 raster cells within a horizontal distance of 400 m (based on a 10-m DEM resolution). The average horizontal length of cable line per cell was 7.9 m (or, in the worst case, $\sqrt{2}/2$ m). Because we found variation in the length of cable line per cell, it was difficult to make general recommendations for choosing δl. However, as shown in Fig. 3d and e, if we chose a δl that was less than the resolution of the DEM/2, then we achieved the absolute minimal objective value.

The height of intermediate supports was defined by parameters $h_{\min}$, $h_{\max}$ and δh. Whereas the first two were specified through the characteristics of the cable system, δh could vary. Here, the influence of δh on calculation speed proved comparable to that of δl described above (Fig. 3b). If δh was altered (cf., Fig. 3a), the objective became minimal for small values of δh and became substantially worse for larger δh. Term δh also had to be sufficiently small to produce support heights with an overall minimum MinV. Therefore, we could recommend that δh be less than 1 m in order to acquire suitable results for practical applications.

Implementation

We evaluated our approach in Matlab by considering first- and second-order elements of Zweifel's Taylor series procedure for catenary equations. Our implementation featured an interface to import a longitudinal section between head and tail spars for a specific cable road, as obtained from a GIS system.

Model application

The purpose of our model application was to (1) compare the CTC and LIN approaches for a real-world cable layout in a test area and (2) investigate the effect of a three-span simplification.

Test area

The test area was located on the northern slopes of the Swiss Alps in the region of Einsiedeln (central Switzerland; UTM Coordinates, 47.127557/8.846569). We randomly chose an area typical for cable yarding that is characterized by a low soil-bearing capacity and slopes between 25 and 50%. The design of the cable required geometric information about the longitudinal profile, which could be obtained in three ways—field survey, manual extraction with a contour map, or output from a digital terrain model. We determined the geometry of the longitudinal sections from the DEM via SwissTopo, which covers all of Switzerland at a 2 m by 2 m resolution. We then generated the 10 m by 10 m resolution through extrapolation to get a smoother ground profile. Table 1 presents the properties for the five longitudinal sections for our mobile application. Profile lengths varied between 230 and 990 m, while the average slope was 18–45%. Table 2 lists the engineering design values used here, which are typical for the type of cable system usually applied.

The following graph parameter values were used for our optimization: $\delta l = 10$ m, $l_{\min} = 30$ m, $l_{\max} = 1{,}000$ m, $h_{\min} = 8$ m, $h_{\max} = 14$ m and $\delta h = 1$ m.

Comparison between LIN and CTC approaches

Figure 4 illustrates the differences between CTC and LIN approaches for a two-span skyline structure. Here, the horizontal component of the tensile force was assumed to be 90 kN. Positioning a 20-kN load at the two mid-span positions resulted in an increase in tensile force of about 30% for the short span and about 60% for the large span (Fig. 4, upper part). At the same time, our CTC approach resulted in a smaller mid-span deflection of the load path, by approximately 10% for span 1 and 30% for span 2. This comparison demonstrated that the CTC approach was more appropriate.

We optimized the intermediate support layout and studied the configuration values for the optimized solution (Tables 3, 4). Values for length profile 1 are shown in Fig. 5. To calculate the LIN solution, we assumed the same basic tensile force ($T^0 = T^{0,\mathrm{opt}}$) that was achieved via CTC.

With CTC, fewer intermediate supports were necessary to cover a particular length, especially for long profiles. The average length of a span increased from 122 to 159 m (+30%) for $Q = 25$ kN and from 164 to 182 m (+11%) for $Q = 20$ kN. If the number of intermediate supports was not reduced, the heights of the intermediate supports had to be decreased. In general, the longer the length profile, the greater the impact of the CTC approach on heights and numbers of intermediate supports.

The optimum basic tensile ($T^{0,\mathrm{opt}}$) for the best solution varied from 98 to 148 kN for load $Q = 25$ kN and from 119 to 144 kN for $Q = 20$ kN. For all cases, the maximum acting tensile force ($T^{\max}$) in the system ranged from 167 to 178 kN, that is, an increase in basic tensile force of about 20–80% while the load was moving over the span. Therefore, the greater the length of the longest span, the higher the tensile force tended to be.

Because equations associated with the CTC approach are nonlinear, they are solved numerically through an iterative method. Although this is implemented efficiently with the bi-section algorithm (Forsythe et al. 1976), calculation times are about 30–60 times higher compared with

Table 1 Properties of the longitudinal sections

Line nr.	1	2	3	4	5
Length (m)	570	470	990	800	230
Height difference (dm)	2,508	1,600	−2,568	1,464	−1,040
Average slope (0..1)	0.44	0.34	−0.26	0.18	−0.45

Table 2 Engineering design values of the cable system

Property	Abbreviation	Unit	Value
Load weight	Q	kN	20; 25
Self-weight of the skyline	q_S	kN/m	0.0228
Self-weight of the mainline	q_m	kN/m	0.0058
Cross-sectional area of the skyline	A	mm^2	380
Design strength	T^a	kN	179
E module	E	kN/mm^2	100
Height of the head spar	h_{HS}	m	11

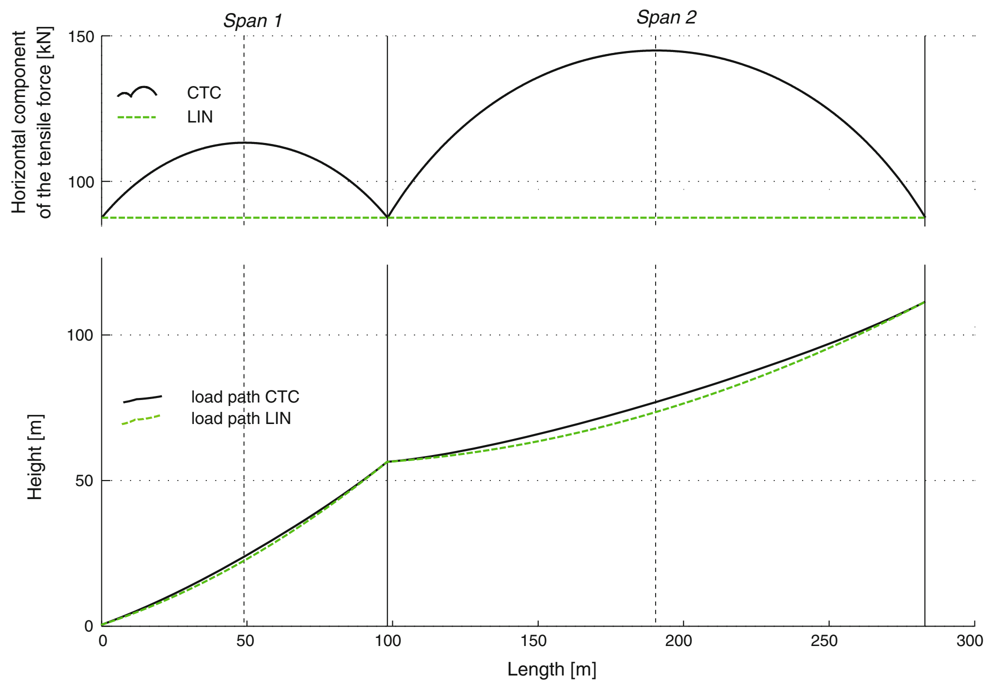

Fig. 4 Effect of two mechanical methods. The linear approach (LIN) assumed constant tensile forces, whereas the close-to-catenary (CTC) method mapped nonlinear behaviour. The main gaps were nonlinear behaviour of the tensile force, resulting in less mid-span deflection and increased tensile force when the load was moving over the span

Table 3 Configuration values for tested length profiles ($Q = 25$ kN)

Line nr.	1	2	3	4	5
$T^{0,\mathrm{opt}}$ lower bound (kN)	98	124	127	148	127
$T^{0,\mathrm{opt}}$ upper bound (kN)	99	128	128	149	128
Calculation time optimization (s)					
CTC	68.6	35.8	150.2	78.5	10.2
LIN	1.1	0.6	3.5	2.8	0.1
Heights of supports (m), intermeidate supports in bold letters					
CTC	[11, **14**, **10**, 11]	[11, **14**, **10**, 10]	[11, **10**, **12**, **14**, **12**, 10]	[11, **12**, **10**, **9**, **12**, **9**, 12]	[11, **11**, 11]
LIN	[11, **9**, **12**, **13**, **13**, 9]	[11, **13**, **14**, **11**, 10]	[11, **11**, **12**, **13**, **14**, **13**, 10]	[11, **13**, **9**, **8**, **9**, **12**, **8**, **9**, 12]	[11, **14**, 13]
Average length of a span (m)					
CTC	190	157	198	133	115
LIN	114	118	165	100	115
$T_{\max}$ (kN)					
CTC	178.0	173.0	177.3	178	173.3
LIN	160.3	168.7	160.5	173.1	173.4

Cable mechanics assumptions are LIN for linearized and CTC for close-to-catenary

the linear method. Nevertheless, we were able to solve all of our CTC applications in less than 1 min.

Effect of three-span representations

To assess how a "three-span representation" can affect results, we calculated the mid-span deflection (y_m) of a load path, for examples, shown in Table 5. Generally, the variations were small, just a few centimetres. However, for a long cable line (e.g. cable road nr. 3), fluctuations in deflection were slightly higher, ranging from 9 to 22 cm (max. difference 4%). Furthermore, the "three-span representation" deflection was always larger than that calculated when using the "all-span representation" due to the incorporation of an additional safety factor.

Discussion and conclusions

Our research was aimed at (1) developing a method for identifying the optimum intermediate support layout for a cable-yarding harvest operation, (2) comparing the optimization procedures for two approaches to cable mechanics—linearized versus close-to-catenary—and (3) investigating

Table 4 Configuration values for tested length profiles ($Q = 20$ kN)

Line nr.	1	2	3	4	5
$T^{0,\mathrm{opt}}$ lower bound [kN]	119	129	137	143	143
$T^{0,\mathrm{opt}}$ upper bound [kN]	120	142	139	144	144
Calculation time optimization (s)					
CTC	74.6	39.3	177.7	100.9	10.4
LIN	1.1	0.7	3.5	2.1	0.1
Heights of supports (m), intermeidate supports in bold letters					
CTC	[11, **10, 9**, 8]	[11, **12, 9**, 10]	[11, **13, 11, 11**, 9]	[11, **11, 12, 11**, 10]	[11, **10**, 10]
LIN	[11, **11, 12**, 13]	[11, **12, 10, 9**, 10]	[11, **9, 14, 12, 12**, 9]	[11, **11, 14, 13**, 14]	[11, **11**, 11]
Average length of a span (m)					
CTC	190	157	248	200	115
LIN	190	118	198	200	115
$T_{\max}$ (kN)					
CTC	177.7	167.0	176.2	177.1	173.3
LIN	177.0	158.7	172.2	176.1	173.3

Cable mechanics assumptions are LIN for linearized and CTC for close-to-catenary

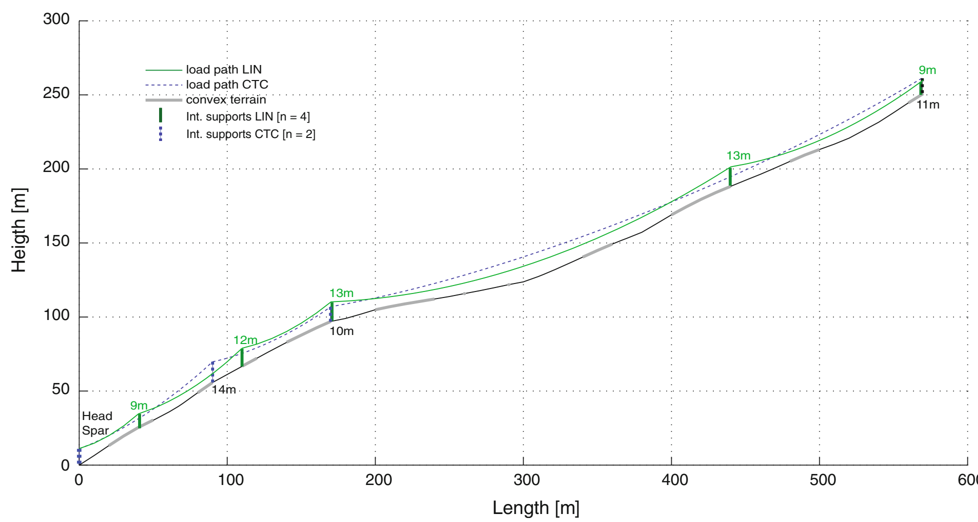

Fig. 5 Effect of applying the close-to-catenary (CTC) approach to an intermediate support layout and load path versus the linear (LIN) approach (Length profile nr. 1). More intermediate supports were required when implementing the latter

the effect of simplifications on the result (three-span representation).

This study produced the following major findings. First, combining these mechanical approaches with a layout representation of intermediate supports (mathematical graph) led to optimality in less than 3 min of calculation time. Second, the CTC approach resulted in larger spans and fewer intermediate supports being required. Here, the average length of a span increased up to 60% for a single cable corridor and by about 10–30% over all tested cable corridors. In most cases, both the number and height of those intermediate supports decreased. Third, simplification via a three-span representation had only a marginal influence on the accuracy of the load path for a skyline. Hence, the deflection was always overestimated, resulting in a "hidden" structural safety. Fourth, the basic tensile force increased significantly (by up to 80%) when the load was located at the mid-span position of the largest span.

To our knowledge, the approach presented here is the first to optimize the intermediate support layout while concurrently considering CTC cable mechanics for multi-span cable road configurations. Although the procedure outlined by Leitner et al. (1994) is based on an exact optimization procedure, it lacks adequate cable mechanics, using the formula of Pestal (1961). There, the outcome is always shorter spans and more intermediate supports. By contrast, the method described by Sessions (1992) and Chung and Sessions (2003) is based on exact cable mechanics (catenary analysis), but relies on simple heuristics that do not identify the real, optimum layout for intermediate support.

Our findings have important implications. First, operations practitioners could benefit from this smarter cable

Table 5 Differences in deflection between the "three-span representation" and the "all-span approach" for several cable roads (design parameters: $Q = 20$ kN, $q_S = 0.02$ kN/m, $q_M = 0$ kN/m, $T^0 = 100$ kN, $E = 160$ kN/mm^2, $A = 209$ mm^2)

Cable road nr.	Span nr.	Mid-span deflection (y_m) (m)		Difference (%)	Difference (m)	Span length a (m)	Span height b (m)
		3-Span	All-span				
3	1	5.13	4.96	3.3	0.16	100	−28
	2	7.75	7.54	2.8	0.21	160	−34
	3	20.00	19.91	0.5	0.09	440	−119
	4	8.91	8.70	2.4	0.21	180	−51
	5	5.74	5.52	4.0	0.22	110	−36
2	1	10.01	9.98	0.3	0.03	260	86
	2	7.72	7.71	0.2	0.01	150	52
	3	2.77	2.76	0.2	0.01	60	18
1	1	5.82	5.80	0.3	0.02	110	67
	2	3.07	3.06	0.2	0.01	60	31
	3	17.05	17.02	0.2	0.03	400	98

Cable mechanics: CTC

road layout that requires lower set-up and dismantling costs. Second, safety codes for skyline systems should be checked for consistency with our findings. Standing skyline configurations typically have fixed anchoring at the head and tail spars. There, tensile force is usually controlled only for the unloaded configuration, and it is assumed that the design considers that this force increases upon loading. However, that heavily depends on the geometric layout of the system, whereas some codes provide only rules of thumb to account for that effect.

Further research is needed to resolve the following tasks.

1. For our objective function, we did not use real costs and did not distinguish between intermediate supports that are artificial or natural (e.g. trees), although that selection of material will lead to completely different optimum solutions. This is important because constructing an artificial support is much more expensive than using an existing tree. Future evaluations should involve the formulation of a real-cost function and a differentiation between artificial and natural supports.
2. The calculation time associated with implementing the CTC approach is about 30 to 60 times longer than for LIN. Nevertheless, that period is sufficient when running a single application. However, to use our model as a component when optimizing for large areas, that speed must be increased.

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